Nonlinear Control of an Exoskeleton Seven Degrees of Freedom Robot to Realize an Active and Passive Rehabilitation Tasks

by

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Brahim Brahmi, 2019
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Contrôle non linéaire d’un robot exosquelette sept degrés de liberté pour réaliser des tâches de réhabilitation active et passive

Brahim BRAHMI

RÉSUMÉ

Cette thèse de doctorat propose le développement d’un robot exosquelette appelé robot ETS-MARSE, utilisé pour réhabiliter les patients ayant une déficience à un membre supérieur. Les développements, inclus dans ce travail, sont la conception et la validation expérimentale d’une solution de cinématique inverse de robot et de contrôle non-linéaire pour le robot ETS-MARSE. Ces approches garantissent une bonne performance de l’exosquelette en mouvement de réhabilitation passive et active en présence d’incertitudes dynamiques et cinématiques et de perturbations indésirables.

Considérant la population croissante de personnes atteintes par ce genre de déficience, il est nécessaire d’améliorer l’accessibilité à la physiothérapie en utilisant une technologie robotique pour la réadaptation. Récemment, la robotique de réadaptation a attiré beaucoup d’attention de la communauté scientifique, car elle est capable de surmonter les limites de la thérapie physique conventionnelle. L’importance du robot de réadaptation réside dans sa capacité à assurer une physiothérapie intensive pendant une longue période. Les données mesurées du robot permettent au physiothérapeute d’évaluer avec précision les performances du patient. Cependant, ces dispositifs font toujours partie d’un domaine émergent et présentent de nombreux défis par rapport aux manipulateurs classiques, tels que la non-linéarité élevée, la dimension élevée (un nombre élevé de degrés de liberté) et la dynamique inconnue (incertitudes). Ces limitations sont provoquées par la structure mécanique complexe conçue pour un usage humain, les types de mouvement d’assistance et la sensibilité de l’interaction avec une grande diversité d’usagers (différentes conditions physiologiques des sujets). En conséquence, ces conditions rendent le système vulnérable aux incertitudes dynamiques et aux perturbations externes. Ces incertitudes et les forces externes peuvent se transformer en une fonction inconnue pouvant affecter les performances du robot exosquelette.

Les principaux défis abordés dans cette thèse consistent tout d’abord à concevoir une solution de cinématique inverse humaine permettant d’effectuer un mouvement fluide similaire au mouvement naturel de l’humaine (mouvement humain). Deuxièmement, à développer des contrôleurs robustes caractérisés par un haut niveau de robustesse et de précision, sans aucune sensibilité aux dynamiques non-linéaires incertaines et aux perturbations inattendues. Cela donnera au système de contrôle plus de flexibilité pour gérer les incertitudes et les variations de paramètres dans les différents modes de mouvement de réhabilitation (passif et actif).

Mots-clés: Cinématique inverse humaine, contrôle adaptatif, les robots de réadaptation
NONLINEAR CONTROL OF AN EXOSKELETON SEVEN DEGREES OF FREEDOM ROBOT TO REALIZE AN ACTIVE AND PASSIVE REHABILITATION TASKS

Brahim BRAHMI

ABSTRACT

This doctoral thesis proposes the developments of an exoskeleton robot used to rehabilitate patients with upper-limb impairment, named ETS-MARSE robot. The developments included in this work are the design, and validation of a kinematic inverse solution and nonlinear control strategy for an upper limb exoskeleton robot. These approaches are used in passive and active rehabilitation motion in presence of dynamics and kinematics uncertainties and unexpected disturbances.

Considering the growing population of post-stroke victims, there is a need to improve accessibility to physiotherapy by using the modern robotic rehabilitation technology. Recently, rehabilitation robotics attracted a lot of attention from the scientific community since it is able to overcome the limitations of conventional physical therapy. The importance of the rehabilitation robot lies in its ability to provide intensive physiotherapy for a long period time. The measured data of the robot allows the physiotherapist to accurately evaluate the patient’s performance. However, these devices are still part of an emerging area and present many challenges compared to the conventional robotic manipulators, such as the high nonlinearity, dimensional (high number of DOFs) and unknown dynamics (uncertainties). These limitations are provoked due to their complex mechanical structure designed for human use, the types of assistive motion, and the sensitivity of the interaction with a large diversity of human wearers. As a result, these conditions make the robot system vulnerable to dynamic uncertainties and external disturbances such as saturation, friction forces, backlash, and payload. Likewise, the interaction between human and the exoskeleton make the system subjected to external disturbances due to different physiological conditions of the subjects like the different weight of the upper limb for each subject. During a rehabilitation movement, the nonlinear uncertain dynamic model and external forces can turn into unknown function that can affect the performance of the exoskeleton robot.

The main challenges addressed in this thesis are firstly to design a human inverse kinematics solution to perform a smooth movement similar to natural human movement (human-like motion). Secondly, to develop controllers characterized by a high-level of robustness and accuracy without any sensitivity to uncertain nonlinear dynamics and unexpected disturbances. This will give the control system more flexibility to handle the uncertainties and parameters’ variation in different modes of rehabilitation motion (passive and active).

Keywords: Human inverse kinematics, adaptive control, rehabilitation robots.
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<tr>
<td>ETS</td>
<td>École de Technologie Supérieure</td>
</tr>
<tr>
<td>MARSE</td>
<td>Motion Assistive Robotic-exoskeleton for Superior Extremity</td>
</tr>
<tr>
<td>GREPCI</td>
<td>Groupe de Recherche en Électronique de Puissance et Commande Industrielle</td>
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<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>VDC</td>
<td>Virtual Decomposition Control</td>
</tr>
<tr>
<td>DoF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>DH</td>
<td>Denavit-Hartenberg parameters</td>
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<tr>
<td>PD</td>
<td>Proportional-Derivative</td>
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<tr>
<td>PI</td>
<td>Proportional-Integral</td>
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<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative</td>
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<tr>
<td>CTC</td>
<td>Computed Torque Control</td>
</tr>
<tr>
<td>DMI</td>
<td>Desired Motion Intention</td>
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<tr>
<td>TDE</td>
<td>Time Delay Estimation</td>
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<tr>
<td>SMC</td>
<td>Sliding Mode Control</td>
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<tr>
<td>RMS</td>
<td>Root-Mean-Square</td>
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<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
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<tr>
<td>EMG</td>
<td>Electromyographic</td>
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<td>HIV</td>
<td>Human Inverse Kinematics</td>
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<td>DLS</td>
<td>Damped Least Square</td>
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LISTE OF SYMBOLS AND UNITS OF MEASUREMENTS

UNITS OF MEASUREMENT

m  meter

cm  centimeter

rad  radian

Nm  Newton meter

s or sec  second

μs  microsecond

○ or deg  degree

N  Newton

kg  Kilogram

g  gram

SYMBOLS

XYZ  Cartesian coordinate system main axes

α_{i−1}  angle from Z_{i−1} to Z_i measured about X_{i−1}

a_{i−1}  length from Z_{i−1} to Z_i measured along X_{i−1}

d_i  distance from X_{i−1} to X_i measured along Z_i

θ_i  angle from X_{i−1} to X_i measured about Z_i

\dot{\theta}_i, \ddot{\theta}_i  joint’s velocity and acceleration, respectively

x, \dot{x}, \ddot{x}  Cartesian position, velocity and acceleration, respectively
$V_i$ Lyapunov function candidate

$J$ Jacobian Matrix

$^{i}_{i+1}T$ transformation matrix

$\tau$ control torque

$\tau_{ex}, f_{ex}$ external disturbances

$F_m$ measured force vector

$M$ inertia matrix

$C$ Coriolis and centrifugal matrix terms

$G$ gravitational vector terms

$Q$ regressor matrix

$\hat{f}_e$ estimated force vector

$\hat{H}$ estimated uncertain vector

$\hat{e}$ estimated time delay error vector
INTRODUCTION

Nowadays, the technological revolution is growing by a fantastical speed. Starting from the first industrial revolution where water and steam were used to mechanize the production to the third one where information technologies are used to automate the production. Today, we stand on the bridge of a new revolution that will change fundamentally our way of working, living and communicating with each other. This revolution known as Industry 4.0 (Xu et al., 2018) will bring changes that humans have never seen before. Robotics is one of the most important manifestations of this revolution (Atkeson et al., 2018). These technologies are used to improve the industrial production by integrating new systems that can replace humans and replicate human activities (Bautista and Wane, 2018). However, to make this revolution succeed, several challenges must be met, among them; technological challenges such as controlling these new nonlinear systems (Atkeson et al., 2018).

Currently, robotics can be used in many fields and for many purposes, for example, military (Liu et al., 2018), manufacturing processes especially in dangerous environment where humans cannot intervene (Clancy et al., 2018), in the medical domain (Ferguson et al., 2018), and so on. Laterally, new robot systems have been introduced to improve the rehabilitation treatment known as "rehabilitation robots" or exoskeleton robots (Xie et al., 2016). The use of rehabilitation robots in the medical rehabilitation field has proved to be of great ability to improve the quality of life of the patient, enhancing its practical motions, and assisting him in daily exercises. The exoskeleton robot is an articulated mechanical structure with several degrees of freedom (DOFs) having the same anatomy of the human arm or leg. Unlike prostheses that replace a limb of the body, the exoskeleton robot clings to it externally and acts in parallel. This fixation allows robot’s wearer to move his/her arm in the workspace. The reachable workspace envelope depends on the number of degrees of freedom (DOFs) available of the exoskeleton robot. It can be dedicated to a specific part of the body such as the hand, arm, leg or several limbs at the same time. Equipped with sensors and actuators, it measures
the movements and forces of the user that allow to the physiotherapist to accurately evaluate the patient’s performance.

This thesis exhibits the development of an exoskeleton robot used to rehabilitate patients with upper-limb impairment, named ETS-MARSE robot (Motion Assistive Robotic-exoskeleton for Superior Extremity). In fact, this thesis presents a continuity of works presented in (Ochoa Luna, 2016; Rahman, 2012). The design of the ETS-MARSE is based on the anatomy of the human upper limb and has been developed to faithfully represent the joints and movements of the upper limb movements, in order to be worn comfortably with the robot’s user during the rehabilitation tasks. The shoulder part is described by three joints: the first two joints are created to produce the vertical and horizontal extension/flexion movement of the shoulder, while the third joint is intended to perform the internal and external rotation of the shoulder. The elbow part is composed of one joint to complete the flexion/extension motion of the elbow. The wrist part is composed of three joints: the first joint is designed to perform pronation and supination motion of the forearm, the second and third joints are dedicated to offer respectively ulnar/radial deviation, and flexion/extension motions. ETS-MARSE is able to provide the different levels of robotic assistance strategies used after neurological accidents. The most urgent, usually the first six weeks after the accident, is passive physical therapy (Sidney et al., 2013; Xie et al., 2016). In this type of therapy, the exoskeleton brings the patient’s limb, which is completely passive, to realize a therapy task. Its advantage lies in the robot’s ability to provide intensive therapy over a long period of time (Brahim et al., 2016b,a). The next types of therapy, active-assisted and active modes, allow the patient to voluntarily initiate movement. Then, the exoskeleton’s wearer can perform a free motion (active mode), or an active-assisted movement where the robot corrects or guides this movement. In the latter case, the robot limits the tremors or corrects the trajectory. After detecting the initiation of a motion, usually voluntary, the exoskeleton will guide the achievement of the activity, often using an impedance and/or admittance control (Li et al., 2017a; Ochoa Luna et al., 2015). Additionally, these strategies can be used
for evaluating or studying subject movements and performance improvement. In these modes, theoretically, the patient should not feel the presence of the exoskeleton robot. Therefore, the subject is completely active and the exoskeleton robot should not affect the movement.

The aim of this thesis is to design and validate experimentally a solution of inverse kinematics and nonlinear control strategy for an upper limb exoskeleton robot to achieve a passive and active rehabilitation purposes. The idea is that improved the performance of the exoskeleton robot, and a new solution of the control system may be realized by maintaining the whole non-linear dynamics model of the robot system in its design. This strategy presents considerable challenges due to: firstly, the unavailability of an analytical solution of nonlinear equations of the robot motion. Secondly, kinematic and dynamic models may be imperfect because of the many difficulties encountered in modeling certain phenomena such as nonlinear friction, kinematics uncertainty caused by visual devices like camera, Kinect..., etc. Dynamic uncertainty or unavailability of feedback signals, necessary to compute the dynamic parameters of the robot system.

The organization of this thesis is given as follows: Chapter 1 describes the research problem, gives the literature review, state the objectives, gives the methodology and claims the originality of the work. Since this document is papers-based thesis, consequently, Chapters 2, 3, 4, 5, and 6 present the main results of the work in the form of published papers. The main contributions of this thesis are summarized as follows:

The first chapter of this thesis presents the problem of research. The identification and justification of the research problem are given in this chapter, as well as the motivation of the current work. A state-of-the-art of the existing literature in this area of research is presented. Then the global and specific objectives of this work are stated. Finally, an overview of the methodology used is provided.
Chapter 2 presents an experimental validation of a new solution of inverse kinematic for 7-DOFs exoskeleton robot (Brahmi et al., 2017c). In this work, we propose a new solution to the inverse kinematic problem, considering human upper limb movement and arm configuration. The philosophy of this approach is to develop a mathematical model based on geometric and analytic solutions of human upper limb motion in order to imitate its movement.

Chapter 3 presents an experimental validation of a new compliant control based on a second-order sliding mode with adaptive-gain incorporating time delay estimation (Brahmi et al., 2018a). In this work, the dynamic parameters of the system were considered uncertain and were estimated by time delay estimation. The main challenge addressed in this research is to ensure that the exoskeleton robot provides an appropriate compliance control that allows it to interact perfectly with humans. In this chapter, the proposed controller uses the human inverse kinematics presented in chapter 2 to complete the active rehabilitation motion.

Chapter 4 presents an experimental validation using backstepping approach integrated with modified time-delay estimation to provide an accurate estimation of unknown dynamics of the exoskeleton robot and to compensate for external bounded disturbances (Brahmi et al., 2018d). The stability of the control system and the convergence of its state errors are established and proved based on Lyapunov–Krasovskii functional theory.

Chapter 5 investigates the passive and active control strategies to provide a physical assistance and rehabilitation by a 7-DOF exoskeleton robot with nonlinear uncertain dynamics and unknown bounded external disturbances due to the robot user’s physiological characteristics (Brahmi et al., 2018c). Besides, The Damped Least Square method is introduced to estimate the desired movement intention of the subject with the objective to provide active rehabilitation motion. The design, stability and convergence analysis are formulated and proven based on the Lyapunov–Krasovskii functional theory.
Chapter 6 presents a new adaptive visual tracking control approach based on sliding mode control in Cartesian space applied in real time to an exoskeleton robot with uncertain kinematics and dynamics, taking into account uncertainties in visual system (camera) parameters (Brahmi et al., 2018b). In this work, the adaptation of kinematic uncertainties is based on a filtered regressor kinematic matrix, whereas, the adaptation of dynamic uncertainties is based on a modified time delay estimation approach. The updated laws are designed using Lyapunov theory to solve the adaptation problem methodically and to show the stability of the robot system.
CHAPTER 1

RESEARCH PROBLEM

Without a doubt, neurological diseases are becoming a challenging problem to the scientific community. According to recent statistics, as many as 15 million people in the world suffer from neurological diseases such as stroke, 6 million people die and 5 million live with persistent disability (Sidney et al., 2013). In Canada, at least 16000 Canadians each year die from a stroke (Sidney et al., 2013). Survivors of a stroke typically don’t have the potential to perform daily activities of their own, such as eating, dressing, and bathing because of the permanent disability often present on one side of the body (De Morand, 2014). This weakness can create many physical and psychological problems related to the behavior of the victim. The rehabilitation program is an effective treatment designed to help the stroke victims recover their lost functional ability, acquire new skills and improve their quality of life (De Morand, 2014). However, this treatment still suffers from several shortcomings, such as the accessibility and fatigue of the therapist (Xie et al., 2016).

Given the growing population of post-stroke victims, there is a need to improve accessibility to physiotherapy. Modern robotic technology can help in many fields even to accomplish the medical tasks such physiotherapy. Recently, rehabilitation robotics has attracted a lot of attention from the scientific community to overcome the limitations of conventional physical therapy (Xie et al., 2016). The importance of the rehabilitation robot lies in its ability to provide intensive physiotherapy for a long period time (Xie et al., 2016). The measured data of the robot allows the physiotherapist to accurately assess the patient’s performance. However, these devices are still part of an emerging area and present many challenges. In fact, these robots have an additional complexity compared to conventional robotic manipulators due to their complex mechanical structure designed for human use, types of assistive motion, and the sensitivity of the interaction with a large diversity of human wearers. As a result, these conditions make the robot system vulnerable to dynamic uncertainties and external disturbances such as saturation, friction forces, backlash, and payload. Likewise, the interaction between
human and the exoskeleton make the system subjected to external disturbances due to different physiological conditions of the subjects like the different weight of upper limb for each patient (Brahmi et al., 2018d,c). During a rehabilitation movement, the nonlinear uncertain dynamic model and external forces can turn into an unknown function that can affect the performance of the exoskeleton robot.

Another problem can be raised with a redundant (in our case 7-DOFs) is the high number of inverse kinematic solutions. That is, the exoskeleton robot can reach some points of its workspace in more than one configuration of its links (redundancy). Finding a solution to the inverse kinematics problem, compatible with human upper limb movement and valid for human arm configuration will help to perform the human-like motion avoiding singularity problems.

Based on above-mentioned problems, the main challenges in this work can be summarized as follows:

- High non-linearity of the system;
- System redundancy (a lot of degree of freedom, in our case 7-DOFs);
- Unknown dynamics, or part of the system is uncertain;
- Uncertain kinematics (when the camera is used to obtain the Cartesian measurement).

The main challenges addressed of this thesis consist first of all in designing an inverse kinematics solution able making the exoskeleton ETS-MARSE robot to perform a smooth movement similar to natural human movement (human-like motion). Secondly, to design robust controllers characterized by a high-level of robustness and accuracy without any sensitivity to uncertain nonlinear dynamics and unexpected disturbances. This will give the control system more flexibility to handle the uncertainties and parameters’ variation in different modes of rehabilitation motion (passive and active).
1.1 Literature review

Most research work in this type of robotic system has so far focused on the following three points: The mechanism of the exoskeleton robot, inverse kinematic solution, and control strategies to achieve different modes of rehabilitation motion.

1.1.1 Rehabilitation robot

In this section, only advanced exoskeleton robots that have demonstrated their clinical efficacy are presented.

InMotion: "InMotion” robot is certainly the most successful rehabilitation robot so far and the most clinically tested. It is a commercialized system. It contains 2 degrees of freedom dedicated to the upper limb. It allows the patient to perform movements in the transverse plane. Most controls that have been developed for this robot are based on the impedance control. This robot also integrated therapeutic games. It demonstrates its effectiveness by its applications in clinical rehabilitation area (Hogan et al., 1992, 1993).

This product delivers several versions of InMotion to different objectives as shown in Fig 1.1:

Figure 1.1 InMotion Arm Robot, InMotion Hand Robot, InMotion Wrist Robot (from the left of the figure to the right, respectively)
Taken from (Hogan et al., 1992, 1993)
**InMotion Arm Robot:** the robot has 2-DOF developed by MIT-Manus. It allows 2D movements in the transverse plane. It is integrated with a therapeutic game that helps in the rehabilitation treatment.

**InMotion Robot Hand:** it is specifically dedicated to the hand. It allows to make gripping movements and relaxation.

**InMotion Wrist Robot:** the robot has 3-DOF designed to the wrist. It allows realizing the movements of pronation and supination, flexion / extension and abduction / adduction. This type of robot also contains therapeutic game interface.

**ARM Guide robot:** The ARM Guide robot (see Figure 1.2) was created by "the Rehabilitation Institute of Chicago and the University of California," and the robot is dedicated to the hand. It contains a splint to fix the patient’s forearm. There are also other passive 2-DOF, equipped with electromagnetic brakes and allow performing pre-defined pointing movements in space. There are several studies (Kahn *et al.*, 2001) demonstrating its effectiveness in the area of traditional therapies. The robot also includes a visual interface that will help in the rehabilitation period.

**ARMin robot:** This robot was developed by Sensory-Motor Systems Lab ETH Zurich (Nef *et al.*, 2007) It is considered one of the most advanced exoskeletons today through clinical validation. ETH began developing this robot in 2005. ARMin I is the first version, it contains 4 DOFs. The latest version of this robot is ARMin IV, with 7 DOFs: 3 DOFs at shoulder joint, 2 DOF for elbow joint and 2 DOFs for wrist joint (see Figure 1.3). The lengths and weights of different parts of the robot are adjustable to suit the patient’s morphology. A spring connected to the upper part of the robot compensates the mechanical gravity of the robot if or when the robot lost the power of actuation. The robot is equipped with position sensors and torque force to measure the interaction between the patient and the robot. Its integration with the environment provides feedback multi-model and different visions for making well-defined movements (Nef *et al.*, 2007).
Several controls techniques have been developed for manipulating this robot to provide different modes of rehabilitation (passive and active). These techniques have been clinically validated on patients using the practice of daily living (Guidali et al., 2011) and therapeutic games with visual feedback. Among the controls approaches applied on the robot, we find the switch to the position control that has been tested on a hemiparesis patient to try to correct pathological synergies arms.

**ABLE robot:** ABLE is an exoskeleton robot developed by CEA-LIST systems, located in the Institute of Intelligent Systems and Robotics in France (Crocher, 2012). It is designed to provide rehabilitation of the upper limb as shown in Figure 1.4. It is characterized by its lightness and mechanical reversibility. The complete system is a structure consisting of a main kinematics parts which has four active DOF, and two secondary kinematics parts; each with four passive DOF (Crocher, 2012). Several controls were applied on this system; among them: proportional-derivative (PD) position control, friction and gravity compensation control to make it as transparent as possible as (Crocher, 2012).
MGA robot: MGA is an exoskeleton robot given in Figure 1.5, it was developed by Georgetown University (Carignan et al., 2007) for the shoulder and elbow joints, this robot has 5 DOF, allowing the patient to exercise flexion/extension, abduction/adduction and internal/external rotation of the shoulder joint, flexion/extension of the elbow joint, and pronation/supination of the forearm joint (Carignan et al., 2007).

SAM robot: SAM is an exoskeleton robot developed by the European Space Agency (ESA). It has 7 degrees of freedom with force feedback and with a mass of 7.4 Kg. It has compact joints with local DC actuation, sensor integration, and virtual reality interface. It is dedicated to complete teleoperation applications. Several controls were applied on the robot; the most important controls are force-position and admittance control (Letier and Preumont, 2010).
1.1.2 Inverse Kinematics

Generally, in most applications using non-redundant exoskeleton robots, the desired trajectories and/or therapeutic tasks are expressed in Cartesian space particularly in active rehabilitation
motion. However, operators prefer to control redundant exoskeleton robots in joint space in order to manage their redundancy (Crocher, 2012). Numerous methods, including numerical (Tolani and Badler, 1996), analytical (Bin et al., 2011), (Tolani et al., 2000), optimization (Xia and Wang, 2001), (Khoogar et al., 2011), vision (Jiang et al., 2014), (Zhang et al., 2014), (Chan et al., 2014), and geometric methods (Tolani and Badler, 1996), (Loh and Rosen, 2013), (Kim et al., 2011a), (Asfour and Dillmann, 2003) have been developed to solve the problem of inverse kinematics. The most frequently and most widely applied solutions for redundant robots are the inverse Jacobian and the Jacobian pseudo-inverse methods (Craig, 2005), (Kelly et al., 2006), (Klein and Huang, 1983). However, these approaches provide multiple solutions in Cartesian space. Moreover, the methods present many downsides, such as singularity problems. Further, in rehabilitation applications, the objective is not only to reach the desired goal; it is also imperative to reach an optimal solution corresponding to a human movement.

![SAM Exoskeleton robot](Taken from (Letier and Preumont, 2010))

1.1.3 Uncertain nonlinear control

The control of uncertain nonlinear dynamics is one of the challenging topics of nonlinear control engineering problems. In particular, a control system should be developed to ensure the stability of the system. In addition, its performance should not be affected by the disturbances
generated from the variation of internal parameters of the system, unmodeled dynamics stimulation, and external disturbances (Slotine et al., 1991; Khalil and Grizzle, 1996). Many studies discussing the problem of modeling and control of exoskeleton robot manipulator based on centralized approaches have been given in (Rahman et al., 2013; Ueda et al., 2010; Lee et al., 2012). Nevertheless, in the previously cited studies, the control design is model-based, in which the control law requires the full dynamic model of the exoskeleton robot. The estimation of the dynamic parameters is one of the open problems in exoskeleton manipulators, notably, with high degrees of freedom (DOFs) (Krstic et al., 1995), and in the presence of human-robot interaction. Conventional control approaches consider that the dynamic model of the upper arm manipulator is known. However, in practice, it becomes very difficult to get the exact model and uncertain may still exist. To overcome this problem, robust control approaches based on the Lyapunov theory are developed to ensure the stability of the full system (Khan et al., 2017, 2016a,b; Huang and Chien, 2010; Luna et al., 2016). However, these controllers are very complicated due to the complexity of the regression matrix (Huang and Chien, 2010). As solution, a time delay estimation (TDE) is proposed (Youcef-Toumi and Shortlidge, 1991; Youcef-Toumi and Ito, 1990; Brahmi et al., 2017a). By this method, it is sufficient to delay the output-input of the system only one step to provide a good approximation of the unknown uncertainties dynamic model of the exoskeleton robot. Nevertheless, TDE approach suffers from the Time Delay Error (TDR) caused by the noisy measurements and hard nonlinear function of the robot model during delay constant, which would degrade the approximation performance.

On the other hand, many other works have used decentralized control for this type of robotic systems as in (Luna et al., 2016; Ochoa Luna et al., 2015). A decentralized adaptive control, based on the virtual decomposition approach, was proposed, where the whole system was decomposed virtually into several individual subsystems. This decomposition makes the parameters, adaptation and the control law very easy. As an example of these works that applied on other type of robots, an adaptive tracking control design for an uncertain mobile manipulator dynamics based on appropriate reduced dynamic model was suggested in (Aviles et al., 2012). An adaptive controller based on the backstepping technique (Brahmi et al., 2016)
was implemented to the trajectory tracking of the wheeled mobile manipulator. Recently, approximation-based control strategies like fuzzy logic and neural networks have been used to learn the exoskeleton dynamic model (Chen et al., 2015; Li et al., 2015a). However, through these approaches only uniformly ultimate boundedness of the tracking errors was achieved. Meanwhile, the estimated weights were not reached to their actual values. This might reduce convergence speed during weights training operation, which stops the approximation-based control for real-time implementation.

It is remarkable from a natural human movement (since the human upper limb is attached with the exoskeleton robot) that the human does not need accurate information about kinematics and dynamics of the arm (or any object carried by upper extremity) to reach an object in space. Due to that, many control strategies have been designed to solve the problem of kinematic and dynamic uncertainties (Arimoto, 1999; Cheah, 2006; Yazarel and Cheah, 2002; Huang and Chien, 2010; Cheah et al., 2005; Hutchinson et al., 1996). The main innovative point of these controllers is that the adaptation of the both kinematic/dynamic uncertainties has been provided, which allows the exoskeleton robot to perform the human-like motion and supplies to the control system more flexibility to handle the uncertainties and parameters variation. However, the above controllers are based on the classical regressor matrix. These types of controllers assume that the robot is linear in a set of physical parameters and find a control law able to ensure the stability of this linear system only around its operating points (Yao, 1996). In fact, the manipulator is highly nonlinear. So, the integration of this adaptation law may affect the stability of the system in the presence of even small disturbances (Yao, 1996). Adaptive visual or image-based tracking control (Hutchinson et al., 1996; Deng et al., 2002; Espiau et al., 1992; Gans et al., 2003; Malis and Chaumette, 2002; Liu et al., 2006) is one of the powerful approaches that has been developed to transact with the kinematic/dynamic uncertainties. This is due to their robustness practically to modeling and calibration errors (Deng et al., 2002). However, these controllers are concentrated on uncertainties in nonlinear transformation functions or image Jacobian matrix but they ignored the uncertain kinematic/dynamic effects. Additionally, few stability analysis are provided in the literature for visual tracking control with the uncer-
tainties of kinematics/dynamics and in the presence of uncertainties in visual system (camera) parameters (Cheah et al., 2006).

1.2 Research objectives and methodology

The main objective of this work is to develop and improve the ETS-MARSE exoskeleton arm to perfectly achieve the passive and active rehabilitation motion to physically disabled people, with full shoulder, elbow and wrist rehabilitation capacities. As discussed above, with the high dimensional of the robot system (7-DOFs), the exoskeleton robot may face a problem to achieve some trajectories in Cartesian space particularly, in active mode or free motion due to the multi-solution or singularity problem provided by the conventional methods of robot inverse kinematics. So, to find an optimal solution of the robot inverse kinematics corresponds to the human arm configuration is a mandatory step to make the exoskeleton mimics the human natural motion.

As mentioned above, these robots are distinguished by a highly nonlinear dynamics because of their complex mechanical design and arduous nonlinearities, such as nonlinear friction forces, backlash, etc. Besides, the collaboration between the human and the robot makes the robot system subject to unknown and external disturbances because of different physiological conditions of each subject. These conditions involve non-linear bio-mechanical properties of the musculoskeletal system, its payload, and the possibility of the existence of spasticity,...etc. It is consequently imperative to design an adaptive controllers that approximates the dynamic model of the exoskeleton robot and minimizes the non-smooth nonlinear constraints effects, while maintaining the stability of the exoskeleton robot at the same time with different mode of rehabilitation motion (passive and active). The methodology described below were applied during this research to achieve these objectives:
1.2.1 Human inverse kinematics

In order to perform rehabilitation tasks, some trajectories must be given in Cartesian space. Because the redundancy problem was not solved, a new approach was designed and implemented to make the exoskeleton robot perform Cartesian trajectories as human-like motion without singularity problem (chapter 2).

1.2.2 Development of the nonlinear control laws

Different adaptive control strategies were investigated. The unknown dynamics or uncertainties dynamics/kinematics, the massive non-linearity, and the high dimensional characterizing such systems considerably complicate the control problem and make it difficult to solve using the conventional approaches cited previously. As a solution to these serious problems, many adaptive control laws were proposed and validated in real time in different mode of rehabilitation motions (passive and active). These techniques can be summarized as follows:

a. A new compliant control based on a second-order sliding mode with adaptive-gain incorporating time delay estimation. In this work, the dynamic parameters of the system were considered uncertain and were estimated by time delay estimation (chapter 3). It is worth mentioning that in this work, the human inverse kinematics solution proposed in chapter 2 is used to complete the active rehabilitation motion;

b. Adaptive backstepping approach integrated with modified time-delay estimation to provide an accurate estimation of unknown dynamics of the exoskeleton robot and to compensate for external bounded disturbances (chapter 4);

c. The passive and active control strategies based on integral backstepping approach to provide a physical assistance and rehabilitation by a 7-DOF exoskeleton robot with nonlinear uncertain dynamics and unknown bounded external disturbances due to the robot user’s physiological characteristics. Besides, The Damped Least Square method is introduced to estimate the desired movement intention of the subject with the objective to provide active rehabilitation motion (chapter 5);
d. Finally, a new adaptive visual tracking control approach based on sliding mode control in Cartesian space applied to an exoskeleton robot with uncertain kinematics and dynamics, taking into account uncertainties in visual system (camera) parameters (chapter 6).

1.2.3 Implementation and experimentation

Besides design and simulation with a diversity of tools (MATLAB, and LabVIEW), all the control strategies and methods investigated in this work were implemented and validated on the ETS-MARSE robot.

Figure 1.7 outlines the methodology explained above and summarizes the structure of this work.

![Figure 1.7 Methodology of the improvement ETS-MARSE rehabilitation system](image-url)
1.3 Originality of the research and contribution

This research focuses on the development of a redundant exoskeleton robot named ETS-MARSE to provide a suitable passive and active rehabilitation motion to physically disabled people, with full shoulder, elbow and wrist rehabilitation capacities. Therefore, the development is focused on two points. The first one is to solve the inverse kinematic problem that allows the exoskeleton robot to perform a human-like motion. The second one is the development of robust nonlinear control laws that allow the exoskeleton robot to achieve the desired physiotherapy treatment, even if the exoskeleton is subject to the dynamic/kinematic uncertainties and external disturbances. Following the literature review, although numerous researches deal with the control of these kinds of robots, few of them are concerned with high nonlinearity and dynamic/kinematic uncertainties where the majority of them consider that the dynamic model of the exoskeleton is fully known. Unfortunately, it is impossible to determine exactly the overall dynamic model of the exoskeleton robot due to its complex mechanical structure and hard nonlinearities that negatively affect the performance of the exoskeleton robot. To solve uncertainty problem, some researchers have proposed an adaptive control approach based on conventional adaptive techniques. In this work, we propose different adaptive strategies. Contrary to what appears in the cited researches, this thesis has the following contributions:

a. Unlike convention inverse kinematics solutions, the proposed solution provides an optimal Cartesian solution resembling the movement of the human upper limb and always presents a valid human arm configuration. In addition, the proposed inverse kinematic algorithm provides a solution that does not pose a singularity problem and characterized by a high level of precision and rapidity of response;

b. The delay time estimation (TDE) strategy is considered to be one of the important contributions employed in this work to account for the unknown uncertainty of the dynamic model of the exoskeleton due to: firstly, its easiness to implement it in real time application. Secondly, TDE is one of the approaches that are not influenced by the size of the estimated dynamics parameters in the case of high DOFs. Nevertheless, TDE approach suffers from the Time Delay Error (TDR) caused by the noisy measurements and hard
nonlinear function of the robot model during delay constant, which would degrade the approximation performance;

c. Using a new recursive control to estimate and reduce the effect of the Time Delay Error (TDR) and improve the robustness of the control system. Usually, this error limits the performance of TDE approach;

d. Considering the unknown kinematics and dynamics with unknown external disturbances (different weight of the arm of each subject), adaptive visual controller incorporating with recursive control is developed to estimate the nonlinear kinematic and dynamic uncertainties with unknown disturbances and to drive the robot to follow the desired functional therapy activity and provide a smooth exoskeleton-aided passive activity;

e. The stability of the system and the convergence of its errors are formulated and demonstrated based on Lyapunov–Krasovskii functional theory to prove the stability of the system in each delayed interval;

f. A new compliant control is developed based on human inverse kinematics. This control ensures an accurate relation between the desired force and the desired position and produces human-like motion;

g. Adaptive gains are incorporated with a second-order sliding mode control to provide an adaptation of the switching gains and to avoid the undesired chattering;

h. Using Damped Least Squares (DLS) technique to easily estimate the user’s intention of movement.

1.4 List of publications

In this section, we present only the published papers related to this thesis:


CHAPTER 2

CARTESIAN TRAJECTORY TRACKING OF A 7-DOF EXOSKELETON ROBOT BASED ON HUMAN INVERSE KINEMATICS

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Abstract

Exoskeleton robots have become an important tool to provide rehabilitation therapy to stroke victims because of their ability to allow rehabilitation exercises, ranging from passive to active-assisted movement, for extended time periods. To generate the desired rehabilitation trajectories and ensure an optimal Cartesian solution, we propose a new solution to the inverse kinematics problem, which is compatible with human upper limb movement and is valid for human arm configuration. In addition, in order to provide passive rehabilitation therapy to the upper extremity of disabled individuals, we implement a robust nonlinear control based on the backstepping technique on the seven-degrees-of-freedom ETS-MARSE robot. The controller was designed to reject the user’s force caused by the subject’s muscular activity. Experimental results validate the stability, robustness, and exactness of the proposed method with the designed tests performed by healthy subjects.

Keywords: Backstepping controller, Exoskeleton robots, Inverse kinematics, Lyapunov function, Robotic rehabilitation.
2.1 Introduction

Neurological diseases have become a challenging problem for the scientific community. According to recent statistics, every year at least 15 million people worldwide suffer neurological diseases such as stroke. Six million of these people die and five million more are living with a persistent disability (Sidney et al., 2013). In Canada each year at least 16,000 Canadians die because of a stroke incident (De Morand, 2014). The survivors from strokes generally do not have the means to perform daily activities such as eating, dressing, and bathing by themselves due to permanent disability, often on one side of the body (De Morand, 2014). The current treatment to aid stroke victims to regain their missing functional capacity, gain new skills, and enhance their quality of life is rehabilitation. However, rehabilitation treatments require intensive and fatiguing work by the therapist (De Morand, 2014), (Brahim et al., 2016a). Modern robotic technology has become an important component in many medical specialization, even to accomplish rehabilitation tasks such as physiotherapy. Recently, robotics rehabilitation has gained attention from the scientific community because robots are able to supplement the treatments provided by conventional physical therapy (de Santé, 2012). The importance of rehabilitation robots lies in their ability to provide intensive physiotherapy for a long period of time (Brahim et al., 2016a), (de Santé, 2012). The measured data of the robot permits the physiotherapist to accurately evaluate the patient’s performance.

Current robotics technologies present us with new devices known as rehabilitation robots, which are used to overcome the limitations of classical physiotherapy approaches and create new methods of rehabilitation treatment (Xie et al., 2016). The most valuable role played by these robots is to help both the therapist and stroke victims improve treatment with rehabilitation therapy. To rehabilitate patients with upper-limb impairment, these exoskeletons habitually are worn on the lateral side of the patients’ upper-limb (Nef et al., 2007). Various research groups have designed this type of manipulator like ARMin IV robot (Nef et al., 2007) designed with (6-DOFs), RUPERT robot (Balasubramanian et al., 2008), (Huang et al., 2016) with (5-DOFs) and (SUEFUL-7) robot (Gopura et al., 2009) consist of (7-DOFs). In order to provide a modern rehabilitation approach for the upper-limb, we have developed a 7-DOF
exoskeleton robot named ETS-MARSE (Rahman et al., 2015). The seven degrees of freedom (DOFs) of the manipulator makes it a redundant robot capable of reaching several arm configurations in its workspace and providing task-oriented exercises in joint space, Cartesian space and free motion (Rahman et al., 2015).

The goal of the assistive automatic control is to assist patients to complete physical therapy. A passive assistive movement is an indispensable thing to do following a stroke accident. In this scenario, the exoskeleton robot carries the dysfunction upper limb of the subject to achieve a passive physical activity (Brahim et al., 2016a); the user’s force is thus considered as an external disturbance. Multiple control strategies have been designed to handle the exoskeleton system in order to provide an assistive passive therapy: such a PID control (Yu and Rosen, 2010), a neural PID controller (Yu and Rosen, 2013), and a nonlinear modified computed torque control (Rahman et al., 2011a). Furthermore, a sliding mode controller combined with exponential reaching law was proposed in (Rahman et al., 2012c) to limit the chattering problem caused by the large control gains. While the above control schemes make use of the dynamic model of the robots, the accuracy of the model, however, directly affects the controller performance. Various nonlinear control strategies developed, which do not need an exact dynamic model of the robot system, like a fuzzy sliding mode adaptive controller proposed in (Sun et al., 1999), and neural sliding mode control (Ciliz, 2005), where both controllers are aimed to overcome the effect of the uncertain nonlinear dynamics and the unexpected external disturbances, which influence the robot performance. However, these methods require heavy computations, making the implementation very difficult (Li et al., 2015a).

Generally, the exoskeleton manipulators originally have a highly nonlinear dynamic model. Unfortunately, it is impossible to determine exactly the overall dynamic model of the exoskeleton robot due to its complex mechanical structure and hard nonlinearities in its parameters, (Li et al., 2016) such as nonlinear friction forces, backlash, etc... . In addition, the subjects exhibit different physiological conditions, such as non-linear bio-mechanical characteristics of the musculoskeletal system, the different weight of the upper-limb for each patient, the presence of spasticity in neurological patients (Brahim et al., 2016b). During a physical move-
ment, the external forces can turn into an unknown function that can affect the exoskeleton performance (Brahim et al., 2016b). In order to address the mentioned problems, we have implemented an adaptive nonlinear control based on the backstepping approach. A robust force observer is implemented to estimate the user’s force. A powerful Lyapunov function is offered to solve the adaptation problem systematically, proving the closed-loop stability and ensuring the asymptotic convergence of the output errors (Li et al., 2015a), (Brahim et al., 2016b), (Jin and Xu, 2013). The principal benefit of the designed control is that an accurate knowledge of the external disturbances is not needed. In addition, it gives outstanding tracking performance similar to the natural human movement despite the presence of external perturbations (Brahim et al., 2016b).

Practically, in most applications using non-redundant exoskeleton robots, the desired trajectories and/or therapeutic tasks are expressed in Cartesian space. However, operators prefer to control redundant exoskeleton robots in joint space in order to manage their redundancy (Crocher, 2012). Numerous methods, including numerical (Tolani and Badler, 1996), analytical (Bin et al., 2011), (Tolani et al., 2000), optimization (Xia and Wang, 2001), (Khoogar et al., 2011), vision (Jiang et al., 2014), (Zhang et al., 2014), (Chan et al., 2014), and geometric methods (Tolani and Badler, 1996), (Loh and Rosen, 2013), (Kim et al., 2011a), (Asfour and Dillmann, 2003), have been developed to solve the problem of inverse kinematics. The most frequently and most widely applied solutions for redundant robots are the inverse Jacobian and the Jacobian pseudo-inverse methods (Craig, 2005), (Kelly et al., 2006), (Klein and Huang, 1983). However, these approaches provide multiple solutions in Cartesian space. Moreover, the methods present many downsides, such as singularity problems. Further, in rehabilitation applications, the objective is not only to reach the desired goal; achieving an optimal solution corresponding to a human movement is also mandatory.

In this paper, a new inverse kinematics solution is proposed to provide Cartesian passive rehabilitation exercises. The philosophy of this approach is to develop a mathematical model based on geometric and analytic solutions of human upper limb motion in order to imitate its movement. To ensure the robustness of this method, we integrate new geometric solutions
consistent with the mechanical design of the ETS-MARSE exoskeleton. The main advantage of this approach is that it provides an optimal Cartesian solution resembling the human upper limb movement and always presents a valid human arm configuration. In addition, the proposed inverse kinematics algorithm provides a solution not having a singularity problem and characterized by a high level of precision and rapidity of response. The controller used to demonstrate the validity of the approach shows that it is capable of following the commanded trajectories due to the smooth output of the inverse kinematics algorithm. The precision and the durability of the control scheme and the algorithm of inverse kinematics was appraised by the implementation of designed exercises conforming to a passive physiotherapy. All therapy tasks were performed with healthy subjects.

The rest of the paper is organized as follows: section 2.2 gives a brief description of the ETS-MARSE and describes the inverse kinematics algorithm. Section 2.3 describes the control strategy. Experiments results and discussion are presented in section 2.4. Finally, the conclusion and future work are presented in section 2.4.

2.2 ETS-MARSE wearable robot

2.2.1 Description of ETS-MARSE robot

The ETS-MARSE is a redundant robot with 7-DOFs, designed to rehabilitate the impaired human upper limb, as shown in Figure 2.1. The designing of the manipulator robot ETSMARSE was originally inspired by the joints and anatomy of the human upper limb in order to be comfortable with the robot users during the rehabilitation tasks. The shoulder part is represented by three joints. The first two joints are designed to perform the horizontal and vertical extension/flexion motion, while the third joint of the shoulder conducts the external/internal rotation of the shoulder movement. The elbow part is formed by one joint to perform flexion/extension movement of the elbow. The last part of the upper limb is the wrist part that consists of three joints. The first joint is designed to achieve pronation/supination movement of the forearm; the second joints and the third joint are designed to perform radial and ulnar deviation and flexion
and extension of the wrist part, respectively (Brahim et al., 2016a), (Rahman et al., 2015). Table 2.1 presents the workspace of the designed robot.

Table 2.1  Workspace ETS-MARSE

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>Motion</th>
<th>Workspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shoulder joint horizontal flexion/extension</td>
<td>0°/140°</td>
</tr>
<tr>
<td>2</td>
<td>Shoulder joint vertical flexion/extension</td>
<td>140°/0°</td>
</tr>
<tr>
<td>3</td>
<td>Shoulder joint internal/external rotation</td>
<td>−85°/75°</td>
</tr>
<tr>
<td>4</td>
<td>Elbow joint flexion/extension</td>
<td>120°/0°</td>
</tr>
<tr>
<td>5</td>
<td>Forearm joint pronation/supination</td>
<td>−85°/85°</td>
</tr>
<tr>
<td>6</td>
<td>Wrist joint ulnar/radial deviation</td>
<td>−30°/20°</td>
</tr>
<tr>
<td>7</td>
<td>Wrist joint flexion/extension</td>
<td>−50°/60°</td>
</tr>
</tbody>
</table>

Figure 2.1  ETS-MARSE with link frame

The modified Denavit–Hartenberg (DH) parameters are given in Table 2.2. These parameters are obtained from the frame reference shown in Figure 2.1 and are used to obtain the homogeneous transformation matrices (Craig, 2005).
Table 2.2 Modified DH parameters

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>$\alpha_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$d_s$</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{\pi}{2}$</td>
<td>0</td>
<td>$d_c$</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{\pi}{2}$</td>
<td>0</td>
<td>$d_w$</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>6</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_6 - \frac{\pi}{2}$</td>
</tr>
<tr>
<td>7</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_7$</td>
</tr>
</tbody>
</table>

The homogenous transformation matrix that associates frames $\{7\}$ to $\{0\}$ can be obtained by multiplying transformation matrices:

$$
\begin{align*}
0_7T = \begin{bmatrix}
0T & 1T & 2T & 3T & 4T & 5T & 6T & 7T
\end{bmatrix}
\end{align*}
$$

(2.1)

2.2.2 Inverse kinematics solution

The design of an exoskeleton like the ETS-MARS robot is inspired by the human arm. As the human extremity has redundant kinematics, exoskeleton redundancy permits it to perform a large number of human movements. However, due to the redundancy, there are infinite solutions to the inverse kinematics problem for any given end-effector position. The main objective of the proposed algorithm is to limit these solutions to an optimal (or unique) solution able to imitate the human motion and to correspond to the human arm configuration. It is interesting to remark that a human uses the minimum joints of the arm with its optimal configuration when he reaches for an object in 3-D space ((De Morand, 2014)). It is significant also that the first three (shoulder) joints of the ETS-MARSE are responsible for the elbow pose. The elbow position and its joint angle value are responsible for the end-effector position. The last (wrist) joints are responsible for the end-effector rotation only. The proposed inverse kinematics solution is inspired directly from the human arm movement. From a known end-effector pose, it is
possible to define the elbow joint ($\theta_4$) geometrically and independently using Figure 2.2 (a):

$$\theta_4 = \pi \pm \arccos \left( \frac{d_e^2 + d_w^2 - \|w - s\|^2}{2d_e d_w} \right)$$ \hspace{1cm} (2.2)

Without a doubt, joint $\theta_4$ has a unique solution; with the help of a geometric relation, we can define the elbow pose. This position is determined by defining the distance ($R$) and the angle ($\alpha$) that is located between the distance $d_e$ and the virtual axis that links between the end-effector position and the fixed shoulder position. From the elbow position, the three joints of the shoulder ($\theta_1$, $\theta_2$, $\theta_3$) can be derived analytically with a unique solution. Thus, the inverse kinematics problem now includes obtaining the configuration of a nonredundant structure where $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$ joints have a unique solution (geometric and/or analytical solution). From the known (desired) end-effector rotation and the above angles ($\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$), the solution of $\theta_5$, $\theta_6$, and $\theta_7$ angles can be derived by the comparison as we will see in the last part. The detail of the solution will be given later. In special positions, when the position of the wrist is fixed with particular constraints, the redundancy of the arm can be parameterized by observing that the elbow is still free to swivel about an axis from the swivel angle ($\phi$) to the shoulder. The axes of this circle are perpendicular to the vector pointing from the shoulder joint to the wrist joint. The swivel angle ($\phi$) is the rotation angle of the elbow around a virtual axis that connects the shoulder and wrist joints. As the swivel angle varies, the elbow traces the arc of a circle on a plane whose normal is parallel to the shoulder-to-wrist axis (Tolani and Badler, 1996), as shown in Figure 2.2 (b). The swivel angle is not a joint of the robot; it is not directly controllable, but a measurable parameter result of the kinematics of the robot, based on the constraints on the wrist orientation. It is important to notice that for a sample or basic rehabilitation motion, the swivel angle is fixed to zero (not influenced the solution of inverse kinematics solution). A simple movement does not contain any constraints on the wrist position and this fixation does not influence the redundancy of the exoskeleton robot.

In order to be able to describe the circle mathematically, the normal vector of the plane is defined as:

$$\hat{n} = \frac{w - s}{\|w - s\|}$$ \hspace{1cm} (2.3)
Then, $\hat{u}$ is set to be a projection of an axis $\hat{z}$ chosen arbitrarily on the circle:

$$\hat{z} = [0 \ 0 \ 1]^T$$  \hspace{1cm} (2.4)

$$\hat{u} = \frac{\hat{z} - (\hat{z} \hat{n}) \hat{n}}{\|\hat{z} - (\hat{z} \hat{n}) \hat{n}\|}$$  \hspace{1cm} (2.5)

and $\hat{v}$ is the last component of the orthonormal base:

$$\hat{v} = \hat{n} \times \hat{u}$$  \hspace{1cm} (2.6)

$R$ and $c$ are the radius and center of the circle, respectively. The radius $R$ is found with simple trigonometric relationships. Let us start by defining the distance $d$ or center of circle $c$ as
follows (see Figure 2.2 (b)):

\[ d = \cos(\alpha) d_e \hat{n} \quad (2.7) \]

with:

\[ \cos(\alpha) = \frac{d^2_w - d^2_e - \|w - s\|^2}{-2de\|w - s\|} \quad (2.8) \]

and:

\[ R = \sin(\alpha) d_e \quad (2.9) \]

Finally, the elbow position can be parameterized as a function of the swivel angle \((\phi)\) about axis \(\hat{u}\):

\[ e(\phi) = d + R [\cos(\phi)\hat{u} + \sin(\phi)\hat{v}] + s \quad (2.10) \]

where \(s\) is the vector shoulder coordinate define such that: \(s_{\text{shoulder}} = [0 ~ 0 ~ d_s]^T\); (see Equation (2.14)).

We see that the elbow position depends on the value of the swivel angle \((\phi)\) and radius \(R\). If the value of the swivel angle \((\phi)\) is defined, we can subsequently solve the problem of inverse kinematics analytically. Finding a value of the swivel angle \((\phi)\) with a small error is very difficult even if it is restricted to the case of one specific and well-defined task (a specific reaching or grasping exercise, for instance), and for the general case, it is impossible to obtain an exact estimation of the swivel angle. Various research works, proposed in (Kim et al., 2011a), (Kim et al., 2011b), (Chua et al., 2013), used estimation methods such as cost functions optimization and a prediction algorithm based on data acquisition to determine the value of the swivel angle for certain specific reaching activities corresponding to rehabilitation exercises. In our case, the elbow cannot rotate along the aforementioned shoulder-wrist circle due to mechanical design limitations of the ETS-MARSE robot. This indicates that even if it were possible to compute a swivel angle from the robot end-effector position, the robot would not be able to achieve it always. However, as we have mentioned, it is very difficult to have the swivel angle computed accurately in direct relation from the manipulator’s end-effector position. It is then not a restriction in terms of inverse kinematics, but it is in terms of the robot working space, which lacks some shared space with a human arm. Thus, the robot lacks
the ability to perform some human motions (these motions consist of a varying swivel angle, outside the robot’s envelope). As we mentioned above, there is no clear method to estimate the swivel angle exactly when we have constraints on the wrist position. However, for the mechanical design of the ETS-MARSE robot, we proposed a solution to solve this problem. Practically, we found that variation of the joint $\theta_6$ provokes a limit variation of swivel angle. This variation is calculated geometrically (see Figure 2.3). In this case, no numerical estimation is needed, and the inverse kinematics solution will remain purely analytical. Therefore, if we have constraints on the wrist, we have a virtual axis that connects between the hand center point ($d_h$) and the new elbow position as shown in Figure 2.3. The variation ($V$) is assumed approximately equal ($V'$). This variation ($V$) is caused by the constraint that is provoked by joint $\theta_6$. Geometrically, the angle ($\vartheta$) is an equal angle ($\vartheta'$). Hence, according to cosines law, it is possible to define angles ($\vartheta$) and angle ($\vartheta'$) such that:

\[
\begin{align*}
V^2 &= a_1^2 + p_1^2 - 2a_1p_1\cos(\vartheta) \\
V'^2 &= a_2^2 + p_2^2 - 2a_2p_2\cos(\vartheta')
\end{align*}
\]

(2.11)

If $V = V'$ we can define $\cos(\vartheta)$ as follow:

\[
\cos(\vartheta) = \cos(\vartheta') = \frac{a_1^2 + p_1^2 - a_2^2 - p_2^2}{2(a_1p_1 - a_2p_2)}
\]

(2.12)

Now $V'$ is available from equation (2.11). Let us now to define the variation of swivel angle ($\phi$) from triangle ($cee'$) such that:

\[
\phi = \frac{2R^2 - V'^2}{2R^2}
\]

(2.13)

The shoulder is translated by a distance $d_s$ on the z-axis as regard to the origin (frame $\{0\}$ in Figure 2.1).

\[
P_{shoulder} = T_{0}^{-1} [0 \ 0 \ 0 \ 1]^T = [0 \ 0 \ d_s \ 1]^T
\]

(2.14)
where \( T \) is the first homogenous transformation matrix calculated by using Table 2.1. The position of the elbow is determined only by the three shoulder joints \( \theta_1, \theta_2 \) and \( \theta_3 \):

\[
P_{\text{elbow}} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}^T = \begin{bmatrix} d_e C_1 S_2 \\ d_e S_1 S_2 \\ d_s + d_e C_2 \end{bmatrix}
\]

(2.15)

so,

\[
P_{\text{elbow}} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} = \begin{bmatrix} d_e C_1 S_2 \\ d_e S_1 S_2 \\ d_s + d_e C_2 \end{bmatrix}
\]

(2.16)

here, \( C_i \) denotes \( \cos(\theta_i) \) and \( S_i \) denotes \( \sin(\theta_i) \).

The wrist joint position depends on \( \theta_1, \theta_2, \theta_3 \) and \( \theta_4 \). The last three joints \( \theta_5, \theta_6 \) and \( \theta_7 \) intervene just to determine the wrist orientation.

\[
P_{\text{elbow}} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}^T = \begin{bmatrix} d_e C_1 S_2 - d_w (S_4 (S_1 S_3 - C_1) C_2 C_3) - C_1 C_4 S_2 \\ d_w (S_4 (C_1 S_3 + C_2 C_3 S_1) + C_4 S_1 S_2) + d_e S_1 S_2 \\ d_s + d_w (C_2 C_4 - C_3 S_2 S_4) + d_e C_2 \end{bmatrix}
\]

(2.17)

2.2.2.1 Solving \( \theta_1 \) and \( \theta_2 \)

Obtained directly from equation (2.16), the elbow position is known. Then:

\[
\frac{e_y}{e_x} = \frac{d_e S_1 S_2}{d_e C_1 S_2} \Rightarrow \theta_1 = \text{atan2}(e_y, e_x)
\]

(2.18)

For \( \theta_2 \) we have two solutions, i.e., an analytic solution and a geometric solution, from equation (2.16):

\[
\cos(\theta_2) = \frac{e_z - d_s}{d_e}
\]

(2.19)
Figure 2.3  (a) Geometrical solution of swivel angle (φ) 3D view. (b) Geometrical solution of swivel angle 2D view

Figure 2.4  Geometrical representation of joint θ₂
In addition:

\[
\begin{align*}
\sin(\theta_2) &= \frac{e_z}{d_e C_1} \quad \text{if } C_1 \neq 0 \\
\sin(\theta_2) &= \frac{e_y}{d_e S_1} \quad \text{Otherwise}
\end{align*}
\]

(2.20)

So,

\[
\theta_2 = \text{atan2} \left( \sin(\theta_2), \frac{e_x - d_x}{d_e} \right)
\]

(2.21)

Since joint 2 is located at the shoulder in Figure 2.4, the coordinate of the origin (joint 1 in Figure 2.2) is \([0 \ 0 \ 0]\), the elbow position is defined by equation (2.16), and the distance between the shoulder and elbow \(d_e\) is constant. We can use the law of cosines to compute \(\theta_2\):

\[
\theta_2^* = \pi \pm \cos \left( \frac{\|e - O\|^2 - d_z^2 - d_e^2}{-2d_edx} \right)
\]

(2.22)

where \(\theta_2^*\) is the geometric solution.

2.2.2.2 Solving \(\theta_3\)

For joint 3 (\(\theta_3\)), we have two solutions as well, analytic and geometric. To find \(\theta_3\) analytically, we multiply both sides of equation (2.17) by \((0 \ T \ 1/2)^{-1}\):

\[
\left(0 \begin{array}{c} T \ 1/2 \end{array}\right)^{-1} \left(0 \begin{array}{c} T \ 1/2 \ 2/3 \ 4/5 \end{array} \right) [0 \ 0 \ 0 \ 1]^T = \left(0 \begin{array}{c} T \ 1/2 \end{array}\right)^{-1} [w_x \ w_y \ w_z \ 1]^T
\]

(2.23)

This leads to:

\[
\begin{bmatrix}
  d_wS_4C_3 \\
  -d_wC_4 - d_e \\
  d_wS_4S_3 \\
  1
\end{bmatrix}
= \begin{bmatrix}
  d_xS_2 - w_zS_2 + w_xC_1C_2 + w_yC_2S_1 \\
  w_xC_1S_2 - w_yS_1S_2 + d_xC_2 - w_zC_2 \\
  w_yC_1 - w_xS_1 \\
  1
\end{bmatrix}
\]

\[
\theta_3 = \text{atan2} (w_yC_1 - w_xS_1, d_xS_2 - w_zS_2 + w_xC_1C_2 + w_yC_2S_1)
\]

(2.24)
Using the law of cosines (see Figure 2.5, we can obtain the geometric solution:

$$\theta_3^* = \pi \pm \arccos \left( \frac{||w - w^*||^2 - 2d_w^2}{-2d_w^2} \right)$$

(2.25)

where $\theta_3^*$ is the geometric solution.

![Geometrical representation of joint $\theta_3$](image)

Figure 2.5  Geometrical representation of joint $\theta_3$

Now, we have two solutions (analytic and geometric) for each joint (joints 2 and 3). The importance of these solutions is that they provide different results for the inverse kinematics, particularly in the case of singularity. These solutions are always valid for human arm movement.

2.2.2.3 Solving $\theta_5$, $\theta_6$ and $\theta_7$

The orientation of the end-effector is given by:

$$R_{\text{end-effector}} = R_1R_2R_3R_4R_5R_6R_7 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

(2.26)
Consider that \( R_w = R_5 R_6 R_7 \):

\[
R_w = \begin{bmatrix}
S_5 S_7 + C_5 C_6 C_7 & C_7 S_5 - C_5 C_6 S_7 & C_5 S_6 \\
C_7 S_6 & -S_6 S_7 & C_6 \\
C_6 C_7 S_5 - C_5 S_7 & C_5 C_7 - C_6 S_5 S_7 & -S_5 S_6
\end{bmatrix}
\]  

(2.27)

Substituting equation (2.27) in equation (2.26):

\[
R_w = R_4^T R_3^T R_2^T R_1^T R_{end-effector}
\]  

(2.28)
where the $R_{\text{end-effector}}$ matrix defines the desired end-effector orientation with respect to the origin. In this case, we use standard matrix rotation and Euler angles.

To find the wrist angles, we apply equation (2.27) and equation (2.28) to do the comparison between them:

$$\theta_6 = \begin{cases} 
\arccos(r_{23}) \\
- [2\pi + \arccos(r_{23})]
\end{cases}$$  \hspace{1cm} (2.29)

If $S_6 \neq 0$

$$\begin{cases} 
\tan(\theta_5) = \frac{r_{33}}{r_{13}} \Rightarrow \theta_5 = \arctan2 \left( \frac{r_{33}}{S_6}, \frac{r_{13}}{S_6} \right) \\
\frac{r_{22}}{r_{21}} = \frac{-S_6S_7}{C_7S_6} \Rightarrow \theta_7 = \arctan2 \left( \frac{-r_{22}}{S_6}, \frac{r_{21}}{S_6} \right)
\end{cases}$$  \hspace{1cm} (2.30)

If $S_6 = 0$

$$\begin{cases} 
\theta_5 = \arctan2(r_{31}, r_{11}) \\
\theta_7 = 0
\end{cases}$$  \hspace{1cm} (2.31)

In summary, the inverse kinematics algorithm of the ETSMARSE is shown in Figure 2.6. The algorithm consists of several steps. First, the desired position must be checked to determine whether or not it is within the robot workspace. If it is in the robot workspace area, the value of joint $\theta_4$ can then be extracted. The position of the elbow is determined by the values of the first four joints of the arm (three joints of the shoulder and one joint of the elbow) with the help of the value of the swivel angle ($\phi$). If the elbow position is within the workspace of the shoulder joint, all the values of the remaining angles can be solved analytically. We can compute joint angle $\theta_2$ analytically; if the solution of this computation is outside $\theta_2$ limits, we use its geometrical solution. Then the value of joint angle $\theta_3$ will be computed in the same manner; if no solution exists, the robot returns to its initial position. If the above angles are available, the algorithm continues to calculate the orientation of the end-effector joints ($\theta_5$, $\theta_6$...
and $\theta$). Finally, if the angle values are outside the joint angle limits, we will set them at the mechanical limit.

### 2.3 Control design

The dynamics behavior of ETS-MARSE manipulator is given by the following expression by using the Lagrangian method:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) + F(\theta, \dot{\theta}) = \tau + \tau_{ex}$$

(2.32)

$$\tau_{ex} = J^T F_{ex}$$

(2.33)

where $\theta$, $\dot{\theta}$ and $\ddot{\theta} \in \mathbb{R}^7$ are, respectively, the joint’s position, velocity, and acceleration vectors, $M(\theta) \in \mathbb{R}^{7 \times 7}$ is the symmetric and positive definite inertia matrix, $C(\theta, \dot{\theta}) \in \mathbb{R}^{7 \times 7}$ is the Coriolis and centrifugal vector, $G(\theta) \in \mathbb{R}^7$ is the gravitational vector, $\tau \in \mathbb{R}^7$ is the torque vector, and $\tau_{ex} \in \mathbb{R}^7$ is the external disturbances vector. $F(\theta, \dot{\theta}) \in \mathbb{R}^7$ is the friction vector considered in this paper and can be written as:

$$F(\theta, \dot{\theta}) = \tau_{friction} = cf \text{ sign}(\dot{\theta})$$

(2.34)

where $cf$ is the friction constant. The stability of the overall system represented by equation (2.32) is confirmed under the following properties and assumptions.

**Property 2.1:** The inertia matrix $M(\theta)$ is symmetric and positive definite for all $\theta \in \mathbb{R}^n$ (Craig, 2005).

**Property 2.2:** $\dot{M}(\theta) - 2C(\theta, \dot{\theta})$ is a skew symmetric matrix, such that: $y^T [\dot{M}(\theta) - 2C(\theta, \dot{\theta})] y = 0, \forall y, \theta, \dot{\theta} \in \mathbb{R}^n$ (Kelly et al., 2006), (Dawson et al., 2003).
Assumption 2.1: For all $t > 0$, there exist constants $\rho_1 > 0$, $\rho_2 > 0$ and $\rho_3 > 0$ where $\|\theta_d(t)\| \leq \rho_1$, $\|\dot{\theta}_d(t)\| \leq \rho_2$ and $\|\ddot{\theta}_d(t)\| \leq \rho_3$. These constants depend on the boundedness of all desired Cartesian trajectory (see Figure 2.11).

Equation (2.32) can be expressed as:

$$\ddot{\theta} = M(\theta)^{-1} (\tau + \tau_{ex}) - M(\theta)^{-1} (C(\theta, \dot{\theta}) \dot{\theta} + G(\theta) + F(\theta, \dot{\theta}))$$  \hspace{1cm} (2.35)

The goal of the control scheme is to ensure that the measured joint positions $\theta$ of the robot track the desired trajectory $\theta_d$, and to limit the effect of the external perturbation by introducing a force observer.

The first step in this approach is to select the errors of the system using the position and velocity of the system, i.e., the state variables. We can define the errors as follows:

$$e_1 = \theta - \theta_d$$  \hspace{1cm} (2.36)

$$e_2 = \dot{\theta} - \gamma$$  \hspace{1cm} (2.37)

where $\theta \in \mathbb{R}^7$ is the measured trajectory, $\theta_d \in \mathbb{R}^7$ is the desired trajectory, and $\gamma$ is a virtual control input. The derivative of equation (2.36) with respect to time is given such:

$$\dot{e}_1 = \dot{\theta} - \dot{\theta}_d$$  \hspace{1cm} (2.38)

Substituting equation (2.37) into equation (2.38), we find:

$$\dot{e}_1 = e_2 + \gamma - \dot{\theta}_d$$  \hspace{1cm} (2.39)
The derivative of equation (2.37) with respect to time such that:

\[
\dot{e}_2 = \ddot{\theta} - \dot{\gamma} \tag{2.40}
\]

The virtual control can be defined as follows:

\[
\gamma = \dot{\theta}_d - k_1 e_1 \tag{2.41}
\]

where \( k_1 \in \mathbb{R}^{7 \times 7} \) is diagonal positive definite matrix. The one-time derivative of the virtual input is: \( \dot{\gamma} = \ddot{\theta}_d - k_1 \dot{e}_1 \).

Consider the first Lyapunov function candidate as follows:

\[
V_1 = \frac{1}{2} e^T_1 e_1 + \frac{1}{2} e^T_2 M(\theta) e_2 \tag{2.42}
\]

The derivative of \( V_1 \) is written as:

\[
\dot{V}_1 = e^T_1 \dot{e}_1 + e^T_2 M(\theta) \dot{e}_2 + \frac{1}{2} e^T_2 \dot{M}(\theta) e_2 \\
= e^T_1 (e_2 + \gamma - \dot{\theta}_d) + e^T_2 M(\theta) (\ddot{\theta} - \dot{\gamma}) + \frac{1}{2} e^T_2 \dot{M}(\theta) e_2 \\
= -e^T_1 k_1 e_1 + e^T_1 e_2 + e^T_2 M(\theta) (M(\theta)^{-1} (\tau + \tau_{ex}) - M(\theta)^{-1} (C(\theta, \dot{\theta}) \dot{\theta} + G(\theta) + F(\theta, \dot{\theta}))) \\
- e^T_2 M(\theta) \dot{\gamma} + \frac{1}{2} e^T_2 \dot{M}(\theta) e_2 \\
= -e^T_1 k_1 e_1 + e^T_1 e_2 + e^T_2 (\tau + J^T F_{ex} - C(\theta, \dot{\theta}) \dot{\theta} - G(\theta) - F(\theta, \dot{\theta}) - M(\theta) \dot{\gamma}) \\
+ \frac{1}{2} e^T_2 \dot{M}(\theta) e_2 \tag{2.43}
\]

From equation (2.37), we can obtain: \( \dot{\theta} = e_2 + \gamma \)

\[
\dot{V}_1 = -e^T_1 k_1 e_1 + e^T_1 e_2 + e^T_2 \left( \tau + J^T F_{ex} - C(\theta, \dot{\theta}) e_2 - C(\theta, \dot{\theta}) \gamma - G(\theta) - F(\theta, \dot{\theta}) - M(\theta) \dot{\gamma} + \frac{1}{2} \dot{M}(\theta) e_2 \right) \tag{2.44}
\]
Using **Property 2.2**, we obtain:

\[
\dot{V}_1 = -e_1^T k_1 e_1 + e_1^T e_2 + e_2^T (\tau + J^T F_{ex} - C(\theta, \dot{\theta}) \gamma - G(\theta) - F(\theta, \dot{\theta}) - M(\theta) \dot{\gamma}) \tag{2.45}
\]

Let the control input that maintains the stability of the robot system as follows:

\[
\tau = -k_2 e_2 - e_1 - J^T F_{ex} + C(\theta, \dot{\theta}) \gamma + G(\theta) + F(\theta, \dot{\theta}) + M(\theta) \dot{\gamma} \tag{2.46}
\]

where \( k_2 \in \mathbb{R}^{7 \times 7} \) is diagonal positive definite matrix. Substituting the control input (2.46) into equation (2.45), we obtain \( \dot{V}_1 \leq -e_1^T k_1 e_1 - e_2^T k_2 e_2 \). However, \( F_{ex} \) is unknown; hence, the control input (2.46) is not suitable. An observer law will be integrated into the control input to estimate the external perturbation.

**Theorem 2.1:** Consider the robot system described by the (2.32). Where, the control law (2.47) and the adaptation law (2.48) ensure: 1) the global asymptotic stability of the system; 2) the convergence of tracking error to zero; and 3) the boundedness of the external force error.

\[
\hat{\tau} = -k_2 e_2 - e_1 - J^T \hat{f}_e + C(\theta, \dot{\theta}) \gamma + G(\theta) + F(\theta, \dot{\theta}) + M(\theta) \dot{\gamma} \tag{2.47}
\]

The adaptation laws are updated by:

\[
\dot{\hat{f}}_e = -k_3^{-T} J e_2 \tag{2.48}
\]

where \( k_3 \in \mathbb{R}^{6 \times 6} \) is diagonal positive definite matrix.

**Proof 2.1:** In the beginning, let us start by defining the estimation error of disturbance as follows:

\[
\hat{f}_e = F_{ex} - \hat{f}_e \tag{2.49}
\]
Consider the second Lyapunov function candidate as follows:

\[ V_2 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T M(\theta) e_2 + \frac{1}{2} \hat{f}_e^T k_3 \hat{f}_e \]  

(2.50)

The derivative of \( V_2 \) with respect to time yields:

\[ \dot{V}_2 = -e_1^T k_1 e_1 + e_1^T e_2 + e_2^T \left( \tau + J^T F_{ex} - C(\theta, \dot{\theta}) \gamma - G(\theta) - F(\theta, \dot{\theta}) - M(\theta) \dot{\gamma} \right) \]

\[ + \hat{f}_e^T k_3 \hat{f}_e \]

(2.51)

Applying the control law (2.47) to equation (2.51) yields:

\[ \dot{V}_2 = -e_1^T k_1 e_1 + e_1^T e_2 + e_2^T \left( \tau + J^T \hat{f}_e \right) + \hat{f}_e^T k_3 \hat{f}_e \]

(2.52)

We can rewrite equation (2.52) as follows:

\[ \dot{V}_2 = -e_1^T k_1 e_1 - e_2^T k_2 e_2 + \left( \hat{f}_e^T k_3 + e_2^T J^T \right) \hat{f}_e \]

(2.53)

Substituting equation (2.48) in equation (2.53), we obtain:

\[ \dot{V}_2 \leq -e_1^T k_1 e_1 - e_2^T k_2 e_2 + \left( -k_3^{-1} k_3 J e_2 + e_2^T J^T \right) \hat{f}_e \]

(2.54)

We obtain:

\[ \dot{V}_2 \leq -e_1^T k_1 e_1 - e_2^T k_2 e_2 \]

(2.55)

with \( k_1 \) and \( k_2 \) being positive gains. Let us take the second derivative of \( \dot{V}_2 \) as follows:

\[ \ddot{V}_2 \leq -2e_1^T k_1 \dot{e}_1 - 2e_2^T k_2 \dot{e}_2 \]

(2.56)
Since $\dot{\theta}$ and $\ddot{\theta}$ are bounded, this means $e_1, \dot{e}_1, e_2$ and $\dot{e}_2$ are bounded. This proves that $\dot{V}_2$ is bounded since $e_1, \dot{e}_1, e_2$ and $\dot{e}_2$ are all bounded. So, $\dot{V}$ is continuous and negative semi-definite; according to Barbalat’s lemma. We have $e_1 \to 0, \dot{e}_1 \to 0, e_2 \to 0$ and $\dot{e}_2 \to 0$ as $t \to \infty$, which mean that the system is asymptotic stable. The closed-loop system of the designed control technique is illustrated in Figure 2.7.

![Figure 2.7 General schematic of the proposed control](image)

### 2.4 Experiments and results

The architecture of the rehabilitation exoskeleton system is presented in Figure 2.8. The LabView (National Instruments) PXI system is the environment where the proposed controller of the system robot was realized. Three blocks are designed to complete the experimental setup. The first one is the user interface, used to select, determine the controller parameters, and define the specification of the rehabilitation exercise. In addition, it provides the measured data of the robot that permits the operator to evaluate accurately the performance of the human exoskeleton system. The second one is a PXI-8108 card, where the control scheme was implemented with a sampling time ($1.25 \, \mu s$). The robot operating system also runs in the PXI processor (Intel Core 2 Duo). The controller output is the torque to the joints; this torque was transformed to current and then to desired voltages to command the motor drivers.
Finally, the last block, a field programmable gate array runs with a sampling time (50 μs). It is slated to execute two loops concurrently. The first loop holds a simple proportional-integral action for controlling the motor’s current, as a function of the calculated reference current. The second loop is designed to obtain the measured data (position angles). For more details see (Rahman et al., 2015), and (Ochoa Luna et al., 2015).

The inverse kinematics algorithm is added to the LabView code, which controls the robot. The inverse kinematics code is inserted into a MATLAB script module of LabView, which simplifies the process since few changes have to be made to the code to fit the new environment. This module allows MATLAB code to be written directly into LabView, with some restrictions.

In this paper, we implemented a passive rehabilitation protocol with two healthy subjects (age: 27–30 years; height: 170–177 cm; and weight: 75–79 kg). Passive rehabilitation therapy com-
prises a set of exercises given by a physiotherapist or clinician to increase muscle power and joint range of movement to the physically disabled individual with impaired upper limb motion. In passive rehabilitation therapy, the subject sits relaxed, and the physiotherapist moves the subject’s arm slowly, usually starting with a small range of movement, and then continuously increasing the joint range. It should be mentioned that in all our experiments, the ETS-MARSE initiated its motion with the elbow joint at $90^\circ$ as can be seen in Figure 2.9. Note that the control gains used for the tests were found experimentally, and are as follows:

$$\begin{align*}
k_1 &= \begin{bmatrix} 18, 16, 18, 18, 18, 18 \end{bmatrix} \\
k_2 &= \begin{bmatrix} 80, 80, 64, 62, 80, 80 \end{bmatrix} \\
k_3 &= \begin{bmatrix} 0.5, 0.5, 0.5, 0.5, 0.5 \end{bmatrix}
\end{align*}$$

In order to validate the human inverse kinematics, the healthy subjects repeated the designed tasks with the standard solution of a redundant robot called pseudo-inverse Jacobian matrix (Rahman et al., 2015), (Rahman et al., 2012a). It is important to notice that for the implementation of a pseudo-inverse Jacobian matrix, we used a point to point technique (Spong et al., 2006a) to avoid the singularity problem. In this case, we compared the performance of the proposed controller and the human inverse kinematic algorithm (PB-HIK) and the proposed controller with the pseudo-inverse Jacobian (B-PIK). The comparison is done by using the average RMS values of the position errors and rotation errors of the end-effector and the execution time of each approach.

In the first trial, the trajectory tracking performance of the PB-HIK in Cartesian space was evaluated. The first exercise starts at the initial position with the elbow joint angle at $90^\circ$. Then, the end-effector follows a path to reach target A; next, it follows path A to B to reach target B, after which it returns to the initial position. The desired rotation of the end-effector here is used. That means the redundancy of the ETS-MARS is employed to perform the rehabilitation activity. The speed of the movement used here is $30 \text{ deg/s}$. Therefore, the objective of this task was first to assess the performance of the inverse kinematics algorithm in reaching different
targets at different locations, such as transferring objects on the surface of a table. Second, it validates the performance of the proposed control in a Cartesian trajectory.

The experimental results for the passive 3D Cartesian trajectory achieved with subject A (age: 28 years; height: 173 cm; and weight: 78.5 kg) are shown in Figure 2.10. From the plot of Figure 2.10(a), we can see that the desired Cartesian trajectory overlaps with the measured Cartesian trajectory. The plot of Figure 2.10(b) compares the desired Cartesian trajectory with the measured Cartesian one. It is clear from Figure 2.10(B) that the designed control presents an excellent trajectory tracking despite the presence of small end-effector errors. Figure 2.11(a) presents the tracking performance of ETS-MARSE in joint space (the outputs of the inverse kinematics algorithm) and it is obvious from the figure that the desired angles are smooth.
We can say that the inverse kinematics algorithm converts trajectories from task space to joint space effectively and precisely. That conversion helps the controller to provide good results as we see in Figure 2.11(b). Figure 2.12 presents the convergence of the user’s force parameters during the therapeutic task that proves the potentiality of the designed controller.

Figure 2.13 shows the experimental results for the same [Figure 2.10(a)] passive 3-D Cartesian trajectory performed with the same subject-A (age: 28 years; height: 173 cm; and weight: 78.5 kg) using the B-PIK. The performed trajectory is very good due to the efficiency of the proposed controller, the trajectory obtained by the proposed inverse kinematics [Figure 2.10(a)]
Figure 2.11 Tracking performance of ETS-MARSE in joint space (the inputs of the control joint-based) corresponding to Figure 2.10(a). (b) Tracking error in joint space is more similar and more flexible to the natural movement than the trajectory obtained by pseudo-inverse kinematics (Figure 2.13). This latter is straight while the natural movement is characterized by the flexibility of the motion. We can conclude that the proposed inverse kinematics provides a good result similar to human movement.
The workspace envelope of the ETS-MARSE with the performed trajectory [illustrated in Figure 2.10(a)] is shown in Figure 2.14. It is manifest from Figure 2.14 that the proposed inverse kinematics provides a good solution with respect to the workspace limits of the ETS-MARSE exoskeleton. The variation of the elbow position is also presented to demonstrate that the variation of the swivel angle provides a good solution. This solution has always belonged to the workspace of the robot. The plot illustrating the variation of the swivel angle is presented in Figure 2.15. As mentioned above, due to the mechanical design of the ETS-MARSE, the elbow cannot rotate along the aforementioned shoulder-wrist circle. For that, we observe a small variation of the swivel angle, not more than 10°.

Let us now confirm the proposed solution by comparing the human inverse kinematics and the pseudo-inverse Jacobian matrix. Table (2.3) summarizes the average RMS errors of the end-effector and time execution of each approach along the 3-D desired trajectory. Where $\delta^\circ$, $\beta^\circ$, and $\gamma^\circ$ are the desired rotation of the end-effector of the robot.

The comparison between the proposed controller with the human inverse algorithm (PB-HIK) and the B-PIK is illustrated in Figure 2.16. It is clear from Fig. 16 that the proposed algorithm gives excellent results for the two subjects notably at the level of the rotation of the end-effector. The execution time of the proposed approach is reduced by more than 50%.
We can infer that the proposed inverse kinematics solution gives an excellent solution compared with the conventional approach. The implementation of this approach with healthy subjects confirms that the proposed algorithm provides a solution similar to the human upper limb.
Table 2.3  Statistical analysis of controllers performance in cartesian space

<table>
<thead>
<tr>
<th>RMS errors</th>
<th>PB with Human-IK</th>
<th>B with Pseudo-IK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Subject-A</td>
<td>Subject-B</td>
</tr>
<tr>
<td>$e_x (cm)$</td>
<td>0.0019</td>
<td>0.0022</td>
</tr>
<tr>
<td>$e_y (cm)$</td>
<td>0.0028</td>
<td>$6.243 \times 10^{-4}$</td>
</tr>
<tr>
<td>$e_z (cm)$</td>
<td>0.0014</td>
<td>0.0016</td>
</tr>
<tr>
<td>$\alpha (deg)$</td>
<td>0.0080</td>
<td>0.0111</td>
</tr>
<tr>
<td>$\beta (deg)$</td>
<td>0.0130</td>
<td>0.0206</td>
</tr>
<tr>
<td>$\gamma (deg)$</td>
<td>0.0207</td>
<td>0.0050</td>
</tr>
<tr>
<td>execution time</td>
<td>0.0106789</td>
<td></td>
</tr>
</tbody>
</table>

![Graph](image_url)  

**Figure 2.15**  Variation of swivel angles of the performed trajectory illustrated in Fig. 11(a)

movement in 3-D space, and that is valid for the human arm configuration in all situations. Additionally, the proposed controller (PB-HIK) provides a good tracking performance even if the dynamic model is affected by different human conditions.
2.5 Conclusion

In this paper, a new inverse kinematics solution was described to manage the redundancy of a 7-DOF exoskeleton robot and provide a passive rehabilitation trajectory imitating the human motion. The proposed algorithm of the inverse kinematics solution is inspired directly from the human arm movement. Therefore, this approach can be extrapolated for all upper limb rehabilitation robots. In addition, the proposed algorithm can be used to limit the multi-solution of the redundancy of this kind of robot. A robust control design, with the adaptation of external force based on backstepping control, is developed by integrating a force observer to estimate the user’s force. The control achieved stability and robustness of the ETS-MARSE exoskeleton robot system with passive Cartesian trajectories. A comparison between the solution obtained by the proposed algorithm and the solution provided by a pseudo-inverse Jacobian matrix is presented to validate the advantages of the proposed human inverse kinematics algorithm. The experimental results have validated the effectiveness and practicability of the proposed algorithm. In future work, we will seek to replace the geometric solution with a visual solution using a visual system such as Kinect and/or camera devices.
Abstract

Rehabilitation robots are a new technology dedicated to the physiotherapy and assistance motion and has aroused great interest in the scientific community. These kinds of robots have shown a high potential in limiting the patient’s disability, increasing its functional movements and helping him/her in daily living activities. This technology is still an emerging area and suffers from many challenges like compliance control and human–robot collaboration. The main challenge addressed in this research is to ensure that the exoskeleton robot provides an appropriate compliance control that allows it to interact perfectly with humans. This article investigates a new compliant control based on a second-order sliding mode with adaptive-gain incorporating time delay estimation. The control uses human inverse kinematics to complete active rehabilitation protocols for an exoskeleton robot with unknown dynamics and unforeseen disturbances. The stability analysis is formulated and demonstrated based on Lyapunov function. An experimental physiotherapy session with three healthy subjects was set up to test the effectiveness of the proposed control, using virtual reality environment.

Keywords: Rehabilitation robots, human inverse kinematics, time delay estimation, second-order sliding mode control, passive and active assistive motion, virtual reality.
3.1 Introduction

Many types of injury such as structural defects, cerebral palsy, brain tumors, spinal injury, multiple sclerosis or other neurological diseases can damage the human nervous system, which means loss of the functional capacity (Sidney et al., 2013; Lundström et al., 2008; Nichols-Larsen et al., 2005). In paralyzed patients, maximum capacity can be restored through physical therapy applications and robotic devices (Sidney et al., 2013; Lundström et al., 2008; Nichols-Larsen et al., 2005; Keller et al., 2015; Xie et al., 2016). The purpose of the physical therapy and the neural rehabilitation program is to help the patient achieve the best possible condition and to gain independence of his functions in order to minimize or to eventually eliminate the problems that might arise from the disease. The employment of robotic devices, known as rehabilitation robots, in assistive domains has demonstrated a high potential to improve the functional movements, and to assist her/him in daily living activities such as self-care skills (Keller et al., 2015). A rehabilitation exoskeleton robot is an articulated mechanical structure with several degrees of freedom (DOFs) having the same anatomy of the human arm (Keller et al., 2015; Xie et al., 2016; Balasubramanian et al., 2008; Gopura et al., 2009; Rahman et al., 2015). Unlike prostheses that replace a limb of the body, the exoskeleton robot clings to it externally and acts in parallel. It can be dedicated to a specific part of the body such as the hand, arm, leg or several limbs at the same time. Equipped with sensors and actuators, it measures the movements and forces of the user and it produces a force to interact dynamically with its wearer.

There are different levels of robotic assistance strategies used after the neurological accidents to provide suitable physical therapy. The most urgent, usually the first six weeks after the accident, is passive physical therapy (Sidney et al., 2013; Xie et al., 2016). In this type of therapy, the exoskeleton brings the patient’s limb, which is completely passive, to realize a therapy task. Its advantage is based on the robot’s ability to provide intensive therapy over a long period of time (Brahim et al., 2016b,a). The next types of therapy, active-assisted and active modes, allow the patient to voluntarily initiate movement. Then, the exoskeleton’s wearer can perform a free motion (active mode), the robot corrects or guides this movement (active-assisted mode).
In the latter case, the robot limits the tremors or corrects the trajectory. After detecting the initiation of a motion, usually predetermined, the robot will guide the achievement of the activity, often using an impedance and/or admittance control (Li et al., 2017a; Ochoa Luna et al., 2015). Additionally, these strategies can be utilized for the evaluation or the study of subject movements. In these modes, theoretically, the patient should not feel the presence of the exoskeleton robot. This is known as robot transparency. Therefore, the subject is completely active and the exoskeleton robot should not affect the movement.

In this research, we focus on active motion control. This type of control is particularly suited to rehabilitation phases where the patient has partially recovered his mobility abilities. The goal is then to improve these abilities by encouraging him to use them. Usually, we focus to interpret the intention of the subject and the abilities that already recovered. A compliant control or indirect force control is an effective approach to accomplish this level of motion assistance. Its successful application appears when the exoskeleton is in free space, and when it is in contact with the known or unknown environment. Admittance control is a typical choice for the rehabilitation application of compliance control (Ochoa Luna et al., 2015; Culmer et al., 2010; Ozkul and Barkana, 2013; Choi et al., 2009; Zanchettin et al., 2016). This control structure aims to create a dynamic relationship between a measured position (and its derivative) and the user’s force. The admittance scheme is implemented by two loops. The first one is the outer force that is responsible for creating the force-position relationship; it usually uses a first order transfer function to define this relation. The second one is the inner position loop which must be as fast as possible (Calanca et al., 2016). Although this control is characterized by its robustness thanks to the existing advanced tracking control (inner position loop), it also suffers from several drawbacks, such as imprecision of the desired performance of the control system or inaccuracy of conversion of the desired force to desired position dynamics (Calanca et al., 2016). The inaccuracy of the performance here is caused by the gain parameters of the first order transfer function which is responsible for establishing a desired force-position relationship as given in (Calanca et al., 2016; Adams and Hannaford, 1999). Many admittance controls have been developed by estimating the user force directly from his biological signals
The most commonly used is electromyography (EMG) signals, which are considered important signals to describe the user’s intention of motion (Khan et al., 2016b; Jamwal et al., 2016; Li et al., 2017b). However, the sensor positioning to obtain the EMG signals is very sensitive, where the thickness of the skin, tiredness and stress of the subject, even the temperature of the body has a big influence on the performance of an EMG-based control (Huang et al., 2015). More than that, the quantity of information from physiological signals and its non-linearity is so large that it takes expensive calculations to process signals, which must be clear input signals for the control (Huang et al., 2015).

The inner position loop may be responsible for the inaccurate performance of the admittance control due to the limitation of the applied control tracking approach (Calanca et al., 2016). This limitation is related to the complicated design of this type of robots and their high number of DOF, where the dynamics of these robots is generally uncertain (Brahim et al., 2016b,a). Furthermore, the contact dynamics of the robot and its wearer increases the unknown nonlinear-uncertainties function. A sliding mode approach is one of the strategies that are widely applied on robotics systems thanks to its attractive characteristics of robustness to nonlinear-uncertainties and external disturbances (Slotine et al., 1991; Utkin et al., 2009). The price to achieve this robustness is to control the undesirable chattering problem (Fridman, 1999). Many conventional approaches were developed to avoid this problem by replacing the discontinuous function by a continuous function (as a saturation function or sigmoid function) to ensure a continuous control (Slotine et al., 1991; Rahman et al., 2013). But in this case, the sliding mode control loses its robustness to disturbances; here the controller forces the sliding trajectories of the system to be close to the sliding surface, not on the sliding surface itself. New approaches have been developed to address the chattering dilemma such high order sliding mode controller (Levant, 2003; Ling et al., 2012) and a second-order sliding mode controller (Bartolini et al., 2001). This latter allows a sliding surface and its derivative to get to zero and to maintain the discontinuous control under an integral function, which can attenuate the undesirable chattering (Bartolini et al., 1998). This approach presents two advantages: the
first one is to keep the robustness characteristic of standard sliding mode control, and the second one is to improve the accuracy of the control performance by attenuating the chattering problem. Nevertheless, the second time-derivative of the sliding surface might amplify the instability of the system, a risk that the nonlinear uncertainties and external disturbances present. Additionally, in physical therapy, usually the exoskeleton robot would perform with different subjects with different levels of neurological injuries. In this case, the control system would be influenced by the variation of parameters of the robot and change of subjects’ characteristics.

3.1.1 Main contribution

Motivated by the previous analysis, we propose new adaptive-gains second-order sliding mode control combined with time delay estimation (TDE) (Youcef-Toumi and Ito, 1990; Jin et al., 2015; Brahmi et al., 2017a,b). This controller deals mainly with the accuracy/robustness problem of a compliant control applied to an exoskeleton robot with a high number (seven) of DOFs. The accuracy of the performance can be decreased within the outer loop (the force loop) due to the admittance function and/or sensibility of biological sensors signals, e.g. EMG. To overcome these drawbacks, the paper proposes a simple force loop based on human inverse kinematics (Brahmi et al., 2017c). Therefore, according to the force applied at the wrist joint by the subject, the force control loop produces a Cartesian displacement with respect to a constant proportional gain. In our case, the required position is the sum of this displacement and the actual Cartesian position. Unlike the existing methods, we use the human inverse kinematics algorithm developed in our previous work (Brahmi et al., 2017c), which is characterized by its accuracy and rapidity of response while avoiding singularities. This makes it possible to provide an accurate relation between desired force and desired position. On the other hand, the robustness of the performance can be negatively influenced within the inner loop (or tracking control loop) thanks to the applied control strategy. Despite the accuracy and robustness of second-order sliding mode control due to its potential to attenuate the undesired chattering dilemma (Bartolini et al., 1998). However, the complicated mechanical structure of the robot and the variation of its parameters (due to the uncertainties function and unforeseen external
forces due to the different subject’s characteristics) require a large switching gain to maintain the stability of the robot system, which again causes the chattering problem. To overcome these limitations, we incorporate the second-order sliding mode control with TDE to achieve an accurate performance of the exoskeleton robot with unknown dynamics and external disturbances. TDE can easily estimate the unknown dynamics and external disturbances with a simple control scheme without being affected by the size of the estimated parameters (high degree of freedom) (Brahim et al., 2016a). It only employs one step time-delayed knowledge about the previous state-response of the system and the previous control input to provide an accurate estimation of unknown dynamics and external perturbations. More than that, TDE is an easy approach in the real-time implementation. In addition, to cope with the variation of the characteristics of the subjects and to avoid the problem of undesirable chattering, a new adaptation law of the commutation gains is proposed. The stability of the robot system and the convergence of its errors are formulated and demonstrated based on Lyapunov function. The contribution of this paper can be summarized in three points:

• A new compliant control is developed based on human inverse kinematics. This control ensures an accurate relation between the desired force and the desired position and produces human-like arm motion. (Outer control loop);

• TDE is used for the evaluation of the unknown dynamics and external disturbances (Inner control loop);

• Adaptive gains are incorporated with a second-order sliding mode control to provide an adaptation of the switching gains and to avoid the undesired chattering (Inner control loop).

The proposed control is evaluated experimentally with healthy subjects using a virtual environment (VE) (Ferrer et al., 2013). This VE is a very attractive tool bringing many benefits (Weiss et al., 2014). It allows the creation of immersive and interactive scenes where the oriented task can be introduced in the form of serious games. This interface also helps in stimulating the subjects by viewing the tasks performed. Finally, we present a comparative study with a conventional control approach that shows the accuracy, robustness and flexibility of the proposed controller dealing with unknown dynamics, external disturbances and parameters variations.
The remainder of the paper is organized as follows. The human inverse kinematics and dynamics of the exoskeleton are presented in the next section. The control scheme is described in section 3.3. Experimental and comparison results are shown in section 3.4; finally, the conclusion is presented in section 3.5.

3.2 Characterization kinetics and dynamics of ETS-MARSE robot

3.2.1 Modeling of ETS-MARSE robot

To make the rehabilitation robot suitable for performing a wide variety of daily-life activities with the ease of human–robot interaction, it must be designed to be in harmony with the human arm configuration. The ETSMARSE is a redundant robot with 7-DOFs, as shown in Figure 3.1. It is designed to assist the impaired human right upper limb. The structure idea of the ETS-MARSE was extracted from the anatomy of the human arm to be in concordance with exoskeleton wearers along the physical therapy activities. The shoulder portion is described by three joints: The first two joints are created to produce the vertical and horizontal extension/flexion movement of the shoulder, while the third joint is aimed to conduct the internal and external rotation of the shoulder. The elbow portion is composed of one joint to complete the flexion/extension motion of the elbow. The wrist portion is composed of three joints: The first joint is shaped to perform pronation and supination motion of the forearm, the second joint and the third joint are dedicated to offering, respectively, ulnar/radial deviation and flexion/extension motions (Rahman et al., 2015, 2013). The robot system is implemented with a virtual interface in which the subject and the therapist can follow the motion of the rehabilitation tasks (Ferrer et al., 2013). Virtual reality is a software environment created to simulate the subject and allow him to perform different physical therapy exercises. This virtual interface can also provide task-oriented activities in task space, Cartesian space, and free motion.

Table (3.1) presents the modified Denavit–Hartenberg (DH) of the exoskeleton robot. Figure 3.1 shows the reference frame attached to the robot which permits to acquire the mentioned
(DH) parameters (Craig, 2005). The workspace of the designed exoskeleton robot is summarized in Table (3.2).

![Reference frames of ETS-MARSE](image)

Figure 3.1  Reference frames of ETS-MARSE

### 3.2.2 Human inverse kinematics solution

In most applications of assistive robots, the required trajectory is given in Cartesian space. The standard transformation from Cartesian space to joint space is done by a non-linear function named Jacobian matrix. Due to the geometry of the robot, the inverse of a Jacobian matrix is not always available. In our case, the exoskeleton robot is redundant with 7 DOFs. The Jacobian pseudo-inverse solution widely used in robotics to solve this invertibility problem can be written as:

\[
\dot{\theta}_d = (J^T (J J^T)^{-1}) \dot{x}_d
\]

(3.1)
where $\dot{x}_d \in \mathbb{R}^{6 \times 1}$ is the desired Cartesian velocity, $\dot{\theta}_d \in \mathbb{R}^{7 \times 1}$ is the calculated joint velocity and $J \in \mathbb{R}^{6 \times 7}$ is the Jacobian matrix of the robot. In addition to the singularity problem, the pseudo-inverse kinematic solution provides a non-unique solution. In general, it implies that it is hard to guarantee a human-like motion. In our previous work (Brahmi et al., 2017c), a new solution of inverse kinematics was developed based on the analysis of human arm motion. This algorithm provides a human-like arm motion without any singularity configuration and it is characterized by its accuracy and rapidity of response. The main purpose of the human inverse kinematics is to limit solutions of the inverse kinematics to an optimal solution capable to mimic the human movement which corresponds to the human arm structure. It is remarkable that the human employs a minimum of arm joints for its optimal shape during him performs a motion in 3D space. It is remarkable also that the first three (shoulder) joints are in charge of the elbow position.

Table 3.1 Modified DH parameters

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>$\alpha_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$d_s$</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{\pi}{2}$</td>
<td>0</td>
<td>$d_e$</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{\pi}{2}$</td>
<td>0</td>
<td>$d_w$</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>6</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_6 - \frac{\pi}{2}$</td>
</tr>
<tr>
<td>7</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_7$</td>
</tr>
</tbody>
</table>

Table 3.2 Workspace ETS-MARSE

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>Motion</th>
<th>Workspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shoulder joint horizontal flexion/extension</td>
<td>$0^\circ/140^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>Shoulder joint vertical flexion/extension</td>
<td>$140^\circ/0^\circ$</td>
</tr>
<tr>
<td>3</td>
<td>Shoulder joint internal/external rotation</td>
<td>$-85^\circ/75^\circ$</td>
</tr>
<tr>
<td>4</td>
<td>Elbow joint flexion/extension</td>
<td>$120^\circ/0^\circ$</td>
</tr>
<tr>
<td>5</td>
<td>Forearm joint pronation/supination</td>
<td>$-85^\circ/85^\circ$</td>
</tr>
<tr>
<td>6</td>
<td>Wrist joint ulnar/radial deviation</td>
<td>$-30^\circ/20^\circ$</td>
</tr>
<tr>
<td>7</td>
<td>Wrist joint flexion/extension</td>
<td>$-50^\circ/60^\circ$</td>
</tr>
</tbody>
</table>
The end-effector position is determined from the elbow pose and its joint angle amplitude, while the end-effector rotation is achieved by the last (wrist) joints. From known end-effector pose, the elbow joint ($\theta_4$) can be calculated geometrically and separately, as we see in Figure 3.2. Based on geometric relationships, we can determine the elbow position easily. This latter is defined by obtaining the distance ($L$) and the angle ($\alpha$) which is situated between the link $d_e$ and the virtual axis that connects between the end-effector position and the shoulder position. While the elbow position is achieved, a unique analytic solution of three joints of the shoulder ($\theta_1$, $\theta_2$ and $\theta_3$) can easily be obtained. Therefore, the inverse kinematic dilemma presently involves getting the arrangement of a non-redundant structure with uniqueness solution of $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$ joints. Meanwhile, the solution of $\theta_5$, $\theta_6$ and $\theta_7$ can be acquired by the identification based on the known end-effector rotation (reference trajectory) and the calculated joints (Brahmi et al., 2017b).

In particular situations, when the position of the wrist is positioned with specific restrictions, the redundancy of the manipulator can be parameterized by remarking that the elbow is yet
free to swivel about an axis from the swivel angle (φ) to the shoulder. The axes of this circle are vertical to the vector guiding from the wrist joint to the shoulder. The swivel angle (φ) is the rotation angle of the elbow around a virtual axis that links wrist joint and the shoulder. The elbow pose draws the arc of a circle on a plane whose usual is parallel to the shoulder-to-wrist axis during the swivel angle changes. It is important to clarify that the swivel angle is not a joint, it is not straight controllable, however a measurable parameter outcome of the kinematics of the exoskeleton, based on the restrictions on the wrist rotation. Another significant point that in primary physical therapy task, the swivel angle is equal to zero, without any impact on the inverse kinematics solution dilemma. A physical therapy motion does not include any restrictions on the wrist posture and this fixation does not affect the redundancy of ETS-MARSE robot. The details can be found in (Brahmi et al., 2017c).

3.2.2.1 Dynamics of ETS-MARSE robot

The dynamic model of ETS-MARSE robot in joint space ($\theta \in \mathbb{R}^{7 \times 1}$) can be described as (Craig, 2005):

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) + F(\theta, \dot{\theta}) = \tau + \tau_{ex}$$  \hspace{1cm} (3.2)

where $\theta$, $\dot{\theta}$ and $\ddot{\theta} \in \mathbb{R}^7$ are, respectively, the joint’s position, velocity, and acceleration vectors, $M(\theta) \in \mathbb{R}^{7 \times 7}$ is the symmetric and positive definite inertia matrix, $C(\theta, \dot{\theta}) \in \mathbb{R}^{7 \times 7}$ is the Coriolis and centrifugal vector, $G(\theta) \in \mathbb{R}^7$ is the gravitational vector, $\tau \in \mathbb{R}^7$ is the applied joint torque to the exoskeleton robot, and $\tau_{ex} \in \mathbb{R}^7$ is the external disturbances vector. Without loss of generality, the matrices of dynamic model (3.2) can be written as follows:

$$\begin{align*}
M(\theta) &= M_0(\theta) + \Delta M(\theta) \\
C(\theta, \dot{\theta}) &= C_0(\theta, \dot{\theta}) + \Delta C(\theta, \dot{\theta}) \\
G(\theta) &= G_0(\theta) + \Delta G(\theta)
\end{align*}$$  \hspace{1cm} (3.3)
where \( M_0(\theta) \), \( C_0(\theta, \dot{\theta}) \) and \( G_0(\theta) \) are respectively the known inertia matrix, the Coriolis centrifugal matrix, and the gravity vector. \( \Delta M(\theta) \), \( \Delta C(\theta) \) and \( \Delta G(\theta) \) are the uncertain parts.

Let us introduce a new variable such that: \( \eta_1 = \theta \) and \( \eta_2 = \dot{\theta} \); hence, the dynamic model expressed in (3.2) can be rewritten as follows:

\[
\begin{align*}
    \dot{\eta}_1 &= \eta_2 \\
    \dot{\eta}_2 &= U(t) + f(t) + H(t)
\end{align*}
\]  

(3.4)

with:

- \( U(t) = M_0^{-1}(\theta) \tau \)
- \( f(t) = M_0^{-1}(\theta) \left[ -C_0(\theta, \dot{\theta}) - G_0(\theta) \right] \)
- \( H(t) = H(\theta, \dot{\theta}, \ddot{\theta}) = M_0^{-1}(\theta) \left[ \tau_{ex} - \Delta M(\theta) \ddot{\theta} - \Delta C(\theta, \dot{\theta}) \dot{\theta} - \Delta G(\theta) \right] \)

### 3.2.2.2 Problem formulation

The problem is the accuracy/robustness dilemma of a compliant control or indirect force control. In this paper, we focus on the implementation of a robust new controller that gives the exoskeleton system a high-level of accuracy of trajectory tracking, and more flexibility and robustness to deal with the unknown nonlinear dynamics, unstructured modeling errors and unknown bounded disturbances. It is important to mention that the controller is formulated and proved based on Lyapunov function using the following Property and Assumptions:

**Property 3.1:** The inertia matrix \( M_0(\theta) \) is symmetric and positive definite for all \( \theta \in \mathbb{R}^n \) (Craig, 2005).

**Assumption 3.1:** The joint position and joint velocity are measured.

**Assumption 3.2:** All kinematic singularities are avoided.

**Assumption 3.3:** Since the dynamic model of the manipulator is continuous, differentiable and bounded, the function \( H(t) \) and its time derivative \( \frac{d}{dt} [H(t)] \) are locally Lipschitz functions.
Assumption 3.4: The desired trajectory is bounded.

Assumption 3.5: The external disturbance $\tau_{ex}$ is supposed to be continuous, has finite energy and satisfies $\|\tau_{ex}\| \leq \omega$, with an unknown positive disturbance boundary $\omega$.

3.3 Control design and its stability

The compliance control objective is to achieve force control via motion control (outer control loop). Therefore, the compliance force is to achieve the required position based on the external force or free trajectory. In such case, the required position is the sum of the current Cartesian position and a certain Cartesian displacement with respect to the gain matrix. This gain matrix consists of proportional gains. The Second-Order Sliding Mode control combined with Time Delay Estimation (inner control loop) objective is to achieve the accuracy/robustness performance of the exoskeleton system. The proposed scheme is illustrated in Figure 3.3, and corresponds to a compliance control strategy; a subcategory of indirect force control as illustrated in Figure 3.4. The desired trajectory in proposed compliance control can be given by the following relation:

$$x_d = x_a + \delta_x$$

(3.5)

where $x_a \in \mathbb{R}^6$ is the measured actual Cartesian position, and $\delta_x \in \mathbb{R}^6$ is the displacement causing by the user’s force. This displacement can be interpreted as the estimate of Desired Movement Intention (DMI) of the subject. If $\delta_x \rightarrow 0$ this means the exoskeleton’s wearer stops to exert forces on the force sensor, so that the exoskeleton robot decreases its motion, when $x_d = x_a$, the exoskeleton rests in its most recent position. In the proposed controller, the exoskeleton changes its position depending on the user’s DMI that is obtained from the user’s measured force. Let us now firstly determine displacement $\delta_x$ from the user’s force as follows (Craig, 2005):

$$\delta_x = k_f F$$

(3.6)
where $k_f \in \mathbb{R}^{6 \times 6}$ is a diagonal positive-definite gain matrix, and $F \in \mathbb{R}^6$ is the measured force vector of the 6-axes force sensor.

Figure 3.3 Diagram of the compliant control scheme

Figure 3.4 Compliance control strategy
For accurate force transmission, forces and moments expressed in the force sensor frame must be transformed into the robot’s base-frame reference using well-known transformation matrix given in (Craig, 2005).

Now let us develop the control tracking of the inner loop. The new approach proposed in this research combines an adaptive-gain second-order sliding mode control and TDE, applied on the dynamic model of the exoskeleton robot presented in equation (3.2). The sliding set of \( n - th \) linked to the surface or equivalent surface is determined by:

\[
S = \dot{S} = \ddot{S} = \ldots = S^{(n-1)} = 0
\]  

Equation (3.7) shows an \( n \)-dimensional condition of the parameter system. In our case, it’s sufficient to differentiate the sliding surface once to obtain the desired control input. Let us choose the switching function or selected surface such that:

\[
S = \dot{e} + \Lambda e
\]  

where \( e = \eta - \eta_1d \in \mathbb{R}^7 \) and \( \dot{e} = \eta_2 - \eta_2d \in \mathbb{R}^7 \) are the position and velocity errors respectively, and \( \eta_1d, \eta_2d \in \mathbb{R}^7 \) are respectively the reference position and velocity, \( \Lambda = \text{diag}(\lambda_{ii}) \) for \( i = 1, \ldots, 7 \) is a diagonal positive matrix. Taking the first derivative of selected surface \( S \) we obtain:

\[
\dot{S} = \ddot{e} + \Lambda \dot{e} = \ddot{\eta} - \ddot{\eta}_2d + \Lambda \dot{e}
\]

\[
= U(t) + f(t) + H(t) - \ddot{\eta}_2d + \Lambda \dot{e}
\]  

In this research, we seek to reduce the chattering phenomena using second-order sliding mode to transform the discontinuous control to continuous signal using integral action. So, the second
The derivative of surface $S$ is given by:

$$
\ddot{S} = \dddot{e} + \Lambda \ddot{e} = \dddot{\eta}_2 - \ddot{\eta}_{2d} + \Lambda \ddot{e} \\
= \frac{d}{dt} [U(t)] + \frac{d}{dt} [f(t)] + \frac{d}{dt} [H(t)] - \ddot{\eta}_{2d} + \Lambda \ddot{e}
$$

(3.10)

The first and second derivative of $S$ leads us to create a new system. Let us before that insert two new variables $\mu_1 = S$, and $\mu_2 = \dot{S}$, hence the new state-space equation is given such that:

$$
\begin{aligned}
\dot{\mu}_1 &= \mu_2 \\
\dot{\mu}_2 &= \frac{d}{dt} [U(t)] + \frac{d}{dt} [f(t)] + \frac{d}{dt} [H(t)] - \ddot{\eta}_{2d} + \Lambda \ddot{e}
\end{aligned}
$$

(3.11)

As we note, the time derivative of the control input $\frac{d}{dt} [U(t)]$ is responsible for handling the second-order sliding mode system presented by equation (3.11). This controller must be developed to contain a discontinuous term. However, to perfectly control the robot system (3.4), we must integrate once $\frac{d}{dt} [U(t)]$ to obtain the control input $U(t)$ with the desired torque $\tau = M_0(\theta)U(t)$. The integration is responsible for transforming the discontinuous control action to a continuous one which helps attenuating the undesirable chattering problem.

To complete the proposed controller procedure, let us introduce a new selected surface for the state-space equation present by (3.11) such that:

$$
\rho = \mu_2 + \varphi \mu_1
$$

(3.12)

where $\varphi = \text{diag}(\varphi_i)$ for $i = 1, \ldots, 7$ is a diagonal positive definite matrix, and:

$$
\dot{\rho} = K \text{sign}(\rho)
$$

(3.13)
where \( K = \text{diag}(k_{ii}) \) for \( i = 1, \ldots, 7 \) is a switching positive gain diagonal matrix, and function \( \text{sign}(\rho) = [\text{sign}(\rho_1), \ldots, \text{sign}(\rho_7)]^T \) is determined such that:

\[
\text{sign}(\rho_i) = \begin{cases} 
1 & \text{for } \rho_i = 0 \\
0 & \text{for } \rho_i > 0 \\
-1 & \text{for } \rho_i < 0 
\end{cases}
\]

Taking the time derivative of equation (3.12), we find:

\[
\dot{\rho} = \dot{\mu}_2 + \varphi \mu_2 \\
= \frac{d}{dt}[U(t)] + \frac{d}{dt}[f(t)] + \frac{d}{dt}[H(t)] - \ddot{\eta}_2d + \Lambda \ddot{e} + \varphi \mu_2 
\]

(3.15)

From equation (3.13) and equation (3.15), we can conclude the time derivative of the control input such that:

\[
\frac{d}{dt}[U(t)] = -\frac{d}{dt}[f(t)] - \frac{d}{dt}[H(t)] - \ddot{\eta}_2d - \Lambda \ddot{e} - \varphi \mu_2 - K\text{sign}(\rho) 
\]

(3.16)

Since \( H(t) \) and \( \frac{d}{dt}[H(t)] \) are uncertain and they may influence the control purpose. In such case, the control law (3.16) is not feasible. To overcome this problem, TDE approach (Youcef-Toumi and Ito, 1990) is used to estimate the uncertainties of the nonlinear robot’s dynamics. So, if Assumption 3.3 is verified, \( \frac{d}{dt}[H(t)] \) can be estimated such that:

\[
\frac{d}{dt}[\dot{H}(t)] \simeq \frac{d}{dt}[H(t-t_d)] = \ddot{\eta}_2(t-t_d) - \frac{d}{dt}[f(t-t_d)] - \frac{d}{dt}[U(t-t_d)] 
\]

(3.17)

where \( t_d \) is very small time-delay constant. Practically, the smallest constant that can be used in real time is the sampling-time period.

As we discussed above, the second-order sliding mode is qualified to attenuate the undesirable chattering. However, the double time derivative of the sliding surface can magnify the risk of the nonlinear uncertainties function that can cause instability of the system. Additionally, in a
rehabilitation protocol, we deal with different subjects which mean different body characteristics. In this case, we still need large switching gains to preserve the stability and the robustness of the robot system, which generates again the chattering problem. In the paper, a solution is proposed to avoid the chattering problem and to provide a smooth human-like motion. In this solution, we tuned the switching gain $K$ adaptively, where the adaptive gains update law is given as follows:

$$\dot{k}_{ii} = \beta_{ii}^{-1} (|\rho_i| - \nu_i \hat{k}_{ii})$$

(3.18)

where the sub-indexes $i$ and $ii$ denote the $i-th$ element of a vector and the $ii-th$ diagonal element of a diagonal matrix, respectively. $\beta_{ii} > 0$ is the adaptation gain. $\hat{k}_{ii}$ is the adapted value of $k_{ii}$ taking into consideration its initial condition.

Finally, $\nu_i$ is determined as $\lim_{t \to \infty} \nu_i = 0$, $\int_0^t \nu_i(w) = Q_i < \infty$. In real time, we choose $\nu_i = \frac{1}{1 + t^2}$. Let us now define the time delay error such that:

$$\varepsilon_i = \frac{d}{dt} [H_i(t)] - \frac{d}{dt} [\hat{H}_i(t)] = \frac{d}{dt} [H_i(t)] - \frac{d}{dt} [H_i(t - t_d)] \leq \delta_i |t - (t - t_d)| \leq \delta_i t_d$$

(3.19)

where $\delta_i$ for $i = 1, \ldots, 7$ is a positive constant known as Lipschitz constant that satisfies the Lipschitz condition in Assumption 3.3.

By substituting the estimated $\frac{d}{dt} [H(t)]$ from equation (3.17) and introduce the adapted $\hat{k}_{ii}$ from equation (3.18) the time derivative of the control input $\frac{d}{dt} [U(t)]$, Equation (3.16) is rewritten such that:

$$\frac{d}{dt} [U(t)] = -\frac{d}{dt} [f(t)] - \frac{d}{dt} [\hat{H}(t)] + \dot{\eta}_2d - \Lambda \ddot{e} - \varphi \mu_2 - \dot{K} sign(\rho)$$

(3.20)

The proposed joint torque law is described in Theorem 3.1.

**Theorem 3.1:** Consider the exoskeleton system presented as the state-space equation (3.4), the proposed joint torque (3.20) ensures the stability of the adaptive gains of the second-order
sliding mode with TDE if the following condition is verified:

\[ k_{ii} > \delta_{td} \text{ for } i = 1, \ldots, 7. \]  

(3.21)

with

\[ \tau = M_0(\theta)U(t) \]  

(3.22)

where \( \int \left( \frac{d}{dt} [U(t)] \right) = U(t) \);

**Proof 3.1:** The proposed Lyapunov function candidate to show the stability of the robot is:

\[ V = \frac{1}{2} \rho^T \rho + \sum_{i=1}^{7} \tilde{k}_{ii}^T \beta_i \tilde{k}_{ii} \]  

(3.23)

where \( \tilde{k}_{ii} = \hat{k}_{ii} - k_{ii} \), The time derivative of equation (3.23) is given by:

\[ \dot{V} = \rho^T \dot{\rho} + \sum_{i=1}^{7} \tilde{k}_{ii}^T \beta_i \dot{\hat{k}}_{ii} \]

\[ = \rho^T \left( \frac{d}{dt} [U(t)] + \frac{d}{dt} [f(t)] + \frac{d}{dt} [H(t)] - \dot{\eta}_{2d} + \Lambda \ddot{e} + \phi \mu_2 \right) + \sum_{i=1}^{7} \tilde{k}_{ii}^T \beta_i \dot{\hat{k}}_{ii} \]  

(3.24)

Substituting \( \frac{d}{dt} [U(t)] \) from equation (3.20) and \( \dot{\hat{k}}_{ii} \) from equation (3.18) into equation (3.24), the derivative of the Lyapunov function (3.24) becomes such that:

\[ \dot{V} = \rho^T \left( \frac{d}{dt} [H(t)] - \frac{d}{dt} [\dot{H}(t)] - \dot{\hat{k}}_{ii} \text{sign}(\rho_i) \right) + \sum_{i=1}^{7} \tilde{k}_{ii}^T |\rho_i| - \nu_i \tilde{k}_{ii}^T \tilde{k}_{ii} \]

\[ = \sum_{i=1}^{7} \rho_i \left( \frac{d}{dt} [H_i(t)] - \frac{d}{dt} [\dot{H}_i(t)] - \dot{\hat{k}}_{ii} \text{sign}(\rho_i) \right) + \tilde{k}_{ii}^T |\rho_i| - \nu_i \tilde{k}_{ii}^T \tilde{k}_{ii} \]  

(3.25)

Using Young’s inequality such that:

\[ \tilde{k}_{ii} \dot{\hat{k}}_{ii} \geq \frac{1}{2} \tilde{k}_{ii}^T \dot{\hat{k}}_{ii} - \frac{1}{2} \tilde{k}_{ii}^2 \]  

(3.26)
where $\tilde{k}_{ii}^T = \tilde{k}_{ii}$. Substituting equation (3.25) into equation (3.26), we find:

$$\dot{V} \leq \sum_{i=1}^{7} \rho_i (\epsilon_i - \tilde{k}_{ii}\text{sign}(\rho_i)) + \tilde{k}_{ii}\rho_i - \frac{1}{2} v_i \tilde{k}_{ii}\tilde{k}_{ii} + \frac{1}{2} v_i k_{ii}^2$$

$$\leq \sum_{i=1}^{7} \rho_i (\epsilon_i - \tilde{k}_{ii}\text{sign}(\rho_i)) + (\tilde{k}_{ii} - k_{ii}) |\rho_i| - \frac{1}{2} v_i \tilde{k}_{ii}\tilde{k}_{ii} + \frac{1}{2} v_i k_{ii}^2$$

$$\leq \sum_{i=1}^{7} -|\rho_i| (k_{ii} - |\epsilon_i|) - \frac{1}{2} v_i \tilde{k}_{ii}\tilde{k}_{ii} + \frac{1}{2} v_i k_{ii}^2$$

$$\leq \sum_{i=1}^{7} -|\rho_i| (k_{ii} - \delta t_d) - \frac{1}{2} v_i \tilde{k}_{ii}\tilde{k}_{ii} + \frac{1}{2} v_i k_{ii}^2$$

(3.27)

Since $\sum_{i=1}^{7} \frac{1}{2} v_i k_{ii}^2 \to 0$ due to the definition of $v$ in equation (3.18), and by verified the following condition:

$$k_{ii} > \delta t_d \text{ for } i = 1, \ldots, 7.$$  

(3.28)

The Lyapunov function $\dot{V}$ is semi-negative definite. To prove the asymptotic stability of the system. Let us integrate both sides of the equation (3.27) gives:

$$V(t) - V(0) \leq -\int_0^t \left( \sum_{i=1}^{7} -|\rho_i| (k_{ii} - \delta t_d) + \frac{1}{2} v_i \tilde{k}_{ii}\tilde{k}_{ii} \right) dw + \int_0^t \left( \frac{1}{2} v_i k_{ii}^2 \right) dw$$

(3.29)

Since $k_{ii}$ is constant and $\int_0^t v_i(\omega) = Q_i < \infty$ due to the definition of $v$ in equation (3.18), one can rewrite equation (3.29) as:

$$V(t) - V(0) \leq -\int_0^t \left( \sum_{i=1}^{7} -|\rho_i| (k_{ii} - \delta t_d) + \frac{1}{2} v_i \tilde{k}_{ii}\tilde{k}_{ii} \right) dw + \int_0^t \left( \frac{1}{2} v_i k_{ii}^2 \right) dw < \infty$$

(3.30)

Therefore, $V$ is bounded which implies that $\rho \in L_\infty$ and $\eta_1, \eta_2 \in L_\infty$ from boundedness of $\eta_{1d}, \eta_{2d}$ (Assumption 3.4). With $k_{ii}$ is constant, we know that $\tilde{k}_{ii}$ is also bounded. Thus, all signals in closed loop are bounded. Thus, the surface $\rho_i$ and its derivative converge to the origin as $t \to \infty$. Hence, the system is stable.
3.4 Experimental and comparative study

3.4.1 Experiment setup

The robot system consists of three processing units, the first is a PC where the top-level commands are sent to the robot using LabVIEW [version 2018] interface namely, the control scheme selection, joint or Cartesian space trajectory, and so on. This PC also receives the data after the robot task is executed to analyze its performance. The other two processing units are part of a National Instruments PXI platform. Firstly, a NI-PXI 8081 controller card with an Intel Core Duo processor. In this card, the main operating system of the robot and the top-level control scheme are executed. In our case, the adaptive-gains second-order sliding mode-based controller as well as the estimation based on time delay approach had a sampling time of 500 ms. The human inverse kinematics algorithm is executed in a MATLAB script module of LabVIEW which can run inside this control loop easily. Finally, at input/output level, a NI PXI-7813 R remote input/output card with a field programmable gate array (FPGA) executes the low-level control, that is, a proportional–integral (PI) current control loop (sampling time of 50 μs) to maintain the current of the motors required by the main controller.

Note that the PI controller runs 10 times faster than the proposed control loop and is executed in the FPGA. Also, in this FPGA, the position feedback via Hall-sensors (joint position) and basic input/output tasks are executed. The force sensor feedback is important to accurately control the movement of the exoskeleton. A high linearity six-axis force sensor (NANO17-R-1.8-M2-M1PCI, ATI industrial Automation, Apex, NC, USA) is chosen to obtain accurate real-time force measurements. This sensor is mounted on the tip of the robot. The joints of the ETS-MARSE are powered by brushless direct current (DC) motors (Maxon EC-45 and Maxon EC-90) combined with harmonic drives (gear ratio 120:1 for motor-1 and motor-2, and gear ratio 100:1 for motors 3–7). Let us summarize the experiments setup by presenting the architecture for the ETS-MARSE system in Figure 3.5. The output of the controller is the joints torque commands. However, the torque commands are converted to motor currents and finally to reference voltage as the voltage value is the drive command for motor drivers. Furthermore, to
realize the RT control of the ETS-MARSE and to ensure that the right control torque command is sent to the joints (as well as the reference voltage commands for the drivers), we also added a PI controller to minimize differences between desired and measured currents (i.e. the error command to PI controller).

A VE (HELIOS) is created in open source Unity 3-D platform (see Figure 3.4). This interface consists of a set of functional movement tasks defined by a therapist. There are two types of communication with the ETS-MARSE system (LabVIEW). A user datagram protocol (UDP)
protocol for transferring position data and a TCP/IP protocol for transferring the operator’s commands (e.g. start/stop the execution of the task) (Ferrer et al., 2013). The choice of these protocols is based on reliability and the speed of transmission of data for each one. Our platform is divided into two profiles: user profile and admin profile. The user profile is under the supervision of a therapist and a biomedical engineer. The latter is responsible for preparing the rehabilitation session (such as customized task, as per the patient’s need). The admin profile is responsible to manage the database of the patient (such as Add/Edit/Delete a patient) and he/she has access to the list of all patients and information concerning the rehabilitation session (Ferrer et al., 2013).

Table 3.4 Controller parameters

<table>
<thead>
<tr>
<th>Gains</th>
<th>Value ($i = 1:7$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_i$</td>
<td>15.7</td>
</tr>
<tr>
<td>$\Lambda_i$</td>
<td>42.2</td>
</tr>
<tr>
<td>$k_{fi}$</td>
<td>0.05 ($i = 1:6$)</td>
</tr>
<tr>
<td>$\beta_{ii}$</td>
<td>2.82</td>
</tr>
<tr>
<td>$\beta_{ii}$</td>
<td>0.01 (initial condition of gains)</td>
</tr>
</tbody>
</table>

3.4.2 Experiments’ results

An experimental physiotherapy session was created to show the effectiveness of the proposed control system. The physical therapy tasks are performed by three different healthy subjects (mean age: 27 + 4.6 years; mean height: 170 + 8.75 cm; mean weight: 75 + 18 kg). Each subject participated in a full physiotherapy session, under the supervision of a therapist and control engineer. The therapist defined the range of motion of each subject and attributed suitable exercise. Within the session, the subject was comfortably seated in a chair (with height adjustment depend to size to each subject) in front of the virtual interface as shown in Figure 3.7. Experimental physiotherapy session was devised to three scenarios. In the first scenario, subject 1 performed a designed task (forearm pronation/ supination) for two repetitions using the proposed controller. The same subject repeated the same tasks using conventional second-order
sliding mode control (constant switching gains) for two consecutive times. The objective of this experiment is to show the accuracy of the proposed controller compared with conventional second-order sliding mode controller (Bartolini et al., 2001). In the second scenario, subject 2 has interacted with the VE trying to follow a proposed trajectory. In the third scenario, subject 3 performed a free motion during 2 min or more. It consisted of reaching an object or perform a daily activity such as eating, and the purpose of the last two experiments is to examine the transparency of the robot with each subject and how the control system permits the robot perform a smooth motion using human inverse kinematics. It is important to mention that each subject was repeatedly asked if he was sensing tired (bored) of the VE and the exoskeleton device and if for any cause intended to pause the trial session. Also, that all the experiments started from the initial position where elbow joint is at $90^\circ$ and all the other joints at $0^\circ$.

![Figure 3.6 Virtual interface (HELIOS) diagram and its communication layout with ETS-MARSE robotic system](image)
3.4.2.1 Scenario 1

Discussion 1: Figure 3.8 presents the workspace of the performance tracking of ETS-MARSE performed by subject 1. It is clear that the proposed controller presents a good result where it keeps the stability of the robot system, small error not more than 2°, and smooth sliding surface. It is remarkable that the applied joint torque input is very smooth and the updated gains converge to a finite value as time going to infinity. Figure 3.9 presents the performance of the robot with the conventional approach. It is obvious that the conventional approach also gives good results, compared with the results (Figure 3.8) that is presented by the proposed control. The designed controller improves the performance of the second-order sliding mode control, where the error position, the sliding surface, and the control input that are provided by the proposed controller are relatively small and smooth than the results (Figure 3.9) provided by the conventional approach.
Figure 3.8  Workspace tracking of the robot performed a forearm pronation/supination by subject 1: (a) (age: 29 years; height: 178 cm; weight: 81 kg) using the proposed controller; and (b) Estimated gains

Figure 3.9  Workspace tracking of the robot performed a forearm pronation/supination by subject 1: (age: 29 years; height: 178 cm; weight: 81 kg) using the conventional controller
3.4.2.2 Scenario 2

**Discussion 2:** Figure 3.10 presents the performance of subject 2 (age: 31 years; height: 183 cm; weight: 83.5 kg) in the virtual interface with the help of ETS-MARSE exoskeleton robot (red line is desired and the green line is the achieved trajectory). Figures 3.10 (b), (3.11), and (3.12) present the workspace of error tracking of the robot in Cartesian space, the estimated gains, and the control input. It is clear from these plots (Figures 3.10 to 3.12) that the control strategy achieved the desired performance with small tracking errors and acceptable control input.
3.4.2.3 Scenario 3

**Discussion 3:** Figures 3.13 to 3.16 present the performance of free motion by subject 3 (age: 27 years; height: 168 cm; weight: 70 kg) with the help of ETS-MARSE exoskeleton robot. Figures 3.13 (b), 3.14, (3.15), and (3.16) present the workspace of error tracking of the robot in Cartesian space, the workspace of the tracking trajectory in joint space, the torque input, and updated gains. It is easy to conclude, from these plots (Figures 3.13 to 3.16) two points. The first one is that the proposed control approach achieved the desired performance with high characteristics (small tracking errors and acceptable control input and convergence of estimated gains). The second one is the algorithm of human inverse kinematics. It provides a good solution and permits to transform the Cartesian task to the joints task by a unique and accurate solution.
Figure 3.13  Workspace tracking of the robot performed a free motion by subject 3: (age: 27 years; height: 168 cm; weight: 70 kg). (b) Cartesian errors

3.5 Conclusion

In this article, we investigated active rehabilitation protocol by presenting a new compliant control based on secondorder sliding mode with adaptive gains incorporating TDE. The control is based on human inverse kinematics to complete active rehabilitation protocols for an exoskeleton robot with unknown dynamics and unforeseen disturbances. The proposed control
is principally aimed to deal with the accuracy/robustness problem of an admittance control. Second-order sliding mode has shown attractive characteristics of accuracy and attenuation of chattering. However, its problem is that the unknown dynamic of the exoskeleton robot and external disturbances can be amplified by the second derivative of sliding surface, which leads to instability of the robot system. Applying a large switching gain is needed to maintain the stability of the robot, which in turn makes the chattering problem existing yet again. Then, employing adaptive gains and TDE will improve the robustness of the second-order sliding mode control while overcoming its main limitation.

Figure 3.14  Tracking trajectory of the robot in joint space corresponding to the free motion performed by subject 3: (age: 27 years; height: 168 cm; weight: 70 kg)

The stability analysis is formulated and demonstrated based on Lyapunov function. An experimental physiotherapy session with healthy subjects using virtual reality was created to test the effectiveness and feasibility of the proposed control.
In the light of these satisfactory results obtained with healthy subjects, we are looking forward to implementing the proposed control strategy with real unhealthy subjects as stroke victims in future work, which permits to evaluate the controller with true case of disturbances such as spasticity/dystonia and muscle weakness in neurological patients.
CHAPTER 4

ADAPTIVE TRACKING CONTROL OF AN EXOSKELETON ROBOT WITH UNCERTAIN DYNAMICS BASED ON ESTIMATED TIME DELAY CONTROL

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Abstract

In this paper, we present a backstepping approach integrated with time-delay estimation to provide an accurate estimation of unknown dynamics and to compensate for external bounded disturbances. The control was implemented to perform passive rehabilitation movements with a 7-DOF exoskeleton robot named ETS-Motion Assistive Robotic-Exoskeleton for Superior Extremity. The unknown dynamics and external bounded disturbances can affect the robotic system in the form of input saturation, time delay errors, friction forces, backlash, and different upper-limb’s mass of each subject. The output of the time delay estimator is coupled directly to the control input of the proposed adaptive tracking control through a feed-forward loop. In this case, the control system ensures a highly accurate tracking of the desired trajectory, while being robust to the uncertainties and unforeseen external forces, and flexible with variation of parameters. Due to the proposed strategy, the designed control approach does not require accurate knowledge of the dynamic parameters of the exoskeleton robot to achieve the desired performance. The stability of the exoskeleton robot and the convergence of its state errors are established and proved based on Lyapunov–Krasovskii functional theory. Experimental results and a comparative study are presented to validate the advantages of the proposed strategy.
Keywords: Backstepping control, rehabilitation robots, time-delay control, time-delay error (TDR).

4.1 Introduction

Neurological diseases have become a growing challenge and a difficult reality for the medical and scientific community, as confirmed by the statistics of the World Health Organization. Each year, at least 15 million people worldwide suffer neurological diseases, such as stroke (Sidney et al., 2013). From them, six million die and five million more are living with a persistent disability (Sidney et al., 2013). At all times, human activities of everyday life, such as balance of legs and arms to move, walk, stand, sit or even relax, take objects, eat, brushing teeth, or cleaning face, depend on a healthy nervous system. Damage caused by nerve cells causes a disruption of communication between the nervous system and the rest of the body, resulting in the inability of the nervous system to function adequately. This condition causes symptoms that can include numbness, pain, balance disorders, etc. Recently, the use of robotic devices known as rehabilitation robots in physiotherapy assistive domains has demonstrated a high potential in preventing the worsening of the subject’s disability, improving its functional movements, ensuring its return to normal life, and helping the subject in daily living activities, such as self-care skills and pick-and-place exercises (Hughes et al., 2016; Keller et al., 2016; Philips et al., 2017; Volpini et al., 2017). The robot’s significance is due to its attractive characteristics such as its ability to provide intensive rehabilitation and its easiness to design a physical therapy activity fitted to the needs of the subject (Xie et al., 2016).

The control of these kinds of robots presents additional complexity over the control of conventional robotic manipulators due to their complex mechanical structure designed for human use, the type of desired tasks, and the sensibility of the interaction with a great diversity of human wearers (Chen et al., 2017; Du and Zhang, 2015; Zhao et al., 2015). To address these problems, different control strategies have been developed for rehabilitation robots. Among them, a simple PID control implemented in (Yu and Rosen, 2013); a nonlinear modified computed torque control that requires a good knowledge of dynamic parameters, implemented in (Rah-
Man et al., 2015)) and (Rahman et al., 2011a). A robust sliding-mode control with exponential reaching law was proposed in (Rahman et al., 2013) to improve the performance of the robot and to limit the chattering problem generated by the high-frequency activity of the control signal. As well, a force controller was proposed in (Ueda et al., 2010) for an exoskeleton that permits to the wearer to achieve motor tasks based on muscle activity data. We can also mention a backstepping control combined with human inverse kinematics (Brahmi et al., 2017c) to provide a human-like motion. Nevertheless, in the previous cited papers, the control scheme is the named model-based controller, in which the control loop requires a dynamic model of the exoskeleton.

In reality, the dynamics of these types of robots is typically uncertain due to their complex and sensitive structure. Particularly, when the number of degrees of freedom (DOFs) of the robot increases, it is not straightforward to find the accurate parameters of the exoskeleton robot; e.g., the parameters’ vector of the robot can be greater than 100 if the number of DOFs of the robot is greater than 4 (Brahim et al., 2016a). Usually, these robots operate under input saturation constraints to ensure the safety to the robot’s user. This latter is one of the most serious nonlinearities of the robotic system (Li et al., 2015b). The effect of the constraint appears when the exoskeleton actuators are unable to provide the required energy to perform a rehabilitation motion, which causes tracking errors. Additionally, a repeating motion can cause the fatigue of the motors (Brahim et al., 2016a), (Brahim et al., 2016b), which provokes many problems, such as a dead zone (Li et al., 2014), friction forces, etc. Moreover, the synergy between the subject and the exoskeleton robot forces the system to operate under unforeseen external forces as the payload caused by the attached upper limb of the subject.

Hence, the estimation of the uncertainties of the nonlinear dynamic parameters of these robots is one of the most challenging problems in the control of high-DOFs robotics exoskeleton systems. Numerous control strategies have been developed to approximate the dynamic parameters. Within these approaches, the linear parameterization of the dynamic equation of motion is used in order to obtain the regressor matrix (Cheah, 2006), (Yazarel and Cheah, 2002). This matrix is required in the design of the updated control law. However, it is not straightforward
to find the parameters of the exoskeleton robot, if it has a high number of DOFs (more than four). Additionally, the integral action of the designed updated law can cause the instability of the robot system in the presence of disturbances, even if it is small. Recently, the approach by fuzzy logic and neural networks presented a significant solution due to their attractive characteristics of the robot’s unknown nonlinear dynamics estimation with minimum feedback from the robotic system (Li et al., 2015b), (Chen et al., 2015; Li et al., 2017a, 2015a). However, these strategies demand a heavy computation cost, which makes their implementation very hard. Adaptive learning control is a robust approach designed to adapt the time-varying uncertainties and disturbances in order to reject them from the rigid body robot (Brahim et al., 2016a), (Jin and Xu, 2013). In this approach, the authors have proposed that the robot performs the same exercises over a fixed period, making this approach limited to repetitive tasks only and actuators fatigue. However, in this paper, a time-delay estimation (TDE) approach (Youcef-Toumi and Ito, 1990; Youcef-Toumi and Shortlidge, 1991; Brahmi et al., 2017a,b) is used to estimate the unknown uncertain parameters of the exoskeleton’s dynamic. The TDE has been implemented in many robotic systems with consistently good performances (Brahmi et al., 2017a), (Jin et al., 2015; Kim et al., 2016; Chen et al., 2016). The choice of use of TDE is that one can easily estimate the unknown dynamics with a simple control scheme. In addition, TDE is one of the strategies that is not affected by the size of the estimated parameters (high DOFs). It can only be employed in time-delayed knowledge from the previous state response of the system and its control input to provide an accurate estimation of unknown dynamics. However, due to noisy measurements and nonlinearity of signals along the sampling time, a time-delay error (TDR) exists, which would deteriorate the robustness and the accuracy of the exoskeleton robot.

The cited constraints (mainly dynamic uncertainties, joint conjunction, friction forces, backlash, mass changing, and TDR), that limit the functionality of the exoskeleton robots, motivate us to design a controller able to provide a highly accurate tracking of the desired trajectory, to be robust to the uncertainties and unforeseen external forces, and to be flexible with the parameters’ variation. In this paper, we present a new adaptive backstepping controller based
on TDE, applied on an upper-limb exoskeleton robot. The backstepping is employed to estimate the unknown uncertainties, unforeseen disturbances, and compensate for the TDR and unexpected disturbances. Furthermore, the theoretical development is complemented by its implementation on an exoskeleton robot. The output of the time-delay estimator is added directly to the control input of the proposed adaptive tracking control via feed-forward loops, which makes the control system more powerful and faster to estimate and compensate for unknown dynamics and external disturbances.

Summarizing, through the proposed strategy, the control approach does not require any accurate knowledge of the dynamic parameters of the exoskeleton robot to reach the desired performance. This controller is designed to be robust and more flexible to deal with the dynamic uncertainties taking into consideration the TDR, and to be more robust to the parameter variations. Moreover, the proposed strategy is not restricted by the repetitive task or periodic desired trajectory. The stability of the exoskeleton system and the convergence of its errors are formulated and demonstrated based on the Lyapunov–Krasovskii functional theory. The validation of the control platform is done by creating a rehabilitation session performed with healthy human subjects. A comparative study is provided and is compared against the conventional approach to show the advantages, feasibility, and the robustness of the proposed approach.

The remainder of the paper is organized as follows. The exoskeleton robot, its kinematics and dynamics are presented in the following section. The control scheme is described in Section 4.3. Experimental and comparison results are shown in Section 4.4. Finally, the conclusion and future work are presented in Section 4.5.

4.2 Characterization of system rehabilitation

4.2.1 Exoskeleton robot development

The developed exoskeleton robot ETS-Motion Assistive Robotic-Exoskeleton for Superior Extremity (ETS-MARSE) is a redundant robot consisting of 7 DOFs, as shown in Figure 4.1. It
was created to provide physical therapy and assisted motion to the injured upper limb. The idea of the designed exoskeleton is basically extracted from the anatomy of the upper limb of the human, to be ergonomic for the user along with the physical therapy session. The shoulder part consists of three joints; the elbow part comprises one joint, and the wrist part consists of three joints. Each part is responsible for performing a variety of upper-limb motions, as shown in Table (4.2). The design of the ETS-MARSE has special features compared with the existing exoskeleton robots. Among them, it has a comparatively low weight, an excellent power/weight ratio, can be easily fitted or removed, and is capable of adequately compensating for gravity.

A new power carrying mechanism was included for supporting the shoulder joint internal/external rotation and for forearm pronation/supination. This robot can be used with a wide range of subjects, due to the length of its adjustable links. This exoskeleton can perform passive (completely support and perform the motion on the subjects’ upper limb) and active assistive
motion (respond to force, electromyography, and/or be compliant with the subject to accompany and assist him/her in the intended motion). All special characteristics of the ETS-MARSE and comparison with similar existing exoskeleton robots are summarized in (Ochoa Luna et al., 2015; Rahman et al., 2014, 2012b; Luna et al., 2016).

Table 4.1 Modified Denavit-Hartenberg parameters

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>$\alpha_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$d_s$</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{\pi}{2}$</td>
<td>0</td>
<td>$d_e$</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{\pi}{2}$</td>
<td>0</td>
<td>$d_w$</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>6</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_6 - \frac{\pi}{2}$</td>
</tr>
<tr>
<td>7</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_7$</td>
</tr>
</tbody>
</table>

Table 4.2 Workspace ETS-MARSE

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>Motion</th>
<th>Workspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shoulder joint horizontal flexion/extension</td>
<td>$0^\circ/140^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>Shoulder joint vertical flexion/extension</td>
<td>$140^\circ/0^\circ$</td>
</tr>
<tr>
<td>3</td>
<td>Shoulder joint internal/external rotation</td>
<td>$-85^\circ/75^\circ$</td>
</tr>
<tr>
<td>4</td>
<td>Elbow joint flexion/extension</td>
<td>$120^\circ/0^\circ$</td>
</tr>
<tr>
<td>5</td>
<td>Forearm joint pronation/supination</td>
<td>$-85^\circ/85^\circ$</td>
</tr>
<tr>
<td>6</td>
<td>Wrist joint ulnar/radial deviation</td>
<td>$-30^\circ/20^\circ$</td>
</tr>
<tr>
<td>7</td>
<td>Wrist joint flexion/extension</td>
<td>$-50^\circ/60^\circ$</td>
</tr>
</tbody>
</table>

4.2.2 Kinematics of ETS-MARSE Robot

The transformation from Cartesian space to joint space is done by a nonlinear function named the Jacobian matrix. In order to maneuver the exoskeleton in Cartesian space, we used the inverse Jacobian matrix method, since the control is executed in the joint space. Due to the redundant nature of the ETSMARSE robot, the inverse kinematics can be achieved using the
pseudoinverse of the Jacobian, which can be expressed as follows:

$$\dot{\theta} = \left( J^T (JJ^T)^{-1} \right) \dot{x}_d$$  \hspace{1cm} (4.1)$$

where $\dot{x}_d \in \mathbb{R}^{6 \times 1}$ is the desired Cartesian velocity, $\dot{\theta} \in \mathbb{R}^{7 \times 1}$ is the calculated joint velocity and $J \in \mathbb{R}^{6 \times 7}$ is the Jacobian matrix of the robot.

The modified Denavit–Hartenberg (DH) parameters (a) are given in Table (4.1). These parameters are obtained from the reference frames shown in Figure 4.1 and are used to obtain the homogeneous transformation matrices.

The workspace of the designed robot is given in Table (4.2). Further detailed information of the parameters and design of ETS-MARSE can be found in (Rahman et al., 2015).

### 4.2.2.1 Dynamics of the ETS-MARSE Robot

The dynamic equation of the ETS-MARSE is expressed in joint space as follow:

$$M(\theta) \ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) + F(\theta, \dot{\theta}) + f_{dis} = \tau$$  \hspace{1cm} (4.2)$$

where $\theta \in \mathbb{R}^7$, $\dot{\theta} \in \mathbb{R}^7$ and $\ddot{\theta} \in \mathbb{R}^7$ are, respectively, the joint’s position, velocity, and acceleration vectors, $M(\theta) \in \mathbb{R}^{7 \times 7}$ is the symmetric and positive definite inertia matrix, $C(\theta, \dot{\theta}) \in \mathbb{R}^{7 \times 7}$ is the Coriolis and centrifugal vector, $G(\theta) \in \mathbb{R}^7$ is the gravitational vector, $\tau \in \mathbb{R}^7$ is the applied joint torque to the exoskeleton robot, and $f_{ex} \in \mathbb{R}^7$ is the external disturbances vector. Without loss of generality, the matrices of dynamic model (4.2) can be written as follows:

\[
\begin{align*}
M(\theta) &= M_0(\theta) + \Delta M(\theta) \\
C(\theta, \dot{\theta}) &= C_0(\theta, \dot{\theta}) + \Delta C(\theta, \dot{\theta}) \\
G(\theta) &= G_0(\theta) + \Delta G(\theta)
\end{align*}
\]  \hspace{1cm} (4.3)
where $M_0(\theta)$, $C_0(\theta, \dot{\theta})$ and $G_0(\theta)$ are respectively the known inertia matrix, the Coriolis centrifugal matrix, and the gravity vector. $\Delta M(\theta)$, $\Delta C(\theta)$ and $\Delta G(\theta)$ are the uncertain parts.

Let us introduce a new variable such that: $\eta_1 = \theta$ and $\eta_2 = \dot{\theta}$; hence, the dynamic model expressed in (4.2) can be rewritten as follows:

$$\begin{cases}
\dot{\eta}_1 = \eta_2 \\
\dot{\eta}_2 = U(t) + f(t) + H(t)
\end{cases} \quad (4.4)$$

where, $U(t) = U(\eta_1)$, $H(t) = H(\eta_1, \eta_2, \dot{\eta}_2)$ and $f(t) = d\eta_1, \eta_2)$. This notation is employed to facilitate handling the control methodology with:

- $U(t) = M_0^{-1}(\theta) \tau$;
- $f(t) = -M_0^{-1}(\theta) [C_0(\theta, \dot{\theta}) + G_0(\theta)]$;
- $H(t) = H(\theta, \dot{\theta}, \ddot{\theta}) = -M_0^{-1}(\theta) [f_{ex} + \Delta M(\theta) \dot{\theta} + \Delta C(\theta, \dot{\theta}) \dot{\theta} + \Delta G(\theta)]$.

### 4.2.2.2 Problem formulation

From (4.2) and (4.4), it is difficult to obtain $H(t)$ due to the uncertainties of the dynamic model of the exoskeleton robot and the aforementioned unknown external effects. Consequently, to solve this dilemma, the proposition of this paper is to obtain a control input able to force the measured trajectory $\eta_d \in \mathbb{R}^7$ to track the desired trajectory even if the exoskeleton robot is under the effect of uncertain and unforeseen external disturbances. In this paper, the aforementioned desired trajectories correspond to those of passive rehabilitation protocol investigated under adaptive control. Before stating the control methodology, the properties and the assumptions that are used in this paper are given as follows:

**Property 4.1:** The inertia matrix $M_0(\theta)$ is symmetric and positive definite for all $\theta \in \mathbb{R}^7$ and satisfying: $\gamma_{\min}(M_0(\theta)) I_{7 \times 7} \leq M_0(\theta) \leq \gamma_{\max}(M_0(\theta)) I_{7 \times 7}$, where $\gamma_{\min}$ and $\gamma_{\max}$ are minimum and maximum eigenvalues, respectively, of the known inertia matrix and $I_{7 \times 7}$ is the $7 \times 7$ identity matrix. (see (Craig, 2005)).
Assumption 4.1: The joint position and joint velocity are measured.

Assumption 4.2: The desired trajectory is bounded.

Assumption 4.3: The external disturbance $f_{ex}$ is supposed to be continuous, has finite energy and satisfies $\|f_{ex}\| \leq \vartheta$, with an unknown positive disturbance boundary $\vartheta$.

Assumption 4.4: The variation of the unknown parameters model $H(t)$ in time is continuous with known delay $t_d$ and globally Lipschitz function. It can be expressed as:

$$H(t) = H(t - t_d) + \varepsilon(t_d)$$  \hspace{1cm} (4.5)

where $\varepsilon(t_d) \in \mathbb{R}^7$ is the TDR vector and $t_d$ is a very-small time-delay constant.

4.3 Adaptive control design

In this section, the design of a tracking control that can estimate the uncertainties and unexpected disturbances and decrease its effects to achieve the desired tracking performance is described. Let us assume first that $H(t)$ is known. Let us define the position error and velocity error as follows:

$$e_1 = \eta_1 - \eta_d$$  \hspace{1cm} (4.6)

$$e_2 = \eta_2 - \xi$$  \hspace{1cm} (4.7)

where $\eta_d \in \mathbb{R}^7$ and $\eta_1 \in \mathbb{R}^7$ are the desired trajectory and measured trajectory, respectively, and $\xi$ is a virtual control input to $e_1$.

Step I: The time derivative of equation (4.6) is given by:

$$\dot{e}_1 = \eta_2 - \eta_d = e_1 + \xi - \eta_d$$  \hspace{1cm} (4.8)
Consider the first Lyapunov function candidate as follow:

\[ V_1 = \frac{1}{2} e_1^T e_1 \]  \hspace{1cm} (4.9)

Taking time derivative of \( V_1 \) as follows:

\[ \dot{V}_1 = e_1^T \dot{e}_1 = e_1^T (e_1 + \xi - \dot{\eta}_d) \]  \hspace{1cm} (4.10)

Let \( \xi = \eta_d - k_1 e_1 \), with \( k_1 \in \mathbb{R}^{7 \times 7} \) being a diagonal positive-definite matrix. Equation (4.10) becomes:

\[ \dot{V}_1 = -e_1^T k_1 e_1 + e_1^T e_2 \]  \hspace{1cm} (4.11)

The first term of (4.11) is stabilizing and the second term will be addressed in the next step.

**Step 2:** Differentiating (4.7), using (4.4), with respect to time yields:

\[ \dot{e}_2 = U(t) - f(t) - H(t) - \dot{\xi} \]  \hspace{1cm} (4.12)

where \( \dot{\xi} = \dot{\eta}_d - k_1 \dot{e}_1 \). Therefore, the proposed model-based control can be given as follows:

\[ U(t) = -k_2 e_2 - e_1 + f(t) + H(t) + \dot{\xi} \]

with \( \tau = M_0(\theta) U(t) \) \hspace{1cm} (4.13)

with \( k_2 \in \mathbb{R}^{7 \times 7} \) being a diagonal positive-definite matrix.

Consider the second Lyapunov function candidate as:

\[ V_2 = V_1 + \frac{1}{2} e_2^T e_2 \]  \hspace{1cm} (4.14)
The time derivative of $V_2$ is given by:

$$\dot{V}_2 = \dot{V}_1 + e_2^T \dot{e}_2$$

$$= \dot{V}_1 + e_2^T \left( U(t) - f(t) - H(t) - \dot{\xi} \right)$$

(4.15)

Considering the model-based control law (4.13), the above equation can be written as follows:

$$\dot{V}_2 = -e_1^T k_1 e_1 - e_2^T k_2 e_2$$

(4.16)

Having the following condition: $k_1 > 0$ and $k_2 > 0$, this implies that $\dot{V}_2 \leq 0$, which means that the robot system is stable.

Practically, as established, all dynamics parameters of the exoskeleton robot are not easy obtained due to the uncertainties and their variation during the robot’s performance. Since $H(t)$ is uncertain, it might influence the control proposition. For now on, we will consider $H(t)$ uncertain. In such case, the model based control law (4.13) is not feasible. To overcome this problem, a new adaptive time-delay controller is proposed as:

$$U(t) = -k_2 e_2 - e_1 + f(t) + \hat{H}(t) + \dot{\xi}$$

with $\tau = M_0(\theta) U(t)$

(4.17)

where $\hat{H}(t)$ is the estimated value of $H(t)$ obtained by the TDE approach (Yousef-Toumi and Ito, 1990). However, due to noisy measurements and nonlinearity of signals along the sampling time, a TDR, $\varepsilon(t_d)$, exists. This TDR would deteriorate the robustness and the accuracy of the robot. To overcome this problem, we proposed the following estimator:

$$\hat{H}(t) = \hat{H}(t - t_d) + \varepsilon(t_d) - k_3 e_2$$

$$\hat{H}(t) = 0, \ \forall \ t \in [-t, \ 0]; \ k_3 = k I_{7 \times 7}$$

(4.18)
where \( k \) is a positive scalar constant and \( I_{7 \times 7} \) is the \( 7 \times 7 \) identity matrix. The proof of (4.18) is given in Appendix I (1). So, if Assumptions 4.4 is verified, \( \hat{H}(t-t_d) \) can be calculated using (4.4) such that (Youcef-Toumi and Ito, 1990):

\[
\hat{H}(t-t_d) = U(t-t_d) - f(t-t_d) - \hat{\eta}_2(t-t_d)
\] (4.19)

where \( t_d \) is a very small time-delay constant. Practically, the smallest constant that can be achieved in real time is the sampling period. According to the Lipschitz condition (Assumptions 4.4), \( \varepsilon(t_d) \) can be calculated as follows:

\[
\varepsilon(t_d) = H(t) - H(t-t_d) \leq \rho t_d
\] (4.20)

where \( \rho > 0 \) is Lipschitz constant. To facilitate the proof of stability, we can write

\[
\frac{d}{dt} \int_{t-t_d}^{t} \hat{H}^T(w) \hat{H}(w) dw
\]

as follows:

\[
\frac{d}{dt} \int_{t-t_d}^{t} \hat{H}(w)^T \hat{H}(w) dw = \hat{H}^T(t) \hat{H}(t) - \hat{H}^T(t-t_d) \hat{H}(t-d)
\] (4.21)

where \( \hat{H}(w) \) is the estimation error of the uncertainties that will be defined latter. Additionally:

\[
\frac{1}{2k} \hat{H}(t)^T \hat{H}(t) - \frac{1}{2k} \hat{H}(t-t_d)^T \hat{H}(t-t_d) = \hat{H}^T(t) e_2 - e_2^T \frac{k_3^T}{2} e_2
\] (4.22)

where \( k_3^T = k_3 \). The details of (4.21) and (4.22) are given in Appendix I (1).

**Theorem 4.1:** Consider the exoskeleton robot system (4.4) that satisfies Assumptions (4.2–4.4), with the proposed adaptive TDE (4.17). If the previous conditions are verified with a bounded initial condition, the robot system (4.4) is stable and its errors \( e_1, e_2, \) and \( \hat{H} \) are bounded.

**Proof:** Consider the following Lyapunov function candidate:

\[
V_3 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2 + \frac{1}{2k} \int_{t-t_d}^{t} \hat{H}^T(w) \hat{H}(w) dw
\] (4.23)
with $\tilde{H}(t) = H(t) - \hat{H}$. The derivative of the proposed Lyapunov function with respect to time, using (4.21), is obtained as:

$$
\dot{V}_3 = -e_1 k_1 e_1 + e_2^T \left( U(t) - f(t) - H(t) - \dot{\xi} \right) + \frac{1}{2k} \tilde{H}(t)^T \tilde{H}(t) - \frac{1}{2k} \tilde{H}(t - t_d)^T \tilde{H}(t - t_d)
$$

(4.24)

Substituting the adaptive time-delay control input (4.17) into (4.24) and using (4.22), we find:

$$
\dot{V}_3 = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \tilde{H}(t) + \tilde{H}(t) e_2 - e_2^T k_3^T e_2
$$

(4.25)

Finally, we obtain:

$$
\dot{V}_3 = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T k_3^T e_2
$$

(4.26)

Since $k_1 > 0$, $k_2 > 0$ and $k_3 = k_3^T > 0$, this implies that $\dot{V}_3 \leq 0$, which means that the robot system is stable.

**Remark 4.1:** We observe that $\dot{V}_3$ is seminegative in the interval $[t - t_d, t]$, which means that $V_3$ is stable, but outside this interval, the stability of $V_3$ cannot be ensured. To guarantee the stability of $V_3$ in the interval $[0, \infty)$, we use the functional Lyapunov-Krasovskii theorem (Fridman, 2014).

To guarantee the asymptotic stability of the delayed system in the interval $[0, \infty)$, we propose the following Lyapunov–Krasovskii function:

$$
V_4 = V_2 + \begin{cases}
\sum_{i=1}^{n} \frac{1}{2k_3i} \int_0^t \tilde{H}_i^2(w) dw, & t \in [0,t_{d1}) ; \\
\frac{1}{2k_3} \int_{t_{d1}}^{t_{d2}} \tilde{H}_i^2(w) dw + \sum_{i=2}^{n} \frac{1}{2k_3i} \int_0^t \tilde{H}_i^2(w) dw, & t \in [t_{d1},t_{d2}) ;
\end{cases}
$$

(4.27)
with:

\[ V_2 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2 \]  

(4.28)

and \( n = 7 \). The derivative of the proposed Lyapunov–Krasovskii function with respect to time is obtained as follows:

\[ \dot{V}_4 = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - \frac{1}{2} \sum_{i=1}^{n} k_3 e_{2i}^2 \]  

(4.29)

It is clear from (29) that \( \dot{V}_4 \leq 0 \), where all gains \( k_1 \), \( k_2 \), and \( k_3 \) are positive. The proof of the stability is detailed in Appendix I (2). The structure of the control scheme is shown in Figure 4.2.

4.4 Experimental and comparative study

4.4.1 Experiment Setup

Implementation was carried out on the ETS-MARSE system described in the following. The system consists of three processing units; the first is a PC, where the top-level commands are
sent to the robot using LabVIEW interface, i.e., the control scheme selection, joint or Cartesian space trajectory, gain adjustments, etc. This PC also receives the data after the robot task is executed to analyze its performance. The other two processing units are part of a National Instruments PXI platform. First is an NI-PXI 8081 controller card with an Intel Core Duo processor; in this card, the main operating system of the robot and the top-level control scheme are executed, in our case, the backstepping based controller as well as the estimation based on TDE approach, at a sampling time of 500 μs. The inverse kinematics algorithm also runs inside this control loop. Finally, at input–output level, an NI PXI-7813R remote input–output card with a field programmable gate array (FPGA) executes the low-level control; i.e., a PI current control loop (sampling time of 50 μs) to maintain the current of the motors required by the main controller. Also, in this FPGA, the position feedback via Hall-sensors (joint position), and basic input output tasks are executed. Force sensor feedback is important to accurately control the movement of the exoskeleton. A high linearity six axis force sensor [NANO17R-1.8-M2-M1PCI, ATI industrial automation] was chosen to obtain accurate real-time force measurements for the ETS-MARSE. This sensor is mounted on the tip of the robot. The joints of the ETS-MARSE are powered by brushless dc motors (Maxon EC-45, EC-90) combined with harmonic drives [gear ratio 120:1 for motor-1, motor-2, and motor-4, and gear ratio 100:1 for motor-3 and motors (5–7)]. The diagram of the architecture of the ETS-MARSE with a healthy subject is shown in Figure 4.3.

![Diagram showing experiment architecture with a subject wearing the ETS-MARSE robot](image)

Figure 4.3   General schematic of experiment architecture (the subject wear the ETS-MARSE robot)
The physical parameters of ETS-MARSE relative to the base reference frame are given in Table 4.3.

<table>
<thead>
<tr>
<th>Joints (i)</th>
<th>Mass (kg)</th>
<th>Centre of mass (m)</th>
<th>Link length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.475</td>
<td>0.0984</td>
<td>0.145</td>
</tr>
<tr>
<td>2</td>
<td>3.737</td>
<td>0.1959</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>2.066</td>
<td>0.163</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0.267</td>
</tr>
<tr>
<td>6</td>
<td>0.779</td>
<td>0.121</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.496</td>
<td>0.0622</td>
<td>0</td>
</tr>
</tbody>
</table>

An experimental session was created to validate the proposed control strategy. The physical therapy tasks are performed by two different healthy subjects (mean age: 27 ± 4.6 years; mean height: 170 ± 8.75 cm; mean weight: 75 ± 18 kg). Each subject participated in a full session, under the supervision of a therapist and a control engineer. The role of the therapist is to define the range of motion of each subject and to determine a suitable exercise. During the session, the subject was ergonomically seated in a chair (height of the chair is adjustable according to the size of each subject) as shown in Figure 4.3. The experimental session was conceived in two scenarios. In the first scenario, each subject performed a basic joint physical therapy task consisting of elbow joint flexion/extension and forearm supination/pronation simultaneously, using the designed control. In this scenario, the subjects repeat the same task with a conventional approach (Khan et al., 2016b) to show the feasibility of the proposed controller. In the second scenario, the proposed controller was tested using a Cartesian task described in the following section. The objective of this task is to show the tracking of the 7-DOFs of the robot in a Cartesian task. In this part, the subject also repeated the same Cartesian trajectory using the conventional controller. It is important to mention that the initial position of the robot is with the elbow joint position at 90° for all experiments, and the external disturbances here are concordantly represented by different physiological conditions of the subjects, the varying mass
of the upper limb with each subject and the TDR. The control gains were chosen arbitrarily as shown in Table (4.4).

<table>
<thead>
<tr>
<th>Gains</th>
<th>Value ($i = 1:7$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{1i}$</td>
<td>15</td>
</tr>
<tr>
<td>$k_{2i}$</td>
<td>150</td>
</tr>
<tr>
<td>$k_{3i}$</td>
<td>0.12</td>
</tr>
</tbody>
</table>

**Remark 4.2:** The conventional controller (Khan *et al.*, 2016b) is characterized by its complex implementation due to the complex regressor dynamic matrix, while the robot has a high degree-of-freedom (7-DOFs).

### 4.4.2 Joint space tests

The experimental results of the proposed controller are illustrated in Figure 4.4. This exercise was performed with subject-1 (age: 30 years; height: 177 cm; weight: 75 kg). In this case, the speed of motion is constant ($48\degree/s$) for the two joints. We can easily see in Figure 4.4 that for the movement of all joints, the desired trajectory, represented by the red line, practically overlaps the measured trajectory, represented by the solid blue line. It is clear from the plots in Figure 4.4 that the proposed controller provides an excellent performance, where the controller has the potential to maintain stability of the system along the designed trajectory with a position error (second column of Figure 4.4) less than $1.5\degree$ for elbow joint and less than $2.0\degree$ for forearm joint. The last column of Figure 4.4 shows the control input, which is clearly smooth and without any chattering effect.

The test result of the conventional controller is given in Figure 4.5. The same subject (subject-1) repeated the task. The conventional controller has good tracking performance, where the controller preserves the stability of the robotic system. However, the proposed controller exhibits a better performance than the conventional controller does. That appears at the level of
tracking errors where the proposed strategy provides tracking errors smaller than the tracking errors given by the conventional approach, especially for the forearm joint.

### 4.4.3 Cartesian space tests

The experimental results with the ETS-MARSE robot in the Cartesian space performed by subject-2 (age: 28 years; height: 173 cm; weight: 72 kg) using the designed controller are shown in Figures 4.6–4.8; as shown in Figures 4.6(a) and (4.7), the desired trajectory (red line) nearly overlapped with the measured trajectory (green line). It can be noted that these results are very satisfactory. Figure 4.6(b) presents the Cartesian errors as functions of time.

From this figure, it is obvious that the Cartesian errors are getting smaller along the desired trajectory. Figure 4.8 shows that the control inputs are bounded without any noticeable control chattering. So, these results confirm that the control strategy is able to achieve the desired robot’s performance even if the nonlinear dynamics of the exoskeleton robot is uncertain.
Figure 4.5  Elbow and forearm motion, trajectory tracking in joint space using the conventional approach. Experiment was conducted with subject-1 (age: 30 years; height: 177 cm; weight: 75 kg)

Figure 4.9 presents the workspace performance of the robot in the Cartesian space (red is the desired trajectory, green is the real trajectory) performed by the same subject-2 using the conventional controller.

In fact, we note from Figures 4.9 and 4.10 that the proposed controller shows a good performance where the Cartesian error [see Figure 4.9(b)] becomes smaller with time. However, the proposed strategy presents an excellent performance compared with the performance presented by the conventional approach where the tracking errors of the proposed approach seen in Figure 4.6(b) are smaller than the tracking errors provided by the classical approach [see Figure 4.9(b)].

Additionally, the control input (see Figure 4.11) presented by the conventional approach is noisier than the control input (see Figure 4.8) presented by the proposed approach. These noises may damage the motors of the robot. From the comparison of the two experimental
scenarios, we can conclude that the proposed strategy provides a high level of precision and robustness against the nonlinear dynamic uncertainties and unknown disturbances.
Figure 4.7  Tracking performance of ETS-MARSE in joint space corresponds to Cartesian tasks using the proposed controller

Figure 4.8  Control input the proposed controller
4.5 Conclusion

In this paper, we have proposed an adaptive control of exoskeleton robots with an unknown parameters model based on a backstepping controller. A new time-delay approach is proposed to estimate the uncertain part of the exoskeleton robot and bounded external disturbances where the TDR, friction forces, backlash, and different upper-limb’s mass of each subject are taken into consideration to improve the robot performance. The main advantage of the proposed adaptive control law is that accurate estimation of robots’ dynamic model is not needed. The output of the time-delay estimator is added directly to the control input through a feed-forward loop, whereby the control scheme provides a highly accurate tracking of the desired performance, robust to the uncertainties and unexpected bounded external forces, and flexible with variations in parameters.

The stability analysis of the proposed control technique with a Lyapunov–Krasovskii function was presented. The robustness of the proposed controller was realized by maneuvering the ETS-MARSE to provide both joint-based and end-effector based rehabilitation exercise to the different subjects. The experimental results demonstrate the excellent performance of the proposed controller compared with the conventional controller.

In future work, we seek to overcome the limitations of this approach, in particular, the value of delayed acceleration for the controller, where the estimation of this variable may deteriorate the accuracy of the controller.
Figure 4.9  (a) Reaching movement exercise, Cartesian trajectory tracking in 3-D space using the conventional controller. (b) Cartesian trajectory tracking error along X-axis, Y-axis, and Z-axis. Experiments was conducted with subject-2 (age: 28 years; height: 173 cm; weight: 72 kg)
Figure 4.10  Tracking performance of ETS-MARSE in joint space corresponds to Cartesian tasks using the conventional controller

Figure 4.11  Control inputs corresponds to Cartesian tasks using the conventional controller
CHAPTER 5

PASSIVE AND ACTIVE REHABILITATION CONTROL OF HUMAN UPPER-LIMB EXOSKELETON ROBOT WITH DYNAMIC UNCERTAINTIES

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Abstract

This paper investigates the passive and active control strategies to provide a physical assistance and rehabilitation by a 7-DOF exoskeleton robot with nonlinear uncertain dynamics and unknown bounded external disturbances due to the robot user's physiological characteristics. An Integral backstepping controller incorporated with Time Delay Estimation (BITDE) is used, which permits the exoskeleton robot to achieve the desired performance of working under the mentioned uncertainties constraints. The Time Delay Estimation (TDE) is employed to estimate the nonlinear uncertain dynamics of the robot and the unknown disturbances. To overcome the limitation of the time delay error inherent of the TDE approach, a recursive algorithm is used to further reduce its effect. The integral action is employed to decrease the impact of the unmodeled dynamics. Besides, The Damped Least Square method is introduced to estimate the desired movement intention of the subject with the objective to provide active rehabilitation. The controller scheme is to ensure that the robot system performs passive and active rehabilitation exercises with a high level of tracking accuracy and robustness, despite the unknown dynamics of the exoskeleton robot and the presence of unknown bounded disturbances. The design, stability and convergence analysis are formulated and proven based on
the Lyapunov–Krasovskii functional theory. Experimental results with healthy subjects, using a virtual environment, show the feasibility and ease of implementation of the control scheme. Its robustness and flexibility to deal with parameter variations due to the unknown external disturbances are also shown.

**Keywords:** Backstepping integral control, Time delay Estimation, Passive and active rehabilitation; Time Delay Error.

### 5.1 Introduction

Neurological diseases have become a challenging problem for the scientific community, as confirmed by the statistics of World Health Organization (WHO). Each year, at least 15 million people worldwide suffer from neurological diseases such as strokes (Sidney et al., 2013). Among them, six million die and five million more are living with a permanent disability (Sidney et al., 2013). The consequences of a stroke can vary from mild to very severe depending on the nature of the stroke, the injured region of the brain, and the degree of damage (De Morand, 2014). Usually, the stroke survivors do not have the means of performing daily-life activities independently, such as eating or dressing, due to permanent disability, frequently on one side of the body (De Morand, 2014). This weakness can generate various physical and psychological problems for the stroke victims.

Rehabilitation programs have proven clinically effective. This treatment is able to support people with stroke to recover their inadequate functional capacity, benefit from the acquisition of new skills, and improve their quality of life. However, these programs need intensive and heavy effort by the therapist (Xie et al., 2016). New devices such as exoskeleton robots, help overcome the limitations of conventional physiotherapy, attracting much attention from the scientific community (Xie et al., 2016). The main benefit of the rehabilitation robots is their ability to provide intensive physiotherapy for the required periods. Another important point is that these robots provide numerous data that allow the physiotherapist to accurately assess the subject’s performance (Xie et al., 2016). However, a major aspect is that the design of
this kind of robot must be harmonious with the human anatomy biomechanics. For this reason and to provide a modern rehabilitation treatment for the upper limb, we have developed an exoskeleton robot named ETS-MARSE (Motion Assistive Robotic-Exoskeleton for Superior Extremity) (Brahim et al., 2016a,b; Rahman et al., 2015, 2013). This robot is compatible with the human arm configuration and can perform various rehabilitation movements and assistive tasks. All the originality of the designed robot is summarized in (Rahman et al., 2015, 2013).

One of the main goals of the control system applied to these robots is to make the exoskeleton robot perform the human-like motion. However, these robots are characterized by highly nonlinear dynamics due to their complicated mechanical construction and hard nonlinearities (Brahim et al., 2017) (such as nonlinear friction forces, backlash, etc.). Furthermore, the synergy between the human and the exoskeleton makes the robot subject to external and unknown disturbances due to different physiological conditions of the subjects. These conditions include non-linear biomechanical characteristics of the musculoskeletal system, the varying weights of the upper-limb for each subject, the presence of spasticity in neurological subjects, etc (Brahim et al., 2017). During a rehabilitation session, the uncertain nonlinear dynamic model and the external disturbances can turn into an unknown function that can deteriorate the robot’s performance (Brahim et al., 2016a,b). In such a case, we need to design a robust controller that provides the system with more flexibility and robustness to deal with these problematic characteristics.

Numerous nonlinear control systems have been designed to overcome the effect of the uncertain nonlinear dynamics and unexpected external disturbances, which influence the robot performance, e.g. conventional adaptive control (Slotine et al., 1991), $H_\infty$ control (Rigatos et al., 2018) and sliding mode control (Slotine et al., 1991; Khalil and Grizzle, 1996; Young et al., 1999). Actually, sliding mode control (SMC) is considered one of the most robust nonlinear controllers developed to control uncertain dynamics. This control is fundamentally based on a larger high-gain switching controller which pushes the system trajectory to converge to the selected sliding surface (Fridman, 1999). Nevertheless, the high-activity switching gain causes a "chattering" problem (Fridman, 1999) which can damage the actuators of the robot. Another
successful nonlinear technique, which has been used to control a robot system with a complex dynamic model, is backstepping control (Slotine et al., 1991; Khalil and Grizzle, 1996; Brahmi et al., 2017c). Backstepping control is characterized by a recursive Lyapunov scheme and a virtual control input, which ensures the stability of the control system. Despite the highly accurate trajectory tracking provided by this approach, backstepping control is very sensitive to the uncertain dynamics and unknown external disturbances (Slotine et al., 1991; Khalil and Grizzle, 1996). Various strategies have been combined with backstepping to improve its robustness against the effect of the nonlinear uncertain dynamics and external disturbances. For example, a conventional adaptive backstepping control was developed in (Zhou and Wen, 2008), assuming that the dynamic model of the manipulator is linear in a set of physical parameters. However, the hard nonlinearities of the manipulator’s dynamic model make the system sensitive to even small disturbances. To overcome this problem, numerous approaches combine backstepping control with fuzzy logic, neural networks, or both (Chen et al., 2015; Li et al., 2017a, 2015a; Yoo and Ham, 2000). However, these strategies need heavy computations that make the implementation very complex. Time delay estimation (TDE) approach may be considered to compensate the uncertainties (Youcef-Toumi and Ito, 1990; Brahmi et al., 2017a,b). The TDE has been implemented in many robotic systems with consistently good performance (Youcef-Toumi and Ito, 1990; Jin et al., 2015; Kim et al., 2016; Karafyllis et al., 2016). TDE utilizes the previous response of the robot system and the previous control input to provide new control actions. However, due to noisy measurements and nonlinearity of signals along the sampling time, a time delay error (TDR) exists, which would deteriorate the robustness and the accuracy of the robot. To the best of our knowledge, no one has proposed a methodic solution to eliminate the influence of this error.

In order to address the mentioned problems, a solution is proposed in this paper by combining a nonlinear integral backstepping control (BI) (Skjetne and Fossen, 2004; Tan et al., 2000) and a time delay estimation (TDE) to estimate the hard nonlinearity of the system introduced by the uncertain nonlinear dynamics and unknown disturbances. TDR is taken into consideration by estimating its value with a recursive estimator. This latter is an intensive or repeating
action used to turn down the influence of this error on the accuracy of the estimation. The
integral control provides a progressive change to reduce the unmodeled error and improve the
robustness of the proposed control against the dynamic uncertainties. The structure of the de-
signed nonlinear Backstepping Integral control based on Time Delay Control (BITDC) aims
to provide a high-level of robustness and accuracy without any sensitivity to uncertain nonlin-
ear dynamics and unexpected disturbances. This will make the exoskeleton robot perform a
smooth movement, similar to the natural human movement, and will supply the control sys-
tem more flexibility to handle the uncertainties and parameters’ variation. The stability of the
robot system and the convergence of its errors are formulated and demonstrated based on Lyap-
unov–Krasovskii functional theory. Moreover, the proposed strategy is characterized as an
easy implementation. The efficiency and the robustness of the proposed approach are validated
with Cartesian trajectory tracking corresponding to passive physical therapy tasks.

The proposed strategy is not limited to perform the passive rehabilitation but also qualified
to perform the active rehabilitation. To complete this protocol, we necessarily rely on the
estimation of the Desired Movement Intention (DMI) of the subject using indirect force control.
This latter is done by Damped Least Square method (DLS), which has been successfully used
in multiple applications (Luo et al., 2013; Liu et al., 2015; Gauthier et al., 2016). The DLS
approach aims to provide a compromise between robustness of the solution and accuracy of
the robot’s performance. Besides, the proposed control is evaluated with healthy subjects using
the virtual environment as an interface (Levant, 2003). This latter is a highly attractive tool,
contributing numerous benefits. It authorized the creation of immersive and interactive scenes
where the oriented task can be introduced in the form of earnest games (Weiss et al., 2014).
This interface also helps in stimulation of the subjects and visualizes the performed tasks. This
interface permits us to record the performance data of the patient to evaluate accurately his
improvement and benefit from its flexibility to adjust the designed functional movement to the
requirement of the patient (Weiss et al., 2014). The comparative study with the conventional
approach shows the accuracy, robustness and flexibility of the proposed controller to deal with
unknown dynamics, external disturbances, and parameters variations.
The outline of the paper is organized as follows. The kinematics and dynamics of the robot are presented in the next section. The control scheme is described in section 5.3. Experimental and comparison results are shown in section 5.4. Finally, the conclusion is presented in section 5.5.

5.2 Description of kinematics and dynamics of ETS-MARSE robot

5.2.1 Exoskeleton robot development

The developed exoskeleton robot ETS-MARSE is a redundant robot consisting of seven DOFs, as shown in Figure 5.1. It was created to provide physical therapy and assisted motion to the injured upper limb. The idea of the designed exoskeleton is basically extracted from the anatomy of the upper limb of the human, to be ergonomic for the user along with the physical therapy session. The shoulder part consists of three joints; the elbow part comprises one joint, and the wrist part consists of three joints. Each part is responsible for performing a variety of upper-limb motions, as shown in Table (5.2). The design of the ETS-MARSE has special features compared with the existing exoskeleton robots. Among them, it has a comparatively low weight, an excellent power/weight ratio, can be easily fitted or removed, and is capable of adequately compensating for gravity. A new power carrying mechanism was included for supporting the shoulder joint internal/external rotation and for forearm pronation/supination. This robot can be used with a wide range of subjects, due to the length of its adjustable links. This exoskeleton can perform passive (completely support and perform the motion on the subjects’ upper limb) and active assistive motion (respond to force, electromyography, and/or be compliant with the subject to accompany and assist him/her in the intended motion). All special characteristics and contribution features of the ETS-MARSE and comparison with similar existing exoskeleton robots are summarized in (Rahman et al., 2015, 2013; Luna et al., 2016; Rahman et al., 2011b).
5.2.2 Kinematics of ETS-MARSE Robot

The transformation from Cartesian space to joint space is done by a non-linear function named the Jacobian matrix $J(\theta) \in \mathbb{R}^{6 \times 7}$ which links the end-effector Cartesian velocities to the joints angular velocity vector $J(\theta)\dot{\theta}$ 35, 36. In order to maneuver the exoskeleton in Cartesian space, we used the inverse Jacobian matrix method, since the proposed control is executed in the joint space. Due to the redundant nature of the ETS-MARSE robot where its Jacobian matrix is not quadratic, the inverse kinematics can be solved using the pseudo-inverse of the Jacobian, which can be expressed as follows (Siciliano et al., 2009):

$$
\begin{align*}
\ddot{\theta}_d &= J^+\ddot{x}_d - J^+JJ^+\dot{x}_d \\
\dot{\theta}_d &= J^+\dot{x}_d
\end{align*}
$$

(5.1)

where $x_d \in \mathbb{R}^{6 \times 1}$, $\dot{x}_d \in \mathbb{R}^{6 \times 1}$ and $\ddot{x}_d \in \mathbb{R}^{6 \times 1}$ are the desired Cartesian position/orientation, velocity and acceleration vectors, respectively. $\ddot{\theta}_d$, $\dot{\theta}_d \in \mathbb{R}^{7 \times 1}$ are the calculated joint acceleration and velocity respectively, and $(J^T(JJ^T)^{-1})$ is pseudo-inverse generalized (Siciliano et al., ...
2009). The proposed joint-space based control in this paper does not need a Jacobian matrix or inversion of a Jacobian matrix, as for a Cartesian space-based controller. The role of the Jacobian matrix and its inverse here is the generation of the desired rehabilitation trajectory. Hence, the singularity is not an issue in this case (Rahman et al., 2011b). Moreover, the singularities of the exoskeleton robot are known to us; they will appear when the ETS-MARSE is straight down ($\theta_2 = 0^\circ$, and/or $\theta_4 = 0^\circ$, and/or $\theta_2 = -90^\circ$). As well, a singularity will happen when the axes of rotation of joint-1 (Z1), and joint-3 (Z3), and/or joint-5 (Z5), and/or joint-7 (Z7) are aligned with each other (Rahman et al., 2011b). By knowing these cases, we can easily define the trajectory by avoiding all kinematics singularities. More details about the singularities of ETS-MARSE can be found in (Rahman et al., 2011b).

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>$\alpha_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$d_s$</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{\pi}{2}$</td>
<td>0</td>
<td>$d_c$</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{\pi}{2}$</td>
<td>0</td>
<td>$d_w$</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>6</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_6 - \frac{\pi}{2}$</td>
</tr>
<tr>
<td>7</td>
<td>$-\frac{\pi}{2}$</td>
<td>0</td>
<td>0</td>
<td>$\theta_7$</td>
</tr>
</tbody>
</table>

The modified Denavit–Hartenberg (DH) parameters (Craig, 2005) are given in Table (5.1). These parameters are obtained from the reference frames shown in Figure 5.1 and are used to obtain the homogeneous transformation matrices. The workspace of the designed robot is given in Table (5.2). The details of the parameters and design of ETS-MARSE are given in (Rahman et al., 2015).
Table 5.2  Workspace ETS-MARSE

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>Motion</th>
<th>Workspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shoulder joint horizontal flexion/extension</td>
<td>$0^\circ/140^\circ$</td>
</tr>
<tr>
<td>2</td>
<td>Shoulder joint vertical flexion/extension</td>
<td>$140^\circ/0^\circ$</td>
</tr>
<tr>
<td>3</td>
<td>Shoulder joint internal/external rotation</td>
<td>$-85^\circ/75^\circ$</td>
</tr>
<tr>
<td>4</td>
<td>Elbow joint flexion/extension</td>
<td>$120^\circ/75^\circ$</td>
</tr>
<tr>
<td>5</td>
<td>Forearm joint pronation/supination</td>
<td>$-85^\circ/85^\circ$</td>
</tr>
<tr>
<td>6</td>
<td>Wrist joint ulnar/radial deviation</td>
<td>$-30^\circ/20^\circ$</td>
</tr>
<tr>
<td>7</td>
<td>Wrist joint flexion/extension</td>
<td>$-50^\circ/60^\circ$</td>
</tr>
</tbody>
</table>

5.2.2.1 Dynamics of the ETS-MARSE Robot

The dynamics behavior of ETS-MARSE is given by the following expression using the Lagrangian approach (Craig, 2005):

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) + f_{\text{dis}} = \tau \quad (5.2)$$

where $\theta \in \mathbb{R}^7$, $\dot{\theta} \in \mathbb{R}^7$ and $\ddot{\theta} \in \mathbb{R}^7$ are, respectively, the joint’s position, velocity, and acceleration vectors, $M(\theta) \in \mathbb{R}^{7 \times 7}$ is the symmetric and positive definite inertia matrix, $C(\theta, \dot{\theta}) \in \mathbb{R}^{7 \times 7}$ is the Coriolis and centrifugal vector, $G(\theta) \in \mathbb{R}^7$ is the gravitational vector, $\tau \in \mathbb{R}^7$ is the applied joint torque to the exoskeleton robot, and $f_{\text{ext}} \in \mathbb{R}^7$ is the external disturbances vector. Without loss of generality, the matrices of dynamic model (5.2) can be written as follows:

$$\begin{cases}
M(\theta) = M_0(\theta) + \Delta M(\theta) \\
C(\theta, \dot{\theta}) = C_0(\theta, \dot{\theta}) + \Delta C(\theta, \dot{\theta}) \\
G(\theta) = G_0(\theta) + \Delta G(\theta)
\end{cases} \quad (5.3)$$

where $M_0(\theta)$, $C_0(\theta, \dot{\theta})$ and $G_0(\theta)$ are respectively the known inertia matrix, the Coriolis centrifugal matrix, and the gravity vector. $\Delta M(\theta)$, $\Delta C(\theta)$ and $\Delta G(\theta)$ are the uncertain parts.
Let us introduce a new variable such that: \( \eta_1 = \theta \) and \( \eta_2 = \dot{\theta} \); hence, the dynamic model expressed in (5.2) can be rewritten as follows:

\[
\begin{align*}
\dot{\eta}_1 &= \eta_2 \\
\dot{\eta}_2 &= U(t) + f(t) + H(t)
\end{align*}
\]  

where, \( U(t) = U(\eta_1), \ H(t) = H(\eta_1, \eta_2, \dot{\eta}_2) \) and \( f(t) = d\eta_1, \eta_2 \). This notation is used for easier handling the control scheme with:

- \( U(t) = M_0^{-1}(\theta) \tau(t) \);
- \( f(t) = M_0^{-1}(\theta) \left[ -C_0(\theta, \dot{\theta}) - G_0(\theta) \right] \);
- \( H(t) = H(\theta, \dot{\theta}, \ddot{\theta}) = M_0^{-1}(\theta) \left[ -f_{\text{ex}} - \Delta M(\theta) \ddot{\theta} - \Delta C(\theta, \dot{\theta}) \dot{\theta} - \Delta G(\theta) \right] \).

### 5.2.2.2 Problem formulation

One of the main research objectives on this type of robots is to design a controller able to make the human-exoskeleton system achieve passive/active physical therapy movement. This should be performed without accurate information about the dynamics of the robotic system while ensuring a smooth movement similar to natural human motion. Therefore, the designed control scheme should ensure that the measured joint position \( \eta_1 \in \mathbb{R}^7 \) of the exoskeleton robot tracks the desired trajectory \( \eta_d \in \mathbb{R}^7 \). Before presenting the control design methodology, we state the properties and the assumptions used in this paper: \( \eta_d \in \mathbb{R}^7 \)

**Property 5.1:** The known part of inertia matrix \( M_0(\theta) \) is symmetric and positive definite for all \( \theta \in \mathbb{R}^7 \) and satisfying: \( \gamma_{\text{min}}(M_0(\theta)) I_{7 \times 7} \leq M_0(\theta) \leq \gamma_{\text{max}}(M_0(\theta)) I_{7 \times 7} \), where \( \gamma_{\text{min}} \) and \( \gamma_{\text{max}} \) are minimum and maximum eigenvalues, respectively, of the known inertia matrix and \( I_{7 \times 7} \) is identity matrix (Spong et al., 2006b).

**Assumption 5.1:** The joint positions are measured and the corresponding velocities are found from the filtered positions.

**Assumption 5.2:** The function \( H(t) \) is globally Lipschitz function.
Assumption 5.3: The variation of the uncertainties parameters $H(t)$ in time is continuous and bounded with known delay $t_d$. It can be written as:

$$H(t) = H(t - t_d) + \varepsilon(t)$$  \hspace{1cm} (5.5)

where $\varepsilon(t)$ is a vector of the delay error of uncertainties variation and $t_d$ is the sampling time period.

Assumption 5.4: The desired trajectory is bounded.

Assumption 5.5: The Jacobian matrix $J(\eta_1)$ is known and non-singular in a finite workspace.

Assumption 5.6: The external disturbance $f_{ex}$ is supposed to be continuous, has finite energy and satisfies $\|f_{ex}\| \leq \vartheta$, with an unknown positive disturbance boundary $\vartheta$.

5.3 Control design

In this section, we seek to design the control that is capable of performing both passive and active rehabilitation movements. Passive rehabilitation mode can be achieved by position control. In such case, the exoskeleton performs a predesigned task. On the other hand, the active assistive motion is achieved by the force exerted by the subject on the tip of the robot (wrist joint) and measured by a force sensor. This force is effective to produce some displacement. In our case, we estimate the desired motion intention of the subject from the measured force using Damped Least Square method (DLS).

5.3.1 Estimation of the Desired Motion Intention (DMI)

This section provides a summary of the estimation method of the Desired Movement Intention (DMI). Let us start with the definition of the desired trajectory in the active assistive motion.
In this protocol, the desired trajectory is updated as follows (Ochoa Luna et al., 2015):

\[ \eta_d = \eta_1 + \Delta\eta_d \]  

(5.6)

where \( \eta_1 \in \mathbb{R}^7 \) is the measured joint position, and \( \Delta\eta_d \in \mathbb{R}^7 \) is the estimated of DMI where this quantity of movement is estimated from the measured user’s force. If \( \Delta\eta_d \rightarrow 0 \), this means that the exoskeleton’s wearer stops exercising forces on the force sensor, making the exoskeleton to decrease its motion and progressively, and whenever \( \eta_d = \eta_1 \), the exoskeleton rests in its most recent position. With this mode, the exoskeleton is permitted to catch the user’s DMI, while the adaptive tracking control ensures an accurate compensation of the robot dynamic uncertainties and the unexpected bounded disturbances submitted to the robot. Let us now estimate the desired movement intention \( \Delta\eta_d \) from the user’s force, we can use the following equation:

\[ F_m = J(\eta_1) + \Delta\eta_d \]  

(5.7)

where \( F_m \in \mathbb{R}^6 \) is the measured user’s force. To solve the equation (5.7), we used Damped Least Squares (DLS) or Levenberg-Marquardt stabilization (Lawson and Hanson, 1995). This approach was originally employed for avoiding the singularity of inverse kinematics solution (Wampler, 1986; Nakamura and Hanafusa, 1986). Rather than merely obtaining the minimum vector \( \Delta\eta_d \) that provides the best solution to equation (5.7), we determine the value of \( \Delta\eta_d \) that minimizes the quantity

\[
\min_{\Delta\eta_d} \| J(\eta) \Delta\eta_d - F_m \|^2 + \lambda^2 \| \Delta\theta_d \|^2
\]  

(5.8)

where \( 0 < \lambda < 1 \) is the damping factor and can be determined to be positive depending upon the accurate estimation specifications. Since the sum of equation (5.8) can be written as:

\[
\left\| \begin{bmatrix} J(\theta) \\ \lambda I_{6 \times 6} \end{bmatrix} \Delta\eta_d - \begin{bmatrix} F_m \\ 0 \end{bmatrix} \right\|^2
\]  

(5.9)
Consequently, the DLS solution is:

\[ \Delta \eta_d = J^T (J J^T + \lambda^2 I_{6 \times 6})^{-1} F_m \]  

(5.10)

**Remark 5.1:** The desired trajectory is the input of the proposed controller that will be detailed in the following subsection. Where, in active mode, the desired trajectory is updated by equation (5.6), while in passive mode, the desired trajectory is predefined by a therapist.

### 5.3.2 Control algorithm

This section presents the proposed tracking control based on the Backstepping integral control that can be expressed as follows: **Step 1:** Let us define the position error as follows:

\[ e(t) = \eta_1 - \eta_d \]  

(5.11)

where \( \eta_d \in \mathbb{R}^7 \) and \( \eta_1 \in \mathbb{R}^7 \) are the desired trajectory and measured trajectory, respectively.

Now, we can choose the regulated errors variables as follows:

\[ e_1 = \eta_1 - \eta_d + \Gamma \int_0^t e \, dt \]  

(5.12)

\[ e_2 = \eta_2 - \xi \]  

(5.13)

where \( \Gamma \in \mathbb{R}^{7 \times 7} \) is a diagonal positive-definite matrix, and \( \xi \) is a virtual control input chosen as follows:

\[ \xi = \dot{\eta}_d - \Gamma \dot{e} - k_1 e_1 \]  

(5.14)

where \( k_1 \in \mathbb{R}^{7 \times 7} \) is a diagonal positive-definite matrix. The derivative of the virtual control input (5.14) is: \( \dot{\xi} = \dot{\eta}_d - \Gamma \ddot{e} - k_1 \ddot{e}_1. \)
The time derivative of equation (5.12) is given by:

\[ \dot{e}_1 = \eta_2 - \dot{\eta}_d + \Gamma e \]  

(5.15)

Substituting equation (5.13) into equation (5.15), we have:

\[ \dot{e}_1 = e_2 + \xi - \dot{\eta}_d + \Gamma e \]  

(5.16)

Consider the first Lyapunov function candidate as:

\[ V_1 = \frac{1}{2} e_1^T e_1 \]  

(5.17)

Taking time derivative of \( V_1 \) as follows:

\[ \dot{V}_1 = e_1^T \dot{e}_1 = e_1^T (e_2 + \xi - \dot{\eta}_d + \Gamma e) \]  

(5.18)

Substituting equation (5.14) into equation (5.18), we obtain:

\[ \dot{V}_1 = -e_1^T k_1 e_1 + e_1^T e_2 \]  

(5.19)

The first term of equation(5.19) is negative and the second term will be addressed in the next step.

**Step 2:** Differentiating (5.13) with respect to time yields:

\[ \dot{e}_2 = U(t) + f(t) + H(t) - \dot{\xi} \]  

(5.20)

Consider the second Lyapunov function candidate as:

\[ V_2 = V_1 + \frac{1}{2} e_2^T e_2 \]  

(5.21)
The time derivative of $V_2$ is given by:

$$
\dot{V}_2 = \dot{V}_1 + e_2^T \dot{e}_2
= \dot{V}_1 + e_2^T \left( U(t) + f(t) + H(t) - \dot{\xi} \right)
$$

(5.22)

Consider the control law $U(t)$ that stabilizes the robot system such that:

$$
U(t) = -k_2 e_2 - e_1 - f(t) - H(t) + \dot{\xi}
$$

with $\tau = M_0(\theta) U(t)$

(5.23)

with $k_2 \in \mathbb{R}^{7 \times 7}$ being a diagonal positive-definite matrix.

Substituting equation (5.23) into equation (5.22), we find:

$$
\dot{V}_2 \leq -e_1^T k_1 e_1 - e_2^T k_2 e_2
$$

(5.24)

Relation (5.24) ensures the global stability of the system. However, $H(t)$ of relation (5.23) is uncertain and the control law of relation (5.23) becomes difficult to compute. To overcome this problem, the TDE approach (Youcef-Toumi and Ito, 1990) may be used to estimate the uncertainties of the nonlinear robot dynamics. So, if Assumption 5.2 is verified, $H(t)$ can be estimated such that:

$$
\hat{H}(t) \simeq H(t - t_d) = \eta_2(t - t_d) - f(t - t_d) - U(t - t_d)
$$

(5.25)

where, $t_d$ is a very-small time delay constant. Practically, the smallest constant that can be achieved in real time is the sampling period. However, due to noisy measurements and non-linearity of signals along the sampling time, a time delay error (TDR) $\epsilon(t)$ exists. This TDR would deteriorate the robustness and the accuracy of the robot. Unfortunately, the TDR is not available. In this case, let us apply an iterative estimator to estimate and reduce its effect and give more flexibility to the control system on dealing with parameters variation and unexpected
disturbances. The iterative estimator is given in scalar form as:

\[
\hat{e}_i(t) = \hat{e}_i(t - t_d) - k_{3i}e_{2i} \\
\hat{e}_i(t) = 0, \quad \forall t \in [-t, 0]; \quad i = 1, \ldots, 7 \tag{5.26}
\]

where, \(k_{3i}\) is a positive constant. The proof of equation (5.26) is given in appendix II (1).

![General schematic of proposed control](image)

Figure 5.2 General schematic of proposed control (for Selector 1 is pass and 0 is stop)

We can now apply a feasible control input able to keep the stability of the system even if the dynamics of the robot is uncertain and the external disturbances exist. The proposed controller is given such that:

\[
U(t) = -k_2e_2 - e_1 - f(t) - \hat{H}(t) + \dot{\xi} - \hat{\epsilon}(t) \tag{5.27}
\]

with the desired torque:

\[
\tau = M_0(\theta) U(t) \tag{5.28}
\]
Before starting the proof of the stabilization of the system, let us define the estimation error of uncertainties using Assumption 5.2 and Assumption 5.3 such that:

\[
\tilde{H}(t) = H(t) - \hat{H}(t) = H(t) - H(t - t_d) = \varepsilon(t)
\] (5.29)

We can now define the estimation error of time delay error \( \varepsilon(t) \) as:

\[
\tilde{\varepsilon}(t) = \hat{\varepsilon}(t) - \varepsilon(t)
\] (5.30)

To guarantee the asymptotic stability of the delayed system in the interval \([0, \infty)\), we propose the following Lyapunov–Krasovskii function:

\[
V_3 = V_2 + \begin{cases}
\sum_{i=1}^{n-1} \frac{1}{2k_{3i}} \int_{t_{d_{i-1}}}^{t_{d_i}} \tilde{\varepsilon}_i^2(s) ds, & t \in [0, t_{d1}); \\
\frac{1}{2k_{31}} \int_{t_{d1}}^{t_{d2}} \tilde{\varepsilon}_1^2(s) ds + \sum_{i=2}^{n} \frac{1}{2k_{3i}} \int_{0}^{t_{d1}} \tilde{\varepsilon}_i^2(s) ds, & t \in [t_{d1}, t_{d2}); \\
\sum_{i=1}^{n-1} \frac{1}{2k_{3i}} \int_{t_{d_{i-1}}}^{t_{d_i}} \tilde{\varepsilon}_i^2(s) ds + \frac{1}{2k_{3n}} \int_{0}^{t_{d_{n-1}}} \tilde{\varepsilon}_n^2(s) ds, & t \in [t_{dn-1}, t_{dn}); \\
\sum_{i=1}^{n} \frac{1}{2k_{3i}} \int_{t_{d_{i-1}}}^{t_{d_i}} \tilde{\varepsilon}_i^2(s) ds, & t \in [t_{dn}, \infty); \\
\end{cases}
\] (5.31)

with:

\[
V_2 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2
\] (5.32)

and \( n = 7 \). The derivative of the proposed Lyapunov–Krasovskii function with respect to time is obtained as follows:

\[
\dot{V}_4 = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - \frac{1}{2} \sum_{i=1}^{n} k_{3i} e_{2i}^2
\] (5.33)

It is clear from (5.33) that \( \dot{V}_4 \leq 0 \), where all gains \( k_1, k_2, \) and \( k_3 \) are positive. The proof of the stability is detailed in Appendix II (2). The structure of the control scheme is shown in Figure 5.2.
5.4 Experimental and comparative study

5.4.1 Experiment Setup

Implementation was carried out on the ETS-MARSE system described below. The system consists of three processing units, the first is a PC where the top-level commands are sent to the robot using LabVIEW interface, i.e. the control scheme selection, joint or Cartesian space trajectory, gain adjustments, etc. This PC also receives the data after the robot task is executed to analyze its performance. The other two processing units are part of a National Instruments PXI platform. Firstly, a NI-PXI 8081 controller card with an Intel Core Duo processor; in this card, the main operating system of the robot and the top-level control scheme are executed. In our case, this is the integral backstepping based controller as well as the estimation based on TDE approach, at a sampling time of 500μs. The inverse kinematics algorithm also runs inside this control loop. Finally, at input-output level, a NI PXI-7813R remote input–output card with an FPGA (field programmable gate array) executes the low-level control; i.e. a PI current control loop (sampling time of 50 μs) to maintain the current of the motors required by the main controller. Also, in this FPGA, the position feedback via Hall-sensors (joint position), and basic input-output tasks are executed. Force sensor feedback is important to accurately control the movement of the exoskeleton. A high linearity 6 axis force sensor [NANO17-R-1.8-M2-M1PCI, ATI industrial Automation] is so chosen to obtain accurate real-time force measurements. This sensor is mounted on the tip of the robot. The joint of the ETS-MARSE is powered by Brushless DC motors ((Maxon EC-45, EC-90 ) combined with harmonic drives (gear ratio 120:1 for motor-1, motor-2, and motor-4 and gear ratio 100:1 for motor-3 and motors 5–7) (Brahmi et al., 2017c; Luna et al., 2016).

The physical parameters of the ETS-MARSE relative to the base reference frame are given in Table (5.3). The diagram of the architecture and overview of the ETS-MARSE system with a human subject is shown in Figure 5.3.
Table 5.3 Physical parameters of ETS-MARSE

<table>
<thead>
<tr>
<th>Joints (i)</th>
<th>Mass (kg)</th>
<th>Centre of mass (m)</th>
<th>Link length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.475</td>
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</tbody>
</table>

The virtual environment HELIOS software was created using the open source "Unity" platform. This interface consists of a set of functional movement tasks defined by a therapist. There are two types of communication with the ETS-MARSE system (LabVIEW). A User Datagram Protocol (UDP) for transferring position data, and a Transmission Control Protocol/Internet Protocol (TCP/IP) for transferring the operator’s commands (e.g. start/stop the execution of the task). The choice of these protocols is based on the reliability and the speed of the transmission of data. Our platform contains two profiles: a user profile and an Admin profile. The user profile is under the supervision of a therapist and of a biomedical engineer. This latter is responsible for preparing the rehabilitation session (such as customizing the activities to the patient’s needs). The Admin profile is responsible for managing the database of the patient (such Add/Edit/Delete a patient) and has access to the list of all patients and information concerning their rehabilitation sessions (Ferrer et al., 2013).

A rehabilitation session was created to prove the effectiveness of the proposed control system. The physical therapy tasks are performed by three different healthy subjects (average age: 27 ± 4.6 years; average height: 170 ± 8.75 cm; average weight: 75 ± 18 kg). Each subject participated in a full session, under the supervision of a therapist and a control engineer. The role of the therapist is the definition of the range of motion of each subject and the attribution of the suitable exercise. Within the session, the subject was comfortably seated in a chair in front of the virtual interface as we show in Figure 5.3(a). The experimental session was divided into two scenarios. In passive rehabilitation, each subject performed the designed task of Figure 5.4(a). This task (Initial position → Target-A → Target-B → Target-C→ Initial
position) is expressed in Cartesian space to evaluate the proposed control. The initial position of the robot is given where elbow joint position is at 90 degrees. This part is followed directly by a comparison study with conventional approaches to show the advantage of the proposed controller. In active rehabilitation, each subject has interacted with the virtual environment and tried to track the proposed trajectory.

It is important to notice that the external disturbances here are represented by different physiological conditions of the subjects, such as non-linear biomechanical characteristics of the musculoskeletal system and the different weights of the upper-limb for each subject. The control gains were tuned manually with a trial-and-error approach as shown in Table (5.4):

---

**Figure 5.3**

(a) General schematic of the experimental architecture.
(b) Overview of the ETS-MARSE with a human subject
Table 5.4  Controller parameters

<table>
<thead>
<tr>
<th>Gains</th>
<th>Value $(i = 1:7)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{1i}$</td>
<td>15</td>
</tr>
<tr>
<td>$k_{2i}$</td>
<td>55.5</td>
</tr>
<tr>
<td>$k_{3i}$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>0.6 for $(i = 6)$</td>
</tr>
</tbody>
</table>

Figure 5.4  (a) Workspace trajectories of the robot in Cartesian space using the proposed controller, performed by Subject-1: (age: 28 years; height: 177 cm; weight: 83 kg). (b) Cartesian errors
Figure 5.5  Position tracking of the robot in joint space corresponding to the movement performed by Subject-1(Fig. 4): (age: 28 years; height: 177 cm; weight: 83 kg)

Figure 5.6  Control inputs of the proposed controller
5.4.2 Experimental results of passive rehabilitation

The experimental results with the ETS-MARSE robot in Cartesian space conducted by Subject-1: (age: 28 years; height: 177 cm; weight: 83 kg) using the proposed control strategy are shown in Figure 5.4 and Figure 5.5. From Figure 5.4(a) and Figure 5.5, we observe that the desired trajectory (red line) nearly overlapped with the measured trajectory (green line). In this case, we can say that these results are reasonably good. Figure 5.4(b) presents the Cartesian tracking errors as functions of time, where it is clear from this figure that the Cartesian errors are converging and smaller along the desired trajectory. Figure 5.6 shows that the control input is bounded without any noticeable chattering. So, these results confirm that the control strategy is suitable to perform the desired task even when the nonlinear dynamics of the robot are uncertain.

5.4.3 Comparative study

In order to prove the feasibility and efficiency of the proposed approach, we compared it experimentally with the conventional adaptive tracking control presented in (Kali et al., 2016). In this latter, an integral backstepping control combined with TDE has been proposed without considering the effect of Time Delay Error (TDR). From this comparison, we can see clearly the impact of TDR on the robot’s performance and the improvement that is provided by the approach proposed in this paper.

Figure 5.7(a) and Figure 5.8 present the Cartesian and joint trajectories tracking (red is the desired trajectory, green is real trajectory) performed by Subject-1: (age: 28 years; height: 177 cm; weight: 83 kg) using the conventional controller. In fact, we remark from figures (Figure 5.7(a and B) and Figure 5.8) that the conventional controller shows a good performance. Where, it is certain to ensure that the Cartesian error is getting smaller with time. However, the control inputs of the conventional controller presented in Figure 5.9 illustrate a noisy signal with chattering phenomenon, while the proposed controller provides a smooth control input Figure 5.6. It is important to remark that we used the same gains’ values that employed in the
proposed approach. So, the TDR is the main cause of the noise in the control input, which may provoke damage in the motors. From the comparison of the two experimental results, we can conclude that the proposed strategy provides a high level of precision and robustness to the nonlinear uncertain dynamics and unknown disturbances without remarkable chattering, compared with the conventional controller.

Figure 5.7  (a) Workspace tracking of the robot in Cartesian space using the conventional controller, performed by Subject-1: (age: 27 years; height: 177 cm; weight: 83 kg). (b) Cartesian errors
Figure 5.8  Position tracking of the robot in joint space corresponding to the movement performed by Subject-1: (age: 28 years; height: 177 cm; weight: 83 kg)

Figure 5.9  Control input of the conventional controller
To show more of the feasibility of the designed strategy, we propose a numerical comparison between the above controllers (conventional controller and proposed controller) by calculating the root mean square (RMS) of the error and the control input of each controller as follows:

$$\|e\|_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \|e(i)\|^2} \quad \text{and} \quad \|\tau\|_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \|\tau(i)\|^2}$$

where $N$ is the number of samples of the signals, corresponding with the time steps of the trial. The evaluation of the controller is given in Table 5.5.

### Table 5.5 Comparative study with conventional control

<table>
<thead>
<tr>
<th>Subjects</th>
<th>Proposed controller</th>
<th>Conventional controller</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$|e|_{RMS_error}$</td>
<td>$|\tau|_{RMS_Torque}$</td>
</tr>
<tr>
<td>Subject-1</td>
<td>0.0149</td>
<td>2.9342</td>
</tr>
<tr>
<td>Subject-2</td>
<td>0.0191</td>
<td>2.9061</td>
</tr>
<tr>
<td>Subject-3</td>
<td>0.0165</td>
<td>2.9132</td>
</tr>
</tbody>
</table>

The results in Table 5.5 show that the proposed controller gives the robot system a high level of robustness to handle parameter variations, nonlinear uncertainties of the dynamics, and unknown disturbances (the different weight of the upper limb for each subject).

To prove the efficiency of the proposed controller, we provide another comparison study of the proposed controller with classical approaches that are previously applied on the ETS-MARSE robot. The subjects repeated the same tasks using conventional (Luna et al., 2016) (PID controller and Computed Torque Control CTC) four consecutive times for each subject, giving a total of 12 experiments for each controller. Figure 5.10 presents the performance result of each controller with different subjects. It is clear from this figure (Figure 5.10) that the proposed control based on TDE presents an excellent performance, much better than PID and Computed Torque Control (CTC) and is not affected by parameters’ variations. Meanwhile, the PID con-
troller exhibited a more variable behavior, while the CTC controller was the most influenced by variation in the subject’s characteristics.

![Figure 5.10 Comparison performance of the proposed controller with PID controller and CTC controller](image)

The interpretation of these results is that the PID controller fundamentally does not need the dynamic parameters of the exoskeleton robot and its wearer to provide good results. On the other hand, the CTC controller needs these parameters to ensure a good performance, so, any variation on these parameters reflects negatively on its performance.

### 5.4.4 Experimental results of active rehabilitation

In this section, the subjects tried to follow the proposed triangle task (red line is the desired trajectory and green line is the actual trajectory) under a virtual interface with different values of damping factor $\lambda$. In this case, the exoskeleton robot is completely passive and the subject is completely active. The Damped Least Square (DLS) algorithm is used to provide the esti-
mation of the desired movement intention (DMI). Figures 5.11–5.13 present the results of this experiment.

Figure 5.11 (a) Workspace tracking of the robot using DLS method performed by Subject-2: (age: 31 years; height: 183 cm; weight: 83.5 kg) (view on Unity platform). (b) Cartesian errors

Figures 5.11 presents the performance of subject-2 (age: 31 years; height: 183 cm; weight: 83.5 kg) in the virtual interface with help of the ETS-MARSE exoskeleton robot (red line is desired and the green line is the achieved trajectory) using different values of damping factor ($\lambda = 0.6$, $\lambda = 0.45$). Figures 5.11–5.13 present the workspace tracking of the robot in Cartesian space, the control input and the measured force exerted by the subject.
It is clear from these plots (Figures 5.11–5.13) that the control strategy achieved the desired performance with small tracking errors and acceptable control input with different values of damping factor. From the good performance of the exoskeleton-subject-2 (Figure 5.11) with $\lambda = 0.6$), we can infer that the Damped Least Square (DLS) algorithm was qualified to estimate
accurately the desired movement intention $\Delta \eta_d$. During the experiment, we concluded that the damping factor $\lambda$ acts as an influential role to realizing an accurate estimation of the user’s intention, as shown in Figures 5.11 with $\lambda = 0.45$. Despite the controller having ensured a good tracking of the desired user’s motion intention, the damping factor affects the estimation accuracy of the desired movement.

5.5 Conclusion

In this paper, we proposed an adaptive control of an exoskeleton robot with uncertain dynamics based on an integral backstepping controller and time delay estimation. Where, the TDR is taken into consideration. The TDE is used to estimate the robot’s nonlinear uncertain dynamics and the unknown bounded perturbations exerted on the exoskeleton robot. To improve the accuracy of the robot performance, the TDR is estimated by a recursive control law to overcome the limitation of the time delay error inherent of the TDE approach. The proposed strategy is designed to provide a high-level of robustness and precision to compensate for nonlinear uncertainties of the dynamics and unknown disturbances, to make the exoskeleton robot achieve smooth motion similar to the natural human movement, and to give to the control system more flexibility to deal with the uncertainties and parameters variation. The Damped Least Square (DLS) method has been employed to estimate the Desired Movement Intention (DMI) of the subjects in order to provide active rehabilitation. The main benefit of the proposed adaptive control approach is that precise knowledge of the dynamic parameters of the robot is not required. Moreover, the proposed adaptive strategy is characterized by the easiness of real-time implementation. The stability analysis of the robot system and the convergence of its errors were proved based on Lyapunov–Krasovskii functional theory. The experimental results prove the effectiveness, facility of implementation, and accuracy of the proposed approach. As a future work, we will seek to overcome the limitations of this approach. In particular, the value of the delayed acceleration for the controller; where the estimation of this variable may deteriorate its accuracy. We are leaning to use this controller with a more complex scenario, such human-robot collaboration.
CHAPTER 6

ADAPTIVE CONTROL OF A 7-DOF EXOSKELETON ROBOT WITH UNCERTAINTIES ON KINEMATICS AND DYNAMICS

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Abstract

In this paper, we propose a new adaptive visual tracking control approach based on sliding mode control in Cartesian space applied to an exoskeleton robot with uncertain kinematics and dynamics, taking into account uncertainties in visual system (camera) parameters. The adaptation of kinematic uncertainties is based on a filtered regressor kinematic matrix, whereas, the adaptation of dynamic uncertainties is based on a Time Delay Estimation approach. This is performed considering the Time Delay Error (TDR) to provide a control action capable of following the designed functional therapy tasks. A new recursive controller is combined with TDE in order to estimate the TDR and limit its effect. The proposed strategy does not need the accurate dynamic and kinematic models of the exoskeleton. The update laws are designed using Lyapunov theory to solve the adaptation problem methodically and to show the stability of the robot system. Experimental results confirm the effectiveness and feasibility of the designed approach.

Keywords: Rehabilitation robots, Time delay control; Time delay error, Uncertainties, recursive control.
6.1 Introduction

Recently, stroke and neurological diseases have become among the most important health-related problems in the world. Stroke survivors bear with disabilities following an accident that affects their quality of life (Lundström et al., 2008; Nichols-Larsen et al., 2005). Annually, worldwide, six million people die and five million live with persistent weakness, from the 15 million people suffering a stroke (Sidney et al., 2013). Physical therapy is the main treatment existing today. It is designed to relieve the patient from the impairment or/and injury and to improve his range of movement, functional capacity, and quality of life (De Morand, 2014).

Thanks to robotics technology, modern rehabilitation treatment is supported by new devices named rehabilitation robots. This kind of robot is able to provide a wide range of physical therapy and overcome some of the limitations of conventional therapy. Many research teams participate in this field, among them, Assisted Rehabilitation and Measurement Guide (ARMin IV) (Philips et al., 2017); Robotic Upper Extremity Repetitive Therapy (RUPERT) (Balasubramanian et al., 2008); Saga University Exoskeleton For Upper Limb (SUEFUL-7) (de Santé, 2012). An important issue is that the design of these robots must be harmonious with the human anatomy configuration. To provide a modern physical therapy for the upper limb, we have developed an exoskeleton robot named ETS-MARSE. This robot is consistent with the human arm configuration and is capable of performing different rehabilitation movements (Brahim et al., 2016b,a; Brahmi et al., 2017c).

Generally, the dynamic parameters of an exoskeleton robot, for instance the ETS-MARSE, are hard to be modeled precisely because of the complexity of the mechanical design such as non-linear friction forces, backlash, and the complexity of the actuators of the robot. In addition, the dynamic characteristics vary due to the different physiological conditions of the subjects, such as an external force caused by subject’s muscular activity (Brahim et al., 2016b). This kind of uncertain nonlinearities can be categorized as both parametric uncertainties and unknown nonlinear functions (Dawson et al., 2003). Additionally, in most applications using rehabilitation robots, the therapeutic tasks are expressed in Cartesian space. In this case, the nonlinear transformation functions or Jacobian matrix that allows the mapping from joint space
to Cartesian space is assumed to be known, to ensure a perfect Cartesian control performance. However, when the feedback position of the robot’s response is provided by a visual system, such as a camera or Kinect, the exoskeleton can be subject to uncertain kinematics. Due to firstly the uncertainties in camera parameters, secondly, when the human and exoskeleton carry/transfer an object with an unknown length and/or orientation (Arimoto, 1999). In this case, it is difficult to derive the solution of inverse kinematics using this standard approach. Moreover, to realize human motion via an exoskeleton robot, it is essential to determine accurately the lengths of links, the joints, and the dimension of the object carried by the robot. Unfortunately, it is difficult to define these physical parameters exactly. Various solutions have been proposed to estimate these parameters (Cheah, 2006; Cheah et al., 1999; Bai et al., 2017; Han, 2009; Tang and Li, 2014). However, almost none of these solutions consider the uncertainties of the camera parameters. This dilemma motivates us to set up a new control system capable of ensuring a sufficient performance in the presence of dynamic and kinematic uncertainties and unknown disturbances exist.

One of the research challenges on this class of robots is to develop a controller that can maneuver the human-exoskeleton system to mimic natural human upper extremity motion. It is remarkable from a natural human movement that the human does not need accurate information about kinematics and dynamics of the arm (or any object carried by upper extremity) to reach an object in space. Due to that, many control strategies have been designed to solve the problem of kinematic and dynamic uncertainties (Arimoto, 1999; Cheah, 2006; Yazarel and Cheah, 2002; Huang and Chien, 2010; Cheah et al., 2005; Hutchinson et al., 1996). The main innovative point of these controllers is that the adaptation of the both kinematic/dynamic uncertainties has been provided, which makes the exoskeleton robot perform the human-like motion and supplies to the control system more flexibility to handle the uncertainties and parameters variation. However, the above controllers are based on the classical regressor matrix. These types of controllers assume that the robot is linear in a set of physical parameters and find a control law able to ensure the stability of this linear system only around its operating points (Yao, 1996). In fact, the manipulator is highly nonlinear. So, the integration of this adaptation law may affect
the stability of the system in the presence of even small disturbances (Yao, 1996). Adaptive vi-
visual or image-based tracking control (Hutchinson et al., 1996; Deng et al., 2002; Espiau et al.,
1992; Gans et al., 2003; Malis and Chaumette, 2002; Liu et al., 2006) is one of the powerful ap-
proaches that has been developed to transact with the kinematic/dynamic uncertainties. This is
due to their robustness practically to modeling and calibration errors (Deng et al., 2002). How-
ever, these controllers are concentrated on uncertainties in nonlinear transformation functions
or image Jacobian matrix but they ignored the uncertain kinematic/dynamic effects. Addition-
ally, a few stability analyses are provided in the literature for visual tracking control with
the uncertainties of kinematics/dynamics and in the presence of uncertainties in visual system
(camera) parameters (Cheah et al., 2006). A Time Delay Estimation (TDE) approach may be
considered to compensate the uncertainties (Youcef-Toumi and Shortlidge, 1991; Efimov et al.,
2015; Rami et al., 2013; ?; Fridman, 2014; Zheng et al., 2010; Brahmi et al., 2017a,b). The
TDE has been implemented in many robotic systems with consistently good performance (Jin
et al., 2015; Kim et al., 2016). The TDE utilizes the previous response of the robot system,
and the previous control input to provide new control actions able to provide an accurate ap-
proximation of uncertainty function. However, due to noisy measurements and nonlinearity of
signals along the sampling time, a time delay error (TDR) exists, which would deteriorate the
robustness and the accuracy of the robot. A through literature review revealed that no research
work has proposed a systematic solution to eliminate the negative influence of this error.

A. Main contribution

All the papers that cited above, except TDE approach, are based on the conventional adaptive
approaches (regressor function) and require a good knowledge of the robot system’s parame-
ters. Practically, it is impossible to define exactly the parameters of the robot system and the
modelisation of the robot is typically uncertain. For these kinds of robots, the adaptation of
the uncertainties function based on full dynamic is very complicated due to the high number of
degrees of freedom (DOFs) of the robot. When the number of DOFs of the robot increases, it
is not straightforward to find the parameters of the robot. Usually, the parameter vector of the
robot can be greater than 100 if its DOFs are greater only than four (Brahmi et al., 2017c).
To address the above problems, we propose a new adaptive visual tracking control for an exoskeleton robot with high number of degrees of freedom (7-DOFs) based on extension of sliding mode, TDE approach and Jacobian transpose taking into consideration the Cartesian and joint spaces. This controller is designed to be robust and more flexible to deal with the kinematic and dynamic uncertainties taking into consideration the uncertainties in the visual system parameters, and to be more robust to the parameter variations. The contribution of this paper can be summarized in three points:

- Considering the unknown kinematics and dynamics with unknown external disturbances (different weight of the arm of each subject), adaptive visual controller incorporating with recursive control is developed to estimate the nonlinear kinematic and dynamic uncertainties with unknown disturbances and to drive the robot to follow the desired functional therapy activity and provide a smooth exoskeleton-aided passive activity;

- The unknown dynamics and external disturbances of the robot system can estimate easily using Time Delay Estimation (TDE) approach. This strategy employs only time-delayed knowledge about the previous control input of the system and its response state to provide an accurate estimation of uncertainties. The main feature of this method is that not influenced by the high degree of freedom of the robot and the size of the estimated parameters;

- Using a new recursive control to reduce the effect of the Time Delay Error (TDR) and improve the robustness of the control system. Usually, this error limits the performance of TDE approach.

The proposed strategy is achieved based on the inner/outer loop structure of robotics system. This latter has some desirable characteristics such as the rapidity of the computation of the control system. In this case, the outer loop is designed to estimate the nonlinear kinematics parameters and uncertainties in the visual system (camera), and the Inner Loop is intended to provide a high-level of precision by compensating the unknown part of the dynamics using TDE approach and while considering the TDR. The recursive control here is designed to reduce the effect or TDR and improve the robustness of the TDE approach. The structure of the designed controller also aims to make the exoskeleton robot perform the human-like movement.
using the predefined trajectories of physical therapy tasks (De Morand, 2014). The stability of the Inner/Outer system and the convergence of its errors are formulated and demonstrated based on Lyapunov function. Compared with conventional approach (Cheah, 2006; Cheah et al., 1999, 2005; Liu et al., 2006) that is applied on only 2-DOFs planar robot, the designed strategy is characterized by the ease of implementation and high precision and robustness to the kinematic/dynamic uncertainties, unforeseen disturbances and uncertainties of camera parameters. The efficiency and the robustness of the proposed approach are validated with Cartesian trajectory tracking corresponding to passive physical therapy tasks (De Morand, 2014).

The outline of the paper is organized as follows. The kinematics and dynamics of the robot are presented in the next section. The control scheme is described in section 6.3. Experimental and comparison results are shown in section 6.4; finally, the conclusion is presented in section 6.5.

6.2 Characterization of ETS-MARSE robot: kinematics and dynamics

6.2.1 Modeling of ETS-MARSE robot

The ETS-MARSE is a redundant robot with 7DOFs, as shown in Figure 6.1. It is designed to rehabilitate the impaired human upper limb. The design of the ETS-MARSE was originally inspired from the anatomy of the human arm. It was ergonomically designed to be comfortable for the subjects (robot users) during the rehabilitation sessions. The shoulder motion part (3-DOF) is consisted of three joints: the first two joints are responsible for shoulder joint’s vertical and horizontal extension/flexion motion, while the third joint is aimed to conduct the internal and external rotation of the shoulder joint. The elbow motion part (1-DOF) is responsible for elbow joint flexion/extension motion. The wrist motion support part of the ETS-MARSE is consisted of three joints: the first joint is designed to achieve pronation and supination movement of the forearm, the second joint and the third joint are designed to perform ulnar/radial deviation, and flexion/extension of the wrist respectively. The design of the ETS-MARSE has special features compared with the existing exoskeleton robots (Rahman et al., 2015). Among
them, it has a comparatively low weight, an excellent power/weight ratio, can be easily fitted or removed, and is capable of adequately compensating for gravity. A new power carrying mechanism was included for supporting the shoulder joint internal/external rotation and for forearm pronation/supination. This exoskeleton robot can be used with a wide range of subjects, due to its adjustable link mechanism. All the key characteristics and contribution features of the ETS-MARSE and comparison with similar existing exoskeleton robots are summarized in (Rahman et al., 2015).

![Figure 6.1 Reference frames of ETS-MARSE](image)

**6.2.1.1 Dynamics of the ETS-MARSE Robot**

The dynamic behavior of ETS-MARSE manipulator is given by the following expression using the Lagrangian method (Dawson et al., 2003):

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = \tau + \tau_{ex}$$  \hspace{1cm} (6.1)
where $\theta \in \mathbb{R}^7$, $\dot{\theta} \in \mathbb{R}^7$ and $\ddot{\theta} \in \mathbb{R}^7$ are, respectively, the joint’s position, velocity, and acceleration vectors, $M(\theta) \in \mathbb{R}^{7 \times 7}$ is the symmetric and positive definite inertia matrix, $C(\theta, \dot{\theta}) \in \mathbb{R}^{7 \times 7}$ is the Coriolis and centrifugal vector, $G(\theta) \in \mathbb{R}^7$ is the gravitational vector, $\tau \in \mathbb{R}^7$ is the applied joint torque to the exoskeleton robot, and $\tau_{ex} \in \mathbb{R}^7$ is the external disturbances vector. Let us denote:

$$\begin{align*}
M(\theta) &= M_0(\theta) + \Delta M(\theta) \\
C(\theta, \dot{\theta}) &= C_0(\theta, \dot{\theta}) + \Delta C(\theta, \dot{\theta}) \\
G(\theta) &= G_0(\theta) + \Delta G(\theta)
\end{align*}$$

(6.2)

where $M_0(\theta)$, $C_0(\theta, \dot{\theta})$ and $G_0(\theta)$ are respectively the known inertia matrix, the Coriolis centrifugal matrix, and the gravity vector. $\Delta M(\theta)$, $\Delta C(\theta)$ and $\Delta G(\theta)$ are the uncertain parts.

Let us introduce a new variable such that: $\eta_1 = \theta$ and $\eta_2 = \dot{\theta}$; hence, the dynamic model expressed in (6.1) can be rewritten as follows:

$$\begin{align*}
\dot{\eta}_1 &= \eta_2 \\
\dot{\eta}_2 &= U(t) + f(t) + H(t)
\end{align*}$$

(6.3)

where, $U(t) = U(\eta_1)$, $H(t) = H(\eta_1, \eta_2, \dot{\eta}_2)$ and $f(t) = d\eta_1, \eta_2)$. This notation is used in order to handle easily later with the control scheme. Where:

- $U(t) = M_0^{-1}(\theta) \tau(t)$;
- $f(t) = M_0^{-1}(\theta) \left[ -C_0(\theta, \dot{\theta}) - G_0(\theta) \right]$;
- $H(t) = H(\theta, \dot{\theta}, \ddot{\theta}) = M_0^{-1}(\theta) \left[ \tau_{ex} - \Delta M(\theta) \ddot{\theta} - \Delta C(\theta, \dot{\theta}) \dot{\theta} - \Delta G(\theta) \right]$.

### 6.2.1.2 Kinematics of ETS-MARSE robot

In most applications of rehabilitation robots, the desired trajectory is expressed in Cartesian space (De Morand, 2014). The transformation from Cartesian space to joint space is done by a non-linear function named the Jacobian matrix. If the position $x$ of the end–effector of the
robot is provided by a visual system such as a camera or Kinect, the standard relation between Cartesian velocity and joint velocity is given as follows:

\[ \dot{x} = J(\eta_1)\eta_2 \]  
(6.4)

where \( J(\eta_1) = J_c(r)J_{RO}(\eta_1) \in \mathbb{R}^{6 \times 7} \) is the total Jacobian matrix of the robot combined with the visual system. \( J_c(r) \in \mathbb{R}^{6 \times 6} \) is the image Jacobian matrix (Cheah et al., 2005; Hutchinson et al., 1996), \( x \in \mathbb{R}^6 \) is the Cartesian position of the end-effector of the robot, \( r \in \mathbb{R}^6 \) is the image feature parameters and \( J_{RO}(\eta_1) \in \mathbb{R}^{6 \times 6} \) is the Jacobian matrix of the manipulator.

### 6.2.1.3 Problem formulation

One of the main objectives of research on this kind of robots is to design a controller able to make the human-exoskeleton system achieve movement without exact information of the kinematics and dynamics of the robotic system and to provide a smooth movement, conforming to physical therapy exercise while the designed control scheme ensures that the measured Cartesian positions \( x \) of the robot tracks the desired Cartesian trajectory \( x_d \). Before giving the control design methodology, we present the properties and the assumptions used in this paper.

**Property 6.1:** The known part of inertia matrix \( M_0(\theta) \) is symmetric and positive definite for all \( \theta \in \mathbb{R}^7 \) (Dawson et al., 2003).

**Property 6.2:** Equation (6.4) can be linear in a set of kinematics parameters like link lengths, which can be written as a linear combination of specified functions as given below (Liu et al., 2006):

\[ \dot{x} = J(\eta_1, \gamma_k)\eta_2 = Q(\eta_1, \eta_2)\gamma_k \]  
(6.5)

where \( Q(\eta_1, \eta_2) \in \mathbb{R}^{6 \times 6} \) is the kinematic regressor matrix and \( \gamma_k \in \mathbb{R}^6 \) is the kinematics parameters vector.
**Assumption 6.1:** The joint position and joint velocity are measured.

**Assumption 6.2:** All kinematic singularities are avoided.

**Assumption 6.3:** Since the dynamic model of the robot is continuous and bounded, we assume that the function $H(t)$ is locally Lipschitz function.

**Assumption 6.4:** The desired trajectory is bounded.

**Assumption 6.5:** The external disturbance $\tau_{ex}$ is supposed to be continuous, has finite energy and satisfies $\|\tau_{ex}\| \leq \vartheta$, with an unknown positive disturbance boundary $\vartheta$.

### 6.3 Control design

The control of a robotic system always needs a precise input measurement to provide a good performance of this system. Particularly, in the case when we have a position feedback from visual devices, like a camera or Kinect. Since no sensor is available to measure the Cartesian velocity input, a linear filter can be used to obtain this variable such that:

$$\left( \frac{d}{dt} + \Lambda \right) z = \Lambda \dot{x}$$

(6.6)

The signal $z \in \mathbb{R}^6$ is obtained from the measured position $x \in \mathbb{R}^6$. $\Lambda$ is a positive constant. Substituting equation (6.5) into (6.6), we can rewrite (6.6) such that:

$$\left( \frac{d}{dt} + \Lambda \right) z = \Lambda Q(\eta_1, \eta_2) \gamma_k$$

(6.7)

where:

$$z = \left( \frac{\Lambda}{\frac{d}{dt} + \Lambda} \right) Q(\eta_1, \eta_2) \gamma_k = \phi(t) \gamma_k$$

(6.8)
where $\phi(t) \in \mathbb{R}^{6 \times 6}$ is a filtered function of $Q(\eta_1, \eta_2)$. Usually the robot begins from the rest position, hence: $z(0) = 0$ and $\phi(t) = 0$ (Cheah et al., 2005).

We can now determine the Cartesian position error, Cartesian velocity error, and estimated Cartesian velocity error as follows:

$$
\begin{align*}
& e_x = x - x_d \\
& \dot{e}_x = \dot{x} - \dot{x}_d \\
& \dot{\hat{e}}_x = \dot{\hat{x}} - \dot{x}_d
\end{align*}
$$

(6.9)

where, $x_d \in \mathbb{R}^6$ is the desired Cartesian trajectory, $x \in \mathbb{R}^6$ is the measured position and $\dot{x} \in \mathbb{R}^6$ is the estimated measured velocity.

Now, we define the vector of required Cartesian velocity as:

$$
\dot{x}_r = \dot{x}_d - \beta e_x
$$

(6.10)

where $\dot{x}_r \in \mathbb{R}^6$ and $\beta$ is a positive constant. Differentiating equation (6.10) with respect to time yields:

$$
\ddot{x}_r = \ddot{x}_d - \beta \dot{e}_x
$$

(6.11)

Considering an uncertain kinematics, the Jacobian matrix is uncertain. In this case equation (6.5) is rewritten as follows:

$$
\dot{\hat{x}} = \dot{f}(\eta_1, \hat{\gamma}_k) \eta_2 = Q(\eta_1, \eta_2) \hat{\gamma}_k
$$

(6.12)

Using equation (6.12), we define the estimated sliding Cartesian surface vector as follows:

$$
\hat{S}_x = \dot{\hat{x}} - \dot{x}_r = \dot{f}(\eta_1, \hat{\gamma}_k) \eta_2 - \dot{x}_r
$$

(6.13)
Differentiating equation (6.13) with respect to time yields

\[ \dot{S}_x = \ddot{x} - \ddot{x}_r = \dot{J}(\eta_1, \hat{\gamma}_k)\dot{\eta}_2 + \dot{J}(\eta_1, \hat{\gamma}_k)\eta_2 - \ddot{x}_r \]  \hspace{1cm} (6.14)

Now, we can define the required joint velocity vector as:

\[ \dot{\eta}_r = \dot{J}^+(\eta_1, \hat{\gamma}_k)\dot{x}_r \]  \hspace{1cm} (6.15)

where \( \dot{J}^+(\eta_1, \hat{\gamma}_k) = J^T(\eta_1, \hat{\gamma}_k)(J^T(\eta_1, \hat{\gamma}_k)J(\eta_1, \hat{\gamma}_k))^{-1} \) is the pseudo-Jacobian matrix.

Differentiating equation (6.15) with respect to time:

\[ \ddot{\eta}_r = \dot{J}^+(\eta_1, \hat{\gamma}_k)\ddot{x}_r + \dot{J}^+(\eta_1, \hat{\gamma}_k)\dot{x}_r \]  \hspace{1cm} (6.16)

It is important also to define the sliding joint surface vector. By using equation (6.10), equation (6.12) and equation (6.15) we have:

\[ S = \eta_2 - \dot{\eta}_r = \dot{J}^+(\eta_1, \hat{\gamma}_k)[(\dot{x} - \dot{x}_d) + \beta e_x] = \dot{J}^+(\eta_1, \hat{\gamma}_k)\dot{S}_x \]  \hspace{1cm} (6.17)

The time derivative of relation equation (6.17) gives:

\[ S = \eta_2 - \dot{\eta}_r = \dot{J}^+(\eta_1, \hat{\gamma}_k)\dot{S}_x + \dot{J}^+(\eta_1, \hat{\gamma}_k)\dot{S}_x \]  \hspace{1cm} (6.18)

Substituting equation (6.18) into equation (6.3), the exoskeleton robot system equation (6.3) can be written as follows:

\[
\begin{cases}
S = \ddot{\eta}_1 - \ddot{\eta}_r \\
\dot{S} = U(t) + f(t) + H(t) - \ddot{\eta}_r
\end{cases}
\]  \hspace{1cm} (6.19)
If all parameters of the robot system given in equation (6.19) are completely known, we can propose the following controller:

\[ U(t) = -\hat{J}^T(\eta_1, \hat{\gamma}_k) (k_1 e_x + k_2 \dot{e}_x + k_3 \dot{S}_x) - f(t) - H(t) + \eta_r - \epsilon(t) \quad (6.20) \]

with:

\[ \dot{S}_x = Q(\eta_1, \eta_2) \hat{\gamma}_k - \dot{x}_r \quad (6.21) \]

and \( k_1, k_2 \) and \( k_3 \in \mathbb{R}^{6 \times 6} \) being diagonal positive matrices. \( \hat{J}^T(\eta_1, \hat{\gamma}_k) \) is the estimated Jacobian transpose based on feedback of Cartesian tracking. The adaptation laws are updated by:

\[ \dot{\hat{\gamma}}_k = k_f^{-1} \phi^T(t) k_2 (\phi(t) \hat{\gamma}_k - z) + k_f^{-1} Q^T(\eta_1, \eta_2) (k_1 + \beta k_2) e_x \quad (6.22) \]

where \( k_f \in \mathbb{R}^{6 \times 6} \) is a diagonal positive matrix. Since \( H(t) \) is uncertain, that may influence the performance of robot tracking. So, if Assumption 6.3 is verified, it is possible to use Time Delay Estimation (Youcef-Toumi and Shortlidge, 1991). In such case, the designed controller is given such that:

\[ U(t) = -\hat{J}^T(\eta_1, \hat{\gamma}_k) (k_1 e_x + k_2 \dot{e}_x + k_3 \dot{S}_x) - f(t) - \hat{H}(t) + \eta_r - \hat{\epsilon}(t) \quad (6.23) \]

where \( \hat{H}(t) \) is obtained using TDE (Youcef-Toumi and Shortlidge, 1991), using equation (6.3) to obtain:

\[ \hat{H}(t) \simeq H(t - t_d) = \eta_2(t - t_d) - f(t - t_d) - U(t - t_d) \quad (6.24) \]

where, \( t_d \) is a positive constant assumed to be very small. Practically, the smallest constant can be reached is the sampling time. However, due to noisy measurements and nonlinearity of signals along the sampling time, a time delay error (TDR) \( \epsilon(t) \) exists, which would deteriorate the robustness and the accuracy of the robot. Unfortunately, the TDR is not available. In
this case, let us apply an iterative estimator to estimate the TDR and to reduce its effect, and
give to the control system more flexibility to deal with parameters’ variation and unexpected
disturbances. The iterative estimator is given as:

\[ \hat{\varepsilon}(t) = \hat{\varepsilon}(t - t_d) - k_4 S \]
\[ \hat{\varepsilon}(t) = 0, \ \forall \ t \in [-t, 0]; \ k_4 = kI_{7 \times 7} \] (6.25)

where, \( k \) is a positive constant. The proof of the above equation (6.25) is given in appendix III.
The closed loop of the global system can be written using equation (6.19) and control input
equation (6.23) as follows:

\[
\begin{align*}
S &= \dot{\eta}_1 - \dot{\eta}_r \\
\dot{S} &= -J^T(\eta_1, \hat{\gamma}_k) (k_1 \dot{e}_x + k_2 \dot{\hat{e}}_x + k_3 \dot{\hat{S}}_x) - \tilde{\varepsilon}(t)
\end{align*}
\] (6.26)

with: \( \tilde{\varepsilon}(t) = \hat{\varepsilon}(t) - \varepsilon(t) \) and \( \varepsilon(t) = H(t) - \hat{H}(t) \) are respectively Time Delay Error, and dynamic uncertainties. Let us know state the main result of the paper.

**Theorem 6.1** The control law for sliding mode with time delay estimation (TDE) of uncertain
robot dynamics determined in equation (6.23) ensures the asymptotic stability of the robot
system. The desired torque input is given as:

\[ \tau = M(\eta_1)U(t) \] (6.27)

where \( U(t) \) is given in equation (6.23).

**Proof** To facilitate the proof of stability, let us define the term of the iterative estimator. First, we can write \( \frac{d}{dt} \int_{t-t_d}^{t} \tilde{\varepsilon}^T(w)\tilde{\varepsilon}(w)dw \) as follows:

\[
\frac{d}{dt} \int_{t-t_d}^{t} \tilde{\varepsilon}^T(w)\tilde{\varepsilon}(w)dw = \tilde{\varepsilon}^T(t)\tilde{\varepsilon}(t) - \tilde{\varepsilon}^T(t - t_d)\tilde{\varepsilon}(t - t_d) \] (6.28)
Additionally,
\[
\frac{1}{2k} \tilde{e}^T(t) \tilde{e}(t) - \frac{1}{2k} \tilde{e}^T(t - t_d) \tilde{e}(t - t_d) = \tilde{e}^T(t) S - \frac{S^T K_4^T}{2} S \tag{6.29}
\]

The details of equation (6.28) and equation (6.29) are given in Appendix III. Consider the following Lyapunov function candidate:
\[
V = \frac{1}{2} S^T S + \frac{1}{2} e_x^T (k_1 + \beta k_2) e_x + \frac{1}{2} \Delta \gamma_k^T k_f \Delta \gamma_k + \frac{1}{2k} \int_{t-t_d}^{t} \tilde{e}^T(w) \tilde{e}(w) dw \tag{6.30}
\]
with: \( \Delta \gamma_k = \gamma_k - \hat{\gamma}_k \), \( \tilde{e}(t) = \hat{e}(t) - e(t) \) and \( e(t) = H(t) - \hat{H}(t) \) are respectively the estimation errors of kinematic uncertainties, Time Delay Error, and dynamic uncertainties. The derivative of the proposed Lyapunov function with respect to time is obtained as:
\[
\dot{V} = S^T \dot{S} + e_x^T (k_1 + \beta k_2) \dot{e}_x - \Delta \gamma_k^T k_f \dot{\gamma}_k + \frac{1}{2k} \tilde{e}^T(t) \tilde{e}(t) - \frac{1}{2k} \tilde{e}^T(t - t_d) \tilde{e}(t - t_d) \tag{6.31}
\]

Substituting \( \dot{S} \) from equation (6.19) and using equation (6.29) into equation (6.31), we find:
\[
\dot{V} = -S^T f^T(\eta_1, \hat{\eta}_k) (k_1 e_x + k_2 \dot{e}_x + k_3 \dot{S}_x) + e_x^T (k_1 + \beta k_2) \dot{e}_x - \Delta \gamma_k^T k_f \dot{\gamma}_k \\
\quad + S^T e(t) - S^T \dot{e}(t) + \tilde{e}^T(t) S - \frac{S^T K_4^T}{2} S \\
\quad = -S^T f^T(\eta_1, \hat{\eta}_k) (k_1 e_x + k_2 \dot{e}_x + k_3 \dot{S}_x) + e_x^T (k_1 + \beta k_2) \dot{e}_x - \Delta \gamma_k^T k_f \dot{\gamma}_k \\
\quad - S^T \frac{K_4^T}{2} S \tag{6.32}
\]

Substituting equation (6.17) and equation (6.22) into equation (6.32), we find:
\[
\dot{V} = -\dot{S}_x^T k_1 e_x - \dot{S}_x^T k_2 \dot{e}_x - \dot{S}_x^T k_3 \dot{S}_x - S^T \frac{K_4^T}{2} S + e_x^T (k_1 + \beta k_2) \dot{e}_x - \Delta \gamma_k^T \phi^T(t) k_2 \phi(t) \Delta \gamma_k \\
\quad - \Delta \gamma_k^T Q^T(\eta_1, \eta_2) (k_1 + \beta k_2) e_x \tag{6.33}
\]
We have from equation (6.5), equation (6.10) and equation (6.13):

\[
\dot{S}_x = \dot{e}_x + \beta e_x - Q(\eta_1, \eta_2)\Delta \gamma_k = \dot{e}_x + \beta e_x \quad (6.34)
\]

with:

\[
Q(\eta_1, \eta_2)\Delta \gamma_k = J(\eta_1)\eta_2 - \dot{J}(\eta_1, \hat{\gamma}_k)\eta_2 = \dot{x} - \hat{x} \quad (6.35)
\]

From equation (6.34) we have:

\[
\dot{\dot{e}}_x = \dot{e}_x - Q(\eta_1, \eta_2)\Delta \gamma_k \quad (6.36)
\]

Substituting equation (6.34) and equation (6.36) into equation (6.33), we find:

\[
\dot{V} = -\dot{e}_x^T k_1 \dot{e}_x + 2\dot{e}_x^T k_2 Q(\eta_1, \eta_2)\Delta \gamma_k - \beta e_x^T k_1 e_x - \dot{S}_x^T k_3 \dot{S}_x - \dot{S}_x^T \frac{k_4}{2} S - \Delta \gamma_k^T \phi^T(t) k_2 \phi(t) \Delta \gamma_k
- \Delta \gamma_k^T Q^T(\eta_1, \eta_2) k_2 Q(\eta_1, \eta_2) \Delta \gamma_k
\quad (6.37)
\]

while \( \dot{e}_x = \dot{e}_x - Q(\eta_1, \eta_2)\Delta \gamma_k \), equation (6.34) can be reduced to:

\[
\dot{V} = -\dot{e}_x^T k_1 \dot{e}_x - \beta e_x^T k_1 e_x - \dot{S}_x^T k_3 \dot{S}_x - \dot{S}_x^T \frac{k_4}{2} S - \Delta \gamma_k^T \phi^T(t) k_2 \phi(t) \Delta \gamma_k
\quad (6.38)
\]

From equation (6.30), we can easily see that \( V \) is positive definite in \( S, \dot{e}, \Delta \gamma_k, \) and \( \hat{e}(t) \). Since \( \dot{V} \) is negative definite from equation (6.38), and \( V \) is bounded, this implies that \( S, e, \hat{\gamma}_k, \) and \( \hat{e}(t) \) are bounded. From equation (6.17) \( \dot{S}_x \) is bounded because \( S \) is bounded. \( x \) is bounded because \( x_d \) is assumed bounded (Assumption 6.4). While \( e_x \) is bounded and \( \dot{x}_d \) is bounded (Assumption 6.4), this implies \( \dot{x}_r \) in equation (6.10) is bounded. The pseudo-inverse of the Jacobian matrix in equation (6.15) is non singular and bounded (all joints of manipulator are revolute), this means that \( \dot{\theta}_r \) is bounded. We observe from equation (6.17) that \( \dot{\theta} \) is bounded, this implies \( \dot{x} \) is bounded while the Jacobian matrix is bounded. We remark from equation (6.11) that \( \ddot{x}_r \) is bounded because \( \dot{e}_x \) and \( \dot{x}_d \) are bounded (Assumption 6.4). It is clear from equa-
tion (6.16) that $\dot{\theta}_r$ is bounded. We see from equation (6.18) that $\dot{S}$ is bounded, that implies $\dot{\theta}$ is bounded. We can conclude from equation (6.14) that $\dot{\hat{S}}_x$ is bounded. The derivative of equation (6.34) is expressed as follows:

$$\dot{\hat{S}}_x = \ddot{e}_x + \beta \dot{e}_x$$

(6.39)

where $\ddot{e}_x = \ddot{\epsilon} - \ddot{x}_d$ is bounded. Since $V$ is bounded and $\dot{V}$ is continuous and negative semi-definite, we can utilize Barbalat’s lemma by differentiating equation equation (6.38) such that:

$$\ddot{V} = -2\dot{e}_x^T k_1 \ddot{\epsilon}_x - 2\beta \dot{e}_x^T k_1 \dot{e}_x - 2\dot{S}_x^T k_3 \dot{\hat{S}}_x - 2S^T k_4 \dot{\Sigma} - \Delta \gamma_k^T \phi^T (t) k_2 (\phi^T (t) \Delta \gamma_k - \phi (t) \gamma_k)$$

(6.40)

Since $\theta$ and $\dot{\theta}$ are bounded, this means $\phi (t), \phi (t)$ are bounded. This proves that $\dot{V}$ is bounded since $e_x, \dot{e}_x, \ddot{e}_x, \dot{\hat{S}}_x, \gamma_k$ are all bounded. So, $\dot{V}$ is continuous and negative semi-definite; according to Barbalat’s lemma. we have $e_x \to 0, \dot{e}_x \to 0, S \to 0$ and $\phi (t) \Delta \gamma_k \to 0$ as $t \to \infty$.

Figure 6.2  Block diagram of the proposed controller for exoskeleton robot with unknown kinematics/dynamics
Now, the derivative of equation (6.34) with respect to time yields:

\[ \ddot{e}_x + \beta \dot{e}_x = \dot{\hat{S}}_x + \dot{Q}(\eta_1, \eta_2)\Delta \gamma_k - \dot{Q}(\eta_1, \eta_2)\dot{\gamma}_k \]  

(6.41)

That means \( \dot{e}_x \) is also bounded. In this case, we have \( \dot{e}_x \to 0 \) as \( t \to \infty \) since \( \ddot{e}_x \) and \( e_x \) are bounded. Hence, the proof is complete. The lock diagram of the proposed controller is given in Figure 6.2.

6.4 Experimental and comparative study

6.4.1 Experiment set-up

The experimental setup of the proposed system is shown in Figure 6.3. The system consists of three processing units. The first is a PC from where the top-level commands are sent to the robot using LabVIEW interface, i.e. the control scheme selection, joint or Cartesian space trajectory, gain adjustments, etc. This PC also receives the data after the robot task is executed to analyze its performance. The other two processing units are part of a National Instruments PXI platform. Firstly, a NI-PXI 8081 controller card with an Intel Core Duo processor; in this card, the main operating system of the robot and the top-level control scheme are executed. In our case, the sliding mode based controller as well as the estimation based on time delay approach, at a sampling time of 500 \( \mu s \). The inverse kinematics algorithm also runs inside
this control loop. Finally, at input-output level, a NI PXI-7813R remote input–output card with an FPGA (field programmable gate array) executes the low-level control; i.e. a PI current control loop (sampling time of 50 \( \mu s \)) to maintain the current of the motors required by the main controller. Also, in this FPGA, the position feedback (Cartesian and joint) via Hall-sensors (joint position) and Kinect (Cartesian End-effector position), basic input-output tasks are executed.

Table 6.1  Modified Denavit-Hartenberg parameters

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>( \alpha_{i-1} )</th>
<th>( a_{i-1} )</th>
<th>( d_i )</th>
<th>( \theta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( d_c )</td>
<td>( \theta_1 )</td>
</tr>
<tr>
<td>2</td>
<td>(-\frac{\pi}{2})</td>
<td>0</td>
<td>0</td>
<td>( \theta_2 )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{\pi}{2})</td>
<td>0</td>
<td>( d_c )</td>
<td>( \theta_3 )</td>
</tr>
<tr>
<td>4</td>
<td>(-\frac{\pi}{2})</td>
<td>0</td>
<td>0</td>
<td>( \theta_4 )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{\pi}{2})</td>
<td>0</td>
<td>( d_w )</td>
<td>( \theta_5 )</td>
</tr>
<tr>
<td>6</td>
<td>(-\frac{\pi}{2})</td>
<td>0</td>
<td>0</td>
<td>( \theta_6 - \frac{\pi}{2} )</td>
</tr>
<tr>
<td>7</td>
<td>(-\frac{\pi}{2})</td>
<td>0</td>
<td>0</td>
<td>( \theta_7 )</td>
</tr>
</tbody>
</table>

The modified Denavit-Hartenberg (DH) parameters are given in Table 6.1. These parameters are obtained from reference frames as shown in Figure 6.1, and are used to obtain the homogeneous transformation matrices.

Table 6.2  Workspace ETS-MARSE

<table>
<thead>
<tr>
<th>joint (i)</th>
<th>Motion</th>
<th>Workspace</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shoulder joint horizontal flexion/extension</td>
<td>( 0^\circ/140^\circ )</td>
</tr>
<tr>
<td>2</td>
<td>Shoulder joint vertical flexion/extension</td>
<td>( 140^\circ/0^\circ )</td>
</tr>
<tr>
<td>3</td>
<td>Shoulder joint internal/external rotation</td>
<td>( -85^\circ/75^\circ )</td>
</tr>
<tr>
<td>4</td>
<td>Elbow joint flexion/extension</td>
<td>( 120^\circ/0^\circ )</td>
</tr>
<tr>
<td>5</td>
<td>Forearm joint pronation/supination</td>
<td>( -85^\circ/85^\circ )</td>
</tr>
<tr>
<td>6</td>
<td>Wrist joint ulnar/radial deviation</td>
<td>( -30^\circ/20^\circ )</td>
</tr>
<tr>
<td>7</td>
<td>Wrist joint flexion/extension</td>
<td>( -50^\circ/60^\circ )</td>
</tr>
</tbody>
</table>
The workspace of the designed robot is given in Table 6.2 and the physical parameters of ETS-MARSE relative to the base reference frame are given in Table 6.3. The details of the parameters and design of ETS-MARSE are given in (Rahman et al., 2015).

In the experiments, the desired Cartesian trajectory corresponds to a prescribed passive physical therapy task performed by three healthy subjects (age: 27 ± 4.6 years; height: 170 ± 8.75 cm; weight: 75 ± 18 kg). This trajectory (Initial position → Target-A → Target-B → Target-C → Initial position) is expressed in Cartesian space to evaluate the proposed control. In this case, the position of the Cartesian End-Effector of the robot is provided by visual system (Kinect). For the carried object, the subject-robot system carried an object with unknown weight and dimensions during the desired trajectory. It is important to notice that the external disturbances here are represented by different physiological conditions of the subjects, such as non-linear biomechanical characteristics of the musculoskeletal system, the different weight of the upper-limb for each subject, the presence of spasticity in neurological patients, etc.

<table>
<thead>
<tr>
<th>Joints (i)</th>
<th>Mass (kg)</th>
<th>Centre of mass (m)</th>
<th>Link length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.475</td>
<td>0.0984</td>
<td>0.145</td>
</tr>
<tr>
<td>2</td>
<td>3.737</td>
<td>0.1959</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>2.066</td>
<td>0.163</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0.267</td>
</tr>
<tr>
<td>6</td>
<td>0.779</td>
<td>0.121</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.496</td>
<td>0.0622</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.3 Physical parameters of ETS-MARSE

The experimental control gains are chosen by trial and error as follows: \( k_1 = 20I_{6 \times 6}, k_2 = 70I_{6 \times 6}, k_3 = 18I_{6 \times 6}, k_f = 0.01I_{6 \times 6}, k_4 = 0.5I_{7 \times 7} \) and \( \beta = 10 \). The experimental results are given in Figure 6.4, Figure 6.5 and Figure 6.6.
6.4.2 Experimental results

6.4.2.1 The main results of the proposed controller with recursive control

The experimental results with ETS-MARSE robot in Cartesian space performed by Subject-1: (age: 27 years; height: 177 cm; weight: 83 kg) using the designed strategy are shown in Figure 6.4. As we see in this figure (Figure 6.4(a)), the desired trajectory (red line) nearly overlapped with the measured trajectory (green line). It can be noticed that these results are fairly good. Figure 6.4(b) presents the Cartesian errors as functions of time. From this figure, it is obvious that the Cartesian errors are getting smaller along the desired trajectory. Figure 6.5 shows that the control input is bounded without any noticeable control chattering. Finally, the convergence of the kinematic (\( \hat{\gamma}_k \)) and dynamic (\( \hat{H}(r) \)) parameters of the exoskeleton robot during the proposed control is shown in Figure 6.6 and Figure 6.7 respectively. These results confirm that the control strategy is able to achieve the desired robot’s performance even if the nonlinear kinematics and dynamics of the exoskeleton robot are uncertain and the parameters of Kinect (camera) device are not completely known.

6.4.2.2 The results of the proposed controller without recursive control

Figure 6.8(a) presents the workspace performance of the robot in Cartesian space (red is the desired trajectory, green is real trajectory) performed with Subject-1: (age: 28 years; height: 177 cm; weight: 83 kg) using the proposed controller without a recursive controller. In fact, we remark from figures (Figure 6.8(a-b)) that the proposed controller without recursive controller shows a good performance. Where the Cartesian error (Figure 6.8(b)) is getting smaller with time. However, the control inputs of the conventional approach, presented in Figure 6.9, illustrate a noisy control input with a small chattering phenomenon, meanwhile, noise and peaks appear also in the estimation parameters of the unknown dynamics parameters as we see in Figure 6.10.
Figure 6.4 (a) Workspace performance of the robot in Cartesian space performed by Subject-1: (age: 27 years; height: 177 cm; weight: 83 kg). (b) Cartesian errors

On the other hand, the proposed controller with recursive control provides a smooth control input (Figure 6.5) and smooth estimation of unknown dynamics parameters (Figure 6.7). Therefore, we can say that the TDR is the main cause of the noise in the control input, which may damage the motors. From the comparison of the two experimental results, we can conclude that the proposed strategy with a recursive action provides a high level of precision and robustness against the nonlinear dynamic uncertainties and unknown disturbances.
6.4.3 Comparative study

In order to evaluate the efficiency and feasibility of the proposed control scheme, we compared it experimentally with the conventional adaptive visual tracking control presented in (Cheah, 2006). This latter is characterized by more complex implementation due to the complex regressor dynamic matrix, while the robot had a high degree of freedom (7-DOFs). To compute the regressor dynamic matrix of the robot, we use the virtual decomposition control (VDC) presented in (Luna et al., 2016).
Table 6.4 Comparative study of the controllers

Figure 6.7 Unknown Dynamics ($\hat{H}(t)\)$ parameters convergence of the exoskeleton robot

Figure 6.11(a) presents the Cartesian trajectory tracking in the 3D workspace (red is the desired trajectory, green is real trajectory) performed by the same subject (Subject-1: age: 27 years; height: 177 cm; weight: 83 kg) using the conventional controller. It is clear from (Figure 6.11(a-b)) that the conventional controller provides a good tracking performance. Where, the error is converging along the desired trajectory as we show in Figure 6.11(b). Nevertheless, there is a presence of chattering phenomenon in the control inputs as shown in Figure 6.12, which may damage the motors of the robot. Compared with the smooth control input that is provided by the proposed strategy (Figure 6.5), we can conclude that the proposed strategy is easily implementable and provides a high precision and robustness to the kinematic/dynamic uncertainties, with unknown disturbances, and uncertainties of the camera parameters.
To show more the feasibility of the designed strategy, we propose a numerical comparison between the above controller (conventional controller and proposed controller) by calculating the root mean square (RMS) of the error and the control input of each controller as follows:

\[ \|e\|_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \|e(i)\|^2} \quad \text{and} \quad \|\tau\|_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \|\tau(i)\|^2} \]

where \( N \) is the number of the sampling time steps of the simulation. The evaluation of the controller is given Table 6.4.
Figure 6.9  Control input.

Figure 6.10  Unknown Dynamic parameters convergence of the exoskeleton robot
Figure 6.11 (a) Workspace performance of the robot in Cartesian space performed by Subject-1: (age: 27 years; height: 177 cm; weight: 83 kg). (b) Cartesian errors
It confirms that the proposed approach gives the robot a high degree of efficiency for dealing well with parameter variations and the nonlinear kinematic/dynamic uncertainties in presence of unknown disturbances (different subjects with different physiological conditions. These conditions include non-linear biomechanical characteristics of the musculoskeletal system, the different weight of the upper limb for each patient, the presence of spasticity/dystonia, muscle weakness in neurological patients, etc) and parameters’ uncertainties of the Kinect compared with the conventional adaptive controller. The proposed controller provides consistent performance with different subjects, keeping the RMS error and general torque input at a small value compared with the conventional controller. Compared with similar tests performed in a previous study with ETS-MARSE robot, the proposed control based on TDE presents an excellent performance as the Virtual Decomposition Control (Luna et al., 2016), and better than PID and Computed Torque Control (CTC) (Luna et al., 2016).

Figure 6.12 Control input
6.5 Conclusion

In this paper, we proposed an adaptive control of a 7DOFs exoskeleton robot with uncertain kinematics and dynamics based on sliding mode controller. Estimated Jacobian matrix is taken into consideration. The control strategy is achieved by inner/outer loops, where the outer loop is designed to estimate the nonlinear kinematic parameters and the inner loop is designed to estimate the unknown dynamics of the robot using TDE approach and recursive control to limit the effect of its time delay error. The main benefit of the proposed adaptive control approach is that precise knowledge of the kinematic/dynamic parameters of the robot is not mandatory. Where, the proposed adaptive strategy is characterized by the ease of real-time implementation and provides a high precision and robustness to the kinematic/dynamic uncertainties, unknown perturbation, and uncertainties of the camera parameters. Additionally, the time delay error is taken into account to improve the accuracy of the robot performance. The stability analysis of inner/outer visual tracking control with kinematics/dynamics uncertainties taken into consideration the uncertainties in the camera device was proved by the Lyapunov function theorem. The robustness of the proposed control was proved with a Cartesian functional therapy task performed by the ETS-MARSE robot. The experimental results show the effectiveness, facility of implementation and accuracy of the proposed approach.
CONCLUSION AND RECOMMENDATIONS

This thesis focused on designing and developing a suitable control strategy for the ETS-MARSE exoskeleton robot (Motion Assistive Robotic-exoskeleton for Superior Extremity) located in the laboratory of GREPCI-ETS. Several adaptive approaches were validated experimentally with the ETS-MARSE and performed by healthy subjects in order to perform a smooth movement, similar to the natural human movement, and to provide different assistive motion: passive and active rehabilitation tasks. The main results in this project can be summarized as follows:

- A new solution of inverse kinematics for 7-DOFs exoskeleton robot has been proposed. It allowed to perform some rehabilitation trajectories in Cartesian space (chapter 2). In this work, we have proposed a new solution to the inverse kinematics problem, compatible with human upper limb movement and valid for human arm configuration. The main advantage of this approach is that it provides an optimal Cartesian solution resembling the human upper limb movement and always presents a valid human arm configuration. In addition, the proposed inverse kinematics algorithm provides a solution avoiding singularity problems and characterized by a high level of precision and rapidity of response.

- A new compliant control based on a second-order sliding mode with adaptive-gain incorporating time delay estimation was applied on ETS-MARSE performed with healthy subjects (chapter 3). In this work, the dynamics parameters of the system were considered uncertain and were estimated by modified time delay estimation. Additionally, the proposed inverse kinematics solution was used with the proposed controller to provide an appropriate compliance control that allows ETS-MARSE to interact perfectly with humans;

- Several adaptive approaches based on Backstepping controller (chapter 4 and 5) integrated with modified time-delay estimation (TDE) to provide an accurate estimation of unknown dynamics of the exoskeleton robot and to compensate for external bounded disturbances.
Unlike the conventional TDE approach, time delay error (TDR) is taken into consideration by estimating its value with a recursive estimator. This latter is designed to reduce the influence of this error on the accuracy of the uncertain estimation. The active rehabilitation tasks were achieved by catching the user’s intention of movement using Damped Least Square method;

- A new adaptive visual tracking control approach based on sliding mode control applied to ETS-MARSE robot with uncertain kinematics and dynamics, taking into account uncertainties in visual system (camera) parameters (chapter 6). The update laws were designed and formulated based on Lyapunov function to solve the adaptation problem methodically and to show the stability of the robot system.

Finally, we conclude that the designed control approaches have ensured a good performance, compensate for the dynamic and kinematic uncertainties during the rehabilitation tasks and eliminated the effect of the bounded disturbances.

Some limitations and problems may be raised in this thesis and can be considered as future work. As a mandatory step, the proposed strategies in this work should validate in clinical trials with real rehabilitation patients. However, before starting this step a profound review of safety protocol and design correction are necessary.

Another significant step for active rehabilitation is mandatory using electromyographic signals. The EMGs signals can be combined with force sensor readings to detect and estimate the subject’s motion intention. It can also be used to transform the stiffness and human force to the control system to allow to evaluate the spasticity of the patient’s upper limb.
1. Proof of (4.18), (4.21), and (4.22)

In such case, considering:

\[
\begin{align*}
  H_i(t) &\in H(t) \\
  \hat{H}_i(t) &\in \hat{H}(t) \\
  \tilde{H}_i(t) &\in \tilde{H}(t) \\
  g(t) &\in \mathbb{R}
\end{align*}
\]

where \(i \in \{1, \ldots, 7\}, g(t)\); it will be determined later. Now, let us propose the following equation:

\[
\begin{align*}
  H_i(t) &= H_i(t - t_d) + \epsilon_i(t_d) \\
  \hat{H}_i(t) &= H_i(t) - \hat{H}_i(t) \\
  \tilde{H}_i(t) &= \hat{H}_i(t - t_d) + \epsilon_i + g(t)
\end{align*}
\]  

Then:

\[
\frac{d}{dt} \int_{t - t_d}^{t} \tilde{H}^T(w) \tilde{H}(w) dw = -2 \tilde{H}^T(w) g(t) - g^T(t) g(t)
\]

Proof:

\[
\frac{d}{dt} \int_{t - t_d}^{t} \tilde{H}^T(w) \tilde{H}(w) dw = \tilde{H}^T(t) \tilde{H}(t) - \tilde{H}^T(t - t_d) \tilde{H}(t - t_d).
\]

Considering \(\hat{H}_i(t) = \hat{H}_i(t - t_d) + \epsilon_i(t) + g(t)\):

\[
\tilde{H}_i^2(t - t_d) = [H_i(t - t_d) - \hat{H}_i(t - t_d)] [H_i(t - t_d) - \hat{H}_i(t - t_d)]
\]
\[
= [H_i(t) - \hat{H}_i(t) + g(t)] [H_i(t) - \hat{H}_i(t) + g(t)]
\]
\[
= \tilde{H}_i^2(t) + 2 \hat{H}_i(t) g(t) + g^2(t)
\]  

Thus, we can obtain:

\[
\tilde{H}_i^2(t) - \tilde{H}_i^2(t - t_d) = -2 \hat{H}_i(t) g(t) - g^2(t)
\]
Let us now define \( g(t) \) function as:

\[
g(t) = -k_{3i}e_{2i} \tag{AI-5}
\]

Substituting equation (AI-5) in (AI-4), we obtain:

\[
\tilde{H}_i^2(t) - \tilde{H}_i^2(t - t_d) = 2\tilde{H}_i(t)k_{3i}e_{2i} - (k_{3i}e_{2i})^2 \tag{AI-6}
\]

Multiplying both sides of (AI-6) by \( \frac{1}{2k_{3i}} \) to obtain:

\[
\frac{1}{2k_{3i}}\tilde{H}_i^2(t) - \frac{1}{2k_{3i}}\tilde{H}_i^2(t - t_d) = \tilde{H}_i(t)e_{2i} - \frac{k_{3i}}{2}e_{2i}^2 \tag{AI-7}
\]


The proposed adaptive time-delay control law (4.17) makes the system converge asymptotically stable. Where \( e_1 \to 0 \) and \( e_2 \to 0 \) as \( t \to \infty \). The proof is formed in three stages. Stage 1 proves the boundedness of \( V_4(t) \) in the interval \([0, t_{dn})\). Stage 2 proves the negativeness of \( V_4(t) \) and asymptotically convergence of tracking errors \( e_1 \) and \( e_2 \) in the interval \([t_{dn}, \infty)\).

stage 1: Boundedness of \( V_4(t) \) in the interval \([0, t_{dn})\). The derivative of equation (4.27) \( V_4(t) \) with respect to time for \( t \in [0, t_{dn}) \) is given by:

\[
\dot{V}_4(t) = V_2(t) + \sum_{i=1}^{7} \frac{1}{2k_{3i}}\tilde{H}_i^2(t) \tag{AI-8}
\]

Differentiating (4.28) \( \dot{V}_2(t) \) with respect to time is given such that:

\[
\dot{V}_2(t) = -e_1^T k_1 \dot{e}_1 + e_1^T e_2 + e_2^T \dot{e}_2 \tag{AI-9}
\]
Substituting equation (4.8) and (4.12) into (AI-9) and applying the adaptive control law (4.17) to find:

\[
\dot{V}_2(t) = -e_1^T k_1 e_1 + e_1^T e_2 + e_2^T (-k_2 e_2 - e_1 + \hat{H}(t) - H(t))
\]

\[
= -e_1^T k_1 e_1 + e_1^T e_2 + e_2^T (-k_2 e_2 - e_1 - (H(t) - \hat{H}(t))) \tag{AI-10}
\]

One obtains:

\[
\dot{V}_2(t) = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \tilde{H}(t) \tag{AI-11}
\]

For \(t \in [0, t_{dn}]\), we have from equation (4.18): \(\hat{H} = -k_3 e_2\). Hence:

\[
\tilde{H}_i^2(t) = (H_i(t) - \hat{H}_i(t))^2
\]

\[
= H_i^2(t) - 2H_i(t)\hat{H}_i(t) + \hat{H}_i^2(t)
\]

\[
= H_i^2(t) - 2\hat{H}_i(t) [\hat{H}_i(t) + \hat{H}_i(t)] + \hat{H}_i^2(t)
\]

\[
= H_i^2(t) + 2k_3 \hat{H}_i(t)e_2^2 - (k_3 e_2^2)^2(t) \tag{AI-12}
\]

Therefore, substituting (AI-11) and (AI-12) in (AI-8) the Lyapunov function \(\dot{V}_4(t)\) is rewritten as follows:

\[
\dot{V}_4(t) = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \hat{H}(t) + \sum_{i=1}^{n} \frac{1}{2k_3} H_i^2(t) + \sum_{i=1}^{n} \tilde{H}_i(t)e_2^2 - \sum_{i=1}^{n} \frac{1}{2k_3} (k_3 e_2^2)^2
\]

\[
= -e_1^T k_1 e_1 - e_2^T k_2 e_2 - \sum_{i=1}^{n} \frac{k_3}{2} \bar{e}_{2i}^2 + \sum_{i=1}^{n} \frac{k_3}{k_3} H_i^2(t) \tag{AI-13}
\]

According to Assumption 4.4, \(H_i(t), i = 1, \ldots, 7\) are bounded. This implies \(\dot{V}_4(t)\) is bounded. Hence, \(\dot{V}_4(t)\) is bounded in time interval \([0, t_{d1}]\).
For \( t \in [t_{dn-1}, t_{dn}) \), conforming to the derivative of \( \dot{V}_4(t) \) with respect to time can be written by

\[
\dot{V}_4(t) = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \dot{H}(t) + \frac{1}{2k_3} H_n^2(t) + \sum_{i=1}^{n-1} \frac{1}{2k_3i} (\dot{H}_i^2(t) - \dot{H}_i^2(t - t_d))
\]

\[
= -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \dot{H}(t) + \frac{1}{2k_3} H_n^2(t) + \sum_{i=1}^{n-1} \frac{1}{2k_3i} H_i^2(t) - e_2^T H_n(t) e_2 - \frac{1}{2} \sum_{i=1}^{n-1} k_3i e_2^2
\]

\[
= -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \dot{H}(t) + \frac{1}{2k_3} H_n^2(t) + \sum_{i=1}^{n-1} \frac{1}{2k_3i} H_i^2(t) - \frac{1}{2} \sum_{i=1}^{n-1} k_3i e_2^2
\]

\[
= -e_1^T k_1 e_1 - e_2^T k_2 e_2 - \frac{1}{2} \sum_{i=1}^{n} k_3i e_2^2 + \frac{1}{2k_3} H_n^2(t)
\]  \hspace{1cm} (AI-14)

It is clear from equation (AI-14) that \( \dot{V}_4(t) \) is bounded for \( t \in [t_{dn-1}, t_{dn}) \) because \( H_i(t) \) is bounded. That means \( \dot{V}_4(t) \) is bounded in time interval \([t_{dn-1}, t_{dn})\).

**Stage 2**: The negativeness of \( \dot{V}_4(t) \) in the interval \([t_{dn}, \infty)\). The derivative of \( \dot{V}_4(t) \) with respect to time for \( t \in [t_{dn}, \infty) \) is given by:

\[
\dot{V}_4(t) = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \dot{H}(t) + \frac{1}{2k_3} H_n^2(t) + \sum_{i=1}^{n-1} \frac{1}{2k_3i} (\dot{H}_i^2(t) - \dot{H}_i^2(t - t_d))
\]  \hspace{1cm} (AI-15)

We have from (AI-6): \( \dot{H}_i^2(t) - \dot{H}_i^2(t - t_d) = 2\dot{H}_i(t)k_3i e_2i - (k_3i e_2i)^2 \). So, (AI-15) can be rewritten as follows:

\[
\dot{V}_4(t) = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \dot{H}(t) + \sum_{i=1}^{n} \dot{H}_i e_2i - \frac{1}{2} \sum_{i=1}^{n} k_3i e_2^2
\]

\[
= -e_1^T k_1 e_1 - e_2^T k_2 e_2 - \frac{1}{2} \sum_{i=1}^{n} k_3i e_2^2
\]  \hspace{1cm} (AI-16)

It is clear from (AI-16) that \( \dot{V}_4(t) \leq 0 \) for \( t \in [t_{dn}, \infty) \), where all gains \( k_1 \), \( k_2 \), and \( k_3i \) are positive, which means that the robot system is stable. Hence, the signals \( e_1 \), \( e_2 \) and \( \dot{H}(t) \) are bounded.
APPENDIX II

1. Proof of the equation 5.26 (Li et al., 2015a):

For very small sampling time period, it is acceptable to assume that the previous time delay error equals the current time delay error. In this case, considering:

\[
\begin{align*}
\varepsilon_i(t) & \in \varepsilon(t) \\
\hat{\varepsilon}_i(t) & \in \hat{\varepsilon}(t) \\
\tilde{\varepsilon}_i(t) & \in \tilde{\varepsilon}(t) \\
g(t) & \in \mathbb{R}
\end{align*}
\]  

(AII-1)

where \(i \in \{1, \ldots, 7\}\), \(g(t)\); it will be determined later. Now, let us propose the following equation:

\[
\begin{align*}
\varepsilon_i(t) & = \varepsilon_i(t - t_d) + \varepsilon_i(t_d) \\
\tilde{\varepsilon}_i(t) & = \varepsilon_i(t) - \hat{\varepsilon}_i(t) \\
\hat{\varepsilon}_i(t) & = \tilde{\varepsilon}_i(t - t_d) + g(t)
\end{align*}
\]  

(AII-2)

Then:

\[
\frac{d}{dt} \int_{t-t_d}^{t} \tilde{\varepsilon}^T(s) \tilde{\varepsilon}(s) ds = -2\tilde{\varepsilon}^T(t)g(t) - g^T(t)g(t)
\]

Proof:

\[
\frac{d}{dt} \int_{t-t_d}^{t} \tilde{\varepsilon}^T(s) \tilde{\varepsilon}(s) ds = \tilde{\varepsilon}^T(t)\tilde{\varepsilon}(t) - \tilde{\varepsilon}^T(t-t_d)\tilde{\varepsilon}(t-t_d).
\]

Considering \(\hat{\varepsilon}_i(t) = \hat{\varepsilon}_i(t - t_d) + g(t)\):

\[
\tilde{\varepsilon}_i^2(t-t_d) = [\varepsilon_i(t-t_d) - \hat{\varepsilon}_i(t-t_d)][\varepsilon_i(t-t_d) - \hat{\varepsilon}_i(t-t_d)]
\]

\[
= [\varepsilon_i(t) - \hat{\varepsilon}_i(t) + g(t)][\varepsilon_i(t) - \hat{\varepsilon}_i(t) + g(t)]
\]

\[
= \tilde{\varepsilon}_i^2(t) + 2\tilde{\varepsilon}_i(t)g(t) + g^2(t)
\]  

(AII-3)
Thus, we can obtain;

\[ \bar{\varepsilon}_i^2(t) - \bar{\varepsilon}_i^2(t - t_d) = -2\bar{\varepsilon}_i(t)g(t) - g^2(t) \]  \hspace{1cm} (AII-4)

Let us now define \( g(t) \) function as:

\[ g(t) = -k_{3i}e_{2i} \]  \hspace{1cm} (AII-5)

Substituting equation (AII-5) in (AII-4), we obtain:

\[ \bar{\varepsilon}_i^2(t) - \bar{\varepsilon}_i^2(t - t_d) = 2\bar{\varepsilon}_i(t)k_{3i}e_{2i} - (k_{3i}e_{2i})^2 \]  \hspace{1cm} (AII-6)

Multiplying both sides of (AII-6) by \( \frac{1}{2k_{3i}} \) to obtain:

\[ \frac{1}{2k_{3i}}\bar{\varepsilon}_i^2(t) - \frac{1}{2k_{3i}}\bar{\varepsilon}_i^2(t - t_d) = \bar{\varepsilon}_i(t)e_{2i} - \frac{k_{3i}}{2}e_{2i}^2 \]  \hspace{1cm} (AII-7)

2. **Stability proof of functional Lyapunov-Krasovskii equation (5.33):**

The proposed adaptive time-delay control law (5.27) ensures the asymptotic stability of the system. Where \( e_1 \to 0 \) and \( e_2 \to 0 \) as \( t \to \infty \). The proof is done in three stages. Stage 1 proves the boundedness of \( V_3(t) \) in the interval \( [0, t_{dn}] \). Stage 2 proves the negativeness of \( V_3(t) \) and asymptotically convergence of tracking errors \( e_1 \) and \( e_2 \) in the interval \( [t_{dn}, \infty) \). Stage 3 concludes the asymptotical convergence of tracking errors \( e_1 \) and \( e_2 \).

**Stage 1:** Boundedness of \( V_3(t) \) in the interval \( [0, t_{dn}] \). The derivative of equation (5.31) \( V_3(t) \) with respect to time for \( t \in [0, t_{d1}) \) is given by:

\[ \dot{V}_3(t) = \dot{V}_2(t) + \sum_{i=1}^{7} \frac{1}{2k_{3i}}\bar{\varepsilon}_i^2(t) \]  \hspace{1cm} (AII-8)
Differentiating (5.32) $\dot{V}_2(t)$ with respect to time is given such that:

$$\dot{V}_2(t) = -e_1^T k_1 \dot{e}_1 + e_1^T e_2 + e_2^T \dot{e}_2$$  \hspace{1cm} \text{(AII-9)}$$

Substituting equation (5.14), (5.16) and (5.20) into (AII-9) and applying the adaptive control law (5.27) to find:

$$\dot{V}_2(t) = -e_1^T k_1 \dot{e}_1 + e_1^T e_2 + e_2^T (-k_2 e_2 - e_1 - \dot{H}(t) + H(t) - \dot{\hat{e}}(t))$$

$$= -e_1^T k_1 \dot{e}_1 + e_1^T e_2 + e_2^T (-k_2 e_2 - e_1 - (H(t) - \dot{H}(t)) - \dot{\hat{e}}(t))$$  \hspace{1cm} \text{(AII-10)}$$

From equation (5.29) and equation (5.30), we find:

$$\dot{V}_2(t) = -e_1^T k_1 \dot{e}_1 - e_2^T k_2 e_2 - e_2^T \tilde{\epsilon}(t)$$  \hspace{1cm} \text{(AII-11)}$$

For $t \in [0, t_{d1}]$, we have from equation (5.26): $\dot{\hat{e}} = -k_3 e_2$. Hence:

$$\tilde{\epsilon}_i^2(t) = (\epsilon_i(t) - \hat{\epsilon}_i(t))^2$$

$$= \epsilon_i^2(t) - 2\epsilon_i(t)\hat{\epsilon}_i(t) + \hat{\epsilon}_i^2(t)$$

$$= \epsilon_i^2(t) - 2\hat{\epsilon}_i(t) [\tilde{\epsilon}_i(t) + \hat{\epsilon}_i(t)] + \hat{\epsilon}_i^2(t)$$

$$= \epsilon_i^2(t) + 2k_3 \tilde{\epsilon}_i(t)e_{2i} - (k_3 e_{2i})^2(t)$$  \hspace{1cm} \text{(AII-12)}$$

Therefore, substituting (AII-11) and (AII-12) in (AII-8) the Lyapunov function $\dot{V}_3(t)$ is rewritten as follows:

$$\dot{V}_3(t) = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \tilde{\epsilon}(t) + \sum_{i=1}^{n} \frac{1}{2k_{3i}} \epsilon_i^2(t) + \sum_{i=1}^{n} \tilde{e}_i(t)e_{2i} - \sum_{i=1}^{n} \frac{1}{2k_{3i}} (k_{3i} e_{2i})^2$$

$$= -e_1^T k_1 e_1 - e_2^T k_2 e_2 - \sum_{i=1}^{n} \frac{k_{3i}}{2} e_{2i}^2 + \sum_{i=1}^{n} \frac{1}{k_{3i}} \epsilon_i^2(t)$$  \hspace{1cm} \text{(AII-13)}$$

According to Assumption 5.2 and Assumption 5.3, $\epsilon_i(t), i = 1, \ldots, 7$ are bounded. This implies $\dot{V}_3(t)$ is bounded. Hence, $\dot{V}_3(t)$ is bounded in time interval $[0, t_{d1})$.
For $t \in [t_{dn-1}, t_{dn})$, conforming to the derivative of $V_3(t)$ with respect to time can be written by

$$
\dot{V}_4(t) = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \ddot{e}(t) + \frac{1}{2} e_n^2(t) + \sum_{i=1}^{n-1} \frac{1}{2k_{3i}} \left( \dddot{e}_i^2(t) - \dddot{e}_i^2(t - t_d) \right)
$$

$$
= -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \ddot{e}(t) + \frac{1}{2k_{3n}} e_n^2(t) + \sum_{i=1}^{n} \dot{e}_i(t) e_{2i} - \frac{1}{2} \sum_{i=1}^{n} k_{3i} e_{2i}^2
$$

$$
= -e_1^T k_1 e_1 - e_2^T k_2 e_2 - \frac{1}{2} \sum_{i=1}^{n} k_{3i} e_{2i}^2 + \frac{1}{2k_{3n}} e_n^2(t)
$$

(AII-14)

It is clear from equation (AII-14) that $\dot{V}_3(t)$ for $t \in [t_{dn-1}, t_{dn})$ because $\dot{e}_i(t)$ is bounded. That means $V_3(t)$ is bounded in time interval $[t_{dn-1}, t_{dn})$.

**Stage 2:** The negativeness of $\dot{V}_3(t)$ in the interval $[t_{dn}, \infty)$. The derivative of $V_3(t)$ with respect to time for $t \in [t_{dn}, \infty)$ is given by:

$$
\dot{V}_4(t) = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \ddot{e}(t) + \sum_{i=1}^{n} \frac{1}{2k_{3i}} \left( \dddot{e}_i^2(t) - \dddot{e}_i^2(t - t_d) \right)
$$

(AII-15)

We have from (AII-6): $\dddot{e}_i^2(t) - \dddot{e}_i^2(t - t_d) = 2\dddot{e}_i(t) e_{2i} - (k_{3i} e_{2i})^2$. So, (AII-15) can be rewritten as follows:

$$
\dot{V}_4(t) = -e_1^T k_1 e_1 - e_2^T k_2 e_2 - e_2^T \ddot{e}(t) + \sum_{i=1}^{n} \dot{e}_i e_{2i} - \frac{1}{2} \sum_{i=1}^{n} k_{3i} e_{2i}^2
$$

(AII-16)

It is clear from (AII-16) that $\dot{V}_3(t) \leq 0$ for $t \in [t_{dn}, \infty)$.

**Stage 3:** It is clear from equation (5.31) that $V_3(t)$ is positive definite in $e_1, e_2, \ddot{e}(t)$ and $\dddot{H}(t)$. Since $\dot{V}_3(t)$ is semi-negative definite, and $V_3(t)$ is bounded, this implies that $e_1, e_2, \ddot{e}(t)$ and $\dddot{H}(t)$ are bounded. In this case, we can utilize Barbalat’s lemma to complete the stability proof.

From **Assumption 5.3** and the boundedness of $e_1$ and $e_2$, and from the boundedness of $\ddot{e}$ in equation (5.14), and boundedness of $\dot{\theta}$ in equation (5.13), we conclude that $\dot{e}_1$ is bounded in
equation (5.14). \( \dot{\theta} \) is bounded from equation (5.2) and \( \dot{\theta}_d \) is bounded (Assumption 5.4), this means that \( \dot{e}_2 \) is bounded. \( \dot{e}_1 \) and \( \dot{e}_2 \) are bounded imply that \( e_1 \) and \( e_2 \) are uniformly continuous. According to Barbalat’s theorem \( e_1 \to 0 \) and \( e_2 \to 0 \) as \( t \to \infty \) and the proof is completed.
APPENDIX III

1. Proof of equations (6.18) and 6.19 (Brahim et al., 2016b,a)

for very small sampling time period, it is acceptable to assume that the previous time delay error equals the current time delay error. In this case, considering:

\[
\begin{aligned}
\varepsilon(t) &= \varepsilon(t-t_d) \\
\tilde{\varepsilon}(t) &= \varepsilon(t) - \hat{\varepsilon}(t) \\
\hat{\varepsilon}(t) &= \hat{\varepsilon}(t-t_d) + g(t)
\end{aligned}
\]  

(AIII-1)

where \(g(t) \in \mathbb{R}^7\); it will be determined later. Then:

\[
\frac{d}{dt} \int_{t-t_d}^{t} \tilde{\varepsilon}^T(w)\tilde{\varepsilon}(w)dw = -2\tilde{\varepsilon}^T(t)g(t) - g^T(t)g(t)
\]  

(AIII-2)

Proof:

It is easy to conclude that:

\[
\frac{d}{dt} \int_{t-t_d}^{t} \tilde{\varepsilon}^T(w)\tilde{\varepsilon}(w)dw = \tilde{\varepsilon}^T(t)\tilde{\varepsilon}(t) - \tilde{\varepsilon}^T(t-t_d)\tilde{\varepsilon}(t-t_d)
\]  

(AIII-3)

Considering the following equation: \(\hat{\varepsilon}(t) = \hat{\varepsilon}(t-t_d) + g(t)\):

\[
\tilde{\varepsilon}^T(t-t_d)\tilde{\varepsilon}(t-t_d) = [\varepsilon^T(t-t_d) - \tilde{\varepsilon}^T(t-t_d)] [\varepsilon(t-t_d) - \hat{\varepsilon}(t-t_d)]
\]

\[
= [\varepsilon^T(t) - \tilde{\varepsilon}^T(t) + g^T(t)] [\varepsilon(t) - \hat{\varepsilon}(t) + g(t)]
\]

\[
= \tilde{\varepsilon}^T(t)\tilde{\varepsilon}(t) + 2\tilde{\varepsilon}^T(t)g(t) - g^T(t)g(t)
\]  

(AIII-4)

Thus, we can obtain:

\[
\tilde{\varepsilon}^T(t)\tilde{\varepsilon}(t) - \tilde{\varepsilon}^T(t-t_d)\tilde{\varepsilon}(t-t_d) = -2\tilde{\varepsilon}^T(t)g(t) - g^T(t)g(t)
\]  

(AIII-5)
Let us now define $g(t)$ function as:

$$g(t) = -k_4 S$$  \hspace{1cm} (AIII-6)

Substituting equation (AIII-6) in (AIII-5), we obtain:

$$\tilde{\epsilon}^T(t)\tilde{\epsilon}(t) - \tilde{\epsilon}^T(t-t_d)\tilde{\epsilon}(t-t_d) = 2\tilde{\epsilon}^T(t)k_4 S - (k_4 S)^T(k_4 S)$$ \hspace{1cm} (AIII-7)

Hence, we have:

$$\frac{1}{2k} \tilde{\epsilon}^T(t)\tilde{\epsilon}(t) - \frac{1}{2k} \tilde{\epsilon}^T(t-t_d)\tilde{\epsilon}(t-t_d) = \tilde{\epsilon}^T(t)S - S^T \frac{k_4^T}{2} S$$ \hspace{1cm} (AIII-8)


Khalil, HK and J Grizzle. 1996. "Nonlinear systems. vol. 3 Prentice hall". *New Jersey*.


Liu, Cai, Chao Song, Qi Lu, Yang Liu, Xuan Feng and Yue Gao. 2015. "Impedance inversion based on L1 norm regularization". *Journal of Applied Geophysics*, vol. 120, p. 7–13.


Luo, Youxin, Qiyuan Liu, Xiaoyi Che and Lingfang Li. 2013. "Damped least-square method based on chaos anti-control for solving forward displacement of general 6-6-type parallel mechanism". *International Journal of Advanced Robotic Systems*, vol. 10, n° 4, p. 186.


