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STRATÉGIE DE PRODUCTION MANUFACTURIÈRE DANS UN ENVIRONNEMENT DE CHAÎNE D'APPROVISIONNEMENT : APPROCHE DYNAMIQUE STOCHASTIQUE

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RÉSUMÉ

Ce projet porte sur le contrôle des activités opérationnelles de la production dans un environnement de chaîne d'approvisionnement. Nous nous adressons aux problèmes de contrôle des rythmes de production, des actions de mise en course ainsi que des stratégies de maintenance préventive de systèmes manufacturiers contraints par un environnement interne et/ou externe non fiables. À cet égard, nous cherchons à déterminer des stratégies intégrées de production et d'approvisionnement, en présence de plusieurs fournisseurs potentiels.

Nous proposons une approche séquentielle de résolution basée sur la modélisation mathématique et la résolution numérique ainsi que la simulation, les plans d'expériences et les algorithmes génétiques. La première partie de l'approche basée sur la théorie de commande optimale et/ou impulsionnelle est indispensable pour avoir une base solide permettant de proposer des stratégies de contrôle qui s'approchent de l'optimum. Quant à la deuxième partie de l'approche, elle vient compléter la première afin de développer des processus décisionnels des activités manufacturières basés sur les politiques développées. De plus, elle permet d'étendre les dites stratégies pour couvrir des systèmes plus complexes.

À un niveau opérationnel de décision, nous démontrons la grande utilité de la combinaison des deux approches susmentionnées qui peut s'avérer incontournable pour amener des solutions à des problèmes *NP*-difficiles. L'application de l'approche aux systèmes étudiés nous a permis de proposer des stratégies de production plus réalistes et plus économiques. La prise en considération du système manufacturier dans son environnement externe, nous a permis de mettre en évidence l'importance d'une gestion intégrée des fonctions de production et d'approvisionnement. À cet égard, les politiques de contrôle proposées nous ont permis de réduire jusqu'à 10 % le coût total, encouru quand il s'agit d'une gestion dissociée.

Cette thèse amène des solutions à une classe de problèmes de modélisation dynamique stochastique de système manufacturier et ce, à plus qu'un niveau de la hiérarchie de décision. De plus, elle met en application une approche globale de résolution permettant de développer des processus décisionnels de gestion. Cette approche peut surmonter les problèmes liés à la résolution des modèles mathématiques quand il s'agit de systèmes de taille réelle.

MANUFACTURING SYSTEM CONTROL IN SUPPLY CHAIN ENVIRONMENT: STOCHASTIC DYNAMIC APPROACH

HAJJI, Adnène

ABSTRACT

This thesis deals with the operational-level control problems in a supply chain environment. We are concerned with the control of production, changeover and maintenance activities of manufacturing systems facing internal and/or external unreliable environment. In this context, we seek to develop integrated production and replenishment strategies for stochastic supply chain with multiple suppliers.

To handle the complexity of the problem, a sequential resolution approach is proposed. It combines mathematical modelling, numerical resolution as well as simulation, design of experiment and genetic algorithms. The first part of the approach is based on optimal and impulsive control theory which is essential to have a rigorous foundation making it possible to propose sub-optimal control strategies. Regarding the second part of the approach, it completes the first part so as to develop manufacturing activities decisional processes. Moreover, it offers more flexibility to handle possible extensions covering more complex systems.

At an operational level of decision, we demonstrate the great usefulness of the proposed combined approach which could be indispensable facing *NP* hard problems. In fact, the application of the aforementioned approach makes it possible to propose more realistic and profitable manufacturing strategies. In a supply chain environment, the consideration of the external environment of the manufacturing system confirms the necessity of considering the interactions present in the system in an integrated model so as to obtain more realistic control policies. In this context, it is shown that it is more profitable to consider in integrated manner the manufacturing and supply control problems. In fact, for the case under study the proposed control policies reduce the total incurred costs up to 10 % compared to the incurred costs under dissociated strategies.

This thesis should bring solutions to a class of manufacturing system modeling under uncertainty. Moreover, it applies a global approach making it possible to develop decision making processes for realistic systems. This approach can overcome the complexity behind mathematical models resolution of big size systems.

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INTRODUCTION ET OBJECTIFS DE LA THÈSE

1.1 Introduction

Ces dernières décennies, l'économie du marché et l'ouverture des frontières ont favorisé la naissance d'un environnement industriel de plus en plus compétitif. Dans ce contexte, la capacité à satisfaire rapidement la demande et atteindre un niveau de service élevé est devenue un avantage concurrentiel important, sans pour autant remettre en question les autres avantages concurrentiels de « coût » et de la « qualité ». Pour faire face à ces exigences et garantir la survie des entreprises, de plus en plus dispersées dans ce contexte de marchés globaux, la notion de chaîne d'approvisionnement et de réseaux d'entreprises a pris une importance considérable.

Plusieurs facteurs ont contribué à considérer qu'une bonne maîtrise des activités de la chaîne d'approvisionnement constitue l'un des éléments les plus déterminants afin d'atteindre les performances souhaitées (Narahari et Biswas (2000)). En effet, une des caractéristiques fondamentales de la chaîne d'approvisionnement est qu'elle se comporte justement comme une chaîne, c'est à dire que chacun des maillons a un impact sur le reste des intervenants, positivement ou négativement. Ainsi, toute rupture de marchandises chez un des fournisseurs se répercutera jusqu'au client final; tandis que tout changement de la demande provoquera une réponse chez les autres joueurs. De ce fait, une gestion efficace de chaque élément ainsi que ses interactions avec les autres intervenants de la chaîne sont indispensables afin de rallier les visions souvent disparates des différents intervenants. Ceci sans oublier la nature dynamique stochastique de l'environnement auxquels la chaîne d'approvisionnement est assujettie.

Dans ce contexte, et en réponse à l'accroissement de l'incertitude et de la complexité de l'environnement industriel, les gestionnaires doivent détenir les outils nécessaires afin de garantir une meilleure intégration de tous les intervenants et atteindre leurs objectifs communs. On pensera à la maximisation de la productivité, la satisfaction de la demande et la minimisation des coûts à prime abord. Ceci se manifeste par l'emploi d'une bonne stratégie d'acquisition et d'acheminement de la matière, de production, de maintenance, de distribution, d'investissement et de coordination, etc. Une des étapes cruciales, pour arriver à ces fins, consiste à un bon choix des outils de modélisation et d'analyse.

Dans la littérature, la modélisation et le contrôle des chaînes d'approvisionnement ont été abordés pour la plupart avec deux visions différentes. Une vision à court terme ne considère que les facteurs et les critères opérationnels. Ces facteurs sont importants lors de la prise de décision mais une vision purement opérationnelle peut conduire les intervenants vers une impasse à long terme. De son côté, une vision à « moyen, long terme » s'intéresse aux facteurs et aux critères qui déterminent la viabilité à moyen et à long terme de la chaîne d'approvisionnement sur un marché concurrentiel. Dans cette thèse, nous nous proposons de rapprocher ces deux visions dans une démarche séquentielle de modélisation et contrôle dynamique stochastique de deux classes de système manufacturiers dans un environnement de chaînes d'approvisionnement.

Cette thèse devra amener des solutions à une classe de problèmes de modélisation dynamique stochastique des systèmes manufacturiers et ce, à plus qu'un niveau de la hiérarchie de décision. De plus, elle met en application une approche globale de résolution permettant de développer des processus décisionnels de gestion. Cette approche peut surmonter les problèmes liés à la résolution des modèles mathématiques quand il s'agit de systèmes de taille réelle.

1.2 Généralités

Dans cette section, nous nous proposons de rappeler quelques concepts et notions de base sur la prise de décision et la commande des chaînes d'approvisionnement directement liées à notre problématique de recherche.

1.2.1 Prise de décision dans une chaîne d'approvisionnement

La prise de décision dans une chaîne d'approvisionnement est un processus complexe. Parmi les raisons les plus importantes derrière cette complexité, nous pouvons citer :

- La nature dynamique des interactions entre tous les éléments de la chaîne,
- L'existence d'événements de nature aléatoire à chaque stage de la chaîne,
- La dimension, souvent large, des systèmes (dimension liée à la structure ou aux décisions).

Les prises de décision dans une chaîne d'approvisionnement peuvent être classées selon des considérations temporelles et / ou fonctionnelles.

Classification temporelle

Les décisions peuvent être classées selon trois niveaux à savoir, le niveau opérationnel, tactique ou stratégique.

- Les décisions stratégiques visent des objectifs à long terme et guide les politiques de la chaîne d'approvisionnement selon des perspectives de conception ou de planification. Généralement, ces décisions ne seront revues qu'après une certaine période de temps qui dépend de plusieurs facteurs et qui peut s'avérer assez longue (plusieurs années).

- Les décisions tactiques visent le moyen terme et sont nécessaires pour une gestion efficace de la chaîne d'approvisionnement, configurée selon les décisions stratégiques. Les intervalles de temps qui régissent ces décisions peuvent s'étendre sur plusieurs semaines voire même plusieurs mois.

- Les décisions opérationnelles visent le court terme et concernent généralement les activités en temps réel des différents acteurs de la chaîne d'approvisionnement.

Classification fonctionnelle

Selon cette classification, il existe quatre domaines de décisions majeures régissant la gestion des chaînes d'approvisionnement : l'approvisionnement, la transformation (fabrication, assemblage…), la distribution et la logistique. De plus, il existe certaines décisions globales qui s'étendent sur plusieurs fonctions. Dans chacune de ces fonctions les décisions peuvent être de nature stratégique, tactique ou opérationnelle. Nous référons le lecteur à Narahari et Biswas (2000) pour une liste des décisions les plus importantes reliées à chacune de ces fonctions.

1.2.2 Nature stochastique des chaînes d'approvisionnement

Dans ce paragraphe, les principales sources de phénomènes aléatoires dans une chaîne d'approvisionnement seront présentées. C'est justement la présence de tels événements dans un contexte dynamique et à tous les niveaux de décision qui explique les difficultés auxquelles les gestionnaires doivent faire face pour contrôler et gérer d'une façon intégrée toutes les activités de la chaîne.

Il existe quatre principales sources de phénomènes aléatoires dans une chaîne d'approvisionnement :

1. L'approvisionnement : d'une manière générale, ces aléas sont liés aux risques d'une rupture d'approvisionnement. Un tel risque peut mettre en cause la viabilité et l'existence même de l'entreprise sur le marché et donc il mérite une grande attention lors de la prise de décision (Gaucher et al. (2000)).

2. Le processus de transformation : les aléas liés au processus de transformation de la matière peuvent regrouper les pannes des systèmes de production, la fiabilité du système de transport…etc. ces aléas doivent aussi être pris en considération lors de la prise de décision.

3. La demande : les aléas liés à la demande sont certainement les éléments de risque les plus étudiés dans la littérature de modélisation et contrôle des chaînes d'approvisionnement. Ces aléas sont, en premier lieu, dus à l'aspect incertain du

marché. De plus, plusieurs recherches ont pu noter qu'un tel aspect en présence de distorsions dans la circulation d'information conduit à un phénomène très perturbant appelé le phénomène de coup de fouet ou « Bullwhip effect ». Cet effet se caractérise par une exagération de la fluctuation de la demande d'un stage à l'autre de la chaîne d'approvisionnement (Lee et al. (1997) et Min (2000)).

4. Sources externes : Braithwaite et Hall (1999) ont noté que les éléments de risque provenant de l'extérieur de la chaîne sont souvent plus importants et plus perturbants que ceux provenant de l'intérieur. Les aléas d'approvisionnement et de la demande peuvent être classés dans ce type de risque. De plus, nous pouvons noter plusieurs autres sources d'aléas externes comme les risques liés à la compétition, aux contrats de partenariat…etc.

Dans ce paragraphe nous avons pu noter quelques aspects stochastiques qui perturbent le fonctionnement d'une chaîne d'approvisionnement. La prise en considération de ces aspects dans un environnement dynamique complique d'avantage les problèmes liés au contrôle des activités de la chaîne. Une bonne partie de ces aspects constituent des éléments clés dans notre problématique de recherche.

1.3 Problématique

La prise en compte des principaux aspects qui font du processus de prise de décision dans une chaîne d'approvisionnement un processus très complexe constitue un défi majeur pour les chercheurs dans le domaine de gestion et contrôle des réseaux d'entreprises. Rappelons que ces aspects peuvent être résumés en :

- La nature dynamique des interactions entre tous les éléments de la chaîne,
- L'existence d'événements de nature aléatoire à chaque stade de la chaîne,
- La dimension, souvent large, des systèmes (dimension liée à la structure ou aux décisions).

Dans la littérature, les efforts qui ont été accomplis pour mieux maîtriser ces aspects sont considérables. Cependant, la diversité des paramètres, le volume des données et les niveaux de décisions impliquées ont limité la plupart des anciens travaux de recherche à des problèmes de taille pas trop larges et touchant un seul niveau de décision.

Dans cette thèse, nous nous proposons d'amener une contribution sous cet angle de vue. En conséquence, une approche séquentielle sera proposée permettant d'offrir une démarche « globale » de contrôle à un niveau opérationnel de décision du processus de transformation tout en prenant en considération son environnement externe. Cette approche permettra de dégager une démarche pour aider les entreprises dans leurs processus de prise décision et commande opérationnelle dans un environnement dynamique stochastique et ce, avec une prise en compte de leur vision tactique-stratégique.

La chaîne d'approvisionnement à laquelle nous nous intéressons consiste en un réseau de partenaires disposés en série et produisant plusieurs types de produits. Comme le montre la figure 1.1, la chaîne d'approvisionnement est constituée de 3 composantes fondamentales soit, les fournisseurs, le processus de transformation et les clients.

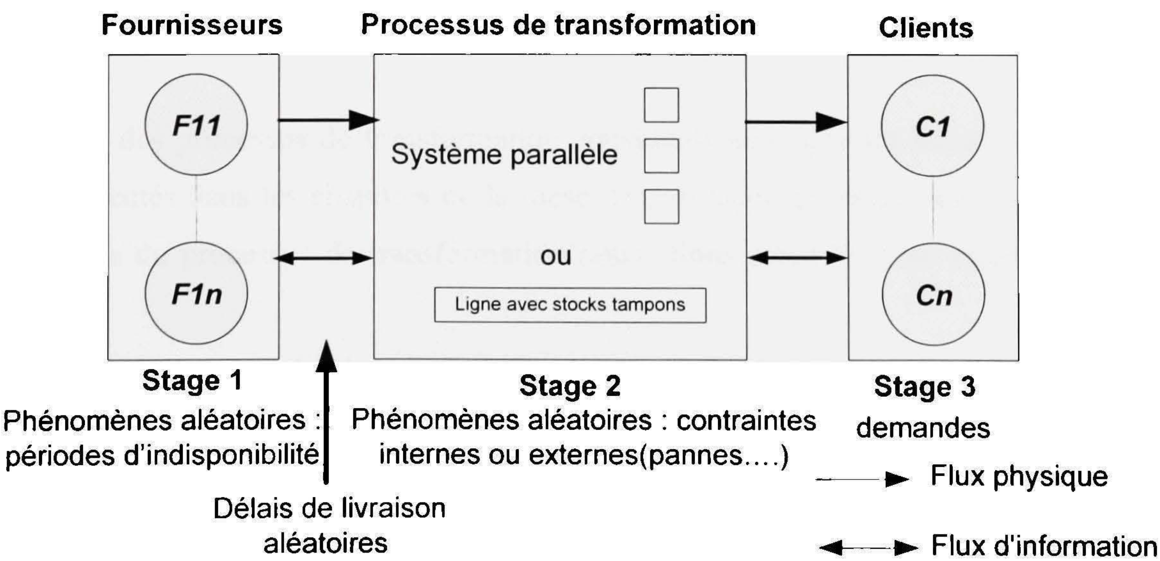


Figure 1.1 Structure du système manufacturier sous étude

Dans le cadre de cette thèse, nous nous proposons de nous rapprocher le plus possible de la réalité afin de garantir la robustesse de notre approche. Pour ce faire nous allons considérer les quatre sources de phénomènes aléatoires qui peuvent exister dans une chaîne d'approvisionnement. Comme illustré dans la figure 1.1, nous allons considérer les principaux phénomènes aléatoires liés aux fournisseurs (capacité et délais de livraison) et au processus de transformation (contraintes internes ou externes). De plus, le fait de considérer, sur un long horizon, un taux de demande constant n'exclut pas l'aspect aléatoire relié aux clients. En effet, l'approche que nous proposons pourra facilement être appliquée pour réévaluer les variables de décisions avec divers taux de demande relatifs à différents horizons.

La commande de la chaîne d'approvisionnement sous étude, dans un contexte dynamique stochastique, doit se faire dans un but bien précis. À tous les niveaux de décision, ce but est incarné dans un problème d'optimisation de plusieurs mesures de performances. Dans notre cas, les mesures de performances considérées sont quantitatives et principalement liés aux coûts. Le fait de considérer les interactions entre les éléments du système dans un contexte dynamique stochastique n'écarte pas les autres aspects aussi importants comme entre autres la capacité et les performances de livraison qui sont considérées implicitement dans la mesure coût.

Les détails des processus de transformation considérés ainsi que les décisions impliquées seront présentés dans les chapitres de la thèse. D'une façon générale, selon la vision d'un gestionnaire du processus de transformation nous allons considérer les prises de décision suivantes :

- Politique de production : stratégie permettant de fixer les rythmes de production fonction de l'état du système.
- Politique de mise en course : stratégie permettant de savoir le moment opportun d'un changement de configuration du système de production afin de lancer la production d'un certain type de produit.

- Politique de maintenance préventive : stratégie permettant de choisir la meilleure politique de maintenance préventive à adopter et la meilleure façon de la gérer.
- Politique d'approvisionnement en présence de plusieurs fournisseurs potentiels : stratégie permettant de répondre à trois questions, quand lancer une commande de réapprovisionnement ? combien faut-il commander ? pour quel fournisseur nous devons opter ? tout en considérant les propriétés et les contraintes du système de production.
- Stratégie de négociation des coûts : partenariat avec les fournisseurs permettant de savoir la marge de manœuvre détenu par rapport à un ensemble d'offre de prix lié à l'approvisionnement en matière première.

1.4 Revue de la littérature

Dans cette section nous nous proposons de fournir une revue critique de la littérature qui touche les aspects d'ordre général de notre problématique. Pour ce faire, nous allons commencer par présenter les approches de modélisation des chaînes d'approvisionnement. La deuxième partie sera dévouée aux approches d'optimisation expérimentale que nous comptons employer dans notre démarche. ***Il est à noter que chaque chapitre comporte une revue détaillée des travaux qui ont touché un des aspects spécifiques de la problématique.***

1.4.1 Approches de modélisation des chaînes d'approvisionnement

La modélisation des chaînes d'approvisionnement exige la prise en compte de tous les éléments de la chaîne entre les fournisseurs et le client final. La diversité des paramètres, le volume des données et les niveaux de décisions impliqués, font qu'il n'existe pas une approche universelle de modélisation des chaînes d'approvisionnement.

Les approches de modélisation des chaînes d'approvisionnement et comme illustré par la figure 1.2 peuvent être classées en quatre catégories (Min et Zhou (2002), Hillier et Liberman (2001) et Beamon (1998)) : déterministe, stochastique, hybrides et modèles basés sur la TI « Technologie de l'Information ». Dans ce qui suit une définition de ces approches et une revue des travaux qui les ont employés.

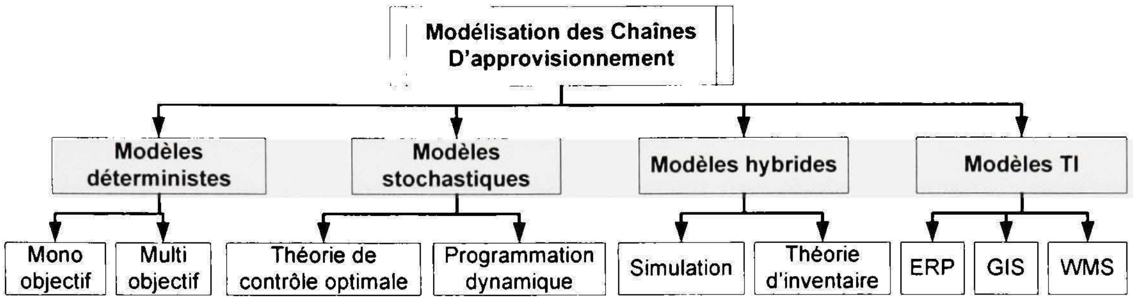


Figure 1.2 Modèles de chaînes d'approvisionnement

1.4.1.1 Modèles déterministes

Les modèles déterministes supposent que les paramètres de la chaîne sont connus et fixés avec certitude. Ils incluent deux types de modélisation à savoir la mono-objective et la multi-objectives. Cette dernière est venue refléter le besoin croissant quant à la modélisation des intérêts conflictuels des différents intervenants de la chaîne. Dans ce qui suit une revue, non exhaustive, des travaux qui ont employé ce type de modèle mathématique pour résoudre des problèmes de planification et contrôle dans les chaînes d'approvisionnement.

Les efforts relatifs au développement d'un modèle couvrant le maximum d'aspect d'une chaîne d'approvisionnement remonte à Glover et al. (1979). Ils ont développé un outil d'aide à la décision intégrant trois segments d'une chaîne d'approvisionnement à savoir l'approvisionnement, la location et la planification de la demande des clients. Le noyau de ce système était basé sur les modèles de réseaux. Leur contribution était importante à la littérature, cependant, le modèle était limité à des problèmes réduits. Ces efforts se sont succédé avec Cohen et Lee (1989), Arntzen et al. (1995), Ashayeri et Rongen (1997) parmi plusieurs autres et plus récemment avec Melachrinoudis et Min (2000) et Nozick et Turnquist (2001). Tous ces travaux avaient les mêmes objectifs, soit, le développement d'un modèle d'aide à la décision qui intègre le maximum d'aspect d'une chaîne d'approvisionnement (éléments de la chaîne, contraintes, objectifs (mono ou multi)). Bien que ces travaux soient considérés comme des apports considérables à la littérature, leurs domaines d'application restent limités du fait que les modèles sont déterministes.

1.4.1.2 Modèles stochastiques

Pour se rapprocher plus des phénomènes réels régissant une chaîne d'approvisionnement à savoir la présence de phénomènes aléatoires, les chercheurs dans ce domaine se sont penchés sur la question. Dans la revue que nous allons présenter nous allons inclure les modèles basés sur la théorie de commande optimale et ceux basés sur la programmation dynamique. Il est à noter que d'autres types de modèle existent notamment les modèles basés sur l'analyse décisionnelle et ceux basés sur la théorie des files d'attente. Dans la présente revue ces derniers types de modèles ne seront pas inclus en raison de leur rare présence dans la littérature (Min et Zhou (2002)).

Un bref rappel sur les travaux de recherche qui ont pris la nature stochastique d'une chaîne d'approvisionnement en considération nous conduit à remonter aux travaux pionniers de Midler (1969). Dans cet article, Midler a développé un modèle en programmation dynamique basé sur la théorie de contrôle optimal. Ce modèle permet de choisir une combinaison optimale des modes de transport, des écoulements de produits et des cheminements des commandes du client au fournisseur et ce, sur un horizon de planification à périodes multiples. Dans la même direction, Tapiero et Soliman (1972) ont développé un modèle permettant de résoudre un problème de planification des inventaires, des transports et de la production dans une chaîne d'approvisionnement faisant face à une demande aléatoire. Ce modèle, du fait qu'il combine un programme linéaire et un autre paramétré, a rencontré des difficultés majeures de résolution. Lee et Bellington (1993) ont intégré le flux de matière des processus des ventes, de la production et de la distribution dans un modèle stochastique. Ce modèle permet de déterminer la politique d'approvisionnement qui maximise le niveau de service relatif à chaque type de produit. Dans la même direction, Lee et Feitzinger (1995) suivi par Swaminathan et Tayur (1999) ont développé des modèles stochastiques permettant de formuler les stratégies de planification différenciée dans le cas d'une demande aléatoire.

Ces efforts se sont succédés par plusieurs chercheurs afin de quantifier les effets qui causent le déséquilibre entre l'approvisionnement et la demande dans une chaîne d'approvisionnement. Parmi ces travaux nous pouvons citer Fisher et al. (1997) qui ont développé un programme stochastique visant la minimisation des effets de ce déséquilibre sur la production (excès d'inventaire ou pénurie). Lee et al. (1997) ont étudié l'effet coup de fouet ou « Bullwhip effect » qui figure parmi les sources du déséquilibre entre la demande et l'approvisionnement. Ils ont ainsi analysé les sources de ce phénomène et ont proposé quelques actions pour y remédier. Dans la même direction, Chen et al. (2000) ont montré que cet effet peut être réduit, mais pas éliminé, et ce, en adoptant une stratégie visant la centralisation de la demande tout au long de la chaîne. Cette approche a été étudiée sur une chaîne d'approvisionnement multi-stages. Plus récemment, Dejonckheere et al. (2003) ont employé une approche de contrôle des systèmes basée sur les fonctions de transfert pour quantifier l'effet « Bullwhip ». Ils ont ainsi réussi à proposer des règles de décision permettant d'éliminer (et/ou de prévenir) l'amplification de la variance causée par l'effet « Bullwhip ».

À travers cette revue, nous tenons à mentionner les efforts considérables qui ont été accomplis par tous ces travaux. Cependant, aucun des travaux présentés dans cette section n'a intégré la vision de commande par retour d'information dans la modélisation et ce, en considérant une ou les trois visions décisionnelles ensemble. Les travaux qui ont touché à cet aspect seront détaillés dans les revues de littérature des différents chapitres de la thèse. Cet aspect constitue un des éléments clés dans le processus de contrôle et de la commande dynamique stochastique des chaînes d'approvisionnement.

1.4.1.3 Modèles hybrides

Les efforts qui ont été accomplis dans la modélisation mathématique des chaînes d'approvisionnement et que ce soit en adoptant une approche purement déterministe ou stochastique sont loin d'être suffisants. En effet, de part la nature même des chaînes d'approvisionnement, plusieurs modèles de contrôle doivent inclure des aspects déterministes et stochastiques simultanément (Budnick et al. (1988) et Zipkin (2000)). Ces modèles sont basés sur la théorie de contrôle des inventaires et/ou la simulation et sont connus sous le nom des modèles hybrides.

Lancioni (2000) a noté que les coûts des inventaires comptent pour environ la moitié des coûts logistiques dans une chaîne d'approvisionnement. C'est pour cette raison que les travaux de recherche qui ont employé la théorie de contrôle des inventaires comme outil de modélisation sont très présents dans la littérature. La revue de la littérature complète en lien avec ce type de modélisation sera intégrée et détaillée plus tard.

Dans ce qui suit nous allons revoir les travaux qui ont employé des approches d'analyse et de modélisation basées sur la simulation ou la simulation combinée avec un outil de modélisation mathématique. Bookbinder et al. (1989) ont employé la simulation et des modèles en programmation linéaire pour évaluer plusieurs alternatives de contrôle d'inventaire et de production et choisir la meilleure d'entres elles. Karabakal et al. (2000) ont combiné la simulation avec des modèles en programmation linéaire mixte pour déterminer le nombre et la localisation des centres de distribution ainsi que les zones de marché auxquelles ils seront affectés et ce, afin d'évaluer des mesures de performances en lien direct avec la satisfaction des clients. Petrovic et al. (1998) ont employé les résultats d'un modèle flou pour servir comme entrée au modèle de simulation d'une chaîne d'approvisionnement et ce, pour calculer les quantités d'approvisionnement et évaluer quelques mesures de performance de la chaîne. Cependant, leurs modèles ont été destinés à un seul type de produit, dans une chaîne sans contraintes de capacité. Récemment, Petrovic (2001) a étendu ces modèles afin d'inclure l'aspect aléatoire des délais de livraison.

Concernant les travaux qui ont employé juste la simulation pour l'évaluation et l'analyse de plusieurs politiques de contrôle, nous pouvons citer Towill (1991) et Towill et al. (1992). Dans ces travaux, ils ont employé la simulation pour évaluer les effets de plusieurs stratégies de contrôle des chaînes d'approvisionnement sur le phénomène d'amplification de la demande. Les stratégies étudiées sont les suivantes :

1- éliminer le stage de distribution de la chaîne d'approvisionnement et inclure la fonction distribution dans le stage du système manufacturier,
2- intégrer le flux d'information dans le modèle global de la chaîne,
3- implanter une politique d'inventaire JIT pour minimiser les délais de livraison,
4- améliorer le mouvement des produits intermédiaires et ce, en modifiant les procédures de commande,
5- modifier les paramètres des procédures de commande existantes.

L'objectif du modèle de simulation était de déterminer les meilleures stratégies capables de suivre les variations de la demande. Les résultats trouvés favorisent la première et la troisième stratégie. Dans la même direction, Wikner et al. (1991) ont examiné 5 stratégies visant l'amélioration des performances d'une chaîne d'approvisionnement et les ont implantées sur une chaîne composée de trois stages.

Il est important de noter que ce type d'approche permet d'investir une large variété de questions du type « what if » à propos d'un modèle complet de chaîne d'approvisionnement réel (dynamique, stochastique). Cependant, cette approche permet d'évaluer et/ou d'optimiser les performances d'une politique de commande ou de contrôle spécifiée à priori. Dans notre travail, et comme mentionné dans la méthodologie, nous comptons employer cette approche (simulation combinée à d'autres outils d'optimisation) mais après avoir déterminé les politiques de contrôle dans un environnement dynamique stochastique.

1.4.1.4 Modèles basés sur la TI

Les modèles basés sur la «TI» (i.e., Technologie de l'Information) sont considérés comme étant les plus récentes innovations (premier travail remonte à Camm et al. (1997)) dans le domaine de contrôle des chaînes d'approvisionnement (Shapiro (2001)). Ils visent l'intégration et la coordination de plusieurs phases de planification dans une chaîne d'approvisionnement avec une vision de commande en temps réel et ce, en utilisant des mécanismes de partage d'information entre les différents partenaires de la chaîne. Pour une revue complète des travaux qui ont basé leurs modèles sous cet angle de vue nous référons le lecteur à Min et Zhou (2002). Sans rentrer dans les détails, nous tenons à préciser que le recours à cette approche ne peut avoir lieu sans une modélisation à priori de la problématique avec les outils mathématiques détaillés précédemment. En effet, un des avantages de la «TI» est qu'elle permet l'intégration d'une politique de production ou de contrôle des inventaires, par exemple, déterminée à priori, dans un support informatique basé sur la «TI». Dans notre cas, cette piste pourra faire l'objet d'une réflexion pour une éventuelle valorisation pratique de nos résultats.

1.4.2 Approches d'optimisation basée sur la simulation

L'approche de modélisation et résolution que nous comptons suivre dans ce travail prévoit le recours aux outils d'optimisation basée sur la simulation. Il est à noter que cette approche diffère de celle détaillée dans la section 1.4.1.3 (modèles hybrides). Du fait que les modèles hybrides basés sur la simulation emploient cette dernière comme outil d'aide à la décision, alors que dans notre cas, nous allons l'employer comme outil de validation et d'optimisation des politiques de contrôle issus de la modélisation dynamique stochastique. Dans la partie qui va suivre nous nous proposons de fournir une revue de la littérature quant à l'utilisation des approches d'optimisation par le biais de la simulation pour résoudre des problèmes de contrôle de système de production stochastique.

La simulation est un puissant outil de modélisation utilisé dans la conception, la planification et le contrôle des systèmes de production complexes. À l'aide de la simulation, nous pouvons décrire en détail le comportement dynamique d'un système de production. Bien qu'elle soit considérée en tant qu'outil d'aide à la décision, qui n'est pas capable de résoudre directement les problèmes mais plutôt d'aider l'analyste à comprendre le comportement du système et de décider en conséquence, plusieurs recherches tentent de lui donner la capacité d'optimisation en la supportant par d'autres approches d'analyse mathématique ou statistique.

Dans la littérature, les techniques d'optimisation par le biais de la simulation peuvent être divisées en six catégories comme le montre la figure 1.3. Pour des définitions détaillées de ces méthodes, nous référons le lecteur à Carson et Maria (1997).

Les trois catégories les plus rencontrées dans la littérature sont les méthodes dérivatives basées sur l'approximation du gradient, la méthodologie des surfaces de réponse et les méthodes heuristiques.

<u>Les méthodes dérivatives</u> :

Elles incluent les méthodes d'approximation stochastique (Azadivar et Talavage, 1980), l'analyse de perturbation infinitésimale (Ho, 1984), la fonction score (Rubinstein, 1991), estimation du ratio (Glynn et al., 1991), frequency domain analysis (Morrice et Schruben, 1987), la méthode des différences finies (Andradòttir (1998)). Toutes ces méthodes visent l'estimation du gradient de la mesure de performance retenue avec respect aux variables de décision.

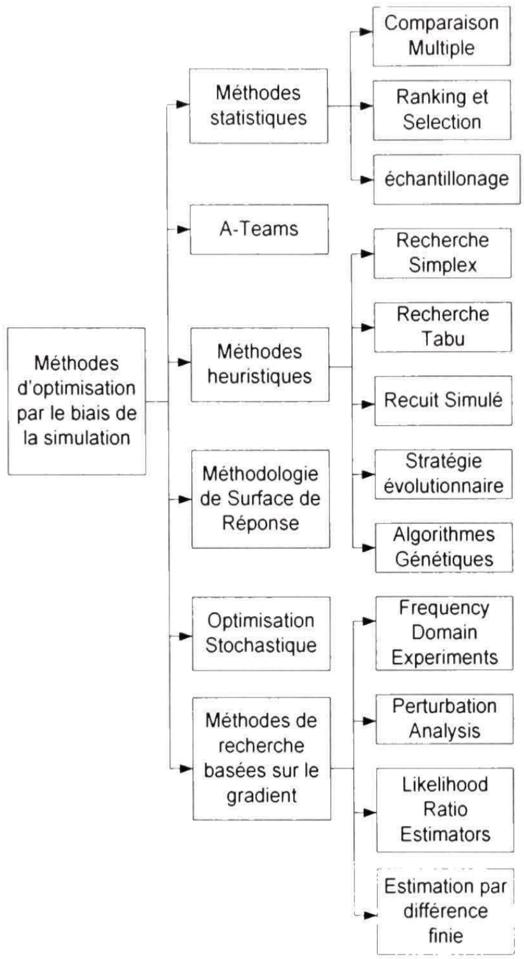


Figure 1.3 Méthodes d'optimisation pas le biais de la simulation

La méthodologie des surfaces de réponse :

D'un point de vue pratique, les méthodes de recherche directe se sont avérées limitées face aux systèmes complexes (Andradòttir (1998)). De là, le besoin de méthodes plus pratiques et faisant intervenir des outils moins compliqués.

La technique la plus connue dans les milieux industriels et académiques est le design expérimental. Cette technique a été élaborée pour optimiser l'organisation des expériences et surtout de les exploiter efficacement. Elle a su par la suite profiter des avantages de la statistique et de la simulation pour modéliser les comportements des systèmes et optimiser ainsi leurs performances. Dans ce contexte d'optimisation, cette technique est surtout

utilisée pour aider à identifier les facteurs et les interactions qui influent sur les performances du système et, par conséquent, permet de dégager un modèle de régression (grâce aux propriétés statistiques du plan d'expérience) exprimant la mesure de performance fonction des paramètres et interactions significatifs. Ce modèle pourra, par la suite, être minimisé ou maximisé pour approximer les meilleures conditions réelles d'opération.
L'approche de simulation combinée aux plans d'expériences pour fin d'optimisation a été employée avec succès par Kenne et Gharbi (1999), Gharbi et Kenne (2000), Kenne et Gharbi (2001).

Kenne et Gharbi (1999) ont utilisé l'approche pour optimiser les paramètres de la politique de production et de maintenance d'un système de production constitué d'une machine traitant un type de produit. Considérant que la dynamique des pannes de la machine dépend de son âge, ils ont montré que les paramètres de la politique de production et de maintenance dépendent également de l'âge. Ils ont ainsi défini la politique de production et de maintenance optimale par trois paramètres qu'ils ont ensuite déterminés expérimentalement par le biais de l'approche ci-haut mentionnée.

Gharbi et kenne (2000) ont utilisé la même approche que Kenne et Gharbi (1999) pour optimiser les paramètres de la politique de production et de maintenance d'un système de production constitué de plusieurs machines en parallèle traitant un type de produit.

Kenne et Gharbi (2001) ont utilisé la même approche pour déterminer les paramètres de la politique de production régissant un système parfaitement flexible, constitué de deux machines en parallèle produisant deux types de produits. Les paramètres de la politique étaient les deux niveaux de stocks optimaux (hedging levels). Kenne et Gharbi (2001) ont aussi abordé des extensions afin d'appliquer la même approche aux systèmes plus larges (plusieurs machines en parallèle produisant plusieurs types de produits).

Dans le cas où le système est régit par des variables de décision ou des paramètres de nature qualitative, cette méthode (plan d'expériences et méthodologie des surfaces de réponse) s'est avérée très limitée (Azadivar et Tompkins (1999)). Dans la problématique que nous nous proposons de résoudre on est confronté à un système ayant une structure variable et dépendant de variables de décision quantitatives et qualitatives. De ce fait, l'utilisation des méthodes heuristiques s'impose.

Les méthodes heuristiques :

Ils consistent en une exploration aléatoire autour de toutes les solutions admissibles interconnectées de l'espace des décisions. La recherche converge quand la solution « optimale » est trouvée. La valeur de la fonction objective du problème à chaque point de la recherche est estimée par simulation. De ce fait, aucune information concernant la forme analytique de la fonction objective n'est exploitée. Les approches de recherche directe par le biais de simulation les plus employer dans notre domaine d'intérêt sont : la recherche simplexe (Azadivar et Lee, 1988), recuit simulé (Ogbu et smith, 1990 ; Lee et Iwata, 1991), la recherche Tabu et les algorithmes génétiques.

Toutes les méthodes présentées dans les sections précédentes, à l'exception du recuit simulé et les algorithmes génétiques, requiert un système ayant une structure fixe et dont les variables de décision sont quantitatives. Dans la problématique que nous nous proposons de résoudre, on est confronté à un système ayant une structure variable et dépendant de variables de décision quantitatives et qualitatives. De ce fait, l'utilisation des algorithmes génétiques ou le recuit simulé s'impose. Dans notre méthodologie, nous avons anticipé notre choix (algorithme génétique) puisque le recuit simulé est rarement employé vue son temps de calcul élevé (Azadivar et Tompkins (1999)).

1.5 Méthodologies

Pour affronter la problématique sus-indiquée, une méthodologie composée de quatre étapes sera envisagée. Les principaux développements de notre démarche sont résumés dans la figure 1.4. Dans ce qui suit, nous nous proposons de détailler les étapes **E1** à **E4** associées à la méthodologie proposée.

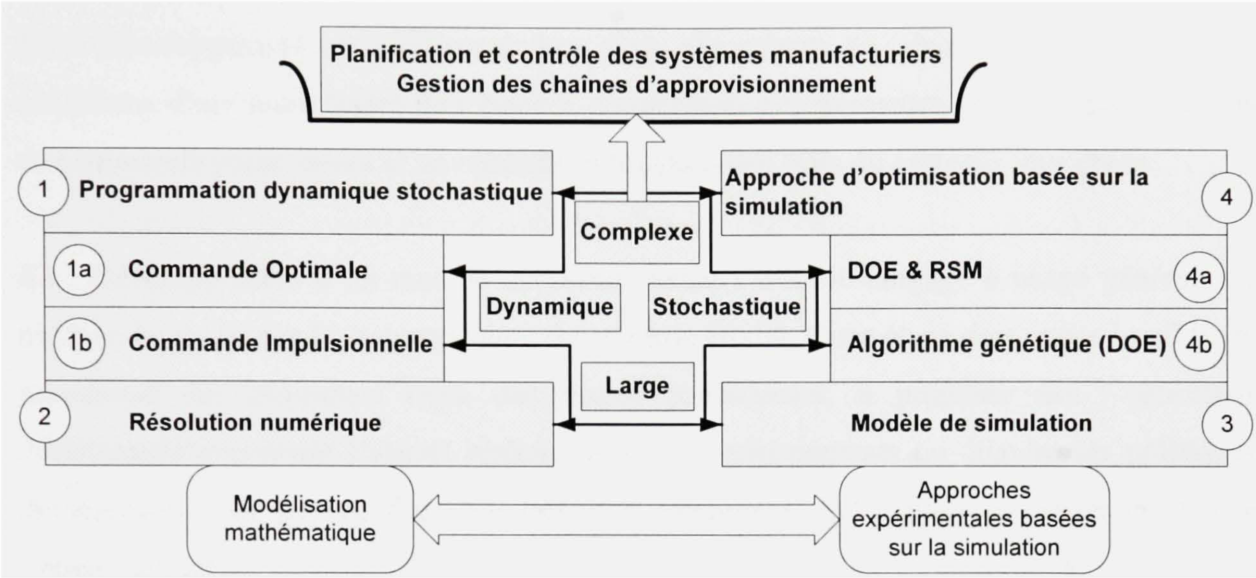


Figure 1.4 Démarche proposée

E1 : Présenter une formulation mathématique du système de production sous étude (i.e., processus de transformation isolé ou en considérant son environnement externe). La prise en considération des interactions et l'aspect aléatoire régissant le système nécessite le recours à la modélisation en programmation dynamique stochastique. Cette étape nous permettra de présenter les conditions d'optimum qui garantissent l'existence et l'unicité des lois de commande optimales. Cependant, la nature de la dynamique du système qui peut être continu et/ou discrète exige le recours à deux théories de modélisation d'une façon distincte ou combinée. C'est ce qui explique les deux points E1a et E1b suivants :

> **E1a :** Recours à la théorie de commande optimale et surmonter les difficultés de modélisation lorsqu'il s'agit d'une dynamique continue du système.

> **E1b :** Recours à la commande optimale impulsionnelle et éventuellement considérer sa combinaison avec la théorie de commande optimale afin de développer les conditions d'optimum qui garantissent l'existence et l'unicité des lois de commande optimales. Ceci lorsqu'il s'agit d'une dynamique combinée du système.

E2 : Développement et implémentation d'un algorithme de résolution numérique des conditions d'optimum issues de l'étape 1. Cette résolution permettra de développer des lois de commande paramétrées et en rétroaction fonction des états du système sous étude.

E3 : Développement d'un modèle de simulation à l'aide de langage à usage général. Le modèle devra décrire le comportement du système étudié. Cette étape doit servir à valider la robustesse des politiques issus des étapes précédentes, à proposer des processus décisionnels exprimant l'aspect réaliste de nos développements ou étendre les politiques développées pour couvrir des systèmes plus complexes. Cette étape est fortement liée à l'étape suivante.

E4 : Recourir aux approches d'optimisation basées sur la simulation comme moyen permettant de compléter l'étape 3 et offrir des stratégies de gestion réalistes. Dans ce contexte, nous devons opter pour l'approche la plus économique permettant de supporter les aspects quantitatifs et qualitatifs du problème d'optimisation. C'est ce qui explique les deux points E4a et E4b suivants.

> **E4a :** Pour faire face à des problèmes d'optimisation faisant intervenir des paramètres évoluant dans un espace réel, c'est les plans d'expériences et la méthodologie des surfaces de réponse qui seront adoptés.
>
> **E4b :** Pour faire face à des problèmes d'optimisation faisant intervenir des paramètres évoluant dans un espace réel et d'autres de nature qualitative, nous allons développer un module d'optimisation basé sur les algorithmes génétiques. À cette étape, notre maîtrise des plans d'expériences et la méthodologie des surfaces de réponse sera mise à profit pour optimiser l'algorithme génétique employé.

Plus précisément, pour résoudre le problème de gestion et contrôle dynamique stochastique d'un processus de transformation dans un contexte de chaîne d'approvisionnement, cette recherche a pour objet de répondre aux questions **Q1** à **Q3** formulées ci-après :

Q1 : Pouvons nous surmonter les difficultés de modélisation mathématiques des problèmes considérés et considérer une dynamique combinée (i.e., continue et discrète) dans un modèle intégré incluant les aspects dynamiques et stochastiques ?

Q2 : Jusqu'à quel point la résolution numérique des conditions d'optimum, issus de nos modèles mathématiques, nous sera utile pour proposer des stratégies de contrôle réaliste ? Et pouvons nous employer ces résultats pour proposer des solutions à des problèmes plus larges et plus complexes ?

Q3 : Jusqu'à quel point la simulation et les approches d'optimisation expérimentale nous seront utiles pour compléter l'approche purement mathématique et proposer des solutions tangibles et pragmatiques aux problèmes considérés ?

1.6 Objectifs de la thèse et contributions

Proposer une approche globale qui prend en considération tous les aspects précédemment soulevés constitue notre principale contribution. Nous pensons qu'un tel objectif ne peut être atteint qu'une fois nous réussirons de donner des réponses aux questions Q1 à Q3.
Les contributions majeures de cette recherche touchent deux aspects. Le premier est d'ordre théorique et se manifeste par l'extension de formulations existantes ou le développement de nouvelles formulations mathématiques des problèmes sous étude, tandis que le deuxième est d'ordre pragmatique et touche le côté pratique des résultats. Cet aspect permet d'étendre les stratégies développées pour couvrir des systèmes plus larges tout en incluant des activités connexes non considérées dans la première partie de l'approche. De plus, il permet de transformer les stratégies développées en processus décisionnels.

- Notre contribution au premier aspect réside dans la réponse à la question **Q1** qui regroupe l'étape **E1 (E1a et E1b)** de la méthodologie proposée. La nouvelle formulation que nous nous proposons de développer va ouvrir les portes à une application plus étendue de la théorie de commande optimale et impulsionnelle aux chaînes d'approvisionnement et aux réseaux d'entreprise.

- Le deuxième aspect se résume dans les questions **Q2** (étape **E2**) et **Q3** (étapes **E3 et E4**). Cette contribution réside dans la résolution des équations d'optimum et la combinaison de l'approche analytique avec l'approche expérimentale pour résoudre la problématique globale. Le fait de montrer que l'approche proposée permet d'aboutir à des solutions logiques et réalistes ouvrira certainement les portes devant le développement d'un outil d'aide à la décision pratique et proche de la réalité (i.e., basé sur des modèles qui considèrent l'aspect dynamique stochastique du système).

Cette thèse a permis de contribuer à l'avancement de la recherche par 8 articles de revue avec comité de lecture (publiés, acceptés et soumis), 6 conférences internationales (publiés dans des actes et soumis) ainsi que 2 Workshops. La section 1.7 présentera un sommaire des articles inclus, détaillera la structure de la thèse, ainsi que la transition entre les différents chapitres.

1.7 Structure de la thèse

La thèse est constituée de sept chapitres incluant le présent chapitre d'introduction. Les six chapitres qui forment le cœur du travail représentent des articles publiés, acceptés ou soumis à des revues scientifiques avec comité de lecture.

Dans la première partie de la thèse (i.e., chapitres 2, 3 et 4), nous nous adressons aux problèmes de contrôle opérationnels de systèmes série parallèle sujets à des pannes aléatoires et produisant plusieurs types de produit. Pour les systèmes parallèles (i.e., chapitre 2, Gharbi et al. (2006) et chapitre 3, Hajji et al. (2007d)), nous proposons des stratégies de production et de mise en course plus réalistes et plus économiques du point de vue coût. Pour les systèmes séries avec stocks tampons (i.e., chapitre 4, Hajji et al. (2007c)), nous démontrons la grande utilité de la combinaison des deux approches susmentionnées qui peut s'avérer incontournable pour amener des solutions à des problèmes *NP* difficile. Dans ce contexte, nous proposons des politiques de production et de mise en course de ligne de production avec stocks tampons produisant *n* familles de produits.

Dans la deuxième partie de la thèse (i.e., chapitre 5, Hajji et al. (2007e) et 6, Hajji et al. (2007a)), nous considérons le système manufacturier dans son environnement externe. Nous cherchons à déterminer des stratégies intégrées de production, de réapprovisionnement en présence de plusieurs fournisseurs potentiels. Dans ce contexte, nous surmontons les difficultés de modélisation en faisant appel à la théorie de commande optimale et impulsionnelle. Les deux cas étudiés nous montrent clairement l'importance de considérer de façon intégrée les fonctions de production et d'approvisionnement et de considérer, dans un environnement aléatoire, plus d'une alternative d'approvisionnement.

La troisième partie de la thèse (i.e., chapitre 7, Hajji et al. (2007f)) sera consacrée au développement d'un module d'optimisation basée sur la simulation, les algorithmes génétiques et les techniques d'optimisation statistiques tels les plans d'expériences et les surfaces de réponse. Le développement de ce module d'optimisation s'est avéré indispensable pour amener une solution aux difficultés liées aux nombres de paramètres des politiques de contrôle à optimiser et l'intégration d'autres activités connexes à la production telle que les stratégies de maintenance préventive. De plus, il nous permettra de considérer d'une façon intégrée le processus de transformation et son environnement externe (i.e., les fournisseurs).

Pour terminer, nous dressons en guise de conclusion le bilan de ce travail et nous présentons les perspectives de cette thèse.

CHAPITRE 2

OPERATIONAL LEVEL-BASED POLICIES IN PRODUCTION RATE CONTROL OF UNRELIABLE MANUFACTURING SYSTEMS WITH SETUPS

Abstract

This paper deals with the control of the production rates and setup actions of an unreliable multiple-machine, multiple-product manufacturing system. Each part type can be processed for a specified length of time on one of the involved machines. When switching the production from one type to another, each machine requires both setup time and setup cost. Our objective is to determine the production rates and a sequence of setups in order to minimize the total setup and surplus cost. Given the fact that an analytical or even a numerical solution of the problem is very difficult to find, a combined approach is presented. The proposed approach is based on stochastic optimal control theory, discrete event simulation, experimental design, and response surface methodology. We will prove experimentally that an extended version of the hedging corridor policy is more realistic and guarantees better performance for two cases of study. The first one consists of the unreliable one machine case with exponential failure and repair time distributions. The second one, which is more complex and where the optimal control theory may not be easily used to obtain the optimal control policy, consists of five machines facing non exponential failure and repair time distributions. To illustrate the contribution of the paper and the robustness of the obtained control policy, numerical examples and sensitivity analysis are presented.

2.1 Introduction

An important class of stochastic manufacturing systems involves non-flexible machines characterized by significant setup time and costs incurred when production is switched from one product type to another. This class of systems belongs to manufacturing systems for which the problem of determining optimal production policies have been considered by many authors. A significant portion of this research is based on the pioneering work of

Kimemia and Gershwin (1983), who suggested a feedback formulation of the control problem in a dynamic manufacturing environment, and showed that the optimal control has a special structure called the *Hedging Point Policy* (HPP). For such a policy, a non-negative production surplus of parts, corresponding to optimal inventory levels, is maintained during times of excess capacity in order to hedge against future capacity shortages caused by machine failures.

For large-scale manufacturing systems (i.e., involving multiple parts and/or multiple machines), different classes of systems have been investigated in several works. An explicit formulation of the optimal control problem for an unreliable flexible machine which produces multiple part types is provided in Sethi and Zhang (1999). In addition, Gharbi and Kenne (2003) provided a sub-optimal control policy for the multiple parts multiple machines problem. The assumption made in the aforementioned classes of systems is that the machines are completely flexible, and thus do not require setup time or cost when production is switched from one part type to another.

Stochastic manufacturing systems with setup costs and/or times have been considered by Sethi and Zhang (1994), Yan and Zhang (1997) and Boukas and Kenne (1997). The proposed models lead to the optimality conditions described by the Hamilton Jacobi Bellman equations (HJB). Such equations are difficult to resolve analytically for more general cases. An explicit solution for such equations was obtained by Akella and Kumar (1986) for a one-machine, one-product manufacturing system. Numerical methods based on the Kushner approach (see Kushner and Dupuis (1992)) were used by Yan and Zhang (1997) and Boukas and Kenne (1997) for a one-machine, two-product manufacturing system. They were able to develop near-optimal control policies for production, maintenance (in Boukas and Kenne (1997)) and setup scheduling in the case of a homogeneous and machine age-dependent Markovian process, respectively.

For the one machine two products case, Yan and Zhang (1997) provide a characterization of the optimal production and setup policy by four exclusive regions as a main result. Under different assumptions, Liberopoulos and Caramanis (1997) also investigate several numerical examples so as to characterize the production and setup policies of the problem. Their results outline important properties of the value function, and those of the optimal control policy, but the structure of such a policy in the overall sample space is yet to be described or quantified. In the same direction, Bai and Elhafsi (1997) focused their contribution on providing a suitable production and setup policy structure, and obtained the so-called *Hedging Corridor Policy* (HCP). The corridor in such a policy guides the surplus trajectory to target positive stock thresholds built up in order to hedge against future capacity shortages caused by machine failures and large setup times. The setup policy in sample space quadrants related to backlog situations is still unknown with the HCP.

This paper's main contribution lies in the development of a production and setup policy for unreliable multiple-machine multiple-part type manufacturing system, for which the production and setup policy is known across the sample space. The resultant control policy, called the *Modified Hedging Corridor Policy* (MHCP) is more realistic and useful in the context of the production planning of manufacturing systems with setup. This paper's contribution is further illustrated by the fact that the proposed MHCP guarantees a lower incurred cost compared to that resulting from the HCP. A simulation-based experimental design approach is combined with the control theory to develop a systematic control approach, as in Gharbi and Kenne (2003), in the case of manufacturing systems involving setup. Once the superiority of the MHCP is proven through such an approach, extension to cover more complex manufacturing systems will be presented (i.e., multiple- machine multiple part type, non-exponential failure and repair time distributions), where the optimal control theory may not be easily used to obtain the control policy.

The proposed control approach consists of estimating the relationship between the incurred cost and the parameters of the control policy considered here as control factors. The Modified Hedging Corridor Policy, parameterized by these factors, is used to conduct simulation experiments. For each configuration of input factor values, the simulation model is used to determine the related output or cost incurred. An input-output data set is then generated through the simulation model. The experimental design is used to determine significant factors and/or their interactions, and the response surface methodology is applied to the input-output data obtained in order to estimate the cost function and the related optimum. Details on the combination of analytical approaches and simulation-based statistical methods can be found in Gharbi and Kenne (2003) and in the references they provide.

This paper is organized as follows: Section 2.2 presents the statement of the optimal production and setup-scheduling problem. The numerical approach and the related control policy are presented in section 2.3. Sections 2.4 and 2.5 describe the combined control approach and the simulation model. Section 2.6 outlines the experimental design approach and the response surface methodology. The usefulness of the proposed control policy and its extension to the multiple machine case with non-exponential failure and repair time distributions is presented in section 2.7. The paper is concluded in section 2.8.

2.2 Problem statement

The manufacturing system under study (Figure 2.1) consists of *m* unreliable machines M_i, i = 1,..., m capable of producing *n* different part types P_j, j = 1,...,n. Machines are not completely flexible in the sense that changes over time (setup activities) between part types are not easily achieved. This setup involves both time and cost to switch from the production of P_i to P_j, denoted by Θ_{ij}^k *and* K_{ij}^k, respectively with i ≠ j, k=1...m.

Figure 2.1 illustrates the system under study, its dynamics and the associated costs to be minimized.

Part type *i* requires an average processing time $p_i > 0, (i = 1,...,n)$ and has an average time between orders $1/d_i$ assumed to be constant. For an *n* part type system, ***x(t)***, ***u(t)*** and ***d*** denote vectors of the inventory/backlog levels $(x_1(t),\dots,x_n(t))'$, production rates $(u_1(t),\dots,u_n(t))'$ such that $u_i(.)=\sum_{j=1}^{m}u_{ij}(.)$, and demand rates $(d_1,\dots,d_n)'$ respectively.

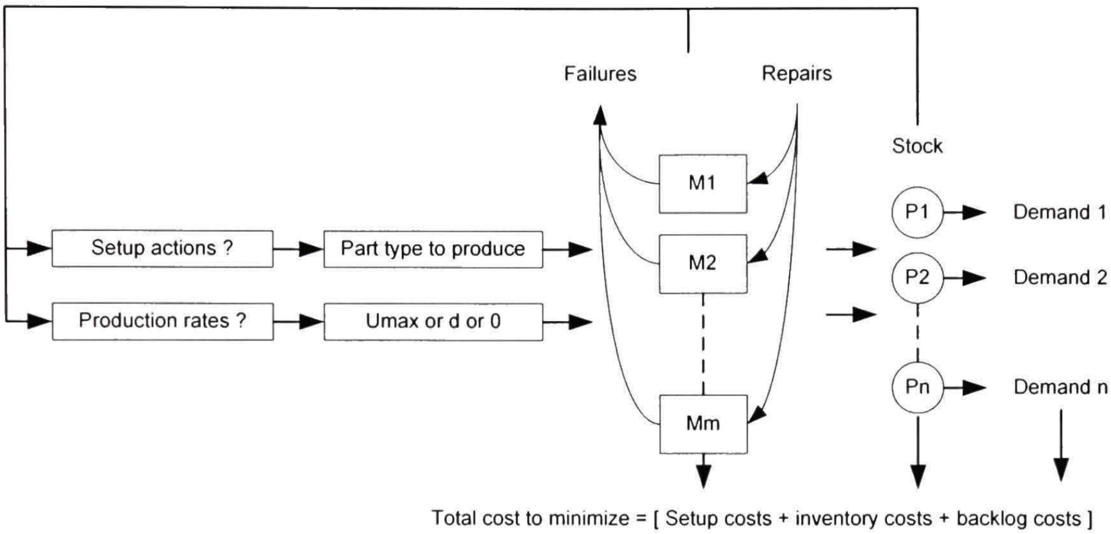


Figure 2.1 Structure of the manufacturing system under study

The state of the system at time t has two components, including a continuous part which describes the cumulative surplus vector, and is measured by $x(t)$ and a discrete part, which describes machines states, and is denoted by $\alpha(\mathrm{t})=(\alpha_1(t),\dots,\alpha_m(t))'$. The state of the stochastic process $\alpha_j(\mathrm{t})$ is equal to 0 if the machine j is under repair and 1 if the machine is operational. For the manufacturing system considered, the state space is given by: $x(\mathrm{t})\in\mathrm{R}^n$, $\alpha(\mathrm{t})\in\mathrm{M}=M_1\times\dots M_m, with\, M_j=\{0,1\}$.

The dynamics of the surplus is given by the following differential equation:

$$\dot{x}(t)=u(t)-d,\quad x(0)=x \tag{2.1}$$

where x denotes the initial vector of surplus levels.

Machine j uptimes and downtimes are assumed to be exponentially distributed with rates p_j *and* r_j, respectively. The machine state evolves according to a continuous-time Markov process with modes in M_j and with a generator matrix Q such that: $Q = \{q_{\alpha\beta}\}$, where $q_{\alpha\beta}$ denotes the transition rates from modes α to β, with $q_{\alpha\beta} \geq 0$ if $\alpha \neq \beta$ and $q_{\alpha\alpha} = -\sum_{\beta \neq \alpha} q_{\alpha\beta}$, $\alpha, \beta \in M$. The transitions rates matrix $\boldsymbol{Q}$ is expressed as follows:

$$Q = \begin{vmatrix} -q_{01} & q_{01} \\ q_{10} & -q_{10} \end{vmatrix} \text{ with } q_{10} = p_j \text{ and } q_{01} = r_j .$$

The production rates at any given time must satisfy the capacity constraint of the system given by the following equation:

$$0 \leq u_{ij}(t) \leq U_{ij}^{\max}(t), \; i = 1...n, \; j = 1..m \tag{2.2}$$

Where $U_{ij}^{\max}(t)$ denotes the maximal production rate of product i on machine j.

For each $\alpha \in M$, the feasible production rates (or capacity) set is given by:

$$\Gamma_i(\alpha) = \{u : u = (\mathrm{u}_1,...,\mathrm{u}_n) \geq 0, 0 \leq \mathrm{u}_{ij} \leq U_{ij}^{\max}(\mathrm{t}), \mathrm{u}_{kj} = 0; \forall k \neq \mathrm{i}\}$$

Our decision variables are production rates $\boldsymbol{u}(.) = (u_1(.),...,u_n(.))$ and a sequence of setups denoted by $\Omega = \{(\tau_0, i_0 i_1), (\tau_1, i_1 i_2), ...\}$. A setup (τ, ij) is defined by the time τ at which it begins and a pair *ij*, denoting that the system was already set up to produce part *i* and is being switched to be able to produce part *j*.

The instantaneous inventory and backlog cost function $g(.)$ is given by the following equation. Where, $x_i^+ = \max(0, x_i)$, $x_i^- = \max(-x_i, 0)$, c_i^- : product type *i* backlog cost and c_i^+ : product type *i* inventory cost.

$$g(x) = \sum_{i=1}^{n} (c_i^+ x_i^+ + c_i^- x_i^-), \tag{2.3}$$

Let i denote the initial setup state of the system and s the remaining setup time. The setup cost is assumed to be charged at the beginning of the setup.

The instantaneous setup cost during s units of time is given by the following equation:

$$R_{ij}(\mathrm{x},s)=\sum_{k=1}^{m}K_{ij}^{k}\,Ind\left\{s=\Theta_{ij}^{k}\right\}+\int_{0}^{S}e^{-\rho t}g(\mathrm{x}-dt)dt,\;\; s\in\left[0,\Theta_{ij}^{k}\right],\ \mathrm{i,j}=1,\ldots,\mathrm{n}\ \text{and}\ i\neq j \qquad (2.4)$$

where ρ denotes the discounted rate of the incurred cost and $Ind\left\{s=\Theta_{ij}^{k}\right\}=\begin{cases}1, & if \;\; s=\Theta_{ij}^{k}\\ 0, & otherwise\end{cases}$.

Using (2.3)-(2.4), the total cost $J(.)$ can be defined by the following expression:

$$J(i,x,\alpha,s,\Omega,u(.))=\int_{0}^{S}e^{-\rho t}g(x(t))dt+E_{i,x-ds,\alpha_{s}}\left[\int_{S}^{\infty}e^{-\rho t}g(x(t))dt+\sum_{k=1}^{m}\sum_{l=0}^{\infty}e^{-\rho\tau_{l}}K_{i_{l}i_{l+1}}^{k}\right] \qquad (2.5)$$

Let A denote the set of admissible decisions (Ω, $\boldsymbol{u}$(.)). The production planning problem considered here is to find an admissible decision or control policy (Ω, $\boldsymbol{u}$(.)) that minimizes J(.), given by (2.5) subject to equations (2.1) to (2.3). Such a feedback control policy, as illustrated in Figure 2.1, determines the production rates and the setup actions as a function of the surplus level $\boldsymbol{x}$ and the state of the system α.

While producing the part type i, the corresponding value function $v_i(.)$ can be given by the following:

$$v_i(x,\alpha,s)=\inf_{(\Omega,u)\in A}J(i,x,\alpha,s,\Omega,u) \qquad \forall x\in R^n,\alpha\in M \qquad (2.6)$$

As in Sethi and Zhang (1994), it can be shown that the value function $v_i(x,\alpha)$ is locally Lipschitz, and is the unique viscosity solution to the following HJB equation:

$$\min\begin{Bmatrix}\min\limits_{u\in\Gamma(\alpha)}\left[(u-d)\,(v_i)_x(x,\alpha)+g(x)+Q.v_i(x,.)(\alpha)\right]-\rho v_i(x,\alpha);\\ \min\limits_{j\neq i}\left[R_{ij}(x,\Theta_{ij})+e^{-\rho\Theta_{ij}}.v_j(x-d\Theta_{ij},1)\right]-v_i(x,\alpha)\end{Bmatrix}=0 \qquad (2.7)$$

where $(v_i)_x(.)$ denotes the gradients of $v_i(.)$ with respect to $\boldsymbol{x}$, $\Theta_{ij} = \underset{k=1,\dots,m}{Sup}\left\{\Theta_{ij}^k\right\}$.

The production and setup policy that we are seeking is obtained when the value function is known. While we cannot analytically solve the HJB equations (2.7), we can however apply numerical methods to obtain the approximation of the value function and the associated control policy as in Yan and Zhang (1997).

2.3 Numerical approach and optimal control policy

In this section, numerical methods are used to approximate the solution of the HJB equations (2.7) corresponding to the stochastic optimal control problem, and to solve the corresponding optimality conditions. This method is based on the Kushner approach (Kushner and Dupuis (1992)). The basic idea behind it consists in using an approximation scheme for the gradient of the value function $v_i(x,\alpha)$.

Let h_j, j =1… n, denote the length of the finite difference interval of the variable x_j. Using the finite difference approximation, $v_i(x,\alpha)$ could be given by $v_i^h(x,\alpha)$, and the gradient $(v_i)_{x_j}(x,\alpha)$ by:

$$(v_i)_{x_j}(x,\alpha)=\begin{cases}\dfrac{1}{h_j}\left(v_i^h\left(x_1,..,x_j+h_j,..,x_n\right)-v_i^h\left(x_1,..,x_j,..,x_n\right)\right) & \text{if } u_j-d_j\geq 0\\ \dfrac{1}{h_j}\left(v_i^h\left(x_1,..,x_j,..,x_n\right)-v_i^h\left(x_1,..,x_j-h_j,..,x_n\right)\right) & \text{if } u_j-d_j<0\end{cases}$$

We could see that:

$$\begin{aligned}\left(u_j-d_j\right)(v_i)_{x_j}(x,\alpha)=&\frac{\left|u_j-d_j\right|}{h_j}v_i^h\left(x_1,..,x_j+h_j,..,x_n\right)Ind\left\{u_j-d_j\geq 0\right\}\\&+\frac{\left|u_j-d_j\right|}{h_j}v_i^h\left(x_1,..,x_j-h_j,..,x_n\right)Ind\left\{u_j-d_j<0\right\}\\&-\frac{\left|u_j-d_j\right|}{h_j}v_i^h\left(x_1,..,x_j,..,x_n\right)\end{aligned}$$

With this approximation, the HJB equations (2.7) are expressed in terms of $v_i^h(x,\alpha)$, as shown in equation (2.8). Such an approximation is used in Hajji et al. (2004) to show that the HJB equations could be represented by the following equation:

$$v_i^h(x,\alpha)=\min\left\{\begin{array}{l}\min\limits_{u\in\Gamma(\alpha)}\left[\begin{array}{l}(\rho+|q_{\alpha\alpha}|+\sum\limits_{j=1}^{n}\frac{|u_j-d_j|}{h_j})^{-1}\\ \left(\begin{array}{l}\sum\limits_{j=1}^{n}\frac{|u_j-d_j|}{h_j}(v_i^h(x(h_j,+))Ind(u_j-d_j\geq 0)+v_i^h(x(h_j,-))Ind(u_j-d_j<0))\\ +g(x)+\sum\limits_{\beta\neq\alpha}q_{\alpha\beta}.v_i^h(x,\beta)\end{array}\right)\end{array}\right];\\ \min\limits_{j\neq i}\left[R_{ij}(x,\Theta_{ij})+e^{-\rho\Theta_{ij}}.v_j^h(x-d\Theta_{ij},1)\right]\end{array}\right\} \quad (2.8)$$

The solution of the numerical approximation of $v_i(x,\alpha)$ may be obtained by either successive approximation or policy improvement techniques (Boukas and Kenne (1997) and Kushner and Dupuis (1992)).

2.3.1 Complexity of the optimal control problem

The dimension of the HJB equations for numerical methods is given by:

$$Dim=2^m\times 3^{m\times n}\times\left[\prod_{i=1}^{n}N_h(x_i)\right]\times 2^{n-1}$$

Where $N_h(x_i)=card[G_h(x_i)]$ with $G_h(x_i)$ describing the numerical grid for the state variable x_i related to product P_i, i=1,…,n. Each machine has two states (i.e., 2^m states for a m-machine manufacturing system) and its production rate can take three values namely maximal production rate, demand rate and zero for each product (i.e., $3^{m\times n}$ states for a *m*-machine, *n*-product manufacturing system). While producing a product type, one of two possible decisions must be taken (i.e., 2^{n-1} for a *n*-product manufacturing system). Based on such dimension, the related numerical algorithm for the five-machine, two-product case is very difficult to implement and to solve. Such system is classified here as complex systems.

Due to the complexity of the HJB equations, the objective of this paper is not to solve them for the complex case, but to determine experimentally the parameters of a modified hedging corridor policy proved to be a best approximation of the optimal control policy than the classical one (i.e., Hedging corridor policy). It will be shown, in the next sections, and without loose of generality that the MHCP guarantees better performance than does the HCP for the one-machine two-product and the five-machine two-product manufacturing systems.

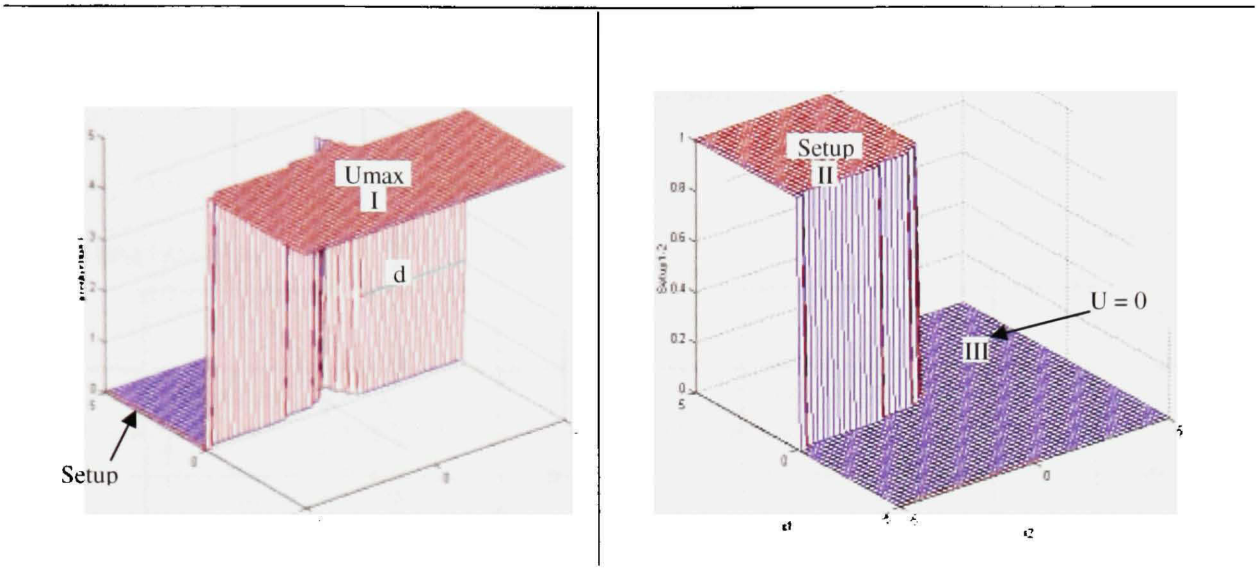
2.3.2 Numerical results of the one-machine two-parts type case

The implementation of the approximation technique requires the use of a finite grid denoted by G_h, where h is a given vector of a finite difference interval. The considered computation domain D is given by:

$$\mathrm{D} = \{(x_1, x_2) : -5 \le x_1 \le 5 \,, -5 \le x_2 \le 5, \, h_1 = h_2 = 0.2\}.$$

To ensure a clear characterization of the control policy, several elements were taken into consideration as part of the implementation process. Indeed, the production and setup policies, in which the machine produces part type i for example, are each observed separately. For each policy, the relevant significant stock threshes are analyzed independently of the others. For each numerical result, the policies are provided as shown in Figure 2.2. The resulting production and setup policies (Figure 2.2 (a) and Figure 2.2 (b), respectively) divide the surplus space into the following three mutually exclusive regions:

- In region I, keep the same setup and produce the part type at the maximum or demand rate,
- In region II, change the configuration of the machine and produce the other part type,
- In region III, keep the same setup of the machine and set the production rate to zero.



(a) Production policy (b) Setup policy

Figure 2.2 Part type 1 production and setup policies

The numerical results used to characterize the optimal production and setup policies are analyzed in this section within several cases. Table 2.1 shows the constant parameters for all numerical examples considered and an illustrative case of variable parameters (i.e., inventory and backlog costs).

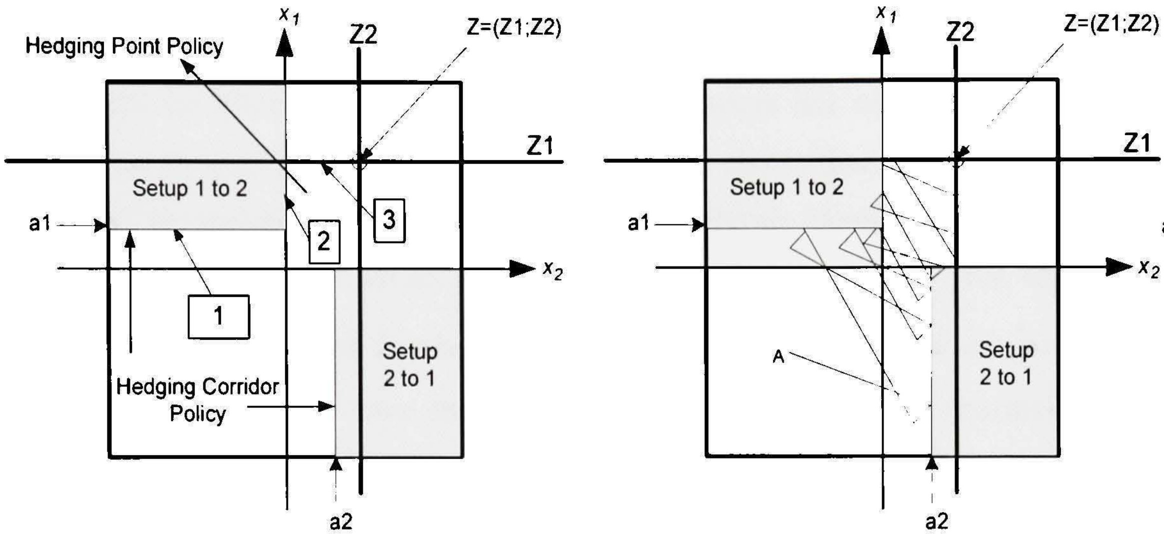
Tableau 2.1

Data parameters

Parameters	(c_i^+, c_i^-)	(K_{12}^1, K_{21}^1)	$(\Theta_{12}^1, \Theta_{21}^1)$	(p, r)	$(u_{11}^{max}, u_{21}^{max})$	(d_1, d_2)	ρ
Values	(1,5)	(0.5,0.5)	(0.16,0.16)	(0.15,0.8)	(5,5)	(2,2)	0.9

It follows from our numerical results that the optimal policy has a particular structure, which we call here the *Modified Hedging Corridor Policy* "MHCP" and illustrated in Figure 2.3. This policy is a combination of the Hedging Corridor Policy (HCP) and the Hedging

Point Policy (HPP). Let Z1 and Z2 define the threshold of products P1 and P2 respectively. Let also and a1 and a2 define the boundary of the corridor.



(a) Setup actions through MHCP (b) Inventory itinerary through MHCP

Figure 2.3 Modified Hedging Corridor policy

The results obtained are different from those found by Yan and Zhang (1997) and Bai and Elhafsi (1997). The proposed MHCP is defined by two hedging levels in the negative and positive areas of each part type. When there is a shortage in the stock level of one part type, the setup action must be performed earlier than the same action when the stock level is still in the positive zone. These actions are conducted with respect to the boundaries of the corridors a_1 *and* a_2 (see arrow 1, Figure 2.3 (a)). However, when the two stock levels are positive, we can proceed with production according to a hedging point policy (see arrow 3, Figure 2.3 (a)), in this case the setup actions are performed when the stock level of the concerned part type reaches the negative area (see arrow 2, Figure 2.3 (a)). The proposed modified hedging corridor policy, as shown in Figure 2.3 (b), states that if starting at point A and reaching point Z, which is the hedging point defined by the intersection of the two hedging levels, the stock trajectory travels through two different corridors, with one in the negative and combined stock area and the other in the positive area.

2.3.3 Sensitivity analysis: parameterized control policy

To illustrate the effect that changing the cost parameters has on the policy observed, a sensitivity analysis (Table 2.2) has been performed. It shows that when backlog costs rise, the value of the hedging thresholds and the boundary levels increase accordingly to ensure the availability of enough stocks to hedge against future backlogs. This observation is confirmed by the numerical threshold levels Z_1^*, Z_2^* and setup boundaries a_1^*, a_2^* presented in Table 2.2 (set I, basic case and cases 1 to 3). Moreover, when the inventory costs increase, the values of the hedging thresholds and the boundary levels decrease to confine the stock accumulation. This observation is confirmed by the numerical threshold levels Z_1^*, Z_2^* and setup boundaries a_1^*, a_2^* presented in Table 2.2 (set II, basic case and cases 1 to 3).

Tableau 2.2

Data parameters for the sensitivity analysis cases

	Cases	c_1^+	c_1^-	c_2^+	c_2^-	(Z_1^*, Z_2^*)	(a_1^*, a_2^*)
Set I	Basic	1	5	1	5	(1.8,1.8)	(0.2,0.2)
	1	1	10	1	10	(2,2)	(0.3,0.3)
	2	1	30	1	30	(2.2,2.2)	(0.4,0.4)
	3	1	60	1	60	(2.6,2.6)	(0.5,0.5)
Set II	Basic	1	60	1	60	(2.6,2.6)	(0.5,0.5)
	1	5	60	5	60	(1.8,1.8)	(0.4,0.4)
	2	10	60	10	60	(1.2,1.2)	(0.3,0.3)
	3	20	60	20	60	(0.6,0.6)	(0.2,0.2)

From the above analysis, it clearly appears that the results obtained make sense, and that the structure of the policy defined by the 4 parameters (a_1 , a_2 , Z_1 *and* Z_2) is always maintained. This allows the development of a parameterized production and setup control policy defined by the following equations:

$$u_1(.) = \begin{cases} u_1^{\max}.Ind\{S_{21} = 1\} & if\ x_1 < Z_1 \\ d_1\ .Ind\{S_{21} = 1\} & if\ x_1 = Z_1 \\ 0 & if\ x_1 > Z_1 \end{cases} \tag{2.9}$$

$$u_2(.) = \begin{cases} u_2^{\max}.Ind\{S_{12} = 1\} & if\ x_2 < Z_2 \\ d_2.Ind\{S_{12} = 1\} & if\ x_1 = Z_2 \\ 0 & if\ x_2 > Z_2 \end{cases} \tag{2.10}$$

$$S_{12} = \begin{cases} 1 \quad if & \begin{cases} x_1 \geq a_1 \\ and \\ x_2 \leq 0 \end{cases} \\ 0 \quad otherwise \end{cases} \tag{2.11}$$

$$S_{21} = \begin{cases} 1 \quad if & \begin{cases} x_2 \geq a_2 \\ and \\ x_1 \leq 0 \end{cases} \\ 0 \quad otherwise \end{cases} \tag{2.12}$$

with the following constraints:

$$0 \leq a_1 \leq Z_1 \text{ and } 0 \leq a_2 \leq Z_2 \tag{2.13}$$

The modified hedging corridor policy presented by equations (2.9) to (2.12) is completely defined for given values of a_i and Z_i (i=1,2), called here design factors. The next sections are aimed at developing a systematic approach for determining optimal values of a_i and Z_i (i=1,2).

2.4 Control approach

In order to bring an approach which could be easily applied to control manufacturing systems at the operational level, the descriptive capacities of discrete event simulation models are combined with analytical models, experimental design, and response surface methodology. A block diagram of the resulting control approach is depicted in Figure 2.4. This approach has been successfully used to control production and perform preventive maintenance activities in the cases of single-machine and multiple-identical-machine flexible manufacturing systems (see Kenne and Gharbi (1999) and Gharbi and Kenne (2000)).

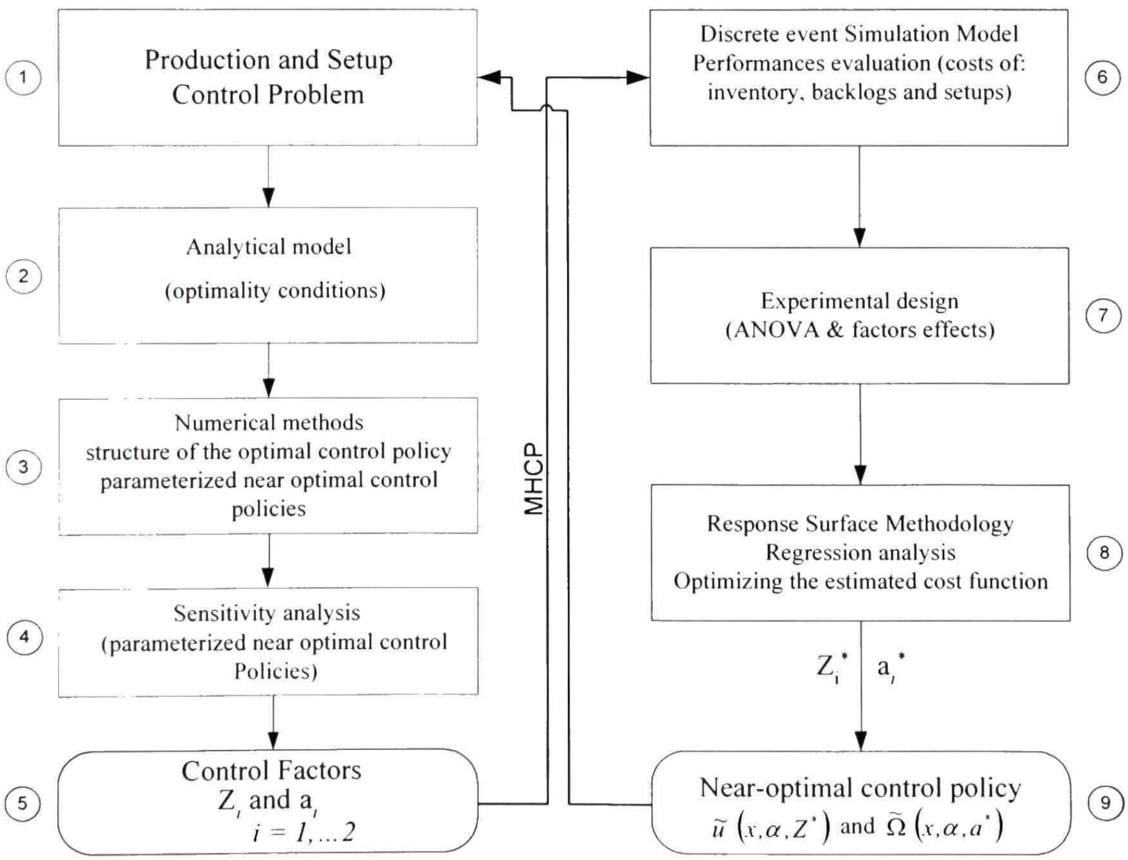


Figure 2.4 Proposed control approach

The structure of the proposed control approach presented in Figure 2.4 consists of the following sequential steps:

1. The *Control problem* statement of the manufacturing system, as shown in section 2.2, consists of a representation of the production and setup control problem through a stochastic optimal control model based on the control theory. Hence, the problem of the optimal flow control for the manufacturing system considered is described in this first step, which contains a specification of the objective of the study. That objective is to find the control variables (***u(x,α), Ω(x,α)***), called the *production rates and setup actions*, in order to improve the related output (i.e., the incurred cost).

2. The *optimality conditions*, described by the HJB equations, are obtained from the problem statement of the previous step. It is shown in this step that the value function, representing the incurred cost, is the solution of the HJB equations, and the corresponding control policy (production rates and setup actions) is optimal.

3. The *numerical methods* are used in this step to solve the optimality equations of the problem, given that there is no way of solving them analytically.

4. The *sensitivity analysis* is conducted to illustrate the effects that changing certain parameters has on the numerical results. It ensures the proper characterization of the control policy structure so as to develop a parameterized policy.

5. The *control factors* Z_i, $i=1,...2$ for production rates control and a_i, $i=1,...2$ for setup actions control, describe the numerical control policy obtained.

6. The *simulation model* uses the near-optimal control policy defined in the previous step as the input for conducting experiments in order to evaluate the performances of the manufacturing system. Hence, for given values of the control factors, the cost incurred is obtained from the simulation model presented later in section 2.5.

7. The *experimental design* approach defines how the control factors can be varied in order to determine the effects of the main factors and their interactions (i.e., analysis of variance or ANOVA) on the cost through a minimal set of simulation experiments.

8. The *response surface methodology* is then used to obtain the relationship between the incurred cost and significant main factors and interactions given in the previous step. The obtained model is then optimized in order to determine the best values of factors called here Z_i^* for production, and a_i^* for setup actions.

9. The *near-optimal control policy* $(\tilde{u}(x,\alpha,Z^*),\tilde{\Omega}(x,\alpha,a^*))$ is thus an improved Modified Hedging Corridor Policy to be applied to the manufacturing system.

The application of the proposed control approach gives the production rates and setup actions described by equations (2.9), (2.10) and (2.11), (2.12) respectively, for the best values of factors Z_i and a_i (i.e., Z_i^* and a_i^*).

2.5 Simulation model

A discrete event simulation model which describes the continuous dynamics of the system (2.1) and its discrete stochastic behaviour is developed using the Visual SLAM language (Pritsker and O'Reilly (1999)). This model consists of several networks, each of which describes a specific task in the system (i.e., demand generation, control policy, states of the machines, inventory control..., etc.). The diagram of the proposed simulation model is shown in Figure 2.5 with the following block notation descriptions:

1. The *INITIALIZATION* block initializes the problem variables (current surplus, production rates, incurred cost, product type to start with…etc.)

2. The *Demand Arrival* block performs the arrival of a demand for product *j* at each d_j^{-1} unit of time. A verification test is then performed on the inventory level of product *j*, and the inventory or the backorder is updated.

3. The *CONTROL POLICY* segment block is defined in section 2.3 (see equations (2.9) to (2.12) for the machine production rates and setup actions). The feedback control policy is defined by the output of the *FLAG* block. This block is used to permanently verify the variation in the stock level $x_j(t)$ in order to specify the best action to carry out (production rate and setup actions).

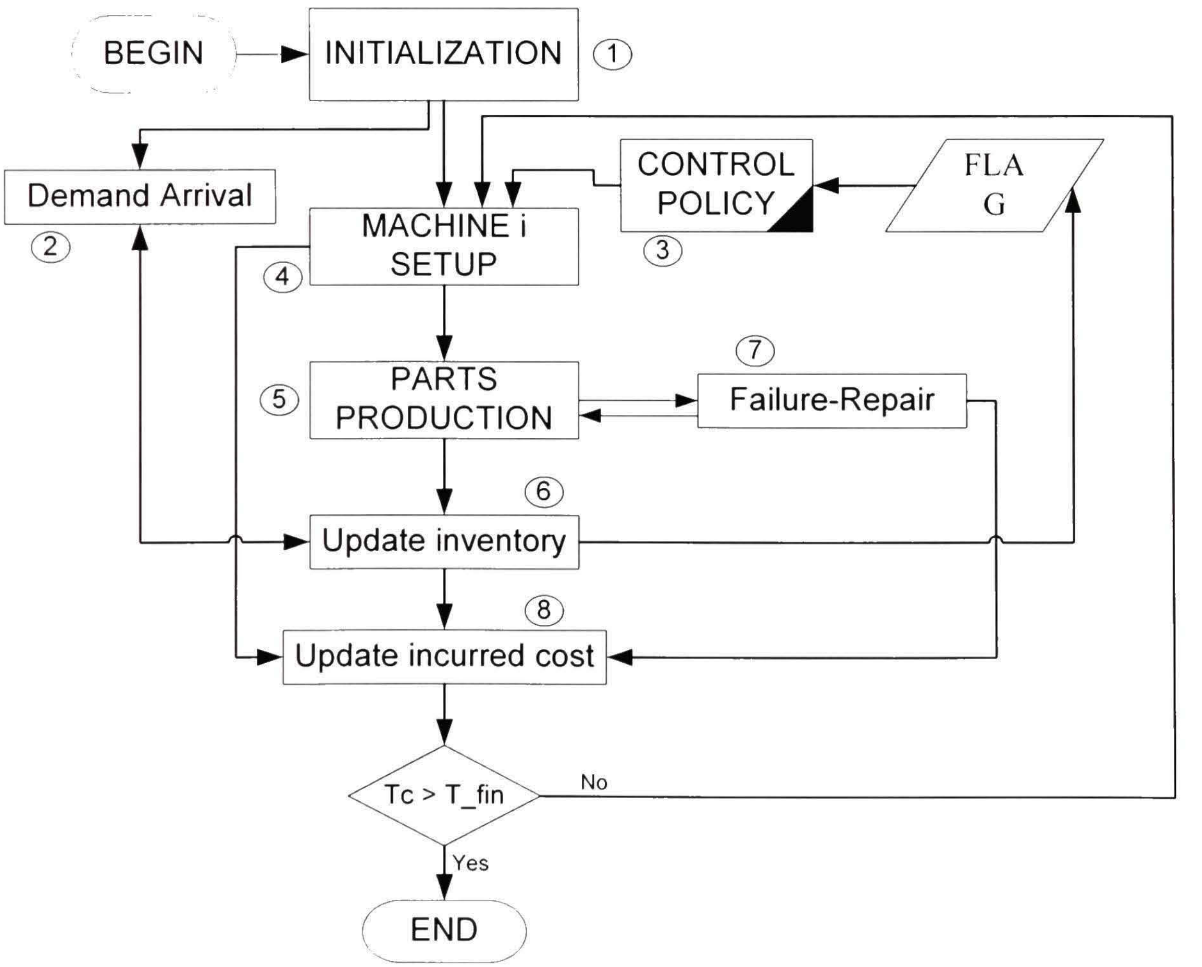


Figure 2.5 Diagram of the simulation model

4. The *MACHINE SETUP* block performs the setup of the machine i according to the policy defined by the *CONTROL POLICY* block.

5. The *PARTS PRODUCTION* block performs the production of finished goods according to the policy defined by the *CONTROL POLICY* block.

6. The *update inventory* block performs the variation of the inventory level when a part is produced or when a demand arrival occurs (i.e., production of finished goods increases inventory if there is no backorder or it satisfies the cumulative demands, and hence decreases backorders). Off-line runs of the simulation model, for a one-machine, two-parts type manufacturing system, using the control policy described by (2.9) to (2.12) for $Z_i = 10$ and $a_i = 5$, $i = 1,2$, is illustrated in Figure 2.6. We should recall that production and setup actions are conducted with respect to the hedging levels Z_1 *and* Z_2 and the boundaries of the corridor a_1 *and* a_2. The setup actions are performed according to a hedging corridor policy in the negative and the combined zones of the inventory (arrow 1, Figure 2.6). When the two stock levels are positive, we can produce according to a hedging level policy (arrow 3, Figure 2.6), in this case the setup actions are performed when the stock level of the concerned part type reaches the negative area (arrow 2, Figure 2.6). It is interesting to note that arrows 1 to 3 in Figure 2.6 represent the same phenomena observed in Figure 2.3 (a) and pointed out by arrows 1 to 3.

7. The *failure-repair* block performs two functions: it defines the time-to-failure of the machine, and repairs a broken one.

8. The *update the incurred cost* block calculates the cost of inventory, backlogs and setup actions.

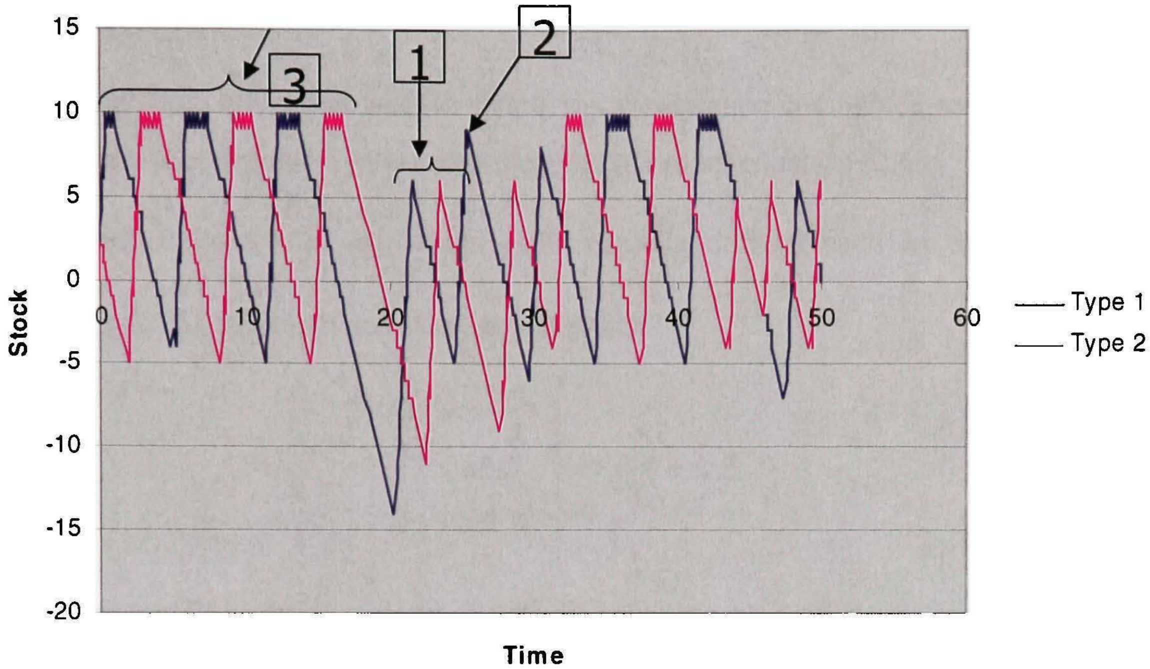


Figure 2.6 Two-products stock trajectory using MHCP (Z1=Z2=10 and a1=a2=5)

In the next section, we present the procedure for varying the control factors simultaneously so as to obtain the appropriate relationship between the incurred cost and significant main factors and interactions. Such a procedure is known as the *experimental design and response surface methodology approach.*

2.6 Experimental Design and Response Surface Methodology

The objectives of this section are to: (i) determine whether the input parameters affect the response, (ii) estimate the relationship between the cost and significant factors, and finally, (iii) compute the optimal values of estimated factors.

For the identical product type case (i.e., $c_1^+ = c_2^+$, $c_1^- = c_2^-$, $K_{12} = K_{21}$, $\Theta_{12} = \Theta_{21}$, $u_1^{max} = u_2^{max}$ and $d_1 = d_2$), we obtained $Z_1 = Z_2$ and $a_1 = a_2$. In this particular case, the control policy is defined by two design factors (a and Z) instead of four.

2.6.1 Experimental design

In this study, we collect and analyze data for a steady state cost which as much as possible approximates that defined by the value function given by equation (2.6).

$\alpha = \frac{a}{Z}$ (with $0 \leq \alpha \leq 1$) is an independent variable defined such as to ensure that the constraints (2.13) are respected. We could see that:

$$\left.\begin{array}{c} \alpha = \frac{a}{Z} \\ and \\ 0 \leq \alpha \leq 1 \end{array}\right\} \Rightarrow 0 \leq a \leq Z$$

The experimental design is concerned with (i) selecting a set of input variables (i.e., factors α and Z) for the simulation model; (ii) setting the levels of selected factors of the model and making decisions on the conditions, such as the length of runs and number of replications, under which the model will be run.

Two independent variables and one dependent variable (the cost) are considered. The levels of independent variables or design factors must be carefully selected to ensure they properly represent the domain of interest. Due to the convexity property of the value function (2.6) (Sethi and Zhang (1994)), the first-order response surface model is rejected. Hence, we selected a 3^2-response surface design since we have 2 independent variables, each at three levels. The levels of the independent variables were selected as in Table 2.3.

Tableau 2.3

Levels of the independent variables

	Low level	Center	High level
α = a/Z	0.1	0.5	0.9
Z	6	18	30

Four replications were conducted for each combination of the factors, and therefore, 36 (3^2 x 4) simulation runs were made. To reduce the number of replications, we used a variance reduction technique called *common random numbers* (Law and Kelton (2000)). We conducted some preliminary simulation experiments using 4 replications, and noticed that the variability allows the effects to be distinguished. It is interesting to note that all possible combinations of different levels of factors are provided by the response surface design considered. The experimental design is used to study the effects that some parameters, namely α and Z, and their interactions have on the performance measure (i.e., the cost).

2.6.2 Statistical analysis

The statistical analysis of the simulation data consists of the multi-factor analysis of the variance (ANOVA). This is done using a statistical software application such as STATGRAPHICS, to provide the effects of the two independent variables on the dependent variable. Table 2.4 illustrates the ANOVA for $c_1^+ = c_2^+ = 5$ and $c_1^- = c_2^- = 15$. From Table 2.4, we can see that the main factors α and Z, their quadratic effects, as well as their interactions are significant at the 0.05 level (i.e., P-value < 0.05; symbol S in the last column). One more result that stands out in the ANOVA table is the *blocks* effect, which appears to be non-significant (symbol NS in the last column). This effect is due to the aforementioned variance reduction technique. The technique guarantees the generation of the same sequence of random numbers, thus the same failure and repair times, within the different runs of one *block* (one replication). However, a different sequence of random numbers is generated from one *block* to another (one replication to another). Consequently, it was expected that the *block* effect would be non-significant.

The residual analysis was used to verify the adequacy of the model. A *residual* versus *predicted* value plot and *normal probability* plot were used to test the homogeneity of the variances and the residual normality, respectively. We concluded that the normality and equality of variance could be improved. Thus, a data transformation was conducted. An analysis of the square of the response variable led to satisfactory plots. Moreover, the R-squared value increased from 0.91 before transformation to 0.951 after transformation, as presented in Table 2.4. This indicates that more than 95% of the total variability is explained by the model (Montgomery (2001)), which is very satisfactory. The model obtained includes two main factors (α and Z), two quadratic effects (α^2 and Z^2), and the interaction effect $\alpha \times Z$.

Tableau 2.4

ANOVA table

Analysis of Variance for (Total cost 3)^(2)

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	
A:Alpha	1,47224E10	1	1,47224E10	335,61	0,0000	S
B:Z	4,74927E9	1	4,74927E9	108,26	0,0000	S
AA	2,2597E9	1	2,2597E9	51,51	0,0000	S
AB	2,08089E8	1	2,08089E8	4,74	0,0383	S
BB	1,34277E9	1	1,34277E9	30,61	0,0000	S
blocks	2,24566E7	3	7,48554E6	0,17	0,9153	NS
Total error	1,18444E9	27	4,38682E7			
Total (corr.)	2,44892E10	35		R-squared = 95,1634 percent		

2.6.3 Response Surface Methodology

The Response surface methodology is a collection of mathematical and statistical techniques that are useful for modeling and analyzing problems in which a response of interest is influenced by several variables, and the objective is to optimize this response (Montgomery (2001)). We assume here that there exists a function Φ of α and Z that provides the value of the cost corresponding to any given combination of input factors, i.e., $Cost^2 = \Phi(\alpha, Z)$.

The function $\Phi(.)$ is called the response surface, and is assumed to be a continuous function of α and Z. The second order model is thus given by:

$$\Phi = \beta_0 + \beta_{11}\alpha + \beta_{12}Z + \beta_{21}\alpha^2 + \beta_{22}Z^2 + \beta_3\alpha.Z + \varepsilon \quad (2.14)$$

where α and Z are the input variables; $\beta_0, \beta_{11}, \beta_{12}, \beta_{21}, \beta_{22}$ *and* β_3 are unknown parameters, and ε is a random error. From STATGRAPHICS, the estimation of unknown parameters is performed, and the following six coefficients achieved. The values of these coefficients are:

$\beta_0 = 138448.0$, $\beta_{11} = -180484$, $\beta_1^2 = -4786.81$, $\beta_{21} = 105041.0$, $\beta_{22} = 89.96$, and $\beta_3 = 751.31$.

The corresponding response surface is presented in Figure 2.7. The optimum is obtained for α* = 0.77 and Z* = 23, and the incurred square cost Φ* is 12540.3. Thus, a*=17 and Z*=23, and the incurred cost is $\sqrt{\Phi^*} = 112$.

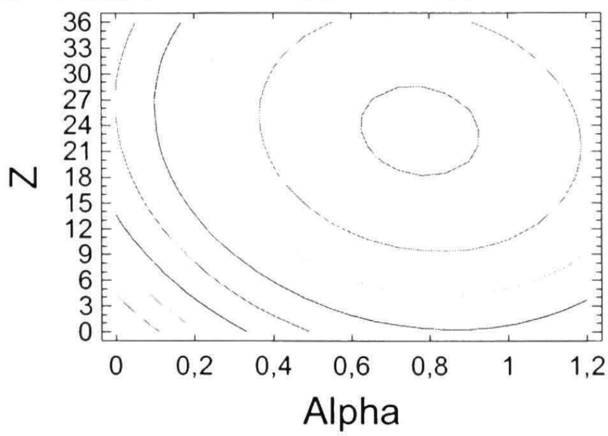


Figure 2.7 Cost response surface

2.6.4 Sensitivity analysis

To illustrate the effect of the cost variation on the design parameters, a sensitivity analysis was conducted. Table 2.5 details the cost variations, and presents the optimal parameters and the incurred optimal costs for the sensitivity analysis cases. It clearly appears that the results obtained make sense and confirm the numerical observation in the sense that when the backlog cost increases, cases 3 to 5, (resp. decreases, cases 3 to 1), the hedging levels and the corridor boundaries increase (resp. decrease).

Tableau 2.5

Optimal design factors and incurred costs with MHCP

Cases	K_{ij}	c_1^+	c_1^-	c_2^+	c_2^-	a*=α.Z	Z*	c^*_{MHCP} §	Remark
1	30	5	8	5	8	11	15	83	$Z^* \downarrow, a^* \downarrow$
2	30	5	10	5	10	13	18	90	$Z^* \downarrow, a^* \downarrow$
3	30	5	15	5	15	17	23	112	Basic case
4	30	5	20	5	20	20	25	119	$Z^* \uparrow, a^* \uparrow$
5	30	5	25	5	25	21	26	125	$Z^* \uparrow, a^* \uparrow$

§ c^*_{MHCP} the optimal incurred cost under MHCP

In the next section, a comparison between the MHCP and HCP is conducted.

2.6.5 Comparison of MHCP and HCP

The hedging corridor policy (Bai and Elhafsi (1997)) is presented in Figure 2.8. The structure of such a policy is defined by two thresholds related to the two-part type. This corridor guides the surplus trajectory to target positive stock thresholds built up to hedge against future capacity shortages brought about by machine failures and setups.

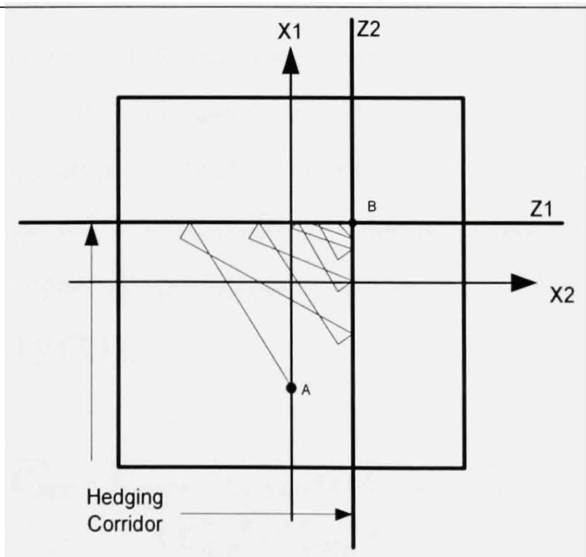


Figure 2.8 Hedging Corridor Policy

We present in Table 2.6 the incurred optimal costs for the same sensitivity analysis input, conducted with the Hedging Corridor Policy (HCP). It is important to note that the results presented in Table 2.6 were obtained under the same conditions (simulation, experimental design and RSM), and following the same approach under which the sensitivity analysis was conducted for the MHCP (table 2.5).

Tableau 2.6

Optimal design factors and incurred costs with HCP

Cases	K_{ij}	c_1^+	c_1^-	c_2^+	c_2^-	Z*	c^*_{HCP} §	Remark
1	30	5	8	5	8	14	89.4	$Z^* \downarrow$, $c^*_{HCP} > c^*_{MHCP}$
2	30	5	10	5	10	16	98.6	$Z^* \downarrow$, $c^*_{HCP} > c^*_{MHCP}$
3	30	5	15	5	15	18	114	Basic case, $c^*_{HCP} > c^*_{MHCP}$
4	30	5	20	5	20	20	123.4	$Z^* \uparrow$, $c^*_{HCP} > c^*_{MHCP}$
5	30	5	25	5	25	21	130	$Z^* \uparrow$, $c^*_{HCP} > c^*_{MHCP}$

§ c^*_{HCP} the optimal incurred cost under HCP

The results obtained show that under HCP, the variation of the design parameter does make sense. Thus, when the backlog cost increases, cases 3 to 5, (resp. decreases, cases 3 to 1), the hedging levels increase (resp. decrease). However, the incurred costs for all the cases are higher than those incurred under MHCP. To confirm the numerical observation and hence the advantage of the proposed MHCP policy compared to that of HCP, a student test was performed in order to compare the performance of the two policies. The confidence interval of $c^*_{HCP} - c^*_{MHCP}$ is given by (2.15).

$$\begin{aligned} &\overline{C}^*_{HCP} - \overline{C}^*_{MHCP} - t_{\alpha/2,n-1} s.e(\overline{C}^*_{HCP} - \overline{C}^*_{MHCP}) \\ &\leq C^*_{HCP} - C^*_{MHCP} \leq \\ &\overline{C}^*_{HCP} - \overline{C}^*_{MHCP} + t_{\alpha/2,n-1} s.e(\overline{C}^*_{HCP} - \overline{C}^*_{MHCP}) \end{aligned} \tag{2.15}$$

where:

$t_{\alpha/2,n-1}$ is the student coefficient function of n and α, with n the number of replications (set at 10) and (1-α), the confidence level (set at 95%).

$s.e(\overline{C}^*_{HCP} - \overline{C}^*_{MHCP}) = \frac{S_D}{\sqrt{n}}$ Standard error,

$$S_D^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(C^*_{HCPi} - C^*_{MHCPi}\right)^2 - n\left(\overline{C}^*_{HCP} - \overline{C}^*_{MHCP}\right)^2\right)$$

$\overline{C}^*_{HCP}$ the average optimal cost incurred under HCP.

$\overline{C}^*_{MHCP}$ the average optimal cost incurred under MHCP

The two configurations under study (HCP and MHCP) were simulated with their optimal design parameters, and the results are presented in Table 2.7.

Tableau 2.7

MHCP Vs. HCP incurred costs for cases 1 to 5, 95% confidence interval

	CASE	1	2	3	4	5
MHCP	C^*_{MHCP}	83	90	112	119	125
HCP	C^*_{HCP}	89.4	98.6	114	123.4	130
Confidence interval	Lower bound	0.5	1.72	1	1.6	0.5
	Upper bound	0.95	2.3	1.5	3.2	1.7

It has been shown that in all cases, it can be concluded that C^*_{HCP} - C^*_{MHCP}>0 at the 95% confidence level. Consequently, the MHCP gives the lower optimal cost, and furthermore, it appears that the MHCP is better than the HCP, and can be used to better approximate the optimal control policy.

2.6.6 Usefulness of the MHCP

As mentioned in the preceding sections, the production and setup policies given by equations (2.9) to (2.12) are completely defined for the given values of a_i and Z_i (i=1,2). These equations explicitly stipulate a feedback policy, and based on the stock levels and the state of the machine, specify the best action to be taken so as to minimize the expected total discounted cost. Thus, the production policy states that when the machine is set up for a one-part type *i*, production must proceed at the maximum rate until the hedging level Z_i is reached, and then proceed at the demand rate at Z_i and stop beyond Z_i. This policy is conditional on the setup policy. It states that when $a_i \leq X_i$ and $X_j < 0$, a setup action must be performed from product type *i* to product type *j*.

In what follows (Figure 2.9), the quantified feedback policy of the basic case of the sensitivity analysis (Table 2.5, section 2.6.4) is presented. This illustration shows the actions that should be taken when the machine is producing part type 1, and is a function of the stock level of product type 1 and type 2 (X_1 and X_2). When the machine is producing part type 2, a mirror schema could be realized so as to achieve a complete production and setup strategy.

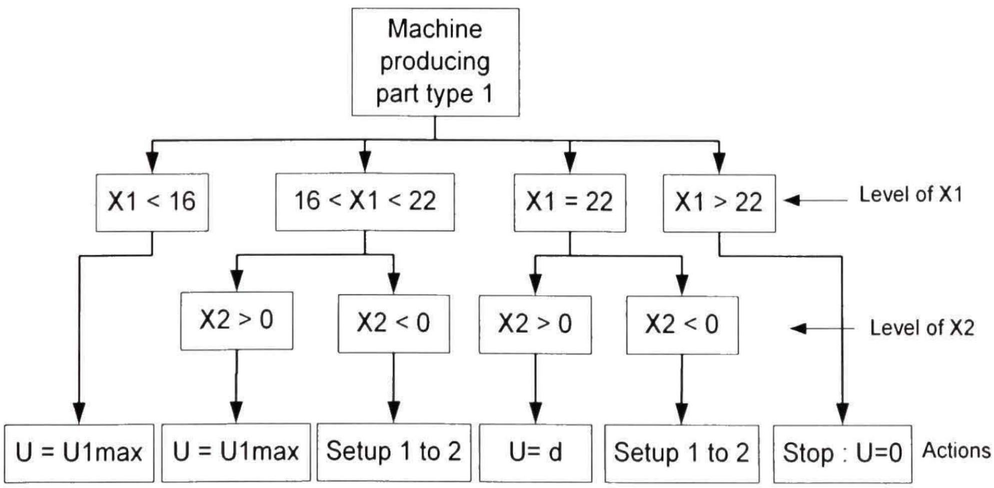


Figure 2.9 Practical solution

2.7 Extension to five-machine manufacturing system with non-exponential failure and repair time distributions

Within the framework of the classical control theory of the last 30 years, no satisfactory method has been devised for the stochastic optimal control of manufacturing systems subject to non-exponential machine up and down times. In fact, the exponential distribution is used to develop optimality conditions, as shown in section 2.2. With non-exponential failure and repair time distributions and/or random demand rates, optimality conditions such as those given by equations (2.7) are very difficult to develop. However, that is the situation that is usually encountered in real manufacturing systems. We refer the reader to Law and Kelton (2000), chapter 6, for details on commonly used demand rates or failure and repair probability time distributions. Despite the demand fluctuations and the types of failure and repair time distributions, a near-optimal control policy could be determined for an unreliable multiple-machine multiple-part type manufacturing system, in a much more complex situation (with non-exponential up and down distribution times for machines and/or random demand rates for products). In the next paragraph, we will explain how this can be done, based on our previous work. In fact, we have already extended the concept of the hedging point policy in the case of one machine producing one kind of product with non-exponential failure and repair time distributions (Kenne and Gharbi (2000)) and in the case of multiple-product multiple-machine manufacturing systems not involving setups (Gharbi and Kenne, (2003)).

For the type of manufacturing system presented in this paper, an exponential failure and repair time distributions and a constant demand rate are first used in order to allow the definition of the optimal setup and production policies, which are then described through a modified hedging corridor policy (MHCP). The structure of such a policy is then parameterized by factors representing the thresholds of the products involved and the boundaries of the corridors. To quantify such a policy, simulation experiments are combined with experimental design and response surface methodology to estimate the optimal values of the MHCP's parameters. In the case of non-exponential machine up and down times and/or random demand rates, the quantification of the MHCP's parameters is possible with the help of the simulation model, which can easily take into account the nature of the machine's failure and repair time distributions and the randomness of the demand. The corresponding output is then given by the simulation model (i.e., cost) which affects the response surface model. Let us now develop the quantified control policy for the five-machine two-product manufacturing systems case. The studied system is subject to non-exponential failure and repair time distribution (i.e., lognormal distribution for example).

Table 2.8 presents the data parameters of two cases (i.e., identical and different machine) describing the five-machine two-product manufacturing system under study. As mentioned previously, the numerical methods for such a system are characterized by a large dimension (see section 2.3.1). In addition, we consider in this section non-exponential distribution machines running and down times. In such a situation the system is known as complex one.

Tableau 2.8

Data table for five-machine two-product manufacturing system

Parameters	$(u_{1j}^{max}, u_{2j}^{max})$, j=1,…,5	(d_1, d_2)	$(\Theta_{12}^{j}, \Theta_{21}^{j})$, j=1,…,5	MTBF			MTTR	
Case1 Identical machines	(1,1)	(2,2)	(1,1)	Lognormal (95,10)			Lognormal (2.5,1.5)	
Case 2 Different machines	(1,1)	(2,2)	(1,1)	M	Lognormal		Lognormal	
					μ	σ	μ	σ
				1	100	10	2.6	1.5
				2	90	10	2.7	1.5
				3	85	10	2.3	1.5
				4	105	10	2.4	1.5
				5	95	10	2.5	1.5

We present in Table 2.9 and Table 2.10 the incurred optimal costs for the same sensitivity analysis input, conducted for the one-machine two- product manufacturing system. It is important to note that the results presented in Table 2.9 and Table 2.10 under the HCP and MHCP policies, respectively, were obtained under the same conditions (simulation, experimental design and RSM).

Tableau 2.9

Optimal design factors and incurred costs with HCP

Cases	K_{ij}^{j}	c_1^+	c_1^-	c_2^+	c_2^-	Z* (case1)	c_{HCP}^*(case1)	Z* (case2)	c_{HCP}^*(case2)
1	6	5	8	5	8	6	68	7	68.5
2	6	5	10	5	10	7	72	8	72.2
3	6	5	15	5	15	8	78	9	78.3
4	6	5	20	5	20	9	82	10	82
5	6	5	25	5	25	9	84	10	84.6

The results obtained show that under HCP, the variation of the design parameter for the two cases (i.e., 1 and 2) does make sense (see section 2.6.4 and 2.6.5 for details). In addition, the incurred costs for all the cases are higher than those incurred under MHCP. This result is shown in Table 2.10. In the same direction, we observe that the incurred costs (under HCP or MHCP) for the five machine case (see Table 2.9 and 2.10) are lower than those for the one machine case (see table 2.5 and 2.6). This observation is explained by the fact that in the five parallel machines case, the system has more capacity than does an equivalent system with only one machine (i.e., the machines do not fall down simultaneously).

Following the same approach than section 2.6.5, it can be concluded that in all cases C_{HCP}^* - C_{MHCP}^*>0 at the 95% confidence level. Consequently, the MHCP gives the lower optimal cost, and furthermore, it appears that the MHCP is better than the HCP, and can be used to better approximate the optimal control policy.

Tableau 2.10

Optimal design factors and incurred costs under MHCP

Cases	K_{ij}	c_1^+	c_1^-	c_2^+	c_2^-	(a*,Z*) Case1	c^*_{MHCP} Case1	(a*,Z*) Case 2	c^*_{MHCP} Case 2
1	6	5	8	5	8	(3,10)	54	(4,11)	54.87
2	6	5	10	5	10	(4,11)	61	(5,12)	61.2
3	6	5	15	5	15	(6,12)	72	(7,13)	72.43
4	6	5	20	5	20	(7,13)	80	(8,14)	80.5
5	6	5	25	5	25	(8,14)	82	(9,15)	82.5

2.8 Conclusion

In this paper, we studied the production and setup control problem for an unreliable multiple-machine multiple-part type manufacturing system and solved the problem in the case of two products. We adopted a numerical approach for solving the HJB equations of the problem, and obtained near-optimal production and setup control policies. The optimal setup policy has been shown in this paper to be described by a Modified Hedging Corridor policy. Based on the numerical solution obtained, a parameterized near-optimal control policy was derived. Such a policy depends on the stock threshold levels and the boundaries of the corridor. To determine the parameters of the control policy, and hence, to achieve a close approximation of the optimal production and setup policies, an experimental approach based on design of experiment, simulation modeling and response surface methodology has been presented. The proposed approach shows that the optimal cost incurred under the developed control policy is lower than that incurred under the hedging corridor policy. Moreover, the proposed combined approach offers an easily applied procedure to control manufacturing systems at the operational level. Based on the parameterized control policy obtained for the one machine two-product manufacturing system case, we presented the extension to the five-machine two-product case subject to non-exponential failure and repair time distributions.

CHAPITRE 3

JOINT PRODUCTION / CHANGEOVER POLICIES FOR *N* FAMILIES OF PRODUCTS IN UNRELIABLE SUPPLY CHAINS

Abstract

This paper considers a two-stage supply chain control problem. The distribution center faces multiple demands type and passes the orders to the transformation stage. This facility is subject to random events such as periods of unavailability due to internal difficulties or market constraints. Our objective is to find information sharing control policies for manufacturing and distribution activities that minimize the expected discounted cost of inventories/backlog, setup and transformation over an infinite horizon. The control policies we are seeking include the optimal production plan and sequence of changeover actions. A continuous dynamic programming formulation of the problem is presented. Then, a numerical scheme is adopted to solve the obtained optimality conditions equations. A complete control policy is finally developed. Based on two and three family products results, an extension to the n family products problem is proposed. The usefulness of such extension as well as application issues are also discussed.

3.1 Introduction

This paper studies the multi-family products control problem arising in a significant class of supply chains. The considered class covers one stage distribution center facing an unreliable downstream transformation system and responding to multi-family demands. To deal with this class of problems in a realistic manner, several issues should be considered such that uncertainty and interactions. Moreover, for a large scale system one has to take into account the complexity associated with the mathematical or numerical resolution. To deal with these topics in the research literature, several approaches have been employed. Among others, diverse mathematical techniques from continuous differential equation systems to mathematical programming models have been attempted (Riddalls et al. (2000)). Yet, we didn`t find in the currently available literature a complete optimal production and setup policy for more than two family products. Following this observation, this paper will extend three previous works (Hajji et al. (2003), (2004) and Gharbi et al. (2006)) to solve the three family products case. Although an optimal policy for n family types is intractable, the two and three family problems are solved and can be used to help formulate heuristic policies for the n family products problem.

This work can be related to two areas of research. The first is the optimal control theory with application to manufacturing systems at an operational level of control. The second is the application to supply chains and production / distribution systems at an operational or tactical level of control. In this context, we are interested in an important class of stochastic production systems which involves significant setup time and costs incurred when production is switched from one product type to another. This class of systems belongs to production systems for which the problem of determining optimal production policies have been considered by many authors. A significant portion of this research is based on the pioneering work of Kimemia and Gershwin (1983), who suggested a feedback formulation of the control problem in a dynamic manufacturing environment, and showed that the optimal control has a special structure called the *Hedging Point Policy* (HPP).

Stochastic manufacturing systems with setups have been considered by Sethi and Zhang (1994), who's study focuses on exact optimal policy via viscosity solutions of Hamilton Jacobi Bellman equations (HJB). They used numerical methods, presented by Kushner and Dupuis (1992), to solve such models. Those results were successfully implemented by Yan and Zhang (1997) and Boukas and Kenne (1997). They were able to develop near optimal control policies for production, maintenance and setup scheduling for a one-unreliable-machine, two-part system. In all those works and principally in Yan and Zhang (1997), the numerical examples were given for the production of identical products. Interestingly, two cases of study, one with identical products and the other with two different products, were presented by Bai and Elhafsi (1997) and Elhafsi and Bai (1996), but no results were observed to define policies in both positive and negative zones of inventory. This observation has launched our two previous works. We have developed a complete production and set-up heuristic policy for a one unreliable stage two-part type production system, which covers all the stock zones namely the positive, negative and combined areas. The obtained policy is called Modified Hedging Corridor Policy (MHCP) and is based on the parameterization of the boundaries of the corridor and the specifications of the optimal relevant policy. Moreover, we have shown in Hajji et al. (2003) that this policy guarantees better performance than the classical policy (HCP). In this work, we will extend the resolution to the three family products case and we will discuss the generalization of the policy to cover the ***n*** family products case.

A stochastic dynamic programming problem is formulated keeping the structure presented in Yan and Zhang (1997). The structure of the solution, under appropriate conditions, is obtained by using the fact that the value function is the unique viscosity solution to the associated Hamilton Jacobi Bellman equations (HJB). Owing that an analytical solution of HJB equations is not in general available; a numerical approach is adopted. To illustrate the structure of the control policy, sensitivity analysis is conducted allowing the development and parameterization of the relevant heuristic policy. Finally, the proposed control heuristic, including production rates and setup strategies, is interpreted to allow addressing the ***n*** family products case.

The paper is organized as follows. Section 3.2 presents a formulation of the optimal production and setup scheduling problem, for the multiple family products supply chain. The HJB equations and the optimality conditions are then derived. Section 3.3, presents the resolution approach. In section 3.4, the derived HJB equations are solved numerically for the two and three family cases. The parameterization of the heuristics are reported in section 3.5, the extension to the ***n*** family products case is also discussed. Applications issues and numerical examples are reported in section 3.6 and 3.7. The paper is concluded in section 3.8.

3.2 Problem statement

The supply chain under study (illustrated in figure 3.1) consists of a distribution center supplied by an unreliable transformation stage. The whole system faces *n* family product demand. Note P_j, j = 1...n, the family product type. The transformation system is not completely flexible in the sense that change over times (set-up activities) are not negligible. Setup activities involve both time and cost. Note that, $\theta_{ij} \geq 0\, and\, K_{ij} \geq 0$, for, i, j = 1...n, and i≠j. Part type *i* requires an average processing time $p_i > 0, (i = 1,...,n)$ and has an average time between orders $1/d_i$ assumed to be constant. The difference between actual production and demand at any time represents the surplus of a part type (backlog if the difference is negative and inventory if the difference is positive). For *n* part type system, ***x(t)***, ***u(t)*** and ***d*** denote vectors $(x_1(t),...,x_n(t))'$, $(u_1(t),...,u_n(t))'$ and $(d_1,...,d_n)'$ respectively.

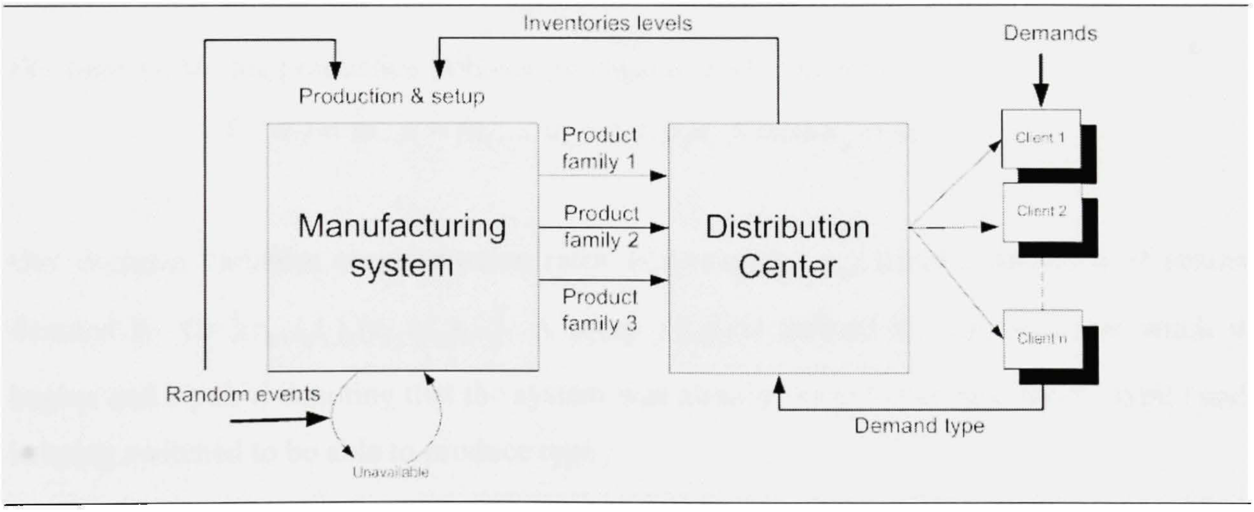


Figure 3.1 Unreliable two stages supply chain

The state of the system at time *t* has two components: A continuous part, which describes the distribution center state and measured by $x(t)$; A discrete part, which describes the manufacturing system state and denoted by $\alpha(t)$. The stochastic process $\alpha(t)$ is equal to 1 if the manufacturing system is available and 2 if it is not.

For the considered multiple family products supply chain the state space is given by: $x(t) \in R^n$, $\alpha(t) \in M = \{1,2\}$. The dynamics of the surplus is given by the following differential equation.

$$\dot{x}(t) = u(t) - d, \quad x(0) = x_0 \tag{3.1}$$

The operational mode of the transformation system at time t can be described by the random variables $\xi(t)$ with value in $M = \{1,2\}$, respectively, where

$$\zeta(t) = \begin{cases} 1 & \text{the transformation stage is available.} \\ 2 & \text{the transformation stage is unavailable.} \end{cases}$$

The transition rates matrix of the stochastic process $\xi(t)$ is denoted by T such that $T = \{q_{\alpha\beta}\}$, with $q_{\alpha\beta} \geq 0$ if $\alpha \neq \beta$ and $q_{\alpha\alpha} = -\sum_{\beta \neq \alpha} q_{\alpha\beta}$, where $\alpha, \beta \in M$.

For each $\alpha \in M$, the production policies (or capacity) set is given by:

$$\Gamma_i(\alpha) = \{u : u = (u_1, ..., u_n) \geq 0, p_i u_i \leq \alpha(t), u_j = 0; \forall j \neq i\}$$

Our decision variables are production rates $u(.) = (u_1(.), ..., u_n(.))$ and a sequence of setups denoted by $\Omega = \{(\tau_0, i_0 i_1), (\tau_1, i_1 i_2), ...\}$. A setup (τ, ij) is defined by the time τ at witch it begins and a pair *ij* denoting that the system was already setup to produce family type *i* and is being switched to be able to produce type *j*.

The instantaneous cost function $g(.)$ is given by the following equation:

$$g(x) = \sum_{i=1}^{n} (c_i^+ x_i^+ + c_i^- x_i^-), \tag{3.2}$$

Where $x_i^+ = \max(0, x_i), x_i^- = \max(-x_i, 0)$, c_i^- and c_i^+ are product type *i* backlog cost and inventory cost respectively.

The overall cost function of production and setup, during s units of times, is given by the following equation:

$$R_{ij}(\boldsymbol{x},s) = K_{ij} Ind\{s = \Theta_{ij}\} + \int_0^s e^{-\rho t} g(\boldsymbol{x} - dt)dt, \quad s \in [0, \Theta_{ij}], \text{i,j} = 1,\ldots,\text{n and } i \neq j \quad (3.3)$$

Using (3.2)-(3.3), the total cost $J(.)$ can be defined by the following expression:

$$J(i,x,\alpha,s,\Omega,u(.)) = \int_0^s e^{-\rho t} g(x(t))dt + E_{i,x-ds,\alpha_s}\left[\int_s^\infty e^{-\rho t} g(x(t))dt + \sum_{l=0}^{\infty} e^{-\rho\tau_l} K_{i_l i_{l+1}}\right] (3.4)$$

Where $E_{i,x-ds,\alpha_s}$ is the conditional expectation given the condition $(i, x - ds, \alpha_s)$ at time s and $\alpha_s = 1$ if s > 0 and $\alpha_s = \alpha$ if s=0.

Let A denote the set of admissible decisions (Ω, $\boldsymbol{u}$(.)). The production planning problem considered herein is to find an admissible decision or control policy (Ω, $\boldsymbol{u}$(.)) that minimizes $J(.)$ given by (3.4) considering equations (3.1) to (3.3). This is a feedback control that specifies the control actions when the system is in a given state ($\boldsymbol{x}$,α). The feedback control determines the production rates and the setup actions as a function of the inventory level in the distribution center and the state of the manufacturing system.

While producing the part type i, the corresponding value function $v_i(.)$ can be given by the following:

$$v_i(x,\alpha) = \inf_{(\Omega,U)\in A} J(i,x,\alpha,\Omega,u) \qquad \forall x \in R^n, \alpha \in M \qquad (3.5)$$

It can be shown, see Hajji et al. (2003) and Hajji et al. (2004) and the references therein, that the value function $v_i(x,\alpha)$ is locally Lipschitz, and is the unique viscosity solution to the following HJB equation:

$$\min\left\{\begin{array}{l}\min\limits_{u\in\Gamma(\alpha)}\left[(u-d)\,(v_i)_x(x,\alpha)+g(x)+Q.v_i(x,.)(\alpha)\right]-\rho v_i(x,\alpha);\\ \min\limits_{j\neq i}\left[R_{ij}(x,\Theta_{ij})+e^{-\rho\Theta_{ij}}.v_j(x-d\Theta_{ij},1)\right]-v_i(x,\alpha)\end{array}\right\}=0 \qquad (3.6)$$

Where $(v_i)_x(.)$ denotes the gradients of $v_i(.)$ with respect to $\boldsymbol{x}$,

The production and setup policy that we are seeking is obtained when the value function is known. While we cannot analytically solve the HJB equations (3.6), we can however apply numerical methods to obtain the approximation of the value function and the associated control policy.

3.3 Numerical approximation

In this section, numerical methods are used to approximate the solution of the HJB equations (3.6) corresponding to the stochastic optimal control problem. This method is based on the Kushner approach (Kushner and Dupuis (1992)). With this approximation, the HJB equations are expressed in terms of $v_i^h(x,\alpha)$ as follow:

$$v_i^h(x,\alpha)=\min\left\{\begin{array}{l}\min\limits_{u\in\Gamma_i(\alpha)}\left[\begin{array}{l}\dfrac{Q_h^\alpha(x,u)}{\rho+Q_h^\alpha(x,u)}\Big\langle\sum\limits_{j=1}^{n}P_h^\alpha(x,x\pm h_j,u)v_i^h(x,\alpha)\\ +\sum\limits_{\beta\neq\alpha}\widetilde{P}_h^\alpha(x,\alpha,\beta,u).v_i^h(x,\beta)\Big\rangle+\dfrac{g(x)}{\rho+Q_h^\alpha(x,u)}\end{array}\right];\\ \min\limits_{j\neq i}\left[R_{ij}(x,\Theta_{ij})+e^{-\rho\Theta_{ij}}.v_j^h(x-d\Theta_{ij},1)\right]\end{array}\right\} \qquad (3.7)$$

Where: h_j, j =1… n, denote the length of the finite difference interval of the variable x_j.

$$Q_h^\alpha(x,u)=|q_{\alpha\alpha}|+\sum_{j=1}^{n}\frac{|u_j-d_j|}{h_j}\;;\; P_h^\alpha(x,x\pm h_j,u)=\frac{|u_j-d_j|}{h_jQ_h^\alpha(x,u)}\;;\;\widetilde{P}_h^\alpha(x,\alpha,\beta,u)=\frac{q_{\alpha\beta}}{Q_h^\alpha(x,u)}$$

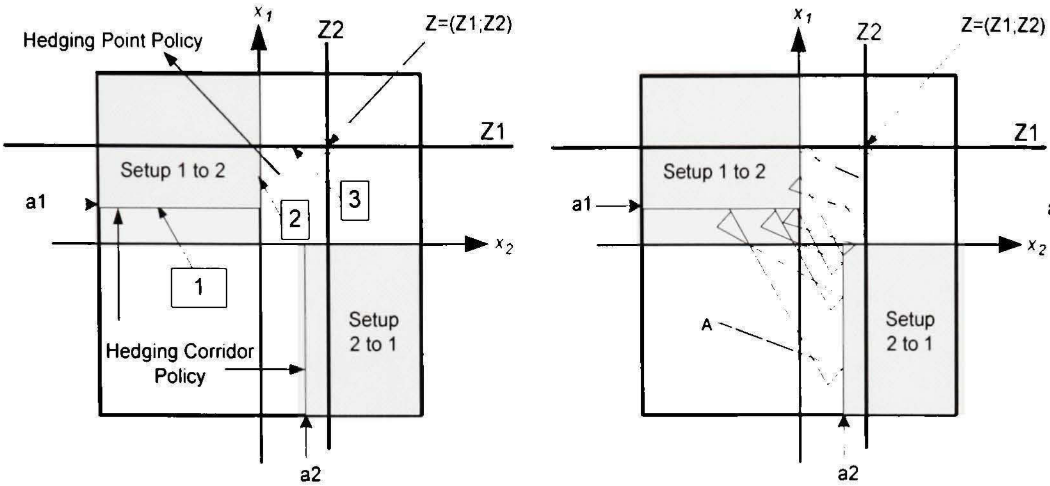
The terms $P_h^{\alpha}(x,x',u)$ and $\widetilde{P}_h^{\alpha}(x,\beta,\beta',u)$ are nonnegative and sum to unity over *x*` and β' for each *x* and β. They can be considered as the transition probability for a controlled Markov chain on the discrete state space $M \times G_h$, where G_h symbolizes a description of the grid. The solution of (3.7) may be obtained by either successive approximation or policy improvement techniques.

3.4 Numerical results

In this section we will recall and present the numerical results for the two and three family products problems, respectively. These results will allow us to propose, in section 3.5, a generalised production and setup policy for the *n* type control problem.

3.4.1 Two family products policy

In Hajji et al. (2004) and Gharbi et al. (2006) it was shown that the optimal policy has a particular structure, which we called the *Modified Hedging Corridor Policy* "MHCP" and illustrated in Figure 3.2. This policy is a combination of the Hedging Corridor Policy (HCP) and the Hedging Point Policy (HPP). Let Z1 and Z2 define the threshold of products P1 and P2 respectively. Let also a1 and a2 define the boundaries of the corridor. The proposed MHCP is defined by two hedging levels in the negative and positive areas of each part type. When there is a shortage in the stock level of one part type, the setup action must be performed earlier than the same action when the stock level is still in the positive zone. These actions are conducted with respect to the boundaries of the corridors a_1 *and* a_2 (see arrow [1], Figure 3.2 (a)). However, when the two stock levels are positive, we can proceed with production according to a hedging point policy (see arrow [3], Figure 3.2 (a)), in this case the setup actions are performed when the stock level of the concerned part type reaches the negative area (see arrow [2], Figure 3.2 (a)).



(a) Setup actions through MHCP (b) Inventory itinerary through MHCP

Figure 3.2 Modified Hedging Corridor Policy

The proposed modified hedging corridor policy, as shown in Figure 3.2 (b), states that if starting at point A and reaching point $Z = (Z_1, Z_2)$, which is the hedging point defined by the intersection of the two hedging levels, the stock trajectory travels through two different corridors, with one in the negative and combined stock area and the other in the positive area. Sensitivity analyses have allowed the development of a parameterized production and setup control policy defined by the following equations:

$$u_1(.) = \begin{cases} u_1^{\max}.Ind\{S_{21} = 1\} & if\ x_1 < Z_1 \\ d_1\ .Ind\{S_{21} = 1\} & if\ x_1 = Z_1 \\ 0 & if\ x_1 > Z_1 \end{cases} \tag{3.8}$$

$$u_2(.) = \begin{cases} u_2^{\max}.Ind\{S_{12} = 1\} & if\ x_2 < Z_2 \\ d_2.Ind\{S_{12} = 1\} & if\ x_1 = Z_2 \\ 0 & if\ x_2 > Z_2 \end{cases} \tag{3.9}$$

$$S_{12} = \begin{cases} 1 \quad if \begin{cases} x_1 \geq a_1 \\ and \\ x_2 \leq b_1 \end{cases} \\ 0 \quad otherwise \end{cases} \tag{3.10}$$

$$S_{21} = \begin{cases} 1 \quad if \begin{cases} x_2 \geq a_2 \\ and \\ x_1 \leq b_2 \end{cases} \\ 0 \quad otherwise \end{cases} \tag{3.11}$$

With the following constraints:

$$0 \leq a_1 \leq Z_1 \text{ and } 0 \leq a_2 \leq Z_2 \tag{3.12}$$

Where: S_{ij} define the setup policy from part type i to part type j.

3.4.2 Three family products policy

The numerical results used to characterize the optimal production and setup policies are presented in this section within five cases. Table 3.1 shows the constant parameters for all the numerical examples and Table 3.2 details the cost variations.

Tableau 3.1

Constant data parameters for cases 1 to 5

Data	d_i	U_i^{max}	q_{12}	q_{21}
Values	0.2	2	0.02	0.2

Tableau 3.2

Data parameters for cases 1 to 5

Cases	C_1^+	C_2^+	C_3^+	C_1^-	C_2^-	C_3	K_{ij}	Θ_{ij}
1	1	1	1	3	3	3	0.1	0.4
2	1	1	1	5	5	5	0.1	0.4
3	1	1	1	20	20	20	0.1	0.4
4	1	1	1	40	40	40	0.1	0.4
5	1	1	1	60	60	60	0.1	0.4

To ensure a clear characterization of the control policy, several elements were taken into consideration as part of the implementation process. Indeed, the production and setup policies are each observed separately. For each policy, the relevant significant stock threshes are analyzed independently of the others. For each numerical result, the policies are provided as shown in Figure 3.3, 3.4 and 3.5. While producing part type 1 we are looking for the production policy given by $u_1(x_1, x_2, x_3)$, the setup policy from part type 1 to part type 2 given by $S_{12}(x_1, x_2, x_3)$ and the setup policy from part type 1 to part type 3 given by $S_{13}(x_1, x_2, x_3)$. For the third case, these policies are shown in figure 3.3, 3.4 and 3.5 respectively.

It follows from our numerical results that the resulting production policy (see figure 3.3) divides the surplus space into two mutually exclusive regions. In region I, produce at the maximal rate and in region II we have to set the production rate to zero. At the boundary of these regions we have to set the production rate equal to the demand rate. Moreover, the results show that the setup policy divides the surplus space into two state dependant regions I and II (see figure 3.4 for the setup policy $S_{12}(x_1, x_2, x_3)$ and figure 3.5 for $S_{13}(x_1, x_2, x_3)$).

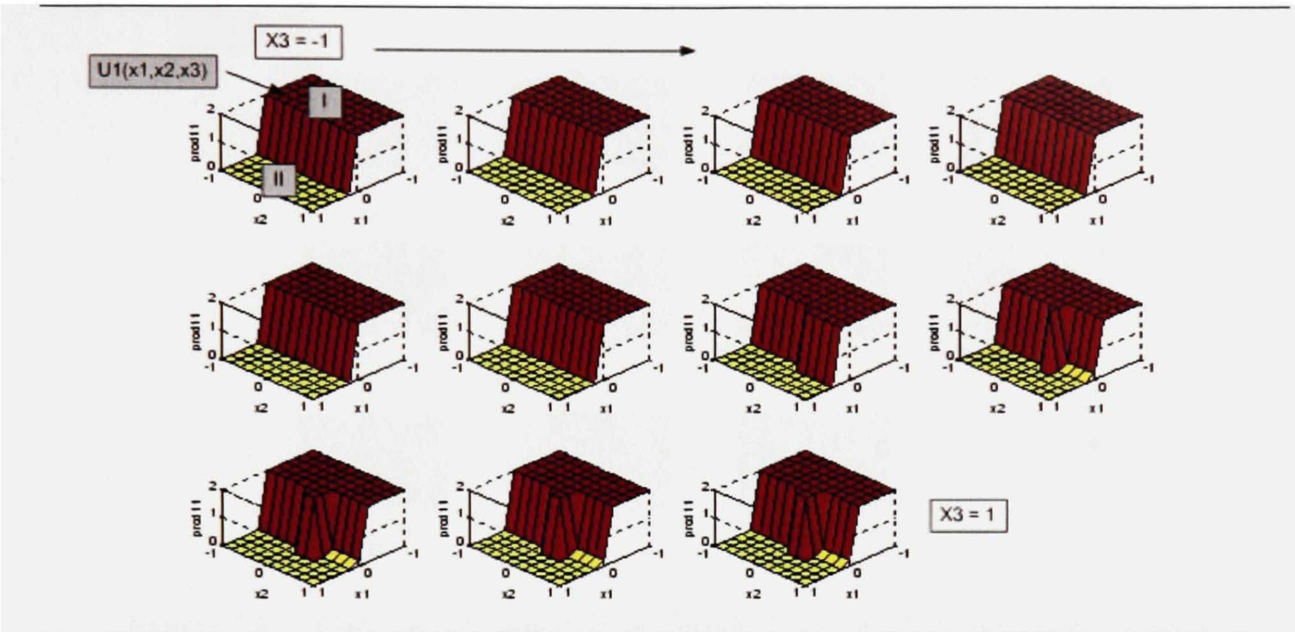


Figure 3.3 Production policy of family 1, case 3

In comparison with the two family products results, it follows that the structure of the production and setup policies are maintained if the third dimension (i.e., x_3) is given a constant value. In this case, the structure of the setup policy from part type one to part type two is maintained and the complementary stock space is attributed to the setup policy from part type one to part type three.

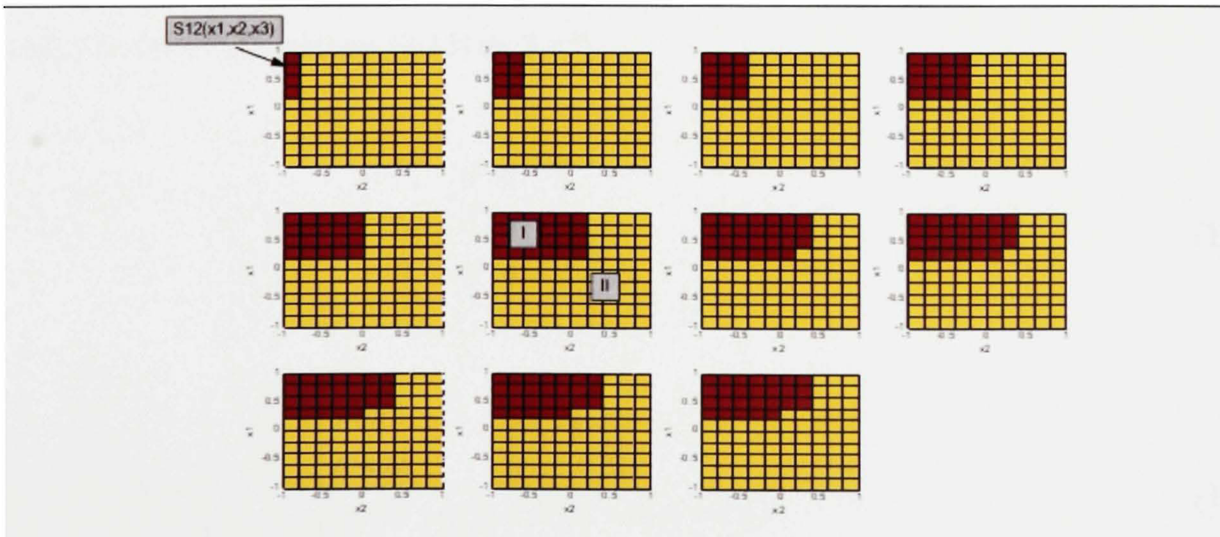


Figure 3.4 Changeover policy from family 1 to 2 while producing 1, case 3

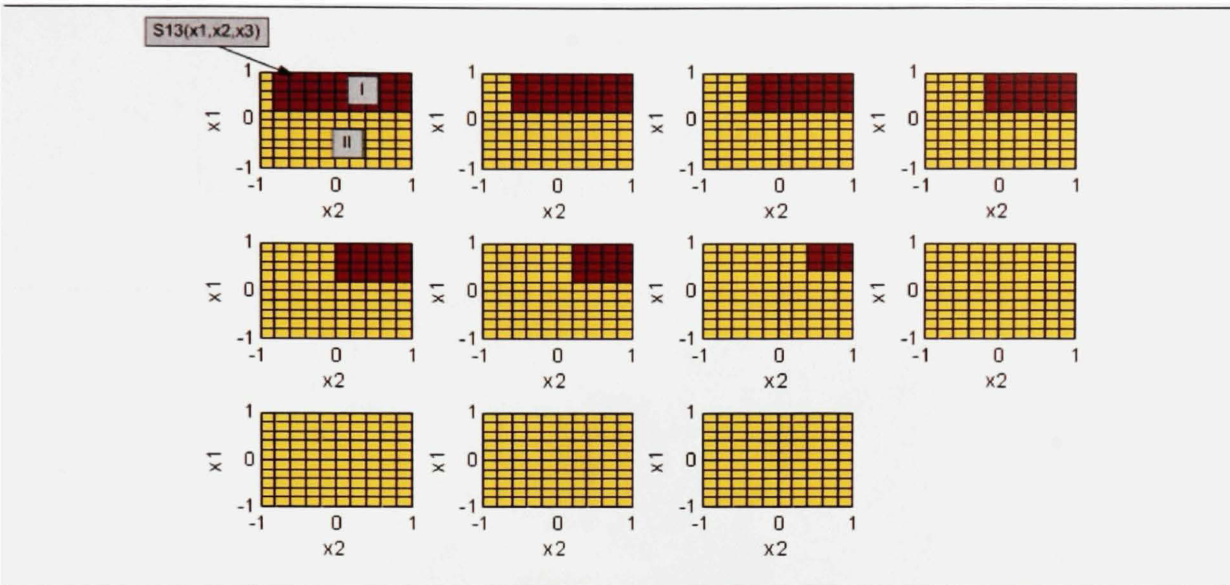


Figure 3.5 Changeover policy from family 1 to 3 while producing 1, case 3

The three dimension representations of $S_{12}(x_1,x_2,x_3)$ and $S_{13}(x_1,x_2,x_3)$ are shown in figure 3.6 and 3.7 respectively. These representations allow a delimitation of the setup policies in the stock space. In this context, three planes allowed us to draw up the boundaries of the setup region. It is interesting to note that plane 1 and plane 3 in figure 3.6 and 3.7 refer to the same boundaries. Moreover, even if plan 2 in figure 3.6 is set to delimit the setup boundary with respect to x_2, it takes the same value with respect to x_3 as shown in figure 3.7. These observations have allowed the development of a parameterized production and setup control policy defined by equations (3.13) to (3.15).

$$u_i(.) = \begin{cases} u_i^{\max} & if\ x_i < Z \\ d_1 & if\ x_i = Z \\ 0 & if\ x_i > Z \end{cases} \tag{13}$$

$$S_{ij} = \begin{cases} 1 \quad if & \begin{cases} plan1 & x_i \geq a & and \\ plan2 & x_j \leq b & and \\ plan3 & \alpha \cdot x_i + \beta \cdot x_j + \gamma \cdot x_k + c \leq 0 & \end{cases} \\ 0 & otherwise \end{cases} \tag{14}$$

$$S_{ik} = \begin{cases} 1 \quad if \begin{cases} plan1 & x_i \geq a & and \\ plan2 & x_k \leq b & and \\ plan3 & \alpha \cdot x_i + \beta \cdot x_j + \gamma \cdot x_k + c \geq 0 \end{cases} \\ 0 \quad otherwise \end{cases} \tag{15}$$

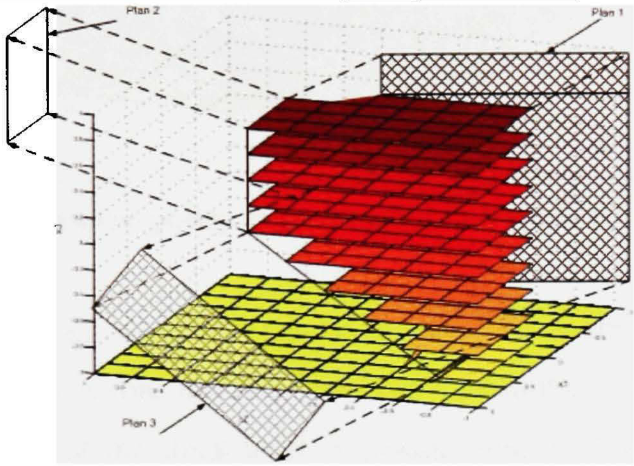


Figure 3.6 Changeover policy from family 1 to 2 while producing 1, case 3

Note that u_i denote the production policy of part type *i*, S_{ij} the setup policy from part type *i* to part type *j* and S_{ik} the setup policy from part type *i* to part type *k*. where $(i, j, k) \in \{1, 2, 3\}$.

After several experimentations and sensitivity analysis, we have clearly observed that the results obtained make sense, and that the structure of the policy defined by equations (3.13) to (3.15) is always maintained. In conclusion, for the « three family » product problem, the production and setup policies can be defined by 7 parameters: $(Z, a, b, \alpha, \beta, \gamma, c)$.

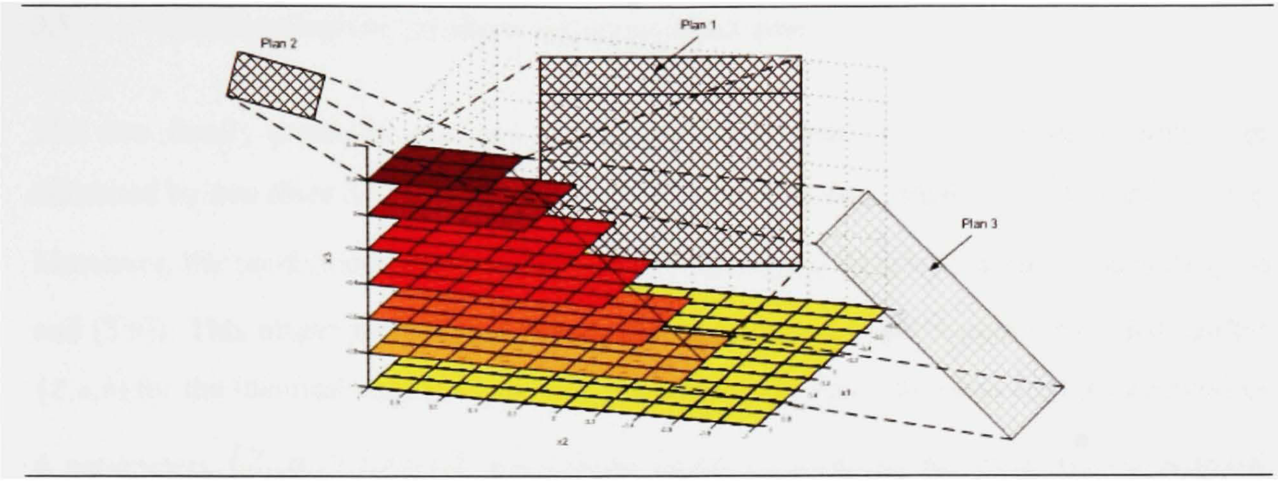


Figure 3.7 Changeover policy from family 1 to 3 while producing 1, case 3

In what follows (Figure 3.8), a practical representation of the feedback policy is presented. This illustration shows the actions that should be taken when the machine is producing part type 1, and is a function of the stock level of product type 1, 2 and 3 (X_1, X_2 and X_3). When the machine is producing part type 2 or 3, a mirror schema could be realized so as to achieve a complete production and setup strategy. In the next section a generalization of the developed control policy to cover n family product cases is proposed and discussed.

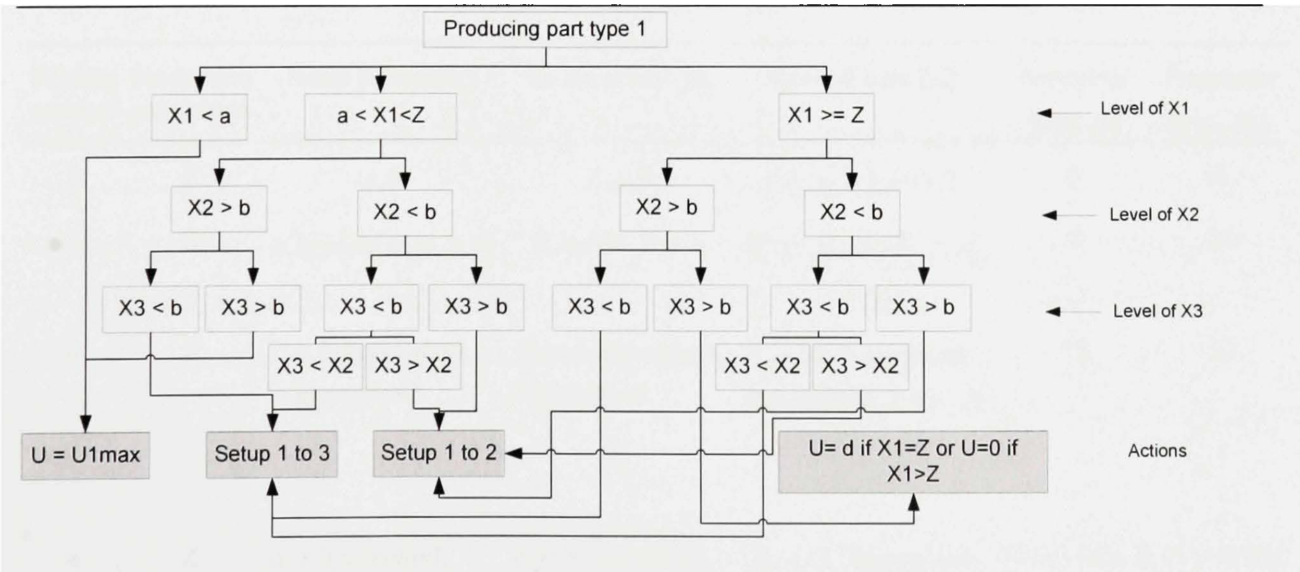


Figure 3.8 Practical solution

3.5 Generalization for the n family product case

The two family products case has showed that the boundaries of the setup policy are delimited by ***two lines*** defined by the parameters a_i and b_i (see equations (3.10) and (3.11)). Moreover, the production policy is defined by the hedging threshold z_i (see equations (3.8) and (3.9)). This observation makes it possible to define a complete policy by 3 parameters (Z,a,b) for the identical case (i.e., the two product family have the same cost parameters) or 6 parameters $(Z_i,a_i,b_i), i=1,2$ for general cases. Considering the three family products case gives rise to an additional dimension in the stock space (i.e., x_3). Before the resolution one can expect that moving the setup boundaries with respect to an additional dimension will lead to a ***plane***. After the resolution of the three family products case, we have observed that this expectation makes sense. Following this idea one can expect that the plan can be generalized to a ***hyper-plane*** in the n dimensions case. Table 3.3 shows the production and setup policies parameters for the two, three and n family products cases.

Tableau 3.3

Policy generalization

Product number	Production parameters	Setup parameters	Identical case (I)	General case (G)	Parameter # for (I)	Parameter # for (G)
2	Z	a,b	Z,a,b	$(Z_i,a_i,b_i), i=1,2$	3	6
3	Z	$a,b,plan(\alpha,\beta,\gamma,c)$	$Z,a,b,\alpha,\beta,\gamma,c$	$Z_i,a_i,b_i,\alpha_i,\beta_i,\gamma_i,c_i$ i=1,2,3	7	21
4	Z	$a,b,hyperplan1,$ $hyperplan2$	$Z,a,b,hyperplan1$ $hyperplan2$	$Z_i,a_i,b_i,hyperplan1_i$ $hyperplan2_i, i=1,..,4$	13	52
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	Z	$a,b,hyperplan1,$ $\ldots,hyperplan(n-2)$	$Z,a,b,hyperplan1,$ $\ldots,hyperplan(n-2)$	$Z_i,a_i,b_i,hyperplan1_i$ $hyperplan(n-2)_i,$ $i=1,..,n$	$3+(n+1)(n-2)$	$(3+(n+1).(n-2)).n$

To complete the policies generalization, a parameterized control policy is presented by the following equations (3.16) and (3.17).

$$u_i(.) = \begin{cases} u_i^{\max} & if\ x_i < Z \\ d_1 & if\ x_i = Z \\ 0 & if\ x_i > Z \end{cases} \tag{3.16}$$

$$S_{ij} = \begin{cases} 1 \quad if \begin{cases} x_i & plan1 & x_i \geq a \ \ and \\ x_j & plan2 & x_j \leq b \ \ and \\ x_k & hyperplan1 & \left(\sum_{i=1}^{n} \alpha_i^k \cdot x_i\right) + c^k \leq 0 \\ \vdots & \vdots & \vdots \\ x_n & hyperplan(n-2) & \left(\sum_{i=1}^{n} \alpha_i^n \cdot x_i\right) + c^n \leq 0 \end{cases} \\ 0 \quad otherwise \end{cases} \tag{3.17}$$

3.6 Application issues

In this section a discussion regarding the usefulness of such results is conducted. As mentioned in the introduction, this class of problem can be related to two areas of research. The first is the optimal control theory with application to manufacturing systems at an operational level of control. The second is the application to supply chains and production / distribution systems at an operational or tactical level of control. In this context, and for a given system, the manager should establish several parameters to govern the production and the changeover feedback control policies. These parameters characterize the inventories levels defining multiple control points. At this point, a main concern arises and it consists in the number of the involved parameters. In this context, we claim that a compromise between the level of optimality and the feasibility of the solution should be made. In the following diagram (Figure 3.9), a sequential approach is proposed. It consists in an appropriate combination between the proposed generalized control policies (step I), practical decision and parameterisation (step II and III) and a simulation based experimental approach (step IV). The latest step could be a combination of discrete / continuous simulation model and an optimization approach; it is a flexible approach to quantify the control policy of the original problem or extended ones. We refer the reader to Gharbi et al. (2006) for more details on the application of step IV and V.

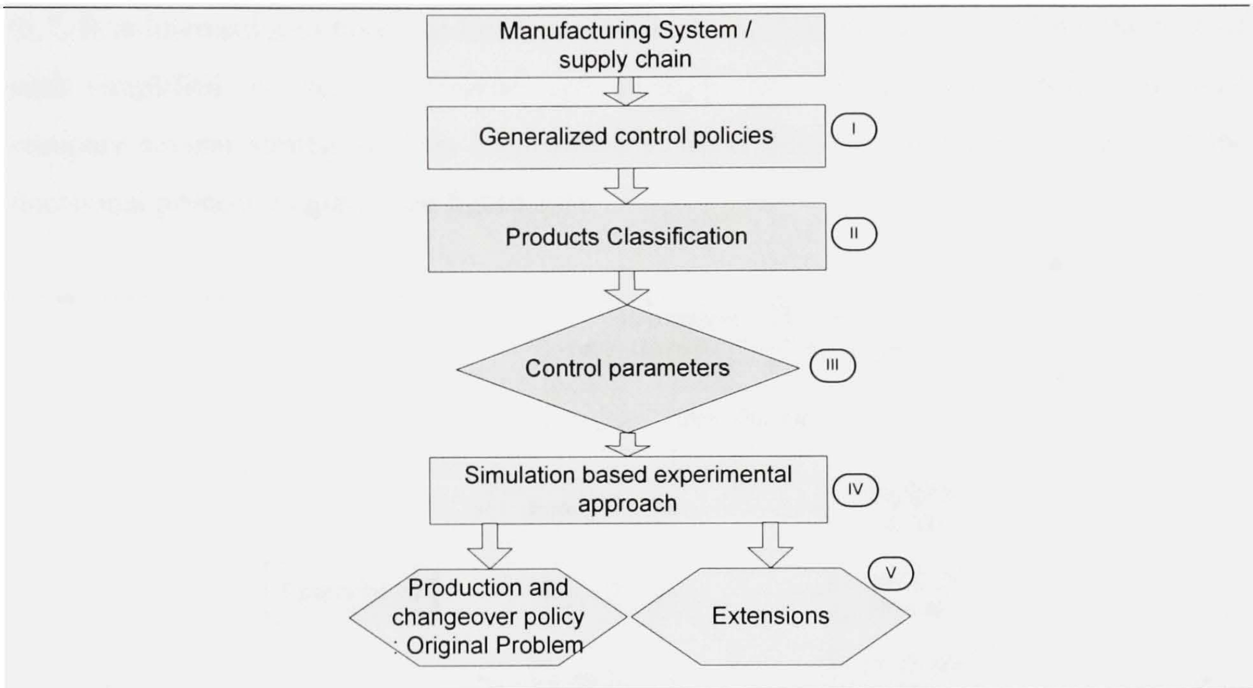


Figure 3.9 Decisional process diagram

Regarding steps II and III, one can resort to the Group Technology principle to categorize and group products having similarities in different classes of products. This classification should decrease the number of control parameters considerably. In fact, one can assume that the setup time and cost are negligible for parts in the same family class and consider the proposed generalized policies to control the production and the changeover actions between the different classes. To apply this strategy one has to combine and follow two of our previous researches: Gharbi et al. (2006) for the application of steps IV and V for systems with setup time and cost and Gharbi and Kenne (2004) for systems with negligible setup time and cost.

To illustrate the simplified policy consider a manufacturing system producing 10 parts type. Following the aforementioned generalization, the control policies should be governed in the best situation by 91 parameters. A group technology classification, as shown in figure 3.10, could help the manger to define three families of products and a global control policies based on the cumulative inventory levels between the different classes defined by 7 parameters. This procedure is able to decrease the number of the control policy parameters to 7. It is interesting to note that this is a simple example presented to highlight the idea of such simplified policies. To measure and quantify the corresponding benefit, one must compare several strategies. This issue could be done following the last two steps of the decisional process diagram (see figure 3.9).

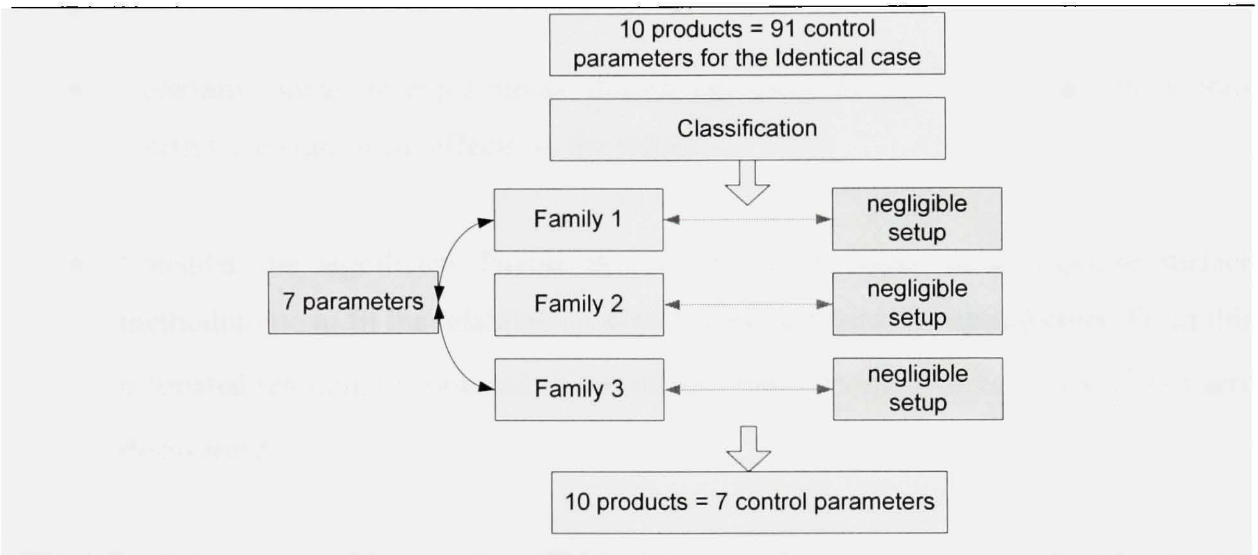


Figure 3.10 Classification example

3.7 Numerical example

Let us apply the simplified control policy developed in section 3.6 for a manufacturing system producing ten different products. If we consider a product classification generating two families of products, the objective of this section is to find the values of the control policy parameters (design factors) which minimize the incurred cost for the production and changeover control policies for identical and different product cases. To follow this purpose we will adopt an experimental approach which is a combination of simulation modeling, experimental design and response surface methodology. The reader is referred to our aforementioned papers for more details on the application of this approach in manufacturing system control. This approach can be summarized by the following steps:

- Develop a simulation model to describe the dynamic of the system using the control policy parameterized by the control parameters defined previously. Those factors are considered as input of such a model and the related incurred cost is defined as its output. Our model was developed using Visual SLAM simulation language (Pritsker and O'Reilly (1999)).

- Determine, using an experimental design approach, the input factors or interactions which have significant effects on the output.

- Consider the significant factors or interactions as input of a response surface methodology, to fit the relationship between the cost and the input factors. From this estimated relation, the optimal values of the input factors, called a_i^*, b_i^* *and* $z_i^*, i = 1,2$ are determined.

We refer the reader to Montgomery (2001) for more details on experimental design and response surface methodology approaches. Table 3.4 presents the data parameters of the ten-products manufacturing system under study. Table 3.5 presents the costs variations.

Tableau 3.4

Data table for ten products manufacturing system

Parameters	Values	Parameters	Values
(k_{12}, k_{21})	(0.5,0.5)	$\left(u_j^{max}\right)$,j=1..10	5
$(\theta_{12}, \theta_{21})$	(0.16,0.16)	$\left(d_j\right)$,j=1..10	2

Tableau 3.5

Cost variation

Groups	Cases	c_1^+	c_1^-	c_2^+	c_2^-
	1	5	10	5	10
I	2	5	15	5	15
(identical products)	3	5	20	5	20
	4	5	25	5	25
	1	5	10	5	30
II	2	5	15	5	30
(different products)	3	5	20	5	30
	4	5	25	5	30

Table 3.6 presents the optimal parameters of the production and changeover policies and the incurred optimal cost for the sensitivity analysis case (table 3.5).

Tableau 3.6

Optimization results

Groups	case	a1	b1	a2	b2	Z = (z1,z2)	Cost
	1	11	1	11	1	(20,20)	83
I	2	13	1	13	1	(23,23)	102
	3	14	1	14	1	(24,24)	110
	4	15	1	15	1	(25,25)	122
	1	10	1	20	2	(20,25)	103
II	2	13	1	19	2	(22,25)	112
	3	15	1	18	2	(24,25)	119
	4	16	1	17	2	(25,25)	130

The obtained results confirm the observation that when varying the costs, the boundaries of the corridors move in the desired directions.

3.8 Conclusion

In this paper, we have developed a complete production and changeover policy for the two stage three family products supply chain. We have solved numerically the Hamilton Jacobi Bellman equations of the problem and carried out several analyses. Based on the obtained numerical results, the optimal control policy of the problem was derived. Even if the numerical method makes it possible the resolution of the optimality conditions, a good approximation of the control policy is hard especially for larger systems. To overcome this issue, based on the two family products results (developed in our previous works) and the three family products results, we have proposed a generalized policy for the n family products problem.

Regarding the usefulness of the obtained results, a discussion was conducted and we claim that, a new direction of research should be taken. It consists of linking the numerical results with experimental approaches such as genetic algorithms or the design of experiment combined with simulation modelling so as to investigate the feasibility of simplified policies to control larger supply chains. Simulation modelling allows a dynamic representation of our system so as to compare the performances of a given control policy with other control strategies. This research is under current investigation as it may interest the reader to know.

CHAPITRE 4

PRODUCTION AND CHANGEOVER CONTROL POLICY OF A CLASS OF FAILURE PRONE BUFFERED FLOW-SHOP

Abstract

This paper deals with a stochastic optimal control problem for a class of buffered multi parts flow-shop manufacturing system. The involved machines are subject to random breakdowns and repairs. The flow shop under consideration is not completely flexible and hence requires setup time and cost in order to switch the production from a part type to another. The considered flow shop class needs change over to be carried on the whole line. Our objective is to find the production plan and sequence of setups that minimize the cost function which penalizes inventory/backlog and setup costs. A continuous dynamic programming formulation of the problem is presented. Then, a numerical scheme is adopted to solve the obtained optimality conditions equations for a two buffered serial machines two parts case. A complete heuristic policy, based on the numerical observations which describe the optimal policies in system states, is developed. It will be shown that the obtained optimal policy is a combination of a KANBAN/CONWIP and a modified hedging corridor policy. Moreover, based on our observations and existent research studies extension to cover more complex flow shop is henceforth possible. The robustness of such a policy is illustrated through sensitivity analysis.

4.1 Introduction

One of the most common problems at an operational decision level consists on finding the best way to process a given number of jobs on a specified number of machines. This problem is referred by various investigators as scheduling, dispatching or sequencing (Gupta and Stafford (2006)). In general, scheduling is a decision making process to determine when, where and how to produce a set of products given requirements in a specific time horizon, a set of limited resources, and processing recipes (Floudas and Lin

(2004)). In the research literature, this problem remains largely open especially for complex manufacturing system and is known to belong to the set of NP-hard problems. This complexity arises if at least one of the following aspects is taken into account: dynamic stochastic behaviour and / or complex (or large) structure of the manufacturing system. One of those systems present in a vast number of industries is the flexible flow lines. They consist on several serial stages with buffers located between them and producing multiple parts type of products. They are especially common in the process industry (Quadt and Kuhn (2007)). Numerous examples are given in the literature, including the electronics manufacturing (Wittrock (1988)), the food and cosmetics (Moursli and Pochet (2000)), the pharmaceutical sector (Guinet and Solomon (1996)) as well as the automotive industry (Agnetis et al., (1997)). In this paper we address the problem of production and changeover control problem in a class of failure prone buffered flow-shop. The considered class requires setup on the whole line when the decision to switch the production from one part type to another is taken.

In the literature, several approaches, mainly heuristic and optimal procedures, are employed to solve the problem. The first approach is very present in the research literature, recent surveys are addressed by Gupta and Stafford (2006) and Quadt and Kuhn (2007). On the other hand, the second approach which consists of a stochastic optimal control problem formulation, seeks to determine optimal control policies for the addressed problem.

The relevant literature dealing with optimal control problems of stochastic flow-shops with limited buffers producing one part type addressed the theory foundation of the optimization problem. In this context, Presman et al. (1995 - 1997) considered a production planning problem in an N-machine flow-shop subject to breakdown and repair of machines and to non-negativity constraints on work-in-process. The objective was to minimize the expected discounted cost of production and inventory / backlog over an infinite horizon. An equivalent problem was addressed in Presman et al. (2002) and Sethi et al. (2000) to minimize the long-run average cost. Basically, they used a stochastic dynamic programming formulation and showed that the value function of the problem is locally Lipshitz and is a solution to a dynamic programming equation together with a certain boundary condition. Stochastic flow-shop manufacturing systems with setups have been considered by Bai et al. (1996) where they studied a manufacturing system consisting of two failure prone machines separated by two internal buffers and producing two different parts. Each machine requires a constant non negligible setup change time from one product to the other. Their results were interesting and they succeeded to determine a good control strategy based on a combination of mathematical modeling and heuristics. Yavuz and Tufekci (2006) studied a real case study of an electronic manufacturing flow-shop. They splited the master problem into two sub-problems which were concerned with determining the batch sizes and production sequences, respectively. They developed a dynamic programming procedure to solve the batching problem and suggested an existing method to solve the sequencing problem. They showed that their solution approach is effective in meeting the JIT goals and is efficient in its computational requirements.

In all aforementioned works, it seems clear that a stochastic optimal control approach (or its variant) has been successful only for simple systems. Moreover, many researchers consider that even if optimal control policies can be found for realistic systems, they risk being too complicated to implement. Optimal control analysis, however, is valuable in that knowledge of the optimal policy or its structure even for small size problems may point to the design or help to assess the performance of simple heuristic policies for more complex systems (Liberopoulos (1997)).

Based on these facts, the main purpose of this paper is to develop a production and set-up heuristic policy for a stochastic multiple machines flow-shop producing multiple parts. Two previous works and interesting observations made after the resolution of the system under study will make it possible to generalize the obtained policy for complex flow-shops.

A stochastic dynamic programming problem is formulated keeping the structure presented in Presman et al. (1995 – 1997) and Hajji et al. (2004). The structure of the solution, under appropriate conditions, is obtained by using the fact that the value function is the unique viscosity solution to the associated dynamic programming equations. Owing that an analytical solution of these equations is not in general available, a numerical approach is adopted to find an approximate value function. To illustrate the structure of the control policy, the problem is solved for two buffered machines two-part flow-shop manufacturing system. It is followed by experimentations and sensitivity analysis, allowing the development and parameterization of the optimal relevant heuristic policy to control the system. Finally, based on our observations and two previous works a generalized control policy to cover m machines n parts type flow-shops is proposed.

The paper is organized as follows. Section 4.2 introduces the notation and presents a formulation of the optimal production and setup scheduling problem, for a m buffered machines multiple products manufacturing system. In section 4.3, the dynamic programming equations and the optimality conditions are derived. In section 4.4, the derived optimality conditions are solved numerically for the case of a two buffered machines two parts manufacturing system. Section 4.5, provides sensitivity analysis to illustrate the optimal control policy structure. The generalization of the heuristic to cover m machines n parts type flow-shops is reported in section 4.6. The paper is concluded in section 4.7.

4.2 Notation and problem formulation

4.2.1 Notation

The following notation will be used in the rest of the paper

- n — number of products
- P_i — product type i, $1 \le i \le n$
- Θ_{ij} — setup duration to switch from P_i to P_j
- K_{ij} — setup cost to switch from P_i to P_j
- d^i — demand rate for part type i, $1 \le i \le n$

$\boldsymbol{d}=\left(d^{1},\ldots,d^{n}\right)'$ vector of demand rates,

$u_k^i(t)$ production rate of product i, $1 \le i \le n$, on machine k, $1 \le k \le m$

$\overset{\max}{U}{}_k^i$ maximal production rate of part type i on machine k

B_k buffer downstream machine k

$x_k^i(t)$ inventory level of product i on B_k, $1 \le k \le (m-1)$

$x_m^i(t)$ inventory / backlog levels of product i on B_m (finished products)

$x^i(t)=\left(x_1^i,\ldots,x_m^i\right)'$ vector of inventory / backlog levels of product i

$x_k(t)=\left(x_k^1,\ldots,x_k^n\right)'$ vector of inventory / backlog levels on B_k, $1 \le k \le m$

$\alpha_k(t)$ continuous time and finite state Markov process of the machine capacity k

$q_{\alpha\beta}^k$ transition rates from modes α to β on machine k

c_{im}^- product type i backlog cost, incurred on finished product (buffer m)

c_{ik}^+ product type i inventory cost incurred on buffer k, $1 \le k \le m$

ρ discounted rate of the incurred cost

$g(.)$ instantaneous cost function

$R(.)$ overall cost function during the setup

$J(.)$ expected and discounted cost function

$v(.)$ value function

4.2.2 Problem formulation

The manufacturing system under study consists of an unreliable buffered flow-shop capable of producing n different part types P_i, $1 \le i \le n$. As shown in figure 4.1, the considered flow-shop consists of a serial buffered m machines. The machines are not completely flexible in the sense that change over time (set-up activities) between part types is not negligible. This setup conducted on the whole line involves both time and cost. Note that, $\theta_{ij} \ge 0$ *and* $K_{ij} \ge 0$, for, $i, j = 1,\ldots,n$, and $i \ne j$.

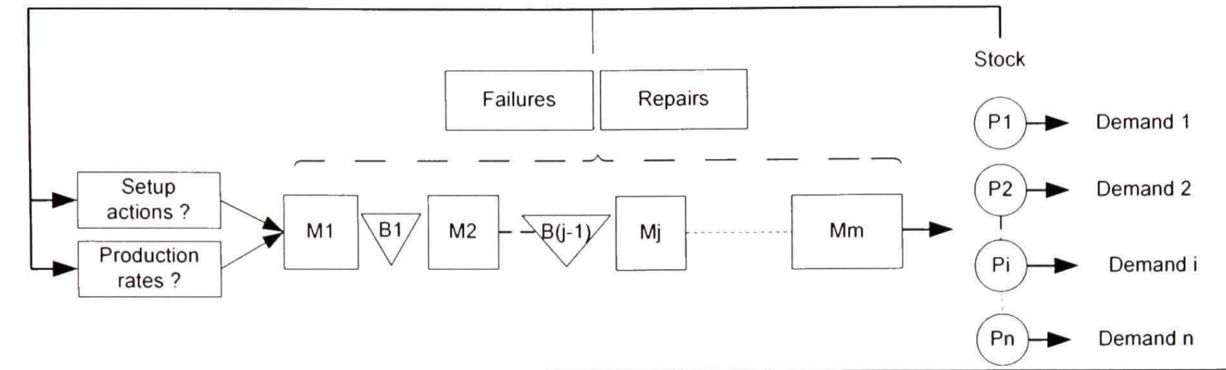


Figure 4.1 Buffered flow-shop manufacturing system

Part type *i* requires an average processing time $p_i^k > 0, (i = 1,\ldots,n; k = 1,\ldots,m)$ on machine *m* and has an average time between orders $1/d^i$. Machines M_k and $M_{k+1}, 1 \le k \le (m-1)$ are separated by a buffer B_k. Each of which is required to store in process products P_i. The level of B_k consists on the sum of $x_k^i(t)$, $1 \le i \le n$. Let L_k the capacity of B_k.

The difference between actual production and downstream demand at any time represents the surplus of a part type. For buffers $B_k, 1 \le k \le (m-1)$ the difference is always positive (i.e, inventory costs c_{ik}^+ are thus charged) or equal to zero (i.e., starvation of machine *k+1*), for buffer B_m the difference is positive (i.e, inventory costs c_{im}^+ are thus charged) or negative (i.e., backlog costs c_{im}^- are thus charged). Note that if the capacity of the buffer $B_k, 1 \le k \le (m-1)$ is reached, machine M_k could be blocked if the downstream demand is equal to zero.

The state of the system at time *t* has two components:

- A continuous part, which describes the cumulative surplus matrix and measured by $x(t)$;
- A discrete part, which describes the whole flow shop state and given by the following processes.

The operational mode of a machines k at time t can be described by the random variables $\xi_k(t), 1 \leq k \leq m$ with value in $M_k = \{1,2\}$, where

$$\xi_k(t) = \begin{cases} 1 & \text{the machine k is available (operational)} \\ 2 & \text{the machine k is unavailable (under repair).} \end{cases}$$

The transition rates matrix of the stochastic processes $\xi_k(t)$ are denoted by T_k such that $T_k = \{q^k{}_{\alpha\beta}\}$, with $q^k{}_{\alpha\beta} \geq 0$ if $\alpha \neq \beta$ and $q^k{}_{\alpha\alpha} = -\sum_{\beta \neq \alpha} q^k{}_{\alpha\beta}$, where $\alpha, \beta \in M_k$. The transitions rates matrix T_k is expressed as follow:

$$T_k = \begin{vmatrix} -q^k{}_{12} & q^k{}_{12} \\ q^k{}_{21} & -q^k{}_{21} \end{vmatrix}$$

The operational mode of the whole system can be described by the random vector $\xi(t) = (\xi_1(t), \ldots, \xi_m(t))$ taking values in $M = M_1 \times \ldots \times M_m$.

Without loose of generality, for the two machine flow shop case, $\xi(t)$ can be expressed as follow.

$$\xi(t) = \begin{cases} 1 & \text{Both } M_1 \text{ and } M_2 \text{ are available.} \\ 2 & M_1 \text{ is available and } M_2 \text{ is unavailable.} \\ 3 & M_1 \text{ is unavailable and } M_2 \text{ is available.} \\ 4 & \text{Both } M_1 \text{ and } M_2 \text{ are unavailable.} \end{cases}$$

The transition rates of the stochastic process $\xi(t)$, (i.e., $T = \{q_{\alpha\beta}\}, \alpha, \beta \in M$) are easily derived from those of $\xi_k(t)$ by using the definition of $\xi(t)$.

For the considered multiple-parts buffered flow-shop manufacturing system, the state space is given by $(x(t), \alpha)$ such that:

$\sum_{i=1}^{n} x_k^i \leq L_k, k = 1, \ldots, (m-1), x_{m+1}^i \in R; \alpha \in M$. Let $S = [0, L_k]^m \times R$; ∂S be the boundary of S and S^0 be the interior of S.

The dynamics of the stock levels is given by the following differential equations:

$$\dot{x}_k^i(t) = u_k^i(t) - u_{k+1}^i(t), \quad k = 1,\ldots,m; i = 1,\ldots,n$$
$$u_{m+1}^i(t) = d_i, \quad i = 1,\ldots,n$$

$u^i(t) = \left(u_1^i,\ldots,u_m^i,u_{m+1}^i\right)'$ extended vector of production rates of product i, $1 \le i \le n$

This equation can also be written in the following vector form:

$$\dot{x}^i(t) = \Lambda^i u^i(t), \quad i = 1,\ldots,n \tag{4.1}$$

Where $\Lambda^i : R^{m+1} \to R^m$ is the corresponding linear operator.

At any given time, the production rates of each machine have to satisfy its capacity constraint. This constraint states that the machine cannot be utilized for more than 100% of its capacity; such a constraint can be represented as:

$$0 \le u_k^i(t) \le U_k^{\max^i} \times \alpha_k(t), \; i = 1,\ldots,n, \; k = 1,\ldots,m \tag{4.2}$$

Note that if $u_k^i(t) \ne 0$ then $u_k^j(t) = 0$ for all $j \ne i; i, j = 1,\ldots,n$.

For each $\alpha_k \in M_k$, the production policies (or capacity) set is given by:

$$\Gamma_k = \left\{ u_k : 0 \le u_k^i(t) \le U_k^{\max^i} \times \alpha_k(t), u_k^j(t) = 0; \forall j \ne i \right\}, k = 1,\ldots,m.$$

Our decision variables are production rates $u_k^i(t), i = 1,\ldots,n; k = 1,\ldots,m$ and a sequence of setups denoted by $\Omega = \{(\tau_0, i_0 i_1), (\tau_1, i_1 i_2), \ldots\}$. A setup (τ, ij) is defined by the time τ at witch it begins and a pair *ij* denoting that the production line was already setup to produce part *i* and is being switched to be able to produce part *j*.

Let *i* denote the initial setup state of the production line and *s* the remaining setup time. The setup cost is assumed to be charged at the beginning of the setup.

The instantaneous cost function $g(.)$ is given by the following equation:

$$g(x_1,x_2,\ldots,x_m)=\left[\sum_{k=1}^{m-1}\sum_{i=1}^{n}\left(c_{ik}^{+}\times x_k^{i\,+}\right)\right]+\sum_{i=1}^{n}\left(c_{im}^{+}\times x_m^{i\,+}+c_{im}^{-}\times x_m^{i\,-}\right) \tag{4.3}$$

Where $x_k^{i\,+}=\max(0,x_k^i)$ and $x_m^{i\,-}=\max(-x_m^i,0)$.

We consider that the system does not incur production or repair cost, the failure penalty is thus the shortage of in process (in this case starvation of the downstream machine is observed) and/or finished products stock (in this case backlog costs are incurred).

The overall cost function of production and setup, during ***s*** units of times, is given by the following equation:

$$R_{ij}(x_1,\ldots,x_m,s)=K_{ij}Ind\{s=\Theta_{ij}\}+\int_0^s e^{-\rho t}g(x_1,\ldots,x_{m-1},x_m-dt)dt,\quad s\in\left[0,\Theta_{ij}\right],\ \forall j\neq i \tag{4.4}$$

The first part of the equation assures that the setup cost is added at the beginning of the setup, while the second part computes the incurred surplus cost. Recall that *s* is the remaining time in the setup.

Using (4.3)-(4.4), the total cost *J*(.) can be defined by the following expression:

$$J(i,x,\alpha,s,\Omega,u(.))=\int_0^s e^{-\rho t}g(x(t))dt+E_{i,x-ds,\alpha_s}\left[\int_s^\infty e^{-\rho t}g(x(t))dt+\sum_{l=0}^{\infty}e^{-\rho\tau_l}K_{i_li_{l+1}}\right] \tag{4.5}$$

where $E_{i,x-ds,\alpha_s}$ is the conditional expectation given the condition $(i,x-ds,\alpha_s)$ at time s and $\alpha_s=1$ if $s>0$ and $\alpha_s=\alpha$ if s=0.

Let *A* denote the set of admissible decisions (Ω, ***u***(.)). The production planning problem considered herein is to find an admissible decision or control policy (Ω, ***u***(.)) that minimizes *J*(.) given by (4.5) considering equations (4.1) to (4.3). This is a feedback control that specifies the control actions when the system is in a given state (***x***,α). The feedback control determines the production rates and the setup actions as a function of the surplus level and the state of the machines.

We make the following assumptions:

For any $i, j, k = 1,\ldots,n, i \neq j, j \neq k$

$$\max(K_{ij}, \Theta_{ij}) > 0 \tag{4.6}$$

$$K_{ij} + K_{jk}e^{-\rho\Theta_{ij}} - K_{ik} > 0, and \quad \Theta_{ij} + \Theta_{jk} - \Theta_{ik} \geq 0 \tag{4.7}$$

The condition (4.6) ensures that the setup changes will not take place with infinitely fast changing production of parts P_i and P_j at some times. The condition (4.7) ensures that if one switches from the production of P_i to the production of P_j and then from P_j to P_k, the related cost and time setups are greater than those incurred while switching directly from the production of P_i to the one of P_k.

While producing the part type i, the corresponding value function $v_i(.)$ is given by the following:

$$v_i(x,\alpha,s) = \inf_{(\Omega,u)\in A} J(i,x,\alpha,s,\Omega,u) \qquad \forall x,\alpha \tag{4.8}$$

For convenience in notation, let $v_i(x,\alpha) = v_i(x,\alpha,0)$. The value function $v_i(x,\alpha,s)$ can then be written in terms of $v_i(x,\alpha)$ and $v_i(x_1,\ldots,x_{m-1},x_m - ds,\alpha)$ as follows:

$$v_i(x,\alpha,s) = \begin{cases} v_i(x,\alpha) & if\ s = 0 \\ \int_0^s e^{-\rho t} g(x(t))dt + e^{-\rho t} v_i(x_1,\ldots,x_{m-1},x_m - ds,1) & if\ s > 0 \end{cases}$$

It is shown, in the next section, that at the optimum the value function $v_i(.)$ given by (4.8) should satisfy a set of equations called optimality conditions.

4.3 Optimality conditions: dynamic programming equations

The properties of the value functions and the dynamic programming equation in terms of directional derivatives (DPEDD) for inner and boundary points are presented in this section. These equations describe the optimality conditions for both production planning and setup scheduling problem.

For convex functions, it is convenient to write a dynamic programming equation in terms of directional derivatives.

A function $f(x), x \in R^N$ is said to have a directional derivative $f'_p(x)$ along the direction $p \in R^N$ if there exists:

$$\lim_{\varepsilon \downarrow 0} \frac{f(x+\varepsilon.p) - f(x)}{\varepsilon} = f'_p(x)$$

If a function $f(x)$ is differentiable at x then $f'_p(x)$ exists for every p and: $f'_p(x) = \langle \nabla f(x), p \rangle$

Where, $\nabla f(x)$ is the gradient of $f(x)$ and $\langle .,. \rangle$ is a scalar product.

Formally, we can write the DPEDD for our problem as follows:

$$\min \left\{ \begin{array}{l} \min\limits_{u^i \in A} \left\{ \sum\limits_{i=1}^{n} \langle \nabla v_i(X_{1m}, \alpha), \Lambda^i u^i(t) \rangle + g(X_{1m}) + \mathrm{T}.v_i(X_{1m},.)(\alpha) \right\} - \rho v_i(X_{1m}, \alpha) \\ \min\limits_{j \neq i} \left\{ R_{ij}(X_{1m}, \Theta_{ij}) + e^{-\rho\Theta_{ij}} v_i(x_1, \ldots, x_{m-1}, x_m - d\Theta_{ij}, 1) \right\} - v_i(X_{1m}, \alpha) \end{array} \right\} = 0 \quad (4.9)$$

Where, $T.v_i(X_{1m},.)(\alpha) = \sum\limits_{\beta \neq \alpha} q_{\alpha\beta} (v_i(X_{1m}, \beta) - v_i(X_{1m}, \alpha))$ and $X_{1m} = (x_1, \ldots, x_m)$.

Because we are faced with a state-constrained problem, we need to shape the value function on the boundary of *S*. To state these boundary conditions we follow the same theory introduced in Capuzzo-Dolcetta and Lions (1990) and applied by Presman et al. (1997). In their work they have shown that, for state constrained problems, we have to consider the solution of the DPEDD equations as viscosity solution inside *S* and viscosity super-solution on the boundaries (i.e., ∂S). The property that $v(.,\alpha)$ is a viscosity super-solution on ∂S plays the role of a boundary condition. We refer the reader to Hajji et al. (2007) and the reference therein for more details.

Let $u^{i^*}(t)$ denote a function that minimizes over A the following expression:

$$\sum_{i=1}^{n} \langle \nabla v_i(X_{1m},\alpha), \Lambda^i u^i(t) \rangle + g(X_{1m}) + \mathrm{T}.v_i(X_{1m},.)(\alpha), \ \forall \alpha \in M$$

Let $S_i(\alpha)$ denote the switching set:

$$S_i(\alpha) = \{X_{1m} : \min_{j \neq i} \{R_{ij}(X_{1m},\Theta_{ij}) + e^{-\rho\Theta_{ij}} v_i(x_1,\ldots,x_{m-1},x_m - d\Theta_{ij},1)\} = v_i(X_{1m},\alpha)\}$$

Using $S_i(\alpha)$ and $u^{i^*}(t)$, an optimal production and setup control can be determined as described below.

Let X_{1m} denote the current surplus. If $X_{1m} \notin S_i(\alpha)$, i.e.

$$\min_{j \neq i} \{R_{ij}(X_{1m},\Theta_{ij}) + e^{-\rho\Theta_{ij}} v_i(x_1,\ldots,x_{m-1},x_m - d\Theta_{ij},1)\} - v_i(X_{1m},\alpha) > 0$$

Then the first part of the DPEDD (4.9) must be equal to 0. In this case, there is no setup needed and the manufacturing system should be operated under the production policy $u^{i^*}(t)$.

However, if $X_{1m} \in S_i(\alpha)$ i.e.,

$$\min_{j \neq i} \{R_{ij}(X_{1m},\Theta_{ij}) + e^{-\rho\Theta_{ij}} v_i(x_1,\ldots,x_{m-1},x_m - d\Theta_{ij},1)\} - v_i(X_{1m},\alpha) = 0$$

Then a setup is required and we need to switch the production from part type *i* to part type *j*, in order to minimize $R_{ij}(X_{1m},\Theta_{ij}) + e^{-\rho\Theta_{ij}} v_i(x_1,\ldots,x_{m-1},x_m - d\Theta_{ij},1)$ over $j = 1,\ldots,n; j \neq i$

To conclude, if the flow-shop is already setup for part type *k*, then choose the control $u^{k^*}(t)$. When the state trajectory X_{1m} reaches the set $S_k(\alpha)$, setup the flow-shop for part *i* which denotes the minimizer of:

$R_{kj}(X_{1m},\Theta_{kj}) + e^{-\rho\Theta_{kj}} v_i(x_1,\ldots,x_{m-1},x_m - d\Theta_{kj},1)$ over $j = 1,\ldots,n; j \neq k$, and produce that part under $u^{i^*}(t)$ and so on.

Let $\left(\Omega^{*}, u^{i^{*}}(.)\right)$ denotes such a production and setup policy, with $\Omega^{*}=\left\{\left(\tau_{k}^{*}, ki\right),\left(\tau_{i}^{*}, im\right), \ldots\right\}$. The production and setup policy that we are seeking is obtained when the value function is known. While we cannot solve analytically the DPEDD (4.9), we can apply numerical methods. However, the obtained value function and the control policy are only approximations. The following description establishes that near optimal control is obtained when the approximated value function is substituted to the true value function to construct the policy.

Let $v_{i}^{\varepsilon}(X_{im}, \alpha)$ denotes a sequence of functions that converges to $v_{i}(X_{1m}, \alpha)$ as $\varepsilon \rightarrow 0$, and $u^{i^{\varepsilon}}(t)$ the function that minimizes the following:

$$\sum_{i=1}^{n}\left\langle\nabla v_{i}^{\varepsilon}(X_{1m}, \alpha), \Lambda^{i} u^{i}(t)\right\rangle+g(X_{1m})+\mathrm{T}.v_{i}^{\varepsilon}(X_{1m}, .)(\alpha), \ \forall \alpha \in M$$

Over $u^{i}(t) \in A$. Let

$$\mathrm{S}_{i}^{\varepsilon}(\alpha)=\left\{X_{1m}: \begin{array}{c}\min\limits_{j \neq i}\left\{R_{ij}(X_{1m}, \Theta_{ij})+e^{-\rho \Theta_{ij}} v_{i}^{\varepsilon}(x_{1}, \ldots, x_{m-1}, x_{m}-d \Theta_{ij}, 1)\right\}-v_{i}^{\varepsilon}(X_{1m}, \alpha) \leq \\ \min\limits_{u^{i} \in A}\left\{\sum_{i=1}^{n}\left\langle\nabla v_{i}^{\varepsilon}(X_{1m}, \alpha), \Lambda^{i} u^{i}(t)\right\rangle+g(X_{1m})+\mathrm{T}.v_{i}^{\varepsilon}(X_{1m}, .)(\alpha)\right\}-\rho v_{i}^{\varepsilon}(X_{1m}, \alpha)\end{array}\right\}$$

As described previously, using $S_{i}^{\varepsilon}(\alpha)$ and $u^{i^{\varepsilon}}(t)$, we define a sequence of setups and an optimal production control. We use $\left(\Omega^{\varepsilon}, u^{i^{\varepsilon}}(.)\right)$ to represent such a policy, with $\Omega^{\varepsilon}=\left\{\left(\tau_{k}^{\varepsilon}, ki\right),\left(\tau_{i}^{\varepsilon}, im\right), \ldots\right\}$.

Under the same assumptions as in Hajji et al. (2004) and the reference therein, it can be shown that the control policy $\left(\Omega^{\varepsilon}, u^{i^{\varepsilon}}(.)\right)$ is asymptotically optimal, i.e.,

$$\lim_{\varepsilon \rightarrow 0} J\left(i, X_{im}, \alpha, \Omega^{\varepsilon}, u^{i^{\varepsilon}}\right)=v_{i}(X_{1m}, \alpha)$$

Based on this fact, the DPEDD can now be solved with numerical approaches to obtain the approximation of the value function and the associated control policy.

4.4 Numerical approach

To approximate the solution of the DPEDD equations corresponding to the stochastic optimal control problem, and to solve the corresponding optimality conditions, numerical methods will be used. Hence, the unbounded domain, typically associated with the infinite horizon control, should be replaced by a large but bounded domain endowed with appropriate boundary conditions. This method is based on the Kushner and Dupuis approach (Kushner and Dupuis (1992)). The basic idea behind it consists of using an approximation schema for the gradient of the value function $v_i(X_{1m},\alpha)$.

4.4.1 Numerical optimality conditions

Let $h_j^i, i=1,..,n; j=1,..,m$, denotes the length of the finite difference interval of the variable x_j^i. Using the finite difference approximation $v_i(X_{1m},\alpha)$ could be given by $v_i^h(X_{1m},\alpha)$ and the gradient $(v_i)_{x_j^i}(.,\alpha)$ by:

$$(v_i)_{x_j^i}(.,\alpha)=\begin{cases}\frac{1}{h_j^i}\left(v_i^h\left(x_1^1,...,x_j^i+h_j^i,...,x_m^n,\alpha\right)-v_i^h\left(x_1^1,...,x_j^i,...,x_m^n,\alpha\right)\right) & if\ u_j^i-u_{j+1}^i\geq 0\\ \frac{1}{h_j^i}\left(v_i^h\left(x_1^1,...,x_j^i,...,x_m^n,\alpha\right)-v_i^h\left(x_1^1,...,x_j^i-h_j^i,...,x_m^n,\alpha\right)\right) & if\ u_j^i-u_{j+1}^i<0\end{cases}$$

And

$$\begin{aligned}(u_j^i-u_{j+1}^i)(v_i)_{x_j^i}(.,\alpha)=&\frac{\left|u_j^i-u_{j+1}^i\right|}{h_j^i}v_i^h(x_1^1,..,x_j^i+h_j^i,..,x_m^n,\alpha)Ind\left\{u_j^i-u_{j+1}^i\geq 0\right\}\\&+\frac{\left|u_j^i-u_{j+1}^i\right|}{h_j^i}v_i^h(x_1^1,..,x_j^i-h_j^i,..,x_m^n,\alpha)Ind\left\{u_j^i-u_{j+1}^i<0\right\}\\&-\frac{\left|u_j^i-u_{j+1}^i\right|}{h_j^i}v_i^h(x_1^1,..,x_j^i,..,x_m^n,\alpha)\end{aligned}$$

Where $Ind\left\{u_j^i-u_{j+1}^i\geq 0\right\}=\begin{cases}1, & if\ u_j^i-u_{j+1}^i\geq 0\\ 0, & otherwise\end{cases}$

Using the finite difference approximation, the DPEDD equations (4.9) are expressed in terms of $v_i^h(X_{1m},\alpha)$ as follows:

$$\min\left\{\begin{array}{l}\min\limits_{u^i}\left\{\begin{array}{l}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}\left[\left(u_j^i-u_{j+1}^i\right)\left(v_i\right)_{x_j^i}(.,\alpha)\right]+g(x_1,..,x_m)+\\ \sum\limits_{\beta\neq\alpha}q_{\alpha\beta}\left(v_i^h(x_1,...,x_m,\beta)-v_i^h(x_1,...,x_m,\alpha)\right)\end{array}\right\}-\rho v_i^h(x_1,..,x_m,\alpha)\\ \min\limits_{j\neq i}\left\{R_{ij}(x_1,...,x_m,\Theta_{ij})+e^{-\rho\Theta_{ij}}v_i^h(x_1,...,x_{m-1},x_m-d\Theta_{ij},1)\right\}-v_i^h(x_1,..,x_m,\alpha)\end{array}\right\}=0 \quad (4.10)$$

The solution of (4.10) may be obtained by either successive approximation or policy improvement techniques.

4.4.2 Implementation for two buffered machines, two products flow-shop

For the two buffered machines two products flow-shop (figure 4.2), the cost equations and the numerical DPEDD (4.10) are given as follow:
$n=m=2, \alpha\in M=\{1,2,3,4\}$.

$$R_{12}\left(x_1^1,x_1^2,x_2^1,x_2^2,\Theta_{12}\right)=K_{12}+\int_0^{\Theta_{12}}e^{-\rho t}g\left(x_1^1,x_1^2,x_2^1-d^1t,x_2^2-d^2t\right)dt,$$

$$R_{21}\left(x_1^1,x_1^2,x_2^1,x_2^2,\Theta_{21}\right)=K_{21}+\int_0^{\Theta_{21}}e^{-\rho t}g\left(x_1^1,x_1^2,x_2^1-d^1t,x_2^2-d^2t\right)dt,$$

$$g\left(x_1^1,x_1^2,x_2^1,x_2^2\right)=\sum_{i=1}^{2}\left(c_{i1}^+\times x_1^{i^+}\right)+\sum_{i=1}^{2}\left(c_{i2}^+\times x_2^{i^+}+c_{i2}^-\times x_2^{i^-}\right)$$

Recall that $x_k^{i^+}=\max(0,x_k^i)$ and $x_m^{i^-}=\max(-x_m^i,0)$.

The discrete dynamic programming equations (4.10) give the following eight equations, which illustrate the optimal value functions for the two products system subject to the four states of the flow-shop.

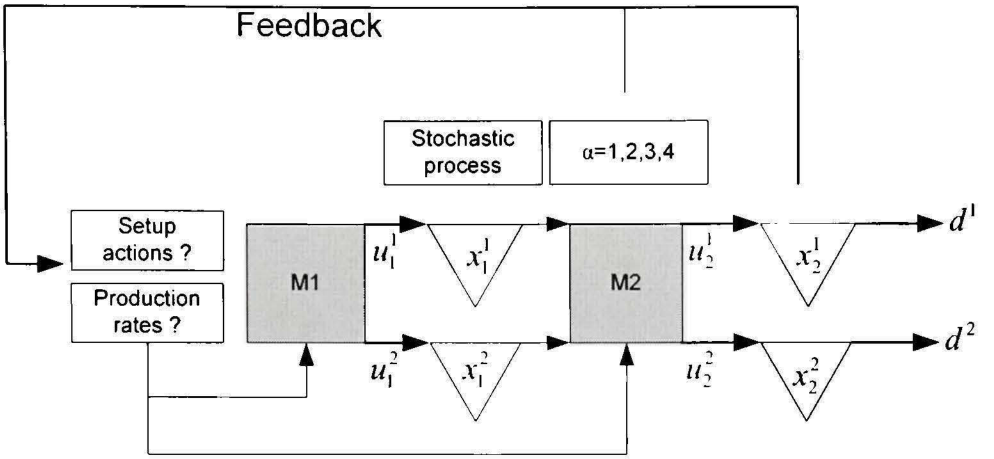


Figure 4.2 Buffered two machines flow-shop

$$
\begin{aligned}
&v_i^h\left(x_1^1, x_1^2, x_2^1, x_2^2, \alpha\right) \\
&= \\
&\min\left\{
\begin{array}{l}
\min\limits_{u'=(u'_1,u'_2)}\left\{
\begin{array}{l}
\left(\rho + \left|q_{\alpha\alpha}\right| + \sum\limits_{i=1}^{2}\sum\limits_{j=1}^{2} \dfrac{u_j^i - u_{j+1}^i}{h_j^i}\right)^{-1} \\
\left(\begin{array}{l}
\sum\limits_{\beta\neq\alpha} q_{\alpha\beta}\left(v_i^h\left(x_1^1, x_1^2, x_2^1, x_2^2, \beta\right)\right) + g\left(x_1^1, x_1^2, x_2^1, x_2^2\right) + \\
\sum\limits_{i=1}^{2}\sum\limits_{j=1}^{2} \dfrac{u_j^i - u_{j+1}^i}{h_j^i}\left[\begin{array}{l} v_i^h\left(x_j^i + h_j^i, \alpha\right) Ind\left\{u_j^i - u_{j+1}^i \geq 0\right\} + \\ v_i^h\left(x_j^i - h_j^i, \alpha\right) Ind\left\{u_j^i - u_{j+1}^i < 0\right\} \end{array}\right]
\end{array}\right)
\end{array}
\right\} \\
\min\limits_{j\neq i}\left\{e^{-\rho\Theta_{ij}} v_j^h\left(x_1^1, x_1^2, x_2^1 - d^1\Theta_{ij}, x_2^2 - d^2\Theta_{ij}, 1\right) + R_{ij}\left(x_1^1, x_1^2, x_2^1, x_2^2, \Theta_{ij}\right)\right\}
\end{array}
\right.
\end{aligned}
$$

, $\forall i = 1,2; \alpha = \{1,2,3,4\}$

The decisions we are seeking consist on the production rates: $u_j^i(.), i, j = 1,2$ and the switching instants. These are feedback control policy functions of the stock states $x_j^i(.)$ and the flow-shop states $\alpha \in M = \{1,2,3,4\}$. The implementation of the approximation technique needs the use of a finite grid denoted herein G_h. Thus some boundary conditions are needed to describe the behaviour of the system at the border of G_h.

The computation domain D is defined as follow:

$$D=\left\{(x_1^1,x_1^2,x_2^1,x_2^2):-a_i^j\le x_i^j\le a_i^j\right\} \tag{4.11}$$

Where $\left\{a_i^j;i,j=1,2\right\}$ are given positive constants.

We refer the reader to Hajji et al. (2004) and Gharbi et al. (2006) for more details on the boundary conditions and the computation algorithm.

4.5 Numerical results

In this section, deep analysis of the numerical results used to characterize the optimal production and changeover policies are presented. Our objective is to characterize the general structure of such policies. Note that for the case under study (i.e., state space) and for the employed computational domain one should illustrate and analyse at least 272, 2D figures to make sure that the control policies are well illustrated. Thus, we will show the most representative numerical results for two basic cases (section 4.5.1) followed by an illustration of the final structure after the conducted sensitivity analysis (section 4.5.2).

4.5.1 Results interpretation for the two basic cases

The computational domain $\mathcal{D}$ given by (4.11) is rewrite here for $\left\{a_i^j=1\,\&\,h_i^j=0.1;i,j=1,2\right\}$.

The transition rate matrix defining the flow-shop stochastic process is defined as follows and corresponds to availabilities rates equal to 90.9 for the two machines.

$$T=\begin{bmatrix}-0.02 & 0.01 & 0.01 & 0\\ 0.1 & -0.11 & 0 & 0.01\\ 0.1 & 0 & -0.11 & 0.01\\ 0 & 0.1 & 0.1 & -0.2\end{bmatrix}$$

Table 4.1 shows the constant parameters for all the numerical examples and Table 4.2 details the cost variations for the two basic cases.

Tableau 4.1

Constant data parameters

PARAMETERS	(d^1, d^2)	$(U_1^{max 1}, U_1^{max 2})$	$(U_2^{max 1}, U_2^{max 2})$	ρ
Values	(0.2,0.2)	(1,1)	(1,1)	0.4

Tableau 4.2

Cost variations

PARAMETERS	(c_{11}^+, c_{21}^+)	(c_{12}^+, c_{22}^+)	(c_{12}^-, c_{22}^-)	(K_{12}, K_{21})	$(\Theta_{12}, \Theta_{21})$
Case1 values	(0.4,0.4)	(1,1)	(20,20)	(0.1,0.1)	(0.4,0.4)
Case2 values	(0.4,0.6)	(1,1.5)	(10,30)	(0.15,0.1)	(0.6,0.4)

In case 1, identical parts are produced while inventory and backlog costs are set so that products are of equal importance. Case 2 shows a scenario for manufacturing two different products. In this case inventory, backlog and setup costs as well as setup durations are set to give more importance to part type 2.

The numerical results for cases 1 and 2 are shown in figure 4.4 to 4.6 and figure 4.7, respectively. For case 1, figure 4.4 and 4.5 show the production policies for part type 2 on machine 1 and 2 respectively. Figure 4.6 shows the changeover policy from part 2 to part type 1. It is interesting to note that for the identical case study (case 1), when the flow-shop is setup for part type 1 the same policies with respect to the stock levels are observed. Based on this fact, the illustration of the control policies when the system is setup for one product is representative and will allow us to dress the final policies structure. For case 2, figure 4.7 shows the changeover policy from part type1 to part type2.

Recall that for the considered system under study we face a four dimension space for the stock levels namely x_1^1, x_1^2, x_2^1 and x_2^2. The following figure (i.e., Figure 4.3) shows how we will illustrate the control policy in the four dimension inventory state space.

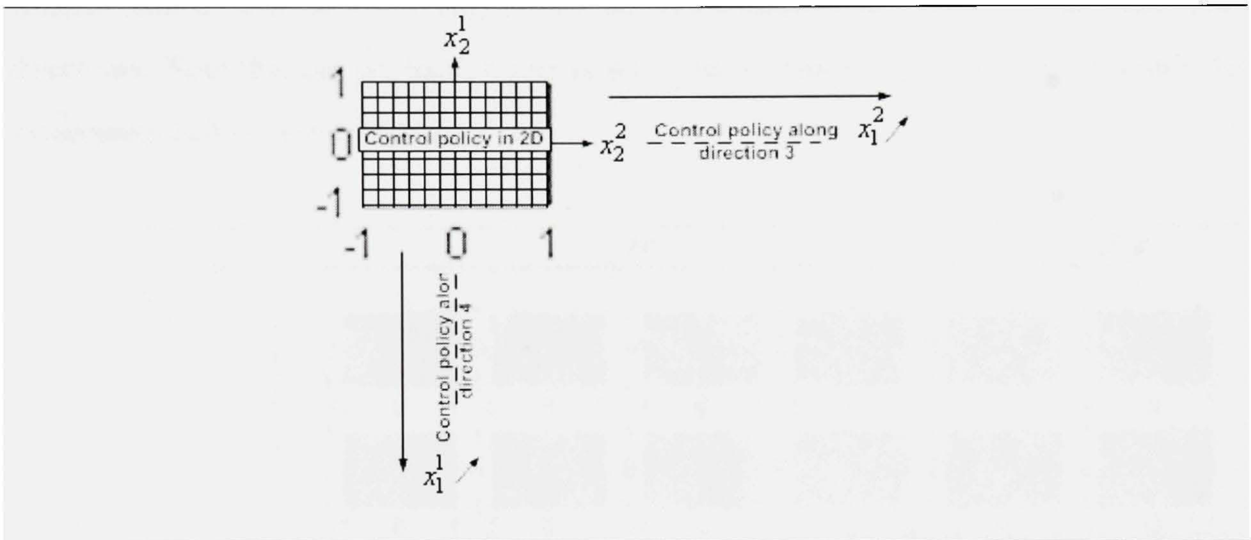


Figure 4.3 Results illustration in 4 dimensions

Case 1 results (figure 4.4, 4.5 and 4.6) show that the resulting production and changeover policies (when the flow-shop is setup for part type 2) divide the surplus space into three mutually exclusive stock regions. From figures 4.4 to 4.6, we can observe the following three regions:

- In region I, keep the same setup and produce this part type at the maximum rate.
- In region II, keep the same setup and set the production rate to zero.
- In region III, change the configuration for the other part type (the setup is from part type 2 to part type 1 in this case 1).

The numerical results of figure 4.4 show the production rate policy of product type 2 on machine 1 function of the stock state space (i.e., $u_1^2 = f(x_1^1, x_1^2, x_2^1, x_2^2)$). It clearly appears that when the line is setup for part type 2, machine 1 must produce at the maximum rate while x_2^2 and x_1^2 are smaller than two different hedging levels. Moreover, the production policy doesn't change with respect to x_1^1 and x_2^1 directions. In the same direction, figure 4.5 shows the production rate policy of product type 2 on machine 2 (i.e., u_2^2), function of the stock state space. It appears that machine 2 must produce at the maximum rate while x_2^2 is smaller than a hedging level. This policy doesn't change with respect to the other three directions. Note that the production rate is set to zero when $x_1^2 = 0$, this point towards the aforementioned starvation situation.

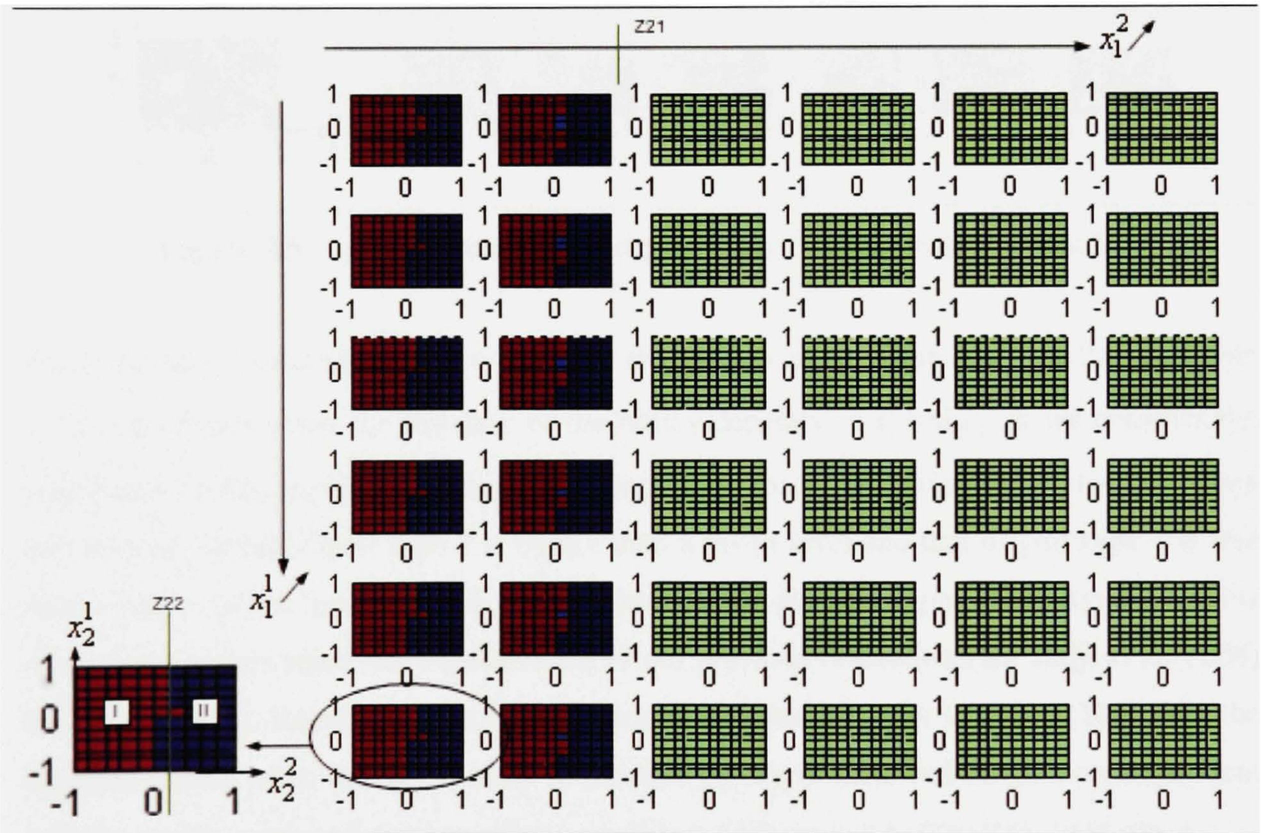


Figure 4.4 Production policy of part type 2 on machine 1, (case 1)

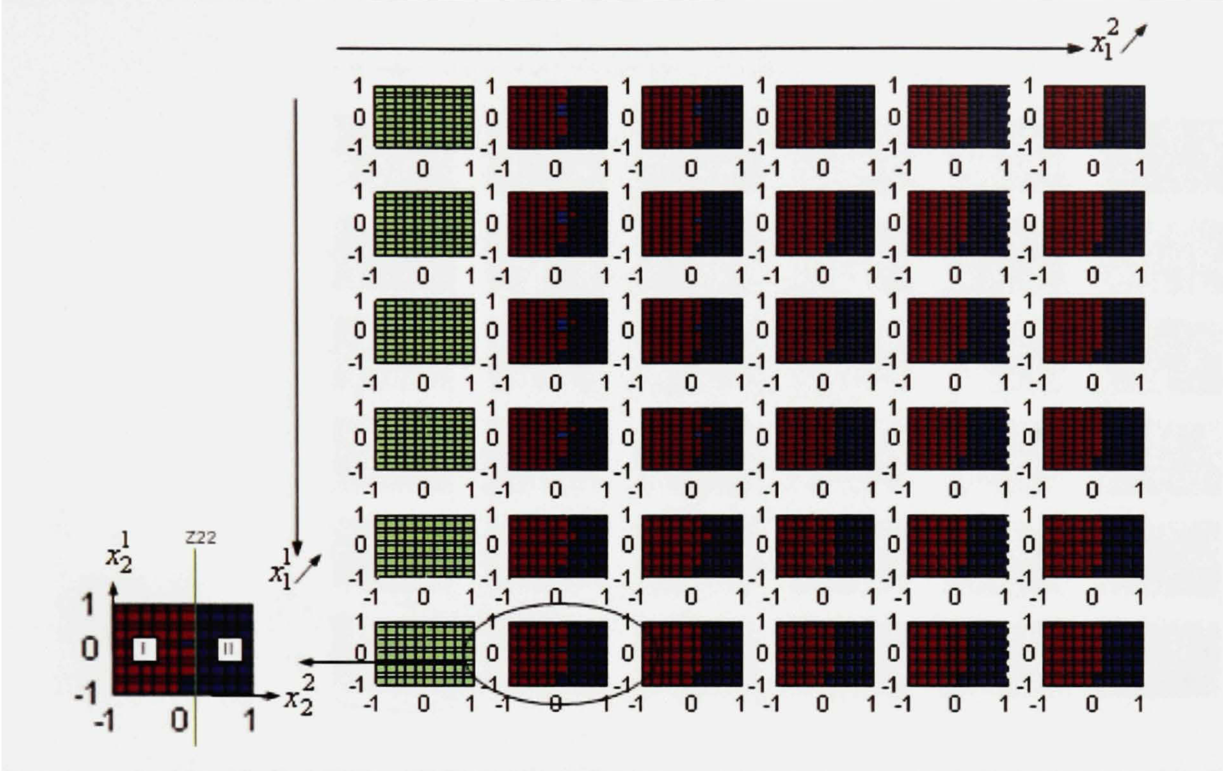


Figure 4.5 Production policy of part type 2 on machine 2, (case 1)

The numerical results of figure 4.6 show the changeover policy from part type 2 to part type 1. We can observe that the structure of the policy depends on x_2^1 and x_2^2. If we consider the boundaries of this region in the corresponding stock space, the policy stipulates that when the inventory level of part type 2 is higher than a given level and that of part type 1 is less than another given level, switch the production to the other part (i.e., type 1). This observation makes sense and confirms one of our previous researches (see Hajji et al (2004) for more details). Recall that this case illustrates an identical parts situation. Therefore the obtained results when the line is setup to produce part type 1 showed that the correspondent policies are the same and one has only to do mirror with respect to the stock state space.

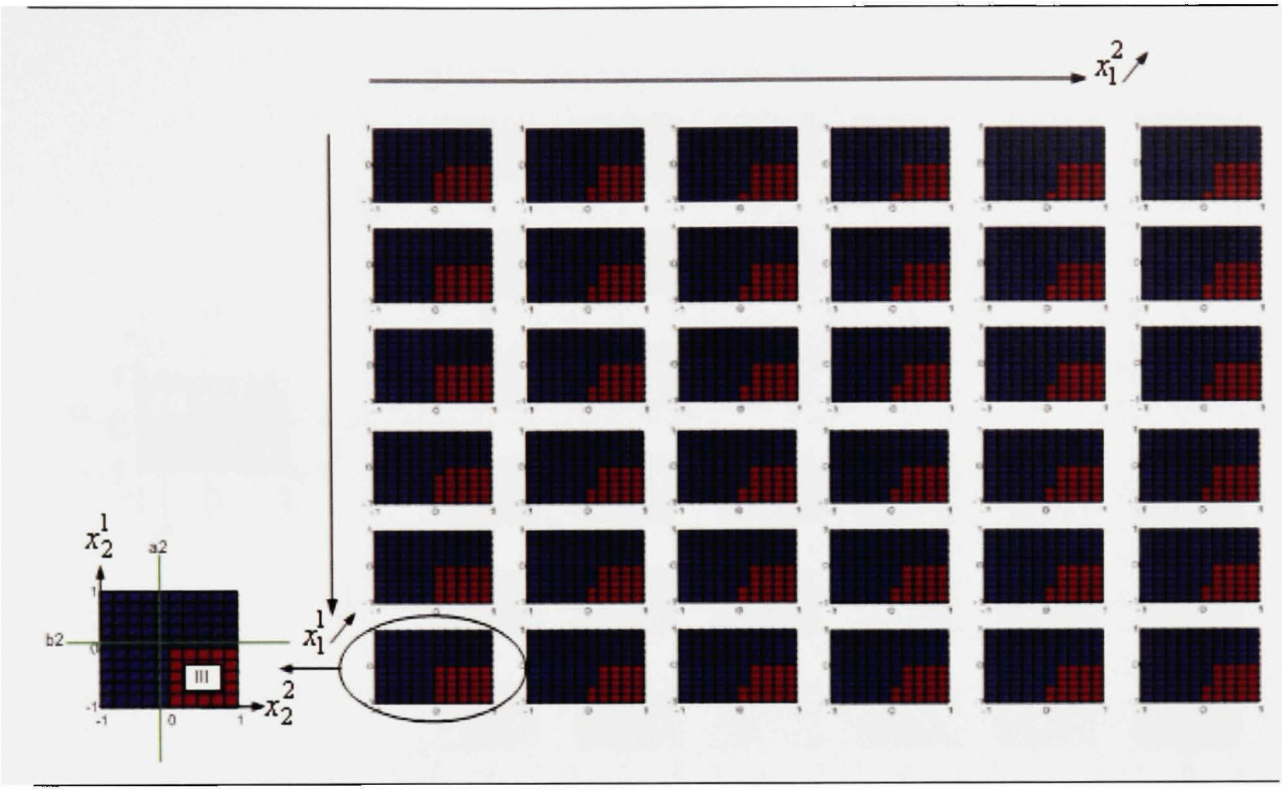


Figure 4.6 Changeover policy from part type 2 to part type 1, (Case 1)

The second case which considers two different parts type showed that the production policies on machine 1 and 2 have preserved the same structure but obviously with different values of the related hedging level. However, the changeover policy illustrated in figure 4.7 reveals a new structure. While this structure depends only on x_2^1 and x_2^2 it gives more importance to the part type owing the highest backlog cost (i.e., part type 2). In fact, even if the inventory level (i.e., x_2^1) of part type 1 is negative and that of part type 2 (i.e., x_2^2) is less than a backlog boundary, one must switch the production to part type 2. Note that the policy has the same structure detailed in the previous section in the other space regions. This observation makes sense and confirms as well one of our previous researches (see Hajji et al (2004) for more details). In the sense that the system should redress the inventory level of the part type owing the highest backlog cost even in detriment of the other part.

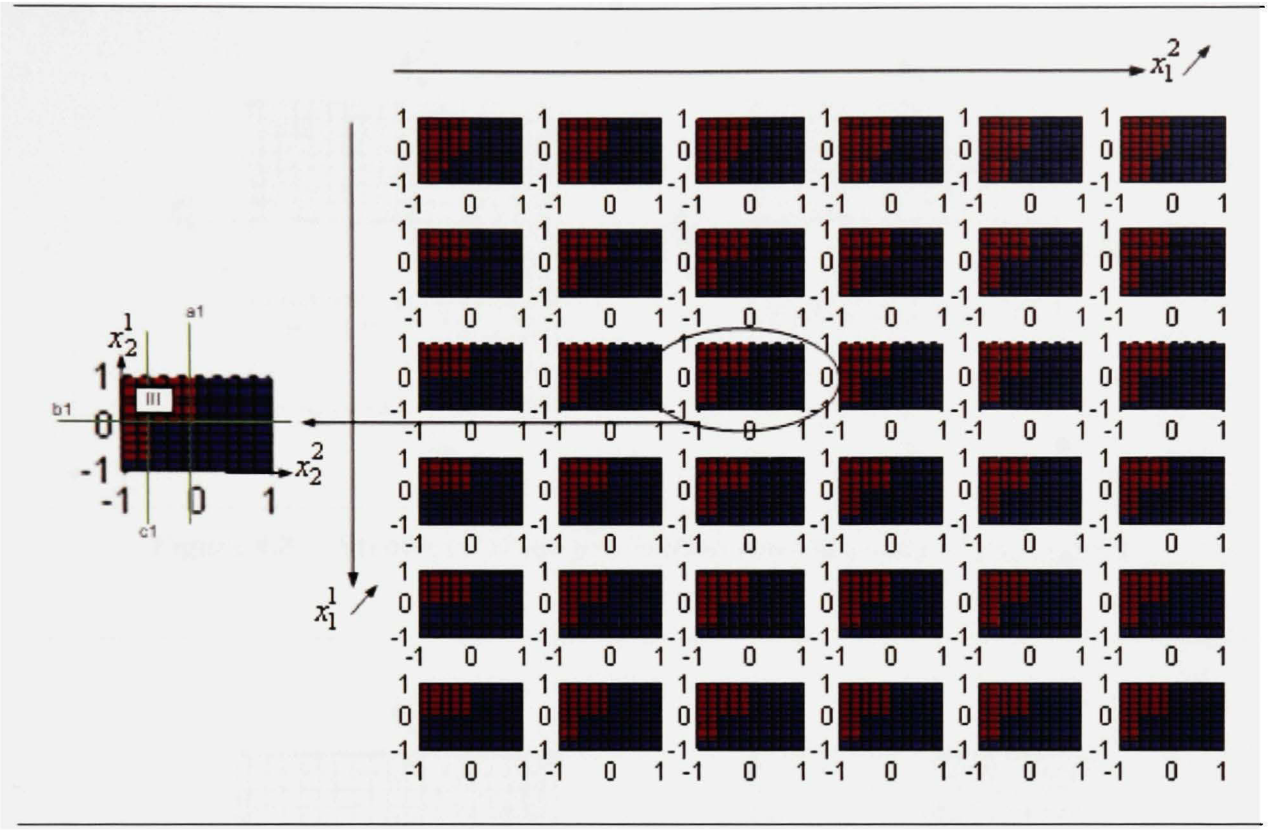


Figure 4.7 Changeover policy from part type 1 to part type 2 (case 2)

The production and setup policies take place in bounded regions; this observation will be useful for the development of a parameterized heuristic of the production and changeover policies so as to approximate the optimal control policies without solving the associated DPEDD equations.

In figure 4.8 and 4.9, we illustrate in the appropriate stock space the hedging levels which govern the production of part type 1 (respectively part type 2) on machine 1 and 2. As explained in the last sections, we must produce this part type to reach a final product threshold Z_1^2 (respectively Z_2^2) and at the same time it is not allowed to accumulate more than Z_1^1 (respectively Z_2^1) in the corresponding «Work In Process» (WIP).

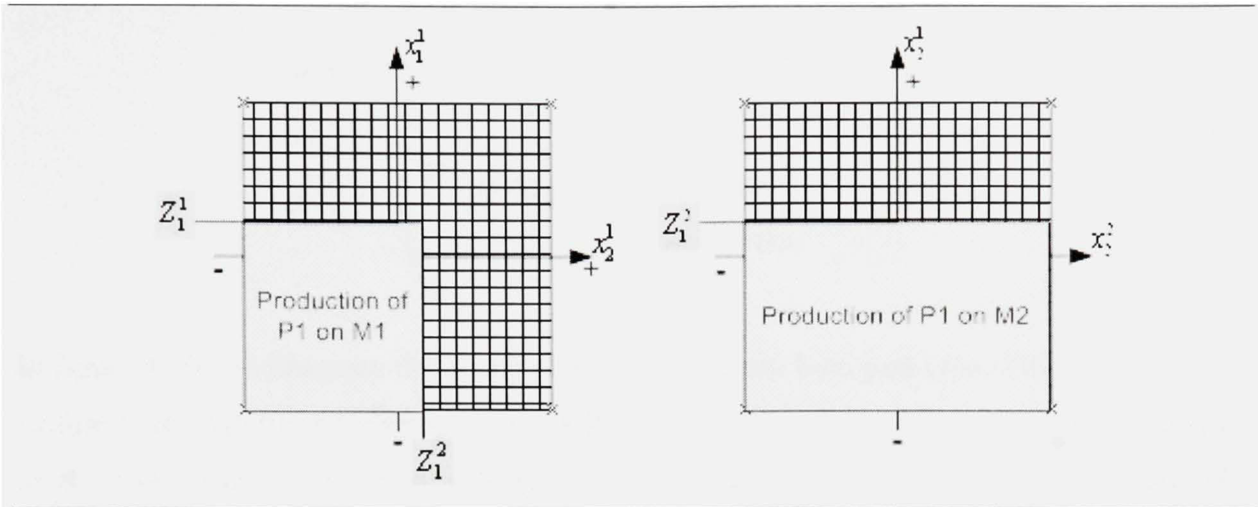


Figure 4.8 Structure of the production control policy of part type 1

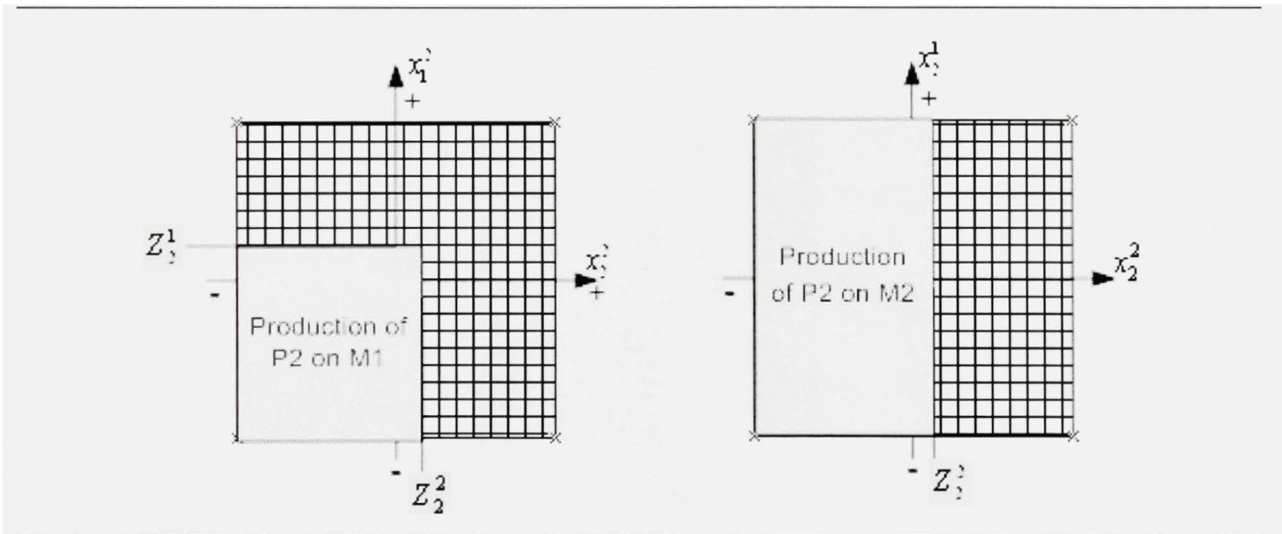


Figure 4.9 Structure of the production control policy of part type 2

If we use the boundaries in the negative and positive zones of the stock space, we can describe and to parameterize the production policies by the following equations.

$$u_1^1(\;) = \begin{cases} U_1^{\max 1} \, IND\{S_1 = 1\} & x_1^1 \le Z_1^1 \;\&\; x_2^1 \le Z_1^2 \\ 0 & otherwise \end{cases}$$

$$u_2^1(\;) = \begin{cases} U_2^{\max 1} \, IND\{S_1 = 1\} & x_2^1 \le Z_1^2 \\ 0 & otherwise \end{cases} \tag{4.12}$$

$$u_1^2(\ )=\begin{cases} U_1^{\max 2}\ IND\{S_2=1\} & x_1^2 \le Z_2^1\ \&\ x_2^2 \le Z_2^2 \\ 0 & otherwise \end{cases}$$
$$u_2^2(\ )=\begin{cases} U_2^{\max 2}\ IND\{S_2=1\} & x_2^2 \le Z_2^2 \\ 0 & otherwise \end{cases} \tag{4.13}$$

In figure 4.10, we illustrate the changeover policies from both part type. This policy is very similar to the Modified Hedging Corridor Policy found in our previous researches (see Hajji et al. (2004) and Gharbi et al, (2006)). Its main characteristic lies in the appearance of a corridor in the positive and negative stock areas. The boundaries of this policy are denoted by, a_i, b_i, c_i and a_j, b_j, with j referring to the part type owing the higher costs.

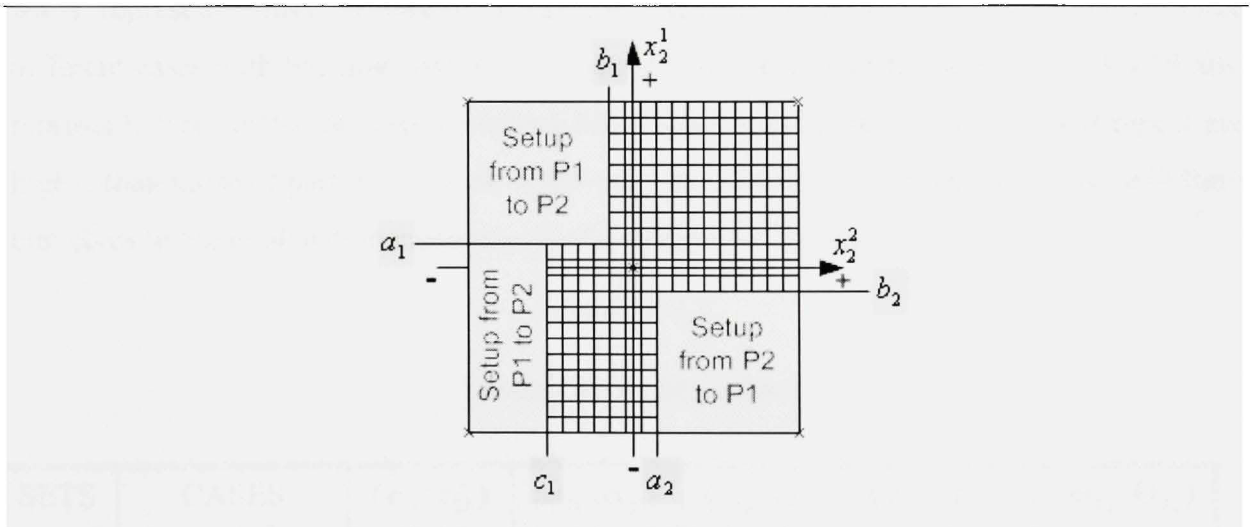


Figure 4.10 changeover control policy structure

If we use the boundaries in the negative and positive zones of the stock space, we can describe and to parameterize the changeover policies by the following equations.

$$S_1(\ )=\begin{cases} 1 & x_2^1 \le b_2\ \&\ x_2^2 \ge a_2 \\ 0 & otherwise \end{cases}$$
$$S_2(\ )=\begin{cases} 1 & \left(x_2^1 \ge a_1\ \&\ x_2^2 \le b_1\right) \| \left(x_2^2 \le c_1\right) \\ 0 & otherwise \end{cases} \tag{4.14}$$

4.5.2 Sensitivity analysis and policy structure validation

The system under study involves operational and system parameters and its complexity is attributable to their wide range of variability. To understand the effect that changing these parameters has on the solution, sensitivity analysis is necessary. However, it`s a hard task to track the effects of all the parameters and their interactions, for that reason our efforts are concentrated on operational parameters judged to be the most appropriate. Hence, costs of surplus and backorders for each product are considered in the study. The sensitivity analysis enables the tracking of variations to the policy boundaries so as to make sure that the parameterized heuristic control policies detailed in section 3.5.1 make sense and are very close to the optimal and general control. Let us now consider the sets presented in Table 4.3. Set I represents three symmetric cases for identical products. Set II, represents three different cases with backlog cost for part type 2 higher than that for part type 1. Set III also represents three different cases for which both backlog and surplus costs for part type 2 are higher than those of part type 1. Due to the great number of the derived figures we will limit ourselves to the explanation and analysis of the observed results.

Tableau 4.3

Sensitivity analysis data

SETS	CASES	(c_{11}^+, c_{21}^+)	(c_{12}^+, c_{22}^+)	(c_{12}^-, c_{22}^-)	(K_{12}, K_{21})	$(\Theta_{12}, \Theta_{21})$
I	1	(0.4,0.4)	(1,1)	(10,10)	(0.1,0.1)	(0.4,0.4)
	2	(0.4,0.4)	(1,1)	(20,20)	(0.1,0.1)	(0.4,0.4)
	3	(0.4,0.4)	(1,1)	(30,30)	(0.1,0.1)	(0.4,0.4)
II	1	(0.4,0.4)	(1,1)	(10,20)	(0.1,0.1)	(0.4,0.4)
	2	(0.4,0.4)	(1,1)	(10,30)	(0.1,0.1)	(0.4,0.4)
	3	(0.4,0.4)	(1,1)	(10,40)	(0.1,0.1)	(0.4,0.4)
III	1	(0.4,0.5)	(1,1.2)	(10,20)	(0.12,0.1)	(0.5,0.4)
	2	(0.4,0.6)	(1,1.5)	(10,30)	(0.15,0.1)	(0.6,0.4)
	3	(0.4,0.7)	(1,1.7)	(10,40)	(0.17,0.1)	(0.7,0.4)

Analysis of set I results

The results of set I cases show the effect of increasing the backlog costs on the production and changeover policies. In fact, the boundaries of the control policies move in the desired direction with respect to the variation of the parameters. This means that when the backlog costs increase, the values of the hedging thresholds increase. It is illustrated by a movement of $(Z_1^1, Z_1^2)\uparrow, (Z_2^1, Z_2^2)\uparrow$ for the production policy boundaries.

For the changeover policy, it appears that a_1, a_2 increase and $c_1 \to -\infty$. These observations make sense since a_1 *and* a_2 play the role of security levels for the part type being produced and the fact that the two parts have the same incurred costs removes c_1 from the changeover control policy.

Analysis of set II results

The results of set II cases show the effect of increasing the backlog cost of part type 2 on the production and changeover policies. In comparison to the results of set I, it seems realistic that some of the aforementioned boundaries despite the others keep the same variations. This means that when the backlog cost of part type 2 increases, only the values of the hedging thresholds of part 2 increases. It is illustrated by a movement of $(Z_1^2, Z_2^2)\uparrow$ for the production policy boundaries.

For the changeover policy, it appears that a_1 decreases, a_2 increases and c_1 increases (i.e., $c_1 \to 0$). These observations make sense since: a_2 plays the role of security levels for the part type 2 and a_1 plays the role of an obstacle facing the changeover to that part type. Moreover, the fact that we are facing two different parts replaces c_1 to the changeover control policy.

Analysis of set III results

The results of set III cases show the effect of increasing all the costs of part type 2 on the production and changeover policies. In comparison to the results of set II, one can observe the same variations but with lower values compared to those of set II. However, the fact that all the costs of part type 2 are higher than those of part type 1 revealed that the hedging threshes of this last part move in an increasing direction (i.e., $\left(Z_1^1, Z_2^1\right)\uparrow$).

These numerical results indicate that in all cases the structure of the optimal production and setup policy is constant. Moreover, the boundaries of the control policies move in the desired direction with respect to the variation of the parameters. These observations show that the developed and reported parameterized heuristic makes sense and can be used to approximate the optimal control policy.

For the two machines two parts case, the parameterized production and setup policy relies on seven parameters, denoted by a_i, b_i, c_i and a_j, b_j, j referring to the part type owing the higher costs and $Z_i^j; i, j = 1,2$. The best control policy for a given manufacturing system is found for the best values of those parameters. To approximate such values, one can resort to design of experiments combined to simulation modelling, such as in Gharbi et al. (2006).

4.5.3 Practical contribution

In the manufacturing system control literature, a great number of research studies have studied different production control policies. A non exhaustive listing includes CONWIP, BASE STOCK, KANBAN, GENERALIZED KANBAN, EXTENDED KANBAN and CONWIP KANBAN control systems. We refer the reader to Boonlertvanich (2005) for detailed analysis of all these policies. It is interesting to note that for diverse types of manufacturing systems a conducted literature survey reveals that the hybrid control system guarantees better performances. Bonvik et al. (1997) addressed and confirmed this issue in a flow-shop context with a simulation based approach. However, the great majority of these studies considered one part type systems and haven't support their results with analytical foundation in a stochastic dynamic context. Based on these facts, the multi parts problem remains an open research issue.

For the system considered in this paper, the developed parameterized heuristic can be considered as a major contribution, since it confirms existing results and addresses the multi parts issue. The developed control policy, illustrated by figure 4.11, point toward a KANBAN/CONWIP, MHCP control policies. Such a heuristic can be employed, after optimization of the corresponding parameters, to control the production and the changeovers on multi parts multi machines flow-shops. This issue is detailed in section 4.6.

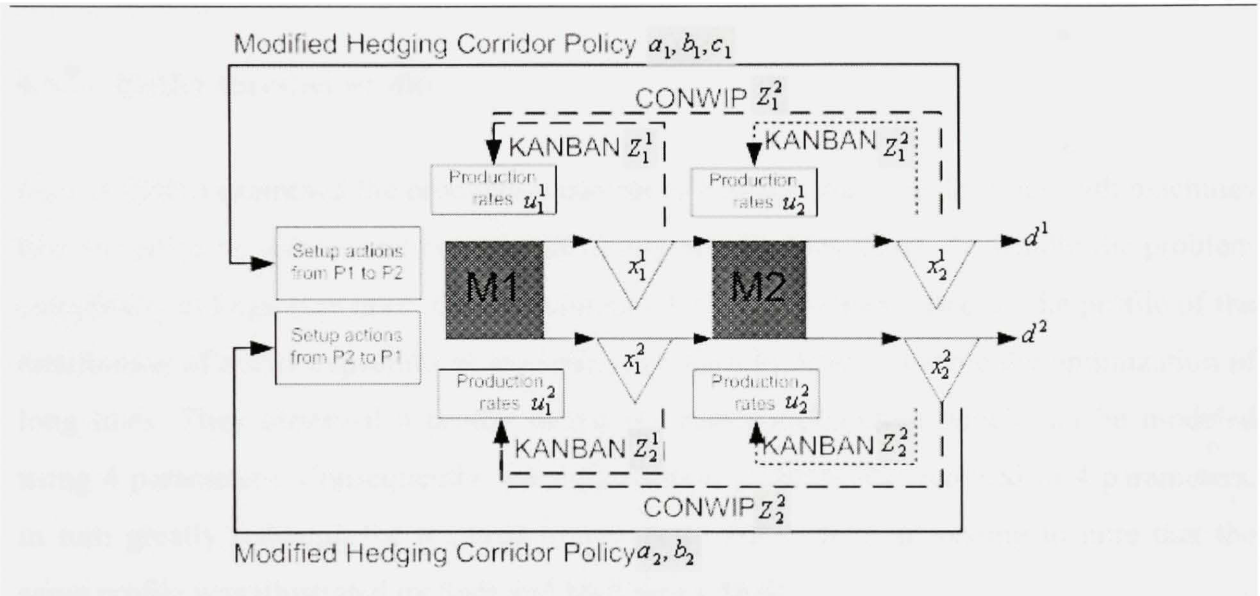


Figure 4.11 Feedback control policies

4.6 Generalization for m machines n products flow-shops

An interesting observation rising from the developed control policy consists of the fact that the changeover policy depends exclusively on the final stock levels. This observation makes it possible to address the generalization issue in two steps. Regarding the production policy, a decision to apply the KANBAN/ CONWIP control system to the whole line seems to be a good option. In fact, this issue has been addressed in previous researches where they proved the advantage of such a control system mechanism. However, from a practical point of view, the great number of thresh levels ($\# Z_i^j = n \times m$) to determine could be an obstacle facing its application. Thus, the question would be to find a way to decrease this number.

Regarding the changeover policy, a solution to the n part type problem is almost impossible. Therefore, we should find a way to approximate the n part type policy from observations made after solving accessible situations.

In this section we review interesting results from two previous works. These results will bring answers to the aforementioned questions and make it possible to propose generalized policies for the m machines n products flow-shops case.

4.6.1 Buffer threshes profile

Lavoie (2006) examined the production control of homogenous transfer lines with machines that are prone to failure, and consider inventory and backlog costs. To handle the problem complexity in large size lines, they developed a heuristic method based on the profile of the distribution of buffer capacities in moderate size lines in order to enable the optimization of long lines. They observed a profile in the parameter distribution which can be modeled using 4 parameters. Consequently, the optimization problem was reduced to 4 parameters, in turn greatly reducing the required optimization effort. It is interesting to note that the same profile was illustrated by Sadr and Malhamé (2004).

Figure 4.12 shows the general profile in the distribution of the hedging levels: while the first and last buffers seem to be more independent, the mid-section of the line seems to vary linearly.

These results are of great importance in our case. In fact, we can reduce the number of parameters to find from $\#Z_i^j = n \times m$ to $\#Z_i^j = n \times 4$. For example in the case of 10 buffered machines flow-shop with 10 products type, the number of parameters is equal to 40 instead of 100. It is important to note that these observations were made for homogeneous flow-shop systems producing one part type. Therefore, to generalize these observations for the multi-parts type case, additional studies must be undertaken. We expect that, for the considered flow-shop class (i.e., dissociated buffer for each part type) and given that the changeover control policies were shown to be dissociated from the production control policy, the same profile could be observed.

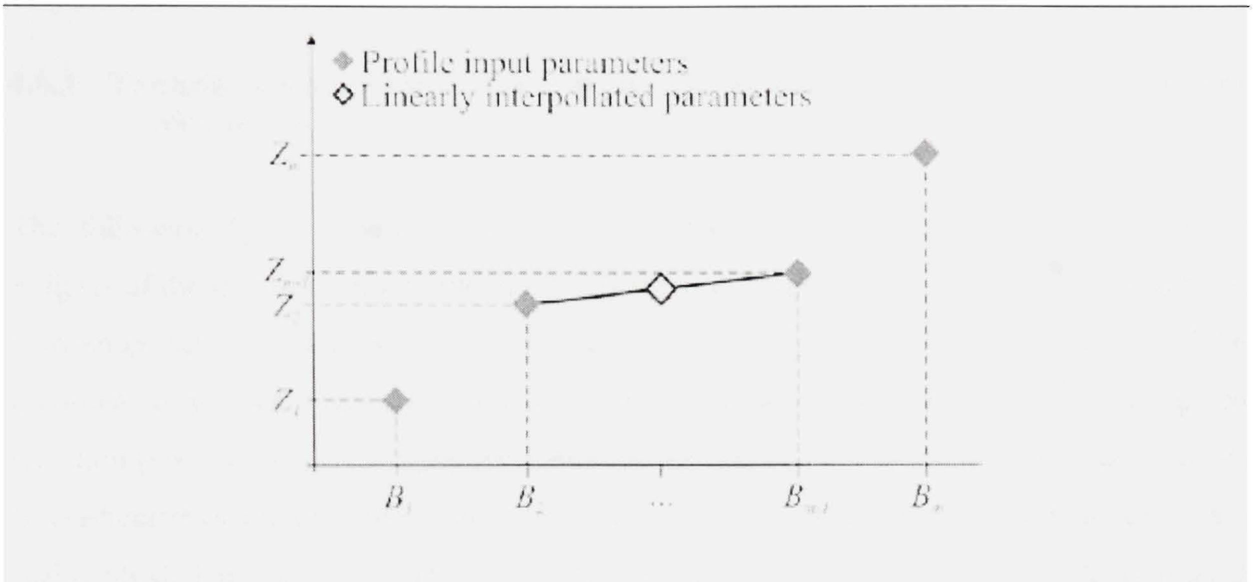


Figure 4.12 Parameterized profile with 4 parameters

4.6.2 Generalized changeover policy for more than two parts type

In Hajji et al. (2007 b, d), we have developed a complete production and setup policy for the two stage three family products manufacturing system. We have solved numerically the Hamilton Jacobi Bellman equations of the problem and carried out several analyses. Based on the obtained numerical results, the optimal control policy of the problem was derived. Base on the two and three family products results, we have proposed a generalized policy for the *n* family products problem.

The two family products case has showed that the boundaries of the setup policy are delimited by ***two lines*** defined by two parameters. Considering the three family products case gives rise to an additional dimension in the stock space (i.e., x_3). Before the resolution one can expect that moving the setup boundaries with respect to an additional dimension will lead to a ***plane***. After the resolution of the three family products case, we have observed that this intuition was confirmed. Following this idea one can expect that the plane can be generalized to a ***hyper-plane*** in the *n* dimensions case. We refer the reader to Hajji et al. (2007 b) for a detailed definition of the parameters governing the proposed generalized policy.

4.6.3 Towards a generalized control policy for m machines n products flow-shops with setups

The following figure shows the mechanisms governing the production and changeover policies of the *m* machines *n* products flow-shop. It is interesting to note that for the general flow-shop case, the number of the parameters governing the proposed policies will be obviously important. Therefore, the aforementioned approach to determine them for a given situation (see section 4.5.2) could be inappropriate and one has to employ other approaches (meta-heuristics for example) combined to simulation modeling to approximate them in a real context. The use of this approach in the general case is under current investigation as it may interest the reader to know.

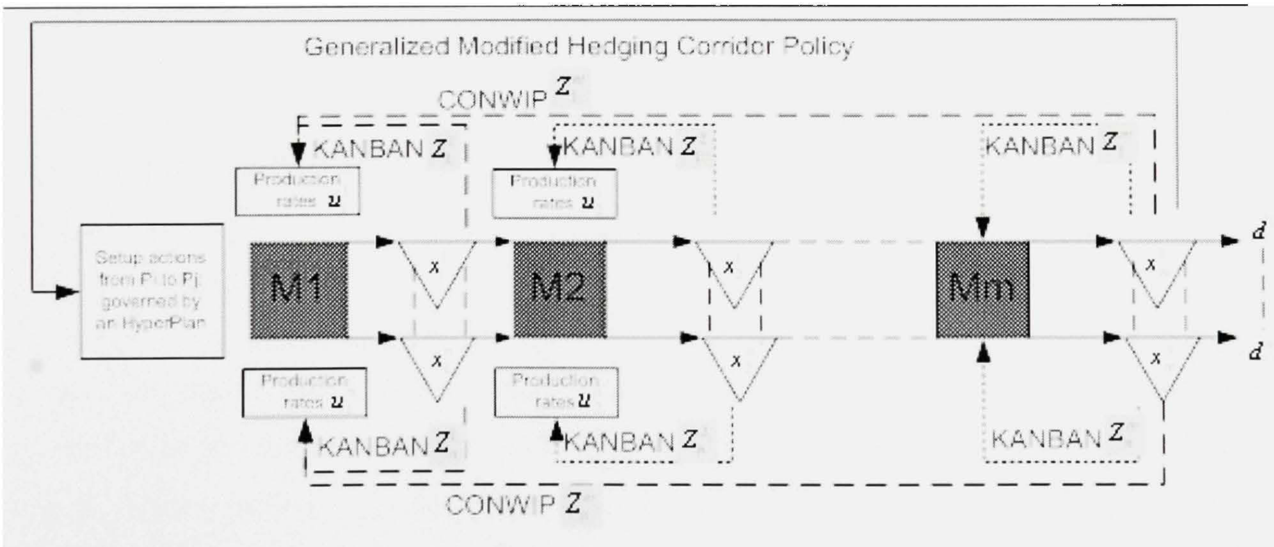


Figure 4.13 m machines n parts flow-shop control mechanism

4.7 Conclusion

In this paper, we studied the production and changeover control problem for buffered flow-shops producing multiple products type. We developed the DPEDD of the problem and adopted a numerical approach to solve them. The optimal production and changeover policies have been shown in this paper to be described by a combined KANBAN/ CONWIP and MHCP policy. Based on the obtained numerical solution, a parameterized heuristic and near optimal control policy are derived. Such a heuristic depends on the stock threshold levels and the boundaries of the corridor. Moreover, based on two previous research studies a generalized production and changeover control policies for the m machines n products flow-shops with setups were derived. As it may interest the reader to know, a generalized simulation model using meta-heuristics aiming to optimize the parameters governing the whole system is under current investigation.

CHAPITRE 5

PRODUCTION AND SUPPLY CONTROL IN UNRELIABLE MANUFACTURING SYSTEMS: IMPACT OF RANDOM DELAY ON PARTNERSHIP AND NEGOTIATION

Abstract

This paper considers a stochastic optimal control problem of unreliable three stages manufacturing systems. The supplier and the transformation stage are both subject to random events. Moreover, due to the periods of unavailability of the supplier, a random delay could postpone the reception of the order. Our objective is to find a control policy for the supply and production activities that minimizes the incurred cost and to propose a practical approach aiming to evaluate and quantify the control policy. Stochastic dynamic programming and numerical methods combined to a simulation based approach are thus proposed to achieve a close approximation of the production and supply policy. To illustrate the usefulness of the combined approach, extensions to cover more complex systems, where optimal control theory may not be easily used, are developed and analyzed. To illustrate the practical usefulness of the approach, an application aiming to develop a quantitative tool to help establishing and negotiating order costs is presented.

5.1 Introduction

In an open market environment, manufacturing systems managers face several random events which should be taken into account in any decision support system. While a good comprehension of the system could help the manager to prevent and face internal difficulties, external phenomena are much more difficult to deal with. Moreover, in a dynamic context, dealing with the interactions between internal and external random events could be also an important issue to consider. In this context, our objective is to propose a practical approach aiming to achieve a close approximation of a joint production and supply control policy in a dynamic stochastic environment.

A review of the relevant control theory literature has showed that the two problems aiming to control production or supply activities call upon different formulations and lead to different kinds of policies. To control the flow rates of parts through an unreliable manufacturing system Kimemia and Gershwin (1983) introduced the hedging point policy (HPP). Within such a policy, a non negative production surplus of part types, corresponding to the optimal inventory levels, is maintained during times of excess capacity availability to hedge against future capacity shortage caused by machine failures. Based on the pioneering work of Kimemia and Gershwin (1983) and the HPP concept, different classes of manufacturing systems have been investigated. Among many others, Akella and Kumar (1986) developed an explicit formulation of the HPP; Boukas and Haurie (1990) investigated the production and preventive maintenance control problem; Feng and Yan (2000) focused their contribution on providing a suitable production policy for unreliable systems facing stochastic demands; Hajji et al. (2004) developed a production and setup policy for unreliable manufacturing systems. The main assumption made in these papers is that the system will never be starved and thus has a reliable supply of raw material. This assumption could simply not be realistic considering the fact that an unreliable supplier or a random delay (due to transport instability for example) leads to a random availability of the raw material. Moreover, the random events were assumed to evolve according to a Markov processes. This assumption leads to a relatively easier formulation of the problem but could be a strong assumption for general cases study. To overcome this issue, Gharbi et al. (2006) proposed a simulation based experimental approach aiming to solve the unreliable multiple parts multiple machines control problem facing non markovian processes. However, the reliable supply belonged as an assumption of the considered problem.

On the other hand, based on different sets of assumptions (e.g., backlog or lost sales in the case of unfilled demand), many works have considered the stochastic aspect of supply. Among others, Bensoussan et al. (1983), Gullu et al. (1999) and Cheng and Sethi (1999). Basically, the dynamic programming approach was employed using the concept of K-convexity to establish the optimality conditions. In the aforementioned works, different proofs of the optimality of (s, S) type policy were provided. Within such a policy an economic lot of raw material is ordered when the upstream inventory level reaches s. It should be mentioned that these results are obtained under a zero lead time condition. It is interesting to note that some recent works (see Lee, 2005) have analyzed and proposed joint policies for the integrated system but in a static manner.

The whole problem we are seeking to consider in a dynamic stochastic context is still an open problem. In addition, the latest literature has shown that the integrated models through the intra-department planning by integrating raw material procurement and its production is more realistic and will result in better performance than that when the planning is performed separately (Lee, 2005). In this context and considering the fact that in the control literature, these two problems leading to the HPP and (s, S) policies are still be considered independently, we believe that our combined approach will be of a great utility to help solving the integrated problem and to propose sub-optimal policy for more complex systems.

The main contribution of this paper lies in the development of an integrated production and delayed supply policy for stochastic manufacturing systems. A stochastic dynamic programming problem is formulated. The structure of the solution, under appropriate conditions, is obtained by using the fact that the value function is the unique viscosity solution to the associated Hamilton Jacobi Bellman equations (HJB). Owing that an analytical solution of HJB equations is not in general available; a numerical approach is adopted to illustrate the structure of the control policy. A simulation-based experimental approach is then combined with the control theory to develop a systematic control approach, as in Gharbi et al. (2006). Once a close approximation of the optimal production and supply control policy is achieved, extension to cover more complex situations, where the optimal control theory may not be easily used, will be presented (i.e. non-exponential distributions of the delay). To illustrate the practical usefulness of the proposed approach, a decision making support is presented. It consists in a quantitative tool to help managers negotiating and establishing order costs and system parameters.

The paper is organized as follows. Section 5.2 presents the manufacturing system under study and the proposed approach. Section 5.3 presents the mathematical formulation of the production and delayed supply problem. Section 5.4 presents the numerical results and the related control policy. Section 5.5 describes the simulation based experimental approach used to quantify, achieve an approximation of the optimal policy and to determine the related cost incurred for the integrated policies. Section 5.6 presents a decisional process offering useful solution of the quantified feedback policy and allows possible extensions to cover more complex systems. The paper is concluded in section 5.7.

5.2 Proposed approach

The manufacturing system under study (figure 5.1) consists of an unreliable transformation system supplied by an unreliable upstream supplier. The whole system faces a one family product demand. Moreover, due to the periods of unavailability of the supplier a random delay could affect the reception instant of an order (figure 5.2).

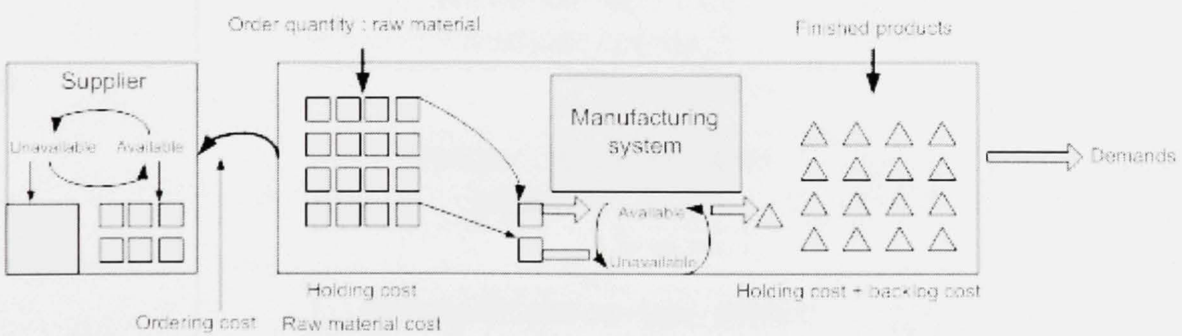


Figure 5.1 Manufacturing system under study

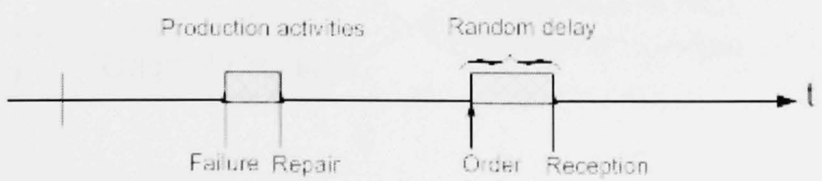


Figure 5.2 Random events

Our objective is to determine the production rates and a sequence of supply decisions in order to minimize the total expected discounted cost of ordering, raw material, holding (raw material and finished products) and backlog over an infinite horizon. Moreover, to overcome the difficulties behind the mathematical characterization of the optimal control policy for complex systems, a combined approach is proposed. As shown in figure 5.3, the approach consists in an appropriate combination between mathematical formulation (step I), numerical resolution and parameterisation (step II and III) and a simulation based experimental approach (step IV). The latest step is a combination of discrete / continuous simulation model, experimental design and response surface methodology; it is a flexible approach which will allow us to quantify the control policy of the original problem and to propose useful extensions for more complex systems.

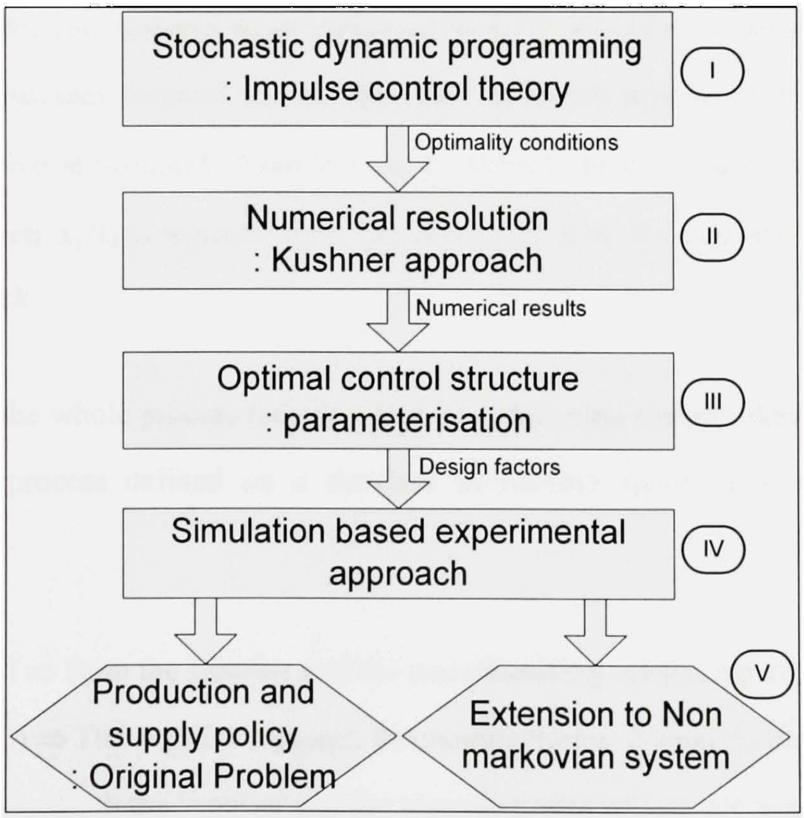


Figure 5.3 Proposed approach

5.3 Mathematical model and problem statement

In a dynamic context, the evolution of the manufacturing system under study changes with the flow of time (i.e., evolves under conditions of uncertainty). To formulate the optimization problem in a dynamic stochastic context one needs to characterize, as a first step, the state of the system at each instant t. In our case we have defined the state of the system by three components.

(1) The level of the finished product stock measured by x_2(t), this is a continuous part which describes the cumulative surplus vector (inventory if positive and backlog if negative).

(2) The level of the raw material stock measured by x_1(t), this is a combined part facing the continuous downstream demand and an impulsive upstream supply when a Q_i lot of raw material is received at instant θ_i. Even if it is operational, the manufacturing system cannot process parts when x_1(t) is equal to zero. Let $0 \leq x_1(t) \leq L$ be the capacity constraint of the raw material stock.

(3) The state of the whole process (supplier and manufacturing system) denoted by $\xi(t)$; this is a stochastic process defined on a standard probability space (Γ, F, P) taking values in $M = \{1,2,3,4\}$.

$\zeta(t) = 1 \Rightarrow$ Both the supplier and the manufacturing system are available.

$\zeta(t) = 2\,(3) \Rightarrow$ The supplier (respect. the manufacturing system) is unavailable.

$\zeta(t) = 4 \Rightarrow$ Both the supplier and the manufacturing system are unavailable.

The transition rates matrix of the stochastic process $\xi(t)$ can be easily derived from those of the supplier and the manufacturing system. Available and unavailable times of these processes are assumed to evolve according to continuous time Markov processes and can be described by the random variables $\xi_1(t)$ and $\xi_2(t)$ with value in $M_1 = \{1,2\}$ *and* $M_2 = \{1,2\}$, respectively, where:

$$\xi_1(t) = \begin{cases} 1 & \text{the supplier is available} \\ 2 & \text{the supplier is unavailable.} \end{cases} \quad \xi_2(t) = \begin{cases} 1 & \text{the manufacturing system is available.} \\ 2 & \text{the manufacturing system is unavailable.} \end{cases}$$

The transition rates matrix of the stochastic processes $\xi_1(t)$ and $\xi_2(t)$ are denoted by T_1 *and* T_2 such that $T_i = \{q^i{}_{\alpha\beta}\}$, with $q^i{}_{\alpha\beta} \geq 0$ if $\alpha \neq \beta$ and $q^i{}_{\alpha\alpha} = -\sum_{\beta \neq \alpha} q^i{}_{\alpha\beta}$, where $\alpha, \beta \in M_i$. The transitions rates matrix T_i is expressed as follow:

$$T_i = \begin{vmatrix} -q^i{}_{12} & q^i{}_{12} \\ q^i{}_{21} & -q^i{}_{21} \end{vmatrix}$$

The transition rates of the stochastic process $\xi(t)$ can be derived as follows:

$$T = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ q_{41} & q_{42} & q_{43} & q_{44} \end{bmatrix} = \begin{bmatrix} -\left(q_{12}^1 + q_{12}^2\right) & q_{12}^2 & q_{12}^1 & 0 \\ q_{21}^2 & -\left(q_{12}^1 + q_{21}^2\right) & 0 & q_{12}^1 \\ q_{21}^1 & 0 & -\left(q_{21}^1 + q_{12}^2\right) & q_{12}^2 \\ 0 & q_{21}^1 & q_{21}^2 & -\left(q_{21}^1 + q_{21}^2\right) \end{bmatrix}$$

For the considered manufacturing system, the state space is given by (x_1, x_2, α) such that: $x_1 \in [0, L]; x_2 \in R; \alpha \in M = \{1,2,3,4\}$

The dynamics of the stock levels $x_1(t)$ and $x_2(t)$ is given by the following differential equations:

$$\begin{aligned} &\dot{x}_2(t) = u(t,\alpha) - d,\ x_2(0) = x_2,\ \forall t \geq 0 \\ &\dot{x}_1(t) = -u(t,\alpha),\ x_1(0) = x_1,\ \forall t \in \left]\theta_i, \theta_{i+1}\right[\\ &x_1\left((\theta+\delta)_i^+\right) = x_1\left((\theta+\delta)_i^-\right) + Q_i(\alpha),\ i = 1,\ldots,N. \end{aligned} \tag{5.1}$$

Where x_1, x_2 denote the stocks levels at time t=0. $x_1(t), x_2(t)$, denote the raw material and finished product stock levels at time t, d the demand rate and u(t,α) the manufacturing system production rate. θ_i^-, θ_i^+, denote negative and positive boundaries of the N receipt instants i. δ denotes a fixed delay in delivery.

At any given time, the production rates and the order quantities have to satisfy the production and supply capacity constraints.

$$\begin{aligned} &0 \le u\ (t,\alpha) \le U_{\max} \times Ind\{\zeta(t) = 1,\ 2\} \\ &0 \le Q_i(t) \le L \times Ind\{\zeta(t) = 1,\ 3\}, i = 1,\ldots, N \end{aligned} \tag{5.2}$$

Where, $Ind\{\zeta(t) = \alpha\} = \begin{cases} 1, & \text{if } \zeta(t) = \alpha \\ 0, & \text{otherwise} \end{cases}$, $U_{\max}$ denote the maximal production rate, and L the raw material buffer capacity.

Our decision variables are the production rate $u(\cdot)$ and a sequence of supply orders denoted by $\Omega = \{(\theta_0, Q_0), (\theta_1, Q_1), \ldots\}$, with (θ_i, Q_i) defined by the time θ_i at witch the order is placed and the order quantity Q_i. Given (2), let $A(\alpha)$ be the set of admissible decisions (Ω, u(.)) given by:

$$A(\alpha) = \begin{Bmatrix} (\Omega, u(.)) : 0 \le u\ (t,\alpha) \le U_{\max} \times Ind\{\zeta(t) = 1,\ 2\}; \\ 0 \le Q_i(t) \le L \times Ind\{\zeta(t) = 1,\ 3\} \end{Bmatrix}$$

The instantaneous production, finished product inventory and backlog cost function $g(.)$ is given by the following equation:

$$g(x_1(t), x_2(t), u(t,\alpha)) = c_1^+ . x_1^+ + c_2^+ . x_2^+ + c_2^- . x_2^- + c_u . u(\cdot),\ t \in]\theta_i, \theta_{i+1}[\tag{5.3}$$

Where, $x_i^+ = \max(0, x_i)$ *and* $x_i^- = \max(-x_i, 0)$, c_2^+ *and* c_2^- denote the inventory and backlog costs of the finish product, c_1^+ *and* c_u are the inventory cost of the raw material and the production cost.

The cost function of the supply order at time θ_i, is given by the following equation:

$$R\ (Q_i, \alpha) = K\ Ind\{t = \theta_i\} + c_A Q_i + E_\alpha \int_0^\delta \left(c_1^+\ x_1^+(t) + c_2^+\ x_2^+(t) + c_2^-\ x_2^-(t) + c_u\ u(t, \alpha)\right) dt \quad (5.4)$$

Where K *and* c_A are the order and raw material cost. Using (5.3)-(5.4), the total cost J(.) can be defined by the following expression:

$$J(x_1, x_2, u, \theta, Q, \alpha) = E\left[\int_0^\infty e^{-\rho t} g(x_1, x_2, u)\ dt + \sum_{l=0}^\infty e^{-\rho\theta_i}\left(K + c_A \times Q_i\right)\right] \quad (5.5)$$

Where ρ denotes the discounted rate.

The production planning problem considered herein is to find an admissible decision or control policy (Ω, u(.)) that minimizes J(.) given by (5.5) considering equations (5.1) to (5.3). This is a control policy that specifies the production rate and the supply decisions when the system is in a given state (x_1, x_2, α).

The corresponding value function $v(.)$ can be given by the following:

$$v(x_1, x_2, \alpha) = \min_{(\Omega, u) \in A} J(x_1, x_2, u, \Omega, \alpha) \quad (5.6)$$

As in Sethi and Zhang (1994), and using the optimal impulsive control theory (Sethi and Thompson, 2000) it can be shown that the value function $v(x_1, x_1, \alpha)$ is the unique viscosity solution to the following HJB equation:

$$\min\left\{\begin{array}{l} \min\limits_{u}\left\{\begin{array}{l}(-u)v_{x_1}+(u-d)v_{x_2}+g(x_1,x_2,u)\\ +\sum\limits_{\beta\neq\alpha}\lambda_{\alpha\beta}\left(v(x_1,x_2,\beta)-v(x_1,x_2,\alpha)\right)\end{array}\right\}-\rho v(x_1,x_2,\alpha);\\ \min\limits_{Q} E_{\alpha}\left\{R(Q,\alpha)+e^{-\rho.\delta}v(x_1+Q-\delta.u,x_2+(u-d).\delta,\alpha)\right\}-v(x_1,x_2,\alpha)\end{array}\right\}=0 \quad (5.7)$$

Where $(v)_x(.)$, denotes the gradients of $v(.)$ with respect to x.

The production and supply policy that we are seeking is obtained when the value function is known. While we cannot solve analytically the HJB equations (5.7), we can apply numerical methods to obtain the approximation of the value function and the associated control policy.

5.4 Numerical results

In this section we present the numerical results and the obtained optimal control policy for the considered system. The numerical methods used to solve the optimality conditions, corresponding to the stochastic optimal control problem, are based on the Kushner approach (Kushner and Dupuis, 1992). The solution of the numerical approximation of $v_i(x,\alpha)$ may be obtained by either successive approximation or policy improvement techniques (Boukas and Haurie, 1990 and Kushner and Dupuis, 1992).

Recall that when the supplier is unavailable the manufacturing system has to wait for a random length of time (random delay) for the supplier to become available. Based on this fact and for a best characterization of the policy, two cases of supplier availability have been studied. These cases of study (i.e., supplier availability), illustrated by equations (5.8) and (5.9), showed us the reaction of the manufacturing system facing such situation. Equation (5.8) (respectively (5.9)) correspond to availabilities rates equal to 83.33 (respectively 33.33) for the supplier, the manufacturing system is available at 90.9 % for the two cases.
The transition rate matrixes defining the manufacturing system and supplier availabilities are as follow:

$$T^1 = \begin{bmatrix} -0.02 & 0.02 \\ 0.1 & -0.1 \end{bmatrix}; T^2 = \begin{bmatrix} -0.01 & 0.01 \\ 0.1 & -0.1 \end{bmatrix}; T = \begin{bmatrix} -0.03 & 0.01 & 0.02 & 0 \\ 0.1 & -0.12 & 0 & 0.02 \\ 0.1 & 0 & -0.11 & 0.01 \\ 0 & 0.1 & 0.1 & -0.2 \end{bmatrix} \quad (5.8)$$

$$T^1 = \begin{bmatrix} -0.2 & 0.2 \\ 0.1 & -0.1 \end{bmatrix}; T^2 = \begin{bmatrix} -0.01 & 0.01 \\ 0.1 & -0.1 \end{bmatrix}; T = \begin{bmatrix} -0.21 & 0.01 & 0.2 & 0 \\ 0.1 & -0.3 & 0 & 0.2 \\ 0.1 & 0 & -0.11 & 0.01 \\ 0 & 0.1 & 0.1 & -0.2 \end{bmatrix} \quad (5.9)$$

Table 5.1 shows the constant data for the numerical example.

Tableau 5.1

Constant data parameters

Parameters	U_{max}	d	ρ	δ
Values	2.5	2	0.4	1

To ensure a clear characterization of the control policy, several elements were taken into consideration as part of the implementation process. Indeed, the production and supply policies are each observed separately. For each policy, the relevant significant threshold levels are analyzed independently of the others. For each numerical result, the policies are provided as shown in Figure 5.4a, 5.4b. $u(t,1)$ and $u(t,3)$ are the production policies of the manufacturing system in system state 1 (manufacturing system and supplier available) and 3 (manufacturing system available and supplier unavailable). $\Omega(x_1, x_1, 1)$ and $\Omega(x_1, x_1, 2)$ are the supply policies in system state 1 and 2 (Figure 5.4c and 5.4d).

It follows from our numerical results that the resulting production policy divides the surplus space into two mutually exclusive regions. In region I, we produce at the maximal rate and in region II we have to set the production rate to zero. At the boundary of these regions we have to set the production rate equal to the demand rate. Moreover, the results show that the supply policy is governed by an order quantity (illustrated by region III in figure 5.4c) and an order point (illustrated by region IV in figure 5.4c). This order point reflects the necessity to have a security raw material stock level to face a possible random delivery delay when the supplier is unavailable. In addition, the second case study for a lower supplier availability rate (i.e., equal to 33.33 and given by (5.9)) show that the order point and the order quantity take higher values than the first case (i.e., supplier availability equal to 83.33). The supply policy for this case study is shown in figure 5.5. In conclusion, the optimal policy is a combination of the Hedging Point Policy and an (s, Q) type inventory Policy. Let s, Q and Z define the policy parameters.

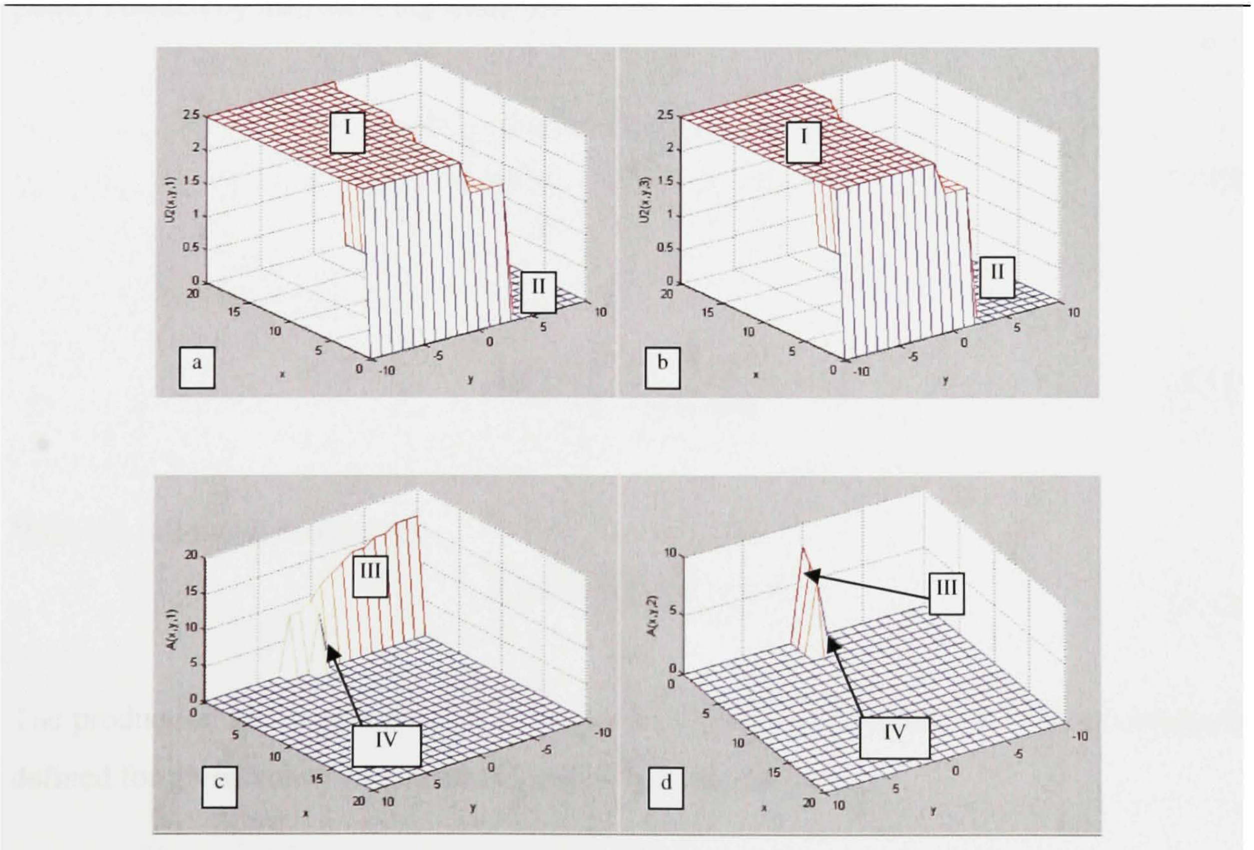


Figure 5.4 Optimal production and supply policy

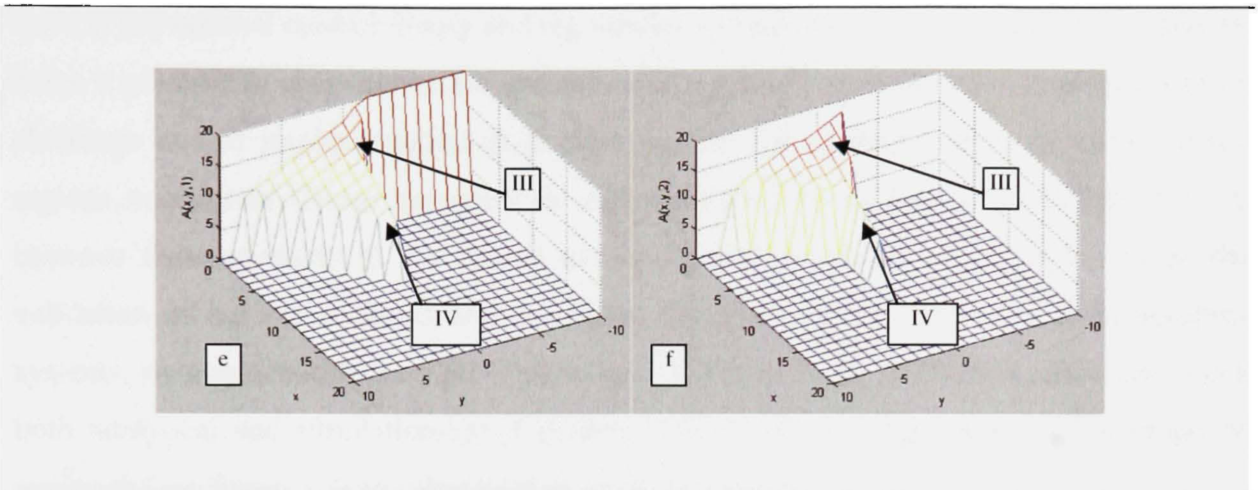


Figure 5.5 Optimal supply policy

After several sensitivity analysis, we have clearly observed that the results obtained make sense, and that the structure of the policy defined by the 3 parameters (s, Q and Z) is always maintained. This allows the development of a parameterized production and supply control policy defined by the following equations:

$$u(.):\begin{cases} U_{\max}.Ind\{\alpha = 1,3\} & if \quad x_2 < Z \\ d.Ind\{\alpha = 1,3\} & if \quad x_2 = Z \\ 0 & if \quad x_2 > Z \end{cases} \tag{5.10}$$

$$\Omega(.):\begin{cases} Q & if \ X_1 \le s \\ 0 & otherwise \end{cases} \tag{5.11}$$

With the following constraints:

$$Z \ge 0; s < Q < L; s \ge 0 \tag{5.12}$$

The production and supply policies presented by equations (5.10) and (5.11) are completely defined for given values of s, Q and Z, called here design factors.

Even if the optimal control theory and the numerical resolution of the optimality conditions make it possible to characterize the optimal policy, a good approximation remains always a challenge due to implementation difficulties and to the existence of irregularities in the regions boundaries. These irregularities lead sometimes to inadequate interpretations. To improve these methods in the sphere of manufacturing system, in order to ensure the validation of our observations and to extend the control policy to cover more complex systems, we augment the descriptive capacities of conventional simulation models by using both analytical and simulation-based models. The fourth and fifth steps of the proposed approach (see figure 5.3) are illustrated in the next sections.

5.5 Simulation based experimental approach

This section is aimed at conducting sensitivity analysis leading to a close approximation of the optimal control policy and for determining the values of s, Q and Z which minimize the incurred cost. Moreover, it allows approximating the control policies of more complex systems, where the optimal control theory may not be easily used. To follow this purpose we will adopt an experimental approach which is a combination of simulation modeling, experimental design and response surface methodology.

5.5.1 Proposed approach

The reader is referred to Gharbi et al. (2006) for more details on the application of this approach and which can be summarized by the following three steps:

I. Develop a simulation model to describe the dynamic of the system governed by the production and supply policy defined previously and parameterized by the 3 parameters s, Q and Z. These factors are considered as input of such a model and the related incurred total cost is defined as its output. Our model was developed using Visual SLAM simulation language with C sub-routines (Pritsker and O'Reilly, 1999). It is interesting to note that the combined discrete/continuous simulation model is more flexible and reduces the execution time (Lavoie et al. (2007)).

II. Develop an appropriate experimental design to be run on the simulation model. The statistical analysis of the obtained results allows determining, from the values of the input factors and the related total cost values, the input factors and / or their interactions which have significant effects on the output.

III. Consider the significant factors or interactions as input of a response surface methodology, to fit the relationship between the cost and the input factors. From this estimated relation called regression equation, the optimal values of the input factors, called s^*, Q^* *and* Z^* are determined. We refer the reader to Montgomery (2001) for more details on experimental design and response surface methodology approaches.

5.5.2 Simulation model

The Visual SLAM portion is composed of various networks describing specific tasks such as random events, production activity, threshold crossing of inventory variables, etc.... The model is shown in Figure 5.6 with the following descriptions of the main blocks.

1) The INITIALIZATION block sets the values of, s, Q, Z, the demand rate, the manufacturing system parameters such as U_{max}, mean time to failure and mean time to repair and the supplier parameters such as the delay, the mean time between unavailability and mean time to become available. The maximum and minimum time step specifications for integration of the cumulative variables and allowable errors are also assigned at this step as well as the simulation time T_fin and the time for the warm up period.

2) The CONTROL POLICY is implemented through the use of observation networks that raise a flag whenever one of the thresholds is crossed. The manufacturing system production rate and/or the supply order are then set according to equations (5.10) and (5.11).

3) The STATE EQUATIONS are equations (5.1) defined as a C language insert. They describe the inventory and backlog variables using the production rates set by the control policy and the binary variables from the failure and repair of the manufacturing system and the availability of the supplier.

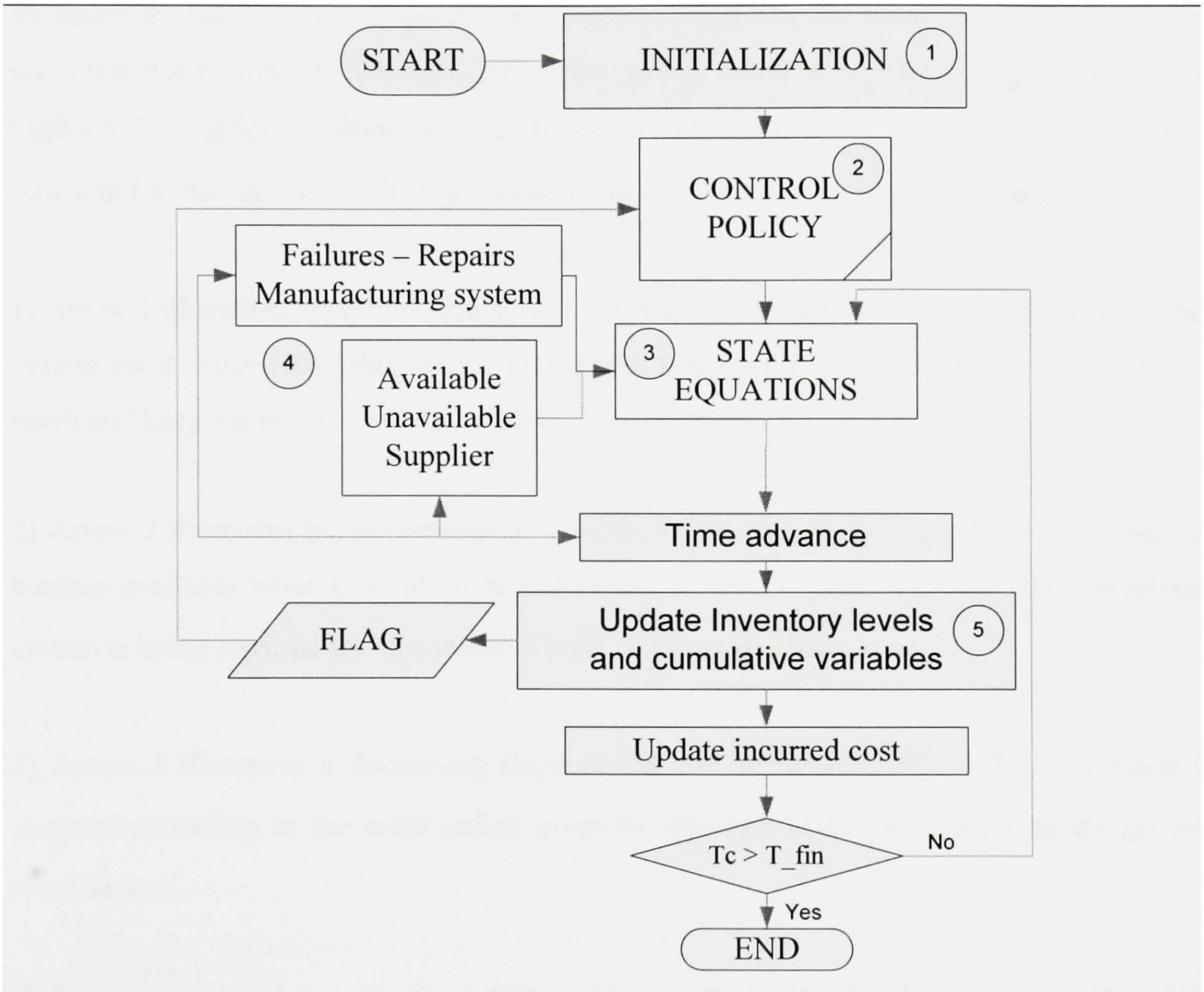


Figure 5.6 Simulation block diagram

4) The AVAILABILITY of the supplier and the FAILURES AND REPAIRS blocks sample the random events from their respective probability distributions. These states are incorporated in the state equations by the means of the stochastic process.

5) The UPDATE INVENTORY LEVELS AND CUMULATIVE VARIABLES blocks are used once the time step is chosen. The cumulative variables are integrated using the Runge-Kutta-Fehlberg (RKF) method as described in Pritsker & O'Reilly (1999).

5.5.3 Validation of the simulation model

To verify the accuracy of the model, we verified graphically the dynamics of the stocks to see if the model works according to the control policy given by equations (5.10) and (5.11). Figure 5.7 is a graphical illustration of the trajectories of the raw material (X_1) and finished products (X_2) stock levels with the following descriptions of the different arrows.

1) Arrow 1 illustrates the production at the maximal and demand rate that the manufacturing system must follow according to the production policy (5.10). It produces at this rates to reach and keep the thresh level Z (equal to 15 in figure 5.7).

2) Arrow 2 illustrates the occurrence of a failure at the manufacturing system. The system became available when $X_2 = -10$. It is interesting to observe that when the manufacturing system is being repaired the raw material level X_1 keeps the same level.

3) Arrow 3 illustrates a decreasing slope of the raw material level X_1. When it reaches $s = 2$ and according to the order policy given by equation (5.11) an order quantity $Q = 10$ must be send.

4) Arrow 4 illustrates the reception of the order quantity Q. The reception event arrives after a random lead time (time period between arrow 3 and 4).

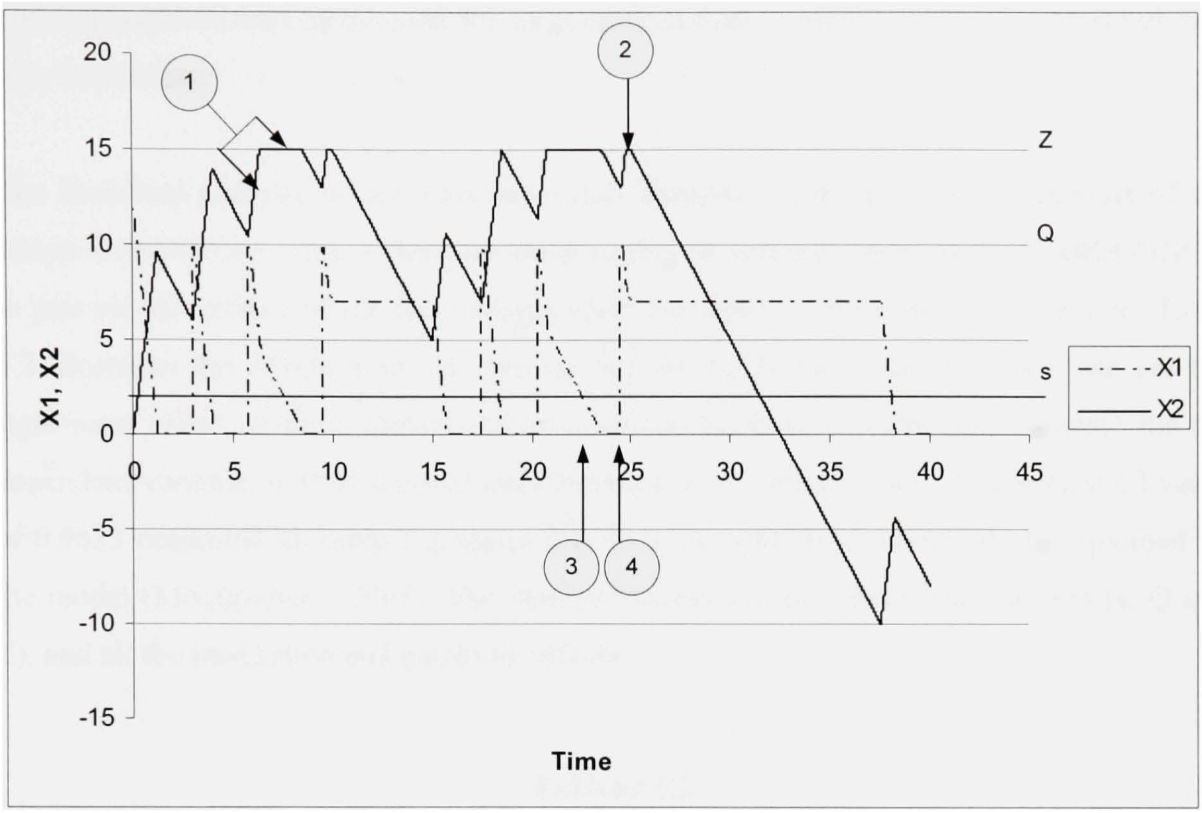


Figure 5.7 Stock dynamics

5.5.4 Experimental design

The experimental design is concerned with (i) selecting a set of input variables (i.e., s, Q and Z) for the simulation model; and (ii) setting the minimal and maximal levels of selected factors of the model and deciding on the conditions, such as length of runs and number of replications, under which the model will be run. In our case, three independent variables and one dependent variable (the cost) were considered. The levels of independent variables or design factors should be chosen carefully so that they represent the domain of interest. Due to the convexity property of the value function (5.6), the first-order response surface model is rejected. In fact, we selected a 3^3 response surface design since we have 3 independent variables at three levels each. It is interesting to note that all possible combinations of different levels of factors are provided by the response surface design considered herein.

Five replications were conducted for each combination; therefore 135 (27 x 5) simulation runs were made.

The statistical analysis of the simulation data consists of the multi-factor analysis of the variance (ANOVA). This is done by using statistical software, such as STATGRAPHICS, to provide the effects of the three independent variables on the dependent variable. Table 5.2 illustrates the ANOVA for the second case of Set IV (see table 5.3). We can see the significant effect of main factors and interactions (symbol S in the last column) for the dependent variable, at 0.05 level of significance (i.e., P-value < 0.05). The R-squared value of 0.9633 presented in Table 5.2, states that 96.33% of the total variability is explained by the model (Montgomery, 2001). The obtained model includes three main factors (s, Q and Z), and all the interaction and quadratic effects.

Tableau 5.2

ANOVA table for the total cost, case 2, Set IV

Source	*Sum of Squares*	*DF*	*Mean Square*	*F-Ratio*	*P-Value*	Significant
A:s	60844,3	1	60844,3	486,46	0,0000	S
B:**Q**	170338,	1	170338,	1361,89	0,0000	S
C:**Z**	29333,4	1	29333,4	234,53	0,0000	S
AA	4390,71	1	4390,71	35,10	0,0000	S
AB	61834,0	1	61834,0	494,38	0,0000	S
AC	15164,7	1	15164,7	121,25	0,0000	S
BB	32994,0	1	32994,0	263,80	0,0000	S
BC	20761,1	1	20761,1	165,99	0,0000	S
CC	1825,08	1	1825,08	14,59	0,0002	S
blocks	140,978	4	35,2446	0,28	0,8893	NS
Total error	15134,0	121	125,074			
Total (corr.)	412760,	134				

R-squared = **96.3335** percent(s)

Response surface methodology is a collection of mathematical and statistical techniques that are useful for modeling and analysing problems in which a response of interest is influenced by several variables and the objective is to optimize this response (Montgomery, 2001). We assume here that there exists a function Φ of s, Q and Z that provides the value of the cost corresponding to any given combination of input factors. That is Cost = Φ(s, Q, Z).

The function Φ(.) is called the response surface and is assumed to be a continuous function of s, Q and Z. We choose the second-order model given by:

$$\begin{aligned} Cost = \beta_0 + \beta_{11}s + \beta_{12}Q + \beta_{13}Z + \beta_{21}s^2 + \\ \beta_{22}Q^2 + \beta_{23}Z^2 + \beta_{31}s.Q + \beta_{32}s.Z + \beta_{33}Z.Q + \varepsilon \end{aligned} \tag{5.13}$$

Where s, Q and Z are the input variables; $\beta_0, \beta_{ij}, i, j = 1,2,3$ are unknown parameters and ε is a random error.

From STATGRAPHICS, the estimation of β_{ij} is performed and the following 10 coefficients achieved. The values of these coefficients for the considered case of the sensitivity analysis are:

$\beta_0 = 1773{,}48; \beta_{11} = -12{,}99; \beta_{12} = -14{,}4; \beta_{13} = -9{,}03; \beta_{21} = 0{,}03; \beta_{22} = 0{,}037; \beta_{23} = 0{,}02; \beta_{31} = 0{,}05;$
$\beta_{32} = 0{,}04; \beta_{33} = 0{,}03$

It follows from the correspondent response surfaces (figure 5.8) that the optimal values of s, Q and Z, are respectively equal to 55, 123 and 78. The optimal cost is equal to 180.59.

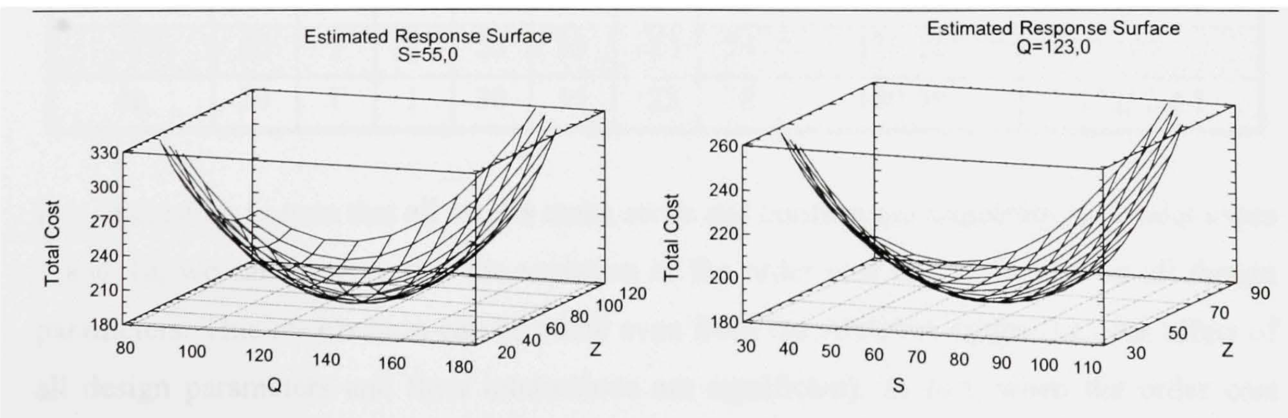


Figure 5.8 Cost response surfaces

5.5.5 Sensitivity analysis

To illustrate the effect of the costs variations and the random delay on the design parameters, three sensitivity analyses are conducted and presented in the next sub sections.

5.5.5.1 Cost variations

Table 5.3 details the cost variations of K, c_1^+, c_2^+ and c_2^- and presents the optimal parameters and the incurred optimal cost for the sensitivity analysis cases under an exponentially distributed random delay.

Tableau 5.3

Sensitivity analysis data and results with δ=exp(2)

CASES	K	c_1^+	c_2^+	c_2^-	OPTIMAL PARAMETERS			OPTIMAL COST	IMPACT ON
					$s*$	$Q*$	$Z*$		
1	100	1	1	20	45	131	69	178.01	----
1a	**150**	1	1	20	42	134	70	185.49	$s\downarrow Q\uparrow Z\uparrow$
2	100	1.1	1	20	37	133	76	185.07	----
2a	100	**1.2**	1	20	30	135	82	191.55	$s\downarrow Q\uparrow Z\uparrow$
3	100	1	1	20	45	131	69	178.01	----
3a	100	1	**1.1**	20	53	130	58	183.67	$s\uparrow Q\downarrow Z\downarrow$
4	50	1	1	25	53	125	74	176.14	----
4a	50	1	1	**30**	55	123	78	180.59	$s\uparrow Q\downarrow Z\uparrow$

It is interesting to note that all results make sense and confirm our expectations. Under cases 1 and 1a, we can observe that the variation of the order cost has an impact on all design parameters. This observation can be made even from the ANOVA tables (i.e., the effect of all design parameters and their interactions are significant). In fact, when the order cost increases; the order point decreases, the economic order quantity increases and the hedging level (security final stock level) increases. Indeed, when the order cost is higher, one has to order more but less frequently. Moreover, one has to keep a higher level of finished product. This observation shows how the system reacts to transform material to the final stock owing that it incurs the same holding cost than the raw material stock. This reaction insures a higher final security stock level to hedge against future capacity shortages caused by the manufacturing system unavailability.

Under cases 2 and 2a, we can also observe that when the inventory cost of raw material increases, the order point decreases, the order quantity increases and the hedging level increases. This observation shows how the system reacts to transform material to the stock incurring the lowest cost (final stock in this case).

This observation is confirmed by cases 3 and 3a and show that the dynamic reaction of the manufacturing system makes sense. In fact, when the inventory cost of final product increases, the order point increases, the order quantity decreases and the hedging level decreases.

Under cases 4 and 4a, we can observe that the variation in the final product backlog cost has a direct impact on the security stock levels. Indeed, when the backlog cost is higher, one has to keep higher stock security levels (i.e., s and Z).

5.5.5.2 Delay mean value variation

Under this sensitivity analysis (table 5.4), a variation in the delay mean value under the same probability distribution (i.e., exponential distribution) is conducted. To illustrate the impact of this variation we use data corresponding to cases 4 and 4a of table 5.3.

Tableau 5.4

Impact of the delay mean value

CASES	$\delta = \exp(2)$			$\delta = \exp(3)$			OTPMAL COST $\delta = \exp(2)$	OTPMAL COST $\delta = \exp(3)$
	s^*	Q^*	Z^*	s^*	Q^*	Z^*		
4	53	125	74	96	205	85	176.14	259.31
4a	55	123	78	98	203	95	180.59	265.49

It is interesting to note that all results make sense and confirm our expectations. Indeed, under the same probability distribution, when the delay mean value increases the system must react and re-evaluate the decision parameters (i.e., s^*, Q^* and Z^*). Under $\delta = \exp(3)$ we can observe that all these parameters increase, in comparison with those under $\delta = \exp(2)$. This re-evaluation must be done to ensure higher security stock levels and a higher order quantity. Moreover, under $\delta = \exp(3)$, the final product backlog cost variation showed the same observation detailed in the previous paragraph.

5.5.5.3 Delay probability distribution and variability variation

Under this sensitivity analysis (table 5.5), a variation in the delay probability distribution and its parameters (i.e., mean and/or standard deviation) is conducted. This analysis shows the importance of such random event and the impact of the probability distribution on the system parameters and cost. To illustrate this issue, three different probability distributions (the exponential, the normal and the uniform distribution) are employed. Under this analysis we use the data of case 4a of table 5.3.

Tableau 5.5

Impact of the probability distribution and variability of the delay

CASES	PROBABILTY DISTRIBUTION	$s*$	$Q*$	$Z*$	OTPMAL COST
1	$\delta = \exp(2)$	55	123	78	180.59
2	$\delta = \exp(3)$	98	203	95	265.49
3	$\delta = normal(3,1)$	35	135	80	133.23
4	$\delta = normal(4,1)$	58	143	88	195.11
5	$\delta = normal(4,2)$	72	167	92	200.83
6	$\delta = uniform(2,4)$	40	117	67	113.88

To confirm the observations of the previous analysis (see table 5.4), the results of table 5.5 show that, under the same probability distribution (case 1 and 2 for the exponential distribution and cases 3 and 4 for the normal distribution), when the delay mean value increases the system must react and re-evaluate the decision parameters (i.e., $s*$, $Q*$ and $Z*$). Moreover, cases 2, 3 and 6 show the impact that the probability distribution has on the optimal parameters and the corresponding optimal cost. In fact, we can observe that the variability of a given probability distribution has a great impact on the system. In this context, we know that the exponential distribution is more variable than the normal distribution which is more variable than the uniform distribution. The corresponding optimal cost and parameters reflect this variability in the sense that the system must keep higher security stock levels and this, results in a higher cost. Under the same circumstances, with the same probability distribution but with a higher standard deviation value (cases 4 and 5), we can make the same observation.

5.6 Impact of delay on partnership and negotiation

In the literature several studies have focused on the qualitative aspects of establishing and negotiating buyer-supplier partnerships. While, reviewing the literature and managerial practices, very few quantitative models and investigations are available in this area (Kelle et al. 2003). In this section we investigate this issue through an illustrative case study. In this context, consider the case were the random delay of the supplier evolves according to an exponential distribution with mean equal to 3 and that the order, holding and backlog costs of the manufacturing system are as follows, K=50, c_1^+=1, c_2^+=1 and c_2^-=30. In this case, the previous analysis (see table 5.4, case 4a) showed that the optimal cost is equal to 265.49 and the optimal control parameters are s*=98, Q*=203 and Z*=95. Now, consider the case where the economic context imposes to re-evaluate our transportation mode for example. This re-evaluation will lead to a more competitive delay, say a delay which evolves according to an exponential distribution with mean equal to 2. Admitting these facts, it is convenient to think that this new mode will lead to higher ordering cost. In this context, it is reasonable that the manager ask the following question: what should be the maximum ordering cost that he can allow (or negotiate) so as he doesn't exceed the total cost under the current practice (i.e., $\delta = \exp(3)$) ? Moreover, he must have the tool to re-evaluate the control parameters under this configuration.

Following the approach presented in the previous sections, our objective is to develop a quantitative model of the total cost, as a function of four design parameters: the order cost K, the order point s, the economic order quantity Q and the hedging point Z. In fact, we selected a response surface design and conducted the required simulations. The statistical analysis (i.e., ANOVA) showed that the obtained model explain 97.8 % of the total variability. The obtained regression model is as follow:

$$Cost = \beta_0 + \beta_{11}K + \beta_{12}s + \beta_{13}Q + \beta_{14}Z + \beta_{21}K^2 + \beta_{22}s^2 + \beta_{23}Q^2 + \beta_{24}Z^2 + \beta_{31}K.s + \beta_{32}K.Q + \beta_{33}K.Z + \beta_{34}s.Q + \beta_{35}s.Z + \beta_{36}Q.Z \quad (5.14)$$

Where K, s, Q and Z are the input variables; $\beta_0, \beta_{ij}, i, j = 1,2,3,4$ are unknown parameters and ε is a random error.

From STATGRAPHICS, the estimation of β_{ij} is performed and the following fifteen coefficients achieved. The values of these coefficients for the considered case are:

$$\beta_0 = 1715{,}63; \beta_{11} = 0{,}377; \beta_{12} = -12{,}28; \beta_{13} = -13{,}92; \beta_{14} = -8{,}66;$$
$$\beta_{21} = 0; \beta_{22} = 0{,}029; \beta_{23} = 0{,}037; \beta_{24} = 0{,}019; \beta_{31} = 0; \beta_{32} = -0{,}0017; \beta_{33} = 0;$$
$$\beta_{34} = 0{,}047; \beta_{35} = 0{,}038; \beta_{36} = 0{,}028.$$

It is important to observe the complexity of such a quadratic model. This complexity could be considered as one reason among others that have lead several studies to focus on qualitative aspects. It is interesting to note that this model is governing the system in a predetermined experimental domain. In our case, we have set the lower and upper bounds of the design parameters as shown in table 5.6.

Tableau 5.6

Levels of design parameters

FACTOR	LOW LEVEL	CENTER	HIGH LEVEL	DESCRIPTION
K	300	500	700	Order cost
s	30	50	70	Order point
Q	80	115	150	Order quantity
Z	50	70	90	Hedging level

The obtained model could be a good tool to help responding to the two questions of the manager. In fact, maintaining the total cost equal to the optimal cost under the current practice (i.e., $\delta = \exp(3)$) leads to the following optimal parameters: *K*=485.38, s*=51, Q*=116 and Z*=70.*

Under these results, the strategy of the manager will be to not exceed an order cost of *485.38*. If this will be the case, the system should be run under *s*=51, Q*=116 and Z*=70.* Otherwise, we fix the order cost and solve equation (5.14) to obtain the other optimal parameters. To crosscheck the validity of the solution, *K*=485.38, s*=51, Q*=116 and Z*=70* were used as input to the simulation model. The cost value obtained was 265.49, which falls in the 95% confidence interval ($\bar{X}(n) \pm t_{n-1,1-\frac{\alpha}{2}} \sqrt{S^2(n)/n}$ = [264.13; 269.48]), obtained using n=10 replications of the simulation model.

In what follows (Figure 5.9), a decision logigram is presented to show the actions that should be taken function of the stock levels (i.e., raw material and finished product), the availability of the supplier and a possible re-evaluation of the supply delay.

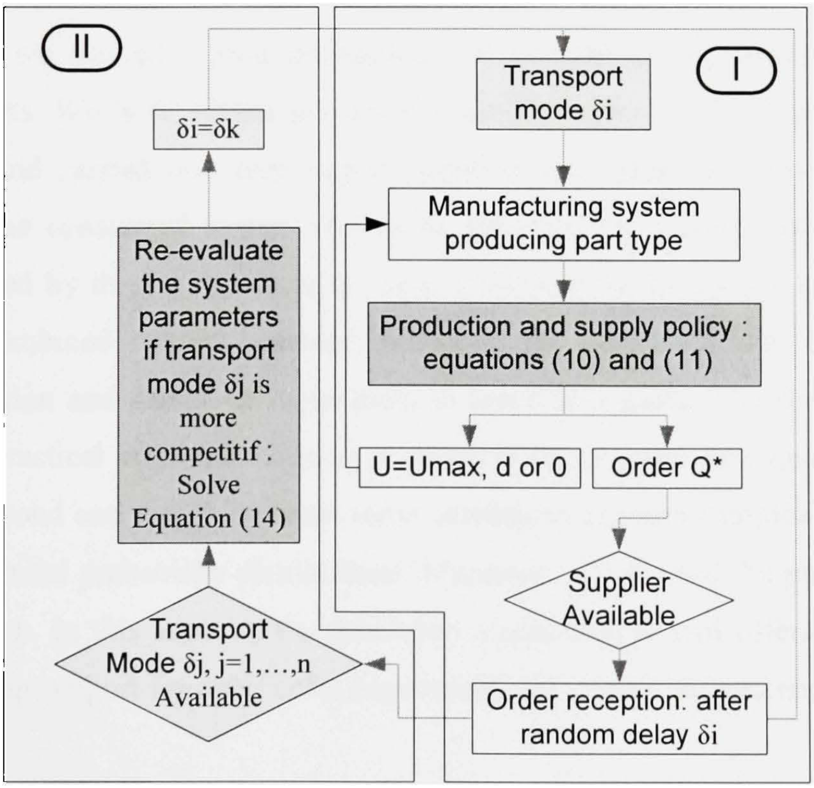


Figure 5.9 Decision support logigram

Figure 5.9 shows that if the manufacturing system is adopting a transport mode offering a given delay δ_i, follow part **I** of the decision support logigram. At this moment in time, the system must be governed by policies given by equations (5.10) and (5.11), where *s*, Q* and Z** are the optimal parameters under the current transport mode ***i***. Admitting that new market realities impose a re-evaluation of the current transport mode and that other modes offering delays $\delta_j, j = 1,...,n$ are available, one has to follow part II of figure 5.9. In fact, negotiations of cost and re-evaluation of the parameters of the system in a binary manner, between the available modes, with respect to equation (5.14) should be done. If a decision to change the current transport mode δ_i to a new mode, say δ_k, is adopted, run the system following part I with mode δ_k.

5.7 Conclusion

In this paper, we studied a joint production and delayed supply control problem under different aspects. We have solved numerically the Hamilton Jacobi Bellman equations of the problem and carried out deep experimentation and sensitivity analysis. The results showed that the considered system should be governed by a combined HPP and (s, Q) policies, defined by three parameters, so as to minimize the incurred costs. We have also proposed a combined control approach based on the numerical characterization of the policy, simulation and design of experiment to reach two goals. The first one consists in proposing a practical approach making it possible to evaluate and quantify the control policy. The second one was to evaluate some extensions of more complex system governed by non-exponential probability distributions. Moreover, we showed the practical usefulness of this approach. In this context, we developed a quantitative tool offering to managers a decision-making support for order costs negotiation and system parameters evaluation.

CHAPITRE 6

REPLENISHMENT AND PRODUCTION ACTIVITIES CONTROL WITH MULTIPLE SUPPLIERS IN STOCHASTIC SUPPLY CHAINS

Abstract

This paper considers the joint supplier selection, replenishment and manufacturing management problem. In a dynamic stochastic context, the considered problem faces conflicting interests basically between the suppliers, the manufacturer and the customers. In this context, coordination and information sharing must be considered in any decision support system to handle the disparate decisions and random phenomena. The main contribution of this paper is the development and resolution of a global mathematical model leading to information sharing strategies for the supplier selection, replenishments and production activities. This is an optimal control problem with state constraints and hybrid dynamics. A dynamic stochastic model is thus proposed and the obtained optimality conditions equations are then solved, numerically. We show that the considered problem leads to a modified state-dependent multi-level (s, S) policy for the supplier selection and replenishment strategy and a base-stock policy for the production activities. We show that these control policies are coupled. This fact confirms the necessity of considering the interactions present in the system in an integrated model so as to obtain more realistic control policies.

6.1 Introduction

In today's ever-changing markets, most manufacturing enterprises operate under highly competitive pressures. This current reality has promoted the establishment of network organisations. Supply chain management (SCM) manages these networks. To accomplish the short and long terms objectives of SCM (e.g., productivity improvement, inventory reduction; customer satisfaction, market share, and profits improvement), tight coordination among the organizations in supply chains is needed (Tan et al. (1998), Lee et al. (2001)).

For manufacturers, the supply (or purchasing) function is widely recognized as a very important key to improve performances in the supply chain Chuang (2004). In fact, since suppliers are organizations external to manufacturers, the coordination with suppliers is not easy unless systems for cooperation and information exchange are integrated (Lee et al. (2001)).

In the research literature, these issues are considered from two points of view. The first one aims at developing new methodologies leading to improved supplier selection processes. The second one seeks new and improved replenishment strategies in the presence of multiple suppliers. It should be noted that the first and the second aspect are generally associated with the long and the short term objectives of the supply chain. To improve the supply chain performance with a long term vision several studies have investigated the supplier selection process.

Among others we refer the reader to Lee et al. (2001), Choi and Hartley (1996) and Verma and Pullman (1998). These studies have explored the current practices in specific sector (e.g., US auto industry in Choi and Hartley (1996), manufacturing companies: specifically metal processing and producers of small machine tools and tooling in Verma and Pullman (1998)) and suggested new methodologies leading to effective supplier management processes (e.g., Lee et al. (2001) proposed the SSMS system which integrates the purchasing and supplier selection systems). Generally, supplier selection is a multi-criteria decision problem (Ghodsypour and O'Brien (1998) and Lee et al. (2001)). The methods suggested in the aforementioned works can be classified into two categories: mathematical programming models and weighting models. The mathematical programming models are principally, goal programming, linear programming or mixed integer programming and the weighting models are the linear scoring model, the AHP (analytic hierarchy process) or the ANP (analytic network process). We refer the reader to Lee et al. (2001), Choi and Hartley (1996), Verma and Pullman (1998) and Ghodsypour and O'Brien (1998) and their references for a complete literature revue of this issue. For a considered class of product, assume that a set of potential suppliers have been determined by an approach such as one of those mentioned previously.

To deal with the suppliers-manufacturer-clients relationships in a mid or short term vision, several issues should be addressed. Namely, from the pre-selected set of suppliers who can respond efficiently to a current order? What is the order quantity to place? At which moment the order must be placed? What is the best production strategy to apply to respond to the clients? These issues are made difficult by the presence of conflicting objectives and by the presence of random phenomena. In the literature, only some of these issues have been addressed. Moreover, they have focused on developing several approaches to formulate the problem with a given replenishment policy. In Chuang (2004), the suppliers-manufacturer relationship has been studied and order allocation problem have been solved with a goal programming approach. Basically, the problem consists in finding out the economic order quantities that should be placed to several suppliers in order to deal with multiple objectives.

On the other hand several researches have attempted to find the optimal control strategy in a dynamic stochastic context. In the class of single stage supply chain facing deterministic lead time and random demands, Zhang (1996) analyzes a model with three suppliers and lead times that differ by one and two periods. The optimal policy is explicitly stated. In addition, simple heuristic ordering policies are discussed and a heuristic framework, based on newsvendor considerations is developed in order to provide decision support for finding appropriate replenishment policy parameters. Within the class of continuous review policies, Moinzadeh and Nahmias (1988) analyze an extension of the (s, Q) policy. Within such a policy an economic lot Q of raw materials is ordered when the down stream inventory level reaches s. In their model, the objective is to minimize long run average costs. The suggested ordering policy is an (s1, s2, Q1, Q2) policy based on the on-hand stock. This policy consists in placing regular order Q1 when level s1 is reached. If, within the replenishment lead time of the regular order, the emergency reorder point s2 is reached, an order of size Q2 is placed. In the same direction Johansen and Thorstenson (1998) analyze a similar model where regular replenishments with a long lead time are controlled by a continuous review (s, Q) policy. In the class of single stage supply chain facing stochastic lead times the research studies have almost exclusively used continuous review (s, Q) policies and focused on the determination of the optimal number of suppliers, the reorder point, the total order quantity and its allocation among the suppliers. These studies have focused on the statistical aspects aiming to argument the advantage of placing orders with multiple suppliers. In Fong et al. (2000) this issue is considered for effective lead times and stock out probabilities for a dual sourcing inventory system facing normally distributed demands and lead times being distributed according to mixtures of Erlang distributions. For a complete review of inventory models with multiple supply options, we refer the reader to Minner (2003).

In this paper, an integrated production, supplier selection and replenishment control problem of a stochastic Supply Chain is considered. Aiming to investigate in a stochastic context, the interaction aspect of this class of supply chains, information sharing control policies are required. While availability, capacity and delivery performances are implicitly considered, a major performance criterion namely the expected discounted cost over an infinite horizon is explicitly considered. This criterion includes ordering, inventories/backlog and transformation costs. A stochastic dynamic programming problem is thus formulated based on the impulsive control theory (Yang (1999)). The developed formulation includes the raw material inventory constraint aspect as well as the hybrid nature of the problem. The structure of the solution, under appropriate conditions, is obtained by using the fact that the value function is the unique viscosity solution to the associated Hamilton Jacobi Bellman (HJB) equations (Yong (1989) and Ramaswamy and Dharmatti (2006)). Owing that an analytical solution of HJB equations is not in general available; a numerical approach is adopted to illustrate the structure of the control policy. Numerical examples and sensitivity analyses are then conducted to achieve a close approximation of the optimal control policy.

The paper is organized as follows. Section 6.2 introduces the notation and presents a formulation of the optimal production and supply problem. The resolution approach is reported in section 6.3. Section 6.4 provides the obtained results aiming to illustrate the optimal control policy structure. A complete characterization of the production and supply policy is reported in section 6.5. The paper is concluded in section 6.6.

6.2 Notation and problem formulation

6.2.1 Notation

The following notation will be used in the rest of the paper.

x : Raw material inventory level	K^j : Ordering cost of supplier j.
y : Finished product inventory / backlog level.	c_R^j : Unit raw material cost from supplier j.
d: Finished product demand rate.	c_R^H : Unit raw material holding cost.
p : Manufacturing production rate.	c_F^H : Unit finished product holding cost.
$p^{\max}$: Maximum manufacturing production rate.	c_F^B : Unit finished product backlog cost.
x_{cap} : Raw material stock capacity.	c_{RF}^T : Unit of raw material transformation cost.
θ_i : Raw material i^{th} order reception instants.	$g(.)$: Instantaneous cost function.
δ^j : Delay between order decision and its reception.	$R(.)$: Overall cost function after ordering.
$Q_i^j(.)$: i^{th} Order quantity corresponding to instant θ_i, ordered from supplier j.	$J(.)$: Overall cost function.
$q_{\alpha\beta}$: Transition rates from modes α to β.	$v(.)$: Value function.
ρ : Discounted rate of the incurred cost.	

6.2.2 Problem formulation

The supply chain under study (illustrated in figure 6.1) consists of an unreliable manufacturing system supplied by multiple unreliable suppliers. The whole system faces a one family product demand.

In this supply chain the manufacturer (stage 2) order raw materials from the more competitive supplier (stage 1). The main criteria for supplier choice are price and supplier service (lead time and reliability). In this study, we assume that the supplier service will cause indirect cost such as costs for holding safety inventory to cover against supply and production variability. Then, through the production processes, the manufacturer converts the raw materials to finished goods which are delivered to the clients (stage 3). The considered supply chain incurred six costs. Between the first and the second stage, there are raw material holding cost, raw material and ordering costs. Between the second and the third stage, there are production, holding and backlog costs.

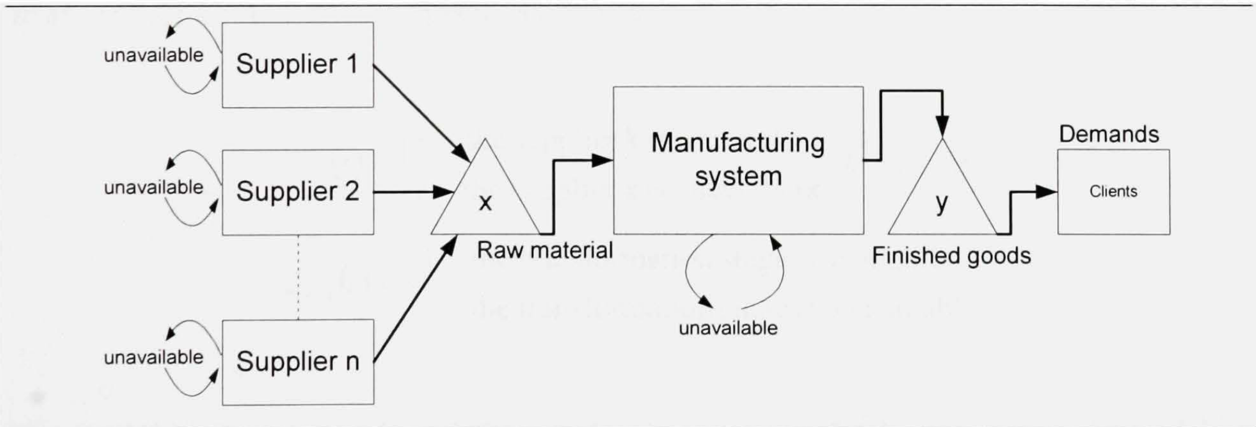


Figure 6.1 Unreliable three stage supply chain

The evolution of the supply chain under study changes with the flow of time (i.e., evolves under conditions of uncertainty). To formulate the optimization problem in a dynamic stochastic context one needs to characterize, as a first step, the state of the system at each instant t. In our case we have defined the state of the system at time t by three components including:

- A continuous part which describes the cumulative surplus level (inventory if positive and backlog if negative) and measured by $y(t)$.
- A piecewise continuous part which describes the raw material level and measured by $x(t)$. This part faces the continuous downstream demand (i.e., manufacturing production rate) and an impulsive upstream supply when a Q_i^j lot of raw material is received from supplier j at instant $\theta_i + \delta^j$. Even if it is available, the transformation manufacturing system cannot proceed parts when $x(t)$ is equal to zero. Let $0 \le x(t) \le x_{cap}$ be the capacity constraint of the raw material stock.
- A discrete part which describes the whole system state (supplier and transformation manufacturing system) and given as follows.

The operational mode of the suppliers and the transformation manufacturing system at time t can be described by the random variables $\xi_k(t), k = 1,\ldots,n$ and $\xi_{n+1}(t)$ with value in $M_k = \{1,2\}, k = 1,\ldots,n+1$, respectively, where

$$\zeta_k(t) = \begin{cases} 1 & \text{the supplier k is available} \\ 2 & \text{the supplier k is unavailable.} \end{cases}, k = 1,\ldots,n$$

$$\zeta_{n+1}(t) = \begin{cases} 1 & \text{the transformation stage is available.} \\ 2 & \text{the transformation stage is unavailable.} \end{cases}$$

The transition rates matrix of the stochastic processes $\xi_k(t), k = 1,\ldots,n$ and $\xi_{n+1}(t)$ are denoted by $\mathrm{T}_i = \{q^i{}_{\alpha\beta}\}, i = 1,\ldots,n+1$, with $q^i{}_{\alpha\beta} \ge 0$ if $\alpha \ne \beta$ and $q^i{}_{\alpha\alpha} = -\sum_{\beta \ne \alpha} q^i{}_{\alpha\beta}$, where $\alpha, \beta \in M_i$. The transitions rates matrix T_i is expressed as follow:

$$\mathrm{T}_i = \begin{vmatrix} -q^i{}_{12} & q^i{}_{12} \\ q^i{}_{21} & -q^i{}_{21} \end{vmatrix}$$

Without loss of generality, for a transformation stage facing two suppliers, the operational mode of the whole system can be described by the random vector $\xi(t) = (\xi_1(t), \xi_2(t), \xi_3(t))$ taking values in $M = M_1 \times M_2 \times M_3$, where : S1, S2 and T denote supplier 1, supplier 2 and the transformation stage respectively.

$$\xi(t) = \begin{cases} 1 & S1: available & S2: unavailable & T: available \\ 2 & S1: available & S2: unavailable & T: unavailable \\ 3 & S1: unavailable & S2: unavailable & T: available \\ 4 & S1: unavailable & S2: unavailable & T: unavailable \\ 5 & S1: available & S2: available & T: available \\ 6 & S1: available & S2: available & T: unavailable \\ 7 & S1: unavailable & S2: available & T: available \\ 8 & S1: unavailable & S2: available & T: unavailable \end{cases}$$

The transition rates of the stochastic process $\xi(t)$, (i.e., $T = \{q_{\alpha\beta}\}, \alpha, \beta \in M$) are easily derived from those of $\xi_k(t)$ by using the definition of $\xi(t)$. Hence, the following transitions rate matrix is derived:

$$T = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{18} \\ q_{21} & q_{22} & \cdots & q_{28} \\ \vdots & \vdots & \ddots & \vdots \\ q_{81} & q_{82} & \cdots & q_{88} \end{bmatrix} =$$

$$\begin{bmatrix} -(q_{12}^3 + q_{12}^1 + q_{21}^2) & q_{12}^3 & q_{12}^1 & 0 & q_{21}^2 & 0 & 0 & 0 \\ q_{21}^3 & -(q_{21}^3 + q_{12}^1 + q_{21}^2) & 0 & q_{12}^1 & 0 & q_{21}^2 & 0 & 0 \\ q_{21}^1 & 0 & -(q_{21}^1 + q_{12}^3 + q_{21}^2) & q_{12}^3 & 0 & 0 & q_{21}^2 & 0 \\ 0 & q_{21}^1 & q_{21}^3 & -(q_{21}^1 + q_{21}^3 + q_{21}^2) & 0 & 0 & 0 & q_{21}^2 \\ q_{12}^2 & 0 & 0 & 0 & -(q_{12}^3 + q_{12}^1 + q_{12}^2) & q_{12}^3 & q_{12}^1 & 0 \\ 0 & q_{12}^2 & 0 & 0 & q_{21}^3 & -(q_{21}^3 + q_{12}^1 + q_{12}^2) & 0 & q_{12}^1 \\ 0 & 0 & q_{12}^2 & 0 & q_{21}^1 & 0 & -(q_{21}^1 + q_{12}^3 + q_{12}^2) & q_{12}^3 \\ 0 & 0 & 0 & q_{12}^2 & 0 & q_{21}^1 & q_{21}^3 & -(q_{21}^3 + q_{21}^1 + q_{12}^2) \end{bmatrix}$$

For the considered supply chain, the state space is given by (x, y, α) such that: $x \in [0, x_{cap}]; y \in R; \alpha \in M$, let $S = [0, x_{cap}] \times R$ and $\partial S = \{0, x_{cap}\} \times R$ and $S^0 =]0, x_{cap}[\times R$ the interior of S.

The dynamics of the stock levels $x(\mathrm{t})$ and $y(\mathrm{t})$ is given by the following differential equations:

$$\begin{aligned}
&\dot{y}(t) = p(t,\alpha) - d,\ y(0) = y,\ \forall t \geq 0 \\
&\dot{x}(t) = -p(t,\alpha),\ x(0) = x,\ \forall t \in \left]\theta_i, \theta_{i+1}\right[\\
&x\left(\left(\theta_i + \delta^j\right)^+\right) = x\left(\left(\theta_i + \delta^j\right)^-\right) + Q_i^j(\alpha),\ i = 1,\ldots,N. j = 1,\ldots,n
\end{aligned} \tag{6.1}$$

Where x, y denote the surplus levels at initial time; $\left(\theta_i + \delta^j\right)^-$ and $\left(\theta_i + \delta^j\right)^+$ denote the negative and positive boundaries of the i^{th} receipt instant from supplier j.

At any given time, the manufacturing production rates and the order quantities have to satisfy the production and supply capacity constraints.

$$\begin{aligned}
&0 \leq p(t,\alpha) \leq p^{\max} \\
&0 \leq x\left(\theta_i + \delta^j\right) + Q_i^j(t) \leq x_{cap}, i = 1,\ldots,N, j = 1,\ldots,n
\end{aligned} \tag{6.2}$$

Our decision variables are the manufacturing production rate $p(\cdot)$ and a sequence of supplier selection and supply orders denoted by $\Omega = \{(\theta_i, \lambda(i), Q_i^{\lambda(i)}), \ldots\}, i = 1,\ldots,N, \lambda(.) = 1,\ldots,n$, see Figure 6.2. With $(\theta_i, \lambda(i), Q_i^{\lambda(i)})$ defined by the time θ_i at which the order is placed, the selected supplier $\lambda(i)$ and the order quantity $Q_i^{\lambda(i)}$. Let $A(\alpha)$ denote the set of admissible decisions $(\Omega, p(.))$ such that:

$$A(\alpha) = \{(\Omega, p(t)), 0 \leq p(t,\alpha) \leq p^{\max}, 0 \leq x\left(\theta_i + \delta^j\right) + Q_i^j(t) \leq x_{cap}, i = 1,\ldots,N, j = 1,\ldots,n, \forall t \geq 0\}$$

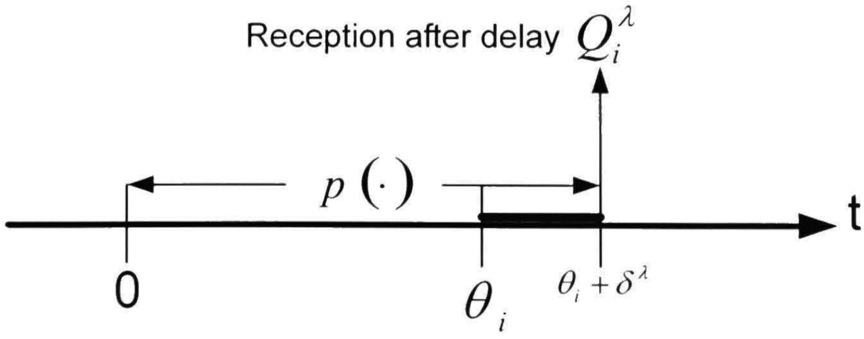


Figure 6.2 Order reception process

In order to penalize the raw material holding cost, the production rate and the finished product inventory (backlog) cost, we define the cost rate function $g(.)$ as follow:

$$g(x(t), y(t), p(t,\alpha)) = c_R^H .x + c_F^H .y^+ + c_F^B .y^- + c_{RF}^T .p(.), \; t \in]\theta_i, \theta_{i+1}[\qquad (6.3)$$

Where, $y^+ = \max(0, y(t))$ *and* $y^- = \max(-y(t), 0)$. Let $z = (x(t), y(t)) \in S$.

In addition, we define the cost function which penalizes the supply order from the selected supplier λ at time θ_i. This function is defined as follow:

$$R(Q_i^\lambda, \lambda, \alpha) = K^\lambda Ind\{t = \theta_i\} + c_R^\lambda .Q_i^\lambda + E_\alpha \int_0^{\delta^\lambda} e^{-\rho t} g(x - p(t,\alpha)t, y + p(t,\alpha) - dt, p(t,\alpha)) dt \qquad (6.4)$$

We make the following assumption on the function $g(.)$.

(A1) g(.,.) is a nonnegative jointly convex function (i.e., convex in either z or p or both). For all $z, z' \in S$ and $p, p' \in A(\alpha)$, there exist constants C_0 and $K_g \geq 0$ such that:

$$|g(z,p) - g(z',p')| \leq C_0 \left[\left(1 + |z|^{K_g} + |z'|^{K_g}\right) \cdot |z - z'| + |p - p'| \right]$$

Using (6.3)-(6.4), the total cost *J(.)* can be defined by the following expression:

$$J(x, y, p, \theta_k, \lambda, Q_k^{\lambda(k)}, \alpha) = E_{x,y,\alpha} \left[\int_0^\infty e^{-\rho t} g(x, y, p)\, dt + \sum_{k=0}^{\infty} e^{-\rho \theta_k} \left(K^{\lambda(k)} + c_R^{\lambda(k)} \times Q_k^{\lambda(k)} \right) \right] \qquad (6.5)$$

Where $E_{x,y,\alpha}$ is the conditional expectation given the condition (x, y, α) at time 0.

The control problem considered herein is to find $(\Omega^*, p^*) \in A(\alpha)$ which minimizes *J(.)* given by (6.5) subject to (6.1) - (6.3). This is a feedback control (see figure 6.3) that determines the production rate and the supply decisions (i.e., supplier selection and supply order) as a function of the system state. The value function of such a stochastic optimal control problem is given by:

$$v(x, y, \alpha) = \inf_{(\Omega, p) \in A(\alpha)} J(x, y, p, \theta_k, \lambda, Q_k^{\lambda(k)}, \alpha) \qquad (6.6)$$

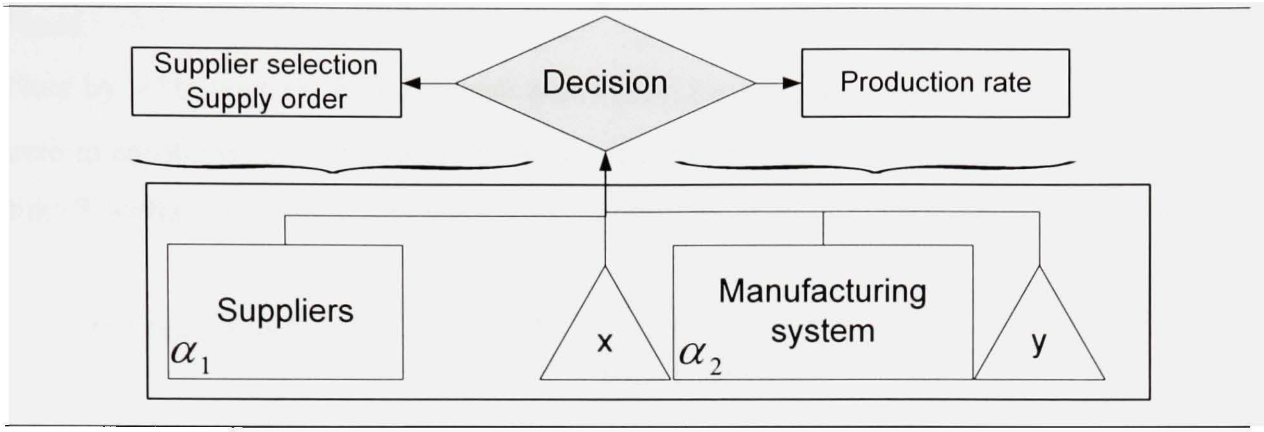


Figure 6.3 Feedback policies

6.2.3 Optimality and boundary conditions

In this section, we apply the dynamic programming approach to derive the optimality conditions (i.e., a set of coupled partial differential equations) of the optimal control problem. Moreover, the presence of state constraints needs to be dealt with separately and it leads to some boundary conditions to be considered at inner points of S.

Proposition 6.1

The value function given by (6.6) is convex and continuous on S^0, and satisfies the condition $|v(z,\alpha)-v(z',\alpha)| \leq C_1\left(1+|z|^{K_g}+|z'|^{K_g}\right)\cdot|z-z'|$

Proof: the proof is similar to Yong (1989) and Lou et al. (1994) given that we consider the interior of the state space S denoted by S^0. □

Theorem 6.1 (The dynamic programming principle)

The value function satisfies $\forall\tau > 0$

$$v(x,y,\alpha,s) = \inf_{(\Omega,p)\in A} E_{x,y,\alpha}\left[\begin{array}{l}\int_0^\tau e^{-\rho t} g(x,y,p)\,dt + \sum_{l=0}^{N} e^{-\rho\theta_l}\left(K^{\lambda(l)} + c_R^{\lambda(l)} \times Q_l^{\lambda(l)}\right) \\ + e^{-\rho\tau} v(x(\tau),y(\tau),\alpha)\end{array}\right] \quad (6.7)$$

Proof:

Note by $v^*(x,y,\alpha,s)$ the right hand side of (6.7), s denotes the initial time and it equals zero in equations (6.5) and (6.6). If we consider the total cost from initial time s to a finish time T with $s < \tau < T$, the following result is obvious:

$$J(x,y,p,\theta_k,\lambda,Q_k^{\lambda(k)},\alpha) \geq E_{x,y,\alpha}\left[\begin{array}{l}\int_s^\tau e^{-\rho t} g(x,y,p)\,dt + \sum_{l=0}^{N} e^{-\rho\theta_l}\left(K^{\lambda(l)} + c_R^{\lambda(l)} \times Q_l^{\lambda(l)}\right) \\ + e^{-\rho\tau} v(x(\tau),y(\tau),\alpha)\end{array}\right]$$

Minimizing each side in respect to an admissible control, it follows:

$$v(x,y,\alpha,s) \geq v^*(x,y,\alpha,s)$$

Conversely, for every $\varepsilon > 0$, there exists an admissible control such that

$$v^*(x,y,\alpha,s) + \varepsilon \geq E\left[\int_s^\tau e^{-\rho t} g(x,y,p)\,dt + \sum_{l=0}^{N} e^{-\rho\theta_l}\left(K^{\lambda(l)} + c_R^{\lambda(l)} \times Q_l^{\lambda(l)}\right) + e^{-\rho\tau} v(x(\tau),y(\tau),\alpha)\right]$$

Following the same development as in Yong and Zhou (1999)(chapter 4, section 3.2) it follows that $v(x,y,\alpha,s) - 2\varepsilon \leq v^*(x,y,\alpha,s)$

Given that ε can be chosen arbitrarily small, we obtain our conclusion. □

For the next development we define the impulse operator which associates for every function $w(.)$ the optimal value after impulse given by:

$$Ow(x,y,\alpha) = \min_{(\lambda,Q^\lambda)\in A}\left\{R(Q^\lambda,\lambda,\alpha) + w(x+Q^\lambda,y,\alpha)\right\} \tag{6.8}$$

Lemma 6.1 for every $\alpha \in M$ the function $Ov(x,y,\alpha)$ is continuous on S^0.

Proof: at the interior of the state space S and if we note *Proposition 6.1*, the proof is similar to Yong (1989) □

Based on the dynamic programming principle, the resulting optimality conditions are formally given by:

$$\min\left\{\begin{array}{l}\min\limits_{p\in A}\left\{\begin{array}{l}(-p)v_x+(p-d)v_y+g(x,y,p)+\\ \sum\limits_{\beta\neq\alpha}q_{\alpha\beta}\left(v(x,y,\beta)-v(x,y,\alpha)\right)\end{array}\right\}-\rho v(x,y,\alpha);\\ \min\limits_{(\lambda,Q^\lambda)\in A}E_\alpha\left\{\begin{array}{l}R(Q^\lambda,\lambda,\alpha)+\\ e^{-\rho.\delta^\lambda}v\left(x+Q^\lambda-\delta^\lambda.p,y+(p-d).\delta^\lambda,\alpha\right)\end{array}\right\}-v(x,y,\alpha)\end{array}\right\}=0 \quad (6.9)$$

Where $(v)_x(.)$, denotes the gradients of $v(.)$ with respect to x.

Definition 6.1

A function $v(.)\in C(S)\equiv\{\text{set of continuous function on S}\}$ is called a viscosity sub-solution (super-solution) of (6.7), if for any $\varphi(.)\in C^1(S)$ with $v(.)-\varphi(.)$ attaining a local maximum (minimum) at $z\in S$, then

$$\min\left\{\begin{array}{l}\min\limits_{p\in A}\left\{\begin{array}{l}(-p)v_x+(p-d)v_y+g(x,y,p)\\ +\sum\limits_{\beta\neq\alpha}q_{\alpha\beta}\left(v(x,y,\beta)-v(x,y,\alpha)\right)\end{array}\right\}-\rho v(x,y,\alpha);\\ \min\limits_{(\lambda,Q^\lambda)\in A}E_\alpha\left\{R(Q^\lambda,\lambda,\alpha)+e^{-\rho.\delta^\lambda}v\left(x+Q^\lambda-\delta^\lambda.p,y+(p-d).\delta^\lambda,\alpha\right)\right\}-v(x,y,\alpha)\end{array}\right\}\leq 0(\geq 0)$$

Definition 6.2

A function $v(.)\in C(S)\equiv\{\text{set of continuous function on S}\}$, is called a viscosity solution if it is both a sub-solution and super-solution.

Theorem 6.2

The value function $v(x,y,\alpha)$ is a viscosity solution (see *definition 6.2*) of the HJB equations (6.9) on S^0.

Proof:

The proof can be developed, as in Yong (1989) and Sethi and Thompson (2000), by considering the *Lemma 6.1* and the replenishment decision as a « stopping decision » with cost given by $E_\alpha\left(O\left(e^{-\rho.\delta}v(x,y,\alpha)\right)\right)$. □

Because we are faced with a state-constrained problem, we need to shape the value function on the boundary of S. To state these boundary conditions we follow the same theory introduced in Capuzzo-Dolcetta and Lions (1990). In their work they have shown that, for state constrained problems, we have to consider the solution of the HJB equations as a viscosity solution inside S and viscosity super-solution on the boundaries (i.e., ∂S). The property that $v(x,y,\alpha)$ is a viscosity super-solution on ∂S plays the role of a boundary condition, which can be given by:

$$\min\left\{\begin{array}{c}\min_{p\in A}\left\{\begin{array}{l}(-p)v_x+(p-d)v_y+g(x,y,p)\\+\sum_{\beta\neq\alpha}q_{\alpha\beta}\left(v(x,y,\beta)-v(x,y,\alpha)\right)\end{array}\right\}-\rho v(x,y,\alpha);\\ \min_{(\lambda,Q^\lambda)\in A}E_\alpha\left\{R(Q^\lambda,\lambda,\alpha)+e^{-\rho.\delta^\lambda}v\left(x+Q^\lambda-\delta^\lambda.p,y+(p-d).\delta^\lambda,\alpha\right)\right\}-v(x,y,\alpha)\end{array}\right\}\geq 0, on \partial S$$

In section 6.3, we present the numerical method used to solve the optimality conditions (6.9), corresponding to the stochastic optimal control problem.

6.3 Numerical resolution

The considered method is based on the Kushner approach Kushner and Dupuis (1992). The solution of the numerical approximation of the optimality conditions (6.9) may be obtained by either successive approximation or policy improvement techniques. The implementation of the approximation technique needs the use of a finite grid denoted herein G_h, where h is a given vector of finite difference intervals. Thus some boundary conditions are needed to describe the behaviour of the system at the border of G_h.

The computation domain G_h is defined as follow:

$$G_h=\{(x,y):0\leq x\leq a\ ,-b\leq y\leq b\} \tag{6.10}$$

Where a and b are given positive constants.

For the numerical implementation, a set of constraints like those presented in Yan and Zhang (1995) are used as boundary conditions.

$$(1)\ v^h(x,-b-h_y,\alpha)=v^h(x,-b,\alpha)+\frac{c_F^B}{\rho}h_y,$$

$$(2)\ v^h(a+h_x,y,\alpha)=v^h(a,y,\alpha)+\frac{c_R^H}{\rho}h_x,$$

$$(3)\ v^h(x,b+h_y,\alpha)=v^h(x,b,\alpha)+\frac{c_F^H}{\rho}h_y,$$

Let h_x and h_y denote the lengths of the finite difference interval of the variable x and y, respectively. Using the finite difference approximation, $v(x,y,\alpha)$ could be given by $v^h(x,y,\alpha)$ and the gradients $(v)_y(x,y,\alpha)$ and $(v)_x(x,y,\alpha)$ by:

$$(v)_y(x,y,\alpha)=\begin{cases}\dfrac{1}{h_y}(v^{h_y}(x,y+h_y,\alpha)-v^{h_y}(x,y,\alpha)) & if\ p-d\geq 0\\ \dfrac{1}{h_y}(v^{h_y}(x,y,\alpha)-v^{h_y}(x,y-h_y,\alpha)) & if\ p-d<0\end{cases}$$

$$(v)_x(x,y,\alpha)=\frac{1}{h_x}(v^{h_x}(x,y,\alpha)-v^{h_x}(x-h_x,y,\alpha))$$

Let:

$$\Lambda^h(p,\alpha,\rho)=\rho+|q_{\alpha\alpha}|+\frac{|-p|}{h_x}+\frac{|p-d|}{h_y}$$

$$Z^x(h_x,x,y,p,\alpha)=\frac{|-p|}{h_x}\left(v^h(x-h_x,y,\alpha)\right)$$

$$Z^y(h_y,x,y,p,\alpha)=\frac{|p-d|}{h_y}\left(v^h(x,y+h_y,\alpha)Ind\{p-d\geq 0\}+v^h(x,y-h_y,\alpha)Ind\{p-d<0\}\right)$$

Where, $Ind\{condition\}=\begin{cases}1, & \text{if condition true}\\ 0, & \text{otherwise}\end{cases}$

With this approximation, the HJB equations (6.9) are expressed in terms of $v^h(x,y,\alpha)$ as follow:

$$v^h(x,y,\alpha)=\left\{\begin{array}{l}\min\limits_{p\in A}\left\{\left(\Lambda^h(p,\alpha,\rho)\right)^{-1}\times\left\{\begin{array}{c}\left[Z^x(h_x,x,y,p,\alpha)+Z^y(h_y,x,y,p,\alpha)\right]\\ +g(x,y,p)+\sum\limits_{\beta\neq\alpha}q_{\alpha\beta}v^h(x,y,\beta)\end{array}\right\}\right\};\\ \min\limits_{(\lambda,Q^\lambda)\in A}E_\alpha\left\{R(Q^\lambda,\lambda,\alpha)+e^{-\rho.\delta^\lambda}v^h\left(x+Q^\lambda-\delta^\lambda.p,y+(p-d).\delta^\lambda,\alpha\right)\right\}\end{array}\right\} \qquad (6.11)$$

The solution of the numerical approximation of the value function may be obtained by either successive approximation or policy improvement techniques (see Hajji et al. (2004) and Boukas and Haurie (1990) for more details). This algorithm will be applied in section 6.4 to solve the numerical optimality conditions (6.11).

6.4 Numerical results and parameterized control policy

To illustrate the supplier selection, replenishment and production policies several elements should be considered.

A. Parameters defining the manufacturing system: namely the maximum production rate (i.e., $p^{\max}$), the demand rate (i.e., d), availability stochastic process (i.e., $T_3=\{q^3{}_{\alpha\beta}\}$), raw material and finished product holding and backlog costs (i.e., $c_R^H, c_F^H, c_F^B, c_{RF}^T$).

B. Parameters defining the suppliers: namely the ordering costs (i.e., K^1, K^2), raw material costs (i.e., c_R^1, c_R^2), supply delay (i.e., δ^1, δ^2) and availability stochastic processes (i.e., $T_1=\{q^1{}_{\alpha\beta}\}$, $T_2=\{q^2{}_{\alpha\beta}\}$).

C. Economic parameter: the discounted rate of the incurred cost (i.e., ρ).

In order to characterize the policies structure, two steps are required. The first step consists in solving in an optimized manner (i.e., computation time and good choice of the computation domain) the optimality conditions for a fixed set of parameters (i.e., A, B and C). The second step consists on carrying sensitivity analysis in order to ascertain the validity of those results. These two steps are presented in the following sections.

6.4.1 Data parameters and results

The numerical results used to illustrate the optimal production and supply policies are presented in this section for four cases of suppliers and manufacturing system availabilities (i.e., Set I, II, III and IV). Given the fact that when the selected supplier is unavailable the transformation stage has to wait for a random length of time (random delay), which is on average equal to the mean time for the supplier to become available in addition to the fixed delay δ^j, these study cases (i.e., suppliers availabilities) showed us the reaction of the transformation stage facing such a situation.

Table 6.1 shows the data parameters, the computational domain G_h given by (6.10) is taken for a=10 and b=10 with $h_x = h_y = 1$ and the discounted rate $\rho = 0.1$.

Tableau 6.1

Data parameters

PARAMETERS	$p^{\max}$	d	c_R^H	c_F^H	c_F^B	c_{RF}^T
Values	2.5	2	0.3	0.35	5	0.1
PARAMETERS	K^1	K^2	c_R^1	c_R^2	δ^1	δ^2
Values	3.5	1.5	1.5	0.5	0.1	0.4

The transitions rates matrices defining the supply chain stochastic process for Sets I, II, III and IV are defined in table 6.2.

To ensure a clear characterization of the control policy, several elements were taken into consideration as part of the implementation process. Indeed, the selection, replenishment and production policies are each observed separately. For each policy, the relevant significant stock threshes are first analyzed independently, then in connection with others thresholds. For each numerical result, $p(x, y, \alpha)$ are the production policies of the transformation stage in the system state α, $A1(x, y, \alpha)$ and $A2(x, y, \alpha)$ are the replenishment policies in system state α from supplier 1 and 2 respectively.
The numerical results for the considered case (table 6.1) are shown in Figures 6.4 to 6.7 for Set I to IV, respectively.

Tableau 6.2

Transition rates matrices

SETS	T_1	AVAIL.	T_2	AVAIL.	T_3	AVAIL.
I	$\begin{bmatrix} -0.01 & 0.01 \\ 0.33 & -0.33 \end{bmatrix}$	97%	$\begin{bmatrix} -0.2 & 0.2 \\ 0.1 & -0.1 \end{bmatrix}$	33%	$\begin{bmatrix} -0.01 & 0.01 \\ 0.33 & -0.33 \end{bmatrix}$	97%
II	$\begin{bmatrix} -0.01 & 0.01 \\ 0.33 & -0.33 \end{bmatrix}$	97%	$\begin{bmatrix} -0.02 & 0.02 \\ 0.1 & -0.1 \end{bmatrix}$	83%	$\begin{bmatrix} -0.01 & 0.01 \\ 0.33 & -0.33 \end{bmatrix}$	97%
III	$\begin{bmatrix} -0.01 & 0.01 \\ 0.33 & -0.33 \end{bmatrix}$	97%	$\begin{bmatrix} -0.2 & 0.2 \\ 0.1 & -0.1 \end{bmatrix}$	33%	$\begin{bmatrix} -0.02 & 0.02 \\ 0.1 & -0.1 \end{bmatrix}$	83%
IV	$\begin{bmatrix} -0.01 & 0.01 \\ 0.33 & -0.33 \end{bmatrix}$	97%	$\begin{bmatrix} -0.02 & 0.02 \\ 0.1 & -0.1 \end{bmatrix}$	83%	$\begin{bmatrix} -0.02 & 0.02 \\ 0.1 & -0.1 \end{bmatrix}$	83%

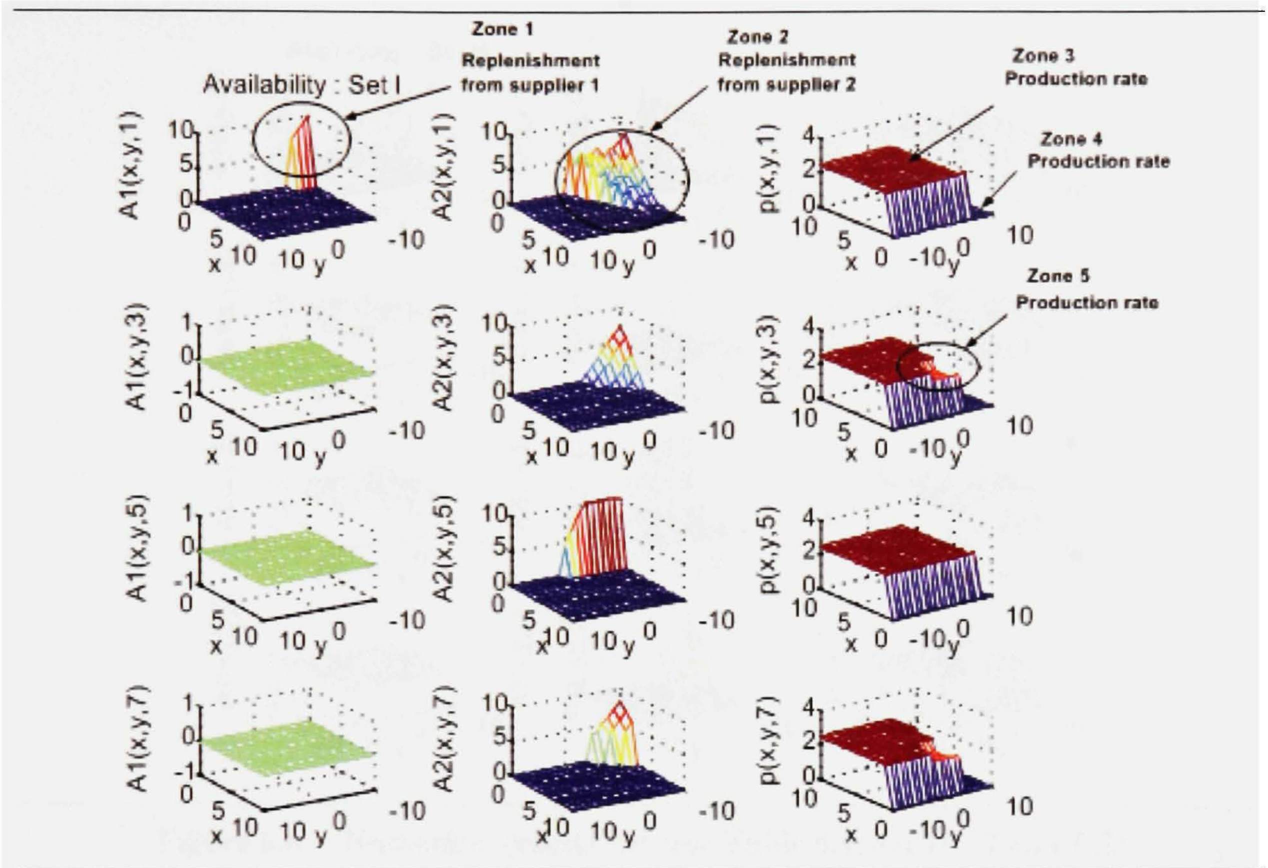


Figure 6.4 Numerical results for case Table 6.1, Set I (Table 6.2)

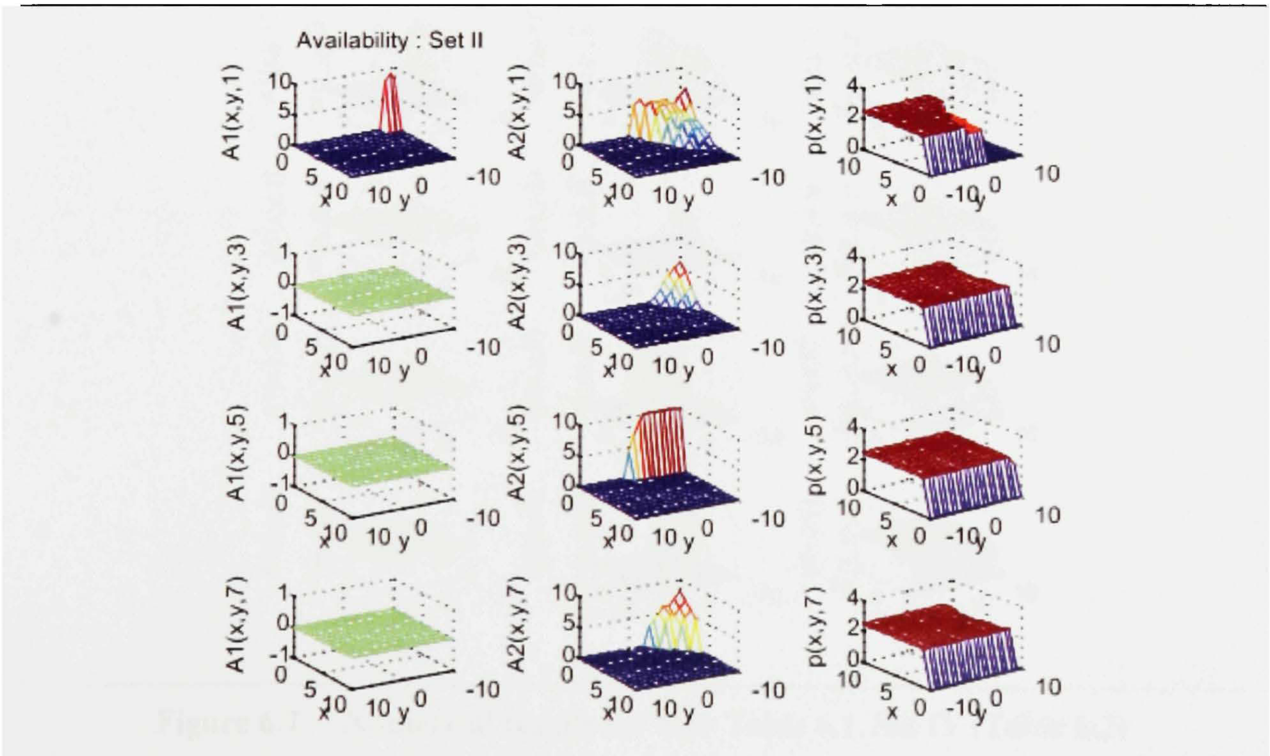


Figure 6.5 Numerical reuslts for case Table 6.1, Set II (Table 6.2)

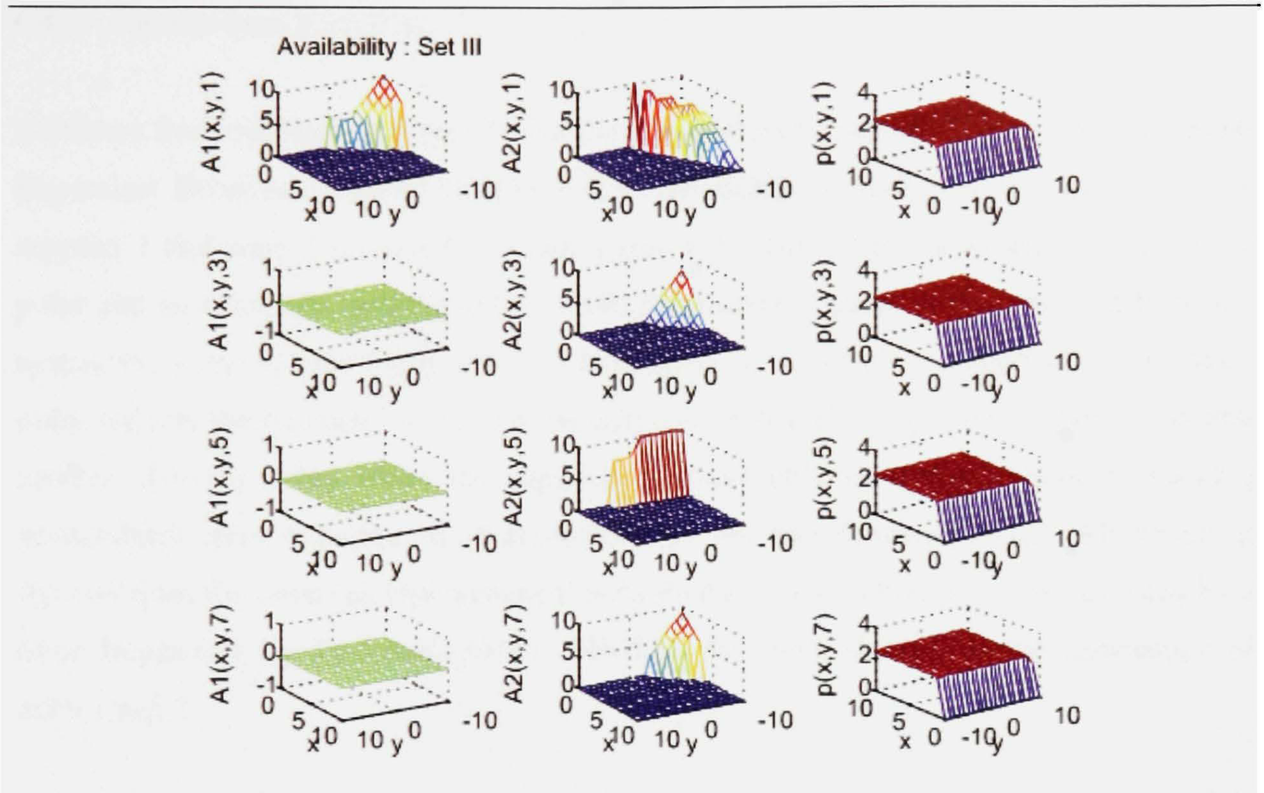


Figure 6.6 Numerical results for case Table 6.1, Set III (Table 6.2)

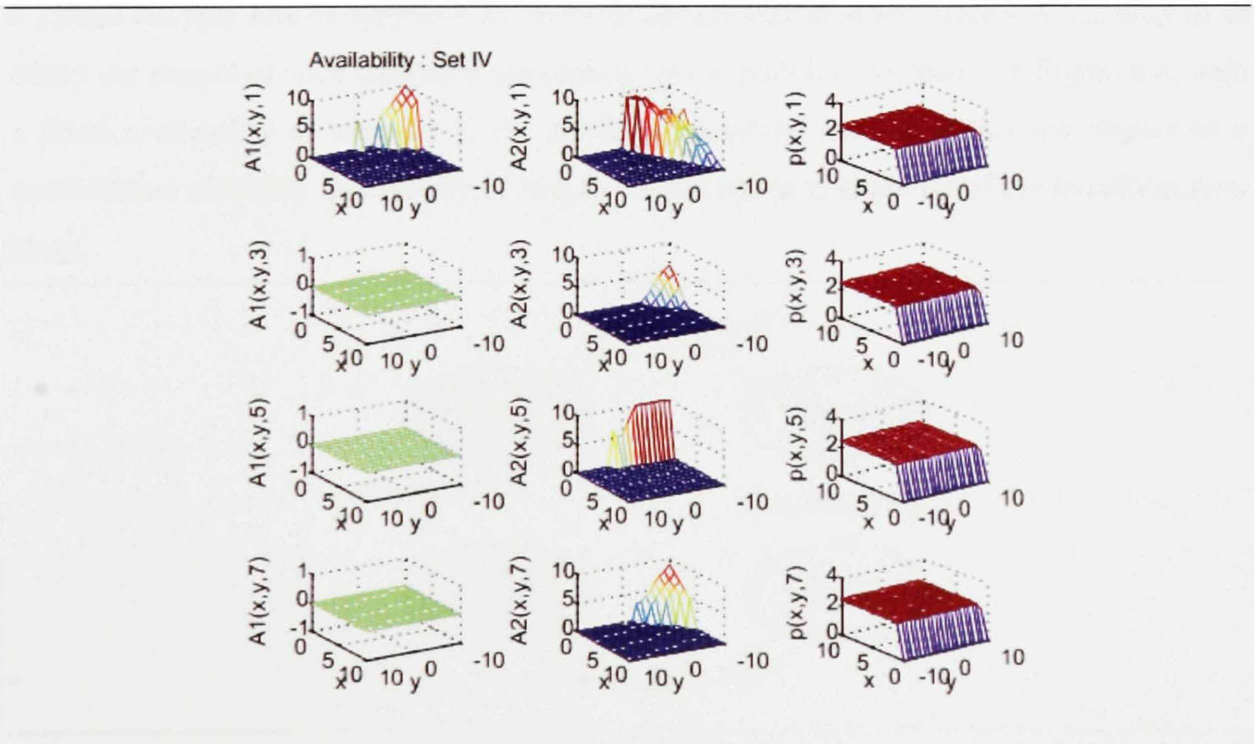


Figure 6.7 Numerical results for case Table 6.1, Set IV (Table 6.2)

6.4.2 Results interpretation

It follows from our numerical results that the replenishment policies are governed by a **S**tate **D**ependant **E**conomic **O**rder **Q**uantity policy, **SD-EOQ** for short, showed by zone 1 for supplier 1 and zone 2 for supplier 2 (see figure 6.4). This policy is governed by an order point and an economic order quantity, these parameters depend on the whole state of the system (x, y and α) (see figure 6.4 to 6.7 for states 1, 3, 5 and 7 respectively). The order point reflects the necessity to have a security raw material stock level to face a possible random delivery delay when the supplier is unavailable or a big amount of backlog accumulated after a period of unavailability of the transformation stage. Moreover, it follows from the observed replenishment policies that the selection policy is governed by a **S**tate **D**ependant **U**p-**T**o-**L**evels policy, **SD-UTL** for short, showed by the intersection of zone 1 and 2.

It should be noted that the preliminary analysis conducted under the fixed costs case (table 6.1) and the four sets of suppliers and transformation availabilities (table 6.2) enabled us to detect the impact of such stochastic parameters on the policies. As shown in Figure 6.8, with a fixed availability of supplier 1, the conducted analysis aims to detect the impact of a combination of higher availability of supplier 2 and lower availability of the transformation stage.

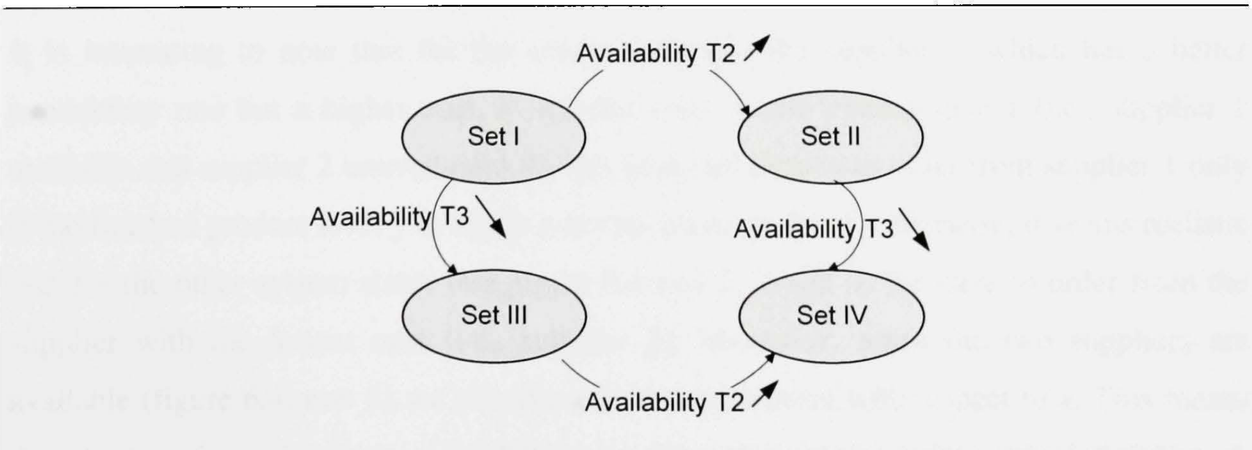


Figure 6.8 Availabilities variation

The results of the four sets (figure 6.4 to 6.7) show that the replenishment policies are governed by three state dependant factors for each supplier. These factors define the economic order quantities and the order points. The results of set I (figure 6.4) show that when the raw material level reaches zone 1 or zone 2 we have to order an economic quantity from supplier 1 or supplier 2 respectively. Let ($S_{F1}^{x,\alpha}$, $S_{F1}^{y,\alpha}$, Q_{F1}^{α}) the order point with respect to x, the order point with respect to y and the order quantity if supplier 1 is selected and ($S_{F2}^{x,\alpha}$, $S_{F2}^{y,\alpha}$, Q_{F2}^{α}) the equivalent parameters if supplier 2 is selected. Figure 6.9 and 6.10 illustrate these factors.

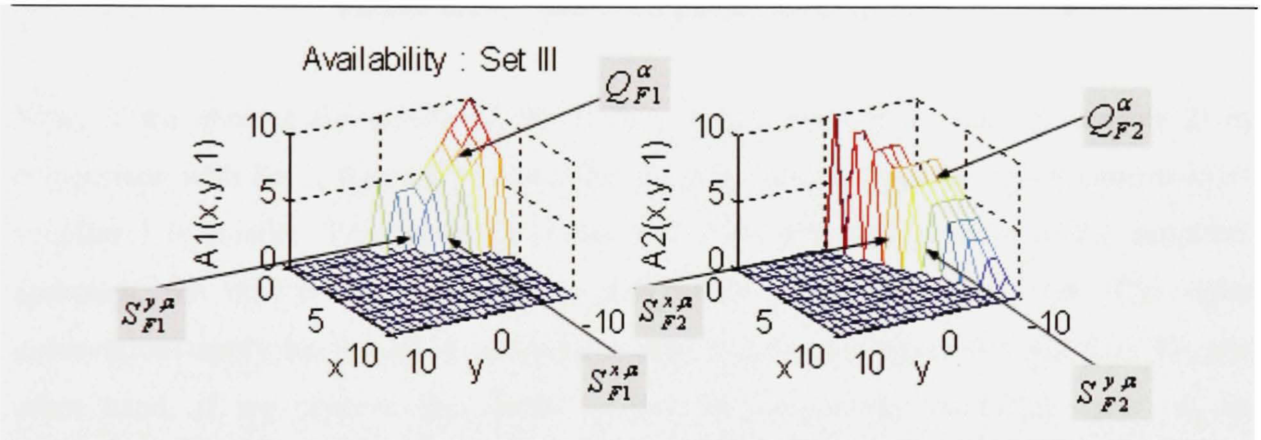


Figure 6.9 Replenishement policies boundaries

It is interesting to note that for the considered case, the supplier 1, which has a better availability rate but a higher cost, is selected only in the system state 1 (i.e., supplier 1 available and supplier 2 unavailable). In this case, we choose to order from supplier 1 only if the finished product level y is below a certain shortage level. Otherwise, it seems realistic that for the other system states (see figure 6.4 raw 2, 3 and 4) we have to order from the supplier with the lowest cost (i.e., supplier 2). Moreover, when the two suppliers are available (figure 6.4, raw 3) we observe a lower order point with respect to x. This means that the transformation stage doesn't have to forecast a large security raw material stock level.

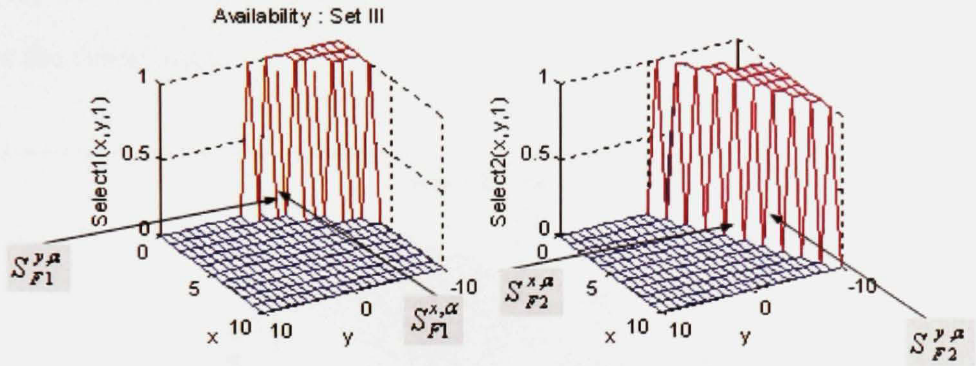


Figure 6.10 a: select supplier 1

Figure 6.10 b: select supplier 2

Figure 6.10 Selection policy boundaries

Now, if we observe the results of Set II (i.e., higher availability rate of supplier 2) in comparison with Set I, figure 6.5 shows that the zone reserved for the replenishment from supplier 1 is smaller. This tendency shows that if the availability ratios of the suppliers approach, the choice will be fixed on that which offers the lower cost. The same observations apply for Set III in comparison with Set IV (see figure 6.6 and 6.7). On the other hand, if we observe the results of Set III (i.e., lower availability rate of the transformation stage) in comparison with Set I, figure 6.6 shows that we must order more while keeping higher security levels (i.e., $S_{F1}^{x,\alpha}$, $S_{F1}^{y,\alpha}$ and $S_{F2}^{x,\alpha}$, $S_{F2}^{y,\alpha}$).

Furthermore, the results show that the resulting production policy divides the surplus space into three mutually exclusive regions (zone 3, 4 and 5 in figure 6.4). In zone 3, produce at the maximal rate, in zone 5 set the production rate equal to the demand rate and in zone 4 set the production rate equal to zero. Moreover, the results show that the area covered by these regions changes depending on the state of the whole system. These results point towards a Modified state dependent multi level Base Stock Policy (MBSP for short) type of production control, given that, at any time, the production rate is either at the lowest demand or maximum level. With some approximations (i.e., we consider the average threshes), to hedge against the existence of irregularities in the regions boundaries, we can define three state dependant hedging levels which characterize the observed optimal production policy.

Let Z_x^α, Z_{y1}^α and Z_{y2}^α define these factors. Figure 6.11 illustrates how these factors delimitates the stock space.

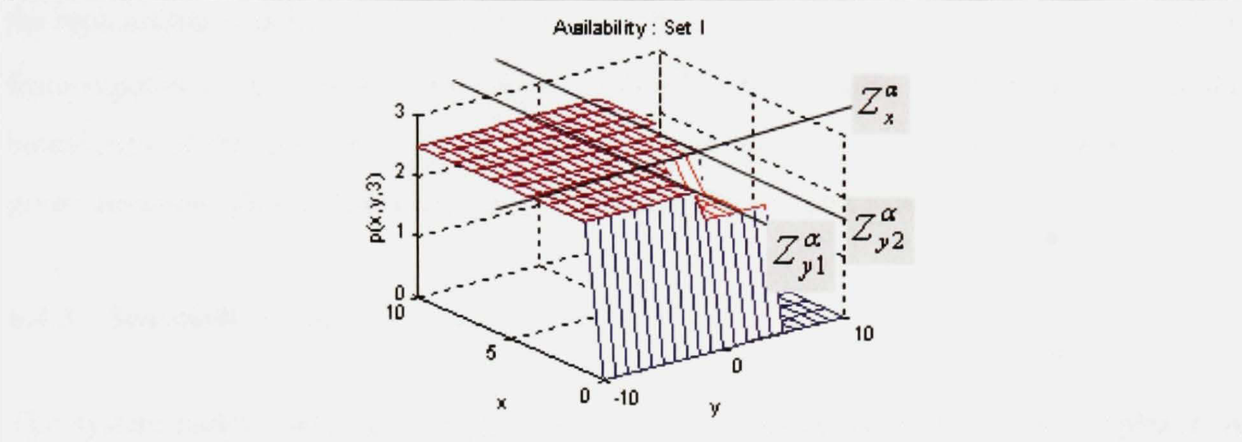


Figure 6.11 Production policy boundaries

The results show that the production policy is governed by three hedging levels which delimitate the areas where the production rate must be set to the maximum or the demand rate. The values of these hedging levels depend on the state of the system. In the class of hedging point policy where the system must keep a finished product security level to hedge against periods of manufacturing system unavailability, our results make sense. Moreover, we observe in figure 6.4 that the production at the demand rate appears in two states (i.e., state 3, raw 2 and state 7, raw 4). As first analysis we can think about the holding costs which are different in the considered case, so it is more profitable to keep the raw material in the up stream area when supplier 1 (i.e., which is the best from availability point of view) is unavailable. However, when supplier 1 is available the transformation stage must keep the stock in the down stream area to prevent the possibility of its next unavailability period. In addition, the results of Set III and IV (see figure 6.6 and 6.7), with the same data parameters but with a lower availability rate of the transformation stage, confirm these observations. Indeed, the hedging levels are higher and the production rate is often in the maximum rate, comparing to those of Set I, allowing the transformation stage to stock more products to hedge against its lower availability rate.

The interpretation of the numerical results has shown that the optimal policy is a combination of **MBSP**, **SD-UTL** and **SD-EOQ** policies. The combined policy is completely defined by nine parameters, Z_x^α , Z_{y1}^α and Z_{y2}^α for the production policy, $S_{F1}^{x,\alpha}$, $S_{F1}^{y,\alpha}$ and Q_{F1}^α for the replenishment policy from supplier 1 and $S_{F2}^{x,\alpha}$, $S_{F2}^{y,\alpha}$ and Q_{F2}^α for the replenishment policy from supplier 2. In order to ascertain the validity of those results, it will be shown that the boundaries of the policies move as predicted when the data parameters are changed in a given direction. This is illustrated through a sensitivity analysis in section 6.4.3.

6.4.3 Sensitivity analysis

The system under study involves operational and system parameters and its complexity is attributable to their wide range of variability. To understand the effect that changing these parameters has on the control policy, sensitivity analysis is necessary. In this paper, we have concentrated our efforts on operational parameters judged to be the most appropriate. Hence, holding, backlog, raw material and ordering costs are considered in the analysis.

Let us now consider the sensitivity analysis cases presented in Table 6.3. Group I and II represent six cases where sensitivity on raw material and ordering costs for the two suppliers are conducted. Group III represents three cases with sensitivity on finished product backlog cost. Group IV and V represent six cases with sensitivity on raw material and finished products holding costs. Note that the same sensitivity analyses were conducted under the aforementioned availability sets (see table 6.2).

Tableau 6.3

Sensitivity analysis data

GROUPS	CASES	c_R^H	c_F^H	c_F^B	c_R^1	c_R^2	K^1	K^2	RESULTS
I	Base	0.3	0.35	5	**2.5**	0.5	**10**	1.5	---
	2	0.3	0.35	5	**1**	0.5	**5**	1.5	$S_{F1}^{x,\alpha}\uparrow, S_{F1}^{y,\alpha}\uparrow, Q_{F1}^{\alpha}\downarrow$
	3	0.3	0.35	5	**5**	0.5	**15**	1.5	$S_{F1}^{x,\alpha}\downarrow, S_{F1}^{y,\alpha}\downarrow, Q_{F1}^{\alpha}\uparrow$
II	Base	0.3	0.35	5	5	**2**	15	**6**	---
	2	0.3	0.35	5	5	**1**	15	**3**	$S_{F2}^{x,\alpha}\downarrow S_{F2}^{y,\alpha}\downarrow Q_{F2}^{\alpha}\uparrow$
	3	0.3	0.35	5	5	**3**	15	**9**	$S_{F1}^{x,\alpha}\downarrow, S_{F1}^{y,\alpha}\downarrow, Q_{F1}^{\alpha}\uparrow$
III	Base	0.3	0.35	**3**	1.5	0.5	3.5	1.5	----
	2	0.3	0.35	**2**	1.5	0.5	3.5	1.5	$Z_x^{\alpha}\ Z_{y1}^{\alpha}\ Z_{y2}^{\alpha}, S, Q\downarrow$
	3	0.3	0.35	**5**	1.5	0.5	3.5	1.5	$Z_x^{\alpha}\ Z_{y1}^{\alpha}\ Z_{y2}^{\alpha}, S, Q\uparrow$
IV	Base	**0.35**	0.5	5	1.5	0.5	3.5	1.5	---
	2	**0.3**	0.5	5	1.5	0.5	3.5	1.5	Ss ↑, Qs ↓, Zs ↓
	3	**0.4**	0.5	5	1.5	0.5	3.5	1.5	Ss ↓, Qs ↑, Zs ↑
V	Base	0.3	**0.35**	5	1.5	0.5	3.5	1.5	---
	2	0.3	**0.3**	5	1.5	0.5	3.5	1.5	Ss ↓, Qs ↑, Zs ↑
	3	0.3	**0.4**	5	1.5	0.5	3.5	1.5	Ss ↑, Qs ↓, Zs ↓

<u>Analysis of Group I-II results</u>

The results of group I (resp. group II) show the effect of decreasing or increasing the raw material and ordering cost of supplier 1 (resp. supplier 2) on the control policies. Recall that supplier 1 admits a higher availability rate. The results have shown that the boundaries of the replenishment and selection policies moved in a convincing direction in respect to the variation of the parameters. In fact, when we increase c_R^1 and K^1, $S_{F1}^{x,\alpha}$, $S_{F1}^{y,\alpha}$ decrease and Q_{F1}^{α} increases. This means that facing a higher cost of supplier 1, the transformation system chooses to select more often the supplier 2 and keep supplier 1 for extreme situations (i.e., high level of finished product backlogs) with higher order quantity. In the other hand, when

c_R^1 and K^1 decrease we observe an opposite reaction. In addition the results of group II show that when c_R^2 and K^2 decrease (resp. increase) the transformation system must favour supplier 2 (resp. supplier 1) in the sense that $S_{F2}^{x,\alpha}$ and $S_{F2}^{y,\alpha}$ decrease and Q_{F2}^{α} increases (resp. $S_{F1}^{x,\alpha}$ and $S_{F1}^{y,\alpha}$ decrease and Q_{F1}^{α} increases).

Analysis of Group III results

The results of group III show the effect of increasing or decreasing the finished product backlog cost on the control policies. It is shown that when c_F^B increases (resp. decreases) the values of the hedging thresholds, the order points and the ordering quantities increase (resp. decrease). This means that facing a higher backlog cost, the system must react to keep higher raw material and finished product levels.

Analysis of Group IV-V results

The results of group IV and V show the effect of varying the raw material and finished product holding costs on the control policies. It is shown that when the holding cost of the raw material increases (resp. decreases), the order points decrease (resp. increase), the order quantities increase (resp. decrease) and the hedging levels increase (resp. decrease). This observation shows how the system reacts to transform material to the stock incurring the lowest cost (final stock in this case). This observation is confirmed by the results of group V and show that the dynamic reaction of the system makes sense.

6.5 Structure of the production, replenishment and supplier selection control policies

In this section, a parameterized control policy based on the analysis of the numerical results of section 6.4 is developed. In order to describe the optimal production, replenishment and supplier selection policies by mathematical equations, the boundaries parameters observed and introduced earlier will be used. These parameters are defined as follows:

Z_x^α , Z_{y1}^α and Z_{y2}^α for the production policy, $S_{F1}^{x,\alpha}$, $S_{F1}^{y,\alpha}$ and Q_{F1}^α for the replenishment policy from supplier 1 and $S_{F2}^{x,\alpha}$, $S_{F2}^{y,\alpha}$ and Q_{F2}^α for the replenishment policy from supplier 2.

6.5.1 Optimal production policy

As shown within the numerical results and the Figure 6.11, the optimal production rate can be described by a Modified Base Stock Policy (MBSP for short) which is state dependent multi levels and can be expressed as follow.

$$p(x,y,\alpha)=\begin{cases} P^{\max} & if \quad \left(y<Z_{y2}^\alpha \;\&\; x>Z_x^\alpha\right) \| \; y<Z_{y1}^\alpha \\ d & if \quad Z_{y1}^\alpha<y<Z_{y2}^\alpha \;\&\; x<Z_x^\alpha \\ 0 & if \quad y>Z_{y2}^\alpha \end{cases} \tag{6.12}$$

Recall that Z_x^α, Z_{y1}^α and Z_{y2}^α represent the threshold parameters with the following constraints.

$$Z_x^\alpha \geq 0; Z_{y2}^\alpha > Z_{y1}^\alpha \geq 0 \tag{6.13}$$

6.5.2 Optimal replenishment policy

As shown in the previous paragraphs and the Figure 6.9, the optimal replenishment policies can be described by a State Dependant Economic Order Quantity policy (SD-EOQ for short) which can be expressed by the following equations.

Replenishment policy from supplier 1:

$$\Omega^1(x,y,\alpha)=\begin{cases} Q_{F1}^\alpha(x,y) & if \quad x<S_{F1}^{x,\alpha} \;\&\; y<S_{F1}^{y,\alpha} \;\&\; \lambda(x,y)=1 \\ 0 & otherwise \end{cases} \tag{6.14}$$

Replenishment policy from supplier 2:

$$\Omega^2(x,y,\alpha)=\begin{cases} Q_{F2}^{\alpha}(x,y) & if \quad x< S_{F2}^{x,\alpha} \ \& \ y< S_{F2}^{y,\alpha} \ \& \ \lambda(x,y)=2 \\ 0 & otherwise \end{cases} \tag{6.15}$$

Recall that $S_{F1}^{x,\alpha}$, $S_{F1}^{y,\alpha}$ and Q_{F1}^{α} (resp. $S_{F2}^{x,\alpha}$, $S_{F2}^{y,\alpha}$ and Q_{F2}^{α}) represent the order points and the economic order quantities for supplier 1 (resp. supplier 2), λ represent the selection policy indicator and defined in section 6.5.3.

6.5.3 Optimal supplier selection policy

It follows from the observed replenishment policies that the selection policy (see figure 6.10) is governed by a State Dependant Up-To-Levels policy, SD-UTL for short, showed by the intersection of zone 1 and 2 (see figure 6.4) which can be expressed by the following equations.

$$\lambda(x,y)=\begin{cases} 1 & if \quad x< S_{F1}^{x,\alpha} \ \& \ y< S_{F1}^{y,\alpha} \\ 2 & if \quad S_{F1}^{x,\alpha} < x < S_{F2}^{x,\alpha} \ \& \ S_{F1}^{y,\alpha} < y < S_{F2}^{y,\alpha} \\ 0 & otherwise \end{cases} \tag{6.16}$$

6.6 Conclusion

In conclusion, it would be interesting to point out the contribution of this paper. Indeed, complete information sharing production, replenishment and supplier selection control policies for an unreliable supply chain was developed. These policies are described by equation (6.12) to (6.16) and illustrated in Figure 6.9, 6.10 and 6.11. The policies are « information sharing » type since the supply chain actors establish their strategies of control in the whole state space.

From a mathematical point of view, we have called upon optimal and impulsive control theory notions to propose a hybrid stochastic model of the considered problem. We have solved numerically the Hamilton Jacobi Bellman equations of the problem and carried out deep sensitivity analysis. Based on the obtained numerical results, optimal control policy of the problem was derived. The obtained control policy has been shown to be described by a combined MBSP and SD-EOQ, SD-UTL policies defined by nine parameters for the case of two suppliers and $3\times(n+1)$ for the case of n suppliers.

CHAPITRE 7

OPERATIONAL LEVEL-BASED POLICIES OF UNRELIABLE FLOW-SHOPS IN A SUPPLY CHAIN ENVIRONMENT

Abstract

This paper deals with the control of the manufacturing activities (i.e., production, setup and maintenance) of an unreliable flow-shop, multiple-product manufacturing system in a supply chain environment. In fact, the transformation system faces an unreliable upstream supply and a random replenishment delay. Our objective is to determine the manufacturing activities planning (i.e., production rates, a sequence of setups and the best maintenance strategy) together with the raw material replenishment strategy in order to minimize the total setup, holding, backlog, failures, preventive maintenance and supply cost. Obviously, an analytical solution of the problem is very difficult to find. Thus a combined approach is proposed and is based on stochastic optimal control theory, discrete/continuous event simulation, genetic algorithm and experimental design. Following two of our previous works (i.e., see introduction) where we proved that: 1- for the production and setup control problem an extended version of the hedging corridor policy combined to a hybrid (KANBAN/CONWIP) production control mechanism are more realistic 2- the integrated manufacturing and supply problem leads to a combined replenishment policy depending on the raw material and the finished products inventory levels; The contribution of this paper consists on developing an optimization module making it possible to find in a stochastic dynamic manner the best control parameters of the production, replenishment and setup actions simultaneously with the best maintenance scheduling between bloc, age or opportunistic strategies. It will be shown that it is more profitable to consider in integrated manner the manufacturing and supply control problems. In fact, for the case under study we found that the total incurred cost can be reduced up to 10 %. Moreover, depending on the economic context, it is more profitable to consider more than one maintenance strategy and to adopt the best one in a given context.

7.1 Introduction

In nowadays industrial context, operations planning and control is gaining much more importance in companies' improvement process. To respond to real case problems three complex realities arises: the number of decisions, the system size / configuration and the dynamic-stochastic aspects of a given manufacturing system. One of those systems present in a vast number of industries is the flexible flow-shops. They consist on several serial stages with buffers located between them and producing multiple parts type of products. They are common in the process industry including the electronics manufacturing, the food and cosmetics, the pharmaceutical sector as well as the automotive industry, see Hajji et al. (2007c) for related references. In the aforementioned work (i.e., Hajji et al. (2007c)), we addressed the problem of production and changeover control in a class of failure prone buffered flow-shop. In the conducted literature revue, it appeared that in a dynamic stochastic context, the joint production and setup control problem has been successfully solved only for simple systems. Although, many researchers consider that even if optimal control policies can be found for realistic systems, they risk being too complicated to implement. We showed that optimal control analysis was valuable to propose joint control policies for complex systems (i.e., failure prone buffered *m* machine *n* parts type flow-shop). Thus, we were able to overcome the complexity behind the size and the dynamic stochastic aspects of a given manufacturing system (buffered flow-shops in our case).

To go further with practical concerns, the issue of decisions diversity should be addressed. Three of the most important tasks carried out in manufacturing systems are production, scheduling control and maintenance planning. In the research literature these three tasks are mostly dealt with separately. Ruiz et al. (2007) made a recent contribution in this context to integrate maintenance planning with the flow-shop sequencing problem. In a dynamic stochastic context, however, the problem remains largely open. A revue of the literature has shown recent studies aiming to jointly control production and maintenance planning in a stochastic dynamic context. They succeeded to propose variants of the two main strategies (age dependent and bloc, see section 6.2.2 for definitions and references) to guarantee better performances. We refer the reader to Boulet et al. (2005) and (2007) for a recent review of the literature addressing the joint production control and maintenance planning problem. The main point which one can note in these two recent studies (Boulet et al.) is the advantage of combining simulation based approaches and mathematical based policies to gain benefit knowledge of a given complex system. However, in large size cases the used optimization approach (i.e., Design of Experiment) fails to keep its force. Moreover, when the system involves quantitative and qualitative parameters, which is the case in the system under study (see section 6.2), the recourse to other approaches become indispensable.

Regarding, the joint production, setup and maintenance planning control in multi parts buffered flow-shops, the problem remains requiring a robust and flexible approach to address the aforementioned three concerns (great number of decisions, system size / configuration and the dynamic-stochastic aspects).

In a supply chain context, one of the main issues arising when dealing with the manufacturing activities control consists on the relationship with the suppliers. This issue was addressed in Hajji et al. (2007) but without considering the details of the manufacturing shop floor. In Hajji et al. (2007) the integrated production and replenishment control problem was considered and it was shown that the optimal replenishment policy depends on the raw material and finished products inventory levels. It is interesting to note that in the literature we haven't found studies taking into account the replenishment control problem together with other manufacturing activities control.
Based on these facts, the main contribution of this paper is to propose a flexible and useful approach making it possible to address in a stochastic dynamic manner the joint replenishment, production, setup and maintenance planning problem in multi parts buffered flow-shops.

The paper is organized as follows. Section 7.2 states the problem and presents the main results of the optimal production and setup scheduling problem, for a m buffered machines multiple products manufacturing system addressed in Hajji et al. (2007c), the main results of the joint production and replenishment control problem addressed in Hajji et al. (2007), as well as the maintenance strategies involved in the considered optimization problem. In section 7.3, a revue of simulation based optimization approaches is presented to introduce the proposed approach detailed in section 7.4. Section 7.5 and 7.6 presents the genetic algorithm and the implemented optimization module. The obtained results and related discussions are reported in section 7.7. The paper is concluded in section 7.8.

7.2 Problem statement

The manufacturing system under study consists of an unreliable buffered flow-shop capable of producing n different part types P_i, $1 \le i \le n$. As shown in figure 7.1, the considered flow-shop consists in a serial buffered m machines. The machines are not completely flexible in the sense that change over time (set-up activities) between part types is not negligible. This setup conducted on the whole line involves both time (i.e., Θ_{ij}) and cost (i.e., K_{ij}). Note that, $\theta_{ij} \ge 0$ *and* $K_{ij} \ge 0$, for, $i, j = 1,\ldots,n$, and $i \ne j$.

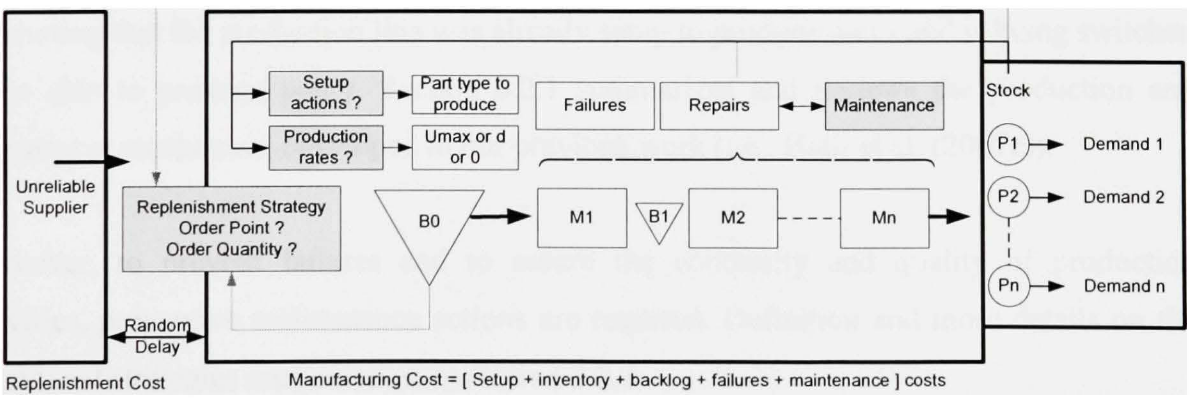


Figure 7.1 *m* machines *n* parts flow-shop system in supply chain environment

Part type i have a production rate $0 \le u_k^i(t) \le U_k^{\max i}$ $(i = 1,\ldots,n; k = 1,\ldots,m)$ on machine k and have an average time between orders $1/d_i$.

Machines M_k and M_{k+1} ,$1 \leq k \leq (m-1)$ are separated by a buffer B_k. Each of which is required to store in process products P_i. The level of B_k consists on the sum of $x_k^i(t)$, $1 \leq i \leq n$ (i.e., inventory level of product i on B_k ,$1 \leq k \leq (m-1)$).

The difference between actual production and downstream demand at any time represents the surplus of a part type. For buffers B_k ,$1 \leq k \leq (m-1)$ the difference is always positive (i.e, inventory costs c_{ik}^+ are thus charged) or equal to zero (i.e., starvation of machine *k+1*), for buffer B_m the difference is positive (i.e, inventory costs c_{im}^+ are thus charged) or negative (i.e., backlog costs c_{im}^- are thus charged). Note that if the capacity of the buffer B_k ($1 \leq k \leq (m-1)$) is reached, machine M_k could be blocked if the downstream demand is equal to zero.

Regarding the production and changeover control problem, our decision variables are production rates $u_k^i(t), i = 1,\ldots,n; k = 1,\ldots,m$ and a sequence of setups denoted by $\Omega = \{(\tau_0, i_0 i_1), (\tau_1, i_1 i_2), \ldots\}$. A setup (τ, ij) is defined by the time τ at witch it begins and a pair *ij* denoting that the production line was already setup to produce part *i* and is being switched to be able to produce part *j*. Section 6.2.1 summarizes and reviews the production and changeover mechanism developed in our previous work (i.e., Hajji et al. (2007c)).

Moreover, to prevent failures and to assure the continuity and quality of production activities, preventive maintenance actions are required. Definition and more details on the considered strategies are presented in section 7.2.2.

When considering the manufacturing system in its external environment, one of the main issues to consider consists on a random raw material supply. As shown in figure 7.1, the manufacturing system under study is facing a random supply due to periods of unavailability of the supplier and/or a random transportation delay. This issue was considered in Hajji et al.

(2007), more details on the joint optimal production and replenishment strategies are presented in section 7.2.3.

7.2.1 Production and changeover mechanisms

For the system considered in Hajji et al. (2007c), the developed parameterized heuristic can be considered as a major contribution, since it confirms existent results and address the multi parts issue. The developed control policy, illustrated by figure 7.2, point toward a KANBAN/CONWIP MHCP control policies. Such a heuristic can be employed, after optimization of the correspondent parameters, to control the production and the changeovers on multi parts multi machines flow-shops.

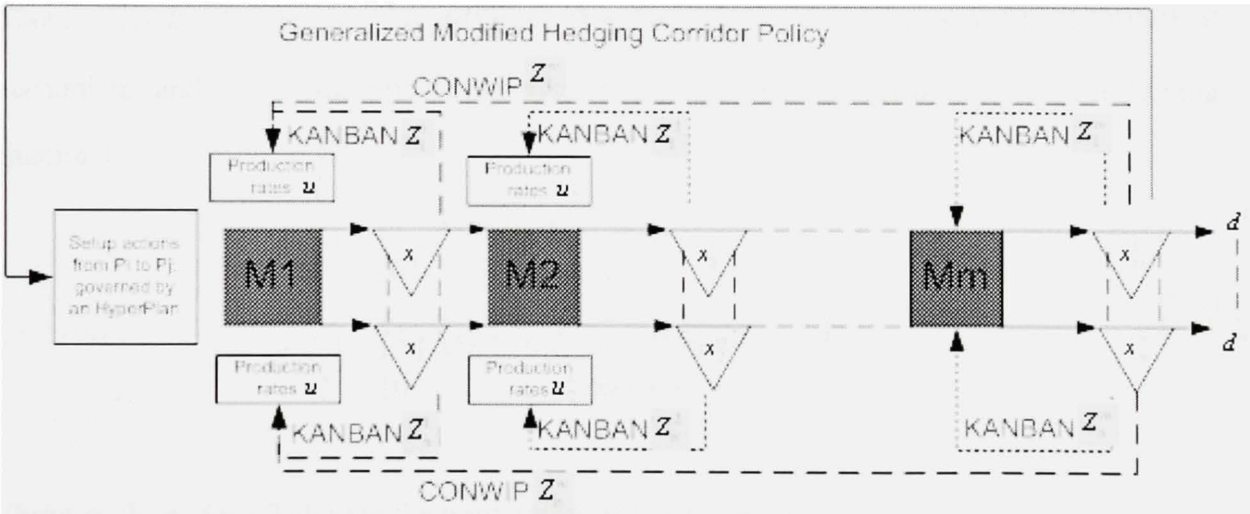


Figure 7.2 m machines n parts flow-shop control mechanism

Without loose of generality (see Hajji et al. (2007c) for the general policies), for the two machines flow-shop two parts type, we can describe and parameterize the production policies by the following equations.

$$u_1^1(\ )=\begin{cases} \overset{\max}{U}{}_1^1\ IND\{S_1=1\} & x_1^1 \le Z_1^1\ \&\ x_2^1 \le Z_1^2 \\ 0 & otherwise \end{cases}$$
$$u_2^1(\ )=\begin{cases} \overset{\max}{U}{}_2^1\ IND\{S_1=1\} & x_2^1 \le Z_1^2 \\ 0 & otherwise \end{cases} \quad (7.1)$$

$$u_1^2(\ )=\begin{cases} \overset{\max}{U}{}_1^2\ IND\{S_2=1\} & x_1^2 \le Z_2^1\ \&\ x_2^2 \le Z_2^2 \\ 0 & otherwise \end{cases}$$
$$u_2^2(\ )=\begin{cases} \overset{\max}{U}{}_2^2\ IND\{S_2=1\} & x_2^2 \le Z_2^2 \\ 0 & otherwise \end{cases} \quad (7.2)$$

Where $Z_k^i, i,k=1,2.$ denote the different threshes involved in the KANBAN/CONWIP mechanism and $S_i, i=1,2.$ define the system configuration described by the following equations.

$$S_1(\ )=\begin{cases} 1 & x_2^1 \le b_2\ \&\ x_2^2 \ge a_2 \\ 0 & otherwise \end{cases}$$
$$S_2(\ )=\begin{cases} 1 & \left(x_2^1 \ge a_1\ \&\ x_2^2 \le b_1\right) \| \left(x_2^2 \le c_1\right) \\ 0 & otherwise \end{cases} \quad (7.3)$$

Where $a_i, b_i, c_i, i=1,2.$ denote the boundaries of the setup zones.

7.2.2 Maintenance strategies

Preventive maintenance strategy can be defined as a set of actions making it possible to maintain or restore a given good to prevent failures. In the literature a large number of strategies are developed and discussed (Duffuaa et al. (1999)). We can group the common strategies in two big families. The first one, called bloc replacement preventive maintenance policy (BRP), consists on replacing the component at failure and at moments $k.T_{BRP}$ (k=1, 2, 3, ...) regardless its age and condition, let c_{BRP} the associated cost when a preventive action is performed and c_F the repair cost after a failure. This strategy depends on the parameter T_{BRP} defining the cycle of preventive actions. The second one, called age replacement preventive maintenance policy (ARP), consists on replacing given equipment by a new one at failure. Otherwise, if the component survives T_{ARP} time units (i.e., uninterrupted operation time) a preventive maintenance is performed. T_{ARP} is known as the age replacement period, let c_{ARP} the associated cost.

In the literature a wide number of modified and improved maintenance strategies are developed. We refer the reader to Boulet et al. (2007) for a recent revue. In this paper our choice is fixed on this two basic maintenance strategies and a third one considered to be the most adapted to our case (i.e., presence of setup time) (Kelly et al. (1997)). In fact, the third considered preventive maintenance strategy, called opportunistic replacement policy (ORP) and consists on adopting a classic strategy and additionally take benefit from changeover times to initiate a maintenance action. In our case, ORP consists on replacing given equipment by a new one at failure or, like ARP after T_{ORP} time units of uninterrupted operation time or, when a setup action is initiated and a percentage of T_{ORP} is run out (i.e., $\alpha.T_{ORP}$), let c_{ORP} the associated cost.

Our objective is to find the best production and changeover control policies parameters (section 7.2.1) as well as the best maintenance strategy for each machine and its parameters to minimize the total cost of inventory, backlog, setups, failures and preventive maintenance. For the considered system the optimization problem include ***quantitative parameters*** (i.e., $Z_k^i, i = 1,..,n, k = 1,..,m$. $a_i, b_i, c_i, i = 1,..,n$. T_{ARP}^k or T_{BRP}^k or (T_{ORP}^k and α^k) for every machine) and ***qualitative parameters*** ($\beta_k, k = 1,...,m$) denoting the selected maintenance strategy for machine k and equal to 1 for BRP, 2 for ARP and 3 for ORP.

7.2.3 Replenishment strategies

The joint production and replenishment control problem addressed in Hajji et al. (2007) and as shown in figure 7.3 consists on finding the optimal policies function of the whole system states.

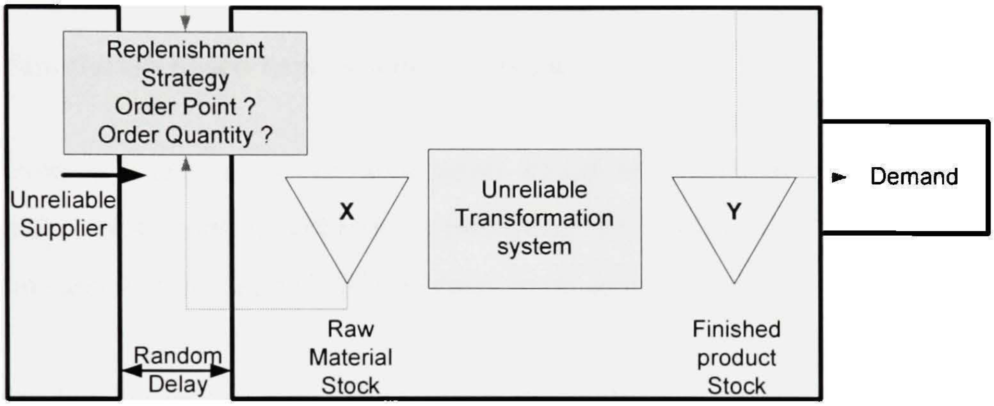


Figure 7.3 Joint production and replenishment problem

The results show that the supply policy is governed by a **S**tate **D**ependant **E**conomic **O**rder **Q**uantity policy, **SD-EOQ** for short. This policy is governed by an order point and an economic order quantity, these parameters depend on the whole state of the system (***x, y*** and ***α***). The order point reflects the necessity to have a security raw material stock level to face a possible random delivery delay when the supplier is unavailable or a big amount of backlog accumulated after a period of unavailability of the transformation stage.
The **SD-EOQ** can be expressed by the following equations.

$$\Omega_{\text{supply}}(x,y) = \begin{cases} Q^i(x,y) & if \quad x < s_R^i \;\&\; y < s_F^i \\ 0 & otherwise \end{cases} \quad (7.4)$$

Recall that s_R^i and s_F^i represent the order points in respect to the raw material and finished product inventory levels x and y of part type i and $Q^i(x,y)$ represents the economic order quantities of part type i with the following constraints.

$$s_R^i \geq 0; s_F^i \geq 0 \, and \, Q^i(x, y) > 0 \tag{7.5}$$

In the following section a revue of simulation based experimental approaches is presented to introduce the proposed approach.

7.3 Simulation based experimental approach

In this section a brief review of simulation based experimental approaches to solve manufacturing system control problems is presented. The following figure (i.e, figure 7.4) adopted from Carson and Maria (1997) resumes these approaches.

Before presenting the literature revue related to the problem under study it is interesting to recall the importance of the simulation modeling in such methodology. In fact, in classical optimization approaches such as mathematical programming, it is indispensable to know in advance the transfer function. Moreover, it is much easier to involve only quantitative variables in the optimization process. This is simply not the case in a stochastic manufacturing system context where the transfer function is difficult to know in advance and which could depend on qualitative parameters. Thus, simulation modeling is a good alternative to describe the dynamic stochastic behaviour of the system. In fact, in simulation based optimization approaches, the objective function and the system constraints are described in a simulation model which consists on several networks, each of which describes a specific task in the system (i.e., demand generation, control policy, states of the machines, inventory control..., etc.). Therefore, the decision variables are the conditions under which the simulation is run, the performance measures are one or multiple responses given by the simulation.

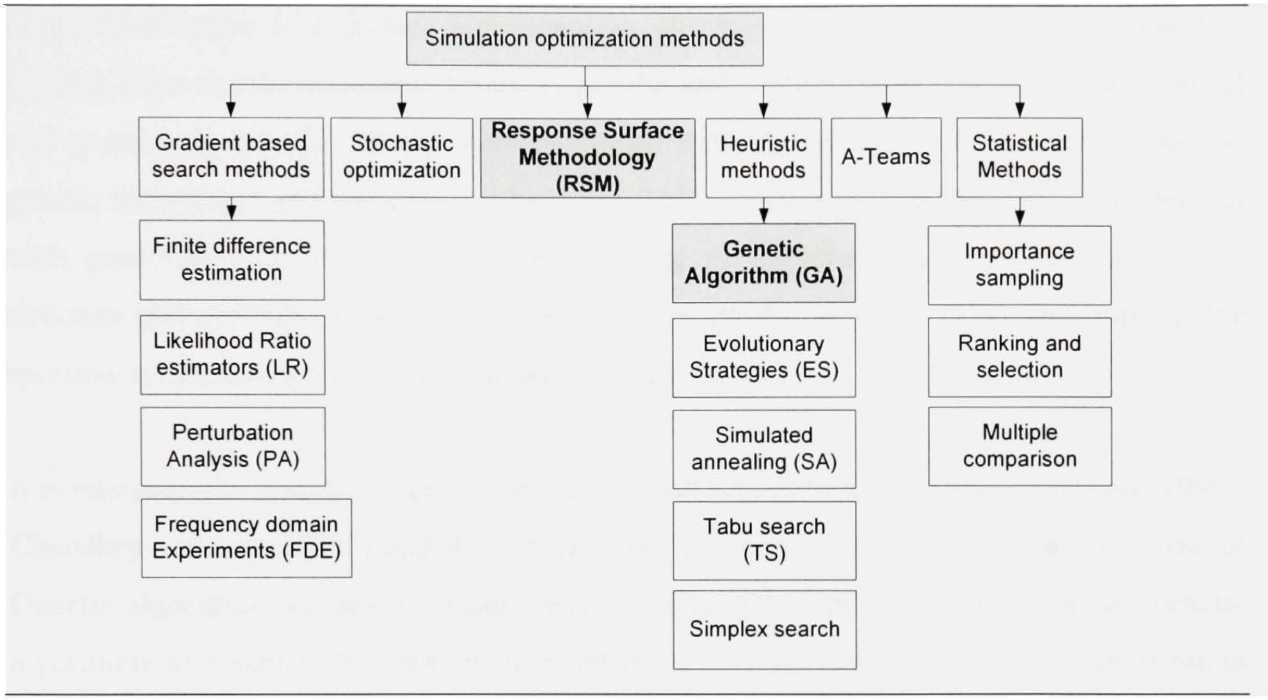


Figure 7.4 Simulation based optimization methods

In the literature, simulation based optimization approaches can be classified in six categories (see figure 7.4). In what follows, the two most encountered categories namely, the gradient based search methods and the heuristic methods, are discussed. For more details on the other methods see Carson and Maria (1997).

Regarding gradient based search methods; they cover finite difference estimation (Andradòttir (1998)), likelihood ratio estimation (Glynn et al. (1991)), perturbation analysis (Ho (1984)) and frequency domain experiments (Morrice and Schruben (1987)). These methods aim is to estimate the retained performance measure with respect to the decision variables.

In the other hand, heuristic methods consist on a random exploration of the admissible solutions in the whole decisions space. The search process ends when the best solution is found. At each point of the search process, the objective function value of the problem is estimated via the simulation model. Thus, no information regarding the analytic form of the objective function is required. This category covers simplex search (Azadivar and Lee (1988)), tabu search, simulated annealing (Ogbu and Smith (1990), Lee and Iwata (1991)) and genetic algorithms. All the aforementioned methods, except simulated annealing and genetic algorithms, require a system having a fixed structure during the search process and with quantitative decision variables. In our case the system under study has a variable structure and depends on both quantitative and qualitative variables. This fact imposes the recourse to simulated annealing or genetic algorithms.

It is interesting to note that previous researches and survey (Azadivar and Tompkins (1999), Chaudhry and Luo (2005) and Ruiz et al. (2007)) have demonstrated the effectiveness of Genetic algorithms solutions. In our case, our approach combines simulation and genetic algorithms to optimize the system. In addition to the implementation of this optimization module, we will propose a solution to the problem of choosing the parameters of a given genetic algorithm, the approach involve design of experiments (DOE) and response surface methodology (RSM).

7.4 Proposed approach

In order to bring an approach which could be easily applied to control manufacturing systems at the operational level, the descriptive capacities of discrete/continuous event simulation models are combined with analytical models, genetic algorithms, design of experiments, and response surface methodology. A block diagram of the resulting control approach is depicted in Figure 7.5.

I. The first part of the approach consists on addressing the optimal control problem mathematically. This issue was addressed in Hajji et al. (2007 & 2007c), the resulting parameterized production, changeover and replenishment control policies were summarized in section 7.2.1 and 7.2.3.

II. The second part consists on building an optimization module supporting the quantitative and qualitative parameters governing the production, changeover, replenishment and preventive maintenance mechanisms. This module link a parameterized simulation model with a genetic algorithm making it possible to run a genetic algorithm search process for the best solution (total incurred cost).

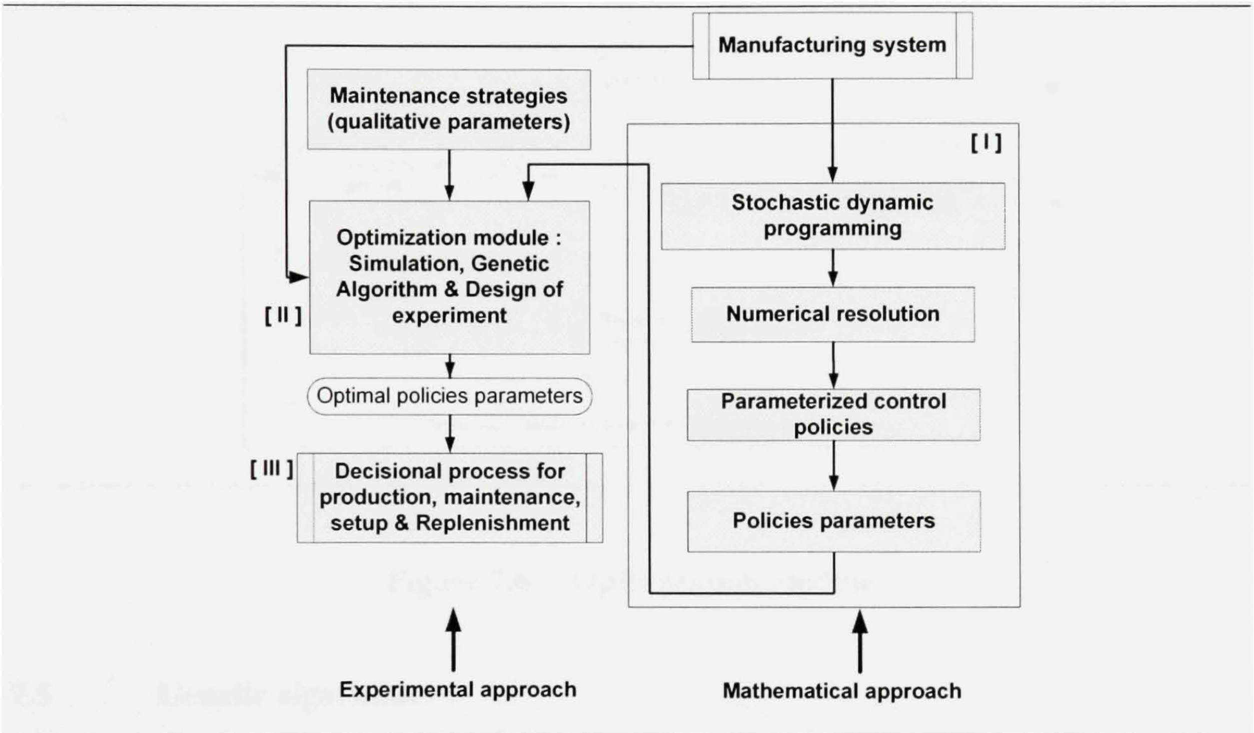


Figure 7.5 Proposed approach

The search process is detailed in figure 7.6. It consists on running the genetic algorithm with respect to its stopping rule and evaluates each desired configuration through the simulation model.

The original point for this optimization module is the optimization of the genetic algorithm parameters using DOE. This issue is detailed in section 7.6.

III. The third part is the resulting decisional process for a given configuration of the flow-shop manufacturing system. The aforementioned configuration includes the technical and economic aspects of the system.

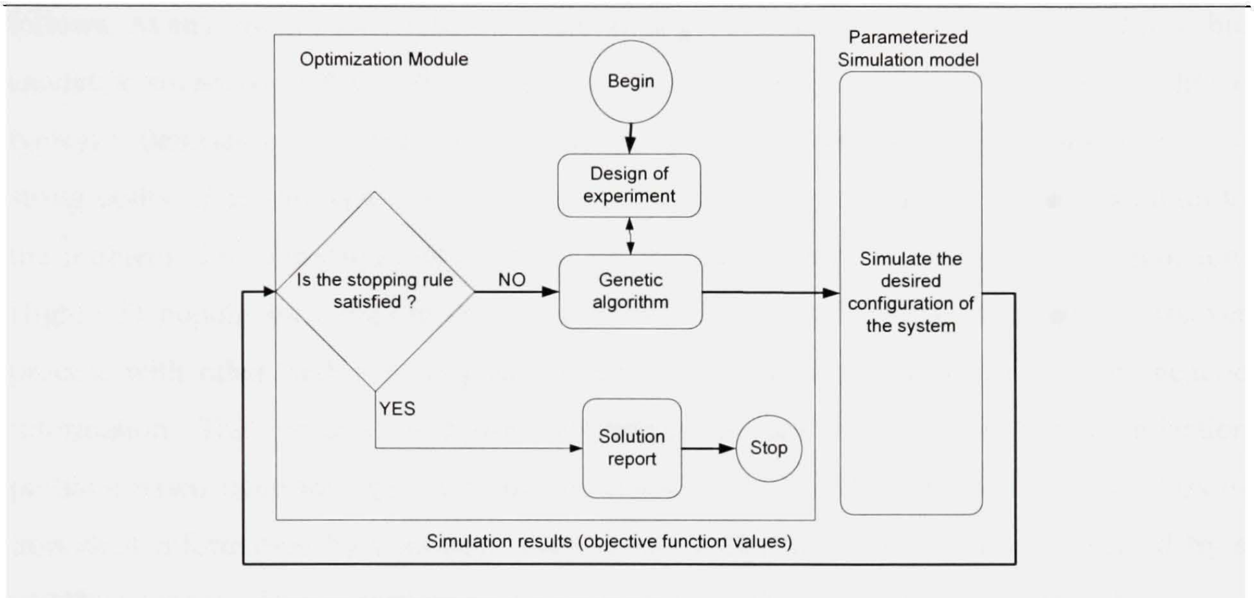


Figure 7.6 Optimization module

7.5 Genetic algorithm

This section is adopted from Legault (1994) and Chaudhry and Luo (2005) to give an overview on genetic algorithm principles. A genetic algorithm, first introduced by Holland (1975), is a heuristic search procedure which is based on the natural process of evolution as in biological sciences. As this highly adaptive evolutionary process progresses, the population genetics evolves in a given environment according to the natural behaviour in which the fittest survive and the weakest is destroyed. Thus, the genes from the adept donor will then propagate to another recipient during each successive generation, hence creating more adept offspring suitable for the defined environment. In optimization terms, the search algorithm improves the solution over generations as it progresses toward the optimum. Genetic algorithms have been successfully applied in solving a variety of optimization problems which are difficult to solve. These problems include the travelling salesperson problem, job-shop scheduling problems and routing problems, among others.

In terms of an optimization problem, the genetic algorithm approach is summarized as follows. At any given point in time, the genetic algorithm generates a population of possible candidate solutions. Initially, the population size is chosen at random. However this choice typically depends on the characteristics of the problem. Each population component is a string entity of chromosome (e.g., (0,1) bit string) which represents a possible solution to the problem. The population components are evaluated based on a given fitness function. Highly fit population components are given the chance to reproduce through a crossover process with other highly fit population elements by exchanging pieces of their genetic information. This process produces «offspring» or new solutions to the optimization problem based upon the high-performance characteristics of the parents. Premature loss of important information by randomly altering bits within a chromosome is prevented by a mutation process. This procedure continues until a satisfactory solution is achieved.

7.6 Optimization module implementation

This section deals with the presentation of the various elements which compose the optimization module.

7.6.1 Overview

When genetic algorithms are employed to carry out an optimization process, each point in the solution space is characterized by a chromosome. Each position in the chromosome characterizes a decision alternative. In our case, the decision alternative could be quantitative (i.e., policies parameters) or qualitative (i.e., maintenance strategies). The optimization process starts with a random sample of the solution space. Each point of this sample is sent to the simulation model to evaluate his objective function (i.e., total cost). After that, based on the objective function values of each point, a *selection* rule is employed to generate a new population. Chromosomes of this population are then *crossover* following fixed rules to construct a new generation. Another decision to be taken consists on a possible introduction of mutation in the process. This is done by random sampling with a given probability, commonly chosen as small value (less than 5 %).

Regarding the search process stopping rule, many of them are used in the literature. A common rule consists on stopping the algorithm when the generated populations become equivalent or when we find that there is no significant improvement. Another rule consists on running the simulation for a fixed number of runs.

To set the optimization module, 3 steps are required. Building a parameterized simulation model; building a genetic algorithm program and connecting these two sub-units to allow the communication between them.

7.6.2 Genetic algorithm: MATLAB Toolbox

The main data structures in the GA toolbox (Chipperfield et al. (1994)) are chromosomes, phenotypes, objective function values and fitness values. The chromosome structure stores an entire population in a single matrix of size $N_{ind} \times L_{ind}$, where, N_{ind} is the number of individuals and L_{ind} is the length of the chromosome structure. Phenotypes are stored in a matrix of dimension $N_{ind} \times N_{\mathrm{var}}$ where, N_{var} is the number of decision variables. A $N_{ind} \times N_{obj}$ matrix stores the objective function values, where N_{obj} is the number of objectives. Finally, the fitness values are stored in a vector of length N_{ind}. In all of these data structures, each row corresponds to a particular individual.

The GA toolbox uses MATLAB matrix functions to build a set of versatile routines for implementing a wide range of genetic algorithm methods. In this section we outline the major procedures of the GA Toolbox and especially those used in our program.

1. Population representation and initialisation: the GA Toolbox supports binary, integer and floating-point chromosome representations. Binary and integer populations may be initialised using the Toolbox function to create populations, **crtbp**. Real-valued populations may be initialised using **crtrp**. Conversion between binary and real-values is provided by the routine **bs2rv**.

2. Fitness assignment: the fitness function transforms the raw objective function values into non-negative figures of merit for each individual. The Toolbox supports the offsetting and **scaling** method of Goldberg (1989) and the linear-**ranking** algorithm of Baker (1985).

3. Selection functions: available routines include roulette wheel selection (Goldberg (1989), routine **rws**) and stochastic universal sampling (Baker (1987), routine **sus**).

4. Crossover operators: the crossover routines recombine pairs of individuals with given probability to produce offspring. Single-point, double-point (Baker (1987)) and shuffle crossover (Caruana et al. (1989)) are implemented in the routines **xovsp**, **xovdp** and **xovsh** respectively. A general multi-point (Syswerda (1989)) crossover routine, xovmp, is also provided.

5. Mutation operators: Binary and integer mutation are performed by the routine **mut**. Real-values mutation is available using the breeder GA mutation function, **mutbga**.

The following steps summarize the employed Genetic Algorithm:

1. Population representation and initialisation: binary representation with « N_{ind} » the number of individuals and « **Preci** » the precision of the binary representation.
2. Fitness: the linear-ranking method of Baker (1985).
3. Selection: stochastic universal sampling of Baker (1987). The technique needs to fix a ratio « **GGAP** » of the best elements to keep.

4. Crossover: Single-point (Baker (1987)) with crossover probability « **Pc** ».
5. Mutation: binary mutation with probability $P_m = 1/L_{ind}$, L_{ind} is the length of the chromosome structure equal $L_{ind} = \Pr eci \times N_{var}$.

Let « **MaxGen** » be the maximum number of generation if the stopping algorithm rule is fixed following this criteria.

7.6.3 Simulation model

The simulation model is build to describe the dynamic of the system governed by the production, changeover and maintenance policies defined previously and parameterized by the aforementioned parameters (section 7.2). These factors are considered as input of such a model and the related incurred total cost is defined as its output. The combined discrete/continuous parameterized simulation model is developed using the Visual SLAM language (Pritsker & O'Reilly (1999)) with C sub-routines. It is interesting to note that the combined discrete/continuous simulation model is more flexible and reduces the execution time (Lavoie et al. (2007)).

The Visual SLAM portion is composed of various networks describing specific tasks (failure and repair events, preventive maintenance cycles, changeover and production threshold variables crossing, data exchange with Genetic algorithm, etc...). The simulation ends when current simulation time Tc reaches the defined simulation period *T_fin*. Figure 7.7 shows a bloc diagram representation of the simulation model.

1) The Exchange data block read the parameters of each individual of the population set by the Genetic Algorithm. To run the model the INITIALIZATION block sets these values and other parameters defining the system (e.g., the demands rates, setup duration, maximal production rates,...) as well as the simulation time *T_fin* and the time for the warm up period after which statistics are cleared.

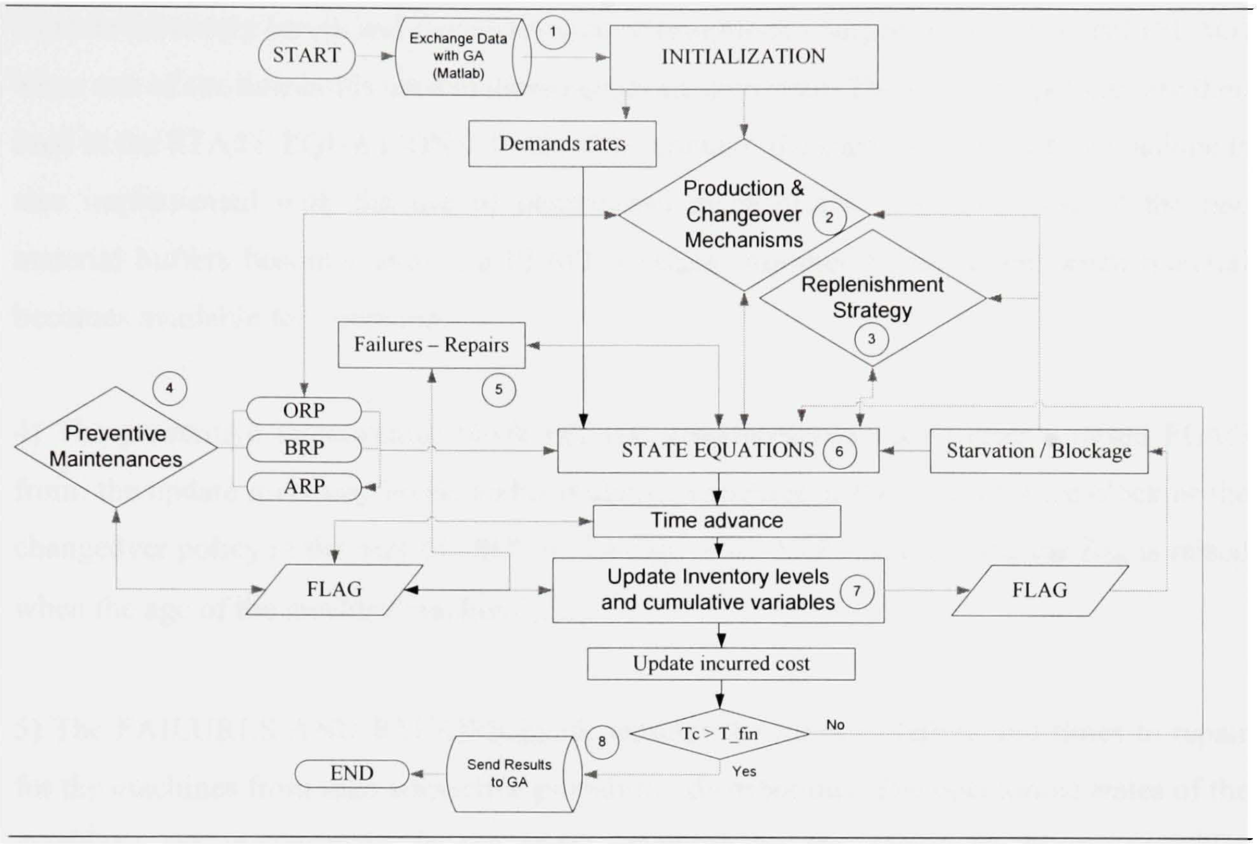


Figure 7.7 Simulation model bloc diagram

2) The production and changeover MECHANISM set the production rates and the part type to produce according to its control equation (i.e., equations (7.1), (7.2) and (7.3) in section 7.2, for the two machine two parts case). This block is in connection with the «update inventory levels and cumulative variables» block charged to send a signal (FLAG) when one of the thresholds used in these equations is crossed. The resulting policies are then used in the STATE EQUATIONS. In the same context, the starvation / blockage of the machines is also implemented with the use of observation mechanisms. Whenever one of the in-process buffers becomes empty or full, a FLAG is raised. Another signal is sent when material becomes available for operation.

3) The replenishment MECHANISM set the order quantities according to its control equation (i.e., equations (7.4) and (7.5) in section 7.2). This block is in connection with the

«update inventory levels and cumulative variables» block charged to send a signal (FLAG) when one of the thresholds used in these equations is crossed. The resulting policies are then used in the STATE EQUATIONS. In the same context, the starvation of the first machine is also implemented with the use of observation mechanisms. Whenever one of the raw material buffers becomes empty, a FLAG is raised. Another signal is sent when material becomes available for operation.

4) The preventive maintenance block initiates a maintenance action after a raised FLAG from: the update inventory levels and cumulative variables or the time advance block or the changeover policy in the case of ORP. In the case of an ARP, for example, the flag is raised when the age of the machine reaches T_{ARP}.

5) The FAILURES AND REPAIRS block samples the times to failure and times to repair for the machines from their respective probability distributions. The operational states of the machines are incorporated in the state equations by the means of binary variables multiplying the production rates. The repair action of a given machine set its age to zero.

6) The STATE EQUATIONS are defined as a C language insert. They describe the inventory, backlog and age variables using the production rates set by the control policy and the binary variables from the failure/repair, blockage/starvation, setup and maintenance networks.

7) The ADVANCE TIME, UPDATE INVENTORY LEVELS AND CUMULATIVE VARIABLES block is used once the time step is provided by the simulation software. The cumulative variables are integrated using the Runge-Kutta-Fehlberg (RKF) method as described in Pritsker & O'Reilly (1999).

8) The Send Results block writes the incurred cost of each individual of the genetic algorithm population in an external file. This file being available to the genetic algorithm program reiterates the optimization process.

7.6.4 GA parameters optimization procedure

When dealing with genetic algorithms, the choice of the GA parameters (i.e., the precision of the binary representation and selection ratio for example) is an important issue to be taken into account since it can affect the optimization process and the final results. In the research literature the choice of these parameters is generally based on experience. One of the few studies addressing the GA parameters optimization is Pongcharoen et al. (2002) where they used design of experiment and statistical analysis approaches. In our case, this approach was used in a significant number of our research studies (see Boulet et la. (2007) and Lavoie et al. (2007) and the references therein) and it can be easily integrated to our approach. Figure 7.8 illustrates the procedure and can be the following points. We refer the reader to Montgomery (2001) and Banks (1998) for more details on DOE and statistical analysis.

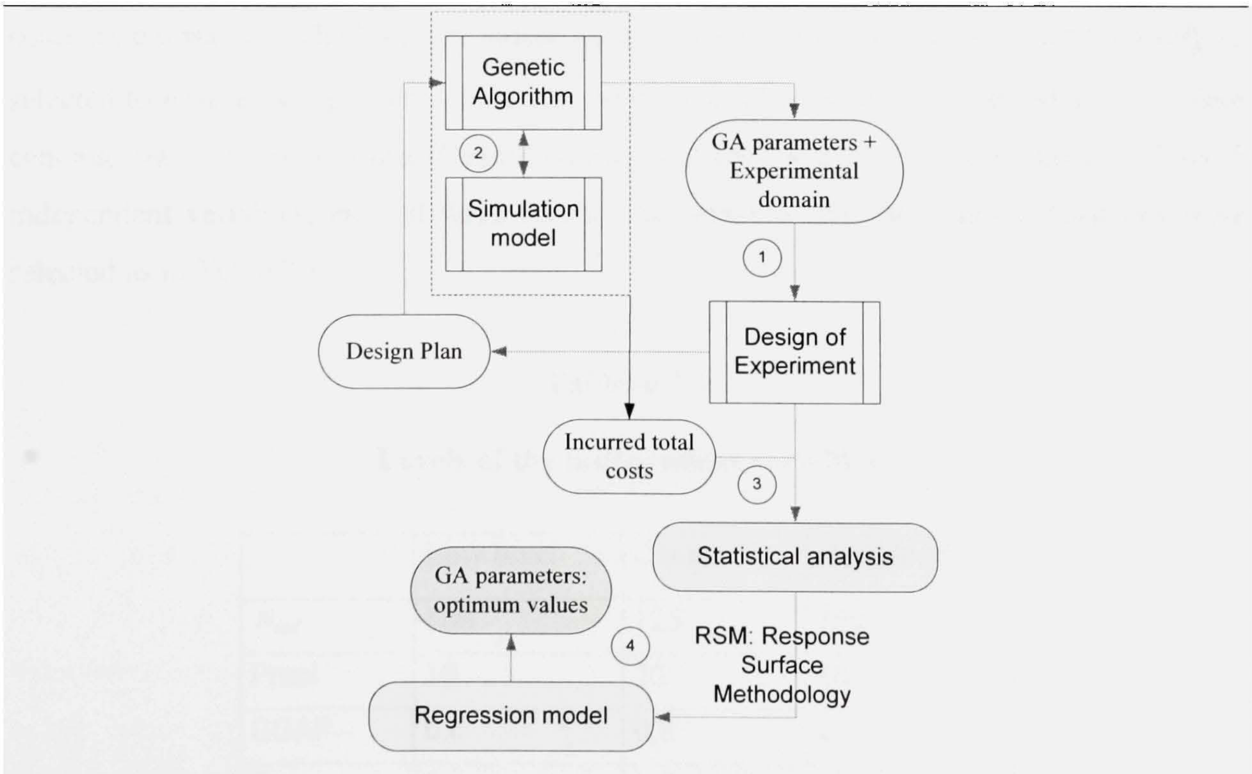


Figure 7.8 DOE & RSM optimization approach

1. The experimental design is concerned with (i) selecting a set of input variables (i.e., GA parameters); (ii) setting the levels of selected factors of the model and making decisions on the conditions, such as the length of runs and number of replications, under which the model will be run.
2. Send the experimental plan to the Optimization module to run all the design.
3. The experimental design and the obtained results are then analysed in order to determine the effects of the main factors and their interactions (i.e., analysis of variance or ANOVA) on the cost.
4. The *response surface methodology* is then used to obtain the relationship between the incurred cost and significant main factors and interactions given in the previous step. The obtained regression model is then optimized in order to determine the best values of the GA parameters.

Five independent variables (GA parameters) and one dependent variable (the total incurred cost) are considered. The levels of independent variables or design factors must be carefully selected to ensure they properly represent the domain of interest. We selected a 2^{5-1} + face centered star + 2 center points (Central composite response surface design) since we have 5 independent variables, each at three levels. The levels of the independent variables were selected as in Table 7.1.

Tableau 7.1

Levels of the independent variables

	Low level	Center	High level
N_{ind}	100	125	150
Preci	10	20	30
GGAP	0,6	0,8	1
Pc	0,6	0,8	1
MaxGen	100	125	150

Three replications were conducted for each combination of the factors, and therefore, the design was composed of 84 experiments.

Table 7.2 shows the optimum parameters obtained after running the simulations and analysing the results. We refer the reader to Gharbi et al. (2006) for more details on the different steps of the statistical analysis leading to the regression model. The optimization of this model in the experimental domain leads to the following optimal values.

Tableau 7.2

Optimal values of the genetic algorithm parameters

	Low level	High level	Optimum
N_{ind}	100	150	150
Preci	10	30	20
GGAP	0,6	1	0.75
Pc	0,6	1	0.77
MaxGen	100	150	150

It is interesting to note that the optimum of two parameters, namely N_{ind} and MaxGen are in the boundary of the experimental domain. Generally, if these kinds of results are obtained we have to review the experimental domain to insure that the optimum will be within this domain. However, this result was expected given that in a heuristic research algorithm these two parameters (i.e., the population size and the number of iterations) are generally selected high to guarantee a result closer to the optimum. Moreover, another experimental design with only three genetic algorithm parameters (i.e., Preci, GGAP and Pc) as design parameters was conducted and were we have fixed the two others to 150. The obtained results were very close to those presented in Table 7.2. Based on these facts, we have fixed the genetic algorithm parameters as shown in Table 7.2 to carry out the following case study.

7.7 Case study: three buffered machines, two parts flow-shop

In this section, the proposed approach is applied to three buffered machines two parts manufacturing system facing an unreliable supplier and a random supply delay. For the considered control problem four decisions have to be taken namely, the production rates of each machine, the changeover actions, the maintenance schedule of each machine and the replenishment strategy. These decisions are governed by the equations and the parameters given in section 7.2.1, 7.2.2 and 7.2.3.

To summarize, our objective is to find the best production, changeover and maintenance control policies parameters as well as the best replenishment strategy to minimize the total cost of inventory, backlog, setups, failures, preventive maintenance and ordering.

For the considered system the optimization problem include ***quantitative parameters:*** Z_k^i (governing the production policy for product i and machine k) ; a_i, b_i, c_i (governing the changeover actions of product i); T_{ARP}^k or T_{BRP}^k or (T_{ORP}^k and α^k) (governing the maintenance strategy of machine k) and s_R^i; s_F^i; Q^i (governing the replenishment policy of product i) and ***qualitative parameters*** ($\beta_k, k = 1,...,m$) denoting the selected maintenance strategy for machine k and equal to 1 for BRP, 2 for ARP and 3 for ORP.

Regarding the comparative study, the same case study but with dissociated controls is conducted. This means that we will consider a classic replenishment strategy depending only on the raw material inventory level and governed by two parameters for each product namely the order point and the order quantity (i.e., s_R^i; Q^i). Our objective is to study the cost profit that one can guarantee if the two problems (manufacturing and replenishment) are considered together.

The involved unit costs are detailed in the following list:

Inventory and setup costs:

K_{ij} Setup cost to switch from P_i to P_j

c_{i3}^{-} Product type *i* backlog cost, incurred on finished product (buffer *3*)

c_{ik}^{+} Product type *i* inventory cost incurred on buffer *k*, $1 \leq k \leq 3$

Corrective and preventive maintenance costs:

c_F Corrective maintenance cost after failure

c_{BRP} Bloc replacement policy cost

c_{ARP} Age replacement policy cost

c_{ORP} Opportunistic replacement policy cost

Replenishment cost:

K^i : Ordering cost of part type *i*.

c_R^i : Unit raw material cost of part type *i*.

c_{RH}^i : Unit raw material holding cost of part type *i*.

c_T^i : Unit of raw material transformation cost of part type *i*.

The following figure (i.e., Figure 7.9) illustrates the control mechanisms and the involved parameters. For the considered system we are concerned with two optimization problem. The first one under the replenishment strategy 1 (joint problem) involve 24 quantitative parameters and 3 qualitative parameters. The second one under the replenishment strategy 2 (dissociated problem) involve 22 quantitative parameters and 3 qualitative parameters.

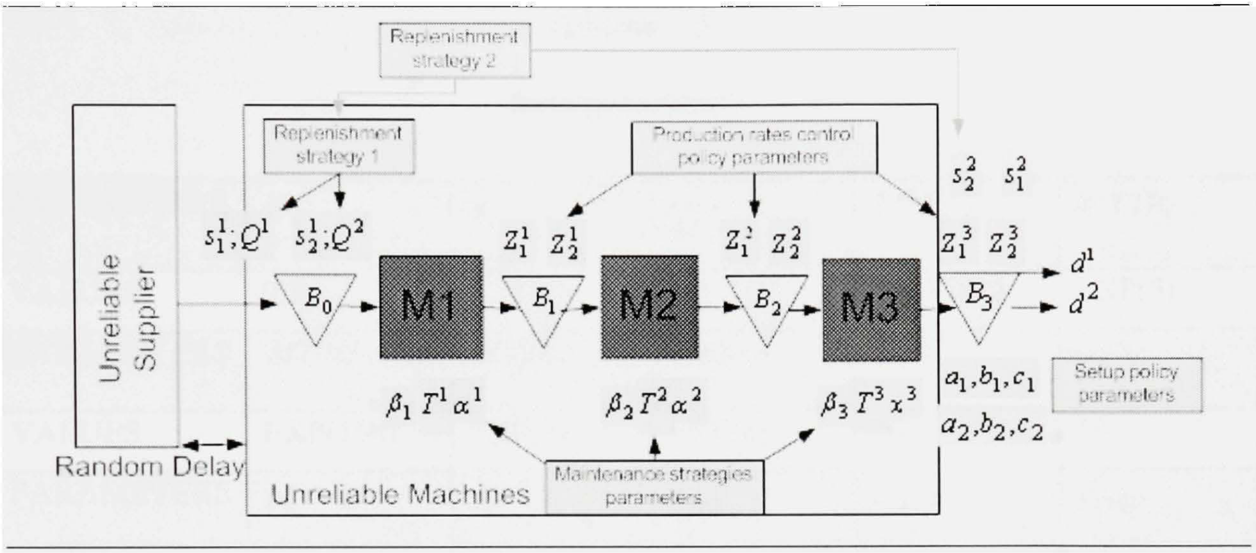


Figure 7.9 ***3*** **machines** ***2*** **parts flow-shop control policies parameters**

The system parameters governing the stochastic processes, the machines configuration and the clients are given as follows:

Θ_{ij} Setup duration to switch from P_i to P_j

d^i Demand rate for part type *i*,

$U_k^{\max i}$ Maximal production rate of part type *i* on machine *k*

$MTBF_k$ Mean time between failures of machine *k* (random)

$MTTR_k$ Mean time to repair of machine *k* (random)

MTBU Mean time between unavailability periods of the supplier (random)

MTTA Mean time for the supplier to become available (random)

DELAY Supply Delay (random)

MAINT maintenance duration (random)

7.7.1 Results analysis under replenishment strategy 1

The system parameters data and the unit costs used to run the optimization module and to characterize the optimal control policies are given in table 7.3.

Tableau 7.3

Data parameters

PARAMETERS	Θ_{ij}	d^i	$U^{\max i}_k$	$MTBF_k$	$MTTR_k$
VALUES	0.3	0.35	1.2	EXP(100)	EXP(5)
PARAMETERS	$MTBU$	$MTTA$	$DELAY$	K_{ij}	c_{i3}^-
VALUES	EXP(150)	EXP(2)	EXP(6)	20	10
PARAMETERS	c_{ik}^+	c_F	c_{BRP}	c_{ARP}	c_{ORP}
VALUES	1	100	50	50	50
PARAMETERS	K^i	c_R^i	c_{RH}^i	c_T^i	MAINT
VALUES	20	3	1	0.1	EXP(3)

The obtained results are given in table 7.4. It is interesting to note, given that we are facing a homogeneous flow-shop and identical parts type with respect to the incurred costs (see table 7.2), that following our expectation the policies parameters are the same for the three machine and the two parts. The raison behind that is to insure a full control of the simulation model and to have a reference point to future case studies (see sensitivity analysis). For this case study, three interesting observations are concluded.

1. Regarding the production policy the values of the hedging levels are increasing from one stage to another. This observation confirms partially the experimental observation of Lavoie (2006).

2. Regarding the changeover policy the values of c_i are infinite. This means that it does not form any more part of the policy which confirms the results of Hajji et al. (2004). In fact, this parameter is involved only in the case of different parts type.

3. In opposition to our expectation the best maintenance strategy is an age replacement policy. This result supports the fact that the age policy is considered by many studies as better than the bloc one. However, it didn't confirm our expectation stipulating that the opportunistic policy could benefit from the setup time to launch a maintenance strategy. As first explanation we think that the considered setup time plays an important role in this issue and there could be a switching time above which the opportunistic strategy will be considered. This issue is taken into account in the sensitivity analysis study.

Tableau 7.4

Control policies parameters

PRAMETERS	$\begin{pmatrix} Z_1^1, Z_1^1 \\ Z_2^1, Z_2^1 \\ Z_3^1, Z_3^1 \end{pmatrix}$	$\begin{pmatrix} a_1, b_1, c_1 \\ a_2, b_2, c_2 \end{pmatrix}$	$\begin{pmatrix} T^1, \alpha^1, \beta_1 \\ T^2, \alpha^2, \beta_2 \\ T^2, \alpha^2, \beta_2 \end{pmatrix}$	$\begin{pmatrix} s_R^1;\ s_F^1; Q^1 \\ s_R^2;\ s_F^2; Q^2 \end{pmatrix}$	AVAERAGE TOTAL COST
Values	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 6,0,-\infty \\ 6,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 5;\ 6;12 \\ 5;\ 6;12 \end{pmatrix}$	38.32

7.7.2 Sensitivity analysis under replenishment strategy 1

To illustrate the effect that some considered system parameters variation have on the control policies, a sensitivity analysis was conducted. Table 7.5 to Table 7.8 details the considered parameters variations, and presents the optimal parameters and the incurred optimal costs for the sensitivity analysis cases. Due to the number of the involved parameters we decided to limit our analysis to 4 parameters, namely the supply delay, the ordering cost, the setup time and the backlog cost. These parameters could be considered as illustrative given that they take into account the three stages of the whole system. Moreover, our objective is to insure the robustness of the approach and the proposed control policies. We claim that, at this point, this objective is reached and it will be reinforced with the comparative study.

Under the first sensitivity analysis (i.e., supply delay variation, table 7.5), it clearly appears that the obtained results make sense. In fact, when the supply delay increases the hedging levels and the order points increase. These variations exhort the system to keep higher security inventory levels for raw material, work in process and finished products to hedge against future shortage due to an increasing random delay.

Tableau 7.5

Sensitivity analysis results (DELAY)

PRAMETERS	*DELAY*	$\begin{pmatrix} Z_1^1, Z_1^1 \\ Z_2^1, Z_2^1 \\ Z_3^1, Z_3^1 \end{pmatrix}$	$\begin{pmatrix} a_1, b_1, c_1 \\ a_2, b_2, c_2 \end{pmatrix}$	$\begin{pmatrix} T^1, \alpha^1, \beta_1 \\ T^2, \alpha^2, \beta_2 \\ T^2, \alpha^2, \beta_2 \end{pmatrix}$	$\begin{pmatrix} s_R^1; s_F^1; Q^1 \\ s_R^2; s_F^2; Q^2 \end{pmatrix}$	COST
I	EXP(3)	$\begin{pmatrix} 6,6 \\ 8,8 \\ 14,14 \end{pmatrix}$	$\begin{pmatrix} 4,0,-\infty \\ 4,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 3;4;12 \\ 3;4;12 \end{pmatrix}$	36.12
II	EXP(6)	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 6,0,-\infty \\ 6,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 5; 6;12 \\ 5; 6;12 \end{pmatrix}$	38.32
III	EXP(10)	$\begin{pmatrix} 8,8 \\ 10,10 \\ 16,16 \end{pmatrix}$	$\begin{pmatrix} 8,0,-\infty \\ 8,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 7;8;12 \\ 7;8;12 \end{pmatrix}$	42.53

Under the second sensitivity analysis (i.e., ordering cost variation, table 7.6), the obtained results are also making sense. However, the only effect was on the ordering quantity. In fact, when the ordering cost increases the transformation system should order a higher economic raw material quantity which is expected and makes sense.

Under the third sensitivity analysis (i.e., backlog cost variation, table 7.7), the same conclusions as in the first analysis are observed. In fact, when the backlog cost increases the hedging levels and the order points increase to hedge against future shortage and a higher backlog costs.

Tableau 7.6

Sensitivity analysis results (Ordering cost)

PRAMETERS	K^i	$\begin{pmatrix} Z_1^1, Z_1^1 \\ Z_2^1, Z_2^1 \\ Z_3^1, Z_3^1 \end{pmatrix}$	$\begin{pmatrix} a_1, b_1, c_1 \\ a_2, b_2, c_2 \end{pmatrix}$	$\begin{pmatrix} T^1, \alpha^1, \beta_1 \\ T^2, \alpha^2, \beta_2 \\ T^2, \alpha^2, \beta_2 \end{pmatrix}$	$\begin{pmatrix} s_R^1; s_F^1; Q^1 \\ s_R^2; s_F^2; Q^2 \end{pmatrix}$	COST
I	15	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 6,0,-\infty \\ 6,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-1 \\ 115,-1 \\ 115,-1 \end{pmatrix}$	$\begin{pmatrix} 5;6;10 \\ 5;6;10 \end{pmatrix}$	37.7
II	20	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 6,0,-\infty \\ 6,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-1 \\ 115,-1 \\ 115,-1 \end{pmatrix}$	$\begin{pmatrix} 5; 6;12 \\ 5; 6;12 \end{pmatrix}$	38.32
III	25	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 6,0,-\infty \\ 6,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-1 \\ 115,-1 \\ 115,-1 \end{pmatrix}$	$\begin{pmatrix} 5;6;14 \\ 5;6;14 \end{pmatrix}$	39.43

The fourth sensitivity analysis illustrating the setup time variation and shown in table 7.7 confirm our aforementioned expectation regarding the connection between the setup time and the opportunistic maintenance strategy. In fact, with a higher setup time (raw 3 of table 7.7) the resulting best preventive maintenance strategy is an opportunistic one. The optimal parameters show that one has to conduct an opportunistic replacement when 60% of the scheduled time is spent and the system is starting a changeover action. Moreover, when the setup time increases the changeover policy parameters a_i are higher. This result makes sense since a_i define the security level of the part type being produced that one has to keep before performing a setup action.

Tableau 7.7

Sensitivity analysis results (backlog cost)

PRAMETERS	c_{i3}^{-}	$\begin{pmatrix} Z_1^1, Z_1^1 \\ Z_2^1, Z_2^1 \\ Z_3^1, Z_3^1 \end{pmatrix}$	$\begin{pmatrix} a_1, b_1, c_1 \\ a_2, b_2, c_2 \end{pmatrix}$	$\begin{pmatrix} T^1, \alpha^1, \beta_1 \\ T^2, \alpha^2, \beta_2 \\ T^2, \alpha^2, \beta_2 \end{pmatrix}$	$\begin{pmatrix} s_R^1; s_F^1; Q^1 \\ s_R^2; s_F^2; Q^2 \end{pmatrix}$	COST
I	7	$\begin{pmatrix} 6,6 \\ 8,8 \\ 14,14 \end{pmatrix}$	$\begin{pmatrix} 4,0,-\infty \\ 4,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 3;4;11 \\ 3;4;11 \end{pmatrix}$	35.82
II	10	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 6,0,-\infty \\ 6,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 5; 6;12 \\ 5; 6;12 \end{pmatrix}$	38.32
III	13	$\begin{pmatrix} 9,9 \\ 11,11 \\ 17,17 \end{pmatrix}$	$\begin{pmatrix} 8,0,-\infty \\ 8,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 7;8;14 \\ 7;8;14 \end{pmatrix}$	42.83

Tableau 7.8

Sensitivity analysis results (setup time)

PRAMETERS	Θ_{ij}	$\begin{pmatrix} Z_1^1, Z_1^1 \\ Z_2^1, Z_2^1 \\ Z_3^1, Z_3^1 \end{pmatrix}$	$\begin{pmatrix} a_1, b_1, c_1 \\ a_2, b_2, c_2 \end{pmatrix}$	$\begin{pmatrix} T^1, \alpha^1, \beta_1 \\ T^2, \alpha^2, \beta_2 \\ T^2, \alpha^2, \beta_2 \end{pmatrix}$	$\begin{pmatrix} s_R^1; s_F^1; Q^1 \\ s_R^2; s_F^2; Q^2 \end{pmatrix}$	COST
I	0.1	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 2,0,-\infty \\ 2,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 5;\ 6;12 \\ 5;\ 6;12 \end{pmatrix}$	38.12
II	0.3	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 6,0,-\infty \\ 6,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 5;\ 6;12 \\ 5;\ 6;12 \end{pmatrix}$	38.32
III	1.5	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 9,0,-\infty \\ 9,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,0.6,3 \\ 115,0.6,3 \\ 115,0.6,3 \end{pmatrix}$	$\begin{pmatrix} 5;\ 6;12 \\ 5;\ 6;12 \end{pmatrix}$	38.83

7.7.3 Comparative study

In this section a comparative study involving the two aforementioned replenishment strategies (see introduction of section 7.7) is conducted. The first strategy (***JS***) consists on replenishment actions taking into account the whole system where the feedback information depends on the levels of raw materials and finished product. The results under this strategy were presented in section 7.7.1 and 7.7.2. The second strategy (***DS***) consists on replenishment actions depending only on raw materials inventory levels. The aim of this study is to confirm the robustness of the approach and at the same time the results of the numerical results of Hajji et al (2007) where the joint production and replenishment problem have led to the first strategy. It is important to note that the results under the second strategy were obtained under the same conditions (simulation and genetic algorithm), and following the same approach under which the sensitivity analysis was conducted for the first strategy (table 7.4 to 7.8).

Table 7.9 shows the base case (table 7.4) optimal control policies parameters under ***JS*** and ***DS***. It is interesting to note that the average total cost under strategy 1 (***JS***) is lower to up 11 % than that under the strategy 2 (***DS***).

Note that the same sensitivity analysis conducted under the ***JS*** was made under ***DS***. The results obtained have shown that the variation of the policies parameters does make sense. However, the incurred costs for all the cases are higher than those incurred under the first strategy (as shown in table 7.9 for the base case). The improvement of the cost lies between 6 to 11 %.

Tableau 7.9

Control policy parameters

PRAMETERS	$\begin{pmatrix} Z_1^1, Z_1^1 \\ Z_2^1, Z_2^1 \\ Z_3^1, Z_3^1 \end{pmatrix}$	$\begin{pmatrix} a_1, b_1, c_1 \\ a_2, b_2, c_2 \end{pmatrix}$	$\begin{pmatrix} T^1, \alpha^1, \beta_1 \\ T^2, \alpha^2, \beta_2 \\ T^2, \alpha^2, \beta_2 \end{pmatrix}$	$\begin{pmatrix} s_R^1; s_F^1; Q^1 \\ s_R^2; s_F^2; Q^2 \end{pmatrix}$ $\begin{pmatrix} s_R^1; Q^1 \\ s_R^2; Q^2 \end{pmatrix}$	AVAERAGE TOTAL COST
Values under strategy 1	$\begin{pmatrix} 7,7 \\ 9,9 \\ 15,15 \end{pmatrix}$	$\begin{pmatrix} 6,0,-\infty \\ 6,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 115,-,1 \\ 115,-,1 \\ 115,-,1 \end{pmatrix}$	$\begin{pmatrix} 5;\ 6;12 \\ 5;\ 6;12 \end{pmatrix}$	38.32
Values under strategy 2	$\begin{pmatrix} 7,7 \\ 9,9 \\ 17,17 \end{pmatrix}$	$\begin{pmatrix} 6,0,-\infty \\ 6,0,-\infty \end{pmatrix}$	$\begin{pmatrix} 105,-,1 \\ 105,-,1 \\ 105,-,1 \end{pmatrix}$	$\begin{pmatrix} 7;13 \\ 7;13 \end{pmatrix}$	43.03

To confirm these observations and hence the advantage of the proposed joint control strategies compared to that of the dissociated control strategies, a student test was performed in order to compare the performance of the two policies. The confidence interval of $c^*_{DS} - c^*_{JS}$ is given by (7.6).

$$\begin{aligned} &\overline{C}^*_{DS} - \overline{C}^*_{JS} - t_{\alpha/2,n-1} s.e(\overline{C}^*_{DS} - \overline{C}^*_{JS}) \\ &\quad \le \overline{C}^*_{DS} - \overline{C}^*_{JS} \le \\ &\overline{C}^*_{DS} - \overline{C}^*_{JS} + t_{\alpha/2,n-1} s.e(\overline{C}^*_{DS} - \overline{C}^*_{JS}) \end{aligned} \tag{7.6}$$

where:

$t_{\alpha/2,n-1}$ is the student coefficient function of n and α, with n the number of replications (set at 10) and (1-α), the confidence level (set at 95%).

$s.e(\overline{C}^*_{DS} - \overline{C}^*_{JS}) = \frac{S_D}{\sqrt{n}}$ Standard error, $S_D^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}\left(C^*_{DSi} - C^*_{JSi}\right)^2 - n\left(\overline{C}^*_{DS} - \overline{C}^*_{JS}\right)^2\right)$

$\overline{C}^*_{DS}$ the average optimal cost incurred under strategy 2.

$\overline{C}^*_{JS}$ the average optimal cost incurred under strategy 1.

It has been shown that in all cases, it can be concluded that C^*_{DS}-C^*_{JS}>0 at the 95% confidence level. Consequently, the first strategy gives the lower optimal cost, and furthermore, it appears that the ***JS*** is better than the ***DS***, and can be used to better approximate the optimal control policy.

7.8 Conclusion

In this paper we studied the control problem of a flow-shop manufacturing system in a supply chain environment. Our objective was to determine the manufacturing activities planning together with the raw material replenishment strategy in order to minimize the total incurred cost. Following two of our previous works, the contribution of this paper consists on developing an optimization module, based on stochastic optimal control theory, discrete/continuous event simulation, genetic algorithm and experimental design, making it possible to find in a stochastic dynamic manner the best control parameters of the production, replenishment and setup actions simultaneously with the best maintenance scheduling between bloc, age or opportunistic strategies. Two interesting results are observed; regarding the best preventive maintenance strategy it was shown that it is more profitable to consider the age and the opportunistic ones and to adopt the best one in a given context. In this context it has been shown that the opportunistic strategy is in connection with the setup time and is better in flow-shop system cases incurring high changeover time. Regarding the supply chain environment, it has been shown that it is more profitable to consider in integrated manner the manufacturing and supply control problems. In fact, we found that the total incurred cost can be reduced up to 11 % under the joint replenishment strategy ***JS***.

As it may interest the reader to know, the same approach is being applied to more complex system of 10 machines flow-shop producing 10 parts type and facing more than one supplier. In this case, another decision should be taken and consists on the selection of the best supplier when the decision to place an order is taken.

CONCLUSION

Motivé par l'importance stratégique d'une gestion intégrée des activités manufacturières dans un environnement de chaîne d'approvisionnement, cette thèse a pour objectif d'amener une approche pragmatique pouvant surmonter la complexité de modélisation et de résolution dans un contexte dynamique stochastique.

Concernant les systèmes de production étudiés, notre choix a été motivé par des futures applications pratiques et aussi par le souci d'amener des contributions à des sujets d'actualité dans la littérature. À cet égard, la présence abondante des lignes de production multi produits avec stocks tampons dans plusieurs secteurs industriels et l'intérêt croissant aux problématiques liées aux chaînes d'approvisionnement ont nourri le besoin d'amener une contribution regroupant ces deux axes de recherche.

À un niveau opérationnel de décision, nous avons considéré trois activités manufacturières fondamentales soient la production, la mise en course et la maintenance préventive. La prise en considération de l'environnement externe a été aussi orientée par des activités stratégiquement très importantes à savoir l'approvisionnement et la collaboration avec les fournisseurs potentiels. Le souci de se mettre dans un contexte réaliste exige une attention particulière aux aspects dynamiques et stochastiques. À cet égard, la nature des systèmes étudiés nous a conduit à faire face à une dynamique continue et / ou discrète et à considérer au moins un phénomène aléatoire à chaque étape du cheminement des produits. Dans ce contexte, c'est la nature aléatoire de la disponibilité des fournisseurs, du délai d'approvisionnement ainsi que la fiabilité du processus de transformation qui ont été pris en considération.

Conscient de l'impossibilité d'amener des solutions exactes à des problèmes héritant leur complexité d'au moins un des aspects suivants : dynamique, stochastique, structure, taille; le recours à une approche séquentielle de résolution basée sur une combinaison de plusieurs approches de modélisation s'est avéré indispensable.

La première étape de l'approche proposée est basée sur la modélisation dynamique stochastique et la résolution numérique. Bien que le recours à l'approche mathématique présente des défis majeurs de modélisation et de résolution, nous avons jugés nécessaires d'avoir des bases solides permettant de caractériser les politiques de gestion des systèmes étudiés. Cette caractérisation servira de base pour proposer des heuristiques de gestion de système plus large.

La deuxième étape de l'approche est basée sur les méthodes d'optimisation basée sur la simulation. À ce sujet, la nature des problèmes sous études et le nombre de paramètres à optimiser ont imposé le recours à deux approches différentes. Les plans d'expériences et la méthodologie des surfaces de réponse (DOE & RSM), statistiquement plus robuste et pouvant proposer un modèle de régression de la mesure de performance à optimiser fonction des paramètres impliqués. Cette méthode s'est avérée puissante lorsque le domaine expérimental est quantitatif et que le nombre de paramètres impliqués n'est pas très élevé. Quant à la deuxième approche, elle est basée sur les algorithmes génétiques plus flexibles face à un nombre élevé de paramètres à optimiser; et pouvant inclure dans l'espace de recherche des paramètres de nature qualitative. De plus, le recours au DOE & RSM, pour optimiser les paramètres de l'algorithme génétique adopté, a constitué une solution aux pratiques actuelles de choix, basées sur l'expérience du décideur.

À la fin de cette thèse nous pouvons considérer que le mandat a été pleinement rempli et que les objectifs fixés ont été atteints. Plusieurs contributions ont vu le jour tout au long de ces années de travail et qui peuvent être classées en trois catégories :

1. Modélisation mathématique : à l'exception du premier système étudié (i.e., système parallèle produisant plusieurs types de produit étudié au chapitre 2), la modélisation des activités manufacturières des systèmes série (i.e., chapitre 4) ainsi que la modélisation des activités d'approvisionnement en présence de plusieurs fournisseurs (i.e., chapitre 5 et 6) constituent des contributions à la littérature. À cet égard, le recours aux théories de

commande optimale et la commande impulsionnelle nous ont permis de surmonter toutes les difficultés de modélisation et de proposer des formulations robustes pouvant amener des solutions aux problèmes d'optimisation considérés.

2. Approche : comme mentionné tout au long de cette thèse, le souci d'amener des solutions réalistes nous a poussé à ne pas nous limiter à une approche purement analytique. Dans ce contexte, la combinaison de la modélisation mathématique, la résolution numérique, la simulation et les approches d'optimisations expérimentales (i.e., DOE & RSM & AG) nous a permis de surmonter les difficultés de résolution menant aux processus décisionnels recherchés. De plus, le recours au DOE & RSM pour optimiser les paramètres de l'algorithme génétique adopté s'est avéré une application originale rarement employé dans la littérature. Le chapitre 4 nous a permis de voir clairement les forces d'une telle approche quand il s'agit de la généralisation des politiques issues de la résolution de système pas trop large. Le chapitre 5 nous a permis d'apprécier la flexibilité de cette approche quand il s'agit de développer des processus décisionnels de gestion et surtout d'avoir une base solide de négociation de coût par exemple. Le chapitre 7 nous a permis de combiner plusieurs aspects, d'inclure des activités connexes de production et de proposer dans un environnement intégré des politiques de gestion améliorées.

3. Applications et nouvelles politiques de gestion : les bénéfices des deux points précédents ont été ressentis à toutes les étapes de cette thèse. Les politiques de gestion proposées ont affiché des améliorations nettes de coût. Cet aspect peut être considéré comme une importante contribution à la littérature et constitue un levier pouvant supporter de futures applications réelles.

PERSPECTIVES DE RECHERCHE

À l'issue de cette thèse, le lecteur va certainement nous partager l'avis que l'approche proposée et les contributions réalisées constituent une base solide à de futures applications et extensions. Ces perspectives peuvent être classées selon quatre catégories :

1. structure et taille des systèmes de production : étendre la résolution à des systèmes plus complexes de point de vue structure et taille constitue une importante piste de recherche à emprunter. Nous pensons que la recherche de partenaires industriels conscients des profits qu'ils peuvent en tirer est primordiale afin de travailler directement sur des cas réels. L'approche d'optimisation basée sur la simulation constitue un atout considérable. Cependant, la résolution numérique des conditions d'optimum issues de la modélisation mathématique doit être améliorée. Le recours à des algorithmes de résolution optimisés et au calcul parallèle pourront constituer des alternatives à considérer.

2. décisions et événements : la prise en considération d'une pratique fondamentale dans le processus de transformation à savoir le contrôle de la qualité est incontournable. Que ce soit au niveau matière première, des encours ou des produits finis, le contrôle de la qualité n'a cessé de prendre de l'ampleur. À ce sujet, tout en considérant l'environnement dynamique stochastique, quelques travaux ont commencé à voir le jour (Gershwin (2006)). Cet aspect constitue aussi une avenue de recherche.

3. mesures de performance : choisir les mesures appropriées afin de quantifier et qualifier les performances d'une chaîne d'approvisionnement est un processus complexe (Beamon (1999)). Cette difficulté est directement liée à la complexité de ces systèmes. Les mesures de performance des chaînes d'approvisionnement peuvent être classées en deux catégories soit : des mesures quantitatives et d'autres qualitatives. Dans son article « Measuring supply chain performance », Beamon a fait une revue intéressante sur les pratiques existantes dans la littérature et qui

consistent en grande majorité au recours à une seule mesure de performance. Il a souligné que ces pratiques sont incomplètes du moment où elles ignorent beaucoup d'aspects critiques des objectifs organisationnels stratégiques de la chaîne. De ce fait, la prise en considération de toutes les caractéristiques et les objectifs d'une chaîne d'approvisionnement exige la prise en compte d'au moins une mesure des trois types séparés de mesures de performance soient : des mesures liées aux ressources afin d'assurer un niveau élevé d'efficacité, d'autres liées au rendement afin de garantir un niveau de service élevé et enfin des mesures liées à la flexibilité afin de garantir une habilité de réponse à la nature aléatoire de l'environnement. Dans le cadre de cette thèse, c'est la mesure coût qui a été employée. La prise en considération d'autres mesure de performance afin d'intégrer les aspects sus indiqués constitue un défis et une piste de recherche.

4. nouveau concept de contrôle : le désir d'une maîtrise optimale des activités manufacturières a été à l'origine des toutes les approches et les méthodes de modélisation et de résolution. La grande majorité de ces pratiques (i.e., mathématiques ou heuristiques) ne laissent pas à l'expérience humaine sa place méritée au début du processus. Pourtant, si on retourne quelques années en arrière, nous pouvons nous rendre compte que la philosophie du Juste à Temps (JÀT) par exemple a été le fruit d'une pratique basée sur l'expérience humaine à la base. En parlant justement du JÀT, de Toyota et du Japon, un séjour de recherche au sein d'un groupe de recherche de l'université Gifu nous a permis de penser à jumeler deux approches afin de développer un nouveau concept de contrôle. Les pratiques japonaises suivent en grande majorité la célèbre expression «la nécessité est la mère de l'innovation» de Taiichi Ohno. Ces pratiques donnent une grande importance à l'expérience humaine afin de proposer des stratégies de gestion. Elles peuvent être intégrées à un niveau, à définir, de notre approche pour surmonter plusieurs difficultés liées à la modélisation et / ou la généralisation des stratégies de gestion.

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