Stratégies manufacturières pour des chaînes d'approvisionnement non fiables intégrant la production, la livraison et le transfert de stock

par

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RÉSUMÉ

Ce mémoire aborde des problèmes de commande optimale stochastique pour des chaînes d'approvisionnement à deux échelons intégrant la livraison des produits finis et le transfert de stock entre localisations. Ces chaînes font face à des évènements aléatoires tels que les pannes et réparations aléatoires. Une approche de programmation dynamique stochastique combinée avec la simulation discrète-continue est utilisée pour modéliser la dynamique complexe du système. Cette approche permet de déterminer les structures des politiques optimales ainsi que les valeurs optimales des paramètres caractérisant ces politiques.

Dans la première partie de ce mémoire, nous étudions le cas d'une chaîne d'approvisionnement composée d'un manufacturier non fiable livrant des produits finis à plusieurs détaillants. Nous développons une politique de commande rétroactive intégrée de production et de livraisons. Nous proposons également une amélioration de cette politique en intégrant l'aspect de priorisation des détaillants dans la politique de livraisons. Les résultats montrent que les politiques proposées assurent une meilleure performance comparée aux politiques adaptées de la littérature. La politique intégrant la priorisation étant la meilleure en termes de coût.

Dans une deuxième partie, nous nous intéressons au cas d'une chaîne d'approvisionnement composée de deux manufacturiers non fiables localisés dans deux régions et qui répondent à différents taux de demandes et qui permettent de transférer le stock entre eux quand c'est nécessaire. L'objectif est de déterminer une politique de commande optimale qui intègre les décisions de production et de transfert de stock entre les localisations. Les résultats montrent que la politique intégrée proposée a une meilleure performance que des politiques adaptées de la littérature incluant le cas où il n'y a pas de transfert de stock entre les détaillants.

Pour résumer, les politiques proposées montrent l'importance de la coordination entre les activités de production, de livraison et de transfert de stock dans des chaînes d'approvisionnement à deux échelons. Elles aident les décideurs à prendre des décisions concernant ces activités de manière simultanée en assurant un coût minimal et une meilleure performance de la chaîne d'approvisionnement.

Mots-clés : commande optimale stochastique, simulation, système de production non fiable, livraison, transfert de stock.

Manufacturing strategies for unreliable supply chains integrating production, delivery and transshipment

Akrem DHAHRI

ABSTRACT

This study addresses the stochastic optimal control problem for two-echelon supply chains that integrate the delivery of finished goods and the transshipment of inventory between locations. These supply chains face random events such as random breakdowns and repairs of the machines. A stochastic dynamic programming approach combined with discrete-continuous simulation is used to model the complex dynamics of the system. This approach makes it possible to determine the structures of the optimal control policies as well as the optimal values of the parameters characterizing these policies.

In the first part of this thesis, we study the case of a supply chain made up of an unreliable manufacturer delivering finished products to several retailers. We develop an integrated production and delivery feedback policy. We also propose an improvement of this policy by integrating the aspect of prioritization of retailers in the delivery policy. The results show that the proposed policies provide better performance compared to policies adapted from the literature. The policy integrating prioritization being the best in terms of cost.

In a second part, we study the case of a supply chain composed of two unreliable manufacturers located in two regions and which face different demand rates, and which allow the transshipment of stock between them. The goal is to determine an optimal control policy that integrates production and transshipment decisions between locations. The results show that the proposed integrated policy outperforms the policies adapted from the literature including the case of no transshipment between retailers.

In summary, the proposed policies demonstrate the importance of coordination between production, delivery and transshipment activities in two-echelon supply chains. They help decision-makers to simultaneously take decisions concerning these activities, ensuring a minimal cost and better supply chain performance.

Keywords: stochastic optimal control, simulation, unreliable manufacturing system, delivery, transshipment.

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$\boldsymbol{x_m}$ *(t)*	Le niveau d'inventaire chez le manufacturier
$\boldsymbol{x_i}$ *(t)*	Le niveau d'inventaire chez le détaillant i
$\boldsymbol{y_i}$ *(t)*	La position d'inventaire chez le détaillant i
y (t)	La position d'inventaire de tout le système
u(t)	Taux de production à l'instant t (produit/unité de temps)
$\boldsymbol{u_m}$	Taux de production maximal (produit/unité de temps)
$\boldsymbol{u_i^m}$	Taux de production maximal du site de production i (produit/unité de temps)
$\boldsymbol{d_i}$	Taux de demande de détaillant i (produit /unité de temps)
$\boldsymbol{Q_i}$	Taille du lot du détaillant i (produits)
$\boldsymbol{Q_{ij}}$	Taille du lot transféré du détaillant i au détaillant j (produits)
$\boldsymbol{\delta_i^j}$	Temps de début de livraison du lot j jusqu'au détaillant i
$\boldsymbol{Q_i^j}$	Taille du lot j livre à l'instant δ_{ij} au détaillant i
$\boldsymbol{Q^{max}}$	Taille du lot maximale
$\boldsymbol{Tr_i}$ *(t)*	Lots en transportation vers le détaillant i à l'instant t
$\boldsymbol{Tr_{ij}}$ *(t)*	Lots en transfert du détaillant i vers le détaillant j à l'instant t
$\boldsymbol{q_{\alpha\beta}}$	Taux de transition du mode α au mode β
MTTF	Temps moyen entre les pannes
MTTR	Temps moyen de réparation
$\boldsymbol{\tau_i}$	Délai de transport du manufacturier vers le détaillant i (unite de temps)
$\boldsymbol{\tau_{ij}}$	Délai de transport du détaillant i vers le détaillant j (unite de temps)
$\boldsymbol{c^+}$	Coût de possession de stock ($/temps/ produit)

$\boldsymbol{c^{-}}$	Coût de pénurie ($/temps/ produit)
$\boldsymbol{c_i^t}$	Coût fixe de transport vers le détaillant i ($/livraison)
$\boldsymbol{c_{ij}^{tr}}$	Coût fixe de transfert de stock du détaillant i vers le détaillant j ($/transfert)
$\boldsymbol{c_v}$	Coût variable de transport ($/produit)
$\boldsymbol{c^v}$	Coût variable de transfert de stock du détaillant i vers le détaillant j ($/produit)
$\boldsymbol{\rho}$	Coût actualisé
HJB	Hamilton-Jacobi-Bellman, équation
HPP	Politique à seuil critique

INTRODUCTION

De nos jours, les entreprises manufacturières font face à une concurrence accrue. La concurrence ne repose pas seulement sur une meilleure conception des produits, l'ingéniosité des stratégies de marketing ou des capacités financières. Mais celle-ci repose aussi sur la capacité de bien gérer les chaînes d’approvisionnement qui sont par nature instables. En effet, un rôle clé de la gestion de la chaîne d’approvisionnement est de minimiser les risques et l'incertitude associés à cette instabilité (Lee, 2002). Une des raisons importantes de cette instabilité est la fiabilité des systèmes manufacturiers qui peut être affectée par des évènements aléatoires caractérisant le système. Parmi les problèmes majeurs rencontrés par les manufacturiers, ce sont les pannes et les réparations aléatoires de ces machines qui peuvent mener à l’incapacité du système à répondre à la demande et peuvent générer des coûts de pénurie très élevés.

Pour atténuer les risques de pénuries possibles, des politiques de commande stochastique ont été proposées par de nombreux chercheurs pour différents types de configurations de systèmes manufacturiers non fiables. Ces politiques sont démontrées efficaces pour les systèmes de production non fiables. Dans ce contexte, beaucoup de travaux ont traité l’intégration de la production avec d’autres échelons de la chaîne d’approvisionnement. Cependant, dans la plupart des cas, ils considèrent l’intégration des fournisseurs et proposent des politiques d’approvisionnement de la matière première. Peu de travaux ont considéré l’échelon en aval de la production qui implique d’autres processus indispensables pour le bon fonctionnement de la chaîne d’approvisionnement. La livraison des produits finis aux détaillants et le transfert de stock entre les détaillants sont des processus existants dans plusieurs chaînes d’approvisionnement, mais qui n’étaient pas précédemment intégrés avec des politiques de contrôle de taux de production dans un contexte de systèmes manufacturiers non fiables.

En se basant sur cette constatation, l’objectif de ce mémoire est de développer des politiques de commande stochastique pour deux chaînes d’approvisionnement, l’une intégrant des activités de livraisons et l'autre intégrant des activités de transfert de stock. Plus précisément, nous proposerons des politiques qui répondent à ces questions suivantes :

Dans une chaîne d'approvisionnement composée d'un manufacturier et plusieurs détaillants, à quel taux de production le manufacturier doit-il produire à chaque instant? Quand doit-il faire la livraison ? À quel détaillant faudrait-il livrer ? Quelle quantité faudrait-il livrer?

Dans une chaîne d'approvisionnement composée de deux manufacturiers localisés dans des régions différentes, à quel taux de production chaque manufacturier doit-il produire à chaque instant? Quand le transfert de stock doit-il se faire entre les détaillants? Quelle quantité doit-elle être transférée d'un détaillant à un autre ?

En reconnaissant l'importance de l'intégration de la production avec les activités de livraison et du transfert de stock, ces politiques peuvent contribuer de façon importante à réduire les coûts de la chaîne d'approvisionnement (incluant les coûts d'inventaire, de pénurie, de transport et de transfert de stock) tout en assurant une grande efficacité et une meilleure satisfaction du client.

Ce mémoire comprend trois chapitres. Dans le premier, nous présenterons la revue de la littérature, ainsi qu'une critique de la littérature. La problématique et les objectifs de notre recherche sont également introduits dans ce chapitre. Le deuxième chapitre présente un premier article scientifique intitulé « *Integrated production-delivery control policy for an unreliable manufacturing system and multiple retailers* » soumis à « *International Journal of Production Economics* ». Le dernier chapitre est également sous forme d'un article intitulé « *Integrated production-transshipment control policy for a two-location unreliable manufacturing system* » soumis à « *International Journal of Production Economics* ».

CHAPITRE 1

REVUE DE LA LITTÉRATURE

1.1 Introduction

Dans ce premier chapitre, nous introduirons les types de chaîne d'approvisionnement qui seront considérés dans notre étude, ainsi que la définition des termes pertinents à notre étude. Nous présenterons, par la suite, les principaux travaux reliés à notre recherche ainsi qu'une critique de la littérature. Dans une deuxième partie, nous définirons la problématique de recherche, les objectifs de la recherche, et nous finirons par présenter la méthodologie de recherche adoptée.

1.2 Configuration d'une chaîne d'approvisionnement et terminologie

Avant de présenter la problématique de recherche, nous définirons dans cette section la terminologie utilisée pour bien comprendre la structure des chaînes d'approvisionnements étudiées.

1.2.1 Terminologie

1.2.1.1 Chaîne d'approvisionnement

La chaîne d'approvisionnement est un réseau d'acteurs (fournisseurs, manufacturiers détaillants; etc.) impliqués dans les activités de transformation d'une matière première en un produit fini délivré à un consommateur final (Chandra et Grabis, 2007).

Une chaîne d'approvisionnement peut être incertaine, s'il y a un phénomène aléatoire qui caractérise un de ses processus. Par exemple, les temps de pannes et de réparations de machines, les délais de livraison, la demande, etc. (Min et Zhou, 2002).

1.2.1.2 Gestion de la chaîne d’approvisionnement

La gestion de la chaîne d’approvisionnement implique les problèmes managériaux et techniques multiples qui doivent être résolus pour que la chaîne d’approvisionnement fonctionne de façon efficace et efficiente (Soni et Kodali, 2013). Les problèmes principaux qui font partie de la gestion de la chaîne d’approvisionnement sont: la compétitivité, le service client, la coordination, la collaboration, la protection d’environnement, la flexibilité, la globalisation, l'intégration, l'externalisation, etc. (Chandra et Grabis, 2007).

1.2.1.3 Intégration de la chaîne d’approvisionnement

L’intégration de la chaîne d’approvisionnement consiste à relier les principales fonctions et les principaux processus qui existent au sein et aussi entre les différents acteurs. Cela permet d'atteindre une meilleure performance de la chaîne d’approvisionnement (CSCMP Glossary of Terms, 2009). L’intégration peut être atteinte à plusieurs niveaux, comme l’intégration des clients, des fournisseurs, de la technologie, de la planification, etc.

1.2.1.4 Système de production

Un système de production peut être défini comme l’ensemble d’êtres humains, de machines et d’équipements liés par un flux commun de matières et d’informations. Les inputs du système de production sont les matières premières et l'énergie. Les informations sont également entrées comme input sous la forme de la demande des clients pour les produits finaux. Les outputs d'un système de production peuvent également être classifiés en matériaux, tels que les produits finis et les déchets, et des informations, telles que des mesures de la performance du système (Chryssolouris, 2006).

1.2.1.5 La logistique de transport et le transfert de stock entre détaillants

Le transport est un processus important de la chaîne d’approvisionnement qui consiste à transporter les produits d’une localisation à une autre.

Le transfert d’inventaire entre les détaillants (en anglais ‘transshipment’) est une politique de collaboration utilisée par les compagnies qui ont des usines et des entrepôts dans différentes localisations. Elle permet de transférer des produits entre les localisations, comme un mécanisme efficace pour corriger les écarts entre les inventaires disponibles et pour éviter des pénuries possibles pour certains détaillants (Herer et al., 2006).

1.2.2 Chaînes d’approvisionnement étudiées

Dans ce projet de recherche, nous étudions deux configurations de chaînes d’approvisionnement. La première est composée d’un seul manufacturier qui fournit des produits à plusieurs détaillants. La deuxième représente une compagnie qui a deux sites de production dans des régions différentes et qui font des transferts de produits entre eux.

1.2.2.1 Une chaîne d’approvisionnement à un manufacturier et plusieurs détaillants intégrant la livraison

Le premier système étudié, présenté dans la figure 1.1, représente une chaîne d'approvisionnement à deux échelons composée d'un manufacturier et de plusieurs détaillants (n détaillants) qui servent les clients finaux.

Le système de production n'est pas fiable et peut faire face à des pannes et des réparations aléatoires. Il produit un seul type de produit et l'entrepose dans un entrepôt local (MW), puis l'expédie aux détaillants, qui sont situés dans différents endroits. Les produits sont expédiés en tailles de lots Q_i (pour le détaillant i).

Le stock du manufacturier se remplit avec un taux égal au taux de la production et les stocks des détaillants qui diminuent avec les taux de demandes correspondantes. Au début de la livraison du lot au détaillant i, le niveau de stock $x_m(t)$ à l'entrepôt local du fabricant diminue par un saut d'une taille de lot Q_i. D'un autre côté, lorsque le lot est reçu par l'entrepôt du détaillant (RW_i), son niveau d'inventaire $x_i(t)$ augmente avec un saut fini d'une taille de lot Q_i. Le délai des livraisons τ_i diffère d’un détaillant à un autre.

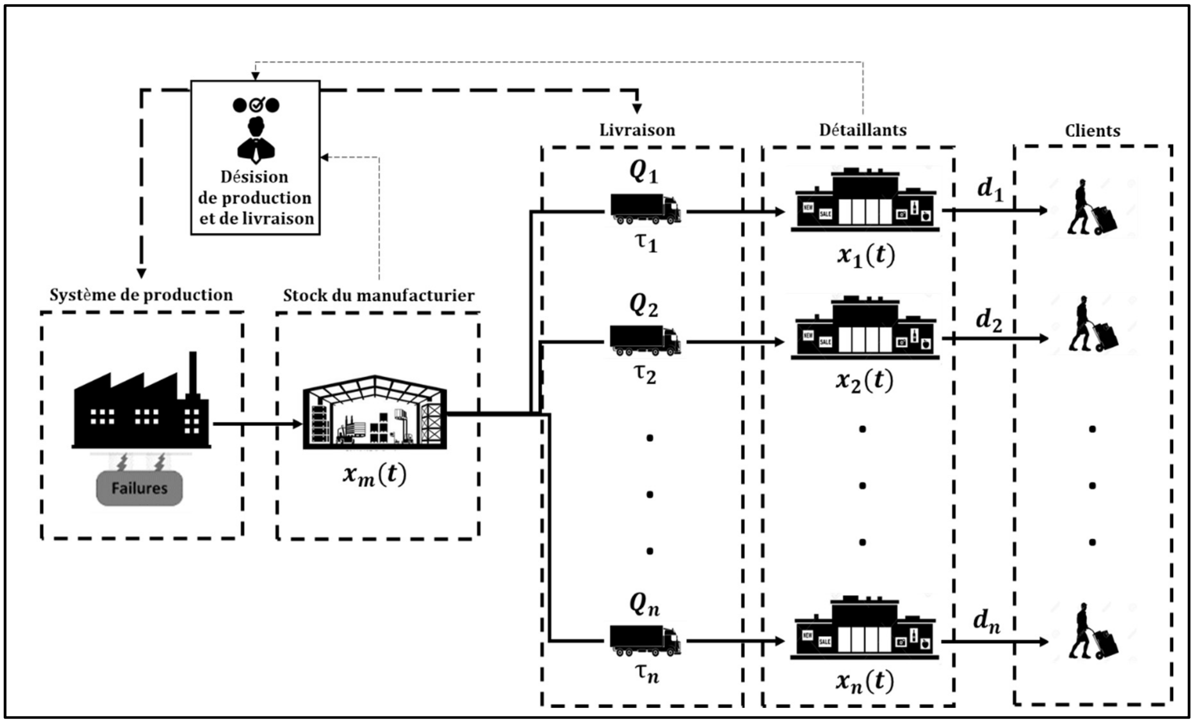

Figure 1.1 Système d'un manufacturier et plusieurs détaillants

La dynamique du système implique une composante continue (taux de la production et la demande) et une composante discrète (livraison des lots aux détaillants).

Les hypothèses suivantes sont prises pour ce système :

- le système manufacturier produit des produits de bonne qualité.
- les taux de demande des clients sont connus et constants.
- les moyens de transport sont fiables et disponibles pour le transport à tout moment.
- les pénuries sont autorisées et nous considérons un coût de pénurie.

1.2.2.2 Une chaîne d'approvisionnement à deux sites de production intégrant le transfert du stock

Le deuxième système étudié représente le cas d'une entreprise qui possède deux sites de production situés dans des régions différentes et qui servent les clients de ces régions (voir figure 1.2). Les deux sites de production produisent le même type de produits et font face à des taux de demande différents, selon la région. Les usines ne sont pas fiables; puisqu'elles font face à des pannes et des réparations aléatoires de leurs machines. Les produits finis sont stockés dans des entrepôts locaux avant de desservir le marché. Le transfert latéral du stock est autorisé entre les deux entrepôts en transférant des lots de produits d'un entrepôt à un autre.

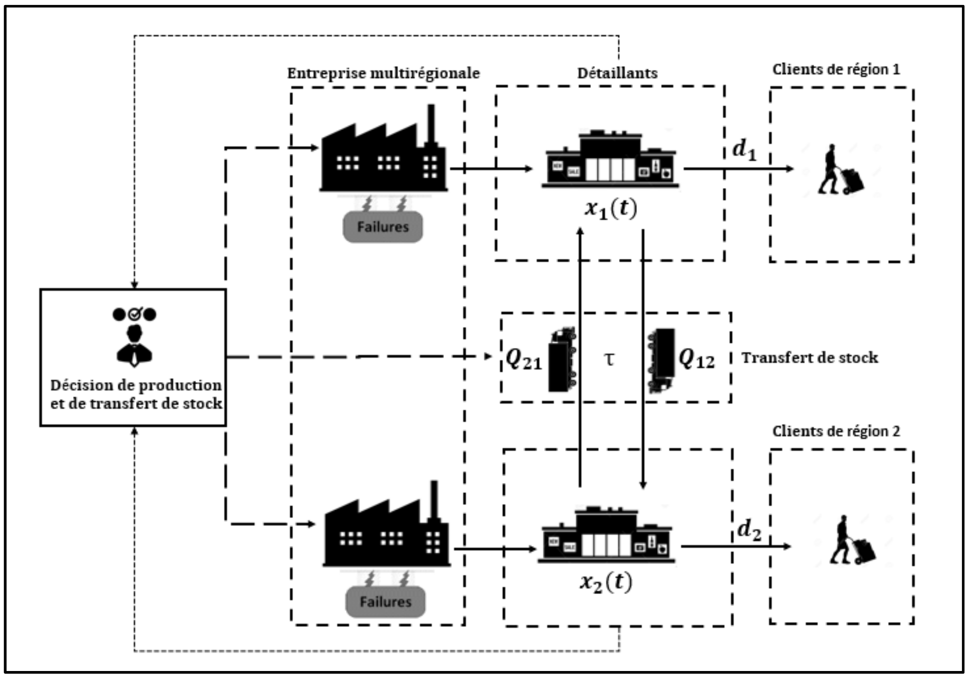

Figure 1.2 Structure de l'entreprise multirégionale étudiée

Au début d'un transfert de stock de l'entrepôt i vers l'entrepôt j, le niveau de stock $x_i(t)$ de l'entrepôt i diminue avec un saut fini égal à la taille du lot Q_{ij}. Lorsque le lot est reçu par l'entrepôt j après un délai τ_{ij}, son niveau d'inventaire $x_j(t)$ augmente d'un saut fini égal à la taille du lot Q_{ij}. Les taux de production des usines peuvent prendre des valeurs comprises entre zéro et les taux de production maximales des machines u_i^m. Les entrepôts sont remplis par le taux de production de l'usine associée et par les transferts latéraux de stock venant d'autres entrepôts.

Les hypothèses suivantes sont prises pour ce système :

- les taux de demande des clients sont connus et constants pour chaque région.

- les coûts de pénurie sont considérés.
- le site de production et l'entrepôt local associé sont situés au même endroit, nous ne tenons donc pas compte des délais de transport entre les deux.
- le transfert du stock peut avoir lieu dans les deux sens entre entrepôts et n'est pas donc unidirectionnel.
- les délais de transfert de stock ne dépendent pas du sens du transfert (le délai de l'entrepôt i à l'entrepôt j est le même que le délai de l'entrepôt j à l'entrepôt i).

1.3 État de l'art

Dans cette section, nous introduirons les principales catégories des travaux de recherche qui sont reliées à notre projet de recherche. Nous présenterons les travaux réalisés dans le domaine de commande optimale stochastique, la politique de commande à seuil critique, l'intégration de production et de la livraison et l'intégration de la production et du transfert du stock.

1.3.1 Commande optimale stochastique

Des politiques de commande optimale stochastique ont été proposées dans la littérature pour de différentes configurations de chaîne d'approvisionnement. Certains travaux intègrent des décisions concernant plusieurs activités et aspects de la chaîne d'approvisionnement, tels que la maintenance, la qualité, la refabrication « Remanufacturing» et l'aspect environnemental.

La considération de la maintenance des systèmes de production était un sujet d'intérêt pour plusieurs chercheurs. Certains ont considéré la maintenance préventive, comme Gharbi et al. (2007) qui ont développé une politique intégrée de contrôle production et de la maintenance préventive dépendante du niveau d'inventaire. Berthaut et al. (2011) ont étudié les effets des coûts et des paramètres du système sur la politique de la maintenance préventive. Plusieurs autres chercheurs ont proposé des politiques de maintenance préventive conjointement avec la politique de production. Dhouib et al. (2012) ont considéré la maintenance préventive basée sur l'âge de la machine. Quant à eux, Rivera-Gomez et al. (2013) ont considéré un système où la maintenance préventive réduit le niveau de détérioration du système et améliore la qualité des produits.

L'intégration de la qualité de produits dans les politiques de contrôle de production a aussi attiré l'attention de beaucoup de chercheurs. Bouslah et al., (2013) ont étudié un système de production par lot où ils ont considéré un plan d'échantillonnage unique pour un système de production imparfait. Hlioui et al., (2015) ont considéré une fraction constante de produits non conformes dans le lot de matière première reçue par le manufacturier qui influence la politique de production. Dans un autre travail, Hlioui et al., (2017) ont proposé une politique à seuil critique modifiée (HPP modifiée) qui prend en considération la proportion de matière première non conforme après avoir fait le contrôle qualité et quand le taux de production est ajusté à la demande.

Une autre direction de recherche dans ce domaine qui attire beaucoup de chercheurs est les chaînes d'approvisionnement qui impliquent la refabrication. Nous pouvons citer comme exemple le travail de Kenné et al. (2012) qui ont considéré un réseau de logistique inverse en boucle fermée, dont les produits retournés du marché sont refabriqués. Ils proposent une politique intégrée de fabrication et de refabrication tout en considérant que les machines sont sujettes à des pannes et des réparations aléatoires. Ce travail est approfondi par Ouaret et al. (2013) pour considérer la demande stochastique. Plusieurs autres auteurs ont envisagé des politiques de refabrication dans différents contextes. Parmi des travaux récents, Assid et al. (2019) ont étudié le setup entre le mode de fabrication et le mode de refabrication dans un système non fiable et ils ont proposé une politique intégrée de production et de setup. Assid et al. (2020) ont étendu ce travail pour considérer des installations mixtes qui peuvent être dédiées pour la fabrication et la refabrication ou partagées. Dans ce dernier type, la fabrication et la refabrication peuvent se faire dans la même installation.

L'intégration des règlementations environnementales avec les politiques de production est aussi un sujet qui a été traité par plusieurs chercheurs. Nous citons, par exemple, les travaux de Ben Salem et al. (2015), Ben Salem et al. (2016), Hajjej et al. (2017), Hajjej et al. (2019), Entezaminia et al. (2020) et dans un travail plus récent, Entezaminia et al. (2021). Ils ont tous proposé des politiques de production pour des systèmes de fabrication sous règlementation environnementale, dans un contexte stochastique.

Dans ce projet, nous considérons deux politiques de commande stochastique pour des chaînes d'approvisionnement à deux échelons. La première intègre la livraison aux détaillants et la deuxième étudie une chaîne d'approvisionnement dans laquelle les détaillants peuvent partager l'inventaire par transfert latéral du stock. Dans le paragraphe suivant, nous étudierons les travaux qui ont considéré des systèmes similaires.

1.3.2 Intégration de la livraison avec les politiques de contrôle de production

Les travaux de la littérature trouvés qui intègrent la livraison avec les politiques de production considèrent généralement un lot de production économique comme une politique de production. Giri et al. (2005) ont été parmi les premiers à présenter deux modèles dans ce contexte. Le premier est un modèle de dimensionnement de lots de production (EMQ) et le second ajoute un stock de sécurité comme variable de décision. Ce travail a été ensuite étendu par Giri et Dohi (2005) pour considérer la maintenance préventive avec des distributions de probabilité générales pour les temps de panne et de réparation. El-Ferik (2008) a considéré la maintenance préventive dans un modèle de EMQ pour un système de production peu fiable. Les travaux de Giri et Dohi (2005) sont prolongés par Sana et Chadhuri (2010) pour considérer les pannes aléatoires. Glock (2010) a étudié un modèle de EMQ pour un système de production et de distribution à deux échelons. Ce travail a été approfondi pour considérer un système de production et de distribution à plusieurs étages par Glock (2011). Chakraborty et Giri (2012) ont proposé un modèle d'EMQ stochastique pour un système de production sujet à des pannes et des réparations aléatoires, en tenant compte de la détérioration du système. Hu et al. (2017) ont proposé un EMQ et ils ont envisagé la maintenance préventive d'un système de production par lots. Chan et al. (2017) ont proposé un modèle de EMQ en tenant compte de la détérioration des produits pendant le transport. Sarkar et al. (2018) ont développé un modèle EMQ pour un système multidétaillants et un seul fabricant et ont considéré la qualité imparfaite des produits.

Kim et Glock (2018) ont étudié un système de production à deux échelons avec des machines parallèles et ils ont pris en compte les coûts de transport et d'expédition des lots de production entre les machines et le stock final. David et Herbon (2020) ont proposé un modèle de EMQ qui optimise la taille du lot de production et la fréquence des livraisons.

Dans la littérature de politiques de contrôle de taux production, nous avons trouvé peu de travaux qui considèrent la livraison. Dans ce paragraphe, nous présentons les travaux qui ont considéré le transport de produits finis du manufacturier à un détaillant ou à un entrepôt lointain. Huang et Irvani (2005) ont envisagé une chaîne d'approvisionnement à deux échelons qui comprend un fabricant et deux détaillants et ils proposent une politique de contrôle de la production dans le contexte du partage sélectif d'informations, cepenant ils ne prennent pas en compte les délais de transport, ni les pannes et réparations aléatoires des machines. Mourani et al. (2008) ont étudié un modèle à flux continu pour un système de production qui produit un seul type de produit et ils ont pris en compte les délais de transport entre la machine et le stock final. Turki et al. (2009) ont proposé un modèle de simulation utilisant l'analyse des perturbations pour un système de production non fiable, en considérant un délai de livraison constant entre le manufacturier et le détaillant. Bouslah et al. (2013) ont élaboré une politique de contrôle du taux de la production pour un système de production par lots non fiables, en tenant compte des délais de transport entre le fabricant et un seul entrepôt local distant (ou un seul détaillant distant). Ils ont proposé une politique qui combine une HPP modifié pour le contrôle du taux de production et une politique de quantité de fabrication économique « economic manufacturing quantity » pour déterminer la taille des lots optimale. Bouslah et al. (2014) ont étendu ce travail en proposant une politique de contrôle qualité pour un système de production par lots non fiable. Turki et al. (2017) ont développé un modèle qui étudie le contrôle du taux de production pour une chaîne d'approvisionnement en boucle fermée de fabrication/refabrication, en tenant compte du transport entre le fabricant et le stock final. Il est à noter que, dans la plupart des modèles discutés ci-dessus, la politique de livraison n'est envisagée que de manière implicite. Les lots produits représentent également le lot de livraison et la décision de livraison consiste simplement à expédier le lot produit au stock final juste après la fin de sa production.

1.3.3 Intégration production et transfert de stock

Il y a beaucoup de travaux dans la littérature qui ont étudié les transferts de stock entre les détaillants. Néanmoins, pas beaucoup de recherches ont été faites qui combinent les politiques de production avec les politiques de transfert de stock. La plupart des travaux faits dans ce contexte considèrent des modèles de revue périodique, comme le travail de Yang et Qin (2007) qui ont considéré le transfert virtuel du stock dans un système de production/inventaire comprenant deux fournisseurs à capacité finie situés dans des régions différentes. Hu et al. (2007) ont étudié un modèle de production/inventaire où les transferts de stock sont autorisés entre deux endroits et dans le but de maximiser les profits des détaillants tout en tenant compte de la capacité du fournisseur. Dans un travail similaire, Hu et al. (2008) ont proposé une quantité de production optimale et une politique de transfert de stock où le système de production est caractérisé par une capacité incertaine. Chen et al. (2015) ont utilisé une nouvelle approche pour le problème conjoint d’inventaire et de transfert de stock, en tenant compte de la capacité incertaine du fournisseur et ils ont prouvé les propriétés structurelles de la fonction de valeur optimale dans l'étude de Hu et al. (2008). Les travaux ci-dessus sont considérés comme des politiques par lots et périodiques et ils ne permettent pas un contrôle en temps réel de la production et de transfert de stock. Abouee-Mehrizi et al. (2015) ont étudié les politiques conjointes de transfert de stock et de réapprovisionnement pour un système d'inventaire à deux détaillants où l'inventaire du détaillant peut être réapprovisionné auprès d'un fournisseur ou par un transfert de stock de l’autre détaillant et en tenant compte des ventes perdues. La plupart des modèles mentionnés ci-dessus sont de type revue périodique et ils considèrent le transfert latéral proactif du stock, ce qui signifie que les transferts sont planifiés à des moments prédéterminés avant que toute la demande soit réalisée pour les périodes correspondantes. Ils ne tiennent pas compte de l'état en temps réel du système.

Cependant, l'un des premiers travaux à aborder l’intégration de la production avec le transfert du stock en considérant un modèle à temps continu est Zhao et al. (2008) qui ont étudié un système à deux sites et ils ont proposé une politique de production/transfert de stock où les transferts sont déclenchés par une demande arrivant en cas de rupture de stock ou en partageant le stock excédentaire avec d'autres détaillants, même en l'absence de rupture de stock. Leur politique de production est une politique qui dépend du niveau de stock et détermine s'il faut produire ou non à chaque emplacement et quel endroit il faut servir. Dans un travail récent, Bhatnagar et Lin (2019) ont étendu le modèle introduit par Zhao et al. (2008) pour considérer un système multilocalisations et ils ont assoupli les restrictions sur les paramètres de coût. Ces auteurs ont étudié un système à un seul échelon qui fait face à une demande discrète et ils ont proposé une politique conjointe de production-transfert de stock qui détermine quand et pour quel site le système doit-il produire et quand effectuer les transferts. Comme beaucoup de travaux de la littérature, ils considèrent uniquement un système de fabrication fiable et ils ne prennent pas en compte les délais de transfert de stock.

Plusieurs travaux de recherche dans la littérature ont considéré aussi la sous-traitance conjointement avec les politiques de contrôle de production. La sous-traitance est une stratégie pour soutenir les usines en cas de pannes. L'un des premiers travaux étudiant la sous-traitance avec contrôle du taux de production est celui de Dror et al. (2009) qui a proposé une politique à seuil critique qui contribue à minimiser le stock de sécurité pour deux produits. Assid et al. (2015) ont proposé une politique conjointe de contrôle du taux de production et de sous-traitance en considérant plusieurs installations de production non fiables fabricant deux produits. Rivera-Gómez et al. (2016) ont étudié un système de production qui se dégrade et ils ont proposé une politique intégrée de contrôle de production et de sous-traitance en utilisant la programmation dynamique stochastique. Ce modèle est étendu par les mêmes auteurs dans le travail de Rivera-Gómez et al. (2018) pour intégrer la maintenance et envisager des machines avec des dégradations progressives et des usures qui réduisent leurs capacités. Dans un travail récent, Assid et al. (2020) ont proposé une politique conjointe de sous-traitance, de production et de setup où le système production est sujet à des pannes et des réparations aléatoires.

1.4 Critique de la littérature

Plusieurs configurations de systèmes de production ont été traitées dans la littérature de la politique de la commande optimale. La chaîne d'approvisionnement à plusieurs échelons a été traitée par de nombreux chercheurs, mais la plupart des travaux se concentrent plus sur la partie en amont du système manufacturier. Dans la littérature, nous trouvons de nombreux travaux qui intègrent des politiques de contrôle de production basées sur HPP avec des politiques d'approvisionnement et de sélection de fournisseurs. Le lecteur est référé aux travaux de Hajji et al., (2009), Hajji et al., (2011), Hlioui et al., (2017) et Assid et al., (2019). Néanmoins, peu de travaux ont proposé des politiques de contrôle de production qui intègrent des activités de la partie en aval du système de production, comme la livraison et le transfert de stock.

En ce qui concerne l'intégration de la livraison avec la production, la plupart des travaux considèrent un lot de production économique pour la politique de production. Peu de travaux présentés dans la littérature considèrent la politique HPP avec la livraison (Huang et Irvani (2005), Mourani et al. (2008), Turki et al. (2009), Bouslah et al. (2013), Bouslah et al. (2014), Turki et al. (2017)). Ces travaux considèrent des politiques à seuil critique, mais aucune politique de livraison explicite n'a été proposée. La livraison est faite directement après la fabrication du lot pour les systèmes de fabrication par lot, ou les produits circulent à travers un flux continu vers le détaillant, comme dans le travail de Mourani et al. (2008). En plus, la majorité considère un seul détaillant, sauf Huang et Irvani (2005) qui considèrent deux détaillants, mais ils ne considèrent pas un système de production non fiable. Les politiques de livraison considérées sont souvent implicites et ne tiennent pas compte de la situation des détaillants. À notre connaissance, aucun travail dans ce contexte n'a considéré la notion de la priorisation des détaillants qui prend en compte la condition des détaillants et priorise ceux qui ont plus de risque de tomber en pénurie.

En ce qui concerne le transfert de stock latéral entre les détaillants, la plupart des travaux qui intègrent la production considèrent un modèle de revue périodique, ce qui implique que la demande et les paramètres à optimiser sont fixés pour toute la période correspondante. Pour les travaux qui ont considéré la modélisation à temps continue, la plupart des modèles considérés s'intéressent plus aux choix de détaillants à qui les usines doivent produire et à la décision de produire ou de ne pas produire. À notre connaissance, aucune politique de contrôle de taux de production n'a été proposée dans ce contexte. En plus, ils ne considèrent pas le délai de transfert dans leur modélisation. Une autre limitation importante, c'est qu'ils ne considèrent pas les pannes et réparations aléatoires des machines qui peuvent influencer les politiques de production et de transfert de stock.

1.5 Problématiques de la recherche

En se basant sur la critique de la revue de la littérature, il est évident que l'intégration de la livraison et du transfert de stock avec la production ont été très peu investigués dans la littérature et plusieurs questions qui les concernent peuvent être posées :

En ce qui concerne l'intégration de la production/livraison :

1. Existe-t-il une politique de production adéquate qui peut contrôler un système manufacturier non fiable qui livre un seul type de produit à plusieurs détaillants sachant qu'ils sont situés dans différentes localisations et qu'ils sont caractérisés par différentes demandes ?

2. Quelle politique de livraison doit être utilisée pour s'assurer que les détaillants sont tous servis au bon moment de façon à minimiser les coûts de pénurie, d'inventaire et de transport ?

3. Existe-t-il une amélioration de la politique de livraison qui tient en compte les détaillants les plus vulnérables aux pénuries ? Est-ce qu'une telle amélioration peut vraiment être plus efficace en termes de coûts ?

En ce qui concerne l'intégration de la production/transfert de stock :

1. Quelle est la meilleure politique de contrôle de production pour un système constitué de deux usines non fiables localisées dans différentes régions, et qui font face à de différents taux de demande et dont les transferts de stock entre les deux localisations sont autorisés ?

2. Quelle politique de transfert de stock doit être déployée pour assurer une meilleure performance du système en termes de coût total de la chaîne ? Puis quand est-ce que c'est plus avantageux de faire les transferts de stock ?

1.6 Objectifs de la recherche

Nous étudions, dans ce projet de recherche, des chaînes d'approvisionnement à deux échelons où les détaillants sont considérés et les systèmes manufacturiers ne sont pas fiables. Ce projet de recherche a deux objectifs principaux :

- Le premier objectif est de développer une nouvelle politique intégrée de contrôle de production et de livraison pour un système de production non fiable. Le système de production produit un seul type de produit et livre les produits par lot à plusieurs détaillants caractérisés par des taux de demandes et des délais de livraison différents. Le but est de minimiser le coût total de la chaîne qui implique les coûts d'inventaire, de pénurie et de transport.

- Le deuxième objectif est de développer une politique intégrée de contrôle de production et de transfert de stock pour une chaîne d'approvisionnement composée de deux usines localisées dans des régions différentes. Elles font aussi face à des différents taux de demande et elles autorisent le transfert de stock entre eux. Le but est de minimiser le coût total d'inventaire, de pénurie et de transfert de stock.

Les deux objectifs sont atteints par la détermination des structures des politiques optimales en utilisant la formulation mathématique et la résolution numérique et la détermination des paramètres optimaux des politiques par l'approche combinée de la simulation et des plans expérimentaux. La méthodologie utilisée pour atteindre ces objectives est décrite dans la section suivante.

1.7 Approche de résolution

Nous utilisons une approche combinée analytique et numérique (figure 1.3) pour déterminer la structure de la politique de contrôle optimale et pour déterminer les paramètres optimaux de ces politiques. Nous décrivons ci-dessous les principales étapes de l'approche adoptée:

1. Formulation du problème et approche analytique : Dans cette étape, nous développons un modèle de programmation dynamique stochastique représentant le système basé sur la théorie du contrôle. La livraison et le transfert de stock sont faits par lots, ce qui introduit un aspect impulsif au problème que les modèles classiques à flux continus ne peuvent pas modéliser. Par conséquent, nous basons notre modèle analytique sur la théorie du contrôle des impulsions (Yong 1989, Yang 1999). Ensuite, en utilisant une approche numérique, les équations HJB obtenues sont résolues.

2. Structure de la politique de contrôle optimale : En se basant sur les résultats de la résolution numérique, la structure de la politique de contrôle est déterminée et les paramètres des politiques considérées sont définis.

3. Plans d'expériences : Dans cette étape, nous déterminons les différentes combinaisons de paramètres de contrôle qui garantissent un nombre minimal d'expériences à conduire.

4. Modèle de simulation : Nous développons un modèle de simulation qui reproduit la dynamique du système en considérant la politique de contrôle comme input et le coût total généré comme output.

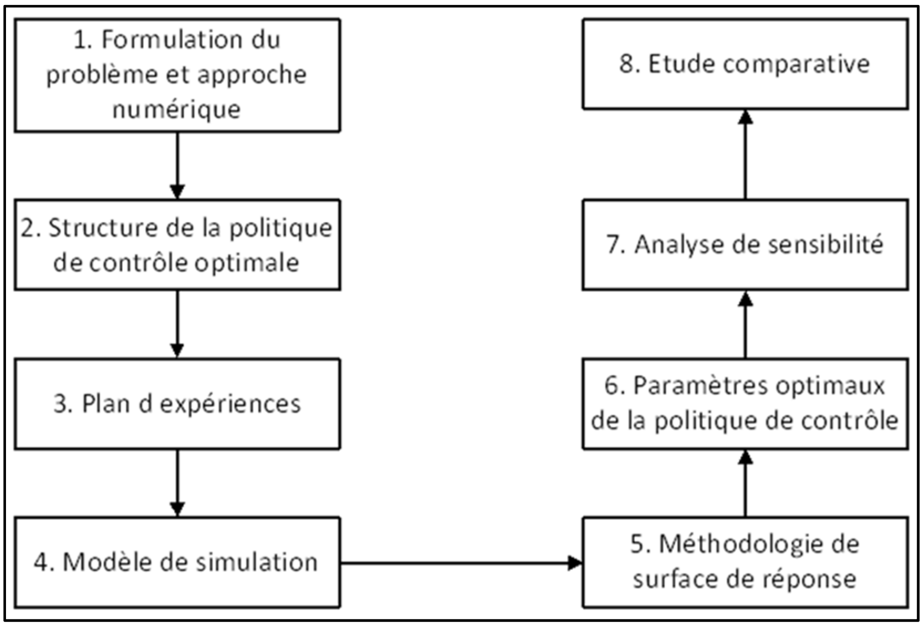

Figure 1.3 Approche de résolution adoptée

5. Méthodologie de surface de réponse : Cette étape explore les effets des variables explicatives (indépendantes) sur la variable de réponse (dépendante) qui est dans notre cas le coût total et elle montre la signifiance de chaque effet.

6. Paramètres optimaux de la politique de contrôle : En se basant sur l'équation de régression déterminée par la méthodologie de surface de réponse, les paramètres optimaux des politiques considérés sont déterminés.

7. Analyse de sensibilité : Cette étape consiste à varier les différents paramètres du coût et les paramètres des systèmes pour étudier leurs effets sur les paramètres optimaux de

la politique et sur le coût total, afin de confirmer la robustesse des politiques considérées.

8. Étude comparative : Une comparaison entre nos politiques proposées et les politiques les plus pertinentes de la littérature qui sont adaptées à notre système est faite pour différentes valeurs de coût (inventaire, pénurie, transport, transfert de stock) et de paramètres de système (disponibilité de système, délai de livraison et délai de transfert de stock), afin de spécifier la politique de contrôle la plus efficace en termes de minimisation du coût total.

1.8 Conclusion

Dans ce chapitre, nous avons présenté les structures des chaînes d'approvisionnement étudiées, ainsi que la terminologie qui définissent le cadre général du projet. Une revue de la littérature est par la suite présentée pour discuter les travaux qui traitent des politiques de contrôle de production pour différents types de systèmes. La revue effectuée montre un manque de travaux qui considèrent la partie en aval du système de production. La livraison et le transfert de stock entre détaillants sont des problèmes qui ont été négligés dans la littérature des politiques de contrôle de production. Même les travaux qui les ont considérés, ont proposé des politiques de livraisons implicites et simplistes. Pour le cas de transfert de stock, ils n'ont pas considéré des systèmes de production non fiables et les délais de transfert de stock. En se basant sur la critique de la littérature faite, nous avons défini les problématiques de la recherche, ses objectifs et finalement, l'approche de résolution adoptée.

CHAPITRE 2

INTEGRATED PRODUCTION-DELIVERY CONTROL POLICY FOR AN UNRELIABLE MANUFACTURING SYSTEM AND MULTIPLE RETAILERS

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Abstract: Production control policies have been studied in the literature for different types of unreliable manufacturing systems. But only few have been addressed to the unreliable manufacturer-retailer systems. We consider the integrated production and delivery control for an unreliable manufacturing system and multiple retailers. The retailers, supplied by the manufacturer through a common warehouse, are located in different sites and face different demand rates. We determine the optimal joint production and delivery control policy with the objective of minimizing the total cost, which comprises holding/ backlog in addition to transportation costs. The stochastic optimal control and the impulse control theory are used to develop the optimal control policy (policy 1). The production is controlled by a hedging point policy and the delivery policy is governed by a state dependent economic order quantity. The optimal parameters of the proposed policy are determined by adopting a simulation-based optimization approach. A sensitivity analysis is conducted to verify the robustness of the results. An improved policy (policy 2) involving priority rules for delivery is proposed using a simulation-based approach. We conduct a comparative study of the new policies (policy 1 and 2) and the most relevant policy found in the literature (policy 3), and the results show that the new proposed improved policy (policy 2) outperforms others.

Keywords: unreliable manufacturing system; feedback production planning; economic order quantity; vendor managed inventory; one-vendor multiple-buyer system; simulation; stochastic dynamic programming.

2.1 Introduction

Nowadays, in a context of highly competitive markets, a high level of coordination between the different actors of the supply chain is essential to be more efficient in the logistic operations and decisions of procurement, transport and warehousing (Wankmüller et al., 2020). Many companies, such as Walmart, Procter & Gamble and National Semiconductor, have had successful experiences showing that supply chain integration has significantly improved their performances and their market shares (Simchi-Levi et al., 2000).

The vendor-buyer cooperation is an example of coordination strategies, which is important to reduce the total cost and improve the responsiveness of the vendor-buyer system (Wee and Yang., 2004). In the past, this type of cooperation mainly consists of optimizing replenishment decisions such as the number and the size of lots transported between the two members, with the main assumption that the vendor is always available. Where in reality vendors may be dealing with several stochastic events, which may result in their unavailability. This is demonstrated in the manufacturer-retailer system, as the manufacturer may face stochastic failures and repairs of his machines. Underestimating these uncertainties could make it difficult to respond properly to the demand and could result in an unsatisfied customer leading to lost sales in the market share (Dadashpour et al., 2020). Moreover, the non-integration of the supply chain under these circumstances may result in high transportation costs due to the absence of a delivery policy that takes into consideration the unreliability of the manufacturer.

In the literature, only few works dealt with this type of integration, where they consider the uncertainties faced by the manufacturer and the transportation of the finished products to the retailers. Previous researchers studied mostly single manufacturer-single retailer system like Bouslah et al., (2013) who developed a production policy jointly with an economic manufacturing quantity model for a batch processing system composed of a manufacturer supplying a remote local warehouse (could be considered as a retailer). However, in real life supply chains, it is common to have a single manufacturer who supplies a product to more than one retailer. These retailers may have different demand rates, and transportation delays. In this context, the manufacturer may face difficulties in adjusting the production rate in order to maintain an inventory level that satisfies all retailers knowing that he would face random failure and repair times of his machine. If the manufacturer does not properly plan his production when dealing with these uncertainties, the total cost which comprises holding, backlog and transportation costs could increase significantly. Therefore, the question that arises is what is the best production control policy that should be applied to minimize these costs? In addition, in case of multiple retailers, the delivery decisions become more challenging since all retailers are supplied by the same manufacturer and compete for the same products. Thus, the decision of delivering to a specific retailer and ignoring others' retailers demand would likely lead them to shortage. So, under these circumstances, what is the best delivery control policy that should be deployed to minimize the total cost? Moreover, if there is more than one retailer competing for a lot; which retailer will be prioritized?

This work falls within this context, as it addresses the abovementioned issues. It proposes an integrated production and delivery control policy for an unreliable manufacturing system that produces single-type product and supplies multiple retailers located in different sites and which have different demand rates and different transportation delays. This joint policy also incorporates the priority to be given to the various retailers aiming to minimize the total incurred cost, including holding, backlog and transportation costs.

We organize the reminder of the paper as follows. The literature is reviewed in section 2.2. In Section 2.3 we present the notations, and we formulate the optimal control problem. In section 2.4 we illustrate the numerical results, and we define the structure of the optimal joint production and delivery policy for a single-manufacturer multiple-retailers system. The optimal control parameters of the policy are determined using a simulation-based optimization approach that we detail in Section 2.5. In section 2.6, we explain and validate the simulation model and the experimental design, and a numerical example is presented. A sensitivity analysis is performed in section 2.7 to verify the effectiveness of the proposed policy. In Section 2.8 we propose an improved control policy that considers priority rules (policy 2) and we adapt the most relevant policy of the literature to our case (policy 3). We compare the proposed policies (Policy 1 and 2) and the nearest policy we found in the literature (policy 3) in section 2.9. Managerial insights are introduced in section 2.10. Finally, the paper is concluded, and future research are presented in section 2.11.

2.2 Literature review

The integration of production with other processes of the supply chain (such as supply and distribution) has been a topic of interest for many researchers in the last few decades. For production in its own, an important concept known as the hedging point policy (HPP) is generally used for the production rate control. This policy, introduced first by Kimemia and Gershwin (1983), considers an optimal safety stock that should be preserved when the manufacturing system is available which aims to protect against the incoming capacity shortages caused by failures. In the literature, we find many works integrating production control policies based on HPP with supply policies for a multistage supply chains. We cite as examples the works of Hajji et al., (2009), Hajji et al., (2011), Hlioui et al., (2017) and Assid et al., (2019). Concerning the distribution, many works proposed integrated production and distribution models that can be presented in two classes based on the type of proposed policy. The first class comprises the works that propose vendor-buyer supply chain models. These works generally consider the problem from a tactical level and have the objective of determining a joint economic-lot size (JELS). The second class represents the works that propose production planning policies for the integrated production-distribution system. Production planning is addressed either by determining an economic manufacturing quantity (EMQ) (category 1) or by proposing a production rate control policy where the rate of production is governed by a hedging point policy (category 2). Our work falls into the category 2 of the second class. In table 2.1 we classify research that integrate production and distribution based on different criteria (columns of the table 2.1) that characterize our work, such as failure prone facilities, the transportation delay of the finished products, multiple retailers, the consideration of prioritization rules for retailers, production planning, production rate control, delivery control policy and the consideration of integrated production-distribution policies.

The first class presents the studies that investigate vendor-buyer supply chain models. These studies address shipment sizing and scheduling aiming to minimize the total cost. Goyal (1976) initiated the idea by proposing a JELS policy that minimizes the total incurred costs for a single-vendor single-buyer (SV-SB) supply chain with the assumption of infinite capacity of the vendor. Then, Banerjee (1986) extended this work for the case of finite capacity of the manufacturer. Goyal (1988) proposed a model for SV-SB supply chain in which the lot size of the vendor is considered as a multiple of the size of the buyer's order. Ertogral et al. (2007) addressed a lot-sizing problem and introduced two models that explicitly integrate the transportation cost in the total operational cost. Pineyro and Viera (2010) investigated the JELS with product returns in the context of two independent demand streams where the substitution of the remanufactured products is allowed. Zhang et Pan (2012) investigated a closed-loop supply chain and considered the capacity constraints and product returns in a lot sizing problem. The aforementioned works considered a SV-SB system. The vendor-buyer supply chain models are extended by many researchers by considering single-vendor and multiple-buyers (SV-MB) systems as in Lu (1995) who presented a model for minimizing the annual cost of the vendor while considering the constraint of an upper limit fort the buyer's cost. Woo et al. (2001) developed a model that optimizes investment and lot sizing decisions to minimize the integrated total cost while considering that the ordering period is similar for all buyers and vendors. Zhang et al. (2007) have considered the problem defined by Woo et al. (2001) for the case of different ordering cycles and assuming that the vendor divides his lot size to multiple lots shipped to the buyer. Rad et al. (2014) investigated a model for a single-vendor two-buyers system with finite vendor capacity. Mateen et al. (2014) studied a SV-MB system under stochastic demand and assuming that buyers are served at the same time. Lee and Park (2016) analyzed a system made of one supplier and two retailers with uncertain demand. They have also included the possibility of transshipment of the surplus of products from a retailer to another who is out of stock. Ben-Daya et al. (2019) investigated the JELS problem for a SV-MB system in a closed-loop supply chain and considering consignment stock partnership. Gharaei and al. (2020) developed a multi-product JELS model for a four-level supply chain, they consider raw materials replenishment and take into consideration real constraints such as space limitations, the number of shipments and the procurement cost. Most of the vendor-buyers supply chain models assume that the vendor is always available. They do not consider the uncertainties due to the unreliability of his machines.

The second class presents the works that proposed production planning policies for the integrated production-distribution systems. In this class we present two categories depending on how the production planning is addressed. The first category represents the works that aim to determine an economic manufacturing quantity (EMQ) which also represents the lot delivered to the final stock. Thus, they implicitly consider the delivery to the retailer. The second category represents the works that propose hedging point policies that allow to vary the production rate based on the optimal stock levels. Concerning the first category, Giri et al. (2005) presented two models of EMQ, the first is production lot sizing model, and the second adds a safety stock as a decision variable. This work is then extended by Giri and Dohi (2005) to consider preventive maintenance and failure and repair times with general probability distributions. El-Ferik (2008) considered the preventive maintenance in an EMQ model while considering the unreliability of the manufacturing system. The work of Giri and Dohi (2005) is extended by Sana and Chadhuri (2010) to consider random breakdowns. Glock (2010) studied an EMQ model for a production-distribution system. This work is extended to consider a multistage production and distribution system by Glock (2011). Chakraborty and Giri (2012) presented a stochastic EMQ model for a failure-prone manufacturing system and considering the process deterioration. Zanoni et al. (2014) analyzed a production facility that produces a single product and ships it in batches to a subsequent production stage and they developed an analytical model that optimizes lot size and the frequency of shipments while incorporating energy costs. Jauhari and Pujawan (2016) investigated a supply chain that comprises one manufacturer who supplies one retailer and proposed a model that aims to optimize the review period and the number of shipments to minimize the joint total cost. Aldurgam et al. (2017) studied a production and distribution system and proposed a model that optimizes the production lot size while considering a stochastic demand. Hu et al. (2017) proposed an EMQ and considered preventive maintenance for a batch processing system. Chan et al. (2017) determine an EMQ model with the consideration of the deterioration of items during transportation. Sarkar et al. (2018) integrated the imperfect quality of products in an EMQ model for a single-manufacturer multiple-buyer system. Kim and Glock (2018) considered parallel machines in a two-stage manufacturing system and integrated the transportation costs of shipping the production lots between the machines and the final stock. David et Herbon (2020) proposed an EMQ model optimizing the frequency of deliveries and considering an upper bound on the whole production and idle cycle. A common assumption of the aforementioned works is that the processed lots can instantly meet the demand without taking into consideration the transportation delays to the final stock serving customers. Such delays can significantly influence these control policies. Moreover, the production rate is considered constant during all the planning horizon without taking into consideration the stock level.

The works of the second category considered a production rate which is controllable depending on the stock levels. Huang and Irvani (2005) studied a two-stage supply chain that comprises one manufacturer supplying two retailers and they proposed a production control policy in the context of selective-information sharing but they do not consider transportation delays. A continuous-flow model was investigated by Mourani et al. (2008) for a single-product manufacturing system considering transportation delays between the machine and the final stock. Turki et al. (2009) used perturbation analysis for a simulation model addressed to a manufacturing system which is unreliable, and they considered constant delivery time. Bouslah et al. (2013) investigated an unreliable batch processing manufacturing system, and they developed a production rate control policy while considering transportation delays between the manufacturer and a single remote local warehouse (or a single remote retailer). They proposed an EMQ-HPP that combines a modified HPP for production rate control and an EMQ for lot sizing. Bouslah et al. (2014) extended this work by proposing a quality control design policy for an unreliable batch processing system. Turki et al. (2017) developed an optimization model that investigates production rate control for a manufacturing/remanufacturing closed-loop supply chain with the consideration of the transportation between the manufacturer and the final stock. It is noteworthy that in most of the models discussed above the delivery policy is only considered implicitly. The produced lots also represent the delivery lot, and the delivery decision is simply to ship the produced lot to the final stock right after its production is finished.

To sum up, we clearly see that the vendor-buyer supply chain models generally consider a vendor that is always available and capable of supplying the buyer. On the other hand, we see that even for the works that addressed production planning, only few considered failure prone facilities and only addressed the single-vendor single-buyer system. Moreover, few papers explicitly considered transportation delays between the manufacturer and the retailer. To the best of our knowledge, no previous work considered all these facets of our problem simultaneously. More precisely, the gap is defined first by the lack of integrated production and distribution control policies for a system that comprises an unreliable manufacturer supplying multiple-retailers and with the consideration of transportations delays. Second, no prioritization rules for the retailers are considered in the literature. Where in reality, prioritizing the most vulnerable retailers to shortages can protect the whole system from high backlog costs.

In this paper, we address the single-manufacturer multiple-retailers system. The manufacturer consists of one unreliable machine that produces one type of product. Transportation delays of the finished products are considered between the manufacturer and the retailers. The retailers are located in different locations and have different demand rates and transportation delays. Prioritization rules of the most vulnerable retailers to shortages are also considered in the delivery decisions.

Our contribution is to propose an integrated production and distribution control policy that considers all the key features discussed above. The proposed policy allows the decision making for production as well as for delivery by determining when to increase/decrease production rate and when to deliver to the retailers and what quantity should be delivered. This proposed control policy allows the supply chains comprised of one manufacturer and multiple retailers to minimize their total cost incurred by the holding /backlog of the inventory and the transportation of the finished products.

Table 2.1 Bibliographic review of the integrated production-distribution systems

	Failure prone facilities	Transportation delay between manufacturer and retailer	Multiple retailer	Retailers' prioritization rule	Production planning	Production rate control policy	Delivery control policy	Integrated production-distribution policy
Class 1: Vendor-buyer supply chain models								
Goyal (1976)							✓	✓
Banerjee et al. (1986)							✓	✓
Goyal (1988)							✓	✓
Lu (1995)			✓				✓	✓
Woo et al. (2001)			✓				✓	✓
Zhang et al. (2007)			✓				✓	✓
Ertogal et al. (2007)			✓				✓	✓
Pineyro and Viera (2010)							✓	✓
Zhang et al. (2012)							✓	✓
Rad et al. (2014)			✓				✓	✓
Mateen et al. (2014)			✓				✓	✓
Lee and Park (2016)			✓				✓	✓
Ben-Daya et al. (2019)			✓				✓	✓
Gharaei and al (2020)			✓				✓	✓
Class 2: Production planning policies								
Category 1: Economic manufacturing quantity (EMQ)								
Giri et al. (2005)	✓				✓		✓	✓
Giri and Dohi (2005)	✓				✓		✓	✓
El-Ferik (2008)	✓				✓		✓	✓
Sana and Chadhuri (2010)	✓				✓		✓	✓
Glock (2010)					✓		✓	✓
Glock (2011)					✓		✓	✓
Chakraborty and Giri (2012)					✓		✓	✓
Zanoni et al. (2014)					✓		✓	✓
Jauhari and Pujawan (2016)					✓		✓	✓
Hu et al. (2017)					✓		✓	✓
Aldurgam et al. (2017)					✓		✓	✓
Chan et al. (2017)					✓		✓	✓
Sarkar et al. (2018)			✓		✓		✓	✓
Kim and Glock (2018)					✓		✓	✓
David et Herbon (2020)					✓		✓	✓
Category 2: Production rate control based on hedging point policies								
Huang and Irvani (2005)			✓		✓	✓		✓
Mourani et al. (2008)	✓	✓			✓	✓		✓
Turki et al. (2009)	✓	✓			✓	✓		✓
Bouslah et al. (2013)	✓	✓			✓	✓	✓	✓
Bouslah et al. (2014)	✓	✓			✓	✓	✓	✓
Turki et al. (2017)	✓	✓			✓	✓		✓
Our proposal	✓	✓	✓	✓	✓	✓	✓	✓

Stochastic dynamic model is used combined with a simulation-optimization model to experimentally define the structure and the optimal parameters of the proposed control policy.

2.3 Description and formulation of the problem

In this section we introduce the problem, and we present its mathematical formulation.

2.3.1 Notations

The notations used for the problem formulation are defined below:

x_m *(t)* The level of inventory at the manufacturer's warehouse

x_i *(t)* The level of inventory at the warehouse of the retailer i

y_i *(t)* The position of inventory of the retailer i

y (t) The position of inventory of the whole system

u(t) Production rate at time t (product/time unit)

u_m Maximum production rate (product / time unit)

d_i Demand rate of the retailer i (product / time unit)

Q_i Lot size for the retailer i (product)

δ_i^j Delivery start time of the *j*th lot to the retailer i

Q_i^j *j*th lot to be delivered at the instant δ_{ij} to retailer i

Q^{max} Maximum delivery lot size

Tr_i *(t)* Lots under transportation to retailer i at time t

$q_{\alpha\beta}$ Transition rate from mode α to mode β

MTTF Mean time to failures

MTTR Mean time to repair

τ_i Transportation delay to retailer i (time unit)

c^+ Holding cost (\$/time/ product)

c^- Backlog cost (\$/time/ product)

c_i^t Fixed transportation cost for retailer i (\$/delivery)

c_v Variable transportation cost (\$/product)

ρ Discounted cost

2.3.2 Description of the system under study

We study a two-stage supply chain presented in Figure 2.1, which is made of a manufacturer and multiple retailers (n retailers) who serve the end customers. The production facility is unreliable and can face random failures and repairs. One type of product is produced and stored in a local warehouse (MW) and then shipped to the retailers, which are located in different locations. The products are shipped in different lots size Q_i (for the retailer i). At the beginning of lot delivery to retailer i, the inventory level $x_m(t)$ at the local warehouse of the manufacturer decreases in a finite jump of a lot size Q_i. On the other side, when the lot is unloaded at the retailer's warehouse (RW_i), its inventory level $x_i(t)$ increases in a finite jump of a lot size Q_i.

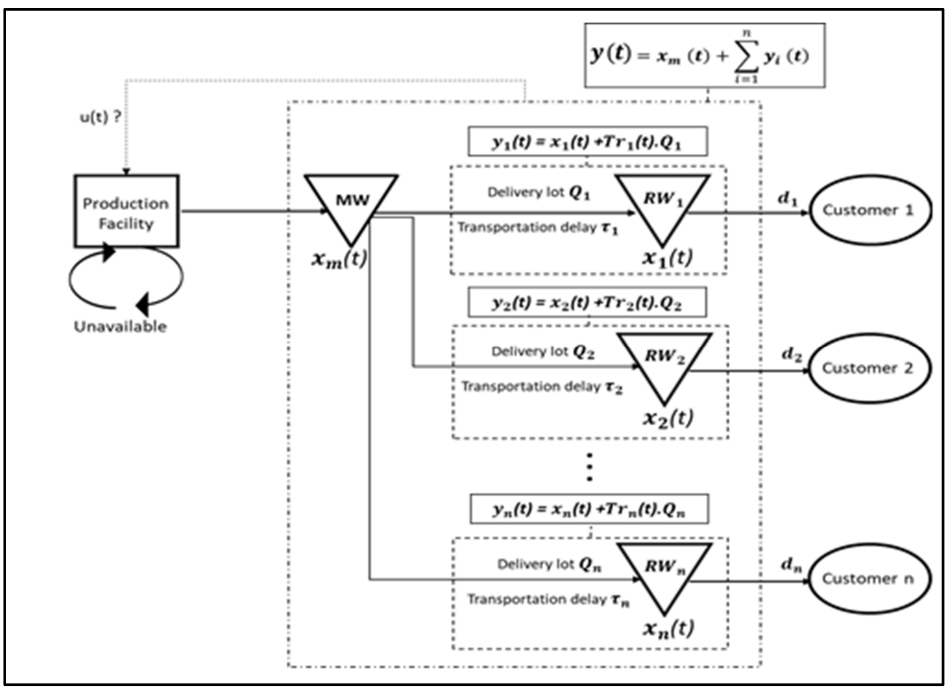

Figure 2.1 The one-manufacturer multiple-retailers system under study

The main assumptions are as follows:

- The manufacturing system produce good quality products.

- The rates of customers demand are known and constant.
- The transportation means are reliable and available to transport at any time.
- Shortages are allowed and we consider a backlog cost.

2.3.3 Optimization problem formulation

The delivery by lots introduces an impulsive aspect to the problem that continuous-flow models cannot reproduce; therefore, we used a stochastic dynamic programming approach using the impulsive control theory (Yong 1989, Yang 1999). Describing the dynamics of the system and formulating the control problem requires defining the system's state at the time t which is composed of four main parts:

- A discrete/continuous variable $x_m(t)$ that represents the level of inventory at time t at the manufacturers warehouse, this quantity increases by a continuous upstream rate equal to the production rate and decreases by a downstream impulse with finite jump Q_i^j at the moment of transportation of the lot j to the retailer *i* at time δ_i^j.

- A discrete/continuous variable $x_i(t)$ that represents the level of inventory at time t at the warehouse of the retailer *i*. This quantity increases with finite jumps of quantity Q_i^j when the lot j arrives at the retailer's warehouse at instant $\delta_i^j + \tau_i$. It also faces a continuous downstream demand d_i.

- A discrete/continuous variable $y_i(t)$ to describe the inventory positions at time t of the retailer *i* and represents the sum of the stock level $x_i(t)$ and the lot under transportation to retailer *i*. Thus, the inventory position for the retailer *i* is: $y_i(t) = x_i(t) + Tr_i(t).Q_i$. Where $Tr_i(t)$ is a binary variable that is equal to 1 when a lot is delivered to retailer i and takes the value 0 otherwise.

- A continuous stochastic process that defines whether the production facility is operational or not at time t, and it is defined by the random variables $\alpha(t)$ with values in $M = \{1,2\}$.

$$\alpha(t) = \begin{cases} 1 & if\ the\ manufacturing\ system\ is\ operational \\ 0 & if\ the\ manufacturing\ system\ is\ down \end{cases}$$

The stochastic process $\alpha(t)$ is characterized by the transition rates matrix which is defined by $T = \{q_{\alpha\beta}\}$. With $q_{\alpha\beta} \geq 0$ if $\alpha \neq \beta$ and $q_{\alpha\alpha} = -q_{\alpha\beta\ (\alpha\neq\beta)}$, where $\alpha, \beta \in M$.
The dynamics of the inventory levels are defined by the differential equations (2.1) presented below:

$$\begin{gathered} \frac{dx_m(t)}{dt} = u(t,\alpha) \quad \forall t \in]\delta_i^j, \delta_i^{j+1}[\\ x_m\big((\delta_i^j)^+\big) = x_m\big((\delta_i^j)^-\big) - Q_i^j \\ \frac{dx_\mathrm{i}(t)}{dt} = -d_\mathrm{i} \quad \forall t \in]\delta_i^j + \tau_\mathrm{i}, \delta_i^{j+1} + \tau_\mathrm{i}] \\ x_\mathrm{i}\big((\delta_i^j + \tau_\mathrm{i})^+\big) = x_\mathrm{i}\big(\delta_i^j + \tau_\mathrm{i})^-\big) + Q_i^j \\ i \in \{1, \dots, n\} \quad , j \in \{1, \dots, \infty\} \end{gathered} \tag{2.1}$$

$(\delta_i^j)^+$ and $(\delta_i^j)^-$ define the left and right limits of the *j*th delivery start time δ_i^j for the retailer i.
$(\delta_i^j + \tau_\mathrm{i})^+$ and $(\delta_i^j + \tau_\mathrm{i})^-$ define the left and right limits of the *j*th receipt time $(\delta_i^j + \tau_i)$ for the retailer *i*.

The production rate and the delivery sizes are constrained by the capacity of the machine and the capacity of the transportation mean.

$$\begin{gathered} 0 \leq u(t,\alpha) \leq u_{max} \\ 0 \leq Q_i^j \leq Q^{max} \\ \{i,j\} \in \{1, \dots, n\} \times \{1, \dots, \infty\} \end{gathered} \tag{2.2}$$

The production rate $u(.)$ and the sequence of deliveries defined below are the decision variables:

$\Omega_i = \{(\delta_i^1, Q_i^1), (\delta_i^2, Q_i^2), \dots, (\delta_i^\infty, Q_i^\infty)\}$, where (δ_i^j, Q_i^j) represents the delivery order by the retailer i of the *j*th lot Q_i^j at time δ_i^j. For $\{i, j\} \in \{1, \dots, n\} \times \{1, \dots, \infty\}$

For the following we define:

$\Pi = (x_1(t), x_2(t), \dots, x_n(t))$, $\Pi_p = \{x_1(t), x_2(t), \dots, x_n(t)\}/\{x_p(t)\}$ and $\Lambda = (\Omega_1, \Omega_2, \dots, \Omega_n)$

The set of admissible decisions (u, Λ) denoted by $\Gamma(\alpha)$ is given by:

$\Gamma(\alpha) = \left\{(u(.), \Lambda) \middle| 0 \le u(t,\alpha) \le u_{max}, 0 \le Q_i^j \le Q^{max}, i \in \{1, \dots, n\}, j \in \{1, \dots, \infty\}\right\}$

For penalizing the inventory (holding and backlog) cost and the production rate we define the following instantaneous cost function *g*(.):

$$g\big(x_m(t), \Pi, u(t,\alpha)\big) = c^+ \begin{pmatrix} x_m + \sum_{i=1}^n x_i^+ \\ + \sum_{i=1}^n ind\{t \in]\delta_i^j, \delta_i^j + \tau_i[\}. Q_i^j \end{pmatrix} + c^- \sum_{i=1}^n x_i^-(t), \ \forall t \in]\delta_i^j, \delta_i^{j+1}[\tag{2.3}$$

where $x_i^+ = \max\big(0, x_i(t)\big)$, $x_i^- = \max(-x_i(t), 0)$; $i \in \{1, \dots, n\}$

Morever, we define the instantaneous cost functions for transportation at times δ_i^j ,$\{i, j\} \in \{1, \dots, n\} \times \{1, \dots, \infty\}$, as follows:

$$R_i(Q_i^j) = Ind\big(t = \delta_i^j\big)\big(c_i^t + c_v Q_i^j\big) + \int_0^{\tau_i} e^{-\rho t} g(x_m + u.t, x_i - d_i.t)dt \tag{2.4}$$

Where $Ind\big(\Theta(.)\big) = \begin{cases} 1 & if\ \Theta(.)\ is\ true \\ 0 & otherwise \end{cases}$

The overall infinite horizon discount cost $J(.)$ is defined based on the equations (2.3) and (2.4) and it is as follows:

$$J(x_m, \Pi, \Lambda, u, \alpha) = E\left\{ \begin{matrix} \int_0^{\infty} e^{-\rho t} g(x, \Pi, u) dt \\ + \sum_{i=1}^{n} \sum_{j=1}^{\infty} e^{-\rho \delta_i^j} (c_i^t + c_v Q_i^j) \end{matrix} \middle| \begin{matrix} x_m(0) = x_0, \\ x_\mathrm{i}(0) = x_{\mathrm{i}_0}; i = \{1, \dots, n\} \\ \alpha(0) = \alpha \end{matrix} \right\} \tag{2.5}$$

Where $E(.)$ is the conditional expectation given the initial conditions of (x_m, Π, α).

The control problem studied herein is to find (u^* , Λ)$\in$ $\Gamma(\alpha)$ that minimizes $J(.)$ From the equation (2.5). The problem is a feedback control policy where the production rate and the delivery decisions are dependent on the system state. The associated value function is given by:

$$v(x_m, \Pi, \alpha) = \inf_{(\Omega_\mathrm{i}, u) \in \Gamma(\alpha)} J(x_m, \Pi, \Lambda, u, \alpha) \quad \forall \alpha \in M \ , \ \ i \in \{1, \dots, n\} \tag{2.6}$$

Based on the optimal impulse control theory (Sethi and Thompson 2005), The value function $v(x_m, x_1, x_2, \alpha)$ is shown to be the unique viscosity solution for the HJB equation (2.8). The proof can be developed as in Yong (1989) by considering the delivery decision as a 'stopping decision' with cost given by equation (2.7).

$$Ow(x_m, \Pi, \alpha) = \min_{i \in \{1, \dots, n\}} \left\{ \min_{(j, Q_i^j) \in \Gamma} \left\{ \begin{matrix} R_i\big(Q_i^j\big) \\ + e^{-\rho \tau_i}\, v\big(x_m - Q_i^j + \tau_i d_i, x_i + Q_i^j - \tau_i d_i, \Pi_i, \alpha\big) \end{matrix} \right\} - v(x_m, \Pi, \alpha) \right\} \tag{2.7}$$

$$\min\left\{\begin{array}{c}\min\limits_{u}\left\{\begin{array}{c}uv_{x_m}-\sum\limits_{i=1}^{n}d_iv_{x_i}+g(x_m,\Pi,u)\\+\sum\limits_{\beta\neq\alpha}q_{\alpha\beta}\big(v(x_m,\Pi,\beta)-v(x_m,\Pi,\alpha)\big)\end{array}\right\}-\rho v(x_m,\Pi,\alpha);\\ \min\limits_{i\in\{1,\ldots,n\}}\{\min\limits_{(j,Q_i^j)\in\Gamma}\left\{\begin{array}{c}R_i\big(Q_i^j\big)\\+e^{-\rho\tau_i}\,v\big(x_m-Q_i^j+\tau_id_i,x_i+Q_i^j-\tau_id_i,\Pi_i,\alpha\big)\end{array}\right\}\\-v(x_m,\Pi,\alpha)\}\end{array}\right\}=0 \quad (2.8)$$

where v_{x_m}, v_{x_i} represents respectively the gradients of $v(.)$ with respect to x_m and x_i . More details of the optimality conditions given by the equation (2.8) and the elementary properties of the value function are presented in the work of Hajji et al. (2009).

Knowing that the analytical resolution of the HJB equations (equation (2.8)) is difficult, we solve them using a numerical approach.

2.3.4 Numerical approach

Because of the Markovian aspect of the studied model, we apply the stochastic dynamic programming approach for a negligible transportation delay case ($\tau_i \approx 0$) to facilitate the implementation process (the lots are immediately received by the retailers). Therefore, the HJB equations become:

$$\min\left\{\begin{array}{c}\min\limits_{u}\left\{\begin{array}{c}uv_{x_m}-\sum\limits_{i=1}^{n}d_iv_{x_i}+g(x_m,\Pi,u)\\+\sum\limits_{\beta\neq\alpha}q_{\alpha\beta}\big(v(x_m,\Pi,\beta)-v(x_m,\Pi,\alpha)\big)\end{array}\right\}-\rho v(x_m,\Pi,\alpha);\\ \min\limits_{i\in\{1,\ldots,n\}}\{\min\limits_{(j,Q_i^j)\in\Gamma}\left\{R_i\big(Q_i^j\big)+\ v\big(x_m-Q_i^j,x_i+Q_i^j,\Pi_i,\alpha\big)\right\}\\-v(x_m,\Pi,\alpha)\}\end{array}\right\}=0 \quad (2.9)$$

Kushner approach (Kushner and Dupuis 1992) is applied to numerically obtain the structure of the optimal control policy. The approach is applied, without losing generality, for the case of two retailers and described in the Appendix I. The algorithm used to solve the problem is presented in the Appendix II.

2.4 Numerical results

2.4.1 Numerical example

A numerical example is presented to characterize the optimal control policy and the problem is solved for the case of two retailers. The data we used are presented in table 2.2.

Table 2.2 The data used in the numerical example

u_{max}	d_1	d_2	ρ	q_{01}	q_{10}	c^+	c^-	c_1^t	c_2^t	c_v
3	2	0.5	0.4	0.3	0.04	0.1	15	50	80	0.01

The computational domain D_h is defined by: $D_h = \{(x_m, x_1, x_2): 0 \leq x_m \leq 50, -25 \leq x_1 \leq 100, -25 \leq x_2 \leq 100\}$ with $h_{x_m} = 1$, $h_{x_1} = 2.5$, $h_{x_2} = 2.5$

For the implementation process, we took into consideration several elements to characterize the control policy. Thus, we observe separately the production and delivery policies. For each numerical result, $u(x_t)$, $Q_1(x_m, x_1)$ and $Q_2(x_m, x_2)$ are the production policy, the delivery policy for the retailer 1 and the delivery policy for the retailer 2, respectively, as shown in the figures 2.2 and 2.3.

The rate of production is observed in function of the sum of the inventory levels of the manufacturer (x_m), the retailer 1 (x_1) and the retailer 2 (x_2) since the manufacturer and the retailers are cooperating in order to serve the demand, thus, the whole system's inventory is facing the demand.

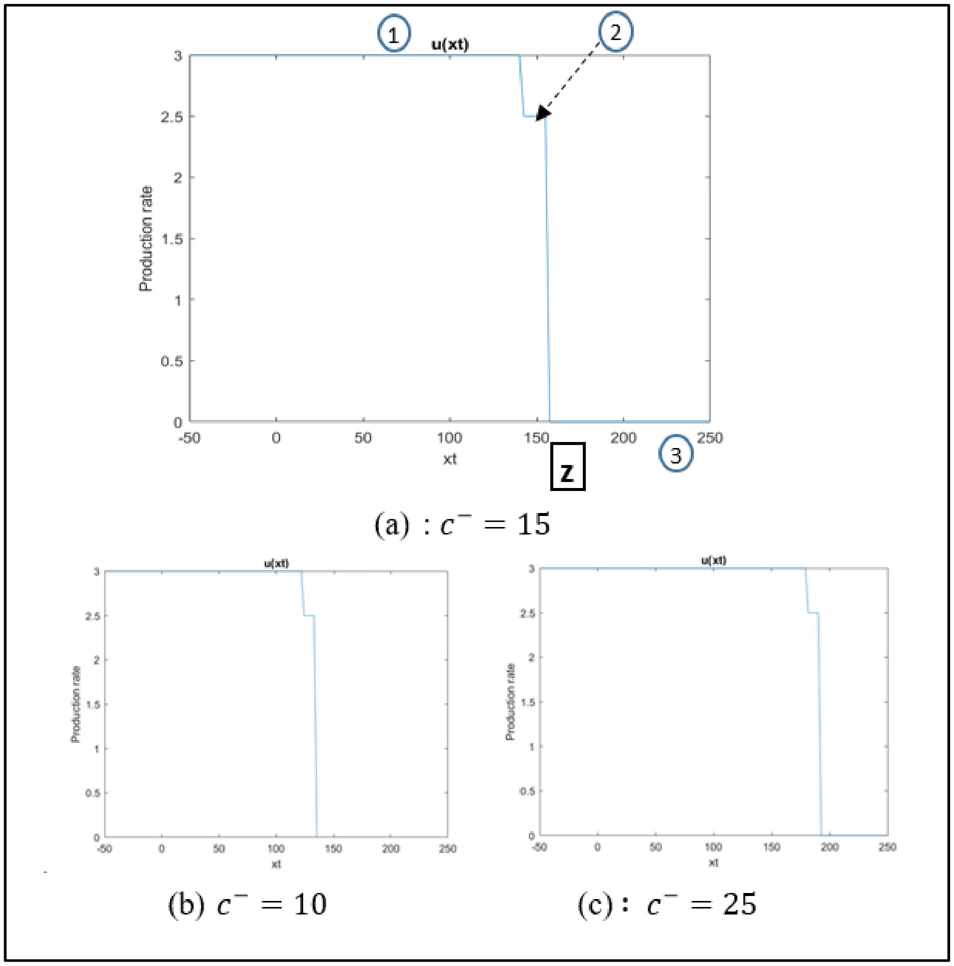

Figure 2.2 Production control policy

The figure 2.2 (a) shows that the surplus space is characterized by two regions which defines the production policy. In region 1, the rate of production is set to its maximum, in region 2, it is set to the sum of the retailers' demand rates $(d_1 + d_2)$ and in region 3, the rate of production is set to zero. The results indicate a type of production control based on a base stock policy (BSP), given that the production rate depends on the total inventory level. The production policy is controlled by hedging level Z as seen in figure 2.2.

Moreover, Figure 2.3 indicates that the delivery policy is controlled by a 'state dependent economic order quantity' policy (SD-EOQ). Two regions divide the surplus space. Region 4 is controlled by an economic order quantity (Q_1or Q_2). The delivery happens when the lot size is available in the manufacturer warehouse x_m and if the finished products' inventory level at the retailers' warehouses is lower than the retailers' reorder point (S_1or S_2); and in region 5, no lot is transferred to the retailers' warehouses ($Q_1 = 0$ or $Q_2 = 0$).

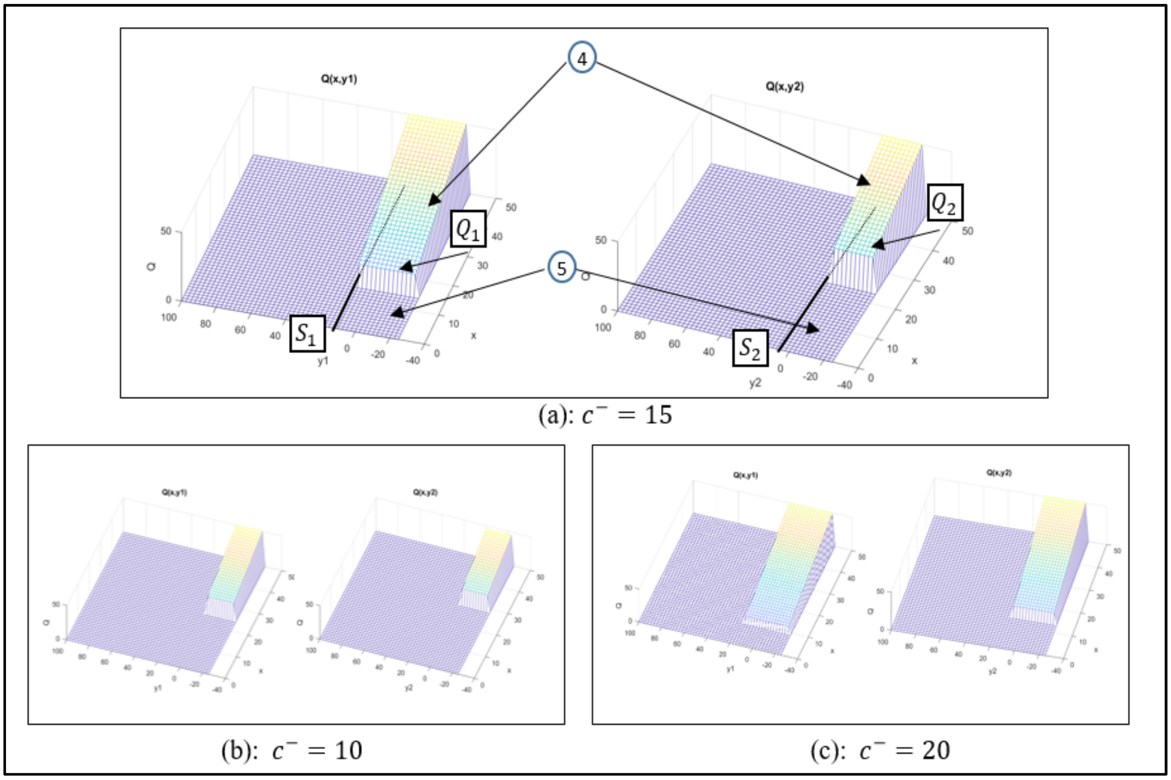

Figure 2.3 Delivery control policy

The structure of the optimal policy is confirmed by conducting a sensitivity analysis that verifies the adequacy of the policy. As an example, we show the effects of varying the backlog cost on the hedging level in figure 2.2. By decreasing the backlog cost in fig. 2.2 (b), the hedging level Z takes lower values to prevent form the high holding costs. The opposite effect happens when the backlog cost is increased (fig. 2.2 (c)) as we can see that the hedging level increases subsequently to protects the system from the high backlog costs caused by shortages. In figure 2.3 we see the effect of varying the backlog cost on the lot size (Q_1 and Q_2) and the reorder points (S_1 and S_2). By increasing the backlog cost in fig. 2.3 (c), the lot sizes decrease to ensure more frequent deliveries and better supply for the retailers, and the reorder points increase to protect the retailers from shortages. The opposite effect is seen when we decrease the backlog cost in fig. 2.3 (b). Therefore, we determine that the optimal control policy combines a BSP and SD-EOQ policies.

2.4.2 Structure of the control policy (Policy 1)

The numerical results show that we can define an approximated optimal control policy for a negligible transportation delay as a combined BSP and SD-EOQ policies, which we can express as follows:

Production policy:

$$u(t,\alpha)=\begin{cases} u_m & if\ \ x_m(t)+\sum_i^n x_i(t) < Z\ and\ \alpha(t)=1 \\ \sum_{i=1}^n d_i & if\ \ x_m(t)+\sum_i^n x_i(t) = Z\ and\ \alpha(t)=1 \\ 0 & if\ \ x_m(t)+\sum_i^n x_i(t) > Z\ or\ \alpha(t)=0 \end{cases} \tag{2.10}$$

Delivery policy for retailer i:

$$\Omega_i(t,Q_i) = \begin{cases} Q_i & if\ x_i(t) \le S_i \\ 0 & otherwise \end{cases} \tag{2.11}$$

$$i \in \{1,\dots,n\}$$

Recall that Z is the threshold parameter, S_i is the reorder points of the retailer *i* and Q_i is the economic lot size for the retailer *i* respect the following constraints:

$Z \ge 0\,; S_i \ge 0\ and\ Q_i > 0\,; i \in \{1,\dots,n\}$

The previous policies are defined for negligible transportation delays, to consider these delays we integrate the notion of inventory position. This notion was introduced by Mourani *et al*. (2008) and Li *et al*. (2009) who studied production planning problems with significant transportation delays and they proposed control policies that depends on the inventory position, which considers the inventory in the warehouses as well as the lots under transportation. The inventory levels $x_i(t)$ at the retailer i can be respectively expressed in terms of his inventory position $y_i(t)$, the demand during the transportation delay, the relationship between these terms is written as follows :

$$x_i(t) = y_i(t-\tau_i) - \tau_i . d_i, \ \forall t \geq \tau_i \tag{2.12}$$

We note from the previous relationships that, for a negligible transportation delay, the inventory position $y_i(t)$ coincide with the inventory level $x_i(t)$.

Moreover, the obtained policies from the numerical results does not show exactly how the delivery decisions are made when the inventory levels at the retailers reach their reorder points at the same time or when one retailer reaches his reorder point while another retailer is waiting for the manufacturer to replenish his stock to be able to serve him. Therefore, to consider all these issues, we introduce an extended policy based on the control policy obtained from the numerical resolution.

Based on the inventory position concept we define the inventory positions at the retailer i, and the whole system, respectively, as follows:

$$\begin{gathered} y_i(t) = x_i(t) + Tr_i(t). Q_i \\ y(t) = x_m(t) + \sum_{i=1}^{n} y_i(t) \end{gathered} \tag{2.13}$$

The proposed control policy is the following:

<u>Production policy:</u>

$$u(t,\alpha) = \begin{cases} u_m & if \ \ y(t) < Z \ and \ \alpha(t) = 1 \\ \sum_{i=1}^{n} d_i & if \ \ y(t) = Z \ and \ \alpha(t) = 1 \\ 0 & if \ \ y(t) > Z \ or \ \alpha(t) = 0 \end{cases} \tag{2.14}$$

<u>Delivery policy for retailer i</u>:

$$\Omega_i(t, Q_i) = \begin{cases} Q_i & if\ y_i(t) \leq S_i\ and\ x_m(t) \geq Q_i \\ 0 & otherwise \end{cases} \tag{2.15}$$

$$i \in \{1, \ldots, n\}$$

The hedging level (Z), the retailers reorder points (S_i) and the lot sizes (Q_i) represent the control parameters that characterize the control policy.

The numerical approach by itself is subject to many limitations in terms of its implementation process and the irregularities found in the boundary of its results (Berthaut et al., 2010). In the same sense, the size of the grid steps h_{x_m} , h_{x_1} and h_{x_2} influence the accuracy of the results; smaller steps give more accurate results, which is too time-consuming. Moreover, in our case we neglect transportation delays in the numerical approach due to the difficulty of its implementation. For these reasons, we chose to proceed with a simulation-based approach that we describe in following section for the advantages that it has compared to the numerical approach. Some of these advantages are the capability of being applicable at an operational level, and that it allows us to consider important system parameters such as transportation delays, any other distribution probabilities and priority rules for retailers.

2.5 Resolution approach

To optimize the control parameters, we use a combined approach using analytical and simulation models with statistical analysis. The used steps for the approach, presented by the block diagram in the figure 2.4, are the following:

- Step 1- Control problem formulation and analytical approach: In this step, we develop a stochastic dynamic programming model representing the system based on the control theory. Then, using a numerical approach, the obtained HJB equations are solved. (See Section 3)

- Step 2 - Control policy: The structure of the optimal control policy (policy 1) is obtained from the numerical results, and the control parameters for production and delivery are determined. (See Section 4)

- Step 3 - Design of experiments: it describes how the parameters of the control policy (Z, Q_i, S_i), are varied to obtain a minimal number of experiments. (See Section 6.1)

- Step 4 - Simulation model: A combined discrete-continuous simulation model that reproduces the behaviors of the manufacturing system is developed. It takes the control policy structure as input and conducts the defined simulation experiments. It provides an evaluation of the performance of the policies. Therefore, for each combination of the control parameters, it provides the total incurred cost. (See Section 6.2)

- Step 5 - Response surface methodology (RSM): The main significant factors and their interactions and their quadratic effects are used in a regression analysis, jointly with the RSM, to define the relationship between the cost and the design parameters (Z, Q_i, S_i). (See Section 6.4)

- Step 6 - Optimal parameters of the control policy: The optimal values of the parameters of the control policy (Z^*, Q_i^*, S_i^*) and the optimal cost are obtained by optimizing the provided regression model. (See Section 6.4)

- Step 7- Sensitivity analysis: We test the proposed policies by varying the system parameters and observe how this affects the optimal parameters to confirm the effectiveness of the policies. (See Section 7)

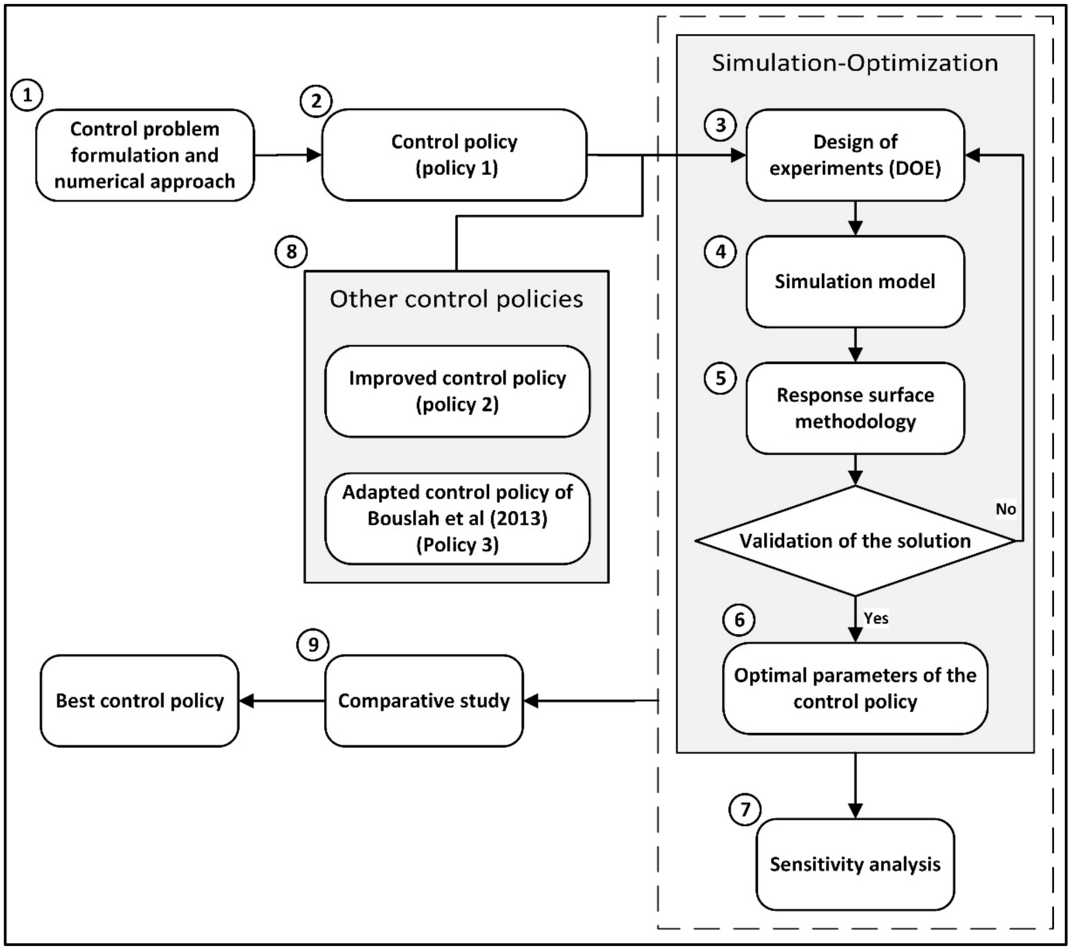

Figure 2.4 Block diagram of the solution approach

- Step 8- Other control policies: we propose an improved control policy that considers priority for retailers (policy 2) and we adapt the policy found in the most relevant research work to our system (policy 3). (See Section 8)

- Step 9- Comparative study: We compare between policy 1, policy 2 and policy 3 with the objective of determining the best control policy cost wise. (See Section 9)

2.6 Simulation optimization

2.6.1 Experimental design

Without losing generality, we consider two retailers in our simulation-based approach. Therefore, we used a complete experimental design that includes four independent variables (Z, Q, S_1, S_2) and one dependant variable (the total expected cost). Q here defines a common lot size for both retailers. This choice is motivated by the slight difference in optimal total cost (<1%) when compared to the case with five independent variables (Z, Q_1, Q_2, S_1, S_2). In fact, we will conduct many simulation optimizations and this choice will considerably reduce the number of simulation runs. We also use the minimum number of replications for this design, which necessitate 162 simulation runs in total (3^4×2) for the simulation optimization of each case.

2.6.2 Simulation model

A combined discrete-continuous simulation model is proposed for the studied system using the SIMAN language and C++ subroutines, and then it was executed through Arena simulation software. The figure 2.5 shows the block diagram that illustrates the main networks and the user routines describing the system.

1. The INITIALIZATION block defines the values of the input parameters including the design factors (Z ,Q, S_1 ,S_2), the maximum production rate (U_m), the demand rate (d), the transitions' values between the different modes, the simulation time (T_{end}).

2. The FAILURES AND REPAIRS block models the time between failures and the time to repair which are calculated using the MTTF and MTTR distributions, it indicates the state of the machine and communicates it to the state equation (2.5).

3. The STATE EQUATION block describes the dynamics of the system in a C++ language by instantaneously calculating the levels of inventory based on the differential equation of equation (2.1).

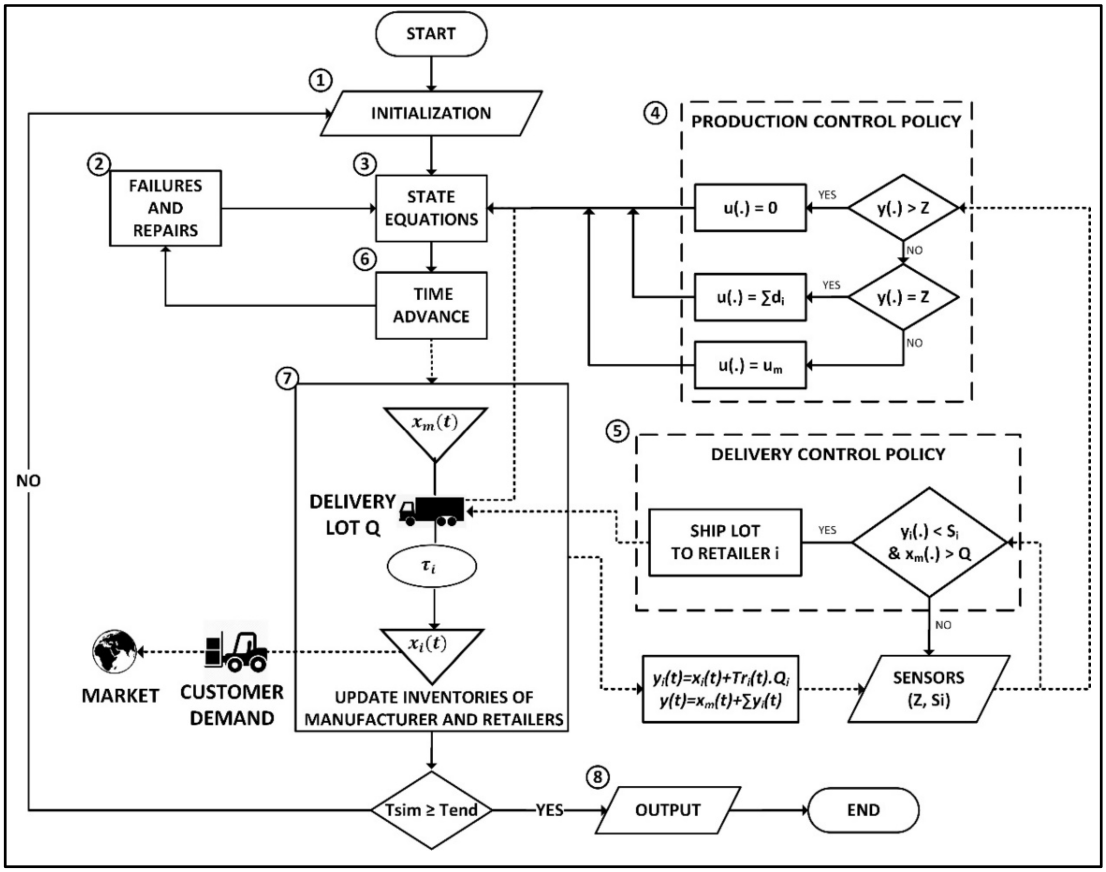

Figure 2.5 Simulation block diagram

4. The PRODUCTION POLICY block sets the right rate of production determined by comparing the current inventory position of the system to the determined threshold.

5. The DELIVERY POLICY block defines the delivery decisions by comparing the inventory positions at the retailers to the reorder points, sensors are noticed when the inventory position of a retailer cross the corresponding reorder point.

6. The TIME ADVANCE block updates the current time.

7. The UPDATE STOCK LEVEL block defines the dynamics of the inventory levels (x_m, x_i, y_i) including the transported lots from the manufacturer to the retailers.

8. The OUTPUT block gives time persistent statistics of the outputs at the end of the simulation, such as positive and negative stock levels and the number of shipments. The total incurred cost is calculated using this information.

If the current simulation time T_{sim} reaches the defined simulation period T_{end}, the simulation ends.

2.6.3 Validation of the simulation model

The accuracy of the simulation model is verified and validated by graphically examining its behavior. Figure 2.6 illustrates the dynamics of the inventory levels for a sample of the simulation run behave according to the production-delivery control policy 1. In fact, it is shown that, at any given time, if the inventory level is under the hedging level Z, the machine is set to produce at its maximum rate (u_m) (area 1 in figure 2.6), if the inventory level reaches the hedging level Z, it is set to produce at a rate equal to the sum of the demand rates of the retailers (d_1+d_2) in area 2.

The manufacturer inventory level follows the rate of the production, it replenishes faster when the production is at its maximum (arrow 3 and 4) and decreases by jumps of Q (arrow 5) when a lot is shipped. The manufacturer inventory is preserved at the same level when the machine is down ($u_m = 0$) (arrow 6). It is worth mentioning that the manufacturer's inventory level can exceed the lot size Q (arrow 7). This allows the manufacturer to ship to both retailers at the same time when they reach their reorder points simultaneously, and when it has the sufficient inventory to ship both orders at the same time. The trajectory of the retailers inventory show that the delivery policy is accurate and follows the (s,Q) policy described in the equation (2.15). In fact, when their inventory level reaches their reorder points (arrow 8 and 9), and when the manufacturer is able to serve them (has enough inventory), the lot is shipped, and it is received after the corresponding transportation delay (arrow 10 and 11).

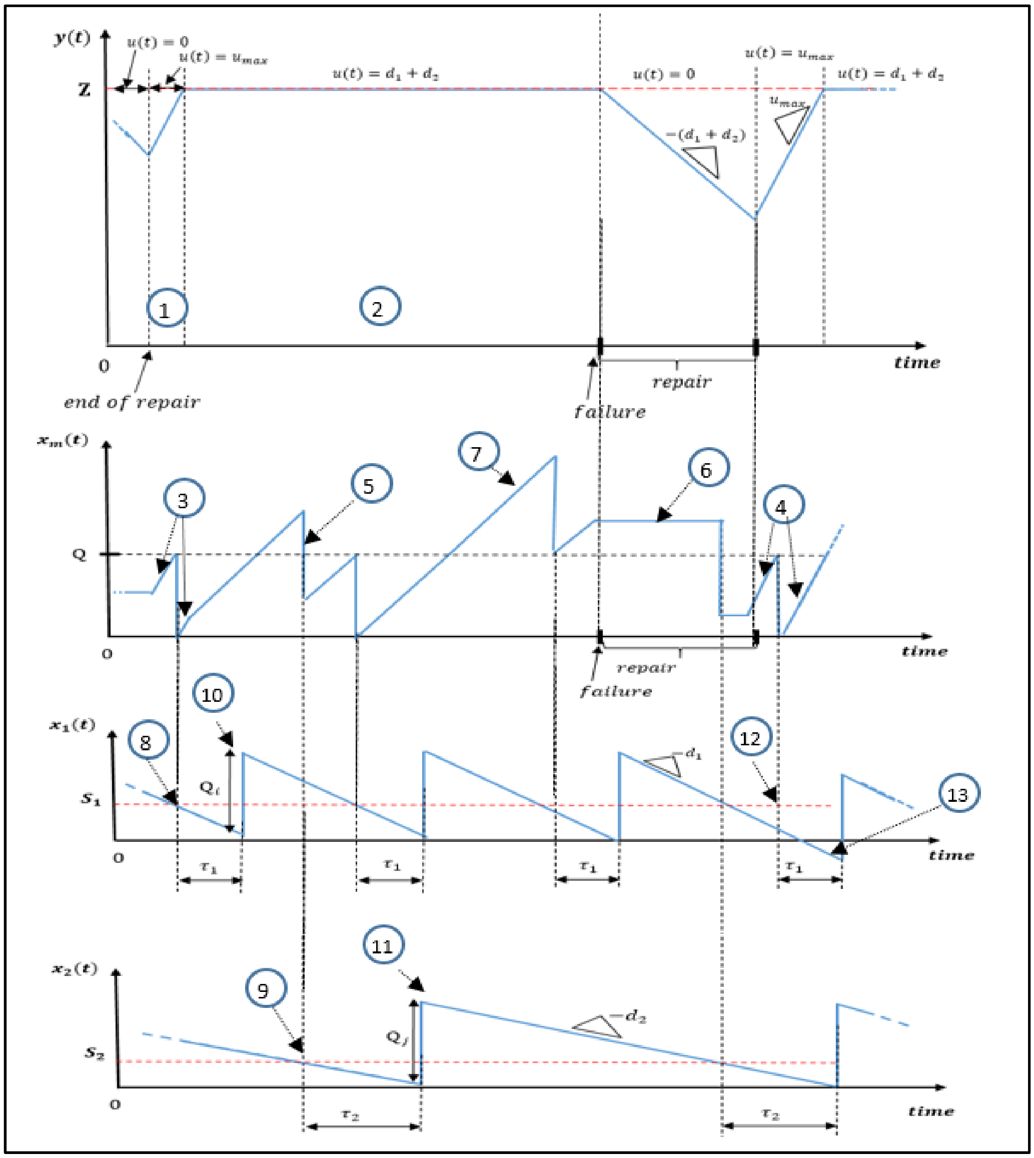

Figure 2.6 Dynamics of the simulation model

When the retailer's inventory levels reach their corresponding reorder point and the inventory of the manufacturer is insufficient (less than Q), the shipment of the lot is postponed till the manufacturer has enough inventory to serve him (arrow 12), this waiting delay may lead to higher risk of a backlog of finished products at the retailers' warehouse (arrow 13). According to the graphics presented in figure 2.6, we verify that the simulation model reproduces accurately the system dynamics.

2.6.4 Numerical example

The optimal parameters (Z^*,Q^*,S_1^*,S_2^*) of the joint production-delivery control policy 1 are determined using a numerical example, the data of the example are defined in table 2.3 below:

Table 2.3 The system parameters

Parameter	$\boldsymbol{u_{max}}$	$\boldsymbol{d_1}$	$\boldsymbol{d_2}$	$\boldsymbol{\tau_1}$	$\boldsymbol{\tau_2}$	$\boldsymbol{MTTF}$
Value	2000	750	400	2	3	$Exp(10)$
Parameter	$\boldsymbol{c^+}$	$\boldsymbol{c^-}$	$\boldsymbol{c_1^t}$	$\boldsymbol{c_2^t}$	$\boldsymbol{c_v}$	$\boldsymbol{MTTR}$
Value	0.5	5	6	8	0.01	$Exp(1.5)$

The values of u_{max} and the demand rates d_1 and d_2 are chosen in respect to the feasibility condition of the system described as follows: $\frac{MTTF}{MTBF+MTTR}u_{max} > d_1 + d_2$

The model is simulated for a duration of to 200,000 time units, which allows to observe more 20000 failures in each replication which ensures that the steady state is reached. The main effects of the design factors (Z, Q, S_1, S_2) and the majority of their quadratic and interactions effects are significant for the dependant variable at a 95% level of significance (as shown in the Pareto chart in figure 2.7). The obtained R-squared adjusted is equal to 95,52, which means that over 95% of the observed variability is explained by the model.

The total expected cost is fitted by the quadratic model in eq. (2.16) obtained from STATGRAPHICS:

$$\begin{gathered}\Phi(Z, Q, S_1, S_2) = 14895.6 - 1.25 . Z - 1.72 . S_1 - 0.18 . S_2 - 0.20 . \\ Q + 1.91. 10^{-4}. Z^2 \ +0.4510^{-4}\ ZS_1 - 2.04 .10^{-4}\ ZS_2 - \\ 5.2. 10^{-4}\ ZQ + 4.69. 10^{-4}\ {S_1}^2 - 2.75 . 10^{-4}. S_1Q - 0.28.10^{-4} S_1\ Q \\ +6.51. 10^{-4}\ {S_2}^2 + 2.52.10^{-4} S_2\ Q + 6.22. 10^{-4}\ Q^2 + \varepsilon\end{gathered} \tag{2.16}$$

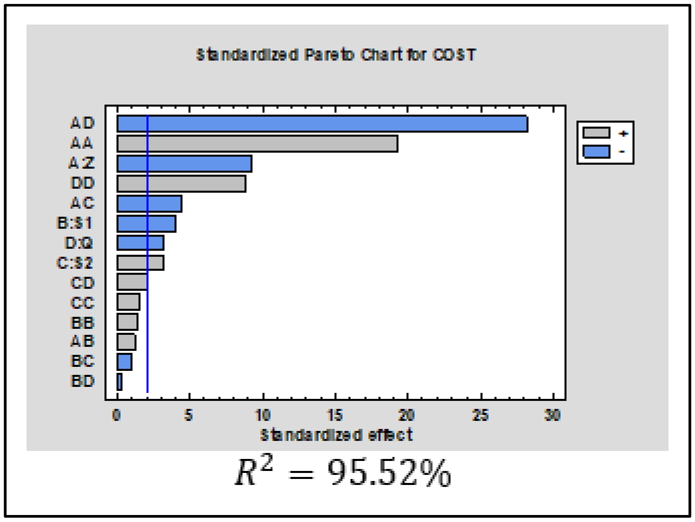

Figure 2.7 Pareto chart

The optimal total cost of 7269$ corresponds to $Z^* = 8829$, $S_1^* = 1874$, $S_2^* = 1205$ and $Q^* = 3705$. The figure 2.8 displays the contour surface plots of the total cost in function of the optimal policy parameters.

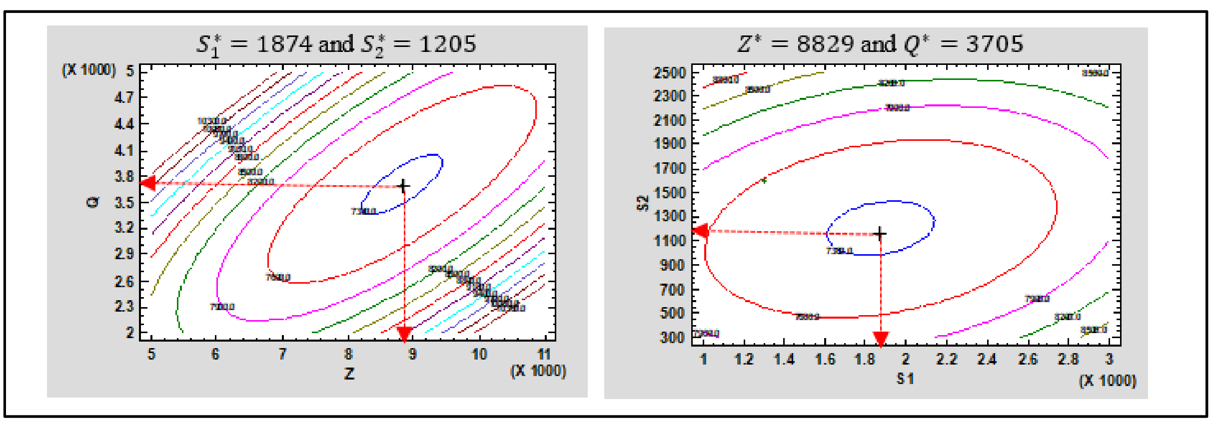

Figure 2.8 Contour surface plots

2.6.5 Validation of the RSM model

To validate the RSM model we conducted a residual analysis to verify its adequacy, the normal probability plot of residuals and the residual versus fitted values plot confirmed the residual normality and the homogeneity of the variances. After the optimization, we performed Student's test to crosscheck the validity of the model. The optimal total cost 7269$ for the policy falls within the 95% confidence interval [7251-7282] (Banks et al., 2005), that we obtained by simulating the model with the optimal parameters as inputs for 40 replications.

We also validate the model by considering the case of similar retailers, in this case we considered that the retailers face the same demand rate d $(d = d_1 = d_2 = 500\ u/h)$, and the transportation delays are the same for both retailers τ $(\tau = \tau_1 = \tau_2 = 2\ hours)$. For this case, since the retailers are similar, we considered a common lot size Q and a common reorder point S. The results showed that for a high machine availability (100%) the reorder point of the retailers is equal to its theoretical value S= $\tau \times d = 1000$ (figure 2.9.b).

2.7 Sensitivity analysis

The effectiveness of the obtained polices is verified by examining the effect of the variation of the different cost parameters and the machine availability on the optimal design parameters and the optimal total expected cost.

2.7.1 Effects of the cost variation

In table 2.4 we present eight different cases, in each case we vary a cost parameter, and we study the optimal parameters and the optimal total cost variations (i.e., respectively, ΔZ^*, ΔS_1^*, ΔS_2^*, ΔQ^* and ΔTC^*) for the proposed policy. We did not include the variable cost of transportation $\boldsymbol{c_v}$ in the table 2.4 because it has no significant effect on the optimal parameters. The effects of the cost variations are described below:

- Effect of the holding cost $\boldsymbol{c^+}$: By increasing c^+ (Case 2), Z^* decreases to avoid high holding costs. Q^* takes lower values subsequently with the decrease of Z^*. The reorder points values S_1^* and S_2^* decrease to avoid higher holding costs for the retailers. Conversely, (case 1), the hedging level Z^*, the reorder points S_1^* and S_2^* increase to allow more stock surplus to avoid the increasing backlog cost. The lot size Q^* increases to avoid transportation costs since there is enough stock to protect against shortages.

Table 2.4 Sensitivity analysis for the proposed policy

	Optimal control Parameters		Z^*	S_1^*	S_2^*	Q^*	TC^*	
	Basic Values		8829	1874	1205	3705	7269	
	Cost changes		ΔZ^*	ΔS_1^*	ΔS_2^*	ΔQ^*	ΔTC^*	Remarks
1	c^+	0.25	+2246	+36	+28	+240	-1406	Z*↑ S1*↑S2*↑Q*↑ C*↓
2		0.75	-1720	-21	-14	-179	+1158	Z*↓ S1*↓S2*↓Q*↓ C*↑
3	c^-	2.5	-3170	-48	-36	+98	-949	Z*↓ S1*↓S2*↓Q*↑ C*↓
4		7.5	+1257	+25	+22	-34	+497	Z*↑ S1*↑S2*↑Q*↓ C*↑
5	c_1^t	6	-465	-6	-4	-430	-1281	Z*↓ S1*↔S2*↔Q*↓ C*↓
6		10	+324	+4	+3	+241	+1115	Z*↑ S1*↔S2*↔Q*↑ C*↑
7	c_2^t	4	-386	-3	-5	-364	-1062	Z*↓ S1*↔S2*↔Q*↓ C*↓
8		8	+297	+2	+3	+182	+868	Z*↑ S1*↔S2*↔Q*↑ C*↑

- Effect of the backlog cost $\boldsymbol{c}^-$: By increasing c^-(case 4), the model reacts by increasing the hedging level Z^* which allows a higher stock level in the system, thus, preventing the system against shortages. The optimal reorder points S_1^* and S_2^* increase to ensure a higher inventory levels at the retailers to protect them against possible shortages. Q^* takes smaller values to allow the lot to be produced faster, therefore, enhance the supply to the retailers to avoid the risk of shortages. Opposite effects are produced by decreasing the backlog cost $\boldsymbol{c}^-$ (case 3).

- Effect of the fixed transportation costs $\mathbf{c}_{t1}$ and $\mathbf{c}_{t2}$: When $\boldsymbol{c}_{t1}$ and $\boldsymbol{c}_{t2}$ increase (case 6 and 8), the frequency of deliveries decreases to reduce the transportation costs, this results in increasing the size of Q^*. The hedging level Z^* increases systematically when the optimal lot size is increased to hedge against possible shortages caused by the less frequent deliveries.

2.7.2 Effects of the machine availability

The effect of the machine availability ($\frac{MTTF}{MTTF+MTTR}$) on the different control policy parameters and the total cost is studied for the case of similar retailers defined in section 6.5 as illustrated in figure 2.9.

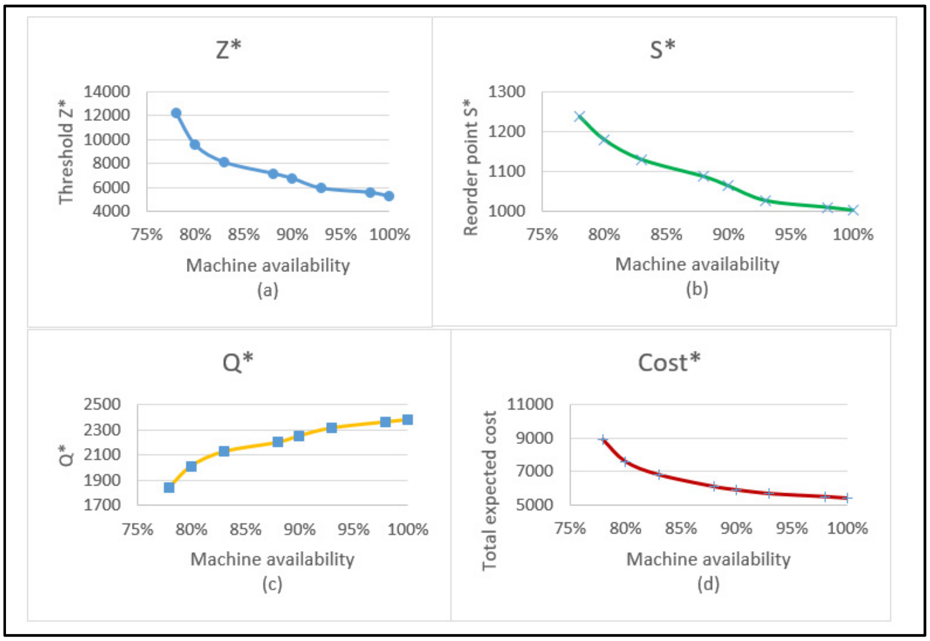

Figure 2.9 Effect of the machine availability
on the optimal parameters and the total cost

For a lower machine availability, the reorder point increases to prevent from the possible shortages. When the machine availability decreases, the system faces more shortages, resulting in a higher cost. Z takes higher values when the machine availability is lower to protect the system from these shortages (Fig. 2.9 (a)) and Q takes lower values when the machine availability is low to ensure deliveries that are more frequent in order to protect against shortages (Fig. 2.9 (c)). S* increases when the machine availability decreases, to ensure enough stock in the retailers' warehouses to protect them against shortages.

The opposite effect happens when the machine availability increases, we see that S* decreases to a value equal to the theoretical reorder point: S$= \tau \times d = 1000$ when the machine is 100% available (Fig. 2.9 (b)).

2.8 Policies considered for comparison

Two questions need to be addressed; first, is there any possible improvement to policy 1 that may give us better performances? And what will occur if we apply one of the closest policies found in the literature to our system? In this section, we respond to these questions by proposing an improvement to policy 1 that involves priority rules (policy 2). We also adapt the most relevant policy we found in the literature to our case (policy 3) and see which one out of these three policies performs the best in terms of cost.

2.8.1 Improved policy with priority rules among retailers (Policy 2)

The policy proposed in the equations 2.14 and 2.15 does not distinguish or prioritize any retailer. When the retailer's reorder point is reached, it allows to his order to be shipped without taking into consideration the other retailers situation. For example, a retailer could face a very high demand rate compared to the other retailers, or he may be located farther from the manufacturer (his transportation delay is much higher), which makes him more vulnerable to shortages, this is translated by a higher reorder point as it involves the interaction between his demand rate and his transportation delay ($S \geq \tau . d$). In this case, if we apply the policy as it is, the prioritized retailer (the one with a higher reorder point) could face more shortages. This situation is experienced especially when the non-prioritized retailer reaches his reorder point while the prioritized retailer has almost reached his reorder point, hence, according to policy 1, the lot is shipped to the non-prioritized retailer anyway. Therefore, the prioritized retailer should wait until the manufacturer finishes replenishing his stock again and have enough inventory on hand to be able to serve him, during this time he may face a high amount of shortages. In these circumstances, some interesting questions should be asked. First, at the moment of shipping an order to a non-prioritized retailer, would it be more valuable if the manufacturer delivers to the prioritized retailer instead, since he is close to his reorder point (in the priority zone)? Second, how could we define this priority zone for the prioritized retailer?

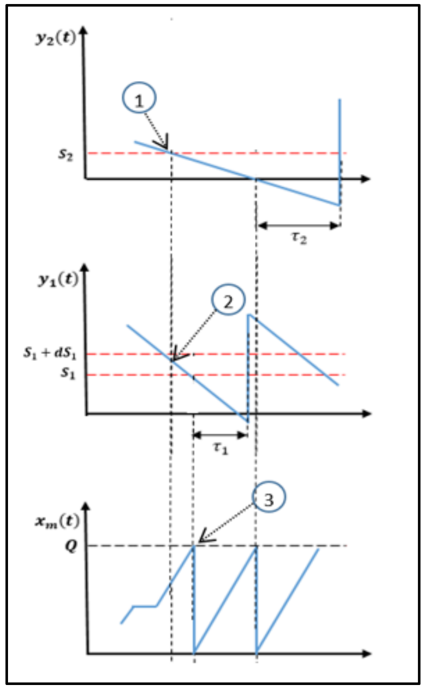

Figure 2.10 Retailer priority zone

We respond to these questions by proposing some improvements to the basic policy (policy 1, eq 2.14- 2.15). In order to consider priority rules, we define a new control policy parameter dS. Therefore, we say that the retailer 1 is in priority zone when: $S_1 \leq y_1 \leq S_1 + dS_1$
In figure 2.10, we illustrate the priority rule by using an example of two retailers and a common lot size Q. In this example, the retailer 1 is the prioritized retailer. If the inventory position of the retailer 2 reaches his reorder point (arrow 1 figure 2.10) while the retailer 1 is in a priority zone, i.e.; $S_1 \leq y_1 \leq S_1 + dS_1$ (arrow 2 in figure 2.10), he should wait for the retailer 1 to be served first (when the manufacturer becomes able to serve him; arrow 3 in figure 2.10). Otherwise, the manufacturer ships the lot for retailer 2 if he has enough inventory on-hand.

By including prioritization rules to the delivery policy, and assuming that, the retailer 1 is the prioritized retailer (has a higher reorder point), the control policy will be redefined as follows:

Production policy: Same as policy 1 (eq 2.14)

Delivery policy:

$$\Omega_1\ (t,Q)=\begin{cases} Q & if\ y_1(t) < S_1 + dS_1 and\ x_m(t) \geq Q \\ 0 & otherwise \end{cases}$$

$$\Omega_2\ (t,Q)=\begin{cases} \mathrm{Q} & if\ y_2(t) < S_2\ and\ [\begin{Bmatrix} Q \leq x_m(t) < 2.Q \\ and\ y_1(t) > S_1 + dS_1 \end{Bmatrix} \\ & or\ \{\ x_m(t) \geq 2.Q\ \}] \\ 0 & otherwise \end{cases} \tag{2.17}$$

2.8.2 Adaptation of a policy from the literature (Policy 3)

The closest control policy of the literature that could be implemented to our system is the policy proposed by Bouslah et al. (2013). They propose a batch production policy. The system produces until the manufacturer's inventory level reaches the batch size Q. Then, the batch is directly shipped to the retailer. However, since they do not propose an explicit delivery policy, we adapt their policy to be applicable on our system by sharing the processed production lot between both retailers proportionally to their reorder point values. (i.e. retailer 1 receives $\frac{d_1\tau_1}{d_1\tau_1+d_2\tau_2}Q$ and retailer 2 receives $\frac{d_2\tau_2}{d_1\tau_1+d_2\tau_2}Q$). And we adapt their production policy to consider the inventory position of the whole system instead of the individual retailers' inventory positions.

The adapted control policy (policy 3) of Bouslah et al. (2013) becomes as follows:

Production policy:

$$u^i(t,\alpha)=\begin{cases} u_m & if\ \langle y(t) < Z\ \rangle\ and\ (\ x_m(t) < Q)\ and\ \alpha(t) = 1 \\ \sum_{i=1}^{n} d_i & if\ (\ y(t) = Z)\ and\ (\ x_m(t) < Q)\ and\ \alpha(t) = 1 \\ 0 & if\ \langle(\ y(t) > Z)\ or\ (\ x_m(t) \geq Q)\rangle or\ \alpha(t) = 0 \end{cases} \tag{2.18}$$

Delivery policy:

$$u^i\Omega_i\ (t,Q)=\begin{cases}\frac{d_i\tau_i}{\sum_{j\neq i}^{n} d_j\tau_j}Q & if\ \ x_m(t)\geq Q\\ 0 & otherwise\end{cases} \tag{2.19}$$

Where $y(t)$ is the inventory position of the whole system at time t and Z is the hedging level.

2.9 Comparative study

We compare the proposed policy (Policy 1) with the improved one (policy 2) and with the adapted one of Bouslah et al. (2013) (policy 3).
We use the same simulation-optimization approach presented in figure 2.4 and in section 6 for the determination of the optimal parameters and the optimal total cost. For policy 2, we introduce the design variable β $(0 < \beta < 1)$ to define a fraction of S_1 which represents the priority zone: $dS_1 = \beta.S_1$.

The obtained response functions of the improved policy 2 (eq (2.20)) and the policy 3 (eq (2.21)) obtained from STATGRAPHICS are respectively as follows:

$$\Phi_{Imp}(Z,Q,S_1,S_2,\beta) = 13652.6 - 1.47 . Z - 1.60 . S_1 - 0.17 . S_2 + 0.61. Q + 320.61 . \beta + 1.97.10^{-4}. Z^2 + 0.48. 10^{-4}. Z S_1 - 1.93 . 10^{-4}. Z S_2 - 5.24 . 10^{-4}.Z Q + 4.41. 10^{-4}. {S_1}^2 - 3.07 . 10^{-4}. S_1 S_2 - 0.35 . 10^{-4}. S_1 Q + 0.12 . S_1 \beta + 6.62. 10^{-4}. \beta^2 + 2.32. 10^{-4}. S_2 Q + 5.44 .10^{-4}. Q^2 - 0.07. Q \beta + \varepsilon \tag{2.20}$$

$$\Phi_{Bou}(Z,Q) = 18091.8 - 1.04. Z - 1.40 .Q + 0.90 .10^{-4}. Z^2 - 0.84 .10^{-4}. ZQ + 1.84 . 10^{-4}. Q^2 + \varepsilon \tag{2.21}$$

The optimal parameters, the optimal total cost and the cost details of the three policies are presented in the table 2.5.

From table 2.5, we notice that Policy 1 and policy 2 outperform the policy of Bouslah et al. (2013) as they give a significant cost reduction. This is mainly due to two reasons; first, as stated before, in Bouslah's policy, the produced lot is delivered directly to the retailers without a delivery policy, which means that at the time of delivery, no decision is made to whom the lot is served. In fact, when the lot is produced, even if one retailer is facing a shortage and the other one has a comfortable inventory level, the produced lot is always going to be shared by the two retailers independently of the retailers' situations and inventory levels, this can lead to a high backlog cost compared to the proposed policies (as seen in table 2.5). Second, we can see that the optimal batch size Q^* of Bouslah's policy is not big enough to serve the two retailers. In the sense that the delivery lot sizes (which are fractions of the production lot size; $Q_1 = \frac{d_1\tau_1}{d_1\tau_1+d_2\tau_2}Q = 3214\ and\ Q_2 = \frac{d_2\tau_2}{d_1\tau_1+d_2\tau_2}Q = 2603$) are smaller than the optimal delivery lot sizes of our proposed policy, noting that Q^* in the proposed policies represent the delivery lot size that each retailer receives after each delivery. The smaller delivery sizes lead to more frequent deliveries and higher transportation cost.

The difference between the values of Z^* and $Q^*$for the policy 1 and policy 2 is due to the fact that the system is better protected from shortages when applying policy 2. Thus, the system needs a lower hedging level Z^* and consequently a lower lot size Q^*. In fact, in table 2.5, we can see the significant advantage of policy 2 in terms of backlog cost compared to policy 1 thanks to the priority given to the retailer who is more vulnerable to shortages. The lot size Q^* of policy 3 is higher than those of policy 1 and policy 2 because it is shared by two retailers. On the other hand, the hedging level Z^* of policy 3 has the same order of value compared to the other policies because it represents a hedging level for the whole inventory system like in the other policies.

Table 2.5 Comparative results

	Optimal variables					Cost details			Optimal total cost
	Z^*	S_1^*	S_2^*	Q^*	$dS_1{}^*$	Inventory cost	Backlog cost	Transportation cost	TC^*
Bouslah et al. (Policy 3)	8548	-	-	5786	-	2437	2957	4146	9540
Basic policy (Policy 1)	8829	1874	1205	3705	-	2577	938	3754	7269
Improved policy (Policy 2)	8550	1913	1235	3355	36	2665	185	3892	6742

As can be seen in figure 2.11, policy 1 and policy 2 outperform policy 3 for the wide range of system and cost parameters (see table 2.5 and the following paragraph for more details). In fact, policy 3 has the worst performance in terms of cost across all the system settings in figure 2.11. The cost reduction of policy 2 compared to policy 1 increases as c^- goes up (fig. 2.11 (b)), because when applying policy 2, the system is more protected against shortages since the prioritized retailer has more comfortable inventory level thanks to the priority that he is given. This means that if we have the two retailers competing for a produced lot, the manufacturer will ship it to the most vulnerable retailer to protect him against a possible high shortage. Therefore, the system is less penalized due to its backlog compared to policy 1. Policy 3 results in more shortages than policy 1 and policy 2 for the lack of a proper delivery policy which increases the cost for the different values of c^-.

The same effect is seen when the transportation costs increase (fig. 2.11 (c) and (d)), this is due to the fact that by increasing the transportation costs the system responds by reducing the frequency of deliveries. This leads to more shortages that are better dealt with by applying policy 2, which protects better against shortages thanks to the priority that it gives to the retailer who is more vulnerable to shortages. On the other hand, when c^+ increases, as seen in fig. 2.11 (a), the cost difference between policy 1 and policy 2 slightly decreases since applying the priority (policy 2) result in higher inventory level at the retailer 1 (because of the higher reorder point), which incur higher holding costs (as seen in table 2.5). The economic advantage of policy 1 and 2 compared to policy 3 is conserved for the different values of holding costs and transportation costs.

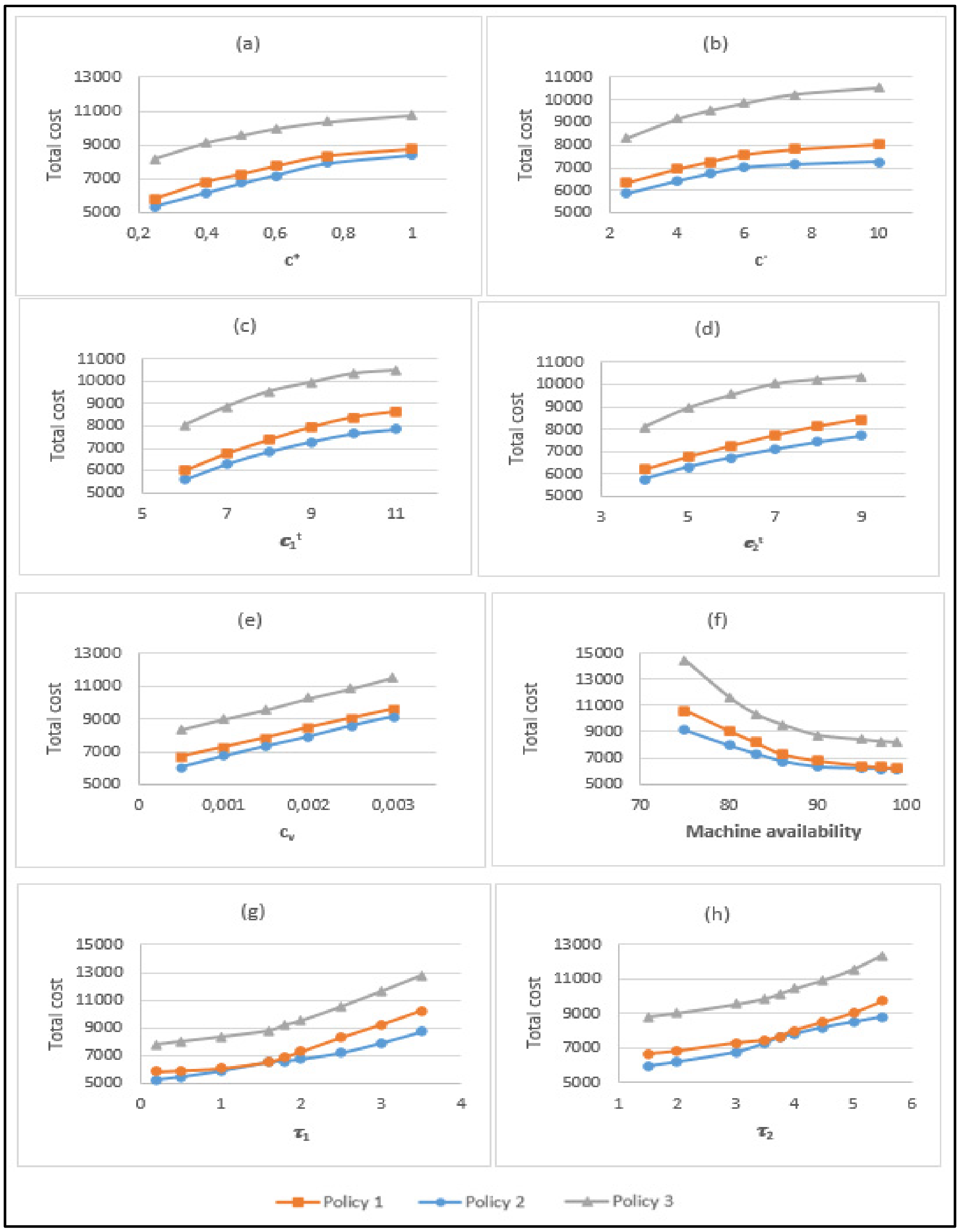

Figure 2.11 Effect of cost parameters, machine availability
and transportation times on the total cost for the proposed control policies

Fig. 2.11 (f) illustrates the effect of the machine availability on the cost of the two proposed policies. We note that by decreasing the machine availability the system faces more shortages. We can see that the cost advantage of policy 2 increases when the machine availability decreases. This is thanks to the priority given to retailer 1 who is more vulnerable to shortages. As stated before, the priority given to retailer 1 ensures that he has a comfortable inventory level, which protects him more from shortages that could occur due to the machine unavailability. The cost advantage of policy 1 and policy 2 compared to policy 3 decreases with higher availability of the machine, however, policy 3 performs the worst even when the machine is always available since the system always produces a shared lot for both retailers before the delivery happens and this results in more shortages faced by the retailers while waiting for the lot to be produced.

Fig. 2.11 (g) and fig. 2.11 (h) show respectively the effect of the variation of the transportation delay τ_1 and τ_2 on the total cost. We see that when we decrease τ_1, we reach a certain value (which corresponds to $\tau_1 = 1.6$) at which the two policies lead to the same total costs, this is explained by the two reorder points becoming equal for this value of τ_1 ($S_1 = 1.6 \times 750 = 1200\ and\ S_2 = 3 \times 400 = 1200$). At this value, the retailer 1 is not prioritized anymore and both policies have the same performance. If we keep decreasing τ_1 , the retailer 2 becomes the prioritized retailer (he has the highest reorder point) so he is given the priority, thus the policy 2 regains its cost advantage compared to policy 1.

In the same sense, when we increase τ_2 we reach a certain value ($\tau_2 = 3.75$) that corresponds to similar reorder points (($S_1 = 2 \times 750 = 1500\ and\ S_2 = 3.75 \times 400 = 1500$) at which policy 1 and policy 2 have the same performance. If we keep increasing the value of τ_2 , the retailer 2 becomes the one with a higher reorder point and therefore is given the priority. The cost advantage of policy 1 and policy 2 is kept when varying the transportation delays as the policy 3 lacks a delivery policy that takes into account these delays.

In summary, the control policy that integrates priority rules (policy 2) outperforms all the considered policies in terms of total cost. In fact, if we do not prioritize the most vulnerable retailer, he may face high shortages because of his high demand or his high transportation delays (i.e. higher reorder point). By giving him the priority in the delivery policy, the whole system would be more protected against shortages and will result in a cost reduction compared to the basic policy (policy 1). As mentioned above, policy 3 performs the worst for the different system parameters due to the lack of a proper delivery policy.

2.10 Managerial insights and practical implementation

An important challenge for the supply chains that comprise a manufacturer supplying multiple retailers is finding the best production and delivery control policies that ensures a better coordination between its members and that allows to reduce high costs of inventory and transportation of the finished products. To achieve this, important decisions that the manager should take are: at which production rate should the manufacturer produce for each moment, when to deliver to each retailer and what quantity should be delivered to each retailer. Moreover, the case of multiple retailers may put the manager in front of a decision of prioritizing a retailer over the others.

Our proposed integrated control policy (policy 2) allows the decision maker to take these decisions to minimize the total cost. The logic chart in figure 2.12 presents the process of decision-making for the manufacturing system presented in in figure 2.1 with the parameters presented in table 2.5 for the basic case of the policy 2. The proposed policy is easy to employ, and only requires the manager to monitor the manufacturer's inventory level, the inventory positions of the retailers and the inventory position of the whole supply chain. Concerning the production policy, the decision maker should take into account the inventory position of the whole supply chain Y, which includes the inventory levels of the manufacturer and the retailers and the lots under transportation. If Y is less than 8550, the production should be set at its maximum rate. If Y is equal to 8550 the rate of production should be set to the sum of the retailer's demand rates (750+400=1150). If the inventory position Y exceeds 8550 production should be stopped.

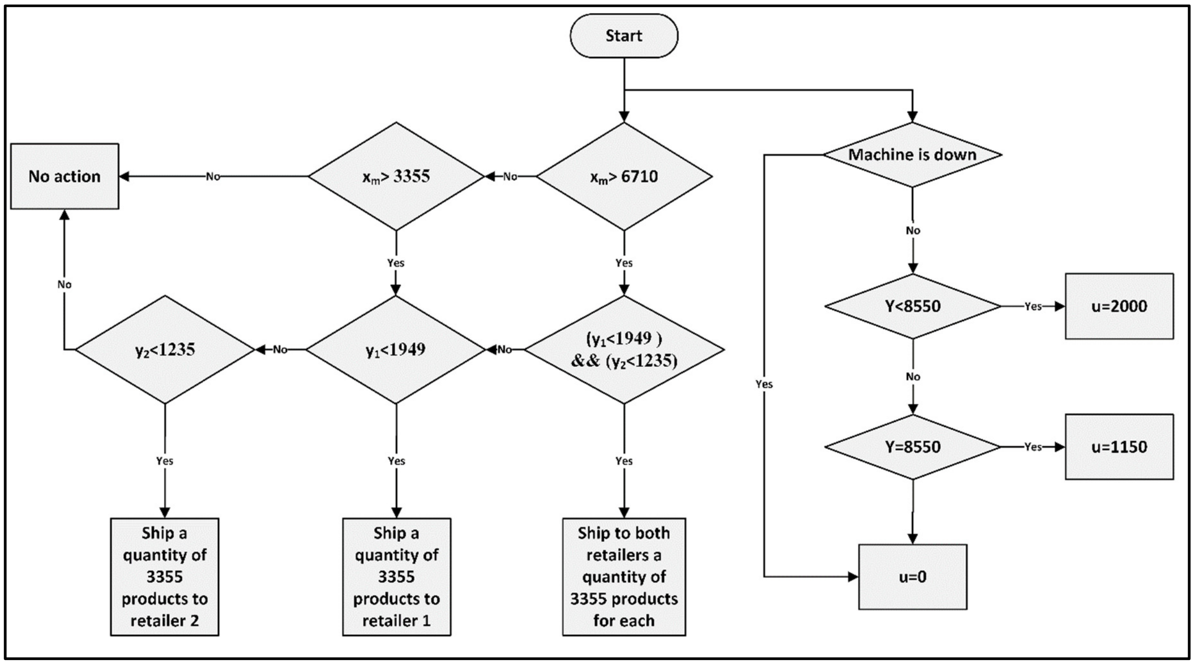

Figure 2.12 Implementation logic chart of the policy 2

On the other hand, the manager should always monitor the inventory level at the manufacturer and the retailers' inventory positions to take the delivery decisions. If the inventory level of the manufacturer is higher than 6710 and one of the retailers inventory position (or both) has reached his reorder point. He should ship a lot of quantity 3355 to the retailer (respectively, to both of them). If the inventory level at the manufacturer is sufficient to serve only one retailer (is higher than 3355 but lower than 6710). The manager should monitor first the inventory position of the prioritized retailer (here retailer 1). If he is in a priority zone, which means his inventory position is less than 1949 (=1913+36). He ships the lot (a quantity of 3355) to the retailer 1. Otherwise, if the inventory position of the retailer 1 is more than 1949, the manager should monitor if the retailer 2 has achieved his reorder point (i.e. his inventory position is less than 1235). If so, the manufacturer ships the lot to the retailer 2. Otherwise, the manager keeps monitoring the inventory positions of the retailers and takes no actions meanwhile.

2.11 Conclusion

In this paper we address the joint production control and delivery control problem for a single-product failure-prone manufacturing system with multiple retailers considering transportation delays.

An integrated production-delivery control policy has been developed based on a combined analytical and simulation-based approach. The obtained policy (policy 1) combines a modified HPP (for production rate control) with a state dependent economic order quantity (to control delivery orders). We proposed an improvement to the policy obtained to include the priority between retailers in policy 2. The optimal parameters for the basic and improved policies were determined using a simulation based experimental approach, and then compared. The results showed an important cost reduction when we consider the prioritization of retailers. A complete sensitivity analysis is performed to confirm the effectiveness of the proposed policies. We also conducted a comparative study that involves policy 1, policy 2 and the nearest policy in the literature that could be adapted to our system. The results of the comparison showed that the proposed policies outperform the existing policies in the literature in terms of total cost incurred by the system and that the policy 2 (with prioritizing rules) is the best control policy as it gives the lowest cost.

Many future research directions could be considered to study more comprehensive manufacturing systems. One of the extensions could consider integrating quality control in the manufacturing system where the produced items have a defective ratio. Integrating preventive maintenance of the manufacturing system is also one other extension that could be considered. It would be interesting to study the unreliability of the transportation networks, as it could affect the control policy. Further studies could also consider multiple products instead of one product.

CHAPITRE 3

INTEGRATED PRODUCTION-TRANSSHIPMENT CONTROL POLICY FOR A TWO-LOCATION UNRELIABLE MANUFACTURING SYSTEM

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Abstract: An important strategy used by two-location firms is lateral transshipment which allows sharing the inventory between the locations to reduce the impact of shortages. Transshipment can particularly be appealing as a policy used between unreliable production facilities as it can reduce the effects of these uncertainties. We study a manufacturing system that allows transshipment between two failure-prone production facilities with different capacities. An integrated production-transshipment control policy is proposed with the aim of minimizing the total cost that comprises holding, backlog and transshipment costs. The structure of the proposed policy is obtained using the stochastic dynamic programming. It consists of a combination of a hedging point policy for production control and a state dependant economic transshipment order quantity. The parameters of the obtained control policy are optimized by adopting a simulation-based optimization approach. The robustness of the results is verified by performing sensitivity analysis. A comparative study that considers our proposed policy with the most relevant policies from the literature that we adapt to our system is conducted and shows that our proposed policy outperforms the others in terms of cost.

Keywords: unreliable manufacturing system; two-location facility, feedback production planning; lateral transshipment; simulation; stochastic dynamic programming.

3.1 Introduction

In a context of localized economy, consumers' interest in local and personalized products has grown significantly. As a response for this trend corporations are building more local "mini-factories" in order to satisfy demands in the different zone markets. This type of implementation has been adopted by many big organisations like Nissan, Adidas, Caterpillar, and Tesla (Bhatnagar and Lin, 2019). A major challenge for this type of supply chains is production uncertainties which can lead to loss in sales and high costs. To counter this, intra-echelon flows are used by these corporations particularly at the retailer level to ensure more flexible and efficient supply chains.

Intra-echelon flows are permitted by lateral transshipment which has been proven to improve the performance of the supply chain in terms of cost and responsiveness (Shao, 2018). This policy ensures better service without necessarily increasing the safety stock at the retailers' warehouses (Burton and Banerjee, 2005). Transshipment is particularly valuable for expensive items that require high service levels and that have high backlog costs (Zhao et al., 2008). However, implementing transshipment policies by their own in a context of unreliable manufacturing systems can be challenging. In fact, the unreliability of the production facilities can lead to shortages and the system may be unable to allow transshipments as there is not enough stock on-hand in many locations to do so. Production control policies are generally used in traditional supply chains to hedge against these shortages. Integrating production control and transshipment policies can be interesting in this context as it can protect these retailers from the high amounts of shortages, and it makes the transshipment feasible and efficient.

Transshipment policies have been addressed in the past jointly with production planning, but not in the context of unreliable manufacturing systems. Knowing that the manufacturers are in different locations and have different demand rates or different capacities of their machines, the non-integration of production and transshipment can result in some retailers having an overstock while others being in shortage. Moreover, transportation delays between retailers have not been considered before in this context, which may affect the transshipment and production decisions. Under these circumstances, the manufacturers may face difficulties in controlling their production rates in order to maintain enough safety stock that protects against shortages and allows the transshipment to be feasible when it is required. Due to the lack of good coordination between production and transshipment decisions, the system may incur high costs of inventory, backlog and transshipment. This work is motivated by the question of how the failure-prone production facilities can achieve this level of coordination. In other terms, how should they adjust their production rates to achieve lower costs of inventory, backlog and transshipment? And when and how many products should be transshipped from one retailer to another? And whether it is better to transship or not?

This research addresses the above-mentioned issues. We develop an integrated production-transshipment control policy for a two-location failure-prone manufacturing system producing a single-type product. Different demand rates and machine capacities characterize each retailer, and we consider transportation delays between retailers for transhipment. This joint policy aims to minimize the total cost including holding, backlog, and transhipment costs.

The outline of this paper is as the following: we present the review of the literature in section 2. In section 3 the production/transshipment system is described, and the problem formulation is presented. We present a numerical approach to define the structure of the joint production/transshipment policy in section 4. We describe the resolution approach adopted to optimize the parameters of the policy in section 5. In section 6 the simulation-optimization model is presented and validated, and a numerical example is presented to optimize the policy's parameters. We perform a sensitivity analysis in section 7. In section 8 we introduce an improvement of the proposed policy in policy 2 and we outline the most relevant control policies that we adapt from the literature, and we conduct a comparative study that involves them and our proposed policy. In section 9 we present the managerial insights and how our proposed policy can be implemented. The conclusion and future research are highlighted in section 9.

3.2 Literature review

Transshipment has attracted the interest of researchers for decades. Different models considering transshipment were developed. The relevant literature is classified in table 3.1 into three main classes based on the type of proposed models. The first class represents periodic-review models where the transshipment decisions are predetermined before the start of the corresponding periods and the second class represent continuous-time models where transshipment decisions depend on the instant state of the system. The third class present the works that proposed hedging point policies in a context of subcontracting, standby machines or transshipment. We classify these works based on different criteria that characterize our work such as failure prone facilities, transshipment delays, multi-facility system, production planning and control, supporting of production facilities, transshipment policy, preventive transshipment and the proposal of a joint production-transshipment policy. In the literature, emergency and preventive transshipments are sometimes referred to respectively as reactive and proactive transshipments (Atan et al. (2018)). For the sake of clarity, and for the remainder of this paper, we will use the term emergency transshipments for the transshipments that happen when a stockout occur and preventive transshipments for the transshipments that have a preventive aspect to them (for example: by occurring before the stockout happens or by considering a holdback level for the sending location).

Concerning the periodic review models (class 1), Gross (1963) introduced the two-location problem with transshipments occurring before the review-period. Das (1975) studied the two-location inventory system in a stochastic context and proposed a joint inventory and transshipment control policies. Tagaras and Cohen (1992) considered the replenishment lead times and studied their effect. Herer and Rashit (1999) studied a single-period two-location problem and incorporated the fixed costs of replenishment. Rudi et al. (2001) investigated a two-location model with the aim of maximizing the joint profit in a single-period. Burton and Banerjee (2005) a studied a two-echelon supply chain and compared transshipments that seek the inventory equalization with transshipments that are based on the availability of stock. Lee et al. (2007) introduced a service level adjustment that determines the optimal transshipment quantity based on the service level. Ramakrishna et al. (2015) proposed a heuristic approach for a two-locations inventory problem considering emergency orders and transshipment between warehouses. Virtual transshipment was also considered in an algorithm for an inventory problem proposed by Liu et al. (2016) which allows virtual stock transfer between multiple locations. Meissner and Senicheva (2018) studied a multi-location inventory problem considering lost sales and determined a near-optimal transshipment policy. Naderi et al. (2020) investigated the single-period transshipment model with deterministic demand and considering capacity constraints for transportation. In a recent work, Dehghani et al. (2021) proposed an inventory model that considers preventive transshipments of blood between hospitals. The abovementioned works did not consider transshipment delays in their models; however, some works considered these delays as in Tagaras and Vlachos (2002) who considered non-negligeable transshipment delays and they noticed that preventive transshipment is beneficial especially when the variability of the demand is high. Rong et al. (2010) also considered the lead times for the preventive transshipments between two retailers in a decentralized system. One common assumption of the aforementioned works is that the suppliers are considered always available and that they have an infinite capacity. However, some periodic-review models have considered capacitated or unreliable suppliers. Yang and Qin (2007) considered virtual transshipment in a production/inventory system comprising two capacitated suppliers situated in different regions. Hu et al. (2007) studied a two-location problem of transshipment and developed a production-inventory model with the aim of maximizing the profits of retailers while considering both certain and uncertain capacity of the supplier. In a similar work Hu et al. (2008) considered an optimal joint inventory control considering transhipment with capacity uncertainty of the supplier. This work is extended by Chen et al. (2015) who employed a new approach to the joint transshipment/inventory problem considering uncertain capacity. Abouee-Mehrizi et al. (2015) investigated the joint transshipment and replenishment policies for a two-retailers inventory system where the retailer’s inventory can be replenished from a supplier or via transshipment and considering lost sales. Most of periodic-review models abovementioned consider transshipments that are planned at predetermined times before each corresponding period, and they do not consider the state of the system.

Continuous-review models (class 2) present another framework for transhipment policies. Lee (1987), Axsater (1990), Sherbrooke (1992) and Alfredsson and Verrijdt (1999) developed continuous-review inventory models with emergency transshipments that are performed when a stockout happens. Grahovac and Chakravarty (2001) developed a continuous-review inventory model considering transshipment for expensive low-demand items. Xu et al. (2003) proposed a continuous review inventory model integrating transshipments and with the consideration of hold-back level defining the amount of inventory that should be preserved in a location after transshipping out. Kukreja and Schmidt (2005) developed a single echelon inventory model where transshipment is considered if an emergency requirement for a part occurs. Qi (2006) investigated the scheduling problem for two-parallel machines considering one way transhipment between machines and single-item and batch mode transshipments. Zhao et al. (2006) proposed an inventory model that considers the decisions of requesting a transshipment or filling a request for other retailers. Van Wijk et al. (2009) considered a continuous review inventory model and developed an optimal threshold type policy for transshipment that minimizes the long-run average costs. Olsson (2015) considered a continuous review-model for a two-location single-echelon system where demands are independent, and backorders are allowed, and they consider transshipment delays. The abovementioned works generally consider emergency transshipment policies which only aim to meet immediate shortages. Shokouhifar et al. (2021) presents an inventory model for blood supply chains where the demand and supply are uncertain and considering lateral transshipment between locations. One of the extensions of transshipment policies is the consideration of hybrid transshipment policies that consider emergency and preventive transshipments at the same time. Paterson et al. (2012) is the first to propose a hybrid transshipment policy in continuous-time model. Yang et al. (2013) studied the benefits of joint pipeline stock and transshipment policy based on a service performance. Glazebrook et al. (2015) propose a hybrid transshipment policy that prevents future shortages, and which is triggered by the same mechanism as in traditional emergency transshipment models. Atan et al. (2018) developed a continuous-review model considering transshipment and characterized by two different priority customers. The abovementioned works consider inventory models and do not integrate production decisions. However, one of the first works that addressed this aspect is Zhao et al. (2008) who studied a two-location system and proposed a production/transshipment policy where transshipments are triggered by a demand arriving during stock-out or by sharing the surplus stock with other retailers even when there is no stockout. Their production policy determines whether to produce or not at each location and what location to serve. Bhatnagar and Lin (2019) extended the model introduced by Zhao et al. (2008) to multilocation system and relaxed the restrictions on the cost parameters and proposed a joint production-transshipment policy that determines when and for what location should the system produce and when to perform transshipments. Both aforementioned works considered production planning, but they did not determine at which rate the production facilities should produce, as they only determine when to produce and for which retailer they should produce. Moreover, they do not consider transhipment delays. They also did not consider the failure-prone aspect of the manufacturing system that can impact the production and transshipment policies.

The class 3 presents the works that have considered hedging point policies jointly with supporting unreliable production facilities such as subcontracting or reserve standby machine. Dror et al. (2009) developed a base stock policy considering subcontracting that helps to minimize the safety stock for two chemical products. Dahane et al. (2011) investigated a system composed of two machines producing a single type of product, and which are unreliable and able perform subcontracting tasks. Gharbi and al. (2011) studied a single-product type manufacturing system composed of a central machine and a reserve machine which is called upon to support the central machine when the inventory level falls below a certain level. Zied et al. (2014) developed an analytical formulation of production and maintenance model for a system made of a principal and a subcontracting machine and considered transportation delays. This work is further extended by Hajej et al. (2014) to consider product returns which are forwarded to the subcontractor to be recycled or remanufactured. Assid et al. (2015) proposed a joint production and subcontracting control policy that considers multiple unreliable production facilities and two product types. Rivera-Gómez et al. (2016) investigated a production facility with deteriorating quality and proposed a joint production control and subcontracting policy using stochastic dynamic programming. This model is extended by

Rivera-Gómez et al. (2018) to incorporate maintenance and consider machines with progressive degradations and wears that reduce their capacities. Hafidi et al. (2018) addressed a similar problem involving subcontracting and proposed a joint production and preventive maintenance policy. Haoues et al. (2019) investigated a system composed of a subcontractor and an outsourcer in a context of cooperation and proposed a mathematical formulation aiming to maximize the total profit. Assid et al. (2020) proposed a joint subcontracting, production and setup policy where the production facility is failure prone. These works consider policies that aim to support the unreliable production facilities, similar to the transshipment policies.

Table 3.1 Bibliographic review of the production and transshipment policies

	Failure prone facilities	Transshipment /transportation delays	Multi-facility system	Production planning	Production control	Supporting of production facilities	Transshipment policy	Preventive transshipment	Joint production-transshipment policy
Class 1: Periodic-review models									
Gross (1963)			✓			✓	✓		
Tagaras and Vlachos (2002)		✓	✓			✓	✓	✓	
Burton and Banerjee (2005)						✓	✓	✓	
Lee et al. (2007)						✓	✓	✓	
Yang and Qin (2007)	✓		✓	✓		✓	✓		✓
Hu et al. (2007)	✓		✓	✓		✓	✓		✓
Hu et al. (2008)	✓		✓	✓		✓	✓		✓
Rong et al. (2010)		✓				✓	✓	✓	
Abouee- Mehrizi et al. (2015)	✓			✓		✓	✓	✓	✓
Chen et al. (2015)	✓		✓	✓		✓	✓		✓
Ramakrishna et al. (2015)			✓			✓	✓		
Liu et al. (2016)			✓			✓	✓		
Meissner and Senicheva (2018)						✓	✓	✓	
Naderi et al. (2020)						✓	✓	✓	
Dehghani et al. (2021)			✓			✓	✓	✓	
Class 2: Continuous-time models									
Lee (1987)						✓	✓		
Grahovac and Chakravarty (2001)						✓	✓		
Xu et al. (2003)						✓	✓		
Kukreja and Schmidt (2005)			✓			✓	✓		
Qi (2006)			✓			✓	✓		
Zhao et al. (2006)			✓			✓	✓		
Zhao et al. (2008)			✓	✓		✓	✓	✓	✓
Van Wijk et al. (2009)		✓	✓			✓	✓		
Peterson et al. (2012)						✓	✓	✓	
Yang et al. (2013)						✓	✓		
Olsson (2015)		✓				✓	✓		
Glazebrook et al. (2015)						✓	✓	✓	
Atan et al. (2018)						✓	✓	✓	
Bhatnagar and Lin (2019)			✓	✓		✓	✓	✓	✓
Shokouhifar et al. (2021)			✓			✓	✓		
Class 3: Hedging point policies with supporting machines									
Dror et al. (2009)			✓	✓	✓	✓			
Gharbi and al. (2011)	✓		✓	✓	✓	✓			
Dahane et al. (2011)	✓		✓	✓	✓	✓			
Hajej et al. (2014)	✓	✓	✓	✓	✓	✓			
Zied et al. (2014)	✓		✓	✓	✓	✓			
Assid et al. (2015)	✓	✓	✓	✓	✓	✓			
Rivera-Gómez et al. (2016)	✓		✓	✓	✓	✓			
Rivera-Gómez et al. (2018)	✓		✓	✓	✓	✓			
Hafidi et al. (2018)	✓		✓	✓	✓	✓			
Haoues et al. (2019)	✓		✓	✓	✓	✓			
Assid et al. (2020)	✓		✓	✓	✓	✓			
Our proposal	**✓**	**✓**	**✓**	**✓**	**✓**	**✓**	**✓**	**✓**	**✓**

The main limitations of the literature described above, are mainly the lack of a proper joint optimal production control and transhipment policy that considers a failure prone manufacturing system and transshipment delays.

The contribution of our work is the proposal of a joint production control and a preventive transshipment policy for unreliable manufacturing systems that comprise two production facilities located in different locations, and which produce the same type of products. Transshipments between these locations are allowed and transshipment costs and delays are considered. Our policy allows to determine when to increase/decrease the production rate and when to enable transshipments and what quantity should be transshipped between the retailers depending on the system state in order to achieve better performance in terms of cost.

3.3 The production/transshipment system description and problem formulation

3.3.1 Notations

The notations defined below are used to formulate the problem.

$x_i(t)$ The level of inventory of the facility i

$y_i(t)$ The position of inventory of the facility i

$u_i(t)$ Rate of production of the facility i at time t (product/time unit)

u_i^{max} Maximum rate of production of the facility i (product / time unit)

d_i Demand rate of the of the facility i (product / time unit)

Q_{ij} Transshipment lot size from facility i to facility j (product)

δ_{ij}^k Transshipment start time of the *k*th lot from facility i to facility j

Q_{ij}^k *k*th lot to be transshipped at the instant δ_{ij}^k from facility i to facility j

Q^{max} Maximum transshipment size

$Tr_{ij}(t)$ Lots being transshipped from facility i to facility j at time t

$q_{\alpha\beta}$ Transition rate from mode α to mode β

TTF_i Time to failures of facility i

TTR_i Time to repair of facility i

τ_{ij}	Transportation delay from facility i to facility j (time unit)
c^+	Holding cost ($/time/ product)
c^-	Backlog cost ($/time/ product)
c_{ij}^{tr}	Fixed transshipment cost from facility i to facility j ($/transshipment)
c_{ij}^{v}	Variable transshipment cost from facility i to facility j ($/product)
ρ	Discounted cost

3.3.2 Two-location facility system description

The considered system consists of a manufacturing company that serves customers in two market zones through two production facilities that produce the same type of product and face different demand rates depending on the region. The production facilities are subject to random failure and repairs of their machines. The final products are stored in the local warehouses adjacent to the production facilities before serving the customers. The lateral transshipment is allowed between the two warehouses by transferring lots of products from one stock location to another.

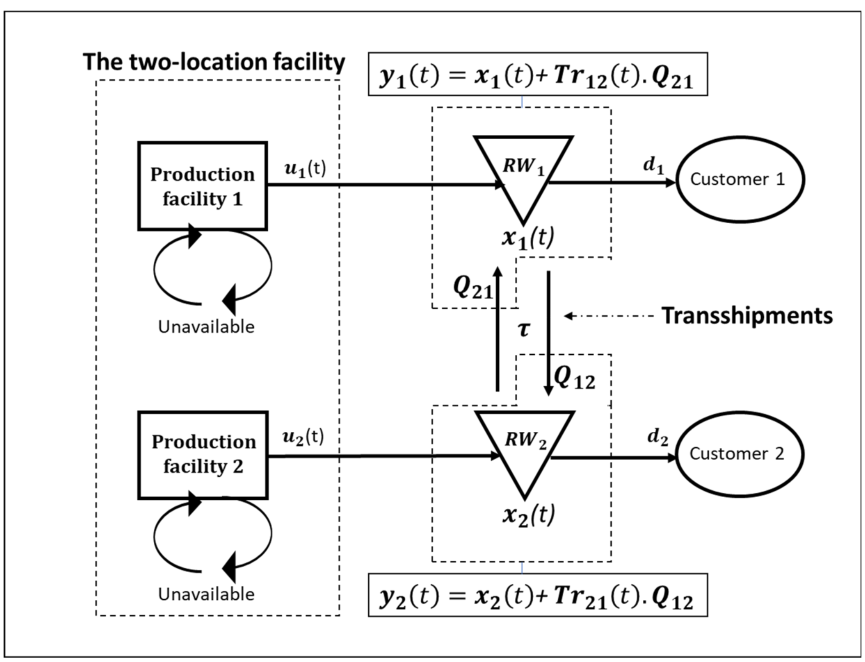

Figure 3.1 The two-location facility system under study

The main assumptions are as follows:

- The rates of customers demand are known and constant for each facility.
- Shortages are allowed.
- Transshipment can take place in both directions between the facilities.

3.3.3 Problem formulation

Knowing that transshipments between the facilities introduce an impulsive aspect to the problem, we base the stochastic dynamic programming on the impulse control theory (Yong 1989, Yang 1999) to reproduce the dynamics of the system. To formulate the control problem, we define the main parts that characterize system's state:

- The discrete/continuous variables:

✓ Inventory level $x_i(t)$ at the facilities' warehouses: This quantity decreases with finite jumps of quantity Q_{ij} when transhipping out to another facility j and it increases with a finite jump Q_{ji} when transhipping in from another facility j. It also faces an upstream continuous production rate $u_i(t)$ and a downstream continuous demand d_i.

✓ Inventory position $y_i(t)$ at the facilities' warehouses: This quantity describes the sum of the inventory levels at the facilities and the lots that are being transshipped to their warehouses. The inventory position of the retailer i is expressed as follows: $y_i(t) = x_i(t) + Tr_{ji}(t).Q_{ji}$. Where $Tr_{ji}(t)$is a binary variable that takes the value 1 when a lot is being transshipped from facility j to facility i and takes 0 otherwise.

The continuous variable that describes the stochastic process of the system's operational mode at time t, it is defined by the random variables $\alpha(t)$. The transition rates matrix of the stochastic process $\alpha(t)$ is denoted by $T = \{q_{\alpha\beta}\}$. The state of the machine i is defined by $\alpha_i(t), i = 1,2$ with values in $B_i = \{1,0\}$, it takes value 1 when it is operational and 0 otherwise. q_{10}^{i} define the failure rates of production facility i and q_{01}^{i} define the repair rates of the facility i.

The system's state is described by the Markov process $\alpha(t) = \alpha_1(t) \times \alpha_2(t)$ with values in $B_1 \times B_2 \equiv M$. The transition rates from state α to state β are denoted such that $q_{\alpha\beta} \geq 0$ if $\alpha \neq \beta$ and $q_{\alpha\alpha} = -q_{\alpha\beta\,(\alpha\neq\beta)}$, where $\alpha, \beta \in M$.
The dynamics of the inventory levels are described in equation 1:

$$\begin{gathered}\frac{dx_{\mathrm{i}}(t)}{dt} = u_{\mathrm{i}}(\mathrm{t}) - d_{\mathrm{i}} \quad \forall t \in]\delta_{ij}^{k}, \delta_{ij}^{k+1}] \cup]\delta_{ji}^{m} + \tau_{\mathrm{ji}}, \delta_{ji}^{m+1} + \tau_{\mathrm{ji}}] \\ x_{\mathrm{i}}\big((\delta_{ij}^{k})^{+}\big) = x_{\mathrm{i}}\big((\delta_{ij}^{k})^{-}\big) - Q_{ij}^{k} \\ x_{\mathrm{i}}\big((\delta_{ji}^{m} + \tau_{\mathrm{ji}})^{+}\big) = x_{\mathrm{i}}\big((\delta_{ji}^{m} + \tau_{\mathrm{ji}})^{-}\big) + Q_{ji}^{m} \\ i \neq j, i, j \in \{1,2\}, \ k, m \in \{1, \dots, \infty\}\end{gathered} \tag{3.1}$$

The following capacity constraints should be satisfied to respect the production capacities and the capacity of the transportation mean used for transshipment:

$$\begin{gathered}0 \leq u_i(t,\alpha) \leq u_i^{max} \\ 0 \leq Q_{ij}^{k} \leq Q^{max} \\ i \neq j, i, j \in \{1,2\}, k \in \{1, \dots, \infty\}\end{gathered} \tag{3.2}$$

$\Omega_{ij} = \{(\delta_{ij}^{1}, Q_{ij}^{1}), (\delta_{ij}^{2}, Q_{ij}^{2}), \dots, (\delta_{ij}^{\infty}, Q_{ij}^{\infty})\}$, where $(\delta_{ij}^{k}, Q_{ij}^{k})$ represents the transshipment from the retailer i to retailer j of the kth lot Q_{ij}^{k} at time δ_{ij}^{k}. For $i \neq j,\ i, j \in \{1,2\}, k \in \{1, \dots, \infty\}$
The set of admissible decisions (u_i, Ω_{ij}) denoted by $\Gamma(\alpha)$ is given by:

$$\Gamma(\alpha) = \left\{ (u(.), \Omega_{ij}) \middle| \begin{matrix} 0 \leq u_i(t,\alpha) \leq u_i^{max}, 0 \leq Q_{ij}^{k} \leq Q^{max}, \\ i \neq j,\ i, j \in \{1,2\}, k \in \{1, \dots, \infty\} \end{matrix} \right\}$$

The inventory costs of facility i that comprise holding and backlog costs are penalized using the following instantaneous cost function g(.):

$$g\big(x_i(t), u_i(t,\alpha)\big) = c^{+}\big(x_{\mathrm{i}}^{+} + ind\{t \in]\delta_{ji}^{m} + \tau_{\mathrm{ji}}, \delta_{ji}^{m+1} + \tau_{\mathrm{ji}}[\,\}. Q_{ji}^{m}\big) + c^{-} x_{\mathrm{i}}^{-}(t) \qquad \forall t \in]\delta_{ij}^{k}, \delta_{ij}^{k+1}] \cup]\delta_{ji}^{m} + \tau_{\mathrm{ji}}, \delta_{ji}^{m+1} + \tau_{\mathrm{ji}}] \tag{3.3}$$

where $x_i^{+} = \max\big(0, x_i(t)\big)$, $x_i^{-} = \max(-x_i(t), 0); i \in \{1,2\}$

We also define the instantaneous cost functions for the transshipment at times δ_{ij}^{k} , $i, j \in \{1,2\}$, $k \in \{1, \dots, \infty\}$ as follows:

$$R_{ij}\big(Q_{ij}^{k}\big) = Ind\big(t = \delta_{ij}^{k}\big)\big(c_{ij}^{tr} + c_{ij}^{v} Q_{ij}^{k}\big) + \int_{0}^{\tau_{ij}} e^{-\rho t} g(x_i + (u_i - d_i).t)dt \tag{3.4}$$

where

$$Ind\big(\Theta(.)\big) = \begin{cases} 1 & if\ \Theta(.)\ is\ true \\ 0 & otherwise \end{cases}$$

The overall infinite horizon discount cost $J(.)$ can be defined as follows:

$$J\big(x_i, \Omega_{ij}, u_i, \alpha\big) = E\left\{ \begin{matrix} \int_0^{\infty} e^{-\rho t} g(x_i, u_i)dt \\ + \sum_{k=1}^{\infty} e^{-\rho \delta_{ij}^{k}} \big(c_{ij}^{tr} + c_{ij}^{v} Q_{ij}^{k}\big) \end{matrix} \middle| \begin{matrix} x_{\mathrm{i}}(0) = x_{\mathrm{i}_0}; i = \{1,2\} \\ \alpha(0) = \alpha \end{matrix} \right\} \tag{3.5}$$

Where $E(.)$ is the conditional expectation given the initial conditions of (x_i, α).

The control problem understudy has the objective of determining $(\mathrm{u}_i^{*}, \Omega_{ij}^{*}) \in \Gamma(\alpha)$ that minimizes $J(.)$ from the equation (5) and based on the equations (1) to (4). The associated value function is given by:

$$v\left(x_i, \Omega_{ij}, \alpha\right) = \inf_{(\Omega_{\mathrm{ij}}, \mathrm{u_i}) \in \Gamma(\alpha)} J\left(x_i, \Omega_{ij}, \mathrm{u_i}, \alpha\right) \quad \forall \alpha \in M \quad , i \in \{1,2\} \tag{3.6}$$

Using the optimal impulse control theory (Sethi and Thompson 2005), the value function $v(x_i, \alpha)$ is shown to be the unique viscosity solution for the following Hamilton-Jacobi-Bellman (HJB) equation (equation 3.8). The proof can be developed as in Yong (1989) by considering the transshipment decision as a 'stopping decision' with cost given by equation (3.7).

$$Ow(x_i, \alpha) = \min_{i,j \in \{1,2\}} \left\{ \min_{(k, Q_{ij}^k) \in \Gamma} \left\{ \begin{matrix} R_{ij}\left(Q_{ij}^k\right) \\ +e^{-\rho \tau_{ij}} \; v \begin{pmatrix} x_i - Q_{ij}^k - \tau_{ij} d_i, x_j \\ +Q_{ij}^k - \tau_{ij} d_j, \alpha \end{pmatrix} \end{matrix} \right\} - v\left(x_i, \Omega_{ij}, \alpha\right) \right\} \tag{3.7}$$

$$\min \left\{ \begin{matrix} \min\limits_{u_i} \left\{ \begin{matrix} \sum\limits_{i=1}^{n} (u_i - d_i) v_{x_i} + g(x_i, u_i) \\ + \sum\limits_{\beta \neq \alpha} q_{\alpha\beta} \left(v\left(x_i, \Omega_{ij}, \beta\right) - v\left(x_i, \Omega_{ij}, \alpha\right) \right) \end{matrix} \right\} \\ -\rho v\left(x_i, \Omega_{ij}, \alpha\right); \\ \min\limits_{i,j \in \{1,2\}} \{ \min\limits_{(k, Q_{ij}^k) \in \Gamma} \left\{ \begin{matrix} R_{ij}\left(Q_{ij}^k\right) \\ +e^{-\rho \tau_{ij}} \; v \begin{pmatrix} x_i - Q_{ij}^k - \tau_{ij} d_i, x_j \\ +Q_{ij}^k - \tau_{ij} d_j, \alpha \end{pmatrix} \end{matrix} \right\} \\ -v\left(x_i, \Omega_{ij}, \alpha\right) \} \end{matrix} \right\} = 0 \tag{3.8}$$

where v_{x_i} represents the gradient of $v(.)$ with respect to x_i.

Given the difficulty of the analytical resolution of the HJB equations (equation (7)), a numerical approach is applied to solve them.

3.3.4 Numerical resolution

The model is characterized by a Markovian aspect. Therefore, we apply the numerical method for a negligeable transportation delay case ($\tau_{ij} \approx 0$) to facilitate the implementation process. Therefore, the HJB equations become:

$$\min \left\{ \begin{array}{c} \min\limits_{u_i} \left\{ \begin{array}{c} \sum\limits_{i=1}^{n} (u_i - d_i) v_{x_i} + g(x_i, u_i) \\ + \sum\limits_{\beta \neq \alpha} q_{\alpha\beta} \left(v(x_i, \Omega_{ij}, \beta) - v(x_i, \Omega_{ij}, \alpha) \right) \end{array} \right\} \\ -\rho v(x_i, \Omega_{ij}, \alpha); \\ \min\limits_{i,j \in \{1,2\}} \{ \min\limits_{(k, Q_{ij}^k) \in \Gamma} \left\{ \begin{array}{c} R_{ij}(Q_{ij}^k) \\ + e^{-\rho \tau_{ij}}\, v(x_i - Q_{ij}^k, x_j + Q_{ij}^k, \alpha) \end{array} \right\} \\ -v(x_i, \Omega_{ij}, \alpha) \} \end{array} \right\} = 0 \tag{3.9}$$

Kushner approach (Kushner and Dupuis 1992) is applied to numerically obtain the structure of the optimal control policy.

3.4 Numerical results

3.4.1 Numerical example

The transition rates that define the stochastic process of a system of two production facilities are defined in the table 3.2 below:

Table 3.2 Transition rates of the considered system

$\alpha_1(t)$	1	0	1	0
$\alpha_2(t)$	1	1	0	0
$\alpha(t)$	1	2	3	4

$q_{12} = q_{10}^1, q_{21} = q_{01}^1 , q_{13} = q_{01}^2, q_{31} = q_{01}^2, q_{34} = q_{10}^1, q_{43} = q_{01}^1, q_{24} = q_{10}^2, q_{42} = q_{01}^2$

The transitions between the states of the system are presented in the diagram of figure 3.2.

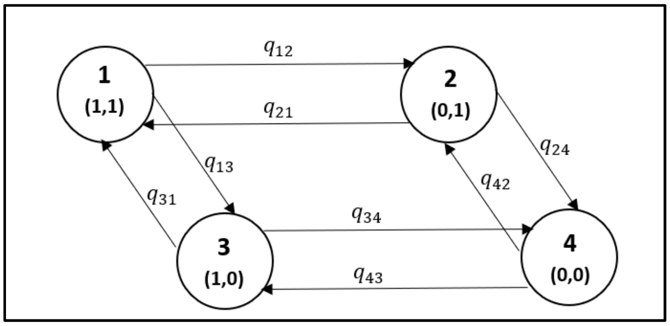

Figure 3.2 Diagram of the states' transition of the system under study

In table 3.3 we present the parameters of the system, the costs and the transition rates values considered for the numerical example.

Table 3.3 Data used for the numerical approach

u_1^{max}	u_2^{max}	d_1	d_2	q_{10}^1	q_{10}^2	q_{01}^1	q_{01}^2	ρ	c^+	c^-	c^{tr}	c^v
2	2	1.6	1	0.01	0.01	0.08	0.08	0.08	0.5	50	100	10

Where here we suppose that the transshipment cost is symmetric between facility 1 and facility 2: c^{tr}=c_{12}^{tr}=c_{21}^{tr} and c^v=c_{12}^v=c_{21}^v.

The computational domain D_h is defined by: $D_h = \{(x_1, x_2): -20 \leq x_1 \leq 60, -20 \leq x_2 \leq 60\}$ with $h_{x_1} = 2$, $h_{x_2} = 2$.

To characterize the control policy, we observe the production and transshipment policy separately. The thresholds that define each policy are observed independently of the others.

The figure 3.3 shows that the production policy for both production facilities is characterized by three regions that divide the surplus space. In region ①, the production facility produces at its maximum rate, and it produces at the rate of the corresponding demand in region ②. And in region ③, the production is stopped. We clearly see that the production policy is a base stock policy (BSP) controlled by a hedging level Z_1 for production facility 1 and Z_2 for the production facility 2.

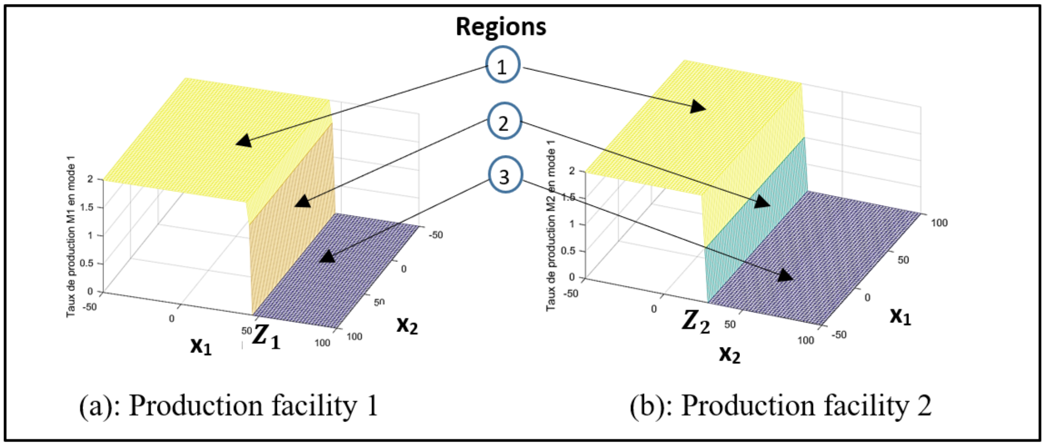

Figure 3.3 Production control policy

The transshipment policy from retailer 1 to 2 defined by $Q_{12}(x_1, x_2)$ and the transshipment policy from retailer 2 to 1 defined by $Q_{21}(x_1, x_2)$ are illustrated in figure 3.4. We see that the transshipment policy is also state dependant. The policy is governed by an order point S_i for each retailer and an economic transshipment quantity Q_{ij} from retailer i to retailer j. These parameters are dependant on the whole state of the system (x_1, x_2, α). Therefore, it can be defined as a state dependant economic transshipment quantity (SD-ETQ).

In our case, we clearly see that the parameters of the transshipment policy from facility 1 to 2 are different from the one from facility 2 to 1. In fact, because of the higher demand that facility 1 faces, it has the higher reorder point S_1 as well as a higher quantity Q_{21} to be transshipped to it from facility 2. Furthermore, we see that these parameters depend on the whole operational state of the system (α). We can see that when a production facility is unavailable its reorder point gets higher. S_1 in mode 2 ($\alpha = 2$, when production facility 1 is unavailable and production facility 2 is available) is higher than S_1 in mode 1 ($\alpha = 1$, when both production facilities are available). We see the same effect with S_2 when the production facility 2 is in the unavailable mode ($\alpha = 3$). This is realistic since an unavailable production facility is more vulnerable to shortages and it needs more transshipment. It is worth mentioning that the transshipment happens when there is enough quantity in the warehouse of the production facility that is transhipping out products (when $x_i \geq Q_{ij}$). Thus, we clearly see that in region ④ no transshipment is allowed because the stock level of the production facility which is transshipping out is negative.

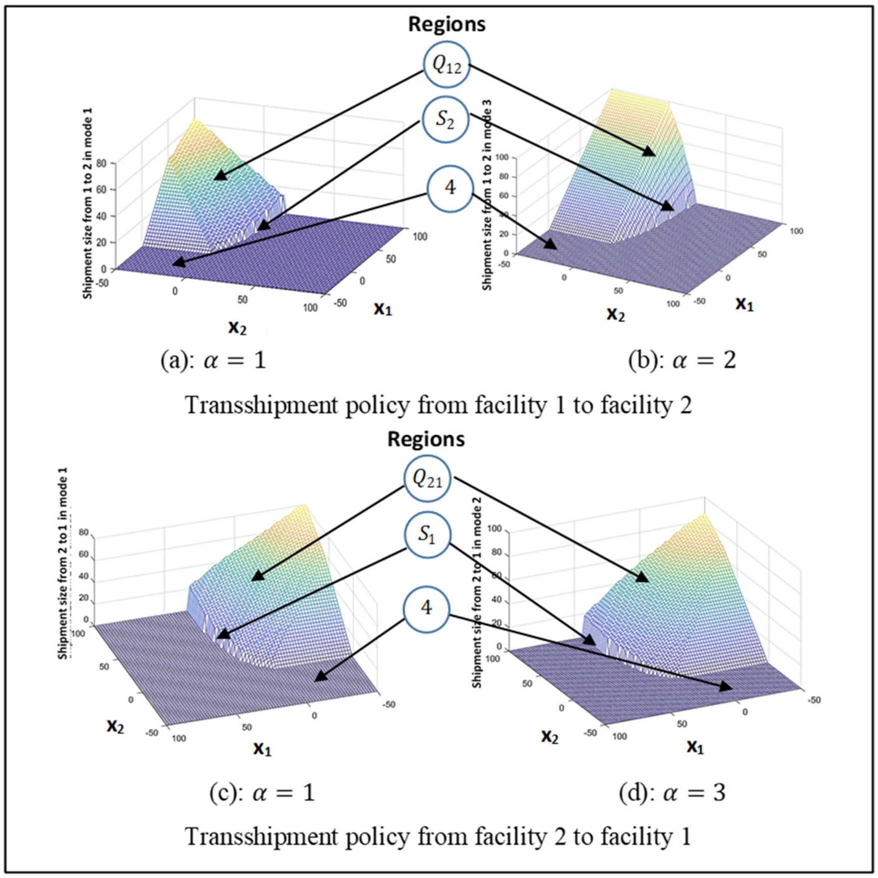

Figure 3.4 Transshipment control policy

We confirm the structure of the optimal policy by performing a sensitivity analysis that verifies the behaviour of the optimal parameters when we vary the costs. We show as an example the effect of the backlog cost on the production and transshipment policies in the mode 1 in figure 3.5.

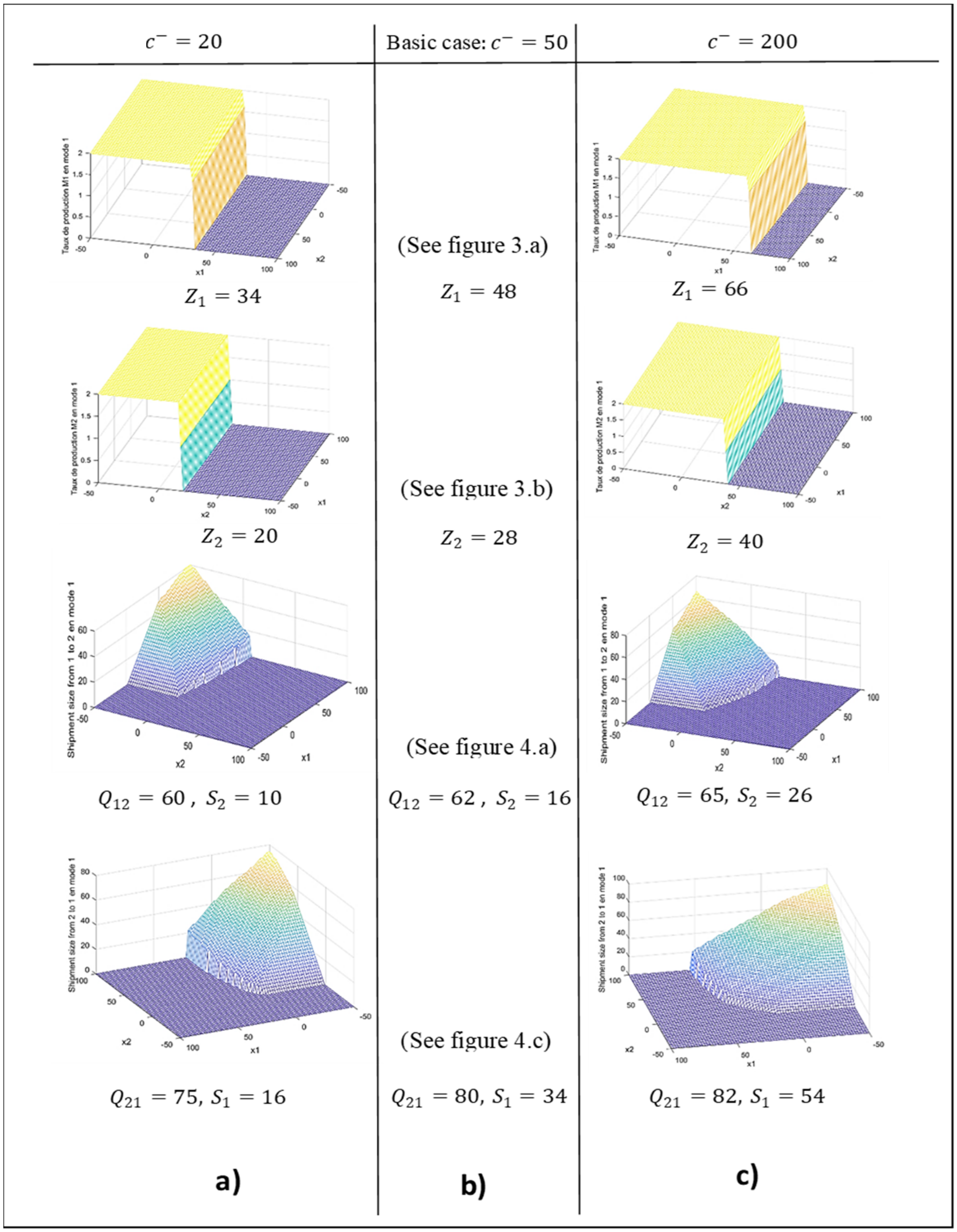

Figure 3.5 Backlog cost effect on the obtained policy

We can see that the increase of the backlog cost (figure 3.5(c)) leads to an increase of the hedging levels Z_1 and Z_2 to hedge against the possible shortages. We see that the reorder points also increase to allow more transshipments between the retailers. This effect is more pronounced for the transshipment policy from facility 2 to facility 1 because the facility 1 is more vulnerable to shortages due to its higher demand. The transshipment quantities also increase with the increase of the backlog cost to ensure that enough quantity of products is sent to the facility vulnerable to shortage to be able to reduce the cost of the shortage. By decreasing the backlog cost (figure 3.5(a)), the opposite effects happen.

3.4.2 Structure of the optimal control policy

It follows up from the numerical results that we can define the optimal control policy as a combined base stock policy (BSP) and state-dependant economic transshipment quantity (SD-ETQ) which can be expressed as follows:

<u>Production policy for production facility i:</u>

$$u_i(t,\alpha)=\begin{cases} u_i^{max} & if\ \ x_i(t) < Z_i\ and\ \alpha_i(t) = 1 \\ d_i & if\ \ x_i(t) = Z_i\ and\ \alpha_i(t) = 1 \\ 0 & if\ \ \ x_i(t) > Z_i\ or\ \alpha_i(t) = 0 \end{cases} \tag{3.10}$$

<u>Transshipment policy from retailer i to retailer j:</u>

$$\Omega_{ij}(t,Q_{ij}) = \begin{cases} Q_{ij} & if\ x_j(t) \le S_j\ \ and\ x_i(t) \ge Q_{ij} \\ 0 & otherwise \end{cases} \qquad i,j \in \{1,2\} \tag{3.11}$$

Recall that Z_i is the threshold parameter, S_i is the transshipment reorder points of the facility i and Q_{ij} is the transshipment quantity from facility i to facility j. The three parameters respect the following constraints:

$$Z_i \ge 0\,; S_i \ge 0\ and\ Q_{ij} > 0\,; i,j \in \{1,2\}$$

The policies introduced above are defined for a negligeable transportation delay, we integrate the notion of inventory position introduced by Mourani *et al*. (2008) and Li *et al*. (2009) which takes into account these delays in the control policy. It considers the existing inventory in the warehouse and the quantity of products being transported at the moment. The position of the inventory of each retailer is expressed as the sum of his inventory level and the lot being transshipped to him at time t and it can be expressed as follows:

$$y_i(t) = x_i(t) + Tr_{ji}(t).Q_{ji} \tag{3.12}$$

Where $Tr_{ji}(t)$ is the binary variable that takes the value 1 when there is a lot being transshipped from retailer j to retailer i at time t. It takes the value 0 otherwise.

The control policy (policy 1) is adjusted by integrating the inventory position. It becomes as follows:

<u>Production policy for production facility i:</u>

$$u_i(t,\alpha)=\begin{cases} u_i^{max} & if\ \ y_i(t) < Z_i \ and\ \alpha_i(t) = 1 \\ d_i & if\ \ y_i(t) = Z_i \ and\ \alpha_i(t) = 1 \\ 0 & if\ \ y_i(t) > Z_i \ or\ \alpha_i(t) = 0 \end{cases} \tag{3.13}$$

<u>Transshipment policy from retailer i to retailer j:</u>

$$\Omega_{ij}(t, Q_{ij}) = \begin{cases} Q_{ij} & if\ y_j(t) \leq S_j \ \ and\ x_i(t) \geq Q_{ij} \\ 0 & otherwise \end{cases}$$
$$i, j \in \{1,2\} \tag{3.14}$$

The parameters that characterize the control policy are the hedging levels Z_i , the transshipment reorder points S_i and the transshipment quantity Q_{ij}. We compare the inventory level $x_i(t)$ (and not the inventory position $y_i(t)$) to Q_{ij} to ensure that the transshipment is only allowed when the required transshipment quantity Q_{ij} is available at the warehouse of the retailer i.

The numerical approach has many limitations when implemented on the operational level because of the irregularities found in the boundaries of its results (Berthaut et al., 2010) and its accuracy requires smaller sizes of the grid steps which makes it time consuming. Moreover, integrating transportation delays is not possible because of the Markovian aspect of the studied model. For these reasons, we proceed with a simulation-based approach.

3.5 Resolution approach

The combined approach of simulation and statistical analysis is used to optimize the parameters of the control policy. The steps used to achieve this is presented in the diagram in figure 3.6 and described below:

1. Step 1: A stochastic dynamic programming model is developed using control theory and the obtained HJB equations are solved by a numerical approach.

2. Step 2: The structure of the parametrized optimal control policy is defined.

3. Step 3: We define the experimental design used to describe how to vary the parameters of the control policy to obtain a minimum set of experiments.

4. Step 4: We develop the combined discrete-continuous simulation model which is used to reproduce the dynamics of the system.

5. Step 5: We determine how the total incurred cost is related to the design parameters by using a regression analysis jointly with the response surface methodology (RSM).

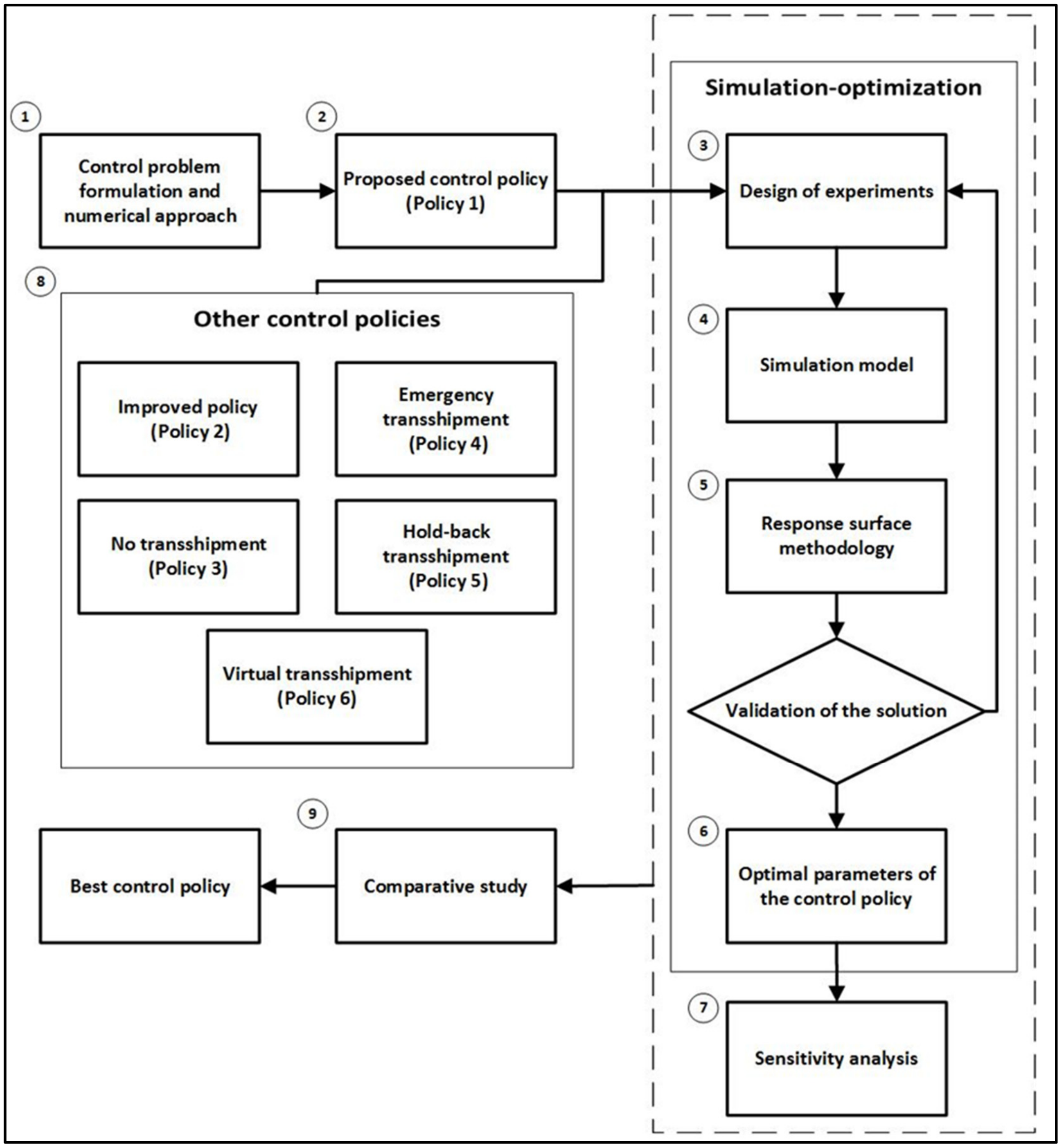

Figure 3.6 Block diagram of the resolution approach

6. Step 6: By optimizing the regression model obtained from the last step, we determine the optimal cost as well as the optimal parameters of the control policy.

7. Step 7: Sensitivity analysis is conducted to show the effect of varying the system parameters and costs aiming to confirm the robustness of the proposed policy.

8. Step 8: We propose an improved policy (policy 2) and we conduct a comparative study that includes the proposed policies and other transshipment policies from the literature.

9. Step 9: We compare between the considered policies to determine the one that performs better in terms of cost.

3.6 Simulation optimization

3.6.1 Experimental design

We use a full experimental design that includes five variables (Z_1, Z_2, S_1, S_2, B) at three levels for each variable (3^5) and 2 replications. B here is a substitute parameter equal to $B_1 = \frac{S_1}{Q_{21}}$ and equal to $B_2 = \frac{S_2}{Q_{12}}$ with B<1. By defining B in the experimental design, we ensure that the quantity transshipped is higher than the reorder points. The choice of a common factor B ($B = B_1 = B_2$) instead of two variables $B_1$and B_2 for each retailer is motivated by the slight difference in optimal total cost when compared to the case with six independent variables $(Z_1, Z_2, S_1, S_2, B_1, B_2)$. Moreover, by proceeding with this choice, we will considerably reduce the number of the simulation runs while leading to the same conclusions.

3.6.2 Simulation model

The simulation model developed to reproduce the behaviour of the studied system is presented in the figure 3.7.

1. The initialization of the simulation by defining the inputs of the model which includes the design parameters $(Z_1, Z_2, S_1, S_2, Q_{21}, Q_{12})$, and the system's parameters such as the production rates, demand rates and transportation delays.

2. The dynamics of the system are modeled using the state equations which allows to instantaneously calculate the inventory levels using C++ language and based on the differential equation (3.1).

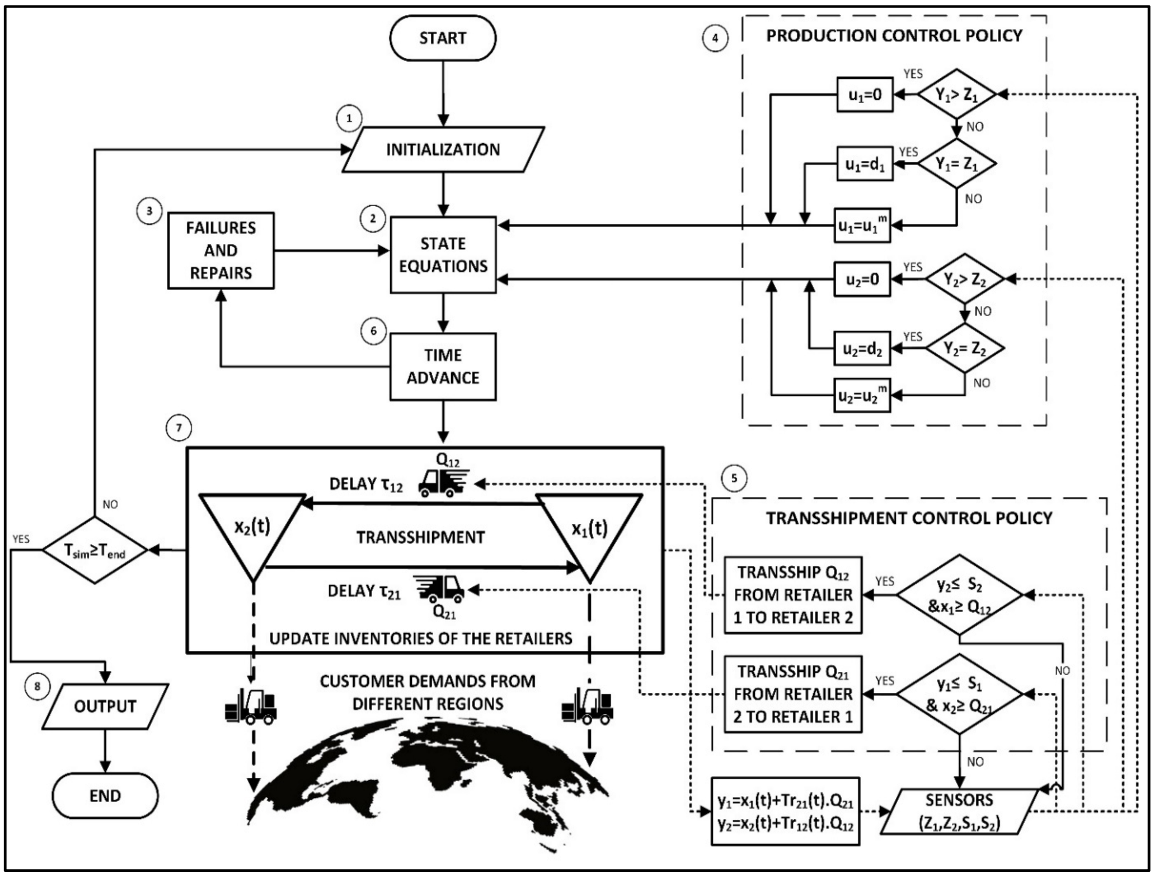

Figure 3.7 Simulation block diagram

3. The time to failure and the time to repair is calculated based on the TTF and TTR distributions to indicate the state of the machine.

4. The production control policy is applied by setting the right production rate by comparing the inventory positions of the retailers with the defined thresholds.

5. The transshipment control policy is applied by comparing the retailers inventory position to their corresponding transshipment reorder points and by comparing their inventory levels to the transshipment quantities.

6. The current time is updated continuously in the time advance block.

7. The dynamics of the inventory levels and positions (x_i, y_i) including the lots under transshipment are updated.

8. The model delivers time persistent statistics of the output variables at the end of the simulation including the average stock and backlog levels and the number of transshipments which allow to calculate the total incurred cost.

The simulation ends when the current time of the simulation T_{sim} reaches the defined simulation duration T_{end}.

3.6.3 Validation of the simulation model

It is shown from the sample of the inventory dynamics and production rates presented in figure 3.8, that the production rate is dependant on the failures and repairs of the machines and on the positions of the inventories of the retailers. It falls to zero when a failure happens (arrow 1 in fig.3.8.a). After the repair, the machines are set to their maximal production rate (arrow 4 in fig.3.8.a). When the inventory position of a retailer reaches the hedging level (arrow 1 in fig.3.8.c) the production rate of the associated production facility is set to the corresponding demand rate (arrow 2 in fig.3.8.a). When it falls under this threshold (arrow 2 in fig.3.8.c) the production is set to its maximum rate (arrow 3 in fig.3.8.a).

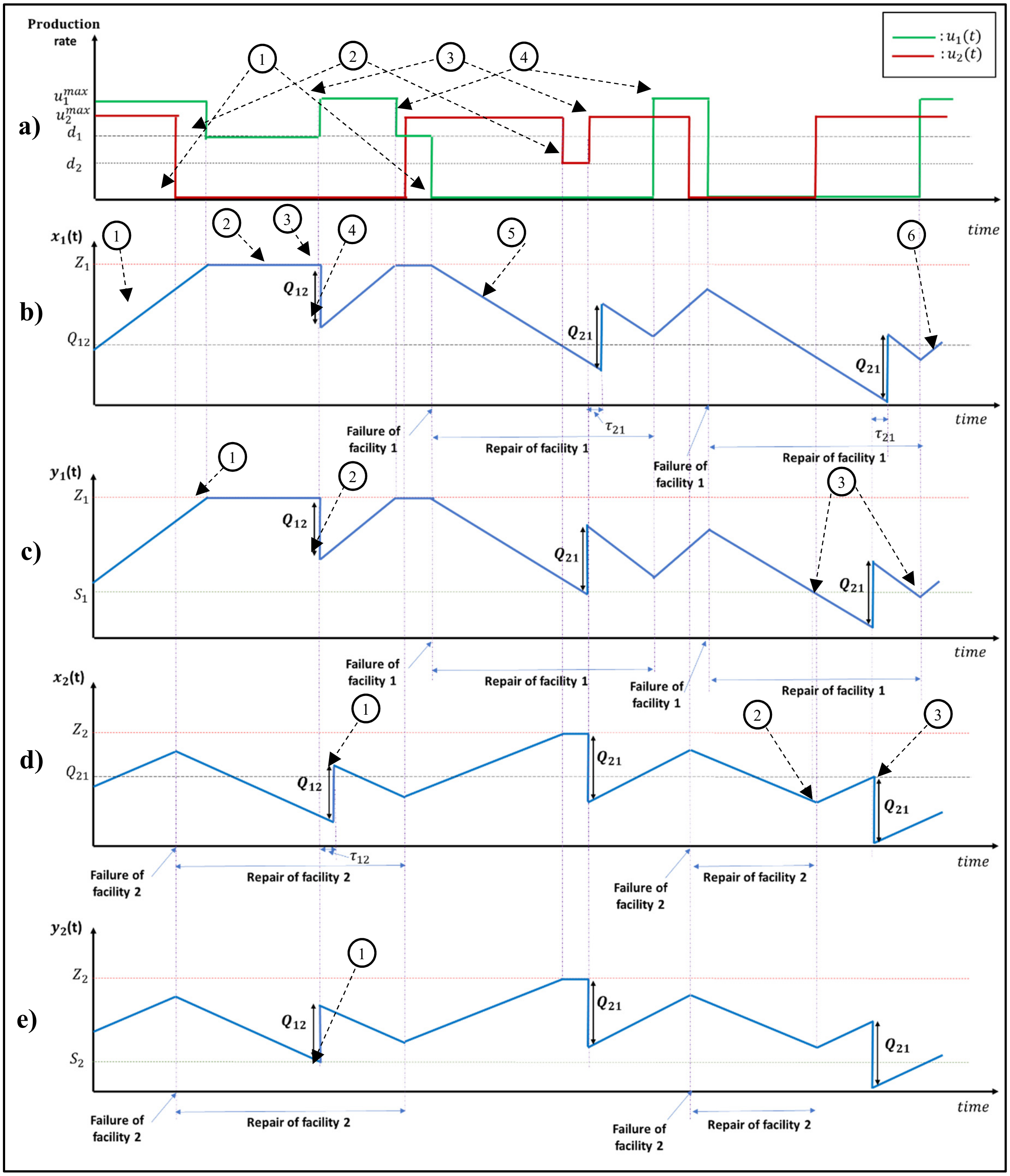

Figure 3.8 Dynamics of the simulation model

We also see that the inventory levels follow the production rate. They are replenished faster when the rate of production is set to its maximum (arrow 1 in fig.3.8.b). When the production is set to the demand rate, they are maintained at the same level (arrow 2 in fig.3.8.b). The inventory levels decrease with a rate equal to the demand rates when the production facility breakdown (arrow 5 in fig.3.8.b).

The trajectories of the inventory position and the inventory level also show that the transshipment policy is respected and is accurate. For example, when the inventory position of retailer 2 reaches its reorder point (arrow 1 in fig.3.8.e) and since retailer 1 has an inventory level higher than the transshipment quantity (arrow 3 in fig.3.8.b) a transshipment of a quantity Q_{12} is made from retailer 1 to retailer 2. Thus, we see a decrease by a quantity Q_{12} (arrow 4 in fig.3.8.b) in the inventory level of the retailer 1 and an increase of a quantity Q_{12} in the inventory level of the retailer 2 after a transshipment delay τ_{12} (arrow 1 in fig.3.8.d). However, if the inventory position of a retailer reaches the corresponding reorder point (arrow 3 in fig.3.8.c), and the inventory level of the other retailer is not enough to proceed the transshipment (arrow 2 in fig.3.8.d), the transshipment is not possible right away. The transshipment is reported till the retailer transhipping out has enough inventory level (arrow 3 in fig.3.8.d). However, the retailer requesting a transshipment can also be replenished by his corresponding production facility when it is repaired (arrow 6 in fig.3.8.b).

3.6.4 Numerical example

An illustrative numerical example is presented in this section; the data used for this example are presented in the table 3.4.

Table 3.4 Data of the numerical example

Parameter	u_1^{max}	d_1	**TTF1**	**TTR1**	c^+	c^-	τ
Value	2000	1500	Exp(15)	Exp(2)	0.15	15	1
Parameter	u_2^{max}	d_2	**TTF2**	**TTR2**	c^{tr}	c^v	
Value	2000	1300	Exp(15)	Exp(2)	1.6	0.01	

In this example we suppose that cost and delays are symmetric between retailer 1 and retailer 2 ($\tau = \tau_{12} = \tau_{21}$, $c^{tr} = c_{12}^{tr} = c_{21}^{tr}$ and $c^v = c_{12}^v = c_{21}^v$).

The considered production rates and demand rates are chosen respecting the feasibility condition described as follows: $\frac{\mathrm{MTTF_i}}{\mathrm{MTTF_i+MTTR_i}} u_i^{max} > d_i$. We simulated the model for a duration of 200,000 time units, which ensures that the steady state is attained. The main effects and most of the interactions and the quadratic effects of the policy parameters are significant for the dependant variable (total cost) at 95% level of significance as illustrated in the Pareto chart in figure 3.9. The obtained R-squared adjusted is equal to 95,14% which means that over 95% of the variability is explained by the model. The adequacy of the model is also verified by a residual analysis. The normality and the homogeneity of the variances is confirmed by analyzing the residual versus fitted values plot as well as the normal probability plot of residuals.

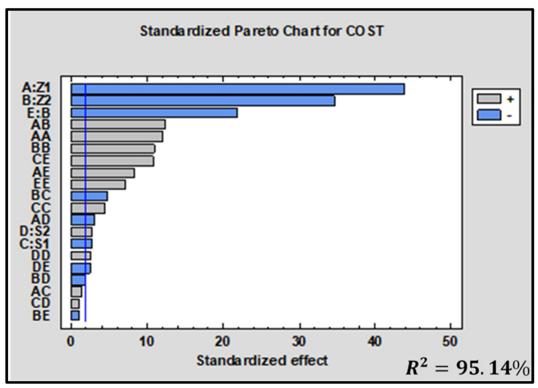

Figure 3.9 Pareto chart

The total expected cost is fitted by the quadratic model in eq. (3.15) obtained from STATGRAPHICS:

$$\begin{aligned}\Phi(\mathrm{Z}_1, \mathrm{Z}_2, \mathrm{S}_1, \mathrm{S}_2, B) = {} & 40097.3 - 2.38.\ \mathrm{Z}_1 - 1.78.\ \mathrm{Z}_2 - 3.68.\ \mathrm{S}_1 + 0.52.\ \mathrm{S}_2 - \\ & 3876.91.\ \mathrm{B} + 6.05.10^{-5}\ {Z_1}^2 + 4.40.10^{-5}.\ \mathrm{Z}_1.\mathrm{Z}_2 - 9.28.\ 10^{-5}.\ \mathrm{Z}_1.\mathrm{S}_2 + \\ & 0.087.\ \mathrm{Z}_1.\mathrm{B} + 5.53.\ 10^{-5}\ {Z_2}^2 - 10.11.\ 10^{-5}\ .\mathrm{Z}_2.\mathrm{S}_1 - 5.38.\ 10^{-5}.\mathrm{Z}_2.\mathrm{S}_2 + \\ & 81.19.\ 10^{-5}\ .{S_1}^2 + 0.69.\ \mathrm{S}_1.\mathrm{B} + 93.29.\ {S_2}^2 - 0.22.\mathrm{S}_2.\mathrm{B} + 321.494.B^2 + \varepsilon\end{aligned} \tag{3.15}$$

The optimal total cost of 4377$ corresponds to $Z_1^* = 12946, Z_2^* = 12883, S_1^* = 1206, S_2^* = 1043,\ B^* = 0.28$. We recall that $B = \frac{S_1}{Q_{21}} = \frac{S_2}{Q_{12}}$ as explained in section 6.1, thus, $Q_{21}^* = 4257$ and $Q_{12}^* = 3681$.

The figure 3.10 displays the contour surface plots of the total cost in function of the optimal policy parameters.

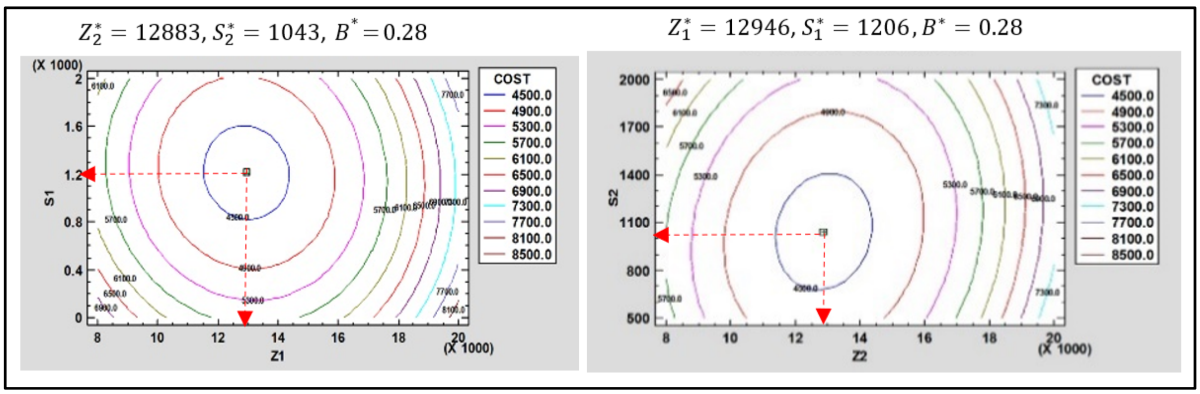

Figure 3.10 Contour surface plots

The validity of the RSM model is cross checked. The optimal total cost falls within the 95% confidence interval [4361-4384] determined by conducting 40 additional replications of the simulation model using as inputs the optimal parameters of the control policy.

3.7 Sensitivity analysis

To confirm the robustness of the results, we conduct an extensive sensitivity analysis to verify how the optimal policy control parameters $(Z_1^*, Z_2^*, S_1^*, S_2^*, Q_{21}^*, Q_{12}^*)$ respond to the variation of the costs and system parameters.

3.7.1 Cost variation

In this subsection, we examine the effect of the cost parameters variation on the optimal parameters and on the optimal cost of the proposed policy as shown in table 3.5.

- Variation of the holding cost c^+: By increasing c^+ (Cases 2), the optimal hedging level Z_1^* and Z_2^* decrease to avoid higher holding costs. The reorder points S_1^* and S_2^* decrease systematically with the decrease of the hedging level since there is less stock to allow transshipments without putting the retailer who sends the transshipment under risk of shortage. The same explanation applies for the transshipment lot sizes Q_{21}^* and Q_{12}^* that decreases with the decrease of the hedging levels. The opposite effects are seen when the holding costs increase (cases 1).

Table 3.5 Sensitivity analysis of the proposed policy (policy 1)

Optimal control Parameters			Z_1^*	Z_2^*	S_1^*	S_2^*	Q_{21}^*	Q_{12}^*	TC^*	
Basic Values			12946	12883	1206	1043	4257	3681	4377	
cases			ΔZ_1^*	ΔZ_2^*	ΔS_1^*	ΔS_2^*	ΔQ_1^*	ΔQ_2^*	ΔTC^*	Remarks
1	c^+	0.1	+815	+642	+7	+4	+15	+6	-1795	$Z_1^*\uparrow Z_2^*\uparrow S_1^*\uparrow S_2^*\uparrow Q_1^*\uparrow Q_2^*\uparrow$
2		0.2	-844	-582	-8	-6	-10	-5	+1696	$Z_1^*\downarrow Z_2^*\downarrow S_1^*\downarrow S_2^*\downarrow Q_1^*\downarrow Q_2^*\downarrow$
3	c^-	10	-723	-83	-119	-21	-594	-244	-261	$Z_1^*\downarrow Z_2^*\downarrow S_1^*\downarrow S_2^*\downarrow Q_1^*\downarrow Q_2^*\downarrow$
4		18	+202	+22	+64	+16	+278	+101	+141	$Z_1^*\uparrow Z_2^*\uparrow S_1^*\uparrow S_2^*\uparrow Q_1^*\uparrow Q_2^*\uparrow$
5	c^{tr}	1.2	-385	-318	+28	+15	-296	-285	-87	$Z_1^*\downarrow Z_2^*\downarrow S_1^*\uparrow S_2^*\uparrow Q_1^*\downarrow Q_2^*\downarrow$
6		2	+219	+196	-14	-12	+142	+124	+82	$Z_1^*\uparrow Z_2^*\uparrow S_1^*\downarrow S_2^*\downarrow Q_1^*\uparrow Q_2^*\uparrow$
7	c^v	0.005	-710	+16	+19	+3	+328	+234	-237	$Z_1^*\downarrow Z_2^*\uparrow S_1^*\uparrow S_2^*\uparrow Q_1^*\uparrow Q_2^*\uparrow$
8		0.015	+577	-66	-16	-3	-371	-285	+213	$Z_1^*\uparrow Z_2^*\downarrow S_1^*\downarrow S_2^*\downarrow Q_1^*\downarrow Q_2^*\downarrow$

- Variation of the backlog cost c^-: By increasing c^-(cases 4), the model reacts by increasing the hedging levels Z_1^* and Z_2^* to allow more stock on-hand which is a measure to protect the system from future shortages and the high backlog costs. The optimal reorder points S_1^* and S_2^* also increase to allow more frequent transshipments between the retailers. The transshipment lot sizes Q_{21}^* and Q_{12}^* also increase to ensure that enough quantity is delivered to the retailer who is requesting the transshipment. The opposite effects are produced by decreasing the backlog cost c^-(cases 3).

- <u>Variation of the transshipment fixed cost c^{tr}</u>: By increasing the transshipment fixed cost c^{tr}(case 6), the model reacts by decreasing the reorder points S_1^* and S_2^* to avoid the frequent transshipments. The increase of the lot sizes Q_{21}^* and Q_{12}^* is due to the fact that the system allows more products to be transferred at each transshipment. The increase of c^{tr} makes the transshipments cost higher therefore the system compensates by increasing the hedging levels Z_1^* and Z_2^* to protect the system against shortages. The opposite effects are observed by decreasing the transshipment cost c^{tr}(case 5).

- <u>Variation of the transshipment variable cost c^v</u>: By increasing c^v(case 8), the reorder points S_1^* and S_2^* decrease to ensure less transshipments. The hedging level Z_1^* of the most vulnerable retailer to shortages increases and the hedging level Z_2^* of the other retailer slightly decrease. This is because with less transshipments, the system responds by protecting the most vulnerable retailer by increasing its hedging level. The other retailer is transshipping less frequently to the vulnerable retailer; thus, we see its hedging level slightly decreases. The reduction of the transshipment lot sizes Q_{21}^* and Q_{12}^* is explained by the higher transshipment unit cost, the smaller the lot size the lower the cost. The opposite effects are seen by decreasing the transshipment variable cost c^v(case 7).

3.7.2 Variation of system availability

We study the effect of the variation of the availabilities of the production facilities on the optimal parameters of the system. As shown in figure 3.11(a), when the availability of the production facility 1 decreases. The corresponding hedging level (Z_1) increases to ensure more safety stock on-hand to reduce the risk of shortages due to breakdowns. The hedging level of the other production facility (Z_2) increases as well but less significantly. This is because transshipments from facility 2 to facility 1 are more required, therefore, the facility 2 needs to have enough safety stock to do so. The more frequent transshipments are shown by the increase of the reorder point (S_1) of the production facility 1 (figure 3.11(b)). A higher transshipment quantity (Q_{21}) received by the facility 1 is also seen in figure 11(c). However, we see a decrease in the reorder point (S_2) of the facility 2 (figure 3.11(b)) and a decrease in the lot size (Q_{12}) (figure 3.11(c)) transshipped to the facility 2 since the production facility 1 is less capable of transshipping out and it may put itself in a risk of shortage when doing so.

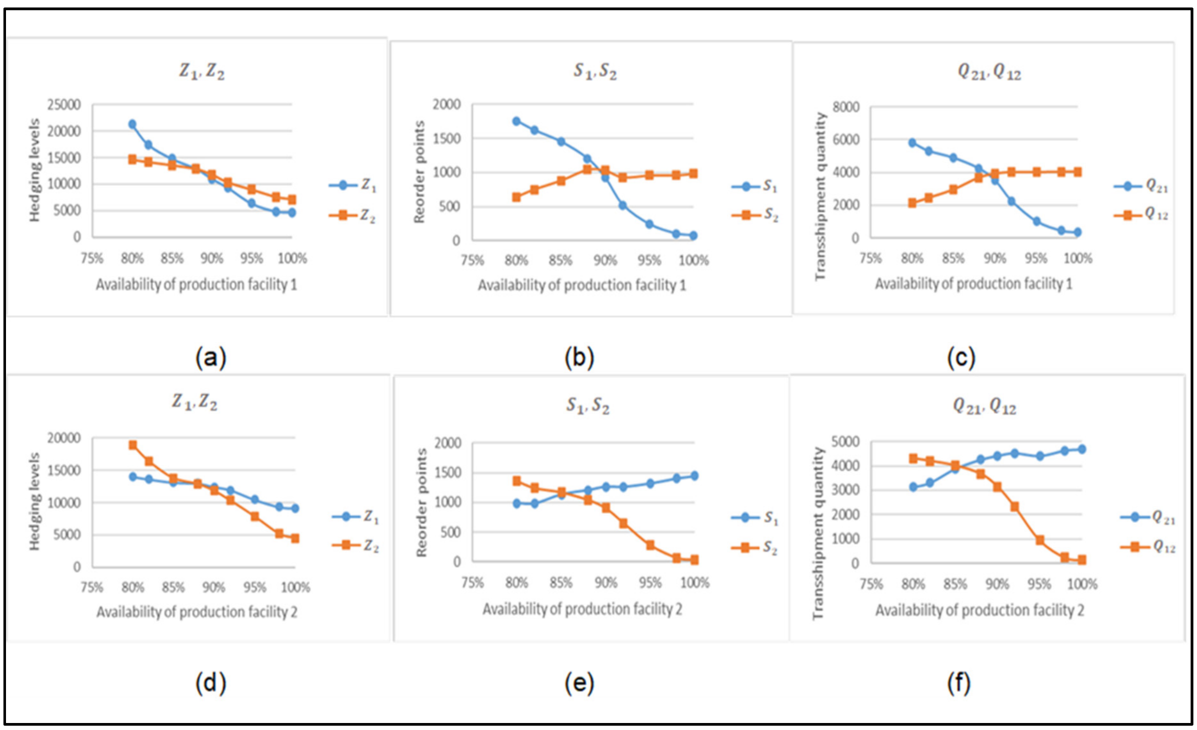

Figure 3.11 Effect of the variation of the production facilities availabilities on the optimal control parameters

When the availability of the production facility 1 is increased, we see the opposite effects. The hedging level Z_1 decreases (figure 3.11(a)) to avoid higher holding costs for the surplus stock. The safety stock Z_2 of the facility 2 also decreases (figure 3.11(a)) since less transshipments are required. This is seen by the lower reorder point (S_1) (figure 3.11(b)) and the lower lot size (Q_{21}) (figure 3.11(c)) and with a very high availabilities transshipments are no more allowed; S_1 and Q_{21} decreases to zero (figure 3.11(b) and figure 3.11(c)). However, transshipment to the facility 2 becomes more frequent since the production capacity of the facility 1 is enough to allow it. Therefore, we see an increase in the reorder point (S_2) (figure 3.11(b)) and the lot size (Q_{12}) (figure 3.11(c)) of the facility 2.

The same logic holds when we decrease or increase the availability of the production facility 2, we can see these effects in the opposite way in the figures (3.11(d), 3.11(e) and 3.11(f)).

3.8 Comparative study

In this section we propose an improved transshipment policy, and we describe the most relevant policies to our work and adapt them to our system. Then we perform a comparative study between these policies and our proposed policies for a wide range of system and cost parameters.

3.8.1 Improved transshipment policy (policy 2)

We enhance our policy by adding a condition to the transshipment policy to protect the facilities when they send the transshipments. We achieve this by adding a condition that ensures the balance of inventory among both retailers when the transshipments are performed. The new enhanced transshipment policy is described as follows:

$$\Omega_{ij}(t, Q_{ij}) \begin{cases} Q_{ij} & if\ y_j(t) \leq S_j \ \ and\ x_i(t) \geq Q_{ij} \\ & and\ y_i(t) - Q_{ij} \geq S_j + dS_j \\ 0 & otherwise \end{cases} \tag{3.16}$$

The condition $y_i(t) - Q_{ij} \geq S_j + dS_j$ implies that right after the transshipment, the inventory position of the sending facility ($y_i(t) - Q_{ij}$) must be greater than the inventory position of the requesting facility (which is S_j at the moment of the transshipment request). Otherwise, the transshipment is not performed. dS_j is a new design variable that defines how much higher the inventory position of the sending facility i should be compared to the inventory position of the requesting facility j.

3.8.2 No transshipment (policy 3)

We consider the case of no transshipment between retailers; in this policy, since there is no transshipment, we only consider the classical HPP policy for the production policy of each facility. Similar to our production policy, the production rate is adjusted depending on hedging levels. The only difference is that we use inventory levels instead of inventory positions since there are no lots transferred between the facilities. The policy can be described as follows:

$$u_i(t)=\begin{cases} u_i^{max} & if\ \ x_i(t) < Z_i \\ d_i & if\ \ x_i(t) = Z_i \\ 0 & if\ \ x_i(t) > Z_i \end{cases} \tag{3.17}$$

3.8.3 Emergency transshipment (policy 4)

Since we did not find in the literature a proper joint production-transshipment control policy to compare it with our policy. We propose a simplified production-transshipment policy by integrating the emergency transshipment policy found in the literature (Peterson et al. 2012) jointly with our proposed production policy (eq.3.13). In this policy the lateral transshipments are triggered when the inventory levels of a retailer fall under zero. Which means that the transshipments reorder point is fixed at the value zero. The emergency transshipment policy can be described as follows:

$$\Omega_{ij}(t, Q_{ij}) = \begin{cases} Q_{ij} & if\ x_j(t) \leq 0\ \ and\ x_i(t) \geq Q_{ij} \\ 0 & otherwise \end{cases} \tag{3.18}$$

3.8.4 Hold-back level transshipment (policy 5)

In the literature, most of the hold-back transshipments are triggered when the inventory level of a retailer falls under zero, but they have a conservative aspect since they also seek to protect the sending location by considering a hold-back level ($\mathrm{H_i}$) that should be maintained at the sending location and below which transshipments are not allowed out. If the inventory level is above this level $\mathrm{H_i}$, the surplus of stock ($x_i(t) - H_i$) can be transshipped out (Xu et al., 2003). We adapt the hold-back transshipments to our system to be triggered when the inventory position $y_j(t)$ (instead of the inventory level) falls under a positive hold-back level ($\mathrm{H_j}$) (instead of zero) which will be more suitable for our system with the transshipment delays. We integrate this adapted transshipment policy jointly with our proposed production policy (eq.13). The hold-back transshipment policy can be described as follows:

$$\Omega_{ij}(t, Q_{ij}) = \begin{cases} Q_{ij} & \text{if } x_j(t) \leq 0 \text{ and } x_i(t) \geq Q_{ij} \\ 0 & \text{otherwise} \end{cases} \tag{3.19}$$

3.8.5 Virtual transshipments (policy 6)

Virtual transshipments implies that if a facility j faces a stockout, the other facility i can produce for it (for facility j). The joint production-virtual transshipment policy used in the literature (Zhao et al., 2008) is expressed as follows:

<u>Production policy of facility i:</u>

$$u_i(t) = \begin{cases} u_i^{max} & \text{if } x_i(t) \leq Z_i \\ 0 & \text{otherwise} \end{cases} \tag{3.20}$$

<u>Transshipment policy from facility i to facility j:</u>

$$\Omega_{ij}(t) = \begin{cases} 1 & \text{if } x_j(t) \leq 0 \text{ and } x_i(t) > K_i \\ 0 & \text{otherwise} \end{cases} \tag{3.21}$$

Where $\Omega_{ij}(t)$=1 means that facility i can produce for facility j until the inventory level of facility j reaches zero or until the inventory level of facility i falls under the level K_i. And $\Omega_{ij}(t)$=0 means that the facility i only produces for itself.

3.8.6 Comparison between our proposed policy and the other policies

We conduct a comparative study between our proposed policies (policy 1 and 2) and the other policies described above. We use the same simulation-optimization approach (figure 21 in section 3.5) used for our proposed policy to optimize the parameters of the policies considered for comparison.

Eq.3.22, Eq.3.23, Eq.3.24, Eq.3.25 and Eq.3.26 represent respectively the response functions obtained from STATGRAPHICS for the improved transshipment policy (ITP), the no transhipment policy (NTR), the emergency transshipment policy (ETR), the hold-back transshipment policy (HBTR) and the virtual transshipment policy (VTR). For the improved policy, we introduce the design variables γ_i (0< γ_i <1) which is a fraction of S_1 and S_2 to define $dS_1 = \gamma_1 . S_1$ and $dS_2 = \gamma_2 . S_2$. Due to the large number of variables, we use a Box-Kehnken design with 8 central points (64 runs) for the improved policy.

$$\begin{aligned}\Phi_{ITP}(Z_1, Z_2, S_1, S_2, B, \gamma_1, \gamma_2) &= 39020.1 - 2.29.\ Z_1 - 1.91.\ Z_2 + 1.10.\ S_1 - \\ &5.32.\ S_2 - 2467.14.\ B - 19863.5\ .\gamma_1 - 1903.3.\ \gamma_2 + 5.14.10^{-5}.\ Z_1{}^2 + \\ &1.26.10^{-5}.Z_1.\ Z_2 + 0.061.Z_1.\ B + 4.84.\ 10^{-5}.Z_2{}^2 + 0.026.\ Z_2.\ B + 21.16. \\ &10^{-5}.\ S_1{}^2 + 52.19.\ 10^{-5}.\ S_2{}^2 + 181.79.\ B^2 + \varepsilon\end{aligned} \tag{3.22}$$

$$\begin{aligned}\Phi_{NTR}(Z_1, Z_2) &= 33132.3 - 2.55.\ Z_1 - 1.26.\ Z_2 + 7.68.10^{-5}\ Z_1{}^2 + 1.03.10^{-5}. \\ &Z_1.\ Z_2 + 4.61.\ 10^{-5}\ Z_2{}^2 + \varepsilon\end{aligned} \tag{3.23}$$

$$\Phi_{ETR}(Z_1, Z_2, Q_{21}, Q_{12}) = 34490.5 - 2.01.\ Z_1 - 1.88.\ Z_2 + 0.18.\ Q_{21} - 1.09.\ Q_{12} + 4.73.\ 10^{-5}\ {Z_1}^2 + 4.54.10^{-5}.\ Z_1.Z_2 - 2.05.\ 10^{-5}.\ Z_1.Q_{21} + 1.48.\ 10^{-5}.\ Z_1.Q_{12} + 5.01.\ 10^{-5}\ {Z_2}^2 - 2.08.\ 10^{-5}.\ Z_2.Q_{21} - 1.45.\ 10^{-5}.\ Z_2.Q_{12} + 2.77.\ 10^{-5}\ {Q_{21}}^2 + 1.69.\ 10^{-5}.\ Q_{21}.Q_{12} + 12.47.\ 10^{-5}\ {Q_{12}}^2 + \varepsilon \quad (3.24)$$

$$\Phi_{HBTR}(Z_1, Z_2, H_1, H_2) = 43127.3 - 3.07.\ Z_1 - 2.65.\ Z_2 + 1.38.\ H_1 + 0.91.\ H_2 + 9.60.\ 10^{-5}\ {Z_1}^2 + 5.79.10^{-5}.\ Z_1.Z_2 + 15.97.10^{-5}.\ Z_1.H_1 + 7.97.\ 10^{-5}\ {Z_2}^2 - 13.14.10^{-5}.\ Z_2.H_2 + 22.21.\ 10^{-5}\ {H_1}^2 + 10.65.10^{-5}.\ H_1.H_2 + 27.39.\ 10^{-5}\ {H_2}^2 + \varepsilon \quad (3.25)$$

$$\Phi_{VTR}(Z_1, Z_2, K_1, K_2) = 19608.9 - 0.29.\ Z_1 - 0.40.\ Z_2 - 0.13.\ K_1 + 0.23.\ K_2 + 0.47.\ 10^{-5}\ {Z_1}^2 + 0.79.\ 10^{-5}\ {Z_2}^2 + 0.11.\ 10^{-5}\ {K_1}^2 + 0.37.10^{-5}.\ K_1.K_2 - 0.94.\ 10^{-5}.\ {K_2}^2 + \varepsilon \quad (3.26)$$

In table 3.6 we present the optimal parameters of the considered policies as well as the cost details.

From table 3.6 we see that the hedging level Z_1^* of the production facility 1, which has the highest demand rate, is the lowest for the proposed policies (policy 1 and 2) compared to the other policies. This is because transshipments are allowed between retailers ahead of the shortage (contrary to the other policies) which also explains why the backlog costs of the proposed policies are lower than the other policies. Transshipment quantities of the improved policy (policy 2) are higher than the basic policy (policy1) since transshipments are more constrained, so the retailers transship more products to take advantage of the fewer transshipments (economy of scale) which also explains the lower transshipment cost of policy 2. Compared to the no transshipment policy, we see that the hedging level Z_2^* of the proposed policies (policy 1 and policy 2) is higher. This is because the facility 2 stores more products in order to be able to transship to the facility 1 when required. The hold-back transshipment policy generates the lowest transshipment cost since it allows fewer transshipments due to the hold-back levels. Virtual transshipment policy generates the highest total cost because its production policy only allows to produce or to stop production, thus, the stock is not maintained at a certain level as in our proposed policies which leads to the highest hedging levels Z_1^* and Z_2^* and to a high backlog cost. Virtual transshipments implies that a facility can produce for the other stocked out facility only if it has already in its warehouse at least a security level of stock (K_1^* and K_2^*) which explains why K_1^* and K_2^* of policy 6 are higher than the reorder points (S_1^* and S_2^*) of the other policies. The virtual transshipment policy incurs the highest cost of transshipment since the products are transshipped item by item to the other facility which incurs higher fixed costs of transshipment.

The superiority of our proposed policies is confirmed for a wide range of cost parameters and system parameters by studying the effects of varying these parameters on the total cost of each policy which are shown in figure 3.12. We did not include the virtual transshipment policy in figure 3.12 since it generates significantly higher cost compared to the other policies.
Figure (3.12.a) shows that our proposed policies (policy 1 and policy 2) outperform the others especially for lower holding costs since the system allows to stock more products which allows more transshipments to be performed leading to better reaction to shortages compared to the other policies.

Table 3.6 Comparative results of the considered policies

	Optimal policy parameters												Cost details			Optimal total cost
	Z_1^*	Z_2^*	S_1^*	S_2^*	Q_{21}^*	Q_{12}^*	dS_1^*	dS_2^*	H_1^*	H_2^*	K_1^*	K_2^*	Inventory cost	Backlog cost	Transshipment cost	TC^*
Basic policy	12946	12883	1206	1043	4257	3681	-	-	-	-	-	-	2521	113	1743	4377
Improved policy	13268	12633	1217	1146	4648	4377	61	607	-	-	-	-	2618	87	1424	4129
No transshipment policy	15806	11980	-	-	-	-	-	-	-	-	-	-	3526	1837	-	5363
Emergency transshipment policy	15396	13699	-	-	6383	3830	-	-	-	-	-	-	3232	585	808	4625
Hold-back transshipment	15011	14049	-	-	-	-	-	-	3233	2315	-	-	2958	851	874	4683
Virtual transshipment	32944	27032	-	-	-	-	-	-	-	-	14250	10265	4265	2140	2558	8963

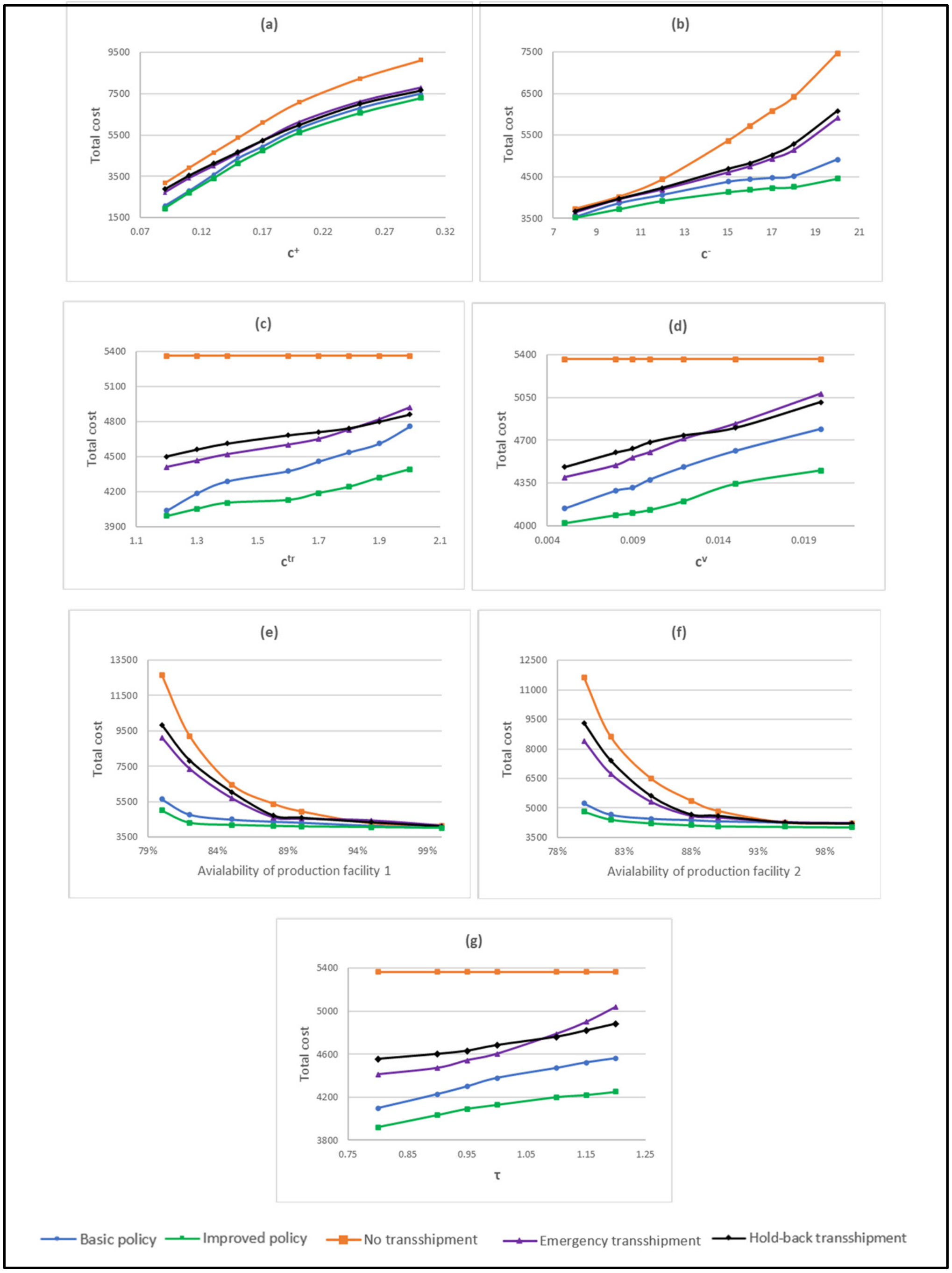

Figure 3.12 The effect of varying the costs and system parameters on the total cost

When we increase the backlog cost in figure (3.12.b) we can see that our proposed policies respond better to shortages by sharing inventory between the facilities at the right time ahead of the shortage since there is a positive reorder point. The improved policy outperforms the basic policy for higher backlog costs since it allows transshipments to be performed while ensuring that the sending facility is not itself at a risk of shortage. This achieved by not only checking if the sending facility has the required transshipment quantity, but also if its inventory position after sending the transshipment will be high enough to protect it against the possible shortages. The same effects of increasing backlog cost are seen when decreasing the availability of the production facilities (figure 3.12.e and figure 3.12.f) since lower availabilities result in higher shortages and therefore higher backlog cost.

By increasing the transshipment fixed costs (c^{tr}) in figure (3.12.c), the transshipment variable costs (c^{v}) in figure (3.12.d) or the transshipment delays (τ) in figure (3.12.g), transshipment operations have a higher cost, and we can see that the cost reduction of the transshipments policies (policy 1,2,4,5 and 6) compared to the no transshipment policy decrease. Emergency policy outperforms the holdback policy for lower transshipment costs and delays since it allows more transshipments as a response for shortages, however, for higher transshipments costs or delays the holdback policy outperforms the emergency policy since it generates fewer transshipments. Our proposed policies (policy 1 and policy 2) generate the lowest cost compared to the others for the wide range of transshipment costs and delays. This is explained by the fact that our policies have reorder points that vary depending on the situation of the facilities and which adapt to the transshipments costs and delays.

To sum up, and as shown in figure 3.12. Our proposed policies outperform the other policies of the literature for the wide range of cost and system parameters and the improved policy is the one that generates the lowest cost among all.

3.9 Managerial insights and practical implementation

The developed joint production-transshipment control policy is designed for the firms that operate in two-locations failure prone production facility that allows cooperation between the facilities by sharing inventories through lateral transshipment. The developed policies allow the decision maker to minimize the total cost by taking decisions at any given time on when and at which rate each production facility should produce as well as the decision of when and how much quantity should be transshipped from one facility to another.

Figure 3.13 shows the decision-making process for the manufacturing system presented in figure 3.1 with the parameters presented in table 3.6 for the basic case of our improved policy (policy 2). The decisions are made by comparing the inventory positions with the hedging levels and the reorder points. In fact, when the inventory position of a production facility is less than the corresponding threshold (13268 for production facility 1 and 12633 for production facility 2), production should be set at its maximum (2000). When the inventory positions are equal to the hedging levels, the production is set to the corresponding demand rates (1500 for production facility 1 and 1300 for production facility 2). The production in a facility is stopped if its inventory position goes above the corresponding hedging level.

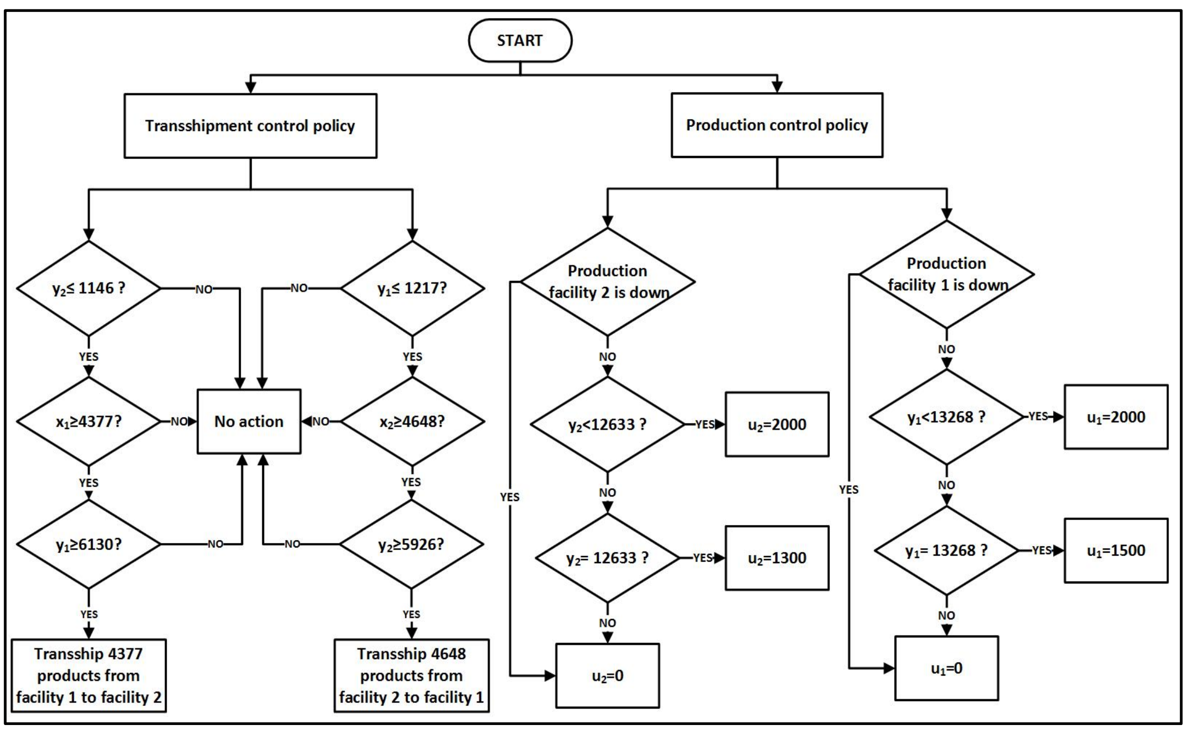

Figure 3.13 Implementation logic chart of the improved policy (policy 2)

Transshipment decisions are also made depending on the inventory positions and the inventory levels, if the inventory position of facility 1 (respectively facility 2) falls under its reorder point 1217 (respectively 1146), the decision maker verifies the inventory level and the inventory position of the other facility; facility 2 (respectively facility 1), if it has in its warehouse the quantity to be transshipped which is 4648 (respectively 4377), and if the inventory position is enough (6130 for facility 1 and 5926 for facility 2) to ensure that the sending facility is protected after the transshipment is performed. A transshipment of a quantity 4648 is launched from facility 2 to facility 1 (respectively a quantity of 4377 is transshipped from facility 1 to facility 2). Otherwise, the system should not transship.

3.10 Conclusion

In the past transshipment has mostly been studied in a context of reliable suppliers and without the consideration of production control policy. In this paper we investigated the two-location failure-prone manufacturing system with the integration of transshipments between the locations and considering transshipment delays. We developed a joint production-transshipment control policy (policy 1) with the aim of minimizing the total cost. We use a combined analytical and simulation-based approach to define the structure of the control policy and to optimize its parameters. The obtained transshipment policy is then enhanced in policy 2 to improve its performance.

The proposed control policies (policy 1 and policy 2) are a combination of an HPP for production control and a state dependant economic transshipment quantity. A comparison is made between the proposed policies and other transshipment policies from the literature. We show that our policies outperform the other policies in terms of cost.

For future research it would be interesting to investigate the case of multi-location failure-prone production facilities (more than two locations). This can be challenging since it requires a decision of selecting the facility from which the transshipment is requested at each time. Routing optimization of transshipments among multiple retailers can also be investigated in the future. Other aspects that exist in many manufacturing systems such as quality control, preventive maintenance or product returns can also be treated jointly with transshipment. The consideration of multiple products is another interesting extension that can be investigated in future research.

CONCLUSION GÉNÉRALE

Pour réussir dans un environnement concurrentiel et imprévisible, il est indispensable que les entreprises manufacturières reconnaissent l'importance cruciale de la bonne gestion de la chaîne d’approvisionnement. La coordination entre les différentes échelles de la chaîne d’approvisionnement est une stratégie fortement répandue et recommandée pour atteindre une meilleure performance globale en termes de coût et de service. La production et la livraison ainsi que les transferts du stock entre les détaillants sont des éléments clés de la chaine d’approvisionnent qui doivent être gérés de façon intégrée par les entreprises pour pouvoir se développer pleinement. Ce travail a permis de répondre à deux objectifs principaux qui touchent à ce contexte. Le premier est de proposer une politique de commande intégrée de production et de livraison vers les détaillants. Le deuxième est de proposer une politique de commande intégrée de production et de transfert de stock entre les détaillants.

Nous avons adopté une approche qui combine l’approche analytique et la simulation pour atteindre nos objectifs. Tout d’abord, nous avons déterminé les structures des politiques optimales en modélisant analytiquement les problèmes et en résolvant numériquement les équations de HJB correspondantes. En deuxième lieu, nous avons utilisé la simulation en combinaison avec les techniques d’optimisation statistiques pour déterminer les valeurs optimales des paramètres des politiques obtenues. Des analyses de sensibilité ainsi que des comparaisons ont été faites pour montrer la robustesse des politiques proposées et leur supériorité en termes de coût par rapport aux politiques de la littérature.

Dans le chapitre 1, nous avons présenté une étude bibliographique qui couvre les notions principales liées à notre mémoire. Nous avons introduit les types de chaîne d’approvisionnement considérées et nous avons défini les termes en relation avec notre étude. Nous avons également présenté une série de revues scientifiques reliées à notre recherche ainsi qu’une critique de la littérature pour montrer l’originalité de notre travail. Dans une deuxième partie, nous avons défini la problématique de recherche, les objectifs de la recherche et nous avons présenté la méthodologie de recherche adoptée.

Dans le chapitre 2, une politique de commande optimale intégrée de production et de livraison a été proposée pour une chaîne d'approvisionnement composée d'un manufacturier non fiable et de plusieurs détaillants. Les détaillants sont localisés dans différentes localisations et nous avons considéré les délais et les coûts de transport. La politique proposée est une combinaison d'une politique à seuil critique pour la production et d'une quantité optimale de commande pour la livraison. La politique proposée est ensuite améliorée en intégrant la notion de priorisation des détaillants. Les comparaisons entre les deux politiques proposées et les politiques de la littérature montrent que nos politiques sont meilleures en termes de coût.

Dans le chapitre 3, nous avons étudié une chaîne d'approvisionnement composée de deux sites de production non fiables qui produisent pour deux détaillants adjacents localisés dans deux localisations différentes et faisant face à de différents taux de demandes. Les transferts de stocks entre les détaillants sont autorisés avec des délais de transport et des coûts de transfert de stock. Nous avons proposé une politique de commande stochastique qui intègre la production et le transfert de stock entre les détaillants. Nous avons amélioré la politique en ajoutant une condition pour le transfert de stock qui garantit que le détaillant qui envoie du stock ne se met pas en risque de pénurie. Les politiques proposées ont une meilleure performance en termes de coût en les comparant avec les autres politiques de la littérature.

Il existe plusieurs voies d'extensions de nos modèles, les recherches futures peuvent intégrer les décisions de production, de livraison et de transfert de stock dans un même modèle. La considération de plusieurs détaillants (plus que deux) pour le système intégrant le transfert de stock peut présenter une extension intéressante pour le futur vu qu'elle nécessite des décisions de choix de détaillant duquel le transfert est demandé. La considération de plus qu'un seul produit serait aussi un axe de recherche à considérer. D'autres phénomènes, comme la qualité et la maintenance préventive, peuvent être intégrés comme extension dans nos systèmes en proposant des politiques adéquates qui les intègrent.

ANNEXE I

NUMERICAL RESOLUTION FOR THE ONE-MANUFACTURER TWO-RETAILER SYSTEM WITH DELIVERY

Kushner approach allows to iteratively approximate the value function $v(x_m, x_1, x_2, \alpha)$ as well as its derivative using the discretization of the state variables (x_m, x_1, x_2, α). The finite grid denoted D_h required for the implementation of the approximation technique is defined as follows:

$$D_h = \{(x_m, \Pi): 0 \leq x_m \leq a\ , -b \leq x_i \leq c\ ; i \in \{1,2\}\}$$

where a, b and c are predefined positive integers.

The value function $v(x_m, x_1, x_2, \alpha)$ is approximated by $v^h(x_m, x_1, x_2, \alpha)$, its partial derivatives are approximated by:

$$(v)_{x_m}(x, y, \alpha) = \frac{1}{h_{x_m}}(v^{h_{x_m}}(x_m + h_{x_m}, x_1, x_2, \alpha) - v^{h_{x_m}}(x_m, x_1, x_2, \alpha))$$

$$(v)_{x_1}(x, y, \alpha) = \frac{1}{h_{x_1}}(v^{h_{x_1}}(x_m, x_1, x_2, \alpha) - v^{h_{x_1}}(x_m, x_1 - h_{x_1}, x_2, \alpha))$$

$$(v)_{x_2}(x, y, \alpha) = \frac{1}{h_{x_2}}(v^{h_{x_2}}(x_m, x_1, x_2, \alpha) - v^{h_{x_2}}(x_m, x_1, x_2 - h_{x_2}, \alpha))$$

In order to obtain the approximated value function $v^h(x_m, x_1, x_2, \alpha)$, the following set of boundary conditions are used as constraints for the numerical implementation (Yan and Zhang 1995) :

$$v^h(a + h_{x_m}, x_1, x_2, \alpha) = v^h(a, x_1, x_2, \alpha) + \frac{c^+}{\rho} h_{x_m}$$

$$v^h(x_m, -b - h_{x_1}, x_2, \alpha) = v^h(x_m, -b, x_2, \alpha) + \frac{c^-}{\rho} h_{x_1}$$

$$v^h(x_m, c + h_{x_1}, x_2, \alpha) = v^h(x_m, c, x_2, \alpha) + \frac{c^+}{\rho} h_{x_1}$$

$$v^h(x_m, x_1, -d - h_{x_2}, \alpha) = v^h(x_m, x_1, -d, \alpha) + \frac{c^-}{\rho} h_{x_2}$$

$$v^h(x_m, x_1, e + h_{x_2}, \alpha) = v^h(x_m, x_1, e, \alpha) + \frac{c^+}{\rho} h_{x_2}$$

ANNEXE II

RESOLUTION ALGORITHM FOR THE ONE-MANUFACTURER TWO-RETAILER SYSTEM WITH DELIVERY

For a defined finite grid steps h, the successive approximation algorithm is obtained by following the seven steps:

Step 1. Initialization: Set a precision $\gamma \in R^+$. Set n:=1, and $(v^h(x_m, x_1, x_2, \alpha))^n$:=0, $\forall \alpha \in M, \forall x_m, x_1, x_2 \in D_h$.

Step 2. Set $(v^h(x_m, x_1, x_2, \alpha))^{n-1} \coloneqq (v^h(x_m, x_1, x_2, \alpha))^n, \forall \alpha \in M, \forall x_m, x_1, x_2 \in D_h$.

Step 3. Determine $u^n(.)$ using the first part of Equation (8).

Step 4. Determine $Q_1^n(.)$ using the second part of Equation (8).

Step 5. Determine $Q_2^n(.)$ using the second part of Equation (8).

Step 6. The corresponding cost function is calculated from the Equation (8). Determine $u^n(.), Q_1^n(.)\ and\ Q_2^n(.)$ for the given value function, $\forall \alpha \in M, \forall x_m, x_1, x_2 \in D_h$

Step 7. Stop test.

$\overline{c} \coloneqq \min_{x_m, x_1, x_2 \in D_h} [\,(v^h(x_m, x_1, x_2, \alpha))^n - (v^h(x_m, x_1, x_2, \alpha))^{n-1}]$

$\underline{c} \coloneqq \max_{x_m, x_1, x_2 \in D_h} [\,(v^h(x_m, x_1, x_2, \alpha))^n - (v^h(x_m, x_1, x_2, \alpha))^{n-1}]$

$c_{min} \coloneqq \frac{\rho}{1-\rho}\overline{c} \qquad c_{max} \coloneqq \frac{\rho}{1-\rho}\underline{c}$

if $|c_{min} - c_{max}| \leq \gamma$ the stop, $(u^*, Q_1^*, Q_2^*) \coloneqq (u^n, Q_1^n, Q_2^n)$

else $n \coloneqq n + 1$ and go to step 2.

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