Optimization of Systems Reliability by Metaheuristic Approach

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ABSTRACT

The application of metaheuristic approaches in addressing the reliability of systems through optimization is of greater interest to researchers and designers in recent years. Reliability optimization has become an essential part of the design and operation of large-scale manufacturing systems. This thesis addresses the optimization of system-reliability for series–parallel systems to solve redundant, continuous, and combinatorial optimization problems in reliability engineering by using metaheuristic approaches (MAs). The problem is to select the best redundancy strategy, component, and redundancy level for each subsystem to maximize the system reliability under system-level constraints. This type of problem involves the selection of components with multiple choices and redundancy levels that yield the maximum benefits, and it is subject to the cost and weight constraints at the system level. These are very common and realistic problems faced in the conceptual design of numerous engineering systems. The development of efficient solutions to these problems is becoming progressively important because mechanical systems are becoming increasingly complex, while development plans are decreasing in size and reliability requirements are rapidly changing and becoming increasingly difficult to adhere to. An optimal design solution can be obtained very frequently and more quickly by using genetic algorithm redundancy allocation problems (GARAPs). In general, redundancy allocation problems (RAPs) are difficult to solve for real cases, especially in large-scale situations. In this study, the reliability optimization of a series–parallel by using a genetic algorithm (GA) and statistical analysis is considered. The approach discussed herein can be applied to address the challenges in system reliability that includes redundant numbers of carefully chosen modules, overall cost, and overall weight.

Most related studies have focused only on the single-objective optimization of RAP. Multiobjective optimization has not yet attracted much attention. This research project examines the multiobjective situation by focusing on multiobjective formulation, which is useful in maximizing system reliability while simultaneously minimizing system cost and weight to solve the RAP. The present study applies a methodology for optimizing the reliability of a series–parallel system based on multiobjective optimization and multistate reliability by using a hybrid GA and a fuzzy function. The study aims to determine the strategy for selecting the degree of redundancy for every subsystem to exploit the general system reliability depending on the overall cost and weight limitations. In addition, the outcomes of the case study for optimizing the reliability of the series–parallel system are presented, and the relationships with previously investigated phenomena are presented to determine the performance of the GA under review. Furthermore, this study established a new metaheuristic-based technique for resolving multiobjective optimization challenges, such as the common reliability–redundancy allocation problem. Additionally, a new simulation process was developed to generate practical tools for designing reliable series–parallel systems. Hence, metaheuristic methods were applied for solving such difficult and complex problems. In addition, metaheuristics provide a useful compromise between the amount of computation time required and the quality of the approximated solution space. The industrial challenges include the maximization of system reliability subject to limited system cost and weight, minimization of system weight subject to limited system cost and the system reliability requirements and increasing of quality components through optimization and system reliability. Furthermore, a real-life situation research on security control of a gas turbine in the overspeed state was explored in this study with the aim of verifying the proposed algorithm from the context of system optimization.

Keywords: Genetic algorithm, optimization, reliability, statistical analysis, redundancy allocation problem, multi-objective optimization, multi-state reliability, fuzzy function, gas turbine, reliability-redundancy allocation, hybrid genetic algorithm

Optimisation de la fiabilité des systèmes par une approche météhéuristique

Essa Abrahim Abdulgader SALEEM

RÉSUMÉ

L'application des approches métaheuristiques pour améliorer la fiabilité des systèmes par l'optimisation est devenue pratique pour les chercheurs ces dernières années. L'optimisation de la fiabilité est devenue un élément essentiel de la conception et de l'exploitation de systèmes de fabrication à grande échelle. Cette thèse aborde l'optimisation de la fiabilité système pour les systèmes série–parallèle afin de résoudre des problèmes d'optimisation redondants, continus et combinatoires en ingénierie de la fiabilité en utilisant des approches métaheuristiques (AM). Le problème consiste à sélectionner la meilleure stratégie de redondance, le composant et le niveau de redondance pour chaque sous-système afin d'optimiser la fiabilité du système sous des contraintes au niveau du système. Ce type de problème implique la sélection de composants à choix multiples et de niveaux de redondance offrant le maximum d'avantages. Il est soumis aux contraintes de coût et de poids au niveau du système. Ce sont des problèmes très courants et réalistes rencontrés dans la conception de nombreux systèmes d'ingénierie. La mise au point de solutions efficaces à ces problèmes devient de plus en plus importante à mesure que les systèmes mécaniques deviennent de plus en plus complexes, que les plans de développement diminuent de taille et que les exigences de fiabilité évoluent rapidement et deviennent de plus en plus difficiles à respecter. Une solution de conception optimale peut être obtenue très fréquemment et plus rapidement en utilisant des problèmes d'allocation de redondance d'algorithme génétique (PARAG). En général, les problèmes d'allocation de redondance (PAR) sont difficiles à résoudre dans des cas réels, en particulier dans des situations de grande envergure. Dans cette étude, l'optimisation de la fiabilité d'une série parallèle en utilisant un algorithme génétique (AG) et une analyse statistique a été considérée. L'approche décrite dans le présent document peut s'appliquer aux problèmes de fiabilité des systèmes, notamment le nombre redondant de modules soigneusement choisis, le coût global et le poids total.

La plupart des études connexes se sont concentrées uniquement sur l'optimisation à objectif unique du PAR. L'optimisation multiobjectif n'a pas encore attiré beaucoup d'attention. Ce projet de recherche a examiné la situation multiobjectif en se concentrant sur la formulation multiobjectif, ce qui est utile pour optimiser la fiabilité du système tout en minimisant son coût et son poids pour résoudre le PAR. La présente étude applique une méthodologie permettant d'optimiser la fiabilité d'un système série–parallèle basé sur une optimisation multiobjective et une fiabilité multi-états en utilisant un AG hybride et une fonction floue. Les objectifs de l'étude à déterminer la stratégie de sélection du degré de redondance de chaque sous-système afin d'exploiter la fiabilité générale du système en fonction des limites globales de coût et de poids. En outre, les résultats de l'étude de cas visant à optimiser la fiabilité du système série–parallèle sont présentés, ainsi que les relations avec les phénomènes précédemment étudiés, afin de déterminer la performance de l'AG examinée. En outre, cette étude a mis au point une nouvelle technique basée sur les métaheuristiques pour résoudre les problèmes a mis d’optimisation multiobjectifs, telle que le problème commun d’allocation fiabilité-redondance. En outre, un nouveau processus de simulation a été développé pour générer des outils pratiques permettant de concevoir des systèmes parallèles série-fiables. Par conséquent, des méthodes métaheuristiques ont été appliquées pour résoudre ces problèmes difficiles et complexes. De plus, les métaheuristiques offrent un compromis utile entre la durée de calcul requise et la qualité de l’espace de solution approché. Les défis industriels incluent la maximisation de la fiabilité du système sous réserve d'un coût et d'un poids système limités, la minimisation de son poids sous le coût système limité et les exigences de fiabilité du système, ainsi que l'augmentation des composants de qualité via l'optimisation et la fiabilité du système. Par conséquent, une étude de situation réelle sur le contrôle de sécurité d'une turbine à gaz en état de survitesse a été explorée dans cette étude dans le but de vérifier l'algorithme proposé du point de vue de l'optimisation du système.

Mots clés: Algorithme génétique, optimisation, fiabilité, analyse statistique, problème d'allocation de redondance, optimisation multi-objectif, fiabilité multi-états, fonction floue, turbine à gaz, allocation de fiabilité-redondance, algorithme génétique hybride

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LIST OF ABBREVIATIONS

ACS	Ant colony system
EBMHSA	Elitism Box-Muller harmony search algorithm
EP	Evolutionary Programming
Fmincon	Find minimum of constrained
Fminimax	Solve minimax constraint problem
GA	Genetic algorithm
HAS	Harmony search algorithm
HGA	Hybrid genetic algorithm
ICA	Imperialist competitive algorithm
LCCs	Life cycle costs
MINLP	Mixed-integer nonlinear programming
MOO	Multi-objective optimization
MVGA	Modified version of the genetic algorithm
NSGA-II	Nondominated sorting genetic algorithm II
OSR	Optimization of system reliability
PBCs	Performance-based contracts
RAP	Redundancy allocation problem
RBCs	Resource-based contracts
RRAP	Reliability redundancy allocation problem

LIST OF SYMBOLS

Abbreviations	Details
a_j	Constraint limit #*j*
$C_i(x)$	Total system cost of the i^{th} subsystem
c_{ij}	Cost of the j^{th} available component type in the i^{th} subsystem
$C_{\max}$	Limit of the cost constraint of the series–parallel system
C_s	Total cost of the series–parallel system
f_1	Objective function for system reliability
f_2	Objective function for system cost
f_3	Objective function for system weight
$g_i(.)$	Constraint function #*j*
i	Index of subsystem, $i \in (1, 2, \ldots, s)$
j	Index of component type in each subsystem
k	Index of redundancy level
m	Number of constraints
m_i	Total number of available component types in the i^{th} subsystem
N	Number of subsystems in the system
$n(x)$	Set of n_i $(n_1, n_2, \ldots, n_s)$
Ni	Set of component types, $N_i = [1, 2, \ldots, m_i]$
$n_i(x)$	Total number of redundant components used in the i^{th} subsystem
P_i	Minimum number of components in parallel required for the i^{th} subsystem to function
PN	Maximum number of components in parallel that can be used in the i^{th} subsystem (user defined)
r, n	Vectors of ri and ni
ri	Reliability of component in subsystem i
$R_i(x)$	Reliability components of the i^{th} subsystem
r_{ij}	Reliability of the j^{th} available component type in the i^{th} subsystem

Rs	Total reliability of the series–parallel system
s	Number of subsystems in the system
$W_i(x)$	Total weight of the i^{th} subsystem
w_{ij}	Weight of the j^{th} available component type in the i^{th} subsystem
$W_{\max}$	Limit of the weight constraint of the series–parallel system
W_s	Total weight of the series–parallel system
x	System configuration matrix
x_{ki}	Number of component types assigned at position k of the i^{th} subsystem, $x_{ki} \in (1, 2, \ldots, m_i, m_{i+1})$

INTRODUCTION

System reliability optimization has become a very important subject matter in industry design and operation of large-scale manufacturing systems. The problem of reliability optimization of mechanical systems is complicated because of the presence of multicriteria constraints, the optimal solution of which is generally a compromise. Presently, reliability is a matter of greater concern than in the past because the increasing complexity of modern engineering and service systems has led to a dramatic rise in their susceptibility to faults. This study focused on the optimization of reliability of mechanical series–parallel systems. Based on the genetic algorithm redundancy allocation problems (GARAPs), a new approach is presented that optimizes the overall reliability of the system while satisfying the constraints in terms of cost, weight, and volume. The advantages of precision, effectiveness, and capacity of the new approach are illustrated through the comparative results of the new technique and other approaches. One of the goals of a reliability engineer is to find the best method to increase system reliability. Recently, system reliability and the need to improve the reliabilities of products and system are increasingly gaining importance. The overall system reliability can be improved by methods, such as the improvement of component reliability, use of redundancy for the less reliable components, repair maintenance of failed components, replacement of substitutable components, and better arrangement of exchangeable components.

In real-world problems, system reliability optimization is a critical issue, which has recently attracted increasing attention in academia and applied engineering research. Generally, reliability optimization problems are categorized into two: (a) the redundancy allocation problem (RAP) and (b) the reliability–redundancy allocation problem (RRAP). When redundancy is used to improve the system reliability and find the optimal number of redundant components in each subsystem to maximize the overall system reliability, subject to some constraints, the corresponding problem is known as RAP. When system reliability is maximized through component reliability choices and component redundancy, the corresponding problem is known as RRAP (Zoulfaghari *et al.*, 2014 and Ardakan *et al.*, 2014).

The objective for solving this problem is to find the number of redundant components maximizing system reliability under several given constraints. It is one of the most researched upon problems in reliability optimization since the 1950s because of its potential for broad applications. When it is difficult to improve the reliability of unreliable components, system reliability can easily be enhanced by adding redundancies to those components. However, design engineers generally prefer improving component reliability over the addition of redundancy because in many cases, redundancy is difficult to add to real systems owing to technical limitations and requirement of relatively large quantities for constraints such as weight, volume, and cost.

The main issue addressed in this study is the reliability optimization of a series–parallel system by using a GA by implementing solutions for the RAP. The goal of this investigation is to improve the dependability of the arrangement of a parallel framework on a GA by solving an RAP. The repetition level for every subsystem and part must be set and the best excess methodology must be selected considering the end goal of boosting the framework quality and ensuring that the quality is consistent for different targets and framework level imperatives, including the cost and weight. The finding of solutions for addressing the issue of framework dependability is vital because mechanical and electrical frameworks and items are becoming increasingly complex, even as development schedules are decreasing in size and reliability requirements are becoming very stringent. This implies a corresponding need to increase the efficiency of the equipment. Frameworks must accomplish their objectives under given working conditions in a specific manner. The level of framework dependability is identified in a straightforward manner with respect to framework cost. Therefore, models must be improved in order to develop a viable and effective system that satisfies the reliability, cost, and weight requirements of the system under investigation. In this study, we applied a GA as a productive strategy to address the optimization of system reliability and related concerns.

Organization of the Thesis

This project presents a manuscript-based thesis and is divided into five chapters. The first chapter provides the requisite information regarding the problem, outlines the research, and states the objectives from an introductory perspective. This chapter also defines the scope of the study and a discussion on the background to this research. In addition, it briefly includes the problem statement and motivation, problem description, study scope and objectives, and literature review. The second chapter describes the methodology developed in this study, emphasizing major assumptions and considerations for each model. Furthermore, it presents a review of the relevant literature. Finally, it lists a number of industrial benefits resulting from the application of the algorithms presented in the subsequent chapters.

The third chapter presents a journal article on the research topic, in which a new approach is presented to optimize the reliability of a series–parallel system by using a GA and statistical analysis that considers system reliability constraints involving the redundant number of selected components, total cost, and total weight.

The fourth chapter presents a journal article that demonstrates the proposed multiobjective optimization of a multistate reliability system for an RAP involving a series–parallel system using a GA and fuzzy function. This chapter first describes the modeling of the proposed methodology, followed by a critical explanation of the formulation of the optimization process and solution using HGA. The findings indicate that the proposed approach can enable designers to determine the number of redundant components and their reliability in a subsystem to develop a system that effectively satisfies the reliability, cost, and weight criteria.

The final chapter presents a journal article that describes an optimal design for control and overspeed protection of a gas turbine by using multiobjective optimization (MOO) on the proposed control to achieve the optimal solution for an RRAP under nonlinear constraints. In this study, the simulation approach and results (curves) can be used as a tool for optimal design of reliability systems for a level of system reliability. Chapters 3, 4, and 5 present published articles of investigations of system reliability optimization using various methodologies and their comparisons.

Finally, concluding remarks are provided from all these studies and some recommendations.

CHAPTER 1

RESEARCH OUTLINES AND OBJECTIVES

1.1 Problem Statement and Motivation

The optimization of a system's reliability requires consideration of the system's reliability constraints, which include the redundant numbers of particular components, overall cost, and overall weight. Modern systems are becoming increasingly complex and automated, and their reliability is a measure of effectiveness that cannot be compromised. Reliability has become a mandatory requirement for customer satisfaction and plays an increasingly important role in determining product competitiveness. Therefore, system-reliability optimization is important in any system design. Essadqi, Idrissi and Amarir (2018) stated that the main goal in the design of industrial systems is to improve system reliability. Hence, in the current thesis, a reliable optimized system was constructed in a manner so as to achieve a consistent quality and address the issues of system-reliability optimization and entangled framework, considering the dependability of every segment as an interim esteemed number. In many practical system designs, the overall system is partitioned into a specific number of subsystems according to the function requirement of the system. Each subsystem comprises different component types with varying reliability, costs, weight, volume, and other characteristics. The overall system reliability depends on the reliability of each subsystem. To maximize system reliability, the approaches used in this research can be considered using more reliable components, adding redundant components in parallel, or a combination of both. For the systems designed using off-the-shelf components, with known cost, reliability, weight, and other attributes, system reliability design can be formulated as a combinatorial optimization problem. The best-known reliability design problem of this type is the reliability and redundancy allocation problem (RRAP). The diversity of system structures, resource constraints, and options for reliability improvement has led to the construction and analysis of several optimization methods with multiple constraints, to find a feasible solution for the RRAPs (Chern, 1992); this can then be identified as the selection of optimal combination of component type and redundancy level for each subsystem to meet various objectives, given constraints on the overall system. The problem can become complicated because of the presence of multiple conflicting objectives, such as minimizing system cost and system weight or volume, while simultaneously maximizing system reliability. The generalized formulation of RAP can be written as

$$Maximize \quad R_s = f(r,n)$$

$$\text{Subject to } g(r,n) \leq l,$$

$$0 \leq r_i \leq 1, \quad r_i \in \Re^+, \quad n_i \in Z^+, \quad 1 \leq i \leq m$$

where *Rs* is the system reliability; *g* is the set of constraint functions usually associated with system weight, volume, and cost; $r = (r_1, r_2, r_3, \ldots, r_m)$ is the vector of component reliabilities of the system; $n = (n_1, n_2, n_3, \ldots, n_m)$ is the vector of the redundancy allocation for the system; r_i and n_i are the reliability and number of components in the i^{th} subsystem, respectively; $f(\cdot)$ is the objective function for the overall system reliability; *l* is the resource limitation; and *m* is the number of subsystems in the system. The most studied design configuration of RAP is a series system of *s* independent *k-out-of-n: G* subsystems. Our goal was to propose an optimization model for the structure of a series–parallel system to determine the number of components and the reliability in each system for maximizing the overall system reliability.

To outline the models, some numerical cases were considered, and their outcomes were examined. As an exceptional case, this manuscript provides an understanding of the related issues and contrasting outcomes, considering the lower and upper limits of the interim esteemed reliabilities of the segment to be the same. Finally, to verify the dependability of the proposed GAs and the diverse GA parameters (such as populace size, crossover rate, and mutation rate, and number of generations), affectability examinations were conducted. Certain GA calculations are simply pursuit calculations to perceive that sexual multiplication and the rule of survival of the fittest empower an organic species to adjust to their condition and successfully contend for their assets. While it is moderately direct, the calculation is a successful stochastic pursuit strategy and is demonstrated as a vigorous critical thinking method that produces superior to irregular outcomes. This perception was first numerically detailed by John Holland in his work titled “Adjustment in Natural and Artificial Systems".

Normally the calculation breeds a foreordained number of ages and each age is populated by a foreordained number of settled length paired strings.

The research problem addressed in the present work can be summarized as follows:

- To investigate an optimization model for determining the structure of a series–parallel system.
- To propose the optimization model for the structure of a series–parallel system to determine the number of components and the reliability of each component in each system to maximize overall system reliability.
- To propose a new simulation process for design of the entire system with the desired level of reliability that enables the designer to determine the reliability of each component corresponding to any value of system reliability *Rs* (e.g., control of a gas turbine in the overspeed mode).

1.2 Problem Description

Global optimization challenges, in which the assessment of the objective function is a costly operation, have recurrently emerged in fields such as engineering, decision-making, and optimum control (Sergeyev, Kvasov and Mukhametzhanov, 2018). The main concern is the optimization of system reliability by using the metaheuristic approach (MA) to solve redundancy, continuity, and combinatorial optimization problems in reliability engineering.

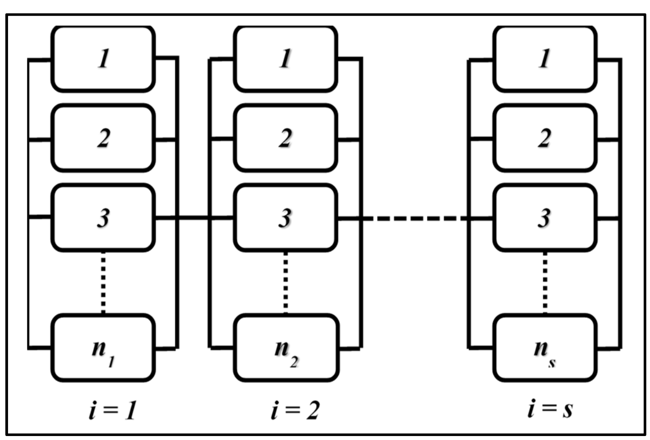

Figure 1. 1 The structure of parallel–series system

This type of reliability-optimization problem determines the nature and value of decision variables such that the system objective function is optimized and all constraints are met. The criterion may be reliability, cost, weight, or volume, and one or more criteria may be considered in an objective function, while the others may be considered as constraints. Although reliability allocation is usually easier than redundancy allocation, the improvement of component reliability is more expensive than the addition of redundant units. Redundancy allocation results in increased design complexity and increased costs through the addition of more components, increased weight, a large amount of space, etc. It also increases the computational complexity of the problem and is classified as NP-hard in literature (Chern 1992). Misra (1975) was the first to introduce the formulation of mixed types of redundancies in the optimal reliability design of a system. The design of new products involves the specification of performance requirements, evaluation and selection of components to perform clearly defined functions, and determination of system-level architecture. Detailed engineering specifications prescribe minimum levels of reliability, maximum weight, maximum volume, etc. If the design must be produced economically or within some specified budget, numerous design alternatives must be considered, resulting in a complex combinatorial optimization problem. In this study, we considered the RAP on a parallel–series system, which has already received intensive investigations (Chern,1992; Kim *et al*., 1993; Ravi *et al*., 1997). Figure 1.1 illustrates a typical structure of a parallel–series system, which consists of s independent subsystems, and the maximal number of components hold for the i^{th} subsystem is n_i. Subsystem i can work properly if at least one of its components is operational. Moreover, for each subsystem, more than one component may also work in parallel.

The system reliability optimization can be broadly classified into continuous function, combinatorial, and mixed-integer programming optimization problems. The generic tasks involve the evaluation of (1) the global optimum cost of complex systems subject to constraints on system reliability; (2) optimum number of redundancies, which maximize the system reliability based on constraints on cost, weight, and volume in a multistage mixed system; or (3) optimum number of redundant units and reliability at each stage in the system to maximize total reliability based on constraints on cost, weight, volume, and stage reliability. The configuration model in the system-design problem usually is the series–parallel system with k-

out-of-n: G subsystems. This is because many systems can be conceptually represented as series–parallel, and because such a configuration can often serve as a bound for other types of system configuration, it has many practical usages. For example, in a gas turbine, the electrical and mechanical system continuously provides overspeed protection to the turbine, as shown in Figure 5-3. That is, when overspeeding occurs, the fuel supply should be cut-off by closing the four stop valves, modeled as four subsystems. To increase the reliability of each subsystem, we can use highly reliable components or/and add redundant components in parallel. Then, such a system becomes a typical series–parallel system with *k-out-of-n: G* subsystems.

1.3 Objectives and Scope of the Study

1.3.1 Objectives

The objectives of this study include the following:

a) To focus on determining the optimal approach using metaheuristic techniques for the solution of the system reliability optimization problem. The exploitation of metaheuristic approaches (MAs) for addressing dependability and repetition assignment issues redundancy allocation problem (RAP) has recently gained interest of analysts. The advances demonstrated in Chapter 3 focus on improving the consistent quality of the framework. The model tends to enhance the framework plan and upkeep exercises during working periods. The computational outcomes were compared to determine which approach is more fitting for understanding complex frameworks that yield enhancement models with consistent quality.
b) To develop a mathematical or technical tool for the best design of the reliability system.
c) To develop a new simulation process based on the hybrid genetic algorithm (HGA) so that alternative solutions required to generate application tools for the optimal design of a reliable series–parallel system are obtained.

1.3.2 Scope

The scope of the research study is as follows:

a) A new mathematical model and formulation for the reliability optimization problem was developed. Ruiz-Rodriguez, Gomez-Gonzalez, and Jurado (2015) presented a method of optimizing the reliability of an electric power system through distributed generation. Additionally, the system's reliability index was calculated and given as the failure probability of the system. The effectiveness of the metaheuristic calculations can be credited to the manner in which they mirror the best highlights in nature, particularly the determination of the appropriate inorganic frameworks that are improved through characteristic choices.

b) To develop a hybrid approach based on genetic algorithm and HGA, which are used to solve similar problems. Wan and Birch (2013) claimed that GAs perform better as a global search method; however, they might frequently take a relatively long amount of time to reach global optimum. Local search (LS) methods have been integrated into GAs to increase their performance as a learning process. A model was simulated using MATLAB in the implementation phase.

c) To apply the results from industrial cases to validate the performance of the proposed approach. A different case study approach will be used to demonstrate the application and reliability study of the optimization using a multiobjective metaheuristic.

d) The results thus obtained would be used to design the optimal configuration of the electromechanical system. Figure 1.2 shows the research framework.

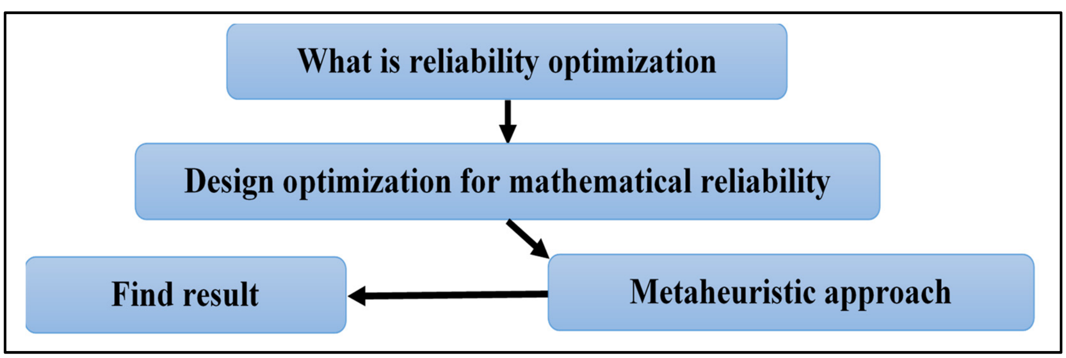

Figure 1. 2 Research framework

1.4 Literature Review

Modern society is largely dependent on technological systems. There is no doubt that these systems have improved our collective productivity, health and affluence; however, our increasing dependence on modern technological systems requires complex operation and sophisticated management. In complex or complicated systems, system reliability plays an important role. The reliability of any system is very important to manufacturers, designers and users alike. During the design phase of a product, reliability engineers and designers are called upon to measure the product's reliability. They may desire to modify the product to improve its reliability in a way that also raises the item's production cost; the increase in production costs, in turn, negatively affects the user's budget. In such a case, the question arises of how to meet the system reliability goal. Therefore, the design reliability optimization problem can be phrased as the problem of providing a reliability improvement at a minimum cost (Zavala *et al.*, 2005). One widely known method for improving the reliability of a system is to introduce several redundant components (Najafi *et al.*, 2013). To better design a system using components with known cost, reliability, weight and other attributes, the corresponding problem can be formulated as a combinatorial optimization problem, where either the system reliability is maximized or the system cost is minimized (Coit & Smith, 1996a; Lyu *et al.*, 2002). Both formulations generally involve constraints on allowable weight, cost, and/or a minimum targeted system reliability level. The corresponding problem is known as the RRAP). The primary objective of the RRAP is to select the best combination of components and levels of redundancy to maximize system reliability and/or to minimize the system cost, subject to several constraints. The RAP is one of the most important reliability optimization problems in the design phase of parallel-series systems, network systems and other systems with various structures. RAP is a complex combinatorial optimization problem that has broad real-world applications, such as in computer network design (Altiparmak *et al.*, 2003), consumer electronics (Painton, and Campbell, 1995), software systems design (Berman, and Ashrafi, 1993), and network design (Deeter and Smith, 1998).

The main goal of reliability design is to improve system reliability. Redundancy allocation is an effective method for maintaining a high level of reliability in the system design phase. While redundancy improves system reliability, it also increases product cost, weight and volume. Thus, it is an important topic for system decision-makers to determine the optimal number of redundant elements under certain system constraints. To address the traditional redundancy optimization problem, several kinds of optimization models have been proposed under the assumption that the lifetimes of the product elements are random variables. Zhao and Liu (2005) proposed three redundancy optimization models that presents the lifetimes of the elements as fuzzy variables. Wang and Watada (2009) developed two fuzzy random redundancy allocation models for a parallel-series system in which the lifetimes of the elements are treated as fuzzy random variables. Recently, researchers have begun to address the reliability optimization designs of some systems by considering interval-valued component reliability in uncertain environments. Roy *et al.* (2014) applied the symmetrical form of interval numbers via an interval-valued parametric functional form to evaluate the optimum system reliability and system cost of the RAP. Zhang and Chen (2016) investigated an interval multi-objective optimization problem for reliability redundancy allocation of a series–parallel system. Other researchers have concentrated on hybrid uncertainty optimization problems for system reliability (Li *et al.*, 2015; Huang *et al.*, 2017).

Generally, there are two approaches that can be used to optimize system reliability: increasing component reliability (reliability allocation) or using redundant components in parallel (redundancy allocation) (Huang *et al.*, 2019). Unfortunately, these two approaches do not always yield competitive results. For example, reliability allocation may incur large costs for only minor improvements to system reliability because of difficulties in design, verification and production. Redundancy allocation not only increases costs, but also adds undesirable extra volume and weight to the system. The RRAP aims overcome these problems (Kuo and Prasad, 2000; Coit and Smith, 1996a). The RRAP is usually formulated as a non-linear optimization problem, which determines the reliability and redundancy levels of components to maximize system reliability under design constraints on, for example cost, volume, or weight. RRAP presents a powerful and attractive method for system reliability optimization; at the same time, it is known as one of the most challenging problems in reliability optimization, due to its high dimension and complexity. Several techniques, especially intelligent optimization algorithms, have been suggested to solve the optimization model arising in RRAP in recent years. For example, artificial bee colony algorithms (Yeh, and Hsieh, 2011; Garg *et al.* 2013; Ghambari and Rahati, 2018), cuckoo search algorithms (Valian and

Valian, 2013; Kanagaraj *et al.*, 2013), genetic algorithms (Kim *et al.*, 2017) and simulation optimization methods (Chang *et al.*, 2018). Muhuri *et al.* (2018) proposed a novel formulation of the RRAP with fuzzy uncertainty. In Peiravi *et al.* (2018), cold-standby strategies for redundant components are used to model the RRAP. Chatwattanasiri *et al.* (2016) studied RRAPs with uncertain stress-based component reliability. Feizabadi and Jahromi (2017) proposed a new model for reliability optimization of series–parallel systems with nonhomogeneous components.

The utilization of redundancy plays an important role in enhancing the reliability of a system. The RAP involves the selection of components and a system-level design configuration to simultaneously optimize some objective functions, such as system reliability, cost and weight, given certain design constraints. The integration of redundant components improves system reliability, but can also increase system cost, weight, etc. Thus, a RAP frequently encounters trade-offs between the maximization of system reliability and the minimization of system cost and weight. Traditionally, the RAP has been solved as a single-objective optimization problem with the goal of maximizing system reliability subject to several constraints. Various methodologies have been proposed to handle it, e.g., heuristic/meta-heuristic approaches such as genetic algorithm (Ardakan and Hamadani, 2014), bacterial-inspired evolutionary algorithm (Hsieh, 2014), swarm optimization (Yeh, 2014; Huang, 2015; Wang and Li, 2014; Kong *et al.*, 2015), and hybrid algorithm (Kanagaraj *et al.*, 2013) methods.

In the last few years a growing number of papers put forward reliability optimization models within the context of service contracts. Jin and Wang (2012) proposed a model to minimize lifecycle costs (LCCs) and maximize the service profit margin under performance-based contracts (PBCs) in the presence of uncertain system usage. Jin and Tian (2012) developed a model to optimize reliability design and inventory level. They minimize LCCs under a nonstationary demand rate and consider a dynamic stocking policy. Jin *et al.* (2015) integrated a spare part inventory, maintenance and repair capacity into one model. They maximize the utility of both the supplier and the customer within a game-theoretic framework, using a gradient-based heuristic and a hybrid algorithm to solve the problem for a single- and multi-item systems, respectively. Kim *et al.* (2017) build a game-theoretic model to study the trade-off between investing in reliability improvement and spare assets under traditional resource-based contracts (RBCs) and PBCs. However, they only incorporated supply chain costs, rather than LCCs. Some papers have jointly optimized the reliability and spare parts inventory of single-component or multi-component systems by minimizing the LCCs or service costs (Öner *et al.*, 2010; Selçuk and A ğralı, 2013; Öner *et al.*, 2013).

The lack of reliability of a product can lead to a number of consequences, such as lack of safety, competitiveness, higher costs of maintenance and repair, and brand name problems. The improvement of reliability is critical in the industrial design. Paganin and Borsato (2017) reported that adopting a design for reliability during new product development is fundamental to ensure the reliability of an item at all stages of its life cycle. The number of studies on reliability design is not very extensive and is relatively dispersed. Solid frameworks are necessary for economic efficiency and aggressiveness in the current industry. To augment profitability, modern frameworks, such as manufacturing frameworks, must be accessible and operational as much as possible. Additionally, as mechanical frameworks comprise various segments, the definitive likelihood of a framework survival specifically relies upon the qualities of the constituent segments. Consequently, the failure of a framework is unavoidable. Therefore, the consistent quality of the framework must be improved through reasonable dependability streamlining techniques to enhance its general efficiency.

The application of a GA for system optimization is similar to a heuristic search method used in artificial intelligence and computing inspired by the process of natural selection belonging to a larger class of evolutionary algorithms (EAs). McCall (2005) states that GAs are a heuristic search and optimization method motivated by natural evolution and have been effectively used in a wide range of real-world applications for solving problems of substantial complexity. As numerous motivations exist, a wide range of calculations could also exist. Each of these calculations tends to apply certain qualities to determine a refreshing formula. For instance, GA calculations were propelled by the Darwinian development qualities of organic frameworks and hereditary administrators, the hybrid transformation and determination of the survival of the fittest are utilized. Numerous variations are currently available and new metaheuristic calculations have been produced. For example, although GA calculations can be exceptionally helpful, they have a few limitations in managing multimodal advancements. GA has successfully been applied to optimization problems in different fields, such as engineering design, optimal control, transportation and assignment problems, job scheduling, inventory control, and other real-life decision-making problems. The most fundamental idea of a GA is to artificially imitate the natural evolutionary process, in which populations undergo continuous changes, through genetic operators such as crossover, mutation, and selection. A GA can easily be implemented with the help of computer programming. In particular, it is very useful for solving complicated optimization problems, which cannot easily be solved using direct or gradient-based mathematical techniques. Large-scale, real-life, discrete, and continuous optimization problems can be effectively managed without making unrealistic assumptions and approximations. By considering the imitation of natural evolution as the foundation, a GA can be designed appropriately and modified to exploit special features of the problem to solve. A GA makes uses of techniques inspired from evolutionary biology such as selection, mutation, inheritance, and recombination to solve a problem. The most commonly employed method in GAs is to randomly create a group of individuals from a given population. The individuals thus formed are evaluated using the evaluation function provided by the programmer. Individuals are then provided with a score, which indirectly highlights the fitness to the given situation. The best two individuals are then used to create one or more offspring, after which random mutations are performed on the offspring. Depending on the needs of the application, the procedure continues until an acceptable solution is derived or until a certain number of generations have passed.

Duan (2013) highlighted the emerging need for an inventory control model for practical real-life applications. The extensively exploited conventional optimization procedures normally necessitate an unequivocal mathematical model expressed based on some assumptions. According to the author, the proposed framework for any effective metaheuristic can function as the optimizer to logically look for the solution space by using a suitable simulation inventory model that functions as the evaluation component. Mechanical frameworks are planned with a few constraints, including cost, weight, and volume of the assets. With restricted assets, exchange dependability and other asset imperatives must be discovered. One of the possible courses is to expand the framework dependability through repetition and segmented consistent quality decisions. Nevertheless, when outlining a quality framework, the fundamental issue is to discover an exchange between dependability improvement and asset utilization. This requires the application of a reasonable multicriteria approach. Data mining has been implemented in several studies by using methods such as statistical regressions, induction decision tree, neural networks, and fuzzy sets and etc. Kou *et al.* (2003) promoted a multiple-criteria linear programming approach to data mining in relation to linear discriminant analysis. In reality, streamlining issues of system reliability comprise various vulnerabilities and challenges. Since the advent of the enhancement approach, various techniques and applications have been proposed to tackle improvement issues, including unclearness and uncertainty. These methodologies treat parameters (coefficients) as uncertain numerical information.

Khorshidi and Nikfalazar (2015) compared two MAs for addressing the reliability optimization of a complex system. They acknowledged that the application of MAs for solving RRAPs has become an attractive approach to researchers in recent years. They presented an optimization model aimed at maximizing system reliability and minimizing the system cost concurrently for “multistate weighted *k-out-of-n* systems” (p. 1). The model used in the study tends to enhance the system design and maintenance activities more than the functioning periods, thus offering a dynamic modeling technique. A newly developed metaheuristic method, called the imperialist competitive algorithm (ICA) together with a GA were applied to improve the resolution of the model’s problem. The computational results were assessed with the aim of ascertaining a more suitable approach for resolving complex-system reliability optimization models. They revealed that although the GA can determine a better answer, the ICA is faster. Additionally, an examination was conducted on various parameters of the ICA.

In contrast to the inconsistent quality of the framework, the nearness of clashing, nonlinear, and questionable destinations further complicates the issue. In such a case, with numerous goals, the synchronous consistent quality boost and cost minimization requires a mindful trade-off approach. The determination of an ideal arrangement is relatively inconceivable. Kuo and Prasad (2000) presented an overview of the approaches established since 1977 as solutions for different reliability-optimization challenges. The authors also explored applications of these approaches to different design challenges. They further addressed concerns on "heuristics, metaheuristic algorithms, exact methods, reliability redundancy allocation, multiobjective optimization, and assignment of interchangeable components in reliability systems" (Kuo and Prasad 2000, p. 31). In relation to other types of applications of system-reliability-optimization methods such as in software development, in a system with common-cause failures, and for maintenance, the exact answers to reliability-optimization challenges are not necessary because the exact solutions are challenging to determine, and even when found, their utility is minimal. Their article reports that modern studies in the field are focused on the development of heuristic and metaheuristic algorithms for resolving optimal redundancy allocation challenges. Therefore, the MA is a potential application strategy for complex issues that a suitable system can implement to locate an arrangement of GA principles. Google's search engine was used to find different databases. During this search, it was discovered that different articles on batch calculation were either simply acknowledged or gathering introductions. This justifies the rationale of the current research to bridge the existing gap in literature regarding system optimization using an MA.

CHAPTER 2

METHODOLOGY

2.1 Previously used Methodologies

Different methodologies have been used in the past in an effort to investigate the aspect of system optimization. In the past two decades, many researchers have solved reliability and redundancy problems under multiobjective formulation. For an overview of the trend of research in this area, readers may refer to the works of Zhao *et al*., (2007), who designed an ant colony algorithm (ACS) with a multiobjective metaheuristic formulation to optimize the reliability of the system to solve the redundancy allocation problem (RAP) of series–parallel *k-out-of-n:G* subsystems (denoted by ACSRAP). This problem couples a dynamic penalty function with a global objective function and a simple local search strategy to obtain efficient solutions for generalized problems such as those of gearbox reliability designs. The algorithm offers distinct advantages compared with the alternative optimization methods and has been mainly used on combinatorial optimization problems. Therefore, the importance of this work lies in its attempt to adapt continuous ant colonies to multiobjective problems. The objectives considered herein were the maximization of system reliability and the minimization of system cost. This study aimed at determining the number and type of the redundant components required for optimizing the objective function under several constraints such as the overall system weight and total number of the components used in all redundancies; the reliability of a system depends on the reliability of each subsystem.

In a related study, Quy (1998) developed a new method to optimize a multiobjective model in certain mechanical systems by using the fuzzy multiobjective method. His approach was based on the algorithm proposed by Rao and Dhingra (1992), and Quy applied it to the modeling and analysis of the overspeed control system of a gas-turbine engine. A system must be designed not only to meet its functional requirement but also to perform its function successfully. A general reliability-design problem involves the determination of both the component reliabilities and number of redundancies required to achieve the best overall reliability. In other words, a multiobjective optimization problem is based on the context of reliability and redundancy apportionment of multiple stages and is subject to several constraints (cost, weight...). The consideration of multiple objective functions is an important aspect in the design of complex engineering systems, particularly a mechanical system. In a design problem, the designer is often forced to state a problem in precise mathematical terms rather than real-world terminology, which is usually imprecise in nature. The impreciseness is not due to the randomness but the inherent fuzziness in the system. Consequently, for a problem involving fuzziness in the design data, the objectives and constraints must be defined or modelled with fuzzy boundaries. The fuzzy objective functions and constraints are characterized according to their membership functions, which are described by a continuous range of values (instead of Boolean values) between 0 and 1. In the design of mechanical systems, particularly complex systems, the relationships and statements used for problem description are normally imprecise or vague. In modeling a real-world design problem, the precise expression of objectives and constraints may not be possible but they may be expressed in a fuzzy way. The multiobjective fuzzy optimization problem can be solved through a four-step procedure similar to solving a single-objective optimization problem, that is, by determining the “best” and “worst” solutions possible for each of the objective functions, using these solutions as boundaries of the optimization problem, and solving the resulting fuzzy optimization problem. In this study, the fuzzy approach was used to solve the optimization problem.

2.2 Methodologies used in this study

Different methodologies were used in the three studies presented in this thesis. The first used a reliability optimization method of a series–parallel system along with a genetic algorithm (GA) and statistical analysis. The GA, inspired by metaheuristics, was used because it searches parallel to various points and is capable of evading being locked into a local optimal solution, as in the case of conventional approaches that launch their searches from a single point. In this investigation, penalty factors were optimized using a reliability fitness function. Different ranges of values were determined for these penalty factors by using a full factorial design with three levels of optimal values of GA reliability. The authors report that following its previous successful applications in solving related real-life RAP challenges, probabilistic searches in GA can resolve the shortcomings of using conventional approaches. In terms of multipoint capability, in the study described in Chapter 3, 10 simulations were performed for each point of the experimental design. From the findings, the average of the 10 reliability values was used to increase the accuracy of the subsequent statistical analysis. The authors determined the best configuration of every point matching the largest value of reliability in terms of cost and weight; thus, this was a better approach. As a result, GAs can be used to solve real-life complex combinatorial problems with exceedingly large search fields. The method used in the first study is important for determining the best combination and redundancy level in research aiming to solve optimization issues through statistical analysis. GA parameters were successfully improved, leading to the best reliability and configuration when using specialized software in the experimental analysis. The first approach in this work uses the GA and statistical analysis based on the redundancy allocation problem to obtain the number of redundant components that either maximize reliability or minimize cost under numerous resource constraints. Our statistical analysis experiment allowed us to choose the penalty factor values that most improved the GA parameters. The important contribution of this work is the decision to use the design and statistical analysis of experiments to optimize two penalty factors in the reliability fitness function of the GA. Therefore, we determined the best combination and the redundancy level for the series–parallel system reliability optimization problem and improved the GA implementation using statistical analysis. The industrial application possibilities of this study include the development of multispeed gearboxes that use different gear pairs in each stage to obtain better solutions for maximizing the overall system reliability, which is subject to total-cost and total-weight constraints.

In this study, we determined the strategy choices for the redundancy level for each subsystem to maximize overall system reliability subject to total cost and total weight constraints, which means that we have to determine the number of components in each subsystem and the values of reliability. Further, we create a tool based on the metaheuristic approach to find the best design with different cost and weight constraints (for example, gearbox). With this tool, we can decide how many gear pairs to use in each stage to maximize the reliability of the gearbox with respect to cost and weight at system levels; this technique provides good results for optimization of redundancy allocation problems (RAPs).

Owing to the existing redundancy problem, the second study used a mathematical model for a series–parallel system as the best approach for optimization. The design and architecture used in this study incorporated a hybrid GA (HGA) with a flexible allowance technique used to fix the problems prevalent in limited engineering design optimization. Chapter 4 explains that the system consists of four subsystems, each of which has a different design component type with similar or different characteristics, including reliability, cost, transmission ratio, material, dimension, and weight. Zhao, Liu, and Dao (2007) stated that on the minimum gear-pair *pi* value of 2 and maximum gear-pair *PN* of 5 were specifications for use in the gearbox at all stages. The report indicates that every subsystem is represented by *PN* positions, with each component listed according to its reliability index; this highlights the necessity of reliability allocation in system design. The allowance of this system enables the design engineers to establish the reliability of a vector of subsystems and components to obtain the optimal overall reliability. The combinatorial optimization challenges in system design result from the system-designed parameters such as identified cost, reliability, weight, and volume. A problem was exploited by conducting a test to determine the capacity of this algorithm to solve RAP, which is considered a gearbox-reliability optimization problem (Zhao, Liu, and Dao, 2007). The method used by these authors was based on the assumption that a minimum and maximum of two and five components exist, respectively, to apply the method at all stages.

The second approach in this work uses a fuzzy function in combination with a HGA as the basis of multiobjective optimization and uses multistate reliability to find the best possible solution for the RAP. This approach enables manufacturers to determine the number and reliability of redundant components in a subsystem in order to develop a system that effectively satisfies the reliability, cost, and weight criteria. This approach can provide system configurations with lower cost or weights without significantly degrading the overall reliability. The results also indicate the robustness of the proposed algorithm and highlight its potential for future application. The important contribution of this approach is that it examines the effectiveness of employing a fuzzy function along with a multiobjective GA for solving the RAP. The opportunities for applying this approach to industrial engineering design include the multistage gearbox problem: many high-performance power transmission applications (e.g., automotive and aerospace) require gear train system design. The optimization of a multispeed gear train system introduces numerous challenges. The main aim of this study is to find the maximum reliability, minimum cost, and minimum weight considering an upper bound on cost and weight and to optimize the reliability of a series–parallel system on the basis of a genetic algorithm (GA) by implementing solutions for the redundancy allocation problem. We decided to investigate a multiobjective optimization problem to find better solutions in terms of maximum reliability, minimum cost, and weight. The study in Chapter 4 presents an optimization method that can objectively find the solution that represents the optimal compromise between the optimization objectives. This is to set the redundancy level for each subsystem and component and to select the best redundancy strategy for each subsystem in order to maximize the system reliability under multiple objectives and system-level constraints, including the cost and weight at the system level. We developed an approach based on fuzzy function and the advantage of choosing the optimal solution (trade-off) from the Pareto-optimal solutions. Our computational results from this technique confirmed the robustness of the proposed algorithm.

In the third study, a nonlinear programming approach was used involving optimal allocation of reliability and redundancy in series systems. The main aim was to solve the problem of multiobjective fuzzy optimization by using the new hybrid MA, leading to an increase in system reliability and a reduction in overall costs. This approach described the use of a reliability–redundancy optimization problem due to overspeed protection along with a multiobjective approach used to maximize system reliability and minimize consumption of resources such as cost, total weight, and volume parameters. According to their report, the approach involves a goal-programming formulation and a goal-achievement method for generating Pareto optimal solutions, in which control and overspeed protection for a gas turbine were nearly the same as those for a steam turbine. Note that a gas turbine operates at a higher temperature than a steam turbine; therefore, it requires close control through control sequencing. The third approach proposed in this thesis is a novel hybrid GA approach based on the RAP for solving the multiobjective optimization design of series–parallel systems to find the number of redundant components that either maximize reliability or minimize cost, weight, and volume under various resource constraints. This approach determines the converged system reliability value until we obtain the values of the number of redundant components n_i and the optimal component reliability levels r_i corresponding to the maximum reliability value. The main advantage of the proposed multiobjective approach is that it offers greater flexibility to system designers for testing problems. The results show the superiority of the HGA over other algorithms we used in terms of searching for a quality and robust solution. The important contribution of this work is the ability to design a new framework for obtaining a whole system with a desired level of reliability. To show one practical use case, we considered a series–parallel overspeed protection system for a gas turbine. Overspeed detection is continuously provided by the electrical and mechanical system. When an overspeed occurs, it is necessary to cut off the fuel supply by closing the four or more parallel control stop valves, which are modeled as four subsystems. Therefore, we can use highly reliable components and/or add redundant components in parallel to increase the reliability of each subsystem.

We proposed a hybrid genetic algorithm and presented a novel system design for the entire system with the desired level of reliability and to develop a new metaheuristic-based approach to solve a multiobjective optimization problem namely the reliability–redundancy allocation problem (RRAP). We evaluated our approach by comparing it with another method in the literature. We used this approach to develop a new simulation process for system design. This a new simulation process is to generate practical tools for designing reliable series–parallel systems and a practical case study regarding security control of a gas turbine in the overspeed is presented to validate the proposed algorithm. to design a new system for how can we obtain a system for the whole system with a level reliability we want. We obtain reliable regression curves, which are of great practical value and enable the designer of the system to determine the values of r_1, r_2, r_3, r_4, Cs, Ws, and Vs corresponding to the value of Rs.

This technique would allow for easier sequencing and more automatic control of the gas turbine. This research, illustrated through studies in Chapters 3–5, used a combination of the research methodologies shown in the framework in Figure 2.1

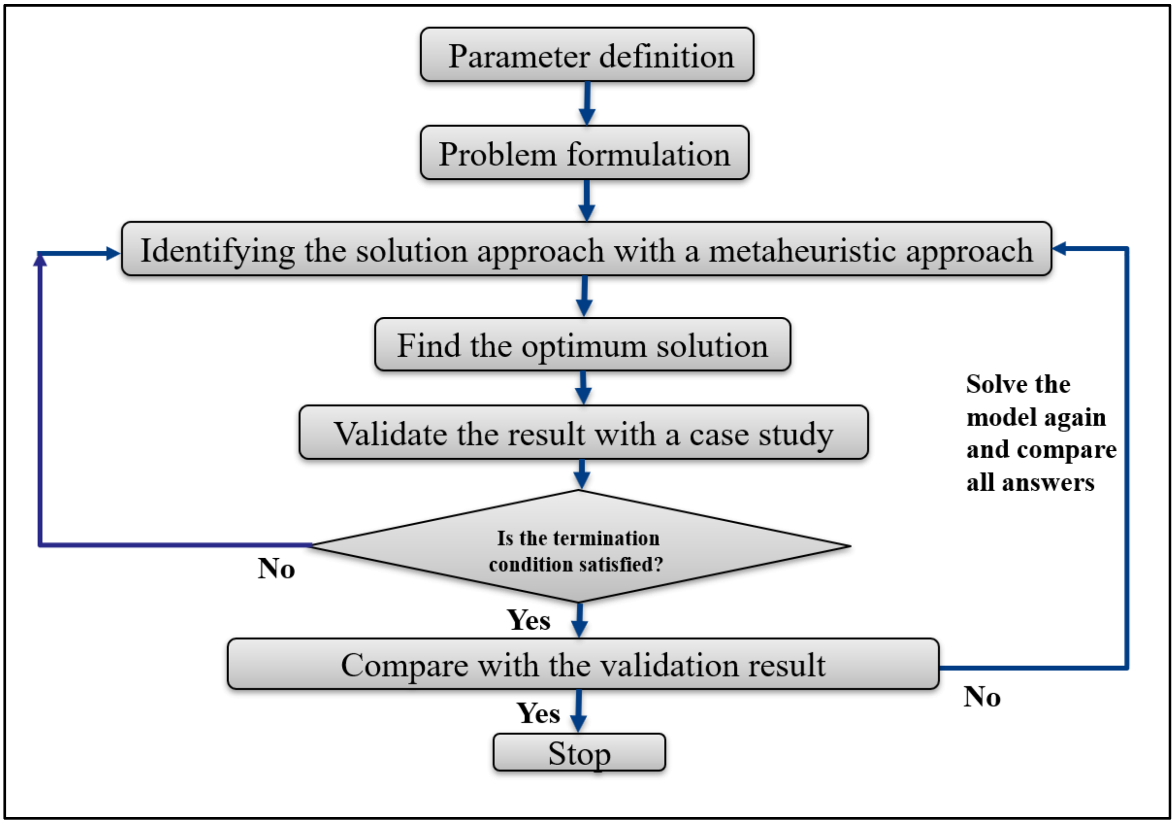

Figure 2. 1 Framework of research implementation methodology

2.3 Difference between Approaches

The previously proposed approaches and those presented in the recent study were compared with the aim of solving system optimization problems. The first approach uses the reliability optimization method of series–parallel systems, whereas the second approach uses a mathematical model for the series–parallel systems. In the first approach, a GA and statistical analysis were used to solve reliability problems, while an HGA with a flexible allowance technique was used in the second approach to solve constrained engineering design optimization problems. The second approach was developed to solve redundancy allocation problems. For the series–parallel system, 10 simulations were conducted for each point in the experimental design, and the average of the 10 reliability values was found to improve the accuracy of the subsequent statistical analysis. The cost and weight parameters were the main determinants of the best system configuration and showed that GA is effective at solving complex combinatorial optimization problems with considerably large and complex search spaces. A previous study proposed the multiobjective ACS, a metaheuristic approach (MA), to solve the reliability optimization problem of series–parallel systems. Through a random search, constructive local search, and long-term dynamic memory strategy, the proposed method efficiently builds good solutions for the RAP. After the ACSRAP was tested, it was compared with GA-RAP. ACS outperformed GA in terms of best solution, reduced variation, and great efficiency. Further, ACSRAP was compared against ACO-RAP, and the results showed a potentially higher efficiency and better capacity by ACS to handle large-scale problems. Thus, ACSRAP has a better constructive strategy than other ant colony algorithms. Through the combination of probabilistic search, multiobjective formulation of local moves, and the dynamic penalty method, the multiobjective ACSRAP allows the obtaining of a frequent and fast optimal design solution. Compared with other methods, ACSRAP can be applied to a more diverse problem domain.

Moreover, the second approach developed effective multiobjective fuzzy optimization techniques for engineering design; it represents an influential approach to solve optimization issues by using fuzzy parameters. In contrast, the credibility of using indeterminate information for reliability allocation needs additional research because the component risk and cost functions have been considered continuously in the past. Therefore, the HGA approach helps identify redundancy allocation challenges to maximize reliability and reductions in cost, weight, and volume. In the past, different studies have explored various techniques to fix redundancy optimization challenges. Nevertheless, the current thesis considers an approach in which different aspects that were overlooked in the past are exploited as foundations for system optimization solutions. The results from computational techniques were not compared to those previously proposed because previous studies did not use a fuzzy function. The mathematical model described in this thesis embodies the multiobjective HGA with a constraint handling strategy to solve optimization problems. The HGA technique is a metaheuristic method used to solve optimization problems efficiently in conjunction with the creation of an initial set of random potential solutions.

Each particle represents a solution to the problem and has a position and velocity that change with each iteration to obtain better solutions. According to Zhao, Liu, and Dao (2007), the multiobjective ACS metaheuristic was established as a method to solve the reliability optimization problem in series–parallel systems. The problem includes the selection of components with multiple choices and redundancy levels that generate many benefits, and is dependent on cost and weight limitations at the system level. These are common challenges and realistic problems faced during the conceptual design stage of several engineering systems in modern technological advancements. The building of efficient solutions to these problems is becoming increasingly significant because the complexity of several mechanical and electrical systems is increasing even as development plans and timelines decrease and reliability requirements seem very strict.

The multiobjective ACS algorithm brings several different benefits to these problems in terms of the alternative optimization methods described in previous research, and can be used in a more diverse problem sphere with regard to the nature or size of prevailing challenges (Zhao, Liu, and Dao, 2007). By using a combination of probabilistic searches, the multiobjective formulation of local moves, dynamic penalty method, and currently proposed multiobjective techniques quite often help reach an optimal design solution faster than some other heuristic techniques. The recommended algorithm was successfully used in an engineering design problem involving a gearbox with many stages. The current development of a new MA to solve a multiobjective optimization problem, known as the reliability-RAP (RRAP) is another milestone differentiating the current approach from the previous approaches.

These technicalities, and the fact that RRAP is an NP-hard problem, would make the solving of optimization problems in an optimal manner difficult by using conventional methods or heuristic approaches. Therefore, it is imperative to build a new simulation process to produce the practical tools required to design reliable series–parallel systems. Zhao, Liu, and Dao (2007) explained these limitations of the conventional methods, inspiring the development of the proposed GA-based hybrid metaheuristic algorithm (HGA) to find an optimum solution, as described in Chapter 5. The previously mentioned application tools were generated from HGA simulation processes that can help design optimally reliable series–parallel systems. A confirmation test was performed for the approach, to enhance the security control system of a gas turbine in the overspeed state. Dhingra (1992), Rao and Dhingra (1992), and Quy (1998) have also developed an application of the reliability–redundancy optimization problem for overspeed protection by using a multiobjective approach. Therefore, no general method has yet been developed to solve the component support problem with discontinuous risk or cost functions. None of the previous relevant studies of Dhingra (1992), Rao and Dhingra (1992), and Quy (1998) used any random-search-based global-optimization method in their choice of methodologies. A major drawback of these works is their lack of establishing a practical instrument to design various components with different physical characteristics such as overall cost, total weight, average volume, and general reliability. As discussed, this application was intended to maximize system reliability and minimize consumption of resources such as cost, weight, and volume. The proposed model for simulating the overspeed control system of a gas-turbine engine allows the emergency reset of the system to be designed independent of the overspeed control used in the approach. Different techniques were used in the three given studies as no researcher has yet used these techniques.

CHAPTER 3

SERIES–PARALLEL SYSTEMS RELIABILITY OPTIMIZATION USING GENETIC ALGORITHM AND STATISTICAL ANALYSIS

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3.1 Abstract

The main objective of this paper is to optimize series–parallel system reliability using Genetic Algorithm (GA) and statistical analysis; considering system reliability constraints which involve the redundant numbers of selected components, total cost, and total weight. To perform this work, firstly the mathematical model which maximizes system reliability subject to maximum system cost and maximum system weight constraints is presented; secondly, a statistical analysis is used to optimize GA parameters, and thirdly GA is used to optimize series–parallel systems reliability. The objective is to determine the strategy choosing the redundancy level for each subsystem to maximize the overall system reliability subject to total cost and total weight constraints. Finally, the series–parallel system case study reliability optimization results are showed, and comparisons with the other previous results are presented to demonstrate the performance of our GA.

Keywords: Genetic algorithm, optimization, reliability, statistical analysis.

3.2 Introduction

THE system reliability optimization has become a very important subject matter area in industry design and operation of large scale manufacturing systems. The main issue that will be dealt with it in this study is the optimizing reliability of a series–parallel system using GA via implementing solutions for the redundancy allocation problem. The problem is to select redundancy level for each subsystem, component, and the best redundancy strategy in order to maximize the system reliability under system-level constraints. This type of problems includes a determination of components with many selections and redundancy levels that create the maximum advantages and are subject to the cost and weight constraints at the system level. These are extremely common problems confronted in the theoretical design of numerous engineering systems. It has become progressively necessary to develop adequate solutions to these issues since various mechanical and electrical systems are becoming more complex, even as developing plans take smaller and reliability requirements display very hard and fast. It is very important that the systems achieve their purpose under circumstances and operating conditions in a certain way. Nevertheless, the reliability level is a function of the investment amounts of a system. Consequently, using the optimization models is required to make an effective decision and perform analysis. The optimization of system reliability (OSR) models has been advanced to resolve the problems in whatever reliability is involved as objective function or constraint. The problem in this research is to optimize a combinatorial engineering design problem by considering the system of reliability constraint, which involves a redundant number of selected components to maximize reliability system or minimize cost system under numerous resources of the constraints.

The series–parallel system considered (Figure 3.1) has M number of subsystems in series, see (Coit *et al.*, 1996a) and (Zhao *et al.*, 2007). In turn, subsystem i contains N_i number of active (i.e., operating) units in parallel. If any one of the subsystems fails, the system fails. Each block in the diagram represents a unit. Reliability allocation is an essential step in system design. It allows determination of the reliability of vector of subsystems and components to obtain targeted overall reliability. For a system with identified cost, reliability, weight, volume, and other system parameters, the corresponding design problem becomes a combinatorial optimization problem, see (Coit *et al.*, 1996b) and (Khorshidi *et al.*, 2015). The best-identified reliability design problem of this type is denoted as the redundancy allocation problem.

Our goal in this paper is to present GA and statistical analysis approach, based on redundancy allocation problem to find the number of redundant components that either maximize reliability or minimize cost under numerous resources of the constraints. The redundancy allocation problem is fundamentally a nonlinear integer programming problem. Most of these problems cannot be answered by direct or indirect or mixed search methods because of separate search space. According to (Chern, 1992), redundancy allocation problem with multiple constraints is somewhat frequently hard to find feasible solutions. This redundancy allocation problem is Non-Deterministic Polynomial-time hard (NP-hard) and it has been well discussed in (Chambari *et al.*, 2012; Kuo and Prasad, 2000; Liang *et al.*, 2007; Sharifi *et al.*, 2015; Tillman *et al.*, 1977).

The penalty function is used in constrained problems optimization, see (Smith and Coit, 1997; Kuri-Morales and Gutiérrez-Garcia, 2002; Yeniay, 2005). Some researchers used statistical analysis to do this work for evolutionary algorithms, see (François and Lavergne, 2001; Mills *et al.*, 2015; Castillo-Valdivieso *et al.*, 2002; Petrovski *et al.*, 2005; Bayabatli and Sabuncuoglu, 2004).
In the next section, we present our solving methodology using GA and statistical analysis.

3.3 Methodology

From the study of the references (Bayabatli and Sabuncuoglu, 2004; Castillo-Valdivieso *et al.*, 2002; François and Lavergne, 2001; Kuri-Morales and Gutiérrez-Garcia, 2002; Mills *et al.*, 2015; Petrovski *et al.*, 2005; Smith and Coit, 1997; Yeniay, 2005), we decided to use the design and statistical analysis of experiments to optimize two penalty factors in our reliability fitness function using GA. We used a full factorial design with three levels. This classification will allow us to determine the ranges of values of these two factors of penalty giving the best values of reliability using GA. We did 10 simulations for every point of our design of experiments and used the average of the ten reliability values found to improve the accuracy of our coming statistical analysis. The best configuration of each point corresponding to the biggest reliability value is given with the corresponding cost and weight values. It is known that GA is effective for solving complex combinatorial optimization problems with large, and complex search spaces.

Assumptions

- All the components r_{ij} have different value, and every branch has a different number of components in series–parallel.
- The failure rate of components in each subsystem is constant.
- Failure rate depends on the number of working elements.
- Components are not repairable; they are changeable only.
- Subsystems have internal linking cost.
- Failed components do not damage the system.

Table 3. 1 Input data for RAP (Zhao *et al.*, 2007)

Gear pair	Stage											
	1			2			3			4		
	r1	c1	w1	r2	c2	w2	r3	c3	w3	r4	c4	w4
1	0.855	3	11	0.743	5	9	0.828	9	15	0.74	6	10
2	0.706	5	12	0.882	6	11	0.842	7	14	0.922	5	10
3	0.931	5	9	0.874	2	14	0.779	7	11	0.855	11	15
4	0.737	7	11	0.783	7	11	0.911	7	12	0.864	9	13
5	0.805	6	14	0.9114	5	7	0.846	3	11	0.816	9	12

Table 3. 2 The nomenclature and notation used to state the mathematical model

R_s	system reliability
C_s	system cost
W_s	system weight
$C_{\max}$	constraint of system cost
$W_{\max}$	constraint of system weight
s	number of subsystems in the system
i	index of subsystem, $i \in (1, 2,\ldots, s)$
j	index of component type

Table 3. 2 (Continued)

k	index of redundancy level
m_i	number of available component types in subsystem i
P_i	minimum number of components in parallel required for subsystem i to function
PN	maximum number of components in parallel (user define)
Ni	a set of component types, $N_i = [1, 2,\ldots,m_i]$
x_{ki}	a component type is assigned at the position k of subsystem i $x_{ki} \in (1, 2,\ldots,m_i,m_{i+1})$
x	system configuration matrix
$n_i(x)$	total number of redundant components used in subsystem i
$n(x)$	$= [n_1, n_2,\ldots,n_s]$
r_{ij}	reliability of the jth component type for subsystem i
c_{ij}	cost of the jth component type for subsystem i
w_{ij}	weight of jth component type for subsystem i
$R_i(x)$	reliability of subsystem i
$C_i(x)$	total cost of subsystem i
$W_i(x)$	total weight of subsystem i

The input data for the reliability optimization of series–parallel systems problem are summarized in Table 3.1. (Zhao *et al.*, 2007) have provided this example problem. The system consists of four subsystems, and each subsystem has different design component type with same or different characteristics as reliability, cost, transmission ratio, material, dimension, weight, etc. The minimum gear pair $pi = 2$ and the maximum gear pair $PN = 5$ will

be used in the gearbox for all stages. Each of the subsystems is represented by PN positions with each component listed according to their reliability index. The input data in Table 3.1 contain component reliability, weight, and cost. The objective is to maximize the system reliability with *k-out-of-n* subsystem connected in the series–parallel system under the given system requirement constraints.

Table 3. 3 Certain system constraints value used

No.	System constraint	
	C_{max}	W_{max}
1	40	115
2	55	125
3	65	130
4	60	120
5	60	130
6	60	140
7	60	150
8	65	120
9	65	140
10	65	150
11	70	120
12	70	130
13	70	140
14	70	150
15	75	120
16	75	130
17	75	140
18	75	150

Figure 3.1 presents a typical example of a series–parallel system configuration with *k-out-of-n* subsystem reliabilities. The system is separated into *s* subsystem indicated by the index *i* (*i = 1, 2,…,s*). *pi* number of effective components is required for the function at least in subsystem *i.* Each subsystem involves one or various components organized in parallel, and it constitutes the lower bound of the redundancy level for subsystem *i*. The upper bound of the component redundancy level in subsystem *i* is *PN*. The system configuration can thus be described as a matrix of size *PN* x *s*: The column index *i* (*i = 1, 2,…,s*) represents subsystem *i*, and the row index *k* (*k =1, 2, …, PN)* of the matrix represents the position where a component will be used in the subsystem. Redundancy allocation problems (RAP) consist of defining the number of components of each type, so that the complete reliability system will be maximized by considering the given constraints such as cost and weight. The content of the case study is shown in Figure 3.2. The problem used in this test to demonstrate the ability of this algorithm for solving RAP is a gearbox reliability optimization problem obtainable in (Zhao *et al.*, 2007). The authors in this reference presumed, in order to apply their method for all stages, that the

minimum number of components is equal to 2, and the maximum number of components is equal to 5. The problem in the reference (Zhao *et al.*, 2007) is to decide how many gear pairs and what kind of gear pair selected to be used in each stage to give maximum reliability of the gearbox with minimization of both system cost and weight. By assuming that all the gear pairs are active components in the stage, then the gearbox is analogous to a series–parallel system with *k-out-of-n*: *G* subsystems.

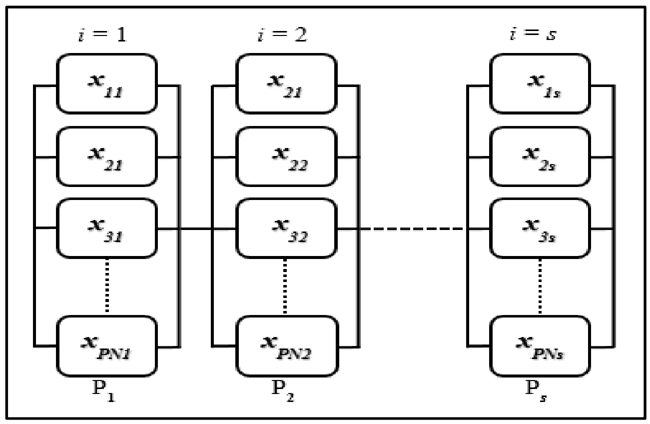

Figure 3. 1 Series–parallel system

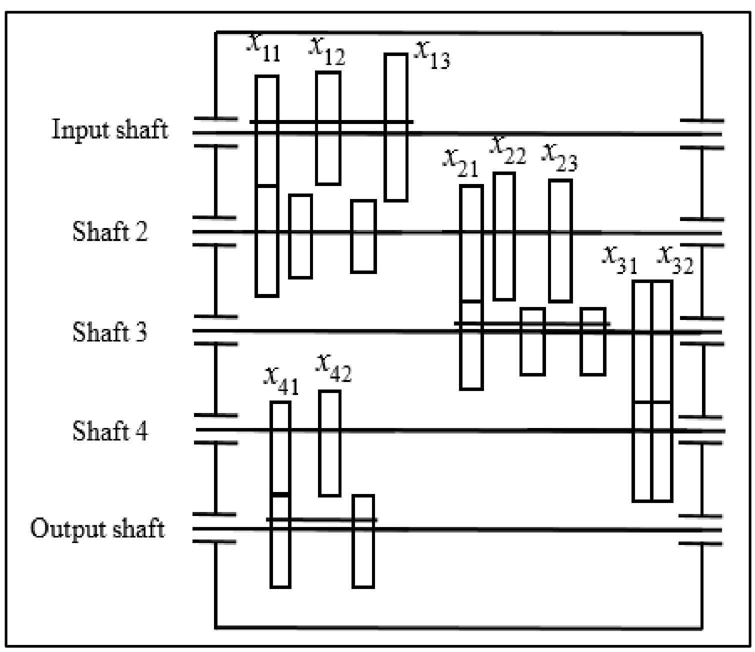

Figure 3. 2 Series–parallel system case study

The studied problem is modeled by (Zhao *et al.*, 2007), and the mathematical model formulated as

$$Maximize\ \ R_s\,(x) = \prod_{i=1}^{s}\left[\left(1 - \prod_{k=1}^{PN}(1 - r_{ix_{ki}}\right)\right] \tag{3.1}$$

Subject to

$$C_s\,(x) = \sum_{i=1}^{s} C_i\,(x) = \sum_{i=1}^{s}\sum_{k=1}^{PN} C_{ix_{ki}}\ \le\ C_{max}\ , \tag{3.2}$$

$$W_s\,(x) = \sum_{i=1}^{s} W_i\,(x) = \sum_{i=1}^{s}\sum_{k=1}^{PN} W_{ix_{ki}}\ \le\ W_{max}\ , \tag{3.3}$$

$$P_i \le n_i \le PN\ \ and\ \ \forall i, i = 1, 2, \dots, s \tag{3.4}$$

A technique based on GA to optimize series–parallel systems reliability is developed (Figure 3.3) in order to find out the best compromise (optimal) solution of the problem. The different steps of the developed technique are given in the chart Figure 3.3.

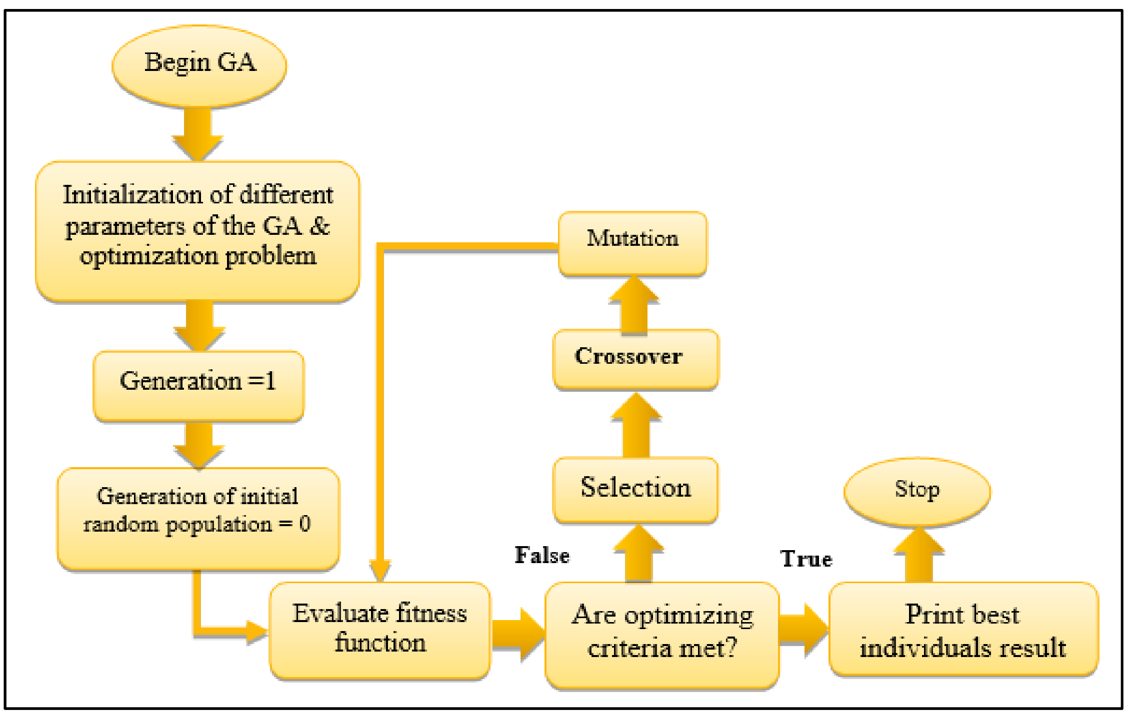

Figure 3. 3 Flow chart of the proposed GA for optimizing system reliability

We used the fitness function *f(i)* to do the reliability optimization of the series–parallel systems using GA in the following form:

$$f(i) = rel(i); \quad \text{if} \; c(i) > C_{max};$$
$$\text{then}: \; f(i) = f(i) - (CPen * (c(i) - C_{max})); \quad \text{if} \; w(i) > W_{max};$$
$$\text{then}: \; f(i) = f(i) - (WPen * (w(i) - W_{max}));$$

where *rel(i)* is the reliability, *c(i)* is the cost, *w(i)* is the weight, C_{max} is the maximum cost, and W_{max} is the maximum weight. *CPen* is the cost penalty factor, and *WPen* is the weight penalty factor. The range of the values in Table 3.4 for cost penalty factor and weight penalty factor was found using trial and error. The dynamic penalty function was defined increasing the penalty for infeasible solutions as the search progresses. The GA implementation is doing with this experimental procedure for determining the initial population size considering the following GA parameters:

- The population size, which determines the size of the population at each generation is 1000, and our maximum number of iteration is 10000.
- We used 20 integers to code our chromosomes (maximum of five gear pair and four stages).
- The value 6 from the configuration means that this position is empty.
- We used four points of crossover generated randomly corresponding to our four subsystems to improve our GA search.
- We could obtain a better result by increase the population size to enable the GA to search for more points.
- Nevertheless, when the population size is large, the GA will take a long time to compute each generation
- Finally, it is very important to note that we set the population size to be at least the value of number of variables, so the individuals in each population span the space being searched.

3.4 Results and Discussion

A numerical application has been demonstrated with the data obtained from test problem 1 of (Zhao *et al.*, 2007), and the obtained results are presented.
We used C_{max} = 65, and the W_{max} = 130 for our GA fitness function cost penalty (*CPen*), and weight penalty (*WPen*) statistical analysis. We used a 3-level full factorial design.

Our statistical analysis in Figures. 3.4 and 3.5 shows that all the residuals are zero, which means that our prediction is very good.

Table 3. 4 GA results of the design of experiments points is used for applying statistical analysis data

No.	CPen	WPen	Average reliability
1	0.1	0.1	0.9961
2	0.1	0.55	0.9965
3	0.1	1	0.9956
4	0.55	0.1	0.9967
5	0.55	0.55	0.9961
6	0.55	1	0.9967
7	1	0.1	0.9961
8	1	0.55	0.9964
9	1	1	0.9960

The contour plot in Figure 3.6 displays the three-dimensional relationship in two dimensions. This plot is on the *x-axis* and *y-axis* scales factors by the predictor and the response values represented by contours. The contour plot can be used to investigate the possible relation between the three variables. We have an average reliability, cost penalty, and weight penalty. This plot shows how cost penalty on the *x-axis* and the weight penalty in *y-axis* affect the quality result. The darker indicates to the higher quality of the average reliability.

The response surface (Figure 3.7) is obtained using the statistical analysis software STATISTICA and it generates the optimal designs. These numbers of the statistical analysis obtained are to choose the best GA for the selection of the optimal designs. The techniques for experimental model design objective are to optimize the response of the output variable (average reliability) which is influenced by cost penalty factor, and weight penalty factor. The response can be represented graphically in the contour plots that help us to visualize the shape of the response. The darker regions indicate higher quality. The response surface plot for the cost penalty strength where the *axis x* is the redder color, the weight penalty for the *axis y* is the less red color, and the *axis z* is the average reliability.

Factor	ANOVA; Var.:Average Reliability; R-sqr=1. (Spreadsheet1.sta) 2 3-level factors, 1 Blocks, 9 Runs DV: Average Reliability SS	df	MS	F	p
(1)CPen L+Q	0.000000	2	0.000000		
(2)WPen L+Q	0.000000	2	0.000000		
1*2	0.000001	4	0.000000		
Error	0.000000	0			
Total SS	0.000001	8			

Figure 3. 4 Model ANOVA result

Case or Run	Observed, Predicted, and Residual Values (Spreadsheet1.sta) 2 3-level factors, 1 Blocks, 9 Runs DV: Average Reliability Observed	Predictd	Resids
1	0.996127	0.996127	0.000000
2	0.996507	0.996507	0.000000
3	0.995593	0.995593	-0.000000
4	0.996716	0.996716	0.000000
5	0.996087	0.996087	-0.000000
6	0.996748	0.996748	0.000000
7	0.996125	0.996125	-0.000000
8	0.996392	0.996392	0.000000
9	0.996016	0.996016	-0.000000

Figure 3. 5 Display observed, predicted, and residual values

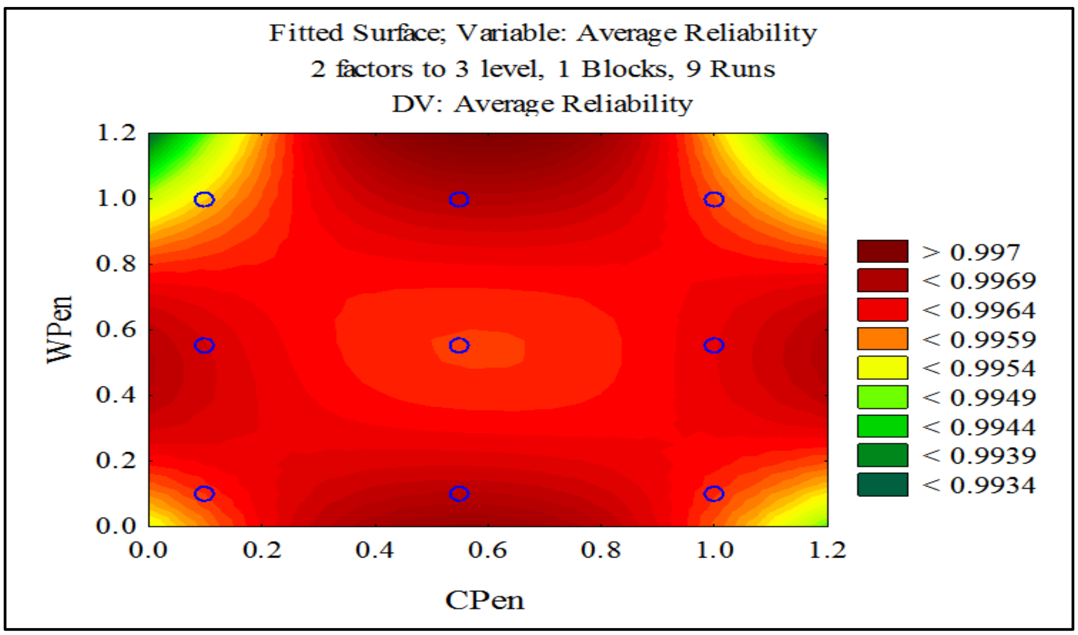

Figure 3. 6 Contour plot of average reliability versus WPen, CPen

The response curves (Figure 3.6) and response surface (Figure 3.7) show that the best parameters are around the cost penalty *CPen* = 0.55 and the weight penalty factor *WPen* = 0.1. We show here one of the ten results running our GA on the center of our design of experiments (*CPen* = 0.55, *WPen* = 0.55): the configuration is 6 3 6 3 3 5 5 6 3 6 5 6 3 4 5 6 2 2 2 6, the reliability = 0.997743, the cost = 62, the weight = 130, and the fitness = 0.997743. The reliability, cost, weight, and fitness graphics of this result are showed respectively on Figures. 3.8-3.11. It can be observed that from the plots which show that GA has already achieved the maximum score at the iteration of 10000.

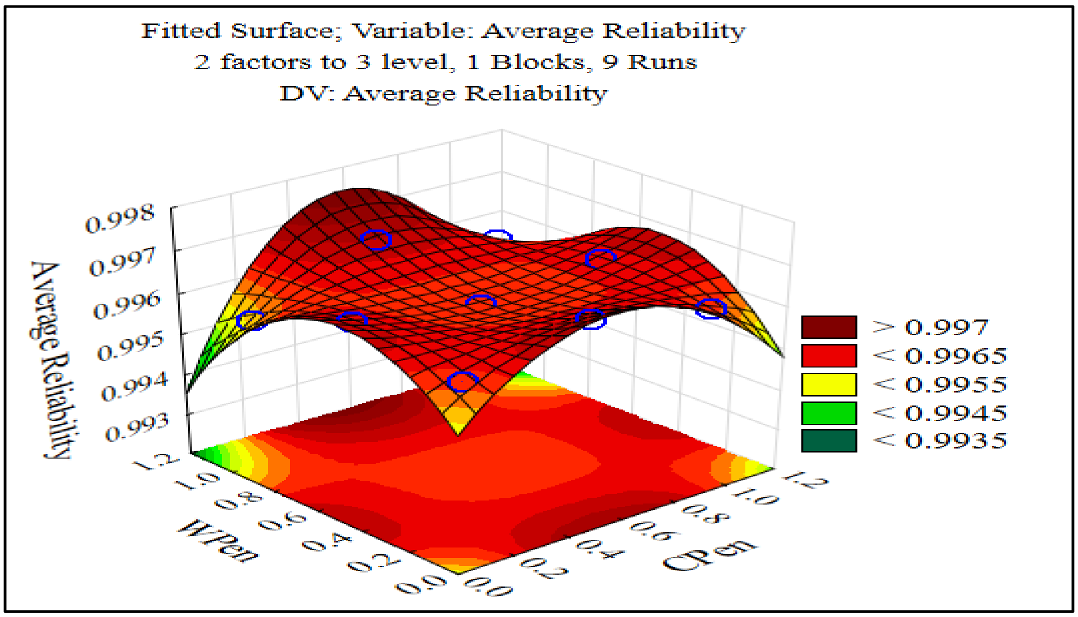

Figure 3. 7 3D Surface plot of average reliability versus CPen and WPen

Table 3.5 reports all the allocated components for each subsystem that we have in our system. For example, at admissible constraint weight = 115 and constraint cost = 40, the best configuration of the 10 simulations that we obtained is = [3, 3, 6, 6, 6, 6, 3, 3, 3, 6, 5, 6, 5, 5, 6, 6, 2, 2, 6, 6], which means that, from the 20 positions, result is illustrated as:

- The first subsystem has two components of type 3.
- The second subsystem has three components of type 3.
- The third subsystem has three components of type 5.
- The fourth subsystem has two components of type 2.

The result obtained in Table 3.5 was just change by the values of the constraints. The cost penalty = 0.55 is constant, and the weight penalty = 0.1 is constant. These results are obtained using the maximum possible improvement with the best feasible solution, which improves the system reliability, cost, and weight.

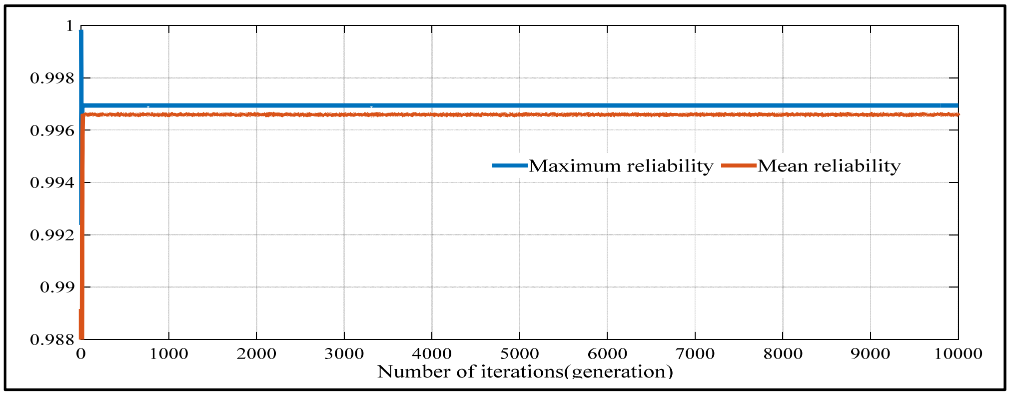

Figure 3. 8 Maximum and mean reliability

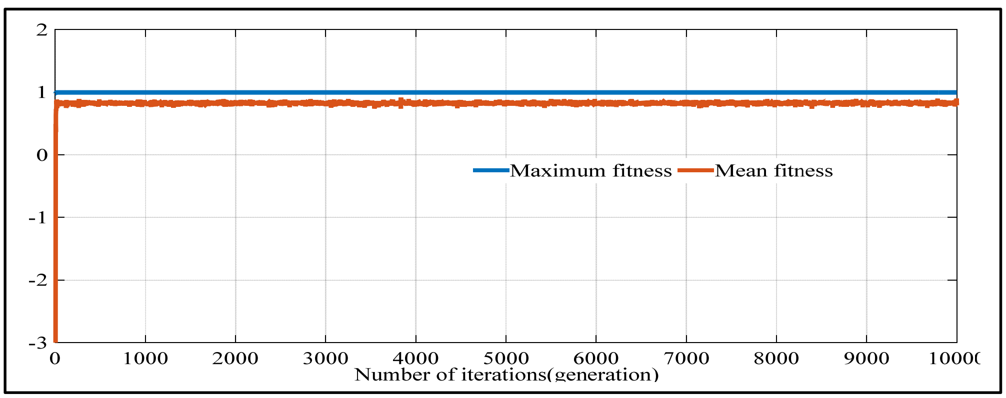

Figure 3. 9 Maximum and mean fitness

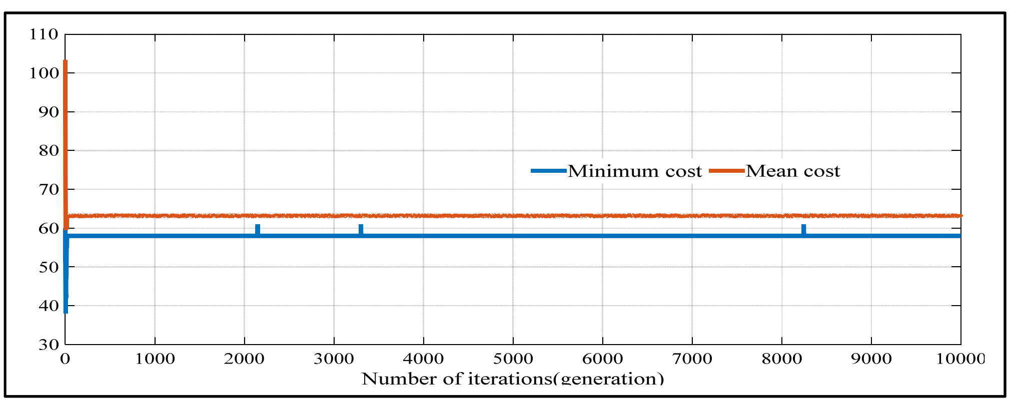

Figure 3. 10 Minimum and mean cost

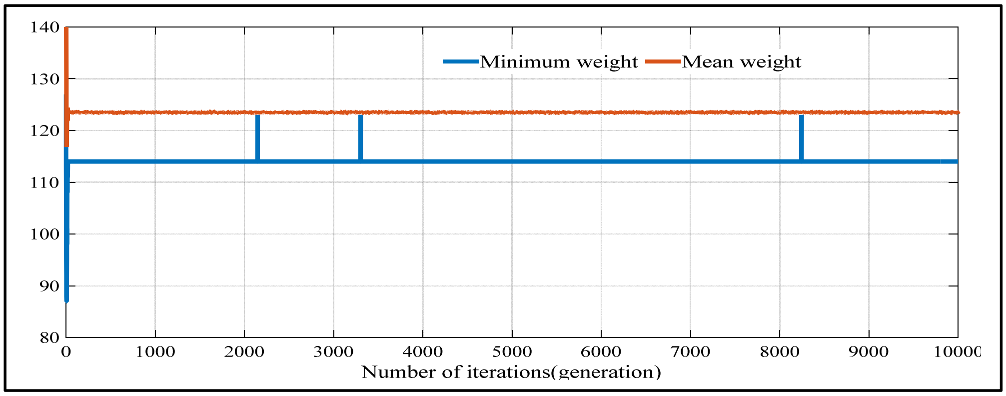

Figure 3. 11 Minimum and mean weight

Table 3. 5 The Results Obtained by GA

No.	C_{Max}	W_{Max}	***Our obtaining result using GA***																							
			Best Configuration of 10 simulations																				**Reliability**	**Cost**	**Weight**	**Fitness**
1	40	115	3	3	6	6	6	6	3	3	3	6	5	6	5	5	6	6	2	2	6	6	0.9836	35	113	0.9836
2	55	125	1	6	3	3	6	5	6	2	6	3	4	5	5	6	6	2	6	2	6	2	0.9954	54	125	0.9954
3	65	130	3	6	6	3	3	5	6	5	3	1	6	6	4	4	5	2	2	6	2	6	0.9977	64	129	0.9977
4	60	120	3	3	6	6	3	3	5	6	5	6	4	4	5	6	6	6	2	1	2	6	0.9959	60	120	0.9959
5	60	130	3	3	6	3	6	5	3	6	5	6	5	5	4	5	6	2	6	6	2	2	0.9979	58	130	0.9979
6	60	140	3	1	3	6	1	2	6	6	5	5	6	5	5	5	5	6	2	2	6	2	0.9979	59	139	0.9979
7	60	150	3	6	3	3	6	5	3	5	3	6	4	5	5	5	6	6	6	2	2	2	0.9987	60	144	0.9987
8	65	120	3	3	6	3	6	5	5	5	6	6	4	6	5	4	6	6	2	2	2	6	0.9973	62	113	0.9973
9	65	140	3	3	3	1	6	5	5	6	6	5	5	4	5	6	5	6	2	2	6	2	0.9985	64	134	0.9985
10	65	150	3	3	6	6	3	6	3	3	5	5	6	5	5	5	4	2	6	2	6	2	0.9987	60	144	0.9987
11	70	120	6	3	3	3	6	6	5	6	2	5	6	4	4	6	4	2	2	2	6	6	0.9976	67	118	0.9976
12	70	130	3	3	3	6	6	1	5	5	6	5	4	6	4	5	6	6	2	2	6	2	0.9978	67	122	0.9978
13	70	140	3	1	6	6	3	3	6	5	5	5	6	4	5	3	4	2	6	2	6	2	0.9985	69	140	0.9985
14	70	150	3	3	1	6	6	3	5	5	5	6	5	6	5	4	5	2	2	1	6	2	0.9988	67	149	0.9988
15	75	120	6	3	3	3	6	5	6	5	6	5	4	4	6	6	4	2	6	6	2	4	0.9974	70	117	0.9974
16	75	130	3	3	6	3	6	5	5	3	6	5	4	4	6	4	6	6	6	2	2	2	0.9984	68	128	0.9984
17	75	140	3	6	6	3	3	5	5	5	3	6	4	6	5	3	4	6	2	2	6	2	0.9988	71	138	0.9988
18	75	150	3	3	6	3	6	5	5	3	6	1	4	5	6	5	4	2	2	2	6	1	0.9991	73	150	0.9991

Table 3. 6 The Comparison of (Zhao *et al.*, 2007) Ant Colony System (ACS) Result and Our GA Result

No.	C_{Max}	W_{Max}	*Our GA result*			*Zhao, J. H., Liu, Z., & Dao, M. T. results using ACS–RAP*		
			Reliability	**Cost**	**Weight**	**Reliability**	**Cost**	**Weight**
1	40	115	0.9836	35	113	0.9861	40	114
2	55	125	0.9954	54	125	0.9973	55	124
3	65	130	0.9977	64	129	0.9977	58	130
4	60	120	0.9959	60	120	0.9968	59	120
5	60	130	0.9979	58	130	0.9977	58	130
6	60	140	0.9979	59	139	0.9985	60	140
7	60	150	0.9987	60	144	0.9987	60	149
8	65	120	0.9973	62	113	0.9968	59	120
9	65	140	0.9985	64	134	0.9988	65	140
10	65	150	0.9987	60	144	0.9990	64	150
11	70	120	0.9976	67	118	0.9968	59	120
12	70	130	0.9978	67	122	0.9988	66	130
13	70	140	0.9985	69	140	0.9990	65	140
14	70	150	0.9988	67	149	0.9992	70	149
15	75	120	0.9974	70	117	0.9968	59	120
16	75	130	0.9984	68	128	0.9988	66	130
17	75	140	0.9988	71	138	0.9992	71	140
18	75	150	0.9991	73	150	0.9995	70	150

3.5 Conclusion

We determined the best combination and the redundancy level for a case study of the series–parallel system reliability optimization problem and improved our GA implementation using statistical analysis. We used STATISTICA software to do our statistical analysis experimental which gave us to choose the best penalty factor values that improved our GA parameters. The best configuration of 10 simulations obtained gave us the best reliability as one can see in Table 3.5.

CHAPTER 4

MULTI-OBJECTIVE OPTIMIZATION OF MULTI-STATE RELIABILITY SYSTEM USING HYBRID METAHEURISTIC GENETIC ALGORITHM AND FUZZY FUNCTION FOR REDUNDANCY ALLOCATION

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4.1 Abstract

This paper proposes a methodology for optimizing the reliability of a series–parallel system on the basis of multi-objective optimization and multi-state reliability using a hybrid genetic algorithm (HGA) and fuzzy function. The considered reliability constraints include the number of selected redundant components, total cost, and total weight. First, we describe the modeling of the proposed methodology. Second, we explain the formulation of the optimization process and the solution using HGA. Most related studies have focused only on single-objective optimization of the redundancy allocation problem (RAP); multi-objective optimization has not attracted much attention thus far. This study investigates the multi-objective scenario. Specifically, multi-objective formulation is considered for maximizing system reliability and minimizing system cost and system weight simultaneously in order to solve the RAP. The objective is to determine the system configuration that achieves the optimal trade-off between reliability, cost, and weight. Finally, the obtained results show that the proposed approach can enable manufacturers to determine the number of redundant components and their reliability in a subsystem in order to develop a system that effectively satisfies the reliability, cost, and weight criteria.

Keywords: Multi-objective optimization; multi-state reliability; hybrid metaheuristic genetic algorithm; fuzzy function.

4.2 Introduction

Optimizing reliability in the design and operation of large- and small-scale systems is an important issue for manufacturers. The objective of this study is to optimize the reliability of a series–parallel system on the basis of a genetic algorithm (GA) by implementing solutions for the redundancy allocation problem (RAP). The problem is to set the redundancy level for each subsystem and component and to select the best redundancy strategy in order to maximize –and weight at the system level.

This problem is extremely common in the theoretical design of various engineering systems. Developing robust solutions to address the issue of system reliability is important because mechanical and electrical systems and products have become increasingly complex over the years. It is crucial for systems to achieve their objectives under given circumstances and operating conditions in a certain manner. However, the level of system reliability is directly related to system cost. Thus, optimization models are required for effective decision-making and analysis. This study focuses on optimizing a combinatorial engineering design problem, i.e., maximizing the reliability and minimizing the cost and weight of a system that involves a redundant number of selected components. The main contribution of this study is that it examines the effectiveness of employing a fuzzy function along with a multi-objective genetic algorithm for solving the redundancy allocation problem.

4.3 Literature review

This paper focuses on multi-objective optimization and multi-state reliability of a series–parallel RAP in which the subsystems are designed in series and the components in each subsystem are organized in parallel. The series–parallel system considered (Figure 4.2) has M subsystems in series (see Coit *et al.*, 1996a), and (Zhao *et al.*, 2007). Further, the i^{th} subsystem consists of N_i active (operating) units organized in parallel. If any subsystem fails, the entire system fails. Each block in the diagram represents a unit. Reliability allocation is an important step in the system design because it allows for the determination of the reliability of a vector of subsystems and components in order to obtain the desired overall reliability. For a system with identified cost, reliability, weight, volume, and other system parameters, the corresponding design problem becomes a combinatorial optimization problem (see Coit *et al.,* 1996b), and (Khorshidi *et al.*, 2015). The best identified reliability design problem of this type is known as the redundancy allocation problem. This paper proposes multi-objective optimization using a hybrid genetic algorithm (HGA)-based optimization methodology for the redundancy allocation problem in order to find the number of redundant components that achieve the highest possible reliability while maintaining the lowest possible cost and weight under numerous resources. The proposed methodology uses a fuzzy function in combination with HGA to find the best possible solution for the redundancy allocation problem. The redundancy allocation problem is fundamentally a nonlinear integer programming problem. In most cases, it cannot be solved by direct, indirect, or mixed search methods because it involves separate search spaces. According to (Chern, 1992), it is often difficult to find feasible solutions for redundancy allocation problems with multiple constraints. Such redundancy allocation problems are non-deterministic polynomial-time hard (NP-hard), and they have been discussed extensively by (Chambari *et al.*, 2012; Kuo and Prasad, 2000; Liang *et al.*, 2007; Sharifi *et al.*, 2015; Tillman *et al.*, 1977). The penalty function is used in constrained problem optimization (see Smith and Coit, 1997; Kuri-Morales and Gutiérrez-Garcia, 2002; Yeniay, 2005). Some researchers have investigated evolutionary algorithms using statistical analysis (see François and Lavergne, 2001; Mills *et al.*, 2015; Castillo-Valdivieso *et al.*, 2002; Petrovski *et al.*, 2005; Abatable and Sabuncuoglu, 2004). Mahaparta and Roy (2006) considered a multi-objective reliability optimization problem for system reliability, in which reliability enhancement involves several mutually conflicting objectives. In this paper, a new fuzzy multi-objective optimization method is introduced, and it is used for effective decision-making with regard to the reliability optimization of series and complex systems with two objectives. Salazar *et al.* (2006) demonstrated a multi-objective optimization technique for solving three types of reliability optimization problems: determining the optimal number of redundant components (redundancy allocation problem), determining the reliability of components (component reliability problem), and determining both the redundancy and the reliability of components (redundancy allocation and component reliability problem) using nondominated sorting genetic algorithm II (NSGA-II). These problems were formulated as single objective mixed-integer nonlinear programming (MINLP) problems with one or several constraints and solved using mathematical programming techniques. Azaron *et al.* (2009) used a genetic algorithm to solve a multi-objective discrete reliability optimization problem involving a non-repairable cold-standby redundant system with k dissimilar units. They employed a double string using continuous relaxation based on reference solution updating. Wang *et al.* (2009) proposed RAP as a multi-objective optimization problem, in which the reliability of the system and the related designing cost are considered as two different objectives. They adopted NSGA-II to solve the multi-objective redundancy allocation problem (MORAP) under a number of constraints. Sahoo *et al.* (2012) formulated four different multi-objective reliability optimization problems using interval mathematics and proposed order relations of interval-valued numbers. Then, these optimization problems were solved using advanced GA and the concept of Pareto optimality. Taboada and Coit (2012) proposed a GA-based multi-objective evolutionary algorithm for reliability optimization of series–parallel systems. They considered three objective functions, namely system reliability, cost, and system weight, to solve RAP; however, they did not use a fuzzy function. In the next section, we present our methodology for solving RAP using HGA and a fuzzy function.

4.4 Methodology Framework

In our experiments, to implement the proposed optimization methodology, we adopted two penalty factors that have been considered by many researchers (Abatable and Sabuncuoglu, 2004; Castillo-Valdivieso *et al.*, 2002; François and Lavergne, 2001; Kuri-Morales and Gutiérrez-Garcia, 2002; Mills *et al.*, 2015; Petrovski *et al.*, 2005; Smith and Coit, 1997; Yeniay, 2005). We used a full factorial design with three levels. The fuzzy function allows the optimization algorithm to identify the solution of the redundancy problem that achieves the optimal trade-off between the optimization objectives from several optimal solutions. We performed 10 simulations for every experiment and used the best result of the 10 reliability values obtained. The best configuration of each point corresponding to the largest reliability value is given with the corresponding cost and weight values. The following assumptions are made in the optimization process:

- All the components r_{ij} have different values, and every branch has a different number of components in series and parallel.

- The failure rate of the components in each subsystem is constant.
- The failure rate depends on the number of working elements.
- The components are not repairable; they are changeable only.
- The subsystems have internal linking costs.
- The failed components do not damage the system.

Figure 4.1 shows the flowchart of the proposed algorithm. The HGA and fuzzy function procedures developed to implement our methodology are illustrated.

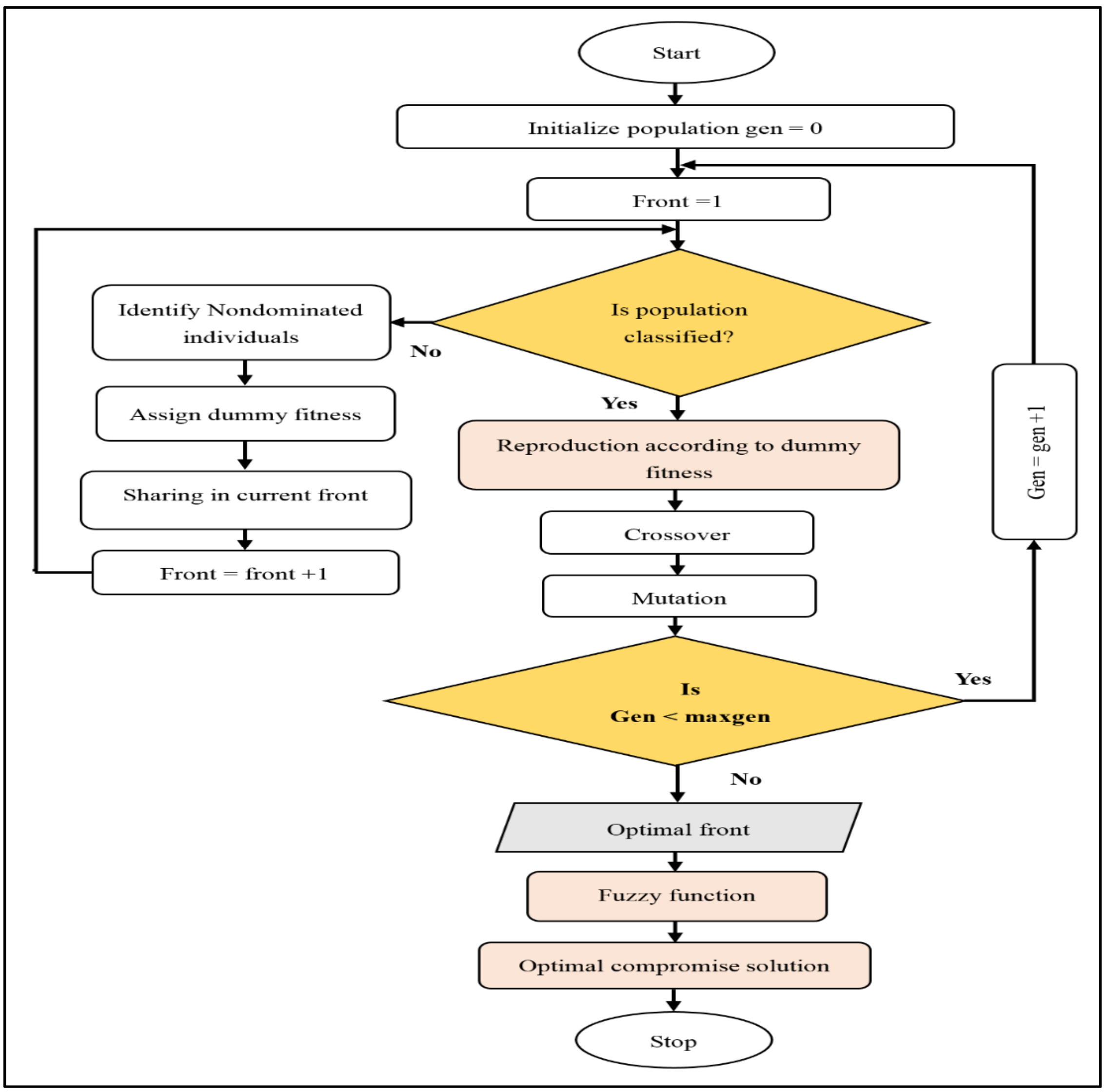

Figure 4. 1 Flowchart of the proposed algorithm.

The proposed method involves the following steps.

Step 1: Generate a population of random individuals.

Step 2: Initialize the front counter to 1.

Step 3: Check the termination condition. If the population is not classified, then identify nondominated individuals, assign large dummy fitness values to them, and to maintain diversity in the population, share these individuals with their dummy fitness values. After sharing, ignore these nondominated individuals temporarily. Then, identify the second nondominated front in the rest of the population and assign a dummy fitness value smaller than the minimum shared dummy fitness of the previous front. Then, increment the front counter by 1.

Step 4: Continue this process until the entire population is classified into several fronts. If the termination condition is satisfied, then reproduction occurs according to the dummy fitness.

Step 5: Use the crossover and mutation genetic operations to generate a new population.

Step 6: Check the termination condition of the proposed algorithm, i.e., if the current generation number is smaller than the maximum generation number, continue the process by going back to the second step until the objectives of the problem are met and increment gen by 1. If the current generation number is not smaller than the maximum generation number, then terminate the generation process. Otherwise, go to the next generation and implement the optimal front and fuzzy function; then, select the solution with the best trade-off and stop.

The flowchart follows the same steps as classical GAs except for the classification of nondominated fronts and the sharing operation. The sharing in each front is achieved by calculating the value of the sharing function between two individuals in the same front. This method is based on several layers of classification of the individuals. Nondominated individuals are assigned a certain dummy fitness value and are then removed from the population, and the process is repeated until the entire population has been classified. To maintain the diversity of the population, the classified individuals are shared (in decision variable space) with their dummy fitness values.

The multi-objective genetic algorithm is implemented using MATLAB® Optimization Toolbox™. First, MATLAB code that represents the fitness function and calculates the values of all the objectives (reliability, cost, and weight) is generated as an M-file. Because RAP is an integer problem, the creation, mutation, and crossover functions of the GA are adapted to generate integer populations that satisfy the problem constraints. The GA is implemented in our experimental procedure to determine the initial population size considering the following parameters:

- The population size in each generation is 1000, and the maximum number of iterations is 10000.
- We used 20 integers to code our chromosomes (maximum of 5 gear pairs and 4 stages).
- The value 6 from the configuration implies that this position is empty.
- We used 4 randomly generated crossover points corresponding to our 4 subsystems to improve our GA search.
- We could obtain better results by increasing the population size in order to enable the GA to search for additional points.
- However, when the population size is large, the GA will take a long time to calculate each generation.
- Finally, it is important to note that we set the population size to be at least the value of a number of variables such that the individuals in each population span the space being searched.

Optimizing the above-mentioned objective functions using a multi-objective genetic algorithm yields a set of solutions that are said to be nondominated or Pareto-optimal. Each of these solutions cannot be improved further without degrading one or more of the other objective values. The aim of the fuzzy function is to choose the optimal solution (trade-off) from the Pareto-optimal solutions. The corresponding linear fuzzy membership function value of the j^{th} objective function, μ_j, is defined as (Brka *et al.*, 2015)

$$\mu_j = \begin{cases} 1 & F_j \leq F_j^{min} \\ \left(F_j^{max} - F_j\right)/\left(F_j^{max} - F_j^{min}\right) & F_j^{min} < F_j < F_j^{max} \\ 0 & F_j \geq F_j^{max} \end{cases} \tag{4.1}$$

Here, for the j^{th} objective functions, F_j , the minimum value is denoted as F_j^{min} and the maximum value is denoted as F_j^{max}, and j takes a value of 1, 2, or 3 because there are three objectives (reliability, cost, and weight). The normalized membership function μ^k for each non-dominant solution is calculated as

$$\mu^k = \sum_{i=1}^{N_{obj}} \mu_j^k / \sum_{k=1}^{M} \sum_{j=1}^{N_{obj}} \mu_j^k \tag{4.2}$$

where N_{obj} is the number of objective functions and M is the number of non-dominated solutions.

4.5 Problem Modeling

We propose HGA-based multi-objective optimization using a fuzzy function for solving multi-state reliability and availability optimization design problems. Considering the system design, we require the simultaneous optimization of more than one objective function. In this optimization problem, there are three objectives: (1) maximizing the system reliability, (2) minimizing the system weight, and (3) minimizing the system cost while satisfying the system requirements. All the components and the system considered have a range of different states, and the fuzzy function technique is used to obtain the system availability. The notations used in our mathematical model for multi-objective optimization and multi-state reliability of RAP are summarized in Table 4.1.

Table 4. 1 Notations used in our mathematical model.

Abbreviations	Details
R_s	Total reliability of the series–parallel system
C_s	Total cost of the series–parallel system
W_s	Total weight of the series–parallel system
$C_{\max}$	Limit of the cost constraint of the series–parallel system
$W_{\max}$	Limit of the weight constraint of the series–parallel system
s	Number of subsystems in the system
i	Index of subsystem, $i \in (1, 2,\ldots, s)$

Table 4. 1 (Continued)

j	Index of component type in each subsystem
k	Index of redundancy level
m_i	Total number of available component types in the i^{th} subsystem
P_i	Minimum number of components in parallel required for the i^{th} subsystem to function
PN	Maximum number of components in parallel that can be used in the i^{th} subsystem (user-defined)
Ni	Set of component types, $N_i = [1, 2, \ldots, m_i]$
x_{ki}	Number of component types assigned at position k of the i^{th} subsystem, $x_{ki} \in (1, 2, \ldots, m_i, m_{i+1})$
x	System configuration matrix
$n_i(x)$	Total number of redundant components used in the i^{th} subsystem
$n(x)$	Set of n_i ($n_1, n_2, \ldots, n_s$)
r_{ij}	Reliability of the j^{th} available component type in the i^{th} subsystem
c_{ij}	Cost of the j^{th} available component type in the i^{th} subsystem
w_{ij}	Weight of the j^{th} available component type in the i^{th} subsystem
$R_i(x)$	Reliability components of the i^{th} subsystem
$C_i(x)$	Total system cost of the i^{th} subsystem
$W_i(x)$	Total weight of the i^{th} subsystem

Based on the notations and basic assumptions, the following performance metrics (namely, system reliability, designing cost, and system weight) are defined.

(1) With regard to the system structure, the reliability of a series–parallel system (R_s) can be calculated as

$$R_s\,(x) = \prod_{i=1}^{s} \left[(1 - \prod_{k=1}^{PN} (1 - r_{ix_{ki}}) \right] \tag{4.3}$$

where s is the number of subsystems in the system, PN is the maximum number of components that can be used in parallel in the i^{th} subsystem, r_i is the reliability of the j^{th} available component in the i^{th} subsystem, and x_{ki} is the number of component types allocated at position k of the i^{th} subsystem $x_{ki} \in (1, 2, \ldots, m_i, m_{i+1})$.

(2) The probable total system design cost (C_s) can be calculated as

$$C_s\,(x) = \sum_{i=1}^{s} C_i(x) = \sum_{i=1}^{s}\sum_{k=1}^{PN} C_{ix_{ki}} \tag{4.4}$$

where C_i is the cost of each available component in the i^{th} subsystem and x_{ki} is the number of component types allocated at position k of the i^{th} subsystem, $x_{ki} \in (1, 2, \ldots, m_i, m_{i+1})$.

(3) Furthermore, we can calculate the weight of the system (W_s) as

$$W_s\,(x) = \sum_{i=1}^{s} W_i(x) = \sum_{i=1}^{s}\sum_{k=1}^{PN} W_{ix_{ki}} \tag{4.5}$$

where W_i is the weight of each available component in the i^{th} subsystem and x_{ki} is the number of component types allocated at position k of the i^{th} subsystem, $x_{ki} \in (1, 2, \ldots, m_i, m_{i+1})$. Multi-objective optimization refers to the solution of problems with two or more objectives to be satisfied simultaneously. Such objectives are often in conflict with each other and are expressed in different units. Because of their nature, multi-objective optimization problems usually have not one solution but a set of solutions, which are referred to as Pareto-optimal solutions or nondominated solutions (see Chankong *et al.* [19] and Hans [20]). When such solutions are represented in the objective function space, the graph obtained is called the Pareto front or the Pareto-optimal set. A general formulation of a multi-objective optimization problem consists of a number of objectives with a number of inequality and equality constraints.

The mathematical model of the problem studied herein is formulated as a multi-objective optimization problem as follows:

$$Max \quad R_s\,(x) \tag{4.6}$$

$$Min \quad C_s\,(x) \tag{4.7}$$

$$Min \quad W_s\,(x) \tag{4.8}$$

Subject to

$$C_s\,(x) \leq \; C_{max} \tag{4.9}$$

$$W_s\,(x) \leq \; W_{max} \tag{4.10}$$

$$P_i \leq n_i \leq PN \;\; and \tag{4.11}$$

$$\forall i, i = \; 1, 2, \ldots, s \tag{4.12}$$

The first constraint is related to minimizing the system design cost (C_s), while the second constraint is related to minimizing the system weight (W_s). C_{max} and W_{max} are the upper bounds of C_s and W_s, respectively.

Figure 4.2 shows a typical example of a series–parallel system configuration with *k-out-of-n* subsystem reliabilities. The system is separated into *s* subsystems indicated by the index *i* (*i = 1, 2,…, s*), and each subsystem consists of one or several components organized in parallel. Further, P_i is the minimum number of active components required for the i^{th} subsystem to function, i.e., the lower bound of the level of component redundancy for the i^{th} subsystem. The upper bound of the level of component redundancy for the i^{th} subsystem is denoted by *PN*. Thus, the system configuration can be defined as a $PN \times s$ matrix. For this matrix, the column index *i* (*i = 1, 2,…, s*) denotes the i^{th} subsystem, and the row index *k* (*k =1, 2,…, PN)* establishes the position where a component will be used in the subsystem. RAP involves defining the number of components of each type such that the total system reliability will be maximized considering the given constraints, such as cost and weight. The content of the case study is shown in Figure 4.3.

The objective of this test is to demonstrate the ability of the proposed algorithm in solving RAP as a gearbox reliability optimization problem, as shown by (Zhao *et al*., 2007), who assumed, in order to apply their method to all stages, that the minimum number of components is equal to 2 and the maximum number of components is equal to 5. In their study, the problem is to decide how many gear pairs and what types of gear pairs are to be selected for use in each stage, which will give the maximum reliability of the gearbox while minimizing both the system cost and the system weight. Because it is assumed that all the gear pairs are active components in each stage, the gearbox is analogous to a series–parallel system with *k-out-of-n G* subsystems.

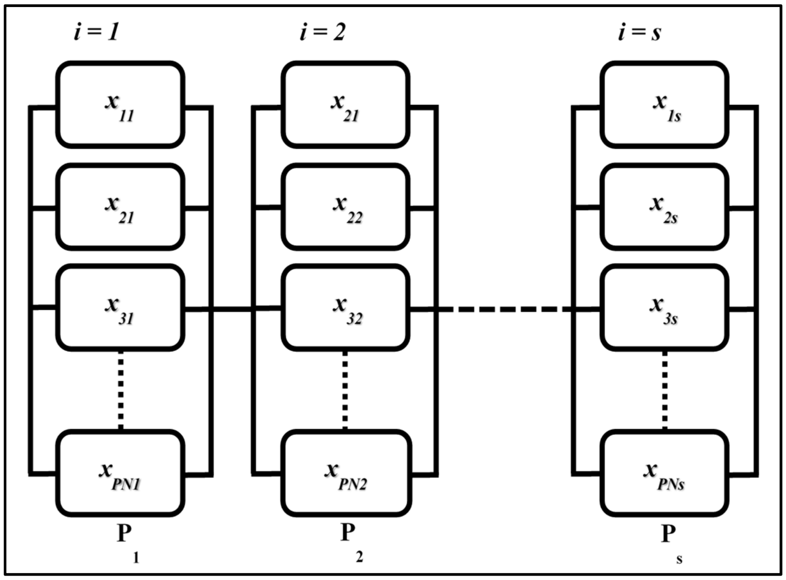

Figure 4. 2 Series–parallel system.

4.6 Gearbox Case Study

Table 4.2 summarizes the input data of component reliability, cost, and weight characteristics for gear pairs in each stage for reliability optimization of the series–parallel systems considered in this problem. The study is based on work conducted previously by (Zhao *et al*., 2007); however, they considered only one objective. Our system consists of 4 subsystems, and each subsystem has a different design component type with similar or dissimilar characteristics, such as reliability, cost, weight, material, dimension, and transmission ratio. Here, we set $P_i = 2$ and $PN = 5$ in the gearbox for all stages. Each of the subsystems is represented by *PN* positions, with each component listed according to its reliability index. The objective is to maximize the system reliability with *k-out-of-n* subsystems connected in the series–parallel system under the given constraints. Table 4.3 lists the values of *Cmax* and *Wmax*. The equivalent scheme of this system is shown in Figure 4.4.

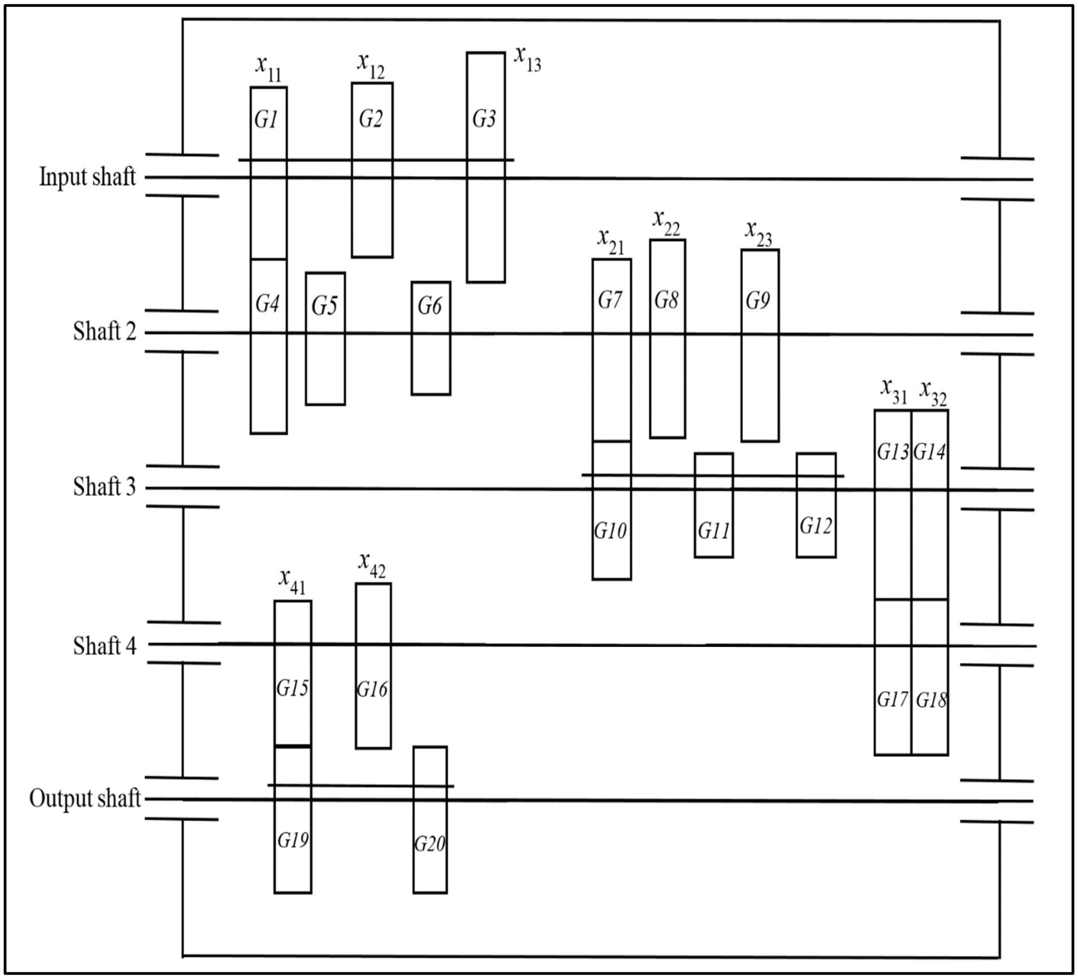

Figure 4. 3 Modeling of gear train system of series–parallel system.

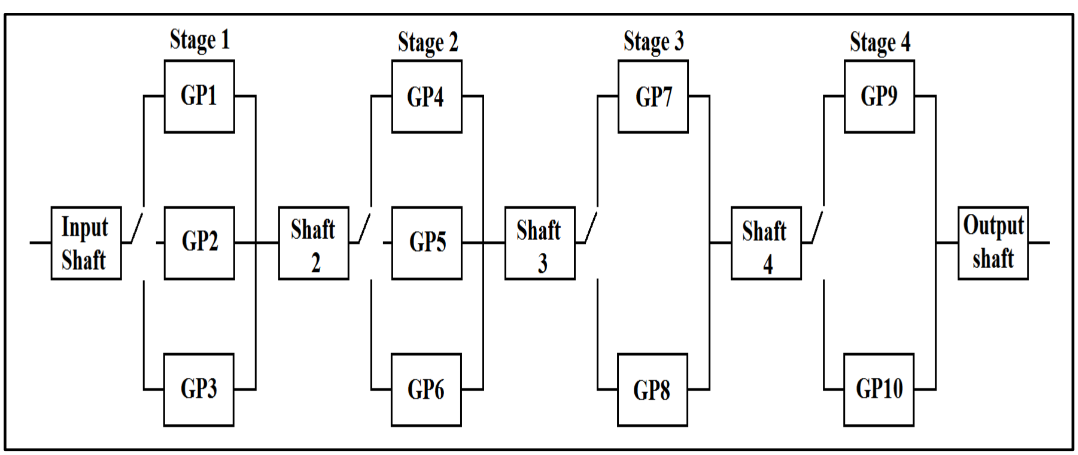

Figure 4. 4 Equivalent scheme for gear train system.

Table 4. 2 Input data for RAP (Zhao *et al.*, 2007).

Gear pair	Stage											
	Stage no 1			Stage no 2			Stage no 3			Stage no 4		
	r1	c1	w1	r2	c2	w2	r3	c3	w3	r4	c4	w4
1	0.855	3	11	0.743	5	9	0.828	9	15	0.74	6	10
2	0.706	5	12	0.882	6	11	0.842	7	14	0.922	5	10
3	0.931	5	9	0.874	2	14	0.779	7	11	0.855	11	15
4	0.737	7	11	0.783	7	11	0.911	7	12	0.864	9	13
5	0.805	6	14	0.9114	5	7	0.846	3	11	0.816	9	12

In Figure 4.4, let Gl, G2, G3, G4,…, G20 represent the number of teeth of each gear. For each stage, the following equations are applicable:

Gl + G4 = G2 + G5 = G3 + G6 (for stage 1 between input shaft 1 and shaft 2).

G7 + G10 = G8 + G11 = G9 + G12 (for stage 2 between shaft 2 and shaft 3).

Gl3 + G17 = G14 + G18 (for stage 3 between shaft 3 and shaft 4).

Gl5 + G19 = G16 + G20 (for stage 4 between shaft 4 and output shaft).

GP1: Gear pair [G1-G4], GP2: Gear pair [G2-G5], GP3: Gear pair [G3-G6], GP4: Gear pair [G7-G10], GP5: Gear pair [G8-G11], GP6: Gear pair [G9-G12], GP7: Gear pair [G13-G17], GP8: Gear pair [G14-G18], GP9: Gear pair [G15-G19], GP10: Gear pair [G16-G20].

Table 4. 3 System constraint values used.

Maximum constraint limit of cost and weight					
No.	C_{max}	W_{max}	No.	C_{max}	W_{max}
1	40	115	10	65	150
2	55	125	11	70	120
3	65	130	12	70	130
4	60	120	13	70	140
5	60	130	14	70	150

Table 4. 3 (Continued)

6	60	140	15	75	120
7	60	150	16	75	130
8	65	120	17	75	140
9	65	140	18	75	150

4.7 Results and Discussion

In this study, we perform multi-objective optimization of a combinatorial redundancy allocation problem for a series–parallel system to solve the formulated reliability optimization multi-objective genetic algorithm (ROMO GA). The reliability optimization design using a multi-objective genetic algorithm for the redundancy allocation problem is presented to determine optimal solutions, where k (k-out-of-n) influences the cost function in series–parallel systems with multiple k–out-of-n subsystems. The objectives are to maximize system reliability and minimize system cost and system weight subject to cost and weight constraints. The constrained k values are considered for all subsystems; some subsystems may require more than one component to function, and the component types are also considered for each subsystem. By using a multi-objective genetic algorithm for solving optimization problems, we can obtain a number of optimal solutions constituting the Pareto-optimal set, and out of these solutions, we can evaluate the best one using an appropriate decision-making technique. The multi-objective optimization methodology is adopted to solve the RAP. Figure 4.5 shows the set of nondominated solutions for the last iteration of the optimization process, where $C_{max} = 40$ and $W_{max} = 115$. Each point in this figure represents an individual solution that has an optimal value of one objective function, and it cannot be improved further without deteriorating at least one of the other objectives. The fuzzy function is employed to define the solution that guarantees an optimal trade-off between the three objectives, and the result is shown in Figure 4.5. The employment of the fuzzy function guarantees consistency and optimality of the selected solution. Figure 4.6 shows the convergence between reliability, cost, and weight.

The optimal trade-off solution shown in Figure 4.7 is [1, 6, 6, 1, 1, 6, 3, 5, 5, 6, 6, 6, 5, 5, 5, 6, 2, 2, 6, 6], and the number of components of each stage of the series–parallel system varies from 2 to 5. Therefore, from the 20 positions, the results are illustrated as follows:

In the first subsystem, there are 3 components of type 1.

In the second subsystem, there are 2 components of type 5 and 1 component of type 3.

In the third subsystem, there are 3 components of type 5.

In the fourth subsystem, there are two components of type 2.

It can be seen that the proposed algorithm is able to obtain a set of uniformly distributed solutions along the Pareto front, as shown in Figure 4.8.

Thus, a new hybrid metaheuristic genetic algorithm and fuzzy function have been successfully demonstrated in this study. Table 4.4 lists the optimal trade-off solutions obtained when different values of the optimization constraints are chosen. From this table, it can be seen that our approach is able to find system configurations with lower cost and weight without significantly degrading the overall reliability.

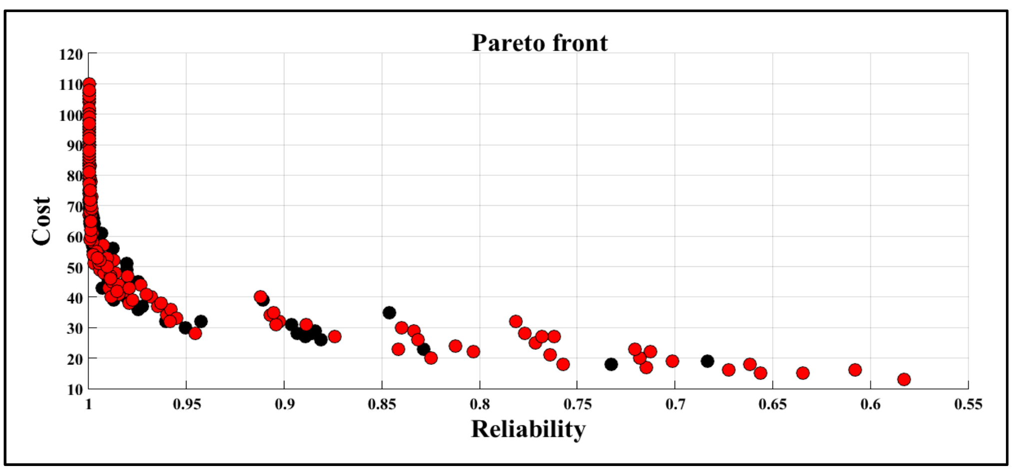

Figure 4. 5 Overall best Pareto front obtained by multi-objective optimization and fuzzy function: cost vs. reliability.

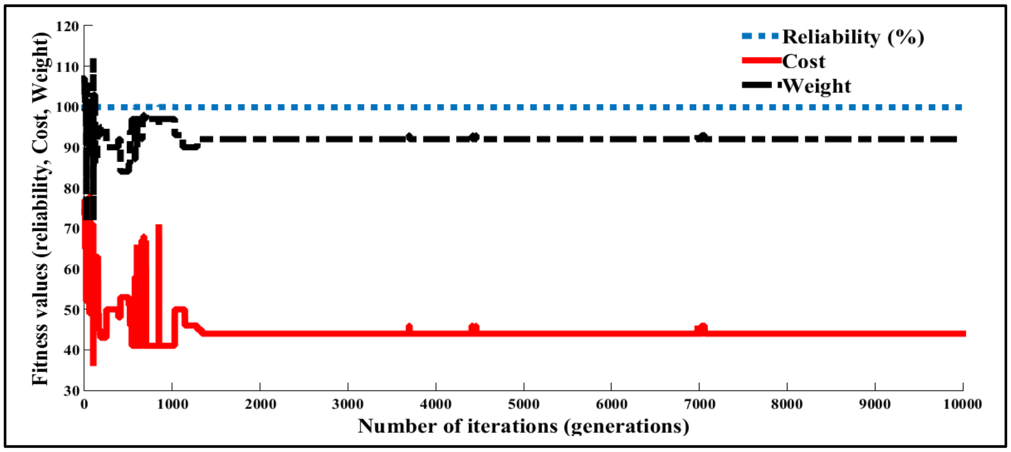

Figure 4. 6 Convergence of reliability, cost, and weight.

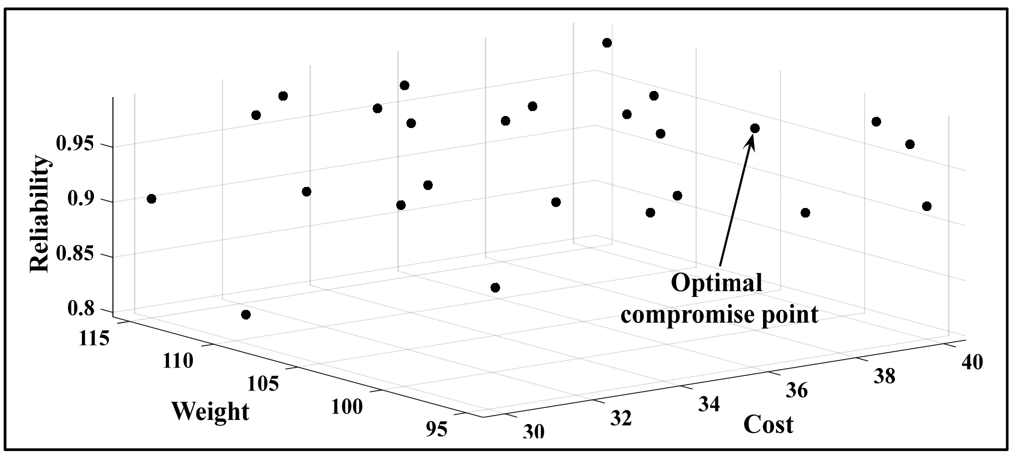

Figure 4. 7 Optimal trade-off point for reliability vs. weight vs. cost in 3D space.

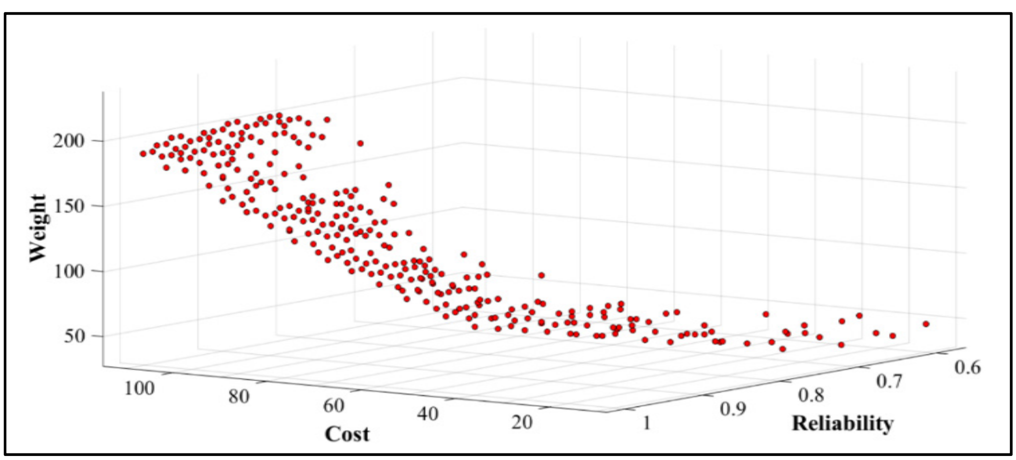

Figure 4. 8 Nondominated solutions obtained from the proposed algorithm for weight vs. cost vs. reliability in 3D space.

Table 4. 4 Optimization results for different cost and weight constraints.

No.	C_{max}	W_{max}	Results obtained via multi-objective optimization of multi-state reliability system using HGA and fuzzy function																						
			Best configuration of 10 simulations																				Reliability	Cost	Weight
1	40	115	1	6	6	1	1	6	3	5	5	6	6	6	5	5	5	6	2	2	6	6	0.9863	40	114
2	55	125	3	1	3	6	6	5	6	5	6	5	5	6	5	5	5	2	6	6	2	2	0.9976	55	124
3	65	130	6	3	6	3	3	5	5	6	5	5	5	5	6	5	5	2	2	2	6	6	0.9986	62	129
4	60	120	3	3	6	3	6	5	6	3	6	5	6	6	4	5	4	6	2	6	2	2	0.9970	59	120
5	60	130	3	6	3	6	3	6	5	5	5	6	5	5	6	5	5	6	2	2	6	2	0.9979	57	122
6	60	140	6	3	6	3	3	5	6	3	5	5	5	6	5	5	5	2	6	2	2	6	0.9985	59	136
7	60	150	1	3	3	6	1	5	5	5	3	6	5	5	5	5	6	2	6	2	6	2	0.9988	60	149
8	65	120	1	6	3	6	3	5	6	5	5	6	6	4	4	6	4	2	6	2	2	6	0.9974	64	116
9	65	140	3	3	6	6	3	5	5	5	6	5	5	5	5	5	5	6	6	2	2	2	0.9990	65	140
10	65	150	3	3	6	6	3	5	5	5	5	6	5	5	5	5	5	6	6	2	2	2	0.9990	65	140
11	70	120	3	6	3	3	6	5	5	6	5	6	4	6	4	6	4	6	2	6	2	2	0.9978	66	114
12	70	130	6	6	3	3	3	5	5	5	6	5	5	5	5	6	4	6	2	2	2	6	0.9988	66	130
13	70	140	3	3	3	6	6	5	5	6	5	5	5	5	5	5	5	6	2	2	2	6	0.9990	65	140
14	70	150	3	6	3	3	6	5	5	5	6	5	5	5	5	5	5	6	2	2	2	2	0.9995	70	150
15	75	120	6	3	3	6	3	5	6	5	5	5	5	6	6	4	4	2	6	6	2	2	0.9979	67	120
16	75	130	3	6	3	6	3	5	6	5	5	5	5	5	6	5	4	2	2	2	6	6	0.9988	66	130
17	75	140	6	3	3	3	3	6	5	5	5	5	4	4	5	6	5	6	6	2	2	2	0.9993	75	140
18	75	150	3	3	3	6	6	5	5	5	5	6	5	5	5	5	5	6	2	2	2	2	0.9995	70	150

4.8 Conclusion

In this study, we proposed multi-objective optimization of a multi-state reliability system for an RAP involving a series–parallel system, based on a genetic algorithm and fuzzy function. Unlike other methodologies, our methodology not only optimizes the cost, weight, and reliability of the system simultaneously but also objectively defines the system configuration that achieves the optimal trade-off between the design objectives. The results showed that our methodology can find better solutions in terms of cost and weight without significantly degrading the overall reliability. The computational results confirmed the robustness of the proposed algorithm and highlighted its potential for future application.

In the future, the proposed technique may be adopted for solving real-life decision-making problems in the form of interval-valued constrained optimization problems. In addition, it can be applied to various areas of engineering, management, and manufacturing.

CHAPTER 5

MULTIPLE-OBJECTIVE OPTIMIZATION AND DESIGN OF SERIES–PARALLEL SYSTEMS USING NOVEL HYBRID GENETIC ALGORITHM META-HEURISTIC APPROACH

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5.1 Abstract

In this study, we develop a new meta-heuristic-based approach to solve a multi-objective optimization problem, namely the reliability-redundancy allocation problem (RRAP). Further, we develop a new simulation process to generate practical tools for designing reliable series–parallel systems. Because the RRAP is an NP-hard problem, conventional techniques or heuristics cannot be used to find the optimal solution. We propose a genetic algorithm (GA)-based hybrid meta-heuristic algorithm, namely the hybrid genetic algorithm (HGA), to find the optimal solution. A simulation process based on the HGA is developed to obtain different alternative solutions that are required to generate application tools for optimal design of reliable series–parallel systems. Finally, a practical case study regarding security control of a gas turbine in the overspeed state is presented to validate the proposed algorithm.

Keywords: Multi-objective optimization, reliability-redundancy allocation, overspeed, gas turbine, hybrid genetic algorithm

5.2 Introduction

Optimization of series–parallel systems is an important aspect of equipment design strategies. The optimized system characteristics, such as reliability, cost, weight, and volume, contribute toward designing the best machine. This approach is challenging because the reliability needs to be maximized whereas the other objective functions need to be minimized. In practice, system reliability optimization is critical, and over the last two decades, considerable effort has been devoted toward the development of reliability criteria for quantifying the nature of generation, transmission, and circulation in composite system frameworks. To improve component reliability and implement redundancy while achieving a trade-off between system performance and resources, reliability design that aims to establish an optimal system-level configuration has long been considered an important advantage in reliability engineering. At present, system reliability is of considerable research significance, as engineering fields involve continual advancements in fixed systems and applications with increasing levels of complexity. Thus, it is imperative for production systems to perform satisfactorily during their expected lifespan. However, failure is an inevitable phenomenon associated with technological advancement of the equipment used in various industries. The reliability-redundancy allocation problem (RRAP) has been studied to optimize system reliability on the basis of the redundancy allocation problem (RAP) (Kuo and Wan, 2007). The RRAP has attracted considerable attention from the viewpoint of developing heuristic optimization algorithms. This paper focuses on an RRAP with the objective of maximizing system reliability under nonlinear constraints, such as system cost, weight, and volume. The RRAP has been shown to be an NP-hard problem, and various optimization approaches have been proposed to solve it. These methods, which are called meta-heuristic methods, have been widely researched and implemented. They can obtain feasible solutions within limited computing time. The main goal of RRAPs is to select the levels of redundancy and component reliability for maximizing and improving system reliability and performance. RRAPs are useful for designing not only systems that are taken together on a large scale but also systems produced in large-scale industrial operation using off-the-shelf components.

5.3 Literature Review

A reliability-redundancy optimization problem can be formulated using components and levels of redundancy to maximize some objective function, given system-level constraints on reliability, cost, and/or weight. The problem of maximizing system reliability through redundancy and component reliability selection is called the reliability-redundancy allocation problem (RRAP). Reliability optimization has been the subject of several studies by Kuo *et al.* (2007), (2000), (1978). (Forsthoffer, 2005; Kundur, 1994; Hejzlar, 1993; Seebgrets, 1995) conducted studies on overspeed protection, such as analysis of the instability of steam turbines and analysis of the reliability of wind turbines. Dhingra (1992) developed an application of the reliability-redundancy optimization problem with regard to overspeed protection by using a multi-objective approach to maximize system reliability and minimize consumption of resources (cost, weight, and volume). This approach involves a goal programming formulation and a goal achievement method for generating Pareto optimal solutions. Control and overspeed protection for a gas turbine are nearly the same as those for a steam turbine. A gas turbine operates at a higher temperature than a steam turbine; hence, it requires closer control, called control sequencing. Sequencing allows automatic control of the gas turbine. Fetanat *et al.* (2012) proposed an optimal design for control and overspeed protection of gas turbine by means of reliability-redundancy optimization achieved using a new type of harmony search algorithm (HSA) known as the elitism Box-Muller harmony search algorithm (EBMHSA). Dhingra and Rao (1992) used goal programming and goal attainment formulations under fuzziness in a multi-objective reliability apportionment problem subject to several design constraints. Rao proposed three methods for finding the optimal solution of each objective function: a method for determining reliability, a method for minimizing cost, and a method for controlling weight. Rao's approach has been developed to optimize redundant series–parallel systems, all components of which are time-dependent. The proposed model for simulation is the overspeed control system for a gas turbine engine. This model, proposed by Dhingra, is a combination of mechanical and electrical systems. Overspeed control is the first step against excessive speed. In general, the emergency reset of the system is designed independent of the overspeed control. Hence, high-reliability operation of control valves is considered. In the normal working mode, the control valves are opened sequentially (Kundur, 1994). Luus (1975) proposed a new non-linear integer programming method that considers the component reliability to be fixed. However, a more general problem is one where the optimal redundancy in each stage is determined to obtain the maximum system reliability. To solve the RRAP, several global optimization methods as well as heuristic and meta-heuristic methods have been proposed in the literature, including the Lagrangian multiplier method, branch and bound method, and linear programming (Kuo and Hwang, 1978; Dhingra, 1992; Hikita *et al.*, 1992; Gopal *et al*., 1978). These approaches do not guarantee exact optimal solutions but achieve reasonably good solutions for complex problems with relatively short computing time. Heuristic techniques, including genetic algorithms, require derivatives for all non-linear constraint functions, which are not derived easily because of the high computational complexity. Yokota *et al.* (1996) and Hsieh *et al.* (1998) applied genetic algorithms (GA) to mixed-integer reliability optimization problems. Zhao *et al.* (2012) developed a hybrid GA with a flexible allowance technique for solving constrained engineering design optimization problems. (Kanagaraj *et al.*, 2013; Ghodrati and Lofti, 2012) developed a hybrid cuckoo search (CS)/GA algorithm to solve reliability-redundancy optimization problems and global optimization problems, respectively. Gen and Yun (2006) developed a soft computing approach for solving various reliability optimization problems. This method combines rough search techniques and local search techniques to prevent premature convergence of the solution. Zou *et al.* (2011) proposed a global harmony search algorithm for solving bridge and overspeed protection system optimization problems by combining the harmony search algorithm with concepts from particle swarm optimization. Different programming and evolutionary optimization techniques have been adopted to optimize different types of RRAPs, e.g., GA (Hsieh *et al.*, 1998) and a new interpretation and formulation of the RRAP (Abouei *et al.*, 2016) using a new mixed strategy and a modified version of the genetic algorithm (MVGA), which shows distinct advantages compared to traditional approaches. Afonso *et al.* (2013) proposed a modified version of the imperialist competitive algorithm (ICA) and demonstrated its capabilities by comparing its results with the best-known results of different benchmarks. Quy (1998) developed a new method to optimize a multi-objective model in certain mechanical systems by using the fuzzy multi-objective method. His approach is based on the algorithm proposed by Rao and Dhingra (1992), and he applied it to the modeling and analysis of the overspeed control system of a gas turbine engine. All the components of this model are time-dependent. The performance of the algorithm was verified by programmed simulation of the above-mentioned model. In summary, Dhingra (1992), Rao and Dhingra (1992), and Quy (1998) developed effective multi-objective fuzzy optimization techniques for engineering design. In particular, they adopted fuzzy programming, which is a powerful technique for solving optimization problems with fuzzy parameters. However, the use of uncertain information for reliability allocation requires further investigation. Moreover, they treated component risk/cost functions as continuous. Thus, no general method for solving the component reinforcement problem with discontinuous risk/cost functions has been proposed thus far. In addition, the three above-mentioned studies did not adopt any random-search-based global optimization methods. In other words, the entire family of meta-heuristics that can efficiently solve highly nonlinear nonconvex mixed-integer optimization problems has been overlooked. The major drawback of these studies is that none of them has developed a practical tool for designing actual components with distinct physical properties, such as cost, weight, volume, and reliability. In this paper, we present a hybrid GA (HGA) approach based on the redundancy allocation problem to find the number of redundant components that either maximize reliability or minimize cost, weight, and volume under various resource constraints. The computational results of our approach are compared with those of previously proposed algorithms.

5.4 Reliability-Redundancy Allocation Problems (RRAPs)

In this study, a reliability-redundancy allocation problem of minimizing the multi-objective function $[-f_1, f_2, f_3]$ subject to several nonlinear design constraints can be stated as a nonlinear mixed-integer programming model. The multi-objective formulation was obtained by applying cost and weight constraints to an objective function. In other words, the general problem of reliability and redundancy is assigned to each of the subsystems such that the system reliability, cost, and weight are optimized. The problem is overspeed protection of a gas turbine system with a time-related cost function, and the multi-objective RRAP model is as follows:

$$Max.Rs\ (r,n)\ \&\ Min.Cs\ (r,n)\ \&\ Min.Ws\ (r,n)$$

$$Subject\ to: \quad g_j\ (r,n) \leq\ a_j\ , j\ = 1,\ldots,m$$

$$1\ \leq\ n_i\ \leq\ 10\ ,\ \ i = 1,2,\ldots,4\ ,\quad n_i\ \in Z^+$$

$$0.5\ \leq\ r_i\ \leq\ 1-10^{-6}, \qquad r_i\ \in r^+$$

Many designers have attempted to improve the reliability of manufacturing systems or product components for greater competitiveness in the market. Typical approaches for achieving higher system reliability include increasing the reliability of system components and using redundant components in various subsystems of the system (Kuo *et al.*, 2000; Hsieh *et al.*, 1998).

5.5 Mathematical Formulation of the Problem

The mathematical model of the optimization problem is given by the equations below. The system reliability, cost, weight, and product of weight and volume are constrained by the design. The resulting multi-objective reliability apportionment problem is as follows: find *n* and *r* that minimize the multi-objective function [$-f_1, f_2, f_3$] subject to $g_j(r, n) \le a_j$, $j = 1,..., m$. Figure 5.1 shows a typical example of a series–parallel system configuration with *k-out-of-n* subsystem reliabilities

where

$$f_1(r,n) \text{ is the system reliability} \tag{5.1}$$

$$f_2(r,n) \text{ is the total system cost} \tag{5.2}$$

$$f_3(r,n) \text{ is the total system weight} \tag{5.3}$$

Subject to

$$g_1(r,n) = \sum_{i=1}^{4} v_i n_i^2 \le 250 \, (Volume) \tag{5.4}$$

$$g_2(r,n) = \sum_{i=1}^{4} w_i n_i \exp\left(\frac{n_i}{4}\right) \le 500 \, (Weight) \tag{5.5}$$

$$g_3(r,n) = \prod_{i=1}^{4} [1-(1-r_i)^{n_i}] \ge 0.95 \, (\text{Reliability}) \tag{5.6}$$

$$g_4(r,n) = \sum_{i=1}^{4} \alpha_i \left(\frac{-t}{\ln r_i}\right)^{\beta_i} [n_i + \exp(\frac{n_i}{4})] \le 400 \, (\text{Cost}) \tag{5.7}$$

$$1 \le n_i \le 10, \; i = 1,2,...,4 [g_5(r,n)], \mathrm{n}_i \in Z^+ \tag{5.8}$$

$$0.5 \le r_i \le 0.999999, [g_6(r,n)], \mathrm{r}_i \in r^+ \tag{5.9}$$

where

$$f_1 = \max.\ R_s(r,n) = \prod_{i=1}^{4} [1-(1-r_i)^{n_i}] \tag{5.10}$$

$$f_2 = \min.\ C_s(r,n) = \sum_{i=1}^{4} \alpha_i \left(\frac{-t}{\ln r_i}\right)^{\beta_i} [n_i + \exp(\frac{n_i}{4})] \tag{5.11}$$

$$f_3 = \min.\ W_s(r,n) = \sum_{i=1}^{4} w_i\, n_i \exp(\frac{n_i}{4}) \tag{5.12}$$

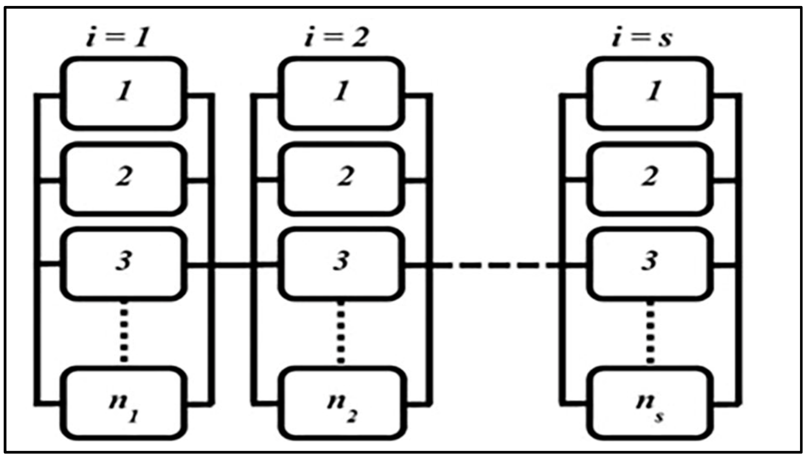


Figure 5. 1 General series–parallel redundancy system

Notation

r_i	Reliability of component in subsystem *i*
n_i	Number of redundant components in subsystem *i*
r, n	Vectors of r_i and n_i
Rs	System reliability
N	Number of subsystems in the system
f_1	Objective function for system reliability
f_2	Objective function for system cost
f_3	Objective function for system weight
$g_i(.)$	Constraint function #*j*
a_j	Constraint limit #*j*
m	Number of constraints

5.6 Methodology Framework

This study aims to propose a new algorithm that can be applied to optimization problems such that system reliability is maximized while system cost and system weight are minimized. The reliability, cost, and weight are subject to four nonlinear resource constraints, and the optimal levels of the reliability of component, r_i, and the number of redundant components, n_i, are to be determined at each stage i of the mechanical system.

Before introducing the RRAP, we present the following assumptions and notations that have been used throughout the entire paper. The hybrid function allows the optimization algorithm to identify the solution of the redundancy problem that achieves the optimal trade-off between the optimization objectives from several optimal solutions. We performed 10 simulations for every experiment and used the best result among the 10 reliability values obtained. The best configuration of each point corresponding to the largest reliability value is given with the corresponding cost, weight, and weight values.

Assumptions

- The supply of components is unlimited.
- The weight and volume of the components are known and deterministic.
- All the redundant components of individual subsystems have different values, and every branch of the system has a different number of components.
- The failure rate of the components in each subsystem is constant.
- Failed components do not damage the system and are not repaired.
- All redundancies are active: the hazard function is the same regardless of whether it is in use.
- Failures of individual components are independent of one another but dependent on the number of working elements.

As mentioned above, few studies have reported the use of HGA for reliability allocation optimization with time-dependent reliability. We need to check whether our approach of using only HGA can guarantee the location of the optimal solution and whether the final solution obtained by the proposed HGA is superior to that obtained by existing methods.

Figure 5.2 shows the flowchart of the proposed algorithm. The HGA procedures that implement our methodology are illustrated. The proposed algorithm involves the following steps:

1. Define the functions of the design problem (*Rs, Ws, Vs*, and *Cs*).
2. Define the nonlinear constraints.
3. Define the lower bound and upper bound for r_i and n_i.
4. Chose the optimization algorithm (fmincon, fminmax, GA, and HGA).
5. Solve the optimization problem.
6. Calculate the optimal values (*Rs, Cs*, and *Ws*).

The hybrid GA is a combination of fmincon and GA. GA is used to find the global optima for optimization problems. "Fmincon" uses gradient information to facilitate rapid convergence. "HybridFcn" allows the GA to find the valley containing the global minimum. Then, fmincon is used to rapidly obtain the minimum of this valley. A hybrid function is an optimization function that runs after the GA terminates in order to improve the value of the fitness function. The hybrid function uses the final point from the GA as its initial point.

This study consists of two parts. In the first part, we identify the approach for solving the problem by using MATLAB code and compare the results with previous results (Quy, 1998) to determine the number of redundant components in stage *i* and the reliability for each component. The second part involves a novel contribution: we develop a model for the entire system with the desired level of reliability. Specifically, we develop a simulation procedure and implement it with different numbers of components for each stage with different values of each component. We use this novel approach to determine the converged value of system reliability until we obtain the values of n_i and r_i corresponding to value of the maximum reliability. Toward this end, we need to perform optimization. For the general structure of the network, we fixed the system reliability to a certain level, i.e., greater than or equal to 0.95.

We implemented a single-objective function with nonlinear constraints and tested it using two methods (n_i is an integer in our problem). The results are summarized in Tables 5.1 and 5.2. In addition, we implemented a multi-objective function. The initial results were obtained for four functions, and r_i and n_i were randomly set to evaluate each function.

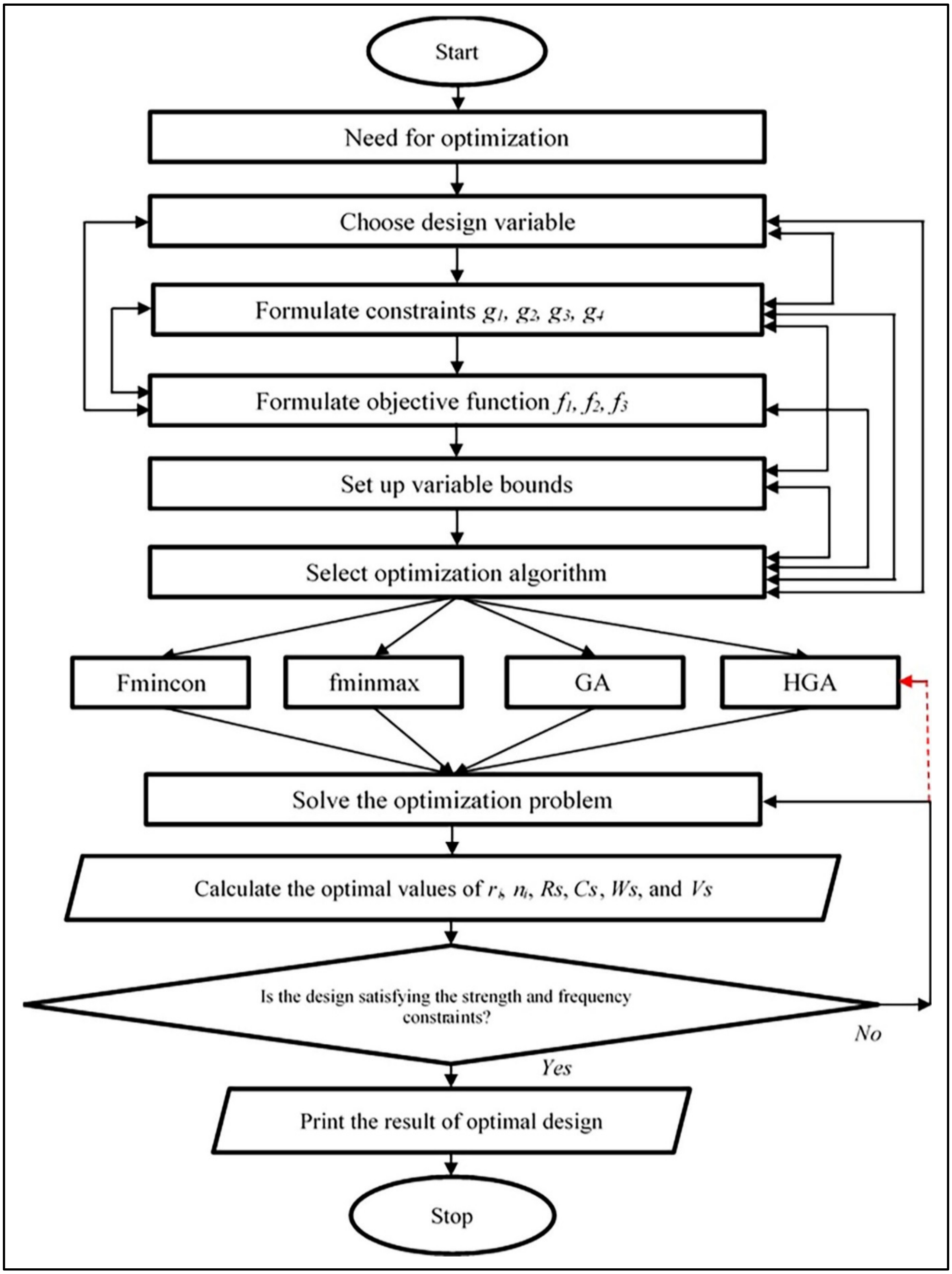


Figure 5. 2 Flowchart of proposed simulation procedure.

Table 5. 1 Simulation results for single-objective function using fmincon optimization method

Objective	Stage	Reliability	Component	Simulation Result
Maximize System Reliability	1 2 3 4	0.8998 0.8680 0.9439 0.8728	5 6 4 6	Rs= 0.9999 Cs= 419.2534 Ws= 541.2671 Vs= 217
Minimize System Cost	1 2 3 4	0.5846 0.5184 0.6988 0.5252	5 6 4 5	Rs= 0.9439 Cs= 36.0616 Ws= 475.1981 Vs= 195
Minimize System Weight	1 2 3 4	0.9534 0.9313 0.9770 0.9351	1 2 1 2	Rs= 0.9232 Cs= 422.7688 Ws= 60.8431 Vs= 20
Multi-objective Functions	1 2 3 4	0.8493 0.7980 0.9147 0.8060	3 3 2 3	Rs= 0.9740 Cs= 109.3931 Ws= 147.0485 Vs= 57

Table 5. 2 Simulation results for single-objective function using fminimax optimization method

Objective	Stage	Reliability	Component	Simulation Result
Maximize System Reliability	1 2 3 4	0.9001 0.8685 0.9431 0.8732	5 6 4 6	Rs= 0.9999 Cs= 420.2802 Ws= 541.2671 Vs= 217
Minimize System Cost	1 2 3 4	0.5846 0.5184 0.6988 0.5252	5 6 4 5	Rs= 0.9439 Cs= 36.0616 Ws= 475.1981 Vs= 195
Minimize System Weight	1 2 3 4	0.9534 0.9313 0.9770 0.9351	1 2 1 2	Rs= 0.9232 Cs= 422.7688 Ws= 60.8431 Vs= 20
Multi-objective Functions	1 2 3 4	0.8493 0.7980 0.9148 0.8059	3 3 2 3	Rs= 0.9740 Cs= 109.3850 Ws= 147.0485 Vs= 57

First step: We implemented a multi-objective function, and we defined the general objective function as follows:

$$f= 10\, f_1+ f_2/400+f_3/500;\ \text{(new definition)}$$

The above-mentioned has three parts: reliability, cost, and weight. This equation maximizes reliability but minimizes cost and weight. It is a normalized form of the objective function because we consider the upper bound of each objective. We penalized the reliability (with a value of 10) for greater emphasis. In addition, we set the upper bounds for *Cs* and *Ws* as 400 and 500, respectively. Therefore, if we divide by these values and take the sum, we will always get a number less than one. Thus, we normalized the functions (f_1, f_2, and f_3).

Second step: We used fmincon and fminmax to solve the objective function.

Third step: We used the GA toolbox and applied this algorithm to our single- and multi-objective function problems. The results are summarized in Table 5.3.

Fourth step: We applied the GA to a new type of multi-objective function and evaluated the results.

Fifth step: We applied the global multi-objective GA to the problem and obtained 70 sets of Pareto optimal solutions.

Last step: We applied HGA optimization to single- and multiple-objective functions on the basis of our first approach. The results are summarized in Table 5.4.

Our multi-objective function aims to minimize cost and weight in the first approach. The results of our optimization give us n_i and r_i for each stage as well as for the entire system, as shown in the final result table. In this study, we performed optimization using GA and HGA. We used the same approach as that for obtaining a constrained minimum of a scalar function of several variables starting at an initial estimate. This is generally referred to as constrained nonlinear optimization or nonlinear programming (fmincon). We used different optimization approaches and finally used HGA. Specifically, we employed GA and fmincon to implement HGA using the first approach. by varying r_i and n_i to achieve the desired system reliability with the objective function. Further, we fixed the system reliability *Rs* to obtain a system with minimum cost and weight in order to determine the structure of our new design in the second approach, which minimizes the worst-case value of a set of multivariable functions, starting at an initial estimate. The values may be subject to constraints. This is generally referred to as the *minimax* problem (fminmax).

We also varied the level of system reliability to show how we can select the desired system reliability; accordingly, we can change the structure of the entire system. In this step, we used GA and MATLAB toolbox. Here, we do not maximize the system reliability *Rs* but we want *Rs* = *A*, and we want to determine the system structure for achieving the minimum cost and weight. We assumed that n_i is a continuous value. In this case, the first method of optimization using fmincon is summarized in Table 5.5. In addition, we can see the result of the second approach of optimization, i.e., fminmax. The results of our contribution are summarized in Tables 5.6, 5.7, and 5.8, which show the different values obtained after we fixed the system reliability.

We tested various algorithms to identify the best ones, which were found to be GA or HGA.

Table 5. 3 Simulation results for single-objective function using GA optimization method

Objective	Stage	Reliability	Component	Simulation Result
Maximize System Reliability	1 2 3 4	0.8902 0.8603 0.9500 0.8806	5 6 4 5	Rs= 0.9999 Cs= 389.3556 Ws= 475.1981 Vs= 195
Minimize System Cost	1 2 3 4	0.6106 0.5550 0.6509 0.5465	5 5 5 5	Rs= 0.9501 Cs= 37.4312 Ws= 471.1963 Vs= 200
Minimize System Weight	1 2 3 4	0.8977 0.9537 0.9732 0.8987	2 2 1 2	Rs= 0.9511 Cs= 414.0766 Ws= 72.9236 Vs= 23
Multi-objective Functions	1 2 3 4	0.8504 0.7956 0.9167 0.8049	3 3 2 3	Rs= 0.9740 Cs= 109.1462 Ws= 147.0485 Vs= 57

Table 5. 4 Simulation results for single-objective function using hybrid optimization method

Objective	Stage	Reliability	Component	Simulation Result
Maximize System Reliability	1 2 3 4	0.8971 0.8659 0.9358 0.8769	5 6 4 5	Rs= 0.9999 Cs= 381.5582 Ws= 475.1981 Vs= 195
Minimize System Cost	1 2 3 4	0.7997 0.7896 0.7154 0.8393	4 4 5 4	Rs= 0.9939 Cs= 133.4582 Ws= 346.2031 Vs= 155
Minimize System Weight	1 2 3 4	0.9668 0.8715 0.9572 0.9382	2 2 2 2	Rs= 0.9769 Cs= 440.5520 Ws= 89.0309 Vs= 32
Multi-objective Functions	1 2 3 4	0.8536 0.7977 0.9189 0.8133	3 3 2 3	Rs= 0.9757 Cs= 114.0175 Ws= 147.0485 Vs= 57

Table 5. 5 Simulation results using fmincon optimization method when system reliability *Rs=A*

Objective	Stage	Reliability	Component	Simulation Result
Minimize System Cost	1 2 3 4	0.5846 0.5184 0.6988 0.5252	5 6 4 5	Rs= 0.9500 Cs= 36.0616 Ws= 475.1981 Vs= 195
Minimize System Weight	1 2 3 4	0.9534 0.9313 0.9770 0.9351	1 2 1 2	Rs= 0.9500 Cs= 422.7688 Ws= 60.8431 Vs= 20
Multi-objective Functions (Cost+ Weight)	1 2 3 4	0.8326 0.7755 0.9053 0.7840	3 3 2 3	Rs= 0.9500 Cs= 91.7003 Ws= 147.0485 Vs= 57

Table 5. 6 Simulation results using fminimax optimization method when system reliability *Rs=A*

Objective	Stage	Reliability	Component	Simulation Result
Minimize System Cost	1 2 3 4	0.5846 0.5184 0.6988 0.5252	5 6 4 5	Rs= 0.9500 Cs= 36.0616 Ws= 475.1981 Vs= 195
Minimize System Weight	1 2 3 4	0.9534 0.9313 0.9770 0.9351	1 2 1 2	Rs= 0.9500 Cs= 422.7688 Ws= 60.8431 Vs= 20
Multi-objective Functions (Cost + weight)	1 2 3 4	0.8325 0.7755 0.9054 0.7841	3 3 2 3	Rs= 0.9500 Cs= 91.7228 Ws= 147.0485 Vs= 57

Table 5. 7 Simulation results using GA optimization method when system reliability *Rs=A*

Objective	Stage	Reliability	Component	Simulation Result
Minimize System Cost	1 2 3 4	0.6317 0.5327 0.6800 0.5980	5 5 5 4	Rs= 0.9500 Cs= 37.5962 Ws= 425.1462 Vs= 182
Minimize System Weight	1 2 3 4	0.8925 0.9369 0.9720 0.9440	2 2 1 2	Rs= 0.9500 Cs= 426.6653 Ws= 72.9236 Vs= 23
Multi-objective Functions (Cost+ Weight)	1 2 3 4	0.8179 0.7812 0.8894 0.7656	3 3 2 3	Rs= 0.9500 Cs= 83.8740 Ws= 47.0485 Vs= 57

Table 5. 8 Simulation results using hybrid optimization method when system reliability *Rs=A*

Objective	Stage	Reliability	Component	Simulation Result
Minimize System Cost	1 2 3 4	0.5846 0.5184 0.6988 0.5252	5 6 4 5	Rs= 0.9500 Cs= 36.0616 Ws= 475.1981 Vs= 195
Minimize System Weight	1 2 3 4	0.9534 0.9313 0.9770 0.9351	1 2 1 2	Rs= 0.9500 Cs= 422.7688 Ws= 60.8431 Vs= 20
Multi-objective Functions (Cost + Weight)	1 2 3 4	0.8326 0.7755 0.9053 0.7840	3 3 2 3	Rs= 0.9500 Cs= 91.7003 Ws=147.0485 Vs= 57

5.7 Hybrid Genetic Algorithm (HGA) for Multi-Objective Optimization

Most previous studies have focused on several methods for solving redundancy optimization problems. In this study, we develop an approach by considering some aspects that have not been considered previously. The mathematical model represents the multi-objective HGA with a constraint-handling strategy for solving the proposed model. HGA is a meta-heuristic method that is used to solve optimization problems efficiently. In this method, first, an initial set of random potential solutions including a number of particles is created. Each particle represents a solution of the problem and has a position and velocity that change in each iteration so that better solutions can be obtained.

5.8 A Case Study: Overspeed Protection System for a Gas Turbine

To evaluate the performance of the HGA in reliability optimization problems, overspeed detection continuously provided by the electrical and mechanical systems is considered in a case study. The benchmark considered is an overspeed protection system for a gas turbine. When overspeed occurs, it is necessary to cut off the fuel supply using control valves, i.e., the four valve controllers (V1–V4) must close. The control system is modeled as a four-stage series–parallel system, as shown in Figure 5.3.

Each stage represents a controller that can be considered as a parallel system. All the components of the system have the same failure rate. The equivalent circuit of the overspeed control system is shown in Figure 5.4.

Here, v_i is the volume of each component in subsystem i, V is the upper limit on the sum of the subsystem products of volume and weight, C is the upper limit on the system cost, and W is the upper limit on the system weight. The parameters α_i and β_i are constants representing the physical characteristics of each component in stage i. T is the operating time during which a component must not fail. The input parameters of the overspeed protection system for a gas turbine are listed in Table 5.9.

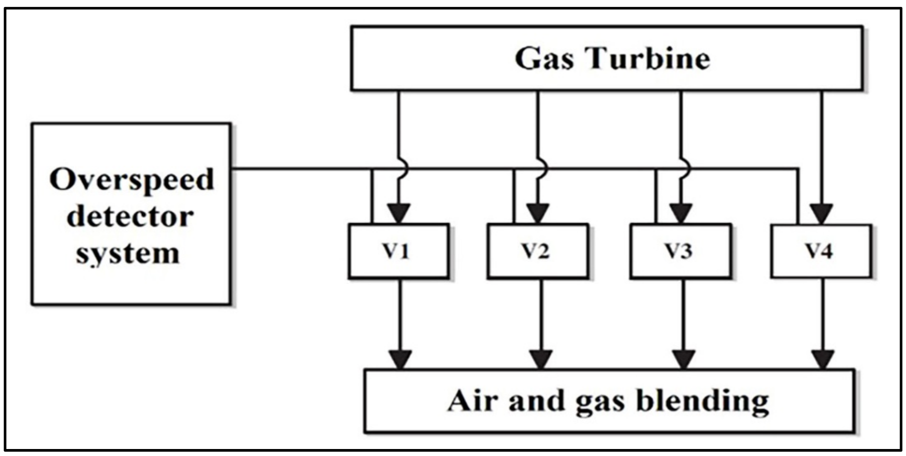


Figure 5. 3 Block diagram of overspeed protection system for gas turbine with four valves

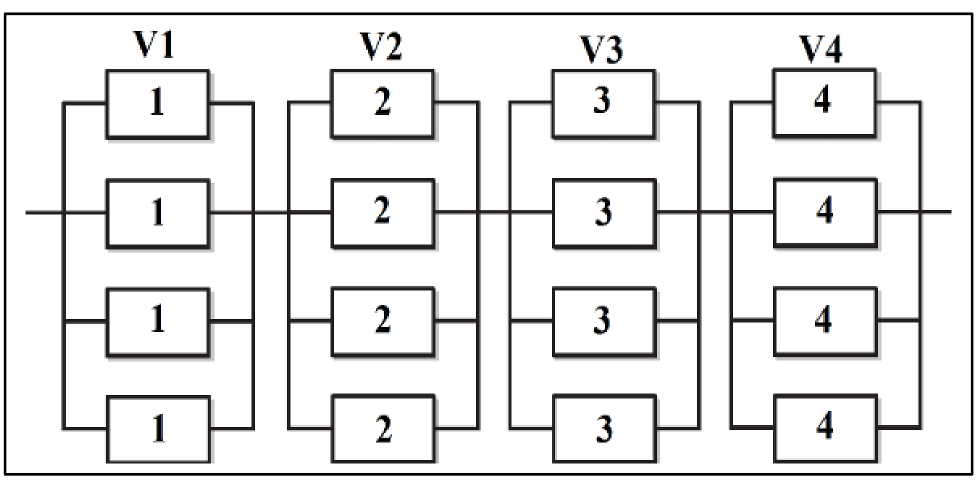


Figure 5. 4 Equivalent circuit: four-stage series–parallel system

Table 5. 9 Design values of different parameters used in overspeed protection system of gas turbine

	Number of stages		**4**	
	Lower limit on R_s		0.95	
	Upper limit on cost		400	
	Upper limit on weight		500	
	Upper limit on volume		250	
	Operating time		1000 hours	
Stage	$10^5\alpha_i$	β_i	v_i	w_i
1	1.0	1.5	1	6
2	2.3	1.5	2	6
3	0.3	1.5	3	8
4	2.3	1.5	2	7

5.9 Computational Results and Discussion

We compared our solutions with those obtained in a previous study (Quy, 1998). From Table 5.10, it is clear that our HGA approach obtains better solutions for the series–parallel system compared to the other approaches presented in the literature. The best fitness and mean fitness of the system cost, system weight, and multi-objective functions are shown in Figures 5.5, 5.6, and 5.7, respectively.

The mathematical model used for calculating the objective function is employed to define the solution that guarantees an optimal trade-off between the two objectives, and the result is shown in Figure 5.8. Figure 5.9 shows the average distance between individuals.

These figures show the number of generations in GA. In addition, the values of each objective function in each iteration are shown. The toolbox is employed to generate these figures, which can be used to determine the most suitable reliability level that minimizes the total cost, weight, and volume subject to various constraints.

The runs of the HGA were continuously monitored throughout the generations (Figures 5.5, 5.6, and 5.7). These plots show the best and mean fitness values of the fitness functions after 100, 100, and 300 generations, respectively. For Figure 5.5, the best fitness is in the range of

38.787 and the mean fitness is in the range of 38.795. For Figure 5.6, the best fitness is in the range of 55.9112 and the mean fitness is in the range of 55.9136. For Figure 5.7, the best fitness is in the range of 0.462836 and the mean fitness is in the range of 0.462929. From these plots, it can easily be observed that the fitness value converges toward the optimal value from generation to generation.

Table 5. 10 Comparison of simulation results of optimal solutions of single- and multi-objective function for series–parallel system using HGA with other results presented in the literature

		Results given in Ref. (Quy, 1998)			***Results given by hybrid genetic algorithm***		
Objective	**Stage**	**Reliability**	**Comp-onent**	**Simulation result**	**Reliability**	**Comp-onent**	**Simulation result**
Maximize System Reliability	1	0.866288	6.0	Rs = 0.999881	0.8971	5	Rs= 0.9999
	2	0.850029	6.0	Cs = 381.12183	0.8659	6	Cs= 381.5582
	3	0.918417	4.0	Ws = 485.77850	0.9358	4	Ws= 475.1981
	4	0.913049	4.0	Vs= 188.0	0.8769	5	Vs= 195
Minimize System Cost	1	0.559777	6.0	Rs = 0.971340	0.7997	4	Rs= 0.9939
	2	0.599392	6.0	Cs = 54.472889	0.7896	4	Cs= 133.4582
	3	0.685273	4.0	Ws = 485.778504	0.7154	5	Ws= 346.2031
	4	0.703375	4.0	Vs = 188.0	0.8393	4	Vs= 155
Minimize System Weight	1	0.864883	3.0	Rs = 0.971597	0.9668	2	Rs= 0.9769
	2	0.944821	2.0	Cs = 295.029388	0.8715	2	Cs= 440.5520
	3	0.905934	2.0	Ws = 107.352295	0.9572	2	Ws= 89.0309
	4	0.880399	2.0	Vs = 370	0.9382	2	Vs= 32
Multi-Objective Optimization	1	0.820009	4.0	Rs = 0.971641	0.8536	3	Rs= 0.9757
	2	0.806433	3.0	Cs = 119.04067	0.7977	3	Cs= 114.0175
	3	0.869349	3.0	Ws = 177.234863	0.9189	2	Ws= 147.0485
	4	0.865680	2.0	Vs = 69.0	0.8133	3	Vs= 57

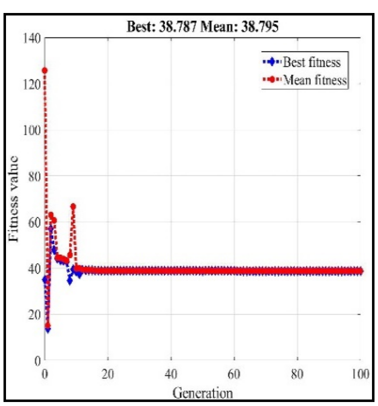


Figure 5. 5 Best fitness and mean fitness of the system cost

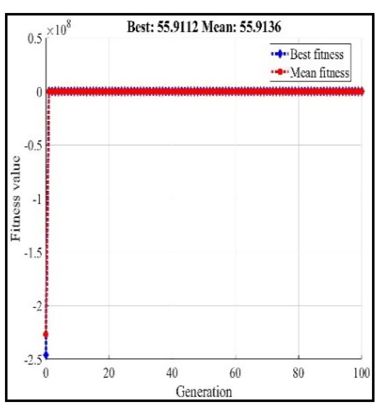


Figure 5. 6 Best fitness and mean fitness of the system weight

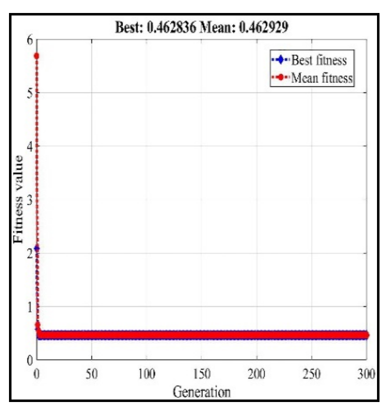


Figure 5. 7 Best fitness and mean fitness of the multi-objective functions

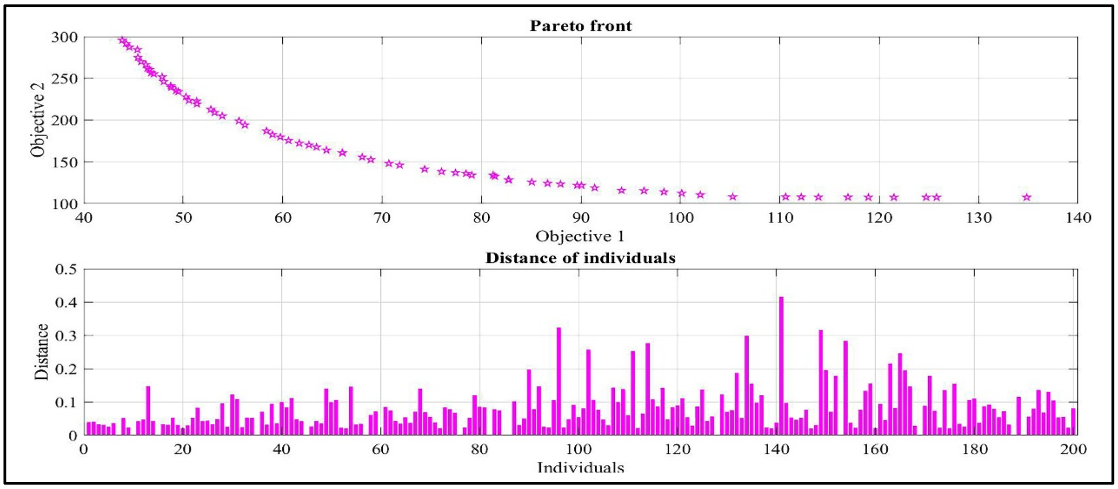


Figure 5. 8 Overall best Pareto front obtained by multi-objective optimization and HGA: cost vs. weight and distance of individuals

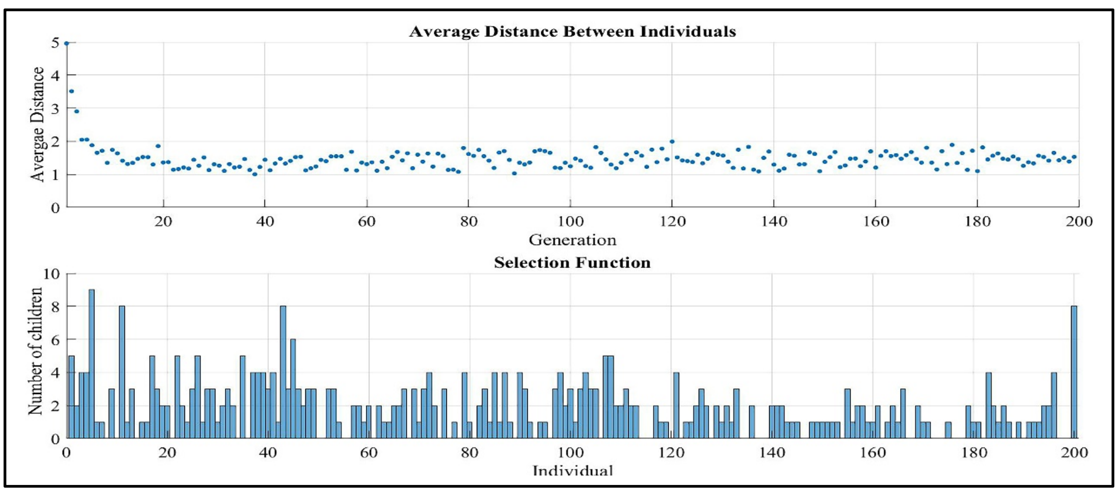


Figure 5. 9 Average distance between individuals

The upper plot function in Figure 5.8 is the HGA Pareto function, which plots the Pareto front (limited to any three objectives) at every generation. This plot shows the trade-off between the two components of *f*. It is plotted in the objective function space. The lower plot shows the histogram distances of individuals.

The upper plot in Figure 5.9 shows the average distance between individuals for each objective, which is a good measure of the diversity of the initial population that affects the performance of the HGA. In general, if the diversity is too high or too low, the HGA might not perform well. Here, it is obvious that the distance does not reach extreme values, so it is considered that the performance is good. The lower plot shows the histogram of the parents, which indicates the parents that contribute to each generation of children populated by each individual.

We considered only the case of multi-objective optimization with the HGA technique for our contribution, and we generated/calculated the values of Ws, Cs, and Vs for 19 values of Rs = A (A = 0.9900, 0.9905, 0.9910, 0.9915, …, 0.9980, 0.9985, and 0.9990). The results are summarized in Table 5.11. On the basis of these tables, we plotted the curves r_i for each stage and Cs, Ws, and Vs as functions of Rs. Further, we determined the mathematical equation of each of these curves. We used the nonlinear regression technique. If the utility of these equations is good, we can use them to estimate/calculate the values of r_1, r_2, r_3, r_4, Cs, Ws, and Vs for any value of Rs (Rs = 0.9900 to 0.9990). Thus, from a practical point of view, these equations are extremely useful. We also obtained the nonlinear regression fitted line plot, which can be used to investigate the relationship between two continuous variables, namely a response variable and a predictor variable. Thus, we can derive a regression equation and plot the regression line. For the copper expansion data, the method determines the type of relationship with these graphs and the line is fitted as per the requirement of data points. Minitab uses the Gauss-Newton algorithm, imposes a maximum of 200 iterations, and employs a tolerance of 0.00001 to achieve convergence. It displays a plot of the data overlaid with a curve illustrating the best-fitting equation based on our expectation function. The plot of the copper expansion data indicates that the specified rational polynomial is a good fit for the data. The points are fairly close to the curve and follow the curve without any systematic deviations from it. If we fit these models, differences will be observed in the desired values as well as in the corresponding points in the graph. This is because the fitted value is given, not the original one. Therefore, it is called the expected value or return of the model. The values of the parameters can be obtained using Minitab. We simply put the values of the parameters in the following regression equation.

$Explained\ variable\ = a + b * Rs + c * Rs^2 + d * Rs^3$. With the parameter estimates in Table 5.13, we obtain r_1New, r_2New, r_3New, r_4New, $CsNew$, $WsNew$, and $VsNew$, as with each value of Rs obtained previously. Then, we obtain a scatter plot between Rs and r_1New, *r*2*New, r*3*New, r*4*New, CsNew, WsNew*, and *VsNew,* as shown in Figure 5.10. There will be non-linear parameters when we fit the models given previously.

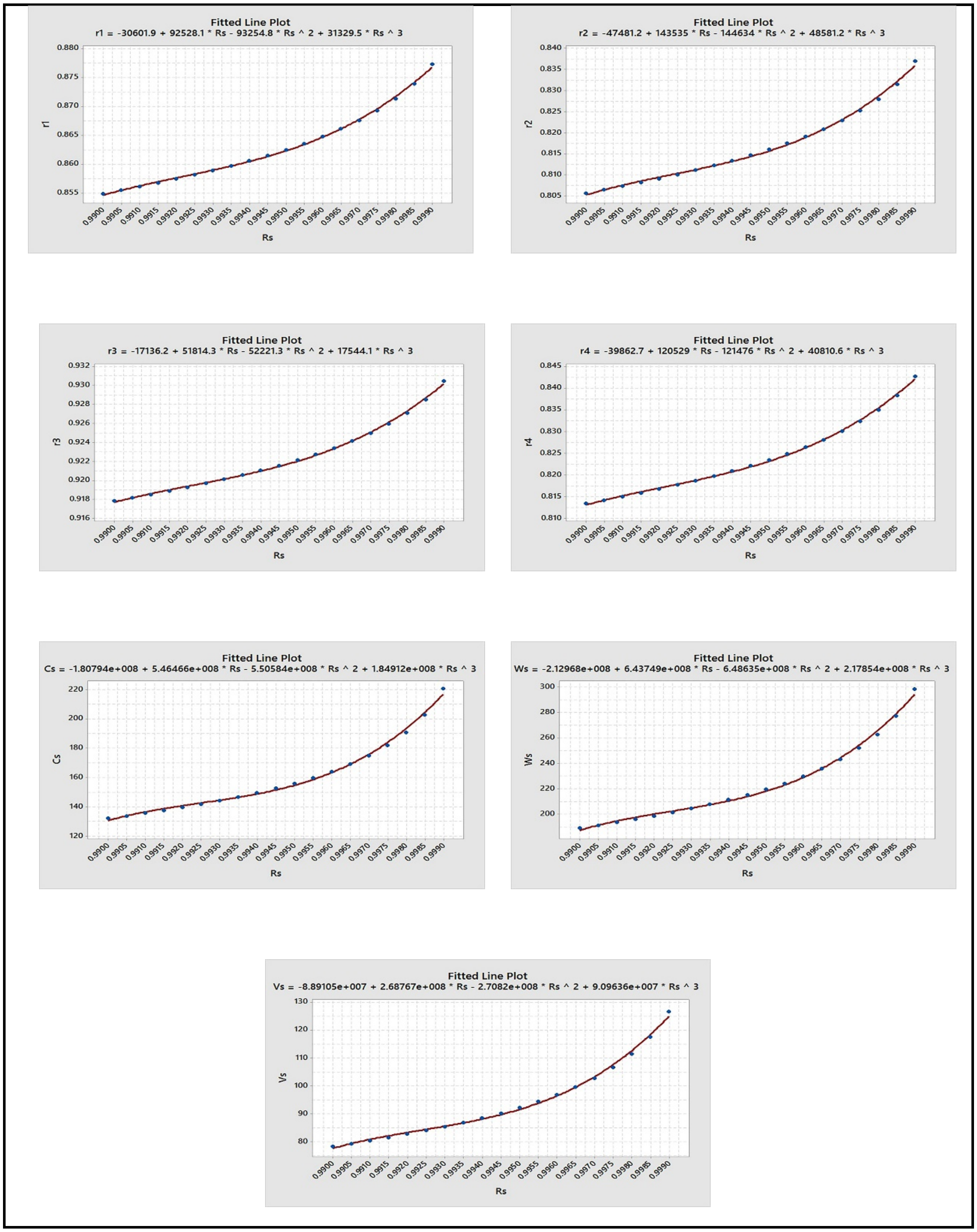


Figure 5. 10 Scatter plot of r_1, r_2, r_3, r_4, *Cs, Ws,* and *Vs* vs. *Rs* - (*Rs*) = 0.9900–0.9990

Table 5. 11 Optimum solutions of HGA for multi-objective optimizations when Rs = 0.990–0.9990

	Reliability r_i for the stage *(1,2,3,4)*				**Component n_i for the stage *(1,2,3,4)***				**Simulation Result**		
R_S	r_1	r_2	r_3	r_4	n_1	n_2	n_3	n_4	Cs	Ws	Vs
0.9900	0.85487685	0.805655761	0.917846852	0.81339754	3.190169321	3.530179896	2.587664286	3.406172744	132.164455	189.0930845	78.39356539
0.9905	0.855473088	0.806463951	0.918182363	0.814181873	3.210073417	3.552423421	2.604289554	3.428104828	133.858123	191.2699882	79.39477332
0.9910	0.856095986	0.807307881	0.918532889	0.815000666	3.230988456	3.575790604	2.62176897	3.451147483	135.6554486	193.5761858	80.45369878
0.9915	0.856748082	0.808191244	0.918899784	0.815857538	3.253032189	3.600408495	2.640202236	3.475425385	137.5689444	196.0275166	81.57726785
0.9920	0.857432193	0.809118161	0.919284538	0.816756654	3.276345812	3.626430945	2.659708108	3.501087952	139.6133923	198.6430896	82.77382002
0.9925	0.858152802	0.810094223	0.919689825	0.817703212	3.301079021	3.654026461	2.68041401	3.528304218	141.8071232	201.4443344	84.05266056
0.9930	0.858914505	0.811125576	0.920118304	0.81870315	3.327424121	3.683409522	2.702481827	3.557285894	144.1720581	204.4583748	85.42555264
0.9935	0.85972226	0.812219748	0.920572273	0.819763928	3.355626654	3.714840684	2.72612569	3.588288077	146.7349448	207.7191939	86.90721932
0.9940	0.860583831	0.813386219	0.921056565	0.820894404	3.385961833	3.748634381	2.751574935	3.621625918	149.5299775	211.2673301	88.51509943
0.9945	0.86150696	0.814636401	0.921575347	0.822105855	3.418809142	3.785198577	2.779150387	3.657697193	152.5998752	215.1564426	90.27224062
0.9950	0.862503387	0.815985304	0.92213516	0.823412564	3.454621766	3.825042849	2.80924355	3.697009653	156.0008801	219.454241	92.20762586
0.9955	0.863587167	0.817452102	0.922744026	0.8248332	3.494016405	3.8688423	2.842375881	3.740227437	159.8074131	224.2515769	94.35993664
0.9960	0.864776784	0.819062299	0.923412064	0.826392467	3.537830595	3.917512894	2.87926149	3.788253165	164.1211822	229.6735445	96.78222413
0.9965	0.866099269	0.820851941	0.924154577	0.828124752	3.587206353	3.972314727	2.920877079	3.842338221	169.0876248	235.895234	99.54831274
0.9970	0.867592417	0.822871926	0.924992706	0.830079694	3.643828911	4.035101433	2.968660812	3.904307868	174.9221985	243.1777639	102.7676572
0.9975	0.869314318	0.825200833	0.925958999	0.832332426	3.710287367	4.108711835	3.024826368	3.976975743	181.9651363	251.930604	106.610654
0.9980	0.871358752	0.827965665	0.927105963	0.8350061	3.790915108	4.197894565	3.093078877	4.065023275	190.7966515	262.8530903	111.3659142
0.9985	0.873899579	0.831400585	0.928530491	0.838326142	3.89372908	4.311439621	3.180304469	4.177144828	202.5337544	277.2792866	117.5782367
0.9990	0.877312537	0.836996655	0.930443934	0.84278106	4.036567983	4.468847795	3.301807032	4.332627137	220.4812934	298.2808393	126.4841872

Table 5. 12 Optimum solutions of HGA for multi-objective optimizations when Rs = 0.990–0.9990 after approximating the values of r_i to four decimals places and adjusting the values of n_i to integer values

	Reliability r_i for the stage (1,2,3,4)				Component n_i				Simulation Result		
Rs	r_1	r_2	r_3	r_4	n_1	n_2	n_3	n_4	***Cs***	***Ws***	***Vs***
0.9900	0.8549	0.8057	0.9178	0.8134	3	4	3	3	133.7041	198.6198	86
0.9905	0.8555	0.8065	0.9182	0.8142	3	4	3	3	134.6502	198.6198	86
0.9910	0.8561	0.8073	0.9185	0.8150	3	4	3	3	135.5692	198.6198	86
0.9915	0.8567	0.8082	0.9189	0.8159	3	4	3	3	136.6158	198.6198	86
0.9920	0.8574	0.8091	0.9193	0.8168	3	4	3	4	150.4994	230.2647	100
0.9925	0.8582	0.8101	0.9197	0.8177	3	4	3	4	151.7831	230.2647	100
0.9930	0.8589	0.8111	0.9201	0.8187	3	4	3	4	153.1025	230.2647	100
0.9935	0.8597	0.8122	0.9206	0.8198	3	4	3	4	154.608	230.2647	100
0.9940	0.8606	0.8134	0.9211	0.8209	3	4	3	4	156.2153	230.2647	100
0.9945	0.8615	0.8146	0.9216	0.8221	3	4	3	4	157.9011	230.2647	100
0.9950	0.8625	0.8160	0.9221	0.8234	3	4	3	4	159.7984	230.2647	100
0.9955	0.8636	0.8175	0.9227	0.8248	3	4	3	4	161.912	230.2647	100
0.9960	0.8648	0.8191	0.9234	0.8264	4	4	3	4	173.4566	257.3974	107
0.9965	0.8661	0.8209	0.9242	0.8281	4	4	3	4	176.295	257.3974	107
0.9970	0.8676	0.8229	0.9250	0.8301	4	4	3	4	179.5854	257.3974	107
0.9975	0.8693	0.8252	0.9260	0.8323	4	4	3	4	183.4539	257.3974	107
0.9980	0.8714	0.8280	0.9271	0.8350	4	4	3	4	188.3124	257.3974	107
0.9985	0.8739	0.8314	0.9285	0.8383	4	4	3	4	194.5076	257.3974	107
0.9990	0.8773	0.8370	0.9304	0.8428	4	4	3	4	204.0998	257.3974	107

Table 5. 13 Explained variable with parameters when Rs = 0.9900–0.9990

Explained variable / **Parameter**	r_1N	r_2N	r_3N	r_4N	***CsN***	***WsN***	***VsN***
a	-30601.9	-47481.2	-17136.2	-39862.7	-180794266	-212968023	-88910519
b	92528.1	143535	51814.3	120529	546466156	643748903	268766970
c	-93254.8	- 144634	-52221.3	-121476	-550583510	-648634663	-270819890
d	31329.5	48581.2	17544.1	40810.6	184911867	217854114	90963580

The results obtained using multi-objective optimization with the HGA are summarized in Table 5.11. It can be seen that the number of components n_i and the individual component reliability r_i in various stages are different. However, in practice, n_i must be an integer.

Therefore, must approximate the values of r_i and adjust the values of n_i to integer values. The new results with this approximation are summarized in Table 5.12. It can be seen that when the numbers of components in different stages, n_i, are modified to integer values, the cost, weight, and volume of the system are reduced slightly.

In this study, we must ensure that the number of simulations (n) for each time is sufficient to achieve convergence. To this end, we changed the value of n (0, 1, 2, 3, ..., 75), and for the simulation with different values of Rs, we can say that for all values of Rs, the simulation process converges at n=30, and for the case of Rs=0.9900, the converged values of r_1, r_2, r_3, and r_4 are 0.8724, 0.9567, 0.8838, and 0.8668, respectively, as shown in the Figure 5.11. In summary, we have discussed our novel approach, i.e., design of system reliability using the simulation process. The advantage of our approach is that the reliable regression curves have been generated using the proposed simulation process (Figure 5.10) and the utility of these curves for the system design is that they can help the designer to determine any level of reliability r_i of the system components, the corresponding value of cost, weight, and volume depending on the chosen value of Rs.

5.10 Conclusions

In this study, we proposed a hybrid genetic algorithm and presented a novel system design for the entire system with the desired level of reliability. Thus, we achieved two objectives. First, we evaluated our approach to determine the robustness of our method by comparing it with another method in the literature. The results indicated that our approach yields better results. Second, we used this approach to develop a new simulation process for system design. We varied Rs and obtained different r_1, r_2, r_3, r_4, Cs, Ws, and Vs. Then, we plotted the curves, which are of great practical significance because they enable the designer of the system to determine the values of r_1, r_2, r_3, r_4, Cs, Ws, and Vs corresponding to the value of Rs. Using Rs = 0.9904, the designer could directly use the curves to obtain all the required values. Some values converge after several iterations in some cases. The performance and robustness of the proposed approach can easily be evaluated. Rapid convergence can be achieved using our model and approach, as shown in Figure 5.11. Moreover, robustness can be confirmed on the basis of similar results obtained under different initial conditions, as shown in Figure 5.11. In addition, Figure 5.10 illustrates the practical utility of our approach, i.e., the designer can determine the reliability of each component corresponding to any value of system reliability *Rs*.

Finally, we fixed the system reliability to obtain a satisfactory system with minimum cost and weight. Comparison of the simulation results indicates the superiority of HGA over other algorithms in terms of searching quality and robustness of the solution. The main advantage of the proposed multi-objective approach is that it offers greater flexibility to system designers for testing problems. Our HGA improves the objective function values and gives the best-known solutions for benchmark suites. Thus, to the best of our knowledge, HGA is an effective algorithm for application to the RRAP. It is especially useful when the optimization problem under consideration is complex.

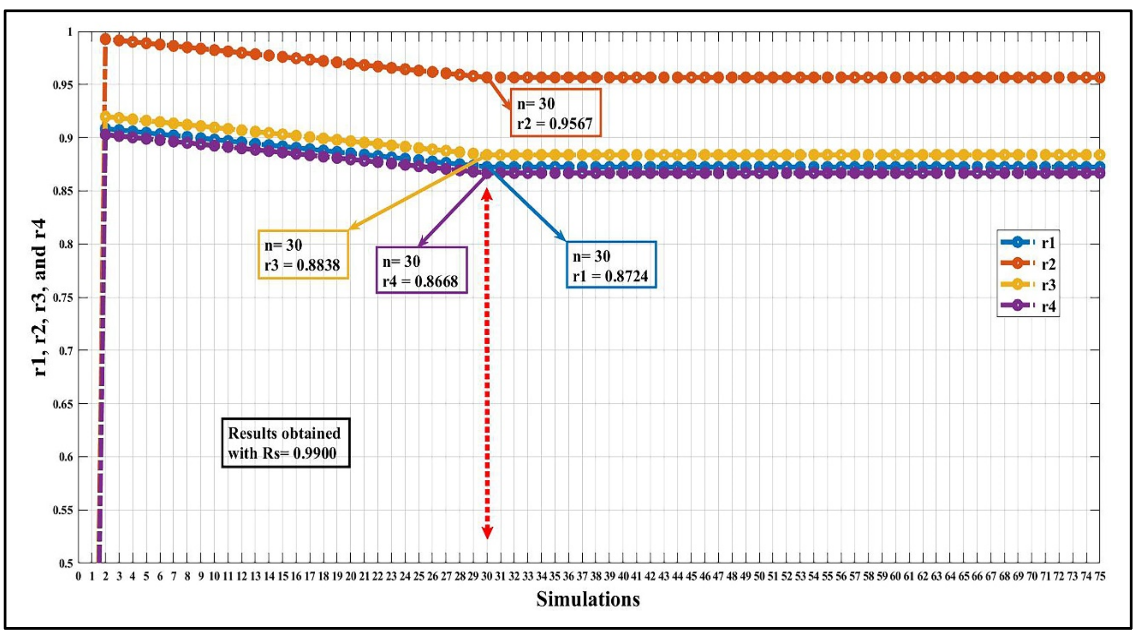


Figure 5. 11 Scatter plot of r_1, r_2, r_3, and r_4 versus number of simulations (n)

In the future, we will focus on extending our approach to other algorithms, such as hybrid, nonlinear, and mixed integer programming, to achieve better results.

CONCLUSION

With current technological developments in the functioning of systems, the improvement of effective solutions to technological problems is gaining importance because of the growing complexity of mechanical and electrical systems. Simultaneously, development schedules are decreasing in size while reliability requirements are becoming stringent. Several system-reliability-optimization techniques are available, and these contain various deterministic, evolutionary, and metaheuristic approaches. The techniques that result in optimal solutions include dynamic, integer, mixed integer, and nonlinear programming, and heuristic methods. This thesis showed that different algorithms were applied to optimize system reliability by using only given cost and weight constraints. An algorithm was used to solve a difficult design problem containing many subsystems and constraint weight and cost, as discussed over five chapters in this thesis with the help of three new algorithms.

Chapter 2 presented a new algorithm aimed to design the most effectively optimized system by using a multiobjective ant colony system (ACS), which is a metaheuristic approach (MA) used to solve reliability optimization challenges in series–parallel systems. The varying problems include the process of selecting components with multiple options and degrees of redundancies to maximize on benefits; this is dependent on the challenges of cost and weight at the system level. The proposed system increased reliability by using an MA to provide solutions for the reliability optimization problem in series–parallel and parallel–series systems. This method was aimed at solving redundancy, continuance, and combinatorial optimization problems in reliability engineering.

Chapter 3 presented the first study, which used a series–parallel system-reliability-optimization method with a genetic algorithm (GA) and statistical analysis. A metaheuristics-inspired GA that performed a parallel search from various points was used as it was capable of evading being locked into a local optimal solution; this phenomenon typically occurs in conventional approaches that launch their searches from a single point. In the new method, penalty factors in the investigation were optimized by using a reliability fitness function. The article also reported that the shortcomings of conventional approaches can be resolved by using probabilistic GA searches, following their previously successful applications for solving real-world related RAP challenges. To examine multipoint capability, 10 simulations were performed in the study in Chapter 3 for each point in the design of the experiments. The findings demonstrated that an average of 10 reliability values were used to increase the accuracy of the subsequent statistical analysis. The authors determined the best configuration of every point that matched the largest reliability value in terms of cost and weight; thus, this approach was an improvement. The determination of the best combination and redundancy level was important for solving optimization issues through statistical analyses. GA parameters were successfully improved using specialized software used in the experimental analysis, leading to the best reliability and configurations. To summarize, we proposed a new approach-based on metaheuristic technique that provided good results for the optimization of RAPs. Through this approach, we designed a practical tool for design of a series–parallel engineering system concerning in the level of series–parallel system.

Chapter 4 presented the second study, which used a mathematical model for a series–parallel system as the best optimization approach. The selected design and architecture incorporated a hybrid GA (HGA) with a flexible allowance technique and was used to address problems prevalent in limited engineering design optimizations. The system comprised four subsystems, each having a different design component type with similar or different characteristics, including reliability, cost, transmission ratio, material, dimension, and weight. The report indicated that every subsystem is represented by *PN* positions, with each component listed according to their reliability index. This highlighted the necessity of incorporating reliability allocation into system design, thus enabling design engineers to establish the reliability of a vector of subsystems and components to obtain the optimal overall reliability. The challenges of optimizing combinations in system design arise from a system where parameters such as cost, reliability, weight, and volume have already been identified. The multi-objective optimization methods available in the literature for this study provide a set of optimal solutions that cannot be improved further without degrading one or more of the other objective values. Generally, the designer chooses the solution that satisfies their needs from the optimal solutions set. However, the designer’s choice may not be objective and or the best available solution owing to human nature. Thus, this study addressed this by presenting an optimization method that can objectively determine the solution that represents the optimal compromise between the optimization objectives.

Chapter 5 presented the third study, which examined a nonlinear programming approach involving the optimal allocation of reliability and redundancy in series systems. The primary aim of this study was to address the issue of multiobjective fuzzy optimization, leading to an increase in system reliability and a reduction of overall costs. This approach described the use of reliability–redundancy optimization in overspeed protection by using a multiobjective approach to maximize system reliability and minimize the consumption of resources such as the cost, total weight, and volume. According to the report, the proposed approach involved a goal-programming formulation and a method for generating the Pareto optimal solutions, in which control and overspeed protection for a gas turbine are nearly identical to those for a steam turbine. A gas turbine operates at a higher temperature than a steam turbine; thus, it must be controlled closely using control sequencing. To summarize, we proposed a hybrid genetic algorithm and presented a novel system design for the entire system with the desired level of reliability. The designer can determine the reliability of each component corresponding to any value of system reliability *Rs*. The curves obtained in this study have great practical value and will enable designers of a system to determine the values of *r1, r2, r3, r4, Cs, Ws*, and *Vs* corresponding to the value of *Rs*.

The studies in the three chapters examined the optimization of reliability of mechanical series–parallel systems. All of these studies face the same type of problem but we applied a different category of constraints in each one depending on the physical characteristics of the system. The three algorithms presented may have great industrial benefits as they offer greater flexibility to system designers for testing problems. The HGA improves the objective function values, yields the best-known solutions for benchmark suites, and is an effective algorithm for application to RRAPs. The approach is especially useful for complex optimization problems.

RECOMMENDATIONS

Optimization is important for all systems requiring reliability. This thesis recommends the use of an altered batch calculation for the dedupe ratio for recording deduplication as a perfection approach and information pressure technique. It is recommended that a changed batch calculation has a superior performance compared with a hereditary programming approach. The main challenges in system optimization is the selection of the best approach, which defines the best redundancy strategy, constituents, and redundancy level for each subsystem so as to capitalize on the system reliability under various system-level limitations. Mathematical modeling helps in gaining better understanding for GA and EP applications. Owing to its complexity, optimally solving reliability problems by applying conventional optimization tools is particularly challenging. GAs have the ability to deliver a good and fast enough solution, thus making them a good approach for future implementations.

The scope of this research includes the following research topics that may be explored in the future.

i) In this thesis, we addressed the optimization problems using a GA and we used the fitness function to perform the reliability optimization of a series–parallel system (Chapter 3). These problems can be solved by other evolutionary/hybrid algorithms. A statistical analysis was used to optimize the GA parameters and the GA was used to optimize the reliability of the series–parallel system. The objective is to determine the strategy of selecting the redundancy level for each subsystem to maximize the overall system reliability, subject to total cost and total weight constraints. We decided to use the design and statistical analysis of experiments to optimize two penalty factors in our reliability fitness function using the GA. A full factorial design was used with three levels. This classification allows us to determine the ranges of values of these two penalty factors, thereby giving the best values of reliability using GA.

ii) In this thesis, the problem of multiobjective reliability optimization of a multistate system was formulated and solved by considering only three objectives: system reliability, cost, and weight. The proposed technique in Chapter 4 may be applied to real-life decision-making problems in the form of interval-valued constrained optimization problems. In addition, it can be applied to various areas of engineering, management, and manufacturing.

iii) For solving the problem in Chapter 5, we strictly focused on a reliability and optimization problem and attempted to solve these problems without extensively considering managerial implications in real industries or cases. Our simulation approach and results (curves) can be used as a tool for the optimal design of reliability systems for a level of system reliability. In the future, we will focus on extending our approach to other algorithms, such as hybrid, nonlinear, and mixed integer programming, to achieve better results. In addition, in the future, the extension of this work can lead to an integration of benchmark databases to test the proposed approach through simulations to determine different parameters of the case study used. The benchmark study would allow us to provide a comprehensive overview of various approaches to provide clear ideas about their capabilities and limitations and draw useful conclusions regarding robustness, efficiency, convergence, and accuracy of the considered methods.

ANNEX I

SUPPLEMENTARY EXPLANATIONS CONCERNING THE RESULTS OBTAINED AND PRESENTED IN CHAPTER 3

This section contains additional information concerning article No. 1 in Chapter 3 to explain the results obtained from the Genetic Algorithm (GA) for the calculated optimal trade-off point. In this information, we present the best configuration results of our approach with the obtained optimal trade-off point.

Maximum cost constraint = 70, maximum weight constraint = 150,

Configuration = **3 3 6 3 6** **5 5 3 6 1** **4 5 6 5 4** **2 2 2 6 1**

Reliability = 0.9991, Cost = 73, weight = 150, Fitness = 0.9991.

The number of component designs with 4 stages having the best reliability are as follows:

- ✓ *S*=4
- ✓ *i*=1 *PN*=3 component type 3,
- ✓ *i*=2 *PN*=2 component type 5, 1 component type 3, and 1 component type 1
- ✓ *i*=3 *PN*=2 component type 4, and 2 component type 5
- ✓ *i*=4 *PN*=3 component type 2, and 1 component type 1

To provide an example, when we say i=1, PN= 3, it indicates the usage of component type 3 for stage 1 as shown in Table-A I-1, which in this case would be 0.931; furthermore, this component must be repeated 3 times in stage 1, as shown in Figures-A I-1 and A I-2. A similar notation is used for the other stages.

Table-A I-1 Series-parallel system input data

Gear pair	**Stage**											
	1			2			3			4		
	r1	c1	w1	r2	c2	w2	r3	c3	w3	r4	c4	w4
1	0.855	3	11	0.743	5	9	0.828	9	15	0.74	6	10
2	0.706	5	12	0.882	6	11	0.842	7	14	0.922	5	10
3	0.931	5	9	0.874	2	14	0.779	7	11	0.855	11	15
4	0.737	7	11	0.783	7	11	0.911	7	12	0.864	9	13
5	0.805	6	14	0.9114	5	7	0.846	3	11	0.816	9	12

Figure-A I-1 shows the transfer of the gear pair from the gearbox to a series–parallel system, and Figure-A I-2 shows the model of the gear train system for a series–parallel system with the optimal trade-off.

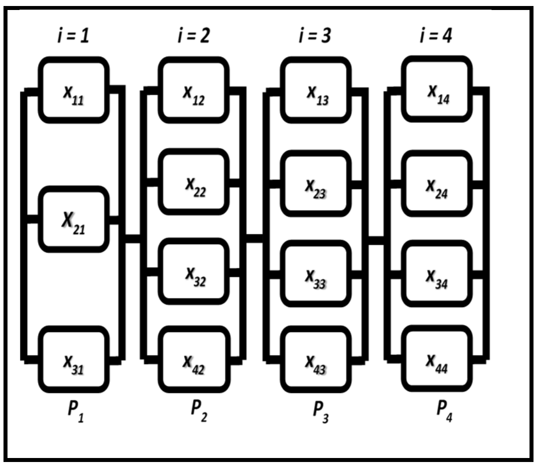


Figure-A I-1 Transfer of the gear pair of the gearbox to a series–parallel system

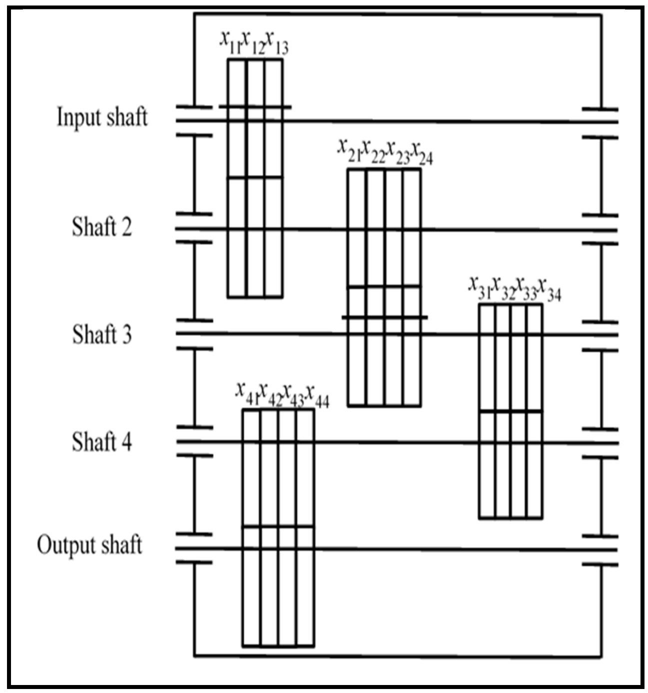


Figure-A I-2 Model of the gear train system for a series–parallel system with the optimal trade-off

ANNEX II

SUPPLEMENTARY EXPLANATIONS CONCERNING THE RESULTS OBTAINED AND PRESENTED IN CHAPTER 4

This section supplements the results obtained from the Hybrid Genetic Algorithm (HGA) for the calculated optimal trade-off point obtained in article No. 2 in Chapter 4. Here, we present the best configuration results using Hybrid Genetic Algorithm with fuzzy function for the optimal trade-off point obtained.

Maximum cost constraint = 70, maximum weight constraint = 150,

Configuration = **3 6 3 3 6** **5 5 5 6 5** **5 5 5 5 5** **6 2 2 2 2**

Reliability = 0.9995, Cost = 70, weight = 150, Fitness = 0.9995.

The number of component designs with 4 stages having the best reliability are as follows:

- ✓ *S*=4
- ✓ *i*=1 *PN*=3 component type 3,
- ✓ *i*=2 *PN*=4 component type 5,
- ✓ *i*=3 *PN*=5 component type 5,
- ✓ *i*=4 *PN*=4 component type 2.

To provide an example, when we say i=3, PN= 5, it indicates the usage of component type 5 for stage 3 as shown in Table-A II-1, which in this case would be 0.846; furthermore, this component must be repeated 5 times in stage 3 as shown in Figures-A II-1 and A II-2. A similar notation is used for the other stages.

Table-A II-1 Series-parallel system input data

Gear pair	Stage											
	1			2			3			4		
	r1	c1	w1	r2	c2	w2	r3	c3	w3	r4	c4	w4
1	0.855	3	11	0.743	5	9	0.828	9	15	0.74	6	10
2	0.706	5	12	0.882	6	11	0.842	7	14	0.922	5	10
3	0.931	5	9	0.874	2	14	0.779	7	11	0.855	11	15
4	0.737	7	11	0.783	7	11	0.911	7	12	0.864	9	13
5	0.805	6	14	0.9114	5	7	0.846	3	11	0.816	9	12

Figure-A II-1 shows the transfer of the gear pair from the gearbox to a series–parallel system, and Figure-A II-2 shows the model of the gear train system of a series–parallel system with the optimal trade-off.

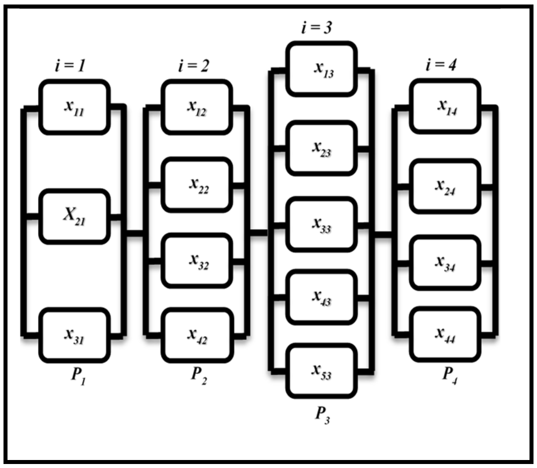


Figure-A II-1 Transfer of a gear pair of the gearbox to a series–parallel system

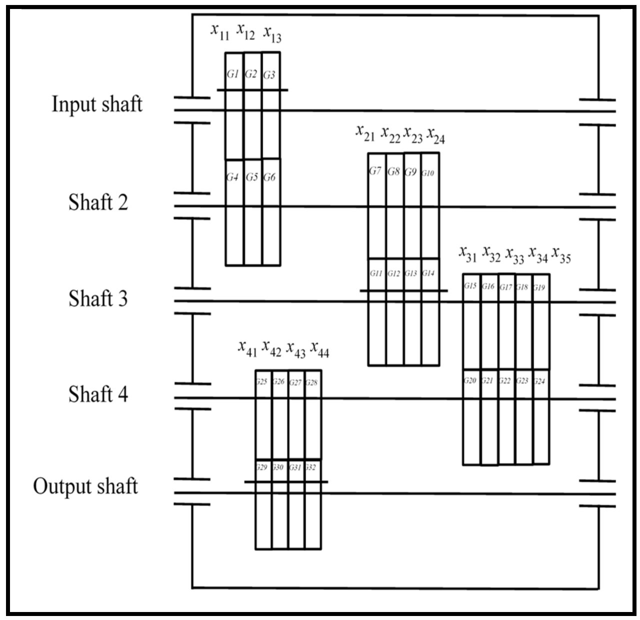


Figure-A II-2 Model of the gear train system of a series–parallel system with the optimal trade-off

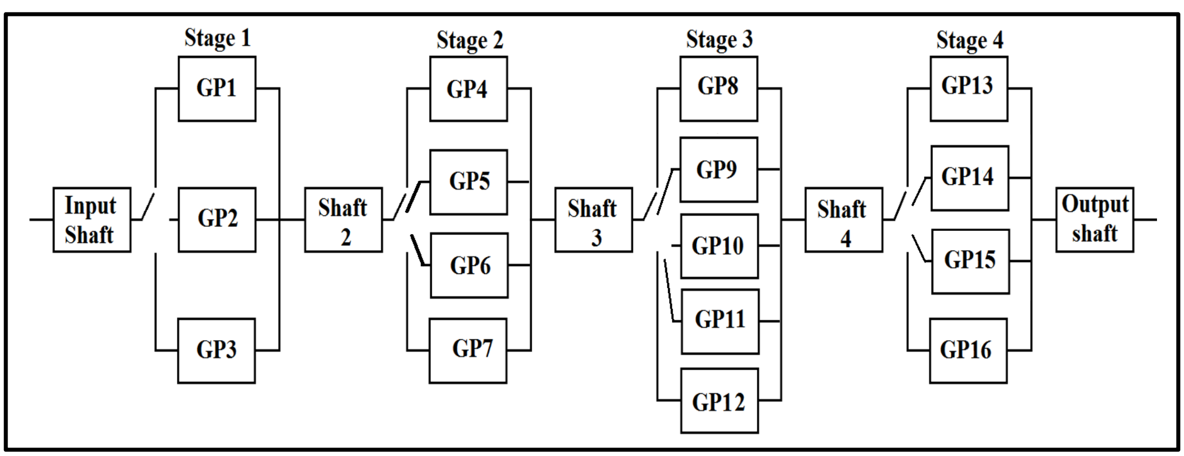


Figure-A II-3 Equivalent scheme for gear train system with the optimal trade-off point.

In Figure-A II-3, let Gl, G2, G3, G4,…, G32 represent the number of teeth of each gear. For each stage, the following equations are applicable:
Gl + G4 = G2 + G5 = G3 + G6 (for stage 1 between input shaft 1 and shaft 2). G7 + G11 = G8 + G12 = G9 + G13 = G10 + G14 (for stage 2 between shaft 2 and shaft 3). Gl5+ G20 = G16 + G21 = G17 + G22 = G18+ G23 = G19 + G24 (for stage 3 between shaft 3 and shaft 4). G25 + G29 = G26 + G30 = G27 + G31 = G28 + G32 (for stage 4 between shaft 4 and output shaft). GP1: Gear pair [G1-G4], GP2: Gear pair [G2-G5], GP3: Gear pair [G3-G6], GP4: Gear pair [G7-G11], GP5: Gear pair [G8-G12], GP6: Gear pair [G9-G13], GP7: Gear pair [G10-G14], GP8: Gear pair [G15-G20], GP9: Gear pair [G16-G21], GP10: Gear pair [G17-G22], GP11: Gear pair [G18-G23] , GP12: Gear pair [G19-G24], GP13: Gear pair [G25-G29] , GP14: Gear pair [G26-G30], GP15: Gear pair [G27-G31], GP16: Gear pair [G28-G32].

The program balances three objectives (reliability, cost, and weight) at a time, as shown in Figure-A II-4, by finding the convergence of the optimal trade-off point that determines the optimal design configuration and maximizes system reliability, minimizes the total cost, and minimizes the system weight for a series-parallel system. This optimal trade-off point is shown in Figure-A II-5.

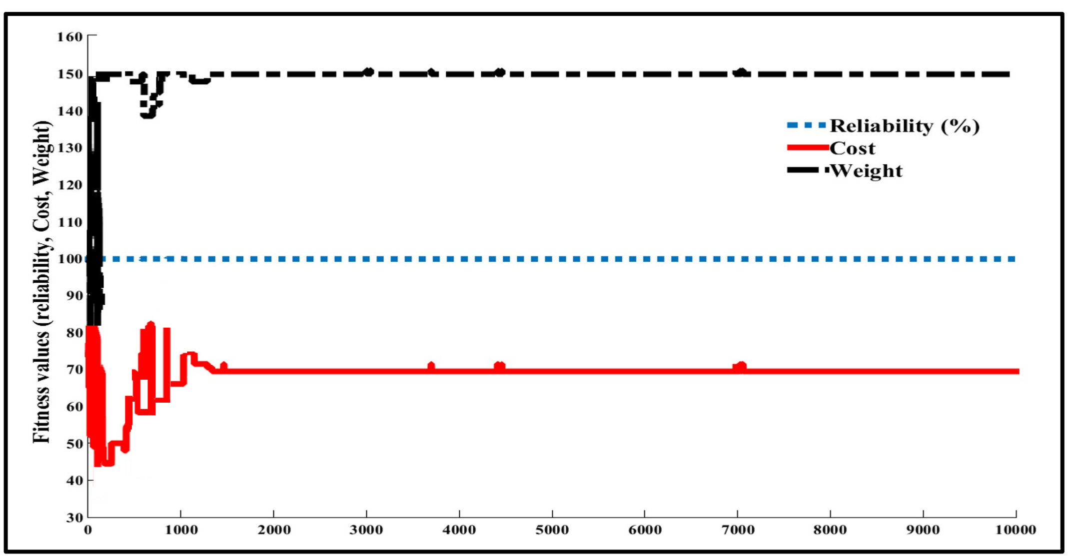


Figure-A II-4 Convergence of optimal trade-off point for reliability, cost, and weight.

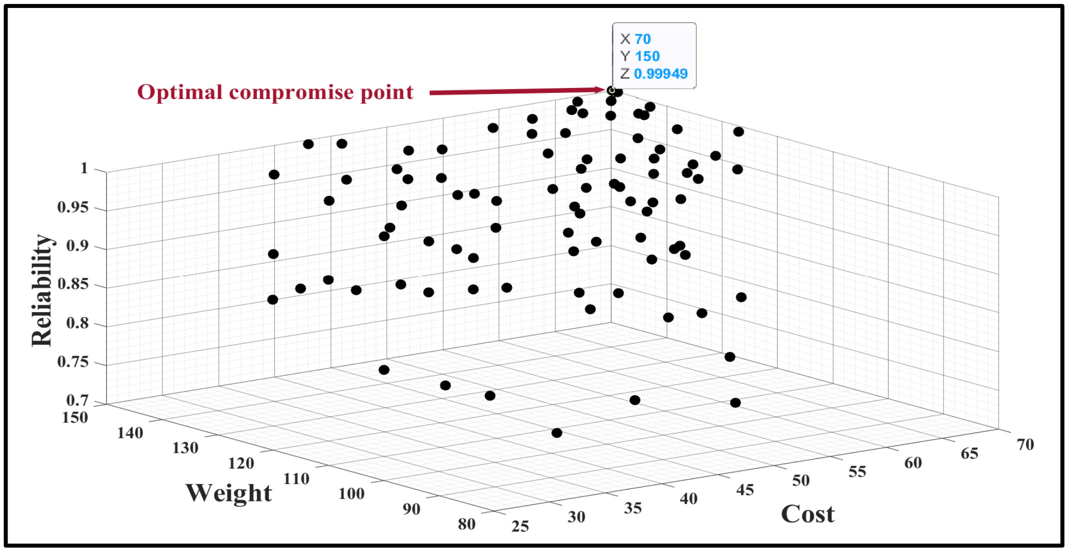


Figure-A II-5 Optimal trade-off point for reliability vs. weight vs. cost in 3D space.

The optimal trade-off solution shown in Figure-A II-5 is [3, 6, 3, 3, 6, 5, 5, 5, 6, 5, 5, 5, 5, 5, 5, 6, 2, 2, 2, 2], with a maximum reliability of 0.9995, cost of 70, and weight of 150.

ANNEX III

SUPPLEMENTARY EXPLANATIONS CONCERNING THE RESULTS OBTAINED AND PRESENTED IN CHAPTER 5

Here, we provide supplementary information to further explain the results obtained from the Hybrid Genetic Algorithm (HGA) in article No. 3 in Chapter 5. Herein, Figure-A III-1 presents the equivalent circuit in greater detail to denote the indexes of the subsystems and the component types in each subsystem.

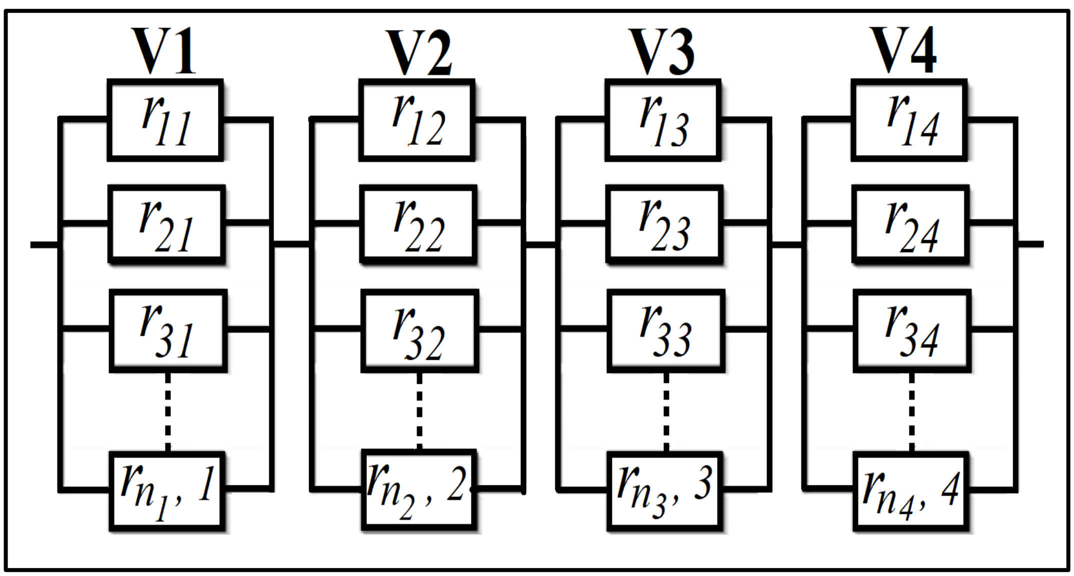


Figure-A III-1 Equivalent circuit: four-stage series-parallel system

$i=1, 2, 3 \ldots, n_j \qquad (n_1, n_2, n_3, n_4),$

$j=1, 2, 3, 4$

$$r_{ij} = \begin{pmatrix} i=1, 2, \ldots, n_k \\ j=1, 2, 3, 4 \end{pmatrix},$$

$$r_{1,j} = r_{2,j} = r_{3,j} \ldots\ldots = r_{k,j}$$

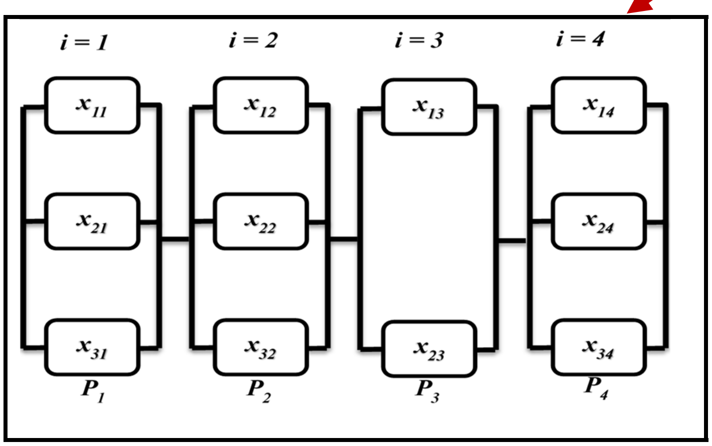
Furthermore, we provide a brief explanation of the approaches used in Chapter 3.

- **Fmincon:** it is used to find the minimum of a constrained nonlinear multivariable function of several variables starting from an initial estimate.
- **Fminmax:** it is used to find a point that minimizes the maximum of a set of objective functions.
- **Genetic algorithm:** it is used to solve difficult engineering problems for solving combinatorial optimization problems with large and complex search spaces.
- **Hybrid GA:** it is a GA combined with **fmincon** to calculate initial values for the **GA**. It is also used to improve the ability of the **GA** to solve optimization problems efficiently.

In Table-A III-1, we compared our solutions with those obtained in a previous study by Quy, N. (1998). From the table, it is clear that our HGA approach obtains better solutions for the series–parallel system than this approach. If we want to obtain the value of the reliability for multi-objective optimization in each stage in this table, the system should have 4 stages, as shown in Figure-A III-2. The first, second, third, and fourth stages contain 3, 3, 2, and 3 components having identical reliability values of 0.8536, 0.7977, 0.9189, and 0.8133, respectively.

Table-A III-1 Comparison of simulation results of optimal solutions of single- and multi-objective function for series–parallel system using HGA with other results presented in the literature

		Results given in Ref. (Quy, 1998)			***Results given by hybrid genetic algorithm***		
Objective	**Stage**	**Reliability**	**Component**	**Simulation result**	**Reliability**	**Component**	**Simulation result**
Maximize	1	0.866288	6.0	Rs = 0.999881	0.8971	5	Rs= 0.9999
System	2	0.850029	6.0	Cs = 381.12183	0.8659	6	Cs= 381.5582
Reliability	3	0.918417	4.0	Ws = 485.77850	0.9358	4	Ws= 475.1981
	4	0.913049	4.0	Vs= 188.0	0.8769	5	Vs= 195
Minimize	1	0.559777	6.0	Rs = 0.971340	0.7997	4	Rs= 0.9939
System	2	0.599392	6.0	Cs = 54.472889	0.7896	4	Cs= 133.4582
Cost	3	0.685273	4.0	Ws = 485.778504	0.7154	5	Ws= 346.2031
	4	0.703375	4.0	Vs = 188.0	0.8393	4	Vs= 155
Minimize	1	0.864883	3.0	Rs = 0.971597	0.9668	2	Rs= 0.9769
System	2	0.944821	2.0	Cs = 295.029388	0.8715	2	Cs= 440.5520
Weight	3	0.905934	2.0	Ws = 107.352295	0.9572	2	Ws= 89.0309
	4	0.880399	2.0	Vs = 370	0.9382	2	Vs= 32
Multi-	1	0.820009	4.0	Rs = 0.971641	0.8536	3	Rs= 0.9757
Objective	2	0.806433	3.0	Cs = 119.04067	0.7977	3	Cs= 114.0175
Optimization	3	0.869349	3.0	Ws = 177.234863	0.9189	2	Ws= 147.0485
	4	0.865680	2.0	Vs = 69.0	0.8133	3	Vs= 57



$$x_{11} = x_{21} = x_{31} = 0.8536$$

$$x_{12} = x_{22} = x_{32} = 0.7977$$

$$x_{13} = x_{23} = 0.9189$$

$$x_{14} = x_{24} = x_{34} = 0.8133$$

Figure-A III-2 Multi-objective result for the number of components transferred into the series–parallel system

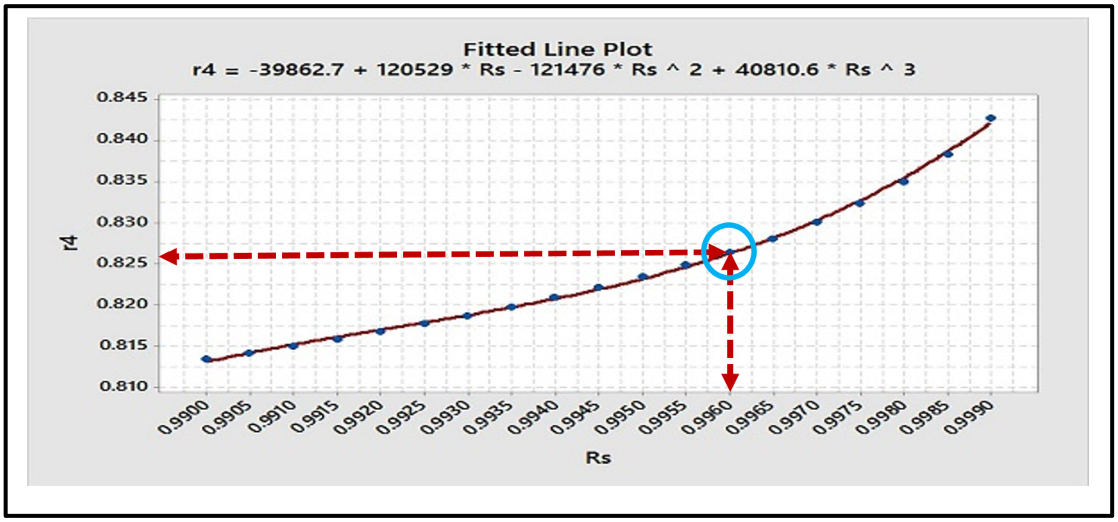


Figure-A III-3 Scatter plot of r_4, vs. *Rs* - (*Rs*) = 0.9900–0.9990

In Figure-A III-3, the reliability of each component corresponding to any value of system reliability *Rs* can be determined. This value *r4* can be obtained from the equation. This can be used to aid specific designs; for example, if an engineer would like to obtain a system with an R_S value of 0.9960, he can determine the value of reliability for *r4* in Figure-A III-3 or *r1, r2, r3, C1, W1,* and *V1* in Figure 5.10.

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