Planning quality and maintenance activities in a closed-loop-serial-multi-stage manufacturing system under different deterioration scenarios

by

Amauri Josafat GOMEZ AGUILAR

THESIS PRESENTED TO ÉCOLE DE TECHNOLOGIE SUPÉRIEURE
IN PARTIAL FULFILLMENT OF A MASTER'S DEGREE WITH THESIS
IN AUTOMATED MANUFACTURING ENGINEERING
M.SC.A.

MONTREAL, LE 04 AOÛT 2025

ÉCOLE DE TECHNOLOGIE SUPÉRIEURE
UNIVERSITÉ DU QUÉBEC

BOARD OF EXAMINERS

THIS THESIS HAS BEEN EVALUATED

BY THE FOLLOWING BOARD OF EXAMINERS

Mr. Jean-Pierre Kenné, Thesis Supervisor
Department of Mechanical Engineering, École de technologie supérieure

Mr. Lucas A. Hof, Chair of the Board of Examiners
Department of Mechanical Engineering at École de technologie supérieure

Mr. Victor Songmene, Member of the jury
Department of Mechanical Engineering at École de technologie supérieure

THIS THESIS WAS PRENSENTED AND DEFENDED

IN THE PRESENCE OF A BOARD OF EXAMINERS AND PUBLIC

LE 17 JUILLET 2025

AT ÉCOLE DE TECHNOLOGIE SUPÉRIEURE

FOREWORD

The following thesis is committed to providing an optimization model that attempts to minimize the total costs of a Serial-Closed-Loop-Multi-Stage Manufacturing System under different deterioration levels by allocating an optimal number of part quality inspection stations and condition-based maintenance strategies along the production lines. Due to the complexity of the problem, numerical methods were used to get a better understanding and optimal policies. This thesis represents a collaborative research endeavor involving the Mechanical Engineering Department, Numerix - Laboratoire de recherche sur l'ingénierie des organisations dans un contexte d'entreprise numérique, and École de technologie supérieure.

ACKNOWLEDGMENT

I would like to express my deepest gratitude to all those who supported me throughout this journey.

First and foremost,1 I am immensely grateful to my professor, Jean-Pierre Kenné, for his unwavering guidance, support and patience. His insightful guidance, constructive criticism and thoughtful feedback have also challenged and inspired me to grow both academically and personally. His mentorship has been a cornerstone of this thesis, and I am honored to have had the privilege of learning from someone of his expertise and integrity.

To my parents, Martin Gomez Anguiano and Isidra Aguilar Gonzalez, I owe an immeasurable debt of gratitude. Their emotional support and belief in me have been a constant source of strength throughout my studies and life endeavors. I could not have come this far without their love and encouragement.

I would like to express my sincere gratitude to the members of my thesis jury, Ph.D. Victor Songmene and Ph.D. Lucas A. Hof for their invaluable support, time, and effort dedicated to reviewing my thesis, for the thoughtful questions that challenged me to think critically. Your expertise and insightful feedback enriched my work. I am truly honored to have had the opportunity to present my work to such esteemed professionals.

Finally, I would like to extend my heartfelt thanks to Erika, Kelly and my friends, who have been my pillars of support throughout this academic journey. Their presence, understanding, and camaraderie have made the challenges more manageable and the successes more meaningful.

Planification de la qualité et des activités de maintenance dans un systèm de fabrication en série à plusieurs étapes et en boucle fermée, selon différents scénarios we détérioration

Amauri Josafat GOMEZ AGUILAR

RÉSUMÉ

Cette thèse étudie les stratégies conjointes d'inspection de la qualité des pièces et de maintenance basée sur l'état pour un système de fabrication en série multi-étapes en boucle fermée dans trois scénarios de détérioration différents. Chaque scénario consiste en un système de fabrication en série vers l'avant et un système de réusinage en série vers l'arrière; les deux partagent une ligne de production de cinq machines qui produisent un type de produits au cours de trois quarts de travail.

L'usure constante, la fatigue et les processus de vieillissement conduisent le système de fabrication en série à plusieurs étapes en boucle fermée à connaître trois scénarios de détérioration différents. Dans le premier scénario, les lignes de production de fabrication et de reconditionnement subissent des niveaux de détérioration uniformes dans toutes les machines, indépendamment de leur complexité opérationnelle (plus l'opération est complexe, plus le facteur de détérioration est élevé).

Dans le second scénario, les lignes de production de reconditionnement avant et arrière sont confrontées à une augmentation qui correspond à la complexité de votre opération. La détérioration des machines augmente de manière exponentielle au cours des trois équipes de travail et des sept jours de travail utilisés comme scénarios d'essai. Dans le cadre de ce scénario, le niveau de détérioration est de l'ordre du million d'euros par jour, ce qui correspond à la complexité de votre opération.

Le troisième scénario présente une situation plus complexe dans laquelle chaque machine des deux échelons du système de fabrication subit des niveaux de détérioration aléatoires qui varient en fonction de l'équipe et de la période de test. Il n'y a pas de modèle prévisible et le facteur de détérioration est directement lié à la complexité du fonctionnement de chaque machine. Cette détérioration entraîne des niveaux de détérioration faibles et élevés, incohérents et imprévisibles, qui ne correspondent pas au type et à la complexité du fonctionnement de la machine.

L'objectif principal de l'étude est de déterminer simultanément le nombre et l'emplacement optimaux pour l'attribution des activités d'inspection de la qualité des pièces et de maintenance basée sur l'état dans chaque scénario afin de minimiser le coût total associé à l'inspection, à la production, à la maintenance, au rebut et aux pénalités pour la production d'articles non conformes. Les outils proposés reposent sur un modèle de programmation mixte entière linéaire (MILP) qui intègre des contraintes de production, de qualité et de maintenance, facilitant ainsi la simulation du comportement du système de fabrication en boucle fermée, en série et en plusieurs étapes.

Pour démontrer l'efficacité de notre modèle, nous avons mené une série d'expériences numériques différentes pour trouver la solution optimale basée sur la qualité des résultats proposés.

Mots-clés: Système en boucle fermée, fabrication en plusieurs étapes, inspection de la qualité, maintenance préventive, programmation linéaire en nombres entiers mixtes, optimisation

Planning quality and maintenance activities in a closed-loop- serial-multi-stage manufacturing system under different deterioration scenarios

Amauri Josafat GOMEZ AGUILAR

ABSTRACT

This thesis investigates joint part quality inspection and condition-based maintenance strategies for a closed-loop-serial-multi-Stage manufacturing system under three different deterioration scenarios. Each scenario comprises a serial-forward manufacturing system and a serial-reverse re-manufacturing system; both share a production line of five working machines that produce one product type throughout three labor shifts.

Constant wear, tear, fatigue, and aging processes lead closed-loop-serial-multi-stage manufacturing to experience three diverse deterioration scenarios. In the first scenario, a serial manufacturing production line suffers three different deterioration levels in all machines based on their operational complexity (The more complex the operation, the higher the deterioration factor). Rework and repair operations are considered for items classified as non-conforming; however, after failing the rework and repair operations, they will scrap.

In the second scenario, reverse re-manufacturing production is fed by collected items at the end of their useful life (EOL) from the same family type as those produced in the forward manufacturing system. The three deterioration levels (stable, incremental, and random) are presented in each working machine and quality inspection error types I and II. Due to the high risk of product re-manufacturing, machine deterioration and inspection errors, repair and rework operations are not allowed to reduce the chances of a high fraction of non-conforming items shipping.

The third scenario presents a more intricate situation where each machine of a closed-loop -serial-multi-stage manufacturing system experiences the three deterioration levels simultaneously in both forward and reverse echelons and quality inspection errors type I and II. However, to reduce the chances of non-conforming items shipping and increase the manufacturing systems surplus, scrap items, as well as EOL collected items, are re-utilized as raw materials in the re-manufacturing process.

The study's primary aim is to determine the optimal number and location for allocating part quality inspection and condition-based maintenance activities in each scenario to minimize the total cost associated with inspection, production, maintenance, scrap and penalties for non-conforming item production. The proposed tools rely on a Mixed-Integer-Linear Programming (MILP) model that incorporates production, quality and maintenance constraints, facilitating simulating the closed-loop-serial-multi-stage manufacturing system's behavior.

To demonstrate the effectiveness of our model, we conducted a series of different numerical experiments to find the optimal solution based on the quality of the proposed results.

X

Keywords: Closed-Loop System, Multi-Stage Manufacturing, Quality Inspection, Preventive Maintenance, Mixed Integer Linear programming, Optimization

TABLE OF CONTENTS

Page

LIST OF TABLES

Page

LIST OF FIGURES

Page

LIST OF ABREVIATIONS

CBM	Condition-Based Maintenance
CM	Corrective Maintenance
CNC	Computer Numerical Control
FIS	Flexible Inspection System
GDP	Gross Domestic Product
MILP	Mixed-Integer-Linear-Programming
PM	Preventive Maintenance
QSM	Quality Management System
QTF	Quality and Cost Transfer Function
RSMSRS	Reverse-Serial-Multi-Stage Remanufacturing System
SMSMS	Serial-Multi-Stage Manufacturing System
TBM	Time-Based Maintenance

LIST OF SYMBOLS

J	Set of forward manufacturing stations
T	Set of forward and reverse re-manufacturing shifts
K	Set of reverse re-manufacturing stations

Forward Manufacturing

$QRM1_t$	Quantity of raw material per unit
$\varepsilon 0_j$	Non-conforming fraction of raw material
ε_{tj}	Deterioration workstation level
de	Forward manufacturing stationary fixed demand
Δ	Forward manufacturing production capacity
γ	Fraction of conforming and non-conforming units passing through QA inspection
α_{tj}	Probability of type-I error
β_{tj}	Probability of type-II error
ω_{tj}	Probability of damage during quality sampling inspection
θ_{tj}	Probability of reworked items does not comply QA during the second inspection
$F1_{tj}$	Fraction of rejected repaired items
$F2_{tj}$	Fraction of rejected scrapped items
$F3_{tj}$	Fraction of rejected items reworked
X	Minimum percentage of acceptable sample units
$X2$	End-of-life recovery products percentage
$X3$	Percentage of collected items
ρ	Unit production cost
i	Unit inspection cost
κ	Unit repair cost
λ	Unit rework cost
S	Unit scrap cost
μ	Unit preventive maintenance cost
ξ	Penalty cost for each non-conforming unit sent to customer

Reverse Remanufacturing

$RQRM1_t$ Quantity of collected raw material
$r\varepsilon 0_k$ Non-conforming fraction of collected raw material
$r\varepsilon_{tk}$ Deterioration workstation level
rde Remanufacturing stationary fixed demand
rΔ Remanufacturing production capacity
$r\gamma$ Fraction of conforming and non-conforming units passing through QA inspection
$r\alpha_{tk}$ Probability of type-I error
$r\beta_{tk}$ Probability of type-II error
$r\omega_{tk}$ Probability of damage during quality sampling inspection
$F2_{tk}$ Fraction of rejected scrapped items
rX Minimum percentage of acceptable sample units
$r\rho$ Unit re-manufacturing cost
ri Unit re-manufacturing inspection cost
$r\xi$ Penalty cost for each non-conforming re-manufacturing unit
rS Unit re-manufacturing scrap cost

INTRODUCTION

Every day, a series of diverse products and services evolve in different dimensions such as performance, design, materials, warranty and sustainability, among others, as a result of new high customer expectations and needs that change rapidly and abruptly; consequently, today's companies face significant challenges not only to produce but also to deliver high-quality, unique and innovative articles and services (Kalpakjian et al., 2016). Therefore, various novel alternatives, such as quality assurance strategies, have been developed to address these challenges.

Quality assurance is considered a key element in the manufacturing sector not only for providing a way to eliminate non-conforming items downstream of the production process, increasing customer satisfaction, but also for minimizing the total production costs associated with quality appraisal and failure (Mandroli et al., 2006) which have a positive impact on achieving the company's business goals. Even with the important role of quality assurance in improving customer experience and reaching business goals, many challenges persist in reducing non-compliant products resulting from anomalies and malfunctioning machines because of different deterioration factors.

Maintenance has emerged as a crucial strategy over the decades, focusing on improving physical assets and preventing unexpected failures in modern manufacturing and re-manufacturing systems (Rivera-Gómez et al., 2020). Preventive Maintenance (PM) is a strategy that has stood due to its cost-benefit results and evolving over time, giving rise to new, reliable philosophies and methods such as Condition-Based Maintenance (CBM) founded in continuous equipment monitoring, for prompt determination of the current performance and health of the production equipment under deterioration (De Jonge & Scarf, 2020).

Quality and maintenance strategies have been showed to have an excellent impact on continuous improvement, customer satisfaction and achievement of business objectives in forward manufacturing traditional systems over the last decade, where only companies placed all their efforts on segregating and reducing non-conforming products, leaving aside the analysis, study, and subsequent treatment of non-conforming products. However, waste reduction is no longer sufficient for today's industry standards.

Customers are now looking for high-quality products from companies that carry out their production processes in a more sustainable way. To reduce the footprint of the manufacturing process, an incremental number of regulations and environmental laws have emerged from government and private institutions worldwide. Therefore, manufacturers have been forced to transition their production models to new circular economy models.

Closed-loop manufacturing systems arise as an innovative production model that seeks to produce the optimal number of high-quality items, reduce production waste, and generate profit for the use and transformation of remaining wastes and EOL collected items. Nonetheless, this manufacturing system has not yet been investigated jointly using part quality inspection and condition-based preventive maintenance.

CHAPTER 1

LITERATURE REVIEW AND RESEARCH OBJECTIVES

1.1 Introduction

The study of manufacturing systems, over time, has undergone a series of major changes in its operational complexity, objectives and challenges to be faced, which generate a great variety of them; however, despite the existence of a wide variety of manufacturing systems, these are based on two main variants. Non-serial and serial manufacturing systems, this second classification, are more common and used in the manufacturing sector, from automotive, aeronautics, textile and food. Due to the wide range of applications of serial systems, they have become an excellent platform for study in the academic and industrial environment, highlighting those systems composed of multiple platforms, such as casting, machining, painting, and assembly processes within the same factory.

Within the wide range of multi-stage flow manufacturing systems, closed-loop production systems have become more relevant today, thanks to their modern production alternative focusing on efficiency, sustainability and waste reduction through integrating new production technologies. This efficiency approach also comes with a series of opportunity areas covering logistics, information technology, production, quality assurance and maintenance, the latter two being the most prevalent elements in any company dedicated to manufacturing goods.

One of the major elements with the most significant impact on companies globally is quality, which encompasses many aspects, from design to expected product or service performance. A good level of quality can generate an incredible feeling of customer satisfaction, bringing with it an excellent reputation or image for a company or, simply, the opposite, generating negative changes in the financial situation of a corporation. As a way to counteract this situation, different methodologies and tools have been developed over time to manage, control, and improve the quality of products and services offered to global markets.

To manage, controlling, and improving quality levels in companies, a growing source of tools, methods and methodologies has emerged that deal with the concept of quality from different perspectives, from the administrative to the physical field. One of these tools is the quality inspection by attributes, which can be developed through robotic or human technologies; being human inspection is one of the most common because of the cost/benefit ratio that it provides to the companies that have it. However, despite providing excellent economic benefits, it is essential to manage and assign optimally since its misuse can generate incessant and higher costs than not having them in some specific cases.

The area of operations research has provided an alternative to the optimal allocation of inspection resources by developing mathematical models, such as the mixed-integer mathematical model used in this research. This model offers a great alternative to modeling real industry problems, provides a high level of flexibility for decision-making, and has excellent potential for resource allocation in logistics and production problems.

Although quality inspection systems present a great option for improving satisfaction levels, producing conforming products, and increasing profitability, more is needed to meet the new challenges to which manufacturing systems are exposed. One of these challenges is the deterioration present in the production lines, an element that has recently become relevant and of a higher level of importance because of the effect it has on the performance and production levels of large manufacturing companies.

The scientific community and the industrial sector have found maintenance a great alternative to counteract the effects of deterioration in the production lines of manufacturing systems, not only to recover the operational status of the machines that make up the production lines when these have fallen into total shutdown as a result of the deterioration, being this the worst and most expensive scenario, due to the costs involved in the expedition of skilled labor to carry out the maintenance, the total repair of machinery or the total replacement of parts.

Considering this situation, a less economically invasive maintenance model has taken relevance, the preventive maintenance based on the current condition of the machinery, whose approach is to be deployed only when the established parameters indicate that it is necessary to have it to avoid a malfunction or shutdown in the available equipment. Similarly, quality inspections and the optimal deployment of maintenance activities are performed through mathematical models of resource use optimization.

The elements mentioned above are an essential part of the research project presented in this thesis, which together presents a new alternative to the solution of problems of resource allocation of quality inspection stations and preventive maintenance activities based on the condition of the production lines of closed-loop multi-satage manufacturing systems, these topics are developed in more detail throughout this literature review.

1.2 Manufacturing Systems in the industry

From a series of different tools, clothes or medicaments, society is surrounded by many products that fulfill specific needs and expectations; some are simple, while others present more sophisticated materials, designs, construction, and usability. Nonetheless, besides the structure of these items, all of them must pass through a sequence of systematic production processes such as machining, welding, molding, and more. These processes transform different kinds of raw materials and information until they become finished products; these arrangements of the production process are also well-known as manufacturing systems (Chryssolouris, 2013).

Besides its purpose, every manufacturing system comprises some key elements that help achieve its goals, such as design, planning, procurement, production, continuous improvement, quality control and maintenance. As expressed in Figure 1.1, these two last topics are the basis of this research project that will be discussed in the following sections.

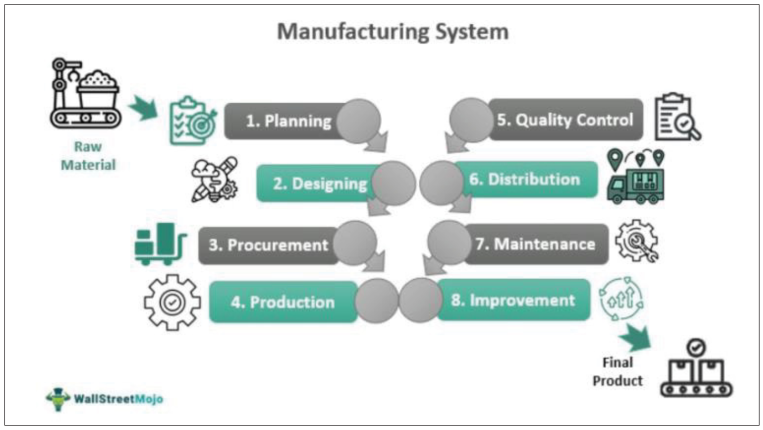

Figure 1.1 Manufacturing system

Taken from (Team, 2023)

In the context of production and industrial engineering, we can encounter two major categories of manufacturing systems (a) Serial-Manufacturing Systems and (b) Non-Serial Manufacturing Systems.

The production and material transformation of Serial-Manufacturing Systems is carried out in a serial sequential way, which means that every working machine can only supply another subsequent operation from its own production line and not another intermediate production line or production cell; hence, the output of one stage becomes the input of its predecessor machine. Since each stage performs singular and specific tasks, the degree of specialization tends to be higher but still offers flexibility and scalability, allowing the entire system to adapt to market fluctuation and take advantage of different economies of scale.

While Serial-Manufacturing Systems carry out all their production processes in a forward sub-sequential way, without intervention between each stage, Non-Serial Manufacturing Systems production lines are designed in a way that not only the manufacturing stage can supply to its predecessor but also alternate production machines or even different production cells, which makes its configuration a more complex and costly manufacturing system (see Figure 1.2). The intrinsic nature of this type of manufacturing system lies in the number of operations each production line within each stage could perform, the number of possible supply combinations each stage could have between each other. (Shi & Zhou, 2009)

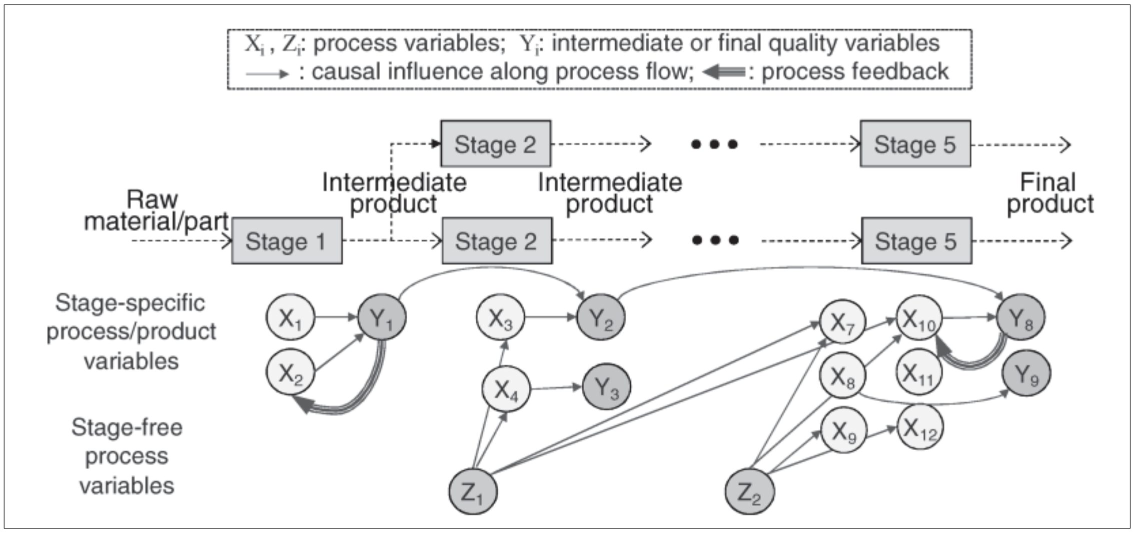

Figure 1.2 Non-serial-multi-stage manufacturing system

Taken from (Shi & Zhou, 2009)

These two manufacturing systems categories serve as a source of inspiration for the presented research project. These systems have been studied recently from different perspectives, looking to achieve various goals because of their wide range of research and development opportunities due to their presence in other manufacturing sectors worldwide today.

1.3 Serial-Single-Stage Manufacturing Systems

A Serial-Single-Stage manufacturing system is a production structure where no subsequent processes provide any additional value or change to their products besides their own one-family type of production process. In other words, their production lines are constituted by only one type of production machine, for example, a production line consisting exclusively of milling machines. Most of the time, single-stage manufacturing systems produce only one product type. Therefore, they require minimal machinery and labor resources, making them less costly to operate (see Figure 1.3).

Many authors have been expanding the existing literature on Single-Stage Manufacturing Systems, such as (Jamal et al., 2004), through the development of batch quantity models; they attempt to determine the optimal batch quantity of a Serial-Single-Stage Manufacturing system that produces a series of defects; therefore, they need to apply perfect reworking operations while trying to minimize the total system costs such as inventory, processing and penalty cost. (Cárdenas-Barrón, 2009) is another author who enriches the previous research by developing an economic production quantity model that not only considers rework activities in the same production cycle but also includes the impact of back order size on the whole performance of the manufacturing system.

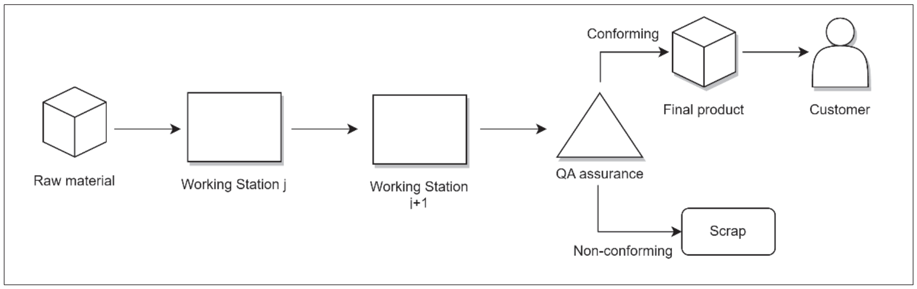

Figure 1.3 Single-Stage manufacturing system

1.3.1 Serial-Multi-Stage Manufacturing Systems

Contrary to a single-stage production scheme where all the operations are carried out in one single production line, with no external interaction or additional contribution from external processes, a Serial-Multi-Stage Manufacturing System comprises a comprehensive series of different working and quality stations, robots, digital, and physical tools, high-skilled labor and other components (see Figure 1.4), that interact jointly in different sequential phases at different times and rhythms to transform raw material into finished products (Shi & Zhou, 2009)

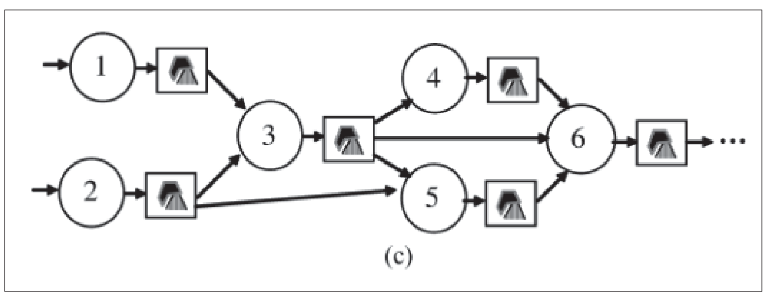

Figure 1.4 Multi-stage manufacturing system

Taken from (Mandroli et al., 2006)

These production systems emerged as a solution for companies seeking an infrastructure that allows them to produce high volumes of sophisticated and complex items requiring numerous and unique high-end manufacturing technologies to supply customer demand and expectations successfully. One of the characteristics of the Serial-Multi-Stage approach is the distinguished efficiency level, as each production process is divided into specialized stages. Continuous improvement and optimization strategies can be applied more precisely, increasing productivity and total production cost reduction.

Due to the elevated degree of Serial-Multi-Stage Manufacturing performance in other aspects, the manufacturing systems are exposed to an excessive and strenuous deterioration factor; therefore, it is essential to have a sharp maintenance and quality assurance mechanism that helps to counteract the adverse effects of machine wear.

Serial-Multi-Stage Manufacturing Systems have been studied from different perspectives, looking to achieve different goals because of their wide range of research and development opportunities and their presence in other manufacturing sectors worldwide today.

1.3.2 Reverse-Serial-Multi-Stage Manufacturing Systems

Traditional manufacturing sector is one of the most essentail element`s in today´s global econonic system, as listed below.

- In 2021, the manufacturing sector, as highlighted by (Thomas, 2023), contributed around 12.0% of the total U.S. GDP, equivalent to about $2.3 trillion, and played a significant role in creating new job opportunities as described in (Sartal et al., 2020).
- Encourage advancing production technologies, such as 3D industrial printing in the healthcare area (see Figure 1.5), to enhance system efficiency and expand product catalogs.

Figure 1.5 3D Metal printing

Taken from (Prashar et al., 2023)

However, over the last few decades, the environment has experienced deterioration because of numerous devastating practices (Jena et al., 2024). Traditional manufacturing structures are widely recognized as significant contributors to the environmental degradation, as a result of their linear models where products are incorrectly discarded after its useful life, generating environmental waste and high level of gas emissions (Govindan & Soleimani, 2017).

Because of the negative impact these harmful practices such incorrect product disposal, government institutions, environmental protection agencies, and other organizations have proposed changes in their way to produce items, including circular economy strategies, such as product reuse (*Apple_Environmental_Progress_Report_2023.Pdf*, 2023), recycling and waste minimization (*Circular Economy Action Plan- for a Cleaner and More Competitive*

Europe, 2020), in order to reduce the impact of its production activities as well as to improve the current environment conditions.

1.3.3 Closed-Loop-Serial-Multi-Stage Manufacturing Systems

Closed-Loop-Serial-Multi-Stage manufacturing systems represent the convergence of the essential elements and benefits of Serial forward and Reverse manufacturing systems that not only attempt to supply at the right time and amount of high-quality items to the most demanding markets but also focus all their efforts to provide a production infrastructure that allows worldwide companies to have more sustainable production processes aligned with the new environment protection laws and regulations imposed by governments and private organizations.

This type of manufacturing system places particular emphasis on the reverse re-manufacturing echelon that, through collecting non-conforming items from the forward production process and end-of-life products, cleaning, disassembly, inspection and repair operations, can generate a new series of products with different characteristics and prices against the ones produced by the forward manufacturing process, as shown the blue reverse scheme in in Figure 1.6.

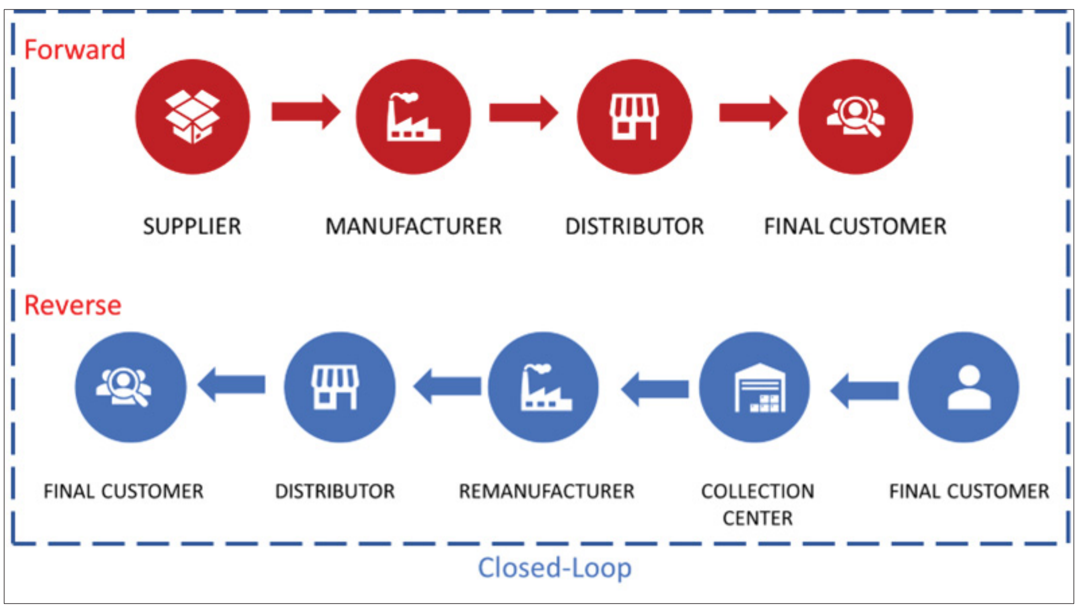

Figure 1.6 Closed-Loop manufacturing system

Due to the tremendous economic, environmental, and operational advantages, more companies worldwide are attempting to transition to closed-loop models and investing in new research and development studies that allow them to better understand the most optimal ways to operate these manufacturing systems. (Bouhchouch et al., 1993) was one of the first research contributions that explored the qualitative and quantitative behavior of a closed-loop manufacturing system with a production line that is divided by different finite capacity buffers between each machine and through an approximate analytical method using a queuing network.

Later works that enriched the existent literature, such as (Werner, 2001) proposed a versatile analytical method that attempts to evaluate the average production rate and the distribution of in-process inventory in a closed-loop manufacturing system with unreliable machines in which the demand is higher than the production capacity, as well as the buffer production and machines available in the system.

1.4 Quality as a way to improve serial manufacturing systems

Quality has been one of the most important elements in the manufacturing sector, whether it produces services or products, for many decades ago. However, its relevance increased after the Industrial Revolution by the cause of numerous companies created to satisfy the demand, which started growing daily until complete globalization emerged. Today, not only local manufacturers and suppliers are involved in the design, production and supply of items, but also new key players worldwide contribute to improving the entire supply chain.

Since the market is continuously growing, manufacturing technologies had to do it simultaneously in different areas; quality was one of the major fields that experienced evolution and improvement over time; methodologies, systems and tools appear to support the upcoming challenges and opportunities that companies face regarding quality (see Figure 1.7)

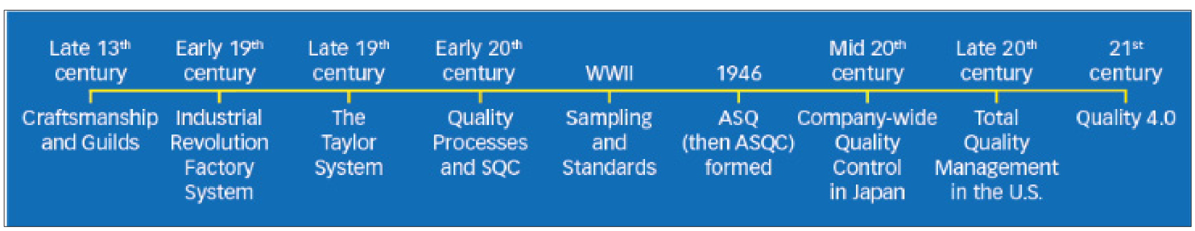

Figure 1.7 Quality history

Taken from (History of Quality - Quality Management History | ASQ, 2023)

1.4.1 The importance of quality in manufacturing systems

Quality has been playing an essential role within the manufacturing field by virtue of the significant benefits these elements have provided to companies worldwide in different ways. Quality has been offered a new set of standards, resulting in the possibility of having products and services that go beyond esthetics and functionality, becoming more ecologically and long-lasting.

However, some significant advantages can be listed for business purposes, such as:

Cost reduction: A correct quality strategy could serve as a solid framework that allows manufacturing corporations to precisely and adequately identify the key areas, processes, methodologies or activities that are not generating any value for the product or service but that incur a cost that is higher than the ones that generate value.

Enhanced reputation: Branding reputation and market presence are two notable factors for a business since these terms can represent the company's growth and success in its primary market and among its different competitors or, on the contrary, its decadency and failure.

Auditing and certifications: Audits are multidimensional evaluations that aid a wide range of entities in ensuring their products, processes, work environment and general practices are supported by high-quality standards such as ISO9001, allowing these companies to reach their business goals.

Standardization is a critical element that can generate unique and well-established procedures or activities that can be replicated in different times, besides atypical factors or environments, producing the same high-quality standards in different places and times.

These benefits have captivated not only companies or research groups in terms of science and technology but also entire nations based on a cultural aspect, such as the Japanese do (Kopp, 2012), textually explains: “In Japanese culture, the ability to follow an example exactly is important and is stressed in various ways in Japanese education.”

1.4.2 Quality assurance

Quality assurance represents the first echelon of a company's quality management systems (see Figure 1.8). It focuses on developing production inspection and assessment strategies to determine whether products meet internal and customer expectations, from performance assurance to correct warranty support. Typically, quality assurance covers all the manufacturing systems and involves general rather than specific and operational decisions. Nonetheless, the decisions and plans generated at this stage significantly impact the global organization's quality of culture.

One of the most significant advantages that quality assurance offers to manufacturers is ensuring internal and external customers that all the quality requirements will be met as expected if the manufacturing system counts with a strong quality control strategy, which is the foundation for the whole quality management system of a company.

Therefore, different manufacturers and researchers worldwide have invested in new and innovative quality control strategies that allow them to reach their goals.

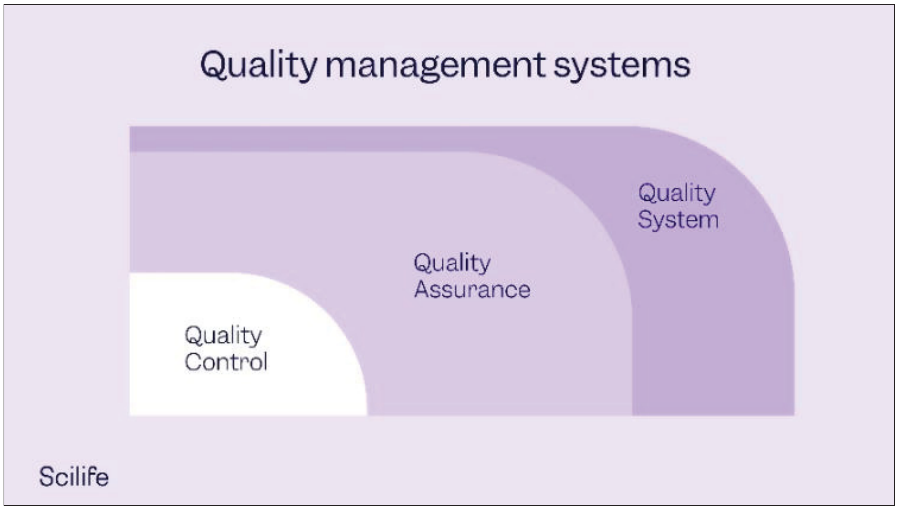

Figure 1.8 Quality echelons

Taken from (Quality Assurance (QA) vs. Quality Control (QC), 2023)

1.4.3 Quality control

Quality control is considered the last leading echelon within the quality management system, providing the whole quality management system with more operational and reactive support rather than tactical, as the quality assurance strategy does. Quality control can be applied using different tools or working methodologies, such as quality inspection, which is considered one of the most widely used in today's manufacturing industries because of the low economic investment, ease and time necessary to implement it. A quality control strategy could occur through the design and implementation of a testing benchmark operated by a skilled, trained operator to perform different types of tests to determine if a service or product has or does not have a specific requirement expected by the company and, even more importantly, by the customer.

1.4.4 Part quality control inspection

Part quality inspection is one of the fundamental strategies for quality control in manufacturing companies that counts on discrete production outputs. Since decades ago, part quality inspection has proven to be a solid support to improve the quality production levels, the reliability of their process, and all the costs associated with these two aspects since its strategy allows prompt detection and segregation of non-conforming items as a result of production errors or lack of maintenance in the equipment. Attributable to the reduction of non-conforming items, the penalty costs imposed by the customer could decrease notably, generating more profit for the business, too.

Data has become an imperative element within every company worldwide, besides their business type, since it provides a method to conclude the state of a process and handy information to perform different production forecasts that are as close to reality as possible. Part quality inspection provides additional low-cost benefits, such as monitoring and feedback alternatives, to determine which phase or process improvements are necessary as countermeasures. Therefore, part quality control inspection could occur in different stages in a manufacturing center (see Figure 1.9).

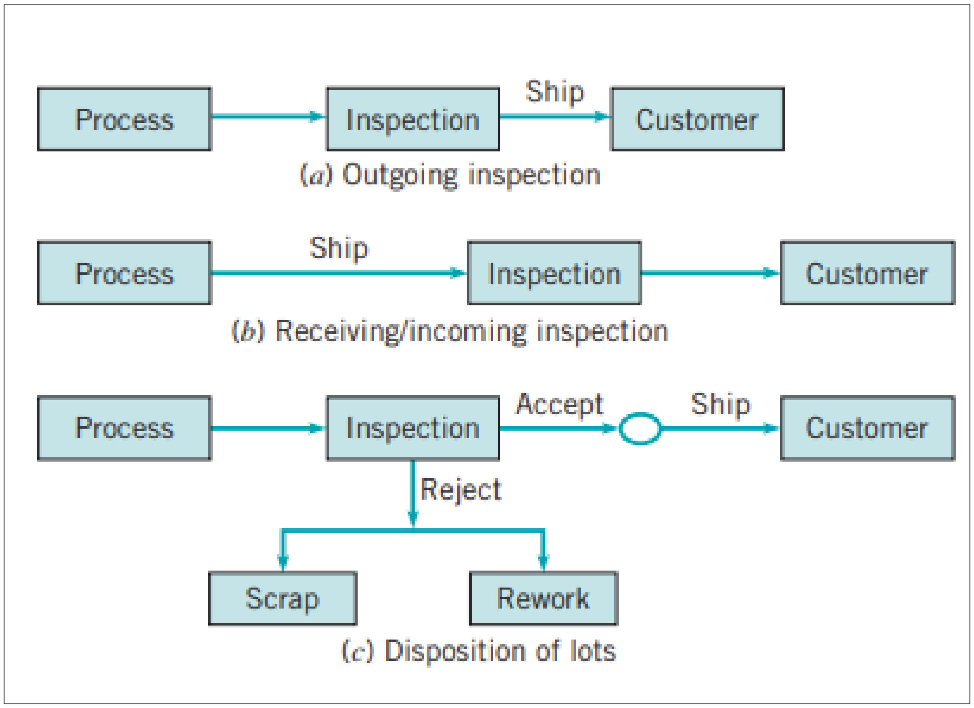

Figure 1.9 Part quality inspection types

Taken from (Montgomery, 2008)

1.5 Maintenance strategies to improve serial manufacturing systems reliability

Today's modern manufacturing systems are in a situation full of technological, ecological, and highly competitive market levels to produce and deliver products that not only meet the customer and business expectations but also exceed them. Thus, plant managers need to increase and improve the efficiency of their production systems to get the most out of them; hence, one of the most evident and straightforward strategies that many of these managers tend to use is to increase the working rate of all their production lines, that is not only limited to manufacturing machinery such as CNC machines, robots but also administrative and information systems.

The plans to increase the performance of production lines strongly affect the correct operation of each of the machines that make them up. This impact is reflected in the level of deterioration that equipment can experience, resulting in partial or complete machinery failure, which represents significant restoration costs and lost sales.

Intending to look for an alternative that allows manufacturers to have highly productive production lines with machinery, facilities and other assets in optimal operating conditions, the industrial and academic sectors have used different maintenance strategies over the years.

In the past, maintenance strategies were limited to only restoring equipment to a minimum operational state that would allow us to meet the production objectives without considering the factors that caused damage. However, maintenance is no longer focused on restoring the assets to their operational status, but rather on integrating a new philosophy that seeks continuous improvement and an ongoing preventive maintenance culture to produce profitable and high-quality products. (Palmer, 2006)

1.5.1 Maintenance strategy benefits

The correct planning of an adequate manufacturing set of strategies tailored to the needs and type of industry brings a series of significant benefits not only limited to restoring the equipment to its operational state, but as well.

- Downtime reduction and equipment efficiency improvement: Good maintenance can help keep the equipment running smoothly and adequately to reach company goals.
- Resources management: Knowledge of labor costs and resources allows tracking and managing all types of resources available in the system to design, plan, and launch more effective maintenance strategies.
- Lower defect rate: One of the major objectives of a well-designed maintenance strategy is to keep the production equipment in an optimal working state to produce the highest percentage of conforming items against the customer and internal demands.
- Occupational safety: If gear runs smoothly, the chances of any operational risk for the employees decrease by a high percentage. However, the cooperative work of the employees is vital to detecting and communicating any malfunction to the maintenance team so they can act in advance.
- Total scrap saving costs: As the number of non-conformities derived from equipment malfunction decreases, the scrap levels and defective overproduction costs decrease similarly.
- Compliance and sustainability: Due to the increasing environmental awareness and the impact of manufacturing activities on the planet, production systems must count on maintenance strategies to reduce the contaminant emissions and inadequate use of energy resources, maintaining equipment operational stability to comply with the regulations imposed by different agencies in charge of environmental care.

1.5.2 Common maintenance challenges in manufacturing

From the paper, food, automotive, or chemical industries, the design and application of maintenance are complex tasks because of different factors that jeopardize the integrity of the system, from the employment mindset and culture to the equipment and tool investment. Among these challenges, we can list some with a higher impact.

- Lack of sufficient training and employee engagement: According to (De Groote, 1995), the competency of the maintenance labor force is an essential factor that affects maintenance performance.
- Budget and resource constraints: Minimum maintenance investment strategies represent damaging production and quality levels for manufacturers.
- Damaged equipment: As deterioration increases, maintenance needs become more frequent, leading to higher labor costs, increased operation time, and potential effects on production quality.

1.5.3 Maintenance Strategies

The design and development of manufacturing maintenance strategies is a process where different factors should be analyzed carefully, from established clear and measurable goals, resources, and budget available and equipment where to apply it, among others, since this information will help to understand the most appropriate maintenance strategy for each type of industry (see Figure 1.10), since there is a vast range of maintenance strategies that serve different purposes and reach various goals. Nonetheless, the fourth elemental maintenance strategy we consider for this study will be mentioned below.

Figure 1.10 Maintenance management steps

1.5.3.1 Corrective maintenance (CM)

Authors such as (Horner et al., 1997) describe corrective maintenance as one of the most straightforward categories since all the repairs and corrections are carried out after a complete shutdown or part failure; therefore, monitoring the equipment's health in the meantime is unnecessary. Once equipment fails, corrective maintenance operations have a high level of prioritization among different productive activities since the equipment should be reinstated to its operational state as soon as possible, which leads to a high amount of labor, time, spare parts and tools investment, as well as production and quality loses (Galar & Kumar, 2017); therefore, the uncertainty associated with this maintenance tends to be as high as their related costs.

It is important to emphasize that corrective maintenance can be determined by three different classifications that cater to different scenarios (see Figure 1.11).

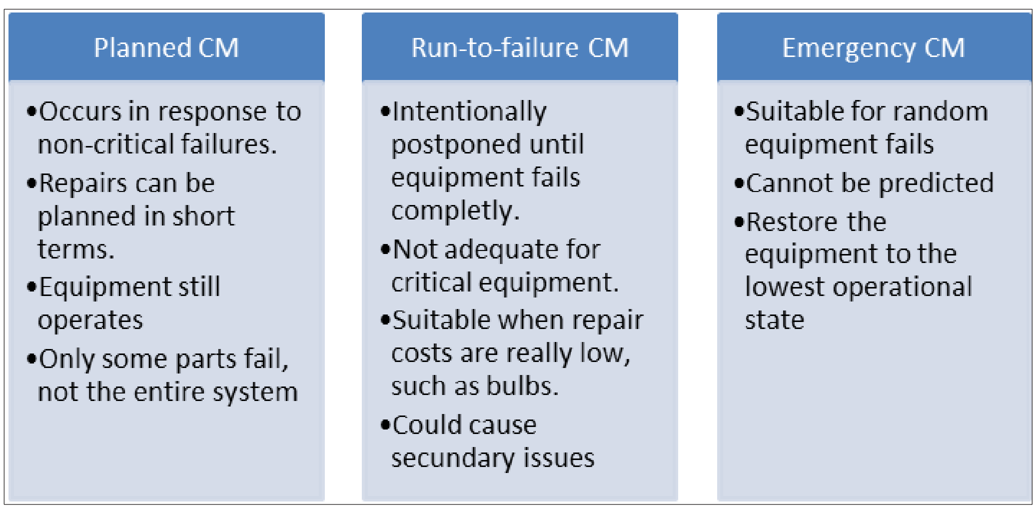

Figure 1.11 Principal CM categories

Corrective maintenance is a measure deployed when unexpected issues arise in the manufacturing systems or when an in-depth repair action is needed. As a result, the production equipment has completely ceased to function. It is generally considered the last maintenance resort due to its high operational costs, the need for skilled labor, production downtime, and decreased production quality. Based on the previous situation, over the years, the manufacturing industry has claimed and developed advanced low-investment manufacturing maintenance strategies that enable companies to maintain total operation efficiency without compromising machine health, thus avoiding economic losses associated with shutdowns.

1.5.3.2 Preventive Maintenance (PM)

Preventive maintenance (PM) has emerged as a support alternative of different manufacturing methods to keep high production rates while maintaining working equipment that is fully operative and stable by carrying out minor repairs or adjustments to the equipment before it fails. (Basri et al., 2017), brings several cost savings related to significant emergency repairs, expedited replacement, and extender overtime labor. As well as corrective maintenance, preventive maintenance can be planned or triggered based on different events, as presented in Figure 1.12.

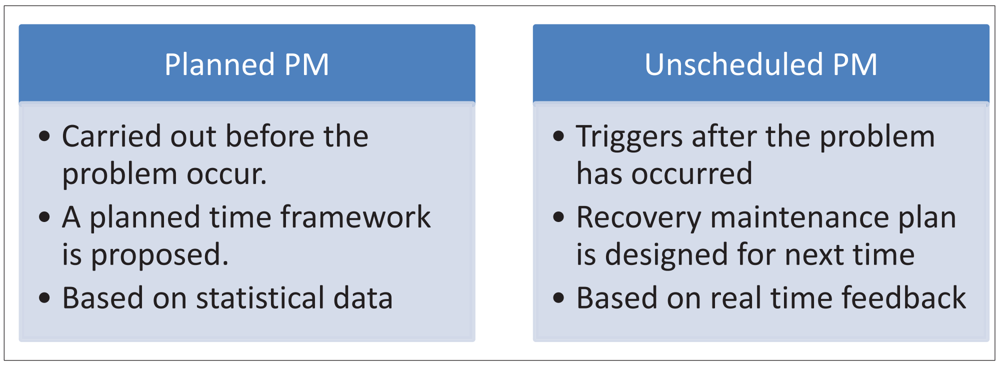

Figure 1.12 Principal PM categories

Even though preventive maintenance can be considered an excellent alternative to reduce random issues, replacement and repair costs, it is necessary to be strategically planned and optimized since performing unnecessary maintenance generates high operation costs and an increased probability of failures because over-maintenance reduces equipment availability. Thus, different preventive maintenance models have emerged over time to serve diverse purposes, particular situations, and industries.

1.5.3.3 Time-Based Maintenance (TBM)

As discussed, incorrect planning of preventive maintenance strategies in a manufacturing company could lead to unnecessary and excessive labor, spare parts and production line downtime costs. On the other hand, if preventive maintenance lacks of adequate maintenance jobs in the assets, it can increase the chances of failing unexpectedly, resulting in elevated corrective maintenance costs. To address the main problems that a preventive maintenance strategy can face, strongly influenced by incorrect scheduling of the necessary equipment maintenance activities, an approach has emerged that tries to carry out these activities on a customer basis that is linked to the condition and behavior of the production equipment on a time basis. (Gits, 1992). Therefore, this maintenance strategy is necessary for some data analysis (see Figure 1.13).

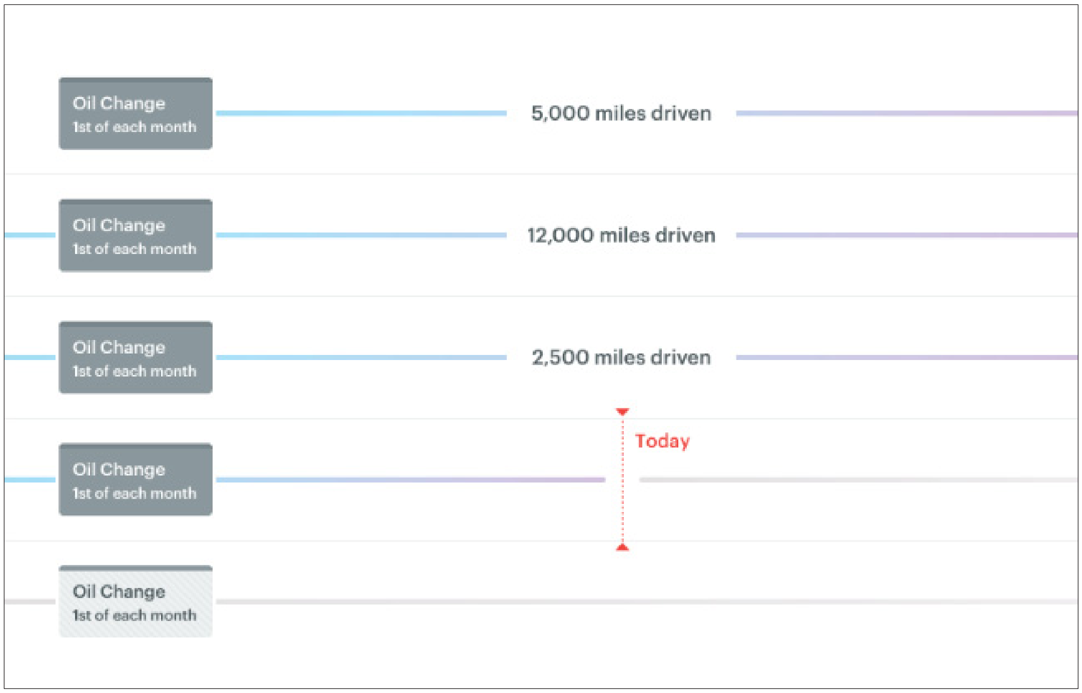

Figure 1.13 Time-based maintenance

Taken from (Why Usage-Based Preventative Maintenance Is Better | Geotab, 2023)

1.5.3.4 Condition-Base Maintenance (CBM)

The importance of real-time feedback and the power it brings to the companies that count on it has been increasing recently. This element plays a crucial role in every operation and its continuous improvement process, output quality, and, most importantly, the decision-making processes for upper management teams. One maintenance strategy that takes advantage of this resource is condition-based maintenance, a strategy that, through condition monitoring techniques and tools, collects data (Ahmad & Kamaruddin, 2012) that is transmitted and informed in real-time in the form of critical indicators to decide what right place, time, and type of maintenance strategy should be applied of an asset.

The data collection can be carried out through sensors, one of the most expensive alternatives, or through physical inspection methods supported by other techniques, such as part quality inspection that results in convenience as result of low investment needed and time to deploy it into production. Condition-based maintenance is a technique that provides a way to react based on the actual condition of the assets; however, based on their strong foundation in data analysis, it gives extra support that goes beyond statistical forecasting of possible new failures on assets in the future.

The focus of condition-based maintenance is to provide a strategy to minimize unscheduled downtimes due to failures in equipment by placing the optimal number of quality control inspections at the right time and right place while also providing valuable quick diagnosis insights about the health of the working equipment based on the data provided in real-time by different monitoring tools or techniques. Due to the diminution of unnecessary maintenance activities and unscheduled failures, manufacturers can experience high maintenance cost savings and production profit improvements.

It is important to emphasize that, as with other maintenance strategies, condition-based also faces a few challenges and complex situations in its implementation, such as equipment deterioration, a topic that has recently been the focus of attention of many research teams.

1.5.3.5 Maintenance and deterioration in manufacturing

Manufacturing companies, whether they work in automotive, agriculture, or pharmacy business, have different production equipment that is exhaustively used to produce an outrageous amount of top-quality products all the time to meet the business and customer requirements; as a result of this high demand pressure on the production equipment, their fatigue level increases, as well as the deterioration level (Kazaz & Sloan, 2013), bringing the production equipment to the point of complete operational shutdown, rendering it unusable. These situations lead to high shutdown costs, corrective maintenance, spare parts expenditures, and high-skill maintenance laborers.

Some of the most common conditions that can affect the state of the gear are regular wear and tear considering high usage level, bad operating conditions, lack of maintenance routines, lack of preventive maintenance, environmental conditions such as humidity or high temperatures, and over maintenance operations. Each piece of equipment can fail in different categories, whether powered, mechanical, or handheld, (see Table 1.1).

Table 1.1 Failure equipment categories

Category	Description	Example
Total Failure	Equipment fails and changes to a useless state. All the system and the components stop working.	The engine breakdown of a CNC Machine
Partial Failure	Some operations are lost; however, the equipment still works. Some components might stop working correctly.	The laptop computer's screen loses signal, but the CPU still works.
Random Failure	The equipment works intermittently, endlessly, and randomly, and all the components stop working,	The force of a press bolster is different each time.

However, it can be highlighted as the fourth most crucial category with the highest impact of deterioration on production equipment.

Aging equipment: Is one of the principal causes of equipment deterioration since no matter what kind of maintenance the equipment gets, the simple age of the equipment can decrease its performance across time; age does not consider the frequency of the maintenance or other factors such as strategies to improve their performance, as long as the equipment is not entirely refurbished with all new components, that in terms of business is as costly as a brand new equipment in the long term since the replacement parts not always are reliable. Additionally, the older the equipment, the more difficult it is to find replacement parts because of original manufacturers discontinuing the production of replacement pieces.

Operator error: Production operations that require human intervention have a high probability of acquiring errors that may decrease the quality of the final product or service (Torres et al., 2021). Sometimes, workers do not operate the equipment following the standard operation procedures, leading to system errors or the beginning of equipment malfunction and a future deterioration factor. On the other hand, because of repetitive activities, employees could develop workshop blindness, leading to an inability to detect the proper operational methodology for the equipment accurately.

1.6 Jointly quality and maintenance strategies to improve manufacturing systems

As previously discussed, part quality inspection and preventive maintenance are essential elements in every manufacturing system worldwide since they provide crucial support to manufacturers' business goals. However, planning the strategies and deployment of these two elements represents a series of challenges that need to be addressed through research.

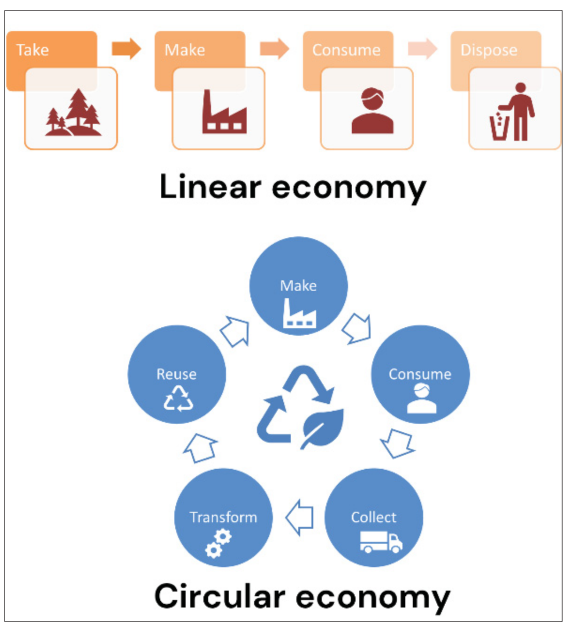

Figure 1.14 Manufacturing economy models

Recently, manufacturing systems have transitioned from traditional forward models that focus on extracting raw materials, transforming them, selling them, and then disposing of them, generating a vast number of waste, to new circular economy models that attempt to increase product life cycles and minimize waste by recycling and reverse re-manufacturing processes, (see Figure 1.14). Yet, because of the youth of these new manufacturing schemes, many challenges remain to be studied and solved, such as quality control and preventive maintenance strategies, two of the key elements of this research project.

1.6.1 Part quality inspection in serial-multi-stage manufacturing systems

From a research and industry context, quality has been an exciting and essential research element as it affects customer satisfaction, which leads to an increase or decrease in customer oversupply. Since its high level of importance nowadays, there is a wide variety of approaches, applications, strategies, models and standards with specific objectives, among which inspection-oriented quality control assurance strategies are found.

Inspection-oriented quality control strategies play an essential role in manufacturing as they provide an alternative to delivering products that meet the required quality standards to various customers along the supply chain by correctly identifying and optimally allocating inspection points stations on production lines. One of the first authors to discuss this area was (Hurst, 1973), who studied the problem of poor quality inspection, whereby quality control measures are only partially reliable in every working station; through a binomial and processing and inspection model, authors studied the probabilistic behavior of sound and defective units, which lead to increased scrap rates, costs and customer dissatisfaction

In the same research area, we find (Eppen & Hurst, 1974) that solved the problem of optimal allocation of inspection stations in a multi-stage manufacturing system by dynamic programming, dealing with poor-quality inspector behavior. One of the assumptions made by the authors is based on the quality history behavior, which means that the inspection decisions are based on the previous quality of the items.the problem types are discriminating in the decision-making process/policy. After a series of numerical tests, the authors determine that processes and cost efficiency can experience improvement over time by accurately recollecting information from the original processes. Additionally, the authors present the error type I (conforming items classified as non-comforming) and II (Non-conforming items classified as conforming) studies in multi-stage manufacturing systems for the first time.

(Ballou & Pazer, 1982) present a model that studies the predictable and unpredictable effects of an inspector's vulnerability on the quality control policies in serial manufacturing systems composed of N number of working stations. Inspection weaknesses refer to the possibility that inspectors make mistakes or inaccuracies during the inspection process because of different factors. The model shows that predictable inspector fallibility has a more significant impact on the number and places of quality stations rather than the inspector's performance, so as a result, the manufacturing system can incur high costs related to inspection in those cases where inspector fallibility is high.

Based on some elemental characteristics of the previous research, the authors (Ballou & Pazer, 1985) now examine the advantages of improving a serial manufacturing process production versus the quality inspection enhancement in terms of cost savings over a long and short period through the previous production-inspection model developed on (Ballou & Pazer, 1982). The research analyzes the interaction of various vital parameters, such as inspection error if the inspection process is presented and quality deterioration only for those items that are not conforming since authors assume they are non-conforming. The results show that product managers should consider the cost ratio process improvement and inspection enhancement become impractical, and a long-run term balance between the two approaches should be used to experience cost savings.

In the following years, authors (Yum† & McDOWELLJ‡, 1987), for the first time, used mixed-integer digit linear programming to find the best inspection strategy for the serial single-stage production system, considering various vital factors, such as productivity level, frequency of inspection and repair capabilities, to minimize total waste, production costs and an appropriate product quality level.

A new cost-effectiveness approach provided by (Barad, 1990) presents a new perspective for addressing the problem of allocation inspection stations in multi-stage manufacturing systems by examining the balance between the percentage of defects in a lot, which represents the economic equality feasibility to neither inspect any item nor to inspect 100% of the lot where the inspection process is perfect. The research problem is addressed by a break-even quality solution-oriented methodological technique that demonstrates that quality inspection can be applied when a cumulative percentage of non-conforming items is lower than the even break quality level.

A flexible inspection system (FIS) that is capable of performing different inspection routines is presented by (Foster et al., 1990) in a new aggregated information environment called AGGIE that is based on a series of different microprocessors placed along the production line, is possible to know the current global state of the working stations over different periods, which can provide accurate information of when might be convenient to place quality inspection benchmarks. The results demonstrate that for new manufacturing schemes, a flexible inspection system can provide a high level of adaptability where different components and parameters are presented in the system.

Another contribution made by (Kang et al., 1990) introduces an expert system prototype supported by a text understanding program that provides the system with relevant data from the analysis of a set of different production scenarios and determines the optimal allocation of inspection stations in a single-stage production system which experience type I and II errors during the inspection process. The project provides a new perspective on the role of data collection and it's used to ensure more accurate system reasoning and robust decision-making in complex manufacturing systems.

In the same field, a new research made by (Raz & Kaspi, 1991) describes the development of a non-linear mathematical model to find the highest number of quality control stations for a multi-stage manufacturing systems. Several inspection procedures, timetable strategies, inspection, production and maintenance errors are part of a new dimension of the problem of inspection allocation. They proposed a new mathematical function called the Quality and Cost Transfer Function (QTF), a powerful tool for analyzing various systems' quality, costs and elements that move through the system model. Each process is described by two transfer functions, one for the proportion of defects and the other for the cumulative unit cost. Finally, the authors found that complex systems can be modeled more easily by transferring functions based on design methods and computational power.

According to previous Foster's dynamic strategy, new research proposed a flexible inspection system in (Foster et al., 1990) and a quality inspection checkpoint strategy in (Villalobos & Foster, 1991), to resolve the optimal allocation of quality inspection strategies supported by quality check routines and inspection points. The strategy is determined in real time based on information collected at each production line stage. However, this information is challenging because it is partly correct due to bad production and testing procedures. In addition, the authors propose two parameters for the project: limited inspection time and fixed inspection routines. All decisions taken at each stage aim to minimize the expected total production costs.

New strategies and methods appear in (Bai & Yun, 1996) capable of dealing with quality audit station allocation problems while attempting to minimize the work, inspection and penalty costs derived from producing at least one non-conforming unit component. A limited number of automatic inspection machines constrains the manufacturing system.

A new study by (Shiau, 2002) describes the development of two heuristic methods that consider production costs and failure rates to determine the best quality inspection plan in a series of multi-stage manufacturing systems, which are usually composed by different workstations with different features; however, they have finite inspection stations and categories available to ensure product quality. Furthermore, the authors aim to minimize the total cost of a single item, such as production costs, inspections, and rework.the authors conclude that both heuristic methods could show acceptable performance on computer processing time and inspection determination planning.

Essential economic aspects are considered, such as general and specific quality costs, facilities and operations. (Oppermann et al., 2003) investigate the solution of a 100% inspection, without sample inspection or control by a statistical method, based on a comparison of different failure rates, without considering the proportion of repairs. The authors conclude that this inspection method depends on the types of industries and the different cost parameters of each sector.

In the same year, (Shiau, 2003a) continued with new research to find the optimal location and type of quality control strategy to be assigned in a serial manufacturing system restricted by a limited number of quality control resources, manufacturing and inspection capabilities that have a strong correlation in the production of non-conforming elements. Authors discovered pattern order methods are more suitable when time efficiency is required and tolerance interval methods when there are significant differences between the tolerance of each workstation, considering manufacturing, inspection, rework, and waste costs.

Another field contribution by (Shiau, 2003b) integrates a methodology of listing quality strategies as a new way to know the appropriate location and time to establish finite quality control stations using a multi-stage manufacturing system under different internal and external costs as well as different tolerance factors that change from time to time. The results show that a sequence/tolerance method has been shown to have an acceptable performance based on time-processing and the quality of the results compared to the quantitative method. However, the quantitative method shows better results based on accuracy and quality.

An exciting new research perspective explores the impact of unreliable measurement systems on manufacturing systems, considering the function of quality loss and the total manufacturing cost inspection errors (Feng & Kapur, 2006). The authors presented two models for unique quality characteristics, inspection errors, and constant as well as variable costs; they found they can be used to make different decisions on inspection allocation. Finally, they present another model in which the simulation procedure does not present inspection errors to analyze the impact of measurement errors on final costs, which can be used when the scrap cost is not high.

Further research was conducted by (Shiau et al., 2007), which introduced genetic algorithms to find the best production process and a control plan for custom manufacturing systems that count only with a few inspection stations and classes. However, each class can perform quality checks at one or more workstations. In addition, production, and inspection capabilities are generally distributed as previous contributions. However, the project's author discovered genetic algorithms are impractical when actual unit costs are embedded and problems are large.

In recent years, mathematical programming has been used not only to solve the problem of assessing inspections in multi-stage manufacturing systems, such as (Vaghefi & Sarhangian, 2009), which proposes the advantages of using simulation techniques to improve the optimization of inspection plans, considering classification errors such as type I and II, while trying to minimize the various inspection-related costs classified in this study. Different numerical examples were used to determine the performance of the simulation algorithm, suggesting that the results are a reliable technique to find the best solution. Although the results are promising, the authors suggested different methods, such as surface reaction methods.

Over the following years, various authors explored the use of new methods to solve inspection allocation problems, such as (Volsem, 2010) that developed a meta-heuristic method, i.e., an evolutionary algorithm that imitates the mechanism of biological evolution, from the author's point of view the research focuses on finding the optimal value of inspection stations, inspection types, inspection limits, and sampling characteristics together that have not been studied before. Furthermore, simulation models are used in this project to calculate the cost of inspection of all potential solutions discovered by algorithms. After the research, the authors concluded evolutionary algorithms could provide reasonable solutions to the limitations of incomplete and asymmetric inspections.

The use of new methods and technologies in the field of quality assurance continues to produce new contributions, such as those proposed by (Azadeh et al., 2015), suggesting the use of particle swarm optimization algorithms (PSOs) to find the best control policy for sample inspection sizes, sample inspection standards, and quality acceptance limits in multi-stage manufacturing systems, cost description using fuzzy numbers, because costs represent uncertainty. Four different examples were used to understand the performance and accuracy of the methodology presented, which showed excellent performance and results when a sample examination was considered a decisive factor.

Different perspectives appear on how to analyze and solve inspection allocation problems in more complex manufacturing systems, from statistical control to the use of sophisticated algorithms, such as (Mohammadi et al., 2015), which proposes a mixed-integer linear mathematical model to calculate the quality characteristics that must be inspected, and to calculate the types of inspection methods that must be used in manufacturing systems simultaneously, and to address high levels of variability using two Taguchi-based and Montecarlo algorithms.

A methodology for solving the problem of inspection allocation, focusing on the flexibility of the inspection system supported by Markov chains for analyzing different manufacturing operations, determining the best inspection routine based on manufacturing process data using two different mathematical models, one with unlimited inspection time and the other limited by finite inspection time, is presented in the research project of (Villalobos et al., 1993), the authors tried to analyze the possibility of avoiding any manufacturing unit even if it was not tested, which produced valuable results for manufacturing systems with intelligent automated inspection systems.

On the other hand, the author (Mousavi et al., 2015) proposes a weighting methodology to solve the challenges of deciding on the distribution of inspections to ensure product quality and improve operational efficiency in the different manufacturing companies presented by multi-attributes, as in previous research projects presented in this document. In addition, fuzzy logic, and intuitionistic fuzzy sets are applied to high-uncertainty scenarios. One of the last contributions was to examine the potential of the mixed linear programming model for accurately detecting and eliminating non-compliance elements through different processes and minimizing the essential inspection costs to improve the economic efficiency of the entire production system proposed by (Mohammadi et al., 2018). The model also provides good-quality results to address the uncertainties and variations that the industry deals with within its regular operations in the real world.

1.6.2 Part quality inspection and preventive maintenance strategies in serial-multi-stage manufacturing systems

The relationship between quality and maintenance is an essential element that all companies must consider, as it dramatically affects customer satisfaction and business objectives. Many enterprises, as well as research communities, are concentrating their efforts on providing solutions in this area such as such as (Bouslah et al., 2018), which focuses on policy development of production, quality and maintenance factors for an aging production line that increases its rate of failure during the time. Using optimization techniques through non-linear programming and simulation modeling, the authors discovered that putting quality inspection stations up to the machines affected by the products' quality will improve overall quality and system reliability compared to any other place on the production line. As a result, inspection costs will be reduced when the number of non-conforming items decreases.

The following contribution is an integrated optimization approach proposed by (Rezaei-Malek, Tavakkoli-Moghaddam, et al., 2018) addressing the quality inspection of parts and preventive maintenance activities in a series of three-stage production system under performance deterioration while reducing the total manufacturing, inspection, guarantee and maintenance costs and highlighting the importance of undetected defects throughout the system. In addition, it provides a new methodology to address this type of problem by developing and linearizing a mixed non-linear programming model with no exact solution methods for optimal performance.

Previous authors (Rezaei-Malek, Siadat, et al., 2018a) propose another approach aimed at optimizing the productivity of a multi-stage manufacturing system and then reducing the total cost of its operations in addition to finding the right place and time for establishing quality inspection spots and performing preventive maintenance activities, as in the previous research project (Rezaei-Malek, Tavakkoli-Moghaddam, et al., 2018). However, in this project, the author assumes the cost parameters and the quantity of demand as uncertain due to the dynamic nature of the market. Nonetheless, a non-linear deterioration of the machinery and non-conforming elements passing through all stations is presented.

Other authors such as (Bahria et al., 2018) presents an unified approach that integrates acceptance sampling by attributes with preventive maintenance to optimize lot sizing and inventory control in systems subject to deterioration. The proposed model minimizes total costs, including inventory holding, backlog, inspection, and maintenance, while ensuring product quality through AOQL (Average Outgoing Quality Limit) constraints. By tightening sampling plans based on system condition, it enhances defect detection and overall reliability. The strategy emphasizes the synergy between maintenance actions and quality assurance in dynamic production environments.

The allocation of preventive maintenance activities and quality control points is studied under various operational uncertainties and cost fluctuations in the cost components produced by the system using a robust possible mixed linear programming model and piecewise linear approximation while analyzing the equilibrium between system productivity and total costs in (Rezaei-Malek et al., 2019).

As manufacturing systems become increasingly complex, new methods, and tools must be explored and designed to improve company processes and profits, so projects such as (Rivera-Gómez et al., 2020), which have developed a policy to optimize production, maintenance and quality inspection activities in common for production systems that suffer from deterioration in and their quality deterioration with machinery that can fail at any time, in order to minimize the expected average costs of these three aspects and at the same time ensure customer satisfaction.

Other studies have considered the utilziation of joint optimization models for serial-parallel multistage complex production systems, integrating quality control, production planning, and preventive maintenance. (Cheng & Li, 2020) presents a hybrid metaheuristic algorithm efficiently minimize total system costs, particularly in quality control and a gamma process to model machine deterioration of a parallel complex manufacturing system.The study reveals that inventory holding costs significantly influence optimal production run lengths and scheduling decisions. The model highlights the need for coordinated decision-making across stages to enhance overall performance and reliability.

Much of recent literature presents presemts integrated planning frameworks that jointly optimizes preventive maintenance, buffer stock levels, and quality control policies in unreliable and imperfect manufacturing systems as the one found in (Hadian et al., 2021). The model in this research accounts for machine degradation, defect generation, and the role of buffer inventory in mitigating production disruptions. It aims to minimize total costs, including maintenance, rework, inventory holding, and quality-related expenses. The results underscore the value of synchronizing maintenance and quality decisions to enhance system resilience and product reliability.

By applying methods based on simulation techniques, mathematical optimization, response surface methodology, experimental design and the average output quality limitations, the authors conclude that distinct quality, production and maintenance parameters represent dynamic behavior that evolves depending on machine deterioration level; therefore, strategies should be flexible enough.

(Hajej et al., 2021) introduced a joint production, quality control and preventive maintenance policy that allows manufacturers to reduce defective components production rate to increase system availability according to the randomness of failure and the random levels of customer satisfaction. In addition to developing a joint policy, the authors also provided an overview of the impact of variations in the production rate during different periods due to each machine's deterioration level. In addition, preventive maintenance activities are insignificant. (Rivera-Gómez et al., 2021) is an extensive work of the existing literature of previous works by (Bouslah et al., 2018) and (Rivera-Gómez et al., 2020) to explore the potential of dynamic quality sampling plans and their level of impact based on economics, as well as the effects of the gradual deterioration of manufacturing equipment.

A number of existing studies in the broader literature have study and implementated genetic algorithms and dynamic optimization models as well an non-linear to solve quality inspection and maintenance problematics, however (Ye et al., 2023) offers a deep reinforcement learning (DRL) model and deterministic policy gradient (DDPG) algorithm to achieve superior adaptability, scalability, and responsiveness compared to conventional approaches.

(Lv et al., 2024) integrates production planning, maintenance scheduling, and quality control in degraded manufacturing systems. Using Cumulative Residual Plot Statistics (CRPS) control charts and stochastic modeling, the approach effectively minimizes product quality variance and total operational costs. The model demonstrates superior performance over traditional disjoint strategies, particularly in systems subject to continuous deterioration. It highlights the importance of dynamic quality variance control in achieving long-term reliability and cost-efficiency.

Most early studies such as (Cao et al., 2025) integrates also CRPS (Cumulative Residual Plot Statistics) for multi-level maintenance and buffer stock planning in degraded, small-batch, and multi-specification production systems. By monitoring product quality variance in real time, the model supports faster, more accurate decisions, reducing operational costs and enhancing production flexibility. It proves especially effective in quality-sensitive and high-mix manufacturing environments. The research emphasizes the role of adaptive quality control in synchronizing maintenance and inventory decisions. Future developments may involve linking CRPS monitoring with IoT-enabled data streams for predictive and autonomous system management.

1.6.3 Part quality inspection and preventive maintenance in serial-closed-loop-multi-stage manufacturing systems

Closed-loop manufacturing systems and green supply management have been one of the most critical areas of research and development over the past decade, but from a logistical point of view (Diallo et al., 2017). However, because of the exponential increase of social awareness among customers who seek to choose companies that not only supply high-quality goods but also use technologies, processes, and materials that reduce environmental impacts, companies and researchers have focused their attention on studying and improving their manufacturing systems using closed-loop designs that consider quality, maintenance, reliability, and production aspects.

Authors (Bhakthavatchalam et al., 2015) place efforts examinating critical challenges in quality assurance, reliability, and preventive maintenance within closed-loop manufacturing systems. It highlights a scarcity of comprehensive models that incorporate warranty management, remanufacturing, and reprocessing strategies. The paper underscores the importance of integrated approaches to enhance system sustainability and lifecycle performance. Notably, it identifies the lack of synergy between quality and maintenance decisions in circular supply chains.

Recent studies such as (Ding et al., 2024) presents an integrated model that jointly optimizes production and maintenance planning in closed-loop assembly systems with machine deterioration. By applying the Weibull distribution to model failure behavior, the approach effectively captures system reliability over time. Results show significant cost reductions and enhanced operational efficiency compared to decoupled planning strategies. The model emphasizes the value of coordination between reuse, remanufacturing, and preventive maintenance.

However despite the effors previously made in this field, where part quality control and preventive maintenance are studied under a closed-loop design, there is still a gap that consider specific quality strategies and preventive maintenance strategies.

In summary, Table 1–Anex 1 highlights the contribution of the leading research areas of this project. Categories I–III classifies the documents discussed by research projects that have significantly contributed to the field. At the same time, other columns identify the key factors in these documents. These factors are characteristic of the current literature. The main contribution of this paper is developing a new integrated model that considers the preventive condition-based maintenance and part quality inspection strategies in a closed-loop-multi-stage production system subject to aging and wear deterioration factors by a mixed-integer-linear mathematical model.

We aim to optimize these interrelated policies while minimizing overall costs and satisfying a series of constraints to provide policies that enable manufacturing companies to make better management decisions to ensure the highest customer satisfaction and the company's internal business needs.

1.7 Summary of literature review

Through the previous introductory literature review, we presented the importance and evolution of different manufacturing systems that currently go beyond producing high-quality products; now seeking environmental, technological, and economical methods to exceed the customer demand as well as the business needs to keep companies competitive and have a presence in the current rapidly growing market. The evolution of manufacturing systems has made their structures more complex to manage and analyze.
This is the case of Closed-Loop-Serial-Multi-Stage manufacturing systems made up of a Serial-Forward Manufacturing and a Serial-Reverse re-manufacturing echelons, which provides top-quality products not only for a primary forward markets but also for a secondary markets that searches for the same quality and support in refurbished products made from collected items at the end of their useful life or scrap while attempting to increase business profitability. Nonetheless, because of the complexity of its design and objectives, it is essential to be supported by different sources of assistance, such as quality control assurance.

Quality is an element that has been present in our life since the past. Yet, its meaning and approach have changed over time following the industry trends, offering a wide range of resources to satisfy customer demand, such as quality management systems that, thanks to their pillars of quality assurance and quality control, customers can be sure that are receiving products or service that meet their needs as well as they will have brand support behind them that can provide solutions to their problems.

The investors can have the confidence that their companies are supported by solid quality foundations that could reduce waste and non-valuable operation costs to avoid economic losses derived from non-compliant operations from manufacturing equipment in poor operating conditions because of different factors, particularly the deterioration factor.

Fortunately, the impact of deterioration can be reduced through well-designed and planned maintenance strategies, which are broken down from preventive to corrective strategies. However, because of the fast-moving and growing market demand, the current trend has pushed manufacturing companies from reactive and corrective maintenance to preventive and planned strategies, such as condition-based preventive maintenance that offers a new, less invasive, less costly, and more effective long-term solution.

To the best of our knowledge, the relationship, behavior and optimization of these critical elements of part quality inspection, condition-based preventive maintenance, aging, and wear deterioration in Closed-Loop-Serial-Multi-Stage manufacturing systems have not been thoroughly investigated yet; therefore, this project is intended to make a new contribution in this field, through a new methodology supported by a mixed-integer-linear mathematical model which will be explained in the following sections of this thesis.

1.7.1 Research Problem

Over time, companies from different productive areas around the world have experienced tremendous productive and economic challenges such as supply chain breakdowns as a result of inconsistencies in the quality of the raw materials, affecting the final product in different dimensions. This kind of challenges emerge from insufficient and adequate planning of quality controls that allow the early and effective detection of non-conforming raw materials, work in process items or final products that do not meet the company and customer's expectations.

On the other hand, it is important to highlight that the production of different non-conforming outputs comes from the inability of manufacturing equipment to produce high standard items. This situation is due to different types of deterioration to which the production equipments are exposed.

Furthermore, with the recent growth in knowledge and awareness of the impact of improper product disposal once they become obsolete, manufacturers have been pushed by governments and private organizations to find new ways to reduce the high consumption and improper product disposal impact on the planet, which represents a significant challenge in the production paradigm and strategies.

Neglecting quality and maintenance issues could severely harm global manufacturers economically, productively, environmentally, and socially. For example, in the automotive field, the production of defective car parts (e.g., engines and transmissions) leads to complex returns, with customers taking vehicles to dealerships for shipment to the factory. Subsequently, the cars will be subjected to various quality tests and necessary adjustments, resulting in economic losses caused by the initial production failure, item returns, quality corrections, maintenance procedures, and customer dissatisfaction. Similarly, neglecting circular economy planning and transitions can lead to substantial economic burdens on various organizations.

Based on the previous challenges discussed, this research project will attempt to solve the problems of quality control and preventive maintenance of a Serial-Closed-Loop-Multi-Stage Manufacturing system through a Mixed-Integer Mathematical model. The aim of the mathematical model is to find the optimal place and time to install part quality inspection benchmarks along a production line to promptly identify and segregate non-conforming items to later classify them according to the activity they need, such as rework or repair to be reinstated in the production line. However, this classification can be compromised because of error types I which means some inspected items are classified as non-conforming while they are truly conforming and II.which represents incorrect classification of non-conforming items as conforming ones.

To reduce the fraction of non-conforming items resulting from working machines subject to different aging, wear, and tear deterioration factors over the time; condition-based preventive maintenance is supported by the quality control inspection stations, which provide information about the current state of the machine based on a series of constraints and rules founded on the quality levels of inspected items and whether they meet the quality levels or not. The quality system functions as a production performance monitor of the manufacturing system, which supports the decision-making process of maintenance deployment.

To cope with the problem of improper disposal of products after the life cycle, our model proposes the reutilization of the remaining non-conforming items from the forward manufacturing process that made up the first echelon of the Closed-Loop manufacturing system as well as a collected fraction of end-of-life products, these two categories of items are utilized as a raw material for a reverse re-manufacturing system which plays the role of the second echelon of the whole Closed-Loop manufacturing system.

1.7.2 Research objectives

Based on the research problem discussed in the previous section, we will list a series of different research objectives this project attempts to reach in every contribution chapter that will be explained in the following sections:

- To study and incorporate part quality inspection and condition-based maintenance strategies in Forward, Reverse, and Closed-Loop-Serial-Multi-Stage manufacturing systems based on three different deterioration levels on the manufacturing machines, quality inspection errors type I and II. As well as to extend the current but minor contributions of general quality and maintenance strategies at all levels in different fields of Closed-Loop-Serial-Multi-Stage Manufacturing Systems.

- Study the economic benefits of reutilization of remaining non-conforming units from forward manufacturing processes and collected items when they are no longer usable to be refurbished and supplied in a second market.

- Study the performance of a Mixed-Integer programming model to perform optimal allocation of part quality inspection and condition-based maintenance strategies as a new tool for managers in charge of strategic decision-making in production lines.

- Comparing the different manufacturing structures, deterioration factors can affect the proportion of conforming items and the total costs of production, inspection and customer penalties.

- Analyze the best cost-benefit balance between applying quality stations vs. maintenance strategies in Forward, Reverse and Serial-Closed-Loop Manufacturing Systems.

1.7.3 Motivation

As seen through the previous extensive literature review, the manufacturing industry is under rapid and constant changes in administrative, economic, and productivity areas as a result of a higher level of demand in terms of quality, performance and warranty of products and services, however, despite all the efforts made by manufacturers and researcher, the speed at which the market and its demands advance still demands even more advances and solutions as quality control and preventive maintenance strategies in more complex manufacturing systems such as Closed-Loop-Serial-Multi-Stage manufacturing systems.

That situation has served as a source of inspiration and motivation to carry out a research project that can serve the industrial and academic sectors in the study of new tools and methods to improve production system reliability performance (Wang, 2002), and such improvement has positive economic, productive and ecological effects.

1.7.4 Methodology

Problem identification: Within this stage, extensive research was carried out on the current problems and challenges that the manufacturing sector was facing at a global level in terms of quality, maintenance, and self-sustainable production. Subsequently, an extensive search was carried out of the tools, philosophies and methods developed to date that try to solve the problems encountered. With the information gathered in the previous points, we analyzed the areas of opportunity in which new research proposals were still needed, of which the development of a tool that would optimize implementing quality controls through inspection stations and preventive maintenance strategies based on condition in a recent circular manufacturing model stood out, (see Figure 1.15).

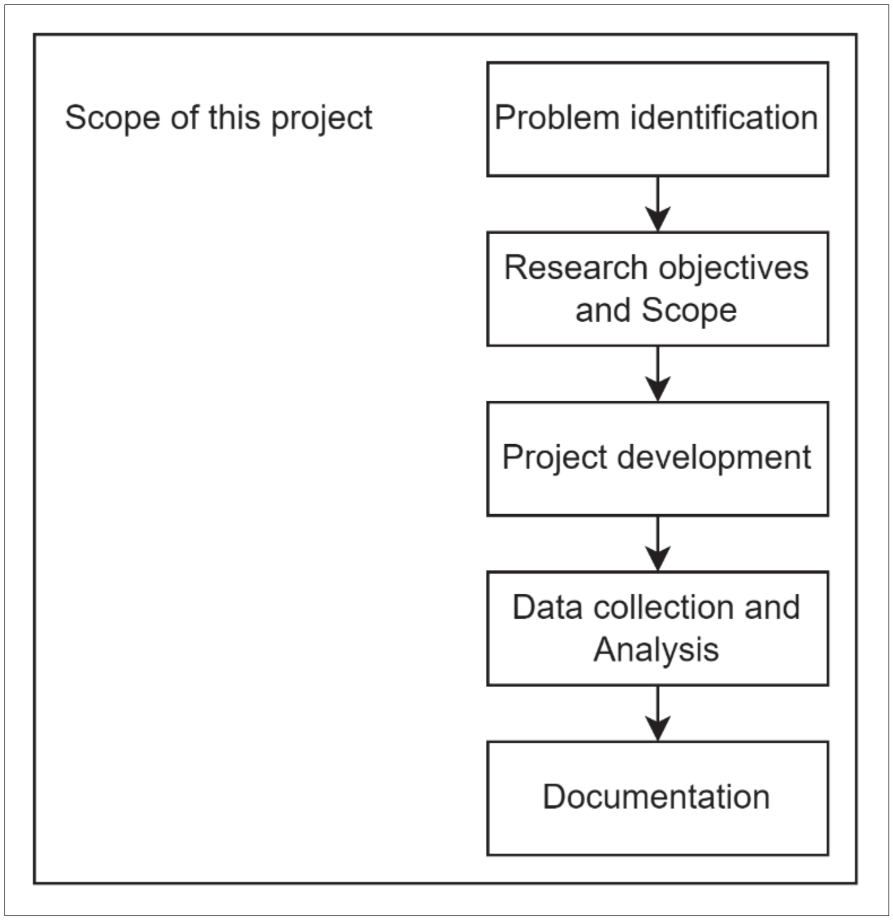

Figure 1.15 Project methodology

Research objectives and scope: In this second phase, the research project's objectives and delimitations were clearly determined to highlight its contribution to the academic and industrial sectors and provide a work scheme to achieve the proposed objectives.

Project development: Throughout this stage, several programming, and mathematical modeling tasks were carried out within the IBM ILOG CPLEX Optimizer suite regarding the variables and constraints that would represent the scenario of the Closed-Loop-Serial Multi-Stage

Data collection and Analysis: In this third stage, data from the functional tests of the optimization program were collected, and a series of analysis were performed to determine the consistency and quality of the results concerning the stations and work shifts where the quality inspections would be placed as well as maintenance and cost reduction that these implementations represent.

Documentation: In this last phase of the elaboration of the research project, the results obtained, and the conclusions based on the previous results are organized and detailed to provide a clearer picture of the objectives achieved through this research project.

1.7.5 Numerical Data

The data collection process in the present research study was structured to align with the theoretical framework this project presents. Data was gathered from multiple resources and numerical testing scenarios, including academic literature such as (Nahmias & Olsen, 2015) (Jacobs et al., 2011; Rezaei & Davoodi, 2008; Rezaei-Malek, Siadat, et al., 2018b), publicly available case studies (Thompson, 2007), simulation models (Curry & Feldman, 2008) and theoretical examples as well. This variety of sources provides a broad perspective, enabling the analysis of common patterns and principles relevant to manufacturing processes.

It is important to note that this study does not involve the use of propietary or confidentail data from any specific industry or company. Given the conceptual nature of the research, the focus remains on generalized manufacturing scenarios derived from validated theoretical models and secondary data sources.

1.8 Conclusion

Through the first chapter of this thesis, a literature review was prepared, where the three fundamental elements of the study that comprise the research project of this thesis were exposed, headed by the study and exploration of the current manufacturing systems in the industrial sector, describing their main characteristics, advantages, disadvantages, examples in the market and their evolution, as well as the challenges that their evolution and structure present day by day.

One of the first challenges of the closed-loop manufacturing system is the aspect of continuous improvement and quality assurance, a topic that is described in the next point of the first chapter and where the different interpretations of quality in the manufacturing sector, the existing methodologies and tools for quality assurance, highlighting their applications, benefits and objectives that each of these available tools provides to closed-loop manufacturing systems; highlighting the allocation and distribution of dedicated quality inspection stations by attributes, due to its economic and productive advantages it provides to manufacturing systems.

The following point describes the third pillar of the research that comprises the areas of opportunity of the manufacturing systems in terms of maintenance, the section where the main tools available in the academic and industrial sector that pretend to offer an improvement solution for the production lines of the manufacturing systems, whose implementation cost and benefit is optimal, are also exposed. In this section, emphasis was placed on the deployment of preventive maintenance activities based on the present condition of the productive equipment because it presents a balance between its implementation costs, the impact it has on the manufacturing systems, and its long-term economic benefits if it is used adequately.

Finally, a summary of the literature review is presented, which explains in detail the problem to be addressed in this research project, presenting the objectives to be achieved with the realization of the project, supported and driven by the motivation described in the following sections, which touches on academic and industrial aspects.

Likewise, the methodology used in the realization of the project is explained to achieve the established objectives based on the areas of opportunity found in the pillars previously described.

CHAPTER 2

PART QUALITY INSPECTION AND CONDITION-BASED PREVENTIVE MAINTENANCE ALLOCATION STRATEGIES IN A DETERIORED SERIAL FORWARD-MULTI-STAGE MANUFACTURING SYSTEM

2.1 Introduction

The production system to be analyzed comprises a serial constant flow production line, which produces only one product type. The production line comprises 5 machines in charge of providing a series of values and attributes to the product as it passes through each of them; however, the last working machine has a higher weighting of responsibility because it is the station in charge of performing the last quality inspection before the product is sent to the final customer.

Each machine that makes up the production line can experience three different levels of deterioration as a result of aging and constant use (Uniform, Incremental, and Random), which compromises the production volume of products conforming to quality standards, causing high penalty costs from each customer for each non-conforming part received.

In order to reduce the fraction of non-conformities sent to the customer, after each workstation, a sample of parts is taken to analyze them and get relevant information to decide to install dedicated inspection stations. The same sampling inspection will be the methodology of verification and analysis of the current state of the production machines to determine the need for a preventive maintenance plan.

It should be noted that during sampling or dedicated inspection, there is a probability of error classification and inspection type I (non-conforming parts classified as compliant) and II (non-conforming parts classified as compliant). Moreover, the production system has three sub-processes in charge of fixing those parts classified as non-conforming:

- Rework: It performs a deep repair and rectification of non-conforming parts and even total component changes.
- Repair: It is performed when the damage to the parts is not deep and does not deteriorate either the functionality or the design to a great extent.
- Scrap: It performs a storage and total scrap process of the non-conforming parts of the process.

The rework process is considered imperfect; therefore, all those reworked parts must go through a second sample or total inspection process in the same quality station of your production process where the part was segregated.

Additionally, during the inspection process, there is a probability that the parts will be damaged by improper handling and inspection process, leading the parts to be automatically classified as scrap, and damage is probable to conforming parts during the inspection process due to a bad inspection procedure.

2.2 Manufacturing System Description

In this section, a production problem of a serial-forward-multi-stage manufacturing system is modeled and analyzed. The manufacturing system comprises 5 machines that produce a single product type. Each machine is subject to three different wear and tear deterioration cases that will serve as test scenarios; the level of deterioration depends on the complexity of the operation performed by each machine, the last machine is the most critical, as it is performs the final inspection before the product is shipped to the customer.

$$M1_{tj} = \left(1 - \varepsilon 0_j\right) * QRM1_t \qquad \forall\, t, j \qquad (2.1)$$

$$M2_{tj} = \varepsilon 0_j * \; QRM1_t \qquad \forall\, t, j \qquad (2.2)$$

The process begins with a quality inspection of conforming (2.1) and non-conforming raw material (2.2), where $QRM1_t$ represents the raw incoming raw material and $\varepsilon 0_j$ the deterministic non-conforming fraction of raw material that enter to the manufacturing system. It represents the proportion of raw materials that do not meet the perdefined quality standars, which in this case is assumed to be fixed and known (see Figure 2.1).

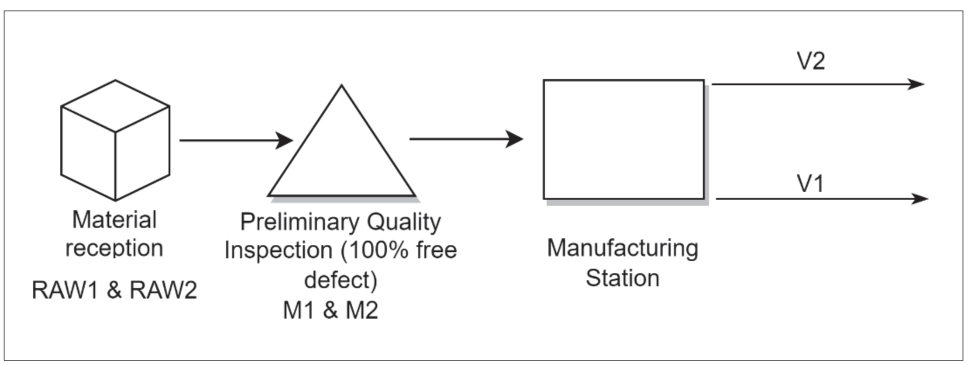

Figure 2.1 Preliminary quality inspection process

To ensure the access of conforming to the manufacturing systems as shown in equation (2.1), a previous quality inspection is carried out. Once the conforming material is processed and supplied to the production line, every workstation $j = 1 \dots 5$, will produce two types of goods:

conforming items (see equation (2.3)) and non-conforming (see equation (2.4)). A random sample is selected from both product type to be inspected using attribute inspection tests.

If material lacks the necessary attributes required by the manufacturer at each workstation, it is classify as non-compliant (see equation (2.6); otherwise, it is considered complaint (equation 2.5) the sample fraction of items to be inspected is denoted by γ. This parameter determines the proportion of produced items that undergo quality inspection. By adjusting this parameter is possible to simulate different inspection policies from minimal to full. While ε_{tj} represents the machine's deterioration level, which influences the overall proportion of conforming and non-conforming items. On other words, the parameter reflects how the progressive wear and degradation of equipment affect the production process. This paramet is crucial for evaluating the impact of maintenance strategies, machine reliability and process performance.

$$V1_{tj} = \left(1 - \varepsilon_{tj}\right) * M1_{tj} + \left(1 - \varepsilon_{tj}\right) * EW1_{tj} \qquad \forall\, t, j \qquad (2.3)$$

$$V2_{tj} = (\varepsilon_{tj} * M1_{tj}) + EW2_{tj} + (\varepsilon_{tj} * EW1_{tj}) \qquad \forall\, t, j \qquad (2.4)$$

$$SI1_{tj} = V1_{tj} * \gamma \qquad \forall\, t, j \qquad (2.5)$$

$$SI2_{tj} = V2_{tj} * \gamma \qquad \forall\, t, j \qquad (2.6)$$

The quality inspection process is performed manually by a qualified operator, due to the cost-benefit ratio that this methodology represents. However, manual inspection introduces the possibility of classification errors: conforming items may be incorrectly classified as non-conforming (α_{tj} error also known as error type I), while a non-conforming item product may be mistakenly classified as complaint (β_{tj}Also known as error type II). As a result, the inspection process ended up with four subcategories of products at the end of the process.

The supposedly conforming classified items $SI1_{tj}$, it is divided into two main subcategories: products classify as quality non-conforming (stated in equation (2.7)) and those classified as conforming ((equation (2.8)).

$$ESR1_{tj} = \alpha_{tj} * SI1_{tj} \qquad \forall\, t, j \qquad (2.7)$$

$$ESR3_{tj} = SI1_{tj} - ESR1_{tj} \qquad \forall\, t, j \qquad (2.8)$$

The supposedly non-conforming items $SI2_{tj}$, it is also branched into non-conforming (see equation (2.10)) and conforming ones (equation (2.9)).

$$ESR2_{tj} = \beta_{tj} * SI2_{tj} \qquad \forall\, t,j \qquad (2.9)$$

$$ESR4_{tj} = \left(1 - \beta_{tj}\right) * SI2_{tj} \qquad \forall\, t,j \qquad (2.10)$$

A fundamental assumption of this project is that conforming parts might be damaged or contaminated during the inspection process, the fraction of these items is deterministically assigned by Ω_{tj} which represents a fixed of items that experience quality risks, demostrated in the following equation:

$$ESR5_{tj} = \Omega_{tj} * ESR3_{tj} \qquad \forall\, t,j \qquad (2.11)$$

Therefore, the parts that are incorporated into the next workstation after meeting the expected quality attributes of that workstation $j = 1 \ldots 5$, are denoted by:

$$FCI_{tj} = (ESR3_{tj} - ESR5_{tj}) + ESR2_{tj} \qquad \forall\, t,j \qquad (2.12)$$

While non-conforming items that do not meet quality requirements will be gathered and categorized as $TSCN_{tj}$, where they may be reworked, repaired, or scrapped, as shown as:

$$TSCN_{tj} = ESR1_{tj} + ESR5_{tj} \qquad \forall\, t,j \qquad (2.13)$$

It is important to emphasize that, during the process of selecting samples to be inspected, this project considers a realistic assumption that a fraction of the non-conforming parts produced will pass to the next production station without being detected by the quality sampling system as stated in the following equation:

$$FNCI_{tj} = \mathrm{V}2_{tj} - SI2_{tj} \qquad \forall\, t,j \qquad (2.14)$$

As mentioned, non-conforming parts will be subjected to reprocessing to establish them in their original state (As New as Original) and be reinstated in the production line. The first process is repair, which happens when it is unnecessary to intervene deeply in the part or replace parts. $F1_{tj}$ denotes the fraction of non-conforming parts, this fraction of item is deterministic. It is important to mention that because of incorrect quality classification by the operators (inspection error type I and II), some conforming parts are repaired unnecessarily, which will be registered as equation (2.15). Since the process is perfect, all parts are recovered and reinstated to the production line.

$$IREP1_{tj} = F1_{tj} * \left(ESR1_{tj} + ESR5_{tj}\right) \qquad \forall\, t,j \qquad (2.15)$$

While the non-conforming ones are denoted as:

$$IREP2_{tj} = F1_{tj} * ESR4_{tj} \qquad \forall\, t,j \qquad (2.16)$$

The second process is rework, which occurs when it is determined that contamination or damage is high; therefore, the items need further intervention or complete part replacement to be restored to as new as the original condition (see equation (2.18)). The fraction of items needing rework is denoted by $F3_{tj}$ which is also a deterministic value. However, some conforming items classified incorrectly may also be exposed to unnecessary rework as stated in equation (2.17).

$$IRWK1_{tj} = F3_{tj} * (ESR1_{tj} + ESR5_{tj}) \qquad \forall\, t,j \qquad (2.17)$$

$$IRWK2_{tj} = F3_{tj} * ESR4_{tj} \qquad \forall\, t,j \qquad (2.18)$$

Since the project considers that rework operations are not perfect due to the degree of complexity of the operation itself, it is assumed that there is a possibility that the reworked parts are not functional and that those originally conforming $IRWK1_{tj}$ now experience some contamination or imperfection. For this reason, a second inspection is proposed which generates three outputs, as shown in Figure 2.2 and the following equations:

The expected fraction of reworked sample articles, accepted as truly compliant during the second quality inspection process, where the percentage of rejection within this inspection process is given by θ_{tj} which assess the effectiveness of the rework and recovery process, based on how many reworked parts are successfully validated.

$$IRWK4_{tj} = (1 - \theta_{tj}) * IRWK3_{tj} \qquad \forall\, t,j \qquad (2.19)$$

While the expected fraction of reworked sample articles rejected as truly non-compliant at quality station for the second time is given by equation (2.20), see the following equation.

$$IRWK5_{tj} = \theta_{tj} * IRWK3_{tj} \qquad \forall\, t,j \qquad (2.20)$$

On the other side the reworked compliant sample articles that became non-compliant because of incorrect handling of the articles during the second inspection at the quality station, for the second time are classified as:

$$IRWK6_{tj} = \theta_{tj} * IRWK4_{tj} \qquad \forall\, t, j \qquad (2.21)$$

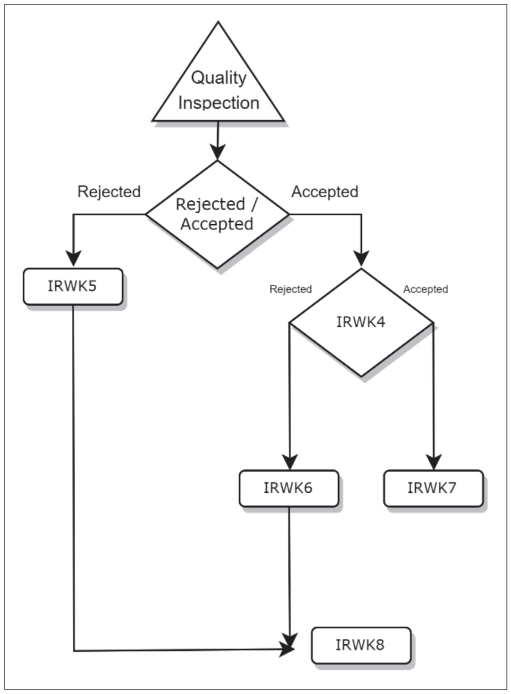

Figure 2.2 Second quality inspection process

After the parts have been re-evaluated, only a fraction of them can be reinstated into the production line as:

$$IRWK7_{tj} = IRWK4_{tj} - IRWK6_{tj} \qquad \forall\, t, j \qquad (2.22)$$

While the remaining ones are considered waste, as expressed in equations (2.23), (2.24) and (2.25), as well as the rejected after the second inspection station.

$$IRWK8_{tj} = IRWK5_{tj} + IRWK6_{tj} \qquad \forall\, t, j \qquad (2.23)$$

$$ISCR1_{tj} = F2_{tj} * (ESR1_{tj} + ESR5_{tj}) \qquad \forall\, t, j \qquad (2.24)$$

$$ISCR2_{tj} = F2_{tj} * ESR4_{tj} \qquad \forall\, t, j \qquad (2.25)$$

$$FSC_{tj} = IRWK8_{tj} + ISCR1_{tj} + ISCR2_{tj} \qquad \forall\, t, j \qquad (2.26)$$

Therefore, the accumulated scrap parts are collected as stated in the previous equation (2.26).

2.3 Problem Description

One of the main constraints within the serialized manufacturing system is to avoid stock; therefore, the same fraction of pieces of raw material represented by $RAW2_{tj}$ should the same fraction of finished products that must be shipped to the customer either the conforming items $V1_{t5}$ and non-conforming $V2_{t5}$ as stated in equation (2.27). Following the same path, the fraction of items produced and shipped to the customer cannot exceed the manufacturing system's production capacity; δ represents each machine's production capacity. Parameter δ is different for each work shift, since shift duration, operation efficency, maintenance schedules or deterioration can cause production capacity to fluctuate. Therefore, each shift is assigned a fixed production limit for each period. (see equation (2.28)).

$$RAW1_{t1} - (V1_{t5} + V2_{t5}) \geq 0 \qquad \forall\, t \qquad (2.27)$$

$$(V1_{t5} + V2_{t5}) \leq \delta \qquad \forall\, t \qquad (2.28)$$

A fundamental part of the assumptions of manufacturing systems is to have penalties issued by the customer due to shipping non-conforming final products. Therefore, in this case study, there is a penalty cost (ξ) for each non-conforming unit shipped to the customer; this cost is assigned to the last workstation ($j = 5$), because it is the station in charge of finalizing the product and should be capable of detecting the defects accepted as conforming ones, (see equation (2.29)). The (ξ) parameter is crucial for evaluating the trade-off between inspection costs and the financial consequences of poor quality, providing a comprehensive perspective on operational and economic performance within the manufacturing system.

$$RNCS2 = \sum_{t}^{T} V2_{t5} * \xi \qquad \forall\, t \qquad (2.29)$$

One of the essential parts of the manufacturing system is knowing when it is feasible and optimal to allocate QA stations with metrics that allow the mathematical model to determine the minimum required specifications, standards or tolerances to either reject or accept a produced item. Within this quality aspect, at least three of five criteria must be met to determine and approve allocating a quality inspection station after any manufacturing station that suffers from quality deterioration, as presented in the following equation:

$$MTX11_{tj} = \begin{cases} 1 & if \quad MT3_{tj} + MTX3_{tj} + MTX4_{tj} \\ & \quad + MTX7_{tj} + MTX10_{tj} \geq 3 \\ 0 & \quad\quad otherwise \end{cases} \qquad \forall\, t, j \qquad (2.30)$$

The first equation (2.31) establishes if the total proportion of conforming and non-conforming items rejected due to the need for an extra process, such as rework, repair, or should be scrapped, is lower than X (30%) of the total fraction of items that passed through inspection, a quality inspection station should be considered after a working machine.

$$MT3_{tj} = \begin{cases} 1 & if \quad (ESR4_{tj} + TSCN_{tj}) \geq ((SI1_{tj} + SI2_{tj}) * X) \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.31)$$

The second equation (2.32) is based on whether the expected fraction of non-conforming units produced in a previous manufacturing operation in its subsequent operation is higher than the half of the fraction denoted as $(X4)$ of conforming items produced in this subsequent operation $j = n + 1$ (see Figure 2.3); a quality inspection should be considered. This half percentage is chosen by the manufacturer as a deterministic parameter to detect trends in defects, quality deviations, consistent level of efficiency on the process. As well provides a practical decision making tool, allowing manufacturers to fine tune their quality control thresholds based on historical performance data.

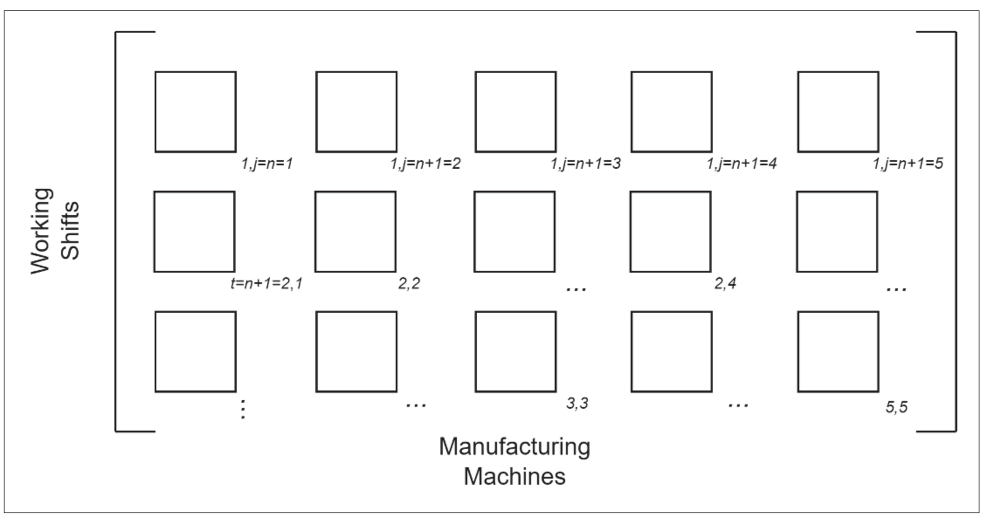

Figure 2.3 Manufacturing machines & working shifts structure

$$MTX3_{tj} = \begin{cases} 1 & if \quad EW2_{tj} \geq (EW2_{tj} * X4) \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.32)$$

The third mathematical statement (2.33) considers installing a quality inspection station if the fraction of conforming items rejected by error type-I, during inspection process is higher or equal to the fraction of sample non-conforming items rejected as truly non-conforming at the quality station j (repair, rework, or scrap).

$$MTX4_{tj} = \begin{cases} 1 & if \quad ESR1_{tj} \geq ESR4_{tj} \\ 0 & \quad otherwhise \end{cases} \qquad \forall\, t, j \quad (2.33)$$

The fourth equation (2.34) shows that a quality inspection station can be allocated if the fraction of sample-non-conforming items accepted as conforming by error type-II (β) at the quality station is higher than the difference from the final fraction of conforming items that will be transferred to the next manufacturing station $j = n + 1$, minus the fraction of sample non-conforming items accepted as conforming by error type-II at the sampling quality station.

The fifth and last equation (2.35) suggest a quality inspection station should be considered if the total scrap cost, which encompasses material losses, production time and disposal costs, (s_{tj} unit scrap cost) from the manufacturing operation is higher than the total inspection cost (i_{tj} unit inspection cost), this inspection cost includes direct costs such as labor, testing materials and time consumed by the inspection process. It reflects the investment required to dected non-conforming items before they proceed to the next stage.

$$MTX7_{tj} = \begin{cases} 1 & if \quad ESR2_{tj} \geq \left(FCI_{tj} - ESR2_{tj}\right) \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.34)$$

$$MTX10_{tj} = \begin{cases} 1 & if \quad \left(ISCR1_{tj} + ISCR2_{tj} * s_{tj}\right) \geq \\ & \quad \left(SI1_{tj} + SI2_{tj} * i_{tj}\right) \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.35)$$

The second phase of the mathematical model analysis is the decision to integrate preventive maintenance strategies in working machines based on their health. In this phase, the decision-making considers five criteria, of which at least one must be fulfilled to apply maintenance.

Compared with the decision-making of quality inspection stations to integrate preventive maintenance, it was decided that at least one criterion should be met, as increasing the number of requirements would increase the production and economic risks of machine failures, (see equation (2.36)).

$$MTZ11_{tj} = \begin{cases} 1 & if \quad MT6_{tj} + MTZ7_{tj} + MTZ8_{tj} \\ & \quad +MTZ9_{tj} + MTZ10_{tj} \geq 1 \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.36)$$

Equation (2.37) sets when the fraction of conforming and non-conforming items rejected as scrap during quality inspection exceed the fraction rejected due to the need for repair or rework. In this case, preventive maintenance should be considered.

$$MT6_{tj} = \begin{cases} 1 \quad if \quad (ISCR1_{tj} + ISCR2_{tj}) \geq \\ \quad (IREP1_{tj} + IREP2_{tj} + IRWK7_{tj}) \\ 0 \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.37)$$

The second equation (2.38) states that if the expected fraction of reworked items that were rejected as truly non-conforming during the second inspection exceeds a deterministic fraction of $X5$ (20%) of conforming items rejected due to the need for repair or rework. Then, preventive maintenance should be considered. In practical terms, X5 acts as a predictive indicator, creating a balance operational costs, quality assurance, and equipment reliability. It provides a proactive mechanism to identify and correct process degradation before it affects downstream operations or customer satisfaction.

$$MTZ7_{tj} = \begin{cases} 1 \quad if \quad \theta_{tj} * IRWK3_{tj} \geq (F3_{tj} * ESR4_{tj}) * X5 \\ 0 \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.38)$$

Equation (2.39) states that preventive maintenance should be performed if the expected fraction of non-conforming items requiring rework and repair, exceed the expected fraction of conforming items that require the same process.

$$MTZ8_{tj} = \begin{cases} 1 \quad if \quad IRWK2_{tj} + IREP2_{tj} \geq IRWK1_{tj} + IREP1_{tj} \\ 0 \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.39)$$

As machines deteriorate over the course of work shifts, the risk of failure or shutdown increases accordingly. When a machine reaches 100% deterioration, the exact moment of failure and the quality of output it can produce become uncertain.

$$MTZ9_{tj} = \begin{cases} 1 \quad if \quad \varepsilon_{tj} \geq 1 \\ 0 \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.40)$$

Therefore, to prevent the machine from operating beyond its critical point, the following equation (2.40) establishes that maintenance must be performed when the machine reaches 100% deterioration or a state where the machine is susceptible to failure at any time, regardless of whether this occurs suddenly or over time.

$$MTZ10_{tj} = \begin{cases} 1 \quad if \quad (\varepsilon_{tj} * M1_{tj}) \geq ((1 - \varepsilon_{tj}) * M1_{tj}) * X6 \\ 0 \quad otherwise \end{cases} \qquad \forall\, t, j \quad (2.41)$$

While the last following equation (2.41) determines whether maintenance should be performed on the working machine and whether the fraction of non-conforming production exceed the manufacturer-specified fraction ($X6$) of conforming items. The X6 parameter is a practical tool for continuous process monitoring, enabling manufacturers to align operational performance with strategic quality and reliability goals.

2.3.1 Case 1–Uniform deterioration factor

In the first case, each of the five different machines that make up the manufacturing system is subject to a deterioration factor that remains constant and unchanged on each machine, regardless of the complexity of machine operation, throughout the first seven days of test production.

This deterioration decreases the machine performance at a steady rate (see Table 2.1), therefore its behavior is linear over the time. It happens as a result of constant and normal routine use, factors such as heat, vibration and mechanical stress among others and not because of random failures. For example, cutting tools (CNC machines) lose sharpness evenly after a set of different parts, impacting the cut quality or a pneumatic press which could slowly lose strength over time, generating different levels of compaction.

Table 2.1 Deterioration level (Day 1 to 7)–(Case 1)

Stage j (Working station)	**Period t**		
	1	**2**	**3**
1	0.020	0.030	0.040
2	0.020	0.030	0.040
3	0.020	0.030	0.040
4	0.020	0.030	0.040
5	0.020	0.030	0.040

During the inspection process, it exists a probability of 5% of type I and type I inspection errors. This same rate is presented in the probability of product damage or contamination due to handling in the quality inspection process Ω. Additionally, this same percentage is present in the probability that a reworked part does not pass the second quality inspection test because the rework process Θ is imperfect, (see Table 2.2, Table 2.3 Table 2.4).

Table 2.2 Quality Inspection error rate I (α) and II (β) - (Case 1)

Period t (Working Shift)	**1**	**2**	**3**
α, β	0.050	0.050	0.050
	0.050	0.050	0.050
	0.050	0.050	0.050
	0.050	0.050	0.050
	0.050	0.050	0.050

Table 2.3 Inspection damage (Ω) - (Case 1)

Period t (Working Shift)	1	2	3
	0.050	0.050	0.050
	0.050	0.050	0.050
Ω rates	0.050	0.050	0.050
	0.050	0.050	0.050
	0.050	0.050	0.050

Table 2.4 Fail rate of second quality inspection (Θ) - (Case 1)

Period t (Working Shift)	1	2	3
	0.050	0.050	0.050
	0.050	0.050	0.050
Θ rates	0.050	0.050	0.050
	0.050	0.050	0.050
	0.050	0.050	0.050

On the other side, it exists a 30% probability that the entire production could be rejected during the quality inspection process and categorized as scrap $F2_{tj}$ because it does not meet the necessary conditions to be repaired or reworked. In comparison, those that only meet the criteria for repair $F1_{tj}$ which represent 20% of the total production, and finally, those that can only be reworked $F3_{tj}$ represent 50% of the total production. (See Figure 2.4)

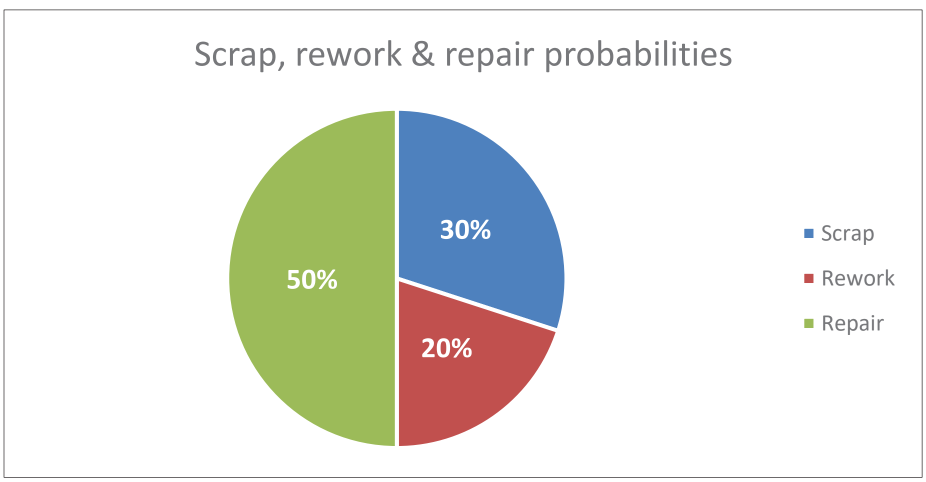

Figure 2.4 Scrap, rework, repair inspection probabilities

As noted earlier, each machine in the production line performs different manufacturing operations, consuming diverse resources at varying rates. Therefore, the cost of rework λ_{tj} and repair κ_{tj} presented in Table 2.5 is considered being different. In this case, the scrap cost is also variable between each machine because the more machines the product advances, the cost increases in proportion to its added value, as well as the scrap cost s_{tj}, (see Table 2.6).

Table 2.5 Repair and rework costs–(Case 1)

Unit repair cost κ_{tj}				Unit rework cost λ_{tj}			
Stage j (Working station)	**Periods t / Working Shifts**			**Stage j (Working station)**	**Periods t / Working Shifts**		
	1	**2**	**3**		**1**	**2**	**3**
1	$0.540	$0.540	$0.540	**1**	$3.600	$3.600	$3.600
2	$0.180	$0.180	$0.180	**2**	$1.080	$1.080	$1.080
3	$0.180	$0.180	$0.180	**3**	$1.080	$1.080	$1.080
4	$0.360	$0.360	$0.360	**4**	$2.160	$2.160	$2.160
5	$0.540	$0.540	$0.540	**5**	$3.240	$3.240	$s3.240

Table 2.6 Scrap cost–(Case 1)

Stage j (Working station)	**Periods t / Working Shifts**		
	1	**2**	**3**
1	$3.324	$3.324	$3.324
2	$1.108	$1.108	$1.108
3	$1.108	$1.108	$1.108
4	$2.216	$2.216	$2.216
5	$3.324	$3.324	$3.324

The condition-based preventive maintenance and quality inspection costs are different for each workstation but remain the same for each work shift; in this case, the cost is associated with the type of machine rather than the performance time, (see Table 2.7).

As mentioned, two test runs were performed on the manufacturing system, one before implementing the optimization strategies of the mathematical model and one after the strategies generated based on the results of the first test run, to analyze the differences between each of the scenarios.

Table 2.7 QA Inspection and CBPM costs

Unit QA inspection cost i_{tj}				Unit CBPM cost μ_{tj}			
Stage j (Working station)	Periods t / Working Shifts			Stage j (Working station)	Periods t / Working Shifts		
	1	2	3		1	2	3
1	$0.216	$0.216	$0.216	1	$20	$20	$20
2	$0.072	$0.072	$0.072	2	$10	$10	$10
3	$0.072	$0.072	$0.072	3	$6.666	$6.666	$6.666
4	$0.144	$0.144	$0.144	4	$10	$10	$10
5	$0.216	$0.216	$0.216	5	$20	$20	$20

Table 2.8 Forward manufacturing costs after QA & PM optimization - (Case 1)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total maintenance cost	$793.31	$560.00	-29.41%	dls
Total penalty cost	$974,842.35	$766,731.03	-21.35%	dls
Total scrap cost	$22,206.69	$20,391.87	-8.17%	dls
Total rework cost	$36,075.86	$33,127.59	-8.17%	dls
Total repair cost	$2,405.06	$2,208.51	-8.17%	dls
Total inspection cost	$30,386.97	$30,403.18	0.05%	dls

In the first scenario, two costs aligned with the objective equation decreased by over 10% when QA and CBPM strategies were applied while 4 up to 5%, compared to prior tests without these strategies. Notably, total maintenance costs for future production runs experienced the most significant changes, dropping by up to 20%. This reduction was attributed to implementing seven different CBPM actions, including two crucial ones on the Type A initial machine, which means critical operation, (see (Table 2.8) and (Figure 2.5)).

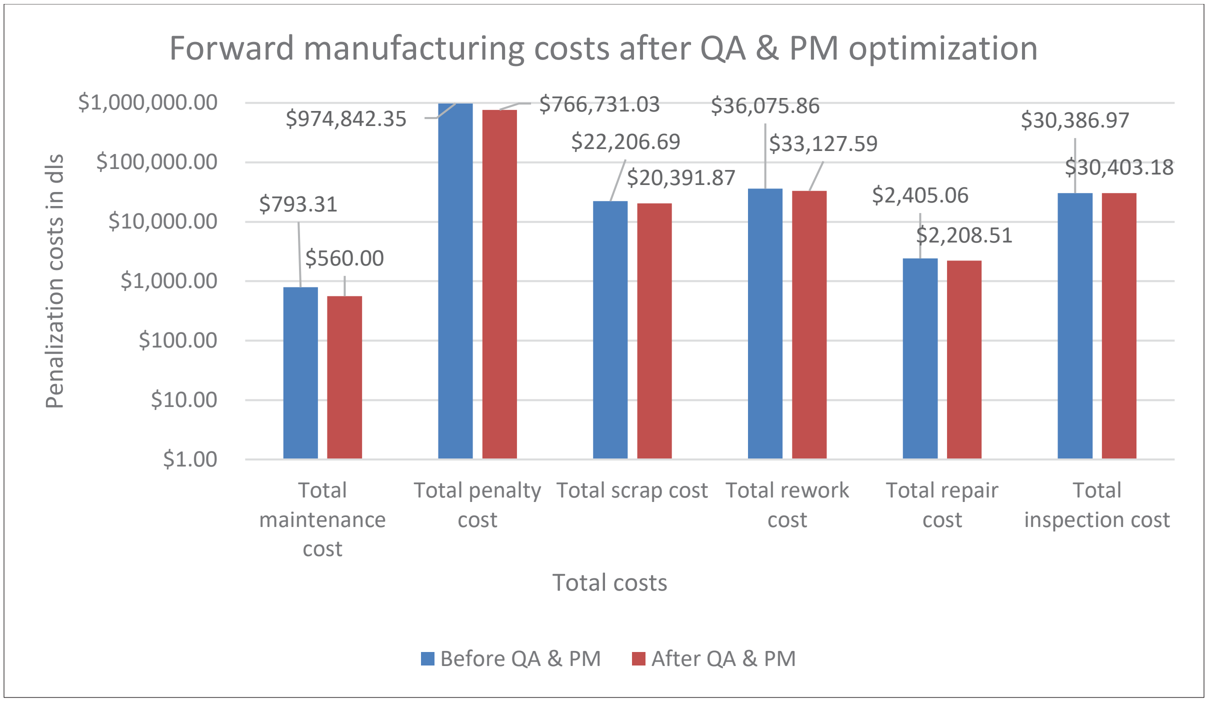

Figure 2.5 Forward manufacturing costs after QA & PM optimization

Table 2.9 Forward manufacturing profit after QA & PM optimization - (Case 1)

Costs	Before QA & PM		After QA & PM		Difference	Unit
Total profit	$	2,978,935.31	$	3,333,718.16	11.910%	dls

Conversely, with the production line yielding more conforming items, the manufacturer slashed penalty costs up to 20% compared to before. Additionally, repair, rework, and scrap costs decreased around 8.172% at the same rate as before optimization. QA inspection increased by 0.053% due to more conforming items selected for sample inspection, as the mathematical model does not assign any quality inspection station. Finally, by incorporating the different maintenance activities, the manufacturing system got an 11.910% of profit (see Table 2.9) and an optimal response from the mathematical model, which could satisfy all the constraints.

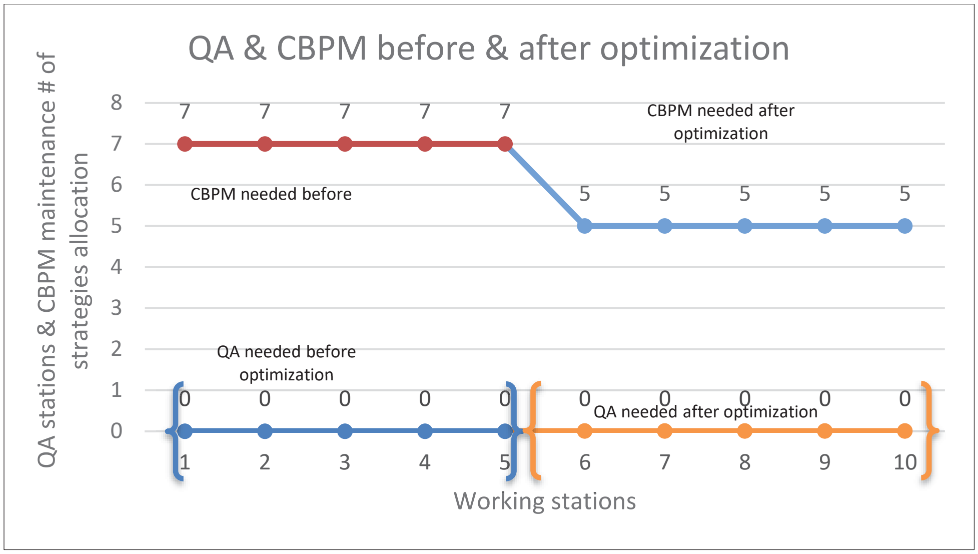

Figure 2.6 QA & CBPM before and after optimization

Of the five criteria imposed as metrics to establish the application of preventive maintenance at the workstations, the manufacturing system met two of them. The first workstation produced defective products, and even after being reworked, over 20% of those products needed a second rework, both in the first and second shifts. While at stations three, four, and five, the fraction of reworked and new reworked non-conforming items was higher than the conforming ones that were also reworked and repaired at the second and third shifts.

After implementing the mathematical model's optimization recommendations during the second production run, it suggested maintaining preventive maintenance for the first machine during the first and second shifts and for the fourth and fifth stations during the third shift. However, due to machinery improvements, maintenance during the second shift for the fourth and fifth stations and the third station in the last shift was deemed unnecessary, (see Figure 2.6).

2.3.2 Case 2–Increasing deterioration factor

One of the fundamental assumptions of the manufacturing system is that raw materials must be free of defects at the beginning of each production shift as shown in Table 2.10; therefore, the input of raw material will be the same in each of the cases and working shifts as presented in Table 2.11.

Table 2.10 Incoming Material Fraction for Serial Multi-Stage Manufacturing per shift ($QRM1_t$)

Period t	Incoming raw material per working shift
1	40000 pcs
2	20000 pcs
3	16000 pcs

Table 2.11 Incoming non-conforming fraction of raw materials for Serial-Multi-Stage-Manufacturing-System per shift

Period t	Incoming raw material per working shift
1	0 pcs
2	0 pcs
3	0 pcs

In this scenario, the manufacturing system was studied under a deterioration factor that increases steadily and exponentially throughout each production day, worsening the condition of the manufacturing machines at an accelerating rate. Unlike the uniform (linear) deterioration, where the deterioration happens steadily, increasing deterioration means each unit of time or usage causes more damage than the last. This deterioration factor is variable depending on the type of operation, complexity and necessary resources of each working machine (seen in Figure 2.7. and Figure 2.8). One of the many characteristics is that machines may show little sign of failure early on, but problems compound quickly near the shifts.

Examples of increasing deterioration withing the manufacturing can be found in production's machine engines when they experience insulation breakdown, leading to arcing and faster wear or when the lubricant oil suffers degradation, friction increases, leading to heat and component damage. This scenario also encompasses explore both type I and II inspection errors at an equivalent rate of 5% as the previous scenario. Moreover, the probabilities associated with potential product damage or contamination due to product handling at the quality inspection process and second inspection error for items already reworked share this same percentage of 5%.

The percentage of conforming and non-conforming items already inspected, which will be classified in three different scenarios based on the severity of the production errors and their potential to be reworked $F3_{tj}$, repaired $F2_{tj}$, or discarded $F1_{tj}$. The cost of these scenarios still varies depending on the type of machine, the resources required to operate it, and its operational complexity, but the data for analysis is shared similarly with the previous case (see Table 2.5 and Table 2.6). Finally, the maintenance and quality inspection costs are still different for each workstation, (see Table 2.12).

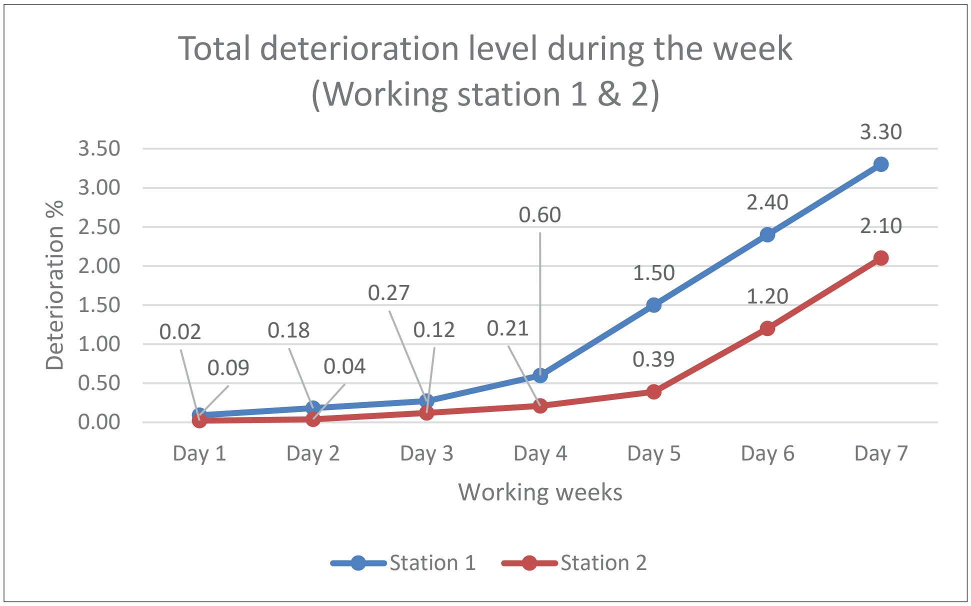

Figure 2.7 Total deterioration level of working station 1 and 2 during the week

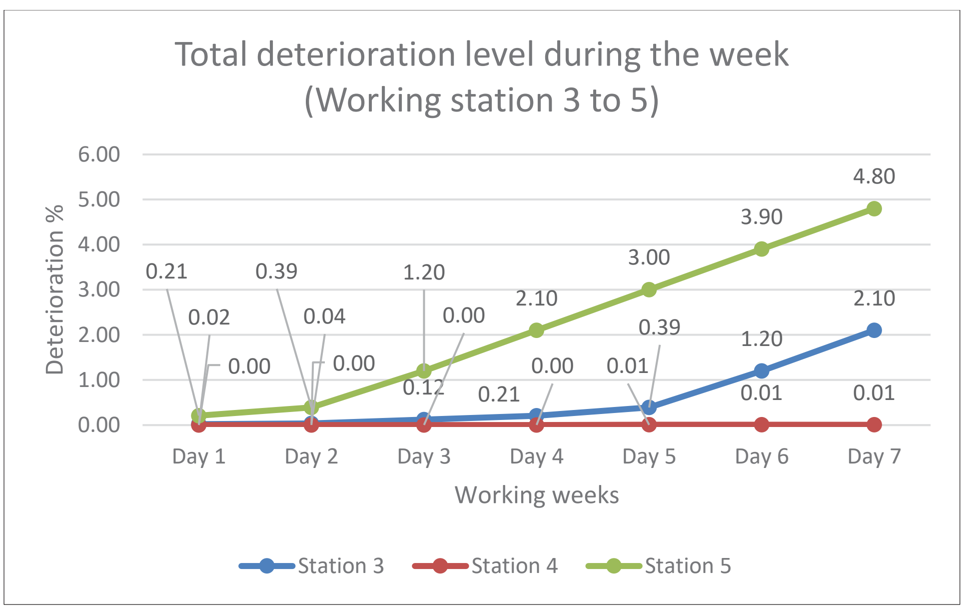

Figure 2.8 Total deterioration level of working station 3 to 5 during the week

Table 2.12 QA Inspection and CBPM costs

Unit QA inspection cost i_{tj}				Unit CBP maintenance cost μ_{tj}			
Stage j (Working station)	Periods t / Working Shifts			Stage j (Working station)	Periods t / Working Shifts		
	1	2	3		1	2	3
1	$0.216	$0.216	$0.216	1	$20	$20	$20
2	$0.072	$0.072	$0.072	2	$10	$10	$10
3	$0.072	$0.072	$0.072	3	$6.666	$6.666	$6.666
4	$0.144	$0.144	$0.144	4	$10	$10	$10
5	$0.216	$0.216	$0.216	5	$20	$20	$20

During the first production test, the manufacturing system was studied with no optimization strategies since the test will provide performance data to the mathematical model to understand the system behavior in areas where an opportunity for improvement exists.

Table 2.13 Forward manufacturing costs after QA & PM optimization - (Case 2)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total inspection cost	$29,990.38	$122,711.51	309.17%	dls
Total maintenance cost	$1,133.33	$986.67	-12.94%	dls
Total repair cost	$8,320.53	$25,035.87	200.89%	dls
Total rework cost	$110,001.53	$381,363.22	246.69%	dls
Total scrap cost	$67,712.05	$231,164.55	241.39%	dls
Total penalty cost	$5,565,940.85	$3,362,521.38	-39.59%	dls

In this instance, two objective equation costs in the manufacturing system could decrease, while others increased by as much as 100% compared to the non-optimized system (see Table 2.13). After optimization, the total maintenance decreases by over 10% since the condition of the machines improves; nonetheless as a result of incremental deterioration level with higher percentages during the last days of the week, the manufacturing system still presents a dependence on CBPM activities in over 50% of the same working machines.

As a result of the CBPM decrement, the mathematical model suggest an increment of allocation of QA inspection stations which surpass the 50% of the previous QA stations, resulting in an exponential cost increment of inspection up to 300% against the first case without optimization. As a consequence of more QA stations, the manufacturing system was capable of analyzing and detecting a higher number of possible non-conforming items, resulting in post-production cost activities of up to 200% for the repair, rework, and even scrap, (see Table 2.14)

Table 2.14 Forward manufacturing shipping items QA & PM optimization - (Case 2)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total fraction of conforming items sent to the customer	183,185.10	288,595.39	57.54%	pcs

due to this new manufacturing non-conforming items detection capability, the penalty gets as a benefit a cost reduction of up to 30% and over 50% of conforming items shipped to customers, improving the profitability of the manufacturer against the first scenario.

Table 2.15 Forward manufacturing profit after QA & PM optimization - (Case 2)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total profit	$ -4,871,404.10	$ -1,951,568.11	-59.94%	dls

Despite the previous increment in QA stations and non-conforming shipments reduction, during the 1st week after the optimization suggestions, the manufacturing system still perceives economic losses after the mathematical model suggestions, the manufacturing system has a higher chance to reach profitability in the next coming production runs, since the increment was 59.94%, considering the improvements keeps for the next time with no changes or situations out of control, (see Table 2.15).

2.3.3 Case 3 – Random deterioration factor

The third case of the forward manufacturing system is a case where the machines in the production line are subjected to a random level of deterioration that refers to the irregular, unpredictable degradation of machines j or its components over the three working shifts, in different magnitudes depending on the complexity of each machine and its work. Such deterioration may exceed the maximum operating point $\varepsilon = 1$ of the machine before shutting down. Unlike uniform or increasing deterioration (which follows a pattern), random deterioration does not follow any pattern. Its timing, rate and severity can vary unpredictably (see Table 2.16).

Random deterioration can be affected by numerous uncontrolled factors, such as material defects, unexpected interactions between components, environmental exposure such as corrosion, temperature or humidity. Sensors and electronics could experience random failures from thermal fatigue, or hydraulic systems may experience internal seal random failures due to unseen cracks or pressure surges.

Table 2.16 Deterioration level (ε_{tj})–(Case 3)

Stage j (Working station)	Day 1			Day 2			Day 3		
	1	2	3	1	2	3	1	2	3
1	0,020	0,030	0,040	0,050	0,060	0,070	0,080	0,090	0,100
2	0,006	0,007	0,008	0,009	0,010	0,020	0,030	0,040	0,050
3	0,006	0,007	0,008	0,009	0,010	0,020	0,030	0,040	0,050
4	0,0004	0,0005	0,0006	0,0007	0,0008	0,0009	0,0011	0,0012	0,0013
5	0,060	0,070	0,080	0,090	0,100	0,200	0,300	0,400	0,500

Stage j (Working station)	Day 4			Day 5			Day 6		
	1	2	3	1	2	3	1	2	3
1	0,100	0,200	0,300	0,400	0,500	0,600	0,700	0,800	0,900
2	0,060	0,070	0,080	0,090	0,100	0,200	0,300	0,400	0,500
3	0,060	0,070	0,080	0,090	0,100	0,200	0,300	0,400	0,500
4	0,0013	0,0014	0,0015	0,0016	0,0017	0,0018	0,0019	0,0020	0,0021
5	0,600	0,700	0,800	0,900	1,000	1,100	1,200	1,300	1,400

Stage j (Working station)	Day 7		
	1	2	3
1	1,000	1,100	1,200
2	0,600	0,700	0,800
3	0,600	0,700	0,800
4	0,0022	0,0023	0,0024
5	1,500	1,600	1,700

As with scenarios one and two, the values of the fraction of inspection errors type I and II, the fraction of probability of contamination and damage of parts during inspection, and reworked parts rejected during their second quality inspection remain similar. This rule also applies to the percentages of parts categorized for rework $F3_{tj}$ (see Table 2.2), repair $F1_{tj}$ (see Table 2.5), or scrap $F2_{tj}$ (see Table 2.6), and their respective operating costs as shown in Table 2.8 and Table 2.9.

The unit costs per quality inspection of the inspection stations to be installed, as well as the cost of maintenance activities, also present the same data as the previous scenarios 1 and 2 (see Table 2.10), following the complexity category of each machine that conforms to the production line (see Table 2.17).

Table 2.17 Machine category

Machine	Category	Operational complexity
1	A	High
2	B	Medium
3	C	Low
4	B	Medium
5	A	High

Throughout the test scenarios before and after the mathematical model's suggestions, results show to have a higher impact on the manufacturing system performance, even if the system experienced random deterioration rates. Since the deterioration factor was presented randomly in this scenario with no distinguishable behavior, the mathematical model found that since the second day was near to surpassing the optimal operation state of the machine, therefore the suggestion pass to allocate QA inspection stations before all the production stations $j = 1 \dots 5$ as well as CBPM, (see (Table 2.18) and (Table 2.19)).

Table 2.18 Second-day deterioration level example

Stage j (Working station)	**Period t**		
	1	**2**	**3**
1	0.43	0.34	0.98
2	0.18	0.77	0.88
3	0.34	0.04	0.37
4	0.71	0.51	0.23
5	0.36	0.87	0.6

Table 2.19 QA & CBPM suggestions after optimization

QA suggestions - Day 1 to Day 7							**CBPM suggestions - Day 1 to Day 7**					
Working Stations/Shifts	**1**	**2**	**3**	**4**	**5**		**Working Stations/Shifts**	**1**	**2**	**3**	**4**	**5**
1	1	1	1	1	1		**1**	1	1	1	1	1
2	1	1	1	1	1		**2**	1	1	1	1	1
3	1	1	1	1	1		**3**	1	1	1	1	1

As in the first scenario where the deterioration factor was uniform throughout the week, scrap, in case three, rework and repair costs increased significantly up to 300% compared to the manufacturing systems with no QA and CBPM support and at least up to 200% when the deterioration factor was incremental every day. However after the first optimization changes, for the next production runs, the manufacturing system decreased the number of QA up to 50% with 43 remaining QA inspection station along the entire week and 28 CBPM fewer activities than first scenario without optimization.

Table 2.20 Forward manufacturing costs after QA & PM optimization - (Case 3)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total inspection cost	$29,463.42	$254,732.53	764.57%	dls
Total maintenance cost	$1,306.67	$1,173.33	-10.20%	dls
Total repair cost	$10,376.25	$43,710.41	321.25%	dls
Total rework cost	$160,011.09	$685,212.23	328.23%	dls
Total scrap cost	$95,807.36	$403,592.81	321.25%	dls
Total penalty cost	$6,844,259.05	$1,263,324.14	81.54%	dls

Although the manufacturing system reduced penalty costs, surpassing an 80% improvement, the penalty cost still affected the manufacturing system during the first week after the optimization suggestions. This penalization cost is translated as complications in obtaining the optimal profit since, after the optimization, the manufacturer still experienced these profit difficulties.

Table 2.21 Forward manufacturing profit after QA & PM optimization - (Case 3)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total profit	$ -7,255,214.41	$ -461,446.14	-93.64%	dls

2.4 Conclusion

After conducting a series of production tests where the suggestions for assigning preventive maintenance activities and quality inspection stations of the mathematical model in question were applied, the following key points were observed.

The manufacturing systems in case number two, whose deterioration was increasing and constant, as well as in the case where the deterioration did not follow a clear pattern, are considerably affected by the deterioration factor because the costs derived from customer penalties are higher than the profits generated by the production of conforming products. However, when the manufacturing system is under a constant level of spoilage over time, it can generate profits because its penalty costs are below 25% of sales, (see (Table 2.24) and Figure 2.9).

Table 2.22 Summary total production costs after optimization - (Case 1, Case 2, Case 3)

Costs	Case 1		Case 2		Case 3	
	Production based	Sales based	Production based	Sales based	Production based	Sales based
Total inspection cost	2.460%	0.912%	6.423%	-6.288%	10.039%	-55.203%
Total maintenance cost	0.045%	0.017%	0.052%	-0.051%	0.046%	-0.254%
Total repair	0.179%	0.066%	1.311%	-1.283%	1.723%	-9.472%
Total rework cost	2.681%	0.994%	19.963%	-19.541%	27.003%	-148.492%
Total scrap cost	1.650%	0.612%	12.100%	-11.845%	15.905%	-87.463%

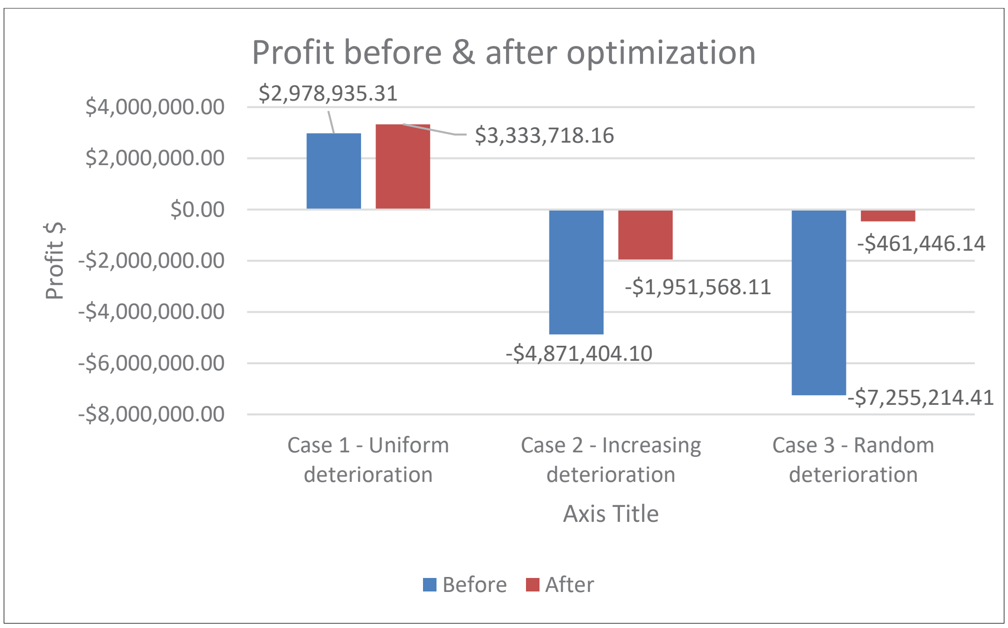

Figure 2.9 Profit after QA and PM optimization

The manufacturing system of the second case could ship 54.247% of conforming products concerning the total percentage of raw material entering the system. Case number three fell 1.936% below case number two (see Table 2.25). This result is mainly derived from the condition of the manufacturing systems from day four for case two and day two for case three, which dictate applying preventive maintenance on all workstations in the three different work shifts and assigning quality inspection stations on more than 50% of the production lines from the second work shift onwards.

The percentage of rework present in each of the three cases affected the level of production and economic losses since it turned out to be greater than even the percentage of repairs and scrap combined, especially in cases two and three (see Figure 2.10). In addition, as explained in Chapter One, rework activities are imperfect, which causes rework costs to increase; however, due to the nature of rework and α as well as β inspection errors, the penalty rate is still present.

Table 2.23 Summary total production after optimization - (Case 1, Case 2, Case 3)

Produced Items	Case 1		Case 2		Case 3	
	Based-raw material	%	Based-raw material	%	Based-raw material	%
Conforming items ship	478,817.433	90.003%	288,595.39	54.247%	278,295.49	52.311%
Nonconforming Items ship	44,969.562	8.453%	197,215.33	37.071%	74,095.29	13.928%
Total items through QA	211,248.929	39.708%	769,956.21	144.729%	1,798,910.41	338.141%
Total items for repair	6,140.460	1.154%	58,568.30	11.009%	120,342.03	22.621%
Total items for rework	15,351.149	2.886%	146,420.74	27.523%	300,855.07	56.552%
Total items for scrap	9,210.690	1.731%	87,852.45	16.514%	180,513.04	33.931%
Total items not recycled	8,213.005	1.544%	46,189.28	8.682%	179,609.23	33.761%

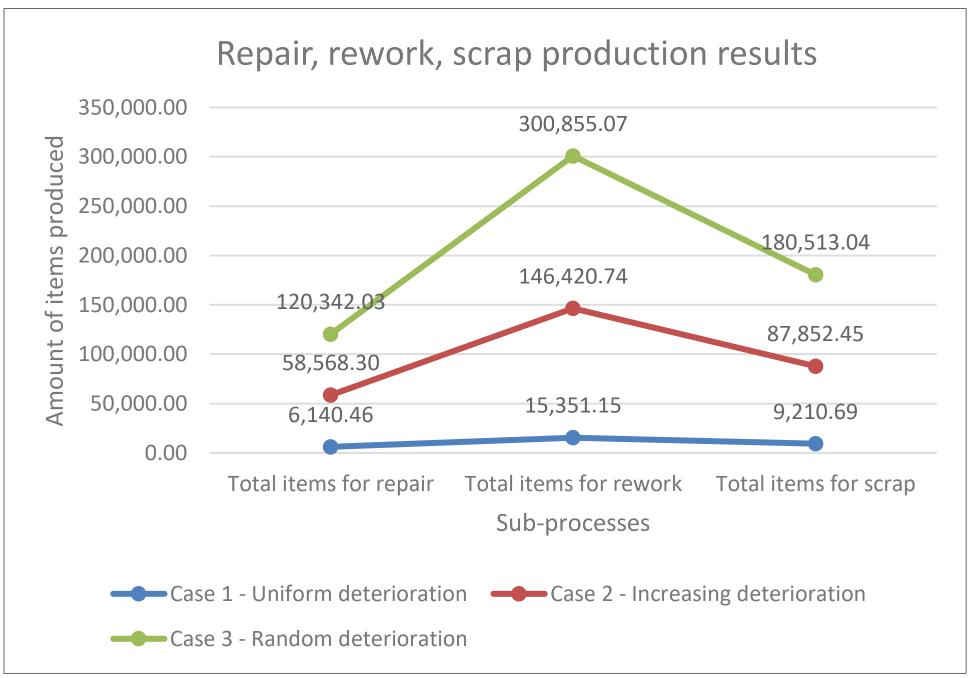

Figure 2.10 Fraction of repair, rework & scrap items produced based on total incoming fraction of raw material supply to the manufacturing system

The high volume of penalty costs is the factor with the most significant impact on the economic performance of the manufacturing systems in each of the cases presented above, representing over 50% of the production costs, (see Table 2.24 and Figure 2.11).

Table 2.24 Summary total penalty costs after optimization - (Case 1, Case 2, Case 3)

Penalty cost	Case 1		Case 2		Case 3	
	Production based	Sales based	Production based	Sales based	Production based	Sales based
	62,043%	22,999%	176,013%	-172,298%	49,785%	-273,775%

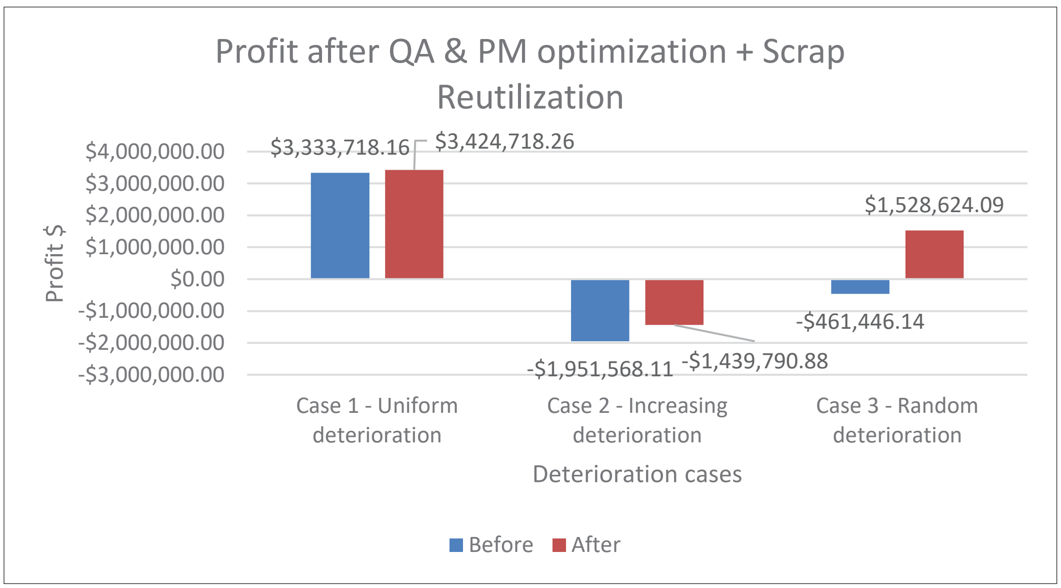

Figure 2.11 Profit after QA & PM optimization + Scrap Reutilization

Table 2.25 Profit + Scrap next forecast - (Case 1, Case 2, Case 3)

Profit + scrap (Difference)	Case 1	Case 2	Case 3
	2,66%	-35,55%	130,19%

Through this first scenario, it was possible to analyze how the proposed mathematical model was able to provide optimal strategies for assigning quality inspection and preventive maintenance stations to traditional manufacturing systems with linear processes, which were under different deterioration factors that changed over the production time invariably from the operational complexity of each machine that makes up the production line.

However, as a result of a lack of a system for handling and re-using non-conforming materials, manufacturing systems still miss an excellent opportunity to generate more economic gains from these materials by reworking and selling them to a secondary market with different characteristics, which is estimated to offset the expenses generated by incorrect manufacturing of these materials (see (Figure 2.11) and Table 2.25)).

Looking ahead, our research delves into the study of a reverse re-manufacturing system. This approach aims to harness the potential of the non-conforming materials generated in the initial forward manufacturing processes by repurposing these materials to supply markets with unique characteristics and needs of the same product family, seeking economic gains.

CHAPTER 3

PART QUALITY INSPECTION AND CONDITION-BASED PREVENTIVE MAINTENANCE ALLOCATION STRATEGIES IN A DETERIORED SERIAL REVERSE-MULTI-STAGE REMANUFACTURING SYSTEM

3.1 Introduction

Chapter three presents a complex and intriguing study of a reverse re-manufacturing system with a linear material flow (see Figure 3.1). Unlike its manufacturing counterpart, this system operates without the intervention of alternate processes or materials. Similar to the forward manufacturing system, the production lines are composed of 5 distinct working machines from the same family.

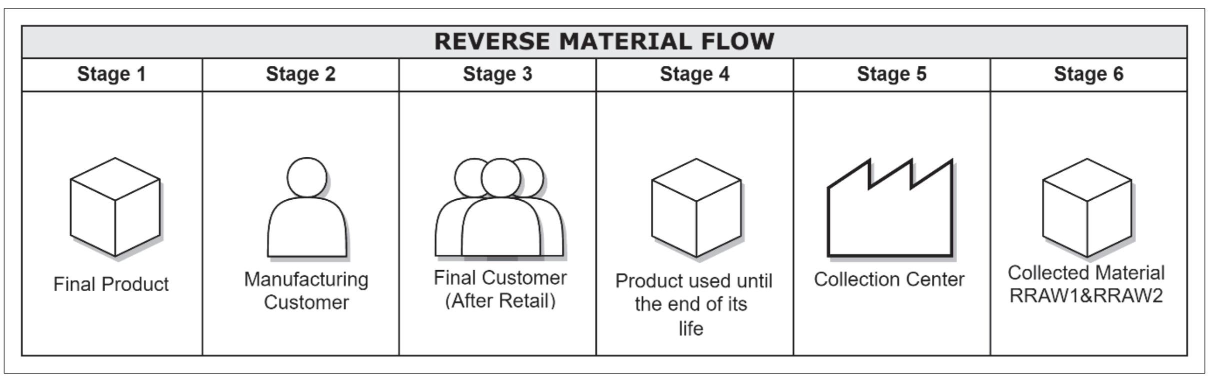

Figure 3.1 Remanufacturing stages

It is crucial to note that each of these machines undergoes a level of deterioration due to their daily operations. This deterioration, which is uniform, incremental, and random, affects the fraction of conforming and non-conforming products. This variation in product quality affects customer satisfaction and the remanufacturer's profit. The fraction of conforming and non-conforming products also fluctuates due to the quality inspection errors operations, leading us to consider the following assumptions.

One of the constraints in the re-manufacturing system is the prohibition of rework activities. This is due to the potential risk it poses to the customer, as the product is re-manufactured and may have had an initial imperfection. As a result, the conforming work factor is low:

- Repair activities are also not allowed in the re-manufacturing system due to the same situation as rework activities.
- The re-manufacturing system provides the same family of products to a second market with unique characteristics and prices than the primary market.

- The scrap generated in the re-manufacturing system is not contemplated for reuse as a raw material; it is discarded.
- The handling and inspection activities of the processed parts can potentially cause contamination and damage to the re-manufacturing system during this inspection process, so these parts are also automatically discarded without the possibility of a second quality inspection to reduce the risk of a non-conforming product being sent to the customer.

3.2 Reverse Remanufacturing System Description

The production lines of the re-manufacturing system $k = 1 \ldots 5$ are composed of the same family of machines used in the forward manufacturing echelons of the previous chapter because both production systems produce the same family of products.

Unlike the previous chapter, where the forward manufacturing system used only virgin raw material coming from external suppliers, the re-manufacturing system, on the contrary, is fed by non-conforming products coming from the same manufacturing system and from products EOL collected from customers $RQRM1_{tk}$. However, since the quality of both materials is variable in terms of quality, the re-manufacturing system has a classification of compliant raw material (see equation (3.1)) and non-compliant too (see equation (3.2)) entering the system. This classification allows the mathematical model to determine the error fractions later on.

$$RV1_{tk} = (1 - r\varepsilon_{tk}) * RM1_{tk} + (1 - r\varepsilon_{tk}) * REW1_{tk} \qquad \forall\, t, k \quad (3.1)$$

$$RV2_{tk} = (r\varepsilon_{tk} * RM1_{tk}) + REW2_{tk} + (r\varepsilon_{tk} * REW1_{tk}) \qquad \forall\, t, k \quad (3.2)$$

After the re-manufacturing operations are completed, the system includes quality inspection sampling operations, which provide data for the mathematical model to determine the fraction of conforming items (3.3) and the fraction of non-conforming products produced during the re-manufacturing process (see equation (3.4))

$$RSI1_{tk} = (1 - r\varepsilon_{tk}) * RM1_{tk} + (1 - r\varepsilon_{tk}) * REW1_{tk} * r\gamma \qquad \forall\, t, k \quad (3.3)$$

$$RSI2_{tk} = (r\varepsilon_{tk} * RM1_{tk}) + REW2_{tk} + (r\varepsilon_{tk} * REW1_{tk}) * r\gamma \qquad \forall\, t, k \quad (3.4)$$

As a result of human interaction in the quality inspection process, whether in sampling or full inspection, the re-manufacturing system faces the possibility of classification errors. The inspector may incorrectly classify materials that meet standards $r\alpha_{tk}$ (type I error) as non-conforming, or conversely, classify defective or contaminated materials $r\beta_{tk}$ (type II error) as conforming. As a result, inspected materials are categorized into two dominant classes: supposedly complain $RSI1_{tk}$ and supposedly non-complaint $RSI2_{tk}$, as illustrated in Figure 3.2.

The supposedly compliant parts $RSI1_{tk}$ is made for two different kinds of classified materials. The first fraction of part incorrectly classified as complaint (see equation (3.5)).

$$RESR1_{tk} = r\alpha_{tk} * RSI1_{tk} \qquad \forall\, t, k \qquad (3.5)$$

And the second fraction of parts correctly classified as non-conforming as stated in equation (3.6)

$$RESR3_{tk} = RSI1_{tk} - RESR1_{tk} \qquad \forall\, t, k \qquad (3.6)$$

For those parts supposedly non-conforming $RSI2_{tk}$, is made based on two different inspected products. Items incorrectly rejected as presented in equation (3.8)

$$RESR3_{tk} = r\beta_{tk} * RSI2_{tk} \qquad \forall\, t, k \qquad (3.7)$$

As well as the truly non-conforming ones $RESR4_{tk}$.

$$RESR4_{tk} = (1 - r\beta_{tk}) * RSI2_{tk} \qquad \forall\, t, k \qquad (3.8)$$

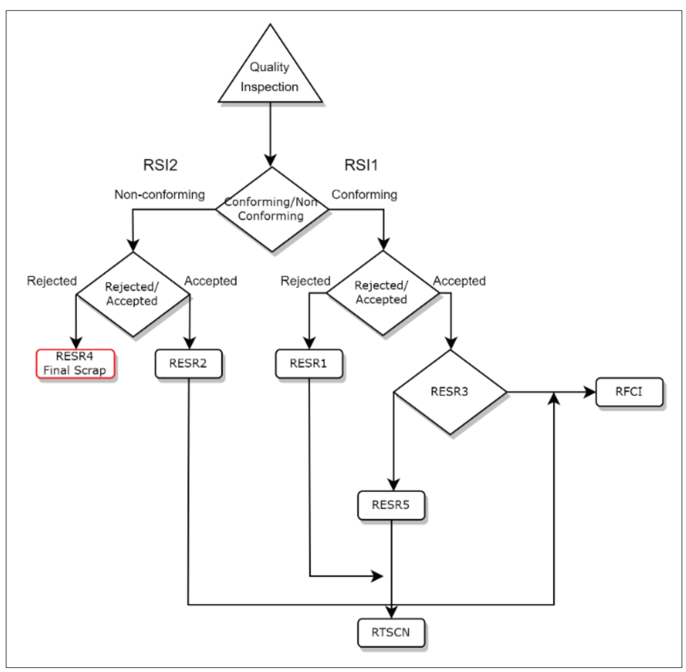

Figure 3.2 Reverse Remanufacturing QA Inspection process

Each stage carries a probability of contamination or damage to the product during the quality inspection and categorization process. Therefore, the mathematical model classifies this fraction using equation (3.9), where $\mathrm{r}\Omega_{tk}$ represents the probability of material contamination.

$$RESR5_{tk} = \mathrm{r}\Omega_{tk} * RESR3_{tk} \qquad \forall\, t,k \qquad (3.9)$$

For those parts that fail to meet the quality standards, they are segregated and categorized as truly non-conforming $RTSCN$. In the previous scenario (Chapter 2), the forward manufacturing system included post-process quality inspection operations to restore the non-conforming parts to the optimal condition trough rework and repair processes, depending on the severity of the damage.

$$RISCR1_{tk} = RF2_{tk} * (RESR1_{tk} + RESR5_{tk}) \qquad \forall\, t,k \qquad (3.10)$$

However, the re-manufacturing system lacks post-process operations, such as repair and rework, to improve final product quality. This is because the raw material may already have pre-existing deterioration or damage. (see equation (3.10)).

$$RISCR2_{tk} = RF2_{tk} * RESR4_{tk} \qquad \forall\, t,k \qquad (3.11)$$

For this reason, all materials categorized as non-conforming are automatically discarded, including the truly non-conforming fraction (see equation (3.12)) and those erroneously rejected (see equation (3.11)).

$$RFCI_{tk} = (RESR3_{tk} - RESR5_{tk}) + RESR2_{tk} \qquad \forall\, t,k \qquad (3.12)$$

Finally, part that were not included in the quality inspection sample, along with those that passed quality tests, continue trough the remaining production stages. Once all stages are completed, they are sent to the final customer (see equation (3.13)).

3.3 Problem description

The storage and management of stock is an economic factor that remarkably affects different companies. That is why some companies in the manufacturing sector choose to not have a physical warehouse that may represent economic losses, as well as the re-manufacturing system of this research study, which does not have a storage system for parts because it is not profitable as a result of the conditions of production and sale of their products.

Having said the above, all fraction of raw material collected from the forward manufacturing system along with end-of-life products used as raw material, must be processed and sent to the customer without storage, as presented in equation (3.13)

$$RRAW1_{t1} - (RV1_{t5} + RV2_{t5}) \geq 0 \qquad \forall\, t \quad (3.13)$$

Therefore, the volume of raw material cannot exceed the production capacity of the re-manufacturing system represented by $r\delta$ (see equation (3.14)).

$$(RV1_{t5} + RV2_{t5}) \leq r\delta \qquad \forall\, t \quad (3.14)$$

The needs and expectations for re-manufactured products often differ from those of the primary market, However, quality standards remain unchanged, meaning customer penalty costs given by $r\xi$ still apply when products fail to meet the agreed quality standards (see equation (3.15)).

$$RNC2_k = \sum_{t}^{T} RV2_{tk} * r\xi \qquad \forall\, k \quad (3.15)$$

To minimize penalties and unnecessary production costs associated with non-conforming products, the mathematical model used five key elements to identify production stations facing difficulties and work shifts requiring quality inspection stations. The first equation (3.16) suggest installing a quality inspection station when the total fraction of reworked items, and those rejected as scrap exceed the minimum required fraction of conforming items that must pass inspection $(RESI1_{tk} + RSI2_{tk})$.

$$RMT3_{tk} = \begin{cases} 1 & if \quad (1 - r\beta_{tk}) * RSI2_{tk} + (RESR1_{tk} + RESR5_{tk}) \geq \\ & \qquad (RESI1_{tk} + RSI2_{tk}) \\ 0 & \qquad otherwise \end{cases} \qquad \forall\, t, k \quad (3.16)$$

The presented equation (3.17) determines the distribution of quality inspection stations by ensuring that the fraction of conforming units at station k is at least half ($X4$) or greater than the production of non-conforming units at the preceding station ($k = n - 1$). If the criterion is not met, integrating an inspection station is recommended. ($k = n - 1$) represents a previous workstation.

$$RMTX3_{tk} = \begin{cases} 1 & if \quad REW2_{tk} \geq (REW2_{tk} * X4) \\ 0 & \qquad otherwise \end{cases} \qquad \forall\, t, k \quad (3.17)$$

Due to the impact of inspection errors type I (α) and II (β) on material sorting and shipment to the final customer, the following equation (3.18) states that if the fraction of rejected conforming products is greater than or equal to the fraction of non-conforming items correctly rejected as defective, an inspection station should be placed.

$$RMTX4_{tk} = \begin{cases} 1 & if \quad RESR1_{tk} \geq RESR4_{tk} \\ 0 & otherwise \end{cases} \qquad \forall\, t,k \quad (3.18)$$

Equation (3.19) supports the quality inspection stations if the fraction of non-conforming items mistakenly accepted as conforming due to error type II (β) is equal to or greater than the difference between the final fraction of inspected conforming items presumed to be compliant minus the fraction of non-conforming items incorrectly accepted as conforming (error type II).

$$RMTX7_{tk} = \begin{cases} 1 & if \quad RESR2_{tk} \geq \ RESR3_{tk} - RESR5_{tk} + RESR2_{tk} \\ 0 & otherwise \end{cases} \qquad \forall\, t,k \quad (3.19)$$

Last equation (3.20) for determining the need for an inspection station is based on economic factors. A quality inspection station is suggested if the total cost of scrap exceed the total cost of quality inspection. Otherwise, the next station is analyzed.

$$RMTX10_{tk} = \begin{cases} 1 & if \quad \left(RISCR1_{tk} + RISCR2_{tk} * rs_{tj}\right) \\ & \quad \geq \left(RSI1_{tk} + RSI2_{tk} * ri_{tk}\right) \\ 0 & otherwise \end{cases} \qquad \forall\, t,k \quad (3.20)$$

Once the mathematical model has analyzed the five criteria described above for each of the workstations, a final evaluation is performed. At least three of the described criteria must be met in order to suggest the distribution of a quality inspection station; otherwise, the mathematical model will not find a potential benefit in suggesting a quality station due to the economic impact this would have on the profit.

The second key aspect of the mathematical model is optimizing the distribution of preventive maintenance activities based on the condition of machines in the re-manufacturing system's production lines. The decision-making process for distributing these maintenance activities relies on four fundamental criteria. The mathematical model evaluates these criteria and recommends a maintenance activity if at least one is met.

The criteria for the distribution and installation of quality stations are stringent due to the risks associated with allowing production, equipment to operate in non-functional conditions and its exposure to deterioration. These factors pose significant economic and production risks. At least one of these criteria must be met. (see equation (3.21)).

$$RMTZ5_{tk} = \begin{cases} 1 & if \quad RMTZ1_{tk} + RMTZ2_{tk} + RMTZ4_{tk} \geq 1 \\ 0 & otherwise \end{cases} \qquad \forall\, t,k \quad (3.21)$$

Equation (3.22) evaluates the need for maintenance when there is a probability that a conforming product may develop a defect or contamination during processing ($r\varepsilon_{tk}$). This information is verified through the quality inspection process. Represents the limit of a contamination.

$$RMTZ1_{tk} = \begin{cases} 1 & if \quad r\varepsilon_{tk} \geq 1 \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t,k \quad (3.22)$$

Maintenance is considered when the fraction of non-conforming raw material entering the re-manufacturing system is equal to or greater than 20% of collected raw material that meets the expect quality standards. (see equation (3.23)).

$$RMTZ2_{tk} = \begin{cases} 1 & if \quad RV2_{tk} \geq ((1 - r\varepsilon_{tk} * RM1_{tk}) \\ & \qquad + (1 - r\varepsilon_{tk} * REW1_{tk})) \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t,k \quad (3.23)$$

$$RMTZ4_{tk} = \begin{cases} 1 & if \quad (RISCR1_{tk} + RISCR2_{tk}) \geq \; (RFCI_{tk} * X3) \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t,k \quad (3.24)$$

The third and final equation (3.24) states that the fraction of conforming and non-conforming items inspected for quality must be at least 25% of the final quantity of conforming products that will proceed to the next station. A 75% conforming rate represents the starting point for industries optimization keeping a balance between high quality and acceptable cost.

3.3.1 Case 1–Uniform deterioration factor

During the first case, the reverse re-manufacturing system is studied under a level of deterioration: it remains constant throughout the three production shifts without unforeseen changes and throughout the seven days of production, (see Table 3.1).

Table 3.1 Deterioration level ($r\varepsilon_{tk}$) from day 1 to 7 - (Case 2)

Stage j (Working station)		**1**	**2**	**3**	**4**	**5**
Period t	**1**	0.02	0.02	0.02	0.02	0.02
	2	0.03	0.03	0.03	0.03	0.03
	3	0.04	0.04	0.04	0.04	0.04

After each production operation, it is possible to carry out a material inspection through sampling or in a dedicated way when the mathematical model determines it. Within each of these inspections, there is a probability of inspection error type I (α) and II (β) as a result of human error as shown in Table 3.2. Concerning the first scenario of the forward manufacturing system, the reverse re-manufacturing system does not contemplate a probability of rework error θ and a second inspection process.

Table 3.2 inspection error I and II–(Case 1)

Period t (Working Shift)	Error type I (α), (β) rate (%)				
1	0.05	0.05	0.05	0.05	0.05
2	0.05	0.05	0.05	0.05	0.05
3	0.05	0.05	0.05	0.05	0.05

One critical difference in the re-manufacturing system is the need for rework and repairs. This means that the fraction of nonconforming parts requiring post-operation is non-existent in this production system. As a result, only a fraction of non-conforming products are categorized as waste $RF2_{tk}$, (see Table 3.3).

Table 3.3 Scrap fraction

Stage k (Working station)		1	2	3	4	5
Periods of time / Working Shifts	1	0.3	0.3	0.3	0.3	0.3
	2	0.3	0.3	0.3	0.3	0.3
	3	0.3	0.3	0.3	0.3	0.3

Since the re-manufacturing system's production line has different production machines with different features, the scrap cost of each machine is variable; however, the scrap cost is higher along each machine because the complexity and risk of failure in each new $k = n + 1$ station are higher as the product's state is closer to its final version. Quality inspection and preventive maintenance costs are also different for each workstation, (see Table 3.4).

Table 3.4 Scrap cost - (Case 1)

Stage k (Working station)		1	2	3	4	5
Periods of time / Working Shifts	1	3.156	1.052	1.052	2.104	3.156
	2	3.156	1.052	1.052	2.104	3.156
	3	3.156	1.052	1.052	2.104	3.156

Two production runs, one before and one after implementing optimization proposals from our developed mathematical model, were carried out to analyze the re-manufacturing system's behavioral, productive, and economic changes.

Table 3.5 Inspection & Maintenance cost - (Case 1)

Unit QA inspection cost i_{tk}				Unit CB maintenance cost μ_{tk}			
Stage k (Working station)	Periods of time / Working Shifts			Stage k (Working station)	Periods of time / Working Shifts		
	1	2	3		1	2	3
1	\$0.216	\$0.216	\$0.216	1	\$20	\$20	\$20
2	\$0.072	\$0.072	\$0.072	2	\$10	\$10	\$10
3	\$0.072	\$0.072	\$0.072	3	\$6.666	\$6.666	\$6.666
4	\$0.144	\$0.144	\$0.144	4	\$10	\$10	\$10
5	\$0.216	\$0.216	\$0.216	5	\$20	\$20	\$20

Table 3.6 Reverse re-manufacturing costs before & after QA & PM optimization - (Case 1)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total inspection cost	\$21,313.71	\$21,313.71	0.000%	dls
Total maintenance cost	\$0.00	\$0.00	0.000%	dls
Total repair cost	\$0.00	\$0.00	0.000%	dls
Total rework cost	\$0.00	\$0.00	0.000%	dls
Total scrap cost	\$15,595.74	\$15,595.74	0.000%	dls
Total penalization cost	\$718,954.70	\$718,954.70	0.000%	dls

The first case presents a reverse re-manufacturing system affected by a uniform deterioration factor in all its workstations. This factor is the same throughout the seven working days and in its three working shifts $t = 1 \dots 3$, regardless of the complexity or type of machine.

During the first production run, the manufacturing system did not present significant production and economic problems despite constantly deteriorating because the total fraction of sales turned out to be higher than the total production costs as a whole, in addition to the penalty costs derived from non-conformities by the final customer.

Through quality sampling inspection, the mathematical model determined that installing quality inspection stations or maintenance activities on the production lines was unnecessary as long as this deterioration factor is not altered over time.

Because the mathematical model did not suggest assigning quality inspection stations or maintenance activities, the production results and associated costs were identical with no change during the second production run. However, there is an opportunity to improve the penalty cost factor through other continuous improvement strategies.

Table 3.7 Reverse Remanufacturing profit after QA & PM optimization - (Case 1)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total profit	$ 2,003,420.74	$ 2,003,420.74	0.000%	dls

Profit likewise remains the same in the two test scenarios, with no change, as do all costs associated with producing the goods, presented in Table 3.7. However, as mentioned, scenario one is still open to possibilities of continuous improvement through some other strategy because although the mathematical model did not follow any optimization option.

Table 3.8 CBPM actions

CBPM–BEFORE						CBPM–AFTER					
Working Stations/Shifts	1	2	3	4	5	Working Stations/Shifts	1	2	3	4	5
1	0	0	0	0	0	1	0	0	0	0	0
2	0	0	0	0	0	2	0	0	0	0	0
3	0	0	0	0	0	3	0	0	0	0	0

3.3.2 Case 2–Increasing deterioration factor

In the second scenario, the re-manufacturing system is studied under a deterioration factor that increases exponentially in each machine throughout the seven days of production and differs in each working shift. Compared to the previous case, the deterioration factor reaches a point where it exceeds the optimal operational limit, which leaves the production equipment in a risky situation because it could unexpectedly stop, (see Table 3.9).

During the first week, when the deterioration factor is presented increasingly and exponentially, the mathematical model found it convenient to install 35 dedicated quality inspection stations and 63 different preventive maintenance activities from the second day; both activities increased until the seventh day for the second process, to improve the condition of the machines and the economic and productive results, because at the end of the first production run, the penalty costs represented the most significant economic loss factor for the re-manufacturing system since the number of non-conforming parts sent to the customer is 43.077% greater than the number of conforming parts received by the customer, (see Table 3.10).

Table 3.9 Increasing deterioration factor ($r\varepsilon$)

Stage K (Working station)	Day 1			Day 2			Day 3		
	1	2	3	1	2	3	1	2	3
1	0,020	0,030	0,040	0,050	0,060	0,070	0,080	0,090	0,100
2	0,006	0,007	0,008	0,009	0,010	0,020	0,030	0,040	0,050
3	0,006	0,007	0,008	0,009	0,010	0,020	0,030	0,040	0,050
4	0,0004	0,0005	0,0006	0,001	0,001	0,001	0,001	0,001	0,001
5	0,060	0,070	0,080	0,090	0,100	0,200	0,300	0,400	0,500
Stage K (Working station)	**Day 4**			**Day 5**			**Day 6**		
	1	**2**	**3**	**1**	**2**	**3**	**1**	**2**	**3**
1	0,100	0,200	0,300	0,400	0,500	0,600	0,700	0,800	0,900
2	0,060	0,070	0,080	0,090	0,100	0,200	0,300	0,400	0,500
3	0,060	0,070	0,080	0,090	0,100	0,200	0,300	0,400	0,500
4	0,0013	0,0014	0,0015	0,0016	0,0017	0,0018	0,0019	0,0020	0,0021
5	0,600	0,700	0,800	0,900	1,000	1,100	1,200	1,300	1,400
Stage K (Working station)	**Day 7**								
	1	**2**	**3**						
1	1,000	1,100	1,200						
2	0,600	0,700	0,800						
3	0,600	0,700	0,800						
4	0,0022	0,0023	0,0024						
5	1,500	1,600	1,700						

Table 3.10 Reverse manufacturing costs after QA & CBPM optimization - (Case 2)

Costs	Cost difference	Difference%	Increment/Decrement	Unit
Total inspection cost	$48,116.950	$1.081	235%	dls
Total maintenance cost	-$106.670	$0.132	-12%	dls
Total repair cost	$0.000	$0.000	0%	dls
Total rework cost	$0.000	$0.000	0%	dls
Total scrap cost	$102,477.560	$1.061	226%	dls
Total penalty cost	$1,608,307.010	-$0.717	-32%	dls

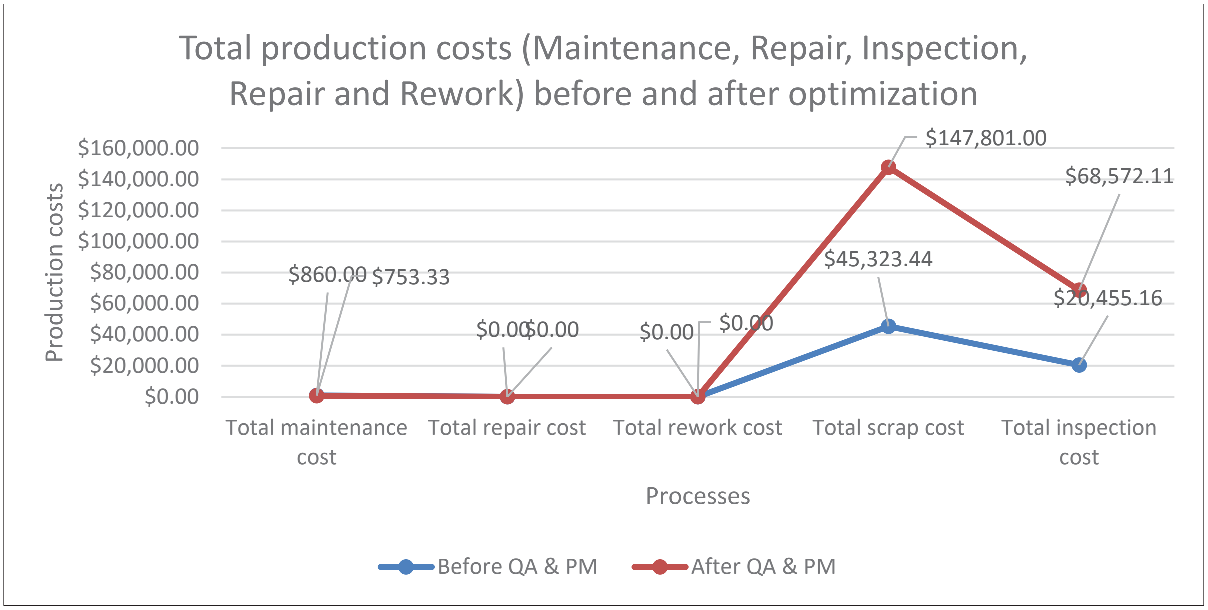

Figure 3.3 Reverse re-manufacturing costs after optimization

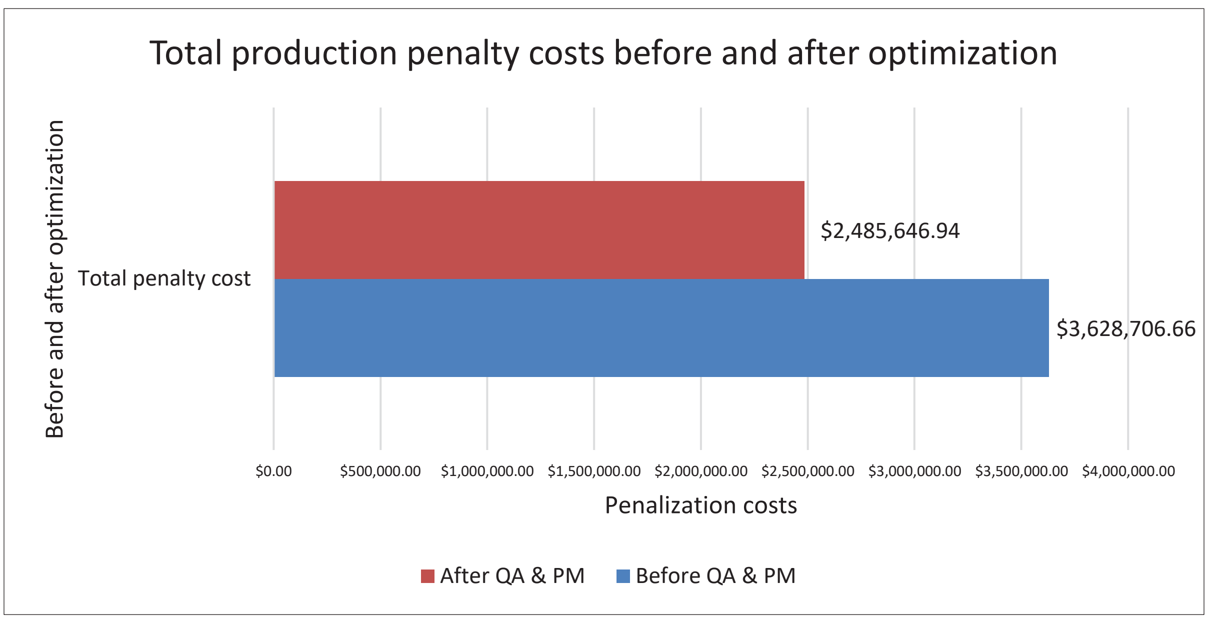

Figure 3.4 Remanufacturing penalty costs after optimization

The large number of non-conforming parts produced by the re-manufacturing system is a consequence of a lack of dedicated quality inspection stations since there are only random samples that allow the separating of some parts, but to provide information to the mathematical model about the operating status of the machines in the production line.

Table 3.11 Total items production results–(Case 2)

Concept	Before QA & PM	After QA & PM
Total conforming items sent to the customer	137,395.15	278,295.49
Total non-conforming items sent to the customer	212,827.37	145,785.74
Total SI1&SI2	150,167.78	429,565.26
Total repair items	0.00	0.00
Total scrap items	20,944.82	55,933.82
Total rework items	0.00	0.00

Therefore, at the end of the second production run under the improvements suggested by the mathematical model, the re-manufacturing system was able to find a substantial improvement thanks to a 31.50% reduction in penalty costs (see Figure 3.3 and Figure 3.4), a reduction in maintenance activities of 12.40%, such improvements also generated expenses 235.23% more in quality inspections. However, despite the above expenses, the manufacturing system improved profit by 52.78%, (see Table 3.12).

Table 3.12 Total profit–(Case 2)

Profit	Case 2	
	Before	After
	$ -3,047,381.10	$ -1,439,074.09

3.3.3 Case 3 – Random deterioration factor

The third case proposes subjecting the re-manufacturing system in reverse to a random deterioration factor that does not present a distinct pattern of behavior. Likewise, the level of deterioration does not prove to be attributed to a particular cause since it does not recur under the same environmental and working conditions. Compared to the previous cases, the consequences, and impact of random deterioration occur from the first two working days to the end of the production week, exposing the re-manufacturing system to failure at any time.

Due to the situation present during the first production run, the mathematical model determined it would be optimal to apply the same number of maintenance activities as inspection stations, 89 stations, and 89 maintenance activities throughout the seven days of the production week.

Table 3.13 Reverse manufacturing costs after QA & PM optimization - (Case 3)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total inspection cost	$19,509.26	$111,168.45	469.82%	dls
Total maintenance cost	$1,180.00	$893.33	-24.29%	dls
Total repair cost	$0.00	$0.00	0%	dls
Total rework cost	$0.00	$0.00	0%	dls
Total scrap cost	$63,444.31	$182,739.52	188.03%	dls
Total penalization cost	$4,484,982.26	$385,029.82	-91.42%	dls

Both optimization suggestions occur from the second day of production, which means that at the end of the week, the production system will send 65.927% of the raw material as a non-conforming product to the end customer, resulting in a penalty cost higher than the manufacturing costs of the target function, coupled with the risk factor of unexpected machine downtime and costs associated with its restoration based on time and labor. Also, the scrap costs generated during the first production run exceeded the inspection costs.

Table 3.14 QA and CBPM results after optimization - (Case 3)

QA before optimization							QA after optimization					
Working Stations/Shifts	**1**	**2**	**3**	**4**	**5**		**Working Stations/Shifts**	**1**	**2**	**3**	**4**	**5**
1	0	0	0	0	0		**1**	1	1	1	1	1
2	0	0	0	0	0		**2**	1	1	1	1	1
3	0	0	0	0	0		**3**	1	1	1	1	1
CBPM before optimization							**CBPM after optimization**					
Working Stations/Shifts	**1**	**2**	**3**	**4**	**5**		**Working Stations/Shifts**	**1**	**2**	**3**	**4**	**5**
1	0	0	0	0	0		**1**	1	1	1	1	1
2	0	0	0	0	0		**2**	1	1	1	1	1
3	0	0	0	0	0		**3**	1	1	1	1	1

During the second production run, even though the level of deterioration was randomly present in each work shift, station, and production day, the mathematical model suggestions generated in the first production run provided remarkable stability to the re-manufacturing system. The stability notably reduces economic losses by 92.74% compared to the first production run. The inspection expenses increased by 469.82%; however, this allowed a remarkable reduction of the penalty costs by 91.42%, which helped the re-manufacturing system to reduce the number of maintenance activities and the distribution of the quality inspection stations by less than 50% compared to the first production run.

Table 3.15 Profit for random deterioration level - (Case 3)

Profit	Case 3	
	Before	After
	$ 4,799,890.94	$ 348,288.28

For case three, the use of the mathematical model as an optimization tool allowed the re-manufacturing system to improve the situation of economic losses because they were reduced by about 92.744% compared to the result of the first scenario, giving way to a series of further improvements for the following production runs, (see Table 3.15).

3.4 Conclusion

A re-manufacturing system has emerged as an excellent option for bringing back to life those products whose condition did not allow them to be reintroduced to their primary market. Through techniques of reuse and renewal of components, the final product is returned to a state of operation but under new aesthetic conditions, warranty and price sought by a second market. However, like traditional manufacturing systems, re-manufacturing systems face similar problems, due to the need for adequate quality and maintenance systems.

Based on the above premise, our research project attempts to provide a new alternative to improve and solve quality problems related to the inspection of materials and the development of preventive maintenance activities, based on the condition of the re-manufacturing system machines, which compromise the quality of their production because of three different deterioration factors which are presented in different ways and intentions, the results are presented as follows.

Table 3.16 Case comparison

Costs	Case 1		Case 2		Case 3	
	Production based	Sales based	Production based	Sales based	Production based	Sales based
Total inspection cost	2.491%	1.064%	6.620%	-4.765%	9.983%	-31.919%
Total maintenance cost	0.000%	0.000%	0.073%	-0.052%	0.080%	-0.256%
Total repair	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Total rework cost	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Total scrap cost	1.823%	0.778%	14.268%	-10.271%	16.411%	-52.468%

It was mentioned that rework and repair activities in the re-manufacturing system under study are not allowed in order to process an item with some condition that may worsen once received by the customer; therefore, repair and rework costs are not represented in any of the different cases of deterioration in the production line.

The inspection costs at the end of the second production run after applying the first optimization suggestions showed to be higher concerning the production level in case number two, where the deterioration occurs incrementally at each workstation until the end of the production line and the third case where the deterioration occurs randomly.

However, this is because many inspection stations were installed throughout the week, and work shifts on the operating machines resulted from a higher volume of inspected products.

Through the study of the re-manufacturing system previously described, it was possible to carry out an analysis and understanding of the behavior of this type of manufacturing system under different conditions of deterioration, absence of rework, and repair activities, which in some instances can help to improve the condition of a part with some defect.

Table 3.17 Reverse re-manufacturing system changes comparison (Case 1, Case 2, Case 3)

Produced Items based on raw material	Case 1		Case 2		Case 3	
	Items	%	Items	%	Items	%
Conforming items ship	338,091.18	84.74%	191,361.39	47.96%	98,690.98	24.74%
Nonconforming Items ship	42,167.43	10.57%	145,785.74	36.54%	22,582.39	5.66%
Total items through QA	156,065.92	39.11%	429,565.26	107.66%	711,734.76	178.38%
Total items for repair	0	0.00%	0	0.00%	2,755.86	0.69%
Total items for rework	0	0.00%	0	0.00%	0	0.00%
Total items for scrap	7,374.97	1.85%	55,933.82	14.02%	74,956.09	18.79%
Total items not recycled	0	0.00%	0	0.00%	0	0.00%

On the other hand, and of great importance is to know the productive and economic benefit that a reverse re-manufacturing system can experience through the use of a mathematical model which, through a series of restrictions, can determine the installation and distribution of dedicated quality inspection stations as well as preventive maintenance activities based on the condition of the manufacturing equipment provided through the information collected by the quality stations based on the fractions of compliant and non-compliant production.

During the different production tests, it was observed that for case number one, even though the system was under a constant level of deterioration, it did not present any problem in meeting the goal of economic gains, product conformity for the customer, and the mathematical model did not contemplate installing the quality inspection stations as well as preventive maintenance activities in any of its production days, which makes it the case with the least negative impact because of deterioration and the least necessary investment regarding quality and maintenance issues.

Cases two and three are those that showed the most significant benefit from the strategic incorporation of maintenance activities and quality inspection stations, regardless that the severity of their deterioration factor was more significant than the first case; however, the production, inspection and maintenance costs were the highest of the three cases.

The above costs, however, brought with them improvements in economic gains since although the manufacturing system after the second production run experienced economic losses after the implemented improvements, these improved by over 40% in both cases, showing economic gains for the next production runs by following the strategies suggested by the mathematical model.

Thanks to the study generated in this third chapter, it was observed that a reverse re-manufacturing system is a great alternative to generate economic gains through the reuse of materials and components of non-conforming final parts of forward manufacturing processes and parts at the end of their useful life, as long as the re-manufacturing systems can count on a tool that helps maintain quality standards and an efficient maintenance system that allows the production equipment to work optimally.

However, despite the economic advantages that a re-manufacturing system can present by itself, there are still great opportunities for improvement through a system or productive structure that can not only take economic advantage of the production of goods or services for a major market but also make use of re-manufacturing systems that allow increasing economic gains through recycling and reuse of unused components in manufacturing systems and those products that completed their life cycle with the final customer.

This through a single closed cycle where both manufacturing systems can act together sharing the same family of products and production equipment, which is currently one of the most innovative alternatives in the industry and which is part of the next chapter.

CHAPTER 4

PART QUALITY INSPECTION AND CONDITION-BASED PREVENTIVE MAINTENANCE ALLOCATION STRATEGIES IN A DETERIORED SERIAL CLOSED-LOOP-MULTI-STAGE MANUFACTURING SYSTEM

4.1 Introduction

Chapter four of the study is based on a closed-loop manufacturing system comprising a forward manufacturing echelon composed of a production line of five different machines in charge of producing a single type of product n for a primary market. upon the expiry of their operational lifespan. Both manufacturing systems linearly perform their processes without carrying out alternate sub-processes.

Since both production echelons produce the same family of products through the same production line, ranking use is assigned to the forward manufacturing echelon because it is important to provide the final product to the primary market with the finality of obtaining raw materials that fulfilled their useful life with the customer and then become raw materials for the reverse re-manufacturing process, however in order to avoid generating waste from these products, these are collected and some of its components that are functional and aesthetically in good condition are re-utilized to generate new products.

The reverse re-manufacturing echelon plays a crucial role in the overall process. It allows for the recycling and re-manufacturing of products that did not meet the quality standards, generating a more significant economic benefit and enhancing customer satisfaction for a second market. This system ensures that both manufacturing systems are constantly active, alternating production in each shift.

The products produced in both manufacturing systems are of the same family. However, the aesthetic and functional characteristics differ to some degree between the two manufacturing systems since the needs and purchasing power are different, being the first market with higher priorities and needs.

Both systems experience the same degradation factor on their production days and shifts, uniformly, incrementally, or randomly. This notably affects production levels, quality and profitability, as the reverse re-manufacturing system depends entirely on the degradation factor's behavior and impact.

Because of such degradation factors, a proportion of conforming and non-conforming production is generated, which can increase or decrease due to the error in the quality inspection system type I and type II due to the human factor on which this inspection system depends.

The manufacturing system has several assumptions, which are listed below:

- The manufacturing system prioritizes using the production line and the supply of its primary market.
- Rework, repair, and scrapping of parts are only present in the forward manufacturing echelon while scrapping of parts is only allowed in the re-manufacturing echelon.
- At the end of each production shift, non-conforming parts from the forward echelon are automatically collected and sent as raw material to the re-manufacturing echelon.
- The re-manufacturing echelon production level is linked to the forward manufacturing system's conforming and non-conforming production levels.
- Items from both echelons can be damaged during the quality inspection process.

4.2 Closed-Loop Manufacturing System Description

The closed-loop manufacturing system begins its production activities with the forward manufacturing process, receiving raw material from various suppliers. This raw material then undergoes an initial quality inspection to ensure that only defect-free material (4.1) enters the production processes. This helps to reduce the fraction of non-conforming pieces (see equation (4.2)) produced at the end of the line or during the process. Once the raw material passes inspection, it is introduced into the production process.

$$M1_{tj} = \left(1 - \varepsilon 0_j\right) * QRM1_t \qquad \forall\ t, j \quad (4.1)$$

$$M2_{tj} = \varepsilon 0_j *\ QRM1_t \qquad \forall\ t, j \quad (4.2)$$

It should be noted that the raw material inspection process is also carried out in the reverse remanufacturing echelon. The raw material comprises non-conforming items from the forward manufacturing echelon $MTF1_{tj} + MTF2_{tj} + MTF3_{tj}$ and those collected from the primary market at the end of their useful life.

Since each machine performs a different operation in both echelons of the closed-loop manufacturing system, the percentage of conforming and non-conforming products generated varies, as does the level of deterioration each machine experiences. As a result, two types of outputs are produced in both production processes: conforming products (see equation (4.3)) and non-conforming products (4.5) in the forward echelon, and conforming (see equation (4.4)) and non-conforming as stated in equation (4.6) in the reverse echelon.

$$V1_{tj} = \left(1 - \varepsilon_{tj}\right) * M1_{tj} + \ \left(1 - \varepsilon_{tj}\right) * EW1_{tj} \qquad \forall\ t, j \quad (4.3)$$

$$RV1_{tk} = (1 - r\varepsilon_{tk}) * RM1_{tk} + \ (1 - r\varepsilon_{tk}) * REW1_{tk} \qquad \forall\, t, k \qquad (4.4)$$

$$V2_{tj} = (\varepsilon_{tj} * M1_{tj}) + EW2_{tj} + (\varepsilon_{tj} * \ EW1_{tj}) \qquad \forall\, t, j \qquad (4.5)$$

$$RV2_{tk} = \ (r\varepsilon_{tk} * RM1_{tk}) + REW2_{tk} + (r\varepsilon_{tk} * \ REW1_{tk}\ ) \qquad \forall\, t, k \qquad (4.6)$$

Both types of processed products (see equations (4.3), (4.4), (4.5) and (4.6)) form a batch from which a random sample is taken for quality analysis based on attributes. The sample is then classified into two categories: conforming (equations (4.7) and (4.8)) and non-conforming (equations (4.9) and (4.10)). The inspection sample is given by λ and it is the same fraction for both manufacturing echelons.

$$SI1_{tj} = \ V1_{tj} * \ \gamma \qquad \forall\, t, j \qquad (4.7)$$

$$RSI1_{tk} = \ (1 - r\varepsilon_{tk}) * RM1_{tk} + \ ((1 - r\varepsilon_{tk}) * REW1_{tk} * \ r\gamma) \qquad \forall\, t, k \qquad (4.8)$$

$$SI2_{tj} = \ V2_{tj} * \ \gamma \qquad \forall\, t, j \qquad (4.9)$$

$$RSI2_{tk} = \ r\varepsilon_{tk} * (RM1_{tk} + REW2_{tk}) + (r\varepsilon_{tk} * \ \ REW1_{tk} * \ r\gamma) \qquad \forall\, t, k \qquad (4.10)$$

The inspection process is carried out by a series of qualified operators in each type of operation due to the cost-benefit that human inspection represents for the closed-loop manufacturing system. However, naturally, because it is a human factor that carries out the inspection process, there is a probability of error during the inspection process, resulting in two types of scenarios or outcomes of inspected products: Supposedly conforming products are classified as non-conforming α_{tj}, and products that do not meet the quality guidelines are classified as conforming β_{tj}.

Because of quality inspection errors α, β, the sample taken from the production batch in both manufacturing and re-manufacturing echelons, results in two classifications of processed products: in the forward echelon, some supposedly conforming items may be incorrectly classified, as shown in equation (4.11), while other are correctly classified, as stated in equation (4.12).

$$ESR1_{tj} = \ \alpha_{tj} * \ SI1_{tj} \qquad \forall\, t, j \qquad (4.11)$$

$$ESR3_{tj} = SI1_{tj} - ESR1_{tj} \qquad \forall\, t, j \qquad (4.12)$$

Similarly, misclassified items are also found in the reverse re-manufacturing echelon, as described in equation (4.13), while correctly classified items are represented in equation (4.14).

$$RESR1_{tk} = r\alpha_{tk} * RSI1_{tk} \qquad \forall\, t, k \qquad (4.13)$$

$$RESR3_{tk} = RSI1_{tk} - RESR1_{tk} \qquad \forall\, t, k \qquad (4.14)$$

Supposedly non-compliant products in the forward echelon can also be categorized in two groups: non-conforming incorrectly classified as conforming (see equation (4.15)) or non-conforming items correctly rejected (4.16).

$$ESR2_{tj} = \beta_{tj} * SI2_{tj} \qquad \forall\, t, j \qquad (4.15)$$

$$ESR4_{tj} = \left(1 - \beta_{tj}\right) * SI2_{tj} \qquad \forall\, t, j \qquad (4.16)$$

The re-manufacturing echelon experiences the same misclassification of non-conforming items (see equation (4.17)), as well as the correct classification and rejection of non-conforming items as expressed in equation (4.18).

$$RESR2_{tk} = r\beta_{tk} * RSI2_{tk} \qquad \forall\, t, k \qquad (4.17)$$

$$RESR4_{tk} = (1 - r\beta_{tk}) * RSI2_{tk} \qquad \forall\, t, k \qquad (4.18)$$

During the quality inspection process, various risk factors such as contamination, mishandling damage, and other external factors can alter the initial quality of the product in the forward manufacturing stage.

$$ESR5_{tj} = \Omega_{tj} * ESR3_{tj} \qquad \forall\, t, j \qquad (4.19)$$

$$RESR5_{tk} = \mathrm{r}\Omega_{tk} * RESR3_{tk} \qquad \forall\, t, k \qquad (4.20)$$

From this point, the manufacturing system categorizes non-conforming products into three distinct classes at the end of the quality inspection process, as shown in Figure 4.1 and (4.21).

$$\begin{aligned} &1.-ESR1_{tj} = \alpha_{tj} * SI1_{tj} && \forall\, t,j \\ &2.-ESR4_{tj} = \left(1-\beta_{tj}\right) * SI2_{tj} && \forall\, t,j \\ &3.-ESR5_{tj} = \Omega_{tj} * ESR3_{tj} && \forall\, t,j \end{aligned} \qquad (4.21)$$

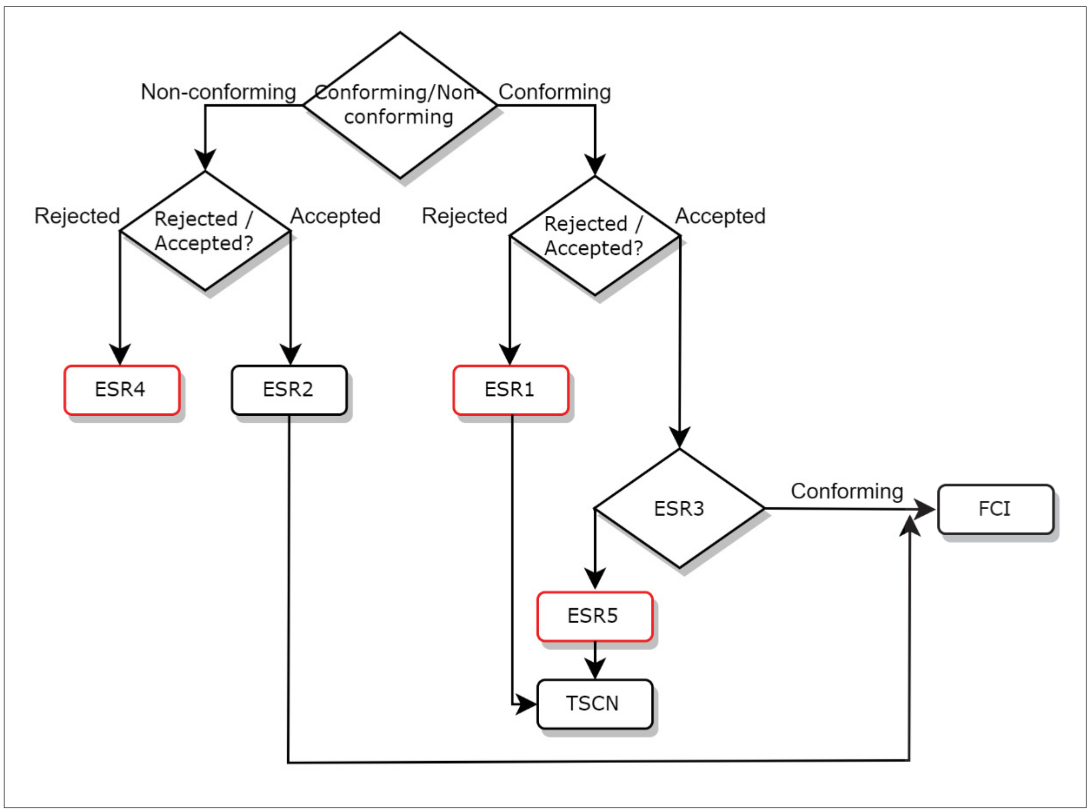

Figure 4.1 Forward manufacturing echelon structure

This research project contemplates that since quality inspection is performed manually, there is a possibility that a fraction of the batches produced at each workstation may pass directly to the next workstation without inspection due to the difficulty in detecting their apparent quality status and therefore may pass unnoticed. This fraction of items is contemplated as an assumption closer to the reality of different industries worldwide, which have hybrid systems where human cooperation is carried out hand in hand with the use of different technological tools, (see equation (4.22)).

$$FNCI_{tj} = \mathrm{V}2_{tj} - SI2_{tj} \qquad \forall\, t,j \qquad (4.22)$$

The fraction of non-conforming items identified by quality inspection as requiring repairs can be reintroduced into the production line after the repair process. Since the repair procedure is relative simple, it allows the product to be restored to its initial state and continue through the subsequent production stages. However, some conforming parts may be incorrectly classified and unnecessarily repaired (see equation (4.23))

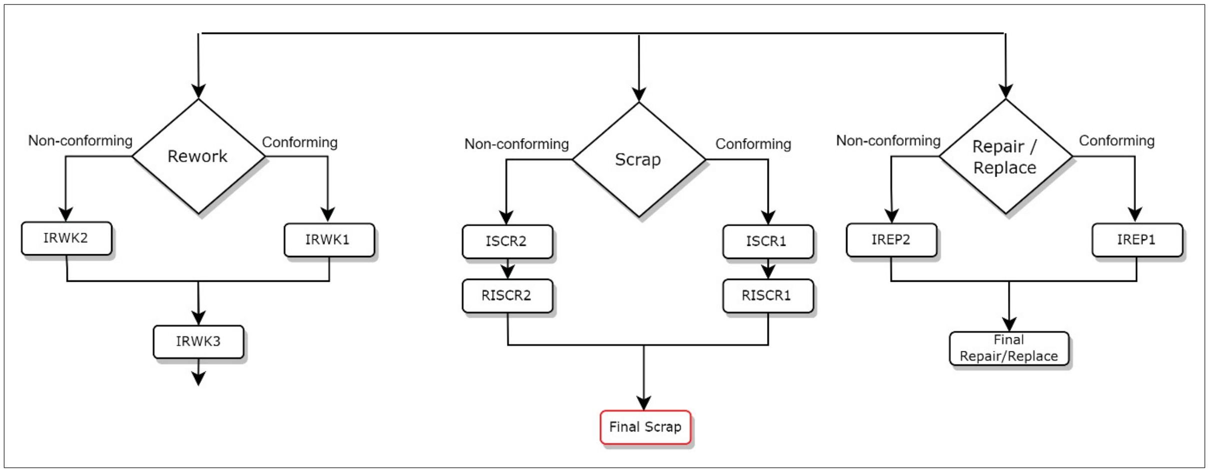

Figure 4.2 Post-production manufacturing processes

$$IREP1_{tj} = F1_{tj} * \left(ESR1_{tj} + ESR5_{tj}\right) \qquad \forall\, t, j \qquad (4.23)$$

Truly non-conforming products occur only in the forward manufacturing echelon, as shown in the following equation:

$$IREP2_{tj} = F1_{tj} * ESR4_{tj} \qquad \forall\, t, j \qquad (4.24)$$

Parts identified during the inspection process as having performance operation beyond simple repairs are sent to the rework process (see Figure 4.2). For these truly non-conforming (reference equation (4.25)), the rework fraction is denoted as $F3_{tj}$.

$$IRWK2_{tj} = F3_{tj} * ESR4_{tj} \qquad \forall\, t, j \qquad (4.25)$$

However, since rework is a more complex process, the probability of failure θ_{tj} in subsequent operations makes the process imperfect, requiring the product to be re-inspected at the same quality station. The re-inspection can yield three outcomes:

$$IRWK4_{tj} = (1 - \theta_{tj}) * IRWK3_{tj} \qquad \forall\, t, j \qquad (4.26)$$

The fraction of reworked items that underwent the quality inspection process a second time and were finally accepted as conforming.

$$IRWK5_{tj} = \theta_{tj} * IRWK3_{tj} \qquad \forall\, t, j \qquad (4.27)$$

The fraction of reworked items that underwent the second quality inspection process and were finally accepted as non-conforming.

$$IRWK6_{tj} = \theta_{tj} * IRWK4_{tj} \qquad \forall\, t, j \qquad (4.28)$$

The fraction of reworked items that were finally classified as non-conforming because of incorrect handling and contamination of materials during the second quality inspection process.

The previous items will finally classified in two major categories: items reworked that will be transferred to the next processing station, as stated in (4.29) and the rejected items during the second quality inspection process that must be scrapped (4.30).

$$IRWK7_{tj} = IRWK4_{tj} - IRWK6_{tj} \qquad \forall\, t, j \qquad (4.29)$$

$$IRWK8_{tj} = IRWK5_{tj} + IRWK6_{tj} \qquad \forall\, t, j \qquad (4.30)$$

After the second quality inspection at each workstation, batches of non-conforming items classified as scrap are collected at the end of each work shift. These materials are then transferred to the reverse re-manufacturing echelon to be used as raw materials, as denoted in equation (4.31).

$$FSC_{tj} = IRWK8_{tj} + ISCR1_{tj} + ISCR2_{tj} \qquad \forall\, t, j \qquad (4.31)$$

While FSC_{tj} are conforming materials that will pass along all the manufacturing stations, the ones that are not part of the quality inspection process and needs to be transferred to the following production stations with no apparent problem.

Those parts that were not part of the quality inspection sample in addition to those that passed the quality tests, are transferred to the following production stations until all stages are completed, they are sent to the final customer.

4.3 Problem Description

Since the closed-loop manufacturing system uses all available raw materials and sells all finished products, it does not maintain long-term storage. Instead, it only holds a minimal stock of materials being processed daily, regardless of the shift. For this reason, in both manufacturing echelons (Forward and reverse), the fraction of products produced cannot exceed the production capacity δ of each echelon. (see equations (4.32) and (4.33)).

$$RAW1_{t1} - (V1_{t5} + V2_{t5}) \geq 0 \qquad \forall\, t \qquad (4.32)$$

$$(V1_{t5} + V2_{t5}) \leq \delta \qquad \forall\, t \qquad (4.33)$$

It is important to note that raw material inspection process is also conducted within the reverse re-manufacturing echelon. The raw material included non-conforming items from the forward manufacturing echelon $MTF1 + MTF2 + MTF3 + (V1_{t5} + V1_{t5} * X2)$ as well as products collected from the primary market at the of their life cycle to begin the re-manufacturing process as described in equations (4.34) and (4.35)).

$$RRAW1_{t1} - (RV1_{t5} + RV2_{t5}) \geq 0 \qquad \forall\, t \qquad (4.34)$$

$$(RV1_{t5} + RV2_{t5}) \leq r\delta \qquad \forall\, t \qquad (4.35)$$

As with the previous cases, the closed-loop manufacturing system is subject to penalties imposed by final customers for receiving non-conforming materials.

As with the previous cases, the closed-loop manufacturing system is subject to penalties imposed by final customers for receiving non-conforming materials. The penalty cost (ξ and $r\xi$) varies depending on the agreements established with the different markets supplied by the closed-loop manufacturing system.

$$(NCS2 = \textstyle\sum_t^T V2_{tj} * \xi) + (RNCS2 = \textstyle\sum_t^T RV2_{tk} * r\xi) \qquad \forall\, t \qquad (4.36)$$

The mathematical model developed in this project must incorporate a set of information and constraints to provide optimal solutions for the closed-loop manufacturing system. One of the primary constraints in the installation and allocation of quality stations at various workstations. This decision-making process is guided by ten different equations, of which at least five must be satisfied. (see equation (4.37)).

$$MTX11_{tj} = \begin{cases} 1 & if \quad MT3_{tj} + MTX3_{tj} + MTX4_{tj} \\ & \quad + MTX7_{tj} + MTX10_{tj} \geq 3 \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \qquad (4.37)$$

The first equation to be satisfied in the forward manufacturing echelon (4.38) states that the total fraction of both conforming parts rejected in error and non-conforming parts–whether reworked, repaired, or scrapped must be less than X of the total fraction of inspected parts. This ensures at least 70% compliance within the inspection.

$$MT3_{tj} = \begin{cases} 1 \quad if\ (ESR4_{tj} + TSCN_{tj}) \geq (SI1_{tj} + SI2_{tj}) * X \\ \quad 0\ otherwise \end{cases} \qquad \forall\ t, j \qquad (4.38)$$

For the forward echelon, the criterion of non-conforming parts is higher, being 50% of the total fraction of the parts that passed through the inspection process, as shown in equation

$$R\ MT3_{tk} = \begin{cases} 1\ if\ (RSI1_{tk} + RSI1_{tk}) * rx\ \geq (RESR4_{tk} + RTSCN_{tk}) \\ \quad 0\ otherwise \end{cases} \qquad \forall\ t, k \qquad (4.39)$$

Following equations (4.40) and (4.41) for the entire closed-loop manufacturing system states that any job n machine that produces more than half of non-conforming products ($X4$) than its subsequent station will be required a dedicated quality inspection station.

$$MTX3_{tj} = \begin{cases} 1 \quad if \quad EW2_{tj} \geq (EW2_{tj} * X4) \\ 0 \quad \quad otherwise \end{cases} \qquad \forall\ t, j \quad (4.40)$$

$$RMTX3_{tk} = \begin{cases} 1 \quad if \quad REW2_{tk} \geq (REW2_{tk} * X4) \\ 0 \quad \quad otherwise \end{cases} \qquad \forall\ t, k \quad (4.41)$$

The following presented equations (4.42) and (4.43) considers installing a quality inspection station as long as the fraction of conforming parts rejected as result of type I error during the inspection process is greater than the number of rejected parts whose condition was actually non-conforming.

$$MTX4_{tj} = \begin{cases} 1 \quad if \quad ESR1_{tj} \geq ESR4_{tj} \\ 0 \quad \quad otherwise \end{cases} \qquad \forall\ t, j \quad (4.42)$$

$$RMTX4_{tk} = \begin{cases} 1 \quad if \quad RESR1_{tk} \geq RESR4_{tk} \\ 0 \quad \quad otherwise \end{cases} \qquad \forall\ t, k \quad (4.43)$$

The fourth indicates to establish a quality inspection station when the fraction of non-conforming products that have been accepted as conforming because of type II inspection error is higher than the final fraction of conforming parts that will be transferred to the next production operation $j = n + 1$, (see equations (4.44) and (4.45)).

$$MTX7_{tj} = \begin{cases} 1 \quad if \quad ESR2_{tj} \geq (ESR3_{tj} - ESR5_{tj}) + ESR2_{tj}) \\ 0 \quad \quad otherwise \end{cases} \qquad \forall\ t, j \quad (4.44)$$

$$RMTX7_{tk} = \begin{cases} 1 \quad if \quad RESR2_{tk} \geq (RESR3_{tk} - RESR5_{tk}) + RESR2_{tk} \\ 0 \quad \quad otherwise \end{cases} \qquad \forall\ t, k \quad (4.45)$$

The last equations (4.46) and (4.47) that allows the mathematical model to determine the feasibility of establishing an inspection station is by diagnosing whether the total scrap costs are greater than the inspection costs, if so, introducing a station is necessary.

$$MTX10_{tj} = \begin{cases} 1 & if \quad (ISCR1_{tj} + ISCR2_{tj}) * s_{tj} \\ & \geq (SI1_{tj} + SI2_{tj}) * i_{tj} \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \quad (4.46)$$

$$RMTX10_{tk} = \begin{cases} 1 & if \quad (RISCR1_{tk} + RISCR2_{tk}) * \; rs_{tj} \\ & \geq (RSI1_{tk} + RSI2_{tk}) * ri_{tk} \\ 0 & \quad otherwse \end{cases} \qquad \forall\, t, k \quad (4.47)$$

The second fundamental aspect of the mathematical model is to help improve the distribution of preventive maintenance activities based on the current state the production lines of the closed-loop manufacturing system, to reduce the fraction of non-conforming parts and the economic losses that this situation entails. The mathematical model performs the optimization operations based on the priority assigned to the different production lines, being in this case the forward echelon which is in first place and then the reverse re-manufacturing echelon.

Under this premise, to evaluate the feasibility of the deployment of the maintenance activities, the mathematical model makes use of five different constraints for the forward echelon, while for the re-manufacturing one, only three elementary constraints are necessary, being in total 8 equations for the closed-loop manufacturing system, (see equations (4.48) and (4.49)).

$$MTZ11_{tj} = \begin{cases} 1 & if \quad MT6_{tj} + MTZ7_{tj} + MTZ8_{tj} + MTZ9_{tj} + MTZ10_{tj} \geq 1 \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \quad (4.48)$$

$$RMTZ5_{tk} = \begin{cases} 1 & if \quad RMTZ1_{tk} + RMTZ2_{tk} + RMTZ4_{tk} \geq 1 \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \quad (4.49)$$

For the forward echelon, equation (4.50) limits the system to not continue operating activities without performing preventive maintenance in the station where the total fraction of rejected parts categorized as scrap is greater than the total fraction of rejected parts that need to be repair or reworked.

$$MT6_{tj} = \begin{cases} 1 & if \quad (ISCR1_{tj} + ISCR2_{tj}) \geq \\ (IREP1_{tj} + IREP2_{tj} + IRWK7_{tj}) & \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \quad (4.50)$$

While for the re-manufacturing echelon, because it has no repair or rework activities allowed, equation (4.51) states to deploy preventive maintenance in that station where exists a probability of a conforming product gaining some type of defect or contamination during the quality inspection process.

$$RMTZ1_{tk} = \begin{cases} 1 & if \quad r\varepsilon_{tk} \geq 1 \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, k \qquad (4.51)$$

In equation (4.52) for the forward manufacturing system is to perform preventive maintenance on stations whose fraction of reworked and rejected parts during the second quality inspection is greater than 20% of the total fraction of rejected conforming which requires rework.

$$MTZ7_{tj} = \begin{cases} 1 & if \quad \theta_{tj} * IRWK3_{tj} \geq F3_{tj} * ESR4_{tj} \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \qquad (4.52)$$

For the re-manufacturing system, the deployment of a maintenance activity occurs when the total fraction of rejected products represents over 20% of the total fraction of raw material collected, as described in equation (4.53).

$$RMTZ7_{tk} = \begin{cases} 1 & if \quad RV2_{tk} \geq ((1 - r\varepsilon_{tk}) * RM1_{tK}) \\ & \quad +((1 - r\varepsilon_{tk}) * REW1_{tk})) \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, k \qquad (4.53)$$

Continuing with the forward manufacturing echelon, equation (4.54) helps in the decision making of providing maintenance activities as long as the fraction of non-conforming reworked or repaired rejected parts, is greater than the fraction of conforming parts that were similarly rejected and that must also be repaired and reworked.

$$MTZ8_{tj} = \begin{cases} 1 & if \quad IRWK2_{tj} + IREP2_{tj} \geq IRWK1_{tj} + IREP1_{tj} \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \qquad (4.54)$$

The third equation (4.55) in the forward manufacturing scheme suggests starting maintenance activities at workstations when the fraction of conforming and non-conforming parts that have passed through quality inspection exceeds $X3$ of the final fraction of conforming products that will proceed to the next station.

$$RMTZ4_{tk} = \begin{cases} 1 & if \quad RISCR1_{tk} + RISCR2_{tk} \geq \; RFCI_{tk} * X3 \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, k \qquad (4.55)$$

Since production lines may unexpectedly experience changes in operational status leading to fatigue and deterioration beyond optimal levels and jeopardizing production quality, the mathematical models shows that preventive maintenance is necessary in the forward manufacturing echelon. This applies whenever production equipment reaches 100% utilization in any of its shifts, as described in the equation.

$$MTZ9_{tj} = \begin{cases} 1 & if \quad \varepsilon_{tj} \geq 1 \\ 0 & \quad otherwise \end{cases} \qquad \forall\, t, j \qquad (4.56)$$

$$MTZ10_{tj} = \begin{cases} 1 & if \quad \varepsilon_{tj} * M1_{tj} \geq \left(\left(1 - \varepsilon_{tj}\right) * M1_{tj}\right) * X6 \\ 0 & otherwise \end{cases} \quad \forall\, t, j \qquad (4.57)$$

If the production line under study has a fraction of non-conforming products exceeding $X6$ (20%) of the conforming ones, the mathematical model recommends implementing preventive maintenance exclusively in the forward production echelon. (see equation (4.57)).

4.3.1 Case 1–Uniform deterioration factor

During case number 1, the closed-loop manufacturing system experiences a deterioration factor in its production equipment that does not manifest any change of improvement or worsening over the three different production shifts in a work week. Because the same machinery that is used in the forward manufacturing echelon is shared and used in the reverse manufacturing echelon, the deterioration factor is the same in both cases. However, their performance and results differ because of their structure and production objectives.

The closed loop manufacturing system provides a quality inspection system by sampling both echelons, carried out manually by a qualified operator in the revision of the critical points of the final products for the main and secondary re-manufacturing markets. However, because the inspection is carried out by a human, there are a number of non-predictable variables within this study, which generate inspection and material classification errors, known as type I and II errors, (see Table 4.1).

Table 4.1 Alpha & Betha error rates

Period t (Working Shift)	**1**	**2**	**3**
Error type II rate (%)	0.05 0.05 0.05 0.05 0.05	0.05 0.05 0.05 0.05 0.05	0.05 0.05 0.05 0.05 0.05

Since the use of a closed-loop manufacturing system requires ensuring the highest fraction of conforming parts for its primary market and enough nonconforming parts to satisfy the demand of the second market belonging to the re-manufacturing echelon, the forward manufacturing system has a series of post-production activities such as rework and repair to recover the highest fraction of nonconforming items. However, these activities are not considered in the re-manufacturing system since the re-manufacturing factor already carries a higher risk of failure than a product made from virgin material parts, so rework and repair increase that factor.

Table 4.2 Closed-Loop unit repair and rework cost - (Case 1)

Unit repair cost					**Unit rework cost**			
Stage j (Working station)	**Periods of time / Working Shifts**				**Stage j (Working station)**	**Periods of time / Working Shifts**		
	1	**2**	**3**			**1**	**2**	**3**
1	\$0.540	\$0.540	\$0.540		**1**	\$3.600	\$3.600	\$3.600
2	\$0.180	\$0.180	\$0.180		**2**	\$1.080	\$1.080	\$1.080
3	\$0.180	\$0.180	\$0.180		**3**	\$1.080	\$1.080	\$1.080
4	\$0.360	\$0.360	\$0.360		**4**	\$2.160	\$2.160	\$2.160
5	\$0.540	\$0.540	\$0.540		**5**	\$3.240	\$3.240	\$3.240

As in the previous chapters, the operating machines of the production system also have a hierarchy, which is linked to the type of operations and their complexity. Therefore, the costs of production, rework, repair, and scrap are different for each of them. These costs are higher in both production echelons, in each subsequent station, because the final product becomes more complex in operation and materials, so its failure is more expensive in each new station.

Table 4.3 Unit forward and reverse scrap cost - (Case 1)

Unit forward scrap cost					**Unit reverse scrap cost**			
Stage j (Working station)	**Periods of time / Working Shifts**				**Stage j (Working station)**	**Periods of time / Working Shifts**		
	1	**2**	**3**			**1**	**2**	**3**
1	\$3.324	\$3.324	\$3.324		**1**	\$3.157	\$3.157	\$3.157
2	\$1.108	\$1.108	\$1.108		**2**	\$1.052	\$1.052	\$1.052
3	\$1.108	\$1.108	\$1.108		**3**	\$1.052	\$1.052	\$1.052
4	\$2.216	\$2.216	\$2.216		**4**	\$2.105	\$2.105	\$2.105
5	\$3.324	\$3.324	\$3.324		**5**	\$3.157	\$3.157	\$3.157

To analyze the challenges, behavior and optimization opportunities of the closed-loop manufacturing system, two production runs were carried out over 7 working days without process optimization actions, with the aim of obtaining preliminary data on the system's current state under the current deterioration factor. (see (Table 4.4) and (Table 4.5)).

Table 4.4 Total forward costs - (Case 1)

Costs	Before QA & PM		After QA & PM		Difference	Unit
Total inspection cost	$	30,386.97	$	30,403.18	0.053%	dls
Total maintenance cost	$	793.33	$	560.00	-29.412%	dls
Total repair cost	$	2,405.06	$	2,208.51	-8.172%	dls
Total rework cost	$	36,998.09	$	33,989.67	-8.131%	dls
Total scrap cost	$	22,206.69	$	20,391.87	-8.172%	dls
Total penalization cost	$	974,842.35	$	766,731.03	-21.348%	dls

Table 4.5 Total reverse costs - (Case 1)

Costs	Before QA & PM		After QA & PM		Difference		Unit
Total inspection cost	$	22,369.84	$	22,299.73		-0.313%	dls
Total maintenance cost	$	-	$	-	$	-	dls
Total repair cost	$	-	$	-	$	-	dls
Total rework cost	$	-	$	-	$	-	dls
Total scrap cost	$	16,410.75	$	16,356.85		-0.328%	dls
Total penalization cost	$	759,542.90	$	756,872.94		-0.352%	dls

The forward echelon is the one that has got the most significant changes in total maintenance costs and penalties because the mathematical model only suggested 49 different preventive maintenance activities throughout the production week, while for the re-manufacturing echelon, none were suggested. This suggestion is derived from the fraction of products manufactured in both echelons, (see Table 4.6).

Table 4.6 Raw material difference between echelons - (Case 1)

Raw material	Forward	Reverse	Difference
Total # of conforming items entering M1	184,507.98	418,793.12	
Total # of non-conforming items entering M2	321,576.88	0.00	
Total	506,084.85	418,793.12	18.876%

While the inspection costs for both echelons were cumulatively reduced by less than 1% after quality optimization and preventive maintenance, (see Figure 4.3)

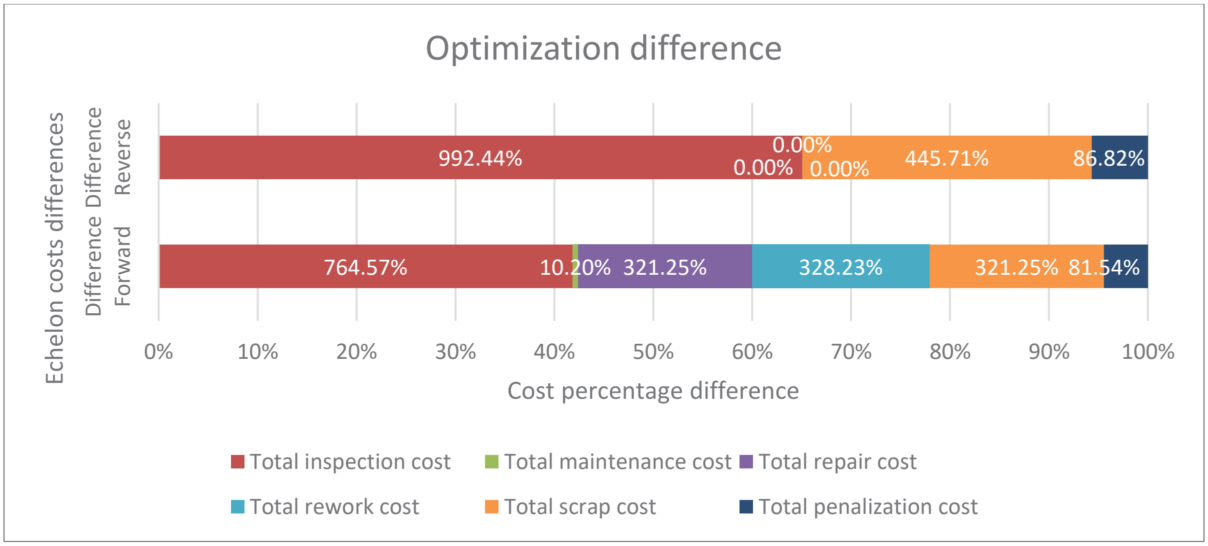

Figure 4.3 Optimization difference between echelons

After application of the suggestions made by the mathematical model, there was an overall 12.206% improvement in profit, with the largest change under the forward manufacturing echelon being 11.915%. This is because the fraction of non-conforming parts shipment decreased by 21.700% in the whole closed loop manufacturing system after optimization, (see **Error! Reference source not found.**) and (Figure 4.4).

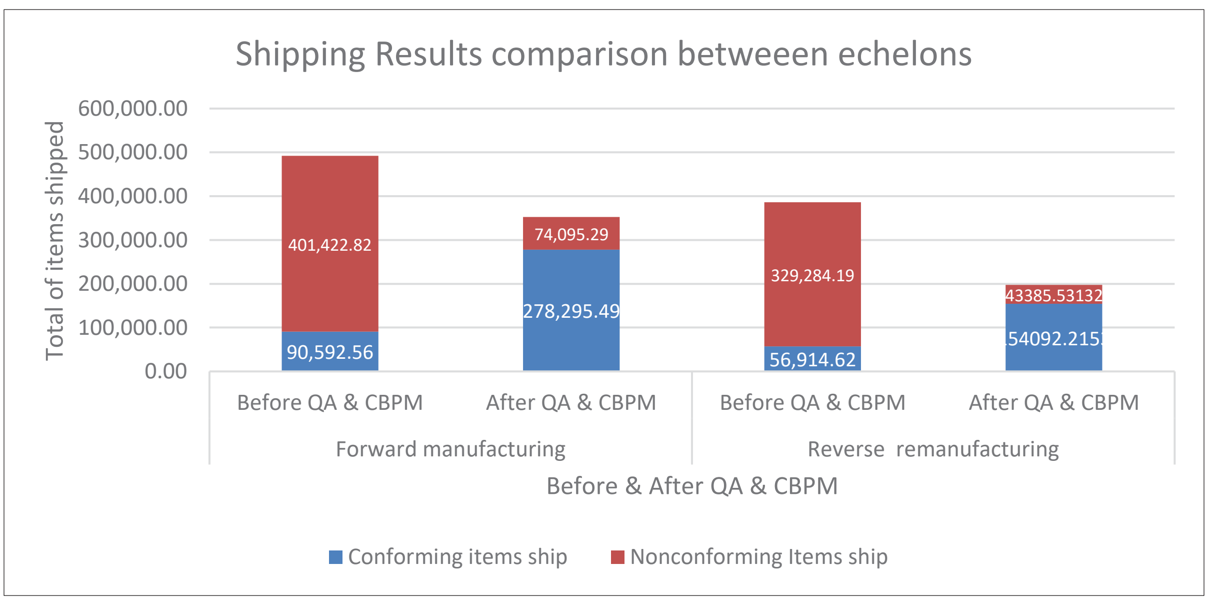

Figure 4.4 Optimization difference between echelons

Table 4.7 Profit results - (Case 1)

Total profit	Before QA & PM	After QA & PM	Difference
Forward Manufacturing	$2,978,013.08	$3,332,856.09	11.915%
Reverse Remanufacturing	$2,094,322.65	$2,088,246.81	0.290%

On the other hand, the distribution and application of quality stations in the closed-loop manufacturing system in this first scenario did not present to be necessary, only the application of over 40 different preventive maintenance activities throughout the production week, which decreased to 35 after its application, being 4 of these activities the ones that changed throughout the week, (see Table 4.8).

Table 4.8 Closed-loop CBPM changes - (Case 1)

Different CBPM activities					
Working Stations/Shifts	**1**	**2**	**3**	**4**	**5**
1	Same	Different	Same	Same	Same
2	Same	Same	Same	Different	Different
3	Same	Same	Different	Same	Same

4.3.2 Case 2–Increasing deterioration factor

One of the effects of the lack of application of maintenance or dedicated quality assurance inspections is an exponential deterioration of the production equipment, bringing them to the point of operational risk due to the fraction of non-conformities that may generate. Within this scenario, the closed-loop manufacturing system presented the following results.

Unlike the previous case, the costs of both inspection costs and rework, repair, and scrap increased by more than 100% when quality inspection stations and preventive maintenance activities were distributed during the week throughout the closed-loop manufacturing system, (see Table 4.9).

Table 4.9 Total forward costs - (Case 2)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total inspection cost	$29,872.09	$119,878.87	301.307%	dls
Total maintenance cost	$1,133.34	$986.66	-12.942%	dls
Total repair cost	$7,154.41	$26,621.99	272.106%	dls
Total rework cost	$112,318.92	$408,927.42	264.077%	dls
Total scrap cost	$67,155.70	$245,809.77	266.030%	dls
Total penalization cost	$5,482,885.73	$3,375,431.53	-38.437%	dls

From the fourth day of production, as a result of the increase in inspection stations, more items that needs rework was captured as well as repair, which together with the scrap generated at each station resulted in a percentage of scrap whose costs were doubled compared to the non-optimized scenario. However, 85.18% of the scrap generated in the forward manufacturing echelon that could be used as raw material in the re-manufacturing process, with which 60.55% of conforming parts could be shipped after optimization, (see Table 4.10).

Table 4.10 Total reverse costs - (Case 2)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total inspection cost	$23,202.03	$106,188.83	357.670%	dls
Total maintenance cost	$853.33	$626.67	-26.562%	dls
Total repair cost	$0.00	$0.00	0.000%	dls
Total rework cost	$0.00	$0.00	0.000%	dls
Total scrap cost	$54,314.57	$197,061.91	262.816%	dls
Total penalization cost	$4,289,343.41	$2,043,282.96	-52.364%	dls

Since all production costs rose by more than 100% compared to the scenario when there were no improvements, the closed-loop manufacturing system still showed economic losses during the first week of improvement, but these economic losses improved since they were reduced by 135.581%. (see Figure 4.5). This improvement is also due to the increase in sales from the re-manufacturing of the scrap generated in the forward manufacturing echelon, (see Table 4.5)

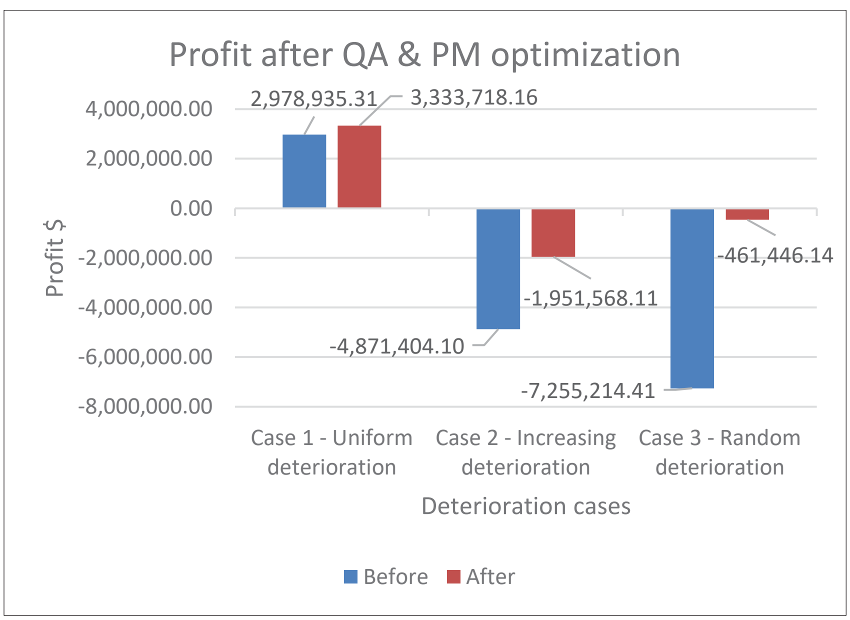

Figure 4.5 Profit improvements of each case after optimization

Table 4.11 Final profit - (Case 2)

Total profit	Before QA & PM	After QA & PM	Difference
Forward Manufacturing	$ -4,774,286.49	$ -2,013,557.18	-57.825%
Reverse Remanufacturing	$ -3,758,469.21	$ -836,041.83	-77.756%

As mentioned above, the closed-loop manufacturing system experienced a higher deterioration starting from the fourth day, especially in the last station, which is the most critical point at the end of the production line and the last step before shipping the product to the customer. For this reason, before optimization, it was suggested that 28 quality stations and 84 different maintenance activities be implemented.

Despite bringing these improvements to the system for the second week of production, it was suggested to increase the number of inspection stations in the last two stages of production to improve the fraction of conforming parts and in order to restore the condition of the machines to maintain the maintenance activities, (please refer to (Table 4.12), (Table 4.13), Table 4.14) and (Table 4.15)).

Table 4.12 Suggested QA inspections before optimization–Forward Echelon

Day 6							Day 7					
Working Stations/Shifts	**1**	**2**	**3**	**4**	**5**		**Working Stations/Shifts**	**1**	**2**	**3**	**4**	**5**
1	0	0	0	0	1		**1**	1	1	1	0	1
2	0	0	0	0	1		**2**	1	1	1	0	1
3	0	0	0	0	1		**3**	1	1	0	0	0

Table 4.13 Suggested QA inspections after optimization–Forward Echelon

Day 6							Day 7					
Working Stations/Shifts	**1**	**2**	**3**	**4**	**5**		**Working Stations/Shifts**	**1**	**2**	**3**	**4**	**5**
1	0	0	1	0	1		**1**	1	0	1	0	1
2	1	1	1	1	1		**2**	1	0	1	0	1
3	1	1	1	0	1		**3**	1	0	1	0	1

Table 4.14 Suggested QA inspections before optimization–Reverse Echelon

Day 6							**Day 7**					
Working Stations/Shifts	**1**	**2**	**3**	**4**	**5**		**Working Stations/Shifts**	**1**	**2**	**3**	**4**	**5**
1	0	0	0	0	1		**1**	1	1	1	1	1
2	0	0	0	0	1		**2**	1	1	1	1	1
3	0	0	1	1	1		**3**	1	1	1	1	1

Table 4.15 Suggested QA inspections after optimization–Reverse Echelon

Day 6							**Day 7**					
Working Stations/Shifts	**1**	**2**	**3**	**4**	**5**		**Working Stations/Shifts**	**1**	**2**	**3**	**4**	**5**
1	0	0	1	0	1		**1**	1	0	1	0	1
2	1	1	1	1	1		**2**	1	0	1	0	1
3	1	1	1	0	1		**3**	1	0	1	0	1

4.3.3 Case 3 – Random deterioration factor

The last test scenario, takes the manufacturing system to one of its operational limits, due to the deterioration factor that is presented in its random production lines, however this deterioration can become less in subsequent stages as well as exceed the operational limit of the production equipment, (see (Figure 4.6), (Figure 4.7) and (Figure 4.8)).

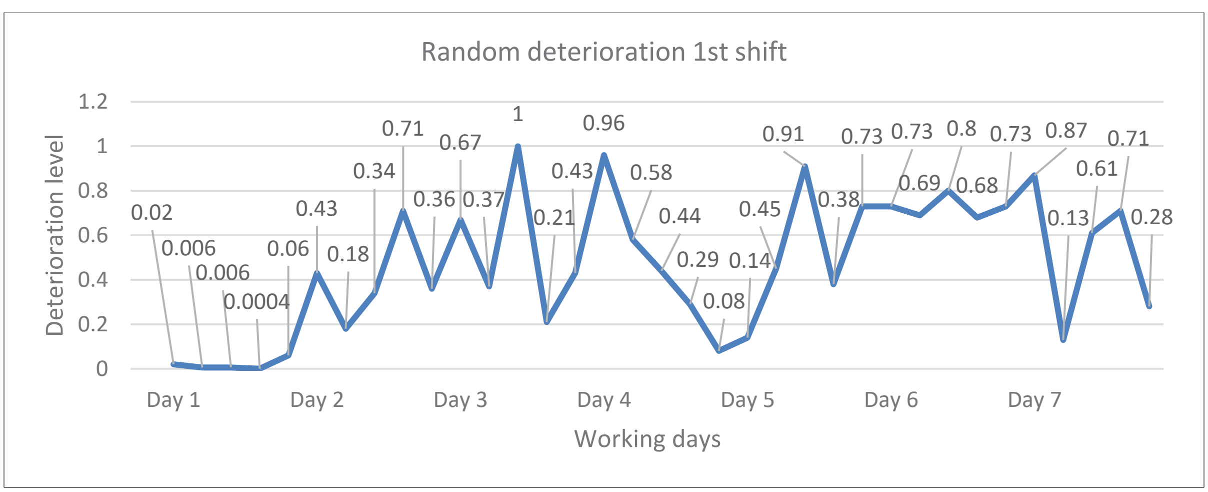

Figure 4.6 Random deterioration behavior 1st shift

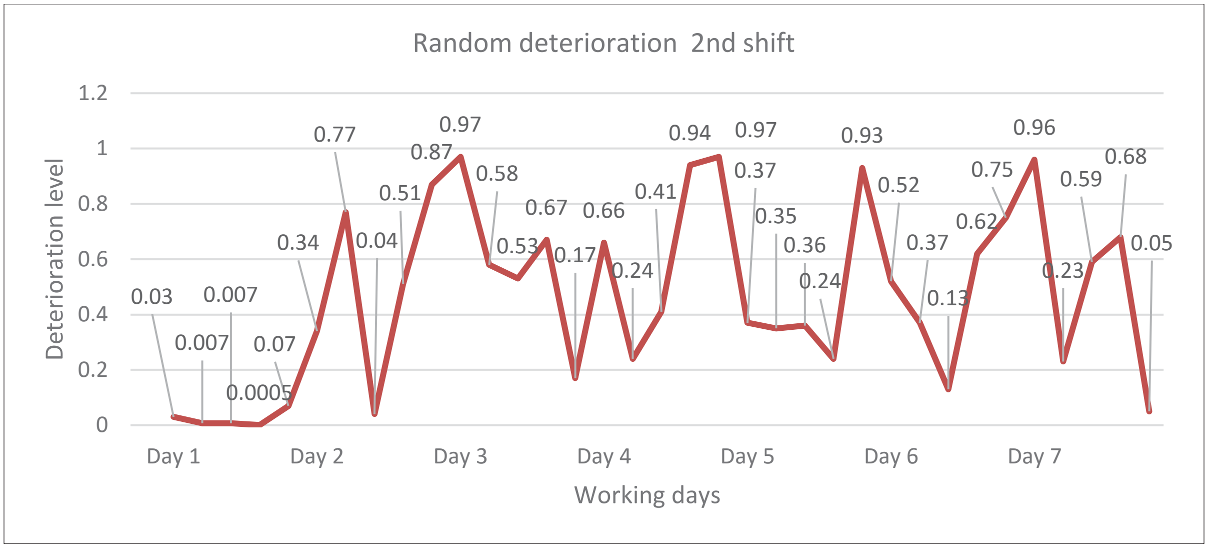

Figure 4.7 Random deterioration behavior 2nd shift

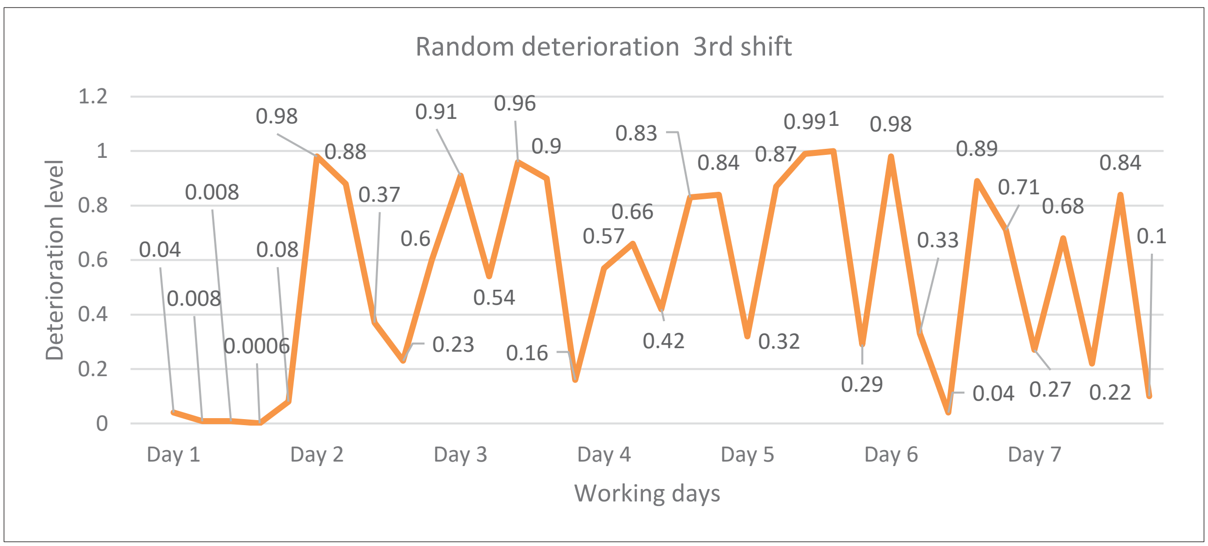

Figure 4.8 Random deterioration behavior 3rd shift

During the first week of production, where maintenance activities were applied to all stations and production shifts from day two through day seven, as well as quality inspection stations for both the forward and reverse manufacturing schemes, the manufacturing system experienced substantial reductions in penalty costs because the number of non-conforming parts shipped was reduced by 133. 906% less than in the quality's absence inspection stations, leading to a profit increase of the entire closed-loop manufacturing system of 160.647% compared to the first week, (see Figure 4.9).

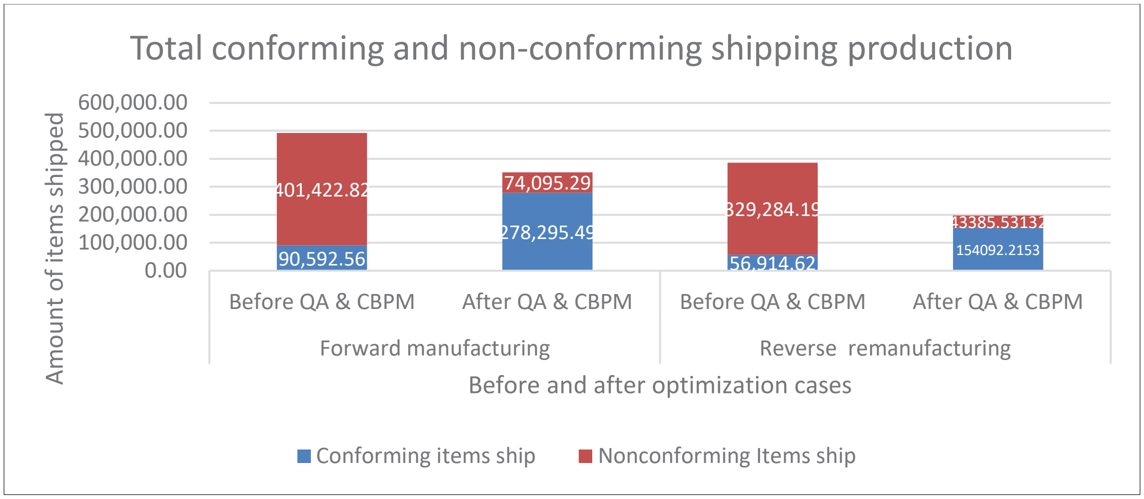

Figure 4.9 Shipping results comparison–(Case 3)

The cost of scrap increased by 321.254% after optimization, whose fraction of nonconforming items were transferred to the re-manufacturing process as raw material increased, of which 64.25% entered as conforming material. The factor that had the greatest economic impact on the closed-loop manufacturing system was the cost of inspection, being the largest of the 3 cases analyzed, since from the second day, all production stations had a station dedicated to inspecting each of the items produced, throughout the three days of production shifts.

Table 4.16 Total forward costs - (Case 3)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total inspection cost	$29,463.42	$254,732.53	764.572%	dls
Total maintenance cost	$1,306.67	$1,173.33	-10.204%	dls
Total repair cost	$10,376.25	$43,710.41	321.254%	dls
Total rework cost	$160,011.09	$685,212.23	328.228%	dls
Total scrap cost	$95,807.36	$403,592.81	321.254%	dls
Total penalization cost	$6,844,259.05	$1,263,324.66	-81.542%	dls

Table 4.17 Total reverse costs - (Case 3)

Costs	Before QA & PM	After QA & PM	Difference	Unit
Total inspection cost	$23,842.83	$260,467.53	992.435%	dls
Total maintenance cost	$1,180.00	$886.67	-24.859%	dls
Total repair cost	$0.00	$0.00	0.000%	dls
Total rework cost	$0.00	$0.00	0.000%	dls
Total scrap cost	$79,393.90	$433,262.00	445.712%	dls
Total penalization cost	$5,614,295.40	$739,723.31	-86.824%	dls

4.4 Conclusion

Through the tests carried out in the three different scenarios in the closed cycle manufacturing system, to find out the improvements that the same manufacturing system could present in terms of production quality, reduction of penalties, improvement of rework costs, repair and scrap among other production costs; this using the forward mathematical model proposed in this research. Substantial changes were observed mainly in case number three, which, despite presenting the highest costs in inspection and maintenance, managed to reduce its economic losses by 155.742%, being the most considerable change of the three cases, followed by case number two with a change of 99.859% where the deterioration is increasing exponentially, (see (Table 4.18), (Table 4.19) and (Figure 4.10).

Table 4.18 Total forward echelon costs comparison

Costs	Case 1		Case 2		Case 3	
	Production based	Sales based	Production based	Sales based	Production based	Sales based
Total inspection cost	1.426%	0.912%	4.016%	-5.954%	5.220%	-55.203%
Total maintenance cost	0.026%	0.017%	0.033%	-0.049%	0.024%	-0.254%
Total repair	0.104%	0.066%	0.892%	-1.322%	0.896%	-9.472%
Total rework cost	1.594%	1.020%	13.700%	-20.309%	14.041%	-148.492%
Total scrap cost	0.956%	0.612%	8.235%	-12.208%	8.270%	-87.463%

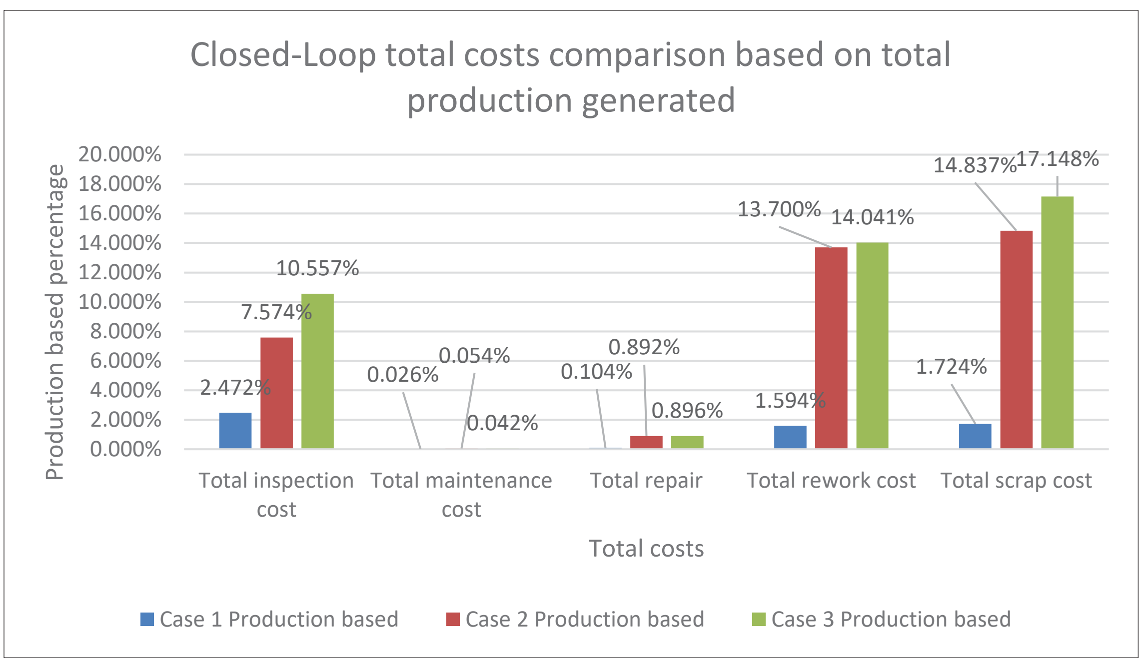

Figure 4.10 Closed-Loop total costs comparison results based on total production

Table 4.19 Total reverse echelon costs comparison

Costs	Case 1		Case 2		Case 3	
	Production based	Sales based	Production based	Sales based	Production based	Sales based
Total inspection cost	1.046%	1.068%	3.558%	-12.701%	5.337%	-21.707%
Total maintenance cost	0.000%	0.000%	0.021%	-0.075%	0.018%	-0.074%
Total repair	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Total rework cost	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Total scrap cost	0.767%	0.783%	6.602%	-23.571%	8.878%	-36.107%

However, despite improving the economic losses in both cases, number one and number two, these closed-loop manufacturing systems still lag behind case number one in the second week of production after installing quality stations and deploying maintenance activities suggested by the model, which increased its profitability, (see Table 4.20).

Table 4.20 Total profit closed-loop manufacturing system

Case 1		Case 2		Case 3	
Before	After	Before	After	Before	After
$5,072,335.74	$5,421,102.90	-$8,532,755.71	-$2,849,599.01	-$13,354,112.73	-$1,661,385.14

Part of the economic losses of case 3, when the manufacturing system was under random deterioration, is due to the discarded percentage of supposedly non-conforming products during the re-manufacturing step, which represents 24.373% of the raw material, due to the lack of rework or repair processes, being part of the re-manufacturing system logic of not having post-processing activities to avoid a more significant number of quality risks while the manufacturing system reaches an optimal level of quality and stability in its productive processes, (see Figure 4.11)

Even though case number three presented a random deterioration factor, it presented a lower fraction of nonconforming parts, 19.330% of the material, than case two, where the deterioration was increasing, with 23.859% of the raw material, (see (Table 4.21) and (Table 4.22)).

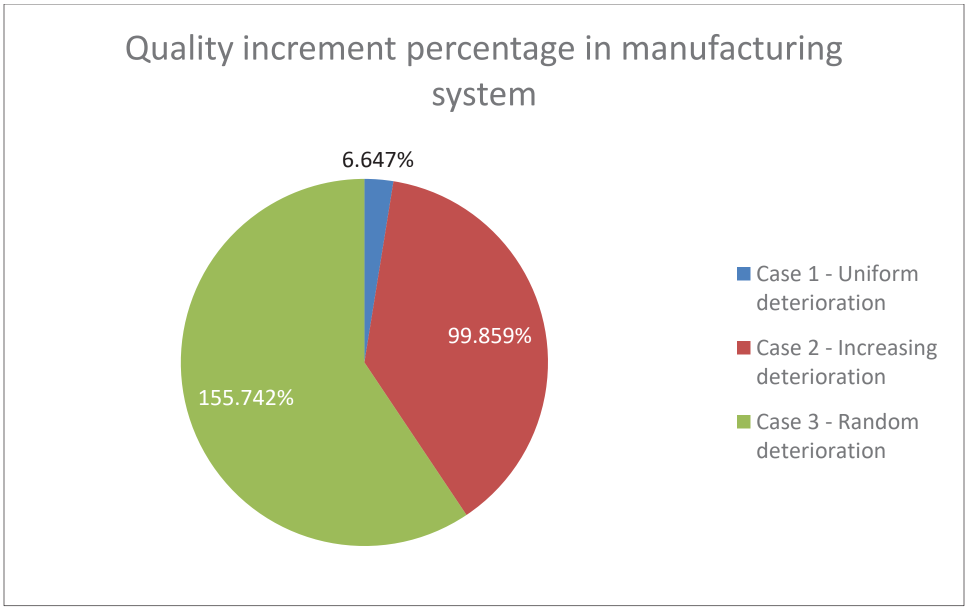

Figure 4.11 Closed-Loop increment profit

Table 4.21 Total forward echelon penalization fraction based on production level

Case 1		Case 2		Case 3	
Production based	**Sales based**	**Production based**	**Sales based**	**Production based**	**Sales based**
35.963%	23.005%	113.086%	-167.635%	25.887%	-273.775%

Table 4.22 Total reverse echelon penalization fraction based on production level

Case 1		Case 2		Case 3	
Production based	**Sales based**	**Production based**	**Sales based**	**Production based**	**Sales based**
35.501%	36.244%	68.455%	-244.400%	15.158%	-61.647%

As a result of the higher number of quality inspection stations and maintenance activities deployed in the third case, the closed-loop manufacturing system was able to detect a higher number of items to rework, repair, and scrap, which generated a higher impact on the costs of these three elements, compared to case one and two

CHAPTER 5

CLOSED-LOOP MANUFACTURING SYSTEM BENEFITS AS RESULT OF QUALITY STATION ALLOCATION AND CONDITION BASE MAINTENANCE

5.1 Introduction

One main element of the mathematical model, is to provide a new alternative for different manufacturing systems, especially those of closed cycle, which seek to improve their quality standards and operational condition of the machines that make up the production lines through the search and optimal allocation of stations dedicated to quality inspection and deployment of preventive maintenance actions based on the current condition of their equipment. However, due to the productive structure of each manufacturing system and the deterioration situation to which they are exposed, the economic and productive benefits are variable between each case, as well as their incidence in the costs implied by the optimization suggestions.

Due to the situation mentioned above, the following chapter jointly analyzes the costs involved install quality inspection stations and maintenance activities of each manufacturing system, in the three scenarios of deterioration proposed, the benefits they get at productive and economic levels, considering the penalty costs for non-compliance with quality standards; ending with a conclusion of the system that got the greatest benefit from the mathematical model.

5.2 Maintenance and inspection costs investments

One of the main elements of the mathematical model is to provide an alternative to different manufacturing systems, which contemplates the optimal allocation of dedicated quality inspection stations and deployment of preventive maintenance actions based on the condition of the production machinery that conforms to the manufacturing system, become these two elements as a reference point to visualize and understand how the mathematical model suggestion affects the manufacturing system under three different and main scenarios of deterioration they were studied.

The closed-loop manufacturing systems were shown to incur the highest quality inspection costs among the forward manufacturing and reverse re-manufacturing systems dedicated when the system is under random deterioration level, since from the second production day, the mathematical model suggests allocating 76 quality inspection station, which 64 changed dynamically through the whole week. Therefore the cost passed from \$53306.25 dls to \$515,200.06 dls, followed by the forward manufacturing system's \$29,463.42 dls to \$254,732.53 dls, while the reverse re-manufacturing system passed from \$19,509.26 dls to \$111,168.45 dls with 38 quality inspection stations where 32 changed dynamically too, (see Figure 5.1)

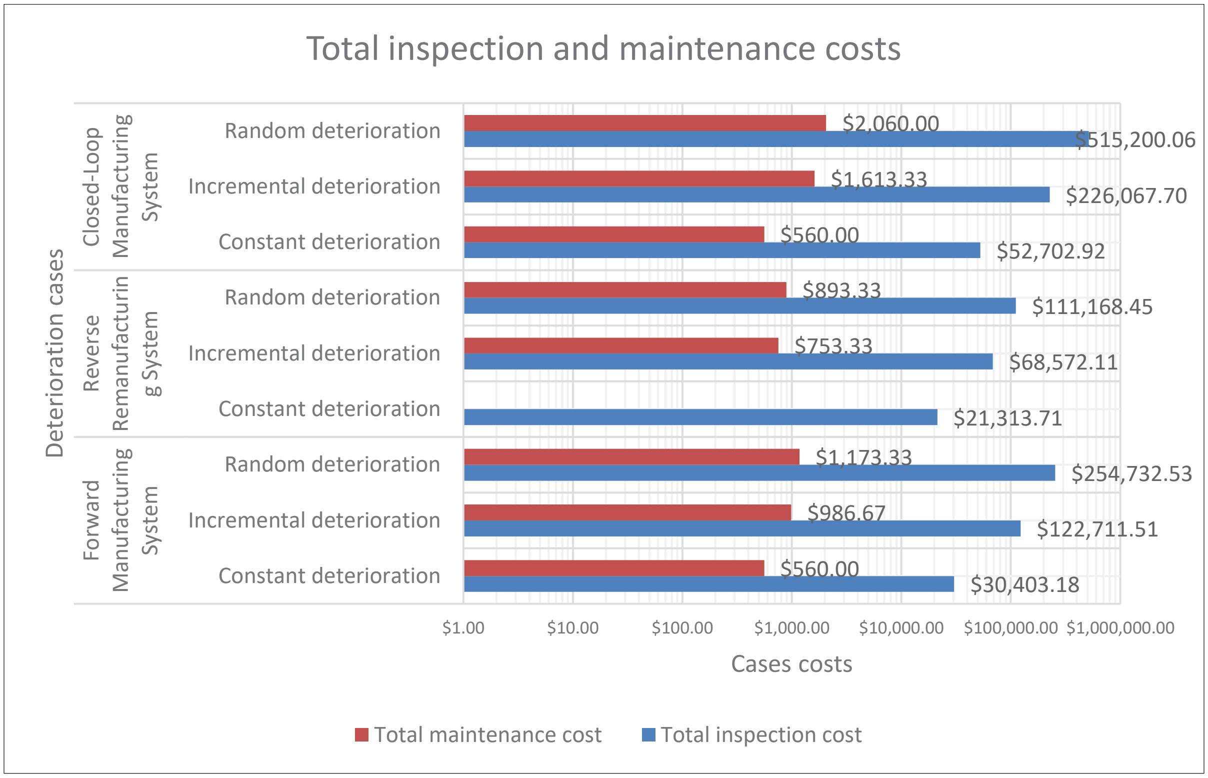

Figure 5.1 Total maintenance and inspection costs comparisons

The maintenance costs, instead of increasing as the quality costs, decrease in all the deterioration scenarios; however, this behavior is only present for the forward manufacturing and closed-loop manufacturing systems since the mathematical model found that only when the re-manufacturing system was under increasing and random deterioration level, deployment of maintenance activities were necessary. In contrast, the deterioration factor was stable over time, and no maintenance was necessary.

In this aspect, the closed-loop re-manufacturing system showed the highest decrement in maintenance costs with 39.504% when the system was under an increasing deterioration factor, followed by the manufacturing system when the deterioration factor was stable over time with a decrement of 29.410% and the re-manufacturing system with a decrement of 24.29% for a random deterioration behavior.

As a result of a higher number of quality inspection stations and the deployment of more preventive maintenance activities, the manufacturing systems showed to be capable of detecting non-conformities to be repaired, reworked, or, in the worst case, scrapped. The highest scrap level generated after optimization was presented in the closed-loop manufacturing system since the amount of material was higher because of non-conforming material

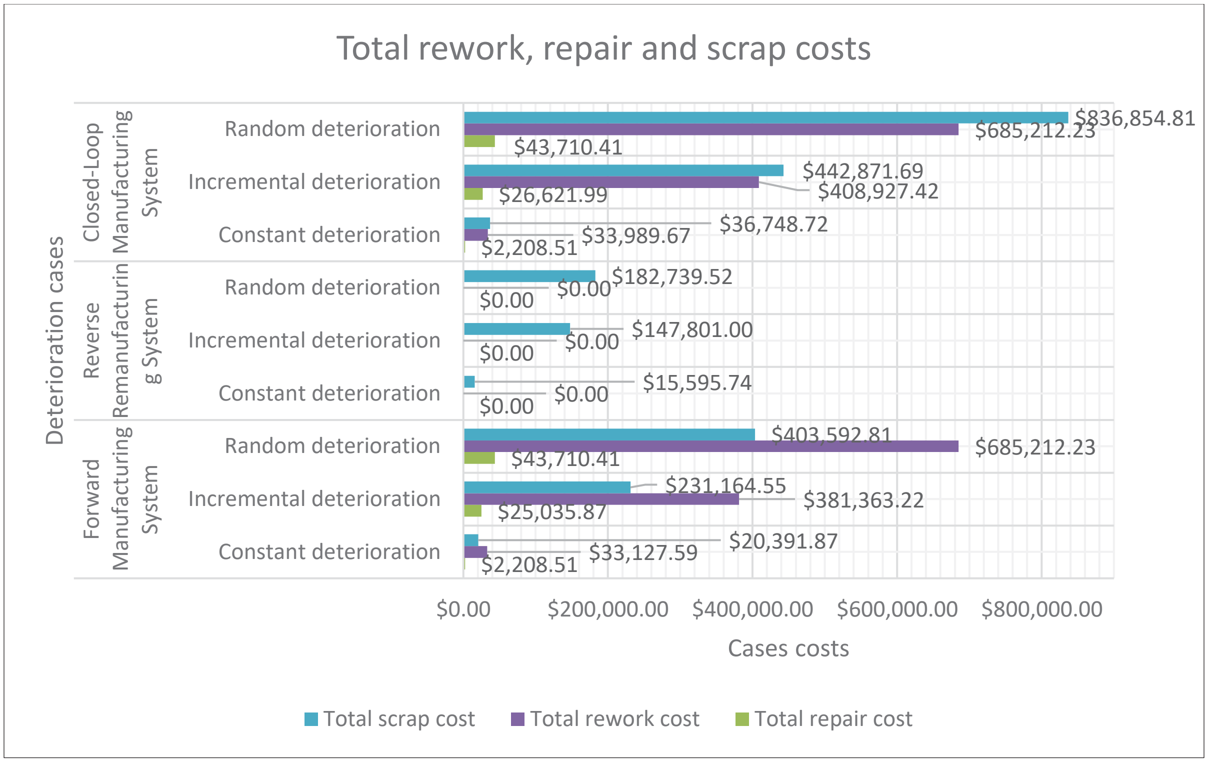

Figure 5.2 Total scrap, rework and repair costs comparison

Re-utilization from the forward echelon in the re-manufacturing echelon. Nonetheless, the most notable change from $95,807.36 to $403,592.81 dls in scrap was generated in the reverse echelon since this echelon is not capable of re-using material through repair or reworked activities. Therefore, the re-manufacturing system incurred neither repair nor rework costs as the forward manufacturing does in every scenario, which showed a decrease of 8.172% again when no optimization was applied, (see Figure 5.2).

5.3 Production changes

In the previous section, we showed the different expenses that the manufacturing systems incurred and how they changed as a result of the maintenance strategies and location of inspection stations, generating an impact primarily in the production of conforming and nonconforming products; however, due to the deterioration factors and inspection errors type I and type II, the fraction of conforming and nonconforming production may vary, in addition to the structure of the manufacturing systems.

In this section, the closed-cycle re-manufacturing system showed more significant changes in the fraction of conforming parts sent to the customer, with an increase of 377.937% and a decrease of 168.366% in the shipment of nonconforming parts for the scenario where the deterioration factor was randomly present, 116% of conforming parts when under growing deterioration and a reduction of 90.801% of non-conformities, (see Figure 5.3).

This was followed by the forward manufacturing system with an increase of 349.196% in conforming parts and a decrease of 207.195% in non-conformance after optimization. In this section, the re-manufacturing system achieved its highest increase in conformity of 83.018% and reduction of non-conformity of 91.415%.

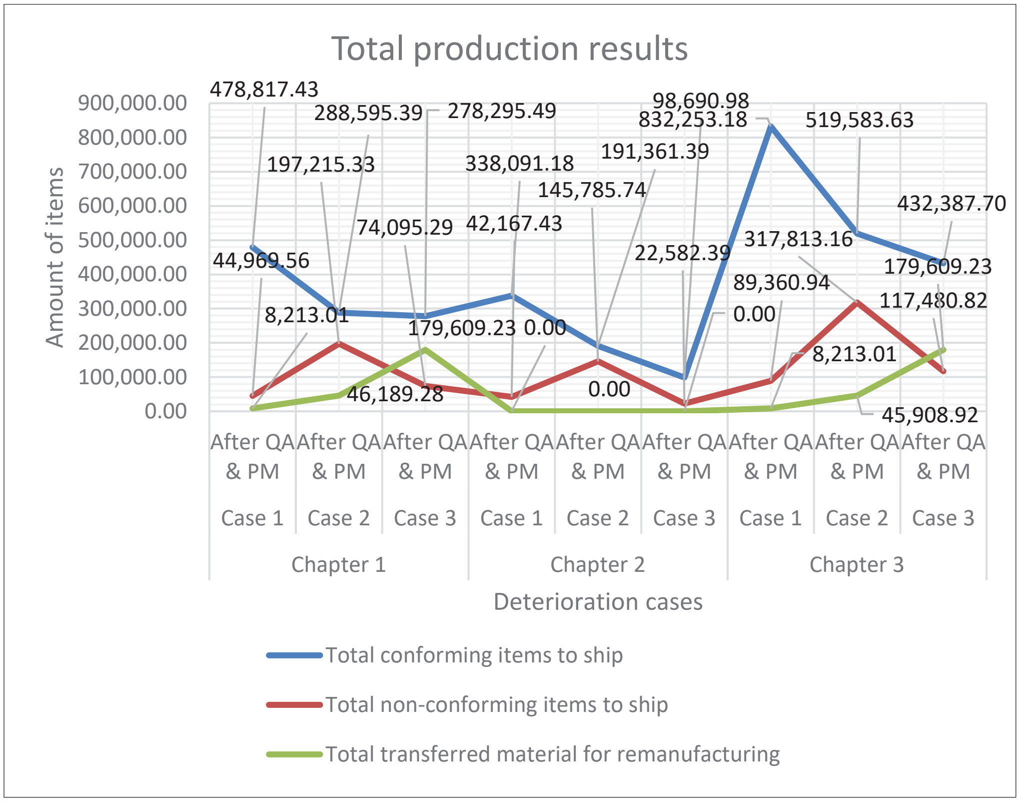

Figure 5.3 Total production results

It is essential to highlight that although the closed-loop manufacturing system had the highest increase in shipments of conforming products after optimization strategies, as well as a considerable decrease in nonconforming parts when the system was under random deterioration, being the most adverse scenario, the forward manufacturing system was able to ship 10.19% more conforming parts when it was under constant deterioration.

The generation of nonconforming items with the potential to be the raw material for a re-manufacturing process decreased only for the forward manufacturing system when it was under random deterioration with a decrease of 81.542% and for the closed cycle manufacturing system by 6.638% when the system was under constant deterioration over time, since when it presented random deterioration, it increased by 349.196%; however it is the only one at which this percentage can generate profit.

5.4 Economic benefits with respect to penalty costs

Regarding the issue of economic benefits, the manufacturing system presented the highest economic income after optimization of 5 million dollars, followed by the forward manufacturing system, which was above 3 million dollars. The last one was the re-manufacturing system, which cost over 2 million dollars; however, the manufacturing systems could only generate economic benefits when they were under stable deterioration throughout the production period. They all showed economic losses when they were under increasing or aleatory deterioration.

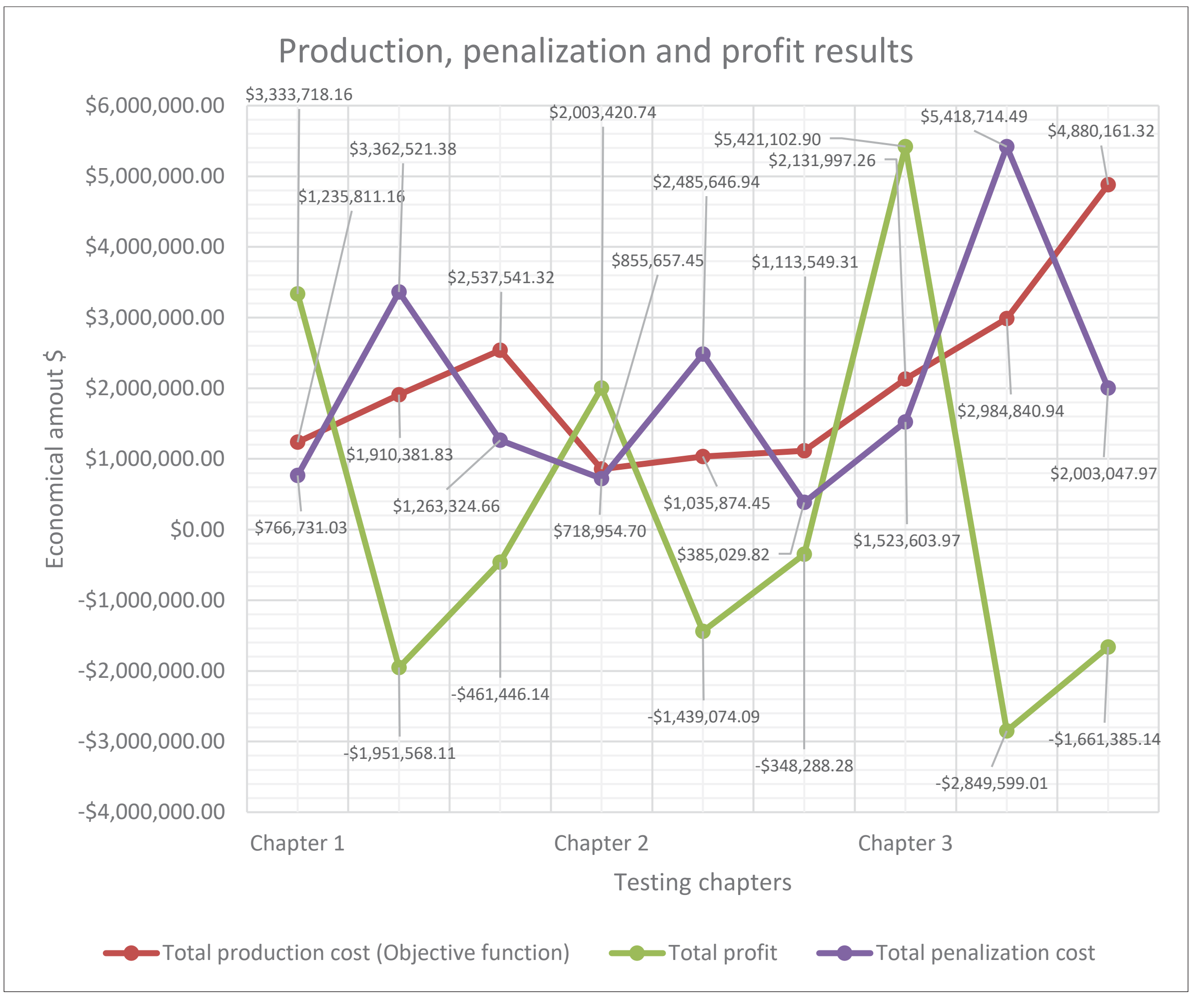

Figure 5.4 Total production, penalization vs profit results

The most significant economic losses in test scenarios two and three were for the closed-loop manufacturing system because the material flow was much higher than each of the isolated cases, which represents a higher amount of nonconforming units produced; however, due to the reuse of reused nonconforming material, the manufacturing system in the first scenario was able to outperform the forward manufacturing system with a difference of 15.59% more profit and 64.2% more than the isolated manufacturing system, (see (Figure 5.4) and (Figure 5.5)).

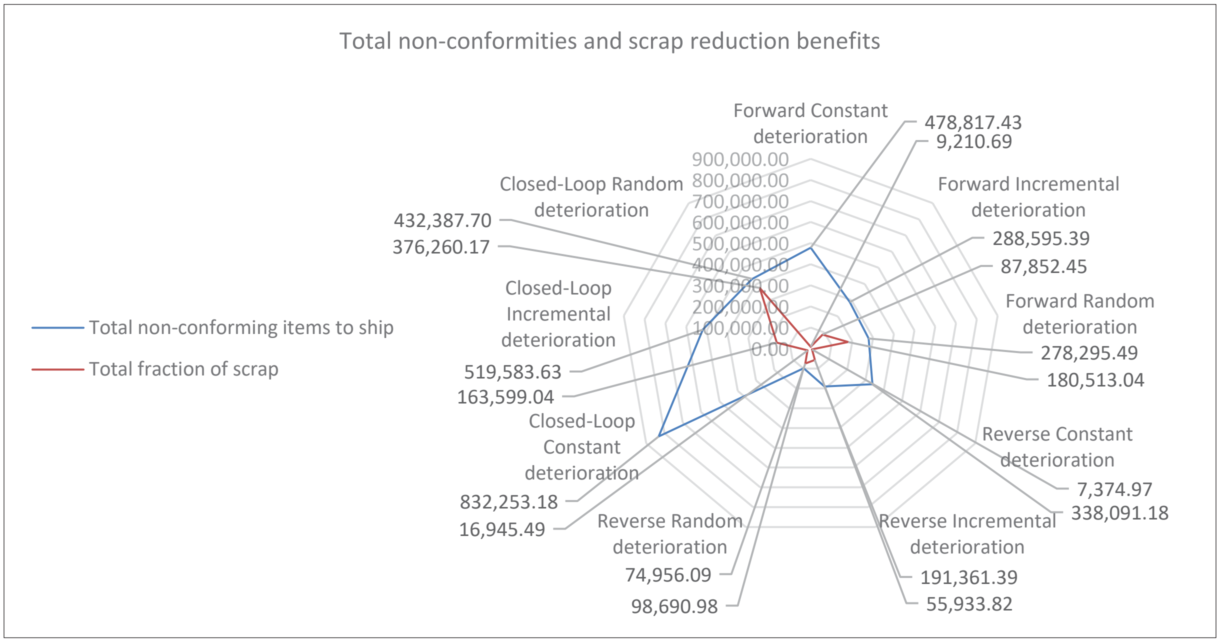

Figure 5.5 Total non-conformities and scrap reduction benefits

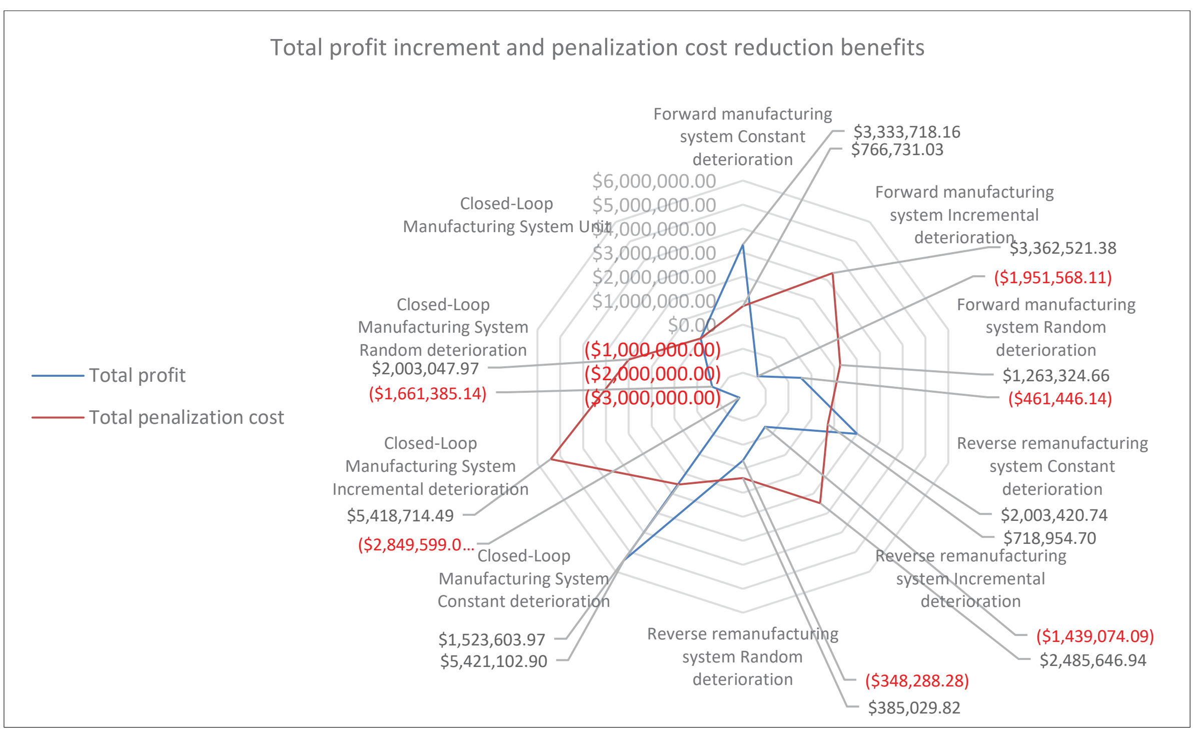

Figure 5.6 Total profit benefits vs penalization cost

It is important to emphasize that even though in the scenarios of increasing and random deterioration, the manufacturing systems experienced economic losses, this was during the first early stage of testing the mathematical model, which proved to provide support to improve the quality and condition of the production machinery, opening an opportunity for improvement over a more extended period, (see Figure 5.6).

5.5 Conclusion

This chapter studied the benefits and changes of each manufacturing system in each of the three cases of deterioration presented after optimization. The analysis considered the percentage of change before and after optimization, both in economic and productive aspects, based on the quality assurance of the items produced and the cost of penalties arising from these changes.

Thanks to the analysis, it was possible to understand the significant impact of the deterioration factor on the manufacturing systems, being this impact is more notable compared to the type I and type II inspection errors that may arise due to the human factor that performs the inspections. Primarily, the random deterioration factor is the one that generates a more substantial economic impact in the different systems because it was found that from the second day for the closed-loop manufacturing systems as well as forward, it is necessary to implement inspection stations in each productive shift, after each station, while for the reverse system, it is from the third day. This increases inspection costs considerably.

However, the balance is generated through the number of maintenance activities deployed; due to the large number of quality inspection stations, the mathematical model found it feasible only to apply maintenance to those stations of high priority and whose deterioration rate was close to or above the optimal production factor, or even not being necessary in cases where the deterioration factor was stable over time, as in the forward manufacturing system, which met production targets and quality standards despite deterioration, even though it had an optimal number of inspection stations.

Just as the forward manufacturing system had the lowest inspection cost when it was under constant and stable deterioration, the analysis found that the dedicated re-manufacturing system incurred the lowest cost after optimization due to its lower raw material flow, its structure free of rework or repairs, and its percentage of raw material dependent on the forward manufacturing process, which if lower, the production level of the re-manufacturing system would be lower as well. However, despite having the lowest costs, the same system experienced the same economic losses in the cases of increasing deterioration and random deterioration.

Just as the re-manufacturing system presented economic losses in the random and incremental deterioration scenario, the forward and closed-loop manufacturing systems presented the same situation independently; the closed-loop manufacturing system was the one that experienced the most significant changes at the production level, increase of conforming parts produced and reduction of penalty costs in each of the deterioration scenarios.

Thanks to its use of non-conforming raw material for rework and conditioning for a second market, which is a fundamental part of its closed-loop structure, it gives way to continuous improvement and more significant profit generation over time, as compared to forward manufacturing or re-manufacturing systems that perform their operations in isolation.

CONCLUSION AND FUTURE RESEARCH

The constant search for different and innovative products on the part of global markets, which satisfy needs that are no longer focused solely on aesthetic or quality aspects but also that they are manufactured through processes with a consciousness of environmental change, such as the reuse of materials through re-manufacturing processes; This leads to the creation of new manufacturing systems, whose objective is to reuse and recondition a series of different products coming from different forward manufacturing systems; however this situation causes a series of improvement opportunities from logistical to economic aspects.

These challenges arise as a consequence of a need for a more organic and fluid connection between the forward manufacturing step and the reverse re-manufacturing step since each one operates in an isolated and independent way. Therefore closed-loop manufacturing systems are born as an alternative to connect both production steps, bringing with them not only the economic and productive benefits that each one represents independently but also generating a source of income and capturing new markets through the reuse of the same family of products internally at home. The potential for capturing new markets through closed-loop manufacturing is an exciting growth opportunity for your business.

However, this manufacturing system is also exposed to different unfavorable factors, such as the deterioration of its operating equipment, leading to generating low-quality levels and high costs in penalties. Because of this situation, the research project developed in this thesis attempts to provide an alternative to improve the quality and condition of the production lines for existing manufacturers and companies in the transition stages to a closed-loop production model.

Our alternative presents the development of a mixed-integer-mathematical model, which supports the optimal deployment of preventive maintenance activities based on the condition of production equipment and quality station facilities dedicated to 100% inspection. The model was tested independently in forward and reverse manufacturing systems and in closed loop manufacturing systems. All manufacturing systems had three distinct deterioration factors: type I and type II quality inspection errors and imperfect rework activities.

The results showed a good performance of the mathematical model in the different manufacturing systems in which it was tested, generating more substantial changes in the closed-loop manufacturing system through increases above 10% when subject to stable deterioration and reduction of economic losses above 160% when under random deterioration. Reduced penalty costs by over 20%. Thanks to the utilization of non-conforming raw material from its forward production step by over 80% when it was under increasing deterioration.

However, these reductions in non-conformities were achieved through investment and inspection costs, which in the first stage of testing reached an increase of over 700% in its forward manufacturing step and maintenance costs of over 25% in its reverse re-manufacturing step. However, despite the positive changes brought about by the proposals made by the mathematical model, not only the closed-loop manufacturing system but also the forward and reverse manufacturing systems showed economic losses when subjected to a deterioration factor that increases exponentially over the days of a production week and over three shifts as well as when under random deterioration.

This research provides a new approach to understanding the interaction of quality inspections and maintenance within complex, closed-loop, multi-stage manufacturing systems, considering human error and diverse deterioration factors. Although we tested the model in a deterministic setting, it can serve as a valuable decision-support tool for manufacturing engineers, scientists, and operations research professionals seeking to improve quality and enhance manufacturing system efficiency. The insights provided could serve as a based for future development of optimization tools. Future empirical studies could apply to this model in a specific industrial context in future research

APPENDIX I
LITERATURE REVIEW CHART

Author	Inspection			Defective type in the system		Defect Strategy
	100 % or none	Sampling	Repeated	Constant /Simple	Random /Simple	Scrapping some
Part quality inspection						
Hurst		✓		✓		
Eppen and Hurst		✓				
Ballou and Pazer (a)		✓		✓		
Ballou and Pazer (b)		✓		✓		
Yum and McDowell		✓		✓		✓
Barad		✓				
Foster et al.	✓			✓		
Kang et al.	✓		✓	✓		✓
Raz and Kaspi	✓		✓	✓		
Villalobos and Foster	✓			✓		
Villalobos et al.	✓			✓		
Bai and Yun		✓				
Shiau (a)		✓			✓	✓
Opperman et al.	✓				✓	
Shiau (b)		✓			✓	✓
Shiau (c)		✓			✓	✓
Feng and Kapur	✓			✓		
Shiau et al. (d)	✓			✓		
Vaghefi and Sarhangian		✓		✓		✓
Volsem	✓					✓
Azadeh et al.		✓		✓		✓
Mousavi et al.		✓		✓		
Mohammadi et al. (a)	✓			✓		
Mohammadi et al. (b)	✓			✓		
Part quality inspection, Preventive Maintenance, Closed-Loop Manufacturing System						
Bouslah et al.		✓			✓	✓
Rezaei-Malek et al. (a)	✓			✓		✓

Rezaei-Malek et al. (b)	✓			✓		✓
Rezaei-Malek et al. (c)	✓			✓		✓
Rivera-Gomez et al. (a)		✓			✓	✓
Hajej et al.		✓			✓	
Rivera-Gomez et al. (b)		✓			✓	✓
Part quality inspection, Preventive Maintenance, Closed-Loop Manufacturing System						
The proposed model	✓	✓	✓	✓	✓	✓

Author	Internal part failure cost				Optimization method
	Rework	Replace/Repair	Reuse	Scrap	Integer Programming
Part quality inspection					
Hurst				✓	
Eppen and Hurst				✓	
Ballou and Pazer (a)				✓	
Ballou and Pazer (b)				✓	
Yum and McDowell	✓	✓		✓	✓
Barad	✓	✓		✓	
Foster et al.				✓	
Kang et al.	✓			✓	
Raz and Kaspi	✓			✓	✓
Villalobos and Foster				✓	
Villalobos et al.				✓	
Bai and Yun				✓	
Shiau (a)	✓	✓		✓	
Opperman et al.				✓	
Shiau (b)	✓	✓		✓	
Shiau (c)	✓	✓		✓	
Feng and Kapur				✓	
Shiau et al. (d)	✓			✓	
Vaghefi and Sarhangian	✓			✓	
Volsem	✓			✓	
Azadeh et al.	✓			✓	
Mousavi et al.				✓	
Mohammadi et al. (a)				✓	✓

Mohammadi et al. (b)				✓	✓
Part quality inspection & Preventive Maintenance					
Bouslah et al.				✓	
Rezaei-Malek et al. (a)	✓	✓		✓	
Rezaei-Malek et al. (b)		✓		✓	✓
Rezaei-Malek et al. (c)		✓		✓	
Rivera-Gomez et al. (a)	✓	✓		✓	
Hajej et al.	✓			✓	
Rivera-Gomez et al. (b)				✓	
Part quality inspection, Preventive Maintenance, Closed-Loop Manufacturing System					
The proposed model	✓	✓	✓	✓	✓

APPENDIX II

PRODUCTION DATA FOR FORWARD, REVERSE AND CLOSED LOOP MANUFACTURING SYSTEMS

Table–A II-1 Quantity of material per unit entering the forward, reverse and closed-loop manufacturing systems in the period t (Working Shift) - $QRM1_t$

Period t (Working Shift)	Number of material that enter each period (Working Shift)
1	40000
2	20000
3	16000

Table–A II-2 Non-conforming fraction of raw materials entering the forward, reverse and closed-loop manufacturing systems in the period t (Working Shift) - $\varepsilon 0_j$, $r\varepsilon 0_j$.

Stage j (Working station)	1	2	3	4	5
% of non-conforming	0	0	0	0	0

Table–A II-3 Non-conforming fraction of collected raw materials into the reverse and closed-loop manufacturing systems in the period t (Working Shift) - $r\varepsilon 0_j$.

Stage j (Working station)	1	2	3	4	5
% of non-conforming	0	0	0	0	0

Table–A II-4 Probability of type-I error at the inspection stations for the forward, reverse and closed-loop manufacturing systems in the period t (Working Shift), αtj & $r\alpha tj$.

Day 1–Day 7			
Stage j, k (Working station)	Period t		
	1	2	3
1	0.05	0.05	0.05
2	0.05	0.05	0.05
3	0.05	0.05	0.05
4	0.05	0.05	0.05
5	0.05	0.05	0.05

Table–A II-5 Probability of type-II error at the inspection station in the forward, reverse and closed-loop manufacturing systems in the period t (Working Shift), βtj & $r\beta tj$.

Day 1–Day 7			
Stage j, k (Working station)	Period t		
	1	2	3
1	0.05	0.05	0.05
2	0.05	0.05	0.05
3	0.05	0.05	0.05
4	0.05	0.05	0.05
5	0.05	0.05	0.05

Table–A II-6 Probability of a conforming sample inspected material at sample inspection station, suffers danger or contamination during the process at period t in the forward, reverse and closed-loop manufacturing systems - ω_{tj} & $r\omega_{tj}$

Day 1–Day 7			
Stage j, k (Working station)	Period t		
	1	2	3
1	0.050	0.050	0.050
2	0.050	0.050	0.050
3	0.050	0.050	0.050
4	0.050	0.050	0.050
5	0.050	0.050	0.050

Table–A II-7 Probability of a reworked item at a production does not comply with the quality standards during period t in the forward, reverse and closed-loop manufacturing systems (Second Quality Inspection Process) - θ_{tj} & $r\theta_{tj}$

Day 1–Day 7			
Stage j, k (Working station)	Period t		
	1	2	3
1	0.050	0.050	0.050
2	0.050	0.050	0.050
3	0.050	0.050	0.050
4	0.050	0.050	0.050
5	0.050	0.050	0.050

Table–A II-8 Fraction of the rejected items repaired/replace at production station, at the time t - $F1_{tj}$

Day 1–Day 7			
Stage j, k (Working station)	Period t		
	1	2	3
1	0.200	0.200	0.200
2	0.200	0.200	0.200
3	0.200	0.200	0.200
4	0.200	0.200	0.200
5	0.200	0.200	0.200

Table–A II-9 Fraction of the rejected items scrapped at the inspection station at the time t - $F2_{tj}$, $RF2_{tj}$

Day 1–Day 7			
Stage j, k (Working station)	Period t		
	1	2	3
1	0.300	0.300	0.300
2	0.300	0.300	0.300
3	0.300	0.300	0.300
4	0.300	0.300	0.300
5	0.300	0.300	0.300

Table–A II-10 Fraction of the rejected items reworked at the production station, at the time t - $F3_{tj}$, $RF3_{tj}$

Day 1–Day 7			
Stage j (Working station)	Period t		
	1	2	3
1	0.500	0.500	0.500
2	0.500	0.500	0.500
3	0.500	0.500	0.500
4	0.500	0.500	0.500
5	0.500	0.500	0.500

Table–A II-11 Unit rework cost at the production station in the period t, λ_{tj}

Day 1–Day 7			
Stage j (Working station)	Period t		
	1	2	3
1	\$3.600	\$3.600	\$3.600
2	\$1.080	\$1.080	\$1.080
3	\$1.080	\$1.080	\$1.080
4	\$2.160	\$2.160	\$2.160
5	\$3.240	\$3.240	\$3.240

Table–A II-12 Unit scrap cost at the production station in the period t, S_{tj}

Day 1–Day 7			
Stage j (Working station)	Period t		
	1	2	3
1	\$3.324	\$3.324	\$3.324
2	\$1.108	\$1.108	\$1.108
3	\$1.108	\$1.108	\$1.108
4	\$2.216	\$2.216	\$2.216
5	\$3.324	\$3.324	\$3.324

Table–A II-13 Unit scrap cost at the production station in the period t, RS_{tj}

Day 1–Day 7			
Stage j (Working station)	Period t		
	1	2	3
1	\$3.157	\$3.157	\$3.157
2	\$1.052	\$1.052	\$1.052
3	\$1.052	\$1.052	\$1.052
4	\$2.105	\$2.105	\$2.105
5	\$3.157	\$3.157	\$3.157

Table–A II-14 Unit condition base preventive maintenance cost at the station in the period t, μ_{tj}, $r\mu_{tj}$

Day 1–Day 7			
Stage j (Working station)	Period t		
	1	2	3
1	\$20.000	\$20.000	\$20.000
2	\$6.667	\$6.667	\$6.667
3	\$6.667	\$6.667	\$6.667
4	\$13.333	\$13.333	\$13.333
5	\$20.000	\$20.000	\$20.000

APPENDIX III

DETERIORATION LEVELS CASE 1

Table–A III-1 Constant deterioration levels

Day 1				Day 2			
Stage j (Working station)	Period t			Stage j (Working station)	Period t		
	1	2	3		1	2	3
1	0.02	0.03	0.04	1	0.02	0.03	0.04
2	0.02	0.03	0.04	2	0.02	0.03	0.04
3	0.02	0.03	0.04	3	0.02	0.03	0.04
4	0.02	0.03	0.04	4	0.02	0.03	0.04
5	0.02	0.03	0.04	5	0.02	0.03	0.04
Day 3				Day 4			
Stage j (Working station)	Period t			Stage j (Working station)	Period t		
	1	2	3		1	2	3
1	0.02	0.03	0.04	1	0.02	0.03	0.04
2	0.02	0.03	0.04	2	0.02	0.03	0.04
3	0.02	0.03	0.04	3	0.02	0.03	0.04
4	0.02	0.03	0.04	4	0.02	0.03	0.04
5	0.02	0.03	0.04	5	0.02	0.03	0.04
Day 5				Day 6			
Stage j (Working station)	Period t			Stage j (Working station)	Period t		
	1	2	3		1	2	3
1	0.02	0.03	0.04	1	0.02	0.03	0.04
2	0.02	0.03	0.04	2	0.02	0.03	0.04
3	0.02	0.03	0.04	3	0.02	0.03	0.04
4	0.02	0.03	0.04	4	0.02	0.03	0.04
5	0.02	0.03	0.04	5	0.02	0.03	0.04
Day 7							
Stage j (Working station)	Period t						
	1	2	3				
1	0.02	0.03	0.04				
2	0.02	0.03	0.04				
3	0.02	0.03	0.04				
4	0.02	0.03	0.04				
5	0.02	0.03	0.04				

APPENDIX IV

DETERIORATION LEVELS CASE 2

Table–A IV-1 Exponential increasing deterioration levels

Day 1				Day 2			
Stage j (Working station)	Period t			Stage j (Working station)	Period t		
	1	2	3		1	2	3
1	0.02	0.03	0.04	1	0.05	0.06	0.07
2	0.006	0.007	0.008	2	0.009	0.01	0.02
3	0.006	0.007	0.008	3	0.009	0.01	0.02
4	0.0004	0.0005	0.0006	4	0.0007	0.0008	0.0009
5	0.06	0.07	0.08	5	0.09	0.1	0.2
Day 3				Day 4			
Stage j (Working station)	Period t			Stage j (Working station)	Period t		
	1	2	3		1	2	3
1	0.08	0.09	0.1	1	0.1	0.2	0.3
2	0.03	0.04	0.05	2	0.06	0.07	0.08
3	0.03	0.04	0.05	3	0.06	0.07	0.08
4	0.001	0.0011	0.0012	4	0.0013	0.0014	0.0015
5	0.3	0.4	0.5	5	0.6	0.7	0.8
Day 5				Day 6			
Stage j (Working station)	Period t			Stage j (Working station)	Period t		
	1	2	3		1	2	3
1	0.4	0.5	0.6	1	0.7	0.8	0.9
2	0.09	0.1	0.2	2	0.3	0.4	0.5
3	0.09	0.1	0.2	3	0.3	0.4	0.5
4	0.0016	0.0017	0.0018	4	0.0019	0.002	0.0021
5	0.9	1	1.1	5	1.2	1.3	1.4
Day 7							
Stage j (Working station)	Period t						
	1	2	3				
1	1	1.1	1.2				
2	0.6	0.7	0.8				
3	0.6	0.7	0.8				
4	0.0022	0.0023	0.0024				
5	1.5	1.6	1.7				

APPENDIX V

DETERIORATION LEVELS CASE 3

Table–A V-1 Random deterioration levels

Day 1				Day 2			
Stage j (Working station)	Period t			Stage j (Working station)	Period t		
	1	2	3		1	2	3
1	0.51	0.29	0.9	1	0.59	0.83	0.41
2	0.33	0.64	0.64	2	0.55	0.72	0.7
3	0.12	0.51	0.44	3	0.28	0.28	0.4
4	0.91	0.68	0.28	4	0.89	0.89	0.8
5	0.58	0.45	0.05	5	0.76	0.21	0.04
Day 3				Day 4			
Stage j (Working station)	Period t			Stage j (Working station)	Period t		
	1	2	3		1	2	3
1	0.98	0.85	0.65	1	0.39	0.41	0.96
2	0.13	0.4	0.87	2	0.3	0.37	0.61
3	0.04	0.36	0.56	3	0.1	0.56	0.8
4	0.93	0.51	0.72	4	0.6	0.96	0.68
5	0.38	0.2	0.63	5	1	0.21	0.22
Day 5				Day 6			
Stage j (Working station)	Period t			Stage j (Working station)	Period t		
	1	2	3		1	2	3
1	0.72	0.05	0.07	1	0.25	0.36	0.85
2	0.65	0.52	0.71	2	0.41	0.28	0.1
3	0.18	0.31	0.15	3	0.59	0.99	0.73
4	0.42	0.79	0.92	4	0.76	0.84	0.98
5	0.89	0.45	0.39	5	0.79	0.21	0.99
Day 7							
Stage j (Working station)	Period t						
	1	2	3				
1	0.29	0.97	0.24				
2	0.72	0.37	0.91				
3	0.54	0.11	0.04				
4	0.76	0.84	0.18				
5	0.93	0.58	0.98				

APPENDIX VI

CPLEX PROGRAMM (CODE)

```
/*********************************************
 * OPL 22.1.0.0 Model
 * Author: Amauri
 * Creation Date: Jan 25, 2024 at 1:34:36 PM
 *********************************************/

//1.- Sets Manufacturing
int n = ...; //Maximum number of forward manufacturing
stations
int m = ...; //Maximum number of forward working shifts

//1.- Sets Remanufacturing
int r = ...; //Maximum number of reverse re-manufacturing
working shifts

//2.- Ranks Manufacturing
range J = 1..n; //Set of forward manufacturing stations
range T = 1..m; //Set of forward and reverse re-manufacturing
shifts

//2.- Ranks Remanufacturing
range K = 1..r; //Set of reverse re-manufacturing stations

//3.1 - General forward manufacturing parameters
float QRM1[T] = ...; //Quantity of raw material per unit (one
unit/minute) entering to the Serial-Forward-Multi-Stage-
Manufacturing-System in the period t (Forward manufacturing
shift/Production line)
float epsilon0[J] = ...; //Non-conforming fraction of raw
material entering the Serial-Forward-Multi-Stage-
Manufacturing- System at station j (Forward manufacturing
station)
float epsilon[T][J] = ...; //Deterioration level of which
working station j (Forward manufacturing station) is exposed
in the period t (Working shift)
```

```
float de = ...; //Forward stationary fixed demand for primary market
float delta = ...; //Forward manufacturing production capacity per period t (Working shift)

//3.1 - General reverse re-manufacturing parameters
//float RQRM1[T] = ...; //Quantity of collected raw material, entering to the Serial-Reverse-Multi-Stage-Remanufacturing-System in the period t (Reverse re-manufacturing shift/Production line)
float repsilon0[K] = ...; //Non-conforming fraction of raw material entering the Serial-Reverse-Multi-Stage-Remanufacturing-System at station k (Reverse re-manufacturing station)
float repsilon[T][K] = ...; //Degradation level of which working station k (Reverse re-manufacturing station) is exposed in the period t (Working shift)
float rde = ...; //Reverse re-manufacturing stationary fixed demand for secondary market
float rdelta = ...; //Reverse re-manufacturing production capacity per period t (Working shift)

//3.2 - Quality forward manufacturing parameters
float gamma = ...; //Fraction of conforming and non-conforming items sampled after station j (Manufacturing Station)
float alpha[T][J] = ...; //Probability of type-I error at sample inspection station j (Forward Manufacturing Station)
float betha[T][J] = ...; //Probability of type-II error at sample inspection station j (Forward Manufacturing Station)
float omega[T][J] = ...; //Probability of a conforming sample inspected material at sample inspection station j (Forward Manufacturing Station) suffers danger or contamination during the process at period t (working shift)
float theta[T][J] = ...; //Probability of a reworked item at station j (Forward manufacturing station) does not comply with the quality standards during period t (working shift)
float F1[T][J] = ...; //Fraction of the rejected items at sample inspection station j (Forward manufacturing station) during period t (working shift) that must be repaired
```

```
float F2[T][J] = ...; //Fraction of the rejected items at
sample inspection station j (Forward Manufacturing Station)
during period t (Working Shift) that must be scrapped
float F3[T][J] = ...; //Fraction of the rejected items at
sample inspection station j (Forward Manufacturing Station)
during period t (Working Shift) that must be reworked
float X = ...; //Minimum percentage of acceptable sample as
quality parameter in sampling quality station j (Quality
inspection decision) (Forward Manufacturing Station)
float X2 = ...; //Percentage of end-of-life products recovery
float X4 = ...; //Percentage of conforming items produced in a
next operation
float X5= ...; //Fraction of sample conforming and non-
conforming items rejected because repair and rework are needed
float X6= ...; //Percentage of a total production of
conforming items during a shift

//3.2 - Quality reverse re-manufacturing parameters
float rgamma = ...; //Fraction of collected conforming and
non-conforming items sampled after station j (Remanufacturing
Station)
float ralpha[T][K] = ...; //Probability of type-I error at
sample inspection station k (Reverse Remanufacturing Station)
float rbetha[T][K] = ...; //Probability of type-II error at
sample inspection station k (Forward Manufacturing Station)
float romega[T][K] = ...; //Probability of a conforming sample
inspected material at sample inspection station k (Reverse
Remanufacturing Station) suffers danger or contamination
during the period t (working shift)
float RF2[T][K] = ...; //Fraction of the rejected items at
sample inspection station k (Reverse Remanufacturing Station)
during period t (Working Shift) that must be scrapped
float RX = ...; //Minimum percentage of acceptable sample as
quality parameter in sampling quality station j (Quality
inspection decision) (Reverse Remanufacturing Station)
float X3 = ...; //Percentage of collected items

//3.3 - Cost forward manufacturing parameters
float rho [T][J] = ...; //Unit production cost for processing
at station j (Forward Manufacturing Station) during period t
(Working Shift)
```

```
float i [T][J] = ...; //Unit inspection cost for inspection at
quality station j (Forward Manufacturing Station) during
period t (Working Shift)
float kappa [T][J] = ...; //Unit repair cost at station j
(Forward Manufacturing Station) during period t (Working
Shift)
float lambda [T][J] = ...; //Unit rework cost at station j
(Forward Manufacturing Station) during period t (Working
Shift)
float s [T][J] = ...; //Unit scrap cost at station j (Forward
Manufacturing Station) during period t (Working Shift)
float mu [T][J] = ...; //Unit preventive maintenance cost at
station j (Forward Manufacturing Station)
float xi = ...; //Penalty cost per non-conforming item
received by the mainly market customer (Forward Manufacturing
System)

//3.3 - Cost reverse re-manufacturing parameters
float rrho[T][K] = ...; //Unit production cost at processing
stage k (Reverse Remanufacturing Station) during period t
(Working Shift)
float ri[T][K] = ...; //Unit inspection cost for inspection at
quality station k (Reverse Remanufacturing Station)
float rs[T][K] = ...; //Unit scrap cost at station k (Reverse
Remanufacturing Station) during period t (Working Shift)
float rmu[T][K] = ...; //Unit preventive maintenance cost at
station k (Reverse Remanufacturing Station)
float rxi = ...; //Penalty cost per non-conforming item
received by the secondary market (Reverse Remanufacturing
Station) (This is a novelty)

//3.4 - forward manufacturing sales cost
float SLC = ...; //Unit sales cost

//3.4 - Reverse re-manufacturing sales cost
float RSLC = ...; //Unit sales cost

//4.-//Constraint
variables****************************************************
*************************************************************
****************
```

```
//4.1 - Forward manufacturing constraint variables
dvar float+ RAW1[T][J]; //Expected quantity of conforming raw
material entering during the 1st period t (Working Shift) at
production line j (Forward Manufacturing Production Line)
dvar float+ RAW2[T][J]; //Expected quantity of non-conforming
raw material entering during the 1st period t (Working Shift)
at production line j (Forward Manufacturing Production Line)

dvar float+ M1[T][J]; //Expected number of conforming items
entering during the 1st period t (Working Shift) at production
line j (Forward Manufacturing Production Line) after prior
100% inspection.
dvar float+ M2[T][J]; //Expected number of non-conforming
items entering during the 1st period t (Working Shift) at
production line j (Forward Manufacturing Production Line)
after prior 100% inspection.

dvar float+ EW1[T][J]; //Expected number of conforming items
entering at stage j (Forward Manufacturing Station) during
period t (Working Shift) (Manufacturing Balance Constraints)
dvar float+ EW2[T][J]; //Expected number of non-conforming
items entering at stage j (Forward Manufacturing Station)
during period t (Working Shift) (Manufacturing Balance
Constraints)

//4.1 - Reverse re-manufacturing constraint variables
dvar float+ RRAW1[T][K]; //Expected quantity of collected
conforming raw material, from collection center entering
during the 1st period t (Working Shift) at station k (Reverse
Remanufacturing Line)
dvar float+ RRAW2[T][K]; //Expected quantity of collected non-
conforming raw material, from collection center entering
during the 1st period t (Working Shift) at station k (Reverse
Remanufacturing Line)

dvar float+ RM1[T][K]; //Expected number of collected
conforming items entering the 1st period t (Working Shift) at
production line k (Reverse Remanufacturing Line) after prior
100% inspection
```

```
dvar float+ RM2[T][K]; //Expected number of collected non-
conforming items entering during the 1st period t (Working
Shift) at production line k (Reverse Remanufacturing Line)
after prior 100% inspection

dvar float+ REW1[T][K]; //Expected number of collected
conforming items entering at stage k (Reverse Remanufacturing
Station) during period t (Working Shift) (Remanufacturing
Balance Constraints)
dvar float+ REW2[T][K]; //Expected number of collected non-
conforming items entering at stage k (Reverse Remanufacturing
Station) during period t (Working Shift) (Remanufacturing
Balance Constraints)

//4.2 - Quality Forward Manufacturing Constraint Variables
dvar float+ V1[T][J]; //Expected number of conforming items
entering at sampling quality station j (Forward Manufacturing
Station) during period t (Working Shift)
dvar float+ V2[T][J]; //Expected number of non-conforming
items entering at sampling quality station j (Forward
Manufacturing Station) during period t (Working Shift)


dvar float+ SI1[T][J]; //Fraction of conforming items for
sampling at sampling quality station j (Forward Manufacturing
Station) during period t (Working Shift)
dvar float+ SI2[T][J]; //Fraction of non-conforming items for
sampling at sampling quality station j (Forward Manufacturing
Station) during period t (Working Shift)

dvar float+ ESR1[T][J]; //Fraction of sample conforming items
rejected by error type-I at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift)
dvar float+ ESR2[T][J]; //Fraction of sample non-conforming
items accepted as conforming by error type-II at sampling
quality station j (Forward Manufacturing Station) during
period t (Working Shift)

dvar float+ ESR3[T][J]; //Fraction of sample conforming items
accepted as truly conforming at sampling quality station j
```

```
(Forward Manufacturing Station) during period t (Working
Shift)
dvar float+ ESR4[T][J]; //Fraction of sample non-conforming
items rejected as truly non-conforming at sampling quality
station j (Forward Manufacturing Station) during period t
(Working Shift) (For repair, rework, or scrap)

dvar float+ ESR5[T][J]; //Fraction of sample conforming items
that became non-conforming as result of mishandling at
sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift)

dvar float+ FCI[T][J]; //Final fraction of sample conforming
items that will be transfer to the next production station j
(Forward Manufacturing Station) during period t (Working
Shift) W1tj
dvar float+ FNCI[T][J]; //Final fraction of sample non-
conforming items that will be transfer to the next production
station j (Forward Manufacturing Station) during period t
(Working Shift) W2tj

dvar float+ TSCN[T][J]; //Total fraction of sample conforming
items that became non-conforming as result of mishandling and
rejection error type-I at sampling quality station j (Forward
Manufacturing Station) during period t (Working Shift) that
must be repaired, reworked or scrapped

dvar float+ IREP1[T][J]; //Expected fraction of sample
conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be repaired
dvar float+ IREP2[T][J]; //Expected fraction of sample non-
conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be repaired

dvar float+ ISCR1[T][J]; //Expected fraction of sample
conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be scrapped
```

```
dvar float+ ISCR2[T][J]; //Expected fraction of sample non-
conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be scrapped

dvar float+ IRWK1[T][J]; //Expected fraction of sample
conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be reworked
dvar float+ IRWK2[T][J]; //Expected fraction of sample
conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be reworked
dvar float+ IRWK3[T][J]; //Total fraction of sample items
reworked at station j (Forward Manufacturing Station) during
period t (Working Shift) that must be inspected for second
time since the process is not perfect.
dvar float+ IRWK4[T][J]; //Expected fraction of sample items
reworked accepted as truly conforming at sampling quality
station j (Forward Manufacturing Station) for second time,
during period t (Working Shift)
dvar float+ IRWK5[T][J]; //Expected fraction of sample items
reworked, rejected as truly non-conforming at sampling quality
station j (Forward Manufacturing Station) for second time,
during period t (Working Shift)
dvar float+ IRWK6[T][J]; //Expected fraction of sample
conforming items reworked that became non-conforming due to
item mishandling during the second inspection at sampling
quality station j (Forward Manufacturing Station) for second
time, during period t (Working Shift)
dvar float+ IRWK7[T][J]; //Expected fraction of sample truly
conforming items reworked that will be transferred to the next
production station j (Forward Manufacturing Station) during
period t (Working Shift) after second sampling inspection
dvar float+ IRWK8[T][J]; //Expected fraction of sample truly
non-conforming items reworked, rejected during second
inspection at sampling quality inspection station j (Forward
Manufacturing Station) during period t (Working Shift) that
must be scrapped
```

```
dvar float+ FSC[T][J]; //Expected fraction of sample non-conforming items reworked and rejected during second sampling quality inspection + Scrapped items at station j (Forward Manufacturing Station) during period t (Working Shift)

//4.2 - Quality Remanufacturing Constrain Variables
dvar float+ RV1[T][K]; //Expected number of collected conforming items entering at sampling quality station k (Reverse Remanufacturing Station) during period t (Working Shift)
dvar float+ RV2[T][K]; //Expected number of collected non-conforming items entering at sampling quality station k (Reverse Remanufacturing Station) during period t (Working Shift)

dvar float+ RSI1[T][K]; //Expected number of collected non-conforming items entering at sampling quality station k (Reverse Remanufacturing Station) during period t (Working Shift)
dvar float+ RSI2[T][K]; //Fraction of collected non-conforming items for sampling at sampling quality station k (Reverse Remanufacturing Station) during period t (Working Shift)

dvar float+ RESR1[T][K]; //Fraction of collected sample conforming items rejected by error type-I at sampling quality station k (Reverse Remanufacturing Station) during period t (Working Shift)
dvar float+ RESR2[T][K]; //Fraction of collected sample non-conforming items rejected by error type-II at sampling quality station k (Reverse Remanufacturing Station) during period t (Working Shift)

dvar float+ RESR3[T][K]; //Fraction of collected sample conforming items accepted as truly conforming at sampling quality station k (Reverse Remanufacturing Station) during period t (Working Shift)
dvar float+ RESR4[T][K]; //Fraction of collected sample non-conforming items rejected as truly non-conforming at sampling quality station k (Reverse Remanufacturing Station) during period t (Working Shift) (For repair, rework, or scrap)
```

```
dvar float+ RESR5[T][K]; //Fraction of sample collected
conforming items that became non-conforming as result of
mishandling at sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift)

dvar float+ RFCI[T][K]; //Final fraction of sample collected
conforming items that will be transfer to the next production
station k (Forward Manufacturing Station) during period t
(Working Shift) W1tj
dvar float+ RFNCI[T][K]; //Final fraction of sample collected
non-conforming items that will be transfer to the next
production station k (Reverse Remanufacturing Station) during
period t (Working Shift) W2tj

dvar float+ RTSCN[T][K]; //Total fraction of sample collected
conforming items that became non-conforming as result of
mishandling and rejection error type-I at sampling quality
station k (Reverse Remanufacturing Station) during period t
(Working Shift) that must be scrapped

dvar float+ RISCR1[T][K]; //Expected fraction of sample
collected conforming items, rejected at sampling quality
station k (Reverse Remanufacturing Station) during period t
(Working Shift) that must be scrapped
dvar float+ RISCR2[T][K]; //Expected fraction of sample
collected non-conforming items, rejected at sampling quality
station j (Reverse Remanufacturing Station) during period t
(Working Shift) that must be scrapped

//4.3 - Material transfer variables (From the manufacturing
process to the re-manufacturing process)
dvar float+ MTF1; //Quantity of scrapped items transferred
from Forward Manufacturing System to 1st shift of Reverse
Remanufacturing System as raw material
dvar float+ MTF2; //Quantity of scrapped items transferred
from Forward Manufacturing System to 2nd shift of Reverse
Remanufacturing System as raw material
dvar float+ MTF3; //Quantity of scrapped items transferred
from Forward Manufacturing System to 3rd shift of Reverse
Remanufacturing System as raw material
```

```
//4.4 - Forward Preventive Manufacturing Maintenance
dvar float+  MAOQL1[T][J]; //Minimum required fraction of
sample conforming items that must pass quality inspection at j
(Forward Manufacturing Station) during period t (Working
Shift) as an internal quality parameter
dvar float+  MAOQL2[T][J]; //Total fraction of samples
conforming and non-conforming items reworked and rejected as
scrap at the sampling quality station j (Forward Manufacturing
Station) during period t (Working Shift) as a quality
parameter

//4.4.1 Quality support variables
dvar boolean MT3[T][J]; //If ((ESR4[t][j] + TSCN[t][j])) >=
(SI1[t][j] + SI2[t][j]) * X
dvar boolean MTX3[T][J]; //If EW2[t][1] >= EW2[t][1]
dvar boolean MTX4[T][J]; //If ESR1[t][j] >= ESR4[t][j]
dvar float+  MTX5[T][J]; // = ESR2[t][j]
dvar float+  MTX6[T][J]; //If FCI[t][j]-ESR2[t][j]
dvar boolean MTX7[T][J]; //If MTX5[t][j] >= MTX6[t][j]
dvar float+  MTX8[T][J]; //If (SI1[t][j] + SI2[t][j]) * i
dvar float+  MTX9[T][J]; //If (ISCR1[t][j] + ISCR2[t][j]) * s
dvar float+  MTX10[T][J]; //If (MTX9[t][j] >= MTX8[t][j]);
dvar boolean MTX11[T][J]; //If (MT3[t][j] + MTX3[t][j] +
MTX4[t][j] + MTX7[t][j] + MTX10[t][j] >= 3)

//4.4.1 - Quality support variables for re-manufacturing
dvar float+  RMAOQL1[T][K]; //Minimum required fraction of
collected sample conforming items that must pass quality
inspection at k (Reverse Remanufacturing Station) during
period t (Working Shift) as internal quality parameter
dvar float+  RMAOQL2[T][K]; //Total fraction of collected
samples conforming and non-conforming items reworked and
rejected as scrap at the sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift) as a
quality parameter
dvar boolean RMT3[T][K]; //If ((RESR4[t][k] + RTSCN[t][k])) >=
(RSI1[t][k] + RSI2[t][k]) *RX
dvar boolean RMTX3[T][K]; //If REW2[t][1] >= REW2[t][1]
dvar boolean RMTX4[T][K]; //If RESR1[t][k] >= RESR4[t][k]
dvar float+  RMTX5[T][K]; //If RESR2[t][k]
dvar float+  RMTX6[T][K]; //If RFCI[t][k]-ESR2[t][k]
```

```
dvar boolean RMTX7[T][K]; // If RMTX5[T][K] >= RMTX6[T][K]
dvar float+  RMTX8[T][K]; // If RSI1[t][k] + RSI2[t][k]) * ri
dvar float+  RMTX9[T][K]; // If RISCR1[t][k] + RISCR2[t][k]) *
rs
dvar float+  RMTX10[T][K]; //If RMTX9[t][k] >= MTX8[t][k]
dvar boolean RMTX11[T][K]; //If RMT3[t][k] + RMTX3[t][k] +
RMTX4[t][k] + RMTX7[t][k] + RMTX10[t][k] >= 3)

//4.5 Maintenance support variables
dvar float+  MT4[T][J]; // = IREP1[t][j] + IREP2[t][j] +
IRWK7[t][j]
dvar float+  MT5[T][J]; // = (ISCR1[t][j] + ISCR2[t][j]);
dvar boolean MT6[T][J]; //(MT5[t][j] >= MT4[t][j]);
dvar boolean MTZ7[T][J]; //(IRWK5[t][j] >=
(IRWK2[t][j]*0.20));
dvar boolean MTZ8[T][J]; //(IRWK2[t][j]+IREP2[t][j] >=
IRWK1[t][j]+IREP1[t][j])
dvar boolean MTZ9[T][J]; //(epsilon[t][j] >= 1);
dvar boolean MTZ10[T][J]; //(V2[t][j] >= (V1[t][j]*0.20))
dvar boolean MTZ11[T][J]; //(MT6[t][j] + MTZ7[t][j] +
MTZ8[t][j] + MTZ9[t][j] + MTZ10[t][j] >= 1)

//4.5 Maintenance support variables
dvar boolean RMTZ1[T][K]; //(repsilon[t][j] >= 1);
dvar boolean RMTZ2[T][K]; //(RV2[t][j] >= (RV1[t][j]*0.20))
dvar float+ RMTZ3[T][K]; // (RISCR1[t][k] + RISCR2[t][k]);
Prior RMT5
dvar boolean RMTZ4[T][K]; //(RMTZ3[t][k] >= (RFCI[t][k] * X3))
dvar boolean RMTZ5[T][K]; //(RMTZ1[t][k] + RMTZ2[t][k] +
RMTZ3[t][k] + RMTZ4[t][k] >= 1);

//4.6 - Forward Manufacturing Shipping
dvar float+ NCS[T][J]; //Fraction of complete non-conforming
items shipped to the mainly market (Forward Manufacturing
System)
dvar float+ NCS2; //Total Fraction of complete non-conforming
items shipped to the mainly market (Forward Manufacturing
System)

//4.5/6 - Reverse re-manufacturing shipping
```

```
dvar float+ RNCS[T][K]; //Fraction of complete non-conforming
items shipped to the secondary market (Reverse Remanufacturing
System)
dvar float RNCS2; //Total fraction of complete non-conforming
items shipped to the secondary market (Reverse Remanufacturing
System)

//4.7 - Total Forward Manufacturing Profit
dvar float PRFT; //Manufacturing profit at each station j
(Forward Manufacturing) during period t (Working Shift)

//4.6/7 - Reverse re-manufacturing profit
dvar float RPRFT; //Manufacturing earnings at each station k
(Reverse Remanufacturing Station) during period t (Working
Shift)


//5.-Decision
variables*****************************************************
**************************************************************
*************

//Forward Manufacturing Costs
dvar float+ TPC; //Total Forward Manufacturing Production Cost
dvar float+ TIC; //Total Forward Manufacturing Production
Inspection Cost
dvar float+ TRPC; //Total Forward Manufacturing Production
Repair Cost
dvar float+ TRWC; //Total Forward Manufacturing Production
Rework Cost
dvar float+ TSC; //Total Forward Manufacturing Production
Scrap Cost
dvar float+ TPMC; //Total Forward Manufacturing Production
Preventive Maintenance Cost

//5.-Decision
variables*****************************************************
**************************************************************
*************

//Remanufacturing Costs
```

```
dvar float+ TRRPC; //Total Reverse Remanufacturing Production
Cost
dvar float+ TRIC; //Total Reverse Remanufacturing Inspection
Cost
dvar float+ TRSC; //Total reverse re-manufacturing scrap cost
dvar float+ TRPMC; //Total Reverse Remanufacturing Preventive
Maintenance Cost - Still in progress

//6.-Objective forward function
computation*************************************************
**************************************************************
***

minimize TPC + TIC + TRPC + TRWC + TSC + TPMC + TRRPC + TRIC +
TRSC + TRPMC;

//7.-//Forward Manufacturing
Constraints*************************************************
**************************************************************
**********************
     subject to {

/*Constrains for forward production and quality rates*/

/*1st block (Forward Manufacturing) - Expected number of raw
material entering to the Serial-Forward-Multi-Stage-
Manufacturing-System*/

     forall (t in T, j in J) //Expected quantity of conforming
raw material entering during the 1st period t (Working Shift)
at production line j (Forward Manufacturing Production Line)
     Constraint1: RAW1[t][j] == (1-epsilon0[j]) * QRM1[t]; //
Forward Manufacturing Done ******

    forall (t in T, j in J) //Expected quantity of non-
conforming raw material entering during the 1st period t
(Working Shift) at production line j (Forward Manufacturing
Production Line)
     Constraint2: RAW2[t][j] == epsilon0[j] * QRM1[t]; //
Forward Manufacturing Done *****
```

```
    forall (t in T, j in J) //Restriction to allow 1st station
j (Forward Manufacturing Station) during period t (Working
Shift) have 100% free defect incoming raw material for
production. (General Calculus)
     Constraint3: M1[t][1] == (1-epsilon0[j]) * QRM1[t]; //
Forward Manufacturing Done *****

     forall (t in T, j in J) //Restriction to allow 1st
station j (Forward Manufacturing Station) during period t
(Working Shift) have 0% defect incoming raw material for
production. (General Calculus)
     Constraint4: M2[t][1] == epsilon0[1] * QRM1[t]; //
Forward Manufacturing Done ******

     forall (t in T, j in J) //Restriction to allow 1st
station j (Forward Manufacturing Station) during period t
(Working Shift) have 100% free defect incoming raw material
for production
     Constraint5: M1[t][j] == (1-epsilon0[j]) * QRM1[t]; //
Forward Manufacturing Done *****

     forall (t in T, j in J) //Restriction to allow 1st
station j (Forward Manufacturing Station) during period t
(Working Shift) have 0% defect incoming raw material for
production
     Constraint6: M2[t][j] == epsilon0[j] * QRM1[t]; //
Manufacturing Done ****

//2nd block (Forward Manufacturing) - Expected number of
conforming & non-conforming items entering at sampling quality
station after production-------------------------------------
-------------------------------------------------------------
-------------------------------------------------------------
------------------------------------------------------

     forall (t in T, j in J) //Expected number of conforming
items entering at sampling quality station j (1) (Forward
Manufacturing Station) during period t (Working Shift)
(General Calculation)
```

```
          Constraint7: V1[t][1] == (1-epsilon[t][1]) *
M1[t][1]; //Forward Manufacturing Done **** //For the V1 data
is possible to get the same results

    forall (t in T, j in J) //Expected number of conforming
items entering at sampling quality station j (2) (Forward
Manufacturing Station) during period t (Working Shift)
(General Calculation)
          Constraint8: V1[t][2] ==(1-epsilon[t][2]) *
EW1[t][2]; // Forward Manufacturing Done ****

    forall (t in T, j in J) //Expected number of conforming
items entering at sampling quality station j (3) (Forward
Manufacturing Station) during period t (Working Shift)
(General Calculation)
          Constraint9: V1[t][3] == (1-epsilon[t][3]) *
EW1[t][3]; // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected number of conforming
items entering at sampling quality station j (4) (Forward
Manufacturing Station) during period t (Working Shift)
(General Calculation)
          Constraint10: V1[t][4] ==(1-epsilon[t][4]) *
EW1[t][4]; // Forward Manufacturing Done ****

    forall (t in T, j in J) //Expected number of conforming
items entering at sampling quality station j (5) (Forward
Manufacturing Station) during period t (Working Shift)
(General Calculation)
          Constraint11: V1[t][5] ==(1-epsilon[t][5]) *
EW1[t][5]; // Forward Manufacturing Done ****

    forall (t in T, j in J) //Expected number of non-
conforming items entering at sampling quality station j (1)
(Forward Manufacturing Station) during period t (Working
Shift) (General Calculation)
          Constraint12: V2[t][1] == (epsilon[t][1] *
M1[t][1]); // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected number of non-
conforming items entering at sampling quality station j (2)
```

```
(Forward Manufacturing Station) during period t (Working
Shift) (General Calculation)
          Constraint13: V2[t][2] == EW2[t][2] + epsilon[t][2]
* EW1[t][2]; // Forward Manufacturing Done ****

    forall (t in T, j in J) //Expected number of non-
conforming items entering at sampling quality station j (3)
(Forward Manufacturing Station) during period t (Working
Shift) (General Calculation)
          Constraint14: V2[t][3] == EW2[t][3] + epsilon[t][3]
* EW1[t][3]; // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected number of non-
conforming items entering at sampling quality station j (4)
(Forward Manufacturing Station) during period t (Working
Shift) (General Calculation)
          Constraint15: V2[t][4] == EW2[t][4] + epsilon[t][4]
* EW1[t][4]; // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected number of non-
conforming items entering at sampling quality station j (5)
(Forward Manufacturing Station) during period t (Working
Shift) (General Calculation)
          Constraint16: V2[t][5] == EW2[t][5] + epsilon[t][5]
* EW1[t][5]; // Forward Manufacturing Done ****

//3rd block (Forward Manufacturing) - Fraction of conforming &
non-conforming items for sampling at sampling quality station
(Sorting and Rejection) (Part 1)------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
---------------------------------------------

     forall (t in T, j in J) //Fraction of conforming items
for sampling at sampling quality station 1 (Forward
Manufacturing Station) during period 1 (Working Shift)
          Constraint17: SI1[1][1] == V1[1][1]; // Forward
Manufacturing Done ****
```

```
     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 1 (Forward
Manufacturing Station) during period 1 (Working Shift)
         Constraint18: SI2[1][1] == V2[1][1]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of conforming items
for sampling at sampling quality station 1 (Forward
Manufacturing Station) during period 2 (Working Shift)
          Constraint19: SI1[2][1] == V1[2][1]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 1 (Forward
Manufacturing Station) during period 2 (Working Shift)
         Constraint20: SI2[2][1] == V2[2][1]; // Forward
Manufacturing Done ****

    forall (t in T, j in J) //Fraction of conforming items for
sampling at sampling quality station 1 (Forward Manufacturing
Station) during period 3 (Working Shift)
          Constraint21: SI1[3][1] == V1[3][1]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 1 (Forward
Manufacturing Station) during period 3 (Working Shift)
         Constraint22: SI2[3][1] == V2[3][1]; //Forward
Manufacturing Done ****

    forall (t in T, j in J) //Fraction of conforming items for
sampling at sampling quality station 2 (Forward Manufacturing
Station) during period 1 (Working Shift)
          Constraint23: SI1[1][2] == V1[1][2]; //Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 2 (Forward
Manufacturing Station) during period 1 (Working Shift)
         Constraint24: SI2[1][2] == V2[1][2]; // Forward
Manufacturing Done ****
```

```
     forall (t in T, j in J) //Fraction of conforming items
for sampling at sampling quality station 2 (Forward
Manufacturing Station) during period 2 (Working Shift)
          Constraint25: SI1[2][2] == V1[2][2]; //Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 2 (Forward
Manufacturing Station) during period 2 (Working Shift)
         Constraint26: SI2[2][2] == V2[2][2]; // Forward
Manufacturing Done ****

    forall (t in T, j in J) //Fraction of conforming items for
sampling at sampling quality station 2 (Forward Manufacturing
Station) during period 3 (Working Shift)
          Constraint27: SI1[3][2] == V1[3][2]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 2 (Forward
Manufacturing Station) during period 3 (Working Shift)
         Constraint28: SI2[3][2] == V2[3][2]; // Forward
Manufacturing Done ****

    forall (t in T, j in J) //Fraction of conforming items for
sampling at sampling quality station 3 (Forward Manufacturing
Station) during period 1 (Working Shift)
          Constraint29: SI1[1][3] == V1[1][3]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 3 (Forward
Manufacturing Station) during period 1 (Working Shift)
         Constraint30: SI2[1][3] == V2[1][3]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of conforming items
for sampling at sampling quality station 3 (Forward
Manufacturing Station) during period 1 (Working Shift)
```

```
          Constraint31: SI1[2][3] == V1[2][3]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 3 (Forward
Manufacturing Station) during period 2 (Working Shift)
         Constraint32: SI2[2][3] == V2[2][3]; // Forward
Manufacturing Done ****

    forall (t in T, j in J) //Fraction of conforming items for
sampling at sampling quality station 3 (Forward Manufacturing
Station) during period 3 (Working Shift)
          Constraint33: SI1[3][3] == V1[3][3]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 3 (Forward
Manufacturing Station) during period 3 (Working Shift)
         Constraint34: SI2[3][3] == V2[3][3]; // Forward
Manufacturing Done ****

    forall (t in T, j in J) //Fraction of conforming items for
sampling at sampling quality station 4 (Forward Manufacturing
Station) during period 1 (Working Shift)
          Constraint35: SI1[1][4] == V1[1][4]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 4 (Forward
Manufacturing Station) during period 1 (Working Shift)
         Constraint36: SI2[1][4] == V2[1][4]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of conforming items
for sampling at sampling quality station 4 (Forward
Manufacturing Station) during period 2 (Working Shift)
          Constraint37: SI1[2][4] == V1[2][4]; // Forward
Manufacturing Done ****
```

```
     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 4 (Forward
Manufacturing Station) during period 2 (Working Shift)
         Constraint38: SI2[2][4] == V2[2][4]; // Forward
Manufacturing Done ****

    forall (t in T, j in J) //Fraction of conforming items for
sampling at sampling quality station 4 (Forward Manufacturing
Station) during period 3 (Working Shift)
          Constraint39: SI1[3][4] == V1[3][4]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 4 (Forward
Manufacturing Station) during period 3 (Working Shift)
         Constraint40: SI2[3][4] == V2[3][4]; // Forward
Manufacturing Done ****

    forall (t in T, j in J) //Fraction of conforming items for
sampling at sampling quality station 5 (Forward Manufacturing
Station) during period 1 (Working Shift)
          Constraint41: SI1[1][5] == V1[1][5]; //Manufacturing
Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 5 (Forward
Manufacturing Station) during period 1 (Working Shift)
         Constraint42: SI2[1][5] == V2[1][5]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of conforming items
for sampling at sampling quality station 5 (Forward
Manufacturing Station) during period 2 (Working Shift)
          Constraint43: SI1[2][5] == V1[2][5]; //Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 5 (Forward
Manufacturing Station) during period 2 (Working Shift)
         Constraint44: SI2[2][5] == V2[2][5]; // Forward
Manufacturing Done ****
```

```
    forall (t in T, j in J) //Fraction of conforming items for
sampling at sampling quality station 5 (Forward Manufacturing
Station) during period 3 (Working Shift)
          Constraint45: SI1[3][5] == V1[3][5]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Fraction of non-conforming
items for sampling at sampling quality station 5 (Forward
Manufacturing Station) during period 3 (Working Shift)
         Constraint46: SI2[3][5] == V2[3][5]; // Forward
Manufacturing Done ****

//4rd block (Forward Manufacturing) - Expected sample-
sorting/rejection number of forward manufactured conforming &
conforming items Part2-------------------------------------------
-----------------------------------------------------------------
-----------------------------------------------------------------
----------------------------------------------------------

     forall (t in T, j in J) //Fraction of sample conforming
items rejected by error type-I at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift).
     Constraint47: ESR1[t][j] == alpha[t][j] * SI1[t][j]; //
Forward Manufacturing Done ****

     forall (t in T, j in J) //Fraction of sample non-
conforming items accepted as conforming by error type-II at
sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift)
          Constraint48: ESR2[t][j] == betha[t][j] * SI2[t][j];
// Forward Manufacturing Done ****

     forall (t in T, j in J) //Fraction of sample conforming
items accepted as truly conforming at sampling quality station
j (Forward Manufacturing Station) during period t (Working
Shift)
          Constraint49: ESR3[t][j] == SI1[t][j] - ESR1[t][j];
//This needs to be changed every stage // Forward
Manufacturing Done
```

```
    forall (t in T, j in J) //Fraction of sample non-
conforming items rejected as truly non-conforming at sampling
quality station j (Forward Manufacturing Station) during
period t (Working Shift) (For repair, rework, or scrap)
         Constraint50: ESR4[t][j] == (1-betha[t][j]) *
SI2[t][j]; // Forward Manufacturing Done ****

    forall (t in T, j in J) //Fraction of sample conforming
items that became non-conforming as result of mishandling at
sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift)
         Constraint51: ESR5[t][j] == omega[t][j] *
ESR3[t][j];  // Forward Manufacturing Done ****

//5th block (Forward Manufacturing) - Expected sample-
sorting/rejection number of forward manufactured conforming &
conforming items Part3-------------------------------------------
-------------------------------------------------------------
-------------------------------------------------------------
----------------------------------------------------

    forall (t in T, j in J) //Final fraction of sample
conforming items that will be transfer to the next production
station j (Forward Manufacturing Station) during period t
(Working Shift) W1tj
         Constraint52: FCI[t][j] == (ESR3[t][j] - ESR5[t][j])
+ ESR2[t][j]; // Forward Manufacturing Done ****

    forall (t in T, j in J) //Final fraction of sample non-
conforming items that will be transfer to the next production
station j (Forward Manufacturing Station) during period t
(Working Shift) W2tj
         Constraint53: FNCI[t][j] ==  V2[t][j] - SI2[t][j];
// Forward Manufacturing Done ****

    forall (t in T, j in J) //Total fraction of sample
conforming items that became non-conforming as result of
mishandling and rejection error type-I at sampling quality
station j (Forward Manufacturing Station) during period t
(Working Shift) that must be repaired, reworked or scrapped
```

```
          Constraint54: TSCN[t][j] == ESR1[t][j] + ESR5[t][j];
// Forward Manufacturing Done ****

//6th block (Forward Manufacturing) - Expected number of items
that will be reworked, repair and scrapped after being
processed-------------------------------------------------------
----------------------------------------------------------------
----------------------------------------------------------------
----------------------------------------

     forall (t in T, j in J) //Expected fraction of sample
conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be repaired
          Constraint55: IREP1[t][j] == F1[t][j] * (ESR1[t][j]
+ ESR5[t][j]); // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected fraction of sample
non-conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be repaired
          Constraint56: IREP2[t][j] == F1[t][j] * ESR4[t][j];
//Done    --- This is part of the 1st objective // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Expected fraction of sample
conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be scrapped
          Constraint57: ISCR1[t][j] == F2[t][j] * (ESR1[t][j]
+ ESR5[t][j]); // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected fraction of sample
non-conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be scrapped
          Constraint58: ISCR2[t][j] == F2[t][j] * ESR4[t][j];
// Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected fraction of sample
conforming items, rejected at sampling quality station j
```

```
(Forward Manufacturing Station) during period t (Working
Shift) that must be reworked
          Constaint59: IRWK1[t][j] == F3[t][j] * (ESR1[t][j] +
ESR5[t][j]); // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected fraction of sample
conforming items, rejected at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be reworked
          Constaint60: IRWK2[t][j] == F3[t][j] * ESR4[t][j];
// Forward Manufacturing Done ****

//7th block (Forward Manufacturing) - Expected number of
conforming & non-conforming reworked items that will be re-
inspected-------------------------------------------------------------
----------------------------------------------------------------
----------------------------------------------------------------
-----------------------------------------

     forall (t in T, j in J) //Total fraction of sample items
reworked at station j (Forward Manufacturing Station) during
period t (Working Shift) that must be inspected for second
time since the process is not perfect
          Constraint61: IRWK3[t][j] ==
(IRWK1[t][j]+IRWK2[t][j]); // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected fraction of sample
items reworked accepted as truly conforming at sampling
quality station j (Forward Manufacturing Station) for second
time, during period t (Working Shift)
          Constraint62: IRWK4[t][j] == (1-theta[t][j]) *
IRWK3[t][j]; // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected fraction of sample
items reworked, rejected as truly non-conforming at sampling
quality station j (Forward Manufacturing Station) for second
time, during period t (Working Shift)
          Constraint63: IRWK5[t][j] == theta[t][j] *
IRWK3[t][j]; //This goes to scrap // Forward Manufacturing
Done ****
```

```
     forall (t in T, j in J) //Expected fraction of sample
conforming items reworked that became non-conforming due to
item mishandling during the second inspection at sampling
quality station j (Forward Manufacturing Station) for second
time, during period t (Working Shift)
          Constraint64: IRWK6[t][j] == theta[t][j] *
IRWK4[t][j]; //This goes to scrap // Forward Manufacturing
Done ****

     forall (t in T, j in J) //Expected fraction of sample
truly conforming items reworked that will be transferred to
the next production station j (Forward Manufacturing Station)
during period t (Working Shift) after second sampling
inspection
          Constraint65: IRWK7[t][j] == IRWK4[t][j] -
IRWK6[t][j]; // Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected fraction of sample
truly non-conforming items reworked, rejected during second
inspection at sampling quality inspection station j (Forward
Manufacturing Station) during period t (Working Shift) that
must be scrapped
          Constraint66: IRWK8[t][j] == IRWK5[t][j] +
IRWK6[t][j]; // Manufacturing Done

     forall (t in T, j in J) //Expected fraction of sample
non-conforming items reworked and rejected during second
sampling quality inspection + Scrapped items at station j
(Forward Manufacturing Station) during period t (Working
Shift)
          Constraint67: FSC[t][j] == IRWK8[t][j] + ISCR1[t][j]
+ ISCR2[t][j]; // Manufacturing Done - This is used just a
calculus tool.

     forall (t in T) //Quantity of scrapped items transferred
from Forward Manufacturing System to 1st shift of Reverse
Remanufacturing System as raw material
          Constraint68: MTF1 == sum (j in J) FSC[1][j] -
FSC[1][5];
```

```
	forall (t in T) //Quantity of scrapped items transferred from Forward Manufacturing System to 2nd shift of Reverse Remanufacturing System as raw material
		Constraint69: MTF2 == sum (j in J) FSC[2][j] - FSC[2][5];

	forall (t in T) //Quantity of scrapped items transferred from Forward Manufacturing System to 3rd shift of Reverse Remanufacturing System as raw material
		Constraint70: MTF3 == sum (j in J) FSC[3][j] - FSC[3][5];

//8th block (Forward Manufacturing) - Material balance constraints-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

	forall (t in T, j in J) //Expected number of conforming items entering at stage j (1) (Forward Manufacturing Station) during period t (Working Shift) (Manufacturing Balance Constraints) (General calculation)
		Constraint71: EW1[t][1] == V1[t][1]; //This should be for the next term -Forward Manufacturing Done ****

	forall (t in T, j in J) //Expected number of conforming items entering at stage j (2) (Forward Manufacturing Station) during period t (Working Shift) (Manufacturing Balance Constraints) (General calculation)
		Constraint72: EW1[t][2] == V1[t][1] - SI1[t][1] + (ESR3[t][1] + ESR2[t][1] - ESR5[t][1]) + (IREP1[t][1] + IREP2[t][1]) + (IRWK7[t][1]); // Forward Manufacturing Done ****

	forall (t in T, j in J) //Expected number of conforming items entering at stage j (3) (Forward Manufacturing Station) during period t (Working Shift) (Manufacturing Balance Constraints) (General calculation)
		Constraint73: EW1[t][3] == V1[t][2] - SI1[t][2] + (ESR3[t][2] + ESR2[t][2] - ESR5[t][2]) + (IREP1[t][2] + IREP2[t][2]) + (IRWK7[t][2]); // Forward manufacturing Done
```

```
     forall (t in T, j in J) //Expected number of conforming
items entering at stage j (4) (Forward Manufacturing Station)
during period t (Working Shift) (Manufacturing Balance
Constraints) (General calculation)
          Constraint74: EW1[t][4] == V1[t][3] - SI1[t][3] +
(ESR3[t][3] + ESR2[t][3] - ESR5[t][3]) + (IREP1[t][3] +
IREP2[t][3]) + (IRWK7[t][3]); // Forward manufacturing Done

     forall (t in T, j in J) //Expected number of conforming
items entering at stage j (5) (Forward Manufacturing Station)
during period t (Working Shift) (Manufacturing Balance
Constraints) (General calculation)
          Constraint75: EW1[t][5] == V1[t][4] - SI1[t][4] +
(ESR3[t][4] + ESR2[t][4] - ESR5[t][4]) + (IREP1[t][4] +
IREP2[t][4]) + (IRWK7[t][4]); //Manufacturing Done

     forall (t in T, j in J) //Expected number of non-
conforming items entering at stage j (1) (Forward
Manufacturing Station) during period t (Working Shift)
(Manufacturing Balance Constraints)
          Constraint76: EW2[t][1] == V2[t][1]; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Expected number of non-
conforming items entering at stage j (2) (Forward
Manufacturing Station) during period t (Working Shift)
(Manufacturing Balance Constraints) (General calculation)
          Constraint77: EW2[t][2] == V2[t][1] - SI2[t][1]; //
Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected number of non-
conforming items entering at stage j (3) (Forward
Manufacturing Station) during period t (Working Shift)
(Manufacturing Balance Constraints) (General calculation)
          Constraint78: EW2[t][3] == V2[t][2] - SI2[t][2]; //
Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected number of non-
conforming items entering at stage j (4) (Forward
```

```
Manufacturing Station) during period t (Working Shift)
(Manufacturing Balance Constraints) (General calculation)
          Constraint79: EW2[t][4] == V2[t][3] - SI2[t][3]; //
Forward Manufacturing Done ****

     forall (t in T, j in J) //Expected number of non-
conforming items entering at stage j (5) (Forward
Manufacturing Station) during period t (Working Shift)
(Manufacturing Balance Constraints) (General calculation)
          Constraint80: EW2[t][5] == V2[t][4] - SI2[t][4]; //
Forward Manufacturing Done ****

//9th block (Forward Manufacturing) - Flow balance
constraints-----------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
-------------------------------------------

     forall (t in T, j in J) //The number of items shipped
from the factory cannot exceed the quantity of raw materials
received (forward manufacturing Production)
          Constraint81: RAW1[t][1] - (V1[t][5] + V2[t][5]) >=
0; // Forward Manufacturing Done ****

     forall (t in T, j in J) //The number of items shipped
from the factory cannot exceed the production capacity
(Forward Manufacturing Production)
          Constraint82: (V1[t][5] + V2[t][5]) <= delta; //
Forward Manufacturing Done ****

     forall (t in T, j in J) //Penalization cost per non-
conforming items ship to the customer (Forward Manufacturing
System)(General calculation)
          Constraint83: NCS[t][1] == V2[t][5] * 0; // Forward
Manufacturing Done ****

     forall (t in T, j in J) //Penalization cost per non-
conforming items ship to the customer (Forward Manufacturing
System) (General calculation)
          Constraint84: NCS[t][2] == V2[t][5] * 0; //
Manufacturing Done ****
```

```
     forall (t in T, j in J) //Penalization cost per non-
conforming items ship to the customer (Forward Manufacturing
System) (General calculation)
          Constraint85: NCS[t][3] == V2[t][5] * 0; //
Manufacturing Done ****

     forall (t in T, j in J) //Penalization cost per non-
conforming items ship to the customer (Forward Manufacturing
System) (General calculation)
          Constraint86: NCS[t][4] == V2[t][5] * 0; //
Manufacturing Done ****

     forall (t in T, j in J) //Penalization cost per non-
conforming items ship to the customer (Forward Manufacturing
System) (General calculation)
          Constraint87: NCS[t][5] == V2[t][5]; //
Manufacturing Done ****

     forall (j in J) //Penalization cost per non-conforming
items ship to the customer (Forward Manufacturing System)
          Constraint88: NCS2 == sum (t in T) NCS[t][5] * xi;
// Manufacturing Done ****

//10th block (Forward Manufacturing) - Forward manufacturing
cost constraints for the objective function-------------------
--------------------------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
----------------------------

     forall (t in T, j in J) //Objective function–Total
Forward Manufacturing Production Cost
          Constraint89: TPC ==  sum (t in T, j in J: j == 1)
(EW1[t][j] + EW2[t][j]) * rho[t][j]; // Forward Manufacturing
Done ****

     forall (t in T, j in J)  //Objective function–Total
Forward Manufacturing Inspection Cost
          Constraint90: TIC == sum (t in T, j in J) (SI1[t][j]
+ SI2[t][j]) * i[t][j]; // Forward Manufacturing Done ****
```

```
    forall (t in T, j in J) //Objective function-Total
Forward Manufacturing Repair Cost
         Constraint91: TRPC == sum (t in T, j in J)
(IREP1[t][j] + IREP2[t][j]) * kappa[t][j]; // Forward
Manufacturing Done ****

    forall (t in T, j in J) //Objective function-Total
Forward Manufacturing Rework Cost
         Constraint92: TRWC == sum (t in T, j in J)
(IRWK1[t][j] + IRWK2[t][j]) * lambda[t][j]; // Forward
Manufacturing Done ****

    forall (t in T, j in J) //Objective function-Total
Forward Manufacturing Scrap Cost
         Constraint93: TSC == sum (t in T, j in J)
(ISCR1[t][j] + ISCR2[t][j]) * s[t][j]; // Forward
Manufacturing Done ****

//11th block (Forward Manufacturing) - Forward manufacturing
inspection decision constraints-------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
------------------------------------------------------------


    forall (t in T, j in J) //Minimum required number of
conforming items passing through inspection after processing
station j (Forward Manufacturing Station) as a quality
parameter
         Constraint94: MAOQL1[t][j] == (SI1[t][j] +
SI2[t][j]) * X; // For the forward manufacturing station is
important to establish new parameters ****

    forall (t in T, j in J) //Total fraction of sample
conforming & non-conforming rejected items reworked, repair
and scrapped at the inspection station j in period t (Working
shift)
         Constraint95: MAOQL2[t][j] == ((ESR4[t][j] +
TSCN[t][j])); // In the second process it is important to take
into account the first data ****
```

```
     forall (t in T, j in J) //1 - If quality inspection
should take place after each stage j (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
          Constraint96: MT3[t][j] == (MAOQL2[t][j] >=
MAOQL1[t][j]); //To know where inspection takes place //
Forward Manufacturing Done ****

     forall (t in T, j in J) //1 - If quality inspection
should take place after each stage j (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
          Constraint97: MTX3[t][1] == (EW2[t][1] >=
EW2[t][1]); //To know where inspection takes place // Forward
Manufacturing Done ****

     forall (t in T, j in J) //1 - If quality inspection
should take place after each stage j (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
          Constraint98: MTX3[t][2] == (EW2[t][2] >=
(EW2[t][1]*X4)); //To know where inspection takes place //
Forward Manufacturing Done ****

     forall (t in T, j in J) //1 - If quality inspection
should take place after each stage j (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
          Constraint99: MTX3[t][3] == (EW2[t][3] >=
(EW2[t][2]*X4)); //To know where inspection takes place //
Forward Manufacturing Done ****

     forall (t in T, j in J) //1 - If quality inspection
should take place after each stage j (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
          Constraint100: MTX3[t][4] == (EW2[t][4] >=
(EW2[t][3]*X4)); //To know where inspection takes place //
Forward Manufacturing Done ****

     forall (t in T, j in J) //1 - If quality inspection
should take place after each stage j (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
          Constraint101: MTX3[t][5] == (EW2[t][5] >=
(EW2[t][4]*X4)); //To know where inspection takes place //
Forward Manufacturing Done ****
```

```
     forall (t in T, j in J) //Third QA testing constraint
          Constraint102: MTX4[t][j] == (ESR1[t][j] >=
ESR4[t][j]);

     forall (t in T, j in J) //Fourth QA testing constraint
          Constraint103: MTX5[t][j] == ESR2[t][j];

     forall (t in T, j in J) //Fifth QA testing constraint
          Constraint104: MTX6[t][j] == (FCI[t][j]-ESR2[t][j]);

     forall (t in T, j in J) //Sixth QA testing constraint
          Constraint105: MTX7[t][j] == (MTX5[t][j] >=
MTX6[t][j]);

     forall (t in T, j in J) //Seventh QA testing constraint -
Inspection cost
          Constraint106: MTX8[t][j] == (SI1[t][j] + SI2[t][j])
* i[t][j];

     forall (t in T, j in J) //8th QA testing constraint
          Constraint107: MTX9[t][j] == (ISCR1[t][j] +
ISCR2[t][j]) * s[t][j];

     forall (t in T, j in J) //If scrap cost is higher than
inspection cost
          Constraint108: MTX10[t][j] == (MTX9[t][j] >=
MTX8[t][j]);

     forall (t in T, j in J) //If quality should be applied
          Constraint109: MTX11[t][j] == (MT3[t][j] +
MTX3[t][j] + MTX4[t][j] + MTX7[t][j] + MTX10[t][j] >= 3);

//12th block (Forward Manufacturing) - Forward maintenance
decision constraints------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
-------------------------------------------------


     forall (t in T, j in J) //Total fraction of samples
conforming, and non-conforming items rejected, repaired, and
```

```
reworked at station j (Forward Manufacturing Station) during
period t (Working shift)
          Constraint110: MT4[t][j] == ((IREP1[t][j] +
IREP2[t][j] + IRWK7[t][j])); // Forward Manufacturing Done
****

     forall (t in T, j in J) //Total fraction of samples
conforming non-conforming items rejected as scrap at the
sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift)
          Constraint111: MT5[t][j] == (ISCR1[t][j] +
ISCR2[t][j]); // Forward Manufacturing Done ****

     forall (t in T, j in J) //Does preventive maintenance
should take place if the fraction of rejected items by scrap
is lower or equal to the MAOQL1 (Desirable quality level) at
station j (Forward Manufacturing Station) during period t
(Working Shift), If yes = 1, Otherwise = 0
          Constraint112: MT6[t][j] == (MT5[t][j] >=
MT4[t][j]); //To know if inspection is necessary // Forward
Manufacturing Done

     forall (t in T, j in J) //If the% (20%) of non-conforming
items already reworked needs rework again, then maintenance
should be applied.
          Constraint113: MTZ7[t][j] == (IRWK5[t][j] >=
(IRWK2[t][j]*X5));

     forall (t in T, j in J) //If the number of non-conforming
items already repaired and reworked is higher than the
conforming ones
          Constraint114: MTZ8[t][j] ==
(IRWK2[t][j]+IREP2[t][j] >= IRWK1[t][j]+IREP1[t][j]);

     forall (t in T, j in J) //If the deterioration factor of
the working machine is higher than operation level, then apply
maintenance
          Constraint115: MTZ9[t][j] == (epsilon[t][j] >= 1);
```

```
     forall (t in T, j in J) //If the fraction of the non-
conforming production is higher than the fraction of
conforming items
          Constraint116: MTZ10[t][j] == (V2[t][j] >=
(V1[t][j]*0.20));

     forall (t in T, j in J) /Maintenance decision
          Constraint117: MTZ11[t][j] == (MT6[t][j] +
MTZ7[t][j] + MTZ8[t][j] + MTZ9[t][j] + MTZ10[t][j] >= 1);

     forall (t in T, j in J) //Total Forward Manufacturing
Preventive Maintenance Cost.
          Constraint118: TPMC == sum (t in T, j in J)
MTZ11[t][j] * mu[t][j];

//14th block (Forward Manufacturing) - Forward profit
constraints-----------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
----------------------------------------

     forall (t in T, j in J) //Total profit at each stage j
(Forward Manufacturing Station) in period t (Working Shift)
        Constraint119: PRFT == (sum (t in T, j in J: j == 5)
V1[t][j] * SLC) - (TPC + TRPC + TRWC + TSC + NCS2);
        //(Beginning Inventory (Raw material) + Manufacturing
Cost (Total Costs) - Ending Inventory ())


//1st block (Remanufacturing constraints) - Expected number of
raw material entering to the Serial-Reverse-Multi-Stage-
Remanufacturing-System*/

      forall (t in T, k in K) //Expected quantity of collected
conforming raw material, from collection center entering
during the 1st period t (Working Shift) at station k (Reverse
Remanufacturing Line) (General calculation)
        Constraint120: RRAW1[t][k] == (1-repsilon0[k]) *
((MTF1+MTF2+MTF3)+((V1[t][5]+V2[t][5])*0.75)); //**** (Reverse
Remanufacturing)
```

```
      forall (t in T, k in K) //Expected quantity of collected
conforming raw material, from collection center entering
during the 3rd period t (Working Shift) at station k (Reverse
Remanufacturing Line) (General calculation)
        Constraint121: RRAW2[t][k] == repsilon0[k] *
((MTF1+MTF2+MTF3)+((V1[t][5]+V2[t][5])*0.75)); //**** (Reverse
Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
conforming items entering the 1st period t (Working Shift) at
production line k (Reverse Remanufacturing Line) after prior
100% inspection - Review
        Constraint122: RM1[t][1] == (1-repsilon0[k]) *
((MTF1+MTF2+MTF3)+((V1[t][5]+V2[t][5])*0.75)); //**** (Reverse
Remanufacturing)

      forall (t in T, k in K) //Expected number of collected
non-conforming items entering during the 2nd period t (Working
Shift) at production line k (Reverse Remanufacturing Line)
after prior 100% inspection
        Constraint123: RM2[t][1] == repsilon0[k] *
((MTF1+MTF2+MTF3)+((V1[t][5]+V2[t][5])*0.75)); //**** (Reverse
Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
conforming items entering the 2nd period t (Working Shift) at
production line k (Reverse Remanufacturing Line) after prior
100% inspection
        Constraint124: RM1[t][k] == (1-repsilon0[k]) *
((MTF1+MTF2+MTF3)+((V1[t][5]+V2[t][5])*0.75)); //**** (Reverse
Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
non-conforming items entering during the 3rd period t (Working
Shift) at production line k (Reverse Remanufacturing Line)
after prior 100% inspection
        Constraint125: RM2[t][k] == repsilon0[k] *
((MTF1+MTF2+MTF3)+((V1[t][5]+V2[t][5])*0.75)); //**** (Reverse
Remanufacturing)
```

```
//2nd block (Reverse Remanufacturing) - Expected number of
conforming & non-conforming items entering at sampling quality
station after production-----------------------------

     forall (t in T, k in K) //Expected number of collected
conforming items entering at sampling quality station k (1)
(Reverse Remanufacturing Station) during period t (Working
Shift)
        Constraint126: RV1[t][1] == (1-repsilon[t][1]) *
RM1[t][1]; //**** (Reverse Remanufacturing)

    forall (t in T, k in K) //Expected number of collected
conforming items entering at sampling quality station k (2)
(Reverse Remanufacturing Station) during period t (Working
Shift)
        Constraint127: RV1[t][2] == (1-repsilon[t][2]) *
REW1[t][2]; //**** (Reverse Remanufacturing)

    forall (t in T, k in K) //Expected number of collected
conforming items entering at sampling quality station k (3)
(Reverse Remanufacturing Station) during period t (Working
Shift)
        Constraint128: RV1[t][3] == (1-repsilon[t][3]) *
REW1[t][3]; //**** (Reverse Remanufacturing)

    forall (t in T, k in K) //Expected number of collected
conforming items entering at sampling quality station k (4)
(Reverse Remanufacturing Station) during period t (Working
Shift)
          Constraint129: RV1[t][4] ==(1-repsilon[t][4]) *
REW1[t][4]; //**** (Reverse Remanufacturing)

    forall (t in T, k in K) //Expected number of collected
conforming items entering at sampling quality station k (5)
(Reverse Remanufacturing Station) during period t (Working
Shift)
          Constraint130: RV1[t][5] ==(1-repsilon[t][5]) *
REW1[t][5]; //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
non-conforming items entering at sampling quality station k
```

```
(1) (Reverse Remanufacturing Station) during period t (Working
Shift)
          Constraint131: RV2[t][1] == (repsilon[t][1] *
RM1[t][1]); //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
non-conforming items entering at sampling quality station k
(2) (Reverse Remanufacturing Station) during period t (Working
Shift)
          Constraint132: RV2[t][2] == REW2[t][2] +
repsilon[t][2] * REW1[t][2]; //**** (Reverse Remanufacturing)

    forall (t in T, k in K) //Expected number of collected
non-conforming items entering at sampling quality station k
(3) (Reverse Remanufacturing Station) during period t (Working
Shift)
         Constraint133: RV2[t][3] == REW2[t][3] +
repsilon[t][3] * REW1[t][3]; //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
non-conforming items entering at sampling quality station k
(4) (Reverse Remanufacturing Station) during period t (Working
Shift)
          Constraint134: RV2[t][4] == REW2[t][4] +
repsilon[t][4] * REW1[t][4]; //**** (Reverse Remanufacturing)

    forall (t in T, k in K) //Expected number of collected
non-conforming items entering at sampling quality station k
(5) (Reverse Remanufacturing Station) during period t (Working
Shift)
          Constraint135: RV2[t][5] == REW2[t][5] +
repsilon[t][5] * REW1[t][5]; //**** (Reverse Remanufacturing)

//3rd block (Reverse Remanufacturing) - Expected sample-
sorting/rejection number of re-manufactured conforming & non
conforming re-manufactured items Part1------------------------
--------------------------------------------------------------
--------------------------------------------------------------
---------------------------------------------------
```

```
    forall (t in T, k in K) //Fraction of collected
conforming items for sampling at sampling quality station 1
(Reverse Remanufacturing Station) during period 1 (Working
Shift)
         Constraint136: RSI1[1][1] == RV1[1][1]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 1
(Reverse Remanufacturing Station) during period 1 (Working
Shift)
         Constraint137: RSI2[1][1] == RV2[1][1]; //****
(Reverse Remanufacturing)

   forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 1 (Reverse
Remanufacturing Station) during period 2 (Working Shift)
         Constraint138: RSI1[2][1] == RV1[2][1]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 1
(Reverse Remanufacturing Station) during period 2 (Working
Shift)
         Constraint139: RSI2[2][1] == RV2[2][1]; //****
(Reverse Remanufacturing)

   forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 1 (Reverse
Remanufacturing Station) during period 3 (Working Shift)
         Constraint140: RSI1[3][1] == RV1[3][1]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 1
(Reverse Remanufacturing Station) during period 3 (Working
Shift)
         Constraint141: RSI2[3][1] == RV2[3][1]; //****
(Reverse Remanufacturing)
```

```
    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 2 (Reverse
Remanufacturing Station) during period 1 (Working Shift)
          Constraint142: RSI1[1][2] == RV1[1][2]; //(Reverse
Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 2
(Reverse Remanufacturing Station) during period 1 (Working
Shift)
          Constraint143: RSI2[1][2] == RV2[1][2]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 2 (Reverse
Remanufacturing Station) during period 2 (Working Shift)
          Constraint144: RSI1[2][2] == RV1[2][2]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 2
(Reverse Remanufacturing Station) during period 2 (Working
Shift)
          Constraint145: RSI2[2][2] == RV2[2][2]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 2 (Reverse
Remanufacturing Station) during period 3 (Working Shift)
          Constraint146: RSI1[3][2] == RV1[3][2]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 2
(Reverse Remanufacturing Station) during period 3 (Working
Shift)
          Constraint147: RSI2[3][2] == RV2[3][2]; //****
(Reverse Remanufacturing)
```

```
    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 3 (Reverse
Remanufacturing Station) during period 1 (Working Shift)
          Constraint148: RSI1[1][3] == RV1[1][3]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 3
(Reverse Remanufacturing Station) during period 1 (Working
Shift)
          Constraint149: RSI2[1][3] == RV2[1][3]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 3 (Reverse
Remanufacturing Station) during period 2 (Working Shift)
          Constraint150: RSI1[2][3] == RV1[2][3]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 3
(Reverse Remanufacturing Station) during period 2 (Working
Shift)
          Constraint151: RSI2[2][3] == RV2[2][3]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 3 (Reverse
Remanufacturing Station) during period 3 (Working Shift)
          Constraint152: RSI1[3][3] == RV1[3][3]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 3
(Reverse Remanufacturing Station) during period 3 (Working
Shift)
          Constraint153: RSI2[3][3] == RV2[3][3];//****
(Reverse Remanufacturing)
```

```
    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 4 (Reverse
Remanufacturing Station) during period 1 (Working Shift)
          Constraint154: RSI1[1][4] == RV1[1][4]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 4
(Reverse Remanufacturing Station) during period 1 (Working
Shift)
          Constraint155: RSI2[1][4] == RV2[1][4]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 4 (Reverse
Remanufacturing Station) during period 2 (Working Shift)
          Constraint156: RSI1[2][4] == RV1[2][4]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 4
(Reverse Remanufacturing Station) during period 2 (Working
Shift)
          Constraint157: RSI2[2][4] == RV2[2][4]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 4 (Reverse
Remanufacturing Station) during period 3 (Working Shift)
          Constraint158: RSI1[3][4] == RV1[3][4]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 4
(Reverse Remanufacturing Station) during period 3 (Working
Shift)
          Constraint159: RSI2[3][4] == RV2[3][4]; //****
(Reverse Remanufacturing)
```

```
    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 5 (Reverse
Remanufacturing Station) during period 1 (Working Shift)
          Constraint160: RSI1[1][5] == RV1[1][5]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 5
(Reverse Remanufacturing Station) during period 1 (Working
Shift)
          Constraint161: RSI2[1][5] == RV2[1][5]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 5 (Reverse
Remanufacturing Station) during period 2 (Working Shift)
          Constraint162: RSI1[2][5] == RV1[2][5]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 5
(Reverse Remanufacturing Station) during period 2 (Working
Shift)
          Constraint163: RSI2[2][5] == RV2[2][5]; //****
(Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected conforming
items for sampling at sampling quality station 5 (Reverse
Remanufacturing Station) during period 3 (Working Shift)
          Constraint165: RSI1[3][5] == RV1[3][5]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected non-
conforming items for sampling at sampling quality station 5
(Reverse Remanufacturing Station) during period 3 (Working
Shift)
          Constraint166: RSI2[3][5] == RV2[3][5]; //****
(Reverse Remanufacturing)

//4th block (Reverse Remanufacturing) - Expected sample-
sorting/rejection number of re-manufactured conforming &
```

```
conforming re-manufactured items Part2------------------------
--------------------------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
-------

    forall (t in T, k in K) //Fraction of collected sample
conforming items rejected by error type-I at sampling quality
station k (Reverse Remanufacturing Station) during period t
(Working Shift)
     Constraint167: RESR1[t][k] == ralpha[t][k] * RSI1[t][k];
//**** (Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected sample
conforming items rejected by error type-I at sampling quality
station k (Reverse Remanufacturing Station) during period t
(Working Shift)
          Constraint168: RESR2[t][k] == rbetha[t][k] *
RSI2[t][k]; //**** (Reverse Remanufacturing)

    forall (t in T, k in K) //Fraction of collected sample
conforming items accepted as truly conforming at sampling
quality station k (Reverse Remanufacturing Station) during
period t (Working Shift)
          Constraint169: RESR3[t][k] == RSI1[t][k] -
RESR1[t][k]; //This needs to be changed every stage //--------
-- //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of collected sample
non-conforming items rejected as truly non-conforming at
sampling quality station k (Reverse Remanufacturing Station)
during period t (Working Shift) (For repair, rework, or scrap)
          Constraint170: RESR4[t][k] == (1-rbetha[t][k]) *
RSI2[t][k]; //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Fraction of sample collected
conforming items that became non-conforming as result of
mishandling at sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift)
          Constraint171: RESR5[t][k] == romega[t][k]  *
RESR3[t][k]; //**** (Reverse Remanufacturing)
```

```
//5th block (Reverse re-manufacturing) - Expected sample-
sorting/rejection number of re-manufactured conforming &
conforming re-manufactured items Part3------------------------
--------------------------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
-------

     forall (t in T, k in K) //Final fraction of sample
collected conforming items that will be transfer to the next
production station k (Forward Manufacturing Station) during
period t (Working Shift) W1tj
          Constraint172: RFCI[t][k] == (RESR3[t][k] -
RESR5[t][k]) + RESR2[t][k]; //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Final fraction of sample
collected non-conforming items that will be transfer to the
next production station k (Reverse Remanufacturing Station)
during period t (Working Shift) W2tj
          Constraint173: RFNCI[t][k] ==  RV2[t][k]-
RSI2[t][k]; //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Total fraction of sample
collected conforming items that became non-conforming as
result of mishandling and rejection error type-I at sampling
quality station k (Reverse Remanufacturing Station) during
period t (Working Shift) that must be scrapped
          Constraint174: RTSCN[t][k] == RESR1[t][k] +
RESR5[t][k]; //**** (Reverse Remanufacturing)

//6th block (Reverse re-manufacturing) - Expected fraction of
sample items that must be scrapped after sampling quality
rejection-----------------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
-----------------

     forall (t in T, k in K) //Expected fraction of sample
collected conforming items, rejected at sampling quality
```

```
station k (Reverse Remanufacturing Station) during period t
(Working Shift) that must be scrapped
         Constraint175: RISCR1[t][k] == RF2[t][k] *
(RESR1[t][k] + RESR5[t][k]); //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected fraction of sample
collected non-conforming items, rejected at sampling quality
station k (Reverse Remanufacturing Station) during period t
(Working Shift) that must be scrapped
          Constraint176: RISCR2[t][k] == ((RF2[t][k] *
RESR4[t][k])); //**** (Reverse Remanufacturing)


//7th block (Remanufacturing) - (Reverse Remanufacturing)
Material Balance Constraints)---------------------------------
-------------------------------------------------------------
-------------------------------------------------------------
-----------------------------------------------------------

     forall (t in T, k in K) //Expected number of collected
conforming items entering at station k (1) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
          Constraint178: REW1[t][1] == RV1[t][1]; //This
should be for the next term //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
conforming items entering at station k (2) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
          Constraint179: REW1[t][2] == RV1[t][1] - RSI1[t][1]
+ ((RESR3[t][1] - RESR5[t][1]) + RESR2[t][1]); // + FSC[t][1];
//**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
conforming items entering at station k (3) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
          Constraint180: REW1[t][3] == RV1[t][2] - RSI1[t][2]
+ ((RESR3[t][2] - RESR5[t][2]) + RESR2[t][2]); // +
(FSC[t][2]); //**** (Reverse Remanufacturing)
```

```
     forall (t in T, k in K) //Expected number of collected
conforming items entering at station k (4) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
          Constraint181: REW1[t][4] == RV1[t][3] - RSI1[t][3]
+ ((RESR3[t][3] - RESR5[t][3]) + RESR2[t][3]); // +
(FSC[t][3]); //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
conforming items entering at station k (5) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
          Constraint182: REW1[t][5] == RV1[t][4] - RSI1[t][4]
+ ((RESR3[t][4] - RESR5[t][4]) + RESR2[t][4]); // +
(FSC[t][4]); //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
non-conforming items entering at station k (1) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
          Constraint183: REW2[t][1] == RV2[t][1]; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
non-conforming items entering at station k (2) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
          Constraint184: REW2[t][2] == RV2[t][1] - RSI2[t][1];
//**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
non-conforming items entering at station k (3) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
          Constraint185: REW2[t][3] == RV2[t][2] - RSI2[t][2];
//**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
non-conforming items entering at station k (4) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
```

```
          Constraint186: REW2[t][4] == RV2[t][3] - RSI2[t][3];
//**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Expected number of collected
non-conforming items entering at station k (5) (Reverse
Remanufacturing Station) during period t (Working Shift)
(Remanufacturing Balance Constraints)
          Constraint187: REW2[t][5] == RV2[t][4] - RSI2[t][4];
//**** (Reverse Remanufacturing)

//8th block (Reverse Remanufacturing) - Flow balancing
constraints---------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
-------------------------------------------

     forall (t in T, k in K) //The number of items shipped
from the re-manufacture process cannot exceed the quantity of
raw materials received (Collected and Scrap Materials)
(Reverse Remanufacturing Production)
          Constraint188: RRAW1[t][1] - (RV1[t][5] + RV2[t][5])
>= 0; //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //The number of items shipped
from the factory cannot exceed the production capacity
(Reverse Remanufacturing Production)
          Constraint189: (RV1[t][5] + RV2[t][5]) <= rdelta;
//**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Penalization cost per non-
conforming items ship to the customer (Reverse Remanufacturing
System) (General calculation)
          Constraint190: RNCS[t][1] == RV2[t][5] * 0; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Penalization cost per non-
conforming items ship to the customer (Reverse Remanufacturing
System) (General calculation)
          Constraint191: RNCS[t][2] == RV2[t][5] * 0; //****
(Reverse Remanufacturing)
```

```
     forall (t in T, k in K) //Penalization cost per non-
conforming items ship to the customer (Reverse Remanufacturing
System) (General calculation)
          Constraint192: RNCS[t][3] == RV2[t][5] * 0; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Penalization cost per non-
conforming items ship to the customer (Reverse Remanufacturing
System) (General calculation)
          Constraint193: RNCS[t][4] == RV2[t][5] * 0; //****
(Reverse Remanufacturing)

     forall (t in T, k in K) //Penalization cost per non-
conforming items ship to the customer (Reverse Remanufacturing
System) (General calculation)
          Constraint194: RNCS[t][5] == RV2[t][5]; //****
(Reverse Remanufacturing)

     forall (k in K) //Total penalization cost per non-
conforming items ship to the customer (Reverse Remanufacturing
System) (General calculation)
          Constraint195: RNCS2 == sum (t in T) RNCS[t][5] *
rxi; //**** (Reverse Remanufacturing)

//9th block (Reverse Remanufacturing) - Remanufacturing cost
constraints for the objective function------------------------
--------------------------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
--------

     forall (t in T, k in K) //Objective function - Total
Reverse Remanufacturing Production Cost
          Constraint196: TRRPC ==  sum (t in T, k in K: k ==
1) (REW1[t][k] + REW2[t][k]) * rrho[t][k]; //**** (Reverse
Remanufacturing)

     forall (t in T, k in K) //Objective function - Total
Reverse Remanufacturing Production Cost
          Constraint197: TRIC == sum (t in T, k in K)
(RSI1[t][k] + RSI2[t][k]) * ri[t][k]; //**** (Reverse
```

```
Remanufacturing) - There are only two factor because this
process is more risky

     forall (t in T, k in K) //Objective function - Total
Forward Manufacturing Scrap Cost
          Constraint198: TRSC == sum (t in T, k in K)
(RISCR1[t][k] + RISCR2[t][k]) * rs[t][k]; //**** (Reverse
Remanufacturing) - There are only two factor because this
process is more risky

//10th block (Reverse Remanufacturing) - Remanufacturing
inspection decision constraints-------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
-

     forall (t in T, k in K) //Minimum required fraction of
collected sample conforming items that must pass quality
inspection at k (Reverse Remanufacturing Station) during
period t (Working Shift) as internal quality parameter
(Review)
          Constraint199: RMAOQL1[t][k] == (RSI1[t][k] +
RSI2[t][k]) * RX; //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Total fraction of collected
samples conforming and non-conforming items reworked and
rejected as scrap at the sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift) as
quality parameter
          Constraint200: RMAOQL2[t][k] == ((RESR4[t][k] +
RTSCN[t][k])); //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //1 - If quality inspection
should take place after each stage k (Reverse Remanufacturing
Station) in period t (Working Shift), 0 - Otherwise
          Constraint201: RMT3[t][k] == (RMAOQL2[t][k] >=
RMAOQL1[t][k]); //To know where inspection takes place //
Remanufacturing Done
```

```
    forall (t in T, k in K) //1 - If quality inspection
should take place after each stage k (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
        Constraint202: RMTX3[t][1] == (REW2[t][1] >=
REW2[t][1]);

    forall (t in T, k in K) //1 - If quality inspection
should take place after each stage k (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
        Constraint203: RMTX3[t][2] == (REW2[t][2] >=
(REW2[t][1]*0.50));

    forall (t in T, k in K) //1 - If quality inspection
should take place after each stage k (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
        Constraint204: RMTX3[t][3] == (REW2[t][3] >=
(REW2[t][2]*0.50));

    forall (t in T, k in K) //1 - If quality inspection
should take place after each stage k (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
        Constraint205: RMTX3[t][4] == (REW2[t][4] >=
(REW2[t][3]*0.50));

    forall (t in T, k in K) //1 - If quality inspection
should take place after each stage k (Forward Manufacturing
Station) in period t (Working Shift), 0 - Otherwise
        Constraint206: RMTX3[t][5] == (REW2[t][5] >=
(REW2[t][4]*0.50));

    forall (t in T, k in K) //Third QA testing constraint
        Constraint207: RMTX4[t][k] == (RESR1[t][k] >=
RESR4[t][k]);

    forall (t in T, k in K) //Fourth QA testing constraint
        Constraint208: RMTX5[t][k] == RESR2[t][k];

    forall (t in T, k in K) //Fifth QA testing constraint
        Constraint209: RMTX6[t][k] == (RFCI[t][k]-
RESR2[t][k]);
```

```
     forall (t in T, k in K) //Sixth QA testing constraint
          Constraint210: RMTX7[t][k] == (RMTX5[t][k] >=
RMTX6[t][k]);

     forall (t in T, k in K) //Seventh QA testing constraint
          Constraint211: RMTX8[t][k] == (RSI1[t][k] +
RSI2[t][k]) * ri[t][k];

     forall (t in T, k in K) //Eight QA testing constraint
          Constraint212: RMTX9[t][k] == (RISCR1[t][k] +
RISCR2[t][k]) * rs[t][k];

     forall (t in T, k in K) //If scrap cost is higher than
inspection cost
          Constraint213: RMTX10[t][k] == (RMTX9[t][k] >=
RMTX8[t][k]);

     forall (t in T, k in K) //If quality should be applied
          Constraint214: RMTX11[t][k] == (RMT3[t][k] +
RMTX3[t][k] + RMTX4[t][k] + RMTX7[t][k] + RMTX10[t][k] >= 3);

//11th block (Reverse Remanufacturing) - Preventive reverse
re-manufacturing maintenance----------------------------------
--------------------------------------------------------------
--------------------------------------------------------------
------------------------------------------------------------



     forall (t in T, k in K) //If the deterioration factor of
the working machine is higher than the operation level
          Constraint215: RMTZ1[t][k] == (repsilon[t][k] >= 1);

     forall (t in T, k in K) //If the fraction of the non-
conforming production is higher than the fraction of
conforming items
          Constraint216: RMTZ2[t][k] == (RV2[t][k] >=
(RV1[t][k]*0.20));

     forall (t in T, k in K) //Total fraction of collected
samples conforming non-conforming items rejected as scrap at
```

```
the sampling quality station k (Reverse Remanufacturing
Station) during period t (Working Shift)
          Constraint217: RMTZ3[t][k] == (RISCR1[t][k] +
RISCR2[t][k]); //**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Does preventive maintenance
should take place if the 25% of first conforming items is
lower than the total scrap
          Constraint218: RMTZ4[t][k] == (RMTZ3[t][k] >=
(RFCI[t][k] * X3)); //To know if inspection is necessary //
//**** (Reverse Remanufacturing)

     forall (t in T, k in K) //Maintenance decision
          Constraint219: RMTZ5[t][k] == (RMTZ1[t][k] +
RMTZ2[t][k] + RMTZ4[t][k] >= 1);

     forall (t in T, k in K) //Total Reverse Remanufacturing
Preventive Maintenance Cost
          Constraint220: TRPMC == sum (t in T, k in K)
RMTZ5[t][k] * rmu[t][k]; //**** (Reverse Remanufacturing)

//12th block (Reverse Remanufacturing) Remanufacturing profit
constraints-----------------------------------------------------------
---------------------------------------------------------------
---------------------------------------------------------------
-------------------------------------------


     forall (t in T, k in K) //Total earnings at each stage j
(Manufacturing Station) in period t (Working Shift)
        Constraint221: RPRFT == (sum (t in T, k in K: k == 5)
RV1[t][k] * RSLC) - (TRRPC + TRSC + RNCS2);
}
```

```
/*********************************************
 * OPL 22.1.0.0 Data
 * Author: Amauri
 * Creation Date: Jan 25, 2024 at 1:34:36 PM
 *********************************************/

//Forward Manufacturing Data

n = 5; //Maximum number of forward manufacturing stations

m = 3; //Maximum number of forward working shifts

de = 76000; //Forward stationary fixed demand for primary
market

delta = 76000; //Forward manufacturing production capacity per
period t (Working shift)

gamma = 0.08; //Fraction of conforming and non-conforming
items sampled after station j (Manufacturing Station)

//rho = 7.20; //Unit production cost for processing at station
j (Forward Manufacturing Station) during period t (Working
Shift)

//i = 0.72; //Unit inspection cost for inspection at quality
station j (Forward Manufacturing Station) during period t
(Working Shift) (10% of the production cost)

//kappa = 1.8; //Unit repair cost at station j (Forward
Manufacturing Station) during period t (Working Shift) (25% of
the production cost)

//lambda = 10.8; //Unit rework cost at station j (Forward
Manufacturing Station) during period t (Working Shift)

//s = 11.08; //Unit scrap cost at station j (Forward
Manufacturing Station) during period t (Working Shift). (Same
as the sales cost)
```

```
//mu = 20; //Unit preventive maintenance cost at station j
(Forward Manufacturing Station)

xi = 17.05; //Penalty cost per non-conforming item received by
the mainly market customer. (Forward Manufacturing System)

X = 0.30; //Minimum percentage of acceptable sample as quality
parameter in sampling quality station j (Quality inspection
decision) (Forward Manufacturing Station)

X2 = 0.75; //Percentage of end-of-life products recovery

X4 = 0.50; //Percentage of conforming items produced in a next
operation

X5 = 0.20; //Fraction of sample conforming and non-conforming
items rejected because repair and rework are needed

X6 = 0.20; //Percentage of a total production of conforming
items during a shift

SLC = 11.08; //Unit sales cost with 35% of profit margin



//ExcelSheet Data

SheetConnection ExcelFile("Project_Data_Closed_Loop.xlsx");

//Results -------------------------------------------------------
---------------------------------------------------------------
---------------------------------------------------------------
-----------------------------------------

QRM1 from SheetRead(ExcelFile,"QRM1_t!C5:C7"); //Quantity of
raw material entering to the Serial-Forward-Multi-Stage-
Manufacturing-System in the period t (Forward manufacturing
shift/Production line)

epsilon0 from SheetRead(ExcelFile,"epsilon0_j!C5:G5");//Non-
conforming fraction of raw material entering the Serial-
```

```
Forward-Multi-Stage-Manufacturing-System at station j (Forward
manufacturing station)

epsilon from SheetRead(ExcelFile,"epsilon_t_j!C6:E10");
//Degradation level of which working station j (Forward
manufacturing station) is exposed in the period t (Working
shift)

alpha from SheetRead(ExcelFile,"alpha_j!C5:E9"); //Probability
of type-I error at sample inspection station j (Forward
Manufacturing Station)

betha from SheetRead(ExcelFile,"betha_j!C5:E9"); //Probability
of type-II error at sample inspection station j (Forward
Manufacturing Station)

omega from SheetRead(ExcelFile,"omega_t_j!C6:E10");
//Probability of a conforming sample inspected material at
sample inspection station j (Forward Manufacturing Station)
suffers danger or contamination during the process at period t
(working shift)

theta from SheetRead(ExcelFile,"theta_t_j!C6:E10");
//Probability of a reworked item at station j (Forward
manufacturing station) does not comply with the quality
standards during period t (working shift)

F1 from SheetRead(ExcelFile,"f1_t_j!C6:E10"); //Fraction of
the rejected items at sample inspection station j (Forward
manufacturing station) during period t (working shift) that
must be repaired

F2 from SheetRead(ExcelFile,"f2_t_j!C6:E10"); //Fraction of
the rejected items at sample inspection station j (Forward
Manufacturing Station) during period t (Working Shift) that
must be scrapped

F3 from SheetRead(ExcelFile,"f3_t_j!C6:E10"); //Fraction of
the rejected items at sample inspection station j (Forward
Manufacturing Station) during period t (Working Shift) that
must be reworked
```

```
rho from SheetRead(ExcelFile,"rho_t_j!C6:E10"); //Unit
production cost for processing at station j (Forward
Manufacturing Station) during period t (Working Shift)

i from SheetRead(ExcelFile,"i_t_j!C6:E10"); //Unit inspection
cost for inspection at quality station j (Forward
Manufacturing Station) during period t (Working Shift) (10% of
the production cost)

kappa from SheetRead(ExcelFile,"kappa_t_j!C6:E10"); //Unit
repair cost at station j (Forward Manufacturing Station)
during period t (Working Shift) (25% of the production cost)

lambda from SheetRead(ExcelFile,"lambda_t_j!C6:E10"); //Unit
rework cost at station j (Forward Manufacturing Station)
during period t (Working Shift)

s from SheetRead(ExcelFile,"s_t_j!C6:E10"); //Unit scrap cost
at station j (Forward Manufacturing Station) during period t
(Working Shift). (Same as the sales cost)

mu from SheetRead(ExcelFile,"mu_t_j!C6:E10"); //Unit
preventive maintenance cost at station j (Forward
Manufacturing Station)


//Results that goes to the ExcelSheet ------------------------
--------------------------------------------------------------
--------------------------------------------------------------
-------------------------------------------

TPC to SheetWrite(ExcelFile,"Results_Forward!E4:E4"); //Total
Forward Manufacturing Production Cost

TIC to SheetWrite(ExcelFile,"Results_Forward!E5:E5"); //Total
Forward Manufacturing Inspection Cost

TRPC to SheetWrite(ExcelFile,"Results_Forward!E6:E6"); //Total
Forward Manufacturing Repair Cost
```

```
TRWC to SheetWrite(ExcelFile,"Results_Forward!E7:E7"); //Total
Forward Manufacturing Rework Cost

TSC to SheetWrite(ExcelFile,"Results_Forward!E8:E8"); //Total
Forward Manufacturing Scrap Cost

TPMC to SheetWrite(ExcelFile,"Results_Forward!L4:L4"); //Total
Forward Manufacturing Preventive Maintenance Cost

RAW1 to SheetWrite(ExcelFile,"Results_Forward!B13:F15");
//Expected quantity of conforming raw material entering during
the 1st period t (Working Shift) at production line j (Forward
Manufacturing Production Line)

RAW2 to SheetWrite(ExcelFile,"Results_Forward!B20:F22");
//Expected quantity of non-conforming raw material entering
during the 1st period t (Working Shift) at production line j
(Forward Manufacturing Production Line)

M1 to SheetWrite(ExcelFile,"Results_Forward!B27:F29");
//Expected number of conforming items entering during the 1st
period t (Working Shift) at production line j (Forward
Manufacturing Production Line) after prior 100% inspection

M2 to SheetWrite(ExcelFile,"Results_Forward!B34:F36");
//Expected number of non-conforming items entering during the
1st period t (Working Shift) at production line j (Forward
Manufacturing Production Line) after prior 100% inspection

EW1 to SheetWrite(ExcelFile,"Results_Forward!I27:M29");
//Expected number of conforming items entering at stage j
(Forward Manufacturing Station) during period t (Working
Shift) (Manufacturing Balance Constraints)

EW2 to SheetWrite(ExcelFile,"Results_Forward!I34:M36");
//Expected number of non-conforming items entering at stage j
(Forward Manufacturing Station) during period t (Working
Shift) (Manufacturing Balance Constraints)

V1 to SheetWrite(ExcelFile,"Results_Forward!I13:M15");
//Expected number of conforming items entering at sampling
```

```
quality station j (Forward Manufacturing Station) during
period t (Working Shift)

V2 to SheetWrite(ExcelFile,"Results_Forward!I20:M22");
//Expected number of non-conforming items entering at sampling
quality station j (Forward Manufacturing Station) during
period t (Working Shift)

SI1 to SheetWrite(ExcelFile,"Results_Forward!P13:T15");
//Fraction of conforming items for sampling at sampling
quality station j (Forward Manufacturing Station) during
period t (Working Shift)

SI2 to SheetWrite(ExcelFile,"Results_Forward!P20:T22");
//Fraction of non-conforming items for sampling at sampling
quality station j (Forward Manufacturing Station) during
period t (Working Shift)

ESR1 to SheetWrite(ExcelFile,"Results_Forward!P27:T29");
//Fraction of sample conforming items rejected by error type-I
at sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift)

ESR2 to SheetWrite(ExcelFile,"Results_Forward!P34:T36");
//Fraction of sample non-conforming items accepted as
conforming by error type-II at sampling quality station j
(Forward Manufacturing Station) during period t (Working
Shift)

ESR3 to SheetWrite(ExcelFile,"Results_Forward!P41:T43");
//Fraction of sample conforming items accepted as truly
conforming at sampling quality station j (Forward
Manufacturing Station) during period t (Working Shift)

ESR4 to SheetWrite(ExcelFile,"Results_Forward!P48:T50");
//Fraction of sample non-conforming items rejected as truly
non-conforming at sampling quality station j (Forward
Manufacturing Station) during period t (Working Shift) (For
repair, rework, or scrap)
```

```
ESR5 to SheetWrite(ExcelFile,"Results_Forward!P55:T57");
//Fraction of sample conforming items that became non-
conforming as result of mishandling at sampling quality
station j (Forward Manufacturing Station) during period t
(Working Shift)

FCI to SheetWrite(ExcelFile,"Results_Forward!W13:AA15");
//Final fraction of sample conforming items that will be
transfer to the next production station j (Forward
Manufacturing Station) during period t (Working Shift) W1tj

FNCI to SheetWrite(ExcelFile,"Results_Forward!W20:AA22");
//Final fraction of sample non-conforming items that will be
transfer to the next production station j (Forward
Manufacturing Station) during period t (Working Shift) W2tj

TSCN to SheetWrite(ExcelFile,"Results_Forward!W27:AA29");
//Total fraction of sample conforming items that became non-
conforming as result of mishandling and rejection error type-I
at sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift) that must be repaired,
reworked or scrapped

IREP1 to SheetWrite(ExcelFile,"Results_Forward!AD13:AH15");
//Expected fraction of sample conforming items, rejected at
sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift) that must be repaired

IREP2 to SheetWrite(ExcelFile,"Results_Forward!AD20:AH22");
//Expected fraction of sample non-conforming items, rejected
at sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift) that must be repaired

ISCR1 to SheetWrite(ExcelFile,"Results_Forward!AD27:AH29");
//Expected fraction of sample conforming items, rejected at
sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift) that must be scrapped

ISCR2 to SheetWrite(ExcelFile,"Results_Forward!AD34:AH36");
//Expected fraction of sample non-conforming items, rejected
```

```
at sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift) that must be scrapped

IRWK1 to SheetWrite(ExcelFile,"Results_Forward!AK13:AO15");
//Expected fraction of sample conforming items, rejected at
sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift) that must be reworked

IRWK2 to SheetWrite(ExcelFile,"Results_Forward!AK20:AO22");
//Expected fraction of sample conforming items, rejected at
sampling quality station j (Forward Manufacturing Station)
during period t (Working Shift) that must be reworked

IRWK3 to SheetWrite(ExcelFile,"Results_Forward!AK27:AO29");
//Total fraction of sample items reworked at station j
(Forward Manufacturing Station) during period t (Working
Shift) that must be inspected for second time since the
process is not perfect

IRWK4 to SheetWrite(ExcelFile,"Results_Forward!AK34:AO36");
//Expected fraction of sample items reworked accepted as truly
conforming at sampling quality station j (Forward
Manufacturing Station) for second time, during period t
(Working Shift)

IRWK5 to SheetWrite(ExcelFile,"Results_Forward!AK41:AO43");
//Expected fraction of sample items reworked, rejected as
truly non-conforming at sampling quality station j (Forward
Manufacturing Station) for second time, during period t
(Working Shift)

IRWK6 to SheetWrite(ExcelFile,"Results_Forward!AK48:AO50");
//Expected fraction of sample conforming items reworked that
became non-conforming due to item mishandling during the
second inspection at sampling quality station j (Forward
Manufacturing Station) for second time, during period t
(Working Shift)

IRWK7 to SheetWrite(ExcelFile,"Results_Forward!AK55:AO57");
//Expected fraction of sample truly conforming items reworked
that will be transferred to the next production station j
```

```
(Forward Manufacturing Station) during period t (Working
Shift) after second sampling inspection

IRWK8 to SheetWrite(ExcelFile,"Results_Forward!AK62:AO64");
//Expected fraction of sample truly non-conforming items
reworked, rejected during second inspection at sampling
quality inspection station j (Forward Manufacturing Station)
during period t (Working Shift) that must be scrapped

FSC to SheetWrite(ExcelFile,"Results_Forward!AK69:AO71");
//Expected fraction of sample truly conforming items reworked
that will be transferred to the next production station j
(Forward Manufacturing Station) minus sampling reworked
conforming items that must be scrapped, during period t
(Working Shift)

MTF1 to SheetWrite(ExcelFile,"Results_Forward!L6:L6");
//Quantity of scrapped items transferred from Forward
Manufacturing System to 1st shift of Reverse Remanufacturing
System as raw material

MTF2 to SheetWrite(ExcelFile,"Results_Forward!L7:L7");
//Quantity of scrapped items transferred from Forward
Manufacturing System to 2nd shift of Reverse Remanufacturing
System as raw material

MTF3 to SheetWrite(ExcelFile,"Results_Forward!L8:L8");
//Quantity of scrapped items transferred from Forward
Manufacturing System to 3rd shift of Reverse Remanufacturing
System as raw material

PRFT to SheetWrite(ExcelFile,"Results_Forward!S4:S4");
//Manufacturing earnings at each station j (Forward
Manufacturing) during period t (Working Shift)

MAOQL1 to SheetWrite(ExcelFile,"Results_Forward!AR13:AV15");
//Minimum required fraction of sample conforming items that
must pass quality inspection at j (Forward Manufacturing
Station) during period t (Working Shift) as an internal
quality parameter
```

```
MAOQL2 to SheetWrite(ExcelFile,"Results_Forward!AR20:AV22");
//Total fraction of samples conforming and non-conforming
items reworked and rejected as scrap at the sampling quality
station j (Forward Manufacturing Station) during period t
(Working Shift) as a quality parameter

MT3 to SheetWrite(ExcelFile,"Results_Forward!AR27:AV29");
//Does quality inspection should take place after each station
j (Forward Manufacturing Station) during period t (Working
Shift) = 1, Otherwise = 0

MT4 to SheetWrite(ExcelFile,"Results_Forward!AR34:AV36");
//Total fraction of samples conforming, and non-conforming
items rejected, repaired, and reworked at station j (Forward
Manufacturing Station) during period t (Working shift)

MT5 to SheetWrite(ExcelFile,"Results_Forward!AR41:AV43");
//Total fraction of samples conforming non-conforming items
rejected as scrap at the sampling quality station j (Forward
Manufacturing Station) during period t (Working Shift)

MT6 to SheetWrite(ExcelFile,"Results_Forward!AR48:AV50");
//Does preventive maintenance should take place if the
fraction of rejected items by scrap is lower or equal to the
MAOQL1 (Desirable quality level) at station j (Forward
Manufacturing Station) during period t (Working Shift), If yes
= 1, Otherwise = 0

MTX11 to SheetWrite(ExcelFile,"Results_Forward!AR55:AV57");
//Final QA decision

MTZ11 to SheetWrite(ExcelFile,"Results_Forward!AR62:AV64");
//Final PM decision

NCS to SheetWrite(ExcelFile,"Results_Forward!AY20:BC22");
//Fraction of complete non-conforming items shipped to the
mainly market (Forward Manufacturing System)

NCS2 to SheetWrite(ExcelFile,"Results_Forward!S5:S5"); //Total
fraction of complete non-conforming items shipped to the
mainly market (Forward Manufacturing System)
```

```
//Reverse Remanufacturing Data

r = 5; //Maximum number reverse re-manufacturing working
stations

rde = 57000; //Reverse re-manufacturing stationary fixed
demand for secondary market (75% of the primary market demand)

rdelta = 57000; //Reverse re-manufacturing production capacity
per period t (Working shift)

rgamma = 0.08; //Fraction of collected conforming and non-
conforming items sampled after station j (Remanufacturing
Station)

//rrho = 7.20; //Unit production cost at processing stage k
(Reverse Remanufacturing Station) during period t (Working
Shift)

//ri = 0.018; //Unit inspection cost for inspection at quality
station k (Reverse Remanufacturing Station)

//rs = 3.60; //Unit scrap cost at station k (Reverse
Remanufacturing Station) during period t (Working Shift)

//rmu = 10; //Unit preventive maintenance cost at station k
(Reverse Remanufacturing Station)

rxi = 17.05; //Penalty cost per non-conforming item received
by the secondary market. (Reverse Remanufacturing Station)
(This is a novelty)

RX = 0.30; //Minimum percentage of acceptable sample as
quality parameter in sampling quality station j (Quality
inspection decision) (Reverse Remanufacturing Station)

X3 = 0.20; //Percentage

RSLC = 10.52; //Unit sales cost with 35% of profit margin
```

```
//ExcelSheet Data
//Results -------------------------------------------------------------
----------------------------------------------------------------------
----------------------------------------------------------------------
-----------------------------------------

//RQRM1 from SheetRead(ExcelFile,"RQRM1_t!C5:C7"); //Quantity
of collected raw material, entering to the Serial-Reverse-
Multi-Stage-Remanufacturing-System in the period t (Reverse
re-manufacturing shift/Production line)

repsilon0 from SheetRead(ExcelFile,"repsilon0_k!C5:G5");//Non-
conforming fraction of raw material entering the Serial-
Reverse-Multi-Stage-Remanufacturing-System at station k
(Reverse re-manufacturing station)

repsilon from SheetRead(ExcelFile,"repsilon_t_k!C6:E10");
//Probability of a conforming item acquires a defect during
processing in the stage k in the period t

ralpha from SheetRead(ExcelFile,"ralpha_k!C5:E9");
//Probability of type-I error at the inspection station k (For
Manufacturing & Remanufacturing)

rbetha from SheetRead(ExcelFile,"rbetha_k!C5:E9");
//Probability of type-II error at sample inspection station k
(Forward Manufacturing Station)

romega from SheetRead(ExcelFile, "romega_t_k!C6:E10");
//Probability of a conforming sample inspected material at
sample inspection station k (Reverse Remanufacturing Station)
suffers danger or contamination during the process at period t
(working shift)

RF2 from SheetRead(ExcelFile,"rf2_t_k!C6:E10"); //Fraction of
the rejected re-manufactured items scraped at the inspection
station k in the period t (Working Shift) (For Manufacturing &
Remanufacturing)

rrho from SheetRead(ExcelFile,"rrho_t_k!C6:E10"); //Reverse
Remanufacturing production cost
```

```
ri from SheetRead(ExcelFile,"ri_t_k!C6:E10"); //Reverse
Remanufacturing inspection cost

rs from SheetRead(ExcelFile,"rs_t_k!C6:E10"); //Reverse
Remanufacturing scrap cost

rmu from SheetRead(ExcelFile,"rmu_t_k!C6:E10"); //Reverse
Remanufacturing preventive maintenance cost

//Results that goes to the ExcelSheet ------------------------
--------------------------------------------------------------
--------------------------------------------------------------
-------------------------------------------

TRRPC to SheetWrite(ExcelFile,"Results_Reverse!E4:E4");
//Total Reverse Remanufacturing Production Cost

TRIC to SheetWrite(ExcelFile,"Results_Reverse!E5:E5"); //Total
Reverse Remanufacturing Inspection Cost

TRSC to SheetWrite(ExcelFile,"Results_Reverse!E8:E8"); //Total
Reverse Remanufacturing Scrap Cost

TRPMC to SheetWrite(ExcelFile,"Results_Reverse!L4:L4");
//Total Reverse Remanufacturing Preventive Maintenance Cost -
Still in progress

RRAW1 to SheetWrite(ExcelFile,"Results_Reverse!B13:F15");
//Expected quantity of collected conforming raw material, from
collection center entering during the 1st period t (Working
Shift) at station k (Reverse Remanufacturing Line)

RRAW2 to SheetWrite(ExcelFile,"Results_Reverse!B20:F22");
//Expected quantity of collected non-conforming raw material,
from collection center entering during the 1st period t
(Working Shift) at station k (Reverse Remanufacturing Line)

RM1 to SheetWrite(ExcelFile,"Results_Reverse!B27:F29");
//Expected number of collected conforming items entering the
1st period t (Working Shift) at production line k (Reverse
Remanufacturing Line) after prior 100% inspection
```

```
RM2 to SheetWrite(ExcelFile,"Results_Reverse!B34:F36");
//Expected number of collected non-conforming items entering
during the 1st period t (Working Shift) at production line k
(Reverse Remanufacturing Line) after prior 100% inspection

REW1 to SheetWrite(ExcelFile,"Results_Reverse!I27:M29");
//Expected number of collected conforming items entering at
station k (Reverse Remanufacturing Station) during period t
(Working Shift) (Remanufacturing Balance Constraints)

REW2 to SheetWrite(ExcelFile,"Results_Reverse!I34:M36");
//Expected number of collected non-conforming items entering
at station k (Reverse Remanufacturing Station) during period t
(Working Shift) (Remanufacturing Balance Constraints)

RV1 to SheetWrite(ExcelFile,"Results_Reverse!I13:M15");
//Expected number of collected conforming items entering at
sampling quality station k (Reverse Remanufacturing Station)
during period t (Working Shift)

RV2 to SheetWrite(ExcelFile,"Results_Reverse!I20:M22");
//Expected number of collected non-conforming items entering
at sampling quality station k (Reverse Remanufacturing
Station) during period t (Working Shift)

RSI1 to SheetWrite(ExcelFile,"Results_Reverse!P13:T15");
//Fraction of collected conforming items for sampling at
sampling quality station k (Reverse Remanufacturing Station)
during period t (Working Shift)

RSI2 to SheetWrite(ExcelFile,"Results_Reverse!P20:T22");
//Fraction of collected non-conforming items for sampling at
sampling quality station k (Reverse Remanufacturing Station)
during period t (Working Shift)

RESR1 to SheetWrite(ExcelFile,"Results_Reverse!P27:T29");
//Fraction of collected sample conforming items rejected by
error type-I at sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift)
```

```
RESR2 to SheetWrite(ExcelFile,"Results_Reverse!P34:T36");
//Fraction of collected sample non-conforming items rejected
by error type-II at sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift)

RESR3 to SheetWrite(ExcelFile,"Results_Reverse!P41:T43");
//Fraction of collected sample conforming items accepted as
truly conforming at sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift)

RESR4 to SheetWrite(ExcelFile,"Results_Reverse!P48:T50");
//Fraction of collected sample non-conforming items rejected
as truly non-conforming at sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift) (For
repair, rework, or scrap)

RESR5 to SheetWrite(ExcelFile,"Results_Reverse!P55:T57");
//Fraction of sample collected conforming items that became
non-conforming as result of mishandling at sampling quality
station k (Reverse Remanufacturing Station) during period t
(Working Shift)

RFCI to SheetWrite(ExcelFile,"Results_Reverse!W13:AA15");
//Final fraction of sample collected conforming items that
will be transfer to the next production station k (Forward
Manufacturing Station) during period t (Working Shift) W1tj

RFNCI to SheetWrite(ExcelFile,"Results_Reverse!W20:AA22");
//Final fraction of sample collected non-conforming items that
will be transfer to the next production station k (Reverse
Remanufacturing Station) during period t (Working Shift) W2tj

RTSCN to SheetWrite(ExcelFile,"Results_Reverse!W27:AA29");
//Total fraction of sample collected conforming items that
became non-conforming as result of mishandling and rejection
error type-I at sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift) that
must be scrapped

RISCR1 to SheetWrite(ExcelFile,"Results_Reverse!AD27:AH29");
//Expected fraction of sample collected conforming items,
```

```
rejected at sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift) that
must be scrapped

RISCR2 to SheetWrite(ExcelFile,"Results_Reverse!AD34:AH36");
//Expected fraction of sample collected non-conforming items,
rejected at sampling quality station k (Reverse
Remanufacturing Station) during period t (Working Shift) that
must be scrapped

RPRFT to SheetWrite(ExcelFile,"Results_Reverse!S4:S4");
//Manufacturing earnings at each station k (Reverse
Remanufacturing Station) during period t (Working Shift)

RMAOQL1 to SheetWrite(ExcelFile,"Results_Reverse!AR13:AV15");
//Minimum required fraction of collected sample conforming
items that must pass quality inspection at k (Reverse
Remanufacturing Station) during period t (Working Shift) as
internal quality parameter

RMAOQL2 to SheetWrite(ExcelFile,"Results_Reverse!AR20:AV22");
//Total fraction of collected samples conforming and non-
conforming items reworked and rejected as scrap at the
sampling quality station k (Reverse Remanufacturing Station)
during period t (Working Shift) as a quality parameter

//RMT3 to SheetWrite(ExcelFile,"Results_Reverse!AR27:AV29");
//Does quality inspection should take place after each station
k (Reverse Remanufacturing Station) during period t (Working
Shift) = 1, Otherwise = 0

//RMTZ3 to SheetWrite(ExcelFile,"Results_Reverse!AR41:AV43");
//(Prior RMT5) Total fraction of collected samples conforming
non-conforming items rejected as scrap at the sampling quality
station k (Reverse Remanufacturing Station) during period t
(Working Shift)

//RMTZ4 to SheetWrite(ExcelFile,"Results_Reverse!AR48:AV50");
//(Prior RMT6)Does preventive maintenance should take place if
the fraction of collected rejected items by scrap is lower or
equal to the MAOQL1 (Desirable quality level) at station j
```

```
(Reverse Remanufacturing Station) during period t (Working
Shift), If yes = 1, Otherwise = 0

RMTX11 to SheetWrite(ExcelFile,"Results_Reverse!AR27:AV29");
//If RMT3[t][k] + RMTX3[t][k] + RMTX4[t][k] + RMTX7[t][k] +
RMTX10[t][k] >= 3)

RMTZ5 to SheetWrite(ExcelFile,"Results_Reverse!AR34:AV36");
//If (RMTZ1[t][k] + RMTZ2[t][k] + RMTZ3[t][k] + RMTZ4[t][k] >=
1);

//RTEC to SheetWrite(ExcelFile,"Results_Reverse!AY13:BC15");
//If (RMTZ1[t][k] + RMTZ2[t][k] + RMTZ3[t][k] + RMTZ4[t][k] >=
1);

RNCS to SheetWrite(ExcelFile,"Results_Reverse!AY20:BC22");
//Fraction of complete non-conforming items shipped to the
secondary market (Reverse Remanufacturing System)

RNCS2 to SheetWrite(ExcelFile,"Results_Reverse!S5:S5");
//Total fraction of complete non-conforming items shipped to
the secondary market (Reverse Remanufacturing System)
```

APPENDIX VII

CONFERENCE: INDUSTRIAL AND MANUFACTURING SYSTEMS ENGINEERING, JUNE 2023, PARIS, (E-BOOK PUBLICATION)

Conference Proceedings, Paris France June 22-23, 2023

Planning Quality and Maintenance Activities in a Closed-Loop Serial Multu-Stage Manufacturing System Under Constant Degradation

Amauri Josafat Gomez Aguilar. Jean Pierre Kenné

Jean Pierre Kenné is with the University of Quebec, ETS, Canada (e-mail: jean-pierre.kenne@etsmtl.ca).

Abstract.
This research presents the development of a self-sustainable manufacturing system from a circular economy perspective, structured by a multi-stage serial production system consisting of a series of machines under deterioration in charge of producing a single product and a reverse remanufacturing system constituted by the same productive systems of the first scheme and different tooling, fed by-products collected at the end of their life cycle, and non-conforming elements of the first productive scheme. Since the advanced production manufacturing system is unable to satisfy the customer's quality expectations completely, we propose the development of a mixed integer linear mathematical model focused on the optimal search and assignment of quality stations and preventive maintenance operation to the machines over a time horizon, intending to segregate the correct number of non-conforming parts for reuse in the remanufacturing system and thereby minimizing production, quality, maintenance, and customer non-conformance penalties. Numerical experiments are performed to analyze the solutions found by the model under different scenarios. The results showed that the correct implementation of a closed manufacturing system and allocation of quality inspection and preventive maintenance operations generate better levels of customer satisfaction and an efficient manufacturing system.

Keywords.
Closed loop, Mixed integer linear programming, Preventive maintenance, Quality Inspection.

APPENDIX VIII

PROOF OF SUBMISSION OF ARTICLE IFAC

-----Message d'origine-----
De : PaperPlaza Conference Manuscript Management System <ifac.101@papercept.net> Envoyé : 10 février 2025 14:30 À : Kenné, Jean-Pierre <jean-pierre.kenne@etsmtl.ca> Cc : Gomez Aguilar, Amauri <amauri.gomez-aguilar.1@ens.etsmtl.ca>; Hof, Lucas <lucas.hof@etsmtl.ca> Objet : Decision on MIM 2025 submission 180

To: Prof. Jean-Pierre Kenné From: IPC and NOC Chairs, MIM 2025 Re: (180) Quality inspection and condition-based preventive maintenance in a closed-loop multi-stage manufacturing system

Dear Colleague,

over 800 contributions were submitted to the 11th IFAC conference MIM2025 in Trondheim. In the peer review, 800+ reviewers have participated, we received 2000+ review reports.

It is a pleasure to inform you that the contribution referenced above, for which you are listed as the corresponding author, has been accepted for presentation at the 11th IFAC Conference on Manufacturing Modelling, Management and Control to be held in Trondheim, Norway during June 30 - July 3, 2025.

Congratulations, IFAC MIM2025 is a very selective scientific event promising to become the "The Event of 2025" among IFAC international scientific forums!

Congratulations on this achievement! We are looking forward to seeing you in Trondheim in June/July 2025.

Best regards,
Prof. Dr. Fabio Sgarbossa, General Chair, Prof. Dr. Alexandre Dolgui, Advisory Chair, Prof. Dr. Dmitry Ivanov, IPC Chair, Associate Prof. Dr. Sotirios Panagou and Prof. Erlend Alfnes, Co-Chairs of National Organizing Committee,

Decision: Accepted as Contributed Paper.
Final submission deadline March 8, 2025.
Submission information
Authors and title:
Amauri Gomez Aguilar, Jean-Pierre Kenné*, Lucas Hof
Quality inspection and condition-based preventive maintenance in a closed-loop multi-stage manufacturing system
Conference: 11th IFAC Conference on Manufacturing Modelling, Management and Control.
Current status: Accepted.

APPENDIX IX

CONFERENCE ARTICLE IFAC

Quality inspection and condition-based preventive maintenance in a closed-loop multi-stage manufacturing system

Gomez Aguilar A.J*, Kenné J.P, Hof L.A*****
**** Mechanical Engineering Department, École de technologie supérieure*
Montreal, QC, QC H3C 1K3, Canada
**Tel: (514) 396-8800; e-mail: amauri.gomez-aguilar.1@ens.etsmtl.ca.*
*** Tel: (514) 396-8549; e-mail: jean-pierre.kenne@etsmtl.ca.*
**** Tel: (514) 396-8800; e-mail: Lucas.Hof@etsmtl.ca*

Abstract: This research presents the development of a Serial-Closed-Loop-Multi-Stage Manufacturing System from a circular economy perspective. The system integrates a production line where five specialized machines operate across both forward and reverse remanufacturing processes. However, due to varying deterioration levels, these machines cannot consistently produce defect-free items, resulting in increased scrap and customer penalty costs. To address this challenge, we propose a mixed-integer linear programming (MILP) model that optimally allocates quality inspection stations and schedules for preventive maintenance across multiple working shifts. The objective is to minimize total production costs, including quality control, maintenance and customer penalties. The model's effectiveness is validated through numerical experiments under three distinct machine deterioration scenarios, demonstrating its potential to enhance efficiency and sustainability in modern manufacturing systems.

Keywords: Closed-Loop Manufacturing, Multi-Stage System, Quality Inspection, Preventive Maintenance, Optimization, Mixed

1. INTRODUCTION

In today's highly competitive global market, manufacturing companies face increasing pressure to deliver high quality products while adhering to evolving customer expectations and sustainability standards. Beyond performance, customers now demand environmentally responsible production methods, such as remanufacturing within Closed-Loop Manufacturing Systems (CLMS) (Melo et al. (2009)). Recognized for their significant environmental and economic benefits, CLMS has gained substantial attention from both industrial and academic communities. Early research, such as:
Bouhchouch et al. (1993) analyzed CLMS behavior using queuing networks, while Werner (2001) developed analytical models to assess the production rate of unreliable CLMS. Despite these advancements, a persistent challenge remains: ensuring product quality while mitigating the effects of machine deterioration.

To address this issue, quality assurance (QA) strategies, particularly inspection-based quality control, have been widely explored to reduce non-conforming products and associated costs (Mandroli et al., 2006). Various studies have investigated quality inspection methodologies, including Hurst (1973), who examined poor-quality inspections using a binomial model, and Mohammadi et al. (2018), who proposed a mixed approach for defect detection.

However, while quality assurance (QA) plays a critical role, maintaining consistent production efficiency remains challenging due to factors such as wear and tear over time. Consequently, preventive maintenance has emerged as a crucial factor in enhancing system performance while simultaneously reducing both quality and maintenance costs.

Several authors have explored the integration of QA and maintenance strategies. Bouslah et al. (2018) introduced a policy for managing production, quality, and maintenance in aging production systems, while Rezaei-Malek et al. (2018) addressed the challenges of QA inspection and maintenance costs by linearizing a mixed non-linear programming model. Notable contributions in this area include Rivera-Gómez et al. (2020), who developed a policy to optimize production, maintenance and QA inspection activities in deteriorating systems, later extending their work to dynamic quality sampling Rivera-Gómez et al. (2021). While these studies provide valuable insights, they primarily focus on single-stage or traditional manufacturing setups, leaving a significant gap in the optimization of complex CLMS.

To bridge this gap, this research introduces an optimization model for CLMS that integrates quality inspection and preventive maintenance planning. Unlike prior studies, which primarily address quality and maintenance separately or within simplified system structures, this work develops a mixed-integer mathematical model that simultaneously determines the optimal allocation of quality inspection stations and maintenance activities across multiple production stages. By systematically balancing quality control and cost efficiency, this approach presents an advancement in manufacturing optimization enhancing production reliability, cost-effectiveness, and sustainability in complex CLMS, an area previously unexplored under this integrated framework.

The structure of this paper is as follows: Section 2 defines the problem in the context of Serial-Closed-Loop-Multi-Stage-Manufacturing Systems (SCLMSMS), followed by an optimization framework and equations in section 3. Section 4 details the simulation of machine deterioration through wear-and-tear scenarios. The results and analysis are presented in Section 5. Conclusions and key insights in Section 6."

2. PROBLEM STATEMENT

This research explores a Serial Closed Loop Multi Stage Manufacturing System, comprising a serial forward production echelon and a reverse remanufacturing echelon. The forward echelon features a five machine production line ($j = 1 \ldots 5$), that fulfills a primary market demand. Given the integrated nature of the system, the same five machines are also utilized in the reverse echelon ($k = 1 \ldots 5$), to re-manufacture non-conforming products from the forward echelon, as well as end-of-life (EOL) products collected from customers. This dual-purpose structure enhances sustainability and operation efficiency. (See Figure 1)

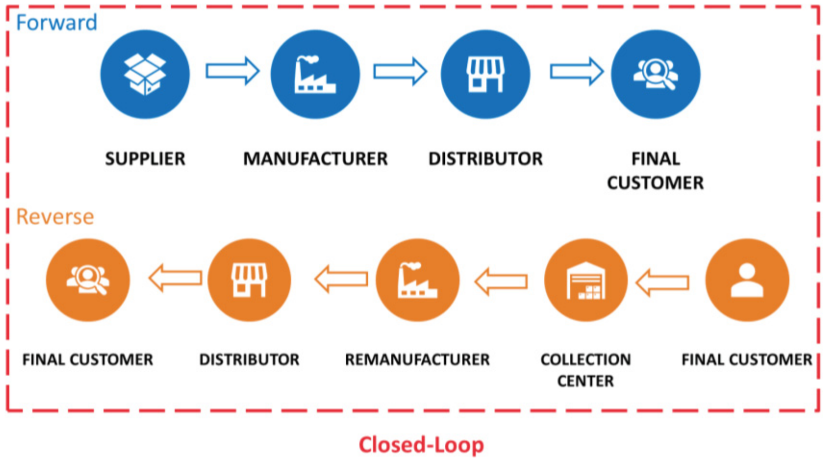

Figure 7Serial-Closed-Loop-Multi-Stage Manufacturing System

Both echelons undergo daily production deterioration across three working shifts ($t = 1,2,3$), leading to the generation of both conforming and non-conforming items. This deterioration directly affects production efficiency, quality and overall profitability of the manufacturing system.

2.1 Assumptions

- Rework and repair occur only in forward echelon, while scrapping is exclusively to the re-manufacturing echelon
- Reverse re-manufacturing relies on scrapped forward manufactured items and EOL products as raw materials.
- Production parameters are deterministic, ensuring a clear system analysis and facilitating future scalability.
- Only 75% of EOL products are viable for re-manufacturing, following a deterministic and industry approach.
- Quality inspections may damage items in both echelons, either due to inspector handling or environmental factors.

2.2 Production structure

The SCLMSMS production process begins with a raw material inspection from multiple suppliers, ensuring that only defect-free material $M1_{tj}$ enters the forward echelon, as below.

$$M1_{tj} = \left(1 - \varepsilon 0_j\right).QRM1_t \quad \forall\, t, j \tag{1}$$

$QRM1_t$ represents the material quantity per unit entering the manufacturing system during period t (working Shift), while ε0 denotes the non-conforming inspected items. A similar process occurs in the re-manufacturing echelon $RM1_{tk}$, as defined in equation (2), utilizing accumulated scrap from the first
($MTF1$), second ($MTF2$) and third shift ($MTF3$) shifts, along with EOL collected items denoted as CI.

$$RM1_{tk} = (1 - r\varepsilon 0_k).(MTF1 + MTF2 + MTF3) + (V1_{t5} + V2_{t5}) * CI \;\; \forall\, t, k \tag{2}$$

Where $V1_{t5}$ and $V2_{t5}$ are given by the following equations in the forward echelon.

$$V1_{tj} = \left(1 - \varepsilon_{tj}\right) * M1_{tj} + \left(1 - \varepsilon_{tj}\right) * EW1_{tj} \quad \forall\, t, j \tag{3}$$
$$V2_{tj} = \varepsilon_{tj} * M1_{tj} + EW2_{tj} + \varepsilon_{tj} * EW1_{tj} \quad \forall\, t, j \tag{4}$$

Where $EW1_{tj}$ and $EW2_{tj}$ are conforming and non-conforming items entering at stage j; with ε_{tj} representing the percentage of contaminated items. For reverse echelon, conforming $RV1_{tk}$ and non-conforming $RV2_{tk}$ items, described by following equations (4) and (6).

$$RV1_{tk} = (1 - r\varepsilon_{tk}) * RM1_{tk} + (1 - r\varepsilon_{tk}) * REW1_{tk} \quad \forall\, t, k \tag{5}$$
$$RV2_{tk} = r\varepsilon_{tk} * RM1_{tk} + REW2_{tk} + r\varepsilon_{tk} * REW1_{tk} \;\; \forall\, t, k \tag{6}$$

Where $r\varepsilon_{tk}$ represents the percentage of contaminated items and $REW1_{tk}$ and $REW2_{tk}$ the fraction of conforming and non-conforming items entering to the reverse system.

For each type of item presented in equations (3-4) for the forward and (5-6) for reverse echelons, a batch of random samples is made to analyze the quality level and performance of the machinery. After the sample inspection, two types of items are generated, including forward $SI1_{tj}$ and reverse $RSI1_{tj}$ compliant items presented in equations (7) and (8).

$$SI1_{tj} = V1_{tj} * \gamma \quad \forall\, t, j \tag{7}$$
$$RV2_{tk} = r\varepsilon_{tk} * RM1_{tk} + REW2_{tk} + r\varepsilon_{tk} * REW1_{tk} \;\; \forall\, t, k \tag{8}$$

Where γ and $r\gamma$ represent the unit rework forward and reverse.Additionally, forward $SI2_{tj}$ and reverse $RSI2_{tk}$for non-compliant items, given by the following equations.

$$SI2_{tj} = V2_{tj} * \gamma \quad \forall\, t, j \tag{9}$$
$$RSI2_{tk} = r\varepsilon_{tk} * RM1_{tk} + REW2_{tk} + r\varepsilon_{tk} * REW1_{tk} * r\gamma \quad \forall\, t, k \tag{10}$$

Since the QA inspection process is not flawless, the inspected items in the forward process ($SI1_{tj}$) include both correctly and incorrectly classified items. Among them, the correctly classified conforming items are denoted as $ESR3_{tj}$. However, some conforming items may be mistakenly identified as non-conforming due to type I inspection error (α_{tj}). These misclassified items are represented by $ESR1_{tj}$, which could either belong to $ESR1_{tj}$ or $ESR3_{tj}$ as applicable.

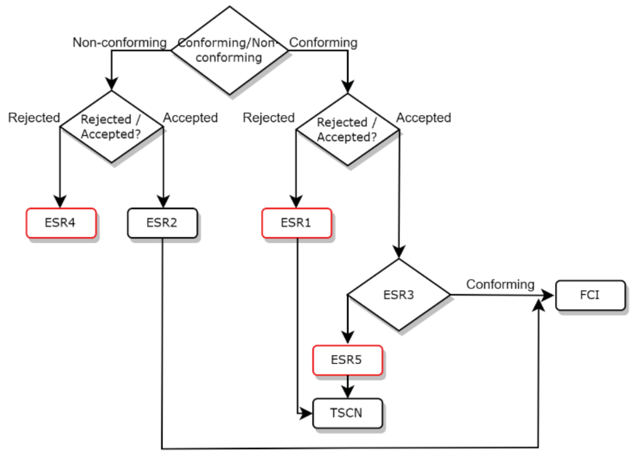

Figure 8 Forward QA inspection process

$$ESR1_{tj} = \alpha_{tj} * SI1_{tj} \quad \forall\, t, j \tag{11}$$

$$ESR3_{tj} = SI1_{tj} - ESR1_{tj} \quad \forall\, t, j$$

On the other hand, inspection leads as well non-conforming items well classified as non-conforming, denoted by $ESR4_{tj}$, and non-conforming items incorrectly classified as conforming ones $ESR2_{tj}$ as result of error type II (β), which could be $ESR2_{tj}$ or $ESR4_{tj}$ (see Figure 2), such that.

$$ESR2_{tj} = \beta_{tj} * V2_{tj} * \gamma \quad \forall\, t, j \tag{12}$$

$$ESR4_{tj} = (1 - \beta_{tj}) * SI2_{tj} \quad \forall\, t, j$$

The same inspection error occurs in the reverse echelon, within the $RSI1_{tj}$ which could be either $RESR1_{tk}$or $RESR3_{tk}$ and $RSI2_{tk}$which could be $RESR2_{tk}$or $RESR4_{tk}$ as in equations (13) and (14).

$$RESR1_{tk} = r\alpha_{tj} * RSI1_{tk} \quad \forall\, t, k \tag{13}$$

$$RESR3_{tk} = RSI1_{tk} - RESR1_{tk} \quad \forall\, t, j$$

$$RESR2_{tk} = r\beta_{tk} * RV2_{tk} * r\gamma \quad \forall\, t, k \tag{14}$$

$$RESR4_{tk} = (1 - r\beta_{tk}) * RSI2_{tk} \quad \forall\, t, k$$

During QA inspection, various risk factors affect the QA process, such as contamination or damage. Therefore, these items are classified as $ESR5_{tj}$ in the forward echelon (see equation (15)) and $RESR5_{tk}$in the reverse echelon (see equation (16)).

$$ESR5_{tj} = \Omega_{tj} * V1_{tj} * \gamma - ESR1_{tj} \; \forall\, t, j \tag{15}$$

$$RESR5_{tk} = \text{r}\Omega_{tk} * \; RESR3_{tk} \;\; \forall \, t, k \tag{16}$$

Where $\Omega_{tj}$and $\text{r}\Omega_{tk}$ represents the probability of an item become contaminated and damaged in forward and reverse echelons. The items that were neither contaminated and damaged during the forward inspection process are re-introduced to the production line as: $FNCI_{tj}$ (17) while in the re-manufacturing as $RFNCI_{tk}$ (18).

$$FCI_{tj} = \left(ESR3_{tj} - \; ESR5_{tj}\right) + ESR2_{tj} \quad\;\; \forall \, t, j \tag{17}$$

$$RFCI_{tk} = (RESR3_{tk} - \; RESR5_{tk}) + RESR2_{tk} \quad\;\; \forall \, t, k \tag{18}$$

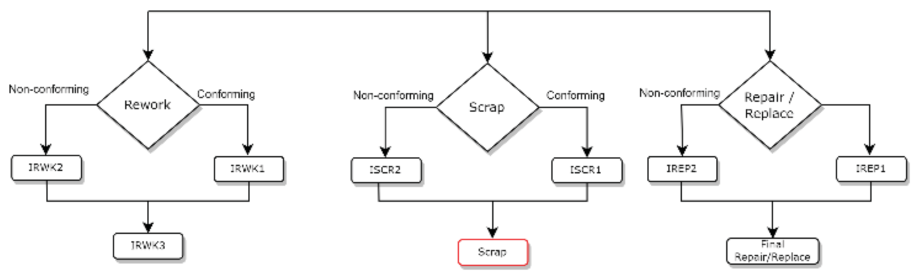

Figure 9 Rework, repair and scrap structure

After the QA inspection, a deterministic fraction of non-conforming items will be repaired ($F1_{tj}$) only in the forward echelon, denoted by $IREP_{tj}$, presented in equation (19).

$$IREP_{tj} = F1_{tj} * \left(ESR1_{tj} + \; ESR5_{tj}\right) + \; \left(F1_{tj} * RS4\right) \forall \, t, j \tag{19}$$

During the QA inspection process, the deterministic fraction of items $F3_{tj}$ with an operational and aesthetic condition complicated to be fixed by repair actions, are transferred to rework denoted as $IRWK2_{tj}$ and represented in the following equation. (See Figure 3).

$$IRWK_{tj} = F3_{tj} * \left(ESR1_{tj} + ESR5_{tj}\right) + \left(F3_{tj} * ESR4_{tj}\right) \quad \forall \, t, j \tag{20}$$

Nonetheless, since the rework process is complex, a failure is probable during its procedure, leading to an imperfect rework. Thus, after rework, an extra QA inspection is required to determine the final conforming fraction of items to re-introduced in the production line $IRWK7_{tj}$ (see equation (21)) and the ones that will be scrap $IRWK8_{tj}$, (see equation (22)).

$$IRWK7_{tj} = IRWK4_{tj} - IRWK6_{tj} \quad \forall \, tj \tag{21}$$

$$IRWK8_{tj} = IRWK5_{tj} + IRWK6_{tj} \quad \forall \, t, j \tag{22}$$

After the second QA inspection, rejected items are classified as scrap, gather with first inspected non-conforming items and transferred to the reverse re-manufacturing echelon as raw material denoted as FSC_{tj}, and illustrated in (23).

$$FSC_{tj} = IRWK8_{tj} + ISCR1_{tj} + ISCR2_{tj} \quad \forall \, t, j \tag{23}$$

Let us present in the next section the mathematical model developed in this paper to provide optimal solutions.

3. OPTIMIZATION AND ALLOCATION CONSTRAINTS

The model developed in this section is made by quality and maintenance constraints described in the following sub-sections.

3.1 Quality inspection allocation constraints

To allocate a QA inspection station indicated as: $MTX11_{tj}$, where at least three of the constraints should be met, as stated in equation (24).

$$MTX11_{tj} = \begin{cases} 1 \; if \; MT3_{tj} + \; MTX3_{tj} + \; MTX4_{tj} + MTX7_{tj} \\ \qquad\qquad\qquad\qquad MTX10_{tj} \; \geq 3, \\ 0 \qquad otherwise \end{cases} \tag{24}$$

The fraction of conforming items rejected by error and the truly non-conforming (under repair, rework, or scrap condition) must be less than 30% (X) of the total fraction of items inspected items as sample quality inspection (see equation (25)).

$$MT3_{tj} = \begin{cases} 1 \; if \; (ESR4_{tj} + ESR1_{tj} + ESR5_{tj}) \; \geq \\ \qquad (SI1_{tj} + V2_{tj} * \; \gamma) * X \; \geq 3, \\ 0 \quad otherwise \end{cases} \tag{25}$$

Where X represents the 30% of the threshold parameter used to keep benchmarking quality set under agreement between customers and the manufacturer for this case.
For the reverse echelon, the system is not allowed to operate without a quality station if more than 50% of items in a sample batch are nonconforming (see equation (26)).

$$RMT3_{tk} = \begin{cases} 1 \; if \; RESR4_{tk} + RESR1_{tk} + RESR5_{tk} * rx \geq \\ \qquad\qquad RSI1_{tj} + RIS2_{tk} * rx \; \geq 3, \\ 0 \qquad otherwise \end{cases} \tag{26}$$

The parameter (rx) represents 50% of the threshold needed to maintain proper system quality and minimize high penalty costs.
The second constraint $MTX3_{tj}$ does not allow to continue machinery operation without QA inspection station when more than half of nonconforming items is higher than conforming production of the next machinery (see equations (27) and (28)).

$$MTX3_{tj} = \begin{cases} 1 \; if \; (EW2_{tj+1} * 50\%) \geq EW2_{tj+1} \\ \; 0 \; otherwise \end{cases} \tag{27}$$

Which $j + 1$ and $k + 1$ represents the next machine, related to machine j or $k + 1$

$$RMTX3_{tk} = \begin{cases} 1\ if\ REW2_{tk+1} \leq (REW2_{tk+1} * 50\%) \\ 0 \qquad otherwise \end{cases} \tag{28}$$

The following constraint evaluates whether the fraction of conforming items rejected because of error type I is greater than the fraction of conforming parts inspected, a quality station must be installed in the forward $MTX4_{tj}$ or reverse $RMTX4_{tk}$ echelon, illustrated in equations (29) and (30).

$$MTX4_{tj} = \begin{cases} 1\ if\ \alpha_{tj} * V1_{tj} * \gamma \geq ESR4_{tj} \\ 0 \qquad otherwise \end{cases} \tag{29}$$

$$RMTX4_{tk} = \begin{cases} 1\ if\ RESR1_{tk} \geq (1 - r\beta_{tk}) * RSI2_{tk} \\ 0 \qquad otherwise \end{cases} \tag{30}$$

The following criteria $MTX7_{tj}$ is established to avoid a quality station when the fraction of non-conforming items accepted as conforming due to inspection error type II is lower than the final fraction of conforming items transferred to the next production station as given by the following equation.

$$MTX7_{tj} = \begin{cases} 1\ if\ ESR2_{tj} \geq \left(ESR3_{tj} - ESR5_{tj}\right) + ESR2_{tj} \\ 0 \qquad otherwise \end{cases} \tag{31}$$

3.2 Preventive maintenance allocation decisions

Five different decision variables are set for the forward (as appearing in equation (32)), while for the re-manufacturing only three (see equation (33)), where at least one should be met to consider maintenance in both echelons.

$$MTZ11_{tj} = \begin{cases} 1if\ MT6_{tj} + MTZ7_{tj} + MTZ8_{tj} + MTZ9_{tj} + MTZ10_{tj} \geq 1 \\ \quad 0 \qquad otherwise \end{cases} \tag{32}$$

$$RMTZ5_{tk} = \begin{cases} 1\ if\ RMTZ1_{tk} + RMTZ2_{tk} + RMTZ4_{tk} \geq 1 \\ 0 \quad otherwise \end{cases} \tag{33}$$

The first decision variable $MT6_{tj}$ limits the forward echelon to continue operating until a maintenance activity is performed if the total fraction of rejected (scrap) is higher than the total fraction of repaired and reworked items, (see equation (34)).

$$MT6_{tj} = \begin{cases} 1\ if\ if\ ISCR1_{tj} + ISCR2_{tj} \geq (IREP1_{tj} + IREP2_{tj} + IRWK1_{tj}) * XR \\ \quad 0 \qquad otherwise \end{cases} \tag{34}$$

Since repair or rework processes are not allowed in the re-manufacturing, thus, maintenance is applied $RMTZ1_{tk}$ when an item acquired a defect or contamination during QA inspection process as result of $r\varepsilon_{tk}$, presented in equation (35).

$$RMTZ1_{tk} = \begin{cases} 1 \; if \; r\varepsilon_{tk} > 1 \; (100\%) \\ 0 \quad\quad otherwise \end{cases} \tag{35}$$

The decision variable $MTZ7_{tj}$considers maintenance in the forward echelon if the fraction of reworked items rejected after the second QA inspection exceeds an imposed 20% of the total fraction of conforming items reworked by error, stated on equation (36).

$$RMTZ1_{tk} = \begin{cases} 1 \; if \; \theta_{tj} * \left(F3_{tj} * ESR1_{tj} + ESR5_{tj}\right) \geq \; \left(F3_{tj} * \left(1 - \beta_{tj}\right) * V2_{tj} * \gamma\right) \; * XS \\ 0 \quad\quad otherwise \end{cases} \tag{36}$$

The XS represented the 20% the threshold imposed to ensure the machinery stop production of items that needs constant rework, while θ_{tj} is the probability of a reworked item fails the second quality inspection. For re-manufacturing, maintenance is applied when the total fraction of rejected items represents more than 20% of the total fraction of collected raw material, represented by $RMTZ2_{tj}$, as in (37).

$$RMTZ2_{tk} = \begin{cases} 1 \; if \; RV2_{tk} \geq (1 - r\varepsilon_{tk}) * (1 - r\varepsilon0_k) * (MTF1 + MTF2 + MTF3) + \left(V1_{tj} + V\right. \\ \quad 0 \quad\quad otherwise \end{cases} \tag{37}$$

In this case, X2 represented the 20% threshold set to prevent machinery from continuing operation under manufacturing standards, thereby reducing the risk of sudden operational shutdowns.

Decision variable $MTZ8_{tj}$ is related to preventive maintenance when the fraction of non-conforming items exceeds the fraction of conforming items that were repaired, reworked, and rejected by error, as in equation (38).

$$MTZ8_{tj} = \begin{cases} 1 \; if \; IRWK2_{tj} + IREP2_{tj} \geq \; IRWK1_{tj} + IREP1_{tj} \\ 0 \quad\quad otherwise \end{cases} \tag{38}$$

The decision variable $RMTZ4_{tk}$ requires maintenance in the forward echelon when the fraction of conforming and non-conforming items inspected, exceeds 25% of the total final fraction of conforming items to be passed to the next production station, (see equation (39)).

$$RMTZ4_{tk} = \begin{cases} 1 \; if \; RISCR1_{tk} + RISCR2_{tk} \geq RFCI_{tk} * X3 \\ 0 \quad\quad otherwise \end{cases} \tag{39}$$

Where X3 represents the 25% of conforming items threshold used to keep the quality benchmark. Additionally, if the forward echelon machinery exceeds its optimal operation status, maintenance should be apply immediately as stated in equation (40).

$$MTZ9_{tj} = \begin{cases} 1 \; if \; \varepsilon_{tj} > 1 \; (100\%) \\ 0 \quad otherwise \end{cases} \tag{40}$$

$$MTZ10_{tj} = \begin{cases} 1 \qquad if \; \varepsilon_{tj} * M1_{tj} \geq \left(1 - \varepsilon_{tj} * M1_{tj}\right) * 20\% \\ 0 \qquad\qquad otherwise \end{cases} \tag{41}$$

If the total fraction of non-conforming items exceeds the conforming items production, maintenance is considered necessary as presented in equation (41).

4. DETERIORATION TESTING SCENARIOS

Three scenarios are designed to rigorously evaluate the accuracy and effectiveness of the developed mathematical model.

4.1 Scenario 1 – Constant deterioration over time

In the first scenario, each machine within the SCLMSMS undergoes a consistent level of deterioration during every working shift throughout a seven-day production week. However, the deterioration progressively intensifies with each consecutive shift, as detailed in Table 1.

Table 23 Machinery deterioration percentage level

Machine	1st Shift	2nd Shift	3rd Shift
1 to 5	0.02%	0.03%	0.04%

The deterioration percentage remains consistent in intensity across both production echelons and occurs simultaneously, in each. productive shift.

4.2 Scenario 2 – Incremental deterioration over time

In the second scenario, the SCLMSMS is subjected to a progressively increasing deterioration level, which escalates both daily and per working shift, as illustrated in Figure 4.
If the deterioration level surpasses 1% for instance, in day 6, where it reaches 1.1% (gray line) it may exceed the optimal operational threshold. Since each machine is classified from A to C based on its operational complexity, as shown in Table 2, the impact of deterioration is categorized accordingly as high, medium or low.

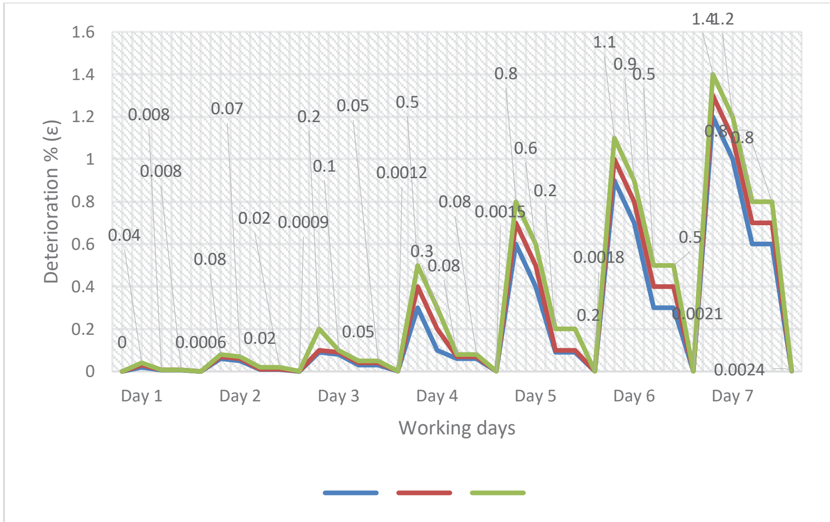

Figure 10 Increasing deterioration factor

Table 24 Machine production complexity level

Machine	Category	Complexity
1 & 5	A	High
2 & 4	B	Medium
3	C	Low

4.3 Scenario 3 – Random deterioration over time

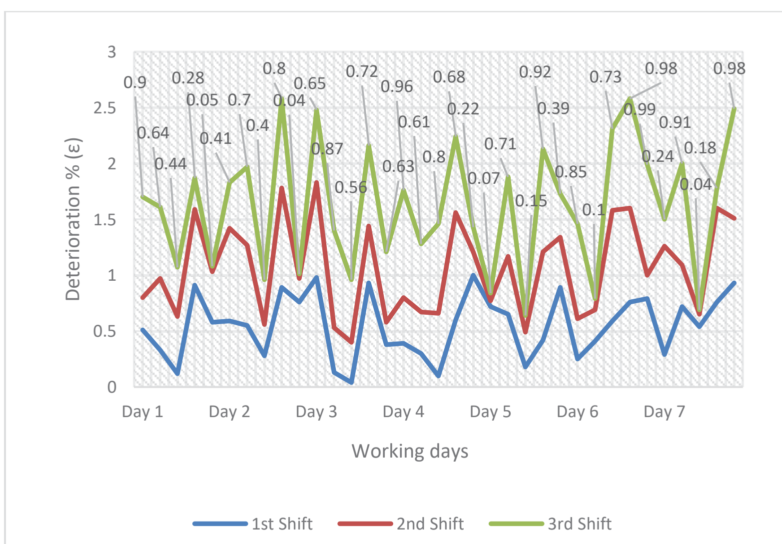

Figure 11 Random deterioration factor

The last case presents a more complex scenario, because the SCLMSMS undergoes a random deterioration factor that varies across different levels throughout the production week. Unlike the previous case, the impact deterioration is not determined by the machine category. Instead, it occurs un-predictable as illustrated in Figure 5.

5. ANALYSIS AND RESULTS

The manufacturing system was evaluated under three distinct deterioration scenarios, analyzing its impact on quality and production outcomes. The same tests were replicated using the mathematical model, revealing the following results.

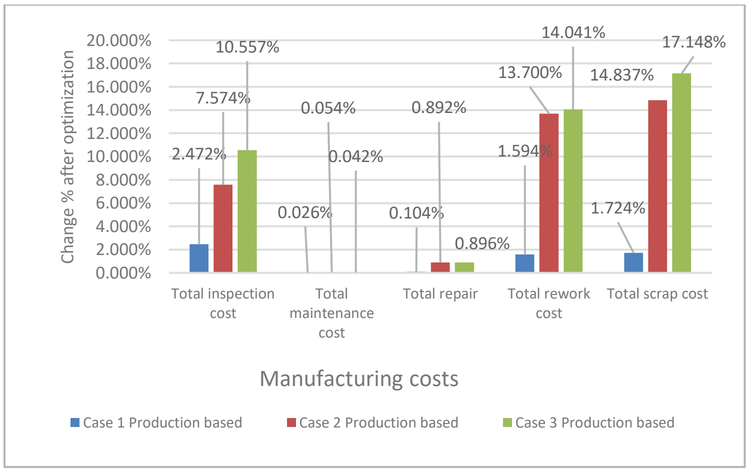

Figure 12 Total manufacturing costs comparison

Across all scenarios, optimizing quality inspection station allocation emerged as a key priority for enhancing early defect detection. However, this improvement comes at cost, with the third scenario incurring the highest inspection expenses (10.5%)

By increasing quality stations, the system effectively segregated non-conforming products for rework or repair, particularly under random deterioration (14%) and incremental deterioration (13.7%). Additionally, quality stations facilitated greater segregation of defective items for re-manufacturing or disposal, even in the first scenario, where 1.7% of products were processed at a higher cost (see Figure 6).

Despite achieving higher defect detection and segregation rates, economic losses persisted under random and incremental deterioration. However, these losses improved by over 65% in both cases, demonstrating the system's ability to mitigate financial impact (see Figure 7)

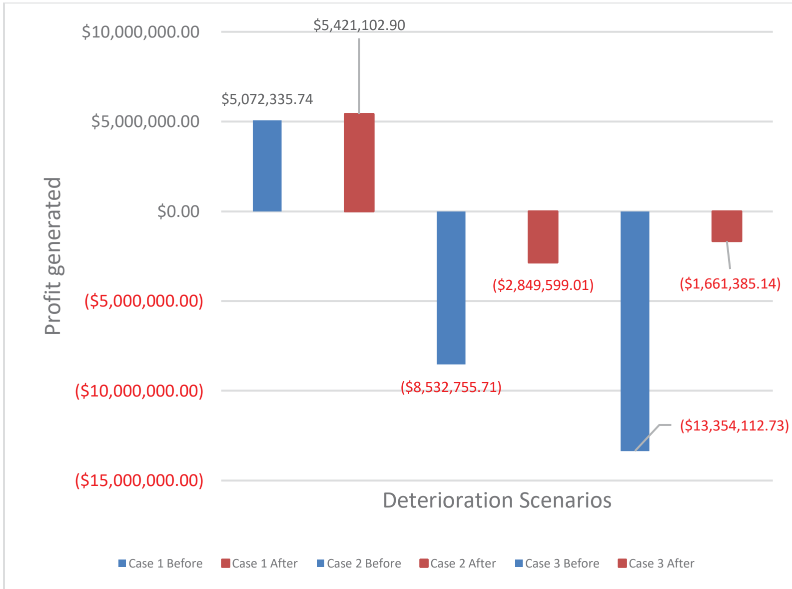

Figure 13 Total profit manufacturing system

Furthermore, the system demonstrated the most significant improvements in penalty cost reduction and profit generation in case one (constant deterioration) and case three (random deterioration). These scenarios outperformed case two, where penalty costs were substantially higher. Notably, case three achieved a 92% reduction in penalty costs compared to case two, highlighting its efficiency as demonstrated in Figure 8.

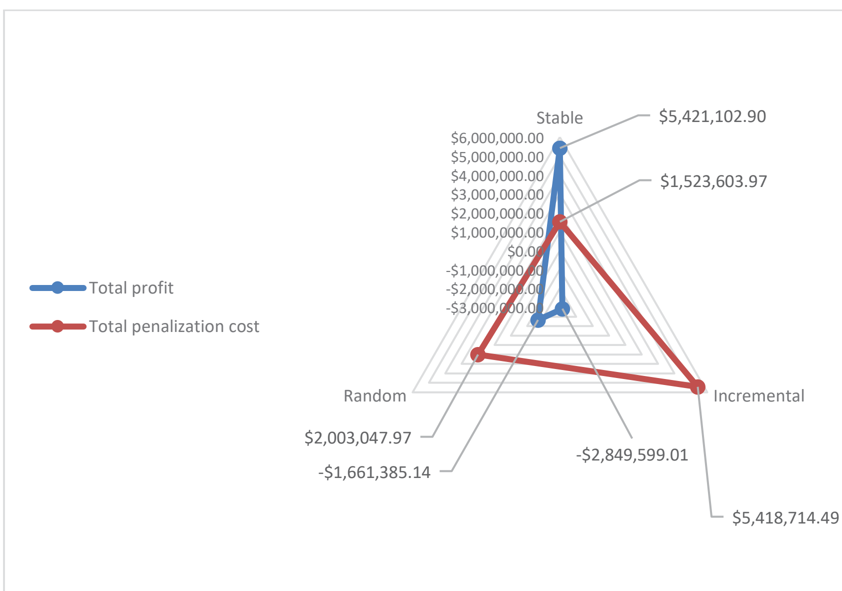

Figure 14 Case benefit comparison

The tests conducted across three scenarios using the proposed mathematical model demonstrated that even under significant deterioration (scenarios 2 and 3), the manufacturing system effectively detected and segregated a higher fraction of non-conforming products. This was largely due to the strategic placement of quality inspection stations. As shown in Figure 5, these improvements led to a 14% to 17% reduction in scrap costs relative to total production, significantly lowering penalty costs for quality non-conformity (Figure 7) while boosting profitability. This analysis highlights the model's effectiveness, particularly in challenging conditions, outperforming cases with constant deterioration (scenario 1), where the impact on profitability was less pronounced.

CONCLUSION

This study demonstrates the effectiveness of the proposed mathematical model as an innovative solution for optimizing quality station assignments and maintenance activities SCLMSMS. By prioritizing quality control over preventive maintenance, the model enables manufacturing systems to detect and segregate non-conforming items more efficiently, facilitating their reprocessing through repair, rework, or re-manufacturing. This approach minimizes waste, maximizes resource utilization, and enhances overall sustainability.

Moreover, the findings reveal that this strategy not only improves product quality but also reduces non-conformance costs, ultimately boosting profitability. By optimizing the allocation of quality and maintenance resources, the system becomes more resilient to operational challenges and better suited for complex production environments.

Future research should focus on refining and expanding optimization tools to further enhance system robustness. Strengthening these strategies will improve long-term maintenance planning, reduce deterioration levels over extended production cycles, and drive more sustainable, cost-efficient operations. These advancements can be applied to even more complex industrial systems, ensuring high performance in demanding production conditions while maintaining superior quality and profitability.

REFERENCES

Bouhchouch, A., Frein, Y., Dallery, Y., 1993. Analysis of closed-loop manufacturing system with finite buffers. Applied Stochastic Models and Data Analysis 9, 111–125.

Bouslah, B., Gharbi, A., Pellerin, R., 2018. Joint production, quality and maintenance control of a two-machine line subject to operation-dependent and quality-dependent failures. International Journal of Production Economics 195, 210–226.

Hurst, E.G., 1973. Imperfect Inspection in a Multistage Production Process. Management Science 20, 378–384.

Mandroli, S.S., Shrivastava, A.K., Ding, Y., 2006. A survey of inspection strategy and sensor distribution studies in discrete-part manufacturing processes. IIE Transactions 38, 309–328.

Melo, M.T., Nickel, S., Saldanha-da-Gama, F., 2009. Facility location and supply chain management – A review. European Journal of Operational Research 196, 401–412.

Mohammadi, M., Dantan, J.-Y., Siadat, A., Tavakkoli-Moghaddam, R., 2018. A bi-objective robust inspection planning model in a multi-stage serial production system. International Journal of Production Research 56, 1432–1457.

Rezaei-Malek, M., Tavakkoli-Moghaddam, R., Siadat, A., Dantan, J.-Y., 2018. A novel model for the integrated planning of part quality inspection and preventive maintenance in a linear-deteriorating serial multi-stage manufacturing system. International Journal of Advanced Manufacturing Technology 96, 3633–3650.

Rivera-Gómez, H., Gharbi, A., Kenné, J.-P., Montaño-Arango, O., Corona-Armenta, J.R., 2020. Joint optimization of production and maintenance strategies considering a dynamic sampling strategy for a deteriorating system. Computers & Industrial Engineering 140, 106273.

Rivera-Gómez, H., Gharbi, A., Kenné, J.-P., Ortiz-Zarco, R., Corona-Armenta, J.R., 2021. Joint production, inspection and maintenance control policies for deteriorating system under quality constraint. Journal of Manufacturing Systems 60, 585–607.

Werner, L.M. (Loren M., 2001. Analysis and design of closed loop manufacturing systems (Thesis). Massachusetts Institute of Technology.

BIBLIOGRAPHY

Ahmad, R., & Kamaruddin, S. (2012). An overview of time-based and condition-based maintenance in industrial application. *Computers & Industrial Engineering, 63(1),* 135–149. https://doi.org/10.1016/j.cie.2012.02.002

Apple_Environmental_Progress_Report_2023.pdf. (2023, October). https://www.apple.com/environment/pdf/Apple_Environmental_Progress_Report_2023.pdf

Azadeh, A., Sangari, M. S., Sangari, E., & Fatehi, S. (2015). A particle swarm algorithm for optimising inspection policies in serial multistage production processes with uncertain inspection costs. *International Journal of Computer Integrated Manufacturing*, *28*(7), 766–780. https://doi.org/10.1080/0951192X.2014.900873

Bahria, N., Chelbi, A., Dridi, I. H., & Bouchriha, H. (2018). Maintenance and quality control integrated strategy for manufacturing systems. *European J. of Industrial Engineering*, *12*(3), 307. https://doi.org/10.1504/ejie.2018.092006

Bai, D. S., & Yun, H. J. (1996). Optimal allocation of inspection effort in a serial multi-stage production system. *Computers & Industrial Engineering*, *30*(3), 387–396. https://doi.org/10.1016/0360-8352(96)00008-3

Ballou, D. P., & Pazer, H. L. (1982). The Impact of Inspector Fallibility on the Inspection Policy in Serial Production Systems. *Management Science*, *28*(4), 387–399. https://doi.org/10.1287/mnsc.28.4.387

Ballou, D. P., & Pazer, H. L. (1985). Process improvement versus enhanced inspection in optimized systems. *International Journal of Production Research*, *23*(6), 1233–1245. https://doi.org/10.1080/00207548508904778

Barad, M. (1990). A break-even quality level approach to location of inspection stations in a multi-stage production process. *International Journal of Production Research*, *28*(1), 29–45. https://doi.org/10.1080/00207549008942682

Basri, E. I., Abdul Razak, I. H., Ab-Samat, H., & Kamaruddin, S. (2017). Preventive maintenance (PM) planning: A review. *Journal of Quality in Maintenance Engineering*, *23*(2), 114–143. https://doi.org/10.1108/JQME-04-2016-0014

Bhakthavatchalam, S., Diallo, C., Venkatadri, U., & Khatab, A. (2015). Quality, Reliability, Maintenance Issues in Closed-Loop Supply Chains: A Review. *IFAC-PapersOnLine*, *48*(3), 460–465. https://doi.org/10.1016/j.ifacol.2015.06.124

Bouhchouch, A., Frein, Y., & Dallery, Y. (1993). Analysis of a closed-loop manufacturing system with finite buffers. *Applied Stochastic Models and Data Analysis*, *9*(2), 111–125. https://doi.org/10.1002/asm.3150090205

Bouslah, B., Gharbi, A., & Pellerin, R. (2018). Joint production, quality and maintenance control of a two-machine line subject to operation-dependent and quality-dependent failures. *International Journal of Production Economics*, *195*, 210–226. https://doi.org/10.1016/j.ijpe.2017.10.016

Cao, Y., Wang, P., Xv, W., & Dong, W. (2025). Joint optimization of quality-based multi-level maintenance and buffer stock within multi-specification and small-batch production. *Frontiers of Engineering Management*. https://doi.org/10.1007/s42524-025-4180-5

Cárdenas-Barrón, L. E. (2009). Economic production quantity with rework process at a single-stage manufacturing system with planned backorders. *Computers & Industrial Engineering*, *57*(3), 1105–1113. https://doi.org/10.1016/j.cie.2009.04.020

Cheng, G., & Li, L. (2020). Joint optimization of production, quality control and maintenance for serial-parallel multistage production systems. *Reliability Engineering & System Safety*, *204*, 107146. https://doi.org/10.1016/j.ress.2020.107146

Circular Economy Action Plan- for a cleaner and more competitive Europe. (2020). https://rcbc.ca/wp-content/uploads/2022/08/EU-European-Union-Circular-Economy-Action-Plan-2020.pdf

Curry, G. L., & Feldman, R. M. (2008). *Manufacturing Systems Modeling and Analysis*. Springer Science & Business Media.

Diallo, C., Venkatadri, U., Khatab, A., & Bhakthavatchalam, S. (2017). State of the art review of quality, reliability and maintenance issues in closed-loop supply chains with remanufacturing. *International Journal of Production Research*, *55*(5), Article 5. https://doi.org/10.1080/00207543.2016.1200152

Ding, P., Wang, Y., Wang, G., Li, X., & Lu, S. (2024). Integrated production and maintenance planning for closed-loop manufacturing assembly system with deteriorating machines. *2024 39th Youth Academic Annual Conference of Chinese Association of Automation (YAC)*, 1164–1169. https://doi.org/10.1109/yac63405.2024.10598534

Eppen, G. D., & Hurst, E. G. (1974). Optimal Location of Inspection Stations in a Multistage Production Process. *Management Science*, *20*(8), 1194–1200. https://doi.org/10.1287/mnsc.20.8.1194

Feng, Q., & Kapur, K. C. (2006). Economic Design of Specifications for 100% Inspection with Imperfect Measurement Systems. *Quality Technology & Quantitative Management*, *3*(2), 127–144. https://doi.org/10.1080/16843703.2006.11673105

Foster, J. W., Malave, C. O., & Villalobos, J. R. (1990). Flexible Inspection within an Aggregated Information Environment. *Computers & Industrial Engineering*, *19*(1), 224–228. https://doi.org/10.1016/0360-8352(90)90110-8

Galar, D., & Kumar, U. (2017). Chapter 6—Prognosis. In D. Galar & U. Kumar (Eds.), *eMaintenance* (pp. 311–370). Academic Press. https://doi.org/10.1016/B978-0-12-811153-6.00006-3

Gits, C. W. (1992). Design of maintenance concepts. *International Journal of Production Economics*, *24*(3), 217–226. https://doi.org/10.1016/0925-5273(92)90133-R

Govindan, K., & Soleimani, H. (2017). A review of reverse logistics and closed-loop supply chains: A Journal of Cleaner Production focus. *Journal of Cleaner Production*, *142*, 371–384. https://doi.org/10.1016/j.jclepro.2016.03.126

Hadian, S. M., Farughi, H., & Rasay, H. (2021). Joint planning of maintenance, buffer stock and quality control for unreliable, imperfect manufacturing systems. *Computers & Industrial Engineering*, *157*, 107304. https://doi.org/10.1016/j.cie.2021.107304

Hajej, Z., Rezg, N., & Gharbi, A. (2021). Joint production preventive maintenance and dynamic inspection for a degrading manufacturing system. *International Journal of Advanced Manufacturing Technology*, *112*(1–2), Article 1–2. https://doi.org/10.1007/s00170-020-06325-3

History of Quality—Quality Management History | ASQ. (2023, November 18). https://asq.org/quality-resources/history-of-quality

Horner, R. M. W., El-Haram, M. A., & Munns, A. K. (1997). Building maintenance strategy: A new management approach. *Journal of Quality in Maintenance Engineering*, *3*(4), 273–280. https://doi.org/10.1108/13552519710176881

Hurst, E. G. (1973). Imperfect Inspection in a Multistage Production Process. *Management Science*, *20*(3), 378–384. JSTOR. http://www.jstor.org/stable/2629729

Jacobs, F. R., Berry, W. L., Whybark, D. C., & Vollmann, T. E. (2011). *Manufacturing Planning and Control for Supply Chain Management*. McGraw Hill Professional.

Jamal, A. M. M., Sarker, B. R., & Mondal, S. (2004). Optimal manufacturing batch size with rework process at a single-stage production system. *Computers & Industrial Engineering*, *47*(1), 77–89. https://doi.org/10.1016/j.cie.2004.03.001

Jena, M. C., Mishra, S. K., & Moharana, H. S. (2024). Environmental Impact of Conventional Manufacturing, Green Manufacturing Initiatives to Mitigate the Impact and Enhancing Sustainability. *Environmental Claims Journal*, *36*(4), 485–504. https://doi.org/10.1080/10406026.2024.2437478

Kang, K., Ebeling, K. A., & La, S. (1990). The optimal location of inspection stations using a rule-based methodology. *Computers & Industrial Engineering*, *19*(1), 272–275. https://doi.org/10.1016/0360-8352(90)90120-B

Kazaz, B., & Sloan, T. W. (2013). The impact of process deterioration on production and maintenance policies. *European Journal of Operational Research*, *227*(1), 88–100. https://doi.org/10.1016/j.ejor.2012.11.052

Kopp, R. (2012, September 24). Hyojunka—The standard procedure in Japan. *Japan Intercultural Consulting*. https://japanintercultural.com/free-resources/articles/hyojunka-the-standard-procedure-in-japan/

Lv, X., Shi, L., He, Y., He, Z., & Lin, D. K. J. (2024). Joint optimization of production, maintenance, and quality control considering the product quality variance of a degraded system. *Frontiers of Engineering Management*, *11*(3), 413–429. https://doi.org/10.1007/s42524-024-3103-1

Mandroli, S. S., Shrivastava, A. K., & Ding, Y. (2006). A survey of inspection strategy and sensor distribution studies in discrete-part manufacturing processes. *IIE Transactions*, *38*(4), Article 4. https://doi.org/10.1080/07408170500327352

Mohammadi, M., Dantan, J.-Y., Siadat, A., & Tavakkoli-Moghaddam, R. (2018). A bi-objective robust inspection planning model in a multi-stage serial production system. *International Journal of Production Research*, *56*(4), Article 4. https://doi.org/10.1080/00207543.2017.1363425

Mohammadi, M., Siadat, A., Dantan, J.-Y., & Tavakkoli-Moghaddam, R. (2015). Mathematical modelling of a robust inspection process plan: Taguchi and Monte Carlo methods. *International Journal of Production Research*, *53*(7), 2202–2224. https://doi.org/10.1080/00207543.2014.980460

Montgomery, D. C. (2008). *Introduction to Statistical Quality Control*. Wiley.

Mousavi, S. M., Mirdamadi, S., Siadat, A., Dantan, J., & Tavakkoli-Moghaddam, R. (2015). An intuitionistic fuzzy grey model for selection problems with an application to the inspection planning in manufacturing firms. *Engineering Applications of Artificial Intelligence*, *39*, 157–167. https://doi.org/10.1016/j.engappai.2014.12.004

Nahmias, S., & Olsen, T. L. (2015). *Production and Operations Analysis: Seventh Edition*. Waveland Press.

Oppermann, M., Sauer, W., Wohlrabe, H., & Zerna, T. (2003). New quality cost models to optimize inspection strategies. *IEEE Transactions on Electronics Packaging Manufacturing*, *26*(4), Article 4. IEEE Transactions on Electronics Packaging Manufacturing. https://doi.org/10.1109/TEPM.2003.822066

Palmer, D. (2006). *Maintenance planning and scheduling handbook* (2nd ed). McGraw-Hill.

Prashar, G., Vasudev, H., & Bhuddhi, D. (2023). Additive manufacturing: Expanding 3D printing horizon in industry 4.0. *International Journal on Interactive Design and Manufacturing (IJIDeM)*, *17*(5), 2221–2235. https://doi.org/10.1007/s12008-022-00956-4

Quality Assurance (QA) vs. Quality Control (QC): Top 5 Differences. (2023, November 25). https://www.scilife.io/blog/qa-vs-qc-top-differences

Raz, T., & Kaspi, M. (1991). Location and sequencing of imperfect inspection operations in serial multi-stage production systems. *International Journal of Production Research*, *29*(8), 1645–1659. https://doi.org/10.1080/00207549108948037

Rezaei, J., & Davoodi, M. (2008). A deterministic, multi-item inventory model with supplier selection and imperfect quality. *Applied Mathematical Modelling*, *32*(10), Article 10. https://doi.org/10.1016/j.apm.2007.07.009

Rezaei-Malek, M., Siadat, A., Dantan, J.-Y., & Tavakkoli-Moghaddam, R. (2018a). An Approximation Approach for an Integrated Part Quality Inspection and Preventive Maintenance Planning in a Nonlinear Deteriorating Serial Multi-stage Manufacturing System. *16th IFAC Symposium on Information Control Problems in Manufacturing INCOM 2018: Bergamo, Italy, 11-13 June 2018*, *51*(11), 270–275. https://doi.org/10.1016/j.ifacol.2018.08.291

Rezaei-Malek, M., Siadat, A., Dantan, J.-Y., & Tavakkoli-Moghaddam, R. (2018b). An Approximation Approach for an Integrated Part Quality Inspection and Preventive Maintenance Planning in a Nonlinear Deteriorating Serial Multi-stage Manufacturing System. *IFAC-PapersOnLine*, *51*(11), Article 11. https://doi.org/10.1016/j.ifacol.2018.08.291

Rezaei-Malek, M., Siadat, A., Dantan, J.-Y., & Tavakkoli-Moghaddam, R. (2019). A trade-off between productivity and cost for the integrated part quality inspection and preventive maintenance planning under uncertainty. *International Journal of Production Research*, *57*(19), 5951–5973. https://doi.org/10.1080/00207543.2018.1556411

Rezaei-Malek, M., Tavakkoli-Moghaddam, R., Siadat, A., & Dantan, J.-Y. (2018). A novel model for the integrated planning of part quality inspection and preventive maintenance in a linear-deteriorating serial multi-stage manufacturing system. *International Journal of Advanced Manufacturing Technology*, *96*(9–12), 3633–3650. https://doi.org/10.1007/s00170-018-1751-1

Rivera-Gómez, H., Gharbi, A., Kenné, J.-P., Montaño-Arango, O., & Corona-Armenta, J. R. (2020). Joint optimization of production and maintenance strategies considering a dynamic sampling strategy for a deteriorating system. *Computers & Industrial Engineering*, *140*, 106273. https://doi.org/10.1016/j.cie.2020.106273

Rivera-Gómez, H., Gharbi, A., Kenné, J.-P., Ortiz-Zarco, R., & Corona-Armenta, J. R. (2021). Joint production, inspection and maintenance control policies for deteriorating system under quality constraint. *Journal of Manufacturing Systems*, *60*, 585–607. https://doi.org/10.1016/j.jmsy.2021.07.018

Sartal, A., Bellas, R., Mejías, A. M., & García-Collado, A. (2020). The sustainable manufacturing concept, evolution and opportunities within Industry 4.0: A literature review. *Advances in Mechanical Engineering*, *12*. https://api.semanticscholar.org/CorpusID:219167865

Shi, J., & Zhou, S. (2009). Quality control and improvement for multistage systems: A survey. *IIE Transactions*, *41*(9), 744–753. https://doi.org/10.1080/07408170902966344

Shiau, Y.-R. (2002). Inspection resource assignment in a multistage manufacturing system with an inspection error model. *International Journal of Production Research*, *40*(8), 1787–1806. https://doi.org/10.1080/00207540110119982

Shiau, Y.-R. (2003a). Inspection Allocation Planning for a Multiple Quality Characteristic Advanced Manufacturing System. *The International Journal of Advanced Manufacturing Technology*, *21*(7), 494–500. https://doi.org/10.1007/s001700300058

Shiau, Y.-R. (2003b). Quick decision-making support for inspection allocation planning with rapidly changing customer requirements. *The International Journal of Advanced Manufacturing Technology*, *22*(9–10), Article 9–10. https://doi.org/10.1007/s00170-002-1521-x

Shiau, Y.-R., Lin, M.-H., & Chuang, W.-C. (2007). Concurrent process/inspection planning for a customized manufacturing system based on genetic algorithm. *The International Journal of Advanced Manufacturing Technology*, *33*(7–8), 746–755. https://doi.org/10.1007/s00170-006-0486-6

Thomas, D. (2023, December). *U.S. manufacturing economy*. National Institute of Standards and Technology of U.S. https://www.nist.gov/el/applied-economics-office/manufacturing/manufacturing-economy/total-us-manufacturing

Thompson, R. (2007). *Manufacturing Processes for Design Professionals*. Thames & Hudson.

Torres, Y., Nadeau, S., & Landau, K. (2021). Classification and Quantification of Human Error in Manufacturing: A Case Study in Complex Manual Assembly. *Applied Sciences*, *11*(2), 749. https://doi.org/10.3390/app11020749

Vaghefi, A., & Sarhangian, V. (2009). Contribution of simulation to the optimization of inspection plans for multi-stage manufacturing systems. *Computers & Industrial Engineering*, *57*(4), 1226–1234. https://doi.org/10.1016/j.cie.2009.06.001

Villalobos, J. R., & Foster, J. W. (1991). Some results from a model of dynamic inspection allocation. *Computers & Industrial Engineering*, *21*(1), 355–358. https://doi.org/10.1016/0360-8352(91)90116-N

Villalobos, J. R., Foster, J. W., & Disney, R. L. (1993). Flexible inspection systems for serial multi-stage production systems. *IIE Transactions*, *25*(3), 16–26. https://doi.org/10.1080/07408179308964287

Volsem, S. V. (2010). Joint optimisation of all inspection parameters for multistage processes: Evolutionary algorithm and simulation. *International Journal of Innovative Computing and Applications*, *2*(3), 170. https://doi.org/10.1504/IJICA.2010.033648

Wang, H. (2002). A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research*, *139*(3), 469–489. https://doi.org/10.1016/S0377-2217(01)00197-7

Werner, L. M. (Loren M. (2001). *Analysis and design of closed loop manufacturing systems* [Thesis, Massachusetts Institute of Technology]. https://dspace.mit.edu/handle/1721.1/82688

Why usage-based preventative maintenance is better | Geotab. (2023, December 6). https://www.geotab.com/blog/usage-based-preventative-maintenance/

Ye, Z., Cai, Z., Yang, H., Si, S., & Zhou, F. (2023). Joint optimization of maintenance and quality inspection for manufacturing networks based on deep reinforcement learning. *Reliability Engineering & System Safety*, *236*, 109290. https://doi.org/10.1016/j.ress.2023.109290

Yum†, B. J., & McDOWELLJ‡, E. D. (1987). Optimal inspection policies in a serial production system including scrap rework and repair: An MILP approach. *International Journal of Production Research*, *25*(10), 1451–1464. https://doi.org/10.1080/00207548708919925