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CADENASSAGE/DÉCADENASSAGE DANS LA GESTION DE LA CAPACITÉ DE  
PRODUCTION

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# **OPTIMISATION DES POLITIQUES DE SURVEILLANCE PAR L'INTÉGRATION DU CADENASSAGE/DÉCADENASSAGE DANS LA GESTION DE LA CAPACITÉ DE PRODUCTION**

Behnam EMAMI-MEHRGANI

## **RÉSUMÉ**

Ce travail vise à développer une stratégie optimale de gestion de la production dans le but de minimiser les coûts de production tout en tenant compte de l'aspect sécurité par le biais du cadencage/décadencage (C/D) durant les étapes de maintenance corrective et préventive. Dans la littérature, des études qui traitent de l'intégration du C/D et de la gestion de production sont presque inexistantes. De plus, dans le contexte actuel de l'industrie, beaucoup de dirigeants et travailleurs pensent, à tort, que la planification et la réalisation des différentes procédures de C/D prend beaucoup de temps. Par conséquent, ce temps de production inactif est perçu comme diminuant la performance de l'entreprise vis-à-vis la cadence de fabrication planifiée. Pour pallier à ce problème le concept novateur de : "mean time to lockout/tagout (MTTTL)" a été développé. Le concept novateur du MTTTL consiste à prendre en considération le temps de C/D en fonction des niveaux d'inventaires pour un système manufacturier afin de trouver une politique optimale. Ce concept permet d'une part de diminuer les coûts totaux de production, comprenant les coûts d'inventaire, de pénurie et de maintenance sur un horizon infini et d'autre part d'augmenter le niveau de sécurité des travailleurs. Cette thèse est présentée en trois (3) phases :

En premier lieu, le projet vise à intégrer le concept novateur du MTTTL dans la gestion de la capacité de production sous forme de deux travaux. Tout d'abord, le MTTTL a été assimilé pour un système en redondance passive (deux machines, non-identiques produisant un seul type de pièce). Une modélisation a été faite par la chaîne de Markov homogène, et une résolution numérique à travers des équations différentielles d'Hamilton-Jacoby-Bellman (HJB) a conduit à la solution du système étudié. Nos résultats ont été validés par le biais d'une analyse de sensibilité.

En deuxième lieu, l'influence de contrôle du MTTTL a été présentée pour une ligne de production constituée de trois machines (deux machines sous forme redondance passive et une troisième machine en série avec les précédentes) produisant un type de pièce. Une modélisation a été faite par la chaîne de Markov homogène. Les outils utilisés à l'étape précédente ont servi à résoudre ce cas. Le modèle de simulation a été également utilisé afin de déterminer les coûts afférents à chaque combinaison obtenue par le biais du plan d'expérience. Partant de ces coûts, une analyse de régression a été faite pour trouver la nouvelle commande optimale du problème d'optimisation considéré. Et pour illustrer l'utilité de nos résultats, une analyse de sensibilité a été effectuée.

## VIII

En troisième lieu, ce travail s'est concentré sur la modélisation du MTTLT et l'erreur humaine pour un système manufacturier flexible (FMS). La modélisation de l'erreur humaine a été effectuée par une chaîne de Markov non-homogène, pour un système en redondance passive produisant une seule pièce. La solution a été exposée par le biais des équations d'HJB et nos résultats ont été confirmés par une analyse de sensibilité.

**Mots clés:** Systèmes manufacturiers, Mean time to lockout/tagout (MTTLT), Gestion de la capacité, Redondance passive, Erreur humaine, Maintenance corrective, Maintenance préventive.



# **OPTIMIZATION OF MONITORING POLICY BY INTEGRATION OF LOCKOUT/TAGOUT IN PRODUCTION CAPACITY CONTROL**

Behnam EMAMI-MEHRGANI

## **ABSTRACT**

This work aims to develop an optimal strategy for production control in order to minimize production costs while taking into account occupational safety aspects through lockout / tagout during corrective and preventive maintenance activities. In the scientific literature, studies that address the integration of the lockout/tagout and production control are almost nonexistent. In addition, currently in industry, many managers and workers mistakenly believe that planning and carrying out the various lockout / tagout procedures takes a long time. Therefore, this idle production time is perceived as reducing the company's performance towards the planned production rate. To overcome this problem the innovative concept of "mean time to logout / tagout (MTTTL)" has been developed. The innovative concept of MTTTL is to be considered the lockout / tagout time according to the inventories levels for a manufacturing system to find an optimal policy. This concept allows on the one hand to reduce total production costs including inventory, shortage and maintenance costs over an infinite horizon and on the other hand to increase the safety worker level. This thesis is presented in three stages:

In the first stage, the project aims to integrate the innovative concept of MTTTL into production capacity control. The MTTTL has been applied to a passive redundancy system (two machines, non-identical producing one type of part). A model has been presented using a homogeneous Markov chain, and the numerical solution of the Hamilton-Jacoby-Bellman (HJB) differential equations has provided the solution for our manufacturing system. Our results have been validated through a sensitivity analysis.

In the second stage, MTTTL control has been applied to a production line consisting of three machines (two machines as passive redundancy and a third machine in series with the previous ones) producing one type of part. A model has been presented using an homogeneous Markov chain. The same tools used in the previous stage have also been used to solve this case. In this case, the simulation model has also been used to determine the costs associated with each combination obtained through the experimental design. Based on these costs, a regression analysis has been conducted to find the new optimal control of the optimization problem under consideration. To illustrate the usefulness of our results, a sensitivity analysis has been performed.

In the third stage, this work focused on MTTTL and human error modeling for a FMS. Human error has been presented using a non-homogeneous Markov chain for a manufacturing system with passive redundancy producing one type of part. The solution has

X

been obtained through the HJB equations and results have been confirmed by a sensitivity analysis.

**Keywords:** Manufacturing systems, Mean time to lockout / tagout (MTTLT), Capacity control, Passive redundancy, Human error, Corrective maintenance, Preventive maintenance.

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## LISTE DES ABRÉVIATIONS, SIGLES ET ACRONYMES

$x(\cdot)$	Niveau d'inventaire
$c^+$	Coût de mise en inventaire par unité de pièce et par unité de temps
$c^-$	Coût de rupture d'inventaire par unité de pièce et par unité de temps
$c^\alpha$	Coût d'intervention sur la machine au mode $\alpha$
$c_{r1}$	Coût de réparation pour la défaillance de type 1
$c_{r2}$	Coût de réparation pour la défaillance de type 2
$c_{\text{tagout}}$	Coût de cadenassage/décadenassage
$g(\cdot)$	Coût instantané
$J(\cdot)$	Coût total actualisé
$v(\cdot)$	Fonction valeur
$\rho$	Taux d'actualisation
$d$	Taux de demande
$u_i(\cdot)$	Taux de production $i$ ( $i=1,2$ )
$u_i^{\max}$	Taux de production maximal ( $i=1,2$ )
$h_x$	Pas de discrétisation suivant l'inventaire du produit
$h_a$	Pas de discrétisation suivant l'âge de la machine
$\xi(\cdot)$	Processus stochastique
$q_{\alpha\beta}$	Taux de transition du mode $\alpha$ au mode $\beta$
$v_{31}^{\min}$	Taux de réparation minimal avec cadenassage/décadenassage pour la défaillance de type 1 (une machine produisant un type de pièce)

$v_{31}^{\max}$	Taux de réparation maximal avec cadénassage/décadenassage pour la défaillance de type 1 (une machine produisant un type de pièce)
$v_{31}(\cdot)$	Taux de réparation avec cadénassage/décadenassage pour la défaillance de type 1 (une machine produisant un type de pièce)
$v_{21}^{\min}$	Taux de réparation minimal avec cadénassage/décadenassage pour la défaillance de type 2 (une machine produisant un type de pièce)
$v_{21}^{\max}$	Taux de réparation maximal avec cadénassage/décadenassage pour la défaillance de type 2 (une machine produisant un type de pièce)
$v_{21}(\cdot)$	Taux de réparation avec cadénassage/décadenassage pour la défaillance de type 2 (une machine produisant un type de pièce)
$\omega_{51}^{\min}$	Taux de réparation minimal avec cadénassage/décadenassage pour la défaillance de type 1 de la machine principale
$\omega_{51}^{\max}$	Taux de réparation maximal avec cadénassage/décadenassage pour la défaillance de type 1 de la machine principale
$\omega_{51}(\cdot)$	Taux de réparation avec cadénassage/décadenassage pour la défaillance de type 1 de la machine principale
$\omega_{41}^{\min}$	Taux de réparation minimal avec cadénassage/décadenassage pour la défaillance de type 2 de la machine principale
$\omega_{41}^{\max}$	Taux de réparation maximal avec cadénassage/décadenassage pour la défaillance de type 2 de la machine principale
$\omega_{41}(\cdot)$	Taux de réparation avec cadénassage/décadenassage pour la défaillance de type 2 de la machine principale
$x_1(\cdot)$	Niveau d'inventaire en-cours
$x_2(\cdot)$	Niveau d'inventaire de produits finis
$c_1^+$	Coût de mise en inventaire d'en-cours par unité de pièce d'en-cours et par unité de temps

$c_2^+$	Coût de mise en inventaire de produits finis par unité de pièce finie et par unité de temps
$c_2^-$	Coût de rupture d'inventaire de produits finis par unité de pièce finie et par unité de temps
$c_{r_1}$	Coût de réparation de la machine principale $M_1$
$c_{r_2}$	Coût de réparation de la machine principale $M_2$
$c_{r_s}$	Coût de réparation de la machine de secours $M_s$
$u_{r_1}$	Taux de réparation avec le cadenassage/décadenassage pour la machine principale $M_1$
$u_{r_2}$	Taux de réparation avec le cadenassage/décadenassage pour la machine principale $M_2$
$u_{r_s}$	Taux de réparation avec le cadenassage/décadenassage pour la machine de secours $M_s$
$u_i(\cdot)$	Taux de production $i(i=1,2,s)$
$u_i^{\max}(\cdot)$	Taux de production maximal $i(i=1,2,s)$
$q_{12}^{1,2}$	Taux de défaillance des machines $M_1$ et $M_2$
$q_{12}^s$	Taux de défaillance de la machine de secours $M_s$
$c_{r_s}$	Coût de réparation de la machine de secours $S$
$u_1$	Taux de production de la machine principale $M$
$u_1^{\max}$	Taux de production maximal de la machine principale $M$
$u_s$	Taux de production de la machine de secours $S$

$u_s^{\max}$	Taux de production maximal de la machine de secours $S$
$w_{42}^{24}$	Taux de maintenance préventive sans erreur humaine
$w_{42}^{24\max}$	Taux de maintenance préventive maximal sans erreur humaine
$w_{42}^{34}$	Taux de maintenance préventive avec erreur humaine
$w_{42}^{34\max}$	Taux de maintenance préventive maximal avec erreur humaine
$q_{14}$	Taux de réparation de la machine de secours $S$
$q_{23}$	Taux de maintenance préventive sans erreur humaine au taux de maintenance préventive avec erreur humaine de la machine de secours $S$
$q_{24}$	Taux de maintenance préventive sans erreur humaine de la machine secours $S$
$q_{34}$	Taux de maintenance préventive sans erreur humaine de la machine secours $S$
$q_{54}$	Taux de réparation de la machine principale $M$
NIOSH	National Institute for Occupational Safety and Health
NOHSC	National Occupational Health and Safety Commission
AFIM	Association Française de Normalisation
IRSST	Institut de Recherche Robert-Sauvé en Santé et Sécurité du Travail
CSST	Commission de la Santé et de la Sécurité au Travail
RSST	Règlement sur la santé et la sécurité du travail
C/D	Cadenassage/décadenassage
CSA	Canadian Standards Association
SST	Santé et Sécurité du Travail
FMS	Flexible Manufacturing System
HJB	Hamilton-Jacobi-Bellman

HPP	Hedging Point Policy
MTTLT	Meant time to lockout/tagout
MTBF	Mean time between failures
MTTR	Mean time to repair
WIP	Work-in-process
DPEDD	Dynamic Programming Equations in Directional Derivative
DD	Directional Derivatives
AQHSST	Association québécoise pour l'hygiène, la santé et la sécurité du travail
GFA	Gesellschaft für Arbeitswissenschaft
ASME	American Society of Mechanical Engineers
FRSQ	Fonds de la Recherche en Santé du Québec
FQRSC	Fonds Québécois de la Recherche sur la Société et la Culture
FQRNT	Fonds de recherche du Québec - Nature et technologies
RRSSTQ	Réseau de Recherche en Santé et en Sécurité du Travail du Québec
ÉREST	Équipe de recherche en sécurité du travail





## INTRODUCTION

Aujourd'hui, l'objectif de l'industriel est de produire le maximum de biens, notamment en minimisant le coût total de fabrication et en respectant les différentes obligations légales auxquelles il est soumis. Cet objectif ne sera atteint qu'avec une bonne gestion de la production, qui prendra en considération toutes les phases de la fabrication, en commençant par la phase de conception jusqu'à la livraison du produit final.

La gestion de la production est efficace et fiable, si nous arrivons à optimiser les diverses ressources de notre système manufacturier. L'environnement des systèmes manufacturiers est stochastique. Parmi ces aspects stochastiques, nous retenons les risques d'accident, les pannes des équipements et la variation de la cadence de fabrication. Afin de maîtriser les phénomènes aléatoires des pannes et des accidents, nous devons trouver une solution, permettant de diminuer leur fréquence ainsi que leur gravité. La nature incertaine de l'environnement manufacturier découle des machines sujettes à des pannes et des réparations aléatoires.

Au cours des dernières années, plusieurs travaux de recherche ont porté sur les problèmes d'optimisation de la production des systèmes manufacturiers, en tenant compte de leur complexité, la concurrence et des enjeux de la mondialisation des marchés, tels que Gharbi et Kenné (2003), Charlot et al. (2006). Malgré tous ces efforts, les fréquences et les gravités des accidents lors des interventions de maintenance restent encore assez élevées au Québec ainsi que dans d'autres pays à travers le monde. Cette situation confirme les résultats publiés par d'autres études au Québec et dans d'autres pays (Pâques 1989, Underwood 1992, NIOSH 1994, Windau 1998, ARIA 2000, Grusenmeyer 2000, NOHSC 2000, Mutawe 2002, AFIM 2004).

Afin d'obtenir une estimation globale de l'impact de la maintenance sur les risques pour les travailleurs au Québec, une étude a été effectuée par l'Institut de Recherche Robert Sauvé en Santé et Sécurité du Travail (IRSST) sur les accidents du travail mortels au Québec entre

1999 et 2003, (R-578 IRSST, 2008). Cette étude a été effectuée à partir des données disponibles à la Commission de la Santé et de la Sécurité du Travail (CSST) pour les années concernées. Entre 1999 et 2003, selon les rapports annuels de la CSST, les accidents du travail ont entraîné 1275 décès. Ces rapports précisent que 163 d'entre eux (soit 13%) ont été produits pendant une activité de maintenance : améliorer la machine, diagnostiquer, essayer, dépanner, localiser une panne, inspecter, modifier, reconstruire, réparer et surveiller le fonctionnement. À la lumière de ces résultats, il est envisageable de penser que les activités de maintenance représentent au Québec une proportion significative des accidents mortels.

Ces aléas, lorsqu'ils se produisent peuvent mettre en danger les différents objectifs d'un système manufacturier. Pour remédier à ce problème, certaines entreprises ont mis en place une méthode de prévention de ces risques nommée cadenassage/décadenassage (C/D). Malheureusement, encore jusqu'à aujourd'hui, certaines entreprises négligent les procédures de C/D lors des différentes tâches de maintenance, ce qui explique les statistiques encore élevées dans ce secteur d'activité. Aujourd'hui, la méthode de C/D est devenue une obligation légale au Québec selon l'article 185 du RSST.

## CHAPITRE 1

### REVUE CRITIQUE DE LA LITTÉRATURE

#### 1.1 Introduction

Bien que la littérature soit riche en travaux sur les systèmes manufacturiers flexibles (FMS), peu d'entre eux traitent l'intégration de C/D en gestion de la production dans un environnement stochastique. Aujourd'hui, le but dans le monde de l'industrie est l'optimisation des systèmes manufacturiers, en répondant aux exigences des clients. Cette optimisation n'est réalisable qu'en diminuant le coût total de production, tout en augmentant la sécurité des divers travailleurs dans un environnement incertain.

Autrement dit, l'optimisation d'un FMS porte sur trois éléments fondamentaux : des facteurs financiers (contient le coût de production ainsi que le coût de la maintenance), des facteurs techniques (la disponibilité, la maintenabilité et la fiabilité des équipements), des facteurs de santé et de sécurité des travailleurs.

Dans la littérature, nous constatons que certains auteurs accordent plus d'importance aux aspects techniques et économiques, tandis que d'autres mettent l'accent sur la sécurité des travailleurs et des équipements dans un système manufacturier bien déterminé. La première catégorie d'auteurs se focalise sur les paramètres qui permettent de contrôler et de déterminer le taux de la production sur un horizon infini. La deuxième catégorie d'études se concentre sur la santé et la sécurité des travailleurs en vue d'une prévention par l'élimination ou le contrôle des aléas susceptibles de produire des accidents ou des maladies professionnelles. Pour cette raison, nous exposons une vision globale de l'optimisation d'un FMS, tout en augmentant la sécurité des travailleurs. Nous incorporons la politique de la maintenance préventive, corrective avec le C/D dans la gestion de la capacité de production, ce qui mène à une recommandation de commande optimale dans un environnement stochastique.

## 1.2 Commande optimale et l'environnement stochastique

Diverses méthodes ont été utilisées pour la recherche de solutions aux problèmes de commande optimale reliés à la gestion de la production pour un FMS. Dans ces différentes méthodes, les chercheurs ont utilisé divers dispositifs comme l'algorithme du Kushner (Kushner et Dupuis, 1992), l'intelligence artificielle en se basant sur les algorithmes génétiques (Basnet et Mize, 1986), l'heuristique (Thesen, 1999) et la simulation (Kenné et Gharbi, 2004). Les résultats élaborés jusqu'ici comportent certaines lacunes: dans ces travaux, certains paramètres comme les pannes, les interventions de maintenance, les accidents et la demande n'ont pas été pris en considérations de façon aléatoires. La majorité de ces méthodes ont été appropriées pour des systèmes purement déterministes. Pour cette raison la théorie de la commande optimale dans un environnement stochastique a été mise au point. Plusieurs chercheurs ont contribué à résoudre ce problème. Rishel (1975) a développé les conditions d'optimum (nécessaires et suffisantes) pour obtenir la solution optimale en utilisant la programmation dynamique. Older et Suri (1980) ont modélisé la commande stochastique pour la planification de la production d'un FMS sujet aux pannes aléatoires suivant un processus markovien homogène. Ils ont obtenu une équation de programmation dynamique de la politique de commande optimale, mais sans obtenir la solution à cause de la complexité du problème. Kimemia et Gershwin (1983) ont modélisé le système stochastique par des processus Markovien homogènes avec un taux de transition constant pour déterminer la politique de production dont le taux de production permet de minimiser le coût d'inventaire et de pénurie. Gershwin (1983) a utilisé l'approximation du seuil critique et de la fonction coût pour l'ordonnancement d'un FMS. La difficulté réside en la résolution des équations d'HJB pour trouver la valeur du seuil critique. Akella et Kumar (1986) ont montré que pour un système Markovien homogène (taux de transitions constants), la politique permettant de maintenir un inventaire de sécurité non négatif pendant les périodes d'excès de capacité pour prévenir les futures insuffisances de capacité est une politique de commande optimale. Cette politique est appelée le seuil critique ou *Hedging Point Policy* (HPP). Dans le cas d'un système manufacturier complexe, la fonction qui réalise le coût optimal appelé fonction valeur doit satisfaire un ensemble d'équations différentielles appelées équation d'HJB, (Boukas et Kenné, 1997). En sachant qu'il n'existe pas de solution analytique connue pour

les équations d'HJB, nous les résolvons par une approche numérique basée sur l'approche numérique de Kushner (Kushner et Dupuis, 1992). Kenné et Gharbi (1999), ont traité le problème de contrôle de flux optimal avec une distribution des pannes de la machine en fonction de l'âge.

En se basant sur la revue de la littérature, nous constatons qu'il existe plusieurs types de méthodes pour l'optimisation de la commande dans un environnement stochastique telles les méthodes : markoviennes, semi-markoviennes, non-markoviennes, les heuristiques, le principe du maximum de Pontryagin et la programmation linéaire. La méthode markovienne peut être regroupée en deux (2) volets comme suit :

- Taux de transition d'un état à un autre de la machine sont constants: Modélisation par la chaîne de Markov homogène;
- Taux de transition d'un état de la machine à un autre ne sont pas constants, mais contrôlables: Modélisation par la chaîne de Markov non-homogène.

Lorsque le système est modélisé par une équation différentielle son avenir est uniquement déterminé par sa situation présente, d'où le nom de dynamique déterministe. Étant donné que l'environnement du système manufacturier est aléatoire ou stochastique, nous avons besoin d'hypothèses pour le traduire. Dans cet environnement, plusieurs évolutions sont possibles à partir de la situation présente, chacune d'elles ayant une certaine probabilité de se réaliser, d'où la nécessité d'une modélisation par la chaîne de Markov (homogène ou non-homogène). De plus la chaîne de Markov peut effectivement bien refléter le comportement dynamique d'un système manufacturier. Avant de définir les processus de chaîne de Markov homogènes et non-homogènes, pour mieux concevoir la suite de ce manuscrit, nous présentons la structure des systèmes manufacturiers dans la Figure 1.1. Nous commençons par la suite la classification des FMS ainsi que les définitions des systèmes en modes redondances.

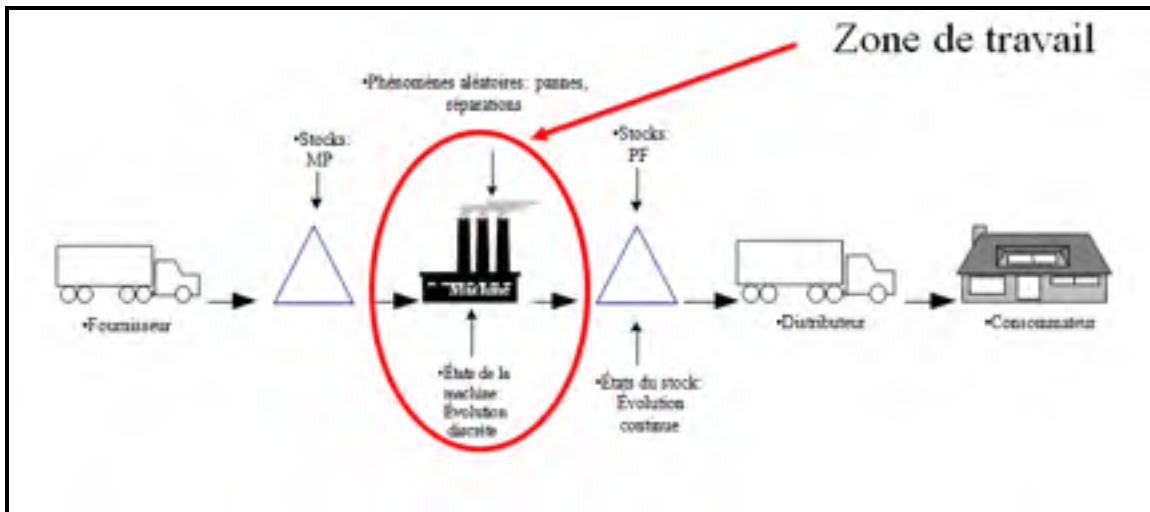


Figure 1.1 Structure globale des systèmes manufacturiers

\*MP : Matière première

\*PF : Produit fini

### 1.3 Système manufacturier et système manufacturier flexible (FMS)

Avant de définir le système manufacturier flexible (FMS), nous commençons par la définition d'un système manufacturier. Un système se définit comme un ensemble d'éléments ou d'entités qui interagissent entre eux selon un certain nombre de principes ou de règles. Si ces entités sont utilisées dans le but de la fabrication d'un bien, nous construisons alors un système manufacturier. Le système manufacturier est un ensemble de machines ou d'équipements, d'éléments de transport, d'unités d'emmagasiner, de personnes, d'ordinateurs ou tout autre élément mis ensemble pour la fabrication (Gershwin, 2002). Un système manufacturier évolue au cours du temps selon les différents facteurs de fabrication tels que les pannes, le nombre d'interventions (maintenance corrective, préventive), l'âge des équipements et les variations de la demande. Tous ces facteurs ont un impact direct sur le rendement d'un système manufacturier. Par conséquent, la variation de ces facteurs dans un système manufacturier exige une flexibilité d'où la nécessité d'un FMS.

### **1.3.1 Classification des systèmes manufacturiers flexibles (FMS)**

Un système manufacturier flexible (FMS) peut-être classifié par rapport à deux (2) critères :

- la circulation du flux (continus, discrets, hybrides);
- la dynamique d'un FMS (stochastique, déterministe, hybride).

#### **1.3.1.1 Système manufacturiers flexibles (FMS) et la circulation du flux**

Il est possible de classifier les FMS en trois (3) catégories selon la circulation de leur flux (le flux est un transfert ou un échange d'une entité à une autre. Cela peut être un échange de données, d'informations, d'équipements, de matières premières, etc.) :

##### **1.3.1.1.1 Système manufacturier flexible continu**

Lorsque le flux des matières ne s'arrête pas entre les postes successifs dans un environnement manufacturier flexible, nous pouvons affirmer que le système manufacturier flexible est continu.

##### **1.3.1.1.2 Système manufacturier flexible discret**

Nous parlons de système discret lorsqu'il y a des points de rupture dans le flux sous forme d'en-cours et d'inventaires intermédiaires (Bironneau, 2000). Le mode discret d'un système peut également se justifier par une suite d'opérations indépendantes réalisées sur des moyens indépendants, par exemple deux (2) machines distinctes (Gharbi et Kenné,2003).

##### **1.3.1.1.3 Système manufacturier flexible hybride**

Les systèmes discrets et continus pris individuellement n'étant pas toujours en mesure de satisfaire les exigences d'une entreprise qui se veut compétitive sur le marché, on peut procéder à leur intégration dans un même système, donnant ainsi naissance aux systèmes de fabrication hybrides (Bhattacharya et Coleman ,1994). Les systèmes manufacturiers flexibles

hybrides en général transforment une matière brute continue à travers des postes qui fonctionnent de manière proportionnellement discrète.

### **1.3.1.2 Dynamique d'un système manufacturier flexible (FMS)**

Il est possible de classer les FMS en trois (3) catégories selon leur dynamique :

#### **1.3.1.2.1 Dynamique stochastique**

Un système manufacturier a une dynamique stochastique si au moins une de ses sorties ou un de ses paramètres est aléatoire (Sader et Sorensen, 2003).

#### **1.3.1.2.2 Dynamique déterministe**

Un système manufacturier a une dynamique déterministe, si ses paramètres peuvent avoir une dynamique évoluant de façon déterministe. (par exemple, la demande, lorsqu'elle est à taux constant et connue)

#### **1.3.1.2.3 Dynamique hybride**

Lorsqu'il n'y a pas d'interaction entre les paramètres d'un système manufacturier flexible, donc le système a une dynamique hybride (Cassandras et al., 1999).

## **1.4 Définition de la redondance**

Il est inévitable qu'avec le temps les choses se détériorent, et cela, même quand elles sont au repos. C'est la raison pour laquelle les entreprises organisent périodiquement le contrôle de leurs équipements. Par ailleurs, nous ne sommes jamais à l'abri d'une défaillance soudaine et imprévue d'où la nécessité des éléments en redondance. D'une manière générale les systèmes en redondance sont divisés en deux (2) types :

- ✓ redondance active;
- ✓ redondance passive.



### **1.4.1 Redondance active**

La redondance active consiste à utiliser plus d'éléments qu'il n'en faut strictement pour remplir une fonction. Nous parlons de redondance active, quand tous les éléments fonctionnent en permanence. Nous distinguons la redondance active totale et partielle.

#### **1.4.1.1 Redondance active totale**

Le système ne devient défaillant qu'avec la défaillance du dernier élément survivant. Par définition, il s'agit d'un système dans lequel les éléments sont associés en parallèle.

#### **1.4.1.2 Redondance active partielle**

Nous parlons de redondance active partielle quand un système comporte  $n$  éléments, dont  $m$  ( $m < n$ ) strictement nécessaires pour qu'il fonctionne. Le système peut donc accepter  $(n-m)$  défaillances.

### **1.4.2 Redondance passive**

La redondance est qualifiée de passive quand les éléments abondants ne sont mis en service qu'au moment du besoin; cela signifie que parmi  $n$  éléments seuls  $m$  sont en service. Ceci implique que certains éléments seront en réserve ou en stock.

## **1.5 Définition de la chaîne de Markov**

La chaîne de Markov contribue à spécifier le processus de production dans un environnement stochastique pour un FMS, tout en tenant compte que l'état du système varie au cours du temps. La chaîne de Markov peut avoir deux allures, homogène et non homogène.

### **1.5.1 Chaîne de Markov homogène et FMS**

Une chaîne de Markov est homogène quand les taux de transition d'un état à un autre de la machine sont considérés constants. Cela suppose que les équipements peuvent tomber en panne même durant l'arrêt de production. Cette présomption est réaliste dans le cas où la

machine subit des facteurs exogènes. Il existe d'autres hypothèses pour la chaîne de Markov homogène tels que :

- le temps moyen de réparation d'une panne est fixe;
- la disponibilité de chaque équipement composant le système est connue à l'avance.

La description des pannes et des réparations des équipements d'un FMS par le processus de Markov homogène, a montré l'importance des pannes aléatoires sur le problème d'optimisation des FMS. Il est à noter que ce processus ne pourrait pas être utilisé à cause de la difficulté de la résolution des équations de programmation dynamique appelées équation d'HJB. Pour résoudre ce problème, les chercheurs (Kimemia et Gershwin, 1983) posent l'hypothèse que l'ensemble de la capacité d'un FMS est un hypercube. En décomposant les équations différentielles couplées du type d'HJB sous forme d'équations normales, il est possible de résoudre séparément afin de trouver les coûts de chaque état de la machine et chaque pièce fabriquée (méthode de la décomposition et approximation quadratique). Dans le même ordre d'idée, les experts précisent que pour un système constitué d'une machine traitant un produit, la politique optimale est caractérisée par le niveau optimal d'inventaire ou seuil critique (Akella et Kumar, 1986). Les experts ont développé une formule analytique permettant de calculer le seuil critique, en supposant un taux de demande constant et que la machine peut se trouver dans deux (2) états (opérationnel, panne). La politique de production détermine le taux de production en fonction de l'inventaire. Cette méthode mène aux trois (3) recommandations suivantes:

- ✓ produire exactement au taux de la demande, si le niveau d'inventaire et l'inventaire optimal sont exactement les mêmes;
- ✓ ne pas produire si le niveau d'inventaire excède l'inventaire optimal;
- ✓ produire au taux maximal si l'inventaire est inférieur au niveau optimal d'inventaire.

Dans le cadre de l'optimisation d'un FMS, l'intégration du contrôle de C/D dans la gestion de la production permet simultanément une vérification des taux de transition des maintenances préventives ainsi qu'une vérification des modes opérationnels ou des modes

pannes. Donc, l'état de l'équipement est considéré comme un processus de la chaîne de Markov non-homogène.

### **1.5.2 Chaîne de Markov non-homogène et FMS**

Dans la majorité de cas, le FMS est modélisé par un processus de la chaîne de Markov non-homogène. La probabilité qu'un équipement tombe en panne augmente avec son âge ainsi que son cycle de fonctionnement. Le taux de transition du mode opérationnel à la maintenance préventive est une variable de contrôle. La fiabilité de la machine est liée à son âge (Boukas et Haurie, 1990). Les travaux de Boukas et Haurie (1990) et de Kenné et Gharbi (1999) prennent en compte une probabilité de pannes des équipements croissante en fonction de l'âge. Ces auteurs procèdent à une restauration de l'âge des équipements à zéro après la maintenance préventive. La politique à seuil critique est obtenue pour contrôler le taux de production en sachant que le taux de maintenance préventive est contrôlé par un âge critique (l'âge optimal) dépendant de l'âge de la machine. L'âge critique est le point de transition du mode opérationnel au mode maintenance préventive. Le contrôle de la maintenance préventive permet d'améliorer la performance d'un FMS, tout en diminuant les coûts de pénurie, les surplus et les réparations en restituant l'âge de la machine à zéro après l'intervention. Gershwin et al. (1996) ont montré l'impact de la maintenance préventive sur la minimisation des coûts de la production et l'assurance d'une meilleure qualité des pièces fabriquées. En se basant sur la division de l'âge de la machine en quatre (4) segments (trois modes opérationnels et un mode panne) et une qualité des pièces fabriquées en mode opérationnel (bon, moyen et mauvais), Kenné (2003) a obtenu une loi de commande sous optimale et a montré que la politique de la commande est asymptotiquement optimale.

### **1.6 Taux de demande variable**

Dans la majorité de cas pour un FMS, les experts utilisent un taux de demande constant où quand elle est variable, elle suit une distribution de la loi exponentielle. En sachant que le taux de demande peut être variable et suit une distribution de Poisson, Feng et Yan (1999) ont montré l'optimisation du seuil critique. Dans le même ordre d'idée, pour un système à

plus de deux (2) états, Sethi et al. (1992) ont résolu le problème par la technique de solution de viscosité, ces derniers ont remarqué que la fonction valeur représentant le coût est convexe. De plus, la fonction valeur est une solution de viscosité des équations d'HJB. En effet, dans certains cas il arrive qu'il ne soit pas possible de représenter les pannes et les réparations des équipements par des processus markoviens. Dans ce cas de figure, les solutions ont été étudiées à partir des solutions obtenues par les processus markoviens, à travers une extension du concept de la politique à seuil critique aux processus non-markoviens. Kenné et Gharbi (2000), Kenné et Gharbi (2003), avec un processus non-markoviens, en considérant une distribution des pannes et des réparations non exponentielles ainsi qu'un taux de demande variable déterminent la solution optimale. Dans ce modèle l'approche utilisée est basée sur les plans d'expérience et les techniques de simulation.

### **1.7 Système de production complexe**

Bien que la littérature soit riche en travaux sur les FMS, peu d'entre eux traitent d'un FMS avec plusieurs machines et plusieurs produits. Dans la plupart des modèles, pour simplifier la résolution, nous considérons des cas simples, une machine et un produit. Dans le cas réel, le FMS est généralement constitué de plusieurs machines. Il est à noter que plus le nombre des machines augmente, plus la résolution du système est complexe, (Kenné et Gharbi, 2000; Kenné et Gharbi, 2003). Les machines peuvent être disposées en série, en parallèle, en redondance (active ou passive), ou même mixte série/parallèle, suivant les besoins de la demande. La disposition des machines dans un FMS a un impact direct sur le coût total de production. Pour les raisons susmentionnées, les auteurs supposent une série d'hypothèses de façon à simplifier la résolution du problème d'optimisation de la production pour un FMS complexe.

### **1.8 Machines en série et FMS**

Dans un FMS, si les machines sont disposées en série le niveau d'inventaire intermédiaire peut être considéré à capacité finie ou infinie. Si le niveau d'inventaire intermédiaire du système a une capacité finie, la panne de la machine en aval peut entraîner le blocage de la machine en amont ainsi que la panne de celle qui est en amont, l'arrêt de celle située en aval.

Sethi et al. (1992) ont exposé une politique de commande hiérarchisée sous optimale pour deux (2) machines en considérant une capacité finie pour les tampons internes. Presman et al. (1997) ont développé une politique sous optimale pour deux machines en série avec l'approche proposée par Zhang et al. (1995). Kenné et Boukas (1998) ont montré une amélioration de la disponibilité du système en introduisant le contrôle de la maintenance corrective, pour un système constitué de plusieurs machines en série avec une capacité des inventaires intermédiaires finies.

### **1.9 Machines en parallèle et FMS**

La complexité due au FMS dont les machines sont disposées en parallèle est reliée à l'ordonnancement quand le système produit plusieurs types de pièces. Dans le cas des systèmes manufacturiers flexibles complexes, la fonction qui réalise le coût optimal appelée fonction valeur doit répondre à un ensemble d'équations différentielles appelées équations d'HJB, (Boukas et Haurie, 1990; Boukas et Kenné, 1997). Dans certains FMS complexes dont les conditions d'optimum sont décrites par les équations d'HJB, la résolution des équations d'HJB est insurmontable analytiquement et parfois même numériquement. Nous référons le lecteur au travail de Kenné et Gharbi (2001). Une autre méthode proposée pour la politique de production optimale pour un système constitué de plusieurs machines en parallèle produisant plusieurs types de pièces est la résolution combinée. La résolution combinée est basée sur les plans d'expérience, les modèles de simulation et la méthodologie de surface de réponse, (Kenné et Gharbi 2000; Kenné et Gharbi 2003).

### **1.10 Différents types de maintenance**

Plusieurs définitions sont attribuées au terme maintenance, notamment Monchy (1991) qui définit la maintenance comme étant un ensemble de moyens de prévention, de correction ou de rénovation suivant l'usage du matériel et suivant sa criticité économique, afin d'optimiser le coût global de possession. La maintenance se décrit comme un ensemble d'opérations d'entretien préventif et curatif destinées à accroître la fiabilité ou de pallier aux défaillances. Supportant principalement la production, la maintenance joue un rôle important sur la qualité, la sécurité et l'accroissement de la productivité dans toute entreprise moderne et

compétitive. Les différentes pratiques de maintenance ont évolué au fil des années en fonction des différents besoins des sociétés industrielles.

Dans les paragraphes qui vont suivre, nous allons aborder la maintenance préventive et corrective.

### **1.10.1 Maintenance préventive**

Durant la Deuxième Guerre Mondiale, les choses ont gravement changé avec l'augmentation de la demande des produits. Ce changement est dû à la pression de la guerre alors que la main d'œuvre baissait considérablement; ceci a entraîné une forte évolution au niveau du développement des mécanismes industriels avec l'avènement des actifs numériques pouvant pallier aux tâches multiples. En vue de cette évolution industrielle, Ben-Daya (2000) définit cette période comme étant le commencement de la forte dépendance industrielle des actifs. Les arrêts des actifs devenant plus considérables, les industries s'engagèrent à accorder plus d'importance à l'aspect préventif, afin d'augmenter la disponibilité et la durée de vie des équipements; d'où la naissance de la deuxième génération de la maintenance appelée maintenance préventive réalisée tout simplement de façon systématique. Ce type de maintenance regroupe la maintenance de type bloc et de type âge. La maintenance préventive type bloc consiste à remplacer des équipements neufs, faits à des intervalles de temps fixes. Ces remplacements ne tiennent pas compte de l'état du système et celui-ci, en cas de panne, est remplacé par un système neuf (Barlow et al., 1996). Par contre, si la politique de remplacement consiste à remplacer le système en cas de panne ou après une période de temps de marche sans panne, nous sommes en face de ce qu'on appelle la maintenance de type âge. L'âge de la machine est remis à zéro après chaque panne (Barlow et al., 1996).

### **1.10.2 Maintenance corrective et curative**

Il y a soixante ans, les industries n'étaient pas bien mécanisées, les équipements étaient surdimensionnés et la main d'œuvre était abondante. Ceci facilitait les réparations et rendait moins coûteux les arrêts de production et la prévention des arrêts des équipements n'était pas une grande priorité. Les tâches de maintenance étaient restreintes aux tâches d'entretien.

La maintenance était donc basée sur la réparation après défaillance. Ceci a donné naissance à la première génération de maintenance appelée maintenance corrective. Une autre définition a été proposée par Stevenson et Benedetti (2006) dans leur livre, la "Gestion des Opérations". Ces auteurs ont distingué entre la maintenance corrective et la maintenance curative. Selon ces auteurs, la maintenance curative consiste à intervenir sur le système pour sa remise en fonctionnement en cas de panne, la maintenance corrective est plutôt une amélioration de l'équipement et des installations en vue de diminuer le temps d'arrêt et les coûts de maintenance.

L'organisation des maintenances corrective et préventive dans un FMS est présentée à la figure ci-dessous :

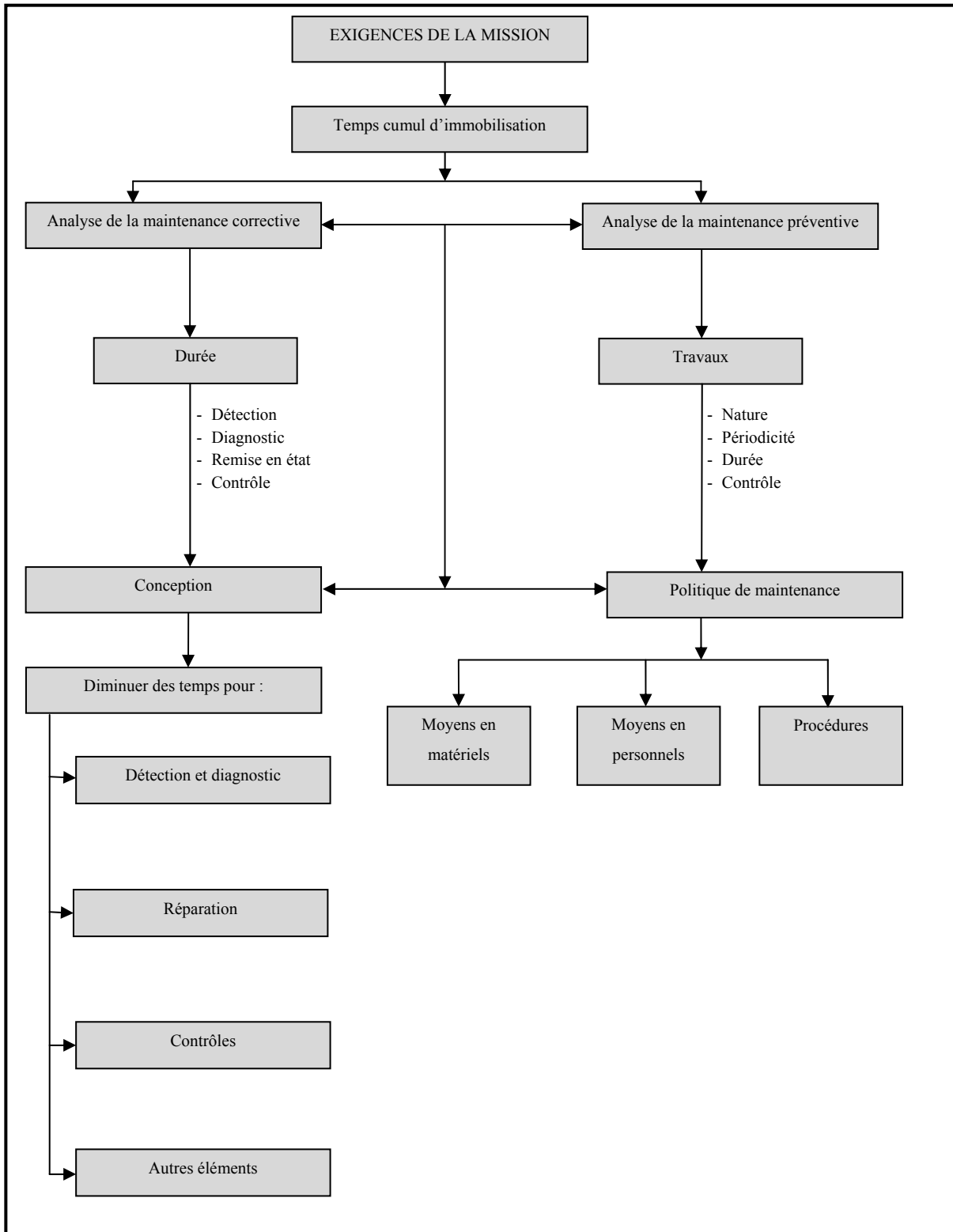


Figure 1.2 Organisation des travaux de maintenance dans un FMS



Nous citons ici quelques études faites par les différents groupes de chercheurs en santé et sécurité du travail (SST) à travers le monde sur l'importance du contrôle et de la gestion des événements indésirables lors de la maintenance.

### **1.11 Maintenance et accident au travail**

Les études faites par l'Occupational Safety and Health Administrations (OSHA) montrent qu'environ 3 millions de travailleurs de maintenance d'équipements sont exposés à des risques graves d'accident. En cas d'accident, le temps moyen perdu par travailleur est d'environ 24 jours. D'autres études menées au Québec par l'IRSST (2008) précisent que près du quart des accidents se produisent au moment des interventions des travailleurs sur les équipements lors d'entretiens et de réparations. Cette situation ajoutée aux douleurs et aux souffrances, liées aux risques ergonomiques (Harichaux et Libert, 2003), représente une perte non négligeable pour les entreprises. D'autres études menées par Chinniah et Champoux (2008) de l'IRSST, en 2005, précisent que les machines dangereuses ont causé la mort d'environ 20 travailleurs au Québec et quelques 13 000 accidents ont pu être reliés aux machines occasionnant des coûts de 70 millions de dollars pour la CSST. De plus, d'après les statistiques de la CSST, au Québec, il y a en moyenne par année 6300 accidents relatifs aux machines dont 17 décès par an. En 2008 seulement, nous avons dénombré 6 pertes en vies humaines, 5225 accidents au cours de travaux d'installation, d'entretien ou de réparation de machines. Pour remédier à ces problèmes, la méthode privilégiée est l'utilisation du C/D ou consignation/déconsignation ou lockout/tagout. Par contre, dans le contexte actuel de l'industrie, beaucoup de dirigeants et travailleurs pensent, à tort, que la planification et la réalisation des différentes procédures de C/D prend beaucoup de temps. Par conséquent, ce temps de production inactif est perçu comme diminuant la performance de l'entreprise vis-à-vis la cadence de fabrication planifiée. (Charlot et al. 2006).

**Nota :** Nous avons cité ci-dessus, ces différents termes pour le C/D car le terme de C/D est connu par les expressions consignation/déconsignation, lockout/tagout en dehors du Québec.

### **1.12 Politique de cadenassage/décadenassage**

Un accident de travail peut être observé comme une perturbation, souvent avec des effets beaucoup plus graves que la panne elle-même. Ces aléas, quand ils se produisent, font beaucoup de dégâts, ce qui empêche l'entreprise de remplir sa mission. L'augmentation des précautions de sécurité fait diminuer la fréquence des accidents, par contre, elles font augmenter le coût total de production.

Parmi les solutions mises en place, pour répondre à ces critères, nous retenons le C/D. Cette solution consiste à verrouiller une machine ou un équipement à l'aide d'un cadenas, puis, de décharger toutes les sources d'énergie résiduelle (hydraulique, électrique, pneumatique et etc.) de façon à éviter la mise en marche prématurée des équipements ou des machines pendant qu'un technicien fait son intervention sur les appareils en question. La méthode de C/D souvent est accompagnée par des fiches de signalisation affichées sur les tableaux de commande et les points de démarrage de manière assez visible pour avertir les employés de l'indisponibilité des appareils sous intervention. Par mesure de sécurité, le cadenas doit avoir une seule clé et celle-ci doit être gardée par l'intervenant lui-même, dans le cas contraire, par le responsable de l'équipe d'intervention. La norme canadienne CSA Z460-05 (2005) définit le C/D ou consignation/déconsignation (Europe) comme «l'installation d'un cadenas ou d'une étiquette sur un dispositif d'isolement des sources d'énergie conformément à une procédure établie, indiquant que le dispositif d'isolement des sources d'énergie ne doit pas être actionné avant le retrait du cadenas ou de l'étiquette conformément à une procédure établie».

L'utilisation actuelle du C/D présente certaines lacunes. Un niveau de risque élevé existe pendant l'intervention des machines en pannes, (Chinniah et Champoux, 2008). Le risque d'erreur humaine au cours des interventions présente une probabilité importante d'impact sous forme d'incidents ou d'accidents du travail chez les maintenanciers et sur la disponibilité du système manufacturier. De plus l'unique utilisation du C/D sous forme procédurale est une solution imparfaite, en effet des problèmes majeurs persistent : non planification des procédures du C/D dans la gestion de la production. Par ailleurs limiter le seul usage du C/D

dans un but de maintenance corrective empêche l'utilisation du C/D en maintenance préventive. Le manque de validation de faisabilité technique est un autre problème du C/D. Afin de remédier à ces lacunes Matsuoka et Muraki (2001) ont proposé une méthode de repérage assisté par ordinateur pour une usine de grande taille afin d'isoler les équipements avec un nombre minimum d'opérations. L'objectif de ce travail a été d'améliorer considérablement le temps de C/D en éliminant la redondance. Saunders et al. (2001) ont préconisé le placardage pour offrir une alternative au C/D au niveau des équipements de distribution de puissance. Charlot et al. (2006) ont montré que l'intégration du contrôle du C/D à la planification de la production facilite le recours à l'implantation de mesures pour diminuer les risques d'accident. Badiane (2010) a considéré la méthode de C/D comme une activité qui peut être planifiée à part entière dans la planification de la production et non conjointement avec l'activité de maintenance. Entre autres, ce travail a démontré que cette façon de faire nous rend moins vulnérables aux inefficacités dans la planification de la maintenance.

Nous avons discuté dans les paragraphes précédents de l'optimisation de la gestion de production et des stratégies de maintenance des systèmes manufacturiers. Dans les cas mentionnés ci-dessus, les auteurs ont peu abordé l'aspect de la sécurité des travailleurs. Pour remédier à cette lacune et afin de diminuer les pertes en vies humaines, les arrêts maladie et les coûts engendrés, nous nous sommes intéressés particulièrement à l'intégration du C/D dans la gestion de la capacité de production. Nous avons également vérifié l'influence de l'erreur humaine sur le C/D ainsi que les activités de maintenance (Figure 1.3).

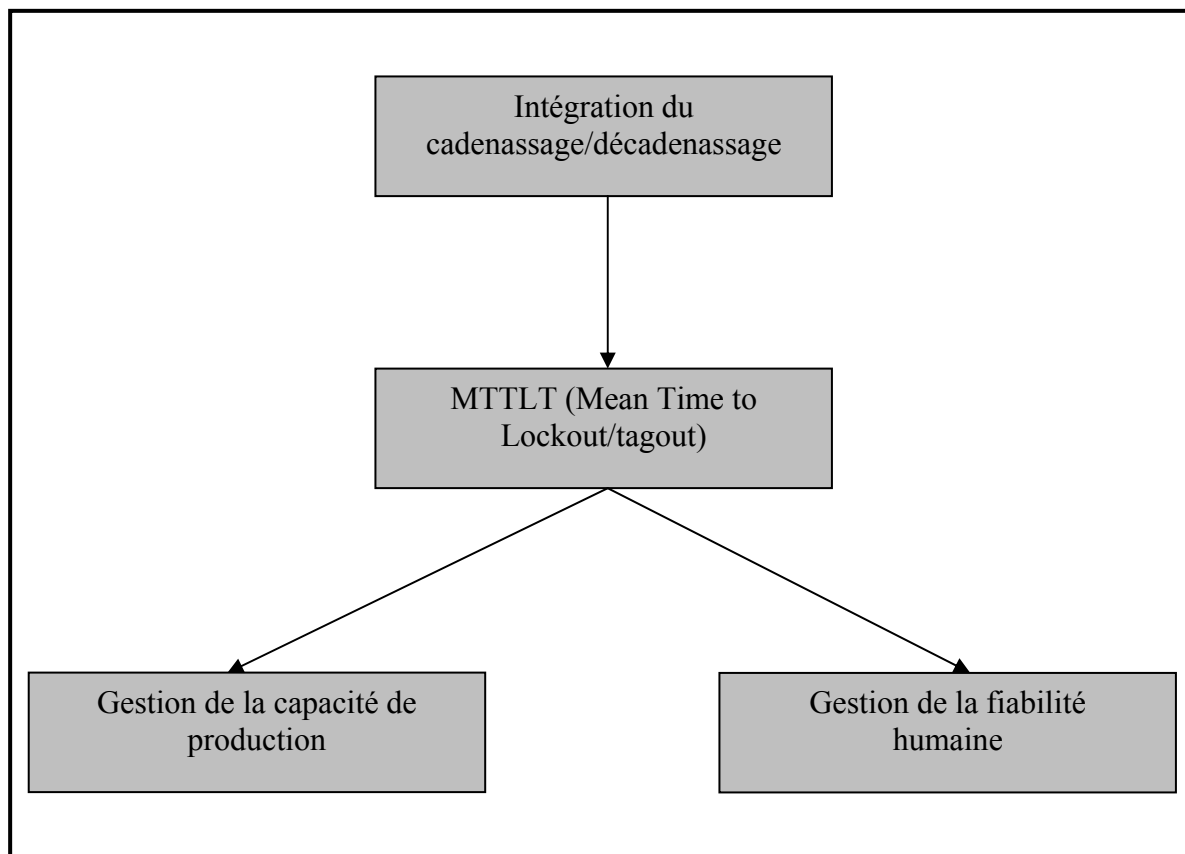


Figure 1.3 Intégration du C/D dans un système manufacturier

## CHAPITRE 2

### PROBLÉMATIQUE ET MÉTHODOLOGIE DE RECHERCHE

#### 2.1 Introduction

Aujourd'hui, la concurrence dans le monde industriel mène les entreprises à viser le maximum de profits en minimisant les coûts liés à la fabrication. Ce but ne peut pas être atteint sans optimisation des diverses ressources de l'entreprise. La compétitivité des entreprises du XXI siècle repose sur trois (3) aspects; la quantité et la qualité des biens ainsi que le respect de la santé et de la sécurité au travail.

Comme nous avons susmentionné, plusieurs travaux de recherche ont porté sur l'optimisation de la gestion de production dans un système manufacturier. De nombreux auteurs s'accordent sur le fait que de tels problèmes sont réalistes lorsqu'ils intègrent conjointement les aspects qualité, inspection, santé et sécurité du travail, dégradation d'équipements, etc., (Gwo-Liang et al., 2009).

L'environnement des systèmes manufacturiers est stochastique. Il est généralement parsemé d'événements dont certains sont prévisibles, d'autres sont seulement contrôlables, d'autres enfin qui ne sont ni prévisibles ni contrôlables (Gershwin, 2002). Ces aléas affectent la gestion de production ainsi que la maintenance des machines et se manifestent à travers les dégradations des machines, des biens, ce qui touchera la marge opérationnelle du système manufacturier.

L'augmentation des pannes et des réparations associées à ces aléas aboutissent à la dégradation des machines, la diminution de la disponibilité ainsi que l'augmentation des risques liés à la maintenance corrective et préventive. Si ces éléments perturbateurs ne sont pas pris en considération, la gestion de production ne permettra pas d'atteindre les trois facteurs concurrentiels (la quantité et la qualité des biens et le respect de la santé et la sécurité des travailleurs) susmentionnés.

Pour faire face aux pannes et aux accidents, les stratégies d'entretien et de contrôle sont divisées en deux (2) volets : celles qui réduisent leurs fréquences et celles qui diminuent leurs gravités (Nollet et al., 1994). Pour cette raison, jusqu'à aujourd'hui, plusieurs études faites par les différents laboratoires de recherche sur les stratégies d'entretien et de contrôle, visent à trouver des méthodes efficaces et fiables, permettant de maîtriser ces éléments de risque et par la suite les rationaliser en production. (Charlot et al., 2006).

## **2.2 Buts de la recherche**

Une fois que nous avons pris connaissance de ce qui est le C/D, nous élaborerons sur les différents problèmes liés à cette méthode de prévention ainsi que les solutions afin de l'améliorer. L'efficacité et l'efficience d'un FMS et sa rentabilité est liée à sa disponibilité et sa maintenabilité (machines et équipements) ainsi qu'à la sécurité de ses travailleurs.

Au cours des dernières années, plusieurs travaux de recherche ont été effectués sur le C/D, mais sans prendre en considération les points suivants :

- 1- La majorité des études proposent des modélisations analytiques, sans valider leur faisabilité technique réelle, sans évaluer leur rentabilité;
- 2- Elles proposent différentes méthodes de C/D sous forme de procédure pour l'intervention sur les équipements, mais non comme un outil fiable pour optimiser la fabrication ou la production;
- 3- Elles élaborent et se centrent sur le C/D lors de la maintenance corrective;
- 4- Remplacer le C/D par d'autres mesures de réduction des risques, entre autres des protecteurs verrouillés ou des dispositifs de sécurité, (R-587 IRSST, 2008).

Cette étude vise à intégrer le C/D dans la gestion de la capacité de production et également à vérifier l'influence de l'erreur humaine sur la sécurité des travailleurs et le coût de production durant les activités de C/D et de maintenance. Cette intégration et vérification a pour objectif

d'augmenter la sécurité des travailleurs, la disponibilité et la maintenabilité des machines, tout en optimisant le coût total de production.

### **2.3 Questions de recherche**

- 1- Comment, peut-on mettre en évidence le C/D dans la gestion de la capacité de production sans la pénaliser?
- 2- Est-il possible de déterminer un modèle théorique qui sera proche de la réalité afin d'intégrer le C/D dans la gestion de la capacité de production?
- 3- Est-ce qu'il existe un modèle mathématique adéquat qui pourra mettre à l'épreuve l'intégration de l'erreur humaine durant les activités de C/D et de maintenance dans un FMS?

### **2.4 Méthodologie proposée**

En premier lieu, le projet a intégré le concept novateur du MTTLT dans la gestion de la capacité de production sous forme de deux travaux. Tout d'abord, nous avons assimilé le MTTLT pour un système en redondance passive (deux machines, non-identiques produisant un seul type de pièce). Les variables de décision ont été les taux de production, le taux de maintenance corrective incluant le C/D. Une modélisation a été faite par la chaîne de Markov homogène (Akella et Kumar, 1986) et une résolution numérique à travers des équations différentielles d'HJB a conduit à la solution du système considéré. L'objectif de ce travail a été d'optimiser le coût de production, d'inventaire, de pénurie sur un horizon infini, tout en augmentant le niveau de sécurité des travailleurs.

En second lieu, nous avons vérifié l'influence de contrôle du MTTLT pour une ligne de production constituée de trois machines (deux machines sous forme redondance passive et une troisième machine en série avec les précédentes) produisant un type de pièce. Les outils de modélisation utilisés à l'étape précédente ont également servi à résoudre ce cas. Les variables de décision ont été les taux de production des machines. Ces variables ont une influence sur le niveau des inventaires intermédiaires et ceux des produits finis. Nous avons

développé les conditions d'optimum en utilisant une approche combinée, se basant sur une combinaison de formalisme analytique, la simulation, le plan d'expérience et la méthodologie de surface de réponse. Une analyse de sensibilité a illustré l'utilité de l'approche proposée. Cette contribution a atteint son objectif en deux (2) volets : 1) trouver les variables de décision permettant de réduire les coûts totaux production, comprenant les coûts d'inventaire et de pénurie sur un horizon infini de planification. 2) libérer un espace-temps essentiel pour minimiser les possibilités de contournement des dispositifs de protection ou d'escamotage des procédures de C/D en intégrant la troisième machine sous forme redondance passive.

Et finalement, notre travail s'est concentré sur la modélisation du MTTLT et l'erreur humaine pour un FMS. La modélisation de l'erreur humaine a été effectuée par une chaîne de Markov non-homogène (Boukas et Haurie, 1990), pour un système en redondance passive produisant une seule pièce. Les variables de décision ont été les taux de production des machines et le taux de maintenance préventive avec ou sans erreurs. Les variables de décision ont une influence sur le niveau des inventaires ainsi que la capacité du système manufacturier. Le taux de défaillance du système manufacturier dépend de sa durée de vie, ce qui signifie que la politique de maintenance préventive dépend de l'âge des machines. En effet, ce travail a prévu une modélisation analytique par chaîne de Markov non homogène, dont la résolution est faite d'une part par la résolution d'équations différentielles d'HJB et d'autre part par analyse numérique. L'objectif de ce travail de recherche est composé de deux (2) volets : 1) trouver les variables de décision permettant de réduire les coûts totaux de production, incluant les coûts d'inventaire et de pénurie sur un horizon de planification infini. 2) vérifier l'influence de l'erreur humaine sur le C/D ainsi que l'activité de maintenance préventive.

Le schéma global de méthodologie est présenté à la figure ci-dessous :



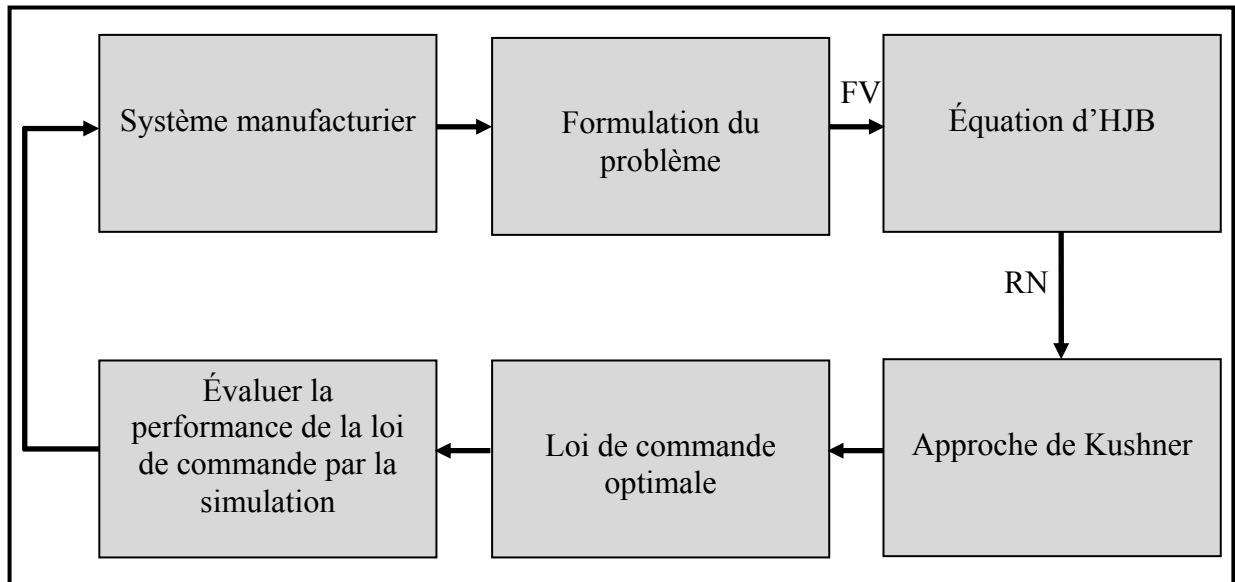


Figure 2.1 Récapitulatif de la méthodologie de la recherche proposée

\*FV : Fonction valeur

\*RN : Résolution numérique

\*HJB : Hamilton-Jacoby-Bellman

## 2.5 Contribution et structure de la these

Les résultats de cette thèse, utilisés dans des cas réels, montreront qu'une planification adéquate du C/D (tel que dans la partie théorique) permet de diminuer le coût de production, en améliorant la sécurité des travailleurs de maintenance et la disponibilité des machines. Les travaux précédents de Charlot et al. (2006) ont permis de construire un modèle analytique de l'intégration du contrôle du C/D à la planification de la production et de l'entretien en utilisant une théorie de contrôle. Un protocole de prises de données en laboratoire et un protocole de prises de données en entreprise ont été conçus par Malek et al.(2009). Le travail de Badiane (2010) a permis de considérer la méthode de C/D comme une activité qui peut être planifiée à part entière dans la planification de la production et non conjointement avec l'activité de maintenance. De plus, ce travail a démontré que cette façon de faire nous rend moins vulnérables aux inefficacités dans la planification de la maintenance. Ce projet de recherche est une suite logique des travaux précédents et a pour objectif de modéliser le MTTLT dans

la gestion de la capacité de production. Il vérifie également l'influence de l'erreur humaine sur la sécurité des travailleurs et le coût de production durant les activités de C/D et de maintenance pour un FMS. Les résultats de ce projet de recherche permettront l'avancement des connaissances, mais surtout la contribution à la réduction des risques de graves accidents dans les milieux de travail et à l'optimisation du coût de production.

Cette thèse est constituée de cinq (5) chapitres comprenant les présents chapitres de la revue critique de la littérature, problématique et méthodologie de recherche. Les trois (3) chapitres qui forment le cœur du travail représentent des articles publiés ou soumis à des revues scientifiques avec comité de lecture. Cette thèse propose également une approche de formulation et de résolution du problème de commande optimale stochastique des systèmes dynamiques en contexte manufacturier. Cette approche permet de trouver la stratégie optimale de gestion de la capacité de production avec les activités de C/D et d'intégrer l'erreur humaine dans la dynamique des systèmes manufacturiers en tenant compte du C/D.

Les contributions de cette thèse sont obtenues à travers la rédaction de trois (3) articles de revues avec comité de pairs, la participation à sept (7) conférences avec comité de pairs et à deux (2) conférences sans comité de pairs.

Les articles de revues sont référencés par :

- **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P., 2011 "Lockout/tagout and operational risks in the production control of manufacturing systems with passive redundancy", *International journal of Production Economics* 132 (2), pp.165-173.
- **Emami-Mehrgani, B.**, Kenné, J.P. and Nadeau, S., 2012 "Lockout/tagout and Optimal Production Control Policies in Failure-prone Non-homogeneous Transfer Lines With Passive Redundancy", Accepted (January), *International Journal of Production Research*. (Acceptance confirmation: TPRS-2012-IJPR-0110).

- **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P., 2012 "Optimal lockout/tagout, preventive maintenance, human error and production policies of manufacturing systems with passive redundancy", Submitted (March), Reliability Engineering & System Safety. (Submission Confirmation: RESS-D-12-00096)

Les articles de conférences par comité de paires sont référencés par :

- **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P., 2012 " Réduction des coûts de production des systèmes manufacturiers sujets aux activités de cadenassage/décadenassage", 34e Congrès de l'Association québécoise pour l'hygiène, la santé et la sécurité du travail (AQHSST), 16 au 18 mai, Gatineau, Québec.
- **Emami-Mehrgani, B.**, Kenné, J.P. and Nadeau, S., 2012 "Lockout/tagout and human error in production control of manufacturing systems with passive redundancy", Gesellschaft für Arbeitswissenschaft (GFA), 22 au 24 février, Kassel, Allemagne.
- **Emami-Mehrgani, B.**, Nadeau, S. and Kenné, J.P. 2011 "Lockout/tagout and operational risks in the production control of production line with passive redundancy", International Mechanical Engineering Congress, American Society of Mechanical Engineers (ASME), 11 au 17 Novembre, Denver, USA.
- **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P., 2011 "Optimisation des politiques de surveillance par le biais du cadenassage/décadenassage", 33e Congrès de l'Association québécoise pour l'hygiène, la santé et la sécurité du travail (AQHSST), 11 au 13 mai, Trois Rivières, Québec.
- **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P. 2011. "Cadenassage et baisse des coûts de production", Travail et Santé 27(2),pp.13-15.

- **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P. 2011. "Temps moyen de cadenassage/décadenassage : un outil d'optimisation des politiques de surveillance et de maintenance", Travail et Santé 27(4),pp.22.
- **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P. 2010 "Integrating lockout/tagout with operational risks: the passive redundancy case", Gesellschaft für Arbeitswissenschaft (GFA), 24 au 26 Mars, Darmstadt, Allemagne, pp.475-482.

Les articles de conférences sans comité de paires sont référencés par:

- **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P., 2011 "Temps moyen de cadenassage/décadenassage: un outil d'optimisation des politiques de surveillance", ÉTS, 14 avril.
- **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P., 2011 "Optimisation des politiques de surveillance par le bais du cadenassage/décadenassage ", ÉTS, 9 décembre.

Les trois articles de revues sont présentés dans les trois (3) prochains chapitres.

L'article du troisième (3) chapitre présente un modèle de détermination conjointe des problèmes d'optimisation de coût, de sécurité des travailleurs, de maintenance corrective pour deux types de défaillances d'un FMS sujet aux pannes et réparations. Le système manufacturier est constitué de deux machines en redondance passive. Les variables de décision sont le taux de production, le taux de maintenance corrective incluant le C/D. Cet article a été publié dans la revue International Journal of production Economics sous la référence : **Emami-Mehrgani, B.**, Nadeau, S and Kenné, J.P., 2011 "Lockout/tagout and operational risks in the production control of manufacturing systems with passive redundancy", International journal of Production Economics 132 (2), pp.165-173.

Dans l'article du chapitre quatre (4), nous avons étudié une ligne de production constituée de trois machines produisant un type de pièce (deux sous forme redondance passive et une troisième en série avec les précédentes). Les machines sont sujettes à des pannes et à des réparations aléatoires. Le problème de contrôle considéré est soumis à des contraintes non-négatives des inventaires en-cours (work-in-process). Les variables de décision sont les taux de production des machines. Les variables de décision influencent le niveau des inventaires intermédiaires et des inventaires de produits finis. Cet article a été accepté dans la revue International Journal of Production Research sous la référence : **Emami-Mehrgani, B.**, Kenné, J.P. and Nadeau, S., 2011 "Lockout/tagout and Optimal Production Control Policies in Failure-prone Non-homogeneous Transfer Lines With Passive Redundancy", Accepted on January 26, 2012, Acceptance Confirmation : TPRS-2012-IJPR-0110.

Dans l'article du chapitre cinq (5), nous avons introduit la notion d'erreur humaine pendant des activités de C/D et de maintenance préventive. Le système manufacturier considéré est constitué de deux machines non-identiques sous forme redondance passive. Les machines sont sujettes à des pannes, à des réparations aléatoires et à des activités de maintenance préventive avec ou sans erreur humaine. Les variables de décision sont les taux de production des machines et le taux de maintenance préventive avec ou sans erreurs. Les variables de décision influencent le niveau des inventaires et la capacité du système manufacturier. Cet article a été soumis à la revue Reliability Engineering & System Safety sous la référence : **Emami-Mehrgani, B.**, Nadeau, S. and Kenné, J.P., 2012 "Optimal lockout/tagout, preventive maintenance, human error and production policies of manufacturing systems with passive redundancy", Submitted on March 7, 2012, Submission Confirmation : RESS-D-12-00096.

Pour terminer, nous récapitulons le bilan de ce travail et nous présentons les travaux futurs.



## CHAPITRE 3

### ARTICLE 1: LOCKOUT/TAGOUT AND PRODUCTION RISKS IN THE PRODUCTION CONTROL OF MANUFACTURING SYSTEMS WITH PASSIVE REDUNDANCY

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#### Résumé

Cet article présente un modèle de détermination conjointe des problèmes d'optimisation de coût, de sécurité des travailleurs, de maintenance corrective pour deux types de défaillances d'un FMS sujet aux pannes et réparations. Des décisions concernant la production ainsi que la maintenance corrective doivent être prises afin de minimiser des coûts de production. Les variables de décision sont le taux de production, le taux de maintenance corrective incluant le cadencage/décadencage (C/D). L'objectif est d'optimiser les coûts de production, d'inventaire et de pénurie sur un horizon infini par une planification efficace et efficiente du C/D lors de la gestion de la production.

Pour atteindre nos objectifs, nous suivrons les étapes suivantes:

- vérification de l'influence de contrôle du C/D ainsi que le temps de réparation pour une machine produisant un type de pièce;
- vérification de l'influence de contrôle du C/D pour un système en redondance passive qui consiste en deux (2) machines (non-identiques) produisant un type de pièce.

Un exemple numérique et des analyses de sensibilité sont présentés pour illustrer l'utilité de l'approche proposée.

### **Abstract**

This paper addresses problems associated with production control and occupational safety in a manufacturing system prone to failure involving two machines working in passive redundancy. Machines turning out one part experience two modes of failure and repair: firstly, where failure occurs when a machine remains in fair condition; and, secondly, where such failure results in outright breakdown. Accordingly, we examine both modes of failure for their impact on a flexible manufacturing system (FMS) with respect to production control in terms of costs associated with lockout/tagout procedures and corrective maintenance. This study seeks to identify optimal costs related to backlogs, inventories and maintenance over an infinite planning horizon, along with levels of occupational risk where production control includes efficient planning of lockouts/tagouts. Our study offers numerical methods which may be employed to achieve optimal conditions in setting control policies. A numerical example and sensitivity analysis support this approach.

**Keyword:** Flexible manufacturing systems; passive redundancy; corrective maintenance; lockout/tagout.

### **3.1 Introduction**

Industry strives to lower production costs and optimize profit through operating methods consistent with various legal requirements. This target can only be achieved through the application of controls governing all stages of production. We need to harness the various resources available throughout a manufacturing system to obtain the benefits that flow from reliable and efficient controls over output. Gharbi and Kenné (2003), and Charlot et al. (2006), and others have produced studies recently addressing optimal production control in flexible manufacturing systems. Despite these efforts, the severity and frequency of accidents remains unacceptably high during maintenance procedures. Results published by a number of



agencies confirm these findings, including the National Occupational Health and Safety Commission (NOHSC, 2000), Mutawe (2002) and the Association Francaise des Ingenieurs de Maintenance (AFIM, 2004). Lockouts/tagouts during such maintenance offer a potential solution to this problem. The machine is padlocked. All sources of residual power (potential, hydraulic, electrical, etc.) are discharged to avoid premature start-up during maintenance. Many managers wrongly assume it takes too much time to plan and carry out a lockout/tagout, fearing such downtime will lower productivity or reduce performance. Researchers cited earlier focused on optimizing production control in manufacturing systems. Many authors concur on the need for approaches that integrate quality control, occupational analysis, health and safety, as well as wear and tear on equipment (Gwo-Liang et al., 2009). It is possible to predict and control certain events while others occur randomly and are beyond control within manufacturing systems (Gershwin, 2002). Control over production, maintenance and profit margins are all adversely affected by events that damage equipment or the goods produced in a manufacturing system. Reliability analysis and maintenance policy optimization for cold-standby system has been studied by many researchers. Zhang and Wang (2006), Zhang and Wang (2007) and Zhang et al. (2006) derived the expected long-run cost per unit time for a repairable system consisting of two identical machines and one repairman when a geometric process depicting working time is assumed. A two machines cold-standby system is composed of a primary machine and a backup machine, where the backup machine is only called upon when the primary machine fails. Cold-standby systems are one of the most important structures in reliability engineering and have been widely used in real manufacturing systems. Manufacturing systems operate in a stochastic environment because machines break down and are repaired randomly. A rising number of breakdowns and repairs degrades structure, reduces availability, and increases occupational hazards associated with corrective maintenance. Left unattended, these disruptive elements erode competitiveness expressed in terms of quantity, quality and occupational health and safety. Maintenance strategies have been developed to address either the frequency or the severity of accidents and breakdowns (Nollet et al., 1994). Occupational health and safety researchers worldwide have carried out studies confirming the importance of monitoring and controlling undesirable incidents during maintenance procedures. The Occupational, Safety

and Health Administration (OSHA) conducted studies concluding that some 3 million workers risk serious injury in work performed to maintain equipment. Workers lose on average about 24 days a year when an accident occurs. Nearly a quarter of all occupational accidents occurring in Quebec result from work to maintain or repair equipment, according to studies carried out by Institut de Recherche Robert Sauvé en Santé et Sécurité du Travail (IRSST, 2008). Pain and suffering along with ergonomic hazards only add to the considerable loss experienced by workers as well as firms in such circumstances (Harichaux and Libert, 2003). An important consideration arises: *How to optimize production yet ensure occupational safety*. The theory put forward makes certain reasonable assumptions in answering this question (rates of demand for various products are held constant; machines are flexible). Hence, we base this theory on the extension of stochastic optimal control theory as in Akella and Kumar (1986). The control policy structure results from a value function that solves the related Hamilton-Jacoby-Bellman (HJB) equations. We chose a numerical approach to determine an approximate value function, rather than a true value function, to construct the control policy. Given appropriate conditions, the control policy constructed becomes asymptotically optimal when variance from the true value function approaches zero (see Kenné et al. (2003). Finally, we present a numerical example and sensitivity analysis demonstrating the validity of this approach. Assumptions and notations are defined in the next section. Section 3.3 states the problem. Section 3.4 offers the Hamilton-Jacoby-Bellman (HJB) equations. We show how a numerical approach yields an approximate value function. Section 3.5 provides a numerical example, sensitivity analysis and discussion of results. Section 3.6 concludes the paper.

## **3.2 Assumptions and Notations**

This paper incorporates the following assumptions and notations:

### **3.2.1 Assumptions**

- 1- Costs of corrective maintenance for repair from an outright breakdown exceed costs of corrective maintenance for repair of a machine in fair condition.

- 2- Mean corrective maintenance time under fair condition is briefer than mean corrective maintenance time for a breakdown.
- 3- The corrective maintenance is carried out with lockout/tagout for two modes of failure.
- 4- The main machine is more robust than the standby machine.
- 5- The main machine returns to production immediately after each repair (corrective maintenance for two modes of failure) and the standby machine stands idle.
- 6- Both machines do not fail simultaneously (with a good maintenance plan).

### 3.2.2 Notations

$x(\cdot)$ : inventory level

$c^+$ : holding cost per unit of item over per unit of time

$c^-$ : backlog cost per unit of item over per unit of time

$c^\alpha$ : cost incurred for the operation on the machine at mode  $\alpha$

$c_{r1}$ : corrective maintenance cost of fair condition

$c_{r2}$ : corrective maintenance cost of outright breakdown

$c_{tagout}$ : lockout/tagout cost

$g(\cdot)$ : instantaneous cost

$J(\cdot)$ : total cost

$v(\cdot)$ : value function

$\rho$ : discount rate

$d$ : demand rate

$u_i(\cdot)$ : production rate of the machine  $i$  ( $i= 1,2$ )

$u_i^{\max}$ : maximal production rate ( $i= 1,2$ )

$h$  : discretization stepsize

$\xi(\cdot)$  : stochastic process

$q_{\alpha\beta}$  : transition rate from mode  $\alpha$  to  $\beta$

$v_{31}^{\min}$  : minimal corrective maintenance rate with lockout/tagout of fair condition (single machine producing one part type)

$v_{31}^{\max}$  : maximal corrective maintenance rate with lockout/tagout of fair condition (single machine producing one part type)

$v_{31}(\cdot)$  : corrective maintenance rate with lockout/tagout of fair condition (single machine producing one part type)

$v_{21}^{\min}$  : minimal corrective maintenance rate with lockout/tagout of outright breakdown (single machine producing one part type)

$v_{21}^{\max}$  : maximal corrective maintenance rate with lockout/tagout of outright breakdown (single machine producing one part type)

$v_{21}(\cdot)$  : corrective maintenance rate with lockout/tagout of outright breakdown (single machine producing one part type)

$\omega_{51}^{\min}$  : minimal corrective maintenance rate with lockout/tagout for fair condition of the main machine

$\omega_{51}^{\max}$  : maximal corrective maintenance rate with lockout/tagout for fair condition of the main machine

$\omega_{51}(\cdot)$  : corrective maintenance rate with lockout/tagout for fair condition of the main machine

$\omega_{41}^{\min}$  : minimal corrective maintenance rate with lockout/tagout for outright breakdown of the main machine

$\omega_{41}^{\max}$  : maximal corrective maintenance rate with lockout/tagout for outright breakdown of the main machine

$\omega_{41}(\cdot)$  : corrective maintenance rate with lockout/tagout for outright breakdown of the main machine

### 3.3 Problem statement

Many fielded systems use cold-standby redundancies (Sainaki, 1994; Pandey et al.,1996 and Kumar et al.,1996) to achieve high reliability levels, since cold-standby redundancies are believed to be more reliable than an analogous system with active redundancies (Subramanian and Anantharaman, 1995; Ebeling, 1997 and Coit, 2001). This section develops a manufacturing system for two machines that are not identical but producing one type of part in passive redundancy. In the system, the main machine operates any time. The other machine, named as the cold-standby redundancy, is in standby position and put online immediately when the main machine fails. Then, the maintenance service is triggered to restore the failed machine. After the failed machine is restored, it will be installed into the standby position. The dynamics of the system are not different from those for one machine producing one type of part. The state of the machine may therefore be characterized in terms of a finite state Markov chain and the homogeneous Markov processes such as in Older and Suri (1980). We consider three modes or sets of states for one machine producing one type of part as follows:

$$\zeta(t) = \begin{cases} 1 & \text{if Operational;} \\ 2 & \text{if Repair with lockout/tagout (corrective maintenance of outright breakdown);} \\ 3 & \text{if Repair with lockout/tagout (corrective maintenance of fair condition).} \end{cases}$$

We do not present the model for single machine producing one type of part in this paper. The reader referred to Charlot et al. (2006) for details of such a system. Thereafter, however, we compare results obtained for a system in passive redundancy with one machine producing one type of part. Hence, the dynamics for two machines in passive redundancy are in a

hybrid state comprised of a discrete state  $\zeta(t) = (\zeta_1(t), \zeta_2(t))$  and a continuous state  $x(t)$ . The discrete state  $\zeta(t) = (\zeta_1(t), \zeta_2(t))$  represents the machine's state and continuous state  $x(t)$  represents the stock level. We have  $\zeta_i(t)=1$  if machine at time-off,  $\zeta_i(t)=2$  if machine operational,  $\zeta_i(t)=3$  if machine in repair with lockout/tagout (corrective maintenance of outright breakdown) and  $\zeta_i(t)=4$  if machine in repair with lockout/tagout (corrective maintenance of fair condition) with  $i = 1, 2$ . A positive value of  $x(t)$  is inventory. A negative value is backlog. This paper considers two machines operating at different rates of production and failure. Consistent with assumptions set out in section 2.1, the Tables 3.1 and 3.2 display diverse modes for our system,  $\zeta(t) = (\zeta_1(t), \zeta_2(t)) \in M = \{1, 2, 3, 4, 5\}$ :

Table 3.1 State transition between the different modes

$\zeta_1(t)$	2	2	2	3	4
$\zeta_2(t)$	1	3	4	2	2
$\zeta(t)$	1	2	3	4	5

Table 3.2 Description of the system modes

Mode	Machines state	Description
1	(2,1)	Main machine operational and standby machine idle
2	(2,3)	Main machine operational and standby machine in repair with lockout/tagout (corrective maintenance of outright breakdown)
3	(2,4)	Main machine operational and standby machine in repair with lockout/tagout (corrective maintenance of faire condition)
4	(3,2)	Main machine in repair with lockout/tagout (corrective maintenance of outright breakdown) and standby machine operational
5	(4,2)	Main machine in repair with lockout/tagout (corrective maintenance of faire condition) and standby machine operational

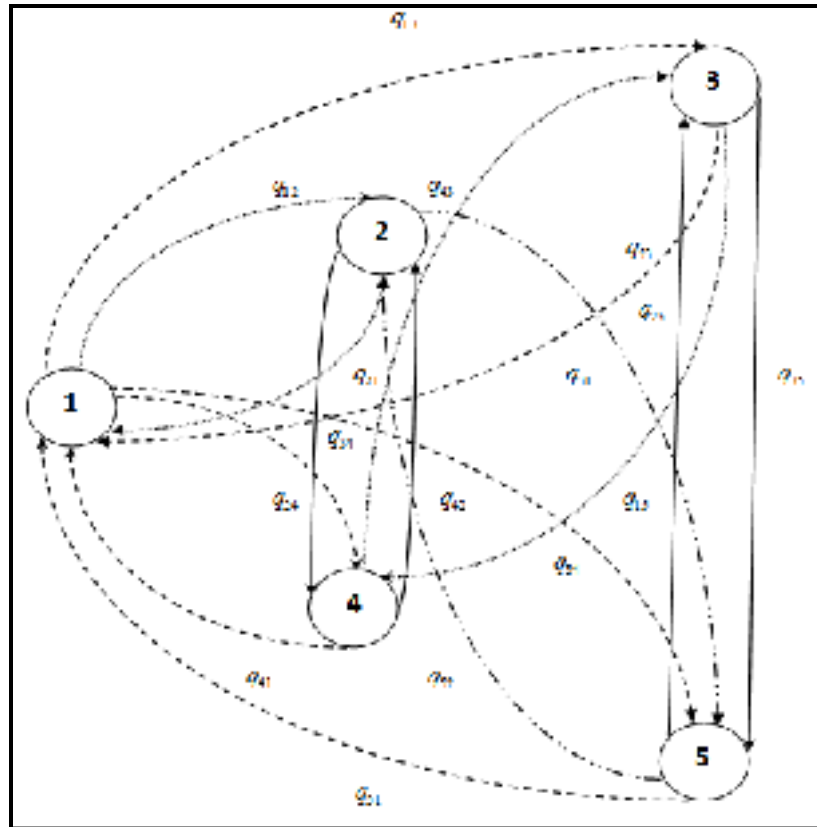


Figure 3.1 Displays the resulting transition diagram.

Mean transition rates can be expressed in Table 3.3, from Table 3.1 and Figure 3.1:

Table 3.3 Mean transition rates between the different modes.

Transition rate	From state	To state
$q_{12}$	(2, 1)	(2, 3)
$q_{13}$	(2, 1)	(2, 4)
$q_{14}$	(2, 1)	(3, 2)
$q_{15}$	(2, 1)	(4, 2)
$q_{21}$	(2, 3)	(2, 1)
$q_{24}$	(2, 3)	(3, 2)
$q_{25}$	(2, 3)	(4, 2)
$q_{31}$	(2, 4)	(2, 1)
$q_{34}$	(2, 4)	(3, 2)
$q_{35}$	(2, 4)	(4, 2)
$q_{41}$	(3, 2)	(2, 1)
$q_{42}$	(3, 2)	(2, 3)
$q_{43}$	(3, 2)	(2, 4)
$q_{51}$	(4, 2)	(2, 1)
$q_{52}$	(4, 2)	(2, 3)
$q_{53}$	(4, 2)	(2, 4)

The 5 X 5 transition matrix  $Q$  of our system follows:

$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} & q_{15} \\ q_{21} & q_{22} & 0 & q_{24} & q_{25} \\ q_{31} & 0 & q_{33} & q_{34} & q_{35} \\ \omega_{41} & q_{42} & q_{43} & q_{44} & 0 \\ \omega_{51} & q_{52} & q_{53} & 0 & q_{55} \end{bmatrix}, \quad (3.1)$$

Hence, the transition matrix  $Q$  depends on:

$\omega_{41}$ : Corrective maintenance rate with lockout/tagout of outright breakdown for the main machine;



$\omega_{51}$ : Corrective maintenance rate with lockout/tagout of fair condition for the main machine;

Where  $\omega_{41} = q_{41}$  and  $\omega_{51} = q_{51}$ .

Thus, the considered stochastic optimal control problem may be characterized by the control dependent transition matrix  $Q(\cdot)$ .

$$q_{\alpha\beta}(\omega_{41}, \omega_{51}) \geq 0 \quad (\alpha \neq \beta) \quad (3.2)$$

$$q_{\alpha\alpha}(\omega_{41}, \omega_{51}) = -\sum_{\alpha \neq \beta} q_{\alpha\beta}(\omega_{41}, \omega_{51}), \quad (\alpha, \beta) \in M \quad (3.3)$$

Transition probabilities are expressed as:

$$p[\xi(t+\delta t) = \beta | \xi(t) = \alpha] = \begin{cases} q_{\alpha\beta}(\cdot)\delta t + o(\delta t) & \text{if } \alpha \neq \beta, \\ 1 + q_{\alpha\alpha}(\cdot)\delta t + o(\delta t) & \text{if } \alpha = \beta. \end{cases} \quad (3.4)$$

The set of admissible decisions at mode  $\alpha(t)$  and control policies (control variables) at mode  $\alpha(t) : u_1(\cdot), u_2(\cdot), \omega_{41}(\cdot)$  and  $\omega_{51}(\cdot)$  is defined by:

$$\Gamma(\alpha) = \left[ \begin{array}{l} ((u_1(\cdot), u_2(\cdot), \omega_{41}(\cdot), \omega_{51}(\cdot)) \in R^4, \\ 0 \leq u_1(\cdot) \leq u_1^{\max}, 0 \leq u_2(\cdot) \leq u_2^{\max}, \\ \omega_{41}^{\min} \leq \omega_{41}(\cdot) \leq \omega_{41}^{\max}, \omega_{51}^{\min} \leq \omega_{51}(\cdot) \leq \omega_{51}^{\max} \end{array} \right], \quad (3.5)$$

In equation (3.5),  $u_1^{\max}$  is the maximal production rate of the main machine,  $u_2^{\max}$  is the maximal production rate of the standby machine,  $\omega_{41}^{\min}$  and  $\omega_{41}^{\max}$  are the minimal and maximal corrective maintenance rates of outright breakdown for the main machine and  $\omega_{51}^{\min}$  and  $\omega_{51}^{\max}$  are the minimal and maximal corrective maintenance rates of fair condition for the main machine.

We turn now to a more complete, though intuitive, definition of the controlled jump process

affecting the operating state of our system.

Let us rewrite equation (3.4) for controlled transition rates (i.e., transitions from modes 4 to 1 and 5 to 1) as follows:

$$q_{41} = \omega_{41}(\cdot) = \lim_{\delta t \rightarrow 0} \left[ 1 / dt (P(\xi(t + \delta t) = 4 | \xi(t) = 1)) \right], \quad (3.6)$$

$$q_{51} = \omega_{51}(\cdot) = \lim_{\delta t \rightarrow 0} \left[ 1 / dt (P(\xi(t + \delta t) = 5 | \xi(t) = 1)) \right], \quad (3.7)$$

The continuous component consists of a continuous variable  $x(\cdot)$  corresponding to the inventory/backlog of products. This state variable is described in the following differential equation:

$$\dot{x}(t) = u(\cdot) - d \quad (3.8)$$

Where :

$$u(\cdot) = u_1 \text{ if } \xi(t) = 1, 2, 3 \text{ and } u(\cdot) = u_2 \text{ if } \xi(t) = 4, 5 \text{ with } x(0) = x$$

Where  $d$  and  $x$  denote respectively the constant demand rate and the initial level of stock.

Objective of the control variable is to minimize discounted cost as follows:

$$J(\alpha, x, u_1, u_2, \omega_{41}, \omega_{51}) = E \left\{ \int_0^{\infty} e^{-\rho t} g(\alpha, x, u_1, u_2, \omega_{41}, \omega_{51}) dt \mid x(0) = x, \xi(0) = \alpha \right\}, \quad (3.9)$$

$$\forall u_1(\cdot) \text{ and } u_2(\cdot) \in \Gamma(\alpha),$$

Where  $g(x, u_1, u_2, \omega_{41}, \omega_{51}) = c^+ x^+(t) + c^- x^-(t) + c^\alpha$ ,  $\forall \alpha \in M$  is the instantaneous cost,  $c^+$ ,  $c^-$  and  $c^\alpha$ , being the cost per unit to produce parts for inventory, backlog as well as intervention cost on the machine.

With:

$$x^+ = \max \{0, x\}$$

$$x^- = \max \{-x, 0\}$$

The system is considered feasible if:

$$\sum \pi u_i^{\max} \geq d \quad (3.10)$$

knowing probability limits can be ascertained from the following equation for a system conforming to a Markov process:

$$\begin{aligned} \pi(\cdot)Q(\cdot) &= 0, \\ \sum \pi &= 1 \end{aligned} \quad (3.11)$$

with:

$\pi(\cdot)$  : Limiting probabilities

$Q(\cdot)$  : Transition matrix rates

Hence, we have  $\pi(\cdot) = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$  representing the vector of limiting probabilities from modes 1 to 5.

The problem is finding the production rates of machines and corrective maintenance rates for both types of failure to minimize the total cost given by equation (3.9). The next section treats this as a dynamic optimization problem.

### 3.4 Optimal conditions and numerical approach

Let  $v(x, \alpha)$  denote the value function or minimum discounted cost for equations (3.9) as expressed in the following equation:

$$v(x, \alpha) = \inf_{u, \alpha \in \Gamma(\alpha)} J(\alpha, x, u_1, u_2, \omega_{41}, \omega_{51}) \quad \forall \alpha \in M, \quad (3.12)$$

With :

$u(\alpha) = u_1$  if  $\alpha = 1, 2, 3$  and  $u(\alpha) = u_2$  if  $\alpha = 4, 5$

This value function  $v(x, \alpha)$  satisfies the set of Hamilton-Jacobi-Bellman (HJB) equations in Kenné et al. (2003) and Charlot et al. (2006). We adapt the HJB equation to the optimal control problem considered as follows:

$$\rho v(x, \alpha) = \min_{u(\alpha) \in \Gamma(\alpha)} \left\{ (u(\alpha) - d) \frac{\partial}{\partial x} v(x, \alpha) + g(x, u, \alpha) + \sum_{\alpha \neq \beta} q_{\alpha\beta} v(x, \beta) \right\}, \forall \alpha, \beta \in M \quad (3.13)$$

With:

$$u(\alpha) = u_1 \text{ if } \alpha = 1, 2, 3 \text{ and } u(\alpha) = u_2 \text{ if } \alpha = 4, 5.$$

Optimal control policy  $(u_1^*, u_2^*, \omega_{41}^*, \omega_{51}^*)$  denotes the minimizer over  $\Gamma(\alpha)$  on the right side of equations (3.9). This policy is consistent with the value function obtained in equation (3.12). Optimal control policy therefore rests in solving equation (3.13). Analytical solution to (3.13) proves almost impossible, however.

Let us now expand the numerical method to solve for optimal conditions identified in the previous section. This method is based on the Kushner approach found in Kushner and Dupuis (1992), Boukas and Haurie (1990) and Kenné et al. (2003).

Kushner's approach lets us use an approximation scheme for the gradient of value function  $v(x, \alpha)$ . Let  $h$  denote the length of the finite difference interval of the variable  $x$ . Hence, using  $h$ ,  $v(x, \alpha)$  is approximated by  $v^h(x, \alpha)$  and  $v_x(x, \alpha)$  as follows:

$$v_x(x, \alpha) \times (u(\alpha) - d) = \left\{ \begin{array}{ll} \frac{1}{h} (v^h(x+h, \alpha) - v^h(x, \alpha)) \times (u(\alpha) - d) & \text{if } (u(\alpha) - d) \geq 0 \\ \frac{1}{h} (v^h(x, \alpha) - v^h(x-h, \alpha)) \times (u(\alpha) - d) & \text{otherwise} \end{array} \right\}, \quad (3.14)$$

With:

$$u(\alpha) = u_1 \text{ if } \alpha = 1, 2, 3 \text{ and } u(\alpha) = u_2 \text{ if } \alpha = 4, 5.$$

We manipulated the approximation arrived at in equation (3.14) to rewrite the HJB equations (3.13) as follows:

$$\begin{aligned}
v^h(x, \alpha) = \min_{u \in \Gamma^h(\alpha)} & \left\{ \left( \rho + |q_{\alpha\alpha}| + \sum_{j=1}^n \frac{|u(\alpha) - d_j|}{h_j} \right)^{-1} \cdot \left[ \sum_{j=1}^n \frac{|u(\alpha) - d_j|}{h_j} (v^h(x+h, \alpha) K^+ \right. \right. \\
& \left. \left. + v^h(x-h, \alpha) K^-) + g(x, \alpha, \cdot) + Q \cdot v^h(x, \alpha) \right] \right\}, \tag{3.15}
\end{aligned}$$

With:

$$u(\alpha) = u_1 \text{ if } \alpha = 1, 2, 3 \text{ and } u(\alpha) = u_2 \text{ if } \alpha = 4, 5.$$

Where  $\Gamma(\alpha)$  is the discrete feasible control space or the so-called control grid and the other term used in equation (3.15) is defined as:

$$K^+ = \begin{cases} 1 & \text{if } (u(\alpha) - d) \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

$$K^- = \begin{cases} 1 & \text{if } (u(\alpha) - d) < 0, \\ 0 & \text{otherwise,} \end{cases}$$

With:

$$u(\alpha) = u_1 \text{ if } \alpha = 1, 2, 3 \text{ and } u(\alpha) = u_2 \text{ if } \alpha = 4, 5.$$

The equation presented in (3.15) can be interpreted as the infinite horizon dynamic programming equation for a discrete-time, discrete-state decision process that addresses problems confronted in optimizing output and controlling maintenance, as in Kenné et al. (2003) and Charlot et al. (2006). The next theorem demonstrates that value function  $v^h(x, \alpha)$  approximates  $v(x, \alpha)$  for a small step size  $h$ .

### Theorem

Let  $v^h(x, \alpha)$  denote a solution to HJB equation (3.15). Assume they are constants  $C_g$  and  $K_g$  as follows:

$$0 \leq v^h(x, \alpha) \leq C_g (1 + |x|^{k_g})$$

Then,

$$\lim_{h \rightarrow 0} v^h(x, \alpha) = v(x, \alpha)$$

### Proof

The proof of this theorem is given in Appendix 3.A.

The subsequent section uses a numerical example to illustrate the structure of the control policies.

### 3.5 Numerical example and sensitivity analysis

We have chosen not to include in this paper any review of the widely known homogenous Markov process  $\alpha \in M = [1, 2, 3]$  used to consider a manufacturing system involving one machine in three states. Markov's homogenous process  $\alpha \in M = [1, 2, 3, 4, 5]$  is followed subsequently to examine a system expanded to two machines that are not identical and operate in five states with passive redundancy. The discrete dynamic programming equation for one machine producing one type of part (M1P1) yields equations (3.16)-(3.18):

$$v^h(x, 1) = \min_{u_1 \in \Gamma^h(1)} \left( \rho + \frac{|u_1 - d|}{h} + q_{12} + q_{13} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h} (v^h(x+h, 1)k^+ + v^h(x-h, 1)k^-) \\ + g(x, 1) + q_{12}v^h(x, 2) + q_{13}v^h(x, 3) \end{array} \right\}, \quad (3.16)$$

$$v^h(x, 2) = \min_{u_1 \in \Gamma^h(2)} \left( \rho + \frac{d}{h} + v_{21} \right)^{-1} \left\{ \frac{d}{h} (v^h(x-h, 2) + g(x, 2) + v_{21}v^h(x, 1)) \right\}, \quad (3.17)$$

$$v^h(x, 3) = \min_{u_1 \in \Gamma^h(3)} \left( \rho + \frac{d}{h} + v_{31} \right)^{-1} \left\{ \frac{d}{h} (v^h(x-h, 3) + g(x, 3) + v_{31}v^h(x, 1)) \right\}, \quad (3.18)$$

Equations (3.19)-(3.23) define a system pairing two machines that are not identical operating in passive redundancy to produce the one part type:

$$v^h(x,1) = \min_{u_1 \in \Gamma^h(1)} \left( \rho + \frac{|u_1 - d|}{h} + q_{12} + q_{13} + q_{14} + q_{15} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h} (v^h(x+h,1)k^+ + v^h(x-h,1)k^-) \\ +g(x,1) + q_{12}v^h(x,2) + q_{13}v^h(x,3) \\ +q_{14}v^h(x,4) + q_{15}v^h(x,5) \end{array} \right\}, \quad (3.19)$$

$$v^h(x,2) = \min_{u_1 \in \Gamma^h(2)} \left( \rho + \frac{|u_1 - d|}{h} + q_{21} + q_{24} + q_{25} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h} (v^h(x+h,2)k^+ + v^h(x-h,2)k^-) \\ +g(x,2) + q_{21}v^h(x,1) + q_{24}v^h(x,4) \\ +q_{25}v^h(x,5) \end{array} \right\}, \quad (3.20)$$

$$v^h(x,3) = \min_{u_1 \in \Gamma^h(3)} \left( \rho + \frac{|u_1 - d|}{h} + q_{31} + q_{34} + q_{35} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h} (v^h(x+h,3)k^+ + v^h(x-h,3)k^-) \\ +g(x,3) + q_{31}v^h(x,1) + q_{34}v^h(x,4) \\ +q_{35}v^h(x,5) \end{array} \right\}, \quad (3.21)$$

$$v^h(x,4) = \min_{u_2 \in \Gamma^h(4)} \left( \rho + \frac{|u_2 - d|}{h} + \omega_{41} + q_{42} + q_{43} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_2 - d|}{h} (v^h(x+h,4)k^+ + v^h(x-h,4)k^-) \\ +g(x,4) + \omega_{41}v^h(x,1) + q_{42}v^h(x,2) \\ +q_{43}v^h(x,3) \end{array} \right\}, \quad (3.22)$$

$$v^h(x,5) = \min_{u_2 \in \Gamma^h(5)} \left( \rho + \frac{|u_2 - d|}{h} + \omega_{51} + q_{52} + q_{53} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_2 - d|}{h} (v^h(x+h,5)k^+ + v^h(x-h,5)k^-) \\ +g(x,5) + \omega_{51}v^h(x,1) + q_{52}v^h(x,2) \\ +q_{53}v^h(x,3) \end{array} \right\}, \quad (3.23)$$

We use the following computational domain:

$$G_x^h = \{x : -10 \leq x \leq 30\}.$$

Note that the computational domain as  $G_x^h = \{x : -10 \leq x \leq 30\}$  for a rather rapid manufacturing system as our system ( $U = 0.4$ ,  $d = 0.2$ ), it allows us to better see the gap between different values as well as different curves. Other parameters for the two cases studied appear in Table 3.4.

Table 3.4 Parameters for the manufacturing systems (M1P1 and M2P1).

Parameter	$c^+$	$c^-$	$c_{tagout}$	$c_{r_1}$	$c_{r_2}$	$u_1^{\max}$	$u_2^{\max}$	$d$
Value	1	10	4	30	40	0.4	0.27	0.2
Parameter (M1P1)	$v_{21}^{\min}$	$v_{21}^{\max}$	$v_{31}^{\min}$	$v_{31}^{\max}$	$q_{12}$	$q_{13}$	$\rho$	$h$
Value (M1P1)	0.1	0.07	0.125	0.2	0.006	0.008	0.001	0.25
Parameter (M2P1)	$\omega_{41}^{\min}$	$\omega_{41}^{\max}$	$\omega_{51}^{\min}$	$\omega_{51}^{\max}$	$q_{14}$	$q_{15}$	$\rho$	$h$
Value (M2P1)	0.1	0.07	0.125	0.2	0.006	0.008	0.001	0.25

The policy iteration technique solves Eqs. (3.16)–(3.23) for optimal conditions. Recall that the control policies are obtained from the numerical resolution of the optimality conditions given by Eqs. (3.16)–(3.23). The structure of production policies ( $u_1$  and  $u_2$ ) states that: if the stock level is lower than a threshold level, then produce at maximum rates; else if the stock level is upper than a threshold level produce nothing; otherwise produce at the demand rate. Such structure in the control literature is called hedging point policy (HPP). We refer the reader to Kimemia and Gershwin (1983) for the structure of this control policy and to Akella and Kumar (1986) for the calculation of the threshold level of this control policy in the case of producing one part type. The structure of corrective maintenance including lockout/tagout policie ( $\omega_{41}$  and  $\omega_{51}$ ) recommend that: if the stock level is lower than a threshold level, the corrective maintenance including lockout/tagout set at maximum rates,



otherwise the corrective maintenance including lockout/tagout set at minimum rates. The joint optimization of production and corrective maintenance policies gives another version of hedging point policy (HPP), which is an extension of so-called hedging point policy (HPP).

This control policy is treated in the equations (3.24)-(3.28):

$$u_i(x, a) = \begin{cases} u_i^{\max} & \text{if } x(\cdot) < Z^*, \\ d & \text{if } x(\cdot) = Z^*, \\ 0 & \text{otherwise,} \end{cases} \quad (3.24)$$

Where:  $u^{\max}(\alpha) = u_1^{\max}$  if  $\alpha = 1, 2, 3$  and  $u^{\max}(\alpha) = u_2^{\max}$  if  $\alpha = 4, 5$ .

Where  $Z^*$  is the optimal threshold value of stock level for each state of the machine.

Resulting equations are:

*For one machine producing one part:*

$$v_{21}(x, 2) = \begin{cases} v_{21}^{\max} & \text{if } x(\cdot) < A^*, \\ v_{21}^{\min} & \text{otherwise.} \end{cases} \quad (3.25)$$

Where  $A^*$  represents the optimal stock level at which the corrective maintenance rate with lockout/tagout needs to switch from  $v_{21}^{\min}$  to  $v_{21}^{\max}$  for outright breakdown.

$$v_{31}(x, 3) = \begin{cases} v_{31}^{\max} & \text{if } x(\cdot) < B^*, \\ v_{31}^{\min} & \text{otherwise.} \end{cases} \quad (3.26)$$

Where  $B^*$  represents the optimal stock level at which the corrective maintenance rate with lockout/tagout needs to switch from  $v_{31}^{\min}$  to  $v_{31}^{\max}$  for fair condition.

*For two machines that are not identical and operating in passive redundancy to produce one part:*

$$\omega_{41}(x, 4) = \begin{cases} \omega_{41}^{\max} & \text{if } x(\cdot) < C^*, \\ \omega_{41}^{\min} & \text{otherwise.} \end{cases} \quad (3.27)$$

Where  $C^*$  is the optimal stock level at which the corrective maintenance rate with lockout/tagout for outright breakdown on the main machine needs to switch from  $\omega_{41}^{\min}$  to  $\omega_{41}^{\max}$ .

$$\omega_{51}(x, 5) = \begin{cases} \omega_{51}^{\max} & \text{if } x(\cdot) < D^*, \\ \omega_{51}^{\min} & \text{otherwise.} \end{cases} \quad (3.28)$$

Where  $D^*$  is the optimal stock level at which the corrective maintenance rate with lockout/tagout for fair condition on the main machine needs to switch from  $\omega_{51}^{\min}$  to  $\omega_{51}^{\max}$ .

Now, we present the influence of the set of control variables on the production threshold and average cost according to the different values of backlog cost, holding cost and corrective maintenance cost for single machine and two machines of passive redundancy in Table 3.5.

Table 3.5 The variations of production threshold and average cost for M1P1 and M2P1.

$c^-$	$c^+$	$C_{r1}$	$C_{r2}$	Z* (M1P1) Maximal time	Z* (M1P1) Minimal time	Z* (M2P1) Maximal time	Z* (M2P1) Minimal time	Average Cost (M1P1) Maximal time	Average Cost (M1P1) Minimal time	Average Cost (M2P1) Maximal time	Average Cost (M2P1) Minimal time
10	5	30	40	3.25	1.5	1.5	0.75	1063.6	503.6	422.5	348.9
20	5	30	40	5.25	2.5	2	1.5	1627.4	743.1	626.1	467.6
30	5	30	40	6.5	3.5	3	2	2043.3	912.6	750.9	564.2
40	5	30	40	7	3.75	3.25	2	2351.5	1050.3	816.7	600.2
50	5	30	40	8	4.25	3.75	2.25	2591	1140.4	981.8	645.9
60	5	30	40	8.5	4.75	3.75	2.25	2809.7	1232.3	1047.8	692.5
70	5	30	40	9	5.25	4	2.5	3014.2	1316.6	1089.7	704.1
80	5	30	40	9.25	5.5	4	2.75	3219.6	1403.3	1173	765.2
90	5	30	40	9.75	5.5	4.25	2.75	3406.6	1478.8	1253.1	763.6
100	5	30	40	10.25	6	4.5	2.75	3599.3	1551.4	1371.2	811.9
30	1	30	40	3.25	1.25	1	0.75	1063.6	503.4	535.5	348.4
30	2	30	40	1.5	0.25	0.75	0.5	1384.1	629.7	545.5	363.8
30	3	30	40	0.5	0.25	0.5	0.25	1553.5	684.1	555.5	371.3
30	4	30	40	0.25	0.25	0.5	0.25	1672.3	764.2	567.5	380.2
30	5	30	40	0	0	0.5	0.25	1827	924.1	682.5	437.1
30	6	30	40	0	0	0.5	0.25	2128	1400.3	712.8	489.6
30	7	30	40	0	0	0.5	0.25	2727.1	1849.1	778.2	541.8
30	8	30	40	0	0	0.5	0.25	3331.3	2214.1	803.5	591.8
30	9	30	40	0	0	0.5	0.25	3832.1	2521.9	884.7	686.8
30	5	30	40	9.25	5.5	4.75	3.5	1063.6	503.6	315.1	239.7
30	5	40	40	9.25	5.5	4.75	3.5	1063.6	513.2	315.1	255.8
30	5	50	40	9.25	5.5	4.75	3.5	1075.4	522.8	328.3	275.7
30	5	60	40	9.25	5.5	4.75	3.5	1087.1	532.4	340.2	285.2
30	5	70	40	9.25	5.5	4.75	3.5	1098.9	542.1	351.7	294.7
30	5	80	40	9.25	5.5	4.75	3.5	1110.6	532.4	363.1	304.2
30	5	90	40	9.25	5.5	4.75	3.5	1122.4	561.3	374.5	314.2
30	5	100	40	9.25	5.5	4.75	3.5	1134.2	570.9	396.4	325.5
30	5	110	40	9.25	5.5	4.75	3.5	1145.9	580.5	414.4	341.5
30	5	120	40	9.25	5.5	4.75	3.5	1157.7	590.1	432.4	361.4
30	5	30	40	9.5	5.75	5	3.75	1063.6	503.6	326.3	256.5
30	5	30	50	9.5	5.75	5	3.75	1063.6	516.5	326.3	274.6
30	5	30	60	9.5	5.75	5	3.75	1078.5	529.5	341.6	297.5
30	5	30	70	9.5	5.75	5	3.75	1093.5	542.4	356.5	309.1
30	5	30	80	9.5	5.75	5	3.75	1108.4	555.4	372.9	322.54
30	5	30	90	9.5	5.75	5	3.75	1123.4	568.4	387.4	334
30	5	30	100	9.5	5.75	5	3.75	1138.3	581.5	402.8	345
30	5	30	110	9.5	5.75	5	3.75	1153.2	594.5	426.7	359.3
30	5	30	120	9.5	5.75	5	3.75	1168.2	607.4	448.6	377.3
30	5	30	130	9.5	5.75	5	3.75	1183.1	620.2	469.6	400.3

The first section of Table 3.5 presents the variations in stock levels based on backlog costs for one machine as well as two dissimilar machines operating in passive redundancy to make one type of part. In this division, the lockout/tagout and corrective maintenance rates are set at minimum and maximum values for fair condition and outright breakdown. Note that varying backlog costs have little impact on stock levels if we increase lockout/ tagout and corrective maintenance rates. Moreover, variations in stock levels drop rapidly when a standby machine is added to our system. As this division shows, control over these variations is enhanced if we maximize lockout/tagout and corrective maintenance rates for both modes of failure. Average costs based on variations in backlog costs appear also in this division of Table 3.5.

It shows that the variations in average cost are directly related to stock levels when lockout/tagout and corrective maintenance rates are set at minimum and maximum values for fair condition and outright breakdown. We can improve this situation by adding the standby machine at maximal lockout/tagout and corrective maintenance rates for both modes of failure.

The second section of Table 3.5 shows the variations in stock levels based on holding costs for one machine as well as two dissimilar machines operating in passive redundancy to make one type of part. This section discloses significant differences in the two systems under study. Stocks drop precipitously as holding costs rise when a lone machine turns out one type of part. We can mitigate this variation by maximizing lockout/tagout as well as corrective maintenance rates for either mode of failure. A far more homogenous variation occurs with passive redundancy, because it makes possible to respond to demand at all times.

The same section of Table 3.5 shows that how average costs and holding costs move in tandem. Passive redundancy improves control over such costs; lockout/tagout and corrective maintenance rates are maximized to prevent both modes of failure. In the third section of Table 3.5, we illustrate the variations in stock levels and the corrective maintenance costs for fair condition occurring in either of the two cases under study. Increased corrective

maintenance costs produce no material variation in stock levels as well as average costs for fair condition. The variations in stock levels and average costs are far lower under passive redundancy than other cases where lockout/tagout and corrective maintenance rates are set at their highest levels to counter either mode of failure.

The last section of Table 3.5 confirm that when corrective maintenance costs rise in response to outright breakdown, the variations in stock levels and average costs are far lower in the passive redundancy than in other cases, where lockout/tagout and corrective maintenance rates are maximized to combat failure modes 1 and 2. These results support the analysis of the previous section.

Optimal costs of production and corrective maintenance including lockout/tagout procedures for both modes of failure may be determined using the analytical model presented in this paper. The numerical approach demonstrates conclusively that the resulting policy is optimal and enhances machine availability. Control policy for our systems considers extension of the hedging point structure. Without limiting in any way the generality of this proposal, this model is based on certain assumptions relating to a pair of machines which are not identical and which operate in passive redundancy. Given certain conditions, extended versions of this model might be adopted across a number of industrial sectors.

### **3.6 Conclusion**

Purpose of this paper is two-fold. We first sought to verify the influence of lockout/tagout and corrective maintenance rates on two modes of failure in a manufacturing system consisting of one machine producing one type of part. Secondly, we added a standby machine which differed from the main machine to monitor the influence of passive redundancy within our system. This work demonstrates clearly that passive redundancy optimizes production and maintenance costs while enhancing occupational safety. Even greater benefits accrue if effective lockout/tagout and maintenance planning occurs in concert with production control. Subsequent research will seek to verify the influence of lockout/tagout and preventive maintenance rates on a production line operating in a flowshop

(such as an assembly line) with internal buffers for two machines working along side a standby machine.

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## APPENDIX 3.A

### Proof

Let us consider :

- $z(x, \alpha) = x + b(x, \alpha)\Delta t^h(x, \alpha)$  as the point which is reachable from  $x$  under control  $\alpha$  in direction  $b(x, \alpha)$  in time  $\Delta t^h(x, \alpha)$ ;
- $\bar{u}_\alpha^h(x)$  ( $u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ ) minimizing value of  $\alpha$  in equation (3.15);
- $\{\xi_n^h, n < \infty\}$  the approximating controlled of our system;
- $\{\bar{u}_n^h, n < \infty\}$  the actual random variables which are the optimal control actions.

We define  $\Delta t_n^h = \Delta t^h(\xi_n^h, \bar{u}_n^h)$  and  $t_n^h = \sum_{i=0}^{n-1} \Delta t_i^h$ .

Then, we have:

$$E\left[\xi_{n+1}^h - \xi_n^h \mid \xi_n^h = x, \bar{u}_n^h = \alpha\right] = b(x, \alpha)\Delta t^h(x, \alpha) = O(h), \quad (3.15.1)$$

$$\text{cov}\left[\xi_{n+1}^h - \xi_n^h \mid \xi_n^h = x, \bar{u}_n^h = \alpha\right] = O(h^2), \quad (3.15.2)$$

Let  $E_n^h$  denote the expectation, given the state and control actions including time  $n$ .

We rewrite:

$$\xi^h(t) = x + \sum_{n: t_{n+1}^h \leq t} \Delta \xi_n^h \quad \text{and} \quad \Delta \xi_n^h = \xi_{n+1}^h - \xi_n^h \quad \text{in form} \quad \Delta \xi_n^h = E_n^h \Delta \xi_n^h + (\Delta \xi_n^h - E_n^h \Delta \xi_n^h)$$

Then, we have:

$$\xi^h(t) = x + \sum_{n: t_{n+1}^h \leq t} E_n^h \Delta \xi_n^h + \sum_{n: t_{n+1}^h \leq t} (\Delta \xi_n^h - E_n^h \Delta \xi_n^h), \quad (3.15.3)$$

By knowing that the right hand sum in (3.15.3) is a continuous time interpolation of a martingale and the variance of (3.15.2) is:

$$E \sum_{n: t_{n+1}^h \leq t} \Delta t_n^h O(h) = O(h)t$$

Hence, we have:

$$E \sup_{t \leq T} \left| \sum_{n: t_{n+1}^h \leq t} (\Delta \xi_n^h - E_n^h \Delta \xi_n^h) \right|^2 = O(h) \text{ for any } t < \infty$$

Thus the effects of that right hand term in (3.15.3) disappear in the limit.

We write the right hand sum in (3.15.3) simply as  $O(h)$ . We define the continuous parameter interpolation  $\bar{u}_\alpha^h(\cdot)$  by  $\bar{u}_\alpha^h(t) = \bar{u}_n^h$  ( $u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ ) on the interval  $[t_{n+1}^h, t_n^h)$ .

Now, using (3.15.1), we have:

$$\begin{aligned} \xi^h(t) &= x + \sum_{n: t_{n+1}^h \leq t} E_n^h \Delta \xi_n^h + O(h) \\ &= x + \int_0^t b(\xi^h(s), \bar{u}_\alpha^h(s)) ds + O(h), \end{aligned} \quad (3.15.4)$$

with:

$u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ .

We now proceed to show that there is some admissible control such that the paths of  $\xi^h(\cdot)$  are actually good approximations to a solution of (3.15). Because  $\Delta t_n^h = O(h)$  and  $\Delta t_n^h \geq k_1 h$ , the piecewise linear interpolations of the paths of process  $\xi^h(\cdot)$  are equicontinuous in  $(w$  and  $h)$ . Thus, for each fixed value of the probability space variable  $w$ , each subsequence of  $\{\xi^h(\cdot)\}$  has a further subsequence which converges to some limit uniformly on each bounded time interval. We now fix the sample space variable  $w$  and let  $h_n(w)$  index a convergent subsequence of the piecewise linear interpolations of  $\{\xi^h(\cdot), \bar{u}_i^h(\cdot)\}$  with limit denoted by  $x(\cdot, w), u_\alpha(\cdot, w)$  ( $u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ ). By the convergence and the compactness of  $U_\alpha$  we have  $u_\alpha(t, w) \in U_\alpha$  ( $u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ ).

The uniform convergence implies that :

$$x(t, w) = x + \int_0^t b(x(s, w), u_\alpha(s, w)) ds, \quad (3.15.5)$$

with:

$u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ .



Thus, the limit path satisfies (3.15) with an admissible control. Also, it is easy seen that  $V^{h,(w)}(x, \alpha)$  converges to  $J(x, u_1(w), u_2(w), \alpha, \omega_{41}, \omega_{51})$ .

Due to the minimality of  $V(x, \alpha)$ , we have:

$$J(x, u_1(w), u_2(w), \alpha, \omega_{41}, \omega_{51}) \geq V(x, \alpha), \quad (3.15.6)$$

Because this holds for each  $w$ ,

$$\lim_{h \rightarrow 0} V^h(x, \alpha) \geq V(x, \alpha), \quad (3.15.7)$$

We now need to get the reverse inequality to (3.15.7). Namely, that there exists an optimal admissible control for finite difference type approximations and that the control can be arbitrarily well approximated by a control which is piecewise constant. That is, given any  $\varepsilon > 0$ , there is an  $\varepsilon$ -optimal control  $u_\alpha^\varepsilon(\cdot)$  ( $u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ ) of the following form: there is  $\delta > 0$  and a finite number of points  $U_\alpha^\varepsilon$  in  $U_\alpha$  ( $u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ ) such that  $u_\alpha^\varepsilon(\cdot)$  is  $U_\alpha^\varepsilon$ -valued and is constant on the intervals  $[j\delta, j\delta + \delta)$ . we now apply this  $\varepsilon$ -optimal control to a passive redundancy system and use the minimality of  $V^h(x, \alpha)$  for the controlled our system to get the reverse inequality to (3.15.7).

We proceed as follows: we fix  $\varepsilon > 0$  and we define a sequence  $u_\alpha^{h,\varepsilon}(\cdot)$  ( $u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ ) of controls for the passive redundancy system by adapting the above  $\varepsilon$ -optimal control  $u_\alpha^\varepsilon(\cdot)$  ( $u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ ) in the following natural way:

Let us consider  $h$  small enough such that  $\delta > \sup_{x,\alpha} \Delta t^h(x, \alpha)$ . We now define the sequences

$u_n^{h,\varepsilon}, \Delta t_n^h = \Delta t^h(\xi_n^{h,\varepsilon}, u_n^{h,\varepsilon})$ , and  $t_n^h = \sum_0^{n-1} \Delta t_i^h$ , recursively by  $u_n^{h,\varepsilon} = u_\alpha^\varepsilon(j\delta)$  for all  $n$  such that  $t_n^h \in [j\delta, j\delta + \delta)$ , for each  $j$ .

Let us  $\xi_n^{h,\varepsilon}(\cdot)$  and  $u_\alpha^{h,\varepsilon}$  ( $u_\alpha = u_1$  if  $\alpha = 1, 2, 3$  and  $u_\alpha = u_2$  if  $\alpha = 4, 5$ ) denote the continuous parameter interpolation (interpolation intervals  $\Delta t_n^h$ ). Note that  $u_\alpha^{h,\varepsilon}$  converges to

$u_\alpha^\varepsilon(\cdot)$  as  $h \rightarrow 0$ , except possibly at the points  $j\delta$ . Now, we fix  $w$  and choose a convergent subsequence of  $\xi^{h,\varepsilon}(\cdot)$  (the sequence need not be the same for each  $w$ ). Let us  $x^\varepsilon(\cdot, w)$  denote the limit. Then, following the analysis which led to (3.15.5), we get that:

$$x^\varepsilon(t, w) = x + \int_0^t b(x^\varepsilon(s, w), u_\alpha^\varepsilon(s, w)) ds$$

with:

$$u_\alpha = u_1 \text{ if } \alpha = 1, 2, 3 \text{ and } u_\alpha = u_2 \text{ if } \alpha = 4, 5.$$

The limit paths  $x^\varepsilon(\cdot, w)$  are all the same, irrespective of the chosen subsequence or of  $w$ , because the solution to (3.15) is unique under the chosen control  $u_i^\varepsilon(\cdot)$ . This implies that the sequence  $J^h(x, u_1^{h,\varepsilon}, u_2^{h,\varepsilon}, \alpha, \omega_{41}, \omega_{51})$  converges to the  $\varepsilon$ -optimal cost  $J(x, u_1^\varepsilon, u_2^\varepsilon, \alpha, \omega_{41}, \omega_{51})$ . Now, using the optimality of  $V^h(x, \alpha)$ , we have:

$$\lim_{h \rightarrow 0} V^h(x, \alpha) \leq \lim_{h \rightarrow 0} J^h(x, u_1^{h,\varepsilon}, u_2^{h,\varepsilon}, \alpha, \omega_{41}, \omega_{51}) = J(x, u_1^\varepsilon, u_2^\varepsilon, \alpha, \omega_{41}, \omega_{51}) \leq V(x, \alpha) + \varepsilon, \quad (3.15.8)$$

Inequalities (3.15.7) and (3.15.8) imply that  $V^h(x, \alpha) \rightarrow V(x, \alpha)$ .

## REFERENCES

- Akella, R. and Kumar, P.R., 1986. Optimal Control of Production Rate in a Failure-Prone Manufacturing System. *IEEE Transactions on Automatic Control* AC 31, pp. 116-12.
- Boukas, E.K. and Haurie, A., 1990. Manufacturing Flow Control and Preventive Maintenance: A Stochastic Control Approach. *IEEE Transactions on Automatic Control* 33, pp. 1024-1031.
- Charlot, E., Kenné, J.P. and Nadeau, S., 2006. Optimal Production, Maintenance and Lockout/tagout Control Policies in Manufacturing Systems. *Int. J. Production Economics* 107, pp. 435-450.
- Coit, D.W., 2001. Cold-standby Redundancy Optimization for Nonrepairable Systems. *IIE Trans*, pp.471–478.
- Ebeling, C.E., 1997. An introduction to reliability and maintainability engineering. New York: McGraw-Hill Companies, Inc, pp.308.
- Gershwin, S.B., 2002. Manufacturing Systems Engineering. Prentice-Hall, Englewood Cliffs, NJ, pp.267.
- Gharbi, A. and Kenné, J.P., 2003. Optimal Production Control Problem in Stochastic Multiple-Product Multiple-Machine Manufacturing Systems. *IIE Transactions* 35, pp. 941-952.
- Gwo-Liang, L., Yen Hung, C. and Shey-Huei, S., 2009. Optimal Economic Production Quantity Policy for Imperfect Process with Imperfect Repair and Maintenance. *European Journal of Operational Research*, vol. 195, pp. 348-357.
- Harichaux, P. and Libert, J., 2003. Ergonomie et Prévention des Risques Professionnels, L'Environnement Physique du Travail et ses Contraintes, vol. 1, Chiron.
- Kenné, J.P, Boukas, E.K. and Gharbi, A. 2003, Control of Production and Corrective Maintenance Rates in a Multiple-Machine, Multiple-product Manufacturing System, *Mathematical and Computer Modelling* 38, pp. 351-365.
- Kimemia, J. and Gershwin, S.B., 1983. An Algorithm for the Computer Control of Production in Flexible Manufacturing Systems. *IEEE Transactions on Automatic Control* 15, pp.353-362.
- Kushner, H.J. and Dupuis, P.G., 1992. Numerical Methods for Stochastic Control Problems in Continuous Time. Springer, New York.
- Kumar, S., and Kumar, D. and Mehta, N.P. 1996. Behavioural Analysis of a Shell Gasification and Carbon Recovery Process in a Urea Fertilizer Plant. *Microelectron Reliab*, pp. 671–673.
- Nollet, J., Kelada, J., O.Diorio, M., Handfield, H., Deschamps, I. and R.Duguay, C., 1994. La Gestion des Opérations et de la Production, Une Approche Systématique. 2<sup>e</sup> Édition, Gaëtan Morin, pp. 163

- Older, G.J and Suri, R., 1980. Time Optimal of Part-Routing in a Manufacturing System with Failure Prone Machines. In: Proceedings of the 19<sup>th</sup> IEEE Conference on Decision Control, vol. 5, pp. 722-727.
- Pandey, D., Jacob, M. and Yadav, J. 1996. Reliability Analysis of a Power Loom Plant with Cold-standby for its Strategic Unit. *Microelectron Reliab*, pp. 115–119.
- Sinaki, G., 1994. Ultra-reliable Fault-tolerant Inertial Reference Unit for Spacecraft. In: Proceedings of the annual rocky mountain guidance and control conference. Univelt Inc., San Diego CA, pp. 239–48.
- Subramanian, R., and Anantharaman V. 1995. Probabilistic Analysis of a Three Unit Cold Standby Redundant System with Repair. *Microelectron Reliab*, pp.1001–1008.
- Yan, H. and Zhang, Q., 1997. A Numerical Method in Optimal Production and Setup Scheduling to Stochastic Manufacturing Systems. *IEEE Transactions on Automatic Control* 42, pp. 1452-1455.
- Zhang, Y.L., Wang, G.J. and Ji, Z.C. 2006. Replacement Problems for a Cold Standby Repairable System, *International Journal of Systems Science*, pp.17–25.
- Zhang, Y.L. and Wang, G.J. 2006. A Bivariate Optimal Repair-replacement Model Using Geometric Process for Cold Standby Repairable System, *Eng. Optim*, pp.609–619.
- Zhang, Y.L. and Wang, G.J. 2007. A Deteriorating Cold Standby Repairable System with Priority in Use, *European Journal of Operational Research*, pp.278–295.

## CHAPITRE 4

### ARTICLE 2: LOCKOUT/TAGOUT AND OPTIMAL PRODUCTION CONTROL POLICIES IN FAILURE-PRONE NON-HOMOGENOUS TRANSFER LINES WITH PASSIVE REDUNDANCY

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#### Résumé

Ce présent article analyse une ligne de production constituée de trois machines produisant un type de pièce (deux sous forme redondance passive et une troisième en série avec les précédentes). Les machines sont sujettes à une dégradation, à des pannes, à des réparations aléatoires. Le problème de contrôle considéré est soumis à des contraintes non-négatives des inventaires en cours (*work-in-process*). La capacité du système est décrite par une chaîne de Markov homogène à états finis. Les variables de décision sont les taux de production des machines. Les variables de décision influencent le niveau des inventaires intermédiaires, des inventaires de produits finis et la capacité du système manufacturier. L'objectif de cette contribution est divisé en deux (2) volets : 1) Trouver les variables de décision permettant de réduire les coûts totaux production, comprenant les coûts d'inventaire et de pénurie sur un horizon infini de planification. 2) Intégrer la troisième machine sous forme redondance passive afin de libérer un espace-temps essentiel pour minimiser les possibilités de contournement des dispositifs de protection ou d'escamotage des procédures de cadenassage/décadenassage (C/D). Afin d'avoir un modèle plus réaliste pour les industries, les conditions d'optimum proposées sont développées en utilisant une approche combinée, qui est présentée sur la base d'une combinaison de formalisme analytique, la simulation, le

plan d'expérience et la méthodologie de surface de réponse. L'utilité de l'approche proposée est illustrée par un exemple numérique et une analyse de sensibilité.

## **Abstract**

This paper considers a production problem for a transfer line subject to random failures and repairs, and differs from other studies on transfer lines. It considers a manufacturing system consisting of three machines (two machines with passive redundancy, and one in series with the previous ones) producing one part type. The control problem is subject to non-negative constraints on work-in-process (WIP). The decision variables are the production rates of two main machines and a standby machine, and influence the WIP levels, the inventory levels and the system's capacity, which is assumed to be described by a finite-state Markov chain. The objective of this paper is to minimize WIP and finished goods inventory costs; it also aims to respect the essential space-time during intervention on machine down, in order to minimize the possibility of the circumvention of protection devices or of the retraction of lockout/tagout procedures through a passive redundancy system. This paper therefore verifies the effect of passive redundancy on optimal stock levels. Given that an analytical or even a numerical solution of the problem is very difficult to find, and that we want to have a more realistic model for industries, we present a combined approach, which is presented based on a combination of analytical formalism, simulation modeling, design of experiments and response surface methodology to optimize a transfer line with passive redundancy, producing one part type. The usefulness of the proposed approach is illustrated through a numerical example and a sensitivity analysis.

**Keywords:** Production control; Lockout/tagout; Passive redundancy; Simulation, Experimental design; Response surface methodology.

## **4.1 Introduction**

To meet customer demand more and more demanding, and in the face of intense competition, companies must be able to produce good quality products at lower cost. This requirement

necessarily makes performance a major concern for all managers. To achieve this goal, we must ensure equipment availability, both functionally and in terms of optimal use. Production systems optimization is a topic that is of interest to researchers as well as to industry. To increase the availability of machines at the functional level, several maintenance strategies have been developed (Kenne et al., 2003; Kenne and Gharbi, 1999; Rezg et al., 2004). The problem of optimally controlling the production rates of a manufacturing system has been widely discussed in the scientific literature. For two decades, significant effort has been devoted to optimizing production systems to meet their complexity, competition and challenges of globalization. However, despite the effectiveness of the techniques developed, other challenges, including control and risk management of adverse events during maintenance, still lie ahead.

Studies conducted in Quebec by the Institut de recherche Robert-Sauvé en santé et sécurité du travail (IRSST), (Chinniah and Champoux, 2008) showed that in 2005, dangerous machinery led to the deaths of about 20 workers in Quebec, and that there were 13,000 accidents linked to them. These accidents also caused \$70 million in damages settled by the Commission de la santé et de la sécurité du travail (CSST). Another study on the safety of lockout/tagout of Sawmill equipment in Quebec conducted by the Institut de recherche Robert-Sauvé en santé et sécurité du travail (IRSST), (Giraud et al., 2008) led the occupational health and safety (OHS) community to recognize that nearly a quarter of all accidents occur during interventions by workers on machines that are down. Manufacturing systems operate in a stochastic environment because machines are subject to random breakdowns and repairs, and in addition, demand for finished goods may vary. It is possible to predict and control certain events while others occur randomly, and are beyond the control of manufacturing systems (Gershwin, 2002). Production systems' dynamics degrade with the number of breakdowns and repairs, and with a rise in the number of breakdowns and repairs comes a reduction in the availability of machines, as well as an increase in the number of occupational hazards associated with maintenance activities; further, they can lead to serious machine-related accidents, which are costly in terms of human life, sick leave days and general financial costs. An important question arises: How can an optimal production policy

be maintained in an uncertain environment, while increasing the safety of maintenance workers? The preferred method used to overcome these problems is the lockout/tagout. It consists in locking a machine with a padlock in order to discharge all sources of residual energy (hydraulic, electrical, etc.) in order to avoid premature starting of equipment throughout a maintenance intervention.

Many managers and workers wrongly believe it takes too long to plan and carry out lockout/tagout, thinking the accompanying downtime reduces productivity or performance. Consequently, lockout/tagout is often deficient because idle production time is seen as a hurdle to planned production rates. In this view, Charlot et al. (2006) considered an analytical model combining lockout/tagout, production and corrective maintenance policies for a single machine producing one part type. This work showed that lockout/tagout time can better be controlled with the proper scheduling in production plan control. Another work integrating lockout/tagout into operational risk in production control is proposed by Emami-Mehrgani et al. (2011). They considered a manufacturing system with passive redundancy consisting of two non-identical machines. Their work demonstrates clearly that passive redundancy optimizes production and maintenance costs while enhancing occupational safety. Even greater benefits occur if effective lockout/tagout maintenance planning occurs in concert with production control.

Many authors have contributed to solving manufacturing systems production planning problems. Based on Rishel's formulation (Rishel, 1975), Older and Suri (1980) devised manufacturing systems having unreliable machine control problems. In their model, breakdowns and repairs are described by a homogeneous Markov process. The main difficulty with this approach is its lack of efficient methods for solving the optimization problem characterized by Hamilton-Jacobi-Bellman (HJB) equations. Akella and Kumar (1986) analytically solved a one machine, one part-type problem. In this view, Lou and Zhang (1994) extended the problem in Akella and Kumar (1986) to a flow control problem for a tandem production system consisting of two unreliable machines, and conducted a rigorous study of the dynamic properties of the system. Similarly, Presman et al. (1995)



considered a production planning problem in an N-machine flow-shop system subject to machine breakdowns and repairs and to non-negativity constraints on work-in-process. Based on the formulation presented in Presman et al. (1995), Hajji et al. (2009) studied production and change-over control production for a buffered flow-shop producing several types of parts. Hajji et al. (2009) have developed dynamic programming equations in terms of problem directional derivative (DPEDD) and adopted a numerical approach to solve them. The purpose of this paper is to control the production rate of the machines in a transfer line with passive redundancy, which is one of the most important structures in reliability engineering, and has been widely used in manufacturing systems. The main contributions of this work are divided into two parts: reduce production costs and provide free space-time to minimize the possibility of circumvention of protection devices or retraction of lockout/tagout procedures for machines under repair. These goals are reached for a transfer line through a passive redundancy system.

In this paper, by making use of the fact that the value function is the unique solution for the associated Hamilton-Jacobi-Bellman (HJB) equations, in terms of directional derivatives (DD), the structure of the solution under appropriate conditions is obtained. Given that an analytical or even a numerical solution of the problem is very difficult to find and that we want to have a more realistic model for industries, we present in this paper an alternative procedure based on the combination of analytical control approach and the experimental design method based on simulation experiments to find an approximation of the optimal control policy. A simulation based experimental design approach is combined with the control theory to develop a systematic control approach as in Gharbi and Kenne (2003) in the case of production line with passive redundancy. The proposed control approach consists of estimating the relationship between the incurred cost and the parameters of the control policy considered in this paper as control factors. The hedging point policy, parameterized by these factors, is used to conduct simulation experiments. For each configuration of input factors values, the simulations model is used to determine the related output (incurred cost). An input-output data set is then generated by simulation model. The significant effects of input factors are determined by experimental design, and a response surface methodology is used

to obtain the relationship between the input and the output factors in order to estimate the cost function. The best values for control factors are determined through this relationship.

This article is organised as follows. Assumptions and notations are defined in the next section. In section 4.3, we provide the problem statement. Section 4.4 provides a numerical example and a sensitivity analysis, and the related production policy is presented. In section 4.5, we present the control approach, the experimental design and response surface methodology. Finally, we conclude the paper in section 4.6.

## 4.2 Assumptions and notations

This paper incorporates the following assumptions and notations:

### 4.2.1 Assumptions

1. Corrective maintenance is carried out with lockout/tagout.
2. The main machine is more robust than the standby machine.
3. The main machine and the standby machine produce the same type of parts for the work-in-process (WIP).
4. The main machine returns to production immediately after each repair (corrective maintenance with lockout/tagout) and the standby machine stands idle.

Assumption 4 is a classical assumption with a passive redundancy system, and is due to the nature of a passive redundancy system.

### 4.2.2 Notations

The following notations are used in the rest of this article:

$x_1(\cdot)$ : inventory level of work-in-process

$x_2(\cdot)$ : inventory/backlog level of finished product

$c_1^+$ : holding cost incurred on buffer

- $c_2^+$  : holding cost incurred on finished product  
 $c_2^-$  : backlog cost incurred on finished product  
 $c^\alpha$  : cost incurred for the operation of the machine under repair at mode  $\alpha$   
 $c_{r_1}$  : corrective maintenance cost of main machine  $M_1$   
 $c_{r_2}$  : corrective maintenance cost of main machine  $M_2$   
 $c_{r_s}$  : corrective maintenance cost of standby machine  $M_s$   
 $c_{tagout}$  : lockout/tagout cost  
 $u_{r_1}$  : corrective maintenance rate with lockout/tagout of main machine  $M_1$   
 $u_{r_2}$  : corrective maintenance rate with lockout/tagout of main machine  $M_2$   
 $u_{r_s}$  : corrective maintenance rate with lockout/tagout of standby machine  $M_s$   
 $g(\cdot)$  : instantaneous cost  
 $J(\cdot)$  : total cost  
 $v(\cdot)$  : value function  
 $\rho$  : discount rate  
 $d$  : demand rate  
 $u_i(\cdot)$  : production rate of machine  $i$  ( $i=1, 2, s$ )  
 $u_i^{\max}(\cdot)$  : machine's  $i$  ( $i=1, 2, s$ ) maximal production rate  
 $q_{12}^{1,2}$  : main machines  $M_1$  and  $M_2$  failure rate  
 $q_{12}^s$  : standby machine  $M_s$  failure rate

## 4.2 Problem statement

In this paper, we consider a flow control problem for a tandem production system with passive redundancy consisting of three unreliable machines. Two machines are in series ( $M_1$  and  $M_2$ ) and one machine is in standby with the main machine  $M_1$ , namely  $M_s$  (standby machine). Recall that in this paper, by using the fact that the value function is the unique solution for the associated Hamilton-Jacobi-Bellman (HJB) equations, in terms of directional derivatives (DD), the structure of the solution under appropriate conditions is obtained. Since either an analytical, a numerical solution of this problem or even an explicit functional relationship between the independent variables of the model (stock level) and performance criteria (cost incurred) is not usually available, an alternative procedure based on the combination of analytical control approach and the experimental design method based on simulation experiments is presented in this paper. A parameterized near-optimal control policy is used in the proposed control approach as input for the simulation model. In order to propose an approach which could be easily applied to control manufacturing systems for a tandem production system with passive redundancy. The manufacturing system is consisting of three unreliable machines (two machines with passive redundancy, and one in series with the previous ones) at the operational level, the descriptive capacities of discrete event simulations models are combined with analytical models, experimental design, and response surface methodology. The system is shown in Figure 4.1. The main machines have two states:  $\xi_{1,2}(t) = 1$  if the main machine  $M_i$  ( $i = 1, 2$ ) is operational and  $\xi_{1,2}(t) = 2$  if the main machine  $M_i$  ( $i = 1, 2$ ) is under repair. The standby machine has three states:  $\xi_s(t) = 1$  if the standby machine  $M_s$  is operational,  $\xi_s(t) = 2$  if the standby machine  $M_s$  is under repair and  $\xi_s(t) = 3$  if the standby machine  $M_s$  is at time-off. Hence, the dynamics for a manufacturing system consisting of three machines (two machines with passive redundancy and one machine in series with the previous ones) is in a hybrid state comprised of a continuous state and a discrete state  $\xi(t) = (\xi(t)_1, \xi(t)_2, \xi(t)_s)$  as follows:

a) *Continuous state*: We denote the number of parts in the work-in-process (WIP) as  $x_1(t)$  and the difference between cumulative production and demand as  $x_2(t)$ . Note that the control problem is subject to non-negative constraints on work-in-process (WIP), meaning that  $x_1(t) \geq 0$ . The surplus  $x_2(t)$  can be positive (i.e., inventory costs  $c_2^+$  are thus charged) or negative (i.e. backlog costs  $c_2^-$  are thus charged). We use  $u_i(t)$  with  $(i=1,2,s)$  to denote the input rate to  $M_i$  and  $x_1(t)$  to denote the number of parts in the buffer between  $M_i$  and  $M_2$  ( $i=1,s$ ).

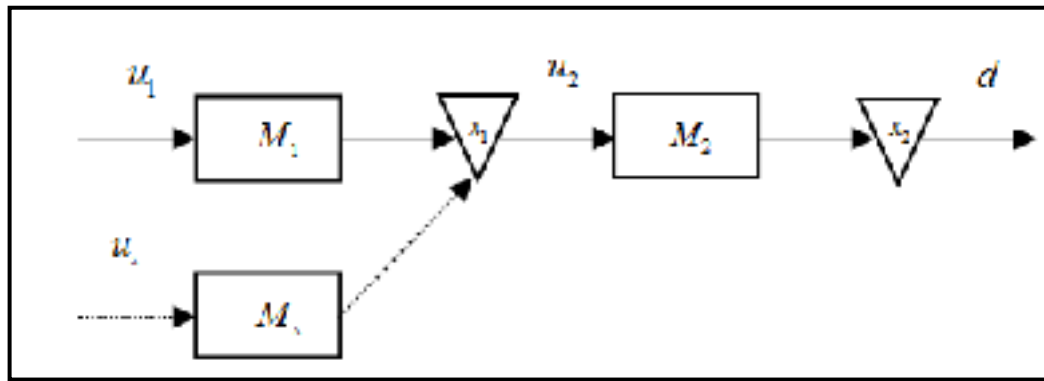


Figure 4.1 Transfer line with passive redundancy producing one part type

The dynamics of the system can be written as follows:

$$\dot{x}_1(t) = \tilde{u}_1(t) - \tilde{u}_2(t) = u_1(t)I_1^\alpha + u_s(t)I_s^\alpha - u_2(t), \quad x_1(0) = x_1, \quad (4.1)$$

with:

$$I_1^\alpha = \begin{cases} 1 & \text{if } \alpha = 3, 4, 5, 6, 7, 8, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad I_s^\alpha = \begin{cases} 1 & \text{if } \alpha = 1, 2, \\ 0 & \text{otherwise.} \end{cases}$$

Note that  $I_1^\alpha$  and  $I_s^\alpha$  present the state of main machines and standby machine in each mode.

$$\dot{x}_2(t) = \tilde{u}_2(t) - \tilde{u}_3(t) = u_2(t) - u_3(t), \quad x_2(0) = x_2, \quad (4.2)$$

with:

$$u_3(t) := d.$$

In matrix notation, the system of equation (4.1)-(4.2) becomes:

$$\dot{x}(t) = A\tilde{u}(t), \quad x(0) = x, \quad (4.3)$$

where  $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ ,  $\tilde{u}(t) = (\tilde{u}_1(t), \tilde{u}_2(t), \tilde{u}_3(t)) = (u_1(t)I_1^\alpha + u_s(t)I_s^\alpha, u_2(t), u_3(t))$  and  $x(t) = (x_1(t), x_2(t))$ ,

with:

$$I_1^\alpha = \begin{cases} 1 & \text{if } \alpha = 3, 4, 5, 6, 7, 8 \\ 0 & \text{otherwise} \end{cases}, \quad I_s^\alpha = \begin{cases} 1 & \text{if } \alpha = 1, 2 \\ 0 & \text{otherwise} \end{cases}, \quad \text{and } u_3(t) := d.$$

b) *Discrete state*: Consistent with assumptions set out in section 4.2.1, the operational mode of the whole system can be described by a random vector  $\xi(t) = (\xi_1(t), \xi_2(t), \xi_s(t))$  taking values in  $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ . Without loss of generality, for the three-machine flow-shop case with passive redundancy,  $\xi(t)$  can be expressed as follows:

$$\xi(t) = \begin{cases} 1 & M_1 \text{ is under repair, } M_2 \text{ is operational and } M_s \text{ is operational;} \\ 2 & M_1 \text{ is under repair, } M_2 \text{ is under repair and } M_s \text{ is operational;} \\ 3 & M_1 \text{ is operational, } M_2 \text{ is operational and } M_s \text{ is under repair;} \\ 4 & M_1 \text{ is operational, } M_2 \text{ is under repair and } M_s \text{ is under repair;} \\ 5 & M_1 \text{ is under repair, } M_2 \text{ is operational and } M_s \text{ is under repair;} \\ 6 & M_1 \text{ is under repair, } M_2 \text{ is under repair and } M_s \text{ is under repair;} \\ 7 & M_1 \text{ is operational, } M_2 \text{ is operational and } M_s \text{ is at time-off;} \\ 8 & M_1 \text{ is operational, } M_2 \text{ is under repair and } M_s \text{ is at time-off.} \end{cases}$$

Figure 4.2 displays the modes of the system associated to the process  $\xi(t)$ .

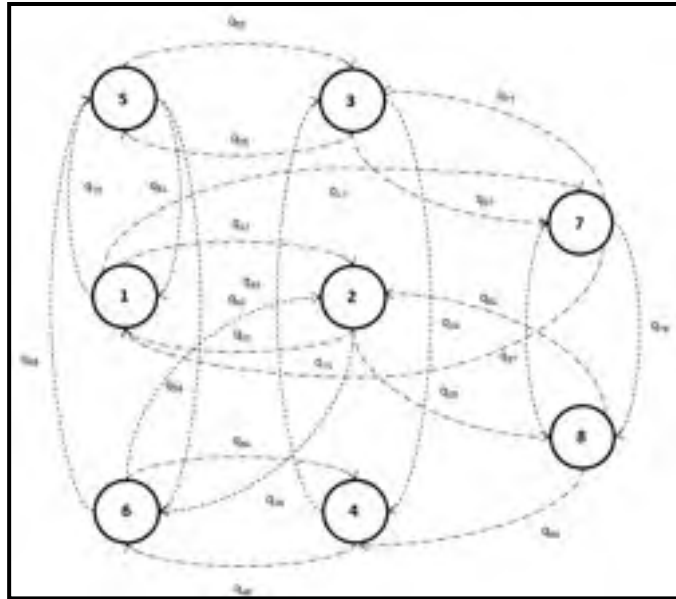


Figure 4.2 State transition diagram

The transition rate matrix of the stochastic processes  $\xi(t)$  is denoted by  $Q$  such that  $Q = \{q_{\alpha\beta}\}$ , with  $q_{\alpha\beta} > 0$  if  $\alpha \neq \beta$  and  $q_{\alpha\alpha} = -\sum_{\beta \neq \alpha} q_{\alpha\beta}$ , where  $\alpha, \beta \in B$ .

The transition probabilities associated to  $q_{\alpha\beta}$  are expressed as:

$$p[\xi(t+\delta t) = \beta | \xi(t) = \alpha] = \begin{cases} q_{\alpha\beta}(\cdot)\delta t + o(\delta t) & \text{if } \alpha \neq \beta, \\ 1 + q_{\alpha\alpha}(\cdot)\delta t + o(\delta t) & \text{if } \alpha = \beta. \end{cases} \quad (4.4)$$

The transition rate matrix  $Q$  is expressed as follows:

$$\begin{bmatrix} q_{11} & q_{12} & 0 & 0 & q_{15} & 0 & q_{17} & 0 \\ q_{21} & q_{22} & 0 & 0 & 0 & q_{26} & 0 & q_{28} \\ 0 & 0 & q_{33} & q_{34} & q_{35} & 0 & q_{37} & 0 \\ 0 & 0 & q_{43} & q_{44} & 0 & q_{46} & 0 & q_{48} \\ q_{51} & 0 & q_{53} & 0 & q_{55} & q_{56} & 0 & 0 \\ 0 & q_{62} & 0 & q_{64} & q_{65} & q_{66} & 0 & 0 \\ q_{71} & 0 & q_{73} & 0 & 0 & 0 & q_{77} & q_{78} \\ 0 & q_{82} & 0 & q_{84} & 0 & 0 & q_{87} & q_{88} \end{bmatrix}, \quad (4.5)$$

The set of admissible decisions at mode  $\alpha(t)$  and control policies (control variables) at mode  $\alpha(t)$ :

$$\Gamma(x, \alpha) = \left[ \begin{array}{l} ((u_1(\cdot), u_2(\cdot), u_s(\cdot)) \in R^3, \\ 0 \leq u_1(\cdot) \leq u_1^{\max}, 0 \leq u_2(\cdot) \leq u_2^{\max}, \\ 0 \leq u_s(\cdot) \leq u_s^{\max} \end{array} \right], \quad (4.6)$$

In equation (4.6),  $u_1^{\max}$  is the main machine's  $M_1$  maximal production rate,  $u_2^{\max}$  is the main machine's  $M_2$  maximal production rate and  $u_s^{\max}$  is the standby machine's  $M_s$  maximal production rate.  $\Gamma(x, \alpha)$  denotes the set of all admissible controls with respect to  $x \in \Lambda$  and  $\alpha(0) = \alpha$ . Let  $\Lambda = [0, \infty) \times R \subset R^m$  denote the state constraint domain.

The control problem consists in finding an admissible control law  $u(\cdot) = (u_1, u_2, u_s)$  that minimizes the cost function  $J(\cdot)$  given by:

$$J(\alpha, x, u) = E \left\{ \int_0^{\infty} e^{-\rho t} g(\alpha, x, \cdot) dt \mid x(0) = x, \xi(0) = \alpha \right\}, \quad (4.7)$$

where  $\rho$  is the discount rate and  $g(x, \alpha, \cdot) = c_1^+ x_1^+ + c_2^+ x_2^+ + c_2^- x_2^- + c^\alpha$  is the instantaneous cost,  $c^+$ ,  $c^-$  and  $c^\alpha$ , being the cost per unit to produce parts for inventory, backlog as well as intervention cost on the machine, respectively.

$$x^+ = \max\{0, x\}, x^- = \max\{-x, 0\}$$

and

$$\begin{aligned} c^\alpha = & ((c_{r_1} + c_{tagout})u_{r_1}) \text{Ind}\{\alpha = 1\} + ((c_{r_1} + c_{tagout})u_{r_1} + (c_{r_2} + c_{tagout})u_{r_2}) \text{Ind}\{\alpha = 2\} \\ & + ((c_{r_s} + c_{tagout})u_{r_s}) \text{Ind}\{\alpha = 3\} + ((c_{r_s} + c_{tagout})u_{r_s} + (c_{r_2} + c_{tagout})u_{r_2}) \text{Ind}\{\alpha = 4\} \\ & + ((c_{r_s} + c_{tagout})u_{r_s} + (c_{r_1} + c_{tagout})u_{r_1}) \text{Ind}\{\alpha = 5\} \\ & + ((c_{r_s} + c_{tagout})u_{r_s} + (c_{r_1} + c_{tagout})u_{r_1} + (c_{r_2} + c_{tagout})u_{r_2}) \text{Ind}\{\alpha = 6\} \\ & + ((c_{r_2} + c_{tagout})u_{r_2}) \text{Ind}\{\alpha = 8\} \end{aligned}$$



With:

$$\text{Ind}\{\Theta(\cdot)\} = \begin{cases} 1 & \text{if } \Theta(\cdot) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

For a given proposition  $\Theta(\cdot)$ . The cost of corrective maintenance with logout/tagout depends on the duration of lockout/tagout and repair activities. This cost is described for main machine  $M_1$  by  $(c_{r_1} + c_{tagout})u_{r_1}$ , for the main machine  $M_2$  by  $(c_{r_2} + c_{tagout})u_{r_2}$  and for the standby machine  $M_s$  by  $(c_{r_s} + c_{tagout})u_{r_s}$ .

Let  $v(x, \alpha)$  denote the value function or minimum discounted cost for equations (4.7) as expressed in the following equation:

$$v(x, \alpha) = \min_{u \in \Gamma(x, \alpha)} J(\alpha, x, u), \forall \alpha \in \mathbf{B} \quad (4.8)$$

In Appendix 4.A, we present the properties of the value function  $v(\cdot)$  given by equation (4.8). It is shown that the value function  $v(\cdot)$  given by (4.8) should satisfy a set of partial differential equations known as the Hamilton-Jacobi-Bellman (HJB) equations in terms of directional derivatives (DD).

### 4.3 Numerical example and sensitivity analysis

Let us consider a transfer line with three non-identical machines (two machines with passive redundancy and one machine in series with the previous ones). The system capacity is described by an eight Markov process with states  $\xi(t) \in \mathbf{B} = [1, 2, 3, 4, 5, 6, 7, 8]$ .

The discrete dynamic programming equation (4.A.6) in Appendix 4.A gives the eight equations which are presented in Appendix 4.B.

We use the following computational domain:

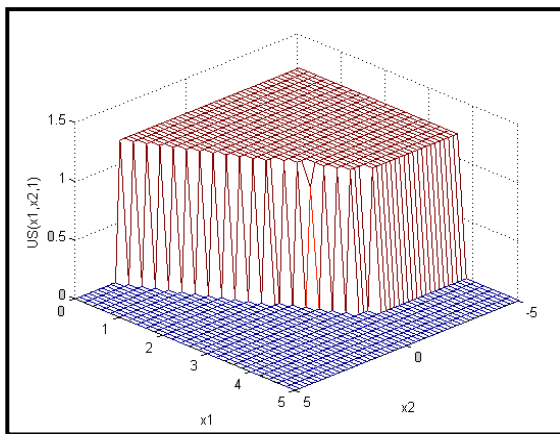
$$G_x^h = \{(x_1, x_2) : 0 \leq x_1 \leq 5; -5 \leq x_2 \leq 5\},$$

The parameters for our case study appear in Table 4.1.

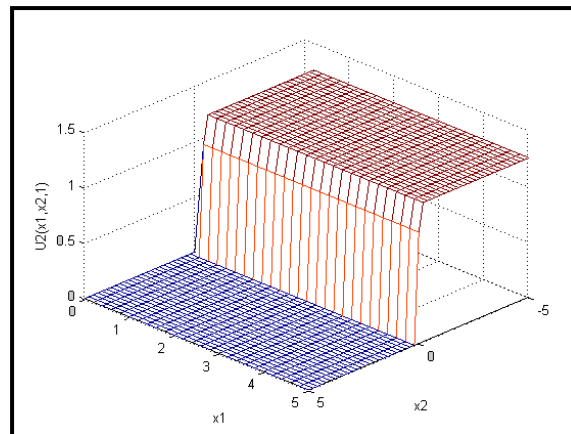
Table 4.1 Parameters of the numerical example

Parameter	$c_1^-$	$c_2^-$	$c_3^-$	$q_{12}^{12}$	$q_{12}^2$	$u_{r_1}$	$u_{r_2}$	$u_{r_3}$	$u_1^{max}$
Value	1	10	150	0.02	0.03	0.08	0.08	0.08	1.3
Parameter	$u_2^{max}$	$u_3^{max}$	$c_{regul}$	$c_t$	$c_{i_2}$	$c_{i_3}$	$\rho$	$d$	
Value	1.25	1.2	50	150	200	250	0.2	1	

The results obtained for the control variables  $u_1, u_2$  and  $u_s$  of a production line with a passive redundancy system are given in Figures (4.3)-(4.9) for illustration purposes.

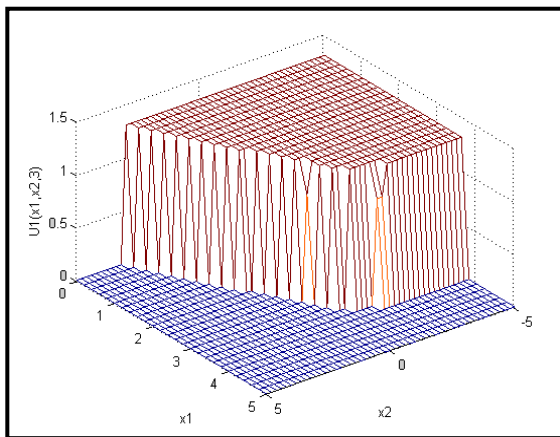


(a) Production rate of  $M_s$

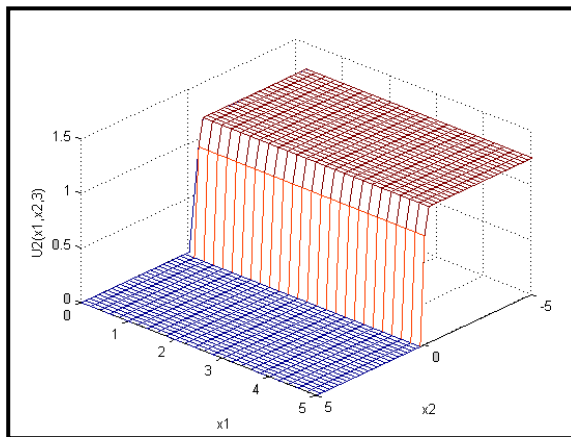


(b) Production rate of  $M_2$

Figure 4.3 Production rate of  $M_s$  and  $M_2$  at mode 1



(a) Production rate of  $M_1$



(b) Production rate of  $M_2$

Figure 4.4 Production rate of  $M_1$  and  $M_2$  at mode 3

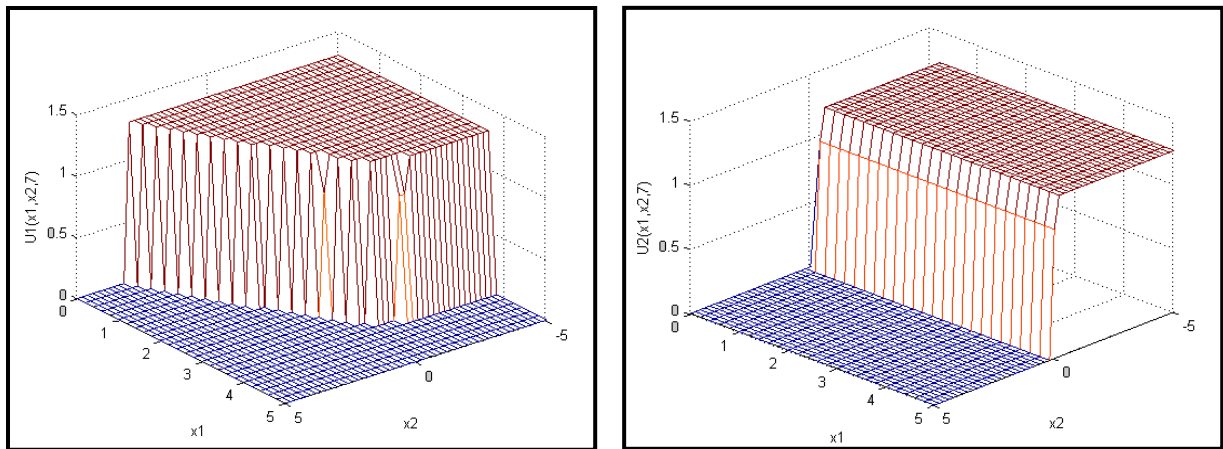
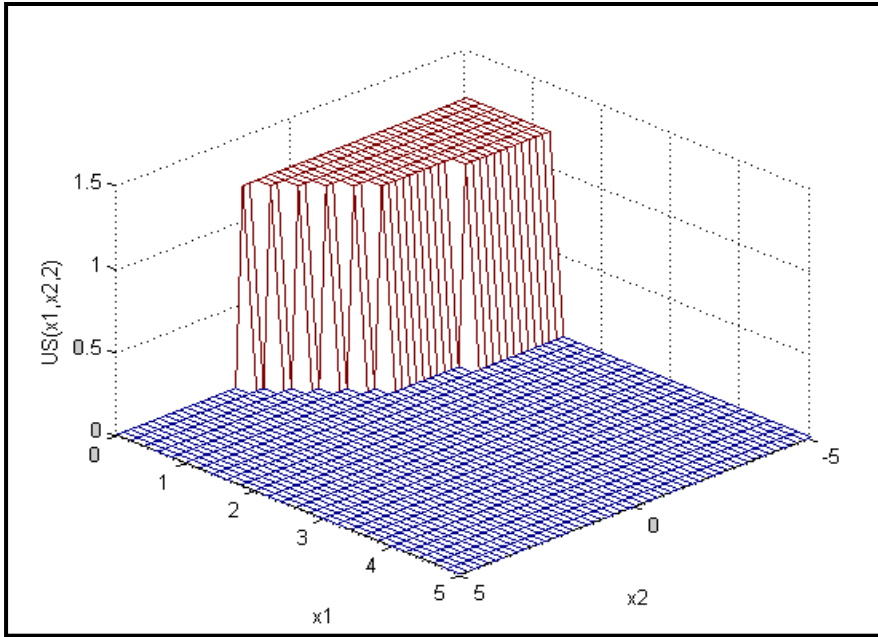
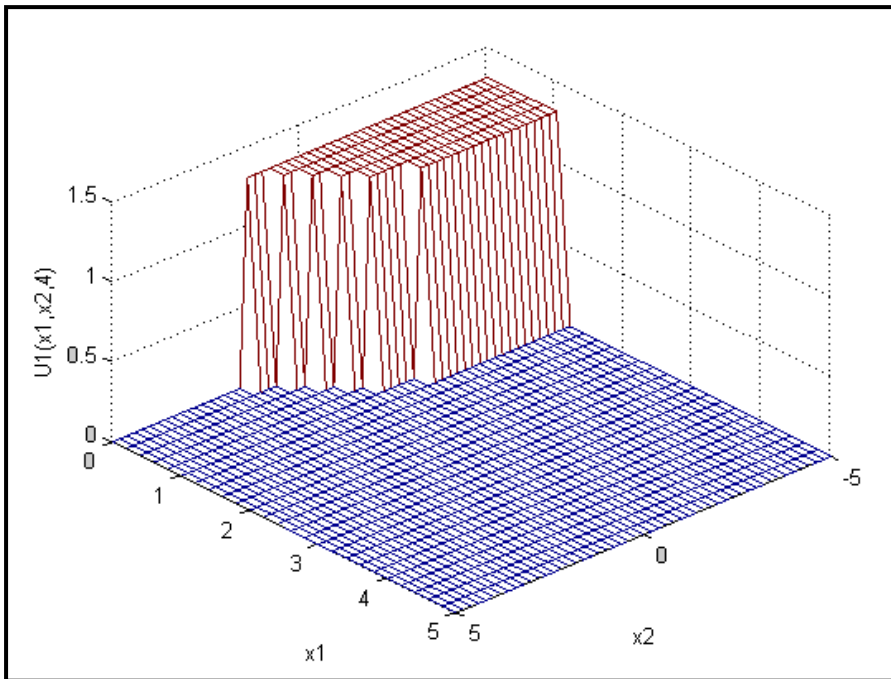
a) Production rate of  $M_1$ b) Production rate of  $M_2$ Figure 4.5 Production rate of  $M_1$  and  $M_2$  at mode 7

Figure 4.3 shows that there is no need to produce the part with sufficient stock levels both in the work-in-process (WIP), described by  $x_1$  and the stock of finished products, described by  $x_2$ . In mode 1, the production rate of the main machine ( $M_1$ ) is described by  $u_1 = 0$ . For small stock levels, the policy obtained properly defines the region in the domain  $(x_1, x_2)$  where a maximal production rate is optimal. In Figure 4.4, we have a similar trend in the optimal production rate as in mode 1. In mode 3, the production rate of the standby machine ( $M_s$ ) is described by  $u_s = 0$ . Figure 4.5 illustrates the same policy as in modes 1 and 3, just that at mode 7, the standby machine is at time-off, meaning that the production rate of standby machine  $u_s = 0$ .



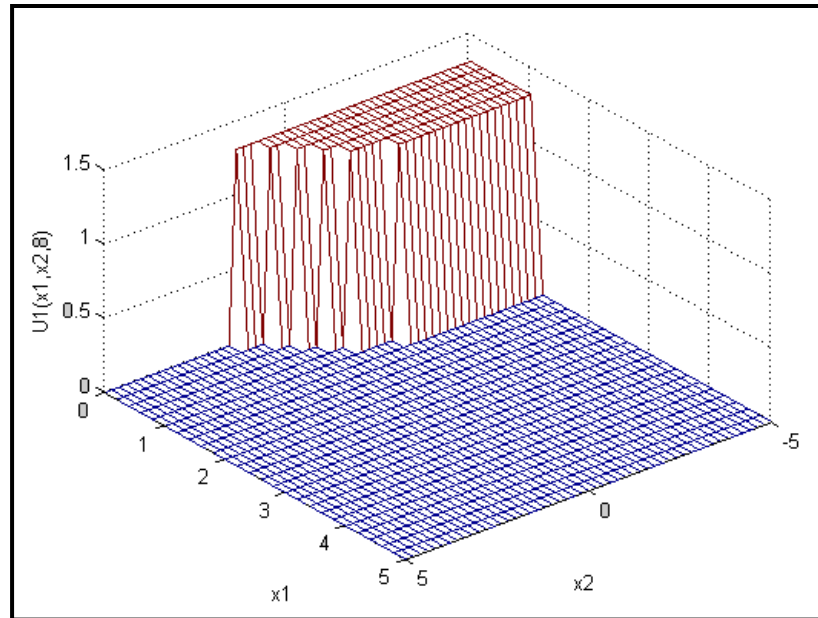
Production rate of  $M_s$

Figure 4.6 Production rate of  $M_s$  at mode 2



Production rate of  $M_1$

Figure 4.7 Production rate of  $M_1$  at mode 4



Production rate of  $M_1$

Figure 4.8 Production rate of  $M_1$  at mode 8

Figure 4.6 illustrates that thanks to a standby machine, we can produce the part for the work-in-process (WIP), as described by  $x_1$ . In mode 2, the production rates of the main machines  $M_1$  and  $M_2$  are described by  $u_1 = u_2 = 0$  respectively. Figure 4.7 shows that the main machine  $M_1$  produces parts for the work-in-process (WIP) described by  $x_1$ . In mode 4, the production rates of main machine  $M_2$  and of standby machine  $M_s$  are described by  $u_2 = u_s = 0$  respectively. We have the same policy in Figure 4.8 as in modes 2 and 4, except that at mode 8, the standby machine is at time-off, meaning that the standby machine's production rate is  $u_s = 0$ . The production rate of the main machine  $M_2$  at mode 8 is described by  $u_2 = 0$ .



Note that the aim of our policy is to find the optimal value of production rates  $u_i$  with  $i = 1, S$  which are dependent on two factors,  $Z_1$  and  $Z_2$ , and the optimal value of production rate  $u_2$  which is dependent on one factor such  $Z_3$ , as illustrated in Figure 4.9.

The next sections are aimed at developing a systematic approach for determining the optimal values of  $Z_1, Z_2, Z_3$ .

#### 4.4 Control approach, experimental design and response surface methodology

In order to propose an approach which could be easily applied to control manufacturing systems at the operational level, the descriptive capacities of discrete event simulations models are combined with analytical models, experimental design, and response surface methodology. Many studies have been covered in the research literature on these subjects. For more details, we refer the reader to Gharbi and Kenné (2000). The structure of the proposed control approach is presented in Figure 4.10.

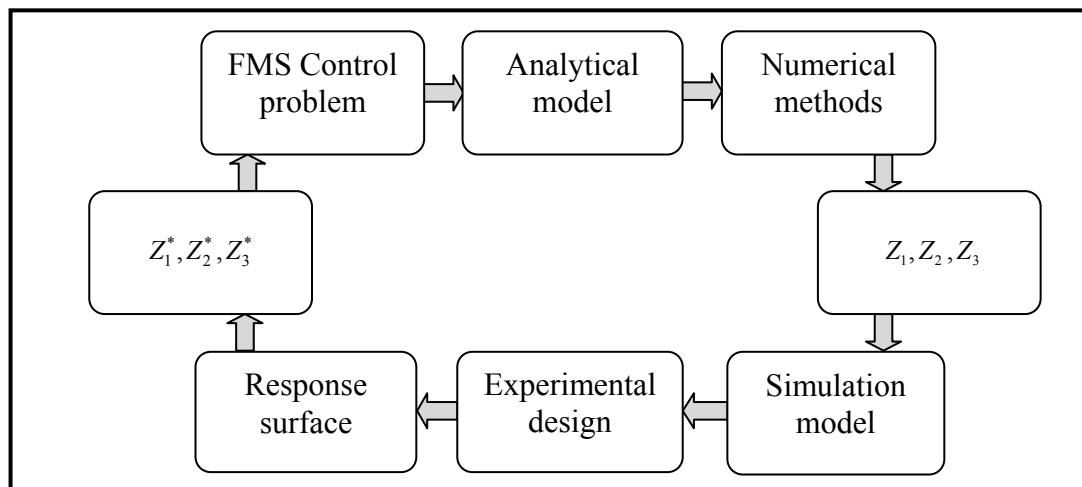


Figure 4.10 Diagram of control approach

1. The manufacturing system's control problem statement, as shown in section 4.3, consists of a production problem presentation for a transfer line with passive redundancy. This problem is presented through a stochastic optimal control model based on control theory.

The aim of this step is to find the control variables  $(u_1(\cdot), u_2(\cdot), u_s(\cdot))$ , called the production rates. The control variables allow an improvement of the incurred cost.

2. The optimality conditions, described by the HJBDD equations, are obtained from the problem statement of the first step. This step shows that the value function, representing the incurred cost, is the solution of the HJBDD equations, and our control policy (production rates) is near-optimal.
3. In this step, we use numerical methods to solve the optimality equations of the problem because it is not possible to solve them analytically.
4. The control factors  $Z_1, Z_2, Z_3$  for the control production rates describe the numerical control policy obtained.
5. The simulation model uses the near-optimal control policy defined in the previous step as the input factor for conducting experiments in order to evaluate the transfer line's with passive redundancy performances. Therefore, the cost incurred is obtained for the given values of the control factors thanks to the simulation model which will be presented in the next section.
6. The experimental design approach defines how control factors can be varied in order to identify the effects of the main factors and their interactions on the cost. These variations must be evaluated through a minimal set of simulation experiments.
7. In this step, we use a response surface methodology to obtain the relationship between the significant main factors and the incurred cost as well as the relationship between the main factors' interactions. Thereafter, the optimized model obtained in order to determine the main factors' best values are called  $Z_1^*, Z_2^*, Z_3^*$  for the production.
8. Using the proposed control approach gives the production rates described by equation (4.9), for the best values of factors  $Z_1, Z_2, Z_3$  meaning  $Z_1^*, Z_2^*, Z_3^*$ .

The performance-estimation tool chosen for this study is a discrete simulation model, and so we used the Arena software, which uses the SIMAN language; we refer the reader to Rossetti (2010).



The SIMAN portion of the software is composed of various networks describing specific tasks (failure and repair events, threshold production crossing of inventory variables, etc.). The simulation model is presented in Figure 4.11.

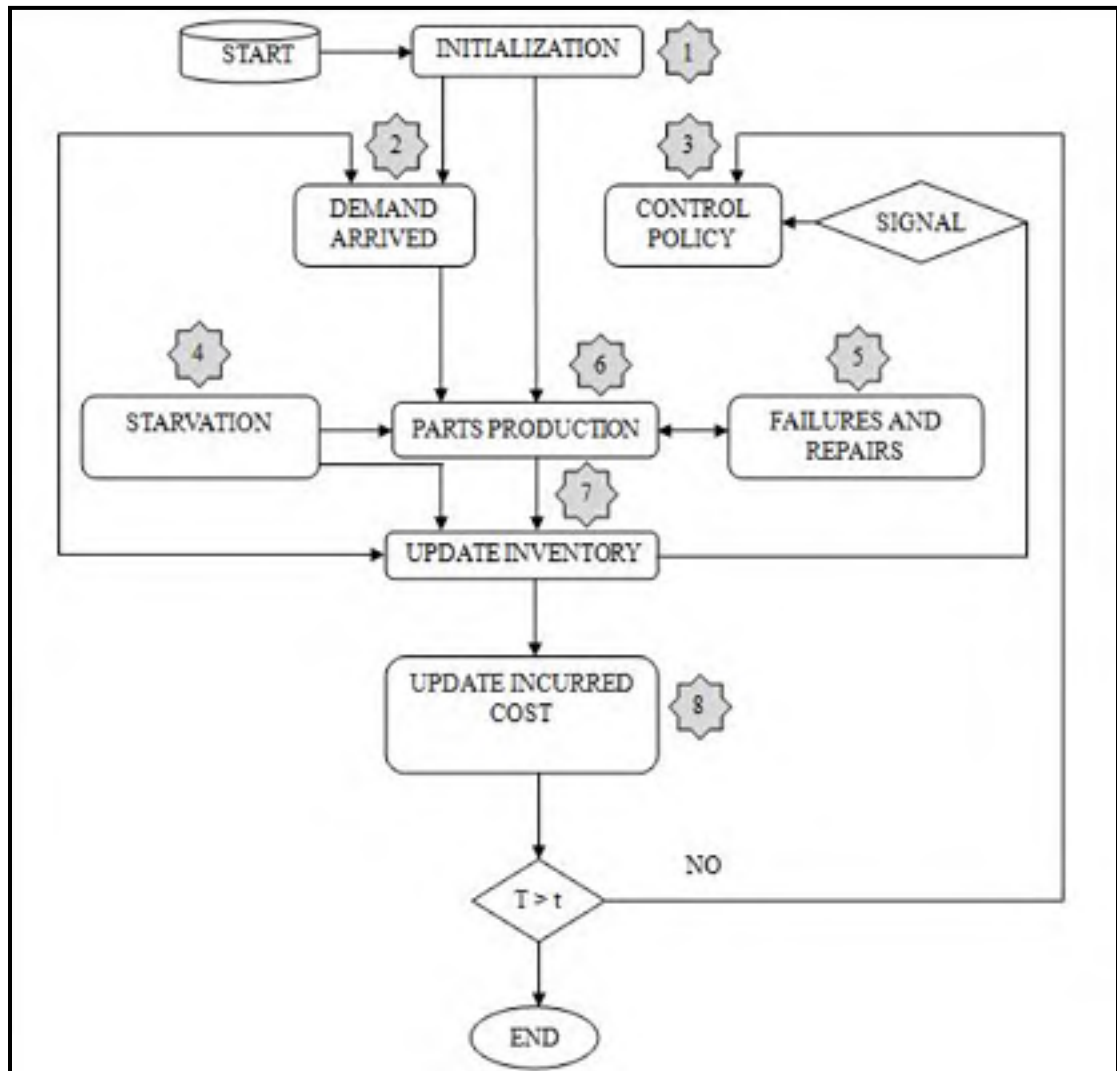


Figure 4.11 Diagram of the simulation model

1. The INITIALIZATION block initializes the problem variables (current surplus, production rates, incurred cost, etc.).
2. The DEMAND ARRIVAL block performs the arrival of a demand for each  $d$  unit of time. A verification test is then performed on the product's inventory level, and the inventory or the backorder is updated.

3. The CONTROL POLICY block is defined in equation (4.9). The feedback control policy is defined by the output of the SIGNAL block, which is used to permanently verify the variation in the stock level  $x(t)$  in order to specify the best action to carry out.
4. The STARVATION of the machines is implemented with the use of observation networks. Whenever the in-process buffer becomes empty, a signal is sent. Another signal is sent when the material becomes available for operation
5. The FAILURES AND REPAIRS block performs two functions: it defines the time-to-failure of the machine as well as the time-to-repair of the machine.
6. The PARTS PRODUCTION block performs the production of finished products according to the policy defined by the CONTROL POLICY.
7. The UPDATE INVENTORY block is used once the time step is chosen. For more details we refer the reader to Pritsker (1999).
8. The UPDATE INCURRED COST block calculates the incurred cost according to the different variable levels and unit costs  $c^+$  and  $c^-$ .

The simulation ends when the current simulation time  $t$  reaches the defined simulation period  $T$ . We thus ran offline simulations to determine the time necessary for the manufacturing system to reach its steady state. We found the simulation time for our manufacturing system at nearly 20000 units of time, and this duration is used for all the simulations.

The simulation parameters used in this paper are the same as in Table 4.1. The input-output data is generated by the simulation model from the variation of independent factors  $Z_1, Z_2$  and  $Z_3$ . Now, we present a procedure to vary these factors simultaneously. Such a procedure uses the experimental design approach. Hence, for three factor problems, as illustrated in the previous sections, we selected a  $3^3$  response surface design since we have three independent variables at three levels. This design leads to the completion of 27 experimental trials. The levels of the independent variables  $Z_1, Z_2, Z_3$  range from a low of 5 to a high of 25. In this paper, we chose three replications, and thus have 108 ( $27*4$ ) simulation

runs. We refer the reader to Montgomery (2005) for more details. We considered all possible combinations of different levels of independent variables by response surface design. The objective of this design is to understand the effects of independent variables on performance measures; in our case, the production average cost. From the ANOVA table, the independent variables  $Z_1, Z_2, Z_3$  and interaction effect as well as the quadratic effect are significant for the dependant variable at 0.05 level of significance. The  $R^2$  value of 0.88, meaning 88% of the total variability, is explained by the model.

The average cost function is given by:

$$\text{Average cost} = \beta_0 + \sum_{i=1}^n \beta_i Z_i + \sum_{i=1}^n \sum_{\substack{j=1 \\ j \geq i}}^n \beta_{ij} Z_i Z_j, \quad (4.10)$$

The estimation of the regression coefficients is performed and ten coefficients achieved in Table 4.2.

Table 4.2 Polynomial coefficients

$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_{11}$	$\beta_{12}$	$\beta_{13}$	$\beta_{22}$	$\beta_{23}$	$\beta_{33}$
860	1.899	-18	4.080	0.174	-0.086	-0.351	0.530	0.272	-0.153

The minimum average cost function is located at  $Z_1^* = 11.4, Z_2^* = 14.5, Z_3^* = 13$ , where  $Z_1^*, Z_2^*$  and  $Z_3^*$  represent the optimal values of independent variables  $Z_1, Z_2$  and  $Z_3$ .

Figures (4.12)-(4.14), illustrate the contour plots of the average cost function or response surface.

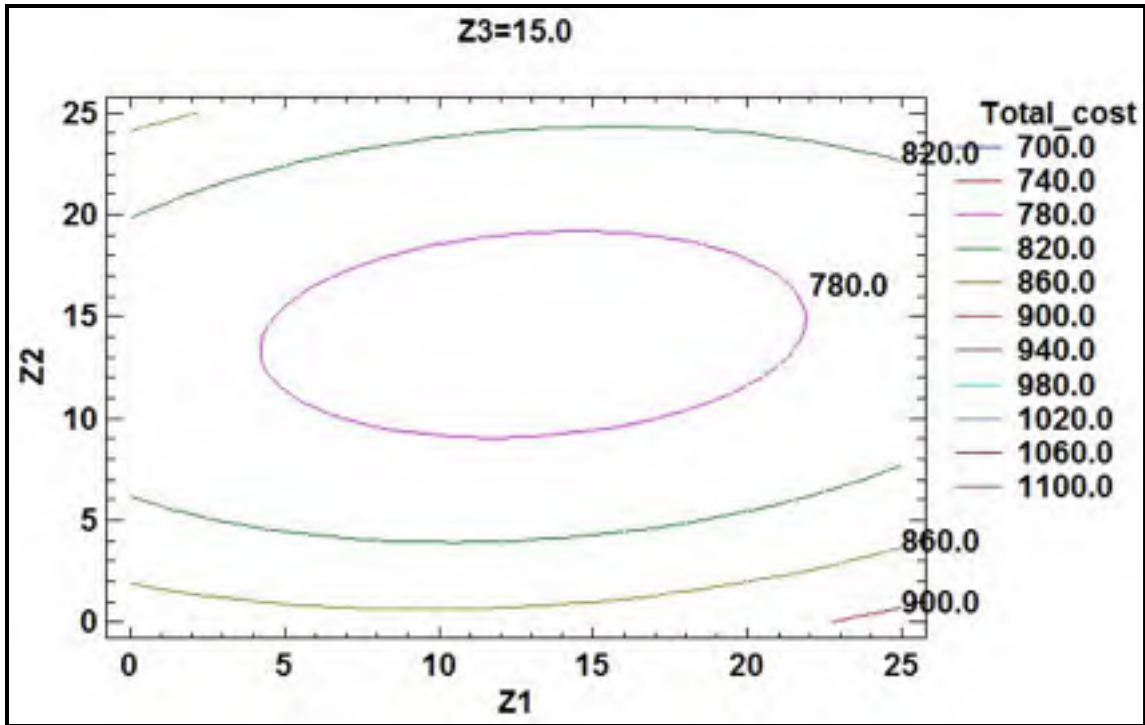


Figure 4.12 Contour plot of the response surface

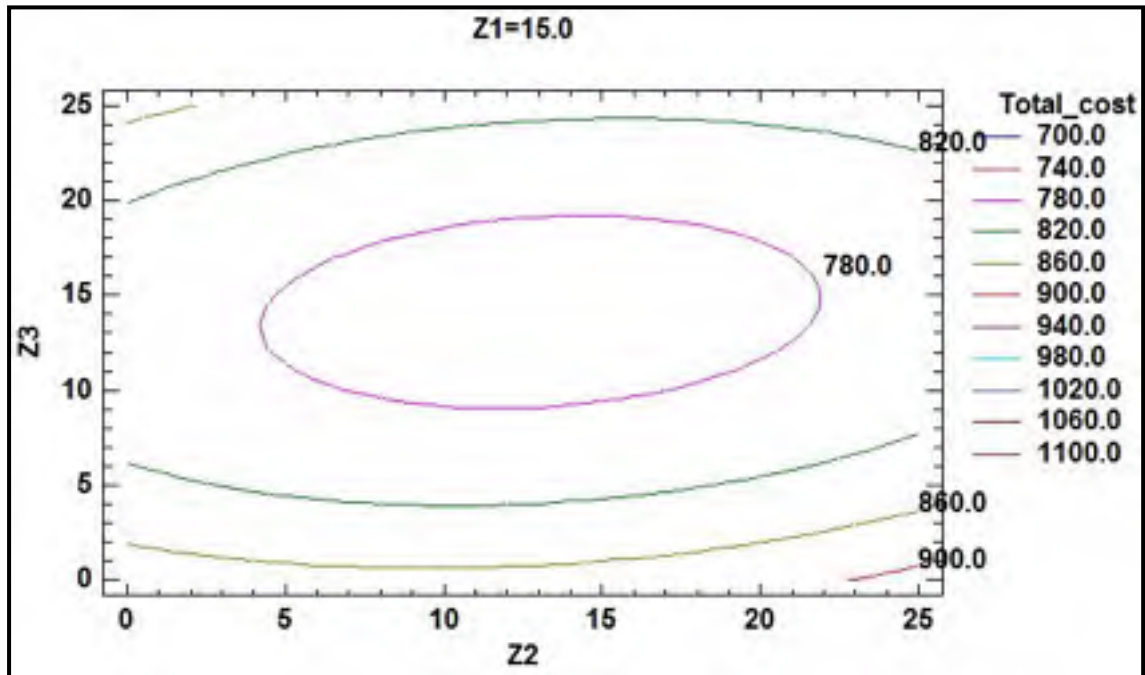


Figure 4.13 Contour plot of the response surface

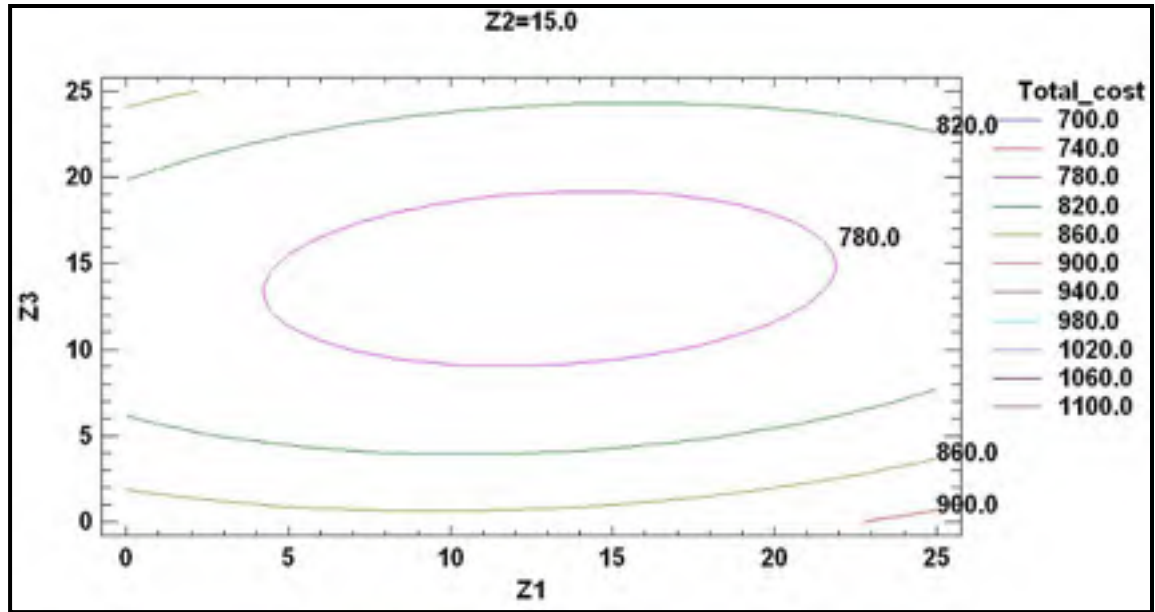


Figure 4.14 Contour plot of the response surface

These values determine the extension of the hedging point policy for the manufacturing system considered, where the average cost is minimised and this control policy is the best approximation of the optimal control. We observe that the cost function is not very sensitive to small variations of finished goods stock levels. The work-in-process (WIP) stock levels appear to be less sensitive to increases or decreases in WIP stock levels' small variations. Because we can produce parts for work-in-process (WIP) at all times thanks to passive redundancy, we can therefore respond to demand permanently.

To illustrate the effect of the variation inventory and backlog costs on the design parameters, a sensitivity analysis was conducted in Table 4.3, with  $c_1^+ = 1$ ,  $c_{r_1} = 150$ ,

$c_{r_2} = 200$ ,  $c_{r_s} = 250$  and  $c_{lagout} = 50$ .

Table 4.3 Variations of optimal design factors based on inventory and backlog costs

$c_2^+$	$c_2^-$	$Z_1^*$	$Z_2^*$	$Z_3^*$
10	60	11.3	14.5	13.1
15	60	11.4	14.5	13.0
20	60	11.4	14.5	13.0
25	60	11.4	14.5	13.0
30	60	11.4	14.5	13.0
35	60	11.4	14.5	13.0
40	60	11.4	14.5	13.0
45	60	11.4	14.5	13.0
50	60	11.4	14.5	13.0
55	60	11.4	14.5	13.0
10	20	11.3	14.5	13.0
10	30	11.4	14.5	13.0
10	40	11.4	14.5	13.0
10	50	11.4	14.5	13.0
10	60	11.4	14.5	13.1
10	70	11.4	14.5	13.1
10	80	11.4	14.5	13.1
10	90	11.4	14.5	13.1
10	100	11.4	14.5	13.1
10	110	11.4	14.5	13.1

The first section of Table 4.3 presents the variations in stock levels based on inventory costs, while the second section shows the variations in stock levels based on backlog costs.

Figures 4.15 and 4.16 plot variations in stock levels based on inventory costs and backlog costs, which are presented in Table 4.3. In these figures, homogenous variation occurs with passive redundancy, because it produces parts for work-in-process (WIP) at all times and makes possible to respond to demand permanently. Therefore, we observe that variations in inventory and backlog cost do not influence the production thresholds of work-in-process (WIP) and finished goods.

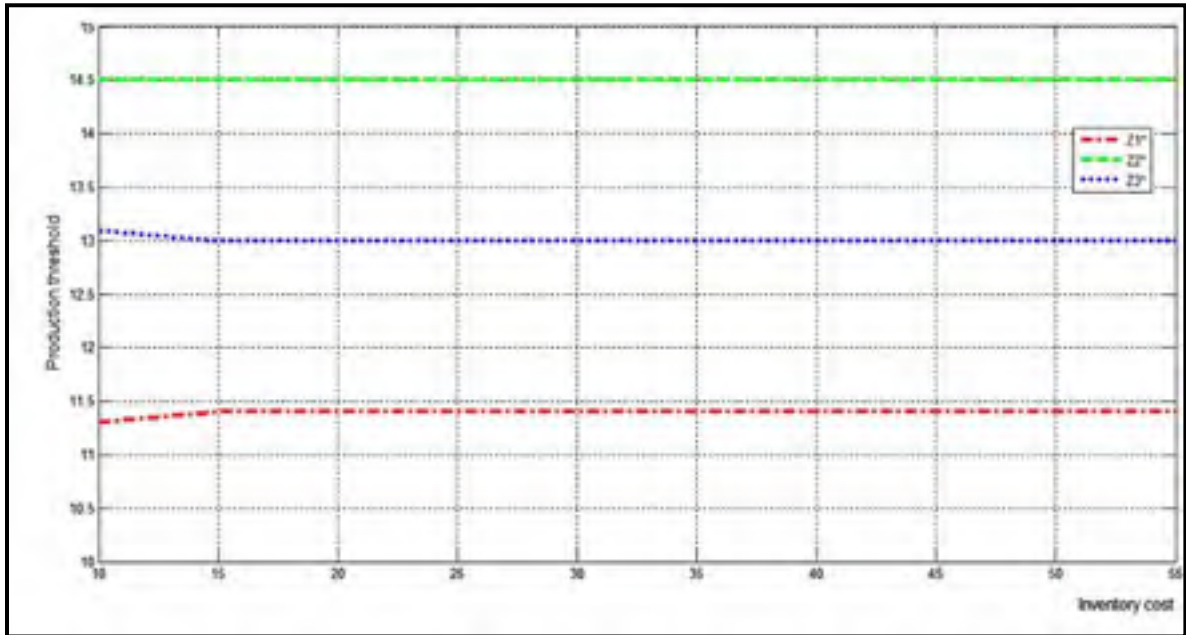


Figure 4.15 Production threshold /Inventory cost

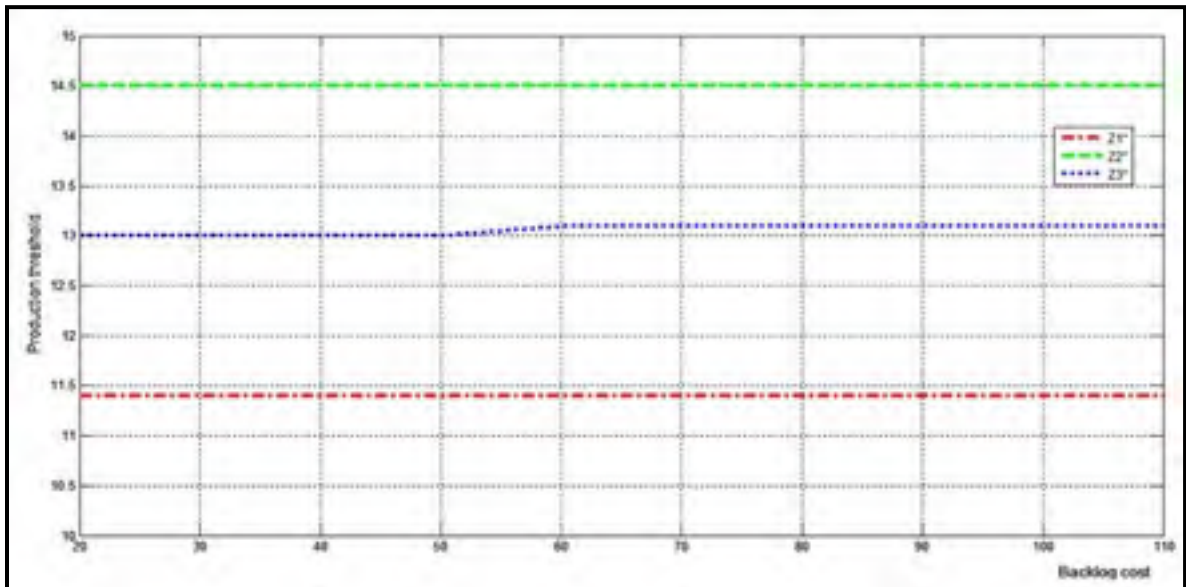


Figure 4.16 Production threshold/Backlog cost

Production a transfer line's optimal costs was determined using the analytical model presented in this paper. The numerical approach, the experimental design and the response

surface methodology consecutively show that the resulting policy is optimal and enhances machine availability. Without loss of generality of this proposal, this model is based on certain assumptions relating to a transfer line consisting of three machines which are not identical, and operate with passive redundancy. We observed that the passive redundancy case allows us to better optimize the production cost of transfer lines while guaranteeing occupational safety. The integration of the second machine as the passive redundancy allows demand to be met permanently. Furthermore, this integration will release the intervention time needed for the machine which is down. It also minimises the possibility of circumvention of device protection or retraction of lockout-tagout procedures.

#### **4.5 Conclusion**

This paper confirms that it is possible to integrate a passive redundancy system in a production line in order to: 1) increase the productivity of workers and material resources, 2) better optimize production costs while guaranteeing the safety of workers. In this paper, the control policy has an extension of hedging point structure. Based on the numerical solution obtained, a parameterized near-optimal control policy was derived. Such a policy depends on stock threshold levels. An experimental design was used to determine the effects of the independent variables on the average cost over the production horizon. We combined an analytical, simulation and statistical method to provide the average cost estimation related to the control problem. Average cost's estimation permits to know the best values of the control parameters. Finally, passive redundancy system improves production costs and worker safety. The former is obtained by meeting the demand whereas the latter is fulfilled by releasing essential space-time for the machines that are under repair.

#### **Acknowledgments**

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(FQRSC); Fonds Québécois de la Recherche sur la Nature et les Technologies (FQRNT); Institut de Recherche Robert-Sauvé en Santé et en Sécurité du Travail (IRSST) and Équipe de Recherche en Sécurité du Travail (EREST).

## APPENDIX 4.A OPTIMAL CONDITIONS AND NUMERICAL APPROACH

The properties of the value functions and the Hamilton-Jacobi-Bellman equation (HJB) in terms of directional derivative (DD) for inner and boundary points are presented in this section. These equations describe the optimality conditions for a transfer line with passive redundancy producing one part type. Hence, we first present the notion of these derivatives and some related properties of convex functions.

A function  $f(x), x \in R^m$ , is said to have a directional derivative  $f'_p(x)$  along direction  $p \in R^m$ , if there exists

$$\lim_{\delta \rightarrow 0} \frac{f(x + \delta p) - f(x)}{\delta} = f'_p(x).$$

If a function  $f(x)$  is differentiable at  $x$ , then  $f'_p(x)$  exists for every  $p$  and

$$f'_p(x) = \langle \nabla f(x), p \rangle$$

Where  $\nabla f(x)$  is the gradient of  $f(x)$  and  $\langle \cdot, \cdot \rangle$  is the scalar product. Furthermore, a continuous convex function defined on a convex domain  $\Sigma$  is differentiable almost everywhere, and has a directional derivative along any direction at any inner point of  $\Sigma$  and along any admissible direction (i.e., a direction  $p$  such that  $x + \delta p \in \Sigma$  for some  $\delta > 0$ ) at any boundary point of  $\Sigma$  (for more details, see Sethi and Zhang, 1994).

Note that  $\{A\tilde{u} : (u_1, u_2, u_s) \in \Gamma(x, \alpha)\}$  is the set of admissible directions at  $x$ .

Regarding the optimality principle, let us write the set of partial differential equations known as the Hamilton-Jacobi-Bellman (HJB) equations in terms of directional derivatives (DD) as follows:

$$\rho v(x, \alpha) = \min_{u \in \Gamma(x, \alpha)} \left\{ v'_{A\tilde{u}}(x, \alpha) + g(\alpha, x, \cdot) + \sum_{\alpha \neq \beta} q_{\alpha\beta} [v(x, \beta) - v(x, \alpha)] \right\}, \quad (4.A.1)$$

$\forall \alpha, \beta \in B$

where:

$$v'_{A\bar{u}}(x, \alpha) = (\tilde{u}_1 - \tilde{u}_2)v_{x_1}(x, \alpha) + (\tilde{u}_2 - \tilde{u}_3)v_{x_2}(x, \alpha).$$

with:

$$\tilde{u}_1 = u_1 I_1^\alpha + u_s I_s^\alpha, \quad I_1^\alpha = \begin{cases} 1 & \text{if } \alpha = 3, 4, 5, 6, 7, 8, \\ 0 & \text{otherwise,} \end{cases}, \quad I_s^\alpha = \begin{cases} 1 & \text{if } \alpha = 1, 2, \\ 0 & \text{otherwise,} \end{cases}, \quad \tilde{u}_2 = u_2 \quad \text{and} \quad \tilde{u}_3 = u_3 := d.$$

The choice of standby machine and the main machine characteristics must be in such a way that respects the feasibility of system.

The system is considered feasible if:

$$\sum \pi u_i^{\max} \geq d, \quad (4.A.2)$$

Where the limitation probabilities can be ascertained from the following equation for a system conforming to a Markov process:

$$\pi(\cdot)Q(\cdot) = 0,$$

$$\sum \pi = 1$$

with:

$\pi(\cdot)$  : Limiting probabilities

$Q(\cdot)$  : Transition matrix rates

Hence, we have  $\pi(\cdot) = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8)$  representing the vector of limiting probabilities from modes 1 to 8.

Let  $\delta\Lambda$  denote the boundary of  $\Lambda$ . If there exists  $x_i = 0 (i=1,2)$ , then  $x \in \delta\Lambda$ . Let the restriction of  $v(x, \alpha)$  on some  $j$ -dimensional face,  $0 < j < m$ , of  $\delta\Lambda$  be differentiable at an inner point  $x_0$  of this face. Hence, there is a vector  $\tilde{\nabla}v(x_0, \alpha)$  such that  $v'_p(x_0, \alpha) = \langle \tilde{\nabla}v(x_0, \alpha), p \rangle$  for any admissible direction at  $x_0$ .

Now, we can write the boundary condition on  $v(\cdot, \cdot)$  from the continuity of the value function by:

$$\begin{aligned} & \min_{\mathbf{u} \in \Gamma(x_0, \alpha)} \left\{ \langle \tilde{\nabla} v(x_0, \alpha), A\tilde{\mathbf{u}} \rangle + g(\alpha, x_0, \cdot) + \sum_{\alpha \neq \beta} q_{\alpha\beta} [v(x_0, \beta) - v(x_0, \alpha)] \right\} \\ & = \min_{\mathbf{u} \in \Gamma(\alpha)} \left\{ \langle \tilde{\nabla} v(x_0, \alpha), A\tilde{\mathbf{u}} \rangle + g(\alpha, x_0, \cdot) + \sum_{\alpha \neq \beta} q_{\alpha\beta} [v(x_0, \beta) - v(x_0, \alpha)] \right\}, \end{aligned} \quad (4.A.3)$$

$\forall \alpha, \beta \in \mathbf{B}$

We refer the reader to Lou and Zhang (1994) for the interpretation of the condition (4.A.3).

The optimal control policy  $(u_1^*, u_2^*, u_s^*)$  denotes a minimizer over  $\Gamma(x, \alpha)$  of the right-hand side of equation (4.A.1). This policy is consistent with the value function obtained in Equation (4.8). The optimal control policy therefore rests in solving Equation (4.A.1). Obtaining an analytical solution of equation (4.A.1) is roughly impossible. The numerical solution of the HJB equation (4.A.1) in terms of directional derivatives (DD) is a challenge considered insurmountable in the scientific literature.

Now, we use numerical methods to solve the optimality conditions presented in this section. This method is based on Kushner's approach (Kushner and Dupuis, 1992). The basic idea behind this approach consists in using an approximation scheme for the directional derivative of the value function  $v(x, \alpha)$ . Let  $h_1$  and  $h_2$  denote the length of the finite difference interval of the variables  $x_1$  and  $x_2$ . Hence, using  $h_1$  and  $h_2$ ,  $v(x, \alpha)$  is approximated by  $v^h(x, \alpha)$ , and  $v_{x_1}$  and  $v_{x_2}$  are approximated by:

$$v_{x_1}(\cdot) = v'_{\tilde{u}_1 - \tilde{u}_2}(\cdot) = \left( \begin{array}{l} \frac{1}{h_1} (v^h(x_1, x_1 + h_1, \alpha) - v^h(x_1, \alpha)) \times (\tilde{u}_1 - \tilde{u}_2) \quad \text{if } (\tilde{u}_1 - \tilde{u}_2) \geq 0, \\ \frac{1}{h_1} (v^h(x_1, \alpha) - v^h(x_1, x_1 - h_1, \alpha)) \times (\tilde{u}_1 - \tilde{u}_2) \quad \text{otherwise,} \\ \text{with:} \\ \tilde{u}_1 = u_1 I_1^\alpha + u_s I_s^\alpha, \quad I_1^\alpha = \begin{cases} 1 & \text{if } \alpha = 3, 4, 5, 6, 7, 8, \\ 0 & \text{otherwise,} \end{cases} \quad , \quad I_s^\alpha = \begin{cases} 1 & \text{if } \alpha = 1, 2, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } \tilde{u}_2 = u_2, \end{array} \right), \quad (4.A.4)$$

$$v_{x_2}(\cdot) = v'_{\tilde{u}_2 - \tilde{u}_3}(\cdot) = \left. \begin{array}{l} \frac{1}{h_2} (v^h(x_1, x_2 + h_2, \alpha) - v^h(x_1, \alpha)) \times (\tilde{u}_2 - \tilde{u}_3) \quad \text{if } (\tilde{u}_2 - \tilde{u}_3) \geq 0, \\ \frac{1}{h_2} (v^h(x_1, \alpha) - v^h(x_1, x_2 - h_2, \alpha)) \times (\tilde{u}_2 - \tilde{u}_3) \quad \text{otherwise,} \\ \text{with:} \\ \tilde{u}_2 = u_2 \quad \text{and} \quad \tilde{u}_3 = u_3 := d, \end{array} \right\} \quad (4.A.5)$$

We manipulated the approximation arrived at in equations (4.A.4) and (4.A.5) to rewrite the HJBDD equations (4.A.1) as follows:

$$v^h(x, \alpha) = \min_{u \in \Gamma^h(x, \alpha)} \left\{ \left( \rho + |q\alpha\alpha| + \frac{(\tilde{u}_4 - \tilde{u}_2)}{h_1} + \frac{(\tilde{u}_2 - \tilde{u}_3)}{h_2} \right)^{-1} \left( \frac{\sum_{\beta \neq \alpha} q\alpha\beta (v^h(x, \beta)) + g(x, \cdot)}{h_1} \left[ (v^h(x_1 + h_1, x_2, \alpha) k_1^+ + v^h(x_1 - h_1, x_2, \alpha) k_1^-) \right] + \frac{(\tilde{u}_2 - \tilde{u}_3)}{h_2} \left[ (v^h(x_1, x_2 + h_2, \alpha) k_2^+ + v^h(x_1, x_2 - h_2, \alpha) k_2^-) \right] \right) \right\} \quad (4.A.6)$$

with:

$$\tilde{u}_4 = u_1 I_1^\alpha + u_s I_s^\alpha, \quad I_1^\alpha = \begin{cases} 1 & \text{if } \alpha = 3, 4, 5, 6, 7, 8, \\ 0 & \text{otherwise,} \end{cases}, \quad I_s^\alpha = \begin{cases} 1 & \text{if } \alpha = 1, 2, \\ 0 & \text{otherwise,} \end{cases}, \quad \tilde{u}_2 = u_2 \quad \text{and} \quad \tilde{u}_3 = u_3 := d.$$

Where  $\Gamma^h(x, \alpha)$  is the discrete feasible control space and the other terms used in equation (4.A.6) are defined as:

$$K_1^+ = \begin{cases} 1 & \text{if } (\tilde{u}_1 - \tilde{u}_2) \geq 0, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad K_1^- = \begin{cases} 1 & \text{if } (\tilde{u}_1 - \tilde{u}_2) < 0, \\ 0 & \text{otherwise,} \end{cases}$$

with:

$$\tilde{u}_1 = u_1 I_1^\alpha + u_s I_s^\alpha, \quad I_1^\alpha = \begin{cases} 1 & \text{if } \alpha = 3, 4, 5, 6, 7, 8, \\ 0 & \text{otherwise,} \end{cases}, \quad I_s^\alpha = \begin{cases} 1 & \text{if } \alpha = 1, 2, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad \tilde{u}_2 = u_2.$$

$$K_2^+ = \begin{cases} 1 & \text{if } (\tilde{u}_2 - \tilde{u}_3) \geq 0, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad K_2^- = \begin{cases} 1 & \text{if } (\tilde{u}_2 - \tilde{u}_3) < 0, \\ 0 & \text{otherwise,} \end{cases}$$

with:

$$\tilde{u}_2 = u_2 \quad \text{and} \quad \tilde{u}_3 = u_3 := d.$$

In this paper, we use the policy improvement technique to derive an approximate optimization problem solution. The algorithm of this technique can be found in Kushner and Dupuis (1992).

## APPENDIX 4.B

The discrete dynamic programming equation (4.A.6) in Appendix 4.A gives the following eight equations:

$$v^h(x,1) = \min_{u \in \Gamma^h(x,1)} \left( \rho + \frac{|u_k - u_s|}{h_1} + \frac{|u_s - d|}{h_2} + q_{12} + q_{15} + q_{17} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_k - u_s|}{h_1} (v^h(x_1 + h_1, x_2, 1) k_1^+ + v^h(x_1 - h_1, x_2, 1) k_1^-) \\ + \frac{|u_s - d|}{h_2} (v^h(x_1, x_2 + h_2, 1) k_2^+ + v^h(x_1, x_2 - h_2, 1) k_2^-) \\ + g(x, 1) + q_{12} v^h(x, 2) + q_{15} v^h(x, 5) + q_{17} v^h(x, 4) \end{array} \right\}, \quad (4.B1)$$

$$v^h(x,2) = \min_{u \in \Gamma^h(x,2)} \left( \rho + \frac{|u_s - u_2|}{h_1} + \frac{d}{h_2} + q_{21} + q_{26} + q_{28} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_s - u_2|}{h_1} (v^h(x_1 + h_1, x_2, 2) k_1^+ + v^h(x_1 - h_1, x_2, 2) k_1^-) \\ + \frac{d}{h_2} (v^h(x_1, x_2 - h_2, 2) k_2^-) + g(x, 2) + q_{21} v^h(x, 1) \\ + q_{26} v^h(x, 6) + q_{28} v^h(x, 8) \end{array} \right\}, \quad (4.B2)$$

$$v^h(x,3) = \min_{u \in \Gamma^h(x,3)} \left( \rho + \frac{|u_1 - u_2|}{h_1} + \frac{|u_2 - d|}{h_2} + q_{34} + q_{35} + q_{37} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - u_2|}{h_1} (v^h(x_1 + h_1, x_2, 3) k_1^+ + v^h(x_1 - h_1, x_2, 3) k_1^-) \\ + \frac{|u_2 - d|}{h_2} (v^h(x_1, x_2 + h_2, 3) k_2^+ + v^h(x_1, x_2 - h_2, 3) k_2^-) \\ + g(x, 3) + q_{34} v^h(x, 4) + q_{35} v^h(x, 5) + q_{37} v^h(x, 7) \end{array} \right\}, \quad (4.B3)$$

$$v^h(x,4) = \min_{u \in \Gamma^h(x,4)} \left( \rho + \frac{|u - u_s|}{h_1} + \frac{d}{h_2} + q_{43} + q_{46} + q_{48} \right)^{-1} \left\{ \begin{array}{l} \frac{|u - u_s|}{h_1} (v^h(x_1 + h_1, x_2, 4) k_1^+ + v^h(x_1 - h_1, x_2, 4) k_1^-) \\ + \frac{d}{h_2} (v^h(x_1, x_2 - h_2, 4) k_2^-) + g(x, 4) + q_{43} v^h(x, 3) \\ + q_{46} v^h(x, 6) + q_{48} v^h(x, 8) \end{array} \right\}, \quad (4.B4)$$

$$v^h(x,5) = \min_{u \in \Gamma^h_{(x,5)}} \left( \rho + \frac{u_2}{h_1} + \frac{|u_2-d|}{h_2} + q_{51} + q_{53} + q_{56} \right)^{-1} \left\{ \begin{array}{l} \frac{u_2}{h_1} (v^h(x_1-h_1, x_2, 5) k_1^-) \\ + \frac{|u_2-d|}{h_2} (v^h(x_1+h_1, x_2, 5) k_2^+ + v^h(x_1, x_2-h_2, 5) k_2^-) \\ + g(x,5) + q_{51} v^h(x,1) + q_{53} v^h(x,3) + q_{56} v^h(x,6) \end{array} \right\}, \quad (4B5)$$

$$v^h(x,6) = \min_{u \in \Gamma^h_{(x,6)}} \left( \rho + \frac{d}{h_2} + q_{62} + q_{64} + q_{65} \right)^{-1} \left\{ \begin{array}{l} \frac{d}{h_2} (v^h(x_1, x_2-h_2, 6) k_2^-) \\ + g(x,6) + q_{62} v^h(x,2) \\ + q_{64} v^h(x,4) + q_{65} v^h(x,5) \end{array} \right\}, \quad (4B.6)$$

$$v^h(x,7) = \min_{u \in \Gamma^h_{(x,7)}} \left( \rho + \frac{|u_1-u_2|}{h_1} + \frac{|u_2-d|}{h_2} + q_{71} + q_{73} + q_{78} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1-u_2|}{h_1} (v^h(x_1+h_1, x_2, 7) k_1^+ + v^h(x_1-h_1, x_2, 7) k_1^-) \\ + \frac{|u_2-d|}{h_2} (v^h(x_1, x_2+h_2, 7) k_2^+ + v^h(x_1, x_2-h_2, 7) k_2^-) \\ + g(x,7) + q_{71} v^h(x,1) + q_{73} v^h(x,3) + q_{78} v^h(x,8) \end{array} \right\}, \quad (4B7)$$

$$v^h(x,8) = \min_{u \in \Gamma^h_{(x,8)}} \left( \rho + \frac{|u_1-u_2|}{h_1} + \frac{d}{h_2} + q_{82} + q_{84} + q_{87} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1-u_2|}{h_1} (v^h(x_1+h_1, x_2, 8) k_1^+ + v^h(x_1-h_1, x_2, 8) k_1^-) \\ + \frac{d}{h_2} (v^h(x_1, x_2-h_2, 8) k_2^-) + g(x,8) \\ + q_{82} v^h(x,2) + q_{84} v^h(x,4) + q_{87} v^h(x,7) \end{array} \right\}, \quad (4B8)$$



## REFERENCES

- Akella, R. and Kumar, P.R., 1986. Optimal Control of Production Rate in a Failure-Prone Manufacturing System. *IEEE Transactions on Automatic Control* AC 31, pp. 116-12.
- Charlot, E., Kenné, J.P. and Nadeau, S., 2006. Optimal Production, Maintenance and Lockout/tagout Control Policies in Manufacturing Systems. *International Journal of Production Economics* 107, pp. 435-450.
- Chinniah, Y., and Champoux, M., 2008. La sécurité des machines automatisées : Analyse des risques et des moyens de protection sur une presse à injection de plastique. R-557. Montréal: IRSST, pp. 93.
- Emami-Mehrgani, B., Nadeau, S and Kenné, J.P., 2011. Lockout/tagout and operational risks in the production control of manufacturing systems with passive redundancy. *International Journal of Production Economics* 132, pp. 165-173.
- Gershwin, S.B., 2002. *Manufacturing Systems Engineering*. Prentice-Hall, Englewood Cliffs, NJ, pp. 267.
- Gharbi, A., and Kenné, J.P. 2000. Production and preventive maintenance rates control for manufacturing system: an experimental design approach. *International Journal of Production Economics*, 65(3), pp. 275-287.
- Giraud, L., Ledoux, É., Tanchoux, S and Pâques, J.J., 2008. La maintenance, État de la connaissance et étude exploratoire. R-578. Montréal: IRSST, pp. 47.
- Hajji, A., Gharbi, A., Kenné, J.P., and Hidehiko, Y. 2009. Production and changeover control policies of a class of failure prone buffered flow-shops. *Production Planning & Control*, Vol. 20, No. 8, pp. 785-800.
- Kenne, J.P., E.K. Boukas and Gharbi, A. 2003. Control of production and corrective maintenance rates in a multiple-machine, multiple-product manufacturing system. *Mathematical and Computer Modelling*, Vol. 38, No. 3-4, pp. 351-365.
- Kenne, J.P., Gharbi, A., 1999. Experimental design in production and maintenance control problem of a single machine, single product manufacturing system. *International Journal of Production Research*, Vol. 37, pp. 621-637.
- Kushner, H.J. and Dupuis, P.G., 1992. *Numerical Methods for Stochastic Control Problems in Continuous Time*. Springer, New York.
- Lou, S.P. and Zhang, Q., 1994. Optimal Feedback Production Planning in a Stochastic Two-Machine Flowshop. *European Journal of Operational Research*, 73, pp. 331-345.
- Montgomery, D.C. 2005. *Design and Analysis of Experiments*. Sixth edition, Wiley, New York, NY.
- Older, G.J. and Suri, R., 1980. Time Optimal of Part-Routing in a Manufacturing System with Failure Prone Machines. In: *Proceedings of the 19<sup>th</sup> IEEE Conference on Decision Control*, Vol. 5, pp. 722-727.

- Presman, E., Sethi, S., and Zhang, Q., 1995. Optimal feedback production planning in a stochastic N-machine flowshop. *Automatica*, Vol. 31, No. 9, pp. 1325-1332.
- Pritsker, A.A.B. and O'Reilly, J.J. 1999. *Simulation with Visual SLAM and Awesim*, second edition. Wiley, West Lafayette, IN.
- Rezg, N., Xie, X., Mati, Y., 2004. Joint optimisation of preventive maintenance and inventory control in a production line using simulation. *International Journal of production Research*, Vol. 42, No. 10, pp. 2029-2046.
- Rishel, R., 1975. Control of Systems With Jumps Markov Disturbances. *IEEE Transactions on Automatic Control*, pp. 241-244
- Rossetti, M.D. 2010. *Simulation Modeling and Arena*. John Wiley and Sons, pp. 573.
- Sethi S.P. and Zhang Q., 1994. *Hierarchical Control Decision Making in Stochastic Manufacturing Systems*. Birkhäuser Boston, p. 418.

## CHAPITRE 5

### ARTICLE 3: OPTIMAL LOCKOUT/TAGOUT, PREVENTIVE MAINTENANCE, HUMAN ERROR AND PRODUCTION POLICIES OF MANUFACTURING SYSTEMS WITH PASSIVE REDUNDANCY

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#### Résumé

Cet article est différent par rapport aux autres articles sur l'intégration de C/D. Ce travail prend en considération le C/D et la maintenance préventive de façon conjointe pour un système manufacturier constitué de deux machines non-identiques sous forme redondance passive. Les machines sont sujettes à des pannes, à des réparations aléatoires et à des activités de maintenance préventive avec ou sans erreur humaine. Les variables de décision sont les taux de production des machines et le taux de maintenance préventive avec ou sans erreurs. Les variables de décision influencent le niveau des inventaires et la capacité du système manufacturier. Le mode de la machine peut être classifié comme étant en opération, en réparation ou en maintenance préventive avec ou sans erreur. La capacité du système est décrite par une chaîne de Markov non-homogène. Dans le modèle proposé, le taux de défaillance du système manufacturier dépend de son âge, ce qui signifie que la politique de maintenance préventive dépend de l'âge des machines. L'objectif de cette contribution est divisé en deux (2) volets : 1) trouver les variables de décision permettant de réduire les coûts totaux de production, comprenant les coûts d'inventaire, de pénurie et de maintenance sur un horizon de planification infini. 2) vérifier l'influence de l'erreur humaine sur le C/D ainsi que

l'activité de maintenance préventive. Des méthodes numériques sont utilisées pour résoudre les conditions d'optimum et obtenir les politiques optimales qui minimisent le coût sur un horizon infini. Des exemples numériques et analyse de sensibilité sont présentés pour illustrer l'utilité de l'approche proposée.

### **Abstract**

The analysis of the optimal production and preventive maintenance with lockout/tagout planning problem for a manufacturing system is presented in this paper. The considered manufacturing system consists of two non-identical machines in passive redundancy producing one type of part. These machines are subject to random breakdowns and repairs. This paper is different compared to other research projects on preventive maintenance and lockout/tagout. The influence of human error on lockout/tagout as well as on preventive maintenance activities are presented in this paper. The decision variables are the production rate and preventive maintenance rate with lockout/tagout which can influence the inventory levels and the system's capacity. The system capacity is described by a finite-state Markov chain. The aim of the study is to minimize production, inventory, backlog and maintenance costs over an infinite planning horizon; in addition, it aims to verify the influence of human reliability on the inventory levels for illustrating the importance of human error during the maintenance and lockout/tagout activities. In this paper, the preventive maintenance policy depends on the machine age. For the considered manufacturing system the optimality conditions are provided, and numerical methods are used to obtain machine age-dependent optimal control policies (production and preventive maintenance rates with lockout/tagout). Numerical examples and sensitivity analysis are presented to illustrate the usefulness of the proposed approach.

**Keywords:** Lockout/tagout, Preventive maintenance, Human error, Production control, Passive redundancy

## 5.1 Introduction

Production systems optimization is an interesting topic for researchers as well as industry. To increase the availability of machines at the functional level, several maintenance strategies have been developed (Lugtigheid et al., 2008; Nahas et al., 2008). The problem of optimally controlling production rates in a manufacturing system has been widely discussed in the scientific literature (Hajji et al., 2009; Kenné and Gharbi, 2008; Kenné et al., 2007). In this view, Rishel (1975) developed the optimal conditions to obtain the optimal solution using dynamic programming. Based on the formalism of Rishel (1975), Sethi et al. (2002) modeled the stochastic control production planning for a flexible manufacturing system (FMS) subject to random failures, which allowed them to obtain the dynamic programming equation of optimal control. In this way, Sethi et al. (2002) modeled the uncertainties of manufacturing system by homogeneous Markov processes and determined the production policy in which the inventory and backlog cost are minimized according to the production rate. The homogeneous process assumes that state transition rates are constant. This is not actually applicable in the manufacturing system in which the failure rate depends on the machine age (Boukas and Haurie (1990)).

Akella and Kumar (1986) modeled a manufacturing system which was consisted of a machine producing one part type with homogeneous Markov chain. The authors found an optimal control policy by using Hamilton-Jacobi-Bellman (HJB) equations which is called Hedging point policy (HPP), (see Dehayem Nodem et al. (2008) for the details). For a manufacturing system composed of two machines producing one part type, Boukas and Haurie (1990) showed that the value function that performs the optimal cost must satisfy a set of differential equations called Hamilton-Jacobi-Bellman (HJB). In the complex manufacturing systems which optimal conditions are described by the HJB equations, obtaining an analytical solution of HJB equations remains impossible. The numerical solution of the HJB equation is a challenge which was considered insurmountable in the scientific literature. Although there is no analytical solution to HJB equations, Yan and Zhang (1997) found a solution for the stochastic optimal control problem. Yan and Zhang used a numerical method based on the Kushner approach (Kushner and Dupuis, 1992) for a

manufacturing system producing several types of parts. These authors showed that the obtained control policy is optimal.

Based on the resolution method and the results of Yan and Zhang (1997) several authors have combined numerical approach with experimental approach based on simulation in order to extend the concept of non-Markov processes. Furthermore, sometimes it is not possible to represent failures and repairs of machines by homogenous Markov processes. This case has been studied by Kenné and Nkeungoue (2008). In this work, authors took into account that the probability of equipment failures increases according to the machine age. Their work has been shown that the hedging point policy was optimal for a manufacturing system consisting of one machine producing one part type. As the failure rate depends on the machine age, the manufacturing system was modeled by non-homogeneous Markov chain. Since the machines are subject to random breakdowns and repairs, manufacturing systems operate in a stochastic environment. Predicting and controlling certain events are possible while others occur randomly, and are consequently beyond the control of manufacturing systems (Gershwin, 2002). An occupational accident can be observed as a disturbance, often with much more severe effects than the breakdown itself. These occupational hazards will cause a lot of damage and prevent the company to perform its tasks.

Occupational safety researchers worldwide have carried out studies confirming the importance of monitoring and controlling undesirable incidents or accidents during maintenance procedures (Charlot et al., 2006). An important question that arises is to know: *How can one find an appropriate solution to optimize production while guaranteeing the safety of workers?* One possible answer is lockout/tagout. It consists of locking a machine with a padlock to discharge all sources of residual energy (hydraulic, electrical, etc.) in order to avoid the premature starting of equipment throughout an intervention. However, the current use of lockout /tagout has several shortcomings such as lack of validation of its feasibility and profitability. There is always a risk during intervention on machines that are down. The risk of human error during these interventions has a direct impact on the availability of manufacturing systems and can lead to occupational incidents or accidents.

Protection devices are often absent or bypassed during these interventions, which explains the statistics of accidents remain high in this sector (Chinniah and Champoux, 2008).

In order to address these problems, Emami-Mehrgani et al. (2011) considered an analytical model combining lockout/tagout, production and corrective maintenance policies for a passive redundancy system, consisting of two non-identical machines. In this work, the authors demonstrated clearly that passive redundancy optimized production and maintenance costs while increasing the security level. Another work integrating lockout/tagout into operational risk in production control is proposed by Emami-Mehrgani et al. (2012). The authors considered in Emami-Mehrgani et al. (2012) a manufacturing system consisting of three machines (two machines with passive redundancy, and one in series with the previous ones) producing one part type. Their work has confirmed that it is possible to integrate a passive redundancy system in a production line in order to reduce the production cost. This also leads to into higher free space-time and minimizes the possibility of circumvention of protection devices or retraction of lockout/tagout procedures for the machines that are under repair.

The original nature of works presented in (Emami-Mehrgani et al. (2011) and Emami-Mehrgani et al. (2012) has allowed them to perceive the importance of human error during maintenance and lockout/tagout activities. Therefore, this research project takes its originality from human error modeling in dynamics of manufacturing system and also verify the influence of human error during maintenance and lockout/tagout activities on the optimal safety stock levels. In this paper, human error is considered as improper operation which increases mean preventive maintenance and lockout/tagout time. Hence, control of the lockout/tagout procedure is integrated into production and maintenance planning. The objectives of this research project are reached by controlling the production rate, preventive maintenance rate with lockout/tagout for a manufacturing system consisting of two non-identical machines in passive redundancy producing one type part. A stochastic dynamic programming problem is formulated for the considered manufacturing system. Generally obtaining the optimal solution analytically is difficult, since the corresponding optimality

conditions are represented by a set of not easy to solve coupled non-linear partial differential equations called HJB. In this paper, the numerical methods are used in order to find a solution of the problem.

This paper is organized as follows: Section 5.2 introduces the notation and assumptions. The model of the problem under consideration states in Section 5.3. A numerical example and a sensitivity analysis are provided in Section 5.4. The paper is concluded in Section 5.5.

## **5.2 Assumptions and notations**

This paper incorporates the following assumptions.

### **5.2.1 Assumptions**

1. The corrective and preventive maintenance are carried out with lockout/tagout.
2. The machine becomes new after each corrective maintenance operation.
3. The machine failure rate is a continuous function of its age. It remains undisturbed by preventive maintenance.
4. The age of machine is rest to zero after each preventive maintenance operation.
5. The preventive maintenance can be achieved with human error or without human error.
6. The mean preventive maintenance time without human error is shorter than mean preventive maintenance time with human error.
7. The production rate of main machine is upper than the standby machine.
8. The main machine and the standby machine produce the same type of parts
9. The main machine returns to production immediately after each repair and the standby machine stands idle.

Assumption 5 takes into account the influence of human reliability on the maintenance activity.



Assumption 9 is a classical assumption in passive redundancy system. It is due to the nature of a passive redundancy system.

### 5.2.2 Notations

The following notations are used in the rest of this paper:

$x(\cdot)$ : inventory/backlog level of finished product

$c^+$ : holding cost incurred on finished product

$c^-$ : backlog cost incurred on finished product

$c^\alpha$ : cost incurred for the operation of the machine under repair at mode  $\alpha$

$c_{r_1}$ : corrective maintenance cost of main machine  $M$

$c_{r_s}$ : corrective maintenance cost of standby machine  $S$

$c_{tagout}$ : lockout/tagout cost

$u_1$ : production rate of main machine  $M$

$u_1^{\max}$ : maximal production rate of main machine  $M$

$u_s$ : production rate of standby machine  $S$

$u_s^{\max}$ : maximal production rate of standby machine  $S$

$w_{42}^{24}$ : preventive maintenance rate without human error

$w_{42}^{24\max}$ : maximal preventive maintenance rate without human error

$w_{42}^{34}$ : preventive maintenance rate with human error

$w_{42}^{34\max}$ : maximal preventive maintenance rate with human error

$g(\cdot)$ : instantaneous cost

$J(\cdot)$ : total cost

$v(\cdot)$ : value function

$\rho$ : discount rate

$d$ : demand rate

$q_{14}$ : repair rate of standby machine  $S$

$q_{23}$ : preventive maintenance rate without human error to preventive maintenance rate with human error of standby machine  $S$

$q_{24}$ : preventive maintenance rate without human error of standby machine  $S$

$q_{34}$ : preventive maintenance rate with human error of standby machine  $S$

$q_{54}$ : repair rate of main machine  $M$

### 5.3 Problem statement

This section develops a manufacturing system for two machines that are not identical in passive redundancy and producing one type of part. These machines are prone to random breakdowns and repairs. The machine could be shut down and locked for preventive maintenance. Repairs will be carried out for machines breakdowns in the context of a year-to-year operation of the system. The system is shown in Figure 5.1. In the system under consideration, the main machine operates any time. The other machine called the cold-standby redundancy goes online immediately when the main machine fails. Then, the maintenance service is triggered to restore the failed machine. After the failed machine is restored, it will be returned into the standby position. The main machine has two states:  $\xi_1(t)=1$  if the main machine  $M$  is operational and  $\xi_1(t)=2$  if the main machine  $M$  is under repair. The standby machine has five states:  $\xi_s(t)=1$  if the standby machine  $S$  is operational,  $\xi_s(t)=2$  if the standby machine  $S$  is under repair,  $\xi_s(t)=3$  if the standby machine  $S$  is under preventive maintenance without human error,  $\xi_s(t)=4$  if the standby

machine  $S$  is under preventive maintenance with human error and  $\xi_s(t) = 5$  if the standby machine  $S$  is at time-off. Hence, the dynamics for a manufacturing system with two machines in passive redundancy is in a hybrid state which consists of a continuous state  $x(t)$  and a discrete state  $\xi(t) = (\xi(t)_1, \xi(t)_s)$  as follows:

a) *Continuous state*: The difference between cumulative production and demand denotes as  $x(\cdot)$ . The surplus  $x(\cdot)$  can be positive (i.e., inventory costs  $c^+$  are thus charged) or negative (i.e. backlog costs  $c^-$  are thus charged). The input rate to  $M$  (main machine) and  $S$  (standby machine) denotes  $u_i(t)$  with  $(i=1, S)$ .

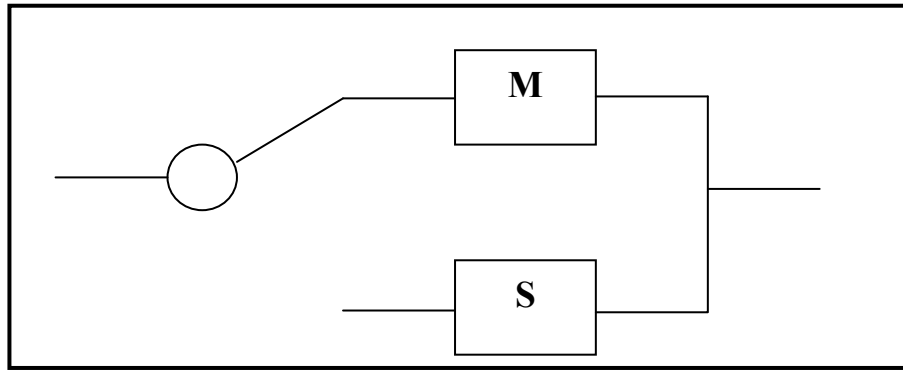


Figure 5.1 Passive redundancy system producing one part type

The dynamics of the system can be written as follows:

$$\begin{aligned} \frac{dx}{dt} &= u_i(\cdot) - d, \quad x(0) = x, \\ &\text{with } i = 1, S \\ \frac{da}{dt} &= f(u(\cdot)), \quad a(0) = a, \\ a(T) &= 0. \end{aligned} \tag{5.1}$$

Where  $f(u_i(\cdot))$  is an increasing function of the machines production rate which represents the machine aging.

b) *Discreet state*: Consistent with assumptions set out in previous section, the operational mode of the whole system can be described by a random vector  $\xi(t) = (\xi_1(t), \xi_s(t))$  taking values in  $B = \{1, 2, 3, 4, 5\}$ . Assuming that in practical terms:

- standby machine cannot be at time-off when the main machine is down;
- standby machine cannot be sent to preventive maintenance, knowing that the main machine is down and vice versa;
- both machine (main and standby) cannot be sent to preventive maintenance at the same time;
- both machines (main and standby) cannot be broken down simultaneously (with a good maintenance plan).

These practical considerations allowed us to describe the dynamics of the system in 5 modes without loss of generality. Hence, for the passive redundancy  $\xi(t)$  can be expressed as follows:

$$\xi(t) = \begin{cases} 1 & \text{M is operational and S is under repair;} \\ 2 & \text{M is operational and S is under preventive maintenance without human error;} \\ 3 & \text{M is operational and S is under preventive maintenance with human error;} \\ 4 & \text{M is operational and S is at time-off;} \\ 5 & \text{M is under rapair and S is operational.} \end{cases}$$

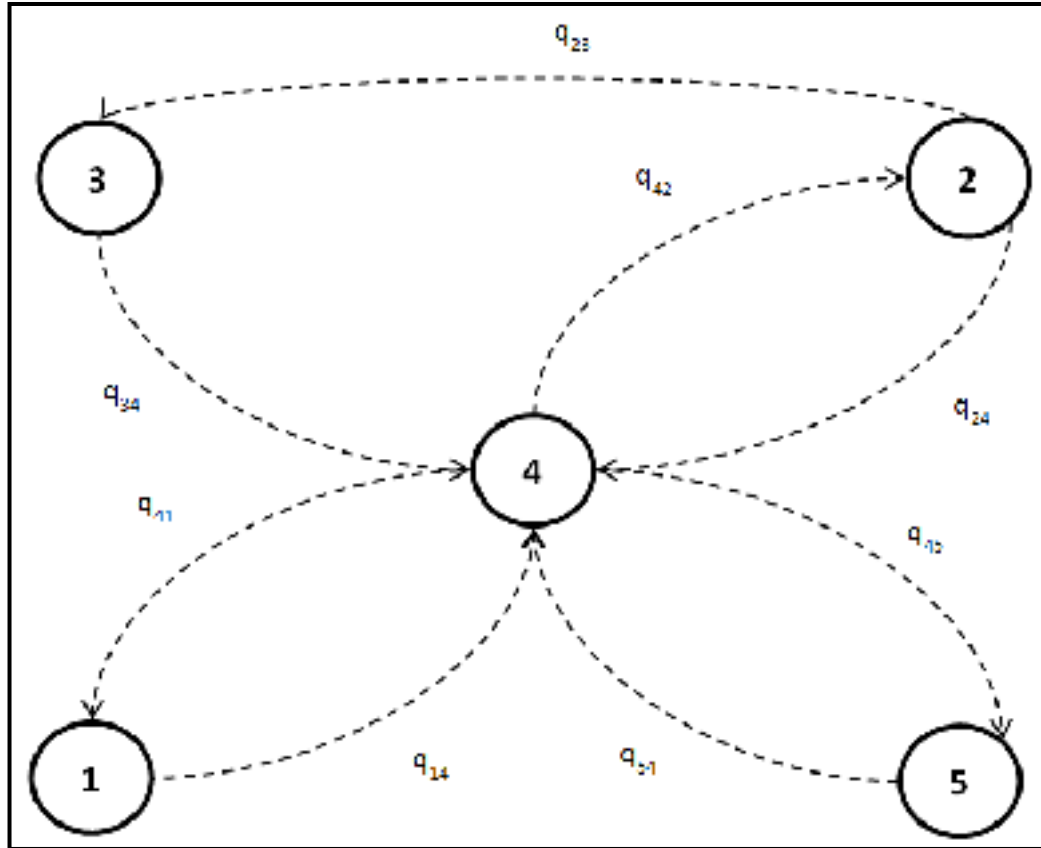


Figure 5.2 State transition diagram

The transition rate matrix of the stochastic processes  $\xi(t)$  is denoted by  $Q$  such that  $Q = \{q_{\alpha\beta}\}$ , with  $q_{\alpha\beta} > 0$  if  $\alpha \neq \beta$  and  $q_{\alpha\alpha} = -\sum_{\beta \neq \alpha} q_{\alpha\beta}$ , where  $\alpha, \beta \in B$ .

The transition probabilities associated to the transition rate  $q_{\alpha\beta}$  are expressed as:

$$p[\xi(t+\delta t) = \beta | \xi(t) = \alpha] = \begin{cases} q_{\alpha\beta}(\cdot)\delta t + o(\delta t) & \text{if } \alpha \neq \beta, \\ 1 + q_{\alpha\alpha}(\cdot)\delta t + o(\delta t) & \text{if } \alpha = \beta. \end{cases} \quad (5.2)$$

The transition rate matrix  $Q$  is expressed as follows:

$$Q(w_{42}^{\alpha\beta}) = \begin{bmatrix} q_{11} & 0 & 0 & q_{14} & 0 \\ 0 & q_{22} & q_{23} & q_{24} & 0 \\ 0 & 0 & q_{33} & q_{34} & 0 \\ q_{41} & w_{42}^{\alpha\beta} & 0 & q_{44} & q_{45} \\ 0 & 0 & 0 & q_{54} & q_{55} \end{bmatrix}, \quad (5.3)$$

Where:

$w_{42}^{\alpha\beta} = w_{42}^{24}$  preventive maintenance rate without human error,

$w_{42}^{\alpha\beta} = w_{42}^{34}$  preventive maintenance rate with human error.

The set of admissible decisions at mode  $\alpha(t)$  and control policies (control variables) at mode  $\alpha(t)$ :

$$\Gamma(\alpha) \left[ \begin{array}{l} ((u_1(\cdot), u_s(\cdot), w_{42}^{24}(\cdot), w_{42}^{34}(\cdot)) \in R^4, \\ 0 \leq u_1(\cdot) \leq u_1^{\max}, 0 \leq u_s(\cdot) \leq u_s^{\max}, \\ w_{42}^{24\min}(\cdot) \leq w_{42}^{24}(\cdot) \leq w_{42}^{24\max}(\cdot), \\ w_{42}^{34\min}(\cdot) \leq w_{42}^{34}(\cdot) \leq w_{42}^{34\max}(\cdot). \end{array} \right] \quad (5.4)$$

In equation (4),  $u_1^{\max}$  is the maximal production rate of the main machine,  $u_s^{\max}$  is the maximal production rate of the standby machine,  $w_{42}^{24\min}$  is the minimal preventive maintenance rate without human error,  $w_{42}^{24\max}$  is the maximal preventive maintenance rate without human error,  $w_{42}^{34\min}$  is the minimal preventive maintenance rate with human error and  $w_{42}^{34\max}$  is the maximal preventive maintenance rate with human error.  $\Gamma(\alpha)$  denotes the set of all admissible controls.

The control problem consists in finding an admissible control law  $u(\cdot) = (u_1, u_s, w_{42}^{24}, w_{42}^{34})$  that minimizes the cost function  $J(\cdot)$  given by:

$$J(a, x, \alpha, u_1, u_s, w_{42}^{24}, w_{42}^{34}) = E \left\{ \int_0^{\infty} e^{-\rho t} g(a, x, \alpha, u_1, u_s, w_{42}^{24}, w_{42}^{34}) dt \mid x(0) = x, \xi(0) = a, \alpha(0) = 0 \right\}, \quad (5.5)$$

Where  $\rho$  is the discount rate and  $g(x, \alpha, \cdot) = c^+ x^+ + c^- x^- + c^\alpha$  is the instantaneous cost,  $c^+$ ,  $c^-$  and  $c^\alpha$ , being the cost per unit to produce parts for inventory, backlog as well as intervention cost on the machine, respectively.

$$x^+ = \max(0, x), x^- = \max(-x, 0)$$

and

$$c^\alpha = (c_{r_1} + c_{tagout})\text{Ind}\{\alpha = 1\} + (c_{pm} + c_{tagout})\text{Ind}\{\alpha = 2\} \\ + (c_{pm} + c_{tagout})\text{Ind}\{\alpha = 3\} + (c_{r_s} + c_{tagout})\text{Ind}\{\alpha = 5\}$$

With:

$$\text{Ind}\{\Theta(\cdot)\} = \begin{cases} 1 & \text{if } \Theta(\cdot) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

For a given proposition  $\Theta(\cdot)$ .

As in Akella and Kumar (1986), the manufacturing system considered has a sufficient capacity to ensure its feasibility, which means the average capacity is not less than the demand rate.

Let  $v(a, x, \alpha)$  denote the value function or minimum discounted cost for equations (5.5) as expressed in the following equation:

$$v(a, x, \alpha) = \min_{(u_1, u_s, w_{42}^{24}, w_{42}^{34}) \in \Gamma(x, \alpha)} J(a, x, \alpha, u_1, u_s, w_{42}^{24}, w_{42}^{34}), \forall \alpha \in B \quad (5.6)$$

In Appendix 5.A, the properties of the value function  $v(\cdot)$  given by equation (5.6) are presented. It is shown that the value function  $v(\cdot)$  given by (5.6) should satisfy a set of partial differential equations known as the Hamilton-Jacobi-Bellman (HJB) equations.

#### 5.4 Numerical example and results analysis

In this section, we present a numerical example for a manufacturing system consisting of two non-identical machines in passive redundancy. The system capacity is described by a five Markov process with states  $\xi(t) \in B = [1, 2, 3, 4, 5]$ . The generator matrix  $Q(\cdot)$  described by equation (5.3) is explicitly defined as follows:

$$Q(w_{42}^{\alpha\beta}) = \begin{bmatrix} -q_{14} & 0 & 0 & q_{14} & 0 \\ 0 & -(q_{23} + q_{24}) & q_{23} & q_{24} & 0 \\ 0 & 0 & -q_{34} & q_{34} & 0 \\ q_{41} & w_{42}^{\alpha\beta} & 0 & -(q_{41} + w_{42}^{\alpha\beta} + q_{45}(a)) & q_{45}(a) \\ 0 & 0 & 0 & q_{54} & q_{55} \end{bmatrix},$$

Where:

$w_{42}^{\alpha\beta} = w_{42}^{24}$  preventive maintenance rate without human error,

$w_{42}^{\alpha\beta} = w_{42}^{34}$  preventive maintenance rate with human error.

and

$$q_{45}(a(t)) = A_{45}^{\infty} \left[ 1 - e^{-(A_t \times a(t)^2)} \right], \quad (5.7)$$

or given constants  $A_{45}^{\infty}$  and  $A_t$ . The transition rate  $q_{45}(\cdot)$  given by equation (5.7), illustrates the impact of a machine age on its dynamics as in Kenné and Nkeungoue (2008). The mean time between failures (MTBF) is machine-age dependent and is given by:

$$\text{MTBF}(a) = \frac{1}{q_{45}(a)} \text{ with } \text{MTBF}(\infty) = \frac{1}{A_{45}^{\infty}}.$$

The discrete dynamic programming equation (5.A.4) gives the following five equations:

$$v^h(\alpha, x, 1) = \min_{(u_1, u_2, w_{42}^{24}, w_{42}^{34}) \in \Gamma(1)} \left( \rho + \frac{|u_1 - d|}{h_x} + \frac{Ku}{h_a} + q_{14} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h_x} (v^h(x+h_x, \alpha, 1)k^+ + v^h(x-h_x, \alpha, 1)k^-) \\ + \frac{Ku}{h_a} (v^h(x+h_x, \alpha, 1) + g(x, \alpha, 1)) \\ + q_{14} v^h(x, \alpha, 1) \end{array} \right\}, \quad (5.8)$$

$$v^h(\alpha, x, 2) = \min_{(u_1, u_2, w_{42}^{24}, w_{42}^{34}) \in \Gamma(2)} \left( \rho + \frac{|u_1 - d|}{h_x} + \frac{Ku}{h_a} + q_{23} + q_{24} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h_x} (v^h(x+h_x, \alpha, 2)k^+ + v^h(x-h_x, \alpha, 2)k^-) \\ + \frac{Ku}{h_a} (v^h(x+h_x, \alpha, 2) + g(x, \alpha, 2)) \\ + q_{23} v^h(x, \alpha, 3) + q_{24} v^h(x, \alpha, 4) \end{array} \right\}, \quad (5.9)$$

$$v^h(\alpha, x, 3) = \min_{(u_1, u_2, w_{42}^{24}, w_{42}^{34}) \in \Gamma(3)} \left( \rho + \frac{|u_1 - d|}{h_x} + \frac{Ku}{h_a} + q_{34} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h_x} (v^h(x+h_x, \alpha, 3)k^+ + v^h(x-h_x, \alpha, 3)k^-) \\ + \frac{Ku}{h_a} (v^h(x+h_x, \alpha, 3) + g(x, \alpha, 3)) \\ + q_{34} v^h(x, \alpha, 4) \end{array} \right\}, \quad (5.10)$$



$$V^h(\alpha, x, 4) = \min_{(u_1, u_s, w_{42}^{\beta}, w_{42}^{\beta}) \in \Gamma(4)} \left( \rho + \frac{|u_1 - d|}{h_x} + \frac{Ku}{h_a} + q_{41} + w_{42}^{\beta} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h_x} (V^h(x+h_x, \alpha, 4)k^+ + V^h(x-h_x, \alpha, 4)k^-) \\ + \frac{Ku}{h_a} (V^h(x+h_x, \alpha, 4) + g(x, \alpha, 4)) \\ + q_{41} V^h(x, \alpha, 1) + w_{42}^{\beta} V^h(x, \alpha, 2) \end{array} \right\}, \quad (5.11)$$

$$V^h(\alpha, x, 5) = \min_{(u_1, u_s, w_{42}^{\beta}, w_{42}^{\beta}) \in \Gamma(5)} \left( \rho + \frac{|u_s - d|}{h_x} + \frac{Ku}{h_a} + q_{54} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_s - d|}{h_x} (V^h(x+h_x, \alpha, 5)k^+ + V^h(x-h_x, \alpha, 5)k^-) \\ + \frac{Ku}{h_a} (V^h(x+h_x, \alpha, 5) + g(x, \alpha, 5)) \\ + q_{54} V^h(x, \alpha, 4) \end{array} \right\}, \quad (5.12)$$

The following computational domain is used:

$$G_{ax}^h = \{(x, a) : -5 \leq x \leq 25; 0 \leq a \leq L_a\}, \text{ in which } h_x = 2, h_a = 3 \text{ and } L_a = 150.$$

$L_a$  should be smaller than the age limit of the machine ( $L_{\max}$ ) with respect to the feasibility conditions.

The characteristics of main and standby machines must be in such a way that respects the feasibility of system.

The system is considered feasible if:

$$\sum \pi(\cdot) u_i^{\max}(\cdot) \geq d \quad (5.13)$$

Where the limitation probabilities can be ascertained from the following equation for a system conforming to a Markov process:

$$\pi(\cdot)Q(\cdot) = 0,$$

$$\sum \pi = 1$$

with:

$\pi(\cdot)$  : Limiting probabilities

$Q(\cdot)$  : Transition matrix rates

$\pi(\cdot) = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$  representing the vector of limiting probabilities from modes 1 to 5.

Using the formula (5.13), we ascertained that the system is not feasible for an age of 287, that means  $L_{\max} = 286$ .

Figure 5.3, illustrates the failure rate of the main machine for each value of its age with values of  $A_{45}^{\infty} = 0.1$  and  $A_t = 1 \times 10^{-5}$  chosen to obtain a failure probabilities trajectory according to the machine age as in Kenné and Nkeungoue (2008).

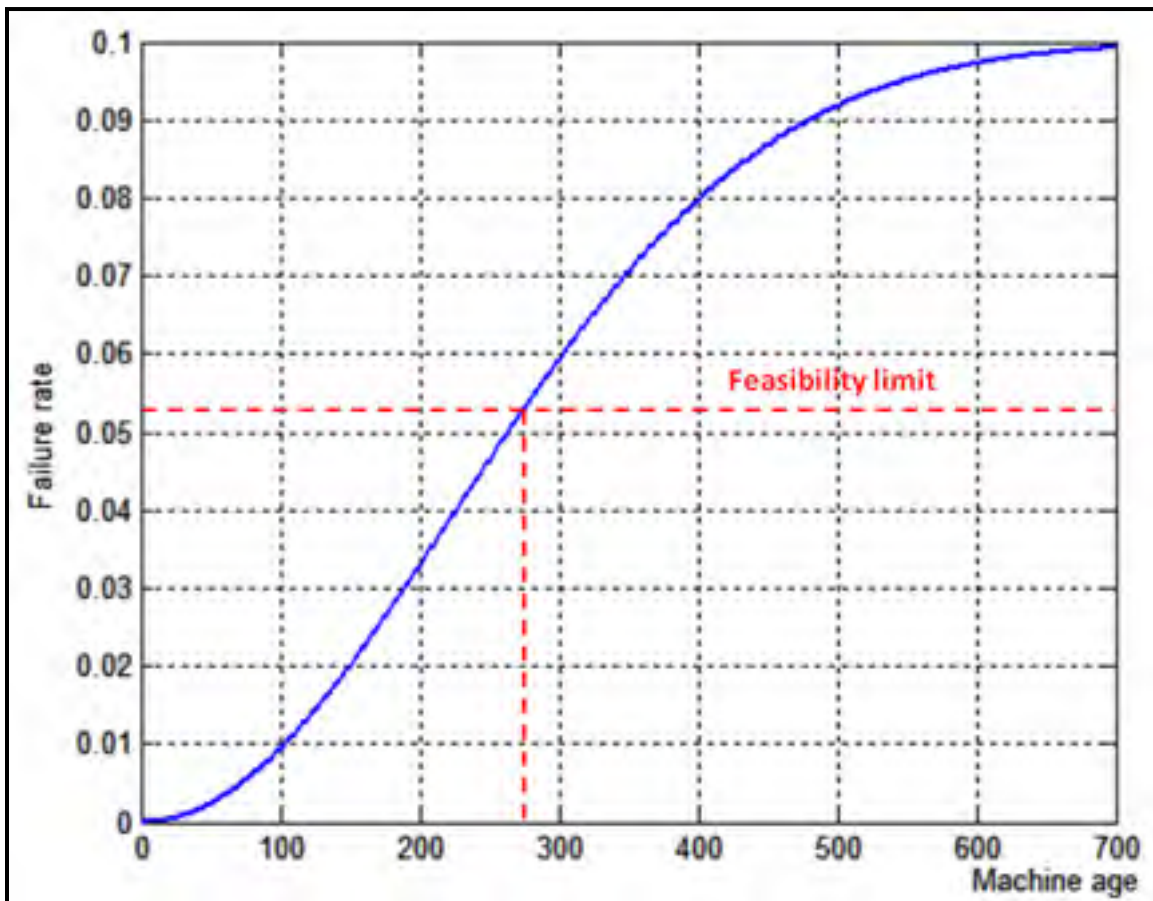


Figure 5.3 Age-dependent failure rate of the main machine

The parameters for our case study appear in Table 5.1.

Table 5.1 Parameters of the numerical example

$u_1^{\max}$	$u_s^{\max}$	$w_{42}^{24\min}$	$w_{42}^{24\max}$	$w_{42}^{34\min}$	$w_{42}^{34\max}$	$q_{14}$	$q_{23}$	$q_{24}$	$q_{34}$	$q_{54}$
0.32	0.28	$10^{-6}$	0.06	$10^{-6}$	0.06	$10^{-5}$	0.02	0.083	0.125	0.053
$d$	$\rho$	$c^+$	$c^-$	$c_r$	$c_{mp}$	$c_{tagout}$				
0.25	0.01	1	30	7900	400	100				

Please note that these parameters have been taken so that the system will be feasible. Further, we will check the influence of these parameters by sensitivity analysis. The policy iteration technique solves equations (5.8)–(5.12) for optimal conditions. Recall that the control policies are obtained from the numerical resolution of the optimality conditions given by equations (5.8)–(5.12). The structure of production policies ( $u_1$  and  $u_s$ ) states that: if the stock level is lower than a threshold level, then produce at maximum rates; else if the stock level is upper than a threshold level produce nothing; otherwise produce at the demand rate. Such structure in the control literature is called hedging point policy (HPP).

The results obtained for the control variables  $u_1, u_s, w_{42}^{24}$  and  $w_{42}^{34}$  of a passive redundancy system are given in Figures (5.4)–(5.8) for illustration purposes.

The machine-age-dependent threshold value, for data presented in Table 1, is defined using the switching trend illustrated in Figures (5.4)–(5.8). Hence, the production policy is given by as follows:

$$u_i(a, x, \alpha) = \begin{cases} u_i^{\max}(\alpha) & \text{if } x(\cdot) < \psi(a), \\ d & \text{if } x(\cdot) = \psi(a), \\ 0 & \text{otherwise,} \end{cases} \quad (5.14)$$

Where:

$$u_i^{\max}(\alpha) = u_1^{\max} \quad \text{if } \alpha = 1, 2, 3, 4 \quad , \quad u_i^{\max}(\alpha) = u_s^{\max} \quad \text{if } \alpha = 5.$$

and  $\psi(a)$  is machine-age dependent function that gives the optimal threshold value for each value of the machine age. Hence, we obtain from numerical result as follows:

$$\psi(a(t)) = \begin{cases} X^*(a) & \text{if } a(t) \geq A^*(a), \\ 0 & \text{otherwise.} \end{cases} \quad (5.15)$$

Figure 5.4 shows the existence of different zones for the production rate of the main machine. Each of these areas follows a production optimal policy. These different production policies reflect the machine age and the inventory level. According to this figure, beyond a certain age the threshold becomes more important. The policy suggests setting the production rate of the machine to its maximal value when the current stock level is under an age-dependent threshold value. In this area the age leads to frequent failure. On the other hand, the production rate of the machine sets to the demand rate when the current stock level is equal to an age-dependent threshold value. Finally the production rate of the machine sets to zero when the current stock level is larger than an age-dependent threshold value.

Figure 5.5 illustrates the same structure as Figure 5.4, but the policy suggests keeping more inventories by increasing the machine age, because the preventive maintenance has been done with the human error. Figure 5.6 plots the same policy for standby machine production rate as main machine production rate, but this figure is illustrated to have more inventories according to machine age. Recall that the standby machine production rate is lower than the main machine production rate.

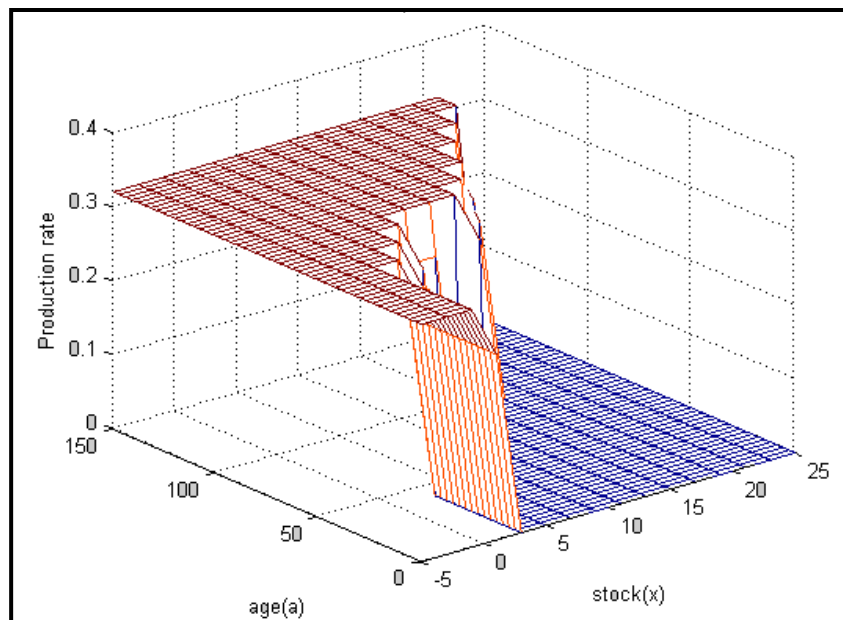


Figure 5.4 Main machine production rate at mode 2

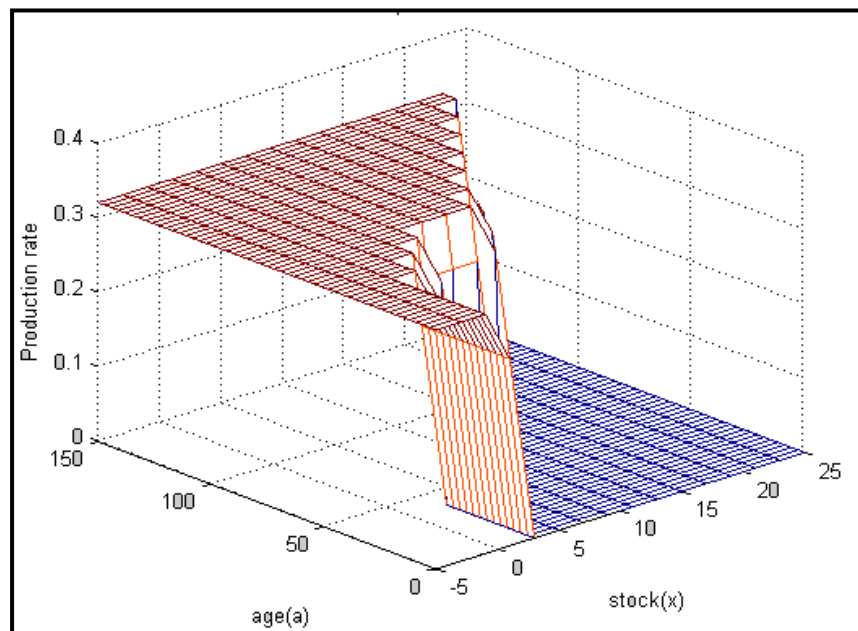


Figure 5.5 Main machine production rate at mode 3

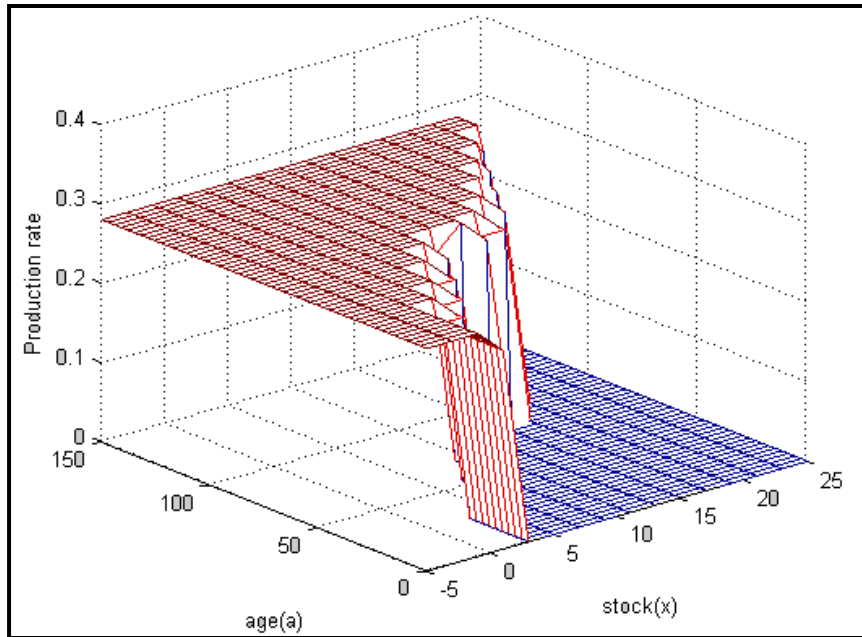


Figure 5.6 Standby machine production rate at mode 5

The preventive maintenance policy with human error and without human error is plotted in Figures 5.7 and 5.8. Figure 5.7 shows the standby machine preventive maintenance rate without human error, plotted in this figure, divides the computational domain  $(x,a)$  into two regions. Hence, the preventive rate is set to its maximal value for backlog situation and to zero for large stock levels. For significant stock levels, the zone in the domain  $(x,a)$  where the preventive maintenance is set to its maximal value increases with the machine age. Figure 5.8 illustrates the same structure as Figure 5.7, but the policy suggests keeping more inventories by increasing the machine age in order not to penalize the demand, for the reason that the preventive maintenance has been done with the human error. The corresponding optimal policy, similar to the previous policy, has a bang bang structure and is given by following equation:

$$w_{42}^{\alpha\beta} = \begin{cases} w_{42}^{\max \alpha\beta} & \text{if } x(\cdot) < B^*(a), \\ w_{42}^{\min \alpha\beta} & \text{otherwise.} \end{cases} \quad (5.16)$$

Where:

$w_{42}^{\alpha\beta} = w_{42}^{24}$  preventive maintenance rate without human error,

$w_{42}^{\alpha\beta} = w_{42}^{34}$  preventive maintenance rate with human error.

and  $B^*(a)$  is machine-age-dependent function that gives the optimal stock level at which it is necessary to switch the preventive maintenance from  $w_{42}^{\alpha\beta}$  to  $w_{42}^{\alpha\beta}$  for given machine age.

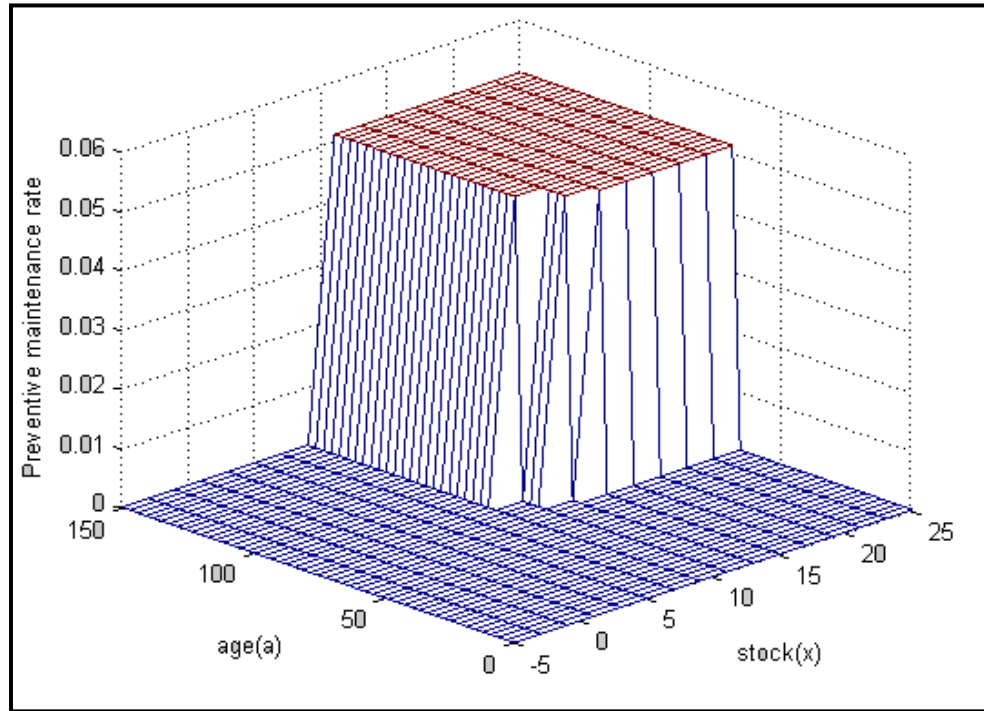


Figure 5.7 Standby machine preventive maintenance rate without human error at mode 2

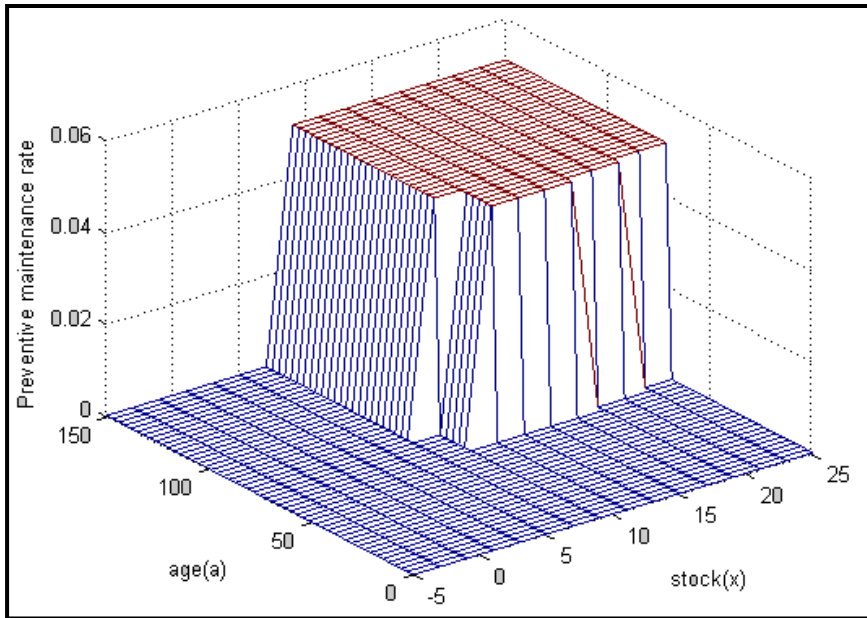


Figure 5.8 Standby machine preventive maintenance rate with human error at mode 3

The influence of the backlog cost on the production threshold according to the machine age as well as preventive maintenance cost according to the machine age for main and standby machine are presented in Figures 5.9 and 5.10.

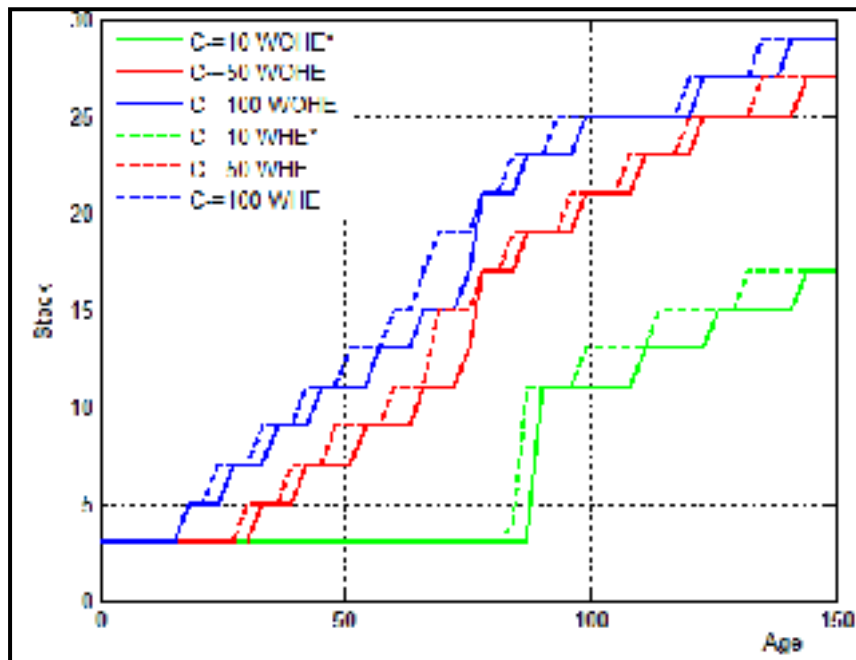


Figure 5.9 Trend of threshold value of main machine according to machine age



WOHE\*: Without human error

WHE\*: With human error

Indeed, Figure 5.9 illustrates that if the production unit is still in its youth phase, it is not essential to store large inventory levels. In this phase, the failure rate is low and the production unit has a high reliability and it is able to meet the demand. As one goes along the machine age increases, storage of finished products inventories are suggested to meet the demand. The quantity of finished products inventories depends on the machine age. When the machine becomes older, the higher level of inventories is needed. Hence, the manufacturing system is preparing for a possible failure whose frequency increases with age. It should be also remarked that if the backlog cost increases, the level of stock must be increased for the same machine age to avoid a possible shortage. This increase is more important, if the maintenance technician makes errors during the maintenance activity.

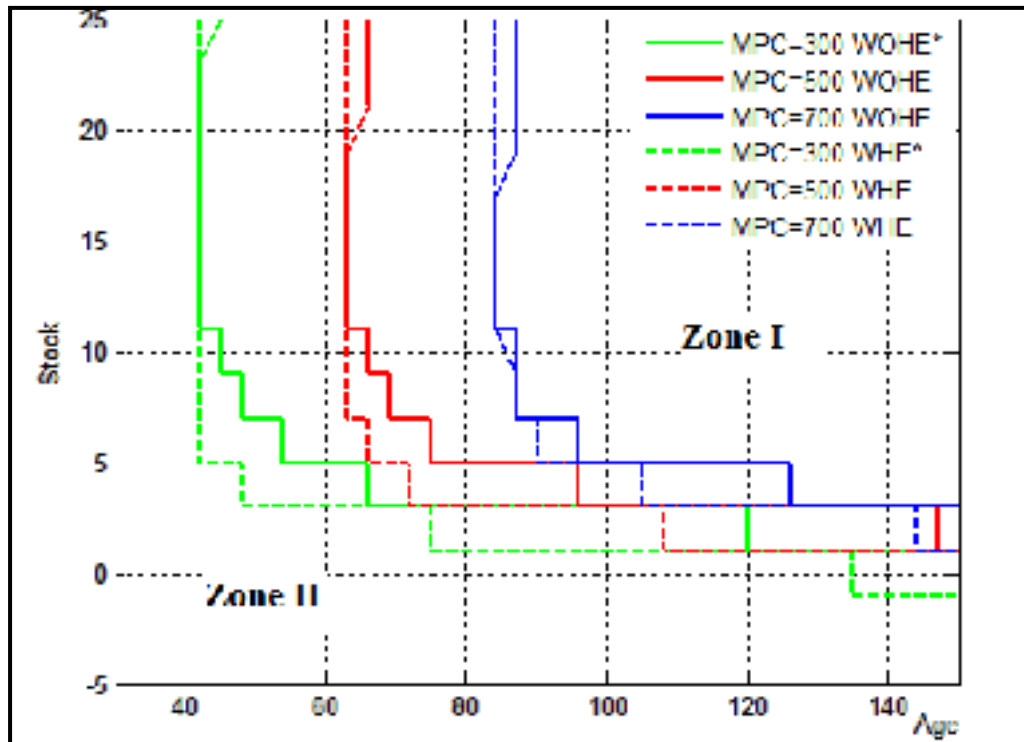


Figure 5.10 Trend of preventive maintenance rate of standby machine according to machine age

WOHE\*: Without human error

WHE\*: With human error

The Figure 5.10 shows the preventive maintenance rate that is based on the machine age and inventory level. This figure can be divided into two main areas I and II. In Zone II, the production policy suggests not to send the machine in preventive maintenance. Indeed, in this area, the production unit is still new and the probability of failure is almost zero. As a result, the manufacturing system is able to meet the demand without fear of failure. Therefore, it is unnecessary to perform preventive maintenance. This maintenance is appropriate only in zone I, as shown in Figure 5.10. Indeed, in this area the production unit is aging and the failure rate increases. Perform preventive maintenance becomes necessary to increase the life of the production system. Therefore, the manufacturing system must ensure a certain inventory level to meet the demand. In addition, if the cost of preventive maintenance increases, the frequency of preventive maintenance must be reduced. It is also observed that if the maintenance technician makes a mistake, a larger inventory level for the same cost of preventive maintenance must be stored in order to meet the demand. Moreover, because of this anomaly the occurrence probability of an incident or accident will be increased.

Optimal costs of production and preventive maintenance including lockout/tagout procedures for two types of preventive maintenance (without human error and with human error) may be determined using the analytical model presented in this paper. In this paper the human error increases repair time which is exponentially distributed. If the repair-time is not exponentially distributed (non-Markovian case), an extension of human error concept developed in this paper can be used. The numerical approach demonstrates that the resulting policy is optimal and enhances machine availability for manufacturing system under consideration. The control policy for manufacturing system considered is based on an extension of the hedging point structure. Without limiting in any way the generality of this proposal, the extension of this model might be adapted to different industrial sectors under certain conditions.

## 5.5 Conclusion

This paper sought to verify the influence of human error on the preventive maintenance activity with lockout/tagout. In this paper, the control policy is based on an extension of the hedging point structure. Based on the numerical solution obtained, a parameterized near-optimal control policy was derived. The proposed approach using a numerical example and a sensitivity analysis are illustrated and validated. This work demonstrates clearly that human error during maintenance activities can increase the production cost while reducing the safety of workers. In order to avoid this problem, the maintenance technicians must be better trained and also the production cadence must not be increased brutally. Because the maintenance technician can make a mistake and the occurrence probability of an incident or accident will be increased. Furthermore, a larger inventory level must be stored in order to meet the demand during the maintenance activity. Lastly, a number of conditions must be met to make effective use of the model presented in this paper.

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## APPENDIX 5.A OPTIMAL CONDITIONS AND NUMERICAL APPROACH

The properties of the value functions and the Hamilton-Jacobi-Bellman equation (HJB) are presented in this section. These equations describe the optimality conditions for a manufacturing system consisting of two non-identical machines in passive redundancy producing one type of part. The HJB equation is adapted to the optimal control problem considered as follows:

$$\rho v(a, x, \alpha) = \min_{(u_1, u_s, w_{42}^{24}, w_{42}^{34}) \in \Gamma(x, \alpha)} \left\{ \begin{array}{l} (u(\alpha) - d) \frac{\partial}{\partial x} v(a, x, \alpha) + Ku \frac{\partial}{\partial \alpha} v(a, x, \alpha) g(\alpha, x, \cdot) \\ + \sum_{\alpha \neq \beta} q_{\alpha\beta} v(x, \beta) \end{array} \right\}, \forall \alpha, \beta \in B \quad (5.A.1)$$

With:

$$u(\alpha) = u_1 \text{ if } \alpha = 1, 2, 3, 4 \text{ and } u(\alpha) = u_s \text{ if } \alpha = 5.$$

The optimal control policy  $(u_1^*, u_s^*, w_{42}^{24*}, w_{42}^{34*})$  denotes a minimizer over  $\Gamma(\alpha)$  of the right-hand side of equation (5.A.1). This policy is consistent with the value function obtained in Equation (5.6). The optimal control policy therefore rests in solving Equation (5.A.1). Obtaining an analytical solution of equation (5.A.1) is roughly impossible. The numerical solution of the HJB equation (5.A.1) is a challenge considered insurmountable in the scientific literature.

The numerical method for solving the optimality conditions is presented in this section. This method is based on the Kushner approach (see Kushner and Dupuis (1992) for details). The main idea behind this approach consists of using an approximation scheme for the gradient of the value function  $v(a, x, \alpha)$ . Let  $h_x$  and  $h_a$  denote the length of the finite difference interval of the variables  $x$  and  $a$  respectively. Using  $h_x$ ,  $v(x, \alpha)$  is approximated by  $v^h(a, x, \alpha)$  and  $v_x(a, x, \alpha)$  is approximated as follows:

$$v_x(a, x, \alpha) \times (u(\alpha) - d) = \left\{ \begin{array}{ll} \frac{1}{h} \left( v^h(a, x + h_x, \alpha) - v^h(a, x, \alpha) \right) \times (u(\alpha) - d) & \text{if } (u(\alpha) - d) \geq 0 \\ \frac{1}{h} \left( v^h(a, x, \alpha) - v^h(a, x - h_x, \alpha) \right) \times (u(\alpha) - d) & \text{otherwise} \end{array} \right\}, \quad (5.A.2)$$

With:

$$u(\alpha) = u_1 \text{ if } \alpha = 1, 2, 3, 4 \text{ and } u(\alpha) = u_s \text{ if } \alpha = 5.$$

Using  $h_a$ ,  $v_a(a, x, \alpha)$  is approximated as follows:

$$v_a(a, x, \alpha) \times f(u) = \frac{1}{h} \left( v^h(a, x + h_a, x) - v^h(a, x, \alpha) \right) \times f(u), \quad (5.A.3)$$

With approximations given by equations (5.A.2) and (5.A.3) and after some manipulations,

the HJB equations (5.A.1) can be rewritten as follows:

$$v^h(a, x, \alpha) = \min_{(u_1, u_s, w_{42}^{24}, w_{42}^{34}) \in \Gamma^h(\alpha)} \left\{ \left( \rho + |q_{\alpha\alpha}| + \frac{|u(\alpha) - d|}{h_x} + \frac{Ku}{h_a} \right)^{-1} \cdot \left[ \frac{|u(\alpha) - d|}{h_x} \left( v^h(a, x + h_x, \alpha) K^+ + v^h(a, x - h_x, \alpha) K^- \right) + \frac{Ku}{h_a} v^h(a, x + h_a, \alpha) + g(a, x, \alpha) + Qv^h(a, x, \alpha) \right] \right\}, \quad (5.A.4)$$

With:

$$u(\alpha) = u_1 \text{ if } \alpha = 1, 2, 3, 4 \text{ and } u(\alpha) = u_s \text{ if } \alpha = 5.$$

Where  $\Gamma(\alpha)$  is the discrete feasible control space and the other term used in equation

(5.A.4) is defined as:

$$K^+ = \begin{cases} 1 & \text{if } (u(\alpha) - d) \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

$$K^- = \begin{cases} 1 & \text{if } (u(\alpha) - d) < 0, \\ 0 & \text{otherwise,} \end{cases}$$

With:

$$u(\alpha) = u_1 \text{ if } \alpha = 1, 2, 3, 4 \text{ and } u(\alpha) = u_s \text{ if } \alpha = 5.$$

The equation presented in (5.A.4) can be interpreted as the infinite horizon dynamic programming equation for a discrete-time, discrete-state decision process that addresses problems confronted in optimizing output and controlling maintenance. (see Boukas and Haurie (1990) for details).

The next theorem shows that value function  $v^h(a, x, \alpha)$  is an approximation to  $v_x(a, x, \alpha)$  for a small step size  $h_x$  and  $h_a$ .

### Theorem

Let  $v^h(a, x, \alpha)$  denote a solution to HJB equation (5.A.4). Assume they are constants  $C_g$  and  $K_g$  as follows:

$$0 \leq v^h(x, \alpha) \leq C_g (1 + |x|^{K_g})$$

Then,

$$\lim_{h \rightarrow 0} v^h(a, x, \alpha) = v(a, x, \alpha)$$

### Proof

The proof of this theorem is given in Emami-Mehrgani et al. (2011).

## REFERENCES

- Akella, R. and Kumar, P.R., 1986. Optimal Control of Production Rate in a Failure-Prone Manufacturing System. *IEEE Transactions on Automatic Control* AC 31, pp. 116-12.
- Boukas, E.K. and Haurie, A., 1990. Manufacturing flow control and preventive maintenance: A stochastic control approach. *IEEE Transactions on Automatic Control* AC 33, pp. 1024–1031.
- Charlot, E., Kenné, J.P. and Nadeau, S., 2006. Optimal Production, Maintenance and Lockout/tagout Control Policies in Manufacturing Systems. *International Journal of Production Economics* 107, pp. 435-450.
- Chinniah, Y. and Champoux, M., 2008. La sécurité des machines automatisées : Analyse des risques et des moyens de protection sur une presse à injection de plastique. R-557. Montréal: IRSST, pp. 93.
- Dehayem Nodem, F.I, Kenné J.P., and Gharbi, A., 2008. Joint Optimization of Capacity Expansion, Production and Maintenance in Stochastic Manufacturing Systems. *Applied Mathematical Science*, Vol. 2, No. 5-8, pp.195–212.
- Emami-Mehrgani, B., Nadeau, S and Kenné, J.P., 2011. Lockout/tagout and operational risks in the production control of manufacturing systems with passive redundancy. *International Journal of Production Economics* 132, pp. 165-173.
- Emami-Mehrgani, B., Kenné J.P., and Nadeau S., 2012. Lockout/tagout and Optimal Production Control Policies in Failure-prone Non-homogeneous Transfer Lines With Passive Redundancy. Accepted (January), *International Journal of Production Research*
- Gershwin, S.B., 2002. *Manufacturing Systems Engineering*. Prentice-Hall, Englewood Cliffs, NJ, pp. 267.
- Hajji, A., Gharbi, A., Kenné, J.P., and Hidehiko, Y. 2009. Production and changeover control policies of a class of failure prone buffered flow-shops. *Production Planning & Control*, Vol. 20, No. 8, pp. 785-800.
- Kenné, J.P., Gharbi, A., and Beit, M., 2007. Age-Dependent Production Planning and Maintenance Strategies in Unreliable Manufacturing Systems with Lost Sale. *European Journal of Operational Research*, Vol.178, pp. 408–420.
- Kenné, J.P., Gharbi A, Najid M. 2009. On the Integrated Production, Inventory, and Preventive Maintenance Problem in Manufacturing Systems with Backorder. *International Journal of Simulation and Process Modelling*, Vol.5, N0. 4 pp. 300-312.
- Kenné, J.P., and Nkeungoue, L.J., 2008. Simultaneous Control of production and corrective maintenance rates of a failure-prone manufacturing system. *Applied Numerical Mathematics*; Vol.58, pp.180-194.

- Kushner, H.J. and Dupuis, P.G., 1992. Numerical Methods for Stochastic Control Problems in Continuous Time. Springer, New York.
- Lugtigheid, D., Banjevic, D., and Jardine A.K.S., 2009. System repairs: when to perform and what to do?. Reliability Engineering and System Safety; Vol.93, No.4; pp.604–15.
- Nahas, N., Khatab, A., Aït-Kadi D., and Nourelfath M., 2008. The extended great deluge algorithm for the imperfect zaintenance optimization problem of multi-state systems. Reliability Engineering and System Safety, Vol. 93, No. 11, pp.1658-1672.
- Rishel, R., 1975. Control of Systems With Jumps Markov Disturbances. IEEE Transactions on Automatic Control, pp. 241-244
- Sethi, S.P, Yan, H., Zhang, H., and Zhang, Q., 2002. Optimal and hierarchical controls in dynamic stochastic manufacturing systems: a survey. Manufacturing & Service Operations Management, Vol.4, No. 2, pp. 133-170.
- Yan, H., and Zhang, Q., 1997. A numerical method in optimal production and setup scheduling to stochastic manufacturing systems. IEEE Transactions on Automatic Control, Vol.42, pp.1452–1455.



## CONCLUSION GÉNÉRALE

L'utilisation actuelle du C/D présente certaines lacunes. Un niveau de risque élevé existe pendant l'intervention sur des machines en pannes. Le risque d'erreur humaine au cours des interventions présente une probabilité importante d'impact sous forme d'incidents ou d'accidents du travail chez les maintenanciers et sur la disponibilité du système manufacturier. Afin de remédier à ces lacunes cette thèse a été développée en trois (3) parties.

Dans la première partie, l'intégration du MTTLT et de la maintenance corrective pour un système manufacturier est combinée à la gestion de la capacité de production. Le système manufacturier est constitué de deux machines non-identiques sous forme redondance passive produisant un seul type de produit. L'objectif de cette intégration est d'augmenter le niveau de sécurité des travailleurs ainsi que la disponibilité des machines. Une modélisation a été faite par chaîne de Markov homogène et une résolution numérique à travers des équations différentielles d'HJB a conduit à la solution du système étudié. Les diverses analyses de sensibilité ont été effectuées afin de confirmer la structure des politiques obtenues.

La deuxième partie aborde également le problème d'intégration du MTTLT dans la gestion de la capacité de production. Le système est modélisé par une chaîne de Markov homogène. L'influence de contrôle du MTTLT a été présentée pour une ligne de production constituée de trois machines (deux machines sous forme redondance passive et une troisième machine en série avec les précédentes) produisant un type de pièce. Afin d'avoir un modèle plus réaliste pour les industries, les conditions d'optimum proposées sont développées en utilisant une approche combinée, en se basant sur une combinaison de formalisme analytique, la simulation, le plan d'expérience et la méthodologie de surface de réponse. Une analyse de sensibilité a illustré l'utilité de nos résultats.

La troisième partie du travail s'est concentré sur la modélisation du MTTLT et l'erreur humaine pour un FMS. Le FMS est constitué deux machines non-identiques sous forme redondance passive produisant un seul type de produit. La théorie de commande est basée sur

des modèles mathématiques. En effet, dans cette partie nous avons prévu une modélisation analytique par chaîne de Markov non homogène, dont la résolution est faite d'une part par la résolution d'équations différentielles d'HJB et par analyse numérique pour illustrer l'utilité de l'approche proposée. Compte tenu du fait que l'erreur humaine allonge les temps de réparation, nous avons montré que l'introduction de l'erreur humaine dans les activités de C/D ainsi que de maintenance peut engendrer un accident grave et une augmentation de coût de production.

Dans cette thèse, notre travail a apporté une contribution scientifique significative en reformulant les modèles mathématiques existant pour intégrer le MTTLT en contexte FMS. Les résultats de nos travaux ont été confirmés à travers des études par modélisation, résolution numérique, l'approche combinée et analyse de sensibilité sur des cas de FMS. Ce travail a confirmé qu'en intégrant le MTTLT dans un système en redondance passive, le système devient moins vulnérable aux variations des coûts de pénurie, d'inventaire et de maintenance en satisfaisant la demande en permanence. Cette intégration dans un système manufacturier en redondance passive a permis de libérer un espace-temps essentiel pour minimiser les possibilités de contournement des dispositifs de protection ou d'escamotage des procédures de C/D. Nos travaux ont été conclus par l'exploration de l'impact de l'erreur humaine sur le C/D ainsi que les activités de maintenance.

## TRAVAUX FUTURS

À l'issue de cette thèse, l'intégration du C/D dans la gestion de la capacité de production a été abordée, ensuite l'influence de l'erreur humaine sur les activités de C/D et de maintenance a été vérifiée. Ces contributions constituent une base solide pour des travaux futurs.

Les résolutions proposées dans cette thèse peuvent être étendus à des systèmes plus complexes de point vue structure et taille afin d'ouvrir une nouvelle piste de recherche. Nos résultats de recherche peuvent être utilisés directement sur des cas réels. Cependant, nous pouvons faire appel à d'autres méthodes de modélisation (semi-Markovien, non-Markovien) et améliorer la résolution numérique des conditions d'optimum. Ceci permettrait de faire une étude comparative des résultats trouvés. Nous pouvons élargir cette étude à d'autres modèles comme M2P2 (deux machines produisant deux types de produit). Dans ces modèles, il faut également définir une politique optimale de maintenance corrective et préventive. Le système manufacturier sujet à des demandes aléatoires peut être considéré afin de trouver une politique optimale de planification de C/D. D'autres prochains développements dans ce projet de recherche peuvent prendre en considération la réduction de capacité et son influence sur les activités de C/D et de maintenance ainsi que l'intégration de la notion des taux de rejets pour le système manufacturier. Cette notion de taux de rejet permettra de mesurer les impacts réelles de tous les facteurs ayant de l'influence sur le MTTLT, la maintenance et la production tels que l'équipement, l'environnement et l'humain. Nous pouvons également prendre en considération la notion de contrôle de la qualité dans l'environnement dynamique stochastique, ce qui est incontournable dans le processus de transformation (matière première, encours, produits finis). Par ailleurs, n'oublions pas l'importance de recueillir les données réelles dans un site industriel et alimenter nos modèles proposés par ces données réelles afin d'évaluer la performance des politiques obtenues. Élaborer ces différents points permettrons d'améliorer la production et le bien-être des travailleurs, qui sont les principaux axes de la gestion des opérations, de l'ergonomie et du génie des facteurs humains.



## RÉFÉRENCES

- Akella, R. and Kumar, P.R., 1986. Optimal Control of Production Rate in a Failure-Prone Manufacturing System. *IEEE Transactions on Automatic Control* AC 31, pp. 116-126.
- Badiane, A., 2010. "L'intégration des stratégies de maintenance à la gestion des opérations des systèmes manufacturiers", Mémoire de maîtrise en génie de la production automatisée, École de technologie supérieure (ÉTS), Montréal, pp.107.
- Barlow, R., Frank, P. and Larry, H., 1996. *Mathematical theory of Reliability*.« SIAM ». New York, Society of applied mathematics, pp. 258.
- Basnet, C. and Mize, J., 1986. An Object-Oriented Framework for Operating Flexible Manufacturing Systems. *Proceedings of the International Conference on Object-Oriented Manufacturing Systems*, pp. 346-341.
- Ben-Daya, M., Salih, D., and Abdul, R., 2000. *Maintenance, Modeling and optimization*. «Norwell». Massachusetts : Kluwer Academic Publisher, pp.474
- Bhattacharya, A.K. and Coleman, J.L., 1994. Linking Manufacturing Strategy to the Design of a Customized Hybrid Production Control System. *International Journal of Computer-Integrated Manufacturing Systems*, vol.7, No.2, pp. 134-41.
- Boukas, E.K. and Haurie, A., 1990. Manufacturing Flow Control and Preventive Maintenance: A Stochastic Control Approach. *IEEE Transactions on Automatic Control* 33, pp. 1024-1031.
- Boukas, E.K. and Kenné, J.P., 1997. Maintenance and Production Control of Manufacturing Systems with Setup. *Lecture in Applied Mathematics*, Vol .33, pp. 55-70.
- Charlot, E., Kenné, J.P. and Nadeau, S., 2006. Optimal Production, Maintenance and Lockout/tagout Control Policies in Manufacturing Systems. *Int. J. Production Economics* 107, pp. 435-450.
- Gershwin, S.B., 2002. *Manufacturing Systems Engineering*. Prentice-Hall, Englewood Cliffs, NJ, pp. 267.
- Gharbi, A. and Kenné, J.P., 2003. Optimal Production Control Problem in Stochastic Multiple-Product Multiple-Machine Manufacturing Systems. *IIE Transactions* 35, pp. 941-952.
- Gwo-Liang, L., Yen Hung, C. and Shey-Huei, S., 2009. Optimal Economic Production Quantity Policy for Imperfect Process with Imperfect Repair and Maintenance. *European Journal of Operational Research*, Vol. 195, pp. 348-357.
- Harichaux, P. and Libert, J., 2003. *Ergonomie et Prévention des Risques Professionnels, L'Environnement Physique du Travail et ses Contraintes*, vol. 1, Chiron, pp.192.
- Kenné, J.P. and Boukas, E.K., 1998. Optimal Control of Failure prone Manufacturing System with Corrective Maintenance. In: *Proceeding of the 18<sup>th</sup> IFIP TC7 Conference*, pp. 468-476.

- Kenné, J.P. and Gharbi, A., 1999. Experimental Design in Production and Maintenance Control Problem of a Single Machine, Single Product Manufacturing System. *International Journal of Production Research*, Vol.37, No.3, pp. 621-637.
- Kenné, J.P. and Gharbi, A., 2000. A Production Planning Problem in Manufacturing Systems with General Failure and Repair Times Distributions. *Production Planning & Control Journal*. Vol.11, No.6, pp. 581-587.
- Kenné, J.P. and Gharbi, A., 2001. A Simulation Optimization Approach in Production Planning of Failure Prone Manufacturing Systems. *Journal of Intelligent Manufacturing* Vol.12, No.5, pp.421-431.
- Kenné, J.P. and Gharbi, A., 2003. Stochastic Optimal Production Planning Control Problem with Corrective Maintenance. *Computers & Industrial Engineering*, Vol. 46, No. 4, pp. 865-875.
- Kenné, J.P., 2004. Commande Optimal Stochastique Appliquée à la Gestion de Production d'une Unité de Fabrication. *Journal Européen des Systèmes Automatisés (JESA)*: pp. 44-64.
- Kenné, J.P. and Gharbi, A., 2004. A Simulation Optimization Based Control Policy for Failure Prone One-Machine, Two-Products Manufacturing System. *Computer and Industrial Engineering*, Vol. 46, No.2, pp. 285-292.
- Kimemia, J. and Gershwin, S.B., 1983. An Algorithm for the Computer Control of Production in Flexible Manufacturing Systems. *IEEE Transactions on Automatic Control* AC 15. pp. 353-362.
- Kushner, H.J. and Dupuis, P.G., 1992. *Numerical Methods for Stochastic Control Problems in Continuous Time*. Springer, New York, pp. 473.
- Mansour, M., Nadeau, S et Kenné, J.P., 2009. "Management and Planning of Lockout Operations : The Case of a High Volume Packing Line", *Gesellschaft für Arbeitswissenschaft (GfA)*, Mars, Dortmund, Allemagne, pp.547-551.
- Matsuoka, S. and Muraki, M. 2001. "Computer aided planning for lockout/tagout program" *Japan, Process safety progress*, Vol. 20, No.2, pp.130-135.
- Monchy, F., 1991. *La fonction maintenance*. « Masson ». Paris, pp. 452.
- Nollet, J., Kelada, J., O.Diorio, M., Handfield, H., Deschamps, I., R.Duguay and C., 1994. *La Gestion des Opérations et de la Production, Une Approche Systématique*. 2<sup>e</sup> Édition, Gaëtan Morin. pp. 163.
- Older, G.J and Suri, R., 1980. Time Optimal of Part-Routing in a Manufacturing System with Failure prone machines. In: *Proceedings of the 19<sup>th</sup> IEEE Conference on Decision Control*, vol. 5, pp. 722-727.
- Presman, E., Sethi, S. and Suo, W., 1997. Existence of Optimal Feedback Production Plans in Stochastic Flowshops with Limited Buffers, *Automatica* 33, pp.1899-1903.
- Rishel, R., 1975. Control of Systems With Jumps Markov Disturbances. *IEEE Transactions on Automatic Control*, pp. 241-244

- Sader, B.H. and Sorensen, C.D., 2003. Deterministic and Stochastic Dynamic Modeling of Continuous Manufacturing Systems Using Analogies to Electrical systems. In: Proceedings of the 2003 Winter Simulation Conference. Vol.2, pp. 1134-1142.
- Sethi, S.P., Zhang, Q. and Zhou, X.P., 1992. Hierarchical Controls in a stochastic Two-machines flowshop with Finite Internal Buffer. In: Proceeding of the 31 st IEEE Conference on decision and Control, pp. 2074-2079.
- Stevenson, W et. C. Benedetti. 2006. La gestion des opérations : Produits et services, 2e éd Montréal : Chenelière/McGaw-Hill, pp.249.
- Thesen, A., 1999. Some Simple, But Efficient, Push and Pull Heuristics for Production Sequencing for Certain Flexible Manufacturing Systems. International Journal of Production Research, Vol. 37, pp.1525-1539.





## ANNEXE 1

### INTEGRATING LOCKOUT/TAGOUT WITH OPERATIONAL RISKS: THE PASSIVE REDUNDANCY CASE

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#### **Abstract**

In this paper, we consider a production control and occupational safety problem for a failure prone manufacturing system consisting of two machines (non-identical) in passive redundancy case. Machines are subject to failures and repairs and can produce one type of part. Therefore we are concerned by the impact of production control including lockout/tagout on production cost and corrective maintenance cost for two type of failures in a flexible manufacturing system (FMS) environment. The aim of this study is to demonstrate the optimal production cost, backlog, inventory and maintenance cost over an infinite planning horizon and the impact on occupational risk level when one offers an efficient planning of lockout/tagout in production control.

#### **1. Introduction**

How can one find an appropriate solution to optimize production while guaranteeing the safety of workers? One possible answer is lockout/tagout. It consists in locking a machine with a padlock, then to discharge all sources of residual energy (potential, hydraulic, electrical, etc.) to avoid the premature starting of equipment throughout an intervention. In the current industrial situation, many managers think wrongfully that planning and realizing lockout/tagout takes a lot of time. This inactive time of production is considered reducing firms' performance. This manuscript demonstrates that it is possible to improve the

performance of flexible manufacturing system (FMS) as for fabrication costs and occupational risk level by offering an efficient planning of lockout/tagout.

This paper is organized as follows:

- firstly, we verify the influence of lockout/tagout activities and time to repair the failure of a machine in a flexible manufacturing system (FMS) environment. In this research, it is assumed that the machine produces one type of part.
- secondly, we integrate the lockout/tagout in production control. We consider lockout/tagout for a system in passive redundancy which consists of two (2) machines (identical and non-identical). Afterword, we propose, by using numerical methods, a production policy for a system constituted of two (2) machines in passive redundancy.
- finally, we make a comparison between the results in this paper and the results of Charlot et al. (2006).

To reach our targets, we follow these steps:

- develop an analytical model of lockout/tagout to be integrated into a flexible manufacturing system (FMS) control;
- use the model of Markov chain (homogeneous and non-homogeneous) by incorporating different parameters, so the model will be as realistic as possible;
- demonstrate the complexity of flexible manufacturing system (FMS) by increasing the number of machines along with number of products;
- present the resolution of a complex flexible manufacturing system (FMS) by the resolution of HJB (Hamilton-Jacoby-bellman) differential equations and numerical approach;
- take the subsequent conclusions.

**Keyword:** Flexible manufacturing systems; Production cost; Passive redundancy; Lockout/tagout.

## 2. Assumptions

In this paper, we use assumptions as followed:

For one machine producing one type of part:

- 1- The costs of corrective maintenance of failure type 2 is more than corrective maintenance cost of failure type 1.
- 2- The mean corrective maintenance time of failure type 1 is shorter than the mean corrective maintenance time of failure type 2.
- 3- The machine, after each repairing becomes new, which means we consider the policy of as good as new (AGAN).
- 4- Throughout this paper, we consider repairing (corrective maintenance of failure type 1 and 2) action with lockout/tagout.

For a system in passive redundancy (two machines non-identical) producing one type of part, we consider previous assumptions and we add other assumptions as follows:

- 5- Machine N1 is the main machine, machine N2 is a standby machine.
- 6- The main machine, after each repairing (corrective maintenance of failure type 1 and 2) returns to production immediately and the standby machine turns back at rest.
- 7- Both machines do not fall down at the same time.
- 8- The standby machine can be broken down through rest. (Exogenous Factors).

### *2.1. One machine producing one part type of a flexible manufacturing system (FMS):*

For one machine producing one part type we refer to Charlot et al. (2006) studies. In this paper we present only the homogeneous Markov chain model and a manufacturing system consisting of two (2) machines (non-identical) in passive redundancy.

### *2.2. Two machines (non-identical) in passive redundancy producing one part type in the flexible manufacturing system (FMS):*

In this case, the dynamics of the system are the same as one machine producing one part type, that is, discrete elements  $\zeta(t)$  and continuous elements  $x(t)$ . The discrete elements  $\zeta(t)$  represents the machine's state and continuous elements  $x(t)$  represent the stock level. Hence, we have  $\zeta(t) \in M = \{1, 2, 3, 4, 5\}$ . A positive value of  $x(t)$  is inventory. A negative value is backlog. It is assumed that the system meets a constant demand rate. In this part, two machines with different production rates were considered as well as different failure rates. ( $\lambda_1 \neq \lambda_2$  and  $U_{\max}^1 \neq U_{\max}^2$ )

Therefore, the transition diagram obtained (Figure 1) is as followed:

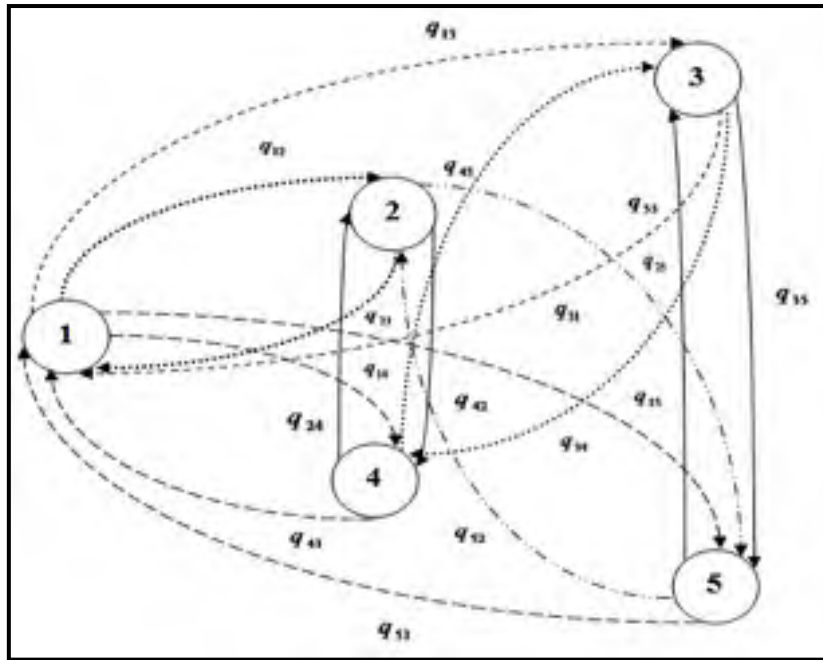


Figure 1 State transition diagram (Two machines (non-identical) in passive redundancy producing one part type)

Table1 Presents the transition modes for figure 1.

$\zeta_1(t)$	2	2	2	3	4
$\zeta_2(t)$	1	3	4	2	2
$\zeta(t)$	1	2	3	4	5

In this paper, the machine can break down randomly.

The 5 X 5 transition matrix  $Q$  of our system is as followed:

$$Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} & q_{15} \\ q_{21} & q_{22} & 0 & q_{24} & q_{25} \\ q_{31} & 0 & q_{33} & q_{34} & q_{34} \\ k_{41} & q_{42} & q_{43} & q_{44} & 0 \\ k_{51} & q_{52} & q_{53} & 0 & q_{55} \end{bmatrix}, \quad (1)$$

Hence, the transition matrix  $Q$  depends on:

$k_{41}$ : Corrective maintenance rate with lockout/tagout of failure type 1 of main machine;

$k_{51}$ : Corrective maintenance rate with lockout/tagout of failure type 2 of main machine;

Where  $k_{41} = q_{41}$  and  $k_{51} = q_{51}$ .

Hence, we have  $\Pi(\cdot) = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$  as representing the vector of limiting probabilities from modes 1 to 5.

Submitted to the following:

$$q_{\alpha\beta}(k_{41}, k_{51}) \geq 0 \quad (\alpha \neq \beta), \quad (2)$$

$$q_{\alpha\alpha}(k_{41}, k_{51}) = - \sum_{\alpha \neq \beta} q_{\alpha\beta}(k_{41}, k_{51}), (\alpha, \beta) \in M \quad (3)$$

The set of acceptable decisions at mode  $\alpha(t)$  are defined by:

$$\begin{aligned} \Gamma(\alpha) = \{ & (u_1(\cdot), u_2(\cdot), k_{41}(\cdot), k_{51}(\cdot)) \in R^4, \\ & 0 \leq u_1(\cdot) \leq u_1^{\max}, 0 \leq u_2(\cdot) \leq u_2^{\max}, \\ & k_{41}^{\min} \leq k_{41} \leq k_{41}^{\max}, k_{51}^{\min} \leq k_{51} \leq k_{51}^{\max} \}, \end{aligned} \quad (4)$$

The control policies (control variables) at mode  $\alpha(t)$  are  $u_1(\cdot), u_2(\cdot), k_{41}(\cdot)$  and  $k_{51}(\cdot)$ ;

The function cost is as followed:

$$\begin{aligned} J(\alpha, x, u_1, u_2, k_{41}, k_{51}) = E \left\{ \int_0^{\infty} e^{-\rho t} g(x, u_1, u_2, k_{41}, k_{51}) dt \mid x(0) = x, \xi(0) = \alpha \right\}, \\ \forall u_1(\cdot) \& u_2(\cdot) \in \Gamma(\alpha), \end{aligned} \quad (5)$$

Hence, we have  $\Pi(\cdot) = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$  as representing the vector of limiting probabilities from modes 1 to 5.

The subject of equations (1)-(5) is minimizing  $J(\cdot)$  given by equations (5) as followed:

$$v(x, \alpha) = \inf_{u \in \Gamma(\alpha)} J(\alpha, x, u_1, u_2, k_{41}, k_{51}), \quad \forall \alpha \in M \quad (6)$$

The main aim of this paper is to optimize the production cost, maintenance cost as well as lockout/tagout cost. This policy is satisfied by the set of Hamilton-Jacobi-Bellman (HJB) equations which are shown in Kenné et al. (2003), Charlot et al. (2006). We write the HJB equation as followed:

$$\rho v(x, \alpha) = \min_{u \in \Gamma(\alpha)} \left\{ (u-d) \frac{\partial}{\partial x} v(x, \alpha) + g(x, u, \alpha) + \sum_{\alpha \neq \beta} q_{\alpha\beta} v(x, \beta) \right\}, \quad \forall \alpha, \beta \in M \quad (7)$$

The optimal control policy  $(u_1^*, u_2^*, k_{41}^*, k_{51}^*)$  denotes the minimizer over  $\Gamma(\alpha)$  of the right hand side of equations (5).

### 3. Numerical approach

In this section, we expand the numerical method for solving the optimality conditions presented in the previous section. This method is based on the Kushner approach (Kushner and Dupuis -1992), Boukas and Haurie (1990), Kenne et al. (2003).

In this paper, we used Kushner's approach for an approximation scheme of the gradient of value function  $v(x, \alpha)$ . Let  $h$  denote the length of the finite difference interval of the variable  $x$ . Hence, using  $h$ ,  $v(x, \alpha)$  is approximated by  $v^h(x, \alpha)$  and  $v_x(x, \alpha)$  as followed:

$$v_x(x, \alpha) \times (u-d) = \left\{ \begin{array}{l} \frac{1}{h} (v^h(x+h, \alpha) - v^h(x, \alpha)) \times (u-d) \quad \text{if } (u-d) \geq 0, \\ \frac{1}{h} (v^h(x, \alpha) - v^h(x-h, \alpha)) \times (u-d) \quad \text{otherwise,} \end{array} \right\}, \quad (8)$$

After approximation given by equation (8) and some manipulations, the HJB equations (7) can be rewritten as:

$$v^h(x, \alpha) = \min_{\mathbf{u} \in \Gamma^h(\alpha)} \left\{ \left( \rho + |q\alpha\alpha| + \sum_{j=1}^n \frac{|u_j - d_j|}{h_j} \right)^{-1} \cdot \left[ \sum_{j=1}^n \frac{|u_j - d_j|}{h_j} (v^h(x+h, \alpha) K^+ + v^h(x-h, \alpha) K^-) + g(x, u, \alpha) + Q v^h(x_1, \dots, x_n, \alpha) \right] \right\} \quad (9)$$

Where  $\Gamma(\alpha)$  is the discrete feasible control space or the so-called control grid and the other term used in equation (9) is defined as:

$$K^+ = \begin{cases} 1 & \text{if } (u-d) \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

$$K^- = \begin{cases} 1 & \text{if } (u-d) < 0, \\ 0 & \text{otherwise.} \end{cases}$$

The system of equation's (9) can be interpreted as the infinite horizon dynamic programming equation of a discrete-time, discrete-state decision process as in Kenné et al. (2003), Charlot et al. (2006), for optimization of production and maintenance control problems. The next theorem shows that the value function  $v^h(x, \alpha)$  is an approximation of  $v(x, \alpha)$  for a small step size  $h$ .

#### 4. Numerical example and sensitivity analysis

We consider a manufacturing system for two (2) machines (non-identical) in passive redundancy case with five (5) states, describing by the homogeneous Markove process  $\alpha \in M = [1, 2, 3, 4, 5]$ . The discrete dynamic programming equation (9) gives the equations (10)-(14) for two (2) machines (non-identical) in the passive redundancy case producing one type part:

$$v^h(x,1) = \min_{u \in \Gamma(1)} \left( \rho + \frac{|u_1 - d|}{h} + q_{12} + q_{13} + q_{14} + q_{15} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h} (v^h(x+h,1)k^+ + v^h(x-h,1)k^-) \\ + g(x,1) + q_{12}v^h(x,2) + q_{13}v^h(x,3) \\ + q_{14}v^h(x,4) + q_{15}v^h(x,5) \end{array} \right\}, \quad (10)$$

$$v^h(x,2) = \min_{u \in \Gamma(2)} \left( \rho + \frac{|u_1 - d|}{h} + q_{21} + q_{24} + q_{25} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h} (v^h(x+h,2)k^+ + v^h(x-h,2)k^-) \\ + g(x,2) + q_{21}v^h(x,1) + q_{24}v^h(x,4) \\ + q_{25}v^h(x,5) \end{array} \right\}, \quad (11)$$

$$v^h(x,3) = \min_{u \in \Gamma(3)} \left( \rho + \frac{|u_1 - d|}{h} + q_{31} + q_{34} + q_{35} \right)^{-1} \left\{ \begin{array}{l} \frac{|u_1 - d|}{h} (v^h(x+h,3)k^+ + v^h(x-h,3)k^-) \\ + g(x,3) + q_{31}v^h(x,1) + q_{34}v^h(x,4) \\ + q_{35}v^h(x,5) \end{array} \right\}, \quad (12)$$



$$v^h(x,4) = \min_{u \in \Gamma(4)} \left( \rho + \frac{|u_2 - d|}{h} + k_{41} + q_{42} + q_{43} \right)^{-1} \left\{ \begin{aligned} & \frac{|u_2 - d|}{h} (v^h(x+h,4)k^+ + v^h(x-h,4)k^-) \\ & + g(x,4) + q_{41}v^h(x,1) + q_{42}v^h(x,2) \\ & + q_{43}v^h(x,3) \end{aligned} \right\}, \quad (13)$$

$$v^h(x,5) = \min_{u \in \Gamma(5)} \left( \rho + \frac{|u_2 - d|}{h} + k_{51} + q_{52} + q_{53} \right)^{-1} \left\{ \begin{aligned} & \frac{|u_2 - d|}{h} (v^h(x+h,5)k^+ + v^h(x-h,5)k^-) \\ & + g(x,5) + q_{51}v^h(x,1) + q_{52}v^h(x,2) \\ & + q_{53}v^h(x,3) \end{aligned} \right\}, \quad (14)$$

Here, we present results which were obtained through the different figures.

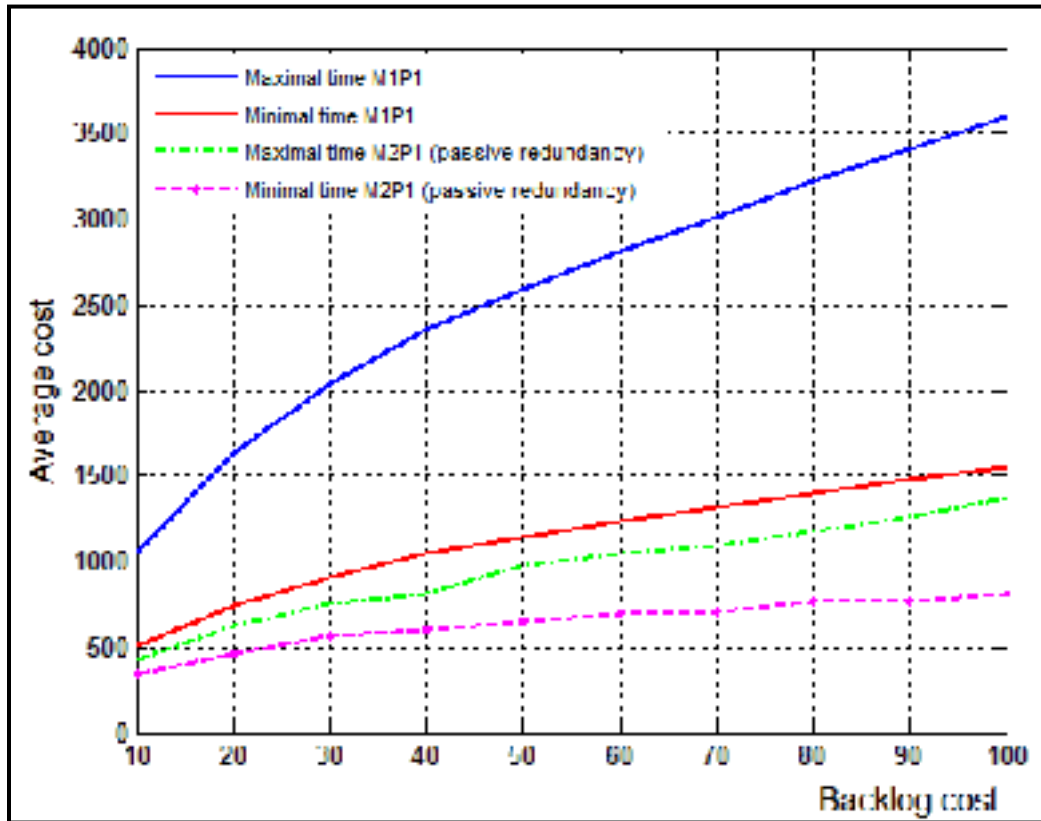


Figure 2 Average cost/backlog cost

In Figure 2, we plotted the average cost variation according to the backlog cost for one machine and two (2) machines non-identical in passive redundancy case producing one part type. In this figure the lockout/tagout, corrective maintenance rate of failure type 1 and type 2 are set to minimum and maximum values. Firstly, we note that the backlog cost variations don't affect much the average cost if we increase the lockout/tagout, corrective maintenance rate for failure type 1 and type 2. Secondly, we observe, if we add the standby machine to our system, we can reduce increasingly this average cost variation. As one can observe from Figure 1, we are able to better control this variation with the maximal lockout/tagout, corrective maintenance rate for failure type 1 and type 2.

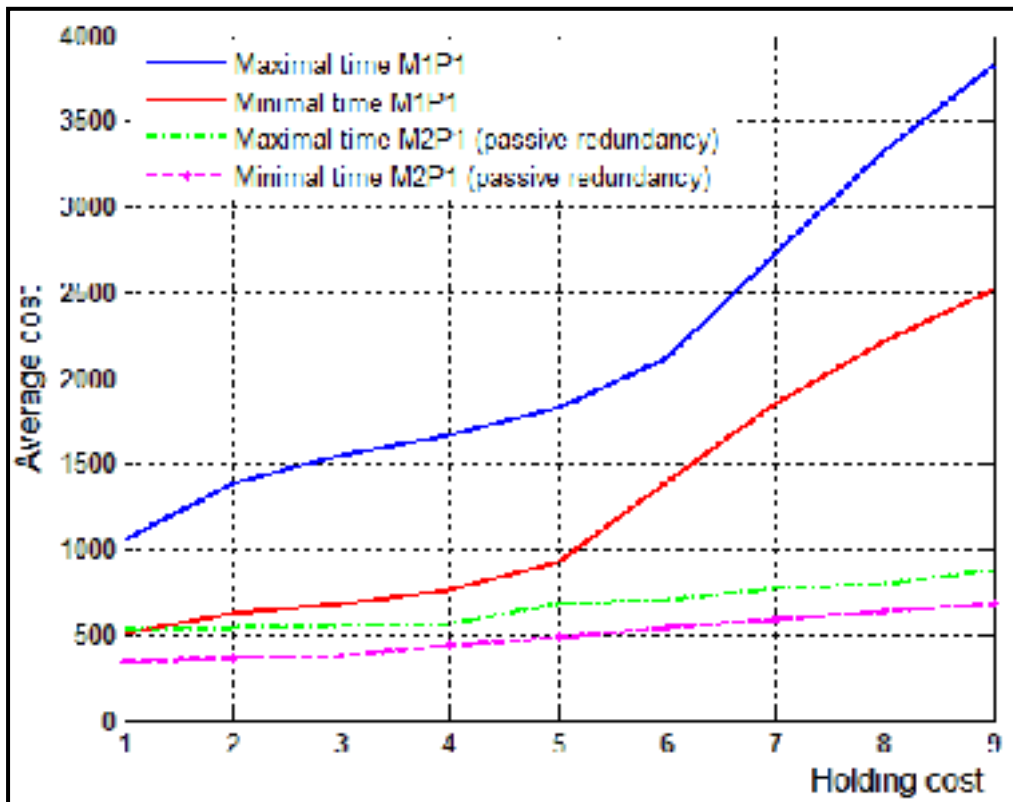


Figure 3 Average cost/holding cost

In Figure 3, we want to verify the influence of holding cost increase according to the average cost. Therefore as shows in the above figure, we can better control this evolution in passive redundancy case where the rates of lockout/tagout, corrective maintenance for failure type 1 and 2 are set to their maximum values.

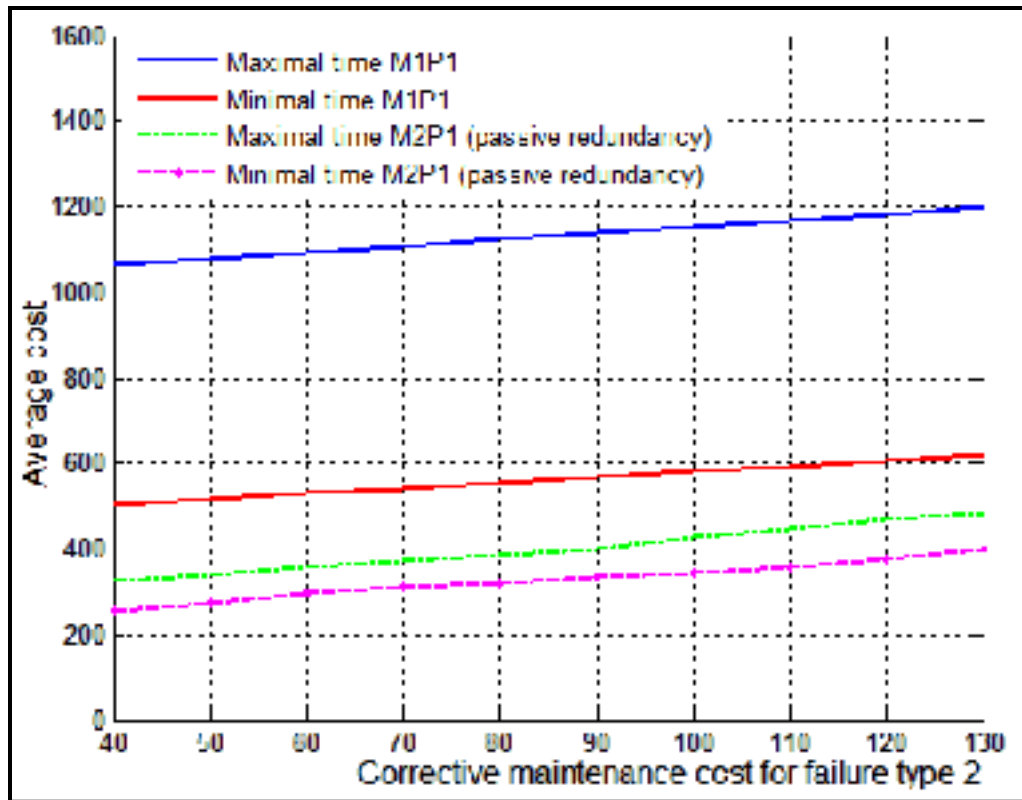


Figure 4 Average cost/corrective maintenance cost for failure type 2

In Figure 4, we plotted corrective maintenance cost of failure type 2 for the two (2) cases under study. We do not have a significant variation of the average cost by increasing the corrective maintenance cost of failure type 2, but we observe that the average cost is much lower in the passive redundancy case compared to the other cases where the rates of lockout/tagout, corrective maintenance for failure type 1 and type 2 are set to their maximum values. We can argue as noticed in the preceding analysis (Figure 4), by increasing the corrective maintenance cost for failure type 1, we observe that the average cost of passive redundancy case is much lower than the other cases where the rates of lockout/tagout, corrective maintenance for failure type 1 and 2 are set to their maximum values.

## **5. Conclusion**

In this paper we verified the influence of lockout/tagout and corrective maintenance rate of two types of failures for a passive redundancy case consisting of two (2) non-identical machines producing one part type. We observed that the passive redundancy case allows us to optimize in a better way the production cost and the maintenance cost while guaranteeing occupational safety. This optimization is most useful, if we introduce an effective planning of lockout/tagout and maintenance during the production control. In this paper, we considered the hedging point structure for the control policy of our systems. We developed an effective solution approach to determine the optimal production cost and corrective maintenance cost including the lockout/tagout control. The model proposed in this manuscript might be used for various industrial sectors. Further researches will verify the influence of lockout/tagout and corrective maintenance rate for a production line consisting of two (2) machines flowshop with internal buffers and one standby machine.

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## References

- Boukas, E.K. and Haurie, A. 1990, Manufacturing Flow Control and Preventive Maintenance: A Stochastic Control Approach, IEEE Transactions on Automatic Control 33, pp. 1024-1031.
- Charlot, E., Kenné, J.P. and Nadeau, S. 2006, Optimal Production, Maintenance and Lockout/tagout Control Policies in Manufacturing Systems, Int. J. Production Economics 107, pp. 435-450.
- Gharbi, A. and Kenné, J.P. 2003, Optimal Production Control Problem in Stochastic Multiple-Product Multiple-Machine Manufacturing Systems, IIE Transactions 35, pp. 941-952.
- Kushner, H.J. and Dupuis, P.G. 1992, Numerical Methods for Stochastic Control Problems in Continuous Time, Springer, New York.
- Kenné, J.P., Boukas, E.K. and Gharbi, A. 2003, Control of Production and Corrective Maintenance Rates in a Multiple-Machine, Multiple-product Manufacturing System, Mathematical and Computer Modelling 38, pp. 351-365.



## ANNEXE 2

### LOCKOUT/TAGOUT AND OPERATIONAL RISKS IN THE PRODUCTION CONTROL OF A TRANSFER LINE WITH PASSIVE REDUNDANCY

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#### **Abstract**

This paper presents an analytical model for the joint determination of optimal production and occupational safety for a failure prone manufacturing system consisting of three machines (two machines as passive redundancy and a third machine in series with the previous ones) producing one type of part. these machines are subject to breakdowns and repairs and the control problem is subject to non-negative constraints on work-in-processes (wip). the decision variables are the production rate of two main machines and a standby machine. the decision variables influence the wip levels, the inventory levels and the system's capacity. the system capacity is assumed to be described by a finite state markov chain. the aim of this paper is to minimize the cost of wip, inventory while respecting occupational safety. the proposed approach is based on the combination of analytical formalism, simulation modeling, design of experiments and response surface methodology to optimize a transfer line in passive redundancy producing one part type. the usefulness of the proposed approach is illustrated through a numerical example.

**Keywords:** Production Control; Lockout/tagout; Passive redundancy; Simulation, Experimental design; Response surface methodology

## 1. Introduction

The problem of optimally controlling the production rates of manufacturing system has been widely discussed in the scientific literature. In the literature, several approaches, mainly heuristic and optimal procedures, are employed to solve the problem. The first approach is present in the research literature, recent surveys is addressed by Gupta and Stafford (2006). On the other hand, the second approach, which consists of a stochastic optimal control problem formulation, to determine optimal control policies for a given problem. The optimal control problems of stochastic flow-shops with limited buffers producing one part type considered as the theory foundation of the optimisation problem. In this context, Presman et al. (1995), considered a production planning problem in an N-machine flow-shop subject to breakdown and repair of machines and to nonnegativity constraints on work-in-process. The authors show that the policy obtained minimizes the expected discounted cost of production and inventory/backlog over an infinite horizon.

The same problem was considered in Sethi et al. (2000), to minimise the long-run average cost. The authors used stochastic dynamic programming formulation and showed that the value function of the problem is locally Lipschitz and the value function is a solution to a dynamic programming equation jointly with a certain boundary condition. In this horizon, Yavuz and Tufekci (2006), studied a real case study of an electronic manufacturing flow-shop. The authors divided the master problem into two sub-problems in order to determine the batch sizes and production sequences. They developed a dynamic programming procedure in order to solve the batching problem and recommended an existing method to solve the sequencing problem. They illustrated the efficiency of their approach to answer the just-in-time (JIT) goals.

Manufacturing systems operate in a stochastic environment because the machines are subject to random breakdowns and repairs. It is possible to predict and control certain events while others occur randomly and are beyond control within manufacturing systems (Gershwin, 2002). A rising number of breakdowns and repairs degrade the dynamic of the production systems, reduces their availability, and increase occupational hazards associated with



maintenance activities. Left unattended, these disruptive elements erode competitiveness expressed in terms of quantity of products, quality of products and occupational safety. Occupational safety researchers worldwide have carried out studies confirming the importance of monitoring and controlling undesirable incidents or accidents during maintenance procedures. An important consideration arises: *How to optimize production yet ensure occupational safety*. One possible answer is lockout/tagout. It consists in locking a machine with a padlock to discharge all sources of residual energy (hydraulic, electrical, etc.) in order to avoid the premature starting of equipment throughout an intervention. Many managers wrongly assume it takes too much time to plan and carry out a lockout/tagout, fearing such downtime will reduce the productivity or the performance.

In this horizon, Emami-Mehrgani et al. (2011), considered an analytical model combining the lockout/tagout, the production and corrective maintenance policies for a passive redundancy system, consisting of two non-identical machines. Their work demonstrates clearly that passive redundancy optimizes production and maintenance costs while enhancing occupational safety. Even greater benefits occur if effective lockout/tagout maintenance planning occurs in concert with production control. The main contributions of this work are reduced the production cost and have free space-time to minimize the possibility of circumvention of protection devices or retraction of lockout/tagout procedures for the machines that are under repair.

In this paper, we resort to a numerical approach to find an approximate value function, instead of the true value function, to construct the control policy. To illustrate the usefulness of the proposed model, we show a numerical example for a transfer line with three non identical machines (two machines as passive redundancy and one machine in series with the previous ones). Based on the results given by numerical methods, a structure of the control production policy is defined and parameterised herein by parameters called control parameters. The proposed control approach consists of estimating the relationship between the incurred cost and the control policy parameters considered as control factors. The extension of hedging point structure, parameterized by these factors. The simulation model is

used to determine the related output (cost incurred), for each configuration of input factor values. The significant effects of input factors are determined by experimental design and the surface methodology is applied to the input-output data obtained in order to estimate the cost function and the related optimum.

## 2. Assumptions and notations

This paper incorporates the following assumptions and notations:

### 2.1 Assumptions

1. The corrective maintenance is carried out with lockout/tagout.
2. The main machine is more robust than the standby machine.
3. The main machine and the standby machine produce the same type of parts for the work-in-process (WIP).
4. The main machine returns to production immediately after each repair (corrective maintenance with lockout/tagout) and the standby machine stands idle.

Assumption 4 is a classical assumption in passive redundancy system. It is due to the nature of a passive redundancy system.

### 2.2 Notations

The following notations are used in the rest of this article:

- $x_1(\cdot)$  inventory levels of work-in-process
- $x_2(\cdot)$  inventory/backlog levels of finished products
- $c_1^+$  holding cost per unit of item over per unit of time for work-in-process
- $c_2^+$  holding cost per unit of item over per unit of time for finished products
- $c_2^-$  backlog cost per unit of item over per unit of time for work-in-process

$c^\alpha$	cost incurred for the operation on the machine under repair at mode $\alpha$
$c_{r_1}$	corrective maintenance cost of main machine 1
$c_{r_2}$	corrective maintenance cost of main machine 2
$c_{r_s}$	corrective maintenance cost of standby machine
$c_{tagout}$	lockout/tagout cost
$u_{r_1}$	corrective maintenance rate with lockout/tagout of main machine 1
$u_{r_2}$	corrective maintenance rate with lockout/tagout of main machine 2
$u_{r_s}$	maintenance rate with lockout/tagout of standby machine
$g(\cdot)$	instantaneous cost
$J(\cdot)$	total cost
$v(\cdot)$	value function
$\rho$	discount rate
$d$	demand rate
$u_i(\cdot)$	production rate of the machine $i$ ( $i=1, 2, s$ )
$u_i^{\max}(\cdot)$	maximal production rate of the machine $i$ ( $i=1, 2, s$ )
$q_{12}^{1,2}$	main machines $M_1$ and $M_2$ failure rate
$q_{12}^s$	standby machine $M_s$ failure rate

### 3. Problem statement

In this paper we consider the flow control problem for a tandem production system with passive redundancy ( $M = 3$ ). The system is shown in Figure 1. The main machines have two states:  $\xi_{1,2}(t) = 1$  if the main machine  $M_i$  ( $i = 1, 2$ ) is operational and  $\xi_{1,2}(t) = 2$  if the main machine  $M_i$  ( $i = 1, 2$ ) is under repair. The standby machine has three states:  $\xi_s(t) = 1$  if the standby machine  $M_s$  is operational,  $\xi_s(t) = 2$  if the standby machine  $M_s$  is under repair and

$\xi_s(t) = 3$  if the standby machine  $M_s$  is at time-off. We use  $u_i(t)$  to denote the input rate to  $M_i$  and  $x_i(t)$  to denote the number of parts in the buffer between  $M_i$  and  $M_2$  ( $i = 1, s$ ). Note that the control problem is subject to non-negative constraints on work-in-process (WIP) that means  $x_1(t) \geq 0$ . For buffer  $x_1(t)$  the difference is always positive (i.e. inventory costs  $c_1^+$  are thus charged) or equal to zero (i.e. starvation of machine  $M_2$ ). The difference between actual production and downstream demand at any time represents by  $x_2(t)$ . For buffer  $x_2(t)$  the difference between actual production and downstream demand is positive (i.e. inventory costs  $c_2^+$  are thus charged) or negative (i.e. backlog costs  $c_2^-$  are thus charged).

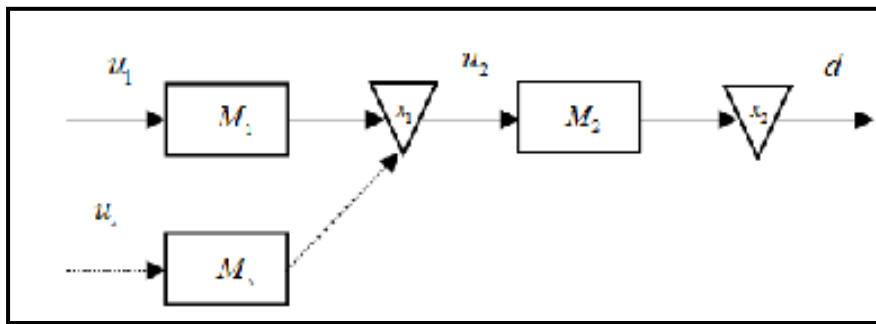


Figure 1 Production line with passive redundancy one part-type producing

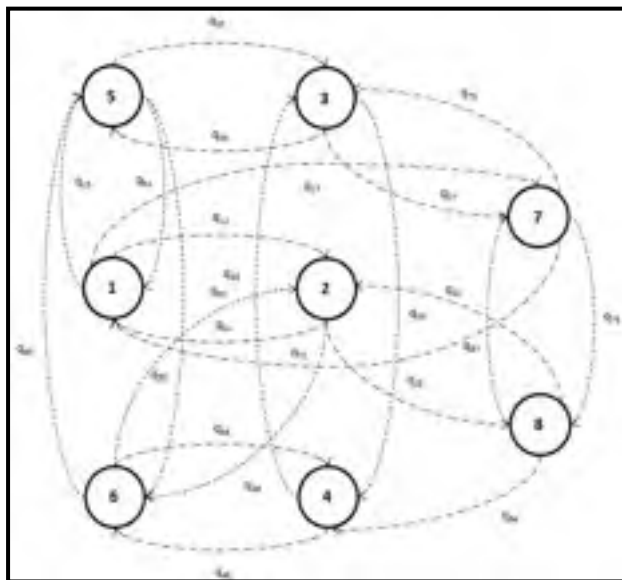


Figure 2 State transition diagram

Figure 2 displays the modes of the system associated to the process  $\xi(t)$ .

The dynamics of the system can be written as follows:

$$\dot{x}_1(t) = \tilde{u}_1(t) - \tilde{u}_2(t) = u_1(t)I_1^\alpha + u_s(t)I_s^\alpha - u_2(t), \quad x_1(0) = x_1, \quad (1)$$

with :

$$I_1^\alpha = \begin{cases} 1 & \text{if } \alpha = 3, 4, 5, 6, 7, 8, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad I_s^\alpha = \begin{cases} 1 & \text{if } \alpha = 1, 2, \\ 0 & \text{otherwise.} \end{cases}$$

$$\dot{x}_2(t) = \tilde{u}_2(t) - \tilde{u}_3(t) = u_2(t) - u_3(t), \quad x_2(0) = x_2, \quad (2)$$

with

$$u_3(t) := d.$$

In matrix notation, the system of equation (1)-(2) becomes:

$$\dot{x}(t) = A \tilde{u}(t), \quad x(0) = x, \quad (3)$$

where  $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ ,  $\tilde{u}(t) = (\tilde{u}_1(t), \tilde{u}_2(t), \tilde{u}_3(t)) = (u_1(t)I_1^\alpha + u_s(t)I_s^\alpha, u_2(t), u_3(t))$

and  $x(t) = (x_1(t), x_2(t))$ ,

with :

$$I_1^\alpha = \begin{cases} 1 & \text{if } \alpha = 3, 4, 5, 6, 7, 8, \\ 0 & \text{otherwise,} \end{cases}, \quad I_s^\alpha = \begin{cases} 1 & \text{if } \alpha = 1, 2, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad u_3(t) := d.$$

The transition rates matrix of the stochastic processes  $\xi(t)$  are denoted by  $Q$  such that

$Q = \{q_{\alpha\beta}\}$ , with  $q_{\alpha\beta} > 0$  if  $\alpha \neq \beta$  and  $q_{\alpha\alpha} = -\sum_{\beta \neq \alpha} q_{\alpha\beta}$ , where  $\alpha, \beta \in B$ .

The transition probabilities associated to  $q_{\alpha\beta}$  are expressed as:

$$p[\xi(t+\delta t) = \beta | \xi(t) = \alpha] = \begin{cases} q_{\alpha\beta}(\cdot)\delta t + o(\delta t) & \text{if } \alpha \neq \beta, \\ 1 + q_{\alpha\alpha}(\cdot)\delta t + o(\delta t) & \text{if } \alpha = \beta. \end{cases} \quad (4)$$

The transitions rates matrix  $Q$  is expressed as follows:

$$\begin{bmatrix} q_{11} & q_{12} & 0 & 0 & q_{15} & 0 & q_{17} & 0 \\ q_{21} & q_{22} & 0 & 0 & 0 & q_{26} & 0 & q_{28} \\ 0 & 0 & q_{33} & q_{34} & q_{35} & 0 & q_{37} & 0 \\ 0 & 0 & q_{43} & q_{44} & 0 & q_{46} & 0 & q_{48} \\ q_{51} & 0 & q_{53} & 0 & q_{55} & q_{56} & 0 & 0 \\ 0 & q_{62} & 0 & q_{64} & q_{65} & q_{66} & 0 & 0 \\ q_{71} & 0 & q_{73} & 0 & 0 & 0 & q_{77} & q_{78} \\ 0 & q_{82} & 0 & q_{84} & 0 & 0 & q_{87} & q_{88} \end{bmatrix}, \quad (5)$$

The operational mode of the whole system can be described by the random vector  $\xi(t) = (\xi_1(t), \xi_2(t), \xi_s(t))$  taking values in  $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ .

Without loss of generality, for the three machines flow-shop case in passive redundancy  $\xi(t)$  can be expressed as follows:

$$\xi(t) = \begin{cases} 1 & M_1 \text{ is under repair, } M_2 \text{ is operational and } M_s \text{ is operational;} \\ 2 & M_1 \text{ is under repair, } M_2 \text{ is under repair and } M_s \text{ is operational;} \\ 3 & M_1 \text{ is operational, } M_2 \text{ is operational and } M_s \text{ is under repair;} \\ 4 & M_1 \text{ is operational, } M_2 \text{ is under repair and } M_s \text{ is under repair;} \\ 5 & M_1 \text{ is under repair, } M_2 \text{ is operational and } M_s \text{ is under repair;} \\ 6 & M_1 \text{ is under repair, } M_2 \text{ is under repair and } M_s \text{ is under repair;} \\ 7 & M_1 \text{ is operational, } M_2 \text{ is operational and } M_s \text{ is at time-off;} \\ 8 & M_1 \text{ is operational, } M_2 \text{ is under repair and } M_s \text{ is at time-off.} \end{cases}$$

The set of admissible decisions at mode  $\alpha(t)$  and control policies (control variables) at mode  $\alpha(t)$  :

$$\Gamma(\alpha) = \left[ \begin{array}{l} ((u_1(\cdot), u_2(\cdot), u_s(\cdot)) \in R^3, \\ 0 \leq u_1(\cdot) \leq u_1^{\max}, 0 \leq u_2(\cdot) \leq u_2^{\max}, \\ 0 \leq u_s(\cdot) \leq u_s^{\max} \end{array} \right], \quad (6)$$

In equation (6),  $u_1^{\max}$  is the maximal production rate of the main machine 1,  $u_2^{\max}$  is the maximal production rate of the main machine 2 and  $u_s^{\max}$  is the maximal production rate of

the standby machine.  $\Gamma(\alpha)$  denote the set of all admissible controls with respect to  $x \in S$  and  $\alpha(0) = \alpha$ . Let  $S = [0, \infty) \times R \subset R^m$  denote the state constraint domain. The control problem consists of finding an admissible control law  $u(\cdot) = (u_1, u_2, u_s)$  that minimize the cost function  $J(\cdot)$  given by:

$$J(\alpha, x, u) = E \left\{ \int_0^{\infty} e^{-\rho t} g(\alpha, x, \cdot) dt \mid x(0) = x, \xi(0) = \alpha \right\}, \quad (7)$$

$$g(x, \alpha, \cdot) = c_1^+ x_1^+ + c_2^+ x_2^+ + c_2^- x_2^- + c^\alpha$$

Where  $g$  is the instantaneous cost,  $c^+$ ,  $c^-$  and  $c^\alpha$ , being the cost per unit to produce parts for inventory, backlog as well as intervention cost on the machine.

$$\begin{aligned} x^+ &= \max\{0, x\}, x^- = \max\{-x, 0\} \quad \text{and } c^\alpha = ((c_{r_1} + c_{tagout})u_{r_1})\text{Ind}\{\alpha = 1\} + ((c_{r_1} + c_{tagout})u_{r_1} \\ &+ (c_{r_2} + c_{tagout})u_{r_2})\text{Ind}\{\alpha = 2\} + ((c_{r_s} + c_{tagout})u_{r_s})\text{Ind}\{\alpha = 3\} \\ &+ ((c_{r_s} + c_{tagout})u_{r_s} + (c_{r_2} + c_{tagout})u_{r_2})\text{Ind}\{\alpha = 4\} \\ &+ ((c_{r_s} + c_{tagout})u_{r_s} + (c_{r_1} + c_{tagout})u_{r_1})\text{Ind}\{\alpha = 5\} \\ &+ ((c_{r_s} + c_{tagout})u_{r_s} + (c_{r_1} + c_{tagout})u_{r_1} \\ &+ (c_{r_2} + c_{tagout})u_{r_2})\text{Ind}\{\alpha = 6\} \\ &+ ((c_{r_2} + c_{tagout})u_{r_2})\text{Ind}\{\alpha = 8\} \end{aligned}$$

with

$$\text{Ind}\{\Theta(\cdot)\} = \begin{cases} 1 & \text{if } \Theta(\cdot) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

Let  $v(x, \alpha)$  denote the value function or minimum discounted cost for equations (7) as expressed in the following equation:

$$v(x, \alpha) = \min_{u \in \Gamma(x, \alpha)} J(\alpha, x, u), \forall \alpha \in B \quad (8)$$

The value function  $v(\cdot)$  given by (8) satisfies a set of partial differential equation known as the Hamilton-Jacobi-Bellman (HJB) equations in terms of directional derivatives (DD).

#### 4. Optimal conditions: dynamic programming equation

A necessary and sufficient condition characterizing the optimal control policy is described by the set of partial differential equation known as the Hamilton-Jacobi-Bellman (HJB) equations in terms of directional derivatives (DD), as in Presman et al. (1995), by:

$$\rho v(x, \alpha) = \min_{u \in \Gamma(x, \alpha)} \left\{ v'_{A\tilde{u}}(x, \alpha) + g(\alpha, x, \cdot) + \sum_{\alpha \neq \beta} q_{\alpha\beta} [v(x, \beta) - v(x, \alpha)] \right\}, \quad (9)$$

$$\forall \alpha, \beta \in B$$

Where:

$$v'_{A\tilde{u}}(x, \alpha) = (\tilde{u}_1 - \tilde{u}_2) v_{x_1}(x, \alpha) + (\tilde{u}_2 - \tilde{u}_3) v_{x_2}(x, \alpha).$$

Note that  $v_{x_1}$  and  $v_{x_2}$  are the partial derivatives of  $v$  compared to  $x_1$  and  $x_2$  respectively. The optimal control policy  $(u_1^*, u_2^*, u_s^*)$  denotes a minimizer over  $\Gamma(\alpha)$  of the right-hand side of equation (9). This policy corresponds to the value function described by Equation (8). Then, when the value function is available, an optimal control policy can be obtained as in Equation (9). However, an analytical solution of Equation (9) is almost impossible to obtain.

#### 5. Numerical approach

In this section, we use the numerical method for solving the optimality conditions presented in the previous section. This method is based on the Kushner approach (Kushner and Dupuis, 1992). The basic idea behind this approach consists of using an approximation scheme for the directional derivative of the value function  $v(x, \alpha)$ . Let  $h_1$  and  $h_2$  denote the length of the finite difference interval of the variables  $x_1$  and  $x_2$ . Hence, using  $h_1$  and  $h_2$ ,  $v(x, \alpha)$  is approximated by  $v^h(x, \alpha)$ ,  $v_{x_1}$  and  $v_{x_2}$  are approximated by:



$$v_{x_1}(\cdot) = v_{\tilde{u}_1 - \tilde{u}_2}(\cdot) = \left. \begin{array}{l} \frac{1}{h_1} \left( v^h(x_1, x_1 + h_1, \alpha) - v^h(x_1, \alpha) \right) \times (\tilde{u}_1 - \tilde{u}_2) \text{ if } (\tilde{u}_1 - \tilde{u}_2) \geq 0 \\ \frac{1}{h_1} \left( v^h(x_1, \alpha) - v^h(x_1, x_1 - h_1, \alpha) \right) \times (\tilde{u}_1 - \tilde{u}_2) \text{ otherwise} \end{array} \right\}, \quad (10)$$

with

$$\tilde{u}_1 = u_1 I_1^\alpha + u_3 I_3^\alpha, \quad I_1^\alpha = \begin{cases} 1 & \text{if } \alpha = 3, 4, 5, 6, 7, 8 \\ 0 & \text{otherwise} \end{cases}, \quad I_3^\alpha = \begin{cases} 1 & \text{if } \alpha = 1, 2 \\ 0 & \text{otherwise} \end{cases} \text{ and } \tilde{u}_2 = u_2.$$

$$v_{x_2}(\cdot) = v_{\tilde{u}_2 - \tilde{u}_3}(\cdot) = \left. \begin{array}{l} \frac{1}{h_2} \left( v^h(x_1, x_2 + h_2, \alpha) - v^h(x_1, \alpha) \right) \times (\tilde{u}_2 - \tilde{u}_3) \text{ if } (\tilde{u}_2 - \tilde{u}_3) \geq 0 \\ \frac{1}{h_2} \left( v^h(x_1, \alpha) - v^h(x_1, x_2 - h_2, \alpha) \right) \times (\tilde{u}_2 - \tilde{u}_3) \text{ otherwise} \end{array} \right\}, \quad (11)$$

with

$$\tilde{u}_2 = u_2 \text{ and } \tilde{u}_3 = u_3 := d.$$

We manipulated the approximation arrived at in equations (10) and (11) to rewrite the HJBDD equation (9) as follows:

$$v^h(x, \alpha) = \min_{\mathbf{u} \in \Gamma^h(x, \alpha)} \left\{ \left( \rho + |q\alpha\alpha| + \frac{(\tilde{u}_1 - \tilde{u}_2)}{h_1} + \frac{(\tilde{u}_2 - \tilde{u}_3)}{h_2} \right)^{-1} \left( \begin{array}{l} \sum_{\beta \neq \alpha} q\alpha\beta (v^h(x, \beta)) + g(x, \cdot) \\ \frac{(\tilde{u}_1 - \tilde{u}_2)}{h_1} \left[ v^h(x_1 + h_1, x_2, \alpha) k_1^+ \right] \\ + v^h(x_1 - h_1, x_2, \alpha) k_1^- \\ + \frac{(\tilde{u}_2 - \tilde{u}_3)}{h_2} \left[ v^h(x_1, x_2 + h_2, \alpha) k_2^+ \right] \\ + v^h(x_1, x_2 - h_2, \alpha) k_2^- \end{array} \right) \right\}, \quad (12)$$

Where  $\Gamma^h(\alpha)$  is the discrete feasible control space and the other terms used in equation (12) are defined as:

$$K_1^+ = \begin{cases} 1 & \text{if } (\tilde{u}_1 - \tilde{u}_2) \geq 0, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad K_1^- = \begin{cases} 1 & \text{if } (\tilde{u}_1 - \tilde{u}_2) < 0, \\ 0 & \text{otherwise,} \end{cases}$$

$$K_2^+ = \begin{cases} 1 & \text{if } (\tilde{u}_2 - \tilde{u}_3) \geq 0, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad K_2^- = \begin{cases} 1 & \text{if } (\tilde{u}_2 - \tilde{u}_3) < 0, \\ 0 & \text{otherwise,} \end{cases}$$

with:

$$\tilde{u}_2 = u_2 \text{ and } \tilde{u}_3 = u_3 := d.$$

In this paper, we use the policy improvement technique to derive a solution of the approximating optimization problem. The algorithm of this technique can be found in Kushner and Dupuis, (1992).

## 6. Numerical example and sensitivity analysis

In this section, we consider a transfer line with three non identical machines (two machines as passive redundancy and one machine in series with the previous ones). The system capacity is described by an eight Markov process with states  $\xi(t) \in \mathbf{B} = [1,2,3,4,5,6,7,8]$ . The discrete dynamic programming equation for our manufacturing system calculated by equation (12), with  $\alpha = [1,2,3,4,5,6,7,8]$ . We use the following computational domain:

$$G_x^h = \{(x_1, x_2) : 0 \leq x_1 \leq 5; -5 \leq x_2 \leq 5\},$$

The parameters for our case study appear in Table 1.

Table 1. Parameters of the numerical example

<b>Parameter</b>	$c_1^+$	$c_2^+$	$c_2^-$	$q_{12}^{1,2}$	$q_{12}^s$	$u_{r_1}$
<b>Value</b>	1	10	150	0.02	0.03	0.08
<b>Parameter</b>	$u_{r_2}$	$u_{r_s}$	$u_1^{\max}$	$u_2^{\max}$	$u_s^{\max}$	$c_{tagout}$
<b>Value</b>	0.08	0.08	1.3	1.25	1.2	50
<b>Parameter</b>	$c_{r_1}$	$c_{r_2}$	$c_{r_s}$	$\rho$	$d$	
<b>Value</b>	150	200	250	0.2	1	

The results obtained for the set of control variables  $u_1, u_2$  and  $u_s$  of a production line with passive redundancy system are given in Figure 3 for an illustration purpose where  $M_1, M_2$  are operational and  $M_3$  (standby machine) is under repair.

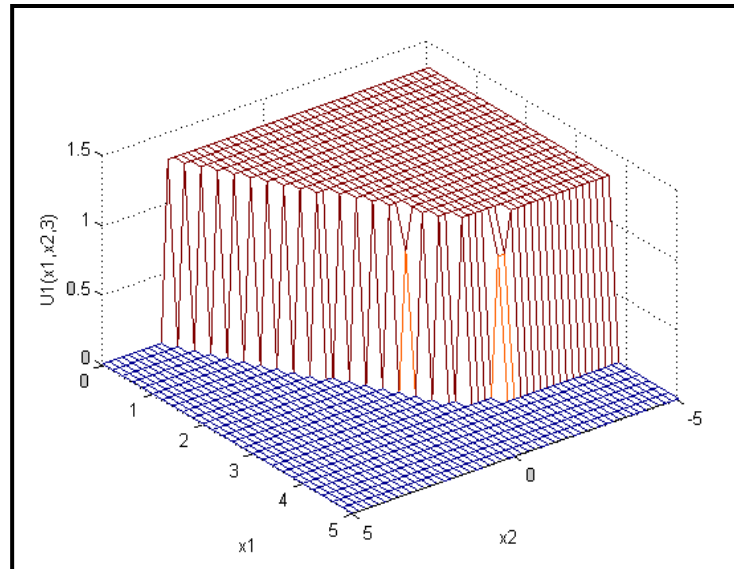
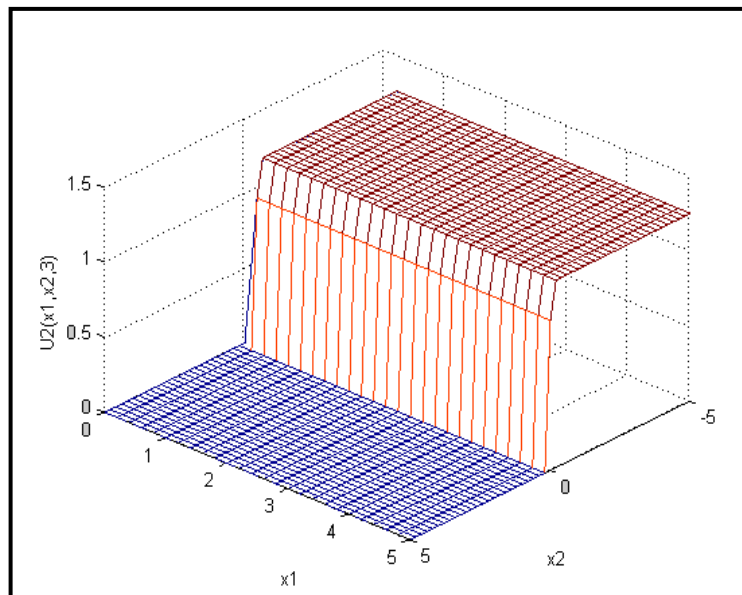
(a) Production rate of  $M_1$ (b) Production rate of  $M_2$ Figure 3 Production rate of  $M_1$  and  $M_2$  at mode 3.

Figure 3 shows that there is no need to produce the part with sufficient stock levels both in the work-in-process (WIP), described by  $x_1$  and in the final stock described by  $x_2$ . For small stock levels, the obtained policy defines well the region in the domain  $(x_1, x_2)$  where a maximal



The structure of the proposed control approach is presented in Figure 4.

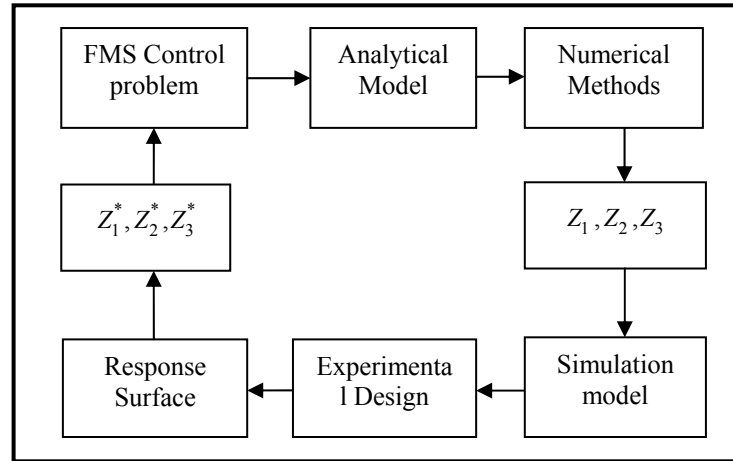


Figure 4 Diagram of control approach

To illustrate the approach presented in this paper we consider a production line with passive redundancy consisting of three machines, producing one part type. The simulation parameters used in this paper are the same as the Table 1. For three factors problems, as illustrated in previous sections, we selected a  $3^3$  response surface design since we have three independent variables at three levels. This design leads to the completion of 27 experimental trials. We have three independent variables  $Z_1, Z_2$  and  $Z_3$  where  $Z_1, Z_2$  and  $Z_3$  present our policy concerning the stock level of  $x_1$  and  $x_2$  for the machine  $M_1, M_2, M_3$ (standby) and one dependent variable (the production average cost). The levels of the independent variables are presented in the Table 2.

Table 2 Levels of independent variables

Factor	Low level	Medium level	High level
$Z_1$	5	15	25
$Z_2$	5	15	25
$Z_3$	5	15	25

In this paper, we chose three replications; hence we have 108 ( $27 \times 4$ ) simulation runs. For more information, we refer the reader to Montgomery (2005). Thereafter, we considered all possible combinations of different levels of independent variables by response surface

design. The objective of this design is to understand the effects of independent variables on performance measures, in our case the production average cost. From the ANOVA table, the independent variables  $Z_1, Z_2, Z_3$  and interaction effect as well as the quadratic effect are significant for the dependant variable at 0.05 level of significant. The  $R^2$  value of 0.88, that means 88% of the total variability, is explained by the model.

The estimation of the regression coefficients is performed and the ten values achieved.

$$\beta_0 = 860, \beta_1 = 1,899, \beta_2 = -18, \beta_3 = 4,080, \beta_{11} = 0,174, \beta_{12} = -0,086, \beta_{13} = -0,351, \\ \beta_{22} = 0,530, \beta_{23} = 0,272, \beta_{33} = -0,153.$$

The average cost function is given by:

$$\text{Average cost} = \beta_0 + \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_{11} Z_1^2 + \beta_{12} Z_1 Z_2 + \beta_{13} Z_1 Z_3 + \beta_{22} Z_2^2 + \beta_{23} Z_2 Z_3 + \beta_{33} Z_3^2$$

The minimum average cost function is located at  $Z_1^* = 14, Z_2^* = 14, Z_3^* = 15$  where  $Z_1^*, Z_2^*$  and  $Z_3^*$  represent the optimal values of independent variables  $Z_1, Z_2$  and  $Z_3$ .

Figure 5 illustrates the contour plot of the average cost function or response surface.

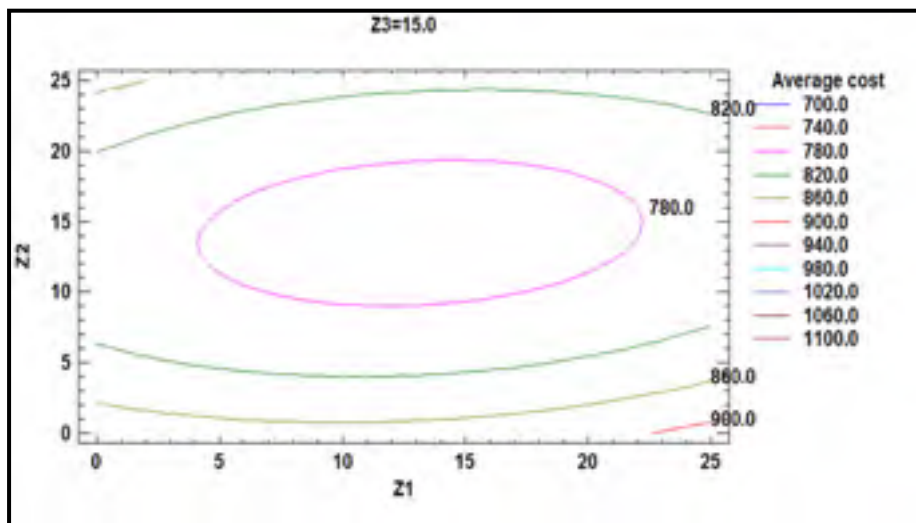


Figure 5 Contour plot of the response surface

These values determined the hedging point policy for our manufacturing system where the average cost is minimised and this control policy is the best approximation of the optimal

control. Without loss of generality of this proposal, this model is based on certain assumptions relating to a transfer line consisting of three machines which are operated in passive redundancy. We observed that the passive redundancy case allows us to optimize in a better way transfer line's production cost while guaranteeing occupational safety. Integration of the second machine as the passive redundancy makes possible to answer to demand permanently. In addition this integration will be released the essential space-time to intervention on the machine under repair, in order to minimize the possibility of circumvention of protection devices or retraction of lockout/tagout procedures.

## **8. Conclusion**

This paper confirms that it is possible to integrate a passive redundancy system in the production line in order to: 1) increase the productivity of workers and material resources, 2) optimize in a better way the production costs while guaranteeing occupational safety. In this paper, we showed that our policy is an extension of the hedging point policy. Analytical and simulation models were developed to describe the dynamic production under the hedging point policy. An experimental design was used to determine the effects of the independent variables on the average cost over the production horizon. We combined the analytical, simulation and statistical methods to provide the estimation of the average cost related to the control problem. The estimation of the average cost permits to calculate the best values of control parameters. Finally, we showed that the passive redundancy systems improve the safety of workers and production cost jointly. Taking into account certain conditions, the proposed approach could be adopted in several industrial sectors.

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(FQRSC); Fonds Québécois de la Recherche sur la Nature et les Technologies (FQRNT) and Institut de Recherche Robert-Sauvé en Santé et en Sécurité du Travail (IRSST).



## References

- Gupta, J.N.D and Stafford, E.F., 2006. "Flowshop scheduling research after five decades". *European Journal of Operational Research*, 169, 699–711.
- Presman, E., Sethi, S., and Zhang, Q., 1995. "Optimal feedback production planning in a stochastic N-machine flowshop". *Automatica*, 31 (9), 1325–1332.
- Sethi, S.P., Zhang, H., and Zhang, Q., 2000. "Hierarchical production control in a stochastic N-machine flowshop with long-run average cost". *Journal of Mathematical Analysis and Applications*, 251, 285–309.
- Yavuz, M. and Tufekci, S., 2006. "Dynamic programming solution to the batching problem in just-in-time flowshops". *Computers & Industrial Engineering*, 51, 416–432.
- Gershwin, S.B., 2002. "Manufacturing Systems Engineering". Prentice-Hall, Englewood Cliffs, NJ, pp.267.
- Emami-Mehrgani, B., Nadeau, S and Kenné, J.P., 2011 "Lockout/tagout and operational risks in the production control of manufacturing systems with passive redundancy". To appear in *International Journal of Production Economics*.
- Kushner, H.J. and Dupuis, P.G., 1992. "Numerical Methods for Stochastic Control Problems in Continuous Time". Springer, New York.
- Kenné, J.P. and Gharbi, A. 2001. "A simulation optimization approach in production planning of failure prone manufacturing systems". *Journal of Intelligent Manufacturing*, New York; Kluwer Academic Publishers.
- Montgomery, D.C. 2005. "Design and Analysis of Experiments". sixth edition, Wiley, New York, NY.



## ANNEXE 3

### LOCKOUT/TAGOUT AND HUMAN ERROR IN PRODUCTION CONTROL OF MANUFACTURING SYSTEMS WITH PASSIVE REDUNDANCY

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#### **Abstract**

Lockout/tagout consists in locking a machine with a padlock to discharge all sources of residual energy (hydraulic, electrical, etc.) in order to block premature starting of equipments during a maintenance intervention. This paper is different compared to other research projects on lockout/tagout. This paper illustrates the human error influence on lockout/tagout as well as on maintenance activities. The aim of our model is to minimize production, inventory, backlog and maintenance costs over an infinite planning horizon. We have integrated lockout/tagout in the context of a manufacturing system using a control theory based on mathematical models. More precisely, we consider a manufacturing system consisting of two non-identical machines in passive redundancy producing one type of part. These machines are subject to breakdowns and repairs. The system capacity is assumed to be described by a finite state Markov chain. The decision variables are the production rate and preventive maintenance rate. The decision variables influence the inventory level and the system's capacity. In the proposed model, the failure rate of the manufacturing system depends on its age that means the preventive maintenance policy is machine age-dependent. A numerical example and sensitivity analysis are presented to illustrate the usefulness of the proposed approach.

**Keywords:** Lockout/tagout, Human error, Production control, Preventive maintenance, Passive redundancy.

## **1. Introduction**

The current use of lockout /tagout has several shortcomings such as lack of validation of its feasibility and profitability. There is always a risk during intervention on machines that are down. The risk of human error during these interventions has a direct impact on the availability of manufacturing systems and can lead to occupational incidents or accidents. Protection devices are often absent or bypassed during these interventions, which explains the statistics of accidents remain high in this sector (Chinniah & Champoux 2008).

In this horizon, Emami-Mehrgani et al. (2011) considered an analytical model combining lockout/tagout, production and corrective maintenance policies for a passive redundancy system, consisting of two non-identical machines. Their work demonstrates clearly that passive redundancy optimizes production and maintenance costs while increasing the security level. Another work integrating lockout/tagout into operational risk in production control is proposed by Emami-Mehrgani et al. (2012). They considered a manufacturing system consisting of three machines (two machines with passive redundancy, and one in series with the previous ones) producing one part type. Their work confirms that it is possible to integrate a passive redundancy system in a production line in order to reduce the production cost and have free space-time to minimize the possibility of circumvention of protection devices or retraction of lockout/tagout procedures for the machines that are under repair. The main contributions of this work are reduced production cost, preventive maintenance cost and also is verified the influence of human error during maintenance activity on the optimal safety stock levels.

## **2. Problem statement**

This section develops a manufacturing system for two machines that are not identical in passive redundancy and producing one type of part. In the system under consideration, the

main machine operates any time. The other machine, named as the cold-standby redundancy, is in standby position and put online immediately when the main machine fails. The main machine has two states:  $\xi_1(t)=1$  if the main machine  $M$  is operational and  $\xi_1(t)=2$  if the main machine  $M$  is under repair. The standby machine has five states:  $\xi_s(t)=1$  if the standby machine  $S$  is operational,  $\xi_s(t)=2$  if the standby machine  $S$  is under repair,  $\xi_s(t)=3$  if the standby machine  $S$  is under preventive maintenance without human error,  $\xi_s(t)=4$  if the standby machine  $S$  is under preventive maintenance with human error and  $\xi_s(t)=5$  if the standby machine  $S$  is at time-off. The dynamics of the system can be written as follows:

$$\begin{aligned} \frac{dx}{dt} &= u_i(\cdot) - d, \quad x(0) = x, \quad \text{with } i = 1, S \\ \frac{da}{dt} &= f(u(\cdot)), \quad a(0) = a, \quad a(T) = 0. \end{aligned} \quad (1)$$

The operational mode of the whole system can be described by a random vector  $\xi(t) = (\xi_1(t), \xi_s(t))$  taking values in  $B = \{1, 2, 3, 4, 5\}$ . Without loss of generality, for the passive redundancy,  $\xi(t)$  can be expressed as follows:

$$\xi(t) = \begin{cases} 1 & \text{M is operational and S is under repair;} \\ 2 & \text{M is operational and S is under preventive maintenance without human error;} \\ 3 & \text{M is operational and S is under preventive maintenance with human error;} \\ 4 & \text{M is operational and S is at time-off;} \\ 5 & \text{M is under repair and S is operational.} \end{cases}$$

The control problem consists in finding an admissible control law  $u(\cdot) = (u_1, u_s, w_{42}^{24}, w_{42}^{34})$  that minimizes the cost function  $J(\cdot)$  given by:

$$J(a, x, \alpha, u, u_s, w_{42}^{24}, w_{42}^{34}) = E \left\{ \int_0^{\infty} e^{-\rho t} g(a, x, \alpha, u, u_s, w_{42}^{24}, w_{42}^{34}) dt \mid x(0) = x, \xi(0) = a, a(0) = 0 \right\}, \quad (2)$$

Let  $v(a, x, \alpha)$  denote the value function or minimum discounted cost for equations (2) as expressed in the following equation:

$$v(a, x, \alpha) = \min_{(u_1, u_s, w_{42}^{24}, w_{42}^{34}) \in \Gamma(x, \alpha)} J(a, x, \alpha, u_1, u_s, w_{42}^{24}, w_{42}^{34}), \forall \alpha \in B \tag{3}$$

Properties of the value function  $v(\cdot)$  given by equation (3) is presented in Boukas and Haurie (1990). It is shown that the value function  $v(\cdot)$  given by (3) should satisfy a set of partial differential equations known as the Hamilton-Jacobi-Bellman (HJB) equations.

### 3. Numerical example and sensitivity analysis

Let us consider a manufacturing system with two non-identical machines in passive redundancy. The system capacity is described by a five Markov process with states  $\xi(t) \in B = [1, 2, 3, 4, 5]$ . The parameters for our case study appear in Table 1.

Table 1 Parameters of the numerical example

$u_1^{\max}$	$u_s^{\max}$	$w_{42}^{24 \min}$	$w_{42}^{24 \max}$	$w_{42}^{34 \min}$	$w_{42}^{34 \max}$	$q_{14}$	$q_{23}$	$q_{24}$	$q_{34}$	$q_{54}$	$d$	$\rho$
.32	.28	$10^{-6}$	.06	$10^{-6}$	.06	.05	.02	.08	.13	.05	.25	.01

The results obtained for the control variables  $u_1, u_s, w_{42}^{24}$  and  $w_{42}^{34}$  of a passive redundancy system are given in Figures (1)-(2) for illustration purposes.

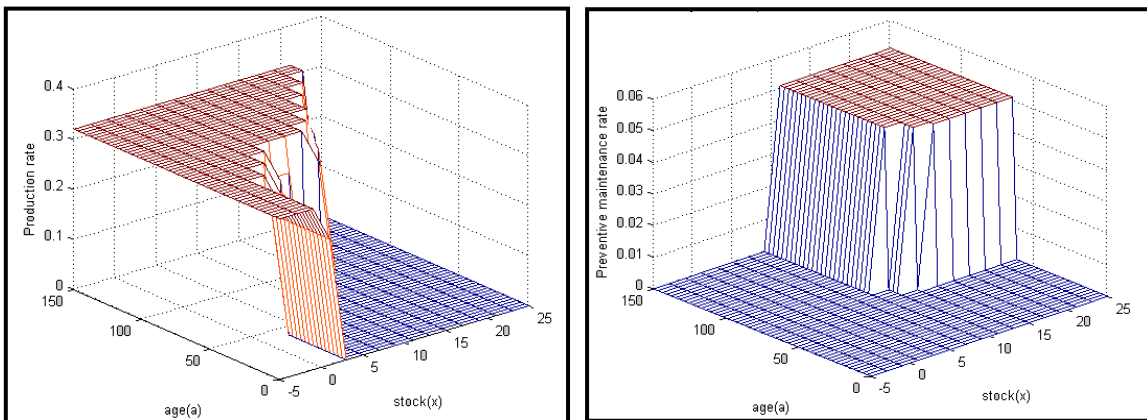


Figure 1 Main machine production rate and standby machine preventive maintenance rate without human error at mode 2

Figure 1 shows the existence of different zones for the production rate of the main machine. According to this figure in the area where the machine age is not yet advanced the production rate must be maintained at zero, despite a low inventory level. On the other side, if the machine age exceeds its youth with the same inventory level, the policy is suggesting to produce with the demand rate. Beyond a certain age, the threshold becomes more important. The policy is suggesting to produce with maximum rate below this threshold. In this area the age leads to frequent failure. Figure 1 also shows the standby machine preventive maintenance rate without human error, plotted in this figure, divides the computational domain  $(x,a)$  into two regions where the preventive rate is set to its maximal value for backlog situation and to zero for large stock levels. For significant stock levels, the zone in the domain  $(x,a)$  where the preventive maintenance is set to its maximal value increases with the machine age.

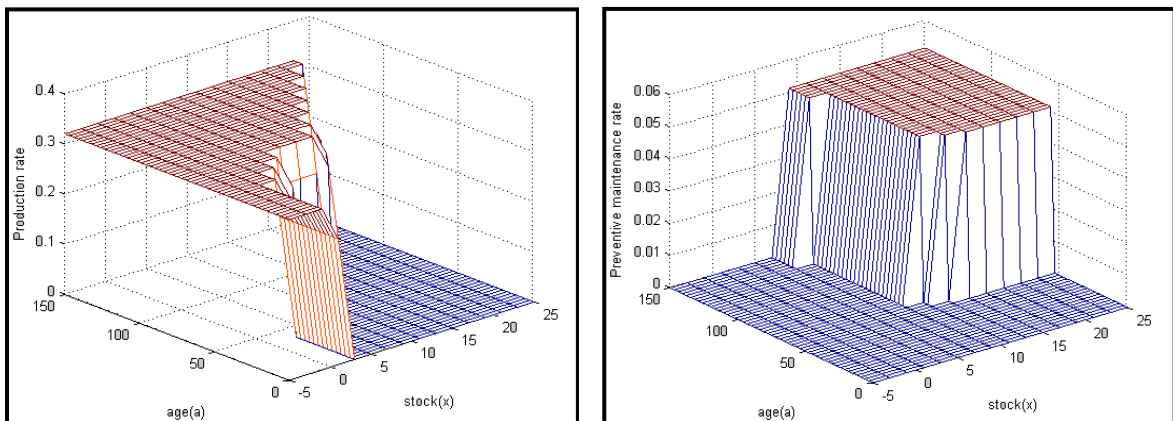


Figure 2 Main machine production rate and standby machine preventive maintenance rate with human error at mode 3

Figure 2 illustrates the same structure as Figure 1, but the policy suggests keeping more inventories by increasing the machine age, because the preventive maintenance has been done with human error.

#### **4. Conclusions**

This paper sought to verify the influence of human error on the preventive maintenance activity with lockout/tagout. In this paper, the control policy is based on an extension of the hedging point structure. This work demonstrates clearly that human error during maintenance activities can increase the production cost while reducing the safety of workers. So to avoid this problem, we must better train our maintenance technicians and also we must not increase the production cadence brutally. Lastly, a number of conditions must be met to make effective use of this model.



## References

- Boukas, E.K. and Haurie, A. 1990, Manufacturing Flow Control and Preventive Maintenance: A Stochastic Control Approach, IEEE Transactions on Automatic Control 33, pp. 1024-1031.
- Chinniah, Y. & Champoux, M. 2008, La sécurité des machines automatisées : Analyse des risques et des moyens de protection sur une presse à injection de plastique, R-557. Montréal: Institut de recherche Robert-Sauvé en santé et en sécurité du travail (IRSST), pp. 93.
- Gershwin, S.B., 2002, Manufacturing Systems Engineering. Prentice-Hall: Englewood Cliffs, pp. 267.
- Emami-Mehrgani, B., Kenné, J.P. & Nadeau, S. 2012, Lockout/tagout and Optimal Production Control Policies in Failure-prone Non-homogeneous Transfer Lines With Passive Redundancy, International Journal of Production Research, Accepted (January 2011).
- Emami-Mehrgani, B., Nadeau, S. & Kenné, J.P. 2011, Lockout/tagout and operational risks in the production control of manufacturing systems with passive redundancy, International Journal of Production Economics, 132, pp.165-173.



## ANNEXE 4

### CADENASSAGE ET BAISSÉ DES COÛTS DE PRODUCTION

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Cet article vise à intégrer le contrôle du C/D dans la gestion de la capacité de production. Cette intégration améliore la rentabilité de l'entreprise, tout en augmentant le niveau de sécurité des divers travailleurs.

#### **1. Quelques statistiques importantes**

Afin d'obtenir une estimation globale de l'impact de la maintenance sur les risques pour les travailleurs au Québec, une étude a été effectuée par l'Institut de Recherche Robert Sauvé en Santé et Sécurité du Travail (IRSST) sur les accidents du travail mortels au Québec entre 1999 et 2003 (Giraud et al., 2008). Les rapports de la CSST révèlent la même période, 1275 décès suite aux accidents du travail. Ils précisent que 163 d'entre eux (13%) ont été produits pendant une activité de maintenance. Depuis quelques années, pour résoudre ce problème, certaines entreprises ont mis en place un système de contrôle des risques par le biais du C/D (Charlot et al., 2006). Leur application réelle est parfois escamotée faute de temps, au nom de la performance de l'entreprise et à cause de difficultés technologiques.

#### **2. Cadenassage/décadenassage**

La norme canadienne CSA Z460-05 (2005) définit le C/D ou consignation/déconsignation (Europe) comme «l'installation d'un cadenas ou d'une étiquette sur un dispositif d'isolement des sources d'énergie conformément à une procédure établie, indiquant que le dispositif d'isolement des sources d'énergie ne doit pas être actionné avant le retrait du cadenas ou de l'étiquette conformément à une procédure établie».

### **3. Démarrage prématuré**

Les accidents liés aux machines coûtent cher en vies humaines, en arrêts maladies et en coûts divers. L'importance de l'intégration du C/D lors des interventions sur les machines n'est plus donc à démontrer. L'augmentation des précautions de sécurité réduit la fréquence des accidents, mais, elles augmentent le coût total de production. Notre question de recherche est de savoir comment trouver une solution adéquate pour optimiser la production, tout en augmentant la sécurité des travailleurs dans une entreprise par le biais du C/D?

### **4. Environnement incertain**

Notre recherche sur le C/D vise à intégrer le contrôle du C/D dans la gestion de la capacité de production en utilisant une théorie de commande, basée sur des modèles mathématiques sophistiqués, mais existants (Emami-Mehrgani et al., 2011). L'environnement des systèmes manufacturiers est incertain. La nature incertaine de l'environnement manufacturier découle des machines sujettes à des pannes et des réparations aléatoires. Ces dernières engendrent des accidents difficilement prévisibles lors des interventions de maintenance. Le projet vise à intégrer le concept novateur promettant des avancées en gestion des activités de C/D dans la gestion des capacités. Ce concept consiste à intégrer un système en redondance passive dans le contexte manufacturier afin de mieux gérer les activités de C/D (Emami-Mehrgani et al., 2010).

### **5. L'influence d'un système en redondance passive**

D'abord, nous exposons l'influence du temps de C/D pour un système manufacturier constitué d'une machine produisant un type de pièce. Ensuite nous assimilons le C/D pour un système en redondance passive produisant un type de pièce. La redondance est qualifiée de passive quand les éléments abondants ne sont mis en service qu'au moment du besoin; cela signifie que parmi un ensemble d'éléments, seul, un sous-ensemble de ces éléments est en service. Ceci implique que certains éléments seront en réserve ou en stock (Basil et Dehombreux, 2002). Finalement, nous faisons une comparaison entre les résultats des deux

systèmes étudiés. Pour une machine produisant un type de pièce, quand la machine est opérationnelle, elle peut être en mode production ou arrêt.

Nous supposons que notre système tombe en panne selon deux types de défaillances, l'une se produit avec une fréquence moins élevée que l'autre. Quand la machine est en panne, pour assurer la disponibilité de machine, on la répare. Nous admettons que le C/D se fait pour effectuer chaque intervention de maintenance sur la machine. Dans le deuxième cas, nous prenons en considération un système de production en redondance passive constitué de deux machines non-identiques. Cette fois-ci, les machines tombent en panne selon deux types de défaillances et le C/D est inclus dans la réparation. Pour établir les données, nous considérons que le temps moyen de C/D varie entre un minimum et un maximum en fonction du niveau de stock. Dans la réalité de ces systèmes, il en est tout autrement : le temps moyen de C/D est considéré constant. La politique de commande obtenue est basée sur la politique à seuil critique telle que définie dans (Akella et Kumar,1989). Cette politique recommande de produire au taux maximal si le niveau d'inventaire est inférieur au seuil critique (3.5 sur la Figure1); produire au taux minimal si le niveau d'inventaire est supérieur au seuil critique; produire au taux de la demande si le niveau d'inventaire est égal au seuil critique.

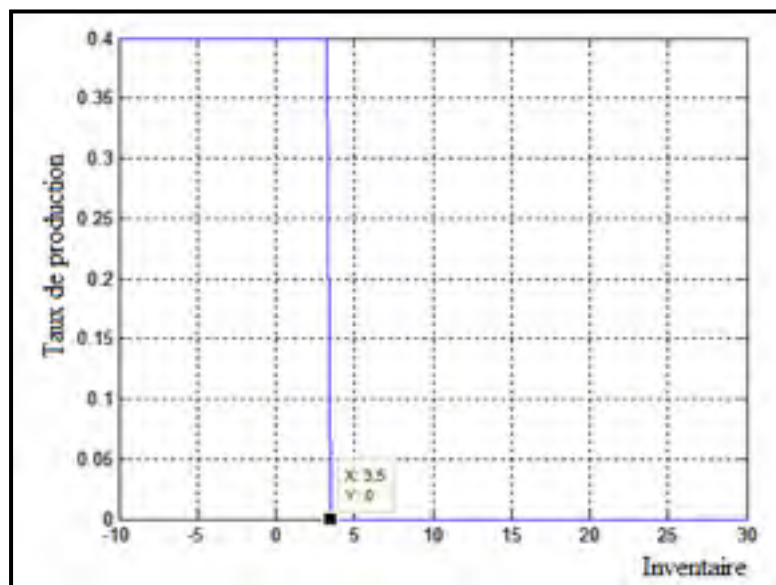


Figure 1 Politique à seuil critique

Premièrement, nous montrons que pour un système manufacturier constitué d'une machine produisant un type de pièce (M1P1), le C/D avec un temps minimal, comparé à un temps maximal, a pour conséquence la diminution du niveau d'inventaire (Figure 2), et diminution du coût de production (Figure 3).

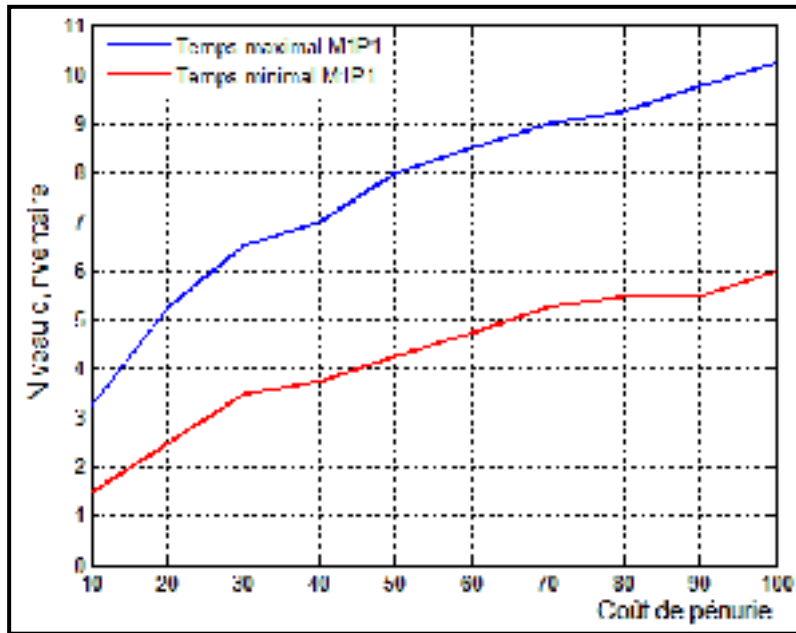


Figure 2 Niveau d'inventaire/coût de pénurie

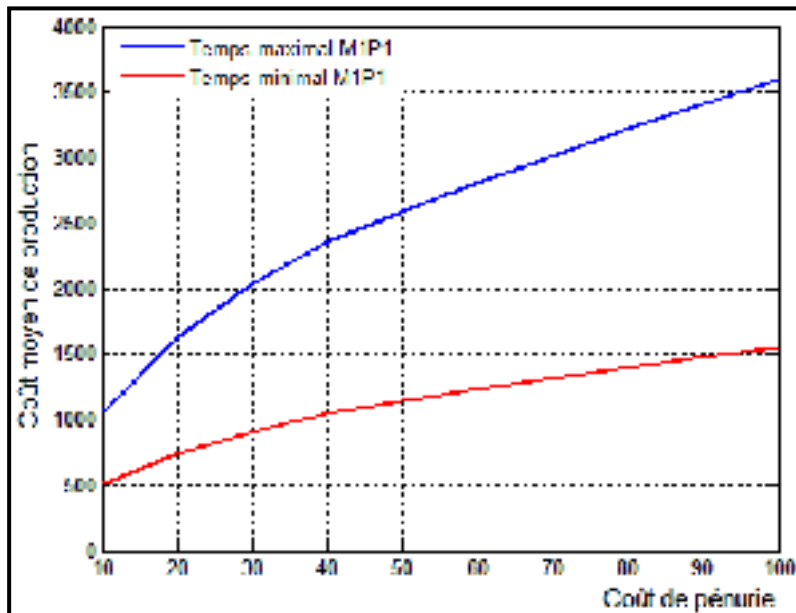


Figure 3 Coût moyen de production/coût de pénurie

Cette stratégie est plus efficace et efficiente lorsque le coût de la pénurie augmente. Deuxièmement, nous montrons l'influence d'un système en redondance passive sur le temps de C/D et nous comparons ces résultats avec le cas précédent (M1P1). Sur la Figure 4, nous constatons qu'en faisant varier le coût de pénurie, le coût moyen de production augmente pour M1P1, mais cette variation est plus faible quand le temps de C/D est fixé à une valeur minimale. Nous pouvons gérer cette situation de façon plus fiable en intégrant la seconde machine sous forme de redondance passive (M2P1), surtout quand le temps de C/D est fixé à une valeur minimale.

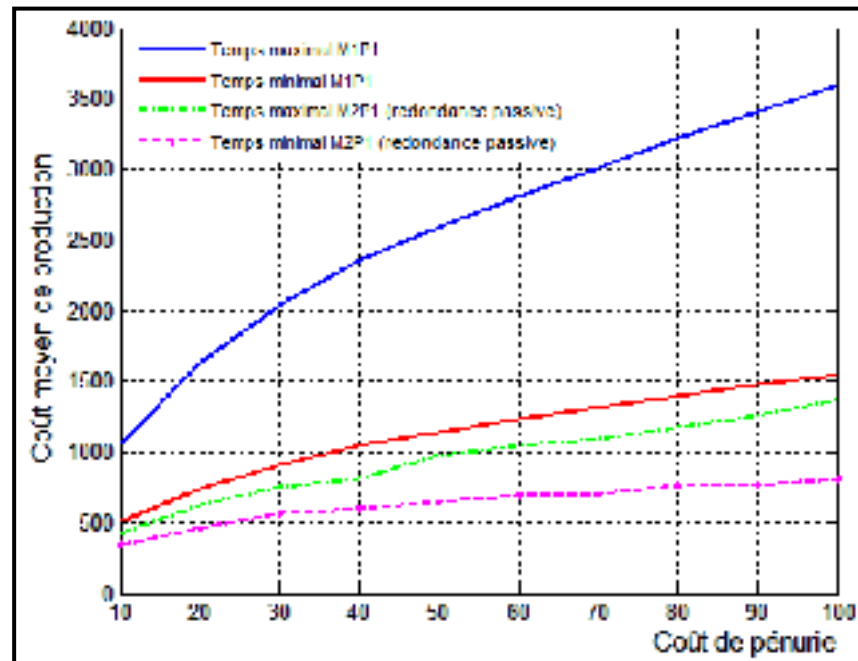


Figure 4 Coût moyen de production/coût de pénurie

On s'intéresse également à la comparaison du coût moyen de production en faisant varier le coût d'inventaire (Figure 5).

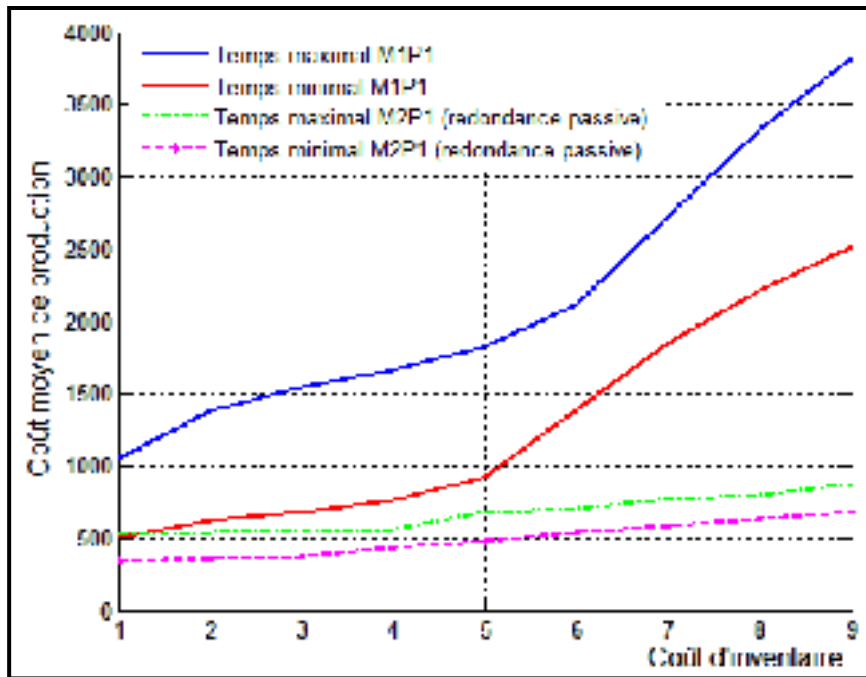


Figure 5 Coût moyen de production/coût d'inventaire

Nous constatons qu'en augmentant le coût d'inventaire, le coût moyen de production croît de façon brutal pour M1P1, mais cette variation est plus faible si le temps du C/D est fixé à une valeur minimale. Nous pouvons maîtriser cette variation en incorporant la seconde machine en redondance passive.

## 6. Conclusion

Nos travaux permettent de confirmer qu'il est possible d'intégrer un système en redondance passive dans le contexte manufacturier afin d'augmenter : 1) la productivité des ressources humaines et matérielles; 2) l'efficacité et l'efficience de la production par une meilleure gestion améliorant la disponibilité du parc machine. Cette intégration libère un espace-temps essentiel pour minimiser les possibilités de contournement des dispositifs de protection ou d'escamotage des procédures de C/D.



## Références

- Akella, R. and Kumar, P.R., 1986. Optimal Control of Production Rate in a Failure-Prone Manufacturing System. IEEE Transactions on Automatic Control AC 31. pp.116-126.
- Giraud, L., Ait-Kadi, D., Ledoux, É., Pâques, J.J. et Tanchoux, S. 2008. " La maintenance, État de la connaissance et étude exploratoire". IRSST. pp.61.
- Charlot, E., Kenné, J.P. and Nadeau, S. 2006. "Optimal Production, Maintenance and Lockout/tagout Control Policies in Manufacturing Systems". International Journal of Production Economics 107, pp. 435-450.
- Emami-Mehrgani, B., Nadeau, S and Kenné, J.P. 2011 "Lockout/tagout and operational risks in the production control of manufacturing systems with passive redundancy". International Journal of Production Economics, accepté.
- Emami-Mehrgani, B., Nadeau, S and Kenné, J.P. 2010. "Integrating lockout/tagout with operational risks: the passive redundancy case", Mars, Darmstadt, Allemagne, pp.475-482.
- Basile, O. et Dehombreux, P. 2002 "Reliabilitix, a Free MATLAB Toolbox for Reliability Analysis", Faculté Polytechnique de Mons. pp.9

## ANNEXE 5

### TEMPS MOYEN DE CADENASSAGE/DÉCADENASSAGE : UN OUTIL D'OPTIMISATION DES POLITIQUES DE SURVEILLANCE ET DE MAINTENANCE

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#### 1. Introduction

L'utilisation actuelle du C/D présente plusieurs lacunes reliées au manque d'évaluation de leurs faisabilité et efficacité. Le risque d'erreur humaine au cours des interventions présente une probabilité importante d'impact sous forme d'incidents ou accidents du travail chez les maintenanciers et sur la disponibilité du système manufacturier. Le concept novateur de «Mean Time to Logout/Tagout (MTTTL)» a été développé pour pallier à ces problèmes pour des systèmes manufacturiers flexibles (FMS).

#### 2. Méthodes

Le MTTTL permet l'intégration du C/D dans la gestion de la capacité de production. Premièrement, le C/D a été modélisé pour un système manufacturier en redondance passive. Deuxièmement, les méthodes numériques basées sur l'approche Kushner ont été utilisées afin de déterminer la politique de production optimale du système modélisé. Finalement, les performances de l'approche ont été évaluées à travers le modèle de simulation et le plan d'expérience du système manufacturier étudié.

### **3. Résultats**

En intégrant le MTTLT dans un système en redondance passive, le système devient moins vulnérable aux variations des coûts de pénurie, d'inventaire et de production en satisfaisant la demande en permanence. Cette intégration dans un système manufacturier en redondance passive a permis de libérer un espace-temps essentiel pour minimiser les possibilités de contournement des dispositifs de protection ou d'escamotage des procédures de C/D.

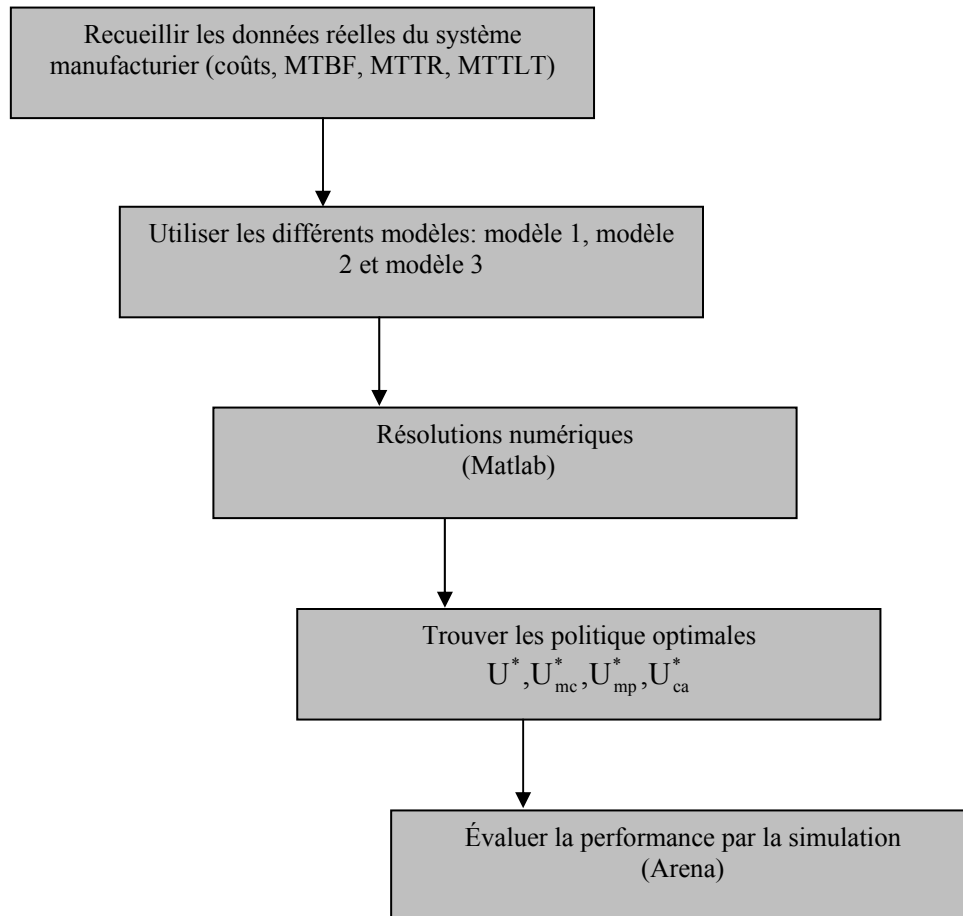
### **4. Conclusion**

Le concept novateur MTTLT a permis d'augmenter à la fois la sécurité, afin d'éviter les accidents graves, ainsi que l'efficacité et l'efficience de la production par une meilleure gestion en améliorant la disponibilité du parc machine.



## ANNEXE 6

### UTILISATION DES DIFFÉRENTS MODÈLES POUR UN CAS RÉEL



MTBF: Mean time between failures

MTTR: Mean time to repair

MTTLT: Mean time to lockouttagout

$U^*$  : Taux de production

$U_{mc}^*$  : Taux de maintenance corrective

$U_{mp}^*$  : Taux de maintenance préventive

$U_{ca}^*$  : Taux de cadencage/décadencage (C/D)