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PRODUCTION AND MAINTENANCE PLANNING OF DETERIORATING MANUFACTURING SYSTEMS TAKING INTO ACCOUNT THE QUALITY OF PRODUCTS

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PLANIFICATION DE LA PRODUCTION ET DE MAINTENANCE DES SYSTÈMES MANUFACTURIERS DÉTÉRIORÉS TENANT COMPTE DE LA QUALITÉ DES PRODUITS

Héctor RIVERA GÓMEZ

RÉSUMÉ

Les travaux de recherche présentés dans cette thèse portent sur l'intégration des aspects de qualité pour le développement de nouveaux modèles de programmation dynamique stochastique. Le but c'est de déterminer conjointement la planification de la production optimale, ainsi que plusieurs stratégies de maintenance pour un système manufacturier pas fiable et sujet à détérioration. En particulier, nous supposons que la détérioration a une influence sévère sur plusieurs aspects de la machine, donc cela conduit à diviser notre travail de recherche en trois (3) phases.

Dans la première phase, nous analysons simultanément la planification de la production et le problème de contrôle de la qualité d'un système manufacturier pas fiable. La machine est soumise à une dégradation dont l'effet est observé principalement sur le débit de qualité. Les décisions relatives à la qualité impliquent une stratégie de révision majeure qui s'oppose à l'effet de la détérioration. Une démarche d'optimisation des modèles de simulation est appliquée pour déterminer la politique de contrôle optimale, permettant une meilleure compréhension de l'influence de la détérioration de la qualité dans ces systèmes.

La deuxième phase de l'étude analyse le fait où la détérioration du système de production est générée par une combinaison de plusieurs facteurs. Nous considérons que le système se détériore en fonction de l'effet de l'usure de la machine et des réparations imparfaites. Plusieurs états opérationnels sont mis en œuvre pour modéliser les variations sur le taux d'unités défectueuses. En outre à la panne, soit une réparation ou une révision majeure peut être effectuée, mais la machine se détériore encore plus avec les réparations suivantes. Nous concevons un modèle de décision semi-markovien, puisque le taux d'unités défectueuses est dépendant de l'histoire de la machine dénoté par le nombre de réparations et de l'ensemble des multiples états opérationnels. Ensuite, le plan simultané de production, et de la stratégie de réparation/révision sont déterminés par des méthodes numériques.

La troisième phase complète les modèles précédents, puisqu'elle considère que la détérioration des systèmes de production a un double effet qui diminue la qualité des pièces produites et augmente également l'intensité des pannes. Nous employons l'âge de la machine pour désigner la détérioration progressive. À la panne, il est mené une réparation minimale qui laisse la machine au même niveau de détérioration qu'avant la défaillance. Pour faire cesser totalement l'effet de la détérioration, on peut effectuer une révision majeure. En outre, cette phase présente des stratégies d'entretien préventif pour réduire partiellement le niveau de détérioration. Ce large éventail de caractéristiques conduit à formuler un modèle semi-markovien que par moyen des méthodes numériques, nous déterminons conjointement le plan de production optimale et les stratégies de révision majeure et de maintenance préventive. Ce modèle précise le rôle des aspects de la qualité dans la politique de contrôle optimale.

De cette façon, notre recherche approfondit sur les effets des aspects de qualité et de la détérioration dans la politique de contrôle optimale, et fournit des contributions intéressantes dans le domaine du contrôle stochastique des systèmes de manufacturiers. Par ailleurs, un certain nombre d'exemples numériques sont effectuées à titre d'illustration. Des analyses de sensibilité approfondies sont présentées dans le but de confirmer la structure et la validité des politiques de contrôle obtenues. Les modèles développés dans cette thèse fournissent de nouvelles connaissances sur les relations entre la politique de production et les aspects de qualité dans le contexte de détérioration, et aussi contribuent également à une meilleure compréhension du comportement des systèmes de production en environnent incertain.

Mots-clés: Planification de la production, Contrôle de qualité, Systèmes manufacturier, Contrôle optimal, Maintenance préventive, Détérioration.

PRODUCTION AND MAINTENANCE PLANNING OF DETERIORATING MANUFACTURING SYSTEMS TAKING INTO ACCOUNT THE QUALITY OF PRODUCTS

Héctor RIVERA GÓMEZ

ABSTRACT

The research work presented in this thesis addresses the integration of quality aspects in the development of stochastic dynamic programming models. The goal is to determine the joint optimal production planning, and several maintenance strategies for an unreliable and deteriorating manufacturing system. In particular, we conjecture that deterioration has a severe influence on various aspects of the machine, thus this leads to divide our research work in three (3) phases.

In the first one, we analyze the simultaneous production planning and quality control problem for an unreliable manufacturing system. The machine is subject to deterioration whose effect is observed mainly on the quality throughput. The quality related decisions involves a major overhaul strategy that counters the effect of deterioration. A simulation optimization approach is applied to determine the optimal control policy, providing a better understanding about the influence of quality deterioration on such system.

The second phase of the research analyzes the fact where the deterioration of the production system is originated by a combination of several factors. We consider that the system deteriorates by the combined effect of the wear of the machine and imperfect repairs. Multiple operational states are implemented to model variations on the rate of defectives. Furthermore at failure, either a repair or a major overhaul can be conducted; however the machine deteriorates even more following repairs. We use a Semi-Markov decision model, since the rate of defectives is depended of the machine's history denoted by the number of repairs and the set of multiple operational states. Then the simultaneous production plan, and repair/overhaul switching strategy are determined through numerical methods.

The third phase complements the previous models by considering that the deterioration of the production systems has a twofold effect that decreases the quality of the parts produced and also increases the failure intensity. We employ the age of the machine to denote the progressive deterioration. At failure it is conducted a minimal repair that leaves the machine at the same level of deterioration before failure. To counter completely the effect of deterioration it can be performed a major overhaul. Moreover, this phase introduces preventive maintenance strategies to reduce partially the level of deterioration. This set of characteristics yields to formulate a Semi-Markov model that thorough numerical methods, we determine the joint optimal production plan and the overhaul and preventive maintenance strategies. This model clarifies the role of quality aspects on the optimal control policy.

In this way our research deepens the effects of quality aspects and deterioration on the optimal control policy, and provides interesting contributions to the domain of stochastic

control of manufacturing systems. Additionally, a number of numerical examples are conducted as illustration, and extensive sensitivity analyses are presented with the purpose to confirm the structure and validity of the obtained control policies. The models developed in this thesis provide further insights into the relations between the production policy and quality aspects in the context of deterioration, and also contribute to a better understanding about the behavior of stochastic manufacturing systems.

Keywords: Production planning, Quality control, Manufacturing systems, Optimal control, Preventive maintenance, Deterioration.

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LIST OF SYMBOLS AND ABREVIATIONS

$x(t)$	Inventory level at time *t*
$a(t)$	Age of the machine at time *t*
$u(t)$	Production rate of the manufacturing system at time *t*
$n(t)$	Current number of failures at time *t*
d	Demand rate
$\xi(t)$	Mode of the machine at time *t*
u_{max}	Maximum production rate
$Q(\cdot)$	Transition rate matrix
$q_{\alpha\alpha'}(\cdot)$, $\lambda_{\alpha\alpha'}(\cdot)$	Transition rate form mode α to mode α'
$\beta(\cdot)$	Rate of defectives
ρ	Discount rate
$\gamma^{\xi(t)}(\cdot)$	Cost rate function
π_i	Limiting probability at mode *i*
v_{min}	Minimum overhaul transition rate
v_{max}	Maximum overhaul transition rate
$g(\cdot)$	Cost rate function
$\omega(\cdot)$	Control variable for the repair/overhaul policy
ω_{min}	Minimum value of the control variable ω
ω_{max}	Maximum value of the control variable ω
$J(\cdot)$	Expected discounted cost function
$V(\cdot)$	Value function
τ	A jump time of the process $\xi(t)$ defined in the reset function
c^{+}	Incurred cost per unit of produced parts for positive inventory
c^{-}	Incurred cost per unit of produced parts for backlog
c_o	Overhaul cost
c_r	Repair cost
c_{pm}	Preventive maintenance cost
c_d	Cost of defectives

N	Maximum number of failures where the system remains operational
n_f	Number of failures needed to perform the overhaul
OP_i^n	Operational state *i* at the n^{th} failure
S	Number of operational states of the aging process for any number of repairs *n*
$\omega_p(\cdot)$	Control variable for the preventive maintenance
$\omega_{\overline{p}}$, $\omega_{\underline{p}}$	Maximal and minimal preventive maintenance rate
$\omega_o(\cdot)$	Control variable for the major overhaul
$\omega_{\overline{o}}$, $\omega_{\underline{o}}$	Maximal and minimal overhaul rate
θ_f	Adjustment parameter for the failure rate
θ_d	Adjustment parameter for the rate of defectives
HJB	Hamilton – Jacobi – Bellman
ABAO	As bad as old conditions
AGAN	As good as new conditions
MTTF	Mean time to failure
MDPHPP	Machine deterioration dependent hedging point policy

INTRODUCTION

Due to the rapid developments in technology, among other factors, there is a constant pressure to change the major drivers that influence economic growth in numerous industrial nations. Actual economic situation is known by everybody as being uncertain, besides markets are becoming more competitive with more exigent clients, and also there are huge challenges with respect to environment issues. This creates the conditions that promote actions to improve our current productive systems.

Some of the topics that attract most of the specialist's attention deal with the problem of how to create or at least maintain levels of wealth given the actual economic condition. In this respect, over the years manufacturing sector has played an important role for economic growth and the development of better technologies, and certainly new productive methods denotes a possibility to overcome some of the present challenges. It is evident that given the more competitive context, permanence in the market is a critical issue. As we can realize, this scenario challenges decision maker to deal with the ability to detect market changes, offer products with higher levels of quality at a competitive price, and procuring highest standards of productivity. Evidently this task is not easy to achieve, however we can dedicate part of the scientific research to propose some preliminary solutions.

In addition, recently it has been detected in the industrial field, the necessity to study in further detail, the relationship of several factors that influence production systems performance, specially the interaction of productivity, quality, and the production systems design. Unfortunately, nowadays there are no sufficient models, guidelines or at least basic principles that consider these factors together explaining their impact as well as their influence. It is known only by experience and preliminary research that product's design determines the greatest part of its quality, but also the quality of the parts may be affected by its production. Therefore, there is an inter-relation among production and quality. Moreover, the impact of quality aspects in the stochastic control of manufacturing systems is far from complete.

Quality is a major area that in line with the ideas of Inman *et al.* (2003), the traditional process used to bring a new product to market, indicates the existence of numerous techniques and research for the product design, product system validation and production system operation. Nevertheless, the phase of production system design is incomplete, since the guidelines and core principles that companies apply to design their production systems, have been created for achieving productivity targets, but actual market situations also demands higher levels of quality. In some cases quality issues is paramount for company permanence in business. Additionally in the area of manufacturing systems control the link with quality aspects has begun to be explored. Therefore a great opportunity for our research is to develop innovative control models that seek to improve the performance of production systems considering the effects of the interaction of productivity, quality and others compelling parameters.

Despite of the recent works in the area of optimal control, definitely it is needed much more research to promote conditions towards industrial applicability. In particular it is worthwhile to state that the objective of this research is to develop stochastic dynamic programming models, and determine the structure of the related optimal policy in order to provide further insight about the effect of quality aspects on the production control problem. Our approach determines simultaneously the optimal production planning, and maintenance strategies for a manufacturing system under uncertainty and several disruptions.

To be more precise, the present dissertation provides a scientific contribution at extending the actual models in the domain, and at analyzing rigorously the obtained control rule that determines simultaneously the optimal production plan and maintenance activities considering the influence of defective throughput and the impact of the progressive deterioration of the production system. The results obtained in this research has been confirmed and validated through optimal control modeling, numerical resolution, simulation optimization and extensive sensitivity analyses for different manufacturing system cases. In the next chapter we present the motivation, and background of our research and the literature review for the field of stochastic control of manufacturing systems.

STOCHASTIC CONTROL OF MANUFACTURING SYSTEMS AND LITERATURE REVIEW

0.1 Motivation

During several years numerous research papers have appeared in the field of stochastic control of manufacturing systems that aim to develop control rules which decide what action to take and when to take it in response to random events and disturbances. In especial, it has been analyzed several factors that occur during the production process such as failures, repairs, delays, set-up, etc.

Stochastic control has received widespread application and has been extensively studied, since it represents an interesting approach that allows taking into account the uncertainty in the dynamics of the production systems. First papers on this domain, such as the one of Rishel (1975), incorporated stochastic uncertainty in their formulation, hence it represents an important contribution, because in this paper it is established the optimally conditions required to obtain an optimal solution.

Subsequently Olsder and Suri (1980) introduced a dynamic programming model in which machine fails and are repaired according to a known Markov process. They determined a feedback control of parts-routing to machines that minimizes the time to complete a specified production target. The aforementioned models are an important source of insight for the set of parameters required at determining optimal production control policies for unreliable manufacturing systems.

In the same direction, the work of Kimemia and Gershwin (1983) has also a great importance, because they proposed a hierarchical control algorithm, where the production management problem consists of meeting production requirements while machines fail and are repaired at random times. It is precisely on this work and the series of papers that followed from it that it is established the systems dynamics, cost structure and variables that are typically used as reference in this domain. More details about the development of this field are provided in the next subsection on the literature review. However it is worthwhile to mention that despite the impressive expansion over the years of this academic subject, real manufacturing systems displays a wider set of events, i.e., preventive maintenance, aging process, engineering changes, defectives and many others. The relevance is that all the above mentioned events certainly may create undesirable disturbances and chaos for the decision maker, evidently making hard to predict the future and more difficult to make accurate plans. Furthermore a major limitation of this domain is that in most of the literature the aspect of quality has been disregarded in the optimal policy, and considerable research remains to be done to investigate in more detail the impact of quality issues and other phenomena such as deterioration for instance.

In a real production context, manufacturing systems may produce defective parts at a given rate. As a consequence, if the machine progressively deteriorates due to a combination of a number of factors including: wear, usage, corrosion, imperfect repairs, etc., then the rate of defectives can be severely affected by the underlying aging process. Deterioration is not unusual in productive systems and there is a very large literature on the subject. However, quality deterioration has not been treated enough in the literature. A key point is that the effects of the deterioration on quality or on other parameters of the production system, can be analyzed from different research directions, and certainly they form the matter of the present research.

Based on the above discussion, we claim that in this domain an integrated model is required to provide further details about how quality aspects and production management are inter-related, and explain how this inter-relation can be influenced by other phenomena such as deterioration, and how can be determined maintenance strategies that counter the arising problems.

0.2 Manufacturing systems applications

In modeling manufacturing systems, the use of predictive and quantitative methods facilitates considerably the understanding of the impact of the series of events that influence such systems. The understanding given by predictive and quantitative methods, is really important because is a required step in order to design and operate production systems more efficiently. Although it is complicated to develop accurate models, the benefit to generate a satisfactory approximation is well rewarded.

Owing to the presence of countless factors such as the frequent introduction of new products, short product and process lifetimes, etc., yields to short factory lifetimes that originates frequent changes at building and rebuilding industrial facilities. Thereby there is a lot of pressure to efficiently operate production systems, because there is no wiggle room for improving those systems after they are operating.

Faced with such a challenging context, it can be observed that there are a number of industrial needs that encourage research in the domain of manufacturing systems. Since production managers require proven methods to predict the performance of these systems, besides they need optimal real-time control policies for their factories, and also they require well trained personnel to operate and design these complex systems. Thus a major goal for this domain is to develop predictive and quantitative methods and models for a proper design and operation of manufacturing systems.

In general, at developing these methods, normally there is an incomplete knowledge of the parameters involved, and frequently appears numerous changes over time because of the presence of random disruptions. Hence, it is core that manufacturing systems must be efficiently operated to reduce the creation and propagation of such disturbances. A simple model can pave the way to gain intuition and assess usefulness to more complex systems, in fact our research is developed in a similar spirit, growing in complexity as we gain expertise with the models proposed.

0.3 Determination of optimal control policies

As has already been mentioned in the previous subsection a major objective in the field of manufacturing systems is to develop efficient control policies for factories based in a complete knowledge of the production system, and to develop analytical methods for evaluate its performance. This is a real necessity because despite it exist many approaches, there is little agreement telling which option is better, frequently policies are not adequately defined leading to create even more problems at implementing them.

One should remark that efficient policies must foster satisfactory performance and provide methods to determine decisions promptly in response to changing conditions. Some performance goals that typically may be considered are the minimization of inventory and backlog of product, maximization of the probability that the demand is satisfied due to on-time delivery or the maximization of predictability, reducing the source of variation and its effect. In practice, there are three types of control policies, which can be defined as follows:

a) Surplus-based: decisions are made based on the amount of production that exceeds demand.
b) Time-based: decisions are made based on how delayed or early a product is.
c) Token-based: decisions are made on the presence or absence of tokens.

The common feature of these policies is to maintain cumulative production as close as possible to cumulative demand as denoted in Kimemia and Gershwin (1983). Even some analytical solution exists for special cases, as reported in Bielecki and Kumar (1988), who studied the case of a single machine, single part type satisfying a constant demand rate. They achieved to determine an analytical solution, in their model they assumed an exponentially distributed time to fail and time to repair, and their problem implied the determination of the production rate at any instant of time to minimize the inventory and backlog cost. These works represent the foundation of the so-called hedging point policy.

In referring to the hedging point policy, it possesses some characteristic traits that for the case of a single machine producing one part type, they imply that the state of the system is determined with the surplus level, which indicates the difference between cumulative production and demand. Also it is used the machine mode, indicating when the machine is operational or at failure. If the surplus is positive it indicates the inventory level, and when the surplus is negative it denotes the backlog of products. It is employed the short term production rate as decision variable, where at the operational state the machine is limited to produce by the maximum production rate, and at failure the machine does not produce at all. The objective function is the minimization of the integral of the running cost over a long time comprising the inventory and backlog cost.

The stock dynamics denotes a differential equation that indicates the difference between the current production rate and the demand of product. The machine can go from the operational state to the failure state and vice versa according to exponential distributions with known parameters. The analytical solution for the single machine problem indicates that the control policy has an interesting structure that can be perfectly characterized by a single control parameter denominated hedging point. This control parameter represents a trade-off between the cost of inventory and the backlog cost. For the single machine case the hedging point is a function of the demand, the maximum production rate, the cost of inventory and backlog, and the parameters of the exponential distributions applied to model the transitions between operational and failure states.

Unfortunately, it does not exist analytical solutions for any larger systems because it is so cumbersome to solve the related optimality conditions equations. Since we are not able to solve exactly complex manufacturing systems, the hint is to obtain a solution structure, and then apply such structure to construct a simple controller that allows us to operate efficiently the production system, and this is exactly the lead we follow in this research. Frequently with problems of real-time control we must be content to devise control policies that provide satisfactory rather than optimal results, but the policies certainly have to ensure reasonable stable behavior.

0.4 Classification of maintenance activities

Research activities addressing maintenance issues have been conducted over the last decades, and from these efforts numerous models have been proposed to determine optimal maintenance polices. In this section we highlight several methods on maintenance activities in order to present a comprehensive overview about the set of maintenance options available in real production.

Normally, maintenance refers to planned or unplanned actions that are performed to maintain or restore a given system in an acceptable operating condition. Maintenance methods aims to improve safety performance, increase availability of the system and reduce the repercussions of random failures, all these at the lowest possible cost. A first classification of maintenance activities denotes two categories: corrective and preventive maintenance. Some authors define corrective maintenance as any action conducted as a result of a failure, seeking to restore the system to a specified condition. Meanwhile preventive maintenance can be defined as any action that is conducted when the system is operational, as an attempt to maintain the system in a specified condition by conducting detection and prevention of incipient failures.

A second classification of maintenance activities is based on the degree to which the system is restored to operation conditions, this leads to define several types of maintenance as indicated by Pham and Wang (1996) and Wang (2002), as follows:

a) Perfect repair: denotes the maintenance action that has the particularity to restore the system to as-good-as-new conditions. Therefore with a perfect repair the system has the same lifetime and failure intensity as a brand-new system. Major overhauls or replacement represent some examples of a perfect repair.
b) Minimal repair: implies a maintenance activity that is not as complete as the previous category, since it focuses only on a part of the system, thus it has the characteristic to

restore the system to as bad-as-old conditions. In other words, the failure intensity remains unchanged in the same conditions it had before the failure.

c) Imperfect repair: this maintenance activity serves to model more realistic situations derived from the two extreme cases of perfect and minimal repair. This type of maintenance may not make the system as-good-as a brand new one, but definitely its performance may be greatly improved with this repair.
d) Worse repair: this type of repair has the characteristic to increase the failure intensity or influence any other parameter of the system, however the system remains operational but to a degraded condition prior to its failure.
e) Worst repair: denotes the repair action that undeliberately upon its conduction makes the system malfunction and fail.

In practical terms there are a number of reasons that we can mention that leads to imperfect, worse or worst repair, for example:

i) When the system breakdowns the damaged elements are repaired, nevertheless other adjacent elements are damaged.
ii) Only it is partially repair the damaged elements.
iii) Existence of hidden damage that is not correctly assessed and corrected during maintenance.
iv) Human errors that leads to incorrect adjustments and further damage in the course of maintenance actions.

Many other reasons can cause imperfect, worse, and worst repairs. According to the second classification, we can sketch out that a preventive maintenance may imply a perfect, minimal, imperfect, worse or worst maintenance. Moreover the same can be stated to corrective maintenance. The type and degree of maintenance that is used in many works, depends on the type of application, the cost arrangement involved, and also to the reliability requirements demanded. Because of the importance of maintenance actions in industrial contexts, in this research we consider different maintenance strategies and we address the development of innovative models that determine optimal maintenance policies.

0.5 Effect of deterioration in manufacturing systems

In the domain of manufacturing systems it is customary to consider that several components are subject to deterioration with usage and age. Fortunately most of these systems are repairable and are possible to be maintained at an acceptable performance level. When the system experience deterioration it is important to avoid further failure during actual operation because it may be disastrous for their performance and also can cause more damage. As mentioned previously in the preceding subsection maintenance actions are necessary since they improve; system reliability, prevents the occurrence of failures, and reduces maintenance cost of this deteriorating systems.

Other factors that explain the reasons to observe deterioration in real life, is because of the impact of the ageing process inherent to such deterioration, the accumulated wearing of various elements of the system and even the influence of the environment, as observed on the analysis of real data in Lam (2006). To model the deterioration phenomena it is possible to exploit the fact that the data of some successive events for deteriorating systems usually exhibit a certain trend in their behavior along the time, and this feature permits to devise an approximation for such pattern. Furthermore, typically two basic characteristics follows from deterioration, first the successive operating times of the production system may decrease, and so this causes that its lifetime is affected. Second, since it is more difficult and it is required more time to correct the accumulated level of deterioration, then the corresponding consecutive repair times increases, until the system is not longer functional.

A general overview about the different methods than can be applied to model deterioration is presented in further detail in the literature review. In many instances it is assumed that deterioration has an influence only on the successive operating times of the system or the consecutive repair times, for example, reducing the time that the machine remains operational or increasing the time required to repair the machine as the level of deterioration of the machine increases. However, in the field of manufacturing systems, it has been started to be sketched out that deterioration also may have an adverse impact on the quality of the parts produced, and certainly in the literature we have found nothing whatsoever regarding the inter-relation between deterioration and quality, and its impact on the control policy.

0.6 Quality issues overview

Since early 1980s quality research topics have captured the attention of production managers and academic researchers. The field of quality has been extensively studied, standing out a number of subjects such as; statistical quality controls, total quality management, six sigma, etc. The considerable grown of the quality domain is given because it is a critical factor for competitiveness, firms that strive for obtaining recognition from their quality efforts, exceed the performance of other companies that disregard quality topics.

In this frame of reference, the quality improvement strategy must start with the determination of relevant quality measures that should be considered in the optimal operation policies. Thus, an important metric is denoted by the quality yield, commonly defined as the fraction of parts produced with acceptable quality, in some cases is related with the rate of defectives. Normally the quality yield is a complex function of how the production system is operated and the set of features of the machines. Effective methods are required to manage effectively the quality yield, and clearly understand how this yield can be affected by disrupting factors.

A number of influencing factors affecting the quality yield can be caused by steady changes in a quality measurement of interest, exhibiting a certain trend as indicated in Besterfield (2009). Progressive changes are very common industrial phenomena and many production systems are subject to this condition. In particular, changes can be caused by the wear of some of the elements of the machine, gradual deterioration of equipment, gradual change in the temperature or humidity, and so on and so forth. As the quality measurement gradually increases it reaches an unacceptable limit where some corrective actions must be conducted to cope with the increment. A common assumption is that the restoration of the system is extremely expensive, and so it is minimized the number of adjustments made to maintain the parts at a certain quality level. At integrating progressive changes on quality to the stochastic control of manufacturing systems, we can realize that they can be investigated from different perspectives, which in fact constitute the matter of our research. In particular, we are interested to investigate the case where the relationship between deterioration and quality is such that the increase on the level of deterioration of the machine decreases the quality of the parts produced. In the next section, we present a literature review about the different topics discussed in the above paragraphs.

0.7 Literature review

Efficient control policies account for the presence of stock and aims to reduce the propagation of disturbances among the manufacturing system. In real context, there are many classes of disruptions with a wide range of frequencies and durations, then is expected that controllers must respond to disruptions in limited but effective ways and manage adequately the production system. The technical advantage of control methods is that they take randomness into account and provide and explicit way of responding to random events. Something that it is not possible in most of optimization deterministic models, because their calculation are so time-consuming, that they are not solved frequently, therefore the decisions are not made in real-time and decision makers must improvise when random events occur. Stochastic control overcome this deficiency and utilizes information to convey commands with instructions on what to do next, and when to do it in response to random events that occur during production. In the remainder of this section several applications are presented that will place in context our research project.

0.7.1 Production planning problem

With the formulation of Kimemia and Gershwin (1983) it was consolidated a theoretical framework that promoted the development of subsequent contributions dealing with the production planning problem of stochastic manufacturing systems. As an extension of their formulation, we can refer to the work of Akella and Kumar (1986), who succeed to obtain a first analytical solution for a special case of a single machine producing a single part type.

Their determined the production rate, where certain costs are specified for both positive and negative inventories and there is a constant demand rate for product. Thorough mathematical approaches they arrived to conjecture the optimal policy and gave a rigorous proof of the optimally of their obtained policy. Moreover they obtained a complete solution for the system and provide a simple explicit formula for the optimal inventory level.

Another extension was provided by Sharifnia (1988), where they derived equations for the steady state probability distributions of the surplus level for the case of a manufacturing system producing a single product with multiple machine failures. In his model the average surplus is easily calculated in function of the inventory levels applied, and then the average cost is minimized to determine the optimal inventory levels. His results are valuable for solving the case of multi-product manufacturing systems.

Recent work in this area was presented by Mok and Porter (2006), who proposed an innovative method to estimate the short-run optimal hedging points for unreliable manufacturing systems producing either single product-types or multiple product-types. Their results have shown to be capable of generating optimal hedging points for systems producing multiple products with different priorities that appear when the cost weightings differ among the different products produced. Furthermore their model is not restricted to exponentially distributed machine failures and repairs, since it can be applicable with other distributions as well. Their results can be extended to the case of unreliable system with variable demands.

A more elaborated application of the hedging point policy can be found in Chan *et al.* (2008), in their model they assumed that the manufacturing system can produce multiple product-types, and that also is possible to use extra production capacity from other machine apart from the original production system. This implies cooperative production since extra capacity can be used only when the demand is larger than the normal capacity. They determined the optimal production plan to satisfy uncertain demands defined with two demand levels for each product-type, thus a two level hedging points are defined for each product-type. Their policy considers all the part types at the same prioritized level and keeps the work in process states on a straight line of the set of states; this facilitates the calculation of the total cost that is a function of the hedging points. This work is useful to the case where demand rates follow a general probability distribution rather than a simple Markov process with two states.

The existence of optimal production policies with adjustable capacity is treated by Gharbi *et al.* (2011), their model consisted of an unreliable manufacturing cell composed of an unreliable central machine responding to a single product demand. They assumed that the central machine may fail to satisfy the long-term demand, and to adjust the production capacity a reserve machine may be called as support. They determined the optimal production control policy for both machines. Their results outperformed production control policies based on classical redundant structures such as stand-by and parallel machines, and offers the possibility to treat the non-exponential failure and repair time distributions.

Dahane *et al.* (2012) study the production planning problem of a manufacturing system conformed of a single repairable machine producing two products satisfying a demand at the end of each period. They consider dependence between the failure rate and the production rate. The main product is produced to fulfill the strategic demand of the principal customer and the secondary product is produced to meet a very profitable demand. Their model determines the production rate during each period for the first product and the duration of the production interval of the second product.

As indicated in the presented works of this subsection, we can notice a common trait of this domain, production systems imply dynamic, uncertain and non-linear problems that are really hard to analyze. As a consequence research in this field is still far from complete, for example the impact of quality aspects has not been considered on them. To stress this fact, in the next section we present some applications that intend to study the inter-relation between quality and production.

0.7.2 Integration of Quality and Production

A number of research works have appeared emphasizing the relevance of incorporating quality aspects in the design and analysis of production systems, as in the paper of Inman *et al.* (2003). Where they identified the industrial necessity to incorporate quality aspects in the production system research and presented several topics where the connection between productivity and quality is evident. They made the emphasis mainly in the design of production systems to increase corporate profitability by increasing quality and maintain the company in competition.

A comprehensive analysis on the inter-relation between quality and productivity is the set of works of Kim and Gershwin (2005, 2008). In the former study they develop a mathematical model based on a Markov process to analyze the performance of small manufacturing systems considering defective production. They obtained analytical expressions for the total production rate, effective production rate and the yield of the system. An extension to larger systems is found in the latter study, where based on the decomposition method they succeed to develop an approximation method for the performance analysis of long manufacturing systems with defective products. Also they analyzed the influence of the location of inspection stations in long flow lines at varying the sets of machines that each inspection station monitors.

Applications considering defective production are found in Bendavid *et al.* (2008), they focused in a single-machine process, producing items with only one quality characteristic that leads to differentiate between conforming and non-conforming parts. In their model the machine produces a finite batch of items, and inspection is conducted to all the units of the batch. They assumed that in the in-control state the system also produces non-conforming parts, and in the out of control state it may also produce conforming items. Their model determines which units to inspect and how to dispose the uninspected parts to minimize the total cost.

Another set of relevant works are provided by Colledani and Tolio (2009, 2011). They propose an analytical method to evaluate the performance of manufacturing systems where techniques of statistical process control are implemented to detect defective products. The quality of the parts produced is monitored by measuring a defined characteristic, with sampling and inspection devices. Also they provided an extension to evaluate the performance of manufacturing lines monitored by statistical control charts, when the lines produce also defectives products. Their method considers the presence of inspection and integrated stations in the line to collect data and compile control charts to facilitate the detection of out of control states.

Even though the aforesaid works have shown that their methods have a good accuracy at evaluating some production logistics performance, and enable to have a better understanding of the real behavior of the production system, lamentably the link with production policies are not treated. Only a few recent contributions have focused on the impact of quality issues on the production control rule.

Concerning the integration of quality issues with production policies, Radhoui *et al.* (2009) introduced a model for a production system producing lots with conforming and non-conforming parts. Their model contains the case of one machine that produces one part type, and they used the proportion of non-conforming parts on each lot to define the threshold that decide the type of maintenance action to perform to the system. Furthermore, their model determines the size of the buffer stock that palliate perturbations caused when the machine is not operational, also it defines the threshold level of the rejection rate that indicates the instant to built that buffer stock.

Mhada *et al.* (2009) extended the Bielecki-Kumar model consisting of a single part, single unreliable machine that satisfies a constant demand rate with the particularity that even when the machine is operational, it produces a fraction of defectives products. They achieved to obtain an analytical solution for the Markovian system, and proposed for the non-Markovian case a control policy, obtained through simulation optimization techniques that use the markovian solution to center the space to search the related control parameters.

The more recent work of Dhouib *et al.* (2012), presents a model where the manufacturing system has general distributions of restoration periods and shifting times to the out control state. Their model analyzes all the possible scenarios for the manufacturing system with constant imperfect production, when it shifts to the out of control state. To reduce the shift rate to the out of control state an age-based maintenance policy is also incorporated. Their model takes into account the link between production, maintenance and quality aspect defining the production phase and the length of the production-restoration cycle. This model provides some basis for study systems with perishable products.

One important remark is that despite the relevance of the foregoing works, it is definitely needed more research in the field of manufacturing system control. Since there are several factors that have not been studied in the inter-relation between the primary components of quality and production, i.e., in these works the effect of deterioration on the manufacturing system has not been addressed yet from the quality point of view. The preceding discussion prompts us to ask whether deterioration models can be extended in somehow to include quality aspects. We present in the next section some applications that highlight this observation.

0.7.3 Optimal policies considering deterioration

Deterioration has capture the attention of various authors such as Love *et al*. (2000), who considered the case of one machine that, is subject to failures, and proposed a repair-replacement policy. Their model indicates if at failure it is convenient to repair the machine, and shift back a small amount the failure intensity, or whether is better to replace the machine that evidently refresh completely the system. They assumed that the failure intensity depends on the age of the machine. However they have not incorporated productions decision in their model.

In another study, Lai and Chen (2006) analyzed the optimal replacement policy of a two-unit system; where there is present a failure rate interaction between units. This interaction implies that when the first machine fails, it causes a certain amount of damage to the second machine at increasing its failure rate by a certain amount, and when the second unit fails induces the first machine to fail immediately. Additionally the failure rates of the machines also increase owing to an ageing process.

Continuing the field of deterioration, Wu and Clements-Croome (2006) described a model for a repairable system for the cases where one maintenance or two maintenance policies are executed. The model that they introduced provides details about how the values of the required parameters are estimated. Moreover, their method permits to model the different phases of deterioration that the system may exhibit during its whole lifetime, focusing in the trend of the failure intensity to illustrate their approach.

Lam (2007) developed a maintenance policy that indicates if the system will be repaired whenever one of the two following conditions occurs first; it fails or its operating time reaches a certain level. The system is replaced by a new one following a certain number of failures. The effect of the deterioration entails that the operation time after repair decreases while the repair times increases after failures. Additionally, they introduced a lifetime distribution taking into account the effect of maintenance activities.

In view of the afore mentioned works, we can infer that in the context of deteriorating systems, clearly is evident that neither of the articles considered involves any appeal to the fact that deterioration also may have an effect on the quality of the parts produced or that it can exist a simultaneous effect on quality and other parameters of the machine. In addition, at considering the link between deterioration and quality, preventive maintenance may have an active role on the optimal control policy as well. Hence, following this line, we present in the next section some works on preventive maintenance strategies.

0.7.4 Preventive maintenance policies

The implementation of preventive maintenance for unreliable manufacturing systems represents an extremely effective strategy to increase their lifetime and reducing the operating cost. The effect of preventive maintenance on production systems has been studied by a number of researcher as Chelbi and Ait-Kadi (2004), who proposed a model for a repairable production system where preventive maintenance are regularly performed at defined periods of time, and a buffer stock is built to palliate perturbations caused by failures. Their mathematical model considers the repair time and preventive maintenance duration to determine an optimal policy.

Other consideration of production planning and preventive maintenance is found in Berthaut *et al.* (2010), they investigated the case consisting of one-machine, one-product manufacturing system where maintenance actions are random and have non-negligible durations. Their obtained policy is a combination of a hedging point policy and a modified periodic preventive maintenance strategy, where this action is conducted when the stock level exceed a determined level. They compared their obtained policy with a simplification where the inventory levels that triggers preventive maintenance and define the hedging point for production control are exactly the same. Also they compared their policy with the strategy that does not skip any preventive maintenance.

For their part Deyahem *et al.* (2011) developed a model that determines the production rate while the degraded manufacturing system is in operation, and when to replace versus repair the system when it fails. They also include a decision variable to determine when to carry out preventive maintenance. In their model there is an increasing probability of failures caused by machine aging and increasing repair times due to imperfect repairs. The benefit of preventive maintenance is to restore completely the condition of the machine.

Ayed *et al.* (2012) study a randomly failing manufacturing system facing a random demand where subcontracting is available to satisfy the required service level. The system operates with a variable production rate and its failure intensify depends on the production rate and the time. Their model determines the production plan for the main production system and for the subcontractor, and that plan is also combined with a preventive maintenance policy, where the optimal policy minimizes the total cost.

We close this subsection with a brief remark, the totality of the works above-presented, focus on the effect of preventive maintenance on the improvement of the availability of the manufacturing system, and there is a scarcely number of works that relates the benefit of the preventive maintenance strategy with the quality of the parts produced. In our view, it can be exploit much more this observation. Consequently, it seems logical to expect a potential extension of current preventive maintenance models from the quality angle. In summary, through the literature review is revealed that this research area is an open field four our conjectures, dealing with quality issues, deterioration, maintenance strategies, etc. We formally introduce in the next subsection, the main objectives for our research and the proposed methodology to attain each of them.

0.8 Objectives of the research

For the sake of clarity, we start by defining that the main objective of this research is to develop stochastic control models that provide efficient control strategies for unreliable manufacturing systems. Fostering their satisfactory performance, procuring methods to determine decisions promptly in response to randomly changing conditions, and reducing the propagation of disturbances among the system. To this end, we consider variables, factors and phenomena that we have identify as relevant in the domain; we pay particular attention to the minimization of the total cost and the maximization of the availability of the manufacturing system. Our models utilize information to convey commands with instructions on how much to produce, the type of maintenance activity that is more convenient to conduct, and when to perform it, in the context of: random failures, progressive deterioration, increasing defectives and changes in the reliability of the system. This primal objective is supported by the following specific goals:

1) Incorporate quality aspects to the stochastic control of manufacturing systems, to gain more insight about the impact of quality on the simultaneous production and maintenance control policies.

2) Investigate in different research directions the role of deterioration on the manufacturing systems, i.e., Markovian models, Semi-markov models (stochastic processes where the information that is available to the decision maker at each instant of time t, includes the state of the production system, as well as the machine's history, therefore the markov property of memorylessness does not hold), decomposition of the operational state, use of several deterioration indices, etc. Especially we focus in the case when it is assumed that deterioration has a continuous severe effect on the quality of the parts produced, and also an effect in the reliability of the system.

3) Extend previous contributions of stochastic control of manufacturing systems to include preventive maintenance strategies, with the focus to improve not only the reliability of the production system, but also the quality of the parts produced.

4) Apply simulation techniques as a way to validate and test the results of the stochastic control models developed in this project, and assess the usefulness of the obtained control policies.

0.9 Methodology

With regard to the methodology applied in this research project, it is constituted by several phases, and it represents an effective and rigorous method to determine efficient control policies for stochastic manufacturing systems. In special, our methodology is useful for situations where the control theory has difficulties of optimality, since the determination of analytical solutions are insurmountable in most of the cases. A block diagram of our research approach is illustrated in Figure 0.1, and is discussed as follows:

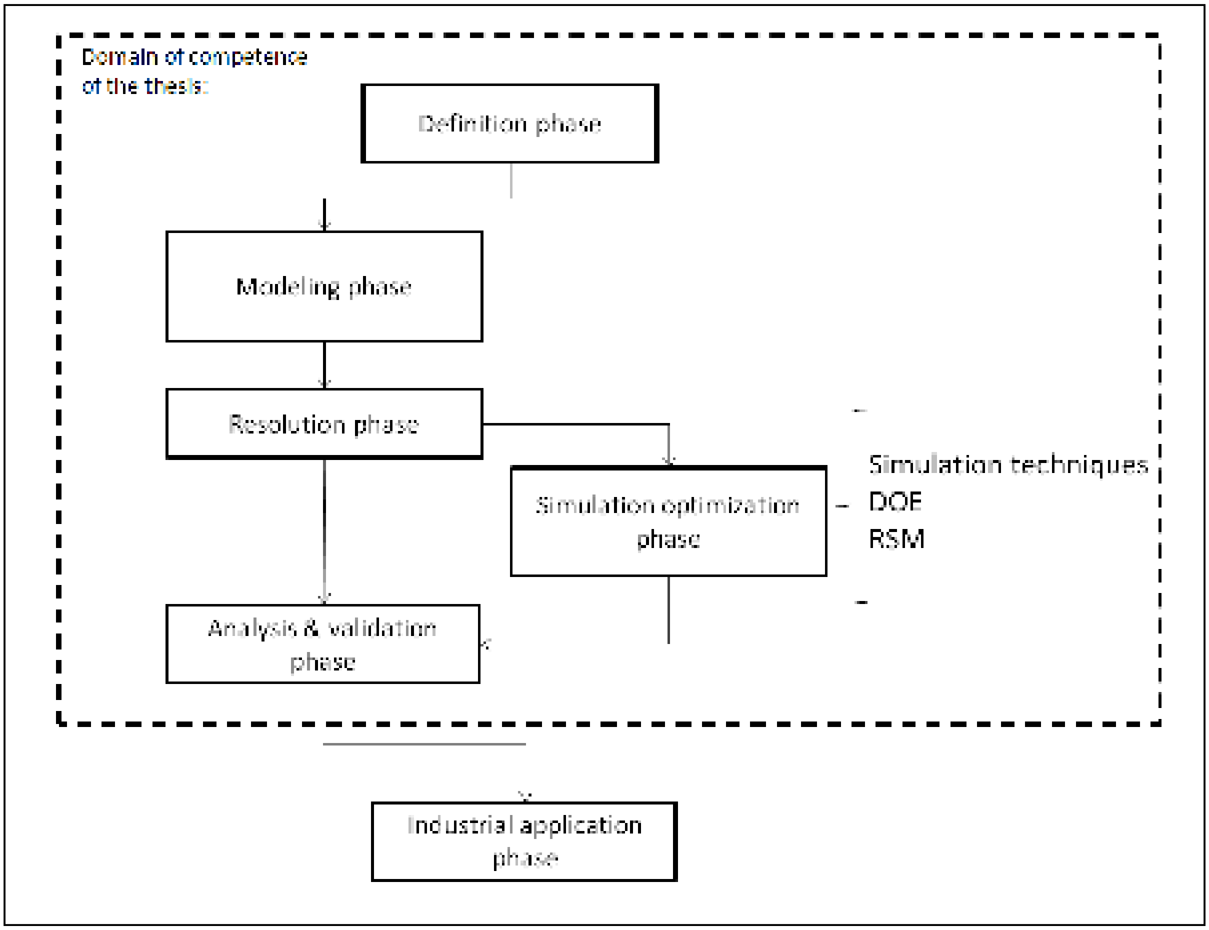

Figure 0.1 Proposed methodology

1) Definition phase: consists in the identification of relevant issues, variables and phenomena through a detail observation of manufacturing systems and examination of pertinent literature to determine the scope of the study.

2) Modeling phase: comprises the formulation of a stochastic dynamic control model based on the control theory, considering the whole collection of features, parameters and phenomena identified in the previous phase. The objective is to develop an effective model to determine an optimal production plan and maintenance strategies in order to palliate the effect of the considered disturbances. Our first model starts with some conjectures, and then once gained intuition about the system, the level of

complexity grows for the rest of the models adding more elaborated assumptions and ideas.

3) Resolution phase: implies the determination of the fundamental manufacturing system control equations derived from the optimality conditions for every studied problem. Then, numerical methods are applied with the purpose to approximate the structure of the control policy and the related control parameters.

4) Simulation optimization phase: for some of our models it is developed a simulation model that reproduces accurately the stochastic behavior of the manufacturing system under study. Such simulation model is combined with statistical analysis based on design of experiments to identify significant factors, and then a parameter optimization is conducted applying response surface methodology. The goal of this phase is to determine in less computational time a measure of the system's performance, denoted by the total cost, as well as calculate the value of the related control parameters.

5) Analysis & validation phase: indicates the examination of the variation of several parameters of the system, and its effect on the control factors and also on the total cost. This phase denotes a sensitivity analysis aimed to ensure the proper characterization of the structure of the control policy, and also intends to demonstrate the efficiency and robustness of the obtained policy.

6) Industrial application phase: once confirmed the robustness and structure of the obtained control policy, finally the best values of the related control parameters are prepared to be implemented to a manufacturing system. Beware that this phase is outside the domain of competence of this thesis.

0.10 Thesis outlined

The remainder of the thesis is organized as follows. The present thesis is composed of four (04) chapters, and its contribution is reflected through one international conference and the redaction of three arbitrated journal papers.

In regard to the academic article presented in the international conference, we gained sufficient expertise at employing simulation techniques and their respective optimization to model the type of manufacturing systems treated in the present thesis. This article is referred as follows:

- H. Rivera-Gómez, A. Gharbi, J. P. Kenné, (2012) "*A comparative study of simulation models for the production control of unreliable manufacturing systems*". 4th International Conference on Information System, Logistics and Supply Chain (ILS 2012), Quebec, Canada, August 26-29, 2012.

The journal papers were submitted to international journals, recognized in the field for their relevance. In the article of chapter two, we employed a simulation optimization approach to study the impact of deterioration on the quality parts, and determine the optimal production plan and overhaul strategy. This article was submitted for the International Journal of Production Economics (IJPE) with the following reference:

- H. Rivera-Gómez, A. Gharbi, J. P. Kenné, (2012) "*Joint production and major maintenance planning policies of a manufacturing system with deteriorating quality*". Accepted with minor remarks on July 29th, 2012. (submission confirmation IJPE-D-12-01365).

The article of the third chapter, extends previous models to address the quality deterioration process as a combination of several factors; mainly we focus on the effect of worse repairs and the natural wear of the machine. We determined the optimal production policy and repair versus overhaul strategy. This article has been accepted for the International Journal of Production Research (IJPR) with the reference:

- H. Rivera-Gómez, A. Gharbi, J. P. Kenné, (2013) *“Production and quality control policies for deteriorating manufacturing”.* Accepted on January 27th, 2013. (submission confirmation Ms Ref. No. TPRS-2012-IJPR-0800).

In chapter four, the article presented addresses the relationship between deterioration and quality from a different perspective, based on the age of the machine. There we proposed that the progressive deterioration has a twofold effect reflected on; the failure intensity of the system and the quality of the parts produced. It is formulated a Semi-Markov model and also preventive maintenance strategies are introduced to struggle against deteriorating conditions. The article has been submitted to the International Journal of Advanced Manufacturing Technology (IJAMT) with the reference:

- H. Rivera-Gomez, A. Gharbi, J. P. Kenné, (2013) “*Joint control of production, overhaul and preventive maintenance for a production system subject to quality and reliability deteriorations”*. Accepted with minor remarks on March 19th, 2013.

We terminate the dissertation with the section of general conclusions, which provides a summary of the contributions of our research, and also presents our future research plan.

CHAPTER 1

ARTICLE 1: DETERMINATION OF THE JOINT PRODUCTION PLANNING AND OVERHAUL SCHEDULING EMPLOYING SIMULATION OPTIMIZATION FOR A QUALITY DETERIORATING MANUFACTURING SYSTEM

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Abstract

We investigate the simultaneous production planning and quality control problem for an unreliable single machine manufacturing system responding to a single product type demand. The machine is subject to deteriorations, and their effect is observed mainly on the rate of defectives, which increases continuously over time. Due to the uncertainty caused by failures, the machine may not meet long-term demand, and an overhaul can be conducted in order to counter the effect of the deterioration. The main objective of this study is to simultaneously determine the optimal production plan and overhaul schedule for the analyzed manufacturing system, in order to minimize the total cost, comprising the inventory, backlog, repair and overhaul cost, over an infinite planning horizon. A stochastic dynamic programming model is proposed, in which a numerical scheme is adopted to solve the optimality condition equations. It is observed that the optimal control policy is described

by a machine deterioration-dependent hedging point policy (MDDHPP). To accurately approximate the related control parameters, a simulation optimization approach based on design of experiments, simulation modeling and response surface methodology is applied. The results obtained provide a better understanding about the influence of the deterioration of quality in the production and overhaul policies. A numerical example and an extensive sensitivity analysis are conducted, and show the robust behaviour and usefulness of the policy obtained.

Keywords: Production control, Numerical Methods, Simulation, Response Surface Methodology, Manufacturing systems, Defective.

1.1 Introduction

Production planning has been studied by several authors, with the common objective of improving productivity. However, a major limitation encountered in most of the literature in this domain lies in the assumption that all the parts produced are conforming items; an assumption which is obviously not realistic in industrial contexts. Fortunately, the inter-relation between productivity and quality has been received growing attention of researchers. We start by examining the need to integrate quality aspects in production policies, since all companies must satisfy high levels of productivity and high standards of quality. Additionally, if we take into account that manufacturing systems progressively degrade over time means that, this factor may have an impact on its operating conditions. Therefore, through this research, we contribute to a better understanding of the inter-relation between production planning and quality, in the case where the manufacturing system is subject to deterioration, which has a negative influence on the quality of parts produced.

In the literature, various authors have covered the production planning problem for unreliable manufacturing systems; for example, in the production systems studied by Kimemia and Gershwin (1983) and Akella and Kumar (1986), the control policy they obtained was found to have a structure called hedging point policy. The importance of such a policy lies in being an efficient way of determining production policies for manufacturing systems. Following these two studies, several extensions to this research area were realized, considering a wide range of aspects such as; transportation delay from the machine to the inventory as in Mourani et al. (2008), multiple-types production satisfying a low and a high demand as in Chan et al. (2008), remanufacturing operations considering a reserve logistic network as in Kenne et al. (2012), etc. Nevertheless, the majority of such extensions of production planning have not covered the influence of quality issues on the control policy. While the importance of quality cannot be ignored, it should be noted that it reflects the need for further analysis of the inter-relation between production and quality aspects. Some papers have highlighted the importance of this inter-relation, such as that by Inman et al. (2003), which presented a comprehensive list of research issues involving the relationship between quality and production system design. However, consideration of quality issues only started growing with the series of works by Kim and Gershwin (2005, 2008), who introduced mathematical models to evaluate the performance of small and large production systems with quality and operational failures. Colledani and Tolio (2006, 2009, 2011) similarly addressed the evaluation of the performance of production systems, with the behaviour of machines monitored by control charts. Although these papers study the influence of quality aspects, their focus is on performance measures, whereas our research approach is different. We focus mainly on the structure of the control policy, and aim to investigate the impact of quality aspects on the production control rule.

A recent area of research has emerged addressing quality issues on the production policy; Radhoui et al. (2009), for instance, used the rate of defectives as a decision variable to determine when to perform preventive maintenance and define the buffer size. The simultaneous determination of maintenance activities and production planning is covered by Njike et al. (2009), who applied several operational states that monitor the system's condition. They used the quantity of defective products as feedback to optimally control the system. Further, Mhada et al. (2011) analyzed the production control problem for an unreliable manufacturing system producing a random fraction of defective items, and succeeded in stating analytical expressions for the production threshold and the optimal cost. Additionally Dhouib et al. (2012) incorporates to the production planning, an age-dependent preventive maintenance policy that reduces the shift rate to the out of control state, where their productive systems produces non-conforming items. Despite the pertinence of these works, we conjecture that the joint production and quality control problem can be studied from a different perspective. For instance, this includes bearing in mind that in real industrial contexts, the production system is subject to deteriorations (because of an infinite set of factors), meaning therefore that the effect of the deterioration may certainly have an impact on the quality of the parts produced. This effect will allow us to extend the concept of deterioration, and relate this factor to the quality yield of the production system. We find some support for our conjecture in the area of deteriorating systems.

Many papers have been published in the area of deteriorating systems, with the typical method used to model deterioration based on the concept of imperfect maintenance. A good discussion on the subject of imperfect maintenance can be found in Pham and Wang (1996). In addition, Wang and Pham (1999) present an interesting method, and propose that after an imperfect repair, the operating time of the system decreases as the number of repairs increases. The idea to treat certain deterioration in the operating conditions was extended by Lam (2007), who proposed that the operating times after a repair decrease, while the consecutive repair times after failure increase. We find another approach for modeling deteriorating systems in the work of Dehayem et al. (2011), who describe a model in which the operating time of the production system follows a decreasing function given by the age of the machine, while the repair time consists of an increasing function with the number of failures. As can be seen from the previous models, it is assumed that deterioration has an effect on the availability of the production system, and that it is used as indicator of deterioration either with the age of the machine or with the number of failures. Nevertheless, these models do not link the degrading process to the parts quality. This observation in turn raises the question of whether it is possible to relate the deterioration phenomenon with the quality yield of the production system. Typically, an efficient alternative for determining the optimal control policies of stochastic manufacturing systems has been the use of simulation optimization approaches.

Simulation modeling has proved to be an effective means to analyze manufacturing systems, as observed in the work of Lavoie et al. (2009a), who compared different pull control mechanisms for homogenous transfer lines. Simulation has also been applied to compare different maintenance strategies, such as in Boschian et al. (2009), where they analyzed the case of two machines working in parallel, applying different maintenance strategies. Other applications of the simulation optimization approach, such as the presented by Berthaut et al. (2010), deal with the determination of production and periodic preventive maintenance rates. Recently, this hybrid methodology was extended by Gharbi et al. (2011), who analyzed the case of the production control problem of a manufacturing cell comprising a central and a reserve machine. Moreover Hajji et al. (2011) used a simulation based approach to determine the production control parameter and product's specification limits that have a direct influence in the rate of non-conforming items. A closer look at these models reveals that this simulation optimization approach is not yet in use in cases where the degrading process of the production system has a continuous deterioration on the parts quality.

Therefore, given this context, we intend to develop a new model for the simultaneous production and quality control policy of a mono-product manufacturing system, composed of a single unreliable machine that is subject to progressive quality deterioration and uncertainties. This is motivated by the need to study the inter-relation between quality issues and the production policy, where phenomena such as deterioration are present. Furthermore, the notion of relating the deterioration of the machine with the parts quality is based on the concept of worse repairs (a maintenance action which increases the rate of defectives). The uncertainties are due to machine failures in a dynamic continuous time stochastic context. We develop a stochastic dynamic programming model with two decision variables, the production rate and the quality decision related to the overhaul strategy, which counters the effect of the deterioration. This specific problem has not been yet addressed in the literature. The resultant control policy called Machine Deterioration Dependent Hedging Point Policy (MDDHPP), adjusts the control parameters according to the level of deterioration of the machine. The contribution of this article is further illustrated by the robust behaviour of the MDDHPP facing several variations of the system parameters in a sensitivity analysis, and providing a better knowledge of the production system behaviour. We propose a simulation optimization approach combined with the control theory to achieve a close approximation of the optimal control policy parameters.

The remainder of the paper is structured as follows. After an overview of the literature in section 1.1, the notations and the system description are presented in section 1.2. The control problem statement is detailed in section 1.3. Numerical methods are applied to characterize the structure of the obtained control policy in section 1.4. The proposed simulation optimization approach is presented in section 1.5, along with a detailing of the simulation model and its validation. In section 1.6, a numerical example is reported to illustrate the system's behaviour. A sensitivity analysis of the control policy is presented in section 1.7, with regards to different cost parameters and trajectories of the rate of defectives. Finally, concluding remarks that illustrate new insights into this topic are given in section 1.8.

1.2 Notation and system description

This section presents the notation and the system description of the manufacturing system under consideration.

1.2.1 Notation

The following notations are used:

$x(t)$	Inventory level at time t
$u(t)$	Production rate of the manufacturing system at time t
$n(t)$	Current number of failures in the interval $[0, t]$
d	Demand rate
$\xi(t)$	Mode of the machine at time t
u_{max}	Maximum production rate
$\beta(n)$	Rate of defectives at the n-failure
ρ	Discount rate
$\lambda_{\alpha\alpha'}(\cdot)$	Transition rate from mode α to mode α'

v_{min}	Minimum overhaul transition rate
v_{max}	Maximum overhaul transition rate
$g(\cdot)$	Cost rate function
$J(\cdot)$	Expected discounted cost function
$V(\cdot)$	Value function
τ	A jump time of the process $\xi(t)$ defined in the reset function
c^+	Incurred cost per unit of produced parts for positive inventory
c^-	Incurred cost per unit of produced parts for backlog
c_o	Overhaul cost
c_r	Repair cost
N	Maximum number of failures where the system is still operational (limit of the system's feasibility)
n_f	Number of failures needed to perform the overhaul

1.2.2 System description

The production system under consideration consists of an unreliable single machine producing one part type. The block diagram of the production system is presented in Figure 1.1. The machine is unreliable, and is subject to random events, such as failures, and different maintenance activities, such as repairs and overhauls. We conjecture that the machine is subject to deterioration, and this phenomenon is tied directly to an increasing defective rate. The quality deterioration leads to the integration of quality related decisions, such as the overhaul strategy that counters the effect of the quality deterioration, into the model. Since the quality of the parts produced is not perfect, the product stock is composed of a mixture of flawless and defective products. In this domain, it is generally assumed that deterioration affects the availability of the system, but in this paper, however, we focus on its effect on the quality of the parts produced, and more specifically on the rate of defectives. When the machine is in a failure state, a worse repair is conducted, having the particular characteristic to deteriorate the machine, thereby increasing the rate of defectives. We thus tie the number of failures to the quality deterioration, and some reasons justifying this condition are discussed later in this paper, in section 1.3. The model's decision variables are related with the production planning and the overhaul strategy. The objective of the model is to determine the simultaneous production and overhaul policies that minimize the total incurred cost, which is composed of the repair cost, the overhaul cost, the inventory cost and the backlog cost of units.

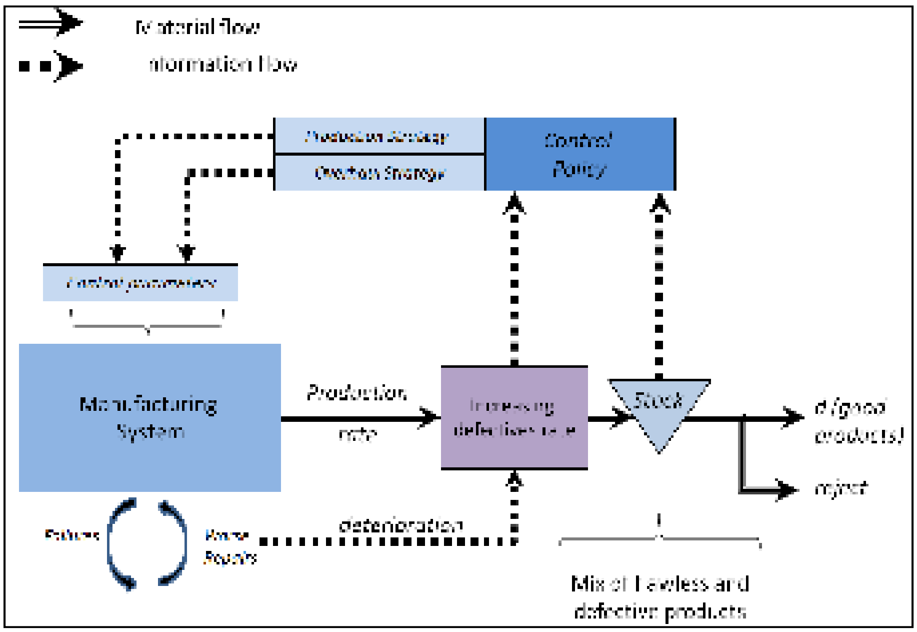


Figure 1.1: Block diagram of the manufacturing system under study

1.3 Control problem statement

In this section, we present the formulation of the control problem for the manufacturing system presented in section 1.2. As mentioned previously, the production system consists of a single machine that produces one type of product to meet client demand. The system is subject to random events (failures, repairs, and overhauls) as well as to quality deterioration. The overhaul implies a perfect maintenance whose benefit is to counter the quality deterioration and restore the machine to as-good-as-new conditions. It is also possible to perform a worse repair, but it has the disadvantage to degrade the quality of the machine. The machine has three modes, described by the stochastic process $\{\xi(t), t \geq 0\}$, with values in $\Omega = \{1,2,3\}$. The machine is available when operational (*ξ(t)=1),* and unavailable when under repair (*ξ(t)=2)* or under overhaul (*ξ(t)=3).* At any given time t, the system is characterized by the machine mode *ξ(t),* the number of failures *n(t)* and the stock level x*(t).* According to standard notation, let d be the demand to be satisfied and $u(t)$ the production rate at time t. At any time instant, the production rate $u(t)$ of the machine has to satisfy the capacity constraint: $0 \leq u(t) \leq u_{max}$, where u_{max} is the maximum production rate.

Our primary concern in this model is that the quality of the parts produced is continuously deteriorated by the degrading process of the manufacturing system, as suggested by Colledani and Tolio (2011) and Kim and Gershwin (2008). These authors conceived a deterioration-quality relationship. To exploit this link in our model, we propose that the deterioration of the machine has the effect of increasing the rate of defectives. Hence, we propose that the dynamics of the inventory/backlog of products $x(t)$ evolves according to the following differential equation:

$$\dot{x}(t) = u(t) - \frac{d}{\left(1 - \beta(n)\right)}, \quad x(0) = x_0 \tag{1.1}$$

where $\beta(n) < 1$, x_0 is the initial stock level, *n* is the current number of failures at time *t,* $\beta(n)$ is a function of the rate of defectives, and the quantity $\frac{d}{(1-\beta(n))}$ represents the adjusted demand that includes defective products. Our remaining problem with this Equation (1.1) is to define how exactly the function $\beta(n)$ relates the deterioration of the machine with the rate of defectives. We find some useful ideas that sketch this condition in the area of deteriorating systems.

We should recall that at failure, the maintenance option available is to conduct a worse repair, defined in Wang (2002) as the maintenance action where the system's operating conditions become worse with this sort of repair. Some reasons accounting for this condition is that at failure, the faulty component is only partially repaired, the influence of human errors, etc. Additionally, imperfect maintenance methods, such as those presented by Wang and Pham (1999) and Lam et al. (2004), use the number of repairs or the number of failures to define the level of deterioration of the system, which allows us to define the failures-deterioration relationship. Even in these models, a certain trend has been observed in the deterioration, and their results have been applied successfully to real industrial data. We therefore extend the concept of worse repairs to model quality deterioration, and based on the relations between failures-deterioration and deterioration-quality, we propose the increasing function (1.2), which defines the rate of defectives as a function of the number of failures, as follows:

$$\beta(n(t)) = \beta_0 + \beta_1 \left(\frac{n(t)}{N}\right)^r \tag{1.2}$$

where β_0 is the value of the rate of defectives at initial conditions (normally with a very low value), N is the maximum number of failures where the system is still operational and feasible to satisfy the demand, and β_1 and r are given parameters obtained from historical data of the machine. It turns out that our formulation is a derivation of a Markov model, because the transitions between modes are denoted by exponential distributions and are not affected by the failures. However, as the rate of defectives follows a defined trajectory given by Equation (1.2), we need the number of failures $n(t)$ in order to properly characterize the state of the system.

Before completing the problem formulation, we would like to draw the reader's attention to an important technical detail. Typically, the manufacturing system will only meet the conditions needed to fulfill the demand rate d over an infinite horizon and reach steady-state, if the system is feasible. In other words, the production system must satisfy the following feasibility condition:

$$u_{max} \cdot \pi_1 \geq d \cdot [1 + \beta(n)] \tag{1.3}$$

where, π_1 is the limiting probability for the operational state. Let $Q(\cdot) = \{\lambda_{\alpha\alpha'}(\cdot)\}$ refer to the related transition matrix. Therefore π_1 can be computed as follows:

$$\pi_i \cdot Q(\cdot) = 0 \quad \text{and} \quad \sum_{i=1}^{3} \pi_i = 1 \tag{1.4}$$

For our case of study, the solution of π_1 for the system of Equations (1.4) yields to the following expression:

$$\pi_1 = \frac{1}{1 + \frac{\lambda_{12}}{\lambda_{21}} + \frac{\lambda_{13}}{\lambda_{31}}} \tag{1.5}$$

where $\lambda_{\alpha\alpha'}$ are the transition rates from mode α to mode α'. In a practical sense, the value of the parameters needed by Equation (1.2) can be determined from the analysis of maintenance service data. The practical advantage of Equation (1.2) is that we can change the value of the parameter *r* to adjust the trend of the defective rate for a specific machine. In Figure 1.2, we present as an illustration, some trajectories of the rate of defectives for *N=20*, applying different values of the parameter *r*.

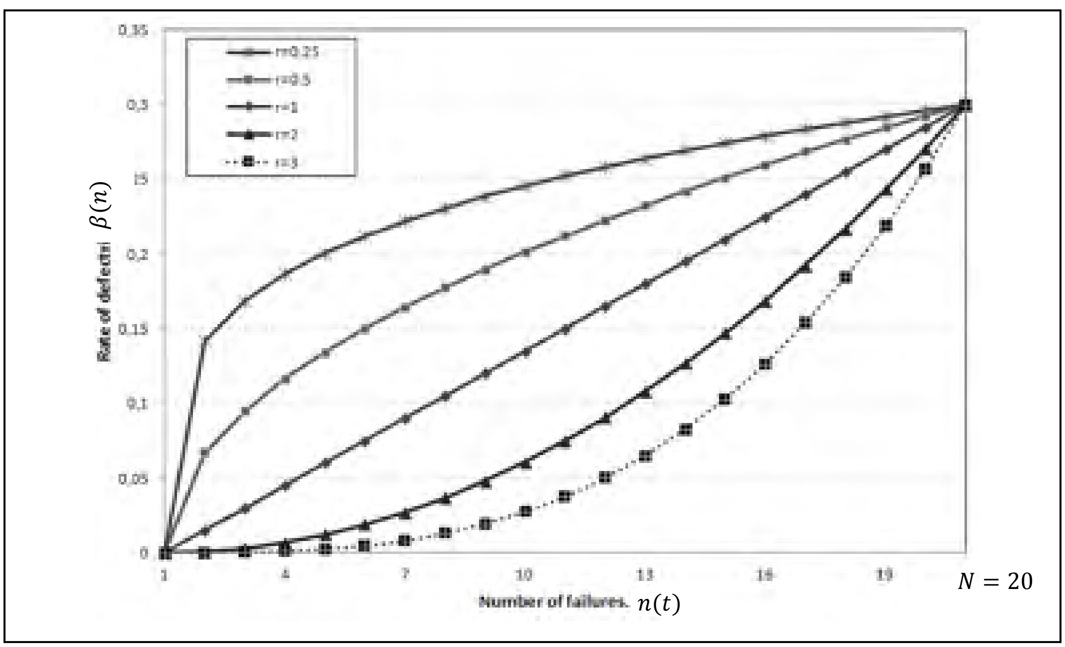


Figure 1.2 Trend of the rate of defectives for different values of the parameter r.

Returning to the model formulation, the machine's mode changes from operation mode to overhaul mode with a transition rate denoted by $\lambda_{13} = v(\cdot)$. The rate $v(\cdot)$ is assumed to be a control variable, and the decision to send the machine for overhaul is taken while the machine is operational. The inverse of $v(\cdot)$ represents the expected mean delay time between the decision to perform the overhaul and the effective switch from operation mode to overhaul mode. In other words, $\frac{1}{v(\cdot)}$ corresponds to the delay between the call of a technician and his/her arrival. Moreover, we assume that the following constraint holds for the overhaul rate:

$$v_{min} \leq v(\cdot) \leq v_{max} \tag{1.6}$$

where v_{min} and v_{max} denote the minimum and maximum overhaul rate, respectively. The instantaneous cost function of the model at mode $\alpha \in \Omega$, is defined by the following equation:

$$g(\alpha, x, n, u, v) = c^{+}x^{+} + c^{-}x^{-} + c_r \cdot Ind\{\xi(t) = 2\} + c_o \cdot Ind\{\xi(t) = 3\} \tag{1.7}$$

with:

$$x^{+} = max(0, x)$$

$$x^{-} = max(-x, 0)$$

$$Ind\{\xi(t) = \alpha\} = \begin{cases} 1 & if \;\; \xi(t) = \alpha \\ 0 & otherwise \end{cases}$$

where c^{+} is the inventory cost, c^{-} is the backlog cost, c_r is the worse repair cost and c_o is the overhaul cost. The control variables of the model are the production rate $u(\cdot)$ and the overhaul rate $v(\cdot)$; the set of admissible decisions (u, v) depends on the stochastic process and, is given by:

$$\Gamma(\alpha) = \{(u(\cdot), v(\cdot)) \in \mathrm{R}^2, \quad 0 \leq u(\alpha,\cdot) \leq u_{max}, \; v_{min} \leq v(\alpha,\cdot) \leq v_{max}\} \tag{1.8}$$

Our objective is to control the production rate $u(\cdot)$ and the overhaul rate $v(\cdot)$ in order to minimize the integral of the expected value of the discounted cost given by:

$$J(\alpha,x,n,u,v) = E\left[\int_{0}^{\infty} e^{-\rho t} g(\cdot)dt \mid \alpha(0)=\alpha, x(0)=x, n(0)=n\right], \qquad \forall\, u(\cdot), v(\cdot) \in \Gamma(\alpha) \tag{1.9}$$

where ρ is the discounted rate and (α,x,n) are the initial state conditions. Optimal policies are obtained by searching in the value function:

$$V(\alpha,x,n) = \inf_{(u,v)\in\Gamma(\alpha)} J(\alpha,x,n,u,v) \tag{1.10}$$

The value function $V(\alpha,x,n)$ satisfies specific properties called optimality conditions. In Appendix A, it is shown that the value function $V(\cdot)$ satisfies the so-called Hamilton-Jacobi-Bellman (HJB) equations. Such equations describe the optimally conditions of the problem, in addition to determining the optimal feedback control (u,v). In this case, the derivation of the optimality conditions leads to the following HJB equations:

$$\rho V(\alpha,x,n) = \inf_{(u,v)\,\in\,\Gamma(\alpha)} \left\{ g[\alpha,x,n,u,v] + \frac{\partial V}{\partial x}[\alpha,x,n]\dot{x} + Q(\cdot)V[\alpha,x,\varphi(\xi,n)](\alpha) \right\} \tag{1.11}$$

where $\frac{\partial V}{\partial x}$ is the derivative of the value function. The control policy (u,v) denotes a minimizer of the right-hand-side of the HJB equations, and therefore, the controls obtained are optimal. Because of the randomness of α, the control policy is a feedback control policy based on the inventory level x, the mode of the machine α, and the number of failures n. Furthermore, as the overhaul activity restores the rate of defectives $\beta(n)$ to initial conditions β_0, at a jump time τ for the process $\xi(t)$, we define a reset function $\varphi(\xi,n)$ by the following relationship:

$$\varphi(\xi,n) = \begin{cases} n+1 & if\ \xi(\tau^+) = 1\ \ and\ \ \xi(\tau^-) = 2 \\ 0 & if\ \xi(\tau^+) = 1\ \ and\ \ \xi(\tau^-) = 3 \\ n & otherwise \end{cases} \tag{1.12}$$

We conclude this section by stating that, when the value function $V(\cdot)$ is available, an optimal control policy can be obtained from the HJB Equations (1.11). The fact is that in general, solving the HJB equations is usually quite difficult, and close-form solutions are only obtained for relatively few simple models. Even finding numerical solutions for the HJB Equations (1.11) is a challenge. Fortunately, Boukas and Haurie (1990) implemented the Kushner' method to solve such a problem in the context of production planning. In the next section, we detail the procedure for determining control policies.

1.4 Structure of optimal control policy

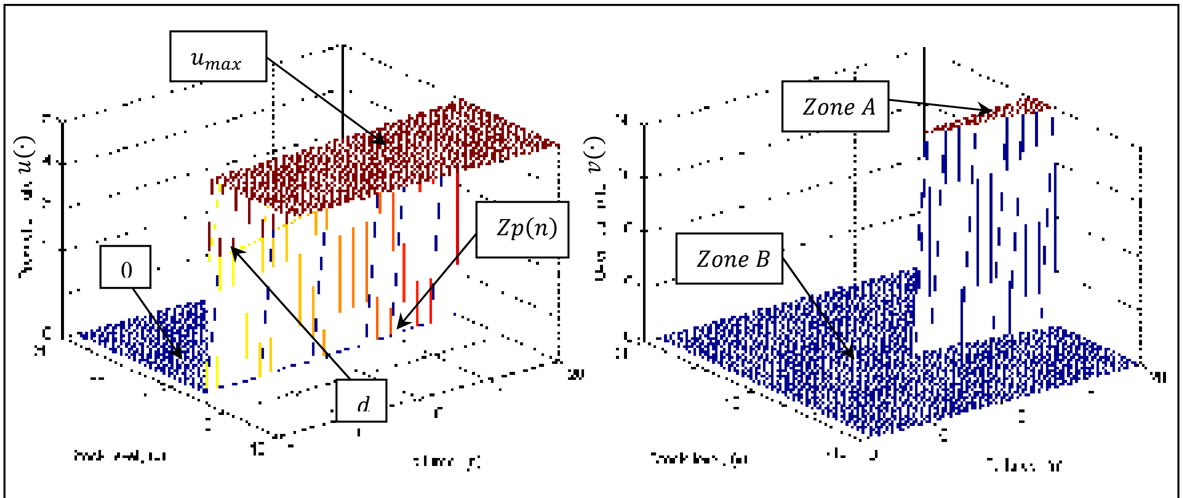
To determine the optimal policy, a solution could be approximated for the HJB equations by the application of numerical methods based on the Kushner technique. The main idea of this approach is to use an approximation scheme for the gradient of the value function $V(\alpha,x,n)$. Then a discrete function $V_h(\alpha,x,n)$ is used to approximate the continuous value function $V(\alpha,x,n)$, and its partial derivative $\frac{\partial V}{\partial x}(\cdot)$ can be expressed as a function of $V_h(\alpha,x,n)$ and the length of the finite difference interval h of the variable x. More details about the numerical method can be consulted in Kushner and Dupuis (1992) and in Hajji et al. (2009), and references therein. Subsequently, $V_h(\alpha,x,n)$ is obtained by solving a discrete dynamic programming using the policy improvement technique. With the parameters presented in Table 1.1, we observe that the condition of feasibility (1.3) is satisfied until the 20th failure, and so it follows that from the HJB Equations (1.11) and their corresponding solution, we obtain the control policy presented in Figure 1.3.

Table 1.1 Parameters of the numerical example

Parameter	u_{max} (units/hr)	d (units/hr)	h	ρ	c+ ($/units/hr)	c- ($/units/hr)
Value	5	3	0.5	0.9	5	250
Parameter	c_r ($)	c_o ($)	N	r	β_o	β_1
Value	5	10	20	1	0	0.35

Parameter	λ_{12} (1/hr)	λ_{21} (1/hr)	λ_{31} (1/hr)	v_{min} (1/hr)	v_{max} (1/hr)
Value	0.1	2	0.6	0.0001	20

The results obtained for the production policy are presented in Figure 1.3a, and for the overhaul policy in Figure 1.3b. Based on these graphics, it is apparent that the optimal production control policy consists of three rules, where the production rate is set to u_{max}, d and 0 respectively, based on the value of the production threshold $Zp(n)$ for every number of repair n. Moreover, the overhaul policy divides the plan (x, n) into two regions, where the overhaul rate is set to v_{min} and v_{max} .



a) Production policy b) Overhaul policy

Figure 1.3 Obtained control policies

The optimal production threshold level is denoted by $Zp(n)$, and defines the limits of the production region. The production control policy obtained is an extension of the hedging point policy, given that it respects the structure presented in Akella and Kumar (1986), in this case due to the effect of the deterioration of the parts quality; it leads to the following Machine Deterioration Dependent Hedging Point Policy (MDDHPP):

$$u(\alpha, x, n)^* = \begin{cases} u_{max} & if\ x(t) < Zp(n) \\ d & x(t) = Zp(n) \\ 0 & x(t) > Zp(n) \end{cases} \qquad (1.13)$$

where $Zp(n)$ is the function that gives the optimal production threshold for each failure n. We note that the production threshold $Zp(\cdot)$ of Figure 1.4, where we present the trace of the production policy, increases progressively; this trend shows the effect of the quality deterioration on the production policy.

The overhaul policy is presented in Figure 1.3b, its analysis is facilitated with the use of its boundary $B_n(\cdot)$. We note that Figure 1.3b identifies two zones in the computational domain delimited by the boundary $B_n(\cdot)$, as follows:

- *Zone* A: in this zone the quality deterioration, denoted by the number of failures, has a high level, which justifies the cost of performing an overhaul, and thus the overhaul rate is set to its maximum value, ($i.e.,\ v(\cdot) = v_{max}$).

- *Zone* B: here the quality deterioration is low, meaning that an overhaul is not recommended, and the overhaul rate is set to its minimum value, ($i.e., v(\cdot) = v_{min}$).

To define the overhaul policy, we will now simultaneously consider the production and the overhaul boundaries as presented in Figure 1.4. Apparently the overhaul trace $B_n(\cdot)$ is represented in Figure 1.4 by the points $\overline{n_o n_{N1} n_{N2}}$. However, it should be noted that the stock level is limited by the production threshold $Zp(\cdot)$, identified by the segment $\overline{Zp_o Zp_N}$. This observation help us to realize that only a part of the overhaul zone A is used, and from what follows, this feasible overhaul region is denoted as the zone A'. This reduction is important as it indicates that we can define the overhaul policy knowing the point n_f^*, which is where the segments $\overline{Zp_o Zp_N}$. and $\overline{n_o n_{N2}}$, intersect.

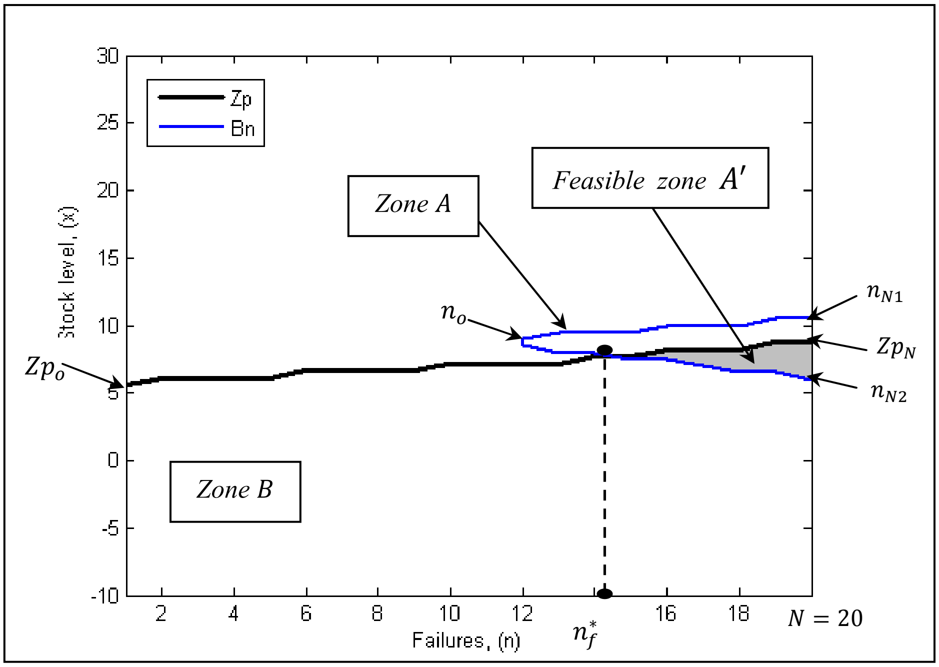


Figure 1.4 Intersection of the production and the overhaul trace

According to the results provided by Figure 1.3b, it is clear that the overhaul policy has a bang-bang structure. Moreover, the overhaul activity is triggered according to the policy described in Figure 1.4, which indicates that the overhaul activity should be performed at rate $v(\cdot)$, with:

$$v(\alpha, x, n)^* = \begin{cases} v_{max} & if \;\; n(t) \geq n_f^* \;\; and \;\; x(\cdot) \in zone \; A' \\ v_{min} & otherwise \end{cases} \tag{1.14}$$

where n_f^* is the number of failures limit required to conduct the overhaul. For a given number of failures n, the parameter n_f^* is provided as illustrated in Figure 1.4, indicating the point where the production threshold $Zp(\cdot)$ intersects the overhaul zone A. The observed trends in the boundaries of the production and overhaul policies are explained by the deterioration phenomenon. On the basis of the previous results, it can be observed that when the number of failures increases, the deterioration of the parts quality also increases, meaning that the machine is sent to overhaul more often. This highlights the fact that the production and overhaul policies are influenced by the quality-deterioration phenomenon. Summing up, we can illustrate the joint production and overhaul policies by Equations (1.13)-(1.14), which are characterized by the control parameters (Zp, n_f), where Zp corresponds to the value of $Zp(\cdot)$ for each n value in Figure 1.4.

Briefly, even though the application of the numerical methods provides the structure of the optimal control policies, the problem is that a satisfactory approximation of the control parameters would be too time-consuming to be applicable at the operational level. This is observed because the accuracy of the numerical results depends on the size of the discrete grid step h, as discussed in Kenné et al. (2003). To overcome this condition in the next section, we propose an alternative simulation optimization approach, to approximate the optimal control parameters (Zp, n_f), and determine the optimal cost. The technical advantage of the simulation optimization is that it is more flexible, and allows us to examine the control policy in a wide range of time and different cost variations.

1.5 Simulation optimization approach

This section presents a simulation optimization approach having the advantage of being applicable at an operational level. The proposed control approach combines analytical and simulation models with statistical analysis, and is based on the works of Gharbi and Kenné (2000) and Berthaut et al. (2010). The block diagram of the proposed approach is presented in Figure 1.5, and consists of the following sequential steps:

- *Mathematical formulation of the optimization problem and numerical resolution*: This step consists of the representation of the simultaneous production planning and the overhaul scheduling problem through an optimal control model. The objective is to determine the control variables (u, v) that minimize the incurred cost. Numerical methods are applied to determine the structure of the control policy, in which we identify the control parameters (Zp, n_f), as discussed in section 1.3 and section 1.4.

- *Development of the simulation model*: A simulation model is developed that uses the control parameters (Zp, n_f) as inputs for conducting several simulations runs. This simulation model accurately reproduces the behavior of the manufacturing system and provides a measure of its performance, denoted by the total incurred cost. A detailed description of the simulation model is presented in section 1.5.1.

- *Statistical analysis:* Data from several simulation runs is collected to perform a design of experiments with the purpose of determining the effects of the main factors and their interactions on the total incurred cost. The design of experiments analyzes the factors with a minimal set of simulation runs.

- *Parameter Optimization*: The response surface methodology is used to express a relationship between the incurred cost and the significant main factors and interactions identified. The resulted expression is then optimized to determine the best values (Zp^*, n_f^*) of the control parameters.

- *Near-optimal control policy*: Finally $\tilde{u}\ (\alpha, x, n, Zp^*)$ and $\tilde{v}(\alpha, x, n, n_f^*)$ define the control policy to be applied to the manufacturing system. The application of the proposed control approach defines the production and overhaul rates described by Equations (1.13) and (1.14) for the best values (Zp^*, n_f^*) of the control factors.

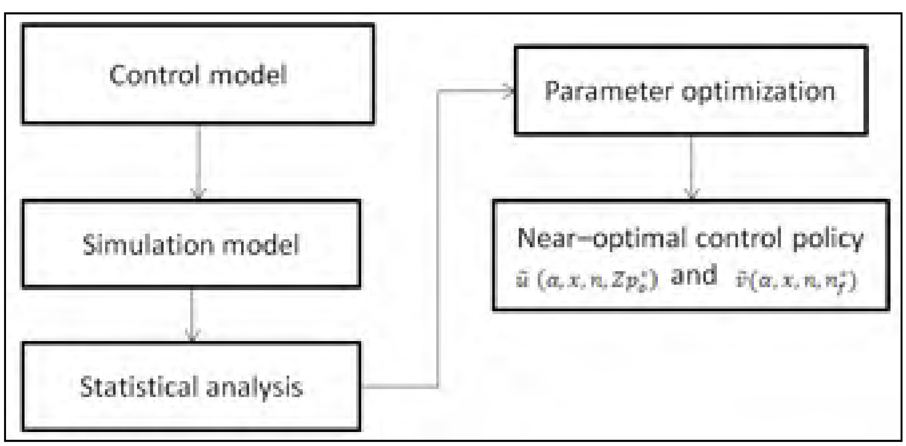


Figure 1.5 Proposed simulation-based control approach

1.5.1 Simulation model

A discrete/continuous simulation model was developed for the manufacturing system under analysis, and the simulation software Arena with C subroutines was applied. The model consisted of several networks and user routines, each of which describes a specific task or event in the system. The block diagram of the simulation model is presented in Figure 1.6, and its description is detailed below:

1. The INITIALIZATION block sets the values of several parameters, such as the input factors (Zp, n_f), the demand rate d, the maximum production rate u_{max}, and the value of the transitions for the different modes. It also defines the step specifications for the time-persistent statistics of the cumulative variables, as well as the simulation time T_{end} and the length of the warm-up period.

2. The FAILURES AND REPAIRS block samples the time to failure and time to repair of the machine from their respective probability distributions. It communicates with the state equation block ⑤ to indicate the operational and the breakdown state; a direct communication also exists with the rate of defectives block ④ to adjust the current rate β with every repair, and subsequently update the production threshold level.

3. The *OVERHAUL POLICY* block, together with observations networks, establishes when to perform the overhaul, as defined in Equation (1.14). It communicates with the failures-repairs block ② and the state equation block ⑤ to properly synchronize different events, such as failures, repairs and overhaul. Moreover, it interacts with the rate of defectives block ④ to indicate when an overhaul has been conducted.

4. The RATE OF DEFECTIVES block receives information from the failures-repairs block ② and the overhaul policy block ③ to correctly update the rate of defectives. Generally, at failure, the rate β, increases as defined in expression (1.3). Conversely, these parameters are restored to initial conditions when an overhaul is performed.

5. The *STATE EQUATION* block defines, in a C language insert, the system dynamics of the production system, which in this case, is the evolution of the inventory level denoted by the differential equation (1.1). For proper operation this block requires the production rate set by the production policy block ⑥, the rate of defectives given in block ④ and the state of the machine defined in the failures-repairs block ③ and the overhaul policy block ②.

6. The *PRODUCTION POLICY* block uses Equation (1.13) to set the proper production rate comparing the current stock level with a pre-determined threshold. When the current stock level crosses the production threshold, a flag is noticed by detection mechanisms. The production rate is then adjusted and used in the differential state equation of block ⑤.

7. The *ADVANCE TIME* block updates the current time according to a time step, it is a combination of discrete event scheduling (consisting of failures, repairs and overhauls), continuous variables threshold crossing events (i.e., to detect when the stock level exceeds the production threshold) and time specifications for the maximum and minimum step.

8. The *UPDATE STOCK LEVEL* block traces any variations of the inventory level for the chosen time step. As well, it integrates the cumulative variables using the Runge-Kutta-Fehlberg algorithm.

9. At the end of the simulation time $T_{end,}$ the *OUTPUT* block provides time persistent statistics of the positive and negative stock, the simulation length, the number of repairs and overhauls conducted, and the number of failure $n_{f,}$ that is where the production threshold intersects the overhaul zone *A.* Subsequently, the total incurred cost is calculated based on the information provided by this block.

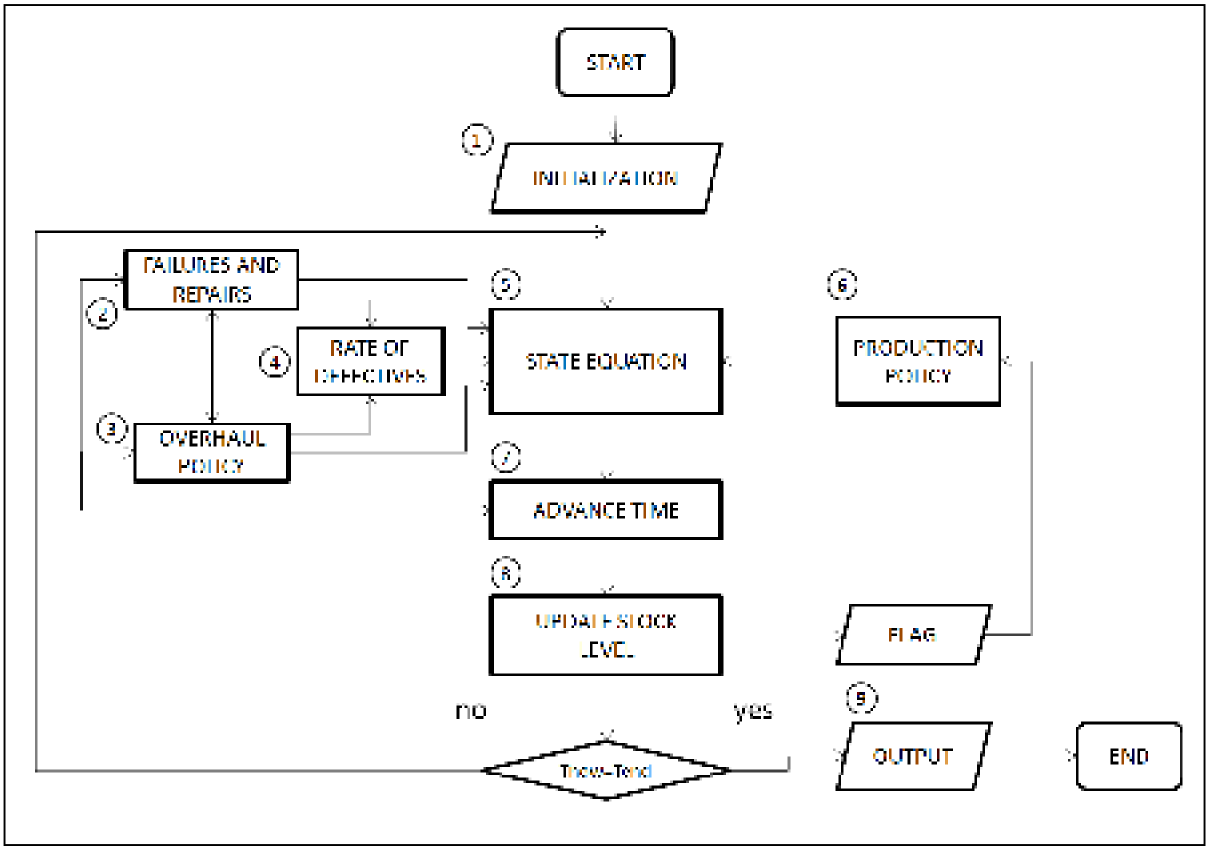


Figure 1.6 Simulation model block diagram

The simulation ends when the current simulation time T_{now} reaches the defined simulation period T_{end}, which is defined as the required time to ensure steady-state conditions. The simulation model reproduces the characteristic features of the proposed manufacturing system, and a more detailed discussion about the validity of the simulation results is provided in the next section.

1.5.2 Validation of the simulation model

To better to verify the accuracy of the simulation model, we examine the evolution of the trajectory of the stock level. This analysis is intended to graphically evaluate whether the simulation model works according to the control policy obtained and the series of proposed assumptions discussed previously. In that regard, Figure 1.7 illustrates the trajectory of the inventory level $x(t)$, when the control parameters are set to $Zp_o=20$, and $n_f=11$. The evolution of the inventory level is as follows: at time $t=0$, the machine maintains a stock level denoted by the production threshold $Zp_o=20$, and then it experiences a series of random failures, and after every failure, the production threshold increases by a certain amount. At time $t=114$, the machine experiences its 11th failure, and at this point the increase in the stock level is notable, as compared to the initial conditions; this point also indicates the moment to carry out an overhaul. At time $t=122.5$, after an overhaul is conducted, the rate of defectives and the production threshold are restored to initial conditions. From this point, the machine deteriorates once again, increasing the production threshold with every failure, (because the worse repairs deteriorates quality) until at time $t=233.5$, where it accumulates eleven other failures. At time $t=242$, after a second overhaul is conducted, the machine is restored to initial conditions. From this point the production system continues its normal deterioration until $t=300$, where the simulation run is completed.

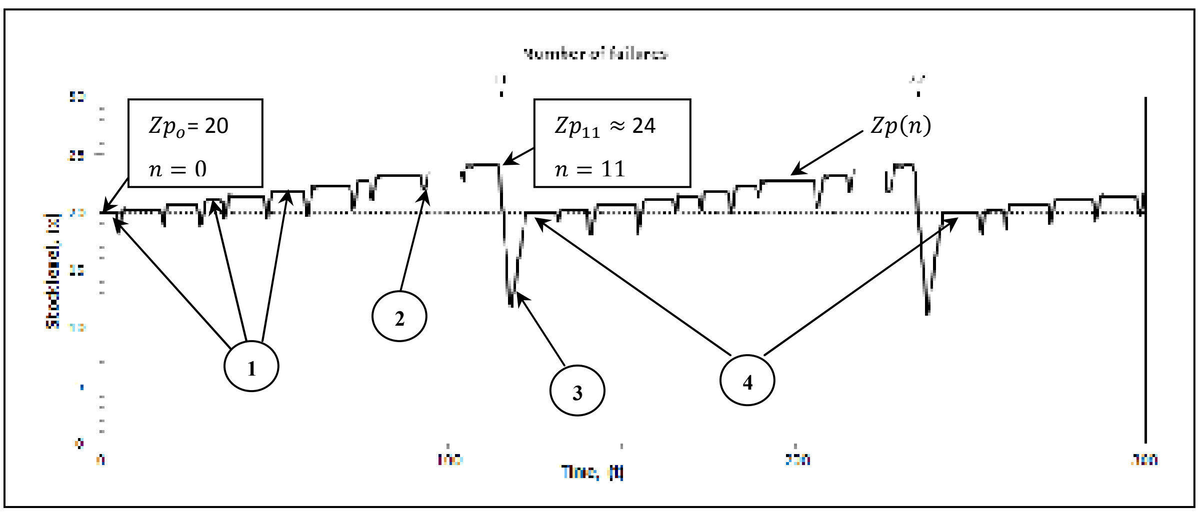


Figure 1.7 Stock trajectory, for $Zp_o=20$ and $n_f=11$

Summing up, from the stock trajectory of Figure 1.7, we note that: i) The inventory level reaches Zp_o and this threshold, as well as the rate of defectives, increases with every failure because of the quality deterioration phenomenon, and arrows ① illustrates the increases in the production threshold. ii) The stock level decreases during repair as indicated by arrow

②, and decreases even more when an overhaul is carried out, as indicated by arrow ③, because an overhaul requires much more time. iii) As expected, every eleven failures, an overhaul is conducted, and this activity restores the rate of defectives and the production threshold to initial conditions, as presented in arrows ④. Based on these observations, we can state that our simulation model works well, according to the assumptions of the proposed manufacturing system, and that it properly reproduces the dynamics of the stock level.

1.6 Numerical example

The determination of the control parameters for the joint production and overhaul policy is illustrated with a numerical example. We combine the simulation model with statistical analyse, based on designs of experiments and parameter optimization, and applying the response surface methodology. In this section, we detailed the procedure for simultaneously and efficiently varying the input variables (Zp, n_f) of the simulation model. In this section we aim to identify the significant main factors and interactions that have a significant effect on the total incurred cost. Then eventually, we determine the best values (Zp^*, n_f^*) of the control parameters and calculate their respective incurred costs.

One point should be noted concerning the control parameters. It is clear from the last section that with each failure, a worse repair is conducted, which leads to different system dynamics, and that implies an increase of the production threshold and the rate of defectives. To determine the control factors, we must first define a limit for the number of failures. Let us, for instances, set $N=20$, as discussed previously in section 1.3. Normally this leads to the definition of 22 parameters, 21 related with the production thresholds (Zp_0 to Zp_{20}) and one related with the number of failures n_f. Given the convexity property of the value function (1.10), and given that an optimal solution of the control problem exists, we can define three levels for each control parameter to obtain a convex estimated cost function. In particular, we consider a second-order model to fit the cost function, and there are various possibilities for addressing this problem. For instance, a complete 3^n factorial design may be conducted, which in the case concerning 22 parameters, leads to a 3^{22} design. If we replicate this design four times, then altogether we will need $(3^{22}x4) = 1.23x10^{11}$ runs, which is a considerable number of calculations. The number of runs could possibly be reduced with a fractional design such as the central composite design (CCD), as reported in Montgomery (2009) and Lavoie et al. (2009b), which in the case of 22 parameters gives a total of $(2^{22} + 2(22) + 3)4 = 1.67x10^7$ runs. The problem is that even in this last scenario, the computational effort needed to perform the calculations would be too time-demanding. Faced with such a situation, it should first be ascertained that each of the designs presented is insurmountable due to time and resource constraints, and mainly because a single run of our simulation model takes 4.6 seconds on average, and an exhaustive simulation of all the combinations would simply be impossible. In average $1.84x10^4$ years would be needed to complete the runs with the 3^{22} factorial design, while the CCD design would require 895 days.

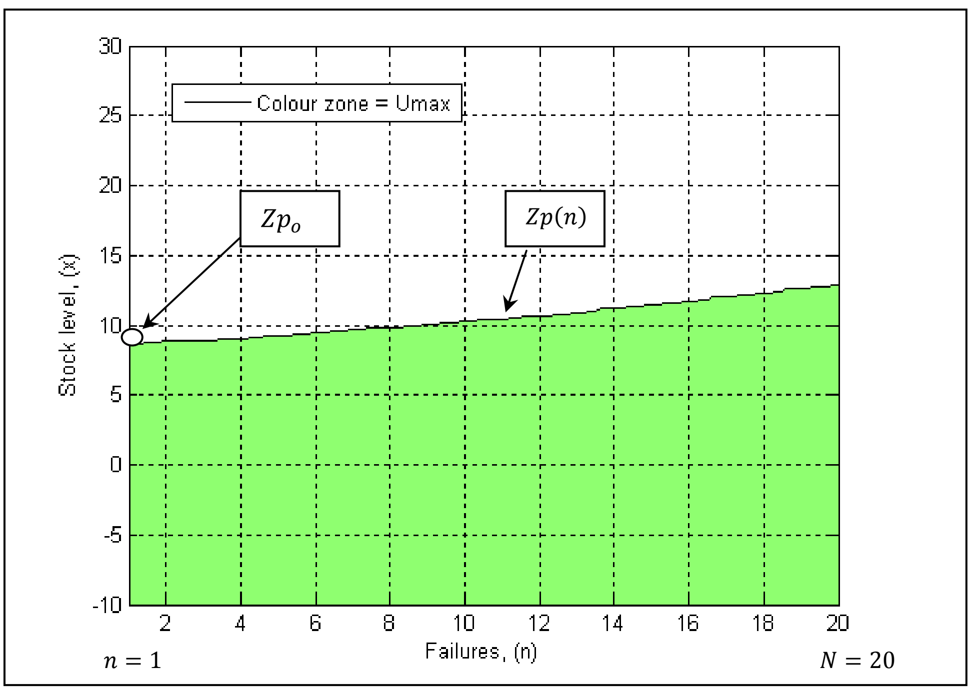


Figure 1.8 Trajectory of the production threshold Z_p as a function of the number of failures n

To tackle this difficulty, we propose a drastic reduction in the number of simulation runs required, while simultaneously maintaining high accuracy in the results. The reduction is based on the observation that the production threshold follows a defined trajectory along the deterioration process. Hence, it is possible to define an analytical expression for the production threshold, when the manufacturing system produces a random fraction of defective products.

Figure 1.8 depicts a trace of the production policy as a function of the number of failures. We conjecture that the trajectory of the production threshold $Zp(\cdot)$ for any number of failures can be determined by only one parameter Zp_o which denotes the value of the production threshold before the first failure. From a modification of the results of Mhada et al. (2011), we can define the whole trajectory of the production threshold $Zp(\cdot)$ according to the following expression:

$$Zp(n) = \begin{cases} \dfrac{Zp_o}{(1-[\beta(n)-\beta_o])} & if\ 0 \leq n \leq N \\ 0 & otherwise \end{cases} \quad (1.15)$$

where Zp_o is the optimal production threshold before the first failure, $\beta(n)$ is the rate of defectives denoted by Equation (1.2) for a given number of failures n, and β_o is the value of the rate of defectives at initial conditions. The application of Equation (1.15) considerably facilitates the determination of the control parameters, since adaptations of this expression can also be applied to define the feasible overhaul zone, based on the control policy discussed in section 1.2. Consequently, the original problem concerning 22 parameters reduces to the determination of only two factors (Zp_o, n_f), leading to the possibility of applying a 3^2 factorial design. Replicating this design four times implies a total of $(3^2 x 4) = 36$ simulation runs, which is much easier to compute, as compared with the complete and fractional designs. The practical advantage of Equation (1.15) is remarkable, and in fact, modifications of this equation are implemented in our simulation model to more efficiently determine the increase of the production threshold and the size of the overhaul zone.

1.6.1 Statistical analysis

The statistical analysis of the simulation data consists in conducting a multifactor analysis of variance (ANOVA). In this section, we determine the control factors (Zp, n_f) using two independent variables (Zp_o, n_o), where Zp_o denotes the production threshold before the first failure as observed in Figure 1.8, and n_o is the number of failures that represents the origin of the overhaul zone A, as shown in Figure 1.4. We identify one dependent variable denoted by the total incurred cost. As discussed previously, a complete experimental 3^2 design is selected to fit the cost function, where each combination of factors is replicated 4 times, requiring 36 simulations runs in total. From off-line simulations, the replication length for each simulation run is set to 1,000,000 time units to ensure that steady-state conditions are achieved. The cost values presented in Table 1.2 are considered in the statistical analysis, and the remaining parameters are defined as indicated in Table 1.1.

Table 1.2 Cost parameters for the statistical analysis

Parameter	c^+ (\$/units/hr)	c^- (\$/units/hr)	c_r (\$)	c_o (\$)
Value	4	250	1000	3000

The ANOVA is performed on the 3^2 experimental design, using the statistical software STATGRAPHICS, in a bid to quantify the effects of the main factors, their interactions and their quadratic effects on the total incurred cost. Based on off-line simulation runs, we select the minimum and the maximum values of the factors (Zp_o, n_o), as presented in Table 1.3.

Table 1.3 Level of the independent variables

Factor	Low level	Center	High level	Description
Zp_o	4	8	12	Production threshold at the first failure
n_o	7	10	13	Failure of origin of the overhaul zone A

The ANOVA table corresponding to the generated data states that at a confidence level of 95%, all the *p–values* are less than 5%. The significant factors are identified in Figure 1.9, where the standardized Pareto plot is presented. The analysis also provides the proportion of the observed variability explained by the model that is denoted by the adjusted coefficient of determination R^2. In this case, we found that the model explains 94.21% of the variability observed in the incurred cost.

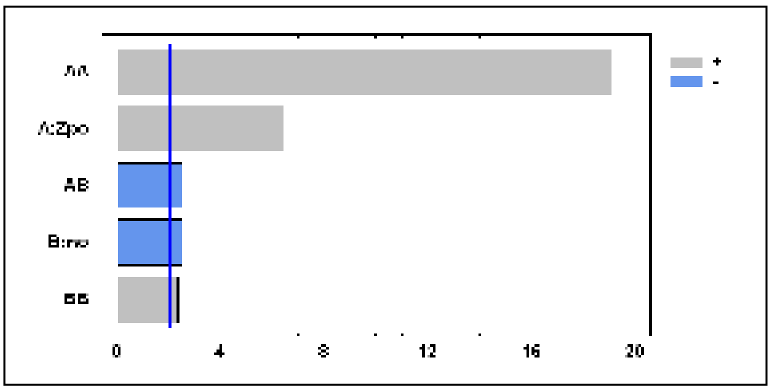


Figure 1.9 Standardized Pareto Plot for the total cost

Based on the ANOVA results, we found that the main factor Zp_o and its quadratic effect Zp_o^2 have the biggest influence in the total cost as depicted in Figure 1.9. However, as the number of failures n_o, the parameter n_o^2 and the interaction $Zp_o n_o$ are also significant; they also must be included in the parameter optimization phase, at a confidence level of 95%. Moreover, the residual versus the predicted valued plot and the normality probability plot were used to test the homogeneity of the variances and the residual normality, respectively. We conclude that the total cost can be determined well by the proposed second-order model.

1.6.2 Parameter optimization

A response surface methodology is applied in order to minimize the incurred cost as a function of the significant variables identified in the last section. To this end, we assume that there exists a continuous function Φ, called the response surface, which defines the total incurred cost corresponding to any given combination of the parameters Zp_o and n_o. In this case, the second-order model obtained is given by:

$$\Phi(Zp_o\,,n_o)=$$

$$194.243\ -7.3398\cdot Zp_o-1.99834\cdot n_o+0.525489\cdot Z_p^2\ -\ 0.0660525\cdot Zp_o n_o+0.115667\ n_o^2+\epsilon \qquad (1.16)$$

The projection of the cost response surface in a two-dimensional plan is presented in Figure 1.10. The minimum total cost is 155.29, and is located at Zp_o^*=7.66 and n_o^*=10.82. With these values, we can determine the missing control parameter n_f^* as provided bellow.

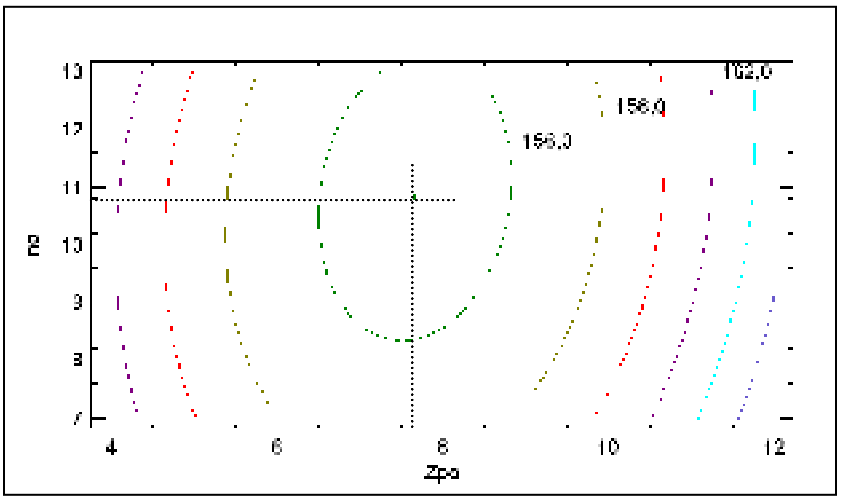


Figure 1.10 Contours of the estimated response surface

In the last section, we stated that from the initial value Zp_o^*, the production threshold follows a defined trajectory denoted by Equation (1.15). Moreover, from point n_o^* the overhaul zone spreads in the plan intersecting the production threshold, as presented in section 1.4. We postulate that a part of the overhaul trace follows an inverse trajectory of the production threshold, as observed in Figure 1.4, and therefore, we use a variation of Expression (1.15) (with an inverse trend) and Expression (1.15), (both expressions incorporated in the simulation model) to define their intersection, and determine the parameter n_f^*. Following this procedure, we identify the intersection of the production threshold and the overhaul zone at failure n_f^*=13.84. Therefore, the values Zp_o^*=7.66 and n_f^*=14 represent the best obtained parameters that should be applied in the simultaneous production and overhaul control policy.

If we round the number of failures to the closest integer n_o^*=11 and minimize the cost Function (1.16), the optimal production threshold is Zp_o^*=7.68, and these parameters lead to a total cost of 155.30. To cross-check the validity of the results obtained, we use these values as input to the simulation model to obtain a 95% confidence interval for the total cost, which in this case, is defined by:

$$\bar{X}(y) \pm t_{n-1,1-\left(\frac{\alpha}{2}\right)}\sqrt{\frac{S^2(y)}{y}} = [154.89;\ 155.70]$$

The interval was calculated with y=35 extra replications of the simulation model. This result confirms that our simulation optimization approach determines the optimal values of the control parameters with high precision, since the minimum cost given by the cost function (1.16) falls inside the confidence interval. With the optimal values of the independent variables, the cost is minimized and the corresponding control policy can subsequently be defined.

1.7 Sensitivity analysis

The sensitivity of the control policy obtained for a set of numerical examples is analysed with respect to different cost variations of the inventory, backlog, repair and overhaul costs (i.e., c^+ , c^-, c_r, c_o). The objective of the analysis is to compare the incurred cost and control parameters for different cost scenarios derived from a basic case. Also, the effect of the variation in the trajectory of the rate of defectives on the control parameters $(Zp_o\ , n_f)$ is analyzed.

1.7.1 Effect of the cost variation

The goal of this analysis is to demonstrate the efficiency and robustness of our simulation optimization approach, and study the behaviour of the control policy obtained, when varying cost parameters such as: c^{+}, c^{-}, c_r, c_o. Thus, Table 1.4 illustrates eight different configurations of cost parameters, derived from a basic case by changing them to higher and lower values. These different configurations are related with variations of the inventory, backlog, repair and overhaul cost.

Table 1.4 Combination of cost parameters of sensitivity analysis

Case	c^{+}	c^{-}	c_r	c_o
Basic case	3	250	1000	3000
Sensitivity of positive inventory cost				
1	**2**	250	1000	3000
2	**4**	250	1000	3000
Sensitivity of backlog stock cost				
3	3	**150**	1000	3000
4	3	**350**	1000	3000
Sensitivity of repair cost				
5	3	250	**500**	3000
6	3	250	**1500**	3000
Sensitivity of overhaul cost				
7	3	250	1000	**2500**
8	3	250	1000	**3500**

The results of the sensitivity analysis, presented in Table 1.5, highlight the consistency between the variation of each cost parameter, the optimal control factors (Zp_o^* , n_f^*) and their respective incurred cost. The first observation from Table 1.5 is that the optimum control factors change in response to variations of the cost parameters. The variation of each cost, the respective control factors and optimal cost are examined and analyzed as follows:

Table 1.5 Sensitivity analysis of different cost parameters

Case	Zp^*	n_f^*	$Cost^*$	$Remark$
Basic case	8.79	13.15	146.35	Base for the comparison
Sensitivity of positive inventory cost				
1	9.95	12.50	136.03	Zp^* increases and n_f^* decreases
2	7.66	13.84	155.29	Zp^* decreases and n_f^* increases
Sensitivity of backlog stock cost				
3	6.60	15.97	140.28	Zp^* decreases and n_f^* increases
4	9.73	12.03	150.16	Zp^* increases and n_f^* decreases
Sensitivity of repair cost				
5	8.83	13.71	102.21	Zp^* almost unchanged and n_f^* increases
6	8.74	12.50	190.33	Zp^* almost unchanged and n_f^* decreases
Sensitivity of overhaul cost				
7	8.74	12.58	143.26	Zp^* almost unchanged and n_f^* decreases
8	8.83	13.65	149.31	Zp^* almost unchanged and n_f^* increases

Variation of the inventory cost, c^+ (cases 1 and 2): From the results obtained and presented in Table 1.5, we observe that the effect of the inventory cost is remarkable on the production threshold. For instance, with an increasing c^+ (case 2), the production threshold Zp^* decreases, because with higher inventory cost, the stock of product is more greatly penalized, leading to smaller production thresholds. The effect of this cost on the overhaul policy is such that when the inventory cost increases, the number of failures n_f^* needed to perform the overhaul also increases, because the production threshold is reduced and the intersection with the overhaul zone moves to a higher number of failures. The final result of this condition is a reduction in the feasible overhaul zone A', (please see Figure 1.4 for an explanation of this reduction). The influence of the inventory cost also is remarkable in the incurred cost, as we observe that the more the inventory cost increases, the more the incurred cost rises as well. An opposite effect is observed on the production and overhaul policies at decreasing c^+ (case 1).

Variation of the backlog cost, c^- *(cases 3 and 4):* The backlog cost influences the production policy, and we observe that at increasing c^- (case 4), the production threshold increases, since, there is more wiggle room to maintain a certain amount of stock, and the increase in the inventory helps protect the system against shortages caused by breakdowns and defectives. With respect to the overhaul policy, the backlog cost also has an effect on its control parameter, since as the backlog cost increases, the production threshold increases, intersecting the overhaul zone at an early number of failures, thus leading to decrease n_f^*. This in turn yields to an increase in the feasible overhaul zone A'. Decreasing c^- (case 3) produces the opposite result in both policies. Furthermore, as can be seen in the results obtained, the effect of the backlog cost on the control parameters is the opposite of the inventory cost effect.

Variation of the repair cost, c_r *(cases 5 and 6):* With respect to the repair cost, the influence of its variation shows that it does not considerably modify the optimal production threshold Zp^*, since this control parameter remained almost at the same level for the analyzed cases. The effect of the repair cost is observed mainly in the overhaul policy, where the underlying pattern shows, that as expected, when the c_r increases (case 6), the number of failures n_f^* decreases, thus implying an increase in the feasible overhaul zone A' with respect to the case with a low repair cost. Moreover, at decreasing c_r (case 5), the overhaul zone A' decreases as well.

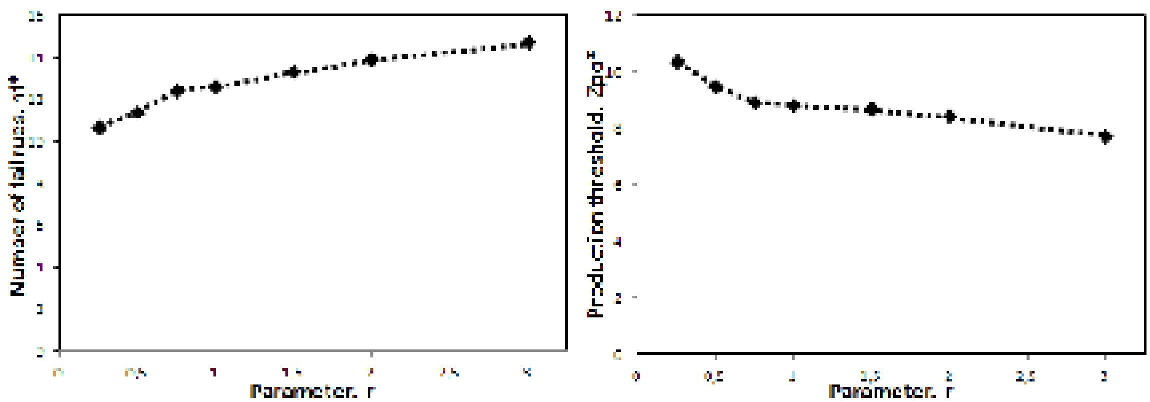
Variation of the overhaul cost, c_o *(cases 7 and 8):* The main effect of this cost parameter is observed on the overhaul policy, since the production threshold remains almost at the same value for the analyzed cases. At decreasing c_o (case 7), more overhauls are conducted, leading to a decrease in the number of repairs n_f^*. Moreover, at increasing c_o (case 8), the overhaul zone A' is reduced, consequently leading to fewer overhauls and an increase in the number of repairs n_f^*. We notice that the effect of the overhaul cost on the overhaul policy is the opposite of the effect of the repair cost.

From the set of numerical examples considered in this sensitivity analysis, it is clear that the results obtained are logical and confirm the structure of our control policy. Generally, any cost variation reflects changes in the control parameters, and we observe that the inventory cost has the opposite effect as the backlog cost, while the repair cost has the opposite effect as the overhaul cost.

1.7.2 Effect of the trajectory of the rate of defectives

In the last section, we discussed the effect of variations of the cost parameters on the control factors and on the total incurred cost. The objective of this section is to allow a better understanding of the control parameters (Zp_o, n_f) when varying the trajectory of the rate of defectives. Hence, in this section, another set of simulation runs were conducted in order to analyze the impact of variations of the trajectory of β on the control factors. This analysis involves the variation of the parameter r in Expression (1.2). The obtained production thresholds Zp_o^* and the number of failures n_f^*, are illustrated in Figures 1.11a and 1.11b, respectively, when varying the parameter r from 0.25 to 3, and the other parameters remain unchanged, as defined in the numerical values of the basic case.

The key observation in this analysis is that the role of the parameter r in Expression (1.2) is to adequately adjust the trajectory of the rate of defectives to a specific manufacturing system, as discussed previously in section 1, and presented graphically in Figure 1.2. An important remark should be made at this point, namely, that changes in the optimal control parameters (Zp_o^*, n_f^*) appear when the path of the rate of defectives does not follow a linear trend. For instance, if the parameter is $r < 1$, then the rate of defectives increases more abruptly, compared to the basic case $r = 1$, and this condition results in a decrease in the number of failures n_f^*, consequently leading to more overhauls, as presented in Figure 1.11a. Additionally, when $r < 1$, the production threshold increases, as depicted in Figure 1.11b, because the rate of defectives increases more rapidly, and so the production system needs more protection to cope with the defective products.



a) Number of failures, n_f^* b) Production threshold, Zp_o^*

Figure 1.11 Effect of the parameter r on the control parameters

There is another interesting observation when the parameter is $r > 1$; it means that the rate of defectives increases more smoothly and slowly compared to the basic case, as can be seen in Figure 1.2. From the obtained results presented in Figure 1.11a, it follows that when $r > 1$, the number of failures n_f^* increases because the overhaul is conducted less frequently, reducing the feasible overhaul zone A'. Furthermore, when the parameter is $r > 1$, this leads to a reduction in the production threshold, since the rate of defectives smooths its increase and thus reducing the need to maintain a certain amount of products as protection for shortages.

Through this sensitivity analysis, it is apparent that the variation of the trajectory of the rate of defectives has a clear effect on the control parameters. In fact, we observed that when the parameter r is less than one, the rate of defectives increases more abruptly, increasing the production threshold and reducing the number of failures n_f^*, thus leading to more overhauls. The opposite effect was observed, when the parameter r is greater than one.

1.8 Conclusions

In this paper, issues pertaining to the inter-relation between production control and quality aspects are investigated in a manufacturing system consisting of an unreliable single machine, single part type manufacturing system. The machine is subject to deteriorations that have a negative effect on the quality of the parts produced, a fact that is especially reflected in the rate of defectives. The quality issues are related with the overhaul strategy, since it counters the effect of the deterioration. A simulation optimization approach is proposed and combines analytical formulation, simulation modeling, statistical analysis and response surface methodology. First, we investigate the structure of the joint production and overhaul control policy through the application of numerical methods designed for stochastic optimal control models. The resulting policy consists of a Machine Deterioration Dependent Hedging Point Policy which controls the production rate, and comprises several increasing thresholds and an overhaul strategy that performs a perfect repair when the number of failures is high. This leads to the identification of the two associated control parameters of the joint control policy as dependent on the deterioration of the production system. A simulation model is developed to reproduce the dynamic of the manufacturing system controlled by a modification of the hedging point policy obtained. An experimental design is applied to investigate the effects of the control factors on the incurred cost over the production horizon, and a cost function is estimated with a response surface from which we determine the best values of the control parameters and their respective incurred costs. The simulation optimization approach is applied on a wide range of time, and a combination of cost parameters is examined in a sensitivity analysis to evaluate their effects on the control policy. It is observed that in general, cost variations reflect changes in the control parameters and that the inventory cost has the opposite effect of the backlog cost, while the repair cost has the opposite effect of the overhaul cost. Also the effect of the variation of the trajectory of the rate of defectives is analyzed, and we observe that when the parameter r is less than one, the rate of defectives increases more rapidly leading to an increase in the production threshold and simultaneously to more overhauls, thus reducing the number of failures n_f^*. The opposite effect on the control parameters is observed when the parameter r is greater than one. The results obtained indicate that the control policy is influenced by the quality deterioration phenomenon, since in this case the hedging point policy is modified to a situation with several thresholds, which increase from one breakdown to the next. Hence, the production threshold increases with the number of failures, and the overhaul is conducted only with high numbers of failures. The use of equation (1.15) and its variations significantly reduces the number of runs needed in the simulation optimization. Finally, our control approach turned out as an interesting alternative for controlling the manufacturing system at the operational level, and our final assessment is that the simulation optimization approach is robust and efficient since it provides accurate results.

Appendix A Optimality conditions

The model denoted in Equations (1.1-1.10) is a stochastic dynamic programming problem because the minimization operation takes into account the randomness of α. Hence, we define the related transition rates λ_{ij} from modes i to j, as follows:

$$\text{prob}[\alpha(t+\delta t)=i|\alpha(t)=j]=\lambda_{ij}\delta t\,,\quad \forall\ \ i,j,i\neq j \tag{A.1}$$

The value function $V(\alpha,x,n)$, defined in Equations (1.10), denotes the value of the cost Function (1.9) when the optimal control policy is applied. Regarding the principle of optimality, we can derive the optimality conditions of the problem; for example, let us assume we know the best possible trajectory during the time interval $[t,\infty]$. However, we know nothing about the problem during the interval $[0,t]$. If $V(\cdot,t)$ denotes a cost-to-go function at time t, then we can break up Equation (1.10) as follows:

$$V(\alpha(0),x(0),n(0),0)=\inf_{\substack{u(t),v(t)\\0\le t\le\infty}} E\left\{\begin{array}{c}\displaystyle\int_0^t e^{-\rho t}g[(\alpha(t),x(t),n(t),u(t),v(t)]dt\\ +\\ \displaystyle\int_t^\infty e^{-\rho t}g[(\alpha(t),x(t),n(t),u(t),v(t)]dt\end{array}\middle|\,\alpha(0),x(0),n(0)\right\} \tag{A.2}$$

However, we know that the integral in the interval $[t,\infty]$ is the value function $V[\alpha(t),x(t),n(t),t]$, so considering the discounted rate ρ, equation (A.2) becomes:

$$V(\alpha(0),x(0),n(0),0)=\inf_{\substack{u(t),v(t)\\0\le t\le\infty}} E\left\{\begin{array}{c}\displaystyle\int_0^t e^{-\rho t}g[(\alpha(t),x(t),n(t),u(t),v(t)]dt\\ +\\ \displaystyle\frac{1}{1+\rho\delta t}V[\alpha(t),x(t),n(t),t]\end{array}\middle|\,\alpha(0),x(0),n(0)\right\} \tag{A.3}$$

Based on this underlying pattern, if we perturb t, we can focus on $V[\alpha(t),x(t),n(t),t]$ to get the one-step counterpart in the interval $[t,t+\delta t]$, as indicated in the following expression:

$$V(\alpha(t), x(t), n(t), t) = \inf_{\substack{u(s),v(s) \\ t\le s\le t+\delta t}} E\left\{ \begin{array}{c} \displaystyle\int_t^{t+\delta t} e^{-\rho t} g[(\alpha(s), x(s), n(s), u(s), v(s)] ds \\ \dfrac{1}{1+\rho\delta t} V[\alpha(t+\delta t), x(t+\delta t), n(t+\delta t), t+\delta t] \end{array} \middle| \alpha(t), x(t), n(t) \right\} \qquad (A.4)$$

where $u(s)$ is treated as constant in the interval $t \le s \le t + \delta t$. Equations (A.4) can be simplified even more if we apply the conditional expectation operation $\widetilde{E}$ (i.e., for any function $H(\alpha)$, $\widetilde{E}\, H(\alpha(t+\delta t)) = E\{H(\alpha(t+\delta t)) | \alpha(t)\}$). Thus, the expectation $\widetilde{E}$ leads to:

$$V(\alpha(t), x(t), n(t), t) = \inf_{\substack{u(s),v(s) \\ t\le s\le t+\delta t}} \tilde{E} \left\{ \begin{array}{c} g[(\alpha(t), x(t), n(t), u(t), v(t)]\delta t \\ + \\ \frac{1}{1+\rho\delta t} V[\alpha(t+\delta t), x(t+\delta t), n(t+\delta t), t+\delta t] \end{array} \right\} + o(\delta t) \qquad (A.5)$$

Then for small δt and after some manipulations, we have:

$$\rho V(\alpha(t), x(t), n(t), t) = \inf_{u(t),v(t)} \tilde{E} \left\{ \begin{array}{c} g[(\alpha(t), x(t), n(t), u(t), v(t)]\delta t \\ + \\ \frac{V[\alpha(t+\delta t), x(t+\delta t), n(t+\delta t), t+\delta t] - V[\alpha(t), x(t), n(t), t]}{\delta t} \end{array} \right\} + o(\delta t) \qquad (A.6)$$

The second term inside the expectation operator $\tilde{E}$ refers to the derivate of $V(\alpha, x, n)$. If the value function is differentiable, then we can apply the full derivative. Also, we can expand the expectation operation $\tilde{E}$, (i.e., using the expansion $\widetilde{E}\, H(\alpha(t+\delta t)) = H(\alpha(t)) + \sum_j H(j)\, \lambda_{j\alpha(t)} \delta t + o(\delta t)$, and so we have:

$$\rho V(\alpha(t), x(t), n(t), t) = \inf_{u(t),v(t)} \left\{ \begin{array}{c} g[(\alpha(t), x(t), n(t), u(t), v(t)]\delta t + \\ \frac{\partial V}{\partial x}[\alpha(t), x(t), n(t), t]\delta x(t) + \frac{\partial V}{\partial t}[\alpha(t), x(t), n(t), t]\delta t \\ + \sum_{\alpha'} V[\alpha', x(t), n(t), t] \lambda_{\alpha'\alpha(t)} \delta t \end{array} \right\} + o(\delta t) \qquad (A.7)$$

We have eliminated the expectation symbol with the summation term. Now, if we replace $\delta x(t)$ by $\delta x(t) = \dot{x}(t)\delta t$, move $\frac{\partial V}{\partial t}$ to the left, let δt approach zero, and do other manipulations, we get:

$$\rho V(\alpha, x, n, t) - \frac{\partial V}{\partial t}[\alpha, x, n, t] = \inf_{u(t), v(t)} \left\{ g[\alpha, x, n, u, v] + \frac{\partial V}{\partial x}[\alpha, x, n, t]\dot{x} + \sum_{\alpha'} V[\alpha', x, n, t]\, \lambda_{\alpha'\alpha} \right\} \tag{A.8}$$

We observe that none of the functions $g(\cdot)$ and $\dot{x}(\cdot)$ are functions of t explicitly. Furthermore, since the time horizon is infinite and a steady-state distribution exists for α, Equation (A.8) is independent of t. Based on this, and replacing the summation term by the generator $\mathrm{Q}(\cdot) = \{\lambda_{\alpha\alpha'}(\cdot)\}$, equations (A.8) can be further simplified to:

$$\rho V(\alpha, x, n) = \inf_{(u,v) \in \Gamma(\alpha)} \left\{ g[\alpha, x, n, u, v] + \frac{\partial V}{\partial x}[\alpha, x, n]\dot{x} + Q(\cdot)V[\alpha, x, \varphi(\xi, n)](\alpha) \right\} \tag{A.9}$$

These are the fundamental manufacturing system control equations called Hamilton-Jacobi-Bellman (HJB) equations, and they are important because they convert the minimization problem, defined over an extended time interval, into a minimization problem at a single time instant. HJB equations allow us to determine the optimal control policy, which in this case, is a real feedback law. Since the problem is stochastic, that means it is specified only when (α, x, n) are determined. Further details about how HJB equations are obtained can be seen in Rishel (1975) and Gershwin (2002).

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CHAPTER 2

ARTICLE 2: PRODUCTION AND QUALITY CONTROL POLICIES FOR DETERIORATING MANUFACTURING SYSTEM

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Abstract

In this paper, a novel model for a deteriorating manufacturing system is analyzed, considering repairs and overhauls of random durations. The machine manufactures one product and the model is further complicated because the quality of the parts' produced deteriorates according to the wear of the machine and human interventions. When a breakdown occurs, either a repair or an overhaul is performed. The machine is restored to as-good-as-new conditions if the overhaul is selected, and conversely, its condition deteriorates following repairs. Multiple operational states are considered to define an aging process. The decision variables of the model are the production rate and the repair/overhaul switching strategy. This paper provides new insights to this research area by considering the simultaneous production and repair/overhaul control policy under the effect of deteriorations. The optimal decision policy minimizes the total incurred cost comprising the inventory,

backlog, repair and overhaul costs over an infinite planning horizon. Our paper differs from other research projects in its consideration of the machine's history, because we use the number of repairs and a set of multiple operational states to control the machine efficiently, and this breaks the markovian property of memoryless. A numerical example is given to illustrate the proposed approach and a sensitivity analysis has been conducted to confirm the structure of the obtained control policies.

Keywords: Quality, Manufacturing system, Optimal control, Numerical methods, Deteriorating systems.

2.1 Introduction

In the area of manufacturing systems, quality is one of the most important factors that define the market survival of a company. Moreover, the quality of a manufacturing system may be affected by the deterioration caused by a series of events, such as breakdowns, repairs, wear, fatigue, corrosion, human errors, etc. Despite the importance of quality, only a few research works have included quality issues in the determination of control policies. These considerations strongly motivate the need of an integrated model that allows us to examine the effect of the inter-relation between production planning and quality issues. Nevertheless other situations may arise, for instance the manufacturing system can be subject to deteriorations, and so at including this factor it is needed further research to have a better understanding of the production system behavior.

The production planning problem of manufacturing systems started attracting increased attention with the work of Akella and Kumar (1986). They proposed an analytical solution to the problem that consisted in controlling the production rate of a failure-prone manufacturing system. This model was extended by Bielecki and Kumar (1988), when they provided another solution for a similar production system. These works helped to consolidate the concept of the so-called Hedging Point Policy (HPP). Later, Sharifnia (1988) established that the HPP is susceptible to generalizations, such as the multi-state property that is used in subsequent works. Over the years, the hedging point policy has grown in complexity; for instance, Gharbi and Kenne (2003) studied the problem of production control for a production system which involves multiple machines producing different part types. Other works, such as Chelbi and Ait-Kadi (2004), have focused on the joint strategy of buffer stock production and preventive maintenance. It has been followed other extensions treating a wide range of aspects such as preventive maintenance as in Rezg et al. (2008), unreliable suppliers as in Hajji et al. (2009) and multi-products production plan as in Dahane et al. (2012). However, it is observed from these extensions that a significant branch of the literature has as main assumption that the manufacturing system produces only conforming parts. Unfortunately, in an industrial context, this assumption is incomplete. The above mentioned papers have therefore not treated the interaction between production and quality issues. This argument led us to develop an integrated model in which the aspects of production and quality are considered, and their interaction is examined.

A limited number of authors have addressed quality issues and the interaction of quality with production planning. The need to include quality in the design of production lines was identified by Inman et al. (2003), who presented several classes of decisions that affect quality and productivity. A comprehensive work on this interaction is the series of papers by Kim and Gershwin (2005, 2008), who mathematically analyzed the performance of manufacturing systems with quality aspects. Also, they extended the mathematical approximation method, called *decomposition,* to evaluate the performance of transfer lines incorporating the effect of quality. The decomposition method has been extensively applied to analyzed long production systems as in Bonvik et al. (2000). In the same direction, the set of works of Colledani and Tolio (2006, 2009, 2011) presented a mathematical method based on discrete Markov chains to evaluate the performance of production systems with manufacturing and inspection machines, where the behavior of the machines were monitored with control charts. Nevertheless, the main difference between these works and our paper is that in the former, Markov chains and the decomposition technique are the main tools used in their analysis, whereas in this paper, we focus on the structure of the control policy derived from a semi-Markov model. Other applications tackle the effect of quality to determine maintenance actions, such as in Radhoui et al. (2009), where they considered production in batches, and used the rejection rate and the buffer size as the decision variables to determine maintenance activities. The maintenance strategy has also been covered by Njike et al. (2009), who developed a model to simultaneously control maintenance activities and production planning, and considered several operational states that monitor the system's health. The idea was to use the quantity of defective products as feedback to optimally control the system. In spite of the above-cited authors, our paper contributes to this growing research area by considering that it is plausible to combine the production planning problem with deterioration in a different direction. While the literature contains many proposals that model deterioration, in our research, however, we conjecture that the manufacturing system is subject to a degrading process, and that this phenomenon is tied directly to the quality of the parts produced. We find some interesting ideas to support this assumption, in the area of deteriorating systems.

Deteriorating systems have been studied by several authors, who have determined optimal policies based on either the age of the machine or its accumulated number of failures. For example, in the series of works presented by Love et al. (1998, 2000), it is considered that at failure, the machine may undergo a repair that partially resets its failure intensity, or be put through a second option, which is to conduct a major repair that restores the machine to an as-good-as-new condition. This model was extended by Dehayem et al. (2011a), who included production planning in the repair/replacement problem. They considered imperfect repair actions, in which the repair time increases with the number of failures and the machine is also age-deteriorated. Lather the authors included preventive maintenance in their model in Dehayem et al. (2011b). The main observation regarding these papers is that, they relate the concept of deterioration with either the time to failure or the repair time, and therefore do not examine the effect of deterioration on the rate of defectives. In contrast, in our research, we want to focus on the interaction between deterioration and the quality of the parts produced. To model the quality-deterioration, in this paper we propose that it is given by the combined effect of two factors; the wear of the machine and human interventions. For this reason we define an internal dynamic for the operational state, which leads to the use of multiple operational states that reflect different quality levels. Furthermore, we complement the deterioration modeling with the consideration of human interventions represented by worse repairs (a maintenance action which makes the rate of defectives increase). We did not find available work in the literature with this approach. The quality deterioration is countered by a major overhaul that restores the production system to initial conditions.

The aforementioned issues are therefore considered in this paper where we present a new model for the simultaneous determination of the production and the repair/overhaul switching policies for a manufacturing system whose produced parts' quality deteriorates over time. The model falls under the class of machine rate of defectives dependent models that aims to incorporate production and quality issues in an integrated model. The optimization problem consists in the joint determination of the production rate and repair/overhaul switching policies in a stochastic environment, where the following considerations are included in the model:

a) The rate of defectives increases with the number of repairs, and so the machine repair's history is needed to model more realistically the deterioration phenomenon and optimally control the manufacturing system.

b) Between failures, there is an aging process defined by multiple operational states that model different quality yields. Where each operational state models a different rate of defectives.

This paper differs from other research projects in that here, due to the degrading process, the production rate and repair/overhaul switching strategy are determined simultaneously with a semi-Markov process, something which has not been yet addressed in the published literature. It is important to note that in this case, Markovian models are not appropriate since the quality of the parts produced, defined by the rate of defectives, depends on the history of the machine. In our model, the machine's history is related to the number of repairs and the set of operational states. The production and repair/overhaul switching policies are determined to minimize the inventory, backlog, repair and overhaul costs over an infinite planning horizon.

The rest of the paper is organized as follows. After an overview of the literature in section 2.1, we present the notations, the problem statement and the mathematical formulation of the problem in section 2.2, also optimality conditions and the numerical approach applied are defined in this section. We present a numerical example to identify the control policy of the problem as well as the respective control factors in section 2.3. An example of an implementation of the obtained results is outlined in section 2.4. Discussions on the behaviors of the production system based on a sensitivity analysis are provided in section 2.5. The paper is concluded in section 2.6.

2.2 Notation and problem statement

This section presents the notation, the problem statement and the formulation of the control problem.

2.2.1 Notation

The following notations are used in this paper:

$x(t)$	Inventory level at time t
$u(t)$	Production rate of the manufacturing system at time t
$n(t)$	Current number of failures at time t
d	Constant demand rate
$\xi(t)$	Mode of the machine at time t
U_{max}	Maximum production rate
$\beta(\cdot)$	Rate of defectives
ρ	Discount rate
π_i	Limiting probability at mode i

$J(\cdot)$	Expected discounted cost function
$\lambda_{\alpha\alpha'}(\cdot)$	Transition rate form mode α to mode α'
$\omega(\cdot)$	Control variable for the repair/overhaul policy
ω_{min}	Minimum value for the control variable ω
ω_{max}	Maximum value of the control variable ω
$g(\cdot)$	Cost rate function
$v(\cdot)$	Value function
τ	Jump time of $\xi(t)$
c^+	Incurred cost per unit of produced parts for positive inventory
c^-	Incurred cost per unit of produced parts for backlog
c_r	Worse repair cost
c_o	Overhaul cost
OP_i^n	Operational state *i* at the n$^{\text{th}}$ failure
S	Number of operational states of the aging process for any number of repairs *n*
N	Maximum number of failures during which the system remains operational
Ω	Modes of the machine

2.2.2 Problem statement

The manufacturing system under study consists of an unreliable single machine producing one part type. The machine is subject to random events, such as failures and maintenance activities; it can produce at maximum capacity or at demand rate to satisfy a constant demand of product. Figure 2.1 presents the block diagram of the production system. Since the machine, as illustrated in (a) in Figure 2.1, is unreliable, there is a buffer stock, as shown in (b), to counter the effect of failures. However, the quality of the products produced is not perfect, as it exists a certain percentage of defectives. The stock is thus a mixture of flawless and defective products. Moreover, we propose that the machine experience a quality deterioration phenomenon, depicted by (c), which leads to an increasing rate of defectives β.

It should be noted though, that in the area of deteriorating systems, the effect of deterioration can be manifested either through increasing repair times or decreasing times to failure. In this paper, we focus on the effect of deterioration on the quality of the parts produced. To model this condition, we propose that deterioration is a combination of: i) an aging process, where quality deteriorates because of the impact of the natural wear of the machine: ii) worse repairs, in which case quality deteriorates due to the influence of human interventions, represented by repairs that leave the machine in a worse condition than before repair. The control policy of the model, represented by (d), implies decision variables related to the production planning and quality control defined by the repair/overhaul strategy. This policy copes with the deterioration phenomenon. When the machine fails, the decision maker has two options:

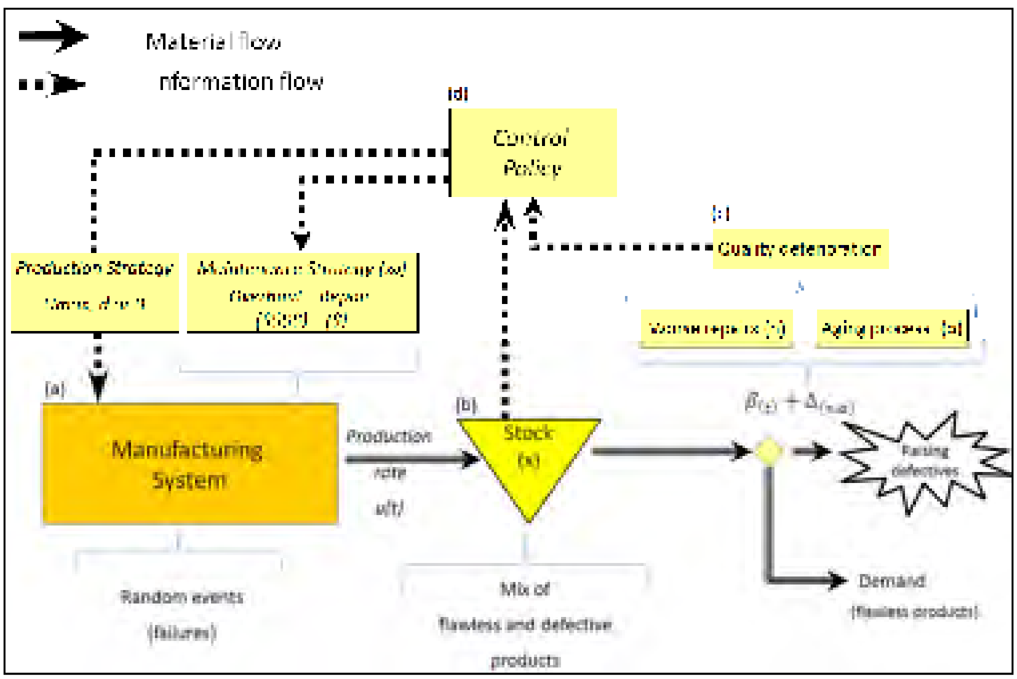


Figure 2.1 Block diagram of the manufacturing system under study

1) Perform an expensive and time-consuming repair called overhaul, which restores the rate of defectives to initial conditions, or
2) Carry out an inexpensive worse repair which lets the machine operate for a while, but with the disadvantage that it deteriorates the machine, increasing the rate of defectives.

We intend to determine the control policy (production rate and the repair/overhaul switching strategy) that minimizes the average total cost, composed of the inventory cost, the backlog cost, the repair and overhaul cost.

2.2.3 Formulation of the control problem

We shall begin the formulation of the control model by presenting the transition diagram of the manufacturing system in Figure 2.2. We emphasize that the machine is subject to deterioration, and that this has a strong effect on the rate of defectives. To model the quality deterioration phenomenon, we used the concept of the aging process, as illustrated in (A) in Figure 2.2. The fact is that the aging process models the natural wear of the machine. To that end, we propose multiple operational states, as depicted in (B) that reflect different quality yields. The operational states OP_i^n are divided into several stages, with the quality of the parts produced deteriorating over time as the machine moves in the operational states. This means that the rate of defectives β increases in every operational state. Furthermore, the machine is subject to random failures. When it is in the failure state F_n it implies a decision point, as presented in (C), where two types of actions can be taken. An expensive repair called overhaul can be performed, as in (D), which counters the effect of the quality deterioration.

The second option is to perform a worse repair, as shown in (E), which is less expensive and faster than the first option, but which deteriorates the machine's conditions; specifically it increases the rate of defectives. In fact, the quality deterioration modeling is complemented by the effect of human interventions represented by the worse repairs. The idea is that a worse repair increases the rate of defectives to a certain extent $(\beta + \Delta)$ for the following reasons: 1) the faulty component is only partially repaired 2) human errors cause further damage, etc., as suggested by Pham and Wang (1996). The combination of the aging process and the worse repairs defines a deterioration cycle: aging-repairs-aging-repairs, etc., as illustrated in Figure 2.2.

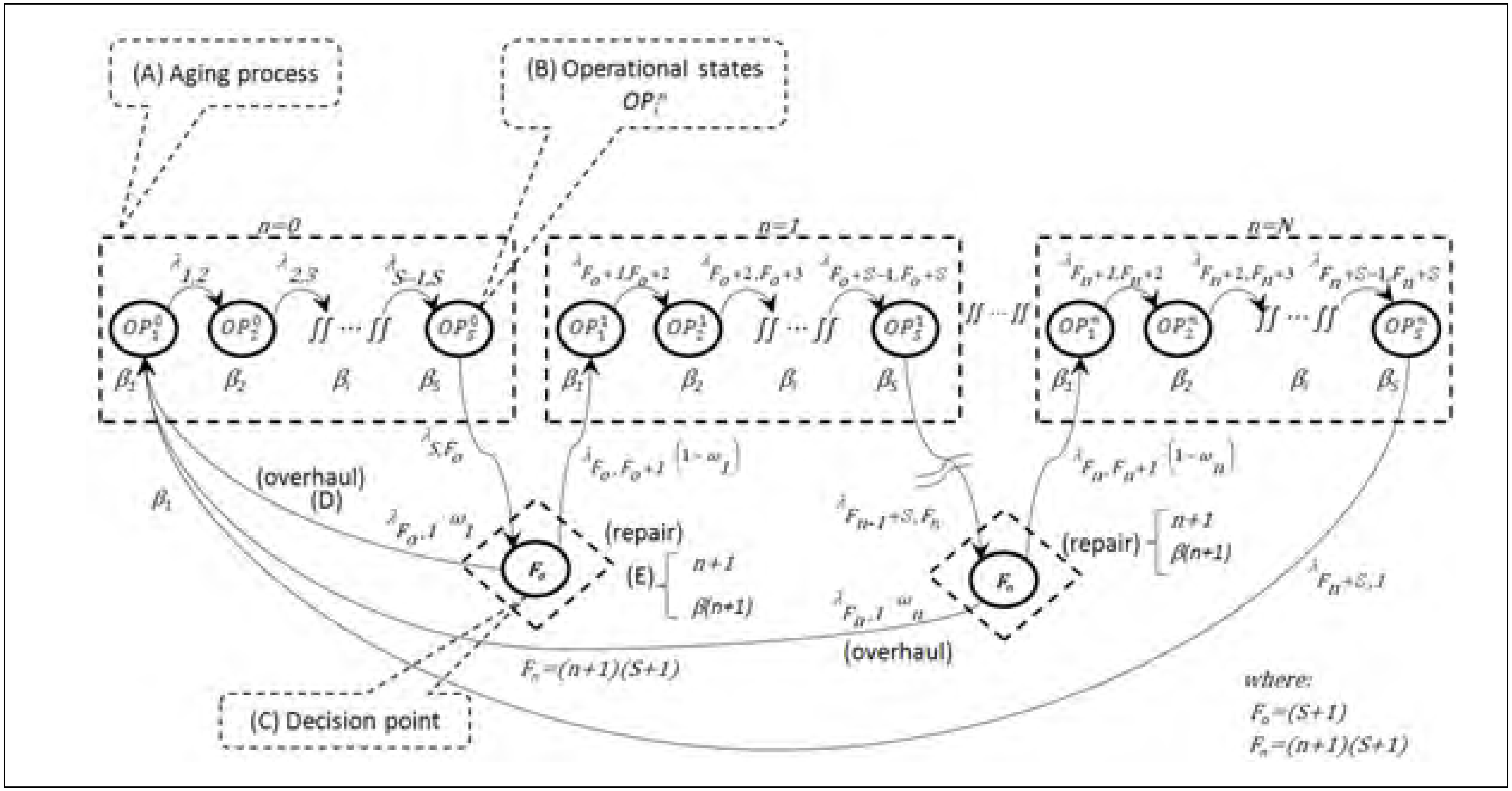


Figure 2.2 Transition diagram of the manufacturing system under study

In the transition diagram, S denotes the number of operational states of the aging process for any number of repairs n, and we use the index $F_n = (n+1)(S+1)$, to identify correctly the transitions between the different states. We can now define the dynamics of the production system as given by a hybrid state consisting of discrete and continuous components.

The discrete components are denoted by $\xi(t)$ and $n(t)$, corresponding to the different states of the system at time t and the number of worse repairs to date, respectively. Thus, the mode of the machine at time t is given by $\xi(t) \in \Omega = \{1,2,\dots,N(S+1)\}$ such that:

$$\xi(t) = \begin{cases} 1 & \text{operational state } OP_1^0 \text{ at no failure} \\ 2 & \text{operational state } OP_2^0 \text{ at no failure} \\ \vdots & \vdots \\ S & \text{operational state } OP_S^0 \text{ at no failure} \\ F_o & \text{the machine is at the } 1^{\text{st}} \text{ failure} \\ 1 & \text{operational state } OP_1^1 \text{ at the } 1^{\text{st}} \text{ failure} \\ \vdots & \vdots \\ F_n & \text{the machine is at the } \text{n}^{\text{th}} \text{ failure} \\ 1 & \text{operational state } OP_1^n \text{ at the } \text{n}^{\text{th}} \text{ failure} \\ 2 & \text{operational state } OP_2^n \text{ at the } \text{n}^{\text{th}} \text{ failure} \\ \vdots & \vdots \\ S & \text{operational state } OP_S^n \text{ at the } \text{n}^{\text{th}} \text{ failure} \end{cases} \tag{2.1}$$

The machine may randomly be at any of the proposed modes over an infinite horizon. The failure/repair/overhaul process is described using the semi-Markov Chain $\xi(t)$ characterized by transition rates $\lambda_{\alpha\alpha'}(\cdot)$, $\alpha, \alpha' \in \Omega$, where the transitions $\lambda_{\alpha\alpha'}$ from modes α to α' satisfy the following conditions:

$$P[\xi(t+\delta t) = \alpha |\, \xi(t) = \alpha', x(t) = x, n(t) = n] =$$

$$\begin{cases} \lambda_{\alpha\alpha'}(\cdot)\delta t + o(x,n,\delta t) & if\ \alpha \neq \alpha' \\ 1 + \lambda_{\alpha\alpha'}(\cdot)\delta t + o(x,n,\delta t) & if\ \alpha = \alpha' \end{cases} \tag{2.2}$$

with:

$$\lambda_{\alpha\alpha'}(\cdot) \geq 0, \quad \lambda_{\alpha\alpha'}(\cdot) = -\sum_{\alpha \neq \alpha'} \lambda_{\alpha\alpha'}(\cdot), \quad \forall\, \alpha, \alpha' \in \Omega \;\; and \;\; \lim_{\delta t \to 0} \frac{o(x, n, \delta t)}{\delta t} = \mathbf{0} \qquad (2.3)$$

The previous Equation (2.2) denote the transition matrix $Q(\cdot) = \{\lambda_{\alpha\alpha'}(\cdot)\}$ of the semi-Markov chain *ξ(t)*. We can improve the performance of the manufacturing system during its life cycle; with the use of the decision variable $\omega(\cdot)$; this variable controls the transition rate to the overhaul or to the worse repair. Hence, the matrix $Q(\cdot)$ depends on the decision variable $\omega(\cdot)$ and is defined by:

$$Q(\omega) = \begin{bmatrix} \lambda_{1,1} & \lambda_{1,2} & 0 & 0 & 0 & & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & & \dots & \dots \\ \lambda_{F_n,1} \cdot \omega & 0 & 0 & \lambda_{F_n,F_n} & & \lambda_{F_n,F_n+1} \cdot (1-\omega) & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & & \dots & \dots \\ \lambda_{N(S+1),1} & 0 & 0 & 0 & 0 & & \dots & \lambda_{N(S+1),N(S+1)} \end{bmatrix} \qquad (2.4)$$

where N represents the maximum number of failures during which the system remains operational. In the next section, we provide a detailed discussion about this parameter N. At the operational state OP_i^n, we assume that the capacity constraint of the manufacturing system is defined by the control variable $u(\cdot)$ as follows:

$$0 \leq u(\cdot) \leq U_{max} \qquad (2.5)$$

where U_{max} is the maximum production capacity and $u(\cdot)$ is the production rate for the operational state OP_i^n. The control variables of the model are the production rate $u(\cdot)$ and the repair/overhaul switching strategy $\omega(\cdot)$. Thus, the set of admissible strategies $\Gamma(\cdot)$ that defines the feasible plan $\big(u(\text{n},\cdot), \omega(\text{n},\cdot)\big)$ depends on the stochastic process *ξ(t),* and is given by the following expression:

$$\Gamma(\alpha) = \left\{ \big(u(\text{n},\cdot), \omega(\text{n},\cdot)\big) \in R^2, \quad 0 \leq u(\text{n},\cdot) \leq u_{max}, \quad 0 \leq \omega(\text{n},\cdot) \leq 1 \right\} \qquad (2.6)$$

We next turn our attention to the continuous component of the hybrid state defined by the variable *x(t)*, which represents the inventory-backlog of parts produced. Since we consider that quality deterioration has the effect of increasing the rate of defectives, it is necessary to increment the demand rate to ensure that the production system satisfies the demand with flawless products. This condition leads to propose that the system dynamics evolves according to the following differential equation:

$$\dot{x}(t) = u(t) - d[1 + \beta(\mathrm{i}, \mathrm{n})] , \;\; x(0) = x_0 \qquad (2.7)$$

where x_0 is the given initial inventory level, d denotes the demand rate and $\beta(\mathrm{i}, \mathrm{n})$ represent the function for the rate of defectives, i describes the stage of the aging process and n is the current number of repairs. Let us define $g(\cdot)$ as the running cost in state $\alpha \in \Omega$, when the current stock level is x and the machine has already had its n^{th} repair, then the cost function is defined by the following equation:

$$g(\alpha, x, n, u, \omega) =$$

$$= c^+ x^+ + c^- x^- + \mathrm{c}^\alpha \qquad (2.8)$$

with

$$x^+ = max(0, x)$$

$$x^- = max(-x, 0)$$

$$\mathrm{c}^\alpha = c_r \cdot \lambda_{F_n, F_n+1} \cdot (1 - \omega) \cdot Ind\{\xi(t) = F_n\} + c_o \cdot \lambda_{F_n, 1} \cdot \omega \cdot Ind\{\xi(t) = F_n\}$$

$$Ind\{\theta(t)\} = \begin{cases} 1 & if\ \theta(t)\ is\ true \\ 0 & otherwise \end{cases}$$

where the constants c^+ and c^- are used to penalize the inventory and backlog of parts, respectively; c_r is the repair cost, and c_o is the overhaul cost. In addition, the objective functional given by the expected discounted cost is:

$J(\alpha, x, n, u, \omega) =$

$$= E\left\{\int_0^{\infty} e^{-\rho t} g(\cdot)dt \,\middle|\, \alpha(0) = \alpha, x(0) = x, n(0) = n\right\} \qquad (2.9)$$

where ρ denotes the discounted rate of the incurred cost. From what has been presented, it is apparent that the problem lies in determining an optimal control policy ($u^*(\cdot)$, $\omega^*(\cdot)$) where we seek to minimize the integral of the discounted cost $J(\cdot)$. Optimal policies are obtained from the value function defined as follows:

$v(\alpha, x, n) =$

$$inf_{(u(n,\cdot),\omega(n,\cdot)) \in \Gamma(\alpha)} J(\alpha, x, n, u, \omega), \quad \forall\ \alpha \in \Omega\ ,\ \ x\ \in\ R,\ n\ \in\ N \qquad (2.10)$$

The value function $v(\alpha, x, n)$ denotes the optimum value (in this case the minimum) of the integral of the discounted cost (2.9) and it satisfies specific properties called *optimality conditions.* In this respect regarding the principle of optimality, generally if $v(\cdot, t)$ denotes a cost-to-go function and if the initial time of the problem is t=0, then the change in the optimal function is made up of two parts: the incremental change of the discounted cost in the time interval $[0, t]$, and the change in the interval $[t, \infty]$. Then we can break up the minimization as follows:

$v(\alpha(t), x(t), n(t), t) =$

$$\inf_{\substack{u(s),\omega(s)\\ 0\le s\le\infty}} E\left\{\begin{matrix}\int_0^t e^{-\rho t} g[(\alpha(s), x(s), n(s), u(s), \omega(s)]ds \\ + \\ \int_t^{\infty} e^{-\rho t} g[(\alpha(s), x(s), n(s), u(s), \omega(s)]ds\end{matrix} \,\middle|\, \alpha(t), x(t), n(t)\right\} \qquad (2.11)$$

The control actions should be chosen to minimize the sum of these two terms. Additionally, the derivation of the optimality conditions must take into account the randomness of α (this explain the expectation operation), and the discounted rate ρ. We can then apply the conditional expectation operation $\widetilde{E}$ (i.e., for any function $H(\alpha)$, $\widetilde{E}\, H\big(\alpha(t + \delta t)\big) =$

$E\{H(\alpha(t+\delta t))|\alpha(t)\}$) and if we perturb t, for any δt, Equation (2.11) can be approximated by:

$$v(\alpha(t), x(t), n(t), t) = \inf_{\substack{u(s),\omega(s)\\ t\le s\le t+\delta t}} \tilde{E} \left\{ \begin{array}{c} g[(\alpha(t), x(t), n(t), u(t), \omega(t)]\delta t \\ + \\ \frac{1}{1+\rho\delta t} v[\alpha(t+\delta t), x(t+\delta t), n(t+\delta t), t+\delta t] \end{array} \right\} + o(\delta t) \tag{2.12}$$

Assuming that $v(\cdot)$ is differentiable, we can expand its derivative, and the expectation operation $\tilde{E}$. Then for small δt and after some manipulations Equations (2.12) becomes:

$$\rho v(\alpha(t), x(t), n(t), t) = \inf_{u(t),\omega(t)} \left\{ \begin{array}{c} g[(\alpha(t), x(t), n(t), u(t), \omega(t)]\delta t \; + \\ \frac{\partial v}{\partial x}[\alpha(t), x(t), n(t), t]\delta x(t) + \frac{\partial v}{\partial t}[\alpha(t), x(t), n(t), t]\delta t \\ + \sum_{\alpha'} v[\alpha', x(t), n(t), t]\, \lambda_{\alpha'\alpha(\mathrm{t})}\delta t \end{array} \right\} + o(\delta t) \tag{2.13}$$

We have eliminated the expectation symbol with the summation term. If we replace $\delta x(t)$ by $\delta x(t) = \dot{x}(t)\delta t$ and do other manipulations, we get:

$$\rho v(\alpha, x, n, t) - \frac{\partial v}{\partial t}[\alpha, x, n, t] = \inf_{u(t),\omega(t)} \left\{ g[\alpha, x, n, u, \omega] + \frac{\partial v}{\partial x}[\alpha, x, n, t]\dot{x} + \sum_{\alpha'} v[\alpha', x, n, t]\, \lambda_{\alpha'\alpha} \right\} \tag{2.14}$$

We observe that none of the functions $g(\cdot)$ and $\dot{x}(\cdot)$ are functions of t explicitly. Furthermore since the time horizon is infinite and a steady state distribution exists for α, Equations (2.14) is independent of t. Based on this and replacing the summation term by the generator $\mathrm{Q}(\cdot) = \{\lambda_{\alpha\alpha'}(\cdot)\}$, Equations (2.14) can be further simplified to:

$$\rho v(\alpha, x, n) = \min_{(u,\omega)\in\Gamma(\alpha)} \left\{ g[\alpha, x, n, u, \omega] + \frac{\partial v}{\partial x}[\alpha, x, n]\dot{x} + Q(\cdot)v[\alpha, x, \varphi(\xi, n)](\alpha) \right\} \tag{2.15}$$

These are the fundamental equations called Hamilton-Jacobi-Bellman (HJB) equations that we use to determine the optimal control policy. Further details about how the HJB equations are obtained can be consulted in Rishel (1975) and Gershwin (2002). In order to complete our formulation, since the overhaul restores the rate of defectives β to initial conditions, and that the effect of worse repairs is to increase this rate, we define, at a jump time τ for the process $\xi(t)$, a reset function $\varphi(\xi, n)$. This reset function describes any discontinuity that may occur at a jump time τ in the modes of the manufacturing system and is defined as follows:

$$\varphi(\xi, n) = \begin{cases} n+1 & if \;\; \xi(\tau^+) = OP_1^n \quad and \;\; \xi(\tau^-) = F_n \\ 0 & if \;\; \xi(\tau^+) = OP_1^0 \quad and \;\; \xi(\tau^-) = F_n \\ n & otherwise \end{cases} \tag{2.16}$$

It remains to determine the optimal policy (u^*, ω^*), when the value function is available, an optimal control policy can be obtained, as presented in the HJB equations. However, the issue is that it is extremely complicated to solve them analytically, because they lead to intractable problems. Also it remains to be specified how exactly the function $\beta(i, n)$ will model the quality deterioration phenomenon. More details about these concerns are provided in the next sections.

2.2.4 Deteriorating systems

The object of this section is to detail the expressions that deal with the deterioration phenomenon. In this respect we find that some authors relate the deterioration of the machine with the quality of the parts produced. For example Kim and Gershwin (2008), proposed a general model that represents the wear of the machine and that also indicates different quality levels, implying a deterioration of quality. In the same direction Colledani and Tolio (2011) suggest that the degrading process of the machine has a continuous deterioration on the parts' quality. Even in the area of quality it exists a control charts for trends, where a gradual change in the part's feature is expected and considered to be normal given for instance by tool wears, as indicated in Besterfield (2009). Therefore, in our paper we conjecture a relationship between deterioration and quality that increases the rate of defectives β by the combination effect of two factors (aging process and human interventions), as denoted by the following expression:

$$\beta(i,n) = \zeta(n) \cdot [1 + \mathrm{p}(i)] \quad (2.17)$$

where i is the current stage of the aging process and satisfies $1 \leq i \leq S$. Additionally n is the current number of repairs, the function $\zeta(n)$ models the effect of worse repairs on the rate of defectives for a given n, and $\mathrm{p}(i)$ represents the effect of the aging process for the current i. To detail how Expression (2.17) works, we concentrate first on the effect of the aging process $\mathrm{p}(i)$. The idea is that the wear of the production system implies certain changes, thereby we model the aging process with a set of discrete operational states OP_i^n with an increasing rate of defectives $(\beta_0 < \beta_1 < \cdots < \beta_N)$ as indicated by Kim (2005), where the movement from state to state represents the deterioration of quality. Thus the increment in the rate of detectives for every operational state OP_i^n is given by the following expression:

$$p(i) = b_b^{(i-1)} \cdot d_b \quad (2.18)$$

where $\mathrm{p}(i)$ represents the percentage that will increment the rate of defectives at a given stage i of the aging process, d_b is a constant with a value near to zero, and b_b denotes the common ratio of the expression. The constant b_b is useful for adjusting the Expression (2.18) to other types of machines. Function (2.18) says that the increment Δ_i in the rate of defectives for every operational state OP_i^n is not fixed; it increases, as presented in Figure 2.3a. The increment Δ_i can be defined from Expression (2.18) as follows:

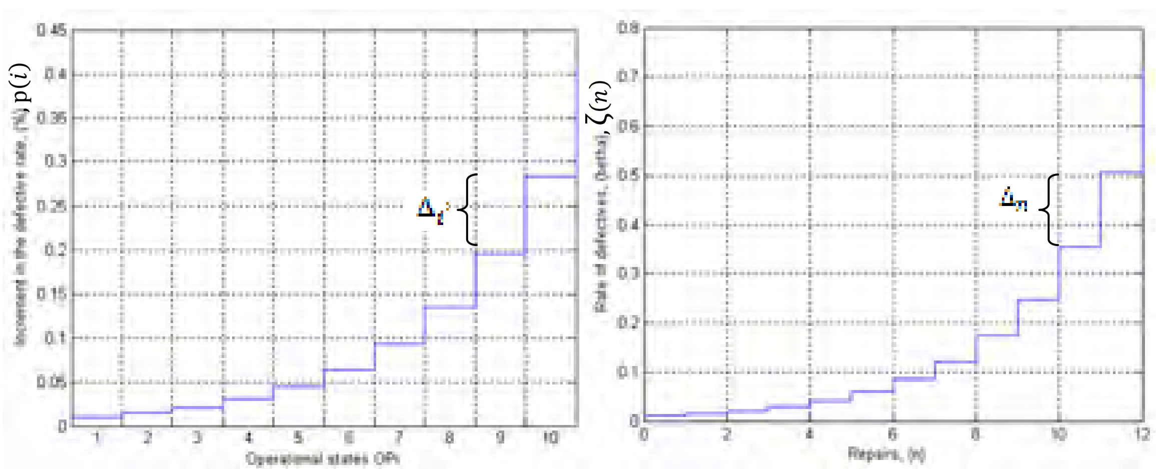
$$\Delta_i = \mathrm{p}(i+1) - \mathrm{p}(i) \quad (2.19)$$

The quality deterioration modeling of Equation (2.17) is complemented with the effect of human interventions, defined by the current number of worse repairs. Several authors have successfully related the number of repairs with the deterioration of the system as in Leung

(2001) and Lam (2007). Moreover they have observed a certain trend in the deterioration, and even their results have been valid in real industrial data as reported in Lam and Chan (1998). Consequently given the relationships between repairs-deterioration and deterioration-quality, we propose an increasing function $\zeta(n)$ to model the effect of the worse repairs on the rate of defectives, according to the following expression:

$$\zeta(n) = \frac{d_b}{a_b^{(n-1)}} \tag{2.20}$$

where d_b is the value of the rate of defectives at initial conditions, n denotes the current number of worse repairs at time t, and a_b describes the common ratio of the expression, which denotes a given constant.



(a) caused by the aging process (b) caused by the worse repairs

Figure 2.3 Trend of the quality deterioration

The advantage of Equation (2.20) is that we can adjust the value of the parameter a_b to modify the trend of the rate of defectives for a specific type of machine. Figure 2.3b presents the graph for this Expression (2.20), where the increment Δ_n can be obtained as follows:

$$\Delta_n = \zeta(n+1) - \zeta(n) \tag{2.21}$$

We claim in Figure 2.3b that after a worse repair the rate of defectives grows a certain amount Δ_n, and due to the accumulated wearing of the machine, this increment Δ_n is not constant; it has a low value for the first repairs and it follows an increasing trajectory as suggested in Lam et al. (2004).

By incorporating Expression (2.18) and (2.20) into $\beta(i,n)$, finally it derives in:

$$\beta(i,n) = \frac{d_b}{a_b^{(n-1)}} \cdot \left[1 + b_b^{(i-1)} \cdot d_b\right] \tag{2.22}$$

We conclude this section with the following remark. The consideration of the deterioration of quality yields to a new model, where the quality of the parts produced is negatively affected. In essence Equation (2.22) says that the quality deterioration has two components; the natural wear of the machine given by the set of operational states and the effect of the worse repairs. These two components summarize the effect of other possible presented factors.

Because of these innovative characteristic, our formulation leads to a semi-Markov model since the information that is available to the decision maker at each instant of time t, includes the state of the production system, denoted by the stock level, as well as the machine's history, with the number of repairs and the set of operational states.

2.2.5 Numerical approach

Notwithstanding the difficulty in solving the HJB Equation (2.15), fortunately it is possible to obtain an approximation of the control policy through the application of a numerical method conceived for optimal control models. This numerical method is based on the Kushner approach, and the crux of this technique consists in approximating the continuous value function $v(\alpha,x,n)$ by the discrete function $v^h(\alpha,x,n)$ and the gradient $\frac{\partial v}{\partial x}(\alpha,x,n)$ by the following expression:

$\frac{\partial v}{\partial x}(\alpha, x, n) =$

$$\begin{cases} \frac{1}{h}(v^h((\alpha, x+h, n) - v^h((\alpha, x, n) & if \quad \dot{x} \geq 0 \\ \frac{1}{h}(v^h((\alpha, x, n) - v^h((\alpha, x-h, n) & if \quad \dot{x} < 0 \end{cases} \tag{2.23}$$

where h is a discrete increment associated with the state variable x. More details about the Kushner approach can be seen in Kushner and Dupuis (1992). The application of the numerical method implies the use of a discrete equation for every mode of the machine that in this case it is defined as follows:

$v^h(\alpha, x, n) =$

$$\min_{(u(\mathrm{n},\cdot),\omega(\mathrm{n},\cdot)) \in \Gamma(\alpha)} \left[\left(\rho + |\mathrm{q}_{\alpha\alpha}| + \frac{|r|}{\mathrm{h}} \right)^{-1} \left(g(\cdot) + v^h(\alpha, x+h, n) \frac{|r|}{\mathrm{h}} \mathrm{Ind}\{r \geq 0\} \right. \right.$$
$$\left. \left. + v^h(\alpha, x-h, n) \frac{|r|}{\mathrm{h}} \mathrm{Ind}\{r < 0\} + Q(\cdot) v(\alpha, x, \varphi(\mathrm{n}, \xi))(\alpha) \right) \right] \tag{2.24}$$

where $r = u(\cdot) - d[1 + \beta(\mathrm{i}, \mathrm{n})]$. In general terms, the numerical method reduces the complexity of the original continuous control problem defined by the HJB Equation (2.15) to a discrete semi-Markov decision process (2.24) with finite state space and finite action space. The technical advantage of the numerical method is that the discrete semi-Markov process (2.24) is much easier to solve than the continuous version. Moreover, we can interpret its coefficients as the transitions probabilities between the different points defined in the computational domain G_{xn}^h. Eventually, the obtained discrete semi-Markov process (2.24) can be solved by the policy improvement technique or value iteration methods.

2.3 Numerical example

This section provides a numerical example of the manufacturing system presented in section 2.2. The computation algorithm applied to solve the discrete semi-Markov process (2.24) is based on the policy improvement technique. Clearly, the solution $v^h(\alpha, x, n)$ of

Equation (2.24) is a discrete approximation that converges to the continuous function $v(\alpha, x, n)$ of Equation (2.10). The algorithm that leads us to the optimal policy requires that we follow a specific sequence of steps, which can be consulted in detail in Kushner and Dupuis (1992) and Kenne et al. (2003). Without loss of generality, we consider three operational states for the stages of the aging process; in other words, this means that $S = 3$. The numerical method is then applied with the data presented in Table 2.1, where the value of each required parameter is defined.

Table 2.1 Parameters of the numerical example

Parameter:	u_{max} (units/hr)	d (units/hr)	h	ρ	c+ ($/units/hr)
Value:	5	3	0.5	0.9	1
Parameter:	c- ($/units/hr)	c_r ($)	c_o ($)	N	a_b
Value:	200	20	660	20	0.815
Parameter:	b_b	d_b	λ_{12} (1/hr)	λ_{23} (1/hr)	λ_{34} (1/hr)
Value:	3.5	0.01	0.5	0.2	0.1
Parameter:	$\lambda_{(F_n+1),(F_n+2)}$ (1/hr)	$\lambda_{(F_n+2),(F_n+3)}$ (1/hr)	$\lambda_{(F_{n-1}+S),\,F_n}$ (1/hr)	$\lambda_{F_n,1}$ (1/hr)	$\lambda_{F_n,(F_n+1)}$ (1/hr)
Value:	0.5	0.2	0.1	0.02	2
Parameter:	ω_{min}	ω_{max}			
Value:	0	1			

Historical production data is the source to determine the proper values of the adjusting paramateres a_b and b_b, fit expressions (2.18) and (2.20) to a given system, and model adequately the deterioration process.

The implementation of the numerical technique also requires the use of a discrete grid for the inventory level x and the number of repairs n. The grid is denoted by G_{xn}^h and defines the computational domain as follows:

$$G_{xn}^h = \{(\mathrm{x}, \mathrm{n}) : -10 \leq \mathrm{x} \leq 30, \quad 0 \leq \mathrm{n} \leq 20, \quad \mathrm{h} = 0.2\} \tag{2.25}$$

The manufacturing system will be in conditions to satisfy the demand rate d over an infinite horizon and reach steady state only if the system is feasible. This implies the satisfaction of the following condition:

$$u_{max} \cdot \pi_1 + \ldots + u_{max} \cdot \pi_{3\mathrm{N}} \geq d \cdot [1 + \beta(i,n)] \quad (2.26)$$

where $\pi_1, \ldots, \pi_{3\mathrm{N}}$ are the limiting probabilities for the operational modes of the machine, which are normally computed as follows:

$$\pi_i \cdot Q(\cdot) = 0 \quad \text{and} \quad \sum_{i=1}^{N(S+1)} \pi_i = 1 \quad (2.27)$$

It is pertinent to note that the feasibility condition (2.26) is satisfied by the selected values of the parameters presented in Table 2.1. With the values of a_b and d_b applied in Expression (2.22), it follows that the limit of feasibility is reached at repair $N = 20$. This implies that the semi-Markov generator $Q(\cdot)$ is composed of 60 operational states OP_i^n and 20 failure states F_n.

The results presented in Figures 2.4 to 2.8 were obtained with the data presented in Table 2.1, and they illustrate the optimal control policies for the production and repair/overhaul switching strategy.

2.3.1 Production policy

The optimal production policy $u^*(\alpha, x, n)$ is illustrated in Figure 2.4, and indicates the production rate of the manufacturing systems applied in the operational states OP_3^n for any number of repairs n and stock level x. From Figure 2.4, it follows that the production policy divides the plan (x, n) into three regions in which the production rate is set to u_{max}, d, and *0,* respectively. Moreover, we observe that the trend in the production thresholds is a clear indicator of the effect of the quality deterioration. It turns out that the more the number of repairs increases, the more the optimal stock level increases as well.

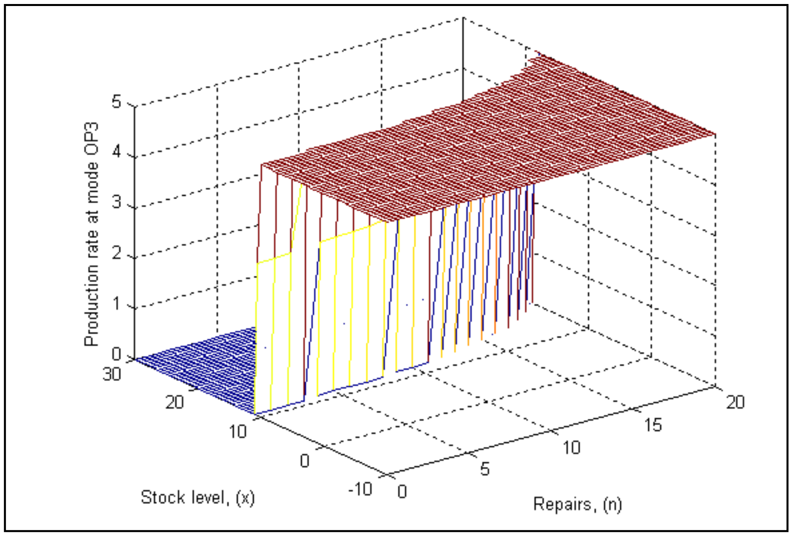
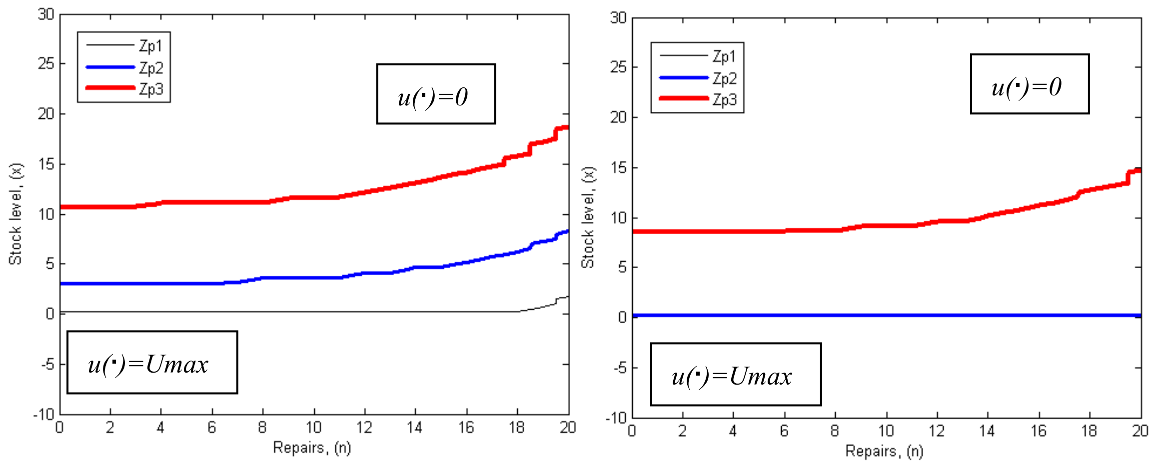


Figure 2.4 Production rate of the manufacturing system for the operational states OP_3^n

To better illustrate the optimal production policy, we use its boundary presented in Figure 2.5a, where we identify the optimal stock level $Z_{pi}^*(\cdot)$ applied in three stages of the operational states. In Figure 2.5a, we observe that $Z_{p1}^*(\cdot) < Z_{p2}^*(\cdot) < Z_{p3}^*(\cdot)$; this is given because as the machine moves in the operational states, the amount of product to hedge against a breakdown increases.

An idealistic case is when the machine has high availability; for instance, 99%. This would imply a general reduction in the production thresholds for all the operational states, as presented in Figure 2.5b. In this figure, the failure takes so long to occur that the production thresholds $Z_{p1}^*(\cdot)$ and $Z_{p2}^*(\cdot)$ are equal to zero and even $Z_{p3}^*(\cdot)$ is lower, compared with Figure 2.5a. We note that when the production system has lower availability, it is necessary to maintain a certain amount of inventory in the operational states, as presented in Figure 2.5a, where according to the data of Table 2.1, the system has an availability of 97%.



(a) With data of Table 1 (b) With high availability data

Figure 2.5 Production trace $Z_{pi}^{*}(\cdot)$ for the three operational states

It is remarkable that the trend in the optimal thresholds is influenced by the function applied to model the quality deterioration. This is apparent because the trend in the production thresholds presented in Figure 2.4 and Figure 2.5a are very similar to the graph of Figure 2.3. To summarize, the optimal production threshold can be defined using the switching trend presented in Figure 2.4 and Figure 2.5a, and the underlying pattern of this policy denotes a machine rate of defectives dependent hedging point policy, which for the numerical illustration, is given by the following equations:

$$u(1,\cdot)^{*} = \begin{cases} u_{max} & if\ x(t) < Z_{p1}^{*}(\cdot) \\ d & x(t) = Z_{p1}^{*}(\cdot) \\ 0 & x(t) > Z_{p1}^{*}(\cdot) \end{cases} \tag{2.28}$$

$$u(2,\cdot)^{*} = \begin{cases} u_{max} & if\ x(t) < Z_{p2}^{*}(\cdot) \\ d & x(t) = Z_{p2}^{*}(\cdot) \\ 0 & x(t) > Z_{p2}^{*}(\cdot) \end{cases} \tag{2.29}$$

$$u(3,\cdot)^{*} = \begin{cases} u_{max} & if\ x(t) < Z_{p3}^{*}(\cdot) \\ d & x(t) = Z_{p3}^{*}(\cdot) \\ 0 & x(t) > Z_{p3}^{*}(\cdot) \end{cases} \tag{2.30}$$

where $Z_{pi}^{*}(\cdot)$ is the function that gives the optimal production threshold for the operational states of the aging process, with the threshold level illustrated in Figure 2.5a. In general, it is observed that the production threshold increases as the machine moves in the operational states, and the following condition holds: $Z_{p1}^{*}(\cdot) \leq Z_{p2}^{*}(\cdot) \leq Z_{p3}^{*}(\cdot)$.

2.3.2 Repair/overhaul switching policy

The repair/overhaul switching policy obtained is presented in Figure 2.6, where we observe that the plan (x, n) is divided into two regions. The decision variable ω is set to its maximum or minimum value, depending on the current stock level and the number of worse repairs. The maximum value, $\omega = 1$, defines the most convenient time to conduct the overhaul, and conversely, the minimum value, $\omega = 0$, suggests when to perform the worse repair. It is apparent that the role of the control variable ω is to synchronize the two available maintenance options.

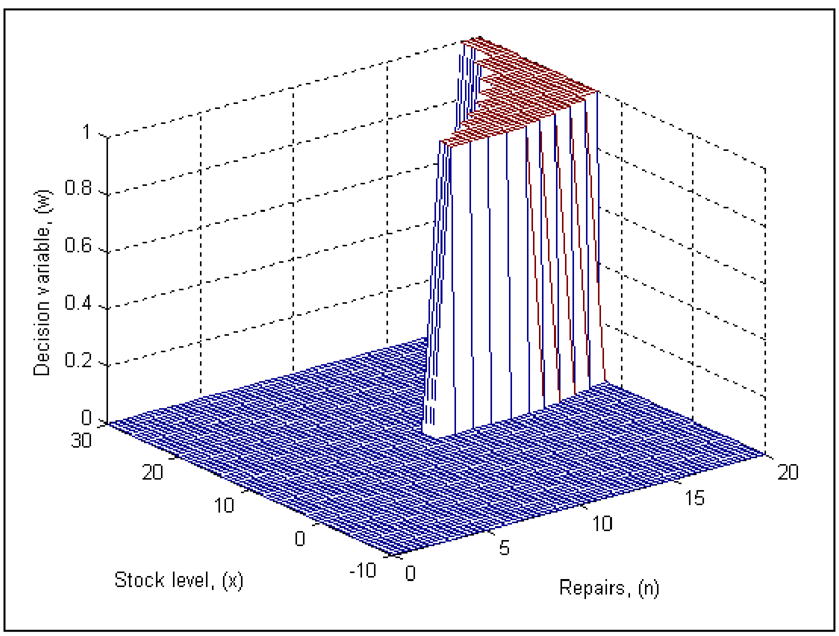


Figure 2.6 Repair/Overhaul policy

To facilitate the analysis of the repair/overhaul switching policy, its boundary denoted by $D_n(\cdot)$ is used. The boundary divides the computational domain G_{xn}^{h} into two zones, according to the type of maintenance performed. The zones are presented in Figure 2.7, and are defined as follows:

- Zone A, here is justified the cost of an overhaul, and therefore this type of maintenance is recommended; furthermore, the decision variable ω is set to its maximum value.
- Zone B, in this zone, the overhaul activity is not recommended, but rather, the worse repair is the maintenance option suggested, and the variable ω is set to its minimum value.

By analyzing Figure 2.7, it seems logical not to perform the overhaul maintenance if the number of repairs is low, since this means that the level of deterioration is also low, and this condition does not justify the high cost of an overhaul. Moreover, we observe that the overhaul zone is located in the positive stock area, which leads to the observation that, much more time and resources are needed to perform an overhaul than to carry out worse repairs. Consequently, some amount of stock is required to circumvent the loss of demand fulfillment while the machine is out of operation, and an overhaul is being performed.

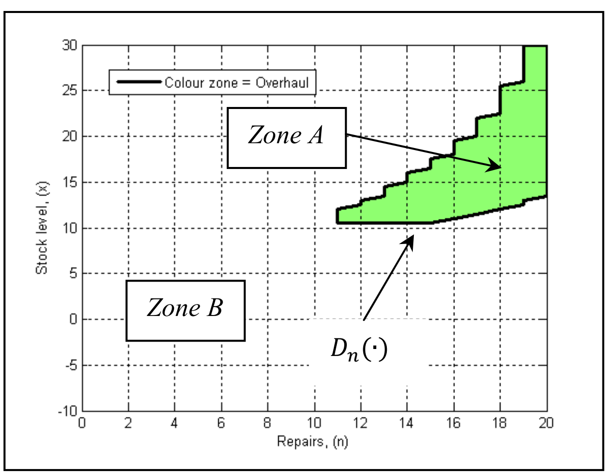


Figure 2.7 Trace of the repair/overhaul policy

In Figure 2.8, we present the intersection of the production and the repair/overhaul policy. As we can observe in this figure, when we include the boundary of the production policy, only a part of the overhaul zone is feasible since the stock level is limited by the production threshold $Z_p^*(\cdot)$. Even though the overhaul zone is bigger, we stress that the intersection of both policies defines the feasible zone A' to implement in the manufacturing system. Thus this condition must be considered in determining the repair/overhaul policy.

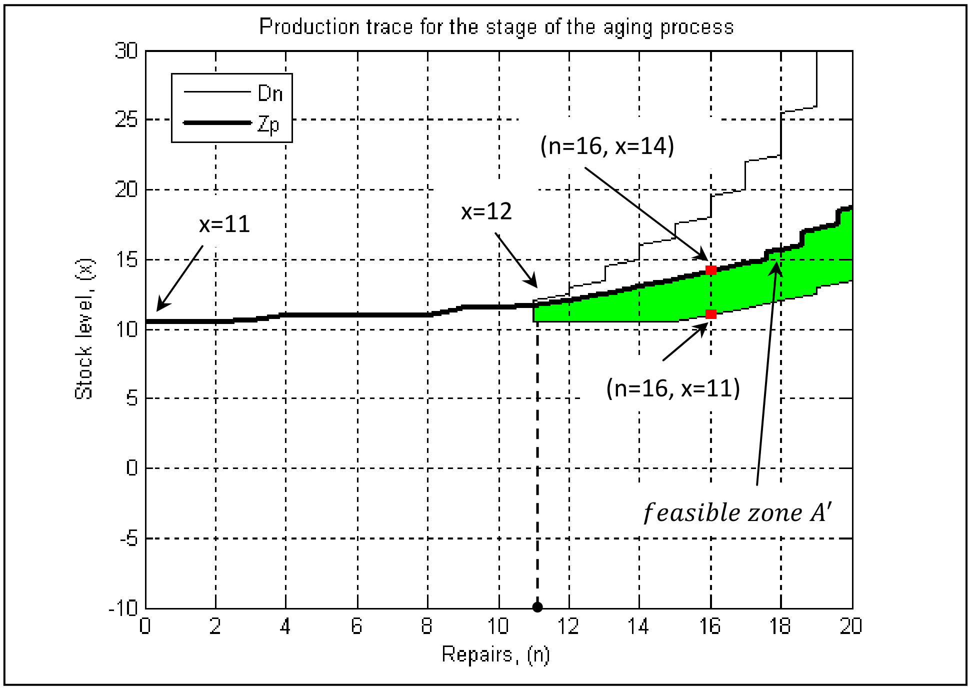


Figure 2.8 Intersection of the production and the repair/overhaul trace

It should be pointed out that repair/overhaul activities are triggered according to the machine rate of defectives dependent policy as observed in Figure 2.8, and that this optimal policy has a bang-bang structure. Let $R_n(\cdot)$ denote a function with value 1 if an overhaul is undertaken after the n[th] failure, and 0 if a repair is performed. Based on the obtained results and with such a notation, the repair/overhaul policy is given by the following expression:

$$R_n(\cdot)^* = \begin{cases} 1 & if\ s(\cdot) \in zone\ A'\ defined\ by\ D_n(\cdot)\ and\ Z_{pi}^* \\ 0 & otherwise \end{cases} \tag{2.31}$$

with

$$s(\cdot) = \big(n(t), x(t)\big) \tag{2.32}$$

where the coordinate $s(\cdot)$ is located in the grid G_{xn}^{h}. From what has been presented, it is valid to say that the joint optimal production and repair/overhaul control policies are defined by the Equations (2.28)-(2.32). Based on the obtained results, it is evident that the production and the repair/overhaul policies can be completely characterized with the control parameters $Z_{pi}(\cdot)$ and $D_n(\cdot)$.

2.4. Model implementation

Figure 2.5a and Figure 2.8 are used in the implementation of the obtained results. At initial time, the machine is in the operational state OP_1^0, and since it is a brand new machine, there is no need to maintain a stock of products, hence the production threshold is $Z_{p1}^*(0)$=0 and the number of repairs is equal to zero. As soon as the machine is put in operation, its rate of defectives starts deteriorating. While awaiting the first failure, the machine will eventually move to the operational state OP_3^0 where the production threshold increases to $Z_{p3}^*(0)$=11. When the first failure occurs, the machine is repaired. This means that the rate of defectives will increase, since the repair does not consider any reduction in the rate of defectives. With the first repair, the machine becomes operational once again, and enters the operational state OP_1^1, from this point, the production system will behave in a similar pattern until the number of repairs reaches $n = 11$. At this point, the production threshold at the operational state OP_3^{11} increases to $Z_{p3}^*(11)$=12, because of the deterioration phenomenon, and also this point indicates that the overhaul is beginning to be carried out.

Figure 2.9 illustrates the implementation of the control policy when the number of repairs is $n = 16$. Let us assume that the machine has already experienced its 16^{th} repair, and is waiting for its next failure in the operational state OP_3^{16}. Meanwhile, the failure does not occur, the production threshold is set to $Z_{p3}^*(16)$=14. This value is used to determine the correct production rate. When the failure arrives, if the current stock level x is inside the interval (11, 14) defined by D_n and $Z_{p3}^*(16)$, then the overhaul is conducted, otherwise the machine is repaired. The benefit of the overhaul is that it restores the machine to initial conditions. Conversely, with the repair, the machine will continue deteriorating since it will eventually experience another failure. In the numerical illustration, the maximum number of repairs that the machine can experience is 20, as discussed in section 2.3, after which it is restored to brand new condition, regardless of its state. Additionally, the rate of defectives following an overhaul is reduced to initial conditions.

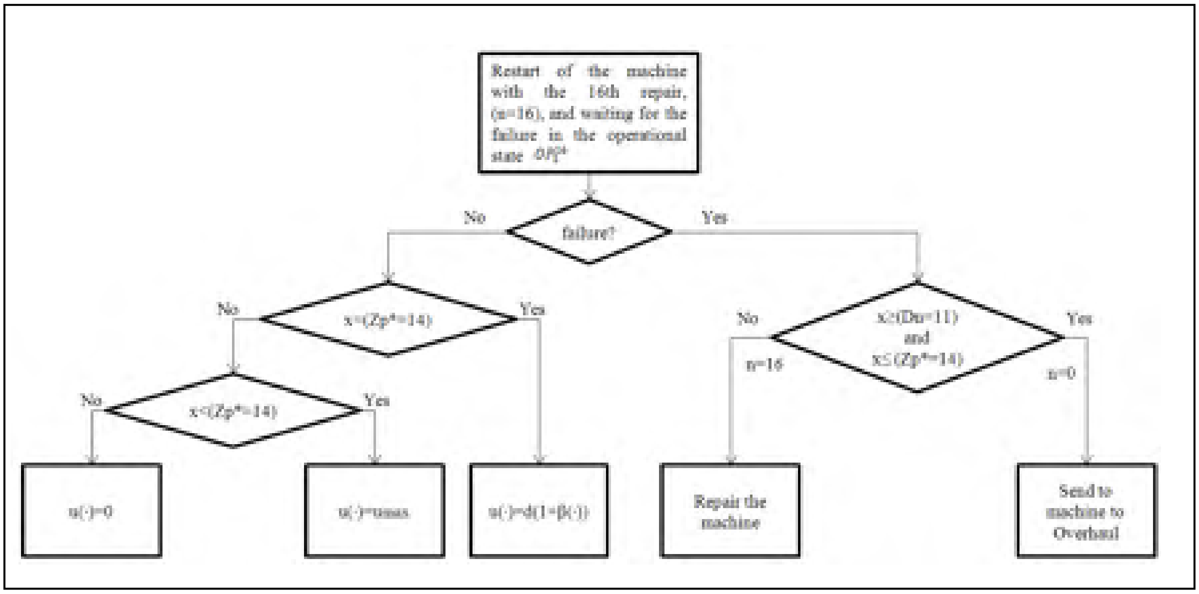


Figure 2.9 Model implementation diagram

To observe how these results are influenced by some of the parameters used in this model, in the next section we verify the structure of the obtained control policy. To that end, a sensitivity analysis is performed to ensure the consistency of the control policy and to illustrate the usefulness of the control approach.

2.5 Sensitivity and results analysis

In this section, we analyze different manufacturing scenarios involving changes in the cost parameters. The purpose is to illustrate the effect of cost variation on the optimal control policies and to determine if it is characterized consistently by the control factors $Z_{pi}(\cdot)$ and $D_n(\cdot)$. The sensitivity of the control policies is analyzed according to the variation of the cost of inventory, the cost of backlog, the cost of overhaul and the repair cost.

2.5.1 Variation of the inventory cost

We shall begin the sensitivity analysis with a discussion about the inventory cost. For this, we analyze the production threshold $Z^{*}_{p3}(\cdot)$, for three different inventory cost values $c^{+} = 1, 3$ and 6. We point out that this threshold $Z^{*}_{p3}(\cdot)$ is applied in the operational states OP^{n}_{3}. From the results of Figure 2.10, it is observed that the more the inventory cost increases, the more the production threshold decreases, since we see that with a higher inventory cost, for instance $c^{+} = 6$, the more the product stock is penalized, and with lower cost, such as $c^{+} = 3$ and $c^{+} = 1$, there is more liberty to maintain stock, and so the production threshold increases. Furthermore, the influence of quality deterioration on the optimal threshold is also evident because it increases as the number of repairs grows.

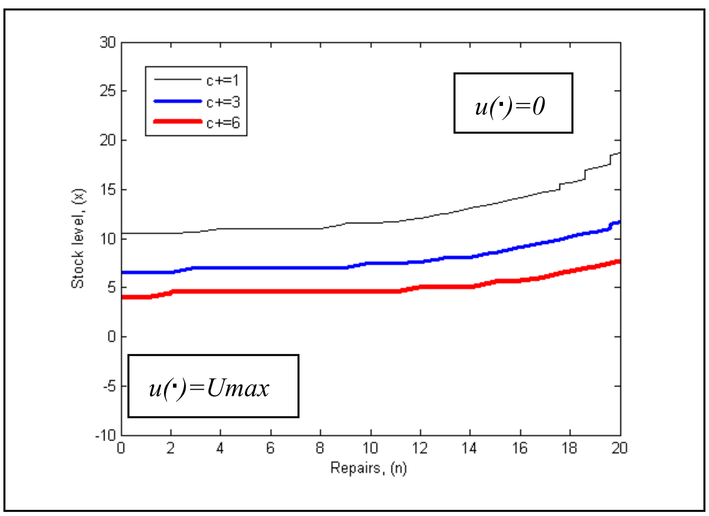
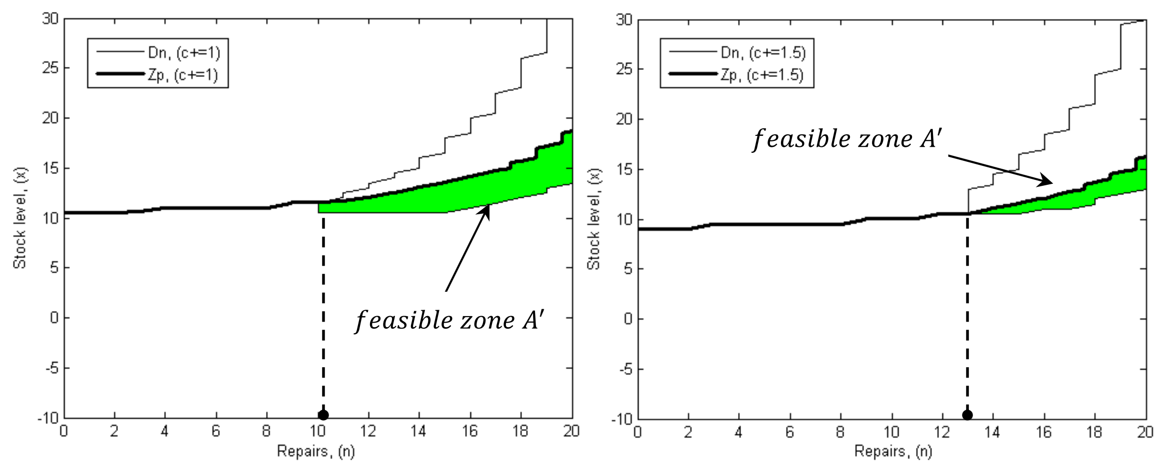


Figure 2.10 Variation of the inventory cost and its effect on the production threshold $Z^{*}_{p3}(\cdot)$

As a matter of interest, in Figure 2.11, we present the effect of the variation of the inventory cost c^{+} on the repair/overhaul policy. We observe in this figure that when the inventory cost is low, for instance $c^{+} = 1$, the overhaul zone A' is confined to a bigger area in the computational domain. From the repair/overhaul trace $D_n(\cdot)$, it is seen that the minimum number of repairs needed before considering the possibility of the overhaul is $n^{*} = 10$. When the inventory cost increases to $c^{+} = 1.5$, the overhaul zone A' reduces, adjusting the number of repairs to $n^* = 13$. This condition is explained by the fact that one of the parameters involved in the determination of the overhaul zone A' is the inventory cost. In general, if the overhaul cost remains constant and we increase the inventory cost, the overhaul zone A' decreases. Moreover, it is readily observed in Figure 2.11 that the repair/overhaul policy is very sensitive to variations of the inventory cost.



a) c^{+}=1 b) c^{+}=1.5

Figure 2.11 Variation of the inventory cost and its effect on the repair/overhaul policy

2.5.2 Variation of the backlog cost

Our next step in the sensitivity analysis involves the backlog cost c^{-}. To examine this parameter, we analyze the production threshold $Z^{*}_{p3}(\cdot)$ applied in the operational states OP^{n}_{3}. From the results of Figure 2.12, we observe that when the backlog cost is c^{-}=100, the production threshold has an initial value close to 8, which grows progressively as the number of repairs increases. When the backlog cost increases to c^{-}=200, the optimal stock level has an initial value close to 10, and it follows a parallel trajectory to the previous case. If we increase the backlog cost to c^{-}=300, the production threshold grows even more, to a value close to 13, and the observed trend is parallel to the other cases. These results tell us that by increasing the backlog cost we increase the production threshold, because with a high backlog cost, product shortages are so severely penalized that, higher amounts of products are allowed to be maintained in order to satisfy the demand.

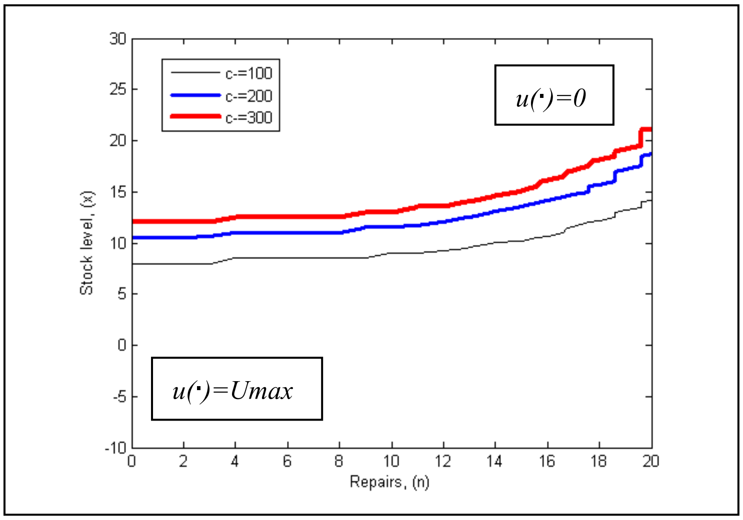


Figure 2.12 Variation of the backlog cost and its effect on the production threshold

To complement the analysis of the backlog cost, we examine its effect on the repair/overhaul policy. This variation is presented in Figure 2.13, where it was compared two cases. In analyzing such scenarios, we notice that when the backlog cost is low, for example c^-=150, the overhaul is less recommended, meanwhile more repairs are performed. Moreover, the minimum number of repairs needed before considering the possibility of the overhaul is $n^* = 14$, as observed in the repair/overhaul trace $D_n(\cdot)$.

When the backlog cost increases to c^-=200, the overhaul zone grows, and the number of repairs thus decreases to $n^* = 7$. The intuition behind this result is that, a variation of the backlog cost is tied directly to the size of the overhaul zone A'. If the overhaul cost is constant and the backlog cost increases, then the overhaul zone A' increases accordingly as well. In the results of Figure 2.13, it can clearly be seen that changes in the backlog cost influence the repair/overhaul policy. Generally, the effect of the backlog cost on the repair/overhaul policy is the inverse of the effect of the inventory cost.

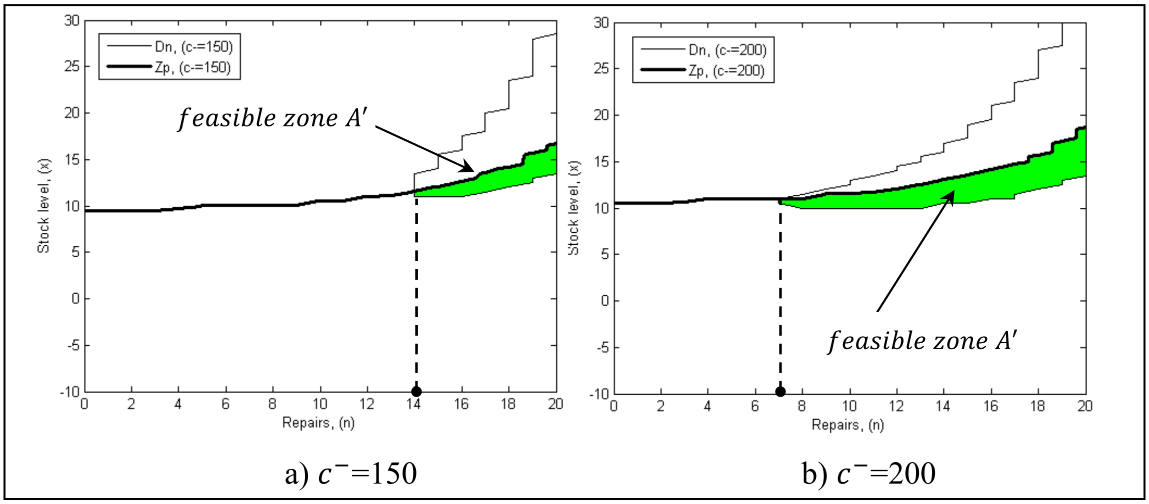
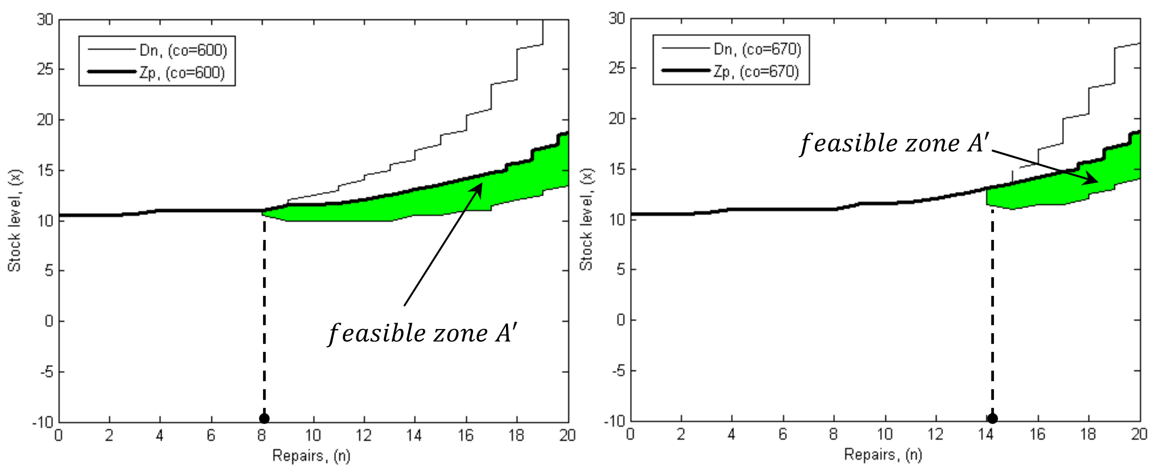


Figure 2.13 Variation of the backlog cost and its effect on the repair/overhaul policy

2.5.3 Variation of the overhaul cost

We shall proceed with the sensitivity analysis, by discussing the overhaul cost c_o. Figure 2.14 is intended to analyze the repair/overhaul trace $D_n(\cdot)$ for two different values $c_o = 600$ and 670. From the obtained results, we notice that when the overhaul cost is low, for instance $c_o = 600,$ the overhaul is more recommended. Additionally, the minimum number of repairs needed before considering the possibility of the overhaul is defined as $n^* = 8$, as indicated in the repair/overhaul trace of Figure 2.14a. If we increase the cost to $c_o = 670$, we see a modification in the maintenance trace, and consequently, the overhaul is less recommended. This increases the number of repairs to $n^* = 14$, as presented in Figure 2.14b. These results lead us to the argument that the variation of the overhaul cost has a strong effect on the repair/overhaul trace $D_n(\cdot)$, and therefore it is evident that, the smaller is the value of the overhaul cost, the more extensive is the overhaul zone A'. Furthermore, we observe that with low levels of quality deterioration, given by low number of repairs, the overhaul is not recommended. This activity is performed only when the rate of defectives is high enough to justify its expensive cost. With respect to the production policy, the overhaul cost has not shown any effect, since the production threshold in both figures is the same.



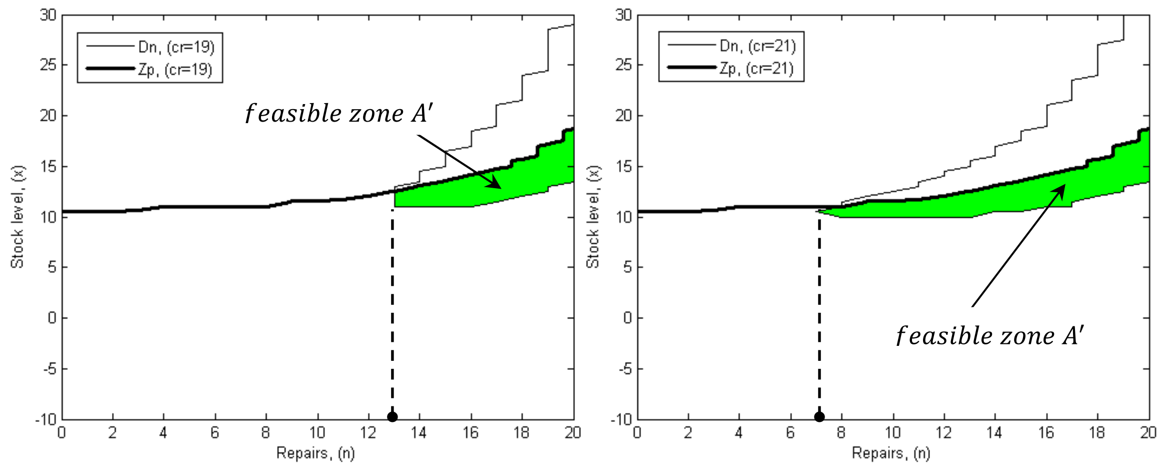
a) c_o=600 b) c_o=670

Figure 2.14 Variation of the overhaul cost and its effect on the repair/overhaul policy

2.5.4 Variation of the repair cost

To complete the sensitivity analysis, we discuss the variation of the repair cost c_r, for two cost scenarios, c_r =19 and 21, as presented in Figure 2.15. We begin the analysis when the repair cost has a low value c_r =19; in this case, the overhaul zone A' is limited to a small area in the computational domain. The repair/overhaul trace of Figure 2.15a implies that the minimum number of repairs needed before considering the possibility of the overhaul is set to $n^* = 13$. Since the repair cost is low, it is recommended to perform fewer overhauls. When the repair cost is c_r =21, the repair/overhaul trace $D_n(\cdot)$ varies notably, and the area for the overhaul increases, as a result of which the number of repairs decreases to $n^* = 7$, as observed in Figure 2.15b.

These results amount to the observation that the repair/overhaul policy is highly sensitive to the repair cost, since only a very small variation in this parameter is needed to influence the optimal repair/overhaul policy. We notice that by decreasing the repair cost, the zone for performing the overhaul decreases as well. With respect to the effect on the production policy, the repair cost has not reported any influence, since the production threshold in both figures is similar.



a) c_r=19 b) c_r=21

Figure 2.15 Variation of the repair cost and its effect on the maintenance policy

From the numerical results presented thus far, in this sensitivity analysis, we observe that the traditional production control policy, which consists in maintaining a certain amount of products to hedge against breakdowns, is modified by the presence of the quality deterioration phenomenon. Considering that the machine deteriorates, it implies a policy with several critical thresholds, which increase from one repair to the next, and which also increases between the operational states of the aging process. From the sensitivity analysis, we can conclude that the structure of the obtained control policy is maintained, and this condition yields to the possibility to characterize the optimal policy with the control factors $Z_{pi}^*(\cdot)$ and $D_n^*(\cdot)$. These factors allow the development of a parameterized control policy for simultaneous production planning and the repair/overhaul switching strategy, which seeks to operate the manufacturing system more efficiently at an optimal cost.

2.6 Conclusions

In this research, we integrated quality issues in the determination of control policies, and this implied that a repair/overhaul planning problem is combined with a production control problem for a manufacturing system subject to deterioration. The effect of the deterioration phenomenon on the machine is reflected in the quality of the parts produced, where the rate of defectives increases due to the combination of two factors: the wear of the machine given by an aging process, and human interventions, tied to worse repairs. We observed that the stock level required to hedge against breakdowns increases with the number of repairs and the evolution in the set of operational states. We found that the performance of overhaul activities depends on the stock level and the number of repairs. Since we integrate the history of the production system into, the number of repairs and the use of multiple operational states, a semi-Markov model was developed. A numerical example was considered to illustrate the utility of the proposed approach, and a sensitivity analysis was conducted to confirm the structure of the control policy. The obtained policy resulted in an interesting alternative for controlling the manufacturing system at the operational level, compared to other works that does not consider quality deterioration. Finally, the assessment of the proposed model shows that quality deterioration has a considerable effect on the production and repair/overhaul policy.

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CHAPTER 3

ARTICLE 3: JOINT CONTROL OF PRODUCTION, OVERHAUL AND PREVENTIVE MAINTENANCE FOR A PRODUCTION SYSTEM SUBJECT TO QUALITY AND RELIABILITY DETERIORATIONS

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Abstract

This research investigates the case of an unreliable manufacturing system subject to quality and reliability deterioration. In particular, we conjecture that the deterioration of the system leads to a continuous increase in the intensity of failures and a decrease on the quality of the parts produced. As such, deterioration implies a twofold effect on the manufacturing system. When the machine fails, minimal repair is conducted, leaving the machine at the same level of deterioration before failure. Hence, the quality of the parts produced and the failure intensity remain unchanged with this repair. Meanwhile, an overhaul refers to a perfect repair that completely restores the quality of the parts and the failure intensity of the machine. This option completely counters all the effects of the deterioration. Preventive maintenance may also be conducted, but it reduces the level of deterioration only partially, improving the

quality of the units produced and the failure intensity just in part. These set of characteristics yield to the formulation of a new control model that simultaneously determines the optimal production plan, the overhaul and preventive maintenance strategies. Such a joint control policy minimizes the total cost including the inventory holding, backlog, overhaul, preventive maintenance and defectives costs over an infinite planning horizon. Since the dynamics of the system change as a function of the level of deterioration, it is necessary to use its history for a proper formulation; therefore a Semi-Markov decision process is used. Numerical methods are applied to determine the control policy and numerical examples are conducted as illustrations. An extensive sensitivity analysis is presented in order to confirm the structure of the control policy obtained and examine the effect of several parameters.

Keywods: Quality, manufacturing systems, optimal control, numerical methods, stochastic processes, preventive maintenance.

3.1 Introduction

Manufacturing system control is a very active domain that has been widely investigated during the years. However, today the influence of several phenomena on the optimal production policy is still not clear. For instance, the impact of quality aspects on the production policy requires more research. Additionally, if we consider the fact that in practice, manufacturing systems are subject to deterioration (caused by a number of factors including the environment, the accumulated wearing, usage, etc), then it is logical to expect that such deterioration is amenable, not only to reduce the normal system operation, but also may have somehow an impact on the quality of the parts produced. In this context, various types of maintenance activities are available to resolve the effects of deterioration, and precisely the determination of the optimal production plan and the most efficient maintenance strategies is the scope of this research.

A detailed overview of the literature reveals that the production control problem has been formulated as a stochastic optimal control model motivated mainly by the pioneering work of

Kimemia and Gershwin (1983). They presented a hierarchical control algorithm for the production management of a failure-prone flexible manufacturing system. Later, based on their formulation, Akella and Kumar (1986) obtained an analytic solution for the case of one machine that produces only one part type. Most of the literature in this research area has its foundation on these works, and recent extensions address the production control problem from different perspectives. For example, Mok and Porter (2006) proposed a stochastic optimization procedure to estimate the production rate of a manufacturing system producing either single product-type or multiple product-types. Sajadi et al. (2008) proposed another production control model, where they determined the production rate of a network of multiple machines, and multiple products with restrictions of the feeding materials. In another study on production control, Gharbi et al. (2011) treated the case of an unreliable central machine where a reserve machine is called upon in support to response to the demand. Despite the relevance and diversity of these papers, a common characteristic is that the aspect of quality is disregarded. Undoubtedly, this is a major limitation, since real manufacturing systems may be disrupted by the existence of defective products.

Some ideas have been proposed over the years to analyze the impact of quality aspects on production systems. For instance, a detailed discussion about the inter-relation between quality and productivity was presented by Inman et al. (2003). In this work they identified relevant industrial research opportunities on the interaction of quality and production system design. In addition, an analytical model was proposed by Tempelmeier and Bürger (2000), where they analyzed the performance of flow production systems with imperfect production. Nevertheless, quality issues have in particular attracted much attention, following the series of works by Kim and Gershwin (2005, 2008) who introduced mathematical models to analyze the performance of manufacturing system also producing defective products. Their approach has been extended by Colledani and Tolio (2009, 2011) who developed mathematical methods to model manufacturing and inspection machines monitored by control charts. Even though these works are relevant, they are unfortunately focused on the determination of performance measures, and they have not paid attention to the control policy that governs the production system. A growing research area intends to deal with this matter of incorporating quality aspects in the production policy, as in Mhada et al. (2011), who studied the production control problem with defective products. They derived analytical expressions for the production threshold and for the optimal cost. Hajji et al. (2012) presented another application that includes defectives. They tackled the joint production control and product specification for an unreliable multiple-parts production system. These contributions notwithstanding, they assumed that the rate of defectives is constant during the whole lifetime of the machine. This is a restrictive assumption in real production. It is clear that production systems experience wear, usage, corrosion, etc, leading to progressive deterioration that may have severe effects on different aspects of the machine, such as quality, reliability, safety etc. Thus, the domain of deteriorating systems provides a useful framework for our research.

Generally, two kinds of approaches to the modeling of variations in the machine's conditions are commonly found in the existing literature on deteriorating systems. In the first approach, some authors, such as Lam (2004), use the number of failures as an indicator of the level of deterioration. In particular, he introduced a lifetime distribution that takes into account the effect of maintenance activities for a deteriorating system whose operating time after repair decreases. In another study, Wu and Clements-Croome (2006) succeeded to model complicated changing failure intensities, with the advantage of capturing the whole system's lifetime. The second approach utilizes the age of the machine as state feedback of the deterioration, for example, Lai and Chen (2005) studied the replacement policy of a two-unit system. The unit's failure rate increases with their age, and a failure rate interaction between units is also presented. Even combinations of both approaches are possible, as in Deyahem et al. (2011), who proposed a joint production and repair-replacement model, where the failure rate of the machine increases with its age, while the repair time increases with the number of failures. It is evident that the aforementioned papers addressed deterioration regardless of quality. By contrast, we conjecture in this research that deterioration may influence the quality of the parts produced. Considering the link with quality, we extend the concept of deterioration to study its effects on the control policy in further detail through age-based maintenance strategies.

In practice, preventive maintenance is widely applied in a number of industrial sectors to counter the effect of random failures and the loss of production. The effective implementation of preventive maintenance policies has been extensively studied as in Yulan et al. (2008) where the joint determination of preventive maintenance and production planning was presented. They considered an objective function, consisted of multiple objectives instead of a single objective as most of the literature. Furthermore Radhoui et al. (2010) develop a model for one machine system producing lots of product. In this model, they utilize the proportion of non-conforming units as decision variable to determine when to perform preventive maintenance. Also they define the size of the buffer stock. On their part, Dhouib et al. (2012) proposed a model that determines the production control and the age-based preventive maintenance policy that reduces the shift rate to the out of control state, where the system produces defective products. Recently, Rivera-Gómez et al. (2013) treated a deteriorating system where the quality of the parts decreases by the influence of the natural wear of the machine and imperfect repairs. In their model they defined an aging process through a set of operational states. In addition, they succeeded to determine the optimal production and repairs policies. As we can notice in the research works mentioned above, a few applications have dealt with preventive maintenance strategies for manufacturing systems producing defectives products. Nevertheless, a key observation is that none of them have considered the simultaneous effect of deterioration on the quality of the parts produced and the reliability of the system, and how their repercussions can be mitigated through preventive maintenance strategies. To an extent, our model addresses this drawback, extending previous models and assumptions.

It is particular worthwhile to mention that this research aims to deepen previous contributions (i.e., Radhoui et al. (2010), Deyahem et al. (2011)) presented in the literature review, by specifically seeking to introduce an integrated model that determines optimal strategies for a manufacturing system subjected to a combined effect of deterioration. In fact, the assertion is that the machine is severely affected by the twofold effect of deterioration, and the objective is to provide a wide overview of such effects that increase the failure intensity, and decrease the quality of the parts produced, something that has not yet been studied in the literature.

Since the machine dynamics are affected by deterioration, we use the age of the machine to denote its history. Markovian models are therefore, not appropriate; instead we formulate a Semi-Markov model. The deterioration decisions considered imply two types of maintenance; a major overhaul, which completely restores the machine, and preventive maintenance that partially renews the level of deterioration. The problem yields to the simultaneous determination of the production, preventive maintenance and overhaul strategies that minimize the total incurred cost over an infinite planning horizon, comprising the inventory, backlog, overhaul, preventive maintenance and defectives cost. An extensive sensitivity analysis is also conducted to illustrate the robustness and effectiveness of the proposed control policy.

The remainder of the paper is organized as follows. In section 3.2, the notations and system description are defined. The control problem formulation is presented in Section 3.3. The numerical techniques applied are detailed in Section 3.4. A numerical example is illustrated in Section 3.5, and a sensitivity analysis is conducted in Section 3.6 to illustrate the usefulness of the obtained control policy. Discussions are given in Section 3.7 and the paper is finally concluded in Section 3.8.

3.2 Notations and manufacturing system description

In this section, we introduce the notations used in this paper, and describe the manufacturing system under consideration.

3.2.1 Notations

The following defines the symbols and notations used in the present research:

$x(t)$	Inventory level at time t
$a(t)$	Age of the machine at time t
$u(t)$	Production rate of the manufacturing system at time t
d	Demand rate

$\xi(t)$	Mode of the machine at time *t*
$Q(\cdot)$	Transition rate matrix
$q_{\alpha\alpha'}(\cdot)$	Transition rate form mode α to mode α'
τ	Jump time of $\xi(t)$
$\beta(\cdot)$	Rate of defectives
ρ	Discount rate
$\gamma^{\xi(t)}(\cdot)$	Cost rate function
$h(\cdot)$	Inventory/backlog cost function
$J(\cdot)$	Expected discounted cost function
$v(\cdot)$	Value function
c^{+}	Incurred cost per unit of produced parts for positive inventory
c^{-}	Incurred cost per unit of produced parts for backlog
c_r	Constant repair cost
c_o	Overhaul cost
c_{pm}	Preventive maintenance cost
c_d	Cost of defectives
u_{max}	Maximum production rate of the manufacturing system
$\omega_p(\cdot)$	Control variable for the preventive maintenance
$\omega_{\bar{p}}$, $\omega_{\underline{p}}$	Maximal and minimal preventive maintenance rate
$\omega_o(\cdot)$	Control variable for the major overhaul
$\omega_{\bar{o}}$, $\omega_{\underline{o}}$	Maximal and minimal overhaul rate
π_i	Limiting probability at mode *i*
θ_f	Adjustment parameter for the failure rate
θ_d	Adjustment parameter for the rate of defectives

3.2.2 Manufacturing system description

We shall begin by stating that the manufacturing system analyzed in this paper concerns the case of a single machine producing one part-type. The machine can produce parts at different rates to satisfy a constant demand for products. These different rate options comprise the production decisions of the control policy. The main issue is, however, that the machine undergoes progressive deterioration which severely degrades its reliability and the quality of the parts produced. This is clearly an undesirable feature. Another significant consideration is that a couple of maintenance activities are available to mitigate the effects of deterioration. A major overhaul consists of a time-consuming and expensive repair that completely restores the machine, mainly its reliability and quality of parts, to initial conditions. Meanwhile preventive maintenance is less expensive, but it only makes a partial restoration of the level of deterioration of the machine. The demand for products is satisfied exclusively by conforming parts, and so is important to maintain a low rate of defectives to reduce the inherent defectives cost.

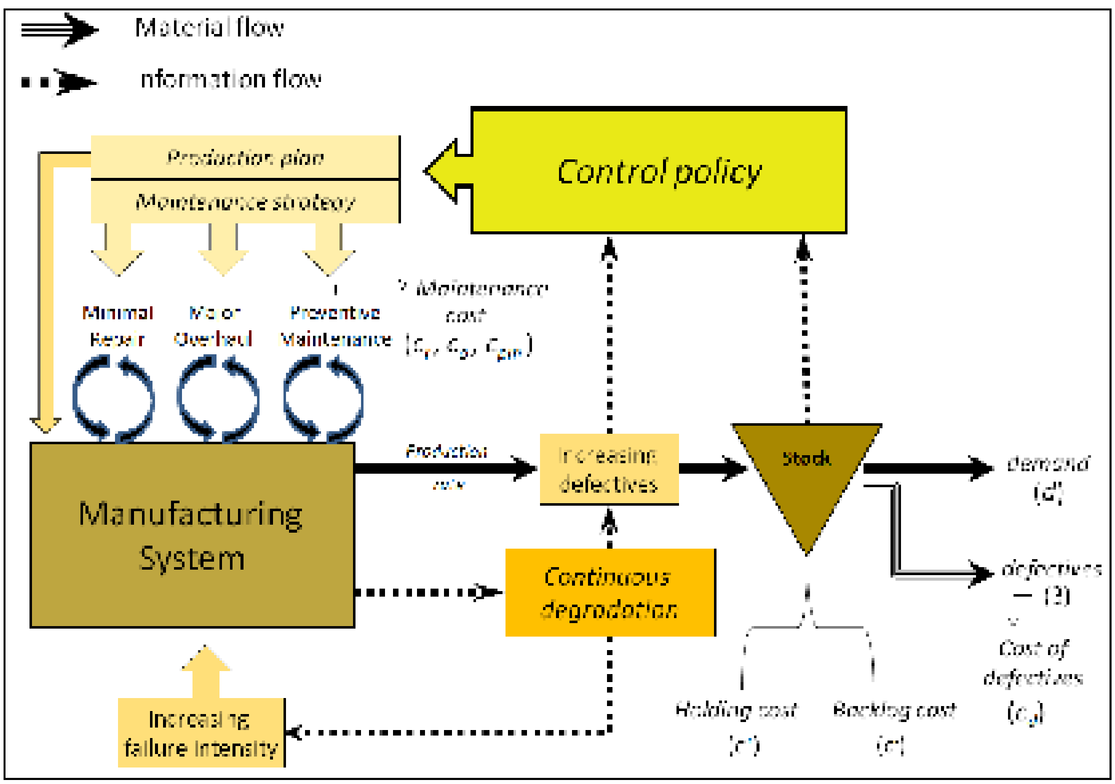


Figure 3.1 Manufacturing system considered

Figure 3.1 illustrates the set of characteristics considered in our model. To make the description complete the objective of the model is to determine, simultaneously, the optimal production plan and the preventive maintenance and overhaul strategies that minimize the total cost. In this case the incurred total cost consists of the inventory, backlog, preventive maintenance, overhaul, repair and defectives costs.

3.3 Control problem formulation

In the following subsections, we will describe the problem statement, the deterioration expressions and optimality conditions for our manufacturing system.

3.3.1 Problem statement

The aim of this subsection is to develop a stochastic dynamic programming model based on the optimal control theory that considers a twofold effect of deterioration as previously mentioned in section 3.2. The system is subject to a number of random events (failures and repairs) and a couple of controlled actions (overhaul and preventive maintenance), thus this defines four different modes denoted by the random variable $\{\xi(t), t \geq 0\}$. The machine's modes can be classified as operational $\xi(t) = 1$, at failure $\xi(t) = 2$, under overhaul $\xi(t) = 3$, and under preventive maintenance $\xi(t) = 4$. Evidently, the mode of the machine at time t denotes a continuous-time discrete state stochastic process $\xi(t) \in \Omega = \{1,2,3,4\}$ such that:

$$\xi(t) = \begin{cases} 1 & \text{operational} \\ 2 & \text{failure} \\ 3 & \text{overhaul} \\ 4 & \text{preventive maintenance} \end{cases} \tag{3.1}$$

The machine may be at any of the four modes over an infinite horizon, as described in the following transition diagram:

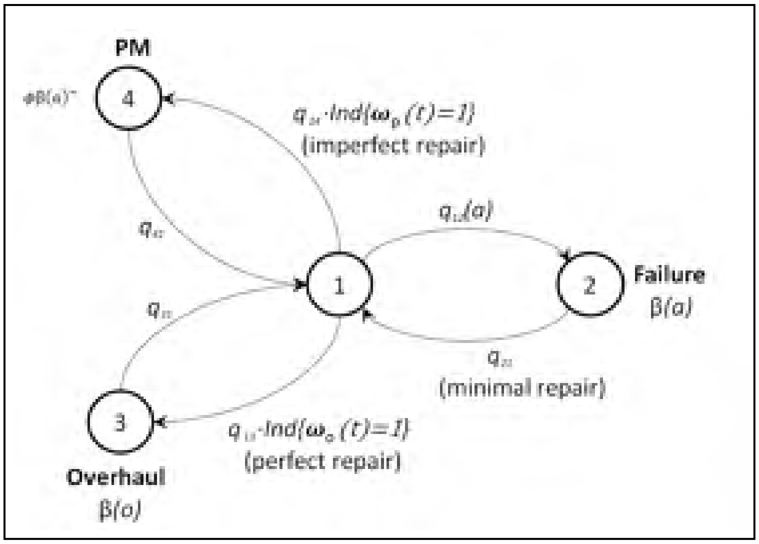


Figure 3.2 State transition diagram of the proposed model

Let d be the constant demand rate to be satisfied, and $\mathrm{u}(t)$ the production rate at the time t, with $\mathrm{u}(t) = 0$, if the machine is not operational (i.e., at failure, under overhaul or under preventive maintenance). When the machine is operational at time t, the production rate $\mathrm{u}(t)$ must satisfy the capacity constraint:

$$0 \leq u(\cdot) \leq u_{max} \quad (3.2)$$

where u_{max} is the maximum production rate. To include the deterioration phenomenon in the model, in what follows we assume that deterioration has a twofold effect on the system, which can be observed on: a) the quality of the parts produced (implying an increasing rate of defectives β), and b) the reliability of the system (reducing the mean time to failure, MTTF, defined as the mean length of time that the machine is expected to last in operation). Thus, when the machine is operational, with $\xi(t) = 1$, it produces a mix of conforming and defective products denoted by the rate of defectives β. However, when the machine is at failure, $\xi(t) = 2$, the corrective maintenance conducted is not perfect, in fact a minimal repair that restores the system to as bad-as-old-conditions (ABAO) is carried out, where both β and MTTF remain at the same values before repair. And when the machine is under overhaul, $\xi(t) = 3$, a perfect maintenance that completely counters the effects of deterioration is performed, restoring the system (both β and MTTF) to as-good-as-new conditions (AGAN), since the deterioration increases the rate of defectives and decreases the MTTF. Clearly these two maintenance options define two extreme cases between minimal and perfect repair. A more general option is when the system is under preventive maintenance, $\xi(t) = 4$, since an imperfect maintenance is performed, restoring the system (mainly β and MTTF) to somewhere between AGAN and ABAO conditions.

In modeling terms, our formulation stresses the fact that the process $\xi(t)$ is characterized by transition rates $q_{\alpha\alpha'}(\cdot)$, $\alpha, \alpha' \in \Omega$, and due to deterioration, the time that the system remains operational decreases progressively, ultimately resulting in a Semi-Markov model. For this reason we need to employ the machine's history to determine appropriate control strategies. The history is denoted by the age of the machine $\mathrm{a}(t)$, therefore the conditional probability that the Semi-Markov process will stay in the same state α or will make a transition to state α' within the next δt time units, incorporates a feedback on the age $\mathrm{a}(t)$, as presented in the following two conditions:

Condition 1:

$$P[\xi(t+\delta t) = \alpha | \xi(t) = \alpha, \quad x(t) = x, \quad \mathrm{a}(t) = \mathrm{a}, \; u(t) = u\,] = 1 + q_{\alpha\alpha}(\mathrm{x}, \mathrm{a}, \mathrm{u})\delta t + o(x, \mathrm{a}, \delta t) \tag{3.3}$$

Condition 2 :

$$P[\xi(t+\delta t) = \alpha' | \xi(t) = \alpha, \quad x(t) = x, \quad \mathrm{a}(t) = \mathrm{a}, \; u(t) = u\,] = q_{\alpha\alpha'}(\mathrm{x}, \mathrm{a}, \mathrm{u})\delta t + o(x, \mathrm{a}, \delta t) \tag{3.4}$$

with:

$$q_{\alpha\alpha}(\mathrm{x}, \mathrm{a}, \mathrm{u}) = -\sum_{\alpha \neq \alpha'} q_{\alpha\alpha'}(\mathrm{x}, \mathrm{a}, \mathrm{u}) \qquad \lim_{\delta t \to 0} \frac{o(x, \mathrm{a}, \delta t)}{\delta t} = 0$$

$$q_{\alpha\alpha'}(\mathrm{x}, \mathrm{a}, \mathrm{u}) \geq 0, \qquad \forall\, \alpha, \alpha' \in \Omega\colon \alpha \neq \alpha'$$

In principle, we can improve the performance of the manufacturing system by the use of the decision variables $\omega_o(\cdot)$ and $\omega_p(\cdot)$, which allow us to control the transition to the major overhaul or preventive maintenance, respectively. To this end, the decision to send the machine to these maintenance activities is undertaken when the machine is operational. For instance, when we set the overhaul decision variable to $[\omega_o(t) = 1]$, the reciprocal of $[\,q_{13} \cdot \{\omega_o(t) = 1\}]$ represents the expected delay time between the decision to perform the overhaul and the effective switch from the operation mode to the overhaul mode. To say it in other words, the reciprocal corresponds to the delay between the call of a technician and its arrival. Meanwhile the machine remains operational when $\{\omega_o(t) = 0\}$. A similar delay is represented by the reciprocal of $\left[\,q_{14} \cdot \{\omega_p(t) = 1\}\right]$, when the machine is sent to preventive maintenance. In addition, it follows that $\omega_o(\cdot)$ and $\omega_p(\cdot)$ define two binary variables that synchronize properly the transitions to the maintenance options available (overhaul and preventive maintenance), as indicated in the following expressions:

$$\omega_o(t) = \begin{cases} 1, if\ Overhaul\ is\ conducted \\ 0, \quad\quad otherwise \end{cases}, \qquad \omega_p(t) = \begin{cases} 1, if\ PM\ is\ conducted \\ 0, otherwise \end{cases} \tag{3.5}$$

The transitions matrix of the Semi-Markov chain $\xi(t)$, is denoted by $Q(\cdot) = \{q_{\alpha\alpha'}(\cdot)\}$, and it depends on the decision variables $\omega_o(\cdot), \omega_p(\cdot)$, as follows:

$$Q(\omega_o, \omega_p) = \begin{pmatrix} q_{11} & q_{12}(a) & q_{13} \cdot Ind\{\omega_o(t) = 1\} & q_{14} \cdot Ind\{\omega_p(t) = 1\} \\ q_{21} & q_{22} & 0 & 0 \\ q_{31} & 0 & q_{33} & 0 \\ q_{41} & 0 & 0 & q_{44} \end{pmatrix} \tag{3.6}$$

with the indicator function defined as:

$$Ind\{\Xi(t)\} = \begin{cases} 1\ if\ \Xi(t)\ is\ true \\ 0\ \ otherwise \end{cases}$$

where $\Xi(t)$ indicates a given proposition. It follows that the control policy of the model is defined by the control variables $u(\cdot)$, $\omega_o(\cdot)$ and $\omega_p(\cdot)$. Hence, the set of the feasible control policies $\Gamma(\alpha)$ which includes ($u(\cdot), \omega_o(\cdot)$ and $\omega_p(\cdot)$), depends on the stochastic process $\xi(t)$ and is given by:

$$\Gamma(\alpha) = \{(u(\mathrm{a},\cdot), \omega_o(\mathrm{a},\cdot), \omega_p(\mathrm{a},\cdot)) \in \mathrm{R}^3, \quad 0 \leq u(\mathrm{a},\cdot) \leq u_{max}, \quad \omega_o(\mathrm{a},\cdot) \in \{0,1\}, \quad \omega_p(\mathrm{a},\cdot) \in \{0,1\}\} \tag{3.7}$$

In the stochastic control setting, normally the dynamics for the stock variable $x(\cdot)$ disregards the existence of defective products, as presented in Gershwin (2002). Nevertheless, our emphasis for this dynamics is to include two distinctive characteristics: i) the existence of defective products, and ii) the influence of deterioration. Hence, in our model the system dynamics evolve based on the following differential equation:

$$\dot{x}(t) = u(t) - d(1 + \beta(a)), \qquad x(0) = \mathrm{x}_0, \tag{3.8}$$

with x_0, as the given initial inventory level, where $x(t) > 0$ represents a positive inventory, and $x(t) \leq 0$ denotes the backlog of products. While $\beta(a)$ represents the rate of defectives as function of the age of the machine $\mathrm{a}(t)$. From the previous expression (3.8), we notice that the impact of defective products is to increase the demand to a higher level, as the quality of the machine deteriorates, to ensure that the demand is satisfied with flawless product. Moreover we define the age of the machine at time t, as an increasing function of its production rate since its last restart, then the cumulative age $\mathrm{a}(t)$ is the solution of the following differential equation:

$$\dot{\mathrm{a}}(t) = k_1 \cdot u(t), \qquad \mathrm{a}(\mathrm{T}) = 0, \tag{3.9}$$

where k_1 is a given positive constant, and T represents the last restart time of the machine. In the next section, we provide further details about the expressions that complement the deterioration modeling, in special we clarify how exactly the age of the machine is linked with the effects of deterioration (namely, the increase of the rate of the defectives β, and the decrease of the MTTF). In closing this section, we note that the mathematical form of the problem actually entails that at any given instant of time t, the system is characterized by the following state variables:

- The machine mode, $\xi(t)$
- The stock level, $x(t)$
- The age of the machine, $\mathrm{a}(t)$

Consequently, the interpretation of the information provided by the vector $\left(\xi(t), x(t), \mathrm{a}(t)\right)$ is fundamental to optimally control the proposed manufacturing system.

3.3.2 Deterioration modeling

In formulating the problem, details of how we model the deterioration phenomenon must be shown. We therefore, concentrate now on the expressions applied in this respect. At first glance, it appears that productive systems are subject to deterioration because of several factors, including usage, wear, aging, and so forth. Some authors have dealt with this matter, for example, Love et al. (2000) proposed that the failure rate depends on the age of the machine, and so they used that age to determine repair activities that reset the failure rate of the system. In the same direction, Dehayem et al. (2011) suggested that the deterioration of the machine is denoted by its age and number of failures, and the effect of deterioration is reflected at increasing several transitions rates. These articles lead to the identification of an age-deterioration relationship, which serves to propose in this paper that the trajectory of the failure rate of the machine q_{12}, is described by an increasing function of its age $\mathrm{a}(t)$, because of the link between the age of the machine and its deterioration, as indicated in the following expression:

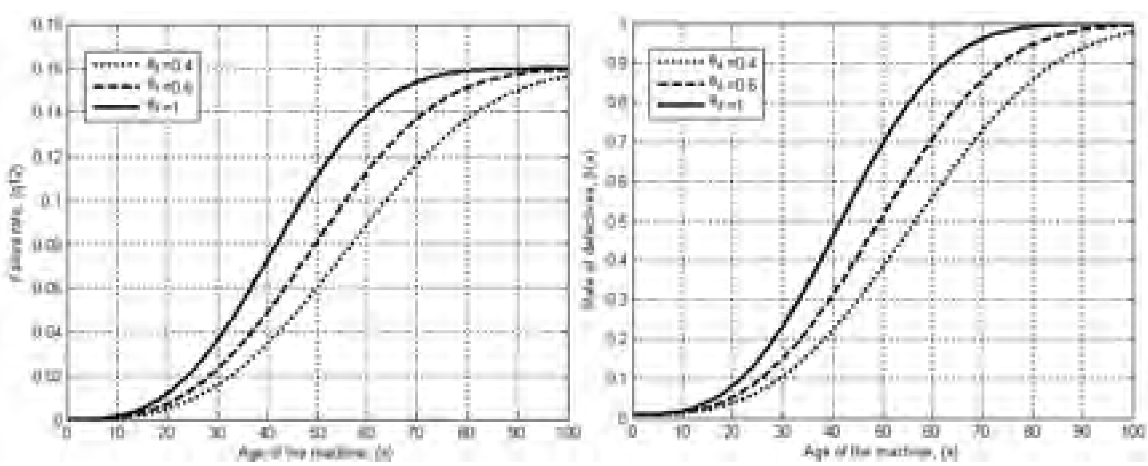
$$q_{12}(\mathrm{a}) = q_1 + q_2\left(1 - e^{-k_2 \cdot \theta_\mathrm{f} \cdot [\mathrm{a}(t)^3]}\right) \tag{3.10}$$

with:

$$0 \leq \theta_\mathrm{f} \leq 1$$

the parameter θ_f is used to adjust the trend of the failure rate, q_1 is the value of the transition q_{12} at initial conditions, q_2 is the boundary considered in the deterioration, and k_2 is a given constant. Expressions such as Equation (10) denotes a machine age increasing failure rate, and have been used by Boukas and Haurie (1990), Kenne and Gharbi (1999), and Gharbi and Kenne (2005) to model manufacturing systems with decreasing reliability. The role of the parameter $a(t)^n$ is to accelerate the variation of the increasing function. Moreover, we introduce the parameters θ_f and θ_d (in Expression 3.11) to provide with a different speed of deterioration to the failure rate and to the rate of defectives. Therefore, these parameters serve us to adjust separately the effects of deterioration on the production system.

The inverse of the transition $q_{12}(a)$ denotes the $MTTF(a)$ in function of the age a, indicating the effect of the age of the machine on more frequent failures. A practical concern is that we can vary the parameter θ_f to adjust the trajectory of the failure rate to a specific system, as presented in Figure 3.3a (where we use $q_1 = 0.01$, $q_2 = 0.16$ and $k_2 = 15x10^{-6}$).



(a) For the failure rate, $q_{12}(a)$ (b) For the rate of defectives, $\beta(a)$

Figure 3.3 Trend of deterioration

Another important aspect is the fact that deterioration also has an effect on the quality of the parts produced. A number of authors have proposed the link between deterioration and quality, such as Kim and Gershwin (2008) who suggested that in modeling the wear of the machine, it can also be used to represent different quality yields. Additionally, Colledani and Tolio (2011) claimed that the existence of a degrading process in production systems may have a continuous deterioration on the quality of parts. These references permit to define an inter-relationship between deterioration and quality. In this way, based on the links of age-deterioration and deterioration-quality, claiming that in our model the rate of defectives β can be modeled by an increasing function of the age of the machine $\mathrm{a}(t)$, is fairly straightforward. In this case we propose an increasing rate of defectives given by:

$$\beta(\mathrm{a}) = b_1 + b_2\left(1 - e^{-k_3 \cdot \theta_\mathrm{d} \cdot [\mathrm{a}(t)^3]}\right) \tag{3.11}$$

with:

$$0 \leq \theta_\mathrm{d} \leq 1$$

where θ_d denotes the adjustment parameter for the trend of the rate of defectives as illustrated in Figure 3.3b, (with $b_1 = 0.01$, $b_2 = 0.99$ and $k_3 = 15x10^{-6}$), b_1 is the value of the rate of defectives at initial conditions, b_2 is the boundary considered in the quality deterioration, and k_3 is a given constant. For the current problem, the parameters θ_f and θ_d, captures one essential feature, namely, that their role is to emphasize whether the deterioration has a stronger effect on quality (rate β), or on the reliability (failure rate q_{12}). The analysis of maintenance service and quality data, is the source to determine the value of the constants needed in Functions 3.10 and 3.11, and it can be useful, any historical information that indicate when the machine failed, how much time it was needed to repair the machine, and if it is observed a pattern of the deterioration that influence certain features of the machine. Additionally, well known increasing functions such as the Weibull distribution can be modeled by selecting suitable values for k_2 and k_3.

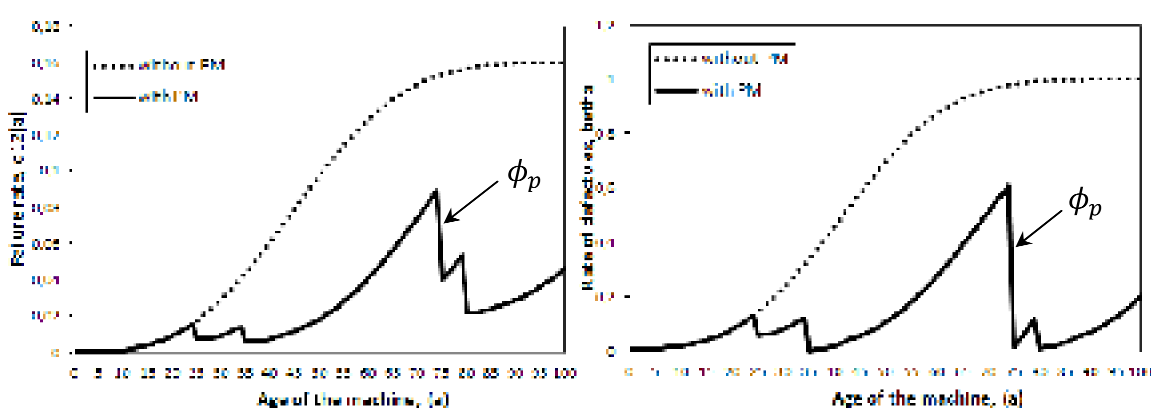
In regard to the preventive maintenance strategy, we state that it rejuvenates the age of the machine of an amount proportional to its age before preventive maintenance, in line with the so-called Arithmetic Reduction of Age (ARA) presented in Doyen and Gaudoin (2004). Hence, we propose the following expression to model the benefit of preventive maintenance:

$$a^{+} = a^{-} - \phi_p a^{-} \quad (3.12)$$

where ϕ_p denotes the efficiency of preventive maintenance and satisfies the condition: $0 < \phi_p < 1$, whereas a^{-} is the age of the machine before preventive maintenance, and a^{+} is the age after preventive maintenance. In practical terms, Expression (3.12) indicates that preventive maintenance reduces the wear out speed of the machine by an amount depending on the preventive maintenance efficiency ϕ_p, and this reduction has a twofold effect on the failure rate and the rate of defectives. The benefit of preventive maintenance also can be altered with the parameters θ_f and θ_d. An additional trait of Expression (3.12) is that we can model different types of maintenance according to the value applied in the preventive maintenance efficiency ϕ_p, as follows:

- $\phi_p = 1$ perfect maintenance, q_{12} and β are pulled back to AGAN conditions.
- $0 < \phi_p < 1$ imperfect maintenance, q_{12} and β reduce partially.
- $\phi_p = 0$ minimal maintenance, q_{12} and β remain in ABAO conditions.

We proceed by presenting in Figure 3.4a, a possible reduction of the failure rate when the efficiency of preventive maintenance is set to ϕ_p=0.6.



(a) On the failure rate, $q_{12}(a)$ (b) On the rate of defectives, $\beta(a)$

Figure 3.4 Benefit of preventive maintenance

Figure 3.4a enables us to realize that preventive maintenance indeed provides a remarkable benefit. The respective effect for the rate of defectives is illustrated in Figure 3.4b. In general, both figures present the same behavior, reducing partially its deterioration level with every preventive maintenance. Even though, we use the same efficiency ϕ_p for both Figures 3.4, we observe that the trajectories are not exactly the same; the differences observed are explained by the values of the parameters θ_f, θ_d, k_2 and k_3 applied in every figure.

3.3.3 Cost function and Optimality conditions

For the sake of completeness, we will now formally declare that the state of the systems has a hybrid condition, formed by a discrete component $\xi(t)$, and continuous components, defined by $x(t)$ and $a(t)$, denoting the vector $\big(\xi(t), x(t), \mathrm{a}(t)\big)$. The cost function $\gamma^{\xi(t)}(\cdot)$ of the model for each initial condition $\big(\xi(t), x(t), \mathrm{a}(t)\big)$, is defined as follows:

$$\gamma^{\xi(t)}(\alpha, x, \mathrm{a}) =$$
$$c^+x^+ + c^-x^- + c_r \cdot Ind\{\xi(t) = 2\} +$$
$$c_o \cdot Ind\{\xi(t) = 3\} + c_{pm} \cdot Ind\{\xi(t) = 4\} + c_d \cdot [\beta(\mathrm{a}) \cdot d] \qquad (3.13)$$

with:

$$x^+ = max(0, x)$$
$$x^- = max(-x, 0)$$

where c^+ and c^- are given constants used to penalize the inventory and backlog of parts, every time that the machine is sent to maintenance, it is incurred a cost depending of the maintenance option conducted. We denoted c_r as the repair cost, c_o is the overhaul cost, c_{pm} is the preventive maintenance cost, and c_d is the defectives cost which is originated by the additional inspection and handling costs related to defective products. The objective of our model is to determine a control policy that minimizes the integral of the following expected discounted cost:

$J(\alpha, x, \mathrm{a}) =$

$$E\left[\int_0^\infty e^{-\rho t}\gamma^{\xi(t)}(\cdot)dt \mid \quad \xi(0) = \alpha, x(0) = x, \mathrm{a}(0) = \mathrm{a}\right], \qquad \forall\left(u, \omega_o, \omega_p\right) \in \Gamma(\alpha) \qquad (3.14)$$

where ρ is the discount rate, and (α, x, a) are the initial conditions of the state variables. The optimal decisions for this problem (u^*,ω_o^*,ω_p^*), minimizes $J(\cdot)$, and simultaneously defines the production, overhaul and preventive maintenance rates, as a function of the mode of the system, the inventory level and the age of the machine. The value function of the problem is defined as follows:

$v(\alpha, x, \mathrm{a}) =$

$$\inf_{(\boldsymbol{u},\omega_o,\omega_P)\in\Gamma(\alpha)} J\left(\alpha, x, \mathrm{a}, u, \omega_o, \omega_p\right), \quad \forall\ \alpha \in \Omega, x\ \in\ R,\ \mathrm{a} \in R \qquad (3.15)$$

The value function $v(\alpha, x, \mathrm{a})$ denotes the optimum value of the cost function (3.14) when the optimal control policy (u^*,ω_o^*,ω_p^*) is applied. One important point to note is that $v(\alpha, x, \mathrm{a})$ satisfies specific properties known as *optimality conditions* which can be derived regarding the *principle of optimality*. Let for instance $v(\cdot, t)$ denote a cost-to-go function at time t, assuming that we know the best possible control trajectory during the time interval $[t, \infty]$, but we know nothing in the interval $[0, t]$, hence we can break up Equation (3.15) into two parts as follows:

$v(\alpha(0), x(0), \mathrm{a}(0), 0\ ) =$

$$\inf_{\substack{u(t),\omega_o(t),\omega_p(t)\\ 0\le t\le\infty}} E\left\{\int_0^t e^{-\rho t}\gamma^{\xi(t)}(\cdot)dt + \int_t^\infty e^{-\rho t}\gamma^{\xi(t)}(\cdot)dt\ \middle|\ \alpha(0), x(0), \mathrm{a}(0)\right\} \qquad (3.16)$$

To handle the randomness of α, the expectation operator E is needed. We can considerably simplify Equation (3.16), noting that the second integral in the interval $[t, \infty]$ is the value function $v(\alpha(t), x(t), \mathrm{a}(t), t\ )$ and reducing the discounted rate. Moreover assuming that the value function is differentiable allows us to apply its full derivative. Therefore, after extensive manipulations, we have:

$$\rho v(\alpha, x, \mathrm{a}, t) - \frac{\partial v}{\partial t}[\alpha, x, \mathrm{a}, t] = \tag{3.17}$$
$$\inf_{u(t),\omega_o(t),\omega_p(t)} \left\{ \gamma^{\alpha}\left[\alpha, x, \mathrm{a}, u, \omega_o, \omega_p\right] + \frac{\partial v}{\partial x}[\alpha, x, \mathrm{a}, t]\dot{x} + \frac{\partial v}{\partial \mathrm{a}}[\alpha, x, \mathrm{a}, t]\dot{a} + \sum_{\alpha'} v[\alpha', x, \mathrm{a}, t]\,\lambda_{\alpha'\alpha} \right\}$$

To simplify matters, since the time horizon is infinite, and a steady-state distribution exists for α, we can state that Equation (3.17) is independent of t, thus eliminating t and $\frac{\partial v}{\partial t}$. By replacing the summation term by the generator $\mathrm{Q}(\cdot) = \{\mathrm{q}_{\alpha\alpha'}(\cdot)\}$, finally Equation (3.17) is reduced to:

$$\rho v(\alpha, x, \mathrm{a}) = \tag{3.18}$$
$$\min_{(u,\omega_o,\omega_p)\,\in\,\Gamma(\alpha)} \left\{ \gamma^{\alpha}\left[\alpha, x, \mathrm{a}, u, \omega_o, \omega_p\right] + \frac{\partial v}{\partial x}[\alpha, x, \mathrm{a}]\dot{x} + \frac{\partial v}{\partial \mathrm{a}}[\alpha, x, \mathrm{a}]\dot{\mathrm{a}} + Q(\cdot)v[\alpha, x, \varphi(\xi, \mathrm{a})](\alpha) \right\}$$

with:

$$\varphi(\xi, \mathrm{a}) = \begin{cases} 0 & if\ \ \xi(\tau^+) = 1 \quad and\ \ \xi(\tau^-) = 3 \\ (1-\phi_p)\mathrm{a}(\tau^-) & if\ \ \xi(\tau^+) = 1 \quad and\ \ \xi(\tau^-) = 4 \\ \mathrm{a}(\tau^-) & otherwise \end{cases}$$

where $\xi(t)= \alpha \in \Omega$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial \mathrm{a}}$ are the partial derivatives of the value function $v(\cdot)$; τ denotes the first jump time of $\xi(t)$, and $\varphi(\xi, \mathrm{a})$ defines a reset function that describes the benefit of the maintenance activities available in the model. The relevance of Expression (3.18) stems from its capacity to define the fundamental manufacturing system control equations so-called Hamilton-Jacobi-Bellman (HJB) equations, which are essentially important because they lead to the optimal feedback control $(\mathrm{u}^*, \omega_o^*, \omega_p^*)$. Further details about the procedure to obtain HJB equations, can be consulted in Gershwin (2002). Nevertheless, the major inherent difficulty in HJB equations is that analytical solutions are cumbersome to obtain, instead numerical methods have been employed since Boukas and Haurie (1990). In the next section, we will look at the approach we adopted, in more detail.

3.4 Numerical approach

The present section provides further detail of the numerical method applied to solve the HJB equations (3.18) presented in section 3.3. The chief difficulty is to find an analytical solution of the HJB equations due to their complex structure which involves the solution of a coupled set of partial differential equations. We overcome such a difficulty by applying of a numerical method based on the Kushner technique. The main idea behind this approach is to use an approximation scheme for the gradient of the value function $v(\alpha, x, \mathrm{a})$, which replaces the unbounded domain of the continuous variables (x, a) by a large but bounded domain, defined in a finite grid of discrete variables G_{xa}. Thus, the Kushner technique approximates the value function $v(\alpha, x, a)$ by a discrete function $v^h(\alpha, x, a)$, and the gradients $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial a}$ are approximated by:

$$\frac{\partial v}{\partial x}(\alpha, x, \mathrm{a}) = \begin{cases} \frac{1}{h_x}[v^h((\alpha, x + h_x, \mathrm{a}) - v^h((\alpha, x, \mathrm{a})] & if \quad \dot{x} \geq 0 \\ \frac{1}{h_x}[v^h((\alpha, x, \mathrm{a}) - v^h((\alpha, x - h_x, \mathrm{a})] & if \quad \dot{x} < 0 \end{cases} \tag{3.19}$$

and:

$$\frac{\partial v}{\partial \mathrm{a}}(\alpha, x, \mathrm{a}) = \frac{1}{h_a}[v^h((\alpha, x, \mathrm{a} + h_{\mathrm{a}}) - v^h(\alpha, x, \mathrm{a})] \tag{3.20}$$

where h_x and h_{a} denote the length of the finite differential interval of the variables x and a, respectively. More details about the Kushner technique can be consulted in Kushner and Dupuis (1992) and Kenné et al. (2003). The HJB Equation (3.18) can be expressed as a function of $v^h(\alpha, x, \mathrm{a})$ with step size h_x and h_{a} on the grid $G_{x\mathrm{a}}$, and this implies discrete dynamic equations for every mode of the machine. Eventually, the solution of the discrete form of the HJB equations will tend to the value of $v^h(\alpha, x, \mathrm{a})$ as h_x and h_{a} tends to zero, and successively, it will also provide the structure of the optimal control policy. In this case the discrete equation derived by the numerical approach is defined as follows:

$$v^h(\alpha, x, \mathrm{a}) = \tag{3.21}$$

$$\min_{(u,\omega_o,\omega_p)\,\in\,\Gamma(\alpha)} \left[\left(\rho + |q_{\alpha\alpha}| + \frac{|r|}{h_x} + \frac{k_1 \cdot u}{h_{\mathrm{a}}} \right)^{-1} \left(\gamma^{\alpha}(\cdot) + v^h(\alpha, \mathrm{x}, \mathrm{a} + h_{\mathrm{a}}) \frac{k_1 \cdot u}{h_{\mathrm{a}}} \right. \right.$$

$$+ v^h(\alpha, x + h_x, \mathrm{a}) \frac{|r|}{h_x} \mathrm{Ind}\{r \geq 0\} + v^h(\alpha, x - h_x, \mathrm{a}) \frac{|r|}{h_x} \mathrm{Ind}\{r < 0\}$$

$$\left. \left. + \sum_{\alpha' \neq \alpha} q_{\alpha\alpha'}(\cdot)\, v(\alpha', x, \varphi(\xi, \mathrm{a})) \right) \right] \qquad \forall\ \alpha \in \Omega,\ x \in R,\ \mathrm{a} \in R$$

where r= $u(t) - d\big(1 + \beta(\mathrm{a})\big)$. From a mathematical point of view, the discrete optimality conditions (3.21) obtained, replace the original infinite horizon problem by a discrete-time and discrete-state Semi-Markov decisions process that is much easier to solve. Actually, we use the policy improvement method to determine the optimal policy of the problem.

3.5 Numerical example

Through the following section, a numerical example is conducted as an illustration of the manufacturing system presented in section 3.2. As mentioned previously, the system capacity is described by a four state Semi-Markov process with mode in $\Omega = \{1,2,3,4\}$. The objective of the numerical example is to analyze the effectiveness of the simultaneous production, overhaul and preventive maintenance policies for a deteriorating system with feedback on the age of the machine and the stock level. We consider the discrete grid $G_{x\mathrm{a}}$ that defines the computational domain for the inventory level *x*, and the age of the machine a, as follows:

$$G_{x\mathrm{a}} = \{(\mathrm{x}, \mathrm{a}) : -10 \leq \mathrm{x} \leq 20, \quad 0 \leq \mathrm{a} \leq 100\} \tag{3.22}$$

We define the value of the required parameters of Equation (3.9) as: k_1= 0.1, for Equation (3.10) as: $k_2 = 15x10^{-6}, q_1 = 0.0001$, $q_2 = 0.16$ and for Equation (3.11) as: $k_3 = 15x10^{-6}$, $b_1 = 0.01$ and $b_2 = 0.99$. Applying these values we can model a wide interval for the rate of defectives and the MTTF. This will facilitate the analysis of several scenarios, from AGAN conditions, until extreme scenarios with high levels of defective products and higher failures intensities. The transition rate from the failure mode to the operational mode is defined as constant with value $q_{21} = 1$, and the transitions rates from overhaul and from preventive maintenance to the operational mode are also constant, with $q_{31} = 3.5$ and $q_{41} = 5$, respectively. Note that the transitions from the operational mode to overhaul or to preventive maintenance imply a shorter delay, with values: $q_{13} = 12$ and $q_{14} = 12$. In addition, we set $u_{max} = 14$, this parameter indicates the number of produced parts per unit of time, and $d = 6$ denotes the demand of product per unit of time. One should remark that the manufacturing system satisfies the condition of feasibility:

$$u_{max} \cdot \pi_1 > d[1 + \beta(a)] \tag{3.23}$$

where π_1 denotes the limiting probability for the operational mode, which in this case is computed as follows:

$$\pi_1 = \frac{1}{1 + \frac{q_{12}}{q_{21}} + \frac{q_{13}}{q_{31}} + \frac{q_{14}}{q_{41}}} \tag{3.24}$$

It turns out that the feasibility condition guarantees the demand satisfaction of products even in the worse scenarios, which are observed when both β and q_{12} are at their peak. The rest of the parameters needed in the numerical example are presented in Table 3.1.

Table 3.1 Parameters for the numerical example

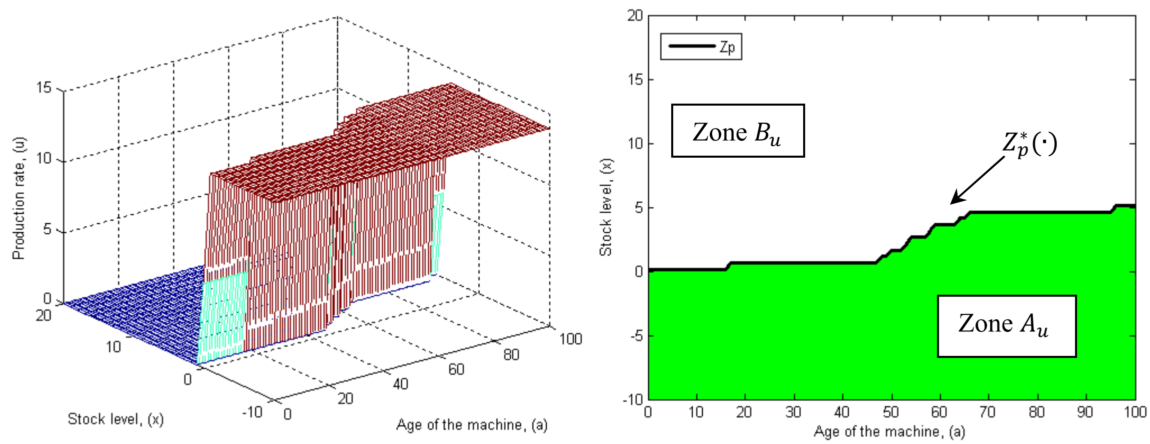
Parameter:	c+ ($/products/time units)	c- ($/products/time units)	c_r ($/repair)	c_o ($/overhaul)	c_{pm} ($/PM)
Value:	11	150	7	10	5
Parameter:	c_d ($/products)	h_x	h_a	ρ	ϕ_p
Value:	2	0.5	0.5	0.9	0.6
Parameter:	θ_d	θ_f	$\omega_{\underline{o}}$	$\omega_{\bar{o}}$	u_{max} (product/time units)
Value:	0.6	0.7	0	1	14
Parameter:	$\omega_{\underline{p}}$	$\omega_{\bar{p}}$	d (products/time units)		
Value:	0	1	6		

The following results were obtained with the data presented in Table 3.1, and are analyzed throughout this section to clearly illustrate the structure of the joint production, overhaul and preventive maintenance policies that optimally control the manufacturing systems of interest.

3.5.1 Production Policy

We concentrate first on the optimal production policy $u^*(\alpha, x, a)$, which indicates the production rate for a certain stock level $x(t)$ and age of the machine $a(t)$. Based on the numerical results presented in Figure 3.5a, the production policy defines three control rules, where the production rate is set to u_{max}, d and 0. More specifically these rules state that: a) If the stock level is inferior to the corresponding optimal stock level $Z_p^*(\cdot)$, then the production rate should be set to the maximum rate. b) Once the stock level is equal to the optimal threshold level, the production rate should be set to the demand rate. c) If the current stock level exceeds the optimal stock level, then the system does not produce at all, and the production rate is set to zero.

From these rules, one can clearly observe an extra characteristic in Figure 3.5a, where the consequence of include deterioration in the model, has the impact of progressively increasing the production threshold. Specifically, we observe that as the age of the machine increases, it also increases the failure rate and the rate of defectives, then the production threshold increases to ensure the demand satisfaction with flawless products. We use the boundary of the optimal production policy presented in Figure 3.5b to better illustrate its pattern, which serves to more easily identify the optimal stock level $Z_p^*(\cdot)$. From this Figure 3.5b we identify two zones: The zone A_u, where the optimal production policy recommends producing at maximum rate to reach the hedging point $Z_p^*(\cdot)$, and the zone B_u, where the recommendation is to not produce at all.



a) Production rate versus stock level, and age of the machine

b) Production Trace $Z_p^*(\cdot)$

Figure 3.5 Optimal production policy

Examining Figures 3.5a and 3.5b, we see that the optimal stock level depends directly on the level of deterioration of the machine, denoted by its age a(t). Consequently, the corresponding production policy is given by the following machine age-dependent hedging point policy:

$$u(1, x, \mathrm{a})^* = \begin{cases} u_{max} & if\ x(t) < Z_p^*(\cdot) \\ d & x(t) = Z_p^*(\cdot) \\ 0 & x(t) > Z_p^*(\cdot) \end{cases} \tag{3.25}$$

where $Z_p^*(\cdot)$ is the function that defines the optimal production threshold at the operational mode as illustrated in Figure 3.5b. This production policy highlights the fact that the optimal stock level $Z_p^*(\cdot)$ is not fixed. In fact it changes depending on the deterioration of the machine. This means that the number of parts to hold in inventory to hedge against more frequent breakdowns and increasing defectives is adjusted to higher values as the deterioration of the machine increases.

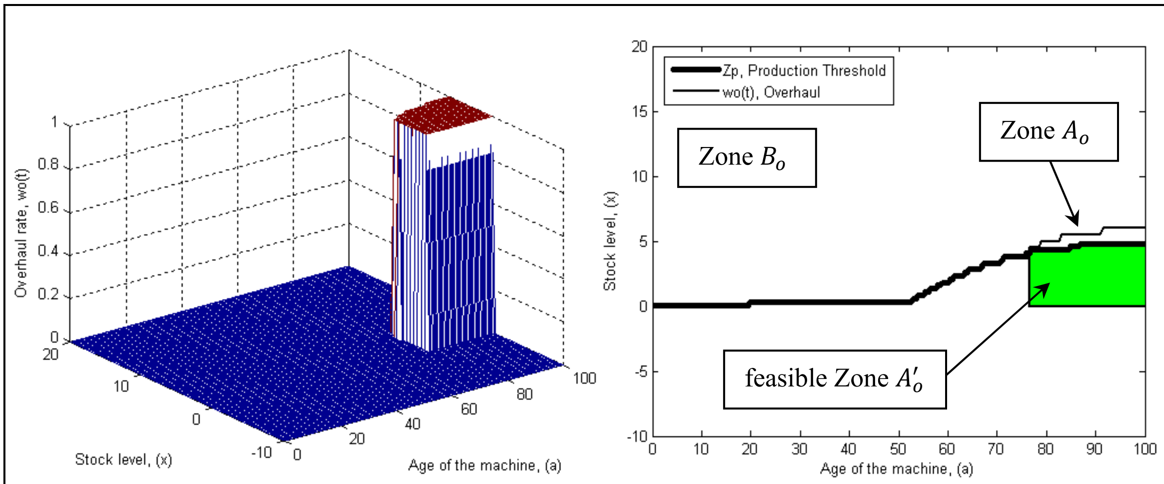
3.5.2 Overhaul control policy

Secondly, we shall discuss in this subsection the optimal overhaul policy obtained from the numerical example. From Figure 3.6a, we can observe that when the age of the machine is moderated, the overhaul activity is not recommended. This is because the rate of defectives β and the transition q_{12} remains in a rather small value with moderated ages, and the demand of products is not seriously affected. Conversely, when the age of the machine increases, the demand satisfaction of products begins to face serious problems, and then the overhaul activity is recommended to resolve the effects of deterioration. From the pattern of Figure 3.6a, we observe that the area to perform the overhaul activities spreads in the grid as a function of the age of the production system. To better illustrate the overhaul policy, we use its trace, presented in Figure 3.6b, where we notice that this policy divides the plane (x, a) in two regions, such that the overhaul rate is set to its minimum or maximum value $(\omega_{\underline{o}}, \omega_{\bar{o}})$ mainly according to the age of the machine. The description of these two zones is as follows:

- Zone A_o: here, the age of the machine has reached such a level that the machine must be sent to overhaul activities, hence the decision variable $\omega_o(\cdot)$ is set to its maximum value.

- Zone B_o: in this zone, the performance of an overhaul is not recommended, and the decision variable $\omega_o(\cdot)$ remains at its minimum value.

It should be noted that upon simultaneous consideration of the production and overhaul boundaries as presented in Figure 3.6b, only a part of the overhaul Zone A_o is utilized, since the stock level is limited by the production threshold $Z_p^*(\cdot)$. This implies a reduction in the zone A_o defining the feasible overhaul $zone\ A'_o$.



a) Overhaul rate versus stock level, and age of the machine

b) Overhaul trace

Figure 3.6 Overhaul policy

The overhaul activity is triggered according to a machine age-dependent policy as presented in Figure 3.6b. This optimal policy states that overhaul activities must be performed at a rate $\omega_o^*(\cdot)$ given by the following equation:

$$\omega_o^*(1, x, \mathrm{a}) = \begin{cases} 1 & if\ \mathrm{a}(t)\ and\ x(t)\ \in zone\ A'_o \\ 0 & otherwhise \end{cases} \tag{3.26}$$

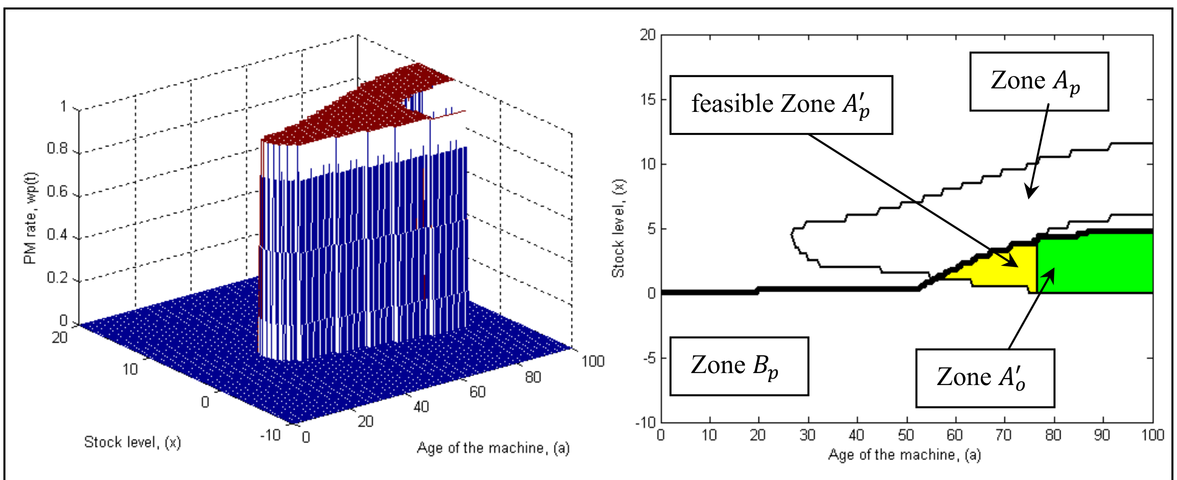
These results may be described verbally by saying that the overhaul policy highlights the fact that as the age of the machine increases, its deterioration becomes worse, and this indicates that the machine must be sent to major overhaul to completely resolve the effects of deterioration. Clearly, the dependence of the age of the machine on the decisions involved indicates the influence of the deterioration on the overhaul policy.

3.5.3 Preventive Maintenance Policy

We terminate this section with the discussion of the preventive maintenance policy. From Figure 3.7a, it is clear that the age of the machine has to reach a certain level to justify the cost of preventive maintenance. To facilitate the analysis of the preventive maintenance policy we use its boundary, as presented in Figure 3.7b. Such a boundary divides the plane (x, a) into two zones:

- Zone A_p: it is recommended to send the machine to preventive maintenance, thus the decision variable $\omega_p(\cdot)$ is set to its maximum value.
- Zone B_p: the policy consist in not conducting preventive maintenance, and so the decision variable $\omega_p(\cdot)$ is set to its minimum value.

A key point about the production threshold $Z_p^*(\cdot)$ is that it limits the stock level, and this defines the feasible preventive maintenance Zone A_p' as presented in Figure 3.7b, where we illustrate the trace of the joint production, overhaul and preventive maintenance policies. From the numerical results, we note that preventive maintenance is always conducted before overhaul, because preventive maintenance is less expensive and takes less time than the overhaul activities. The major overhaul is performed only when the deterioration of the machine has reached a much higher level that justifies its more expensive cost.



a) PM rate versus stock level and age of the machine b) Trace of the PM policy

Figure 3.7 Preventive maintenance policy

Based on the above results, we claim that the optimal preventive maintenance policy follows a switching function based on the age of the machine $\mathrm{a}(t)$ for a given stock level $x(t)$ and it has a bang-bang structure, denoted as follows:

$$\omega_p^*(1, \mathrm{a}, x) = \begin{cases} 1 & if\ \mathrm{a}(t)\ and\ x(t)\ \in zone\ A_p' \\ 0 & otherwise \end{cases} \tag{3.27}$$

Bearing in mind the numerical results obtained so far, we can state that the simultaneous production, overhaul and preventive maintenance policy are perfectly defined by Equations (3.25)-(3.27) and implementing this control policy is possible to govern the manufacturing system by monitoring the stock level $x(t)$ and the deterioration of the machine, denoted by its age $\mathrm{a}(t)$. The production policy can be completely parameterized by the control factor $Z_p^*(\cdot)$, meanwhile the overhaul and preventive maintenance policies can be defined by the zones A_o' and A_p', respectively. In order to confirm and validate the structure of the obtained optimal control policy, an extensive sensitivity analysis is performed in the next section. This analysis also will illustrate the usefulness of the obtained control policy.

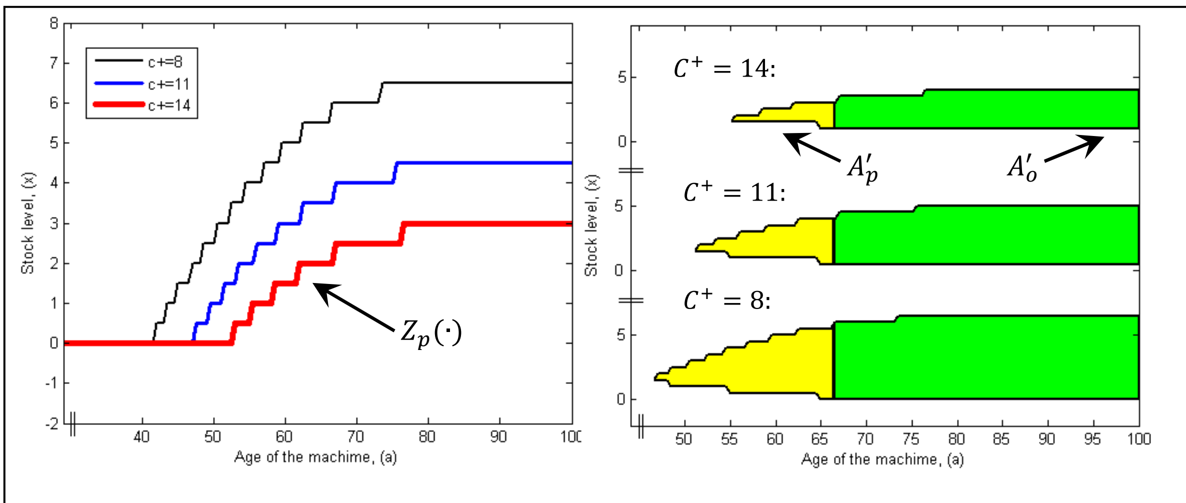
3.6 Sensitivity and results analysis

The present section provides further evidence of the usefulness of the obtained control policy. We performed an extensive sensitivity analysis to illustrate the contribution of the joint policy, and also it permits to confirm its structure. The sensitivity analysis is conducted according to the variation of several parameters such as: the inventory cost, backlog cost, overhaul cost, preventive maintenance cost and defectives cost. Furthermore we analyze the effect of other parameters as well, including: the efficiency ϕ_p of the preventive maintenance and the adjustment parameters θ_f and θ_d, related to the trend of the failure rate and the rate of defectives.

3.6.1 Variation of the inventory cost

The sensitivity analysis begins with the discussion about the effect of the inventory cost. We present the production trace Z_p^* in Figure 3.8a, for different inventory cost values c^+=8, 11 and 14. The results have been obtained by applying the numerical methods presented in the previous section. From Figure 3.8a we notice that when the inventory cost is moderate, i.e., c^+=8, the production threshold has the highest values of the analyzed cases and it starts increasing at around age $a = 42$. When the inventory cost increases to c^+=11, the optimal stock level follows a similar trend but below the previous case, in this scenario the production threshold begins its increase at age $a = 47$. If we set the inventory cost to c^+=14, we observe a considerable reduction in the production threshold Z_p, since it begins its increase at around age $a = 52$.

From the results presented in Figure 3.8a, we can state that the more the inventory cost increases, the more the optimal stock level decreases. The reason for this condition is that with a higher inventory cost, the stock of the product is more penalized, hence reducing the production threshold. Conversely with lower inventory cost there is more liberty to maintain stock, and so the production threshold increases



a) Effect on the production threshold Z_p b) Effect on the zones A'_p and A'_o

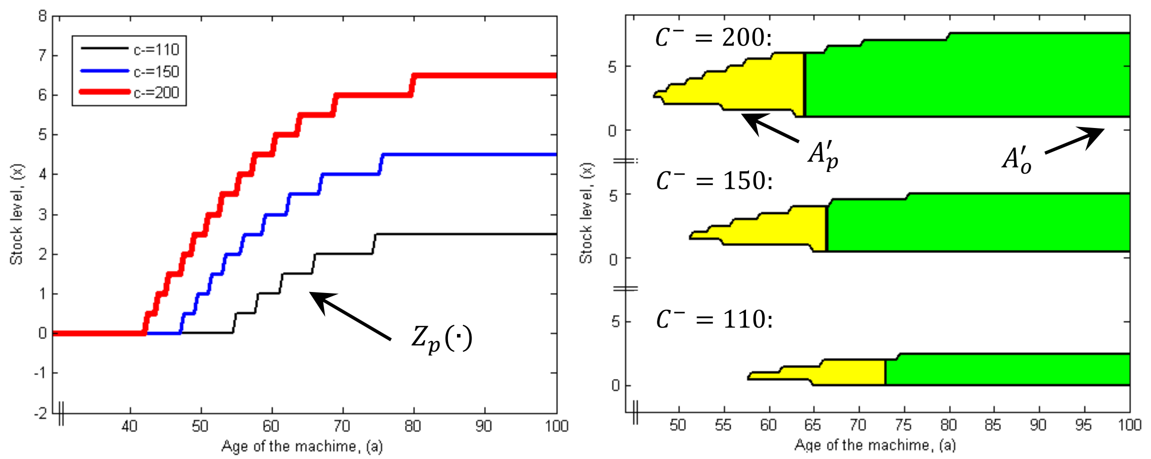
Figure 3.8 Sensitivity to the variation of the inventory cost

Moreover, we note that the deterioration of the machine has a significant effect on the production policy, because the production threshold increases as the age of the machine increases. In particular, our results show that considerable ages denote higher rates of defectives and failures that are more frequent. Therefore, the production threshold increases as protection against the twofold effect of deterioration.

To complement the analysis of the inventory cost, we discuss its effect on the preventive maintenance and overhaul policies. We present in Figure 3.8b three cases, using the same data cost as earlier, from them we observe that when the inventory cost is low, i.e., c^+=8, the zone for preventive maintenance A'_p and the zone for overhaul A'_o, cover a more extensive area in the computational domain. When the inventory cost increases to c^+=11, the zones A'_p and A'_o reduce, and they decrease even more when the inventory cost increases to c^+=14. The reductions are explained because with higher inventory cost, the production threshold reduces, intersecting the zone A_p and A_o at a lower position, thus reducing the feasible zones A'_p and A'_o. Recall that the production threshold is utilized to define the feasible zones for preventive maintenance and overhaul as mentioned previously in Figure 3.7b.

3.6.2 Variation of the backlog cost

The variation of the backlog cost c^-, affects considerably the optimal production threshold as presented in Figure 3.9a, where the production trace for three different cost values c^-=110,150 and 200 are analyzed. Analyzing the results of Figure 3.9a, we notice that when the backlog cost is c^-=110, the production thresholds are at their lowest levels for the entire analysis, and starts to increase at age $a = 54$. If we increase the backlog cost to $c^- = 150$, the production threshold also increases, but in this case it begins to increase considerably at age $a = 47$. Additionally when $c^- = 200$, the production threshold Z_p increases even more than the previous cases, a notable increment from age $a = 42$ is observed. Based on the analysis of these results, we can infer that the underlying pattern for the backlog cost implies that when the backlog cost increases, the production threshold also increases. This pattern suggests that since the backlog of product is more penalized with higher backlog cost, more products must be kept to protect the system from shortages and defectives. The increase in the stock level also indicates the effect of the deterioration in the production level, since to ensure demand satisfaction, the stock level increases when the age of the machine increases as well. Consequently higher ages indicate the presence of more disturbances due to more frequent breakdowns, and the presence of more defectives.



a) Effect on the production threshold Z_p b) Effect on the zones A'_p and A'_o

Figure 3.9 Sensitivity to the variation of the backlog cost

We next turn our attention to the effect of the variation of the backlog cost on the preventive maintenance and overhaul policies. This variation is illustrated in Figure 3.9b, where we compare three cases. From these results, we observe that when the backlog cost is moderate, i.e., $c^- = 110$, preventive maintenance and overhaul are less recommended. When the backlog cost increases to $c^- = 150$, the zone for preventive maintenance A'_p and the zone for overhaul A'_o cover a larger area in the plane (a, x). In addition, if we increase the backlog cost to $c^- = 200$, both zones A'_p and A'_o increase even more. The intuition behind this pattern is that if the backlog cost increases, the production threshold increases as well, thus intersecting the zone A_p of preventive maintenance and the zone A_o of overhaul at a higher position, and this in consequence, increases the feasible zones A'_p and A'_o. Thus, we can infer that the backlog cost is directly linked to the size of the zones of preventive maintenance and overhaul, since both zones increase according to the value of this cost. Besides, as expected from the results of Figure 3.8 and Figure 3.9, we observe that the effect of the backlog cost on the control policy is the inverse of the effect of the inventory cost.

3.6.3 Variation of the preventive maintenance cost

The results of three different cases are presented in Figure 3.10 to examine the variation of the preventive maintenance cost and its influence in the optimal policy. The analysis is made with the values $C_{pm} = 2, 4$ and 6. From the numerical results, we observe that when the preventive maintenance cost is low, for instance $C_{pm} = 2$, the preventive maintenance performance zone is the most extended of the analyzed cases. As the preventive maintenance cost increases to $C_{pm} = 4$, the zone A'_p on the grid is reduced, and reduces even further with a higher cost of $C_{pm} = 6$. From these results, we can deduce that the variation of the preventive maintenance cost mainly affects the preventive maintenance and overhaul policy as observed in Figure 3.10. Additionally this cost C_{pm}, has not reported any effect on the production policy, since the production threshold remained the same for the three analyzed cases.

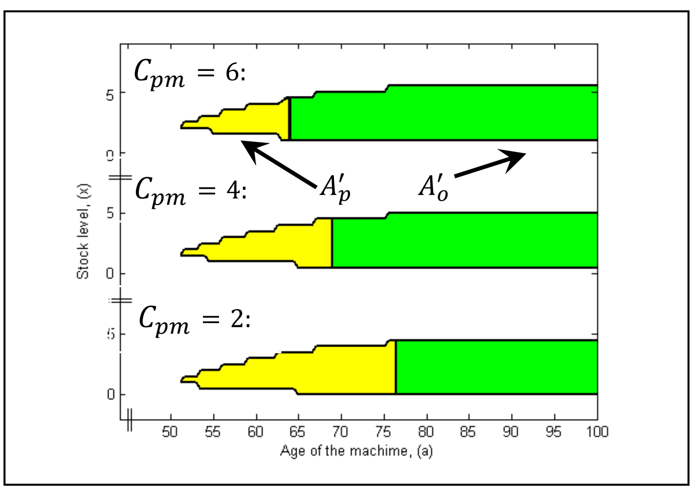


Figure 3.10 Sensitivity to the variation of the preventive maintenance cost and its effect on the preventive maintenance and overhaul policies

To an extent, the observed pattern implies that for a high preventive maintenance cost, the zone A'_p reduces considerably; indicating more overhauls, as should be logically expected. In addition, we notice that, since preventive maintenance is less expensive and takes less time, it is always conducted before major overhauls. Preventive maintenance is only carried out for an intermediate level of deterioration, when the age of the machine has reached a certain level to justify the cost of the activity, but if the age reaches a higher level of deterioration, it is more convenient to perform the overhaul instead.

3.6.4 Variation of the overhaul cost

The sensitivity analysis of the overhaul cost is presented in Figure 3.11, and is performed based on the numerical results of three different cases with values defined as $C_o = 10, 12$ and 14. From the numerical results it follows that when the overhaul cost is low, i.e., $C_o = 10$, we observe that the overhaul zone A'_o is the most prominent of the analyzed scenarios. When the overhaul cost increases to $C_o = 12$, we notice a significant reduction in the zone A'_o, therefore fewer overhauls are conducted. With a higher overhaul cost of $C_o = 14$, the zone A'_o decreases more considerably.

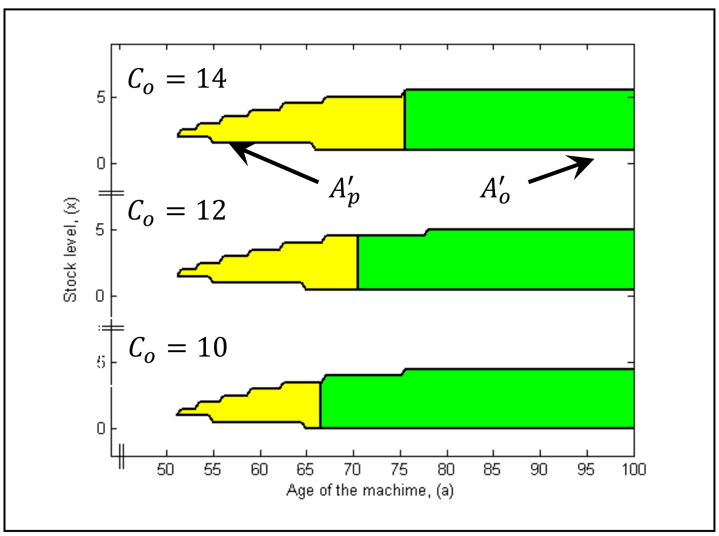


Figure 3.11 Sensitivity to the variation of the overhaul cost and its effect on the preventive maintenance and overhaul policies

From Figure 3.11, the main issue is clearly that the variation of the overhaul cost has a significant effect on the preventive maintenance and overhaul policies, where we note upon increasing this cost C_o, the zone A'_o is reduced, thus less overhauls are conducted. With respect to the production policy, we observe that the overhaul cost does not change the production threshold, since they remained unchanged for the three cases. Moreover, it is clear that as the major overhaul is more expensive and takes more time than the preventive maintenance, the overhaul is carried out only at a high level of deterioration, when the age of the machine is high enough to justify the higher cost of the overhaul.

3.6.5 Variation of the defectives cost

In this subsection, we highlight the preventive maintenance and overhaul policies under the effect of the variation of the defectives cost. Figure 3.12 indicates these polices with the values $C_d = 1, 4$ and 7. The size of the zone where preventive maintenance is recommended is the most considerable when $C_d = 1$. Nonetheless, it can be observed that increasing the defectives cost to $C_d = 4$, has a significant effect on these policies, reducing the preventive maintenance zone A'_p. Furthermore, we note that preventive maintenance is less often initiated when the defectives cost increases to $C_d = 7$.

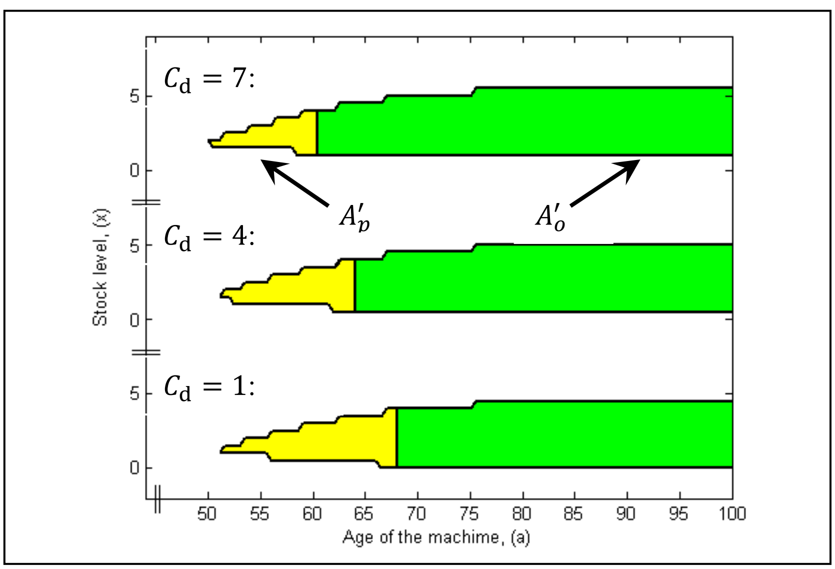


Figure 3.12 Sensitivity to the variation of the overhaul cost and its effect on the preventive maintenance and overhaul policies

These results show clearly that as the defectives cost increases, the preventive maintenance policy exhibits a pattern that recommends fewer numbers of this maintenance, and more overhauls. Additionally we note that this cost C_{d} does not affect the production threshold. To make things clear, defective products are penalized more severely with higher defectives cost, and it follows that on increasing C_{d}, the overhaul is more recommendable because it restores the level of deterioration of the machine more efficiently and rapidly. On conducting an overhaul, the rate of defectives and the MTTF are restored to AGAN conditions, whereas the imperfect preventive maintenance rejuvenates the machine only to a certain percentage. This logic explains the reasons why more overhaul should be performed as the defectives cost increases.

3.6.6 Variation of the preventive maintenance efficiency

An equally significant parameter is the variation of the preventive maintenance efficiency ϕ_p, since it has a remarkable effect mainly on the overhaul and preventive maintenance policies, as illustrated in Figure 3.13. The value of such efficiency ϕ_p has to be attractive enough to encourage preventive maintenance activities, otherwise this activity will not be recommended. This parameter ϕ_p is very useful since it enables the development of more realistic maintenance policies, modeling scenarios between the extreme cases of minimal and perfect repair, as discussed previously in section 3.2. In Figure 3.13, the variation of the preventive maintenance efficiency is presented, with values defined as; $\phi_p = 0.55, 0.6, 0.65$ and 0.7. From these results we observe that when the efficiency ϕ_p is moderate, $\phi_p = 0.55$, the zone for preventive maintenance A'_p is the least extensive on the plane (a, x). If we increase the efficiency to $\phi_p = 0.6$, the zone A'_p increases, recommending more preventive maintenance, and when the efficiency further increases to $\phi_p = 0.65$, the zone A'_p increases further. For the current numerical example, the results tell us that when the maintenance efficiency increases, the zone for preventive maintenance is significantly increased, because the incentive of conducting this activity is closer to the benefit of performing a perfect repair, reducing the level of deterioration to almost AGAN conditions. It can also be said that when the maintenance efficiency reaches a certain level, for example, $\phi_p = 0.7$, preventive maintenance completely displaces the major overhaul because with this efficiency level and given the cost of both activities, it is a better option to conduct preventive maintenance.

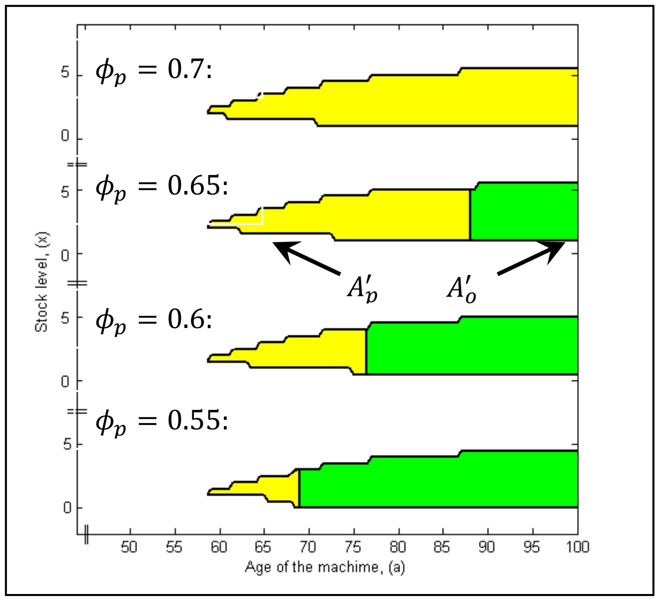
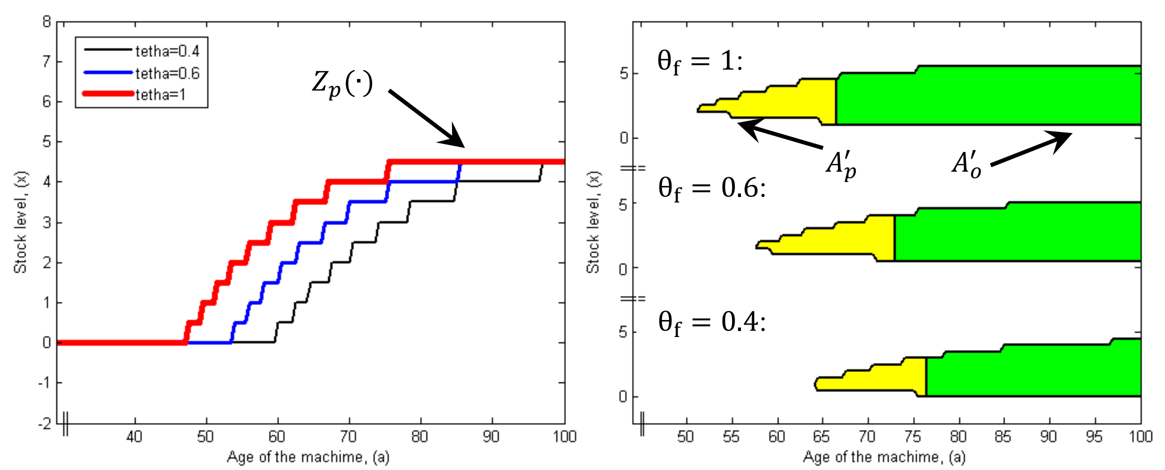


Figure 3.13 Sensitivity to the variation of the preventive maintenance efficiency ϕ_p and its effect on preventive maintenance and overhaul policies

Conversely, it is no surprise to note that when maintenance efficiency decreases, the zone A'_p is considerably reduced, because the benefit of preventive maintenance becomes closer to ABAN conditions, and the reductions on the rate of defectives and MTTF are negligible. Therefore, the main point regarding the efficiency ϕ_p is that the difference in reduction on the age of the machine (and their corresponding change in the rate of defectives β and MTTF), is the fundamental reason to explain the variations observed in the preventive maintenance zones of Figure 3.13. The effect of the efficiency ϕ_p is observed only on the preventive maintenance and overhaul policies, while the production thresholds remain the same for the four cases.

3.6.7 Variation of the adjustment parameter for the failure intensity

In Figure 3.14a we illustrate the effect of the variation of the adjustment parameter θ_f on the production policy. As discussed previously in section 3.2, this parameter θ_f allows to adjust the trend of the failure intensity, denoted by the transition q_{12}. In Figure 3.14a the adjustment parameter takes three values $\theta_f = 0.4, 0.6$ and 1. When the parameter is low, $\theta_f = 0.4$, it means that the system experiences fewer failures, thus the production threshold Z_p reduces. There are more frequent failures when $\theta_f = 0.6$, hence the production threshold increases as protection.

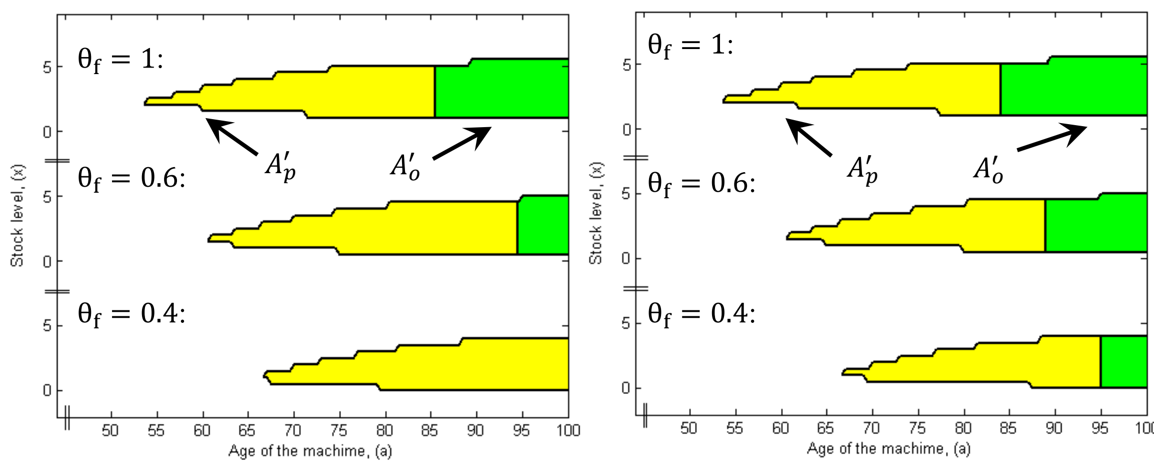


a) Effect on the production threshold Z_p b) Effect on the zones A'_p and

Figure 3.14 Sensitivity to the variation of the adjustment parameter θ_f

If the parameter is set to $\theta_f = 1$, the machine needs more protection against the more frequent failures, leading to an even greater increase in the production threshold. It is worthwhile to mention that when the parameter θ_f increases, the emphasis of deterioration is on more frequent failures, increasing more abruptly transition q_{12}. This affects the MTTF in such a way that reduces the reliability of the production system, promoting the increase of the production threshold as protection for shortages.

As a matter of interest let us now analyze the effect of θ_f on the preventive maintenance and overhaul policies. From the results of Figure 3.14b we notice that when the parameter is set to a low value such as $\theta_f = 0.4$, both the preventive maintenance zone A'_p and the overhaul zone A'_o are the smallest in the illustration. When the parameter increases to $\theta_f = 0.6$, both zones expand along the plane (a, x). And these zones continue their expansion when the parameter increases to $\theta_f = 1$. From this pattern we can draw a first inference that any variation in the production threshold Z_p will certainly affect the total zone for preventive maintenance and overhaul, precisely because the zones A'_p and A'_o are delimited by their intersection with the stock level as presented previously in Figure 3.7b.



a) Zones A'_p and A'_o (with $\phi_p = 0.65$) b) Zones A'_p and A'_o (with $c_o = 14$)

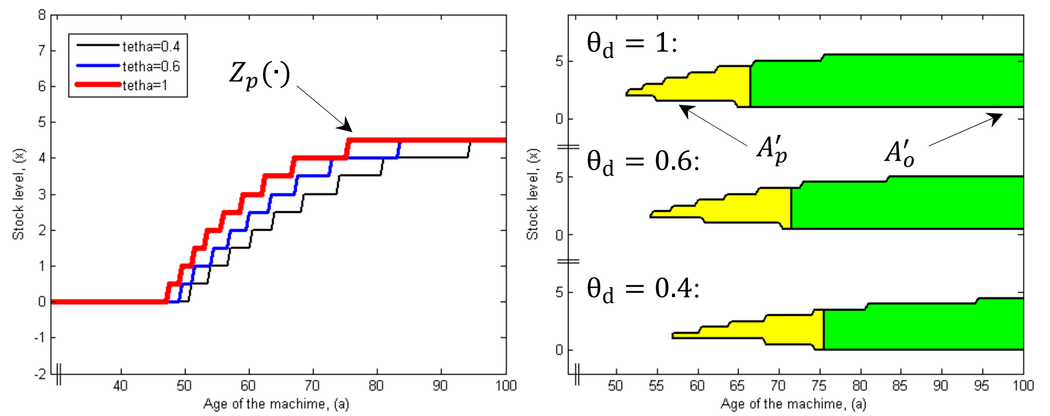
Figure 3.15 Sensitivity to the variation of the adjustment parameter θ_f

A second observation is that with higher values of θ_f we make more emphasis on failures rather than defectives, and from Figure 3.14b, it is clear that higher values of θ_f encourage the conduction of more overhaul to more rapidly counter the effects of deterioration. It is not difficult to observe that there is a set of parameters with a strong influence on preventive maintenance and overhaul policies, with the potential to vary the obtained results. For example, looking at the maintenance efficiency employed in Figure 3.14b, (we used $\phi_p = 0.6$ for the three cases), if we increase the efficiency to $\phi_p = 0.65$, then we will observe a general reduction in the overhaul zone A'_o, as reported in Figure 3.15a, because the preventive maintenance will be more attractive in such cases. However the trend is the same as in Figure 3.14b, promoting more overhaul as the value of θ_f increases. This pattern is confirmed when we increase the overhaul cost from $c_o = 10$ (in Figure 3.14b) to $c_o = 14$, (in Figure 3.15b). With a higher c_o, the overhaul zone A'_o reduces considerably, and there appears then to be an interaction of several parameters that define the preventive maintenance and overhaul policies. Nevertheless a pattern is observed, the zone A'_o of overhaul increases, when the parameter θ_f increases to cope more efficiently the presence of more frequent failures.

3.6.8 Variation of the adjustment parameter of the rate of defectives

In order to get an idea of the sensitivity of the adjustment parameter θ_d on the production control policy, we analyze three cases. Recall that this parameter θ_d modifies the trend in the rate of defectives as a function of the age of the machine, as presented previously in Figure 3.3. There it can also be noted that when the age of the machine is significant enough, the rate of defectives will converge to the same limit value regardless of the amount of θ_d. We obtain the results presented in Figures 3.16a and Figure 3.16b, when the parameter θ_d takes the values $\theta_d = 0.4, 0.6$ and 1. Whenever this parameter is set to a not-excessive value of $\theta_d = 0.4$, it means that the production threshold Z_p does need to be so considerable because the deterioration of the machine does not imply a huge amount of defectives. Then, when $\theta_d = 0.6$, the production threshold increases because the rate of defectives reaches higher values more rapidly.

Additionally, if we increase the parameter to $\theta_d = 1$, the rate of defectives increases so abruptly that in consequence the production threshold must increase further. On the basis of the results presented in Figure 3.16a, we can come to the observation that the role of the increase of the production threshold is to protect the system against the shortage of flawless products. When the parameter θ_d increases, the rate of defectives increases more abruptly, in consequence the demand of product increases as protection, and this causes an increment in the production threshold Z_p.

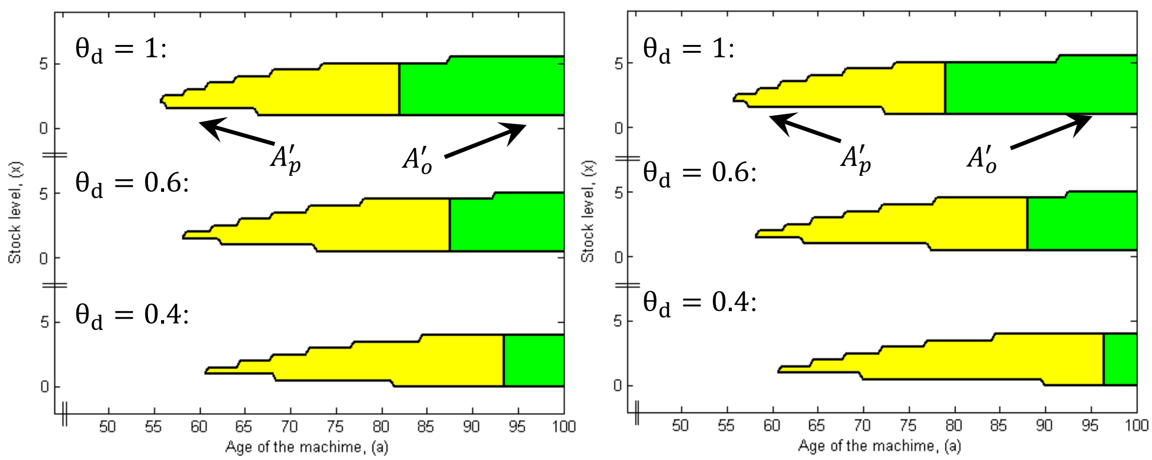


a) Effect on the production threshold Z_p b) Effect on the zones A'_p and A'_o

Figure 3.16 Sensitivity to the variation of the adjustment parameter θ_d

In essence, according to Figure 3.16b, the adjustment parameter θ_d also has a clear effect on the preventive maintenance and overhaul policies. For instance, when $\theta_d = 0.4$, the zones in which preventive maintenance A'_p and overhaul A'_o are feasible are the smallest of the analyzed cases. On increasing the parameter to $\theta_d = 0.6$, the zone A'_o is more extensive on the grid. Moreover, when the parameter increases to $\theta_d = 1$, the zones A'_o expands further on the plane (a, x). As a consequence, we remark firstly, that any variation of the stock level modifies the total region of preventive maintenance and overhaul, denoted by the zones A'_p and A'_o, since they are defined by their intersection with Z_p, as presented previously in Figure 3.7b. Secondly, the major difference in this case is that with higher levels of θ_d, the emphasis is on more defectives instead of failures. More overhaul is conducted when the value of θ_d increases, because overhaul completely restores the rate of defectives with more efficiency than preventive maintenance. This pattern is confirmed, when we increase the preventive maintenance efficiency from $\phi_p = 0.60$ (in Figure 3.16b) to $\phi_p = 0.65$ (in Figure 3.17a), where we observe a similar pattern; the preventive maintenance zone A'_p decreases when θ_d increases. Conversely, the overhaul zone A'_o grows when θ_d increases.

For closure, when we increase the overhaul cost from $c_o = 10$ (in Figure 3.16b) to $c_o = 14$, (in Figure 3.17b), the underlying pattern is the same, the overhaul zone A'_o increases when θ_d increases, replacing the preventive maintenance zone A'_p that decreases.



a) Zones A'_p and A'_o (with $\phi_p = 0.65$)   b) Zones A'_p and A'_o (with $c_o = 14$)

Figure 3.17 Sensitivity to the variation of the adjustment parameter θ_d

3.7 Discussions

From what has been presented on the above sensitivity analysis, it is clearly enough to state that the simultaneous production, preventive maintenance and overhaul control policy is well characterized by the aforementioned control parameters (Z_p, A'_o and A'_p). For the manufacturing system considered, the joint control policy is given by Equations (3.25)-(3.27) and Figures 3.5 to 3.7. It is also important to note that the deterioration phenomenon influences the dynamics of several variables of the system, since on denoting deterioration with the age of the machine, its variation implies changes in the rate of defectives and the MTTF. As analysed in the above results with respect to the production policy, considering the effect of deterioration, it leads to a control policy that modifies the traditional hedging point policy to a situation with several threshold values that increase as a function of the age of the machine. Meanwhile, preventive maintenance and overhaul exhibit an age-dependent policy; they are triggered for a certain age level.

Through the sensitivity analysis conducted, we observe that the production threshold Z_p decreases when the inventory cost increases, because the stock of the product is more penalized, and the contrary effect is observed for the backlog cost. There is also an increase in the production threshold Z_p when the adjustment parameters θ_f and θ_d increase, this as protection against more frequent failures and the presence of more defective products. With regard to the preventive maintenance policy, when the preventive maintenance cost increases, the zone A'_p decreases, since this activity becomes more expensive. Furthermore, when we increase the preventive maintenance efficiency, there is a considerable increase in the zone A'_p, because the incentive to promote this activity is more attractive, and on increasing the parameters θ_f and θ_d the zone A'_p decreases, since in such cases overhaul is more convenient. As we have seen in the previous sensitivity analysis, the overhaul zone A'_o reduces when the overhaul cost increases, and more overhauls are done when the defectives cost increases to completely cope the effects of deterioration. Further, when the parameters θ_f and θ_d increase, more overhaul is also performed. The relevance of the sensitivity analysis is apparent, since it seems that our results are logical and consistent, and this enables us to confirm the structure of the obtained joint control policy.

3.8 Conclusion

It has been observed that the simultaneous production planning, preventive maintenance and overhaul control problem was addressed in this research work for the case of a single machine subject to random failures and deterioration. The main issue of the paper is to integrate quality aspects on the control policy, thus extending the concept of deterioration to create a connection with the rate of defectives and its reliability. We used a maintenance efficiency parameter to decrease the level of deterioration if preventive maintenance is carried out. We formulate a stochastic dynamic programming problem that integrates several types of maintenance; such as perfect, imperfect and minimal repair. Furthermore, on considering the machine's history, with the age of the production system, we devise a Semi-Markov decision model that enables us to take into account the twofold effect of deterioration: on the quality of the parts produced and the MTTF. Another important aspect is the fact that, our research analyzes the interaction of quality issues and the production control from the perspective of deteriorating systems. We illustrate the proposed approach thorough a numerical example, in which we observed that the stock level required as protection against failures and defectives increases as a function of the age of the production system. Meanwhile we found that the performance of preventive maintenance and overhaul activities depends on the stock level and the age of the machine. An extensive sensitivity analysis was also conducted. This serves to confirm the structure of the joint control policy. Finally, we conclude that the continuous deterioration of the machine has considerable effects on the joint production, preventive maintenance and overhaul policies, and the results obtained in this research are very satisfactory and encourage us to extend the study to more complex manufacturing systems.

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GENERAL CONCLUSION

This thesis proposes several models, for the stochastic control of unreliable manufacturing systems subject to deteriorations. The essential modeling question concerns the fact that various parameters of the production system are severely affected by the deterioration. Therefore, the effects of deterioration has been analyzed and modeled from different perspectives throughout this thesis. The focus was to provide a better understanding about the inter-relation between quality and the production policy, in the context of deterioration. The main issue is that deterioration is amenable of a decrement on the quality of the parts produced, although other repercussions on the reliability of the system are investigated as well. The developed models have succeeded to determine the optimal production plan and maintenance strategies which resolve the effects of deterioration. For practical purposes, we proceeded in three phases.

In the first phase, we combined the production management problem with the introduction of major maintenance strategies to counter the effect of deterioration. The analyzed manufacturing system faces deterioration whose principal effect is observed on the quality of the parts produced, denoted by the rate of defectives. Inappropriate maintenance activities worsen the quality of the parts, whereas major maintenance restores completely the production systems to perfect conditions. A stochastic dynamic programming model is proposed where a numerical scheme is adopted to determine the joint optimal control policy. To accurately approximate the related control parameters a simulation optimization approach is applied, consisting in simulation modeling, statistical analysis based on design of experiments, and parameter optimization with response surface methodology. A numerical example is provided, and an extensive sensitivity analysis has confirmed the structure and robustness of the obtained joint policy. The effect of deterioration on the system is evident, since it led to a machine-deterioration dependent policy, where several control parameters are adjusted in function of the level of deterioration of the machine.

With regard to the second phase, the proposed model is further complicated because the deterioration is conceived as a combination of a number of factors, namely the wear of the machine and imperfect repairs. We extended the concept of deterioration at proposing a link with the quality part. The rate of defectives deteriorates following repairs, however multiple operational states are implemented to define and aging process, this entail also to model variations in the rate of defectives. At failure a major maintenance can be conducted to resolve completely the effect of deterioration. Since the rate of defectives is dependent of the history of the system, defined in this case by the numbers of repairs and the multiple operational states, a Semi-Markov decision process is utilized to model the production system. The simultaneous production policy and the repairs/overhaul switching strategy are determined through numerical methods. A numerical example is conducted as illustration of the proposed approach. The results obtained provide new insights into the relation between quality deterioration and the production policy, and we observed that the stock level required to hedge against disruptions varies in function of the number of repairs and the set of operational states. Further, something similar is observed for the control parameter of the repair/overhaul policy. These observations have been validated with a rigorous sensitivity analysis.

The major emphasis in the third phase of our research stems on the introduction of preventive maintenance strategies to extend the control policies obtained in the first and second phases. In this model the deterioration of the production system entails a twofold effect on several parameters of the machine, decreasing the quality of the parts, and also decreasing the reliability of the system, yielding to increase its failure intensity. It is utilized the age of the machine to denote its level of deterioration. Perfect repairs are conducted to cope completely with the effects of deterioration, while preventive maintenance reduces only partially the level of deterioration, improving just in part the quality of the parts and the failure intensity. The model has been formulated as a Semi-Markov model, since the dynamics of the system varies with its level of deterioration, leading to obtain age-dependent policies. The results of a numerical example suggest that the benefit of preventive maintenance improves the quality of the parts produced and also increases the lifetime of the machine, thus this activity enhances its reliability. The obtained joint production planning, overhaul and preventive maintenance policies have been assessed with an extensive sensitivity analysis.

The research work conducted in this dissertation proposes stochastic models that integrate the influence of quality aspects on the production control policy. The developed models can be applied to industries subject to random disruptions and uncertainties that can generate a serious decrease in productivity. Our models serve to decision makers to determine effective control actions involving production and maintenance strategies. Concerning the modeling framework of the present thesis, the area of research that addresses the inter-relation between quality and production is very rich and vast in topics that need further investigation. Future developments will study the generalization of our modeling approach, where several research directions can be follow:

- The simulation optimization approach presented in the first model, dealing with the production planning and overhaul strategy can be extended to the second and third model and also to study more general manufacturing systems which lifetime is denoted by non-exponential distributions.
- The second model can consider the age of the machine, instead of the number of repairs to denote the level of deterioration of the machine, and also it can be introduced a probability for the trend of the rate of defectives after repairs. Since in real production the rate of defective may increase after a repair, but not always.
- We can extend the obtained control policies to analyze more complex manufacturing systems, in special long line production systems implying two or multiple machines, and also it can be analyzed the case of multi-type products. The simulation optimization approach can be useful in this respect.

In addition, our guidelines can be used to develop models for the optimal design of control charts; with this we will include quality control in the optimal control policy, also we can include rework of parts in the models to avoid any loss in production.

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