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OPTIMIZATION OF THE VERTICAL FLIGHT PROFILE ON THE FLIGHT MANAGEMENT SYSTEM FOR GREEN AIRCRAFT

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OPTIMIZATION OF THE VERTICAL FLIGHT PROFILE ON THE FLIGHT MANAGEMENT SYSTEM FOR GREEN AIRCRAFT

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ABSTRACT

To reduce aircraft's fuel consumption, a new method to calculate flight trajectories to be implemented in commercial Flight Management Systems has been developed. The aircraft's model was obtained from a flight performance database, which included experimental flight data. The optimized trajectories for three different commercial aircraft have been analyzed and developed in this thesis.

To obtain the optimal flight trajectory that reduces the global flight cost, the vertical and the LNAV profiles have been studied and analyzed to find the aircraft's available speeds, possible flight altitudes and alternative horizontal trajectories that could reduce the global fuel consumption. A dynamic weather model has been implemented to improve the precision of the algorithm. This weather model calculates the speed and direction of wind, and the outside air temperature from a public weather database.

To reduce the calculation time, different time-optimization algorithms have been implemented, such as the Golden Section search method, and different types of genetic algorithms. The optimization algorithm calculates the aircraft trajectory considering the departure and arrival airport coordinates, the aircraft parameters, the in-flight restrictions such as speeds, altitudes and WPs. The final output is given in terms of the flight time, fuel consumption and global flight cost of the complete flight.

To validate the optimization algorithm results, the software FlightSIM® has been used. This software considers a complete aircraft aerodynamic model for its simulations, giving results that are accurate and very close to reality.

OPTIMISATION DU PROFIL VERTICAL DE VOL DANS LES SYSTÈMES DE GESTION DE VOL POUR LES AVIONS VERTS

Roberto Salvador FÉLIX PATRÓN

RÉSUMÉ

Une nouvelle méthode pour calculer des trajectoires de vol pouvant être implémentée dans un système de gestion de vol a été développée pour réduire la consommation de carburant des avions. Le modèle des avions a été obtenu à partir d'une base de données de performances, composée de données expérimentales de vol. Cette thèse présente l'analyse et le développement de trajectoires optimisées pour trois types d'avions commerciaux.

Afin d'obtenir la trajectoire de vol optimale réduisant le coût global du vol, les profils vertical et latéral de navigation ont été étudiés. Une analyse complète des vitesses disponibles a été effectuée, ainsi qu'une analyse des altitudes de vol possibles et des trajectoires horizontales alternatives qui peuvent aider à réduire la consommation globale de carburant. Un modèle dynamique de la météo a été implémenté afin d'améliorer la précision de l'algorithme. Ce modèle de la météo calcule la direction et la vitesse du vent, ainsi que la température de l'air à partir d'une base de données publique.

Dans le but de réduire le temps de calcul, différents algorithmes d'optimisation ont été implémentés, tels que la méthode de la section d'or ainsi que différents types d'algorithmes génétiques. Les algorithmes d'optimisation calculent la trajectoire de l'avion en considérant les coordonnées des aéroports de départ et d'arrivée, les paramètres de l'avion, et les restrictions pendant le vol comme la vitesse, l'altitude et les points de cheminement. Le résultat global de l'algorithme est donné en termes de temps de vol, carburant consommé et coût global de vol pour un vol complet.

Le logiciel FlightSIM® a été utilisé pour valider les résultats obtenus par les algorithmes d'optimisation. Ce logiciel considère un modèle aérodynamique complet des avions, ce qui produit des résultats précis et très proches de la réalité.

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LIST OF ABREVIATIONS

3D PAM	3D Path Arrival Management
ATC	Air Traffic Control
BADA	Base of Aircraft Data
CDA	Continuous Descent Approach
CI	Cost Index
CO_2	Carbon dioxide
ETMS	Enhanced Traffic Management System
FAA	Federal Aviation Administration
FMS	Flight Management System
GA	Genetic Algorithm
GDPS	Global Deterministic Prediction System
GRIB2	General Regularly-Distributed Information, Binary form version
IAS	Indicated AirSpeed
IATA	International Air Transport Association
ICAO	International Civil Aviation Organization
ISA	International Standard Atmosphere
LNAV	Lateral NAVigation
NGATS	Next Generation Air Transportation System
PDB	Performance DataBase
PTT	Part-Task Trainer
RTA	Required Time of Arrival
SC	Step Climb

SESAR	Single European Sky ATM Research
TAS	True AirSpeed
TOC	Top Of Climb
TOD	Top Of Descent
UTC	Coordinated Universal Time
VNAV	Vertical NAVigation
WP	WayPoint

INTRODUCTION

0.1 Statement of the problem

The global aviation industry produced 676 million tons of CO_2 in 2011, 689 million in 2012, and 701 million in 2013 (ATAG, 2012; 2013; 2014). This amount represents around 2% of the total emissions produced worldwide. CO_2 emissions contribute to global warming, that is one of the biggest environmental problems encountered today.

In Canada, the Green Aviation Research & Development Network (GARDN) was founded in 2009, undertaking different research and development projects to reduce greenhouse gas emissions. One of the first projects in this network was called "Optimized Descents and Cruise". The new proposed optimization algorithm was developed in this project, in collaboration between the École de Technologie Supérieure and the avionics company CMC Electronics – Esterline.

A Flight Management System (FMS) is a fundamental avionics element in actual aircraft. This is a system with the main function to reduce the crew workload during flight time, through the automation of many aircraft tasks; the aircraft path planning is one of them.

A FMS receives inputs such as the aircraft's speed, cruise altitude, the distance to travel and the weather conditions that will provide information for the trajectory creation. The reduction of the fuel burn impact of an aircraft is not limited to consume the least fuel possible, but other variables must also be considered. The Cost Index (CI) is a constant used by the airlines to determine the operating cost of the flight, which includes variables such as the fuel price, the number of crew members working during the flight, and the flight time. It influences directly on the global cost of the flight. A CI close to zero indicates that the operation costs for the flight are low, and thus the flight time importance would also be low. A high CI indicates that the operation cost is high, and the flight time would have to be reduced in order to economize in operation costs.

Figure 0.1 shows the influence of the CI in a climb trajectory in which the FMS calculates the ascent to the TOC (Top Of Climb) position (Airbus, 1998). It should be noted that the higher the CI is, the shorter the trajectory is to the destination point.

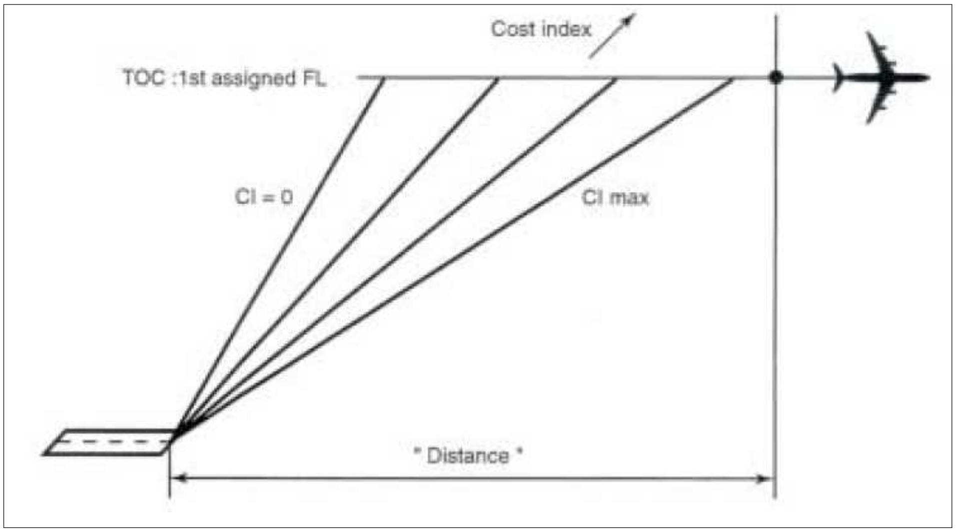

Figure 0.1 CI influence on the climb technique (Airbus, 1998)

Another important factor to consider when optimizing a trajectory is the aircraft weight; the heavier the aircraft is, the lower the optimal altitude is located. Figure 0.2 shows an example of the relationship between the optimal flight altitude and the aircraft weight. In the absence of winds, this relationship is close to linear. The influence of the CI is also represented on Figure 0.2, where as the CI increases, the optimal flight altitude decreases.

Figure 0.3 shows the relationship between the aircraft speeds and altitudes in terms of the CI for an Airbus A310 aircraft.

As seen on Figure 0.3, the speed, the altitude, the gross weight and the CI are entirely codependent in the search of optimal flight conditions.

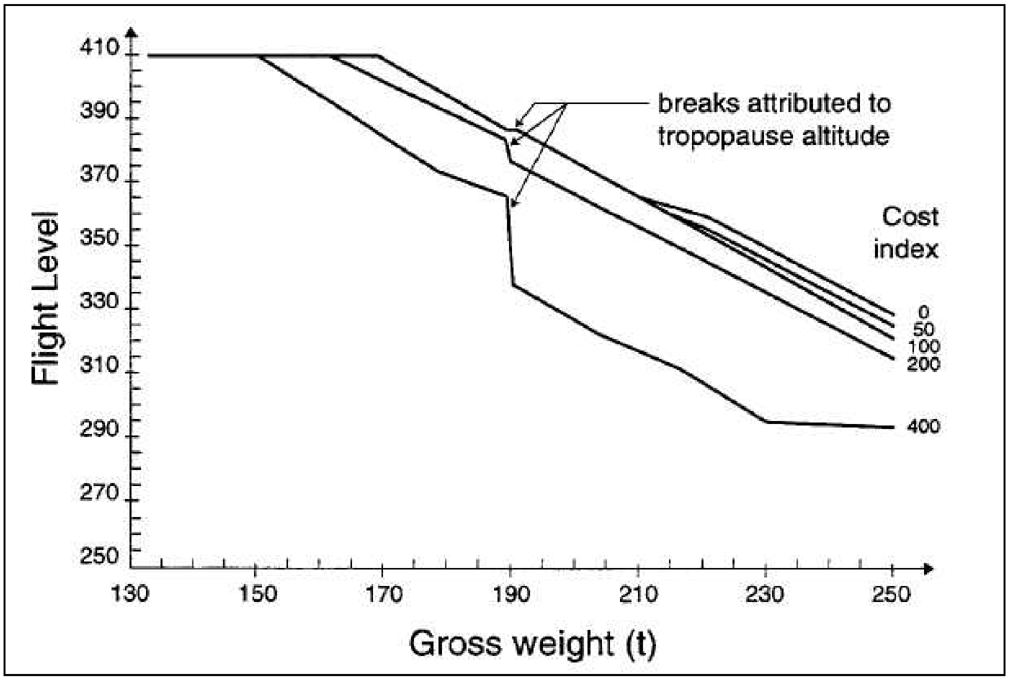

Figure 0.2 Optimal altitude variation with the aircraft gross weight (Airbus, 1998)

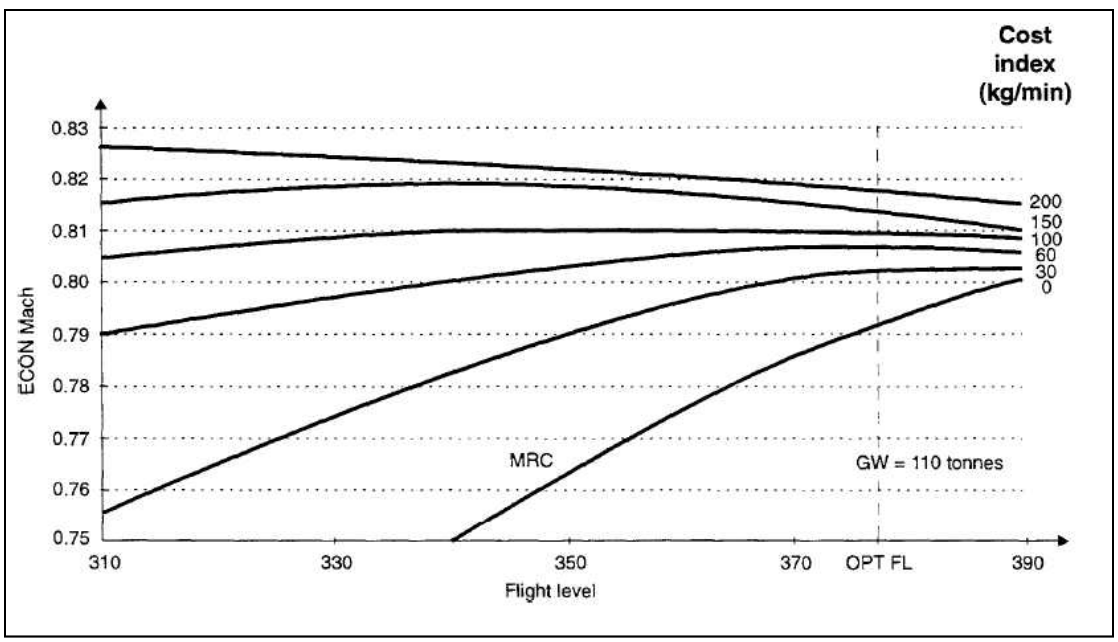

Figure 0.3 Speed and altitude variation with the CI (Airbus, 1998)

For the initial climb, a constant aircraft speed is selected to climb at a specific altitude, frequently 10,000ft (Airbus, 2004, p. 27). The optimal climb speed is selected in terms of the CI. A slower climb speed will result in a shorter traveled distance and a longer time to reach the final destination, while a higher speed will reduce the time (Figure 0.4) but increase the fuel consumption, and this is the reason why the CI determines the choice of the profile to be used.

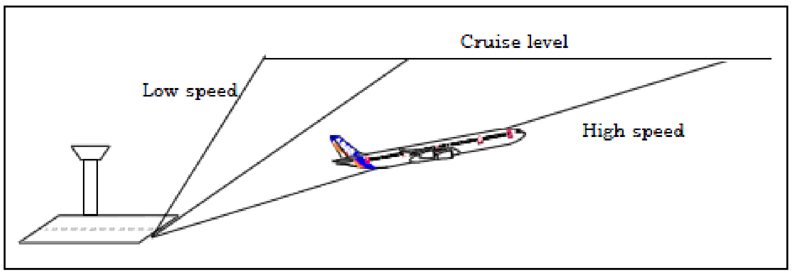

Figure 0.4 Climb profiles (Airbus, 2004)

Once the aircraft is in the cruise phase, a Step Climb (SC) or a continuous climb could be made. The continuous climb will provide the maximal fuel economization, as the optimal altitude will be reached quickly. The SC technique is shown on Figure 0.5. SC consists in ascending in steps of 1000ft, 2000ft or 4000ft, each time followed by a cruise phase, instead of climbing in a straight line to a specific altitude.

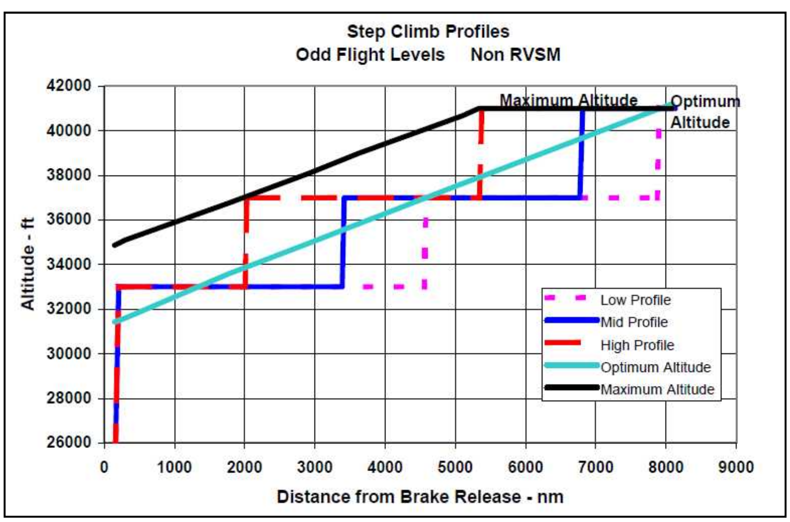

Figure 0.5 SC versus continuous climb (Airbus, 2004)

Wind influence is also an important factor to be considered. In Figure 0.6, the fuel consumption and the flight time are shown for a 2000 nm segment with fixed aircraft gross weight, by taking the Airbus A321 as reference. Positive speed values in knots <kt> are considered as tailwinds, while negative wind speed should be taken as headwinds.

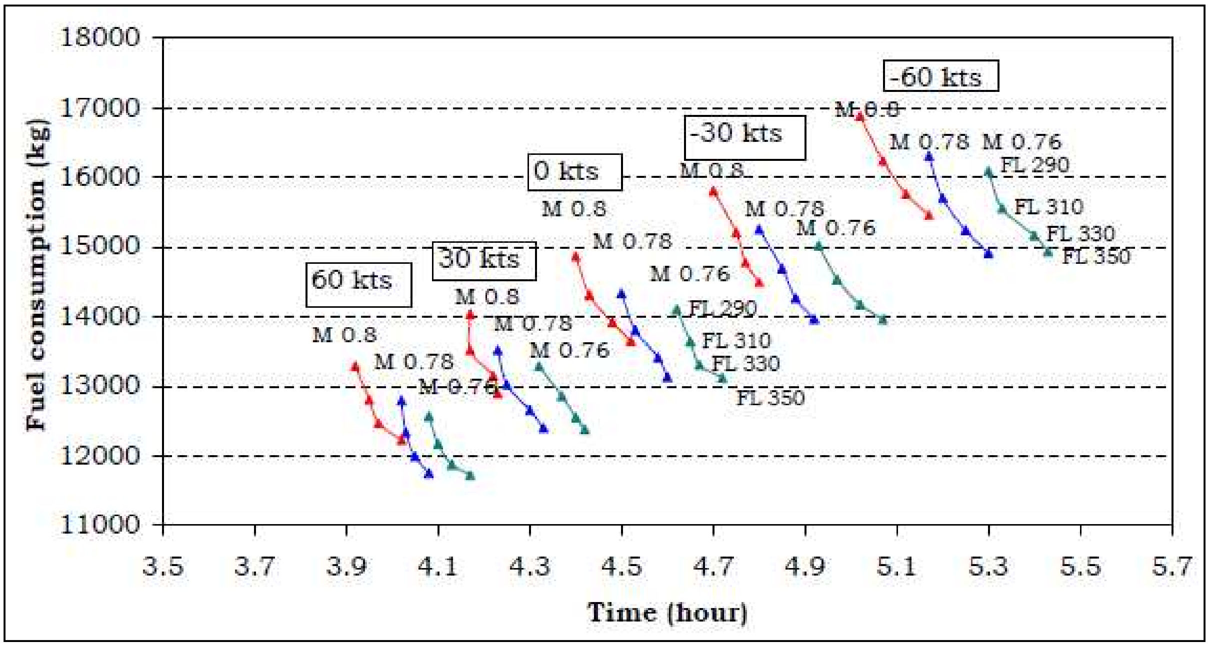

Figure 0.6 Wind influence on fuel consumption and flight time (Airbus, 2004)

It can be observed that the stronger the tailwind is, the minimum flight time and fuel burn are obtained. In case of headwinds, it would be necessary to increase the aircraft's speed to reduce the flight time, thus increasing the fuel burn.

Current FMS platforms do not present a complete optimization of the vertical and lateral flight profiles (VNAV and LNAV).

0.2 Objectives

The global objectives of this project concern the calculation of the optimal LNAV and VNAV profiles in terms of aircraft's speeds and altitudes, by considering the CI, a complete analysis of the winds and the variation of the aircraft weight during the flight, in order to reduce the global flight cost.

The global objectives could be divided into the following sub-objectives:

1. Validation of the numerical model of three different commercial aircraft, by comparing its results expressed in terms of aircraft's fuel burn and flight time with FlightSIM® and the Part-Task Trainer (PTT) from CMC Electronics – Esterline.
2. Calculation of the flight cost for all the possible flight trajectories by performing an exhaustive computation of the parameters included in the PDB of each aircraft to obtain the speeds and altitudes that reduce the global flight cost (VNAV profile).
3. Development and implementation of a dynamic wind model to calculate the influence of the wind and outside air temperature during a flight trajectory.
4. Analysis of alternative flight trajectories to optimize the LNAV profile.
5. Implementation of different time-optimization algorithms to reduce the global computation time of the algorithm.
6. Comparison of the flight trajectory optimization algorithm with real flight trajectories in order to reduce the global flight cost.

0.3 Methodology

In this section, an introduction to the numerical aircraft model is defined, followed by the implementation of the dynamic wind model to calculate the cost of the flight trajectories.

0.3.1 Aircraft model – Performance database

The algorithms in this thesis were developed in Matlab®, using the PDB provided by CMC Electronics – Esterline. The PDB is a database of over 30,000 lines containing information on the real performance of different commercial aircraft. The inputs and the outputs contained in these databases are described in Table 0.1. The flight time is calculated from the aircraft's TAS (True AirSpeed), and the wind influence is calculated with a wind triangle methodology which is explained in the next section. The PDB contains a large quantity of very detailed aircraft information; however, there are five main tables that are used in the development of the algorithms. This information gives the performance (outputs) of each aircraft for different parameters (inputs), at each phase of the flight.

Table 0.1 Inputs and outputs of a typical commercial PDB

Type of table	Inputs	Outputs
Climb	Center of gravity Speed Gross weight ISA deviation Altitude	Fuel burn (kg) Horizontal distance (nm)
Climb acceleration	Gross weight Initial Speed Initial Altitude Delta speed	Fuel burn (kg) Horizontal distance (nm) Delta altitude (ft)
Cruise	Speed Gross weight ISA deviation Altitude	Fuel flow (kg/hr)
Descent deceleration	Vertical speed Gross weight Initial speed Final altitude Delta speed	Fuel burn (kg) Horizontal distance (nm) Delta altitude (ft)
Descent	Speed Gross weight Standard deviation Altitude	Fuel burn (kg) Horizontal distance (nm)

To obtain the performance information from the database, the Lagrange linear interpolation method is applied, as shown in Equation (0.1).

$$x = \frac{y - y_1}{y_0 - y_1} * f_0 + \frac{y - y_0}{y_1 - y_0} * f_1 \qquad (0.1)$$

A complete flight trajectory can be calculated precisely in terms of flight time, distance and fuel burn from Equation (0.1).

0.3.2 Dynamic weather model

The wind data used in this algorithm is extracted from Environment Canada (2013). The information is presented under a Global Deterministic Prediction System (GDPS) format. The GDPS model provides a 601×301 latitude-longitude grid with a resolution of 0.6×0.6 degrees. At each point of this grid, information such as the wind direction, speed, temperature, and pressure can be obtained for different altitudes, in 3-hour time blocks.

The wind directly affects the horizontal distance traveled with respect to ground level, and indirectly affects the fuel consumption. The ground speed is calculated with Equation (0.2) so that it can be considered in the horizontal distance calculation, and is expressed in knots <kt>.

$$\text{Ground speed} = \text{Airspeed} + \text{Effective wind speed} \tag{0.2}$$

The airspeed is an aircraft's speed relative to the air mass, and the wind is the horizontal motion of this air mass relative to the ground. The effective wind is the wind's component of the aircraft's trajectory, and the crosswind is that component perpendicular to the effective wind speed. The effective wind speed is expressed with Equation (0.3).

$$\text{Effective wind speed} = \text{Real wind} - \text{Crosswind} \tag{0.3}$$

In the flight cost optimization program, the influence of the wind is calculated dynamically, i.e., it is updated as the aircraft moves in space and time.

0.3.3 Time-optimization algorithms

To reduce the number of calculations, two different methods are implemented in this project: the Golden Section search and Genetic Algorithms (GAs).

The "Golden Section" method is a nonlinear optimization method that reduces the search interval by the same fraction, with each iteration, at a golden section ratio, which is commonly known in mathematics as the "golden ratio" (Venkataraman, 2009). This method is applied to calculate the VNAV profile without performing an exhaustive search of all the possible flight parameters found in the PDB, but still obtaining the optimal climb and cruise combination.

GAs were used to reduce the calculation time during the cruise, for the LNAV profile. Alternative trajectories were analyzed through a grid (2D and 3D). Within the grid, the number of possible alternative trajectories increases exponentially as its size increases. The calculation of all the possible alternative trajectories is not only impractical, it is also time-consuming.

CHAPTER 1

LITERATURE REVIEW

1.1 Aviation's fuel burn reduction

Multiple solutions to reduce aircraft emissions have been put forward. These solutions can be divided in three major categories : aircraft technology improvement, alternative fuels, and improvements in air traffic management and airline operation (Pan, Huang and Wang, 2014). Each of these categories could increase aircraft efficiency and thereby reduce fuel burn and emissions.

One of the research areas in the aircraft technology improvement category is focused on increasing engine efficiency through lighter designs (Williams and Starke, 2003), increased compression rates (Salvat, Batailly and Legrand, 2013) or optimized aerodynamic patterns (Panovsky J, 2000), to name a few. Airlines have been constantly reducing aircraft weight by changing to lighter seats (AirTransat, 2014). Techniques to install more efficient electrical wiring have also been studied (Wattar et al., 2013). Design studies were performed to reduce drag through wing elasticity improvements (Nguyen et al., 2013), wing morphing (Grigorie, Botez and Popov, 2013) or through the aircraft efficiency increasing through the addition of winglets (Freitag and Schulze, 2009).

To reduce its impact on climate change, the aviation industry has been studying sustainable biofuels to provide a cleaner source of fuel (Sandquist and Guell, 2012). Today, the aviation sector uses petroleum-derived liquid fuels, which is not only a limited fuel resource, it also contributes to CO_2 emissions. Hendricks, Bushnell and Shouse (2011) performed a study on biofuels in which they conclude that there is a large productive capacity for biofuels, and also the potential for carbon emission neutrality and reasonable costs. Airline companies, such as Porter (2012), already use a 50:50 biofuel/Jet A1 fuel blend to perform a complete flight, which showed that biofuels are an important option for a greener aviation sector.

1.2 Flight trajectory optimization

Air traffic management and airline operation improvement would also reduce aviation's environment footprint. Air Traffic Control (ATC) is in charge of assigning the trajectories to airlines; once in-flight, authorization from the ATC is required to perform a trajectory deviation. The FMS is an in-flight device that can be used to identify optimal trajectories to propose to the ATC.

In the mid 1970s, Lockheed developed an FMS to be implemented in their aircraft Tristar-L-1011. Later in the 1980s, other companies started adding the FMS as standard equipment (Avery, 2011). Since then, FMSs have been continuously upgraded and presently all aircraft are equipped with an FMS. The main function of an FMS is to assist the pilot in several tasks, such as navigation, guidance, trajectory prediction and flight path planning.

Even if researchers have been working impetuously on improving FMS, recent studies demonstrated that improvement areas are still vast. Herndon, Cramer and Nicholson (2009) found that many different FMS act differently in terms of optimization and trajectories generation.

Researchers have tried to improve the performance of the FMSs for decades. Lidén (1992) first mentioned that "with no wind, optimal altitude increases nearly linearly with distance as fuel is burned off". He proposed to include wind and temperature variations on the FMSs to obtain an accurate optimization of the vertical flight profile. A couple of years later, Lidén (1994) defined that FMS would be improved by the optimization of aircraft trajectories in order to avoid air traffic issues. These studies were confirmed later by other researchers.

Hagelauer and Mora-Camino (1998) proposed a dynamic programming algorithm for FMSs to calculate the fuel burned by the aircraft during the flight, in order to obtain an accurate estimation for the creation of onboard trajectories. They were able to solve this problem with an acceptable processing time. Dancila, Botez and Labour (2012; 2013) studied a new method to estimate the fuel burn from aircraft to improve the precision of flight trajectory calculations.

In order to have a more substantial impact on the environment, it is much more indicated to conceive and analyze a trajectory optimization for a full flight considering the climb, cruise and descent.

Many research groups have focused specifically on the descent phase, where the goal is to reduce pollution close to air terminals in terms of both noise pollution and fuel burn emissions. Clarke et al. (2004) introduced the Continuous Descent Approach (CDA) method to reduce noise, which consisted of the deceleration and descent of an aircraft at its own vertical profile from the TOD (Top Of Descent). They presented the design and implementation of an optimized profile descent in high-traffic conditions, as for example at the Los Angeles International Airport (LAX), which increased operational efficiency from traffic management and reduced fuel, emissions and noise (Clarke et al., 2013). Dancila, Botez and Ford (2013) created an analysis tool to estimate the fuel and emissions cost produced by aircraft during a missed approach. Reynolds, Ren and Clarke (2007) concluded that the CDA effectively reduced fuel burn and noise near airports simply by keeping the aircraft at the highest possible altitude before its descent.

For long flights, however, the cruise is the phase where the most significant fuel reduction can be obtained (ATAG, 2014). In fact, 80% of the CO_2 emissions produced by aviation come from long flights (more than 1,500 km or 810 nm). To improve the VNAV (Vertical NAVigation) profile in the cruise phase, the pilot has frequently the possibility in-flight to climb to a different flight level in order to reach the optimal altitude.

Lidén studied the variation of the optimal altitude as fuel is burned during the flight (1992). Murrieta (2013) analyzed the cruise phase to determine a pre-optimal vertical profile and to evaluate the altitudes and speeds around its vicinity. Chakravarty (1985), from a flight aerodynamics perspective, described the variation of the optimal cruise speed with flight operation costs. Lovegren (2011) analyzed how the fuel burn could be reduced during the cruise phase by choosing the appropriate cruise altitudes and speeds and by performing SCs. Jensen et al. (2013) presented a speed optimization method for the cruise with fixed lateral movement by analyzing radar information from the United States Federal Aviation Administration's (FAA) Enhanced Traffic Management System (ETMS) (Palacios and Hansman, 2013). Their results show that most flights in the United States do not take place at an optimal speed, which increases their fuel consumption.

The influence of weather on aircraft flight has been considered as part of strategies to take advantage of winds to reduce flight time and/or to avoid headwinds that could also increase global flight costs. Murrieta (2013) presented an algorithm which optimized the vertical and horizontal trajectories by taking into account the wind forces and patterns as well as the variation of the CI. Filippone (2010) analyzed the influence of the cruise altitude on the creation of contrails and on the flight cost. Gagné et al. (2013) performed an exhaustive research of all possible speeds and altitudes to obtain the optimal trajectory and to reduce fuel burn. Bonami et al. (2013) studied a trajectory optimization method capable of guiding an aircraft through different WPs (WayPoints) by considering the wind factors and reducing fuel burn through a multiphase mixed-integer control. Fays and Botez (2013) developed a 4D algorithm treating meteorological conditions or air traffic restrictions in a specified air space, defining them as obstacles in order to improve the FMS's trajectory-creation capabilities. Franco and Rivas (2011) analyzed the minimal fuel consumption for an airplane at a fixed cruise altitude, using a variable arrival-error cost that penalizes both late and early arrivals. They showed that the minimal cost is obtained when the arrival-error cost is null, and found that the use of different optimal cruise altitudes would obtain the minimal cost/lowest fuel consumption with a fixed estimated arrival time.

However, in order to achieve maximal optimization using all these proposed techniques, an improved method of communication between the FMS and the ATC must be established. Mayer (2006) studied the benefits of an integrated aviation modeling and evaluation platform, in which ATC and the FMS could be coupled to obtain better flight path planning.

For both the Next Generation Air Transportation System (NGATS) in the USA, and the Single European Sky ATM Research (SESAR) in Europe, the implementation of Required Time of Arrival (RTA) as a part of the FMS and ATC was an important step towards a better air traffic control. De Smedt and Berz (2007) studied the characteristics of different FMS' performance to determinate the accuracy of their RTA and the influence it could have on ATC. Friberg's (2007) study showed that promising results in terms of the environment could be achieved by establishing communication between the FMS' RTA function and ATC. Air traffic conditions have also been identified as the cause of aircraft missed approaches (Murrieta Mendoza, Botez and Ford, 2014).

In this thesis, a complete flight profile is analyzed, including the climb, cruise and descent, considering both the LNAV and VNAV profiles. These algorithms have been developed to propose an optimal flight trajectory through the FMS to ATC for authorization. The optimization results obtained in this project would still require ATC's permission to execute the proposed optimal trajectory.

1.3 Calculation time optimization algorithms

Searching among all the different possible trajectories that an aircraft could choose would be ideal to find the optimal trajectory that minimizes the fuel burn, but it would eventually result in a long processing time algorithm. In order to reduce the computing time, different time optimization methods have been applied. These methods reduce the computing time by analyzing a smaller portion of the possible flight trajectories that would converge to the minimal fuel burn trajectory. Stochastic methods were considered as possible solution for solving our optimization problem. The Monte Carlo optimization algorithm proposed by Visintini et al. (2006) applied in ATC systems, would be a reasonable approach to find the aircraft optimal trajectories for fuel burn reduction on aircraft. The Monte Carlo method explores the entire range of solutions (that as it was mentioned before, those are practically infinite) while following a random path to converge towards the optimum value of the study, in this case, the maximal fuel efficiency trajectory.

The Golden Section search optimization algorithm has been applied to the calculation of the optimal cruise. This is a nonlinear optimization method that reduces the search interval by the same fraction, with each iteration, at a golden section ratio, which is commonly known in mathematics as the golden ratio (Venkataraman, 2009).

GAs have been used in aviation to resolve high complexity problems, and are useful when a solution involving many imposed restrictions is searched. Kanury and Song (2006) used GAs to find the optimal trajectory under the presence of unknown obstacles, obtaining satisfactory results in a short computing time; the optimal route was obtained using their algorithm and the calculation time was reduced. Turgut and Rosen (2012) used GAs to obtain the optimal aircraft descent in terms of the fuel flow values and altitudes to reduce the global descent cost. These algorithms are useful when searching for a solution involving multiple imposed restrictions. Kouba (2010) studied GAs as a means to incorporate several constraints into a trajectory optimization problem, where the objective was to find the shortest route while considering different restrictions.

Different versions of the GAs have been used throughout this thesis in order to reduce the calculation time for the trajectory optimization problem (Félix Patrón, Berrou and Botez, 2014; Félix Patrón et al., 2013; Félix Patrón et al., 2013).

The flight trajectory optimization algorithms proposed in this thesis reduce the fuel burn while the calculation time is optimized.

CHAPTER 2

APPROACH AND ORGANIZATION OF THE THESIS

The research project presented in this thesis could be divided in four main phases:

- Statement of the problem and model validation
- Optimization of the VNAV profile
- Optimization of the LNAV profile
- Coupling of both the VNAV and LNAV profiles

During the first phase, all the possible flight trajectories were calculated using the aircrafts PDB through MATLAB®, and the aircraft's fuel consumption and flight time were obtained. The results obtained with the algorithm were compared with the results obtained by the flight simulator FlightSIM® from Presagis. The results showed that the results were close to reality and the aircraft models were validated.

During the second phase, after the validation of the aircraft models, for a given flight segment, all the possible flight trajectories for a single path (no horizontal deviations) were calculated using an exhaustive search, analyzing all the different available parameters given by the PDB such as aircraft weight, altitude and speeds. The trajectory representing the lowest flight cost was obtained and defined as the "optimal trajectory for the VNAV profile".

The third phase consisted in the implementation of a weather model into the algorithm. With a complete weather model around the flight route, the algorithm was capable of analyzing possible alternative trajectories to take advantage of the winds aiming to reduce the flight cost. As the trajectory was larger, the number of calculations increased and different time-optimization algorithms were applied. The algorithm analyzed only the cruise phase for long flights, since it is in this phase where an alternative trajectory, even if increasing the actual flight distance, could help the aircraft to reduce de fuel burn by a correct interpretation of the winds. At a fixed altitude during cruise, the LNAV profile was optimized.

In the fourth and final phase, the optimization for both the VNAV and LNAV were coupled in order to obtain the maximal flight optimization.

As the main author, the research in this thesis was diffused in four journal papers and six conference papers. Three of the journal articles have been published and one is currently under review for publication in peer-review scientific journals. These papers are presented from Chapter 3 to Chapter 6.

Dr. Ruxandra Botez, as a co-author, supervised the realization of all the presented research. In the second paper, the internship student Aniss Kessaci worked as a co-author by implementing the GA used to reduce calculation time. In the third paper, the internship student Yolène Berrou created the method to calculate weather dynamically and the coupling of both LNAV and VNAV algorithms. Mr. Dominique Labour, a co-author from the company CMC Electronics – Esterline, worked in-house on the experimental validation of the project developed by our academic team.

In **Chapter 3**, the research paper entitled "New altitude optimization algorithm for a Flight Management System platform improvement on commercial aircraft" (Félix Patrón, Botez and Labour, 2013), was published in The Aeronautical Journal, in August 2013.

This paper presents an introduction to the trajectory optimization subject. In this paper, the aircraft has been numerically modeled through the PDB using Matlab®. It includes a description of the parameters considered during the flight to calculate the flight trajectories, and includes a complete calculation of each flight trajectory. A description is made of how the climb, cruise and descent are calculated.

The flight cost results obtained by the proposed algorithm are compared with the simulator FlightSIM® results to validate the model of the aircraft. Then, the results obtained with the in-house algorithm were compared with the PTT results, which represent a commercial FMS from the company CMC Electronics. At this point, only the VNAV profile was analyzed.

The algorithm calculated the cost for short and long flights differently, and the golden section search method was applied to reduce the calculation time.

In **Chapter 4**, the research paper entitled "Horizontal flight trajectories optimization for commercial aircraft through a Flight Management System" (Félix Patrón, Kessaci and Botez, 2014) was published in The Aeronautical Journal in December 2014.

In this paper, the LNAV profile was optimized during the cruise for long flights, since for short flights (fewer than 500 nm), a horizontal optimization was not profitable.

In this paper, a set of alternative trajectories was created around the original flight path to analyze if by considering the winds, a horizontal deviation is possible. A real weather model was implemented. If the wind influence indicated that a deviation should have been made, the flight cost was calculated for an alternative trajectory and a flight cost reduction was obtained. At this point, the aircraft was held at a constant in-cruise altitude.

The grids in which the alternatives trajectories were traced were variable in size. As the size of these grids increases, the number of possible trajectories also increases. To maintain a low calculation time, a GA using a roulette wheel selection method was implemented.

The third research paper is presented in **Chapter 5**, with the title "New methods of optimization of the flight profiles for performance database-modeled aircraft" (Félix Patrón, Berrou and Botez, 2014) was published in the Journal of Aerospace in December 2014.

After calculating separately the VNAV and LNAV profiles in the two previous research papers, both profiles were coupled to analyze the flight trajectories more deeply. The weather model was calculated dynamically, and included a better implementation of the aircraft model to increase the calculation precision. It was now allowed to optimize the LNAV profile during cruise, while altitudes changed through the VNAV profile. At this point, the highest flight cost reduction was obtained while the algorithm results were compared with real flight information. To reduce calculation time, a GA were applied.

Finally, in **Chapter 6**, the research paper entitled “Flight trajectory optimization through genetic algorithms coupling vertical and lateral profiles” was submitted to the Journal of Computational and Nonlinear Dynamics in August 2014, and it is under review for its publication.

In this paper, only the cruise phase was analyzed. Previously in Chapter 4 and Chapter 5, the LNAV profile considered a 2D grid, in which the optimal horizontal profile was calculated. A 3D grid has been implemented to improve the cruise phase calculation. By analyzing only the cruise results, the calculation time was reduced while a better analysis of the alternative trajectories was performed. The algorithm’s calculation time was also reduced by implementing a GA.

Following the aforementioned structure, a complete flight trajectory was optimized in this thesis, from the validation of the model to the comparison of the model’s trajectories with real flight trajectories, and a significant costs reduction was obtained.

In addition to the four previously mentioned journal papers, six conference papers about this research project were also published and presented, but are not included in this Thesis for reasons of clarity and length of the document. However, the research performed in these conference papers is described next.

The first conference paper "Vertical profile optimization for a Flight Management System using the golden section search method" defined a methodology that optimized the vertical flight profile in terms of speeds and altitudes, through which a trajectory that reduces the global flight cost was obtained (Félix Patrón, Botez and Labour, 2012). It was presented at IECON 2012 – 38th Annual Conference on IEEE Industrial Electronics Society, in Montreal, Quebec, Canada, on October 28th, 2012.

The second conference paper "Low calculation time interpolation method on the altitude optimization algorithm for a commercial FMS" defined an interpolation procedure to calculate fuel burn, distance traveled and flight time, thus the lowest algorithm execution time possible (Félix Patrón, Botez and Labour, 2013). It was presented at the AIAA Aviation 2013 conference, in Los Angeles, California, United States, on August 14th, 2013.

The third conference paper "Speed and altitude optimization on a commercial using genetic algorithms" considered a GA to reduce the calculation time of a vertical profile optimization algorithm (Félix Patrón et al., 2013). It was presented at the AIAA Aviation 2013 conference, in Los Angeles, California, United States, on August 14th, 2013.

The fourth conference paper "Flight trajectories optimization under the influence of winds using genetic algorithms" analyzed the LNAV profile using GAs to obtain the flight trajectory which considered the influence of the wind to reduce fuel burn and flight time (Félix Patrón et al., 2013). It was presented at the AIAA Guidance, Navigation and Control conference, in Boston, Massachusetts, United States, on August 20th, 2013.

The fifth conference paper "Climb, Cruise and Descent 3D Trajectory Optimization Algorithm for a Flight Management System" presented the combination of a LNAV and VNAV optimization algorithms (Félix Patrón, Berrou and Botez, 2014). It was presented at the AIAA Aviation 2014 conference, in Atlanta, Georgia, United States, on June 19th, 2014.

The sixth conference paper "Flight trajectory optimization through genetic algorithms coupling vertical and lateral profiles" presented a combination of LNAV and VNAV optimization during the cruise phase, creating alternative trajectories and analyzing the possibility of making a deviation to reduce fuel burn (Félix Patrón and Botez, 2014). It was presented at the Proceedings of the ASME 2014 International Mechanical Engineering Congress and Exposition, in Montreal, Quebec, Canada, on November 18th, 2014.

CHAPTER 3

ARTICLE 1: NEW ALTITUDE OPTIMIZATION ALGORITHM FOR A FLIGHT MANAGEMENT SYSTEM PLATFORM IMPROVEMENT ON COMMERCIAL AIRCRAFT

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Résumé

Cet article définit une méthodologie pour optimiser un système de gestion de vol commercial en analysant les vitesses et altitudes pour le profil vertical, en obtenant une trajectoire qui réduit le coût global du vol.

La base de données de performances (PDB) fournie par CMC Électronique – Esterline est actuellement utilisée à bord de plusieurs avions commerciaux. La PDB est utilisée comme référence pour la conception de différents algorithmes d'optimisation afin d'obtenir l'altitude optimale à laquelle l'économie de carburant de l'avion est maximale. Les résultats obtenus par ces algorithmes d'optimisation sont comparés avec les résultats obtenus avec le PTT (de l'anglais *Part-TaskTrainer*), simulateur qui représente un système de gestion de vol commercial, fourni aussi par CMC Électronique – Esterline.

Pour valider les résultats, le logiciel FlightSIM® est utilisé. Ce logiciel considère un modèle aérodynamique de vol complet pour ses simulations, et ainsi permet d'obtenir des résultats très proches de la réalité.

Abstract

This article defines a methodology that optimizes a commercial FMS by analyzing the speeds and altitudes for the vertical profile, obtaining a trajectory that reduces the global flight cost.

The PDB (Performance DataBase) provided by CMC Electronics – Esterline is presently used on different commercial airplanes. The PDB is used as the reference to design different trajectory optimization algorithms to obtain the altitude where the aircraft fuel efficiency is the best. These algorithms are compared with the PTT, simulator that represents a commercial FMS, supplied by CMC Electronics – Esterline as well.

To validate the results, the FlightSIM® software is used, which considers a complete aircraft aerodynamic model for its simulations, giving accurate results and very close to reality.

3.1 Introduction

The reduction of fuel consumption on aircraft has taken different tendencies: the development of more efficient engines to decrease the production of pollutant emissions, improvements to the frame to make the aircraft more fuel efficient, or the optimization of the flight trajectories. This article will focus on the FMS capability of creating optimal flight trajectories.

Since the first FMS was added as standard equipment to an aircraft in 1982 (Avery, 2011), FMS have been continuously upgraded, and presently all aircraft are equipped with one. The primary functions of a FMS are to assist the pilot in several tasks, such as navigation, guidance, trajectory prediction and flight path planning.

Even if researchers have been working impetuously on improving the performance of FMS, recent studies demonstrate that improvement areas are still vast. Herndon, Cramer and Nicholson (2009) found that different FMS act differently in terms of optimization and trajectories generation. It is then important to mention that this article focuses on the improvement for a commercial FMS.

The studies of optimal trajectories in aviation have incremented considerably over the last ten years. Many different tendencies have appeared to reduce the fuel burn. Studies to include aircraft traffic control as one of the FMS functions, without the assistance of the ATC, have been analyzed (Schoemig et al., 2006). The main purpose of the ATC is to keep aircraft separated by a safe distance. The ATC will decide if the trajectory proposed by the FMS can be followed by the aircraft.

Other studies have focused specifically in the descent phase, where the goal is to reduce pollution near to air terminals in terms of noise pollution and fuel burn emissions. Different descent techniques have been proposed. Clarke et al. (2004) introduced the CDA method to reduce noise, which consisted in the deceleration and descent of the aircraft at its own vertical profile from the TOD. This method, however, depends on the ATC to proceed, since it needs to have a clear path to the runway. Tong et al. (2007) explained that the CDA can only be used in low air traffic conditions, since "ATC lacks the required ground automation to provide separation assurance services during CDA operations". He then proposed a 3D Path Arrival Management (3D PAM) algorithm to predict 3D descent trajectories and be able to apply CDA in high traffic conditions. Reynolds, Ren and Clarke (2007) concluded after different tests in the Nottingham East Midlands Airport that CDA effectively reduced fuel burn and noise near the terminals simply by keeping the aircraft at the higher altitude possible before creating the descent. Stell (2010) used an Efficient Descent Advisor, which is a method to predict the latest descent point (equivalent to the TOD) in order to apply the 3D PAM technique, but it still needs an improved ATC in order to operate at its maximal efficiency.

To obtain a more substantial impact on the environment, all the flight phases –climb, cruise and descent- have to be analyzed.

The cruise is the most important phase of the flight in terms of fuel economization. Lovegren (2011) analyzed how the fuel burn could be reduced during the cruise if the appropriate speed and altitude is selected, or if SCs are performed on this phase. The selection of the optimal climb, cruise and descent on a FMS will definitely reduce fuel burn.

Campbell (2010) studied the influence of weather imposed obstacles, such as thunderstorms and contrails, in the analysis of air pollution and fuel burn augmentation. He modeled these climatic conditions as obstacles, and created an algorithm capable of creating trajectories to avoid these obstacles with the minimal fuel consumption.

Ideally, to obtain the optimal flight trajectory that minimizes the global flight cost, all the possible flight trajectories would have to be analyzed. However, this would result in a high calculation time process. Instead of calculating all the possibilities, an optimization method is applied. Different optimization methods have been used on aviation systems, such as the Monte Carlo method used by Visintini et al.(2006) to avoid air traffic conflicts and increase air safety, or the GA used by Kouba (2010) to create flight trajectories based on aircraft modeled in six different dimensions. The GA allowed the author to impose several restrictions and still optimize the trajectories.

The trajectory optimization new algorithm proposed in this article is developed using the aircraft PDB data collected by CMC Electronics – Esterline with the aim to be adapted on their FMS; nevertheless, speed and altitude restrictions can be imposed at each WP of the flight trajectory. The maximal optimized trajectory is obtained when all the speeds and altitudes are used; and even if the ATC sets certain restrictions, the algorithm will still find the optimal trajectory within these restrictions. In our algorithm, with respect to other algorithms, a complete flight analysis is performed, and all the phases of the flight can be adapted to ATC's requirements to obtain the maximal optimization and emissions reduction.

During its first phase, only the vertical profile is optimized. Wind conditions are also considered in the calculation of the costs, and the methodology is explained in the following sections of this article. The next versions of the algorithm should include the analysis of the lateral profile, and the obtaining of the weather automatically.

All the available speeds and altitudes are calculated for the climb and cruise, but since the descent start point varies in terms of aircraft weight and remaining flying distance, it would be inefficient to calculate every descent. Optimization methods such as Monte Carlo or GA are expensive in terms of calculation time and not effective when the search space is reduced. Therefore, an interval reducing method was selected. The golden section search is the best of the interval-reducing methods and it is useful on this project because of its simplicity for implementation (Venkataraman, 2009). This method will be later explained in this article.

Aircraft fuel burn is an important contributor for Carbon dioxide (CO_2) emissions to the atmosphere, the principal greenhouse gas. Total CO_2 emissions dues to aircraft traffic represent between 2.0% and 2.5% of all CO_2 emissions to the atmosphere (ICAO, 2010). Greenhouse gases contribute to the global warming effect, which is one of the most important environmental problems encountered nowadays. The creation of more efficient trajectories for aircraft would contribute in the reduction of fuel burn, therefore in the reduction of CO_2 emissions to the atmosphere.

In Canada, the Green Aviation Research & Development Network (GARDN) was founded in 2009. The first project in this network was called Optimized Descents and Cruise. The new proposed optimization algorithm is developed in this project, where the data needed for validation was provided by the well known avionics company CMC Electronics – Esterline.

3.2 Global cost

In aviation, not only the fuel burn is considered in order to plan a flight trajectory. Variables such as the flight time and operation costs must be taken into account. The CI is the term used by the airlines to calculate the operation costs for each flight.

To calculate the global cost of the flight, the fuel cost should be obtained first:

$$\text{Fuel Cost} = \text{Fuel Price} * \sum \text{Fuel burned} \tag{3.1}$$

Where the Fuel Cost is expressed in $, the Fuel Price in $/Kg and the Fuel Burned in Kg.

$$\text{Operation Cost} = \text{Fuel Price} * \text{CI} * \text{Flight Time} * 60 \tag{3.2}$$

Where the Operation Cost is given in $, the CI in Kg/min and depends on each company. The Flight Time is expressed in hours (h), and the number 60 is a constant to convert minutes to hours. The global cost is the sum of the operation and fuel costs, then:

$$\text{Global cost} = \text{Fuel Cost} + \text{Operation Cost} \tag{3.3}$$

$$\text{Global Cost} = \text{Fuel Price} * [\sum \text{Fuel burned} + \text{CI} * \text{Flight Time} * 60] \tag{3.4}$$

It turns to be illogical to consider the Fuel Price, since it changes every time, therefore, to simplify the equation the Global Cost will be given in Kg of fuel, that would have to be multiplied by the fuel price at the moment of the utilization of the algorithm in order to obtain a quantity in terms of Money ($).

$$\text{Global Cost} = \sum \text{Fuel burned} + \text{CI} * \text{Flight Time} * 60 \tag{3.5}$$

The goal of this algorithm is to reduce the global cost of the flight.

3.3 Methodology

Currently, commercial FMS provide a speed optimization, which is calculated from the PDB. It also determines an optimal altitude for the initial values of the aircraft, which can be inaccurate because the fuel reduction is not updated during the flight, thus, the given altitudes and speeds are not truly optimal.

In this paper, the new proposed algorithm will be explained in details. This algorithm improves considerably the FMS trajectory planning by:

- A complete analysis of the variation of speeds and altitudes for the climb phase.
- The search of possible SCs to be executed during the cruise phase to reduce the flight cost.
- The calculation of the optimal descent speed in terms of global cost reduction.

All flight phases are considered in order to obtain the best possible optimization results. The new algorithm improves the path planning and reduces flight cost. Additional altitude, speed and time restrictions are also considered in the development of this optimization algorithm. The new algorithm was developed in Matlab® based on the PDB for different commercial aircraft, and it is capable of reducing the fuel burn with an average of 2.57% (to the date).

Fundamental research data for this project is given by the PDB. The numerical model of the aircraft provides all the necessary information to create the algorithm. The PDB is a database of approximately 30,000 lines, which gives the information about real aircraft performances. It indicates the fuel consumption and the distance flown for a specific flight profile (climb, cruise or descent). For example, the fuel burn and distance for an aircraft cruising with a center of gravity of 28% of the mean aerodynamic chord, flying at 0.8 Mach with a total gross weight of 100 tons, at an altitude of 30,000 ft and a standard deviation temperature of -10ºC. Such an example is shown on Figure 3.1:

```
MODE CRUISE_PROFILE_MACH
! * * * * * * * * * * * * * * * * *
! Last Update     : 2011/01/23 8h2
! Reference CG    : 28
! * * * * * * * * * * * * * * * * *
   SPEED  0.8
      GROSS_WEIGHT  100000
         ISA_DEV  -10
            !Altitude   FuelFlow
              25000     5355
              26000     5146
              27000     4948
              28000     4756
              29000     4574
              30000     4402
              31000     4243
              32000     4091
              33000     3946
              34000     3809
              35000     3685
              36000     3574
              37000     3495
              38000     3441
              39000     3405
              40000     3378
              41000     3376
```

Figure 3.1 Example of information given on the PDB

The PDBs includes as inputs the aircraft weight, altitude, speed, center of gravity and air temperature, and the outputs are the traveled distance and the fuel burn. The traveled time is calculated from the aircraft's TAS, while the wind influence is calculated with a wind triangle methodology, providing a traveled distance correction factor depending on the wind angle and speed. The wind speed and direction are entered manually into the algorithm, at four different altitudes, at each flight WP, in the same way as on the FMS.

The PDB contains very detailed aircraft information; however, there are five main tables that are used in this program and can be observed in Table 3.1.

The wind influence on the trajectory will be calculated using the wind triangle method (Figure 3.2). As the aircraft flights on a straight path, the wind affects the aircraft's speed. Depending on the direction and speed of the wind, the distance traveled by the aircraft will be either reduced or augmented in a time segment.

Table 3.1 Inputs and outputs for the PDB for a commercial aircraft

Type of table	Inputs	Outputs
Climb	Center of gravity Speed Gross weight ISA deviation Altitude	Fuel burn Horizontal distance
Climb acceleration	Gross weight Initial Speed Initial Altitude Delta speed	Fuel burn Horizontal distance Delta altitude
Cruise	Speed Gross weight ISA deviation Altitude	Fuel flow
Descent deceleration	Vertical speed Gross weight Initial speed Final altitude Delta speed	Fuel burn Horizontal distance Delta altitude
Descent	Speed Gross weight Standard deviation Altitude	Fuel burn Horizontal distance

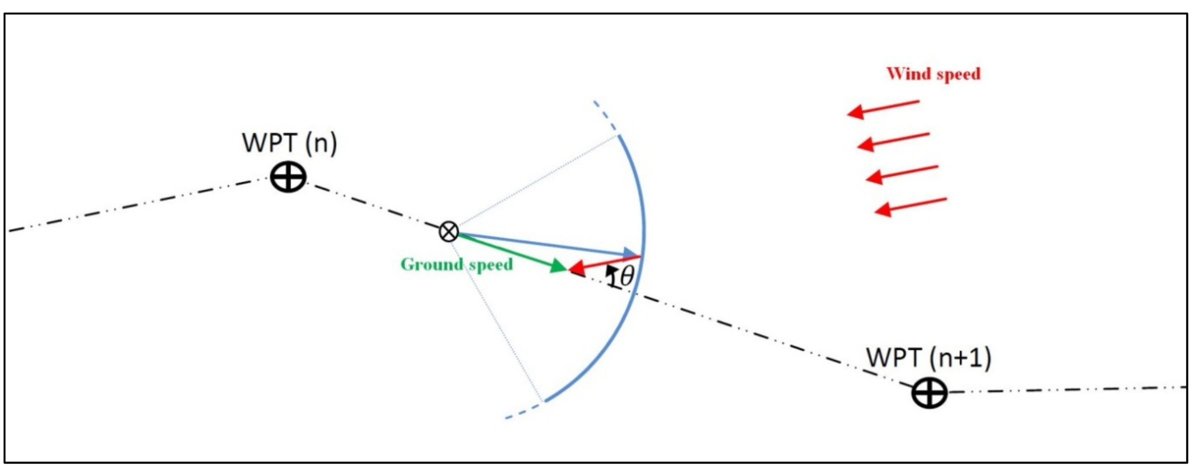

Figure 3.2 Wind factor calculation (Langlet, 2011)

The wind factor can be calculated in the following way (Langlet, 2011):

$$Wind\_factor = \cos\left[\arcsin\left(\frac{\sin(\theta) * \| \overrightarrow{Wind_{speed}} \|}{\| \overrightarrow{Air_{speed}} \|}\right)\right] - \frac{\| \overrightarrow{Wind_{speed}} \|}{\| \overrightarrow{Air_{speed}} \|} * \cos(\theta) \tag{3.6}$$

3.4 Climb

The PDB divides the TAS values in two different types of speeds: IAS (Indicated AirSpeed) and Mach number. The TAS varies with the altitude. For the IAS case, the TAS increases with the altitude, while Mach decreases with the altitude. The altitude for which the TAS due to IAS is equal to the TAS due to Mach is called the crossover altitude. Table 3.2 represents an example for a 300/0.82 speed schedule (composed from an IAS/Mach pair), with an altitude step of 1,000 ft.

The climb phase consists of four different phases:

- Initial climb. Aircraft is located initially at 2,000 ft, and will climb up to 10,000 ft at a constant predefined speed (normally 250 IAS).
- Acceleration phase. Aircraft will accelerate to the selected optimal IAS speed.
- IAS climb. Aircraft will climb at a constant IAS speed after the acceleration phase until the crossover altitude.
- Mach climb. Once the aircraft reaches the crossover altitude, it will climb at a constant Mach speed.

For the purpose of this project and in order to reduce processing time, the Mach speed selected during the cruise phase remains constant through the complete flight. Speed variation during the cruise phase will be considered for future work.

To select the optimal climb for the flight, all available speed schedules will be calculated. Each speed schedule expressed as IAS/Mach has its own crossover altitude that can be seen in Table 3.3. For each IAS/Mach couple, the crossover altitude is calculated using a 1,000 ft altitude step.

Table 3.2 Crossover altitude example for a 300/0.82 speed schedule

Altitude (ft)	TAS due to an IAS of 300 knots (knots)	TAS due to a Mach number of 0.82 (knots)
10,000	345.4	523.2
11,000	350.4	521.3
12,000	355.6	519.4
13,000	360.8	517.4
14,000	366.1	515.5
15,000	371.6	513.5
16,000	377.1	511.6
17,000	382.7	509.6
18,000	388.4	507.7
19,000	394.3	505.8
20,000	400.2	503.8
21,000	406.3	501.4
22,000	412.5	499.4
23,000	418.8	497.5
24,000	425.2	495.6
25,000	431.7	493.6
26,000	438.3	491.2
27,000	445.1	489.2
28,000	452.0	487.3
29,000	459.0	485.4
30,000	466.2	482.9
31,000	473.4	481.0
32,000	**480.8**	**479.0**
33,000	488.4	476.6
34,000	496.1	474.7
35,000	503.9	472.7
36,000	512.5	470.3
37,000	521.8	470.3
38,000	531.6	470.3
39,000	542.0	470.3

The aircraft climbs at a constant 250 IAS from 2,000 ft to 10,000 ft. At 10,000 ft, the acceleration tables are created for each IAS speed. At the final acceleration altitude, the climb for each available IAS is calculated up to the maximal climb altitude (normally, 40,000 ft). The aircraft will only cruise at the Mach speed. After the IAS climb table is calculated, the Mach climb is calculated from the crossover altitude and up to the maximal altitude.

From the crossover altitude and for each 1,000 ft over the crossover altitude, the cruise cost is calculated for the entire length of the flight and is saved in the flight cost table. The flight cost table contains all the possible speed schedules and all the possible cruise altitudes. From the minimal cruise altitude (20,000 ft) to the maximal altitude, only the lowest cost speed schedule for each altitude is selected. Figure 3.3 represents the climb phase.

Table 3.3 Crossover altitudes table (ft)

IAS/ Mach	250	260	270	280	290	300	310	320	330	340	350	360
0.78	38,000	36,000	35,000	33,000	31,000	30,000	28,000	27,000	25,000	24,000	22,000	21,000
0.785	38,000	36,000	35,000	33,000	32,000	30,000	29,000	27,000	26,000	24,000	23,000	21,000
0.79	39,000	37,000	35,000	34,000	32,000	30,000	29,000	27,000	26,000	24,000	23,000	22,000
0.795	39,000	37,000	35,000	34,000	32,000	31,000	29,000	28,000	26,000	25,000	23,000	22,000
0.8	39,000	37,000	36,000	34,000	33,000	31,000	30,000	28,000	27,000	25,000	24,000	22,000
0.805	39,000	38,000	36,000	34,000	33,000	31,000	30,000	28,000	27,000	25,000	24,000	23,000
0.81	39,000	38,000	36,000	35,000	33,000	32,000	30,000	29,000	27,000	26,000	24,000	23,000
0.815	40,000	38,000	37,000	35,000	34,000	32,000	30,000	29,000	28,000	26,000	25,000	23,000
0.82	40,000	39,000	37,000	35,000	34,000	32,000	31,000	29,000	28,000	26,000	25,000	24,000
0.825	40,000	39,000	37,000	36,000	34,000	33,000	31,000	30,000	28,000	27,000	25,000	24,000
0.83	40,000	39,000	38,000	36,000	34,000	33,000	31,000	30,000	29,000	27,000	26,000	24,000
0.835	41,000	39,000	38,000	36,000	35,000	33,000	32,000	30,000	29,000	27,000	26,000	25,000
0.84	41,000	39,000	38,000	36,000	35,000	33,000	32,000	31,000	29,000	28,000	26,000	25,000

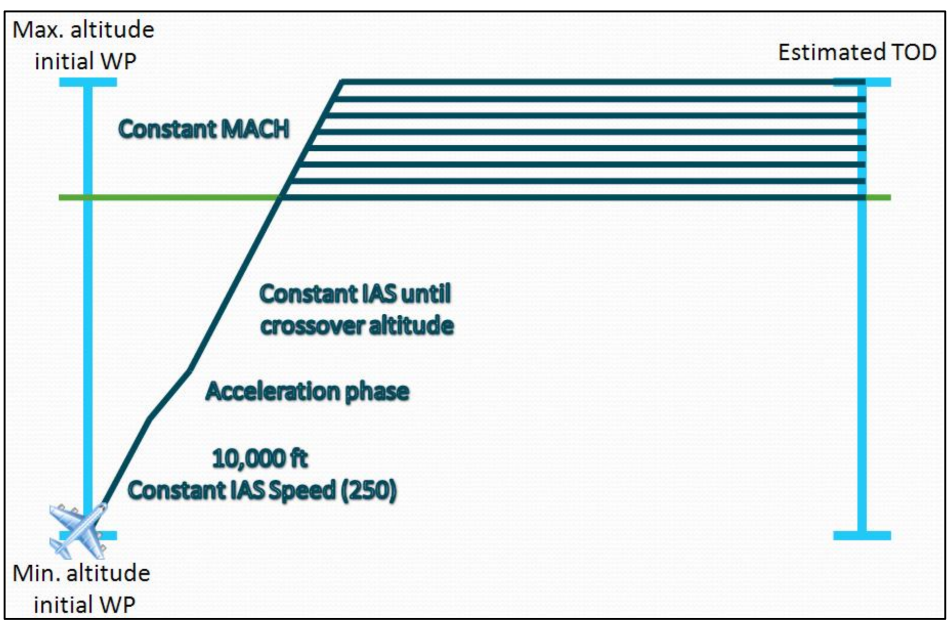

Figure 3.3 Climb phase

3.5 Cruise

The cost optimization algorithm calculates the optimal trajectory depending on the flight length. For short flights (under 500 nm), where usually flight restrictions are not changed during the flight, the algorithm obtains the lowest cost altitude and speed schedule from the flight cost table. For short flights, the descent phase has high influence on the global cost of the flight. Since the descent is the lowest cost phase during a flight, it is possible that would be better if the aircraft would climb higher (higher cost) in order to have a longer descent and a shorter cruise. The cost optimization algorithm uses the Golden Section search optimization algorithm for the cruise. Calculating all the possible descents for the flight cost table would result in an excessive (and unnecessary) calculation time, therefore, the Golden Section method is applied. The Golden Section method is a non linear optimization method that reduces the search interval by the same fraction, with each iteration, at a golden section ratio, which is commonly known in mathematics as the golden ratio (Venkataraman, 2009). The golden section search was selected over other interval reducing methods, such as the dichotomous search or the Fibonacci method, because its efficiency and ease of implementation. The dichotomous search calculates two new evaluations at each iteration, while the golden section search and the Fibonacci method only calculate one new evaluation at each new iteration. The Fibonacci method, however, reduces the size of the interval by the Fibonacci series, which changes the reduction size with each iteration. The golden section search uses a fixed interval reduction, making it simpler to implement.

Applied to the trajectory optimization algorithm, the Golden Section search is the most adequate of the interval reducing methods. The fewest number of iterations are obtained, and its simplicity reduces the algorithm processing time.

The algorithm obtains the lowest cost speed schedule and altitude, which may not be the maximal altitude. Since it could be possible that climbing at a higher altitude (to have a longer descent phase) would result in a lowest global cost trajectory, the method should calculate the descent for all possible altitudes over the cost altitude selected from the flight cost table. Calculating all the possibilities, as it was mentioned before, would result in an excessive calculation time for the algorithm.

The Golden Section method selects a search range, which in this case is from the lowest cost altitude *a* to the maximal available cruise altitude *b* [a,b]. The algorithm divides the search range applying the gold ratio rule, creating two intersections within [a,b], that are named x_1 and x_2 and are calculated as follows:

$$x_1 = \gamma * \mathrm{a} + (1 - \gamma) * \mathrm{b} \tag{3.7}$$

$$x_2 = (1 - \gamma) * \mathrm{a} + \gamma * \mathrm{b} \tag{3.8}$$

Where γ is the golden ratio (0.618), and x_1 and x_2 are the altitudes within the search range, and are rounded to the nearest thousand (the algorithm calculates at each 1,000 ft). The descent is calculated for altitudes x_1 and x_2, and both complete trajectories are compared to continue with the optimization algorithm in the next way:

$$\begin{gathered} Iff(x_1) < f(x_2) \\ \mathrm{b} = x_2 \\ x_2 = x_1 \\ x_1 = \gamma * \mathrm{a} + (1 - \gamma) * \mathrm{b} \end{gathered} \tag{3.9}$$

$$\begin{gathered} Iff(x_2) < f(x_1) \\ \mathrm{a} = x_1 \\ x_1 = x_2 \\ x_2 = (1 - \gamma) * \mathrm{a} + \gamma * \mathrm{b} \end{gathered} \tag{3.10}$$

Where $f(x_1)$ and $f(x_2)$ are the global cost for the trajectories at x_1 and x_2.

In case that because of the rounding of the altitudes, x_1 and x_2 are the same, the algorithm calculates the global cost values for a and b, and eliminates the trajectory with the highest cost. Variable *a* or *b* is replaced.

The Golden Section method stops at a desired tolerance. In this case, it will stop when the search interval is reduced to 2,000 ft (the algorithm cannot calculate two intersections in this interval).The algorithm gives the final trajectory, which is the lowest cost trajectory for the desired flight.

The Golden Section method, applied to the trajectory optimization method, can be better explained by the flow chart in Figure 3.4.

With this methodology, not all possible descents are calculated, but only those for the lowest cost climb and cruise, reducing the number of iterations for the algorithm.

For long flights, the descent phase has a low influence on the global cost. The optimal trajectory is then selected using WPs (Figure 3.5). The trajectory is divided in a number *n* of WPs, where the first WP is used for the climb, and the last one for the descent. In between, WPs allow the imposition of constraints during the flight, such as altitude and speed restrictions, deviation angles, and even time restrictions. After the selection of the optimal climb (flight cost table), at each cruise WP, the possibility to climb at a higher altitude to reduce the flight cost is evaluated. The algorithm evaluates the cost of the climb and the cruise above current altitude, and determines if it is better to climb at a higher altitude to reduce the flight cost. At the last WP, the optimal descent is calculated.

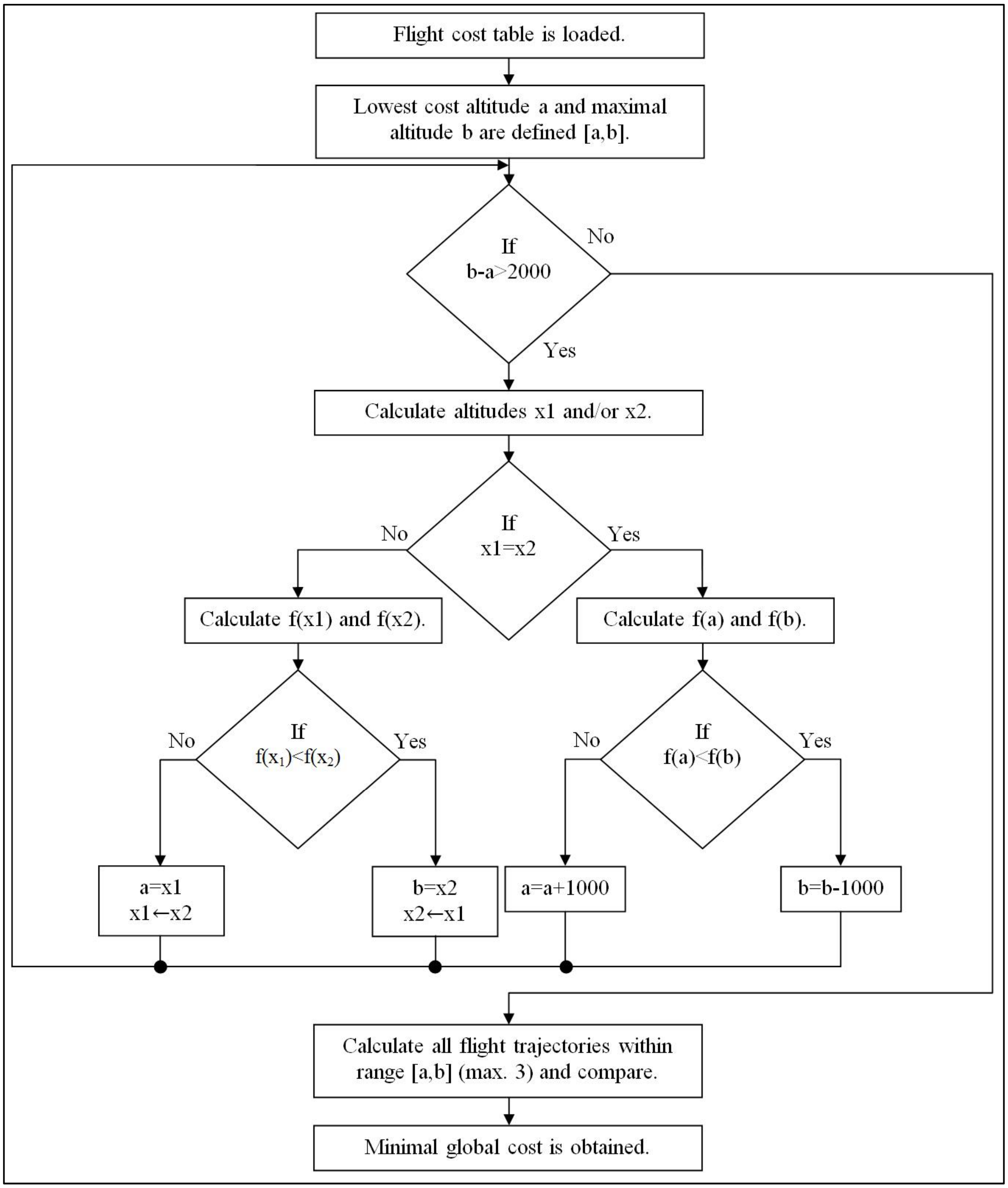

Figure 3.4 Golden section method flowchart

Figure 3.5 Cruise phase (flights over 500 nm)

3.6 Descent

To calculate the descent, the algorithm uses the Mach speed that the aircraft has at the TOD. The descent has the same phases as the climb, but calculated backwards. The descent is represented by Figure 3.6.

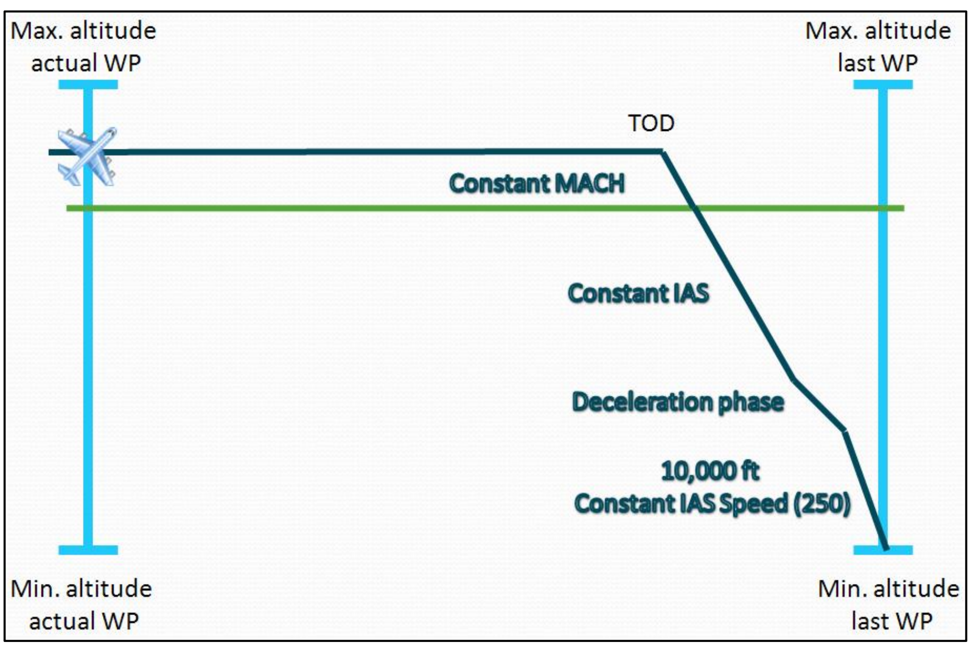

Figure 3.6 Descent

In order to calculate correctly the descent, the horizontal distance has to be estimated.

- The descent from 10,000 ft to 2,000 ft is made at constant 250 IAS, and it is calculated first to estimate the horizontal distance traveled.
- The deceleration is calculated afterwards to obtain the altitude at which the deceleration process should start for each IAS speed.
- Since there is only one Mach speed available (current aircraft speed), the speed schedules will be those Mach/IAS pairs that have current Mach speed. The IAS descent from the crossover altitude and up to the deceleration altitude is calculated, followed by the Mach descent until the crossover altitude.
- The approximated descent horizontal distance is now known for each Mach/IAS pair, and the descent that consists in the lowest fuel per nautical mile ratio is selected as the optimal descent. The cruise distance to arrive to the estimated TOD, is therefore, also known.
- Since the descent is estimated, the horizontal distance is not exact. If the aircraft does not arrive to the final coordinate, the distance difference is applied to the cruise distance, and the optimal descent is recalculated.

The descent methodology is explained by flowchart in Figure 2.7.

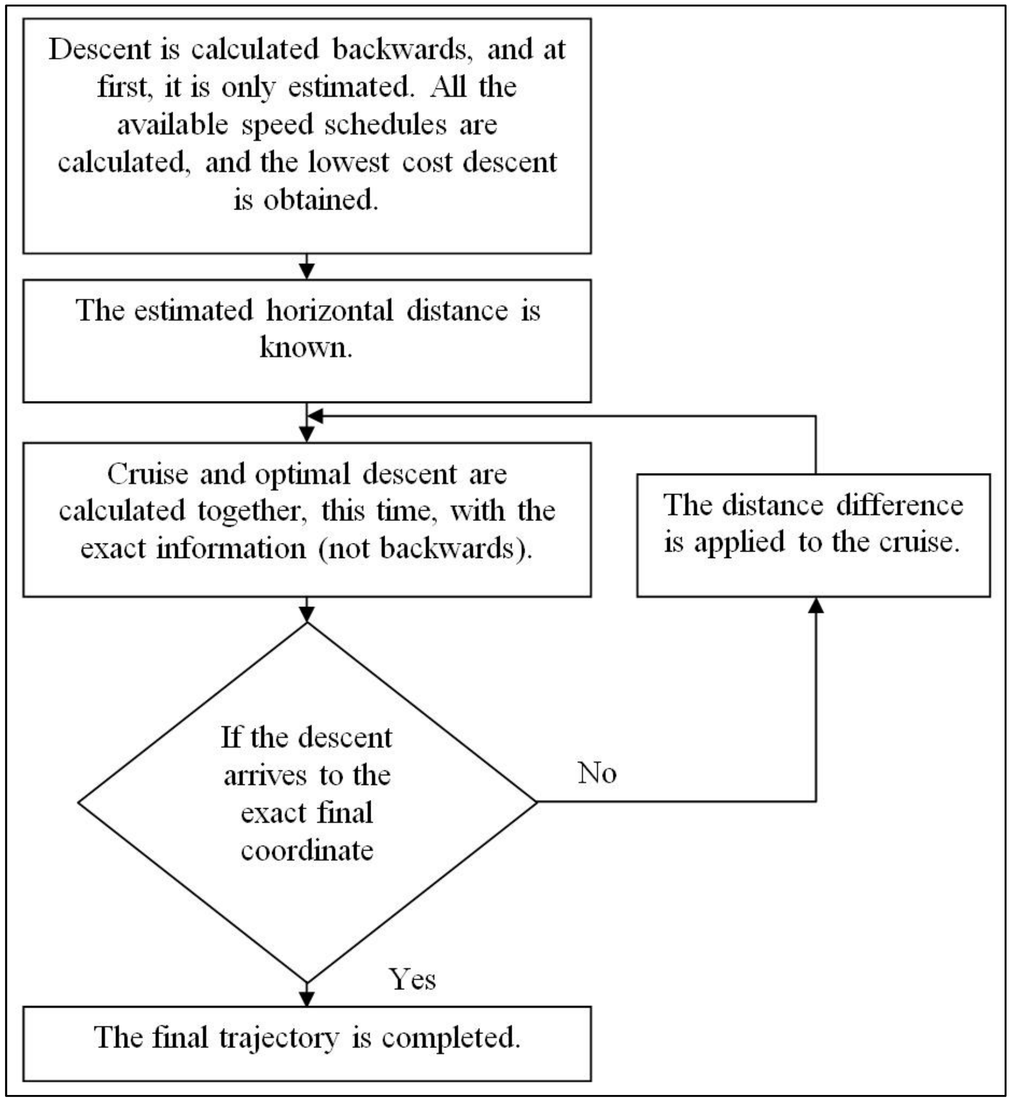

Figure 3.7 Descent flowchart

3.7 Results

The results are presented for two different analyses. Firstly, the tests to verify the algorithm precision and consistency were shown, where the algorithm was found to be more precise than the actual FMS. Secondly, a comparison between the algorithm and the FMS results was done to be able to quantify the advantages of the trajectory selected by the algorithm with respect to the trajectory of the FMS.

The results obtained have been validated with the flight simulator FlightSIM®, code developed by the Presagis Company. This software considers a complete aircraft aerodynamic model for its simulations, giving results in terms of fuel burn, flight time and traveled distance, which are accurate and very close to reality. For the purpose of this project, FlightSIM® represents the reference of reality.

The PTT is the software that represents a commercial FMS. In this section, PTT will be used for clarity of the results presentation. There is no difference between the PTT and a commercial FMS.

The new optimization algorithm is applied for different commercial aircraft. Nine different trajectories for a long-range aircraft were tested on FlightSIM®, using the same speeds, altitudes and distance. Both, the PTT and the proposed algorithm, were compared to FlightSIM® to determine which method has the more precise results. These results are shown on Table 3.4 and Table 3.5.

Table 4 shows the fuel burn analysis and Table 3.5 shows the flight time analysis. The first five columns represent the flight trajectory selected, with the speed, altitude and destination flown. It can be seen on both tables that the optimization algorithm performs better than the PTT, with a 1.75% against a 2.55% error in terms of fuel burn, and 0.49% against 1.03% in terms of flight time. The algorithm gave more precise results.

Table 3.4 Fuel burn precision analysis with FlightSIM®

Flight	Altitude (ft)	Speed schedule (IAS/Mach/IAS)	Depart airport code	Arrival airport code	FLSIM fuel (kg)	Algorithm fuel (kg)	PTT fuel (kg)	Algorithm error fuel (%)	PTT error fuel (%)
1	36000	300/0.78/300	YUL	YYZ	4518.1	4559.8	4554.74	0.92%	0.81%
2	32000	300/0.78/320	YUL	YYZ	4608.9	4648.6	4634.42	0.86%	0.55%
3	34000	300/0.78/300	YUL	YYZ	4544.5	4590.5	4688.97	1.01%	3.18%
4	38000	300/0.78/300	YUL	YYZ	4528.8	4581.6	4700.56	1.17%	3.79%
5	36000	310/0.79/290	YUL	YYZ	4528.8	4574.4	4657.1	1.01%	2.83%
6	40000	340/0.82/260	YUL	YYZ	4103.9	4223.7	4240.9	2.92%	2.19%
7	36000/38000 (SC)	310/0.83/340	YUL	YVR	29133.8	29677.5	29770.9	1.87%	2.43%
8	38000	310/0.82/340	YUL	YVR	29083	29693.1	29790.73	2.10%	3.82%
9	40000	340/0.82/260	YUL	YWG	11939.7	12404.7	12396.36	3.89%	3.34%
Average								**1.75%**	**2.55%**

Table 3.5 Flight time precision analysis with FlightSIM®

Flight	Altitude (ft)	Speed schedule (IAS/Mach/IAS)	Depart airport code	Arrival airport code	FLSIM time (hr)	Algorithm time (hr)	PTT time (hr)	Algorithm error time (%)	PTT error time (%) (abs)
1	36000	300/0.78/300	YUL	YYZ	0.69	0.69	0.7	0.48%	1.49%
2	32000	300/0.78/320	YUL	YYZ	0.68	0.68	0.7	0.74%	3.13%
3	34000	300/0.78/300	YUL	YYZ	0.69	0.69	0.69	0.43%	0.26%
4	38000	300/0.78/300	YUL	YYZ	0.69	0.69	0.69	0.50%	0.50%
5	36000	310/0.79/290	YUL	YYZ	0.69	0.69	0.69	0.51%	0.06%
6	40000	340/0.82/260	YUL	YYZ	0.69	0.69	0.71	0.34%	1.94%
7	36000/38000 (SC)	310/0.83/340	YUL	YVR	4.3	4.28	4.26	0.46%	0.93%
8	38000	310/0.82/340	YUL	YVR	4.34	4.32	4.37	0.48%	0.76%
9	40000	340/0.82/260	YUL	YWG	2.21	2.2	2.22	0.50%	0.17%
Average								**0.49%**	**1.03%**

Since on the global cost formula the time is important, and so is the CI, it should be considered in order to calculate an accurate optimization percentage. For a CI of 0, the time has no influence on the global cost, opposite to a high CI of 100, when the time has a lot of influence in the total cost of the flight. Figure 3.8 displays the error variation depending on the CI.

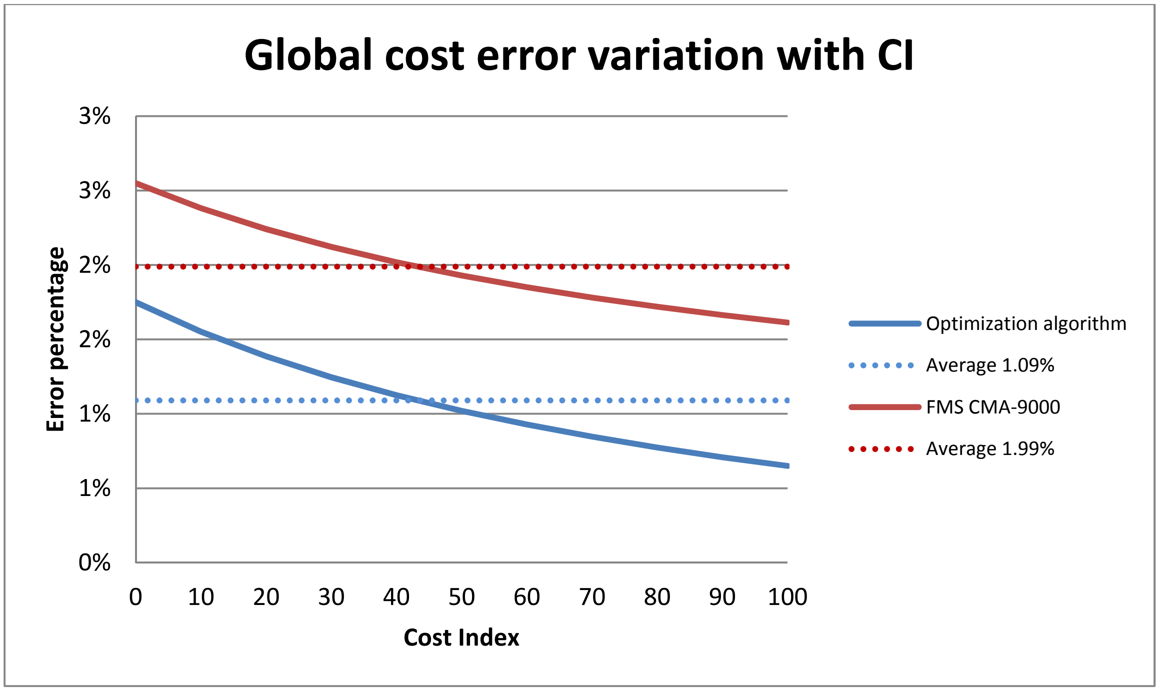

Figure 3.8 Global cost error variation with CI

Results from Figure 3.8 indicate that the proposed algorithm results are closer to the results obtained with FlightSIM®, which as it was indicated before, is the reference used to validate the results. A 1.09% flight cost difference between the new algorithm and FlightSIM® was found, while a 1.99% flight cost error was obtained when compared with the PTT. Therefore, the proposed algorithm gave more precise results than a commercial FMS from CMC Electronics – Esterline.

Previous results show only the precision of the optimization algorithm and the PTT compared to our reality reference, FlightSIM®. To verify that a fuel burn reduction can be obtained in respect to the PTT, a different set of test has been made.

To analyze the fuel burn, 56 tests for a mid-range aircraft were performed, where:

- 20 tests where the same altitude and distance was imposed, looking to compare speed only optimization.

- 36 tests where only the same distance was imposed, looking to compare speed and altitude optimization.

Table 3.6 shows the first 20 tests. In all cases, the same distance and altitude was traveled, and each method was allowed to select its own optimal speed. Results show that the speed selected by the optimization algorithm produced trajectories with a lower cost than those selected by the PTT. In average, a 0.15% cost reduction was obtained. However, these tests only optimized the speed of the flight, since the altitude was imposed. In order to improve results, a speed and altitude optimization is presented next.

Table 3.6 Speed only optimization comparison for the mid-range aircraft

Flight	CI	Altitude	Algorithm cost (kg)	PTT cost (kg)	Algorithm optimization
1	0	32000	9532.4	9603.1	0.74%
2	10	32000	11183.5	11186.7	0.03%
3	20	32000	12800.9	12808.9	0.06%
4	30	32000	14396.3	14415.7	0.13%
5	40	32000	15932.7	15933.4	0.00%
6	50	32000	17430.4	17464.8	0.20%
7	60	32000	18893.0	19020.6	0.68%
8	70	32000	20362.1	20425.7	0.31%
9	80	32000	21830.6	21889.8	0.27%
10	90	32000	23270.2	23286.7	0.07%
11	0	36000	9147.5	9147.0	0.00%
12	10	36000	10728.4	10740.5	0.11%
13	20	36000	12280.0	12263.9	-0.13%
14	30	36000	13804.7	13789.0	-0.11%
15	40	36000	15296.3	15356.9	0.40%
16	50	36000	16765.7	16790.4	0.15%
17	60	36000	18225.7	18245.0	0.11%
18	70	36000	19685.6	19707.0	0.11%
19	80	36000	21145.5	21138.5	-0.03%
20	90	36000	22605.4	22585.4	-0.09%
Average					**0.15%**

Table 3.7 Speed and altitude optimization comparison for the mid-range aircraft

Flight	Trajectory	CI	Aircraft weight (kg)	Algorithm cost (kg)	PTT cost (kg)	Difference
1	**Montreal-Vancouver**	0	138,000	19933.7	20437.2	2.46%
2			141,000	20378.6	20894.6	2.47%
3			144,000	20904.6	21141.8	1.12%
4		20	138,000	25412.1	26452	3.93%
5			141,000	25582.5	26678.5	4.11%
6			144,000	26091.8	26861.9	2.87%
7		40	138,000	30727.8	31761.4	3.25%
8			141,000	31156.6	32430.6	3.93%
9			144,000	31664	32568.6	2.78%
10		60	138,000	36028.1	37545.1	4.04%
11			141,000	36432.4	38220.8	4.68%
12			144,000	36917.6	38292.7	3.59%
13		80	138,000	41297.7	42718	3.32%
14			141,000	41703.7	42718	2.37%
15			144,000	42171.1	43668.6	3.43%
16		99	138,000	46303.9	48208.6	3.95%
17			141,000	46711.3	48259.6	3.21%
18			144,000	47162	48785.8	3.33%
19	**Montreal-Winnipeg**	0	138,000	10503.2	10561.9	0.56%
20			141,000	10706.1	10824.8	1.10%
21			144,000	10877.9	10940.9	0.58%
22		20	138,000	13221.4	13392.8	1.28%
23			141,000	13456.5	13687.2	1.69%
24			144,000	13724.5	13778.4	0.39%
25		40	138,000	15921.5	16237.7	1.95%
26			141,000	16167.1	16551.7	2.32%
27			144,000	16415.9	16621.5	1.24%
28		60	138,000	18575.7	19132.3	2.91%
29			141,000	18821.7	19444	3.20%
30			144,000	19056.4	19487.5	2.21%
31		80	138,000	21229.9	21731.1	2.31%
32			141,000	21469.7	22042.4	2.60%
33			144,000	21695.4	22170.9	2.14%
34		99	138,000	23767.7	24481.1	2.91%
35			141,000	24001.3	24533.1	2.17%
36			144,000	24202.4	24764.9	2.27%
Average						**2.57%**

It can be seen that the optimization algorithm has better performance when it can select its own altitude along with the optimal speed.

Two different trajectories were traveled: from Montreal to Winnipeg and from Montreal to Vancouver. The CI was varied from 0 to 99, and three different aircraft weights were tested. In all of 36 cases, the optimization algorithm gave a lower cost flight trajectory. An average of 2.57% cost reduction was obtained within these 36 tests.

3.8 Conclusions

"Cruise Control" has been an important aspect of civil jet operations since the introduction of the Comet 1 in 1952. The original Comet used some relatively simple calculations to ensure it always flew at the performance limits of the engine airframe. However it was the only aircraft of its type flying and was no subject to the increasing conflict of other airframes operating in a similar environment.

The very large increases in jet propelled aircraft has made it much more difficult to accommodate small adjustments in different airline operating techniques and, in fact, the more pressing demands for collision avoidance and air traffic control and similar events mean that ATC requirements are often dominant in cruise control areas.

Even when certain flight restrictions are imposed by the ATC, such as speed and altitude limits, these restrictions can be defined in the new algorithm and it will search the optimal trajectory within these restrictions, to reduce fuel burn and emissions to the atmosphere. However, the maximal optimization is obtained when the trajectory is entirely defined by the algorithm.

Better results were obtained in terms of precision than current FMS technologies from CMC Electronics-Esterline, obtaining an error of 1.09% compared with FlightSIM®, while the FMS had a 1.99% error.

When the comparison was made between the trajectories proposed by the algorithm, and those proposed by the FMS, the proposed algorithm from this paper improved the global flight cost on 2.57%.

CHAPTER 4

ARTICLE 2: HORIZONTAL FLIGHT TRAJECTORIES OPTIMIZATION FOR COMMERCIAL AIRCRAFT THROUGH A FLIGHT MANAGEMENT SYSTEM

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Résumé

Afin de réduire les émissions dans l'atmosphère, la consommation de carburant des avions doit être réduite. Pour les vols longs, la croisière est la phase dans laquelle on peut obtenir la réduction la plus importante. Une nouvelle méthodologie implémentée sur le profil horizontal de vol afin de diminuer les émissions est décrite dans cet article. L'impact du vent sur un avion pendant le vol peut réduire le temps de celui-ci, en profitant des vents de dos ou en évitant des vents de face. Un ensemble de trajectoires alternatives est évalué pour déterminer le temps de vol le plus court, et ainsi la consommation de carburant la plus faible. Dans le but de déterminer la quantité de carburant attendue, les bases de données de performances dans des systèmes de gestion de vol ont été utilisées. Ces bases de données représentent les performances en vol des avions commerciaux.

Abstract

To reduce aircraft emissions to the atmosphere, the fuel burn from aircraft has to be reduced. For long flights, the cruise is the phase where the most significant reduction can be obtained. A new horizontal profile optimization methodology to achieve lower emissions is described in this article. The impact of wind during a flight can reduce the flight time, either by taking advantage of tailwinds or by avoiding headwinds. A set of alternative trajectories are evaluated to determine the quickest flight time, and therefore, the lowest fuel burn. To

determine the expected amount of fuel reduction, the PDBs used on actual FMS devices, were used. These databases represent the flight performance of commercial aircraft.

4.1 Introduction

The total CO_2 emissions due to aircraft traffic represents between 2.0 and 2.5% of all CO_2 emissions to the atmosphere (ICAO, 2010). Aircraft fuel burn is an important contributor to CO_2 emissions, the principal greenhouse gas. Greenhouse gases contribute to the global warming effect, which is one of the most important environmental problems encountered today.

Various approaches have been utilized to reduce the environmental impact of aviation: the use of biofuels to improve aircraft environmental performance, the development of more efficient engines to decrease emissions and to reduce noise, improvements to the aircraft's frame, and the optimization of flight trajectories. This article describes an algorithm to be implemented in a FMS to improve the horizontal profile to create fuel efficient trajectories and reduce fuel burn. The creation of more efficient trajectories for aircraft would contribute to the reduction of fuel burn, and therefore to the reduction of CO_2 emissions to the atmosphere.

In the mid 1970s, Lockheed developed an FMS to be implemented in their aircraft TriStar L-1011. Later in the 1980s, other companies started adding the FMS as standard equipment (Avery, 2011). Since then, FMSs have been continuously upgraded and presently all aircraft are equipped with an FMS. The main function of an FMS is to assist the pilot in several tasks, such as navigation, guidance, trajectory prediction and flight path planning.

Even though researchers have been working continuously to improve the performance of FMSs, recent studies demonstrate that several avenues remain to be explored and enhanced. The CDA, introduced by Clarke et al. (2004), is a method to reduce noise and fuel burn in the descent phase of a flight. It consists of the aircraft flying its own optimal vertical profile from the TOD. This method, however, depends on the ATC giving their authorization to proceed, since the aircraft must have a clear path to the runway. Studies on including aircraft traffic control as one of the FMS functions have been analyzed (Schoemig et al., 2006). However, most of the methods developed to improve FMS depend on the approval of ATC. Tong et al. (2007) explained that the CDA can only be used in low air traffic conditions, since "ATC lacks the required ground automation to provide separation assurance services during CDA operations". They then proposed a 3D Path Arrival Management (3D PAM) algorithm to predict 3D descent trajectories that could make it possible to apply CDA in high traffic conditions. Reynolds et al. (2007) concluded, after a series of tests in the Nottingham East Midlands Airport, that CDA effectively reduced fuel burn and noise near terminals by keeping aircraft at a higher altitude longer before initiating their descent. Kent and Richards (2013) studied an approach in which aircraft are placed in formation in order to reduce drag and fuel burn, while taking advantage of profitable winds. Nangia and Palmer (2007) reduced overall drag of the order of 15-20% for commercial aircraft flying in formation.

To obtain the best available fuel reduction on the vertical flight profile, the climb, cruise and descent phases were analyzed (Félix Patrón, Botez and Labour, 2013). A 2.57% flight cost reduction was obtained by introducing a performance enhancement of a FMS. Dancila, Botez et Labour (2013) studied a new method to estimate the fuel burn from aircraft to improve the precision in its trajectory calculations for an FMS platform. Gagné et al. (2013) determined the optimal vertical profile by performing an exhaustive search of all the available speeds and altitudes.

The cruise is the most important phase of a flight in terms of potential fuel savings. Lovegren (2011) analyzed the performance of SCs during cruise to optimize fuel burn. Murrieta (2013) analyzed the cruise phase to determine a pre-optimal vertical profile and to evaluate the altitudes and speeds around its vicinity. Chakravarty (1985), from a flight aerodynamics perspective, described the variation of the optimal cruise speed with flight operation costs (CI).

The wind effects during a flight are a very important factor to consider in the creation of flight trajectories. Franco and Rivas (2011) studied the influence of the wind, the CI, and the estimated arrival time to calculate the optimal cruise for flight cost reduction. Campbell (2010) studied the influence of weather conditions, such as thunderstorms and contrails, and modeled them as obstacles in order to create a trajectory to avoid it, to reduce air pollution and fuel burn. These climatic conditions were modeled as obstacles, and then an algorithm created trajectories to avoid these obstacles with the minimal fuel consumption. Sridhar, Ng and Chen (2011) used weather information to model contrails and to avoid them, with a variable penalty coefficient to reduce the fuel burn. Gagné (2013) developed a new method to download meteorological information directly from Environment Canada. Murrieta (2013) analyzed the horizontal flight profile with a new 5-route algorithm that determines if the optimal trajectory will be given by the great circle, or by selecting one of four alternative trajectories, utilizing weather data from Environment Canada. Dijkstra's algorithm was used as a base in that work, but the search space was reduced to five trajectories to develop a faster method. Bonami et al. (2013) applied optimal control to improve flight trajectories and minimize fuel consumption. They obtained a model of an aircraft, defined the airspace with a precise wind forecast and a predefined set of WPs as their inputs for the optimization algorithm.

Since the implementation of these types of algorithms in an FMS requires a reduced calculation time, different time optimization methods have been utilized. A low calculation time interpolation approach was used to calculate flight trajectories on a FMS (Félix Patrón, Botez and Labour, 2013). Miyazawa et al. (2013) developed a four dimensional algorithm using dynamic programming in order to reduce fuel burn from aircraft in a congested airspace. They modeled air traffic as obstacles to avoid during a trajectory, and used real flight coordinates to perform their tests. The Monte Carlo optimization algorithm proposed by Visintini et al. (2006), applied in ATC systems, would be a reasonable approach for determining the optimum trajectories for aircraft fuel burn reduction. The Monte Carlo method explores the entire range of solutions while following a random path to converge towards the optimum value of the study; in this case, the maximal fuel efficiency trajectory.

Fays and Botez (2013) used meta-heuristic methods to follow 4D trajectories and avoid no-fly zones, which could be defined by weather constraints or high airspace traffic. Kanury and Song (2006) used GAs to look for the optimal trajectory in the presence of unknown obstacles, and obtained satisfactory results in a short computing time; the optimal trajectory was obtained and the calculation time in their simulation was reduced. GAs are useful when solving for a problem where many restrictions are imposed. Kouba (2010) studied GAs as a way to include several constraints in a trajectory optimization problem, where the main goal was to find the shortest route while considering different restrictions. The GA proved to be very reliable at solving these types of optimization problems. GAs were used to reduce the number of calculations in the flight cost reduction algorithm (Félix Patrón et al., 2013).

The algorithm described in this article analyzes the horizontal flight profile to create optimum trajectories to reduce fuel burn, however, not considering actual restrictions that may be imposed by the air traffic management. The weather information is downloaded from Environment Canada (2013). The horizontal profile is divided into a variable number of WPs, and a GA is applied to improve the calculation time with respect to other algorithms. The improved grid used to solve this optimization problem gives a complete analysis of the cruise phase, while avoiding points where the aircraft would not likely fly. The influence of the wind's speed and direction is studied to determine if the aircraft should follow the great circle, or if an alternative trajectory can be followed to reduce the flight time, and therefore, the fuel burn. The inputs of the new algorithm are given in terms of the flying altitude, the Mach number, the TOC coordinates, the TOD coordinates, and the number of WPs, as well as the deviation angle, to create the alternative trajectories to the great circle. This new algorithm uses the flight information from PDBs (PDB) for different commercial aircraft. The algorithm described here was developed in a project entitled "Optimized Descents and Cruise", which is part of the Canadian Green Aviation Research & Development Network (GARDN), founded in 2009.

4.2 Methodology

The aircraft's performance model is explained first. These PDBs represent the model of our aircraft and are used to calculate the fuel flow during the cruise. The second step in the methodology is the creation of a grid in which the set of trajectories can be evaluated. Next, the inputs and outputs for the trajectories' optimization algorithm are defined. The weather model created to obtain the wind speeds and directions is then explained. Finally, the GA implemented to reduce the number of calculations is presented.

4.2.1 Aircraft PDB

This project uses information from different types of commercial aircraft. Fundamental research data for this project is given by the PDB. The numerical model of the aircraft provides all the necessary information to create the algorithm. The PDB is a database of approximately 30,000 lines, which gives the information about real aircraft performances. It indicates the fuel consumption and the distance flown for a specific flight profile (climb, cruise or descent).For example, the fuel burn and distance for an aircraft in cruise with a center of gravity of 28% of the mean aerodynamic chord, flying at 0.8 Mach with a total gross weight of 100 tons, at an altitude of 30,000ft and a standard deviation temperature of -10ºC. Such an example is shown in Figure 4.1. At the start of the cruise, the algorithm calculates the fuel flow of the aircraft at the specified parameters of the flight. The inputs required into the PDB to obtain the fuel flow are:

- Mach number
- Aircraft gross weight
- Air temperature
- Altitude

The travel time is calculated from the aircraft's TAS, while the wind influence is calculated with a wind triangle methodology, providing a distance traveled correction factor depending on the wind angle and speed, obtained with information about the wind's speed and direction explained later in this chapter.

```
MODE CRUISE_PROFILE_MACH
! * * * * * * * * * * * * * * * * * *
! Last Update    : 2011/01/23 8h2
! Reference CG   : 28
! * * * * * * * * * * * * * * * * * *
   SPEED  0.8
      GROSS_WEIGHT  100000
         ISA_DEV  -10
            !Altitude  FuelFlow
              20000     0
              21000     0
              22000     0
              23000     0
              24000     0
              25000     5355
              26000     5146
              27000     4948
              28000     4756
              29000     4574
              30000     4402
              31000     4243
              32000     4091
              33000     3946
              34000     3809
              35000     3685
              36000     3574
              37000     3495
              38000     3441
              39000     3405
              40000     3378
              41000     3376
```

Figure 4.1 PDB format

4.2.2 The grid

To analyze different possible trajectories in the horizontal flight profile, four parallel trajectories are added to the great circle. The cruise starts at the TOC and ends at the TOD. From the TOC, a deviation angle is set to create the parallel trajectories (5º in Figure 4.2 and 10º in Figure 4.3). The number of WPs, *n*, defines the distance at which a possible trajectory deviation can be performed (*n* =9 in Figure 4.2 and Figure 4.3). Figure 4.2 and Figure 4.3 represent two different grids with the same TOC and TOD coordinates (the flight cruise from Montreal to Paris and London).

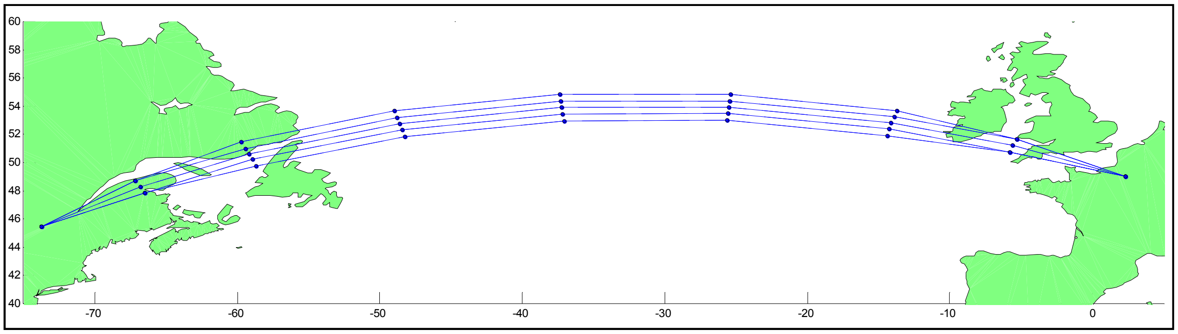

Figure 4.2 Montreal to Paris, 9 WPs and deviation angle set to 5º

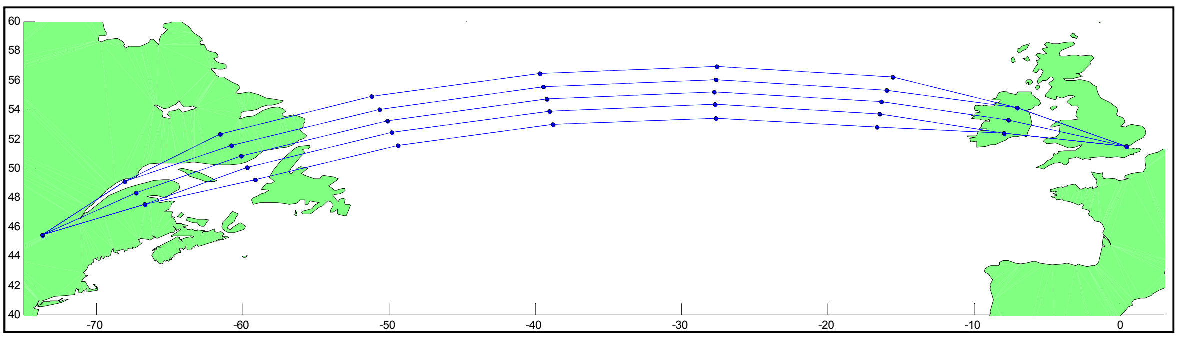

Figure 4.3 Montreal to London, 9 WPs and deviation angle set to 10º

For the purpose of this article, n will be set to 9, while the separation angle will be set to 5º (as shown in Figure 4.2). With these parameters, the distances between WPs, for trajectories such as Montreal to Paris and New York to London, which will be used in the results sections, will be of around 300 nm.

Figure 4.4 shows another example of a grid. The route number represents the ID of each route, and the TOC and TOD are defined. Route 3 represents the great circle. The grid information presented in Table 4.1 shows that each of the nine points of the five routes has its own coordinates, given in terms of latitude and longitude.

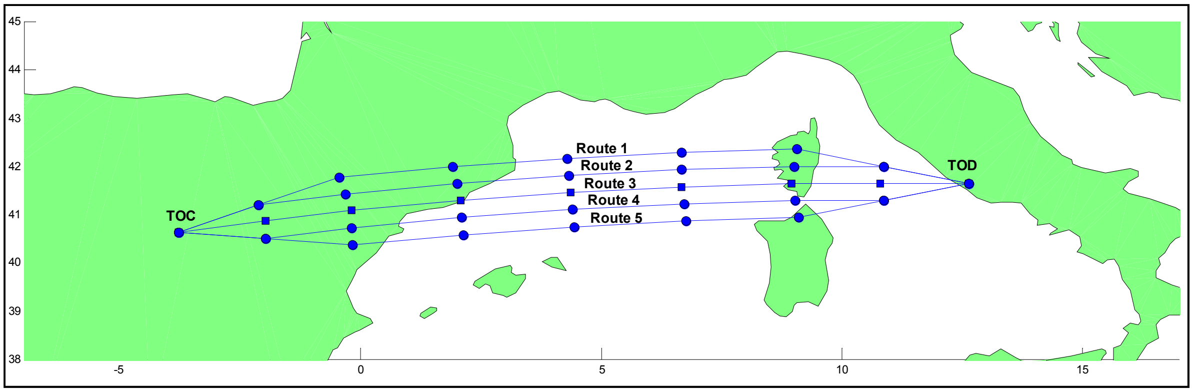

Figure 4.4 Grid example for a flight from Madrid to Rome

Table 4.1 Example of the latitudes and longitudes for a grid

LATITUDES

ID	WP1 (TOC)	WP 2	WP 3	WP 4	WP 5	WP 6	WP 7	WP 8	WP9 (TOD)
1	40.6	41.2	41.8	42.0	42.2	42.3	42.4	42.0	41.6
2	40.6	41.2	41.4	41.6	41.8	41.9	42.0	42.0	41.6
3	40.6	40.9	41.1	41.3	41.5	41.6	41.7	41.6	41.6
4	40.6	40.5	40.7	40.9	41.1	41.2	41.3	41.3	41.6
5	40.6	40.5	40.4	40.6	40.7	40.9	40.9	41.3	41.6

LONGITUDES

ID	WP1 (TOC)	WP 2	WP 3	WP 4	WP 5	WP 6	WP 7	WP 8	WP9 (TOD)
1	-3.8	-2.1	-0.4	1.9	4.3	6.7	9.1	10.9	12.6
2	-3.8	-2.1	-0.3	2.0	4.3	6.7	9.0	10.9	12.6
3	-3.8	-2.0	-0.2	2.1	4.4	6.7	9.0	10.8	12.6
4	-3.8	-2.0	-0.2	2.1	4.4	6.7	9.0	10.9	12.6
5	-3.8	-2.0	-0.2	2.1	4.4	6.8	9.1	10.9	12.6

The rows represent the ID of the route, while the columns represent the WP number. It can be observed that the first and last WPs have the same coordinates.

A route is defined by a vector of dimension 9, where the numbers inside the vector represent the position on each route. For example, the route (3,3,3,2,2,3,4,4,3) is shown in Figure 4.5.

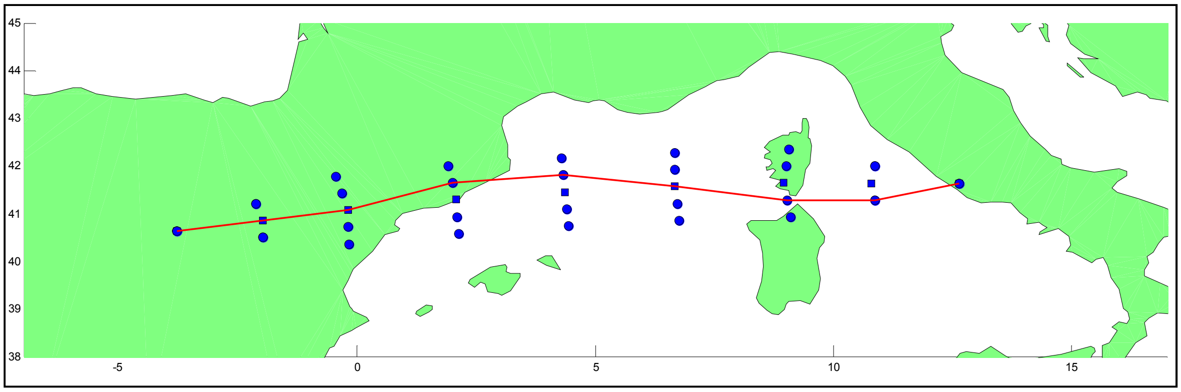

Figure 4.5 Example of a flight trajectory presented on the grid (3,3,3,2,2,3,4,4,3)

These example trajectories, however, would need to be approved by ATC.

The size of the grid can be varied by changing the number of WPs or the deviation angle, but if the number of WPs is increased, the number of possible trajectories would also increase, adding to the calculation time.

From one WP to another, the aircraft can only fly through successive routes. Figure 4.6 represents an example of a route which is not valid, because the aircraft is not supposed to fly from route 5 to route 2, without first flying to route 4 and then route 3.

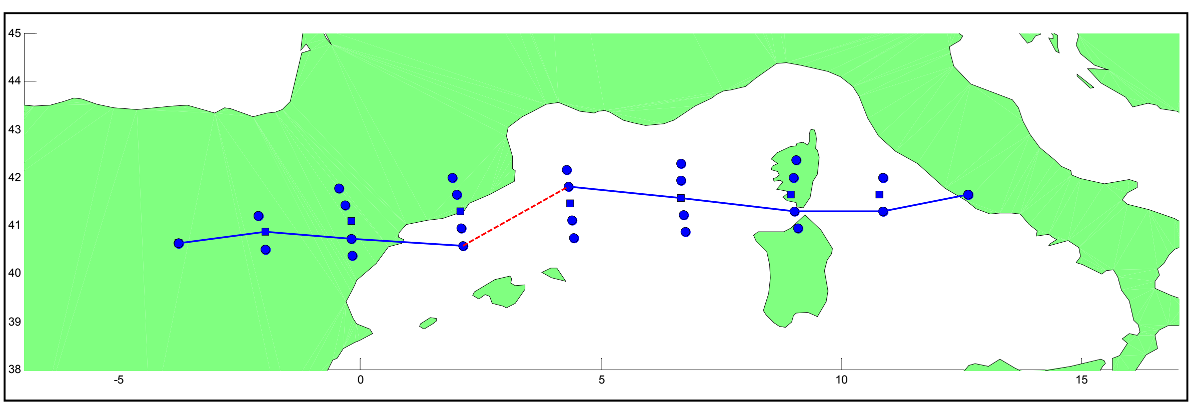

Figure 4.6 Example of an invalid trajectory on the grid

4.2.3 Inputs and outputs

The proposed algorithm calculates the optimal trajectory that most reduces the flight cost. The inputs and outputs required for the complete analysis of the trajectories are presented in Table 4.2.

Table 4.2 Inputs and outputs for the trajectories' optimization algorithm

	Variable	**Units**
Inputs	TOC	Coordinates
	TOD	Coordinates
	Altitude	Feet
	Speed	Mach number
	Initial time	Hours
	Separation angle	Degrees
	Number of WPs per trajectory	N/A
	Number of generations for the GA	N/A
	Size of the initial population	N/A
	CI	Kilograms per hour
	Fuel flow	Kilograms per hour
Outputs	Optimal trajectory	Coordinates
	Optimal trajectory's cost	Kg of fuel
	Great circle's cost	Kg of fuel
	Cost reduction	Percentage

The CI is an input which influences the global cost of a flight. In aviation, it is not only the fuel burn that is considered in planning a flight trajectory - other variables such as the flight time and operation costs must also be taken into account. The CI is the term used in current FMS technologies to calculate the operation costs per hour for each flight.

The global cost is defined as:

$$\text{Global Cost} = \sum \text{Fuel Burned} + \text{CI} * \text{Flight Time} \tag{4.1}$$

Where the Fuel Burned is given in <kg>, the CI in <kg/hr> and the Flight Time in <hours>.

It becomes impractical to consider the fuel price (in <$/kg>), since it changes continuously. The global cost in Equation (4.1) is therefore given in <kg of fuel>, which will be multiplied by the fuel price to obtain a global cost in terms of money (<$>).

Therefore, the global cost can be defined as:

$$\text{Global cost} = \text{Fuel flow} * \text{Flight Time} + \text{CI} * \text{Flight Time} \tag{4.2}$$

where the Fuel Flow is given in <kg/hr>, and is obtained directly from the aircraft's PDB. The Global Cost equation then becomes:

$$\text{Global Cost} = \text{Flight Time} * (\text{Fuel flow} + \text{CI}) \tag{4.3}$$

The cruise optimization algorithm is expressed in terms of the global flight cost.

4.2.4 Weather model

To obtain precise weather for the calculation of the horizontal profile, the weather model from Environment Canada (2013) was utilized. This model creates a grid for the Earth with the current weather and weather predictions. Environment Canada uses the GDPS as their model, which is provided in a binary format called General Regularly-Distributed Information (GRIB2). This model provides a 601x301 latitude-longitude grid with a resolution of 0.6 x 0.6 degrees. The time standard is given by the Coordinated Universal Time (UTC), and the predictions are obtained updated each 12 hours, in 3-hour period blocks. The flight time is then interpolated for a specific time. This model is used in the present algorithm for its precision, and because it can be obtained for free and downloaded directly from Environment Canada (2013).

The downloaded files provide wind and temperature information for different altitudes. The information required for this project is the speed and direction of the wind at a specific altitude. The temperature of the flight is also obtained and introduced on the PDB to obtain the aircraft fuel flow. At each WP, the wind and temperature information is introduced to calculate the influence of the wind with the wind triangle methodology, which provides a distance correction factor depending on the direction and speed of the wind. The outside temperature has an influence on the TAS of the aircraft. However, in order to reduce calculation time, standard air temperatures on the International Standard Atmosphere (ISA deviation 0°C) are used to calculate the speed of sound, TAS and fuel flow, since the algorithm should be implemented in a limited processing device such as an FMS platform.

4.2.5 The genetic algorithm

In order to be able to incorporate this flight optimization algorithm in a FMS, an optimization algorithm has been applied to reduce the number of calculations.

This new cruise fuel burn reduction algorithm was designed to take advantage of the presence of tail winds and to avoid head winds. Since wind is a random process, an optimization algorithm would have to adapt to this randomness.

A GA has been used to reduce the number of calculations. Since there are many possible trajectories that an aircraft can follow in a grid, an optimization algorithm to reduce the calculation time was needed. GAs were selected because of the nature of the problem.

GAs are stochastic algorithms which allow good solutions to be found when a problem encounters randomness and non-linear data, in a reasonable calculation time. Their principles are based on Darwin’s theory of evolution, where the fittest survive to reproduce. A GA mimics the natural evolution process. Starting with an initial population (the parents), a group of selected individuals will either mutate or crossover to create a second generation of individuals (children). Mutation is defined as the alteration of one or more of a set of genes, which would change the composition of a given chromosome. The crossover takes a part of one chromosome, and combines it with a different part of a second chromosome. These processes create diversified individuals.

Only some of the individuals will survive to define the next population. A fitness evaluation function determines which individuals will survive. The process is repeated for a fixed number of generations. Finally, one of the solutions given by the algorithm should be optimal for a specific problem.

Since the process includes random non-linear data, it is possible that a suboptimal solution could be found instead of an optimal one.

GAs comprise the following steps: the definition of individuals and the creation of the initial population, the evaluation of individuals, the selection of the individuals most fitted to create the next generations, the reproduction and the process termination conditions; each of these are explained in the following sections.

4.2.5.1 Individuals and initial population

The individuals for the GA are defined in terms of randomly-created trajectories. The solutions grid was defined in the previous section; an individual must be created within the confines of this grid. The creation of an individual should respect the following constraints:

- The aircraft can only fly to adjacent WPs; and
- The initial and final WPs (TOC and TOD) should be respected.

To start the GA, a specific number of individuals are created. Figure 4.4, presented earlier, could be an example of a randomly-created trajectory.

4.2.5.2 Evaluation

The evaluation process consists of calculating the flight cost of each individual. The flight distance, the aircraft speed and the flying time must be obtained before calculating the flight cost.

- Distance: Obtained directly from the flight trajectory.
- Aircraft TAS: Calculated from the aircraft Mach number and the flying altitude.
- Wind speed: Obtained from the weather model.
- Flight time: Calculated using the distance and the global aircraft speed, which is given by the aircraft TAS and the wind speed.
- Fuel burned: Calculated with the fuel flow and the flight time.

The flight cost is then calculated using Equation (4.1).

4.2.5.3 Selection

According to Darwin's theory of evolution, the best-fitted individuals are those that are more likely to survive and have more chances to reproduce and preserve their genetic heritage. This does not means that less-fitted individuals do not have the right to reproduce; they bring diversity to the process and allow the algorithm to more efficiently avoid local optima and achieve the global optimum.

There are many selection methods for GA, some of which give priority to a faster convergence to a local optimum, if that is one of the constraints of the optimization problem. Other methods produce a slower convergence to the global optimum. The choice of the method depends on the problem to be solved.

Selecting the parents that will create the second generation can be done in different ways, for example, by direct selection of the best-fitted individuals. However, sorting through the possible solutions and immediately selecting the best will reduce the diversity and can cause a faster convergence to a suboptimal solution. Selection by tournament, where the parents compete with each other, or a proportional selection, such as the roulette wheel method, are other options. The roulette method has been chosen because it allows the algorithm to perform in a random way, but still gives every individual a chance to be selected. The roulette does, however, offer more possibilities to the most-fitted individuals. This allows the next generations to be diversified, which allows the algorithm to avoid quick convergence towards a suboptimal solution.

A graphical representation of the roulette wheel selection method can be seen in Figure 4.7, and the performance of this algorithm is given in the results section.

Figure 4.7 represents the roulette wheel methodology. On the left side, the individuals are represented by circles; the size of the circle represents the fitness of that individual. It can be observed that the most-fitted individuals have more possibilities to be selected in the roulette, while the less-fitted individuals still have the possibility of being selected.

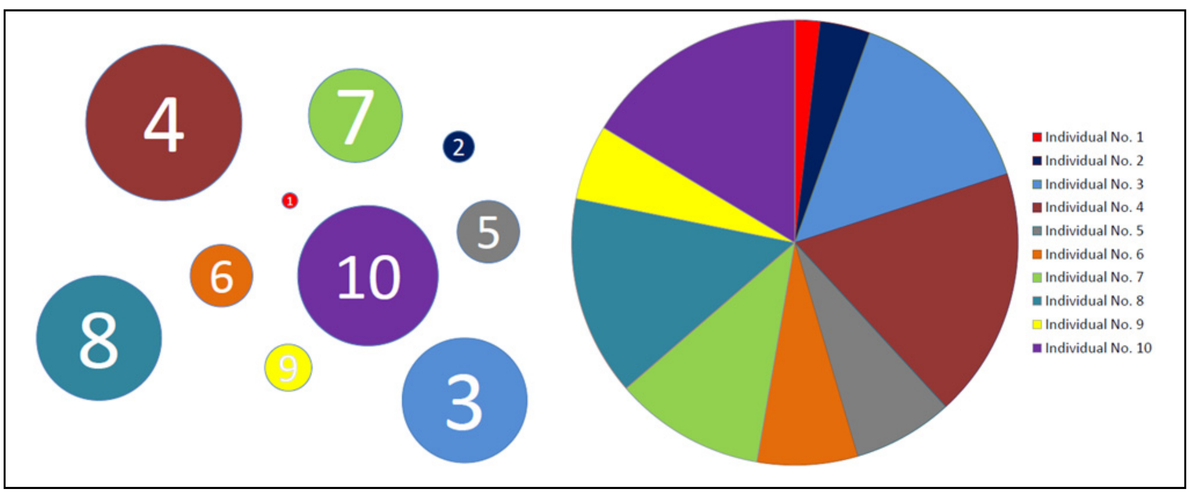

Figure 4.7 Representation of the roulette wheel selection

4.2.5.4 Reproduction

The selected individuals should be reproduced to create a new generation. Since the data used in the cruise problem is given in terms of WPs, which can be divided, it is more practical to select the crossover method here rather than mutation.

Each selected individual is divided in two, and each part of that individual is crossed with a part of a different individual to create two new trajectories. Father 1 is defined as (3,2,2,1,2,3,4,4,3) and Father 2 as (3,3,3,3,2,2,2,2,3). When these two individuals reproduce, Son 1 will be represented by (3,2,2,1,2,2,2,2,3), and Son 2 by (3,3,3,3,2,3,4,4,3). This is shown graphically in Figure 4.8.

If the random factor in the creation of the trajectories produces a crossover with an invalid trajectory by not respecting the adjacent WPs restriction, an adaptation is done to obtain a valid trajectory. An example can be found in Figure 4.9.

To add more diversity to each new generation, a number of randomly selected individuals are added. The problem is then analyzed in the diagram shown in Figure 4.10. The process is repeated until a specific number of generations is reached.

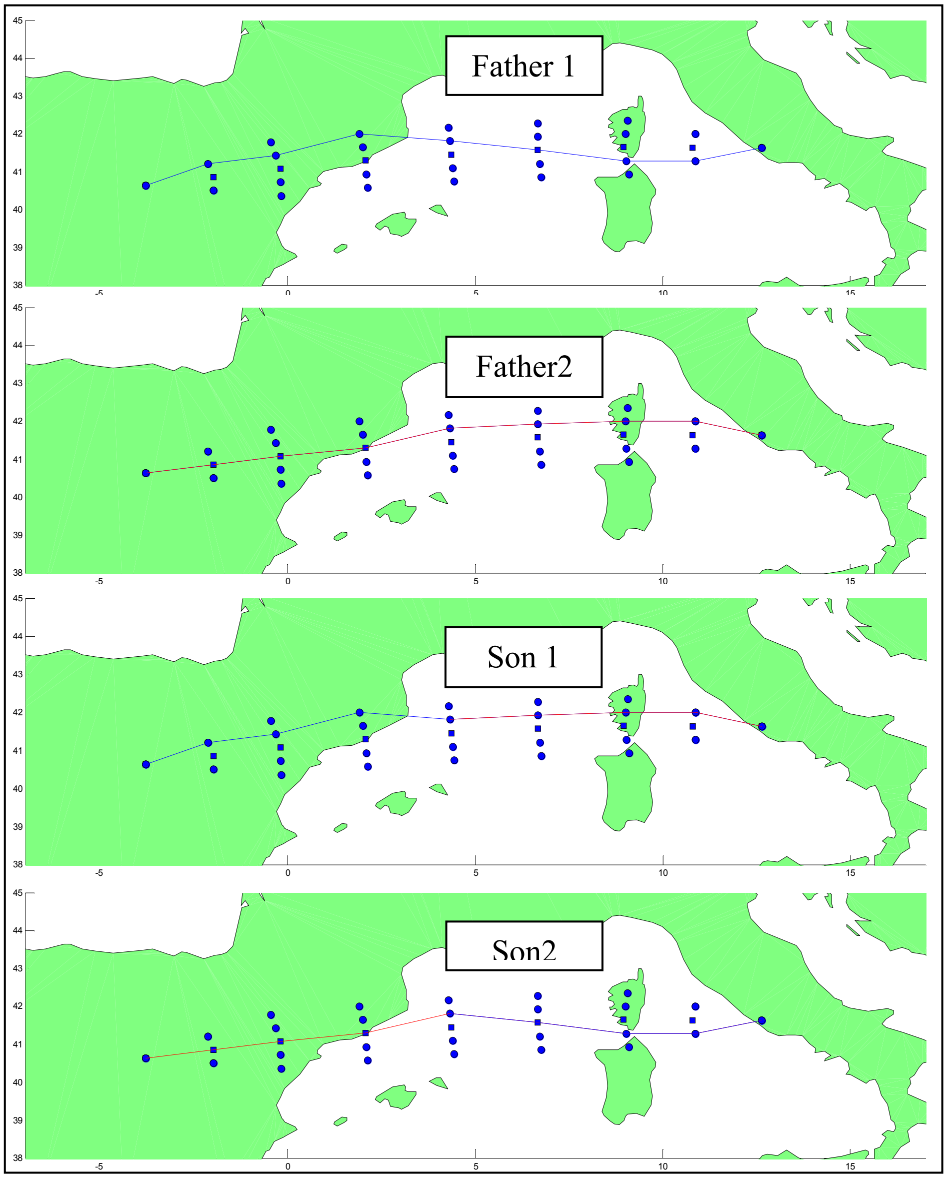

Figure 4.8 Example of reproduction with a GA

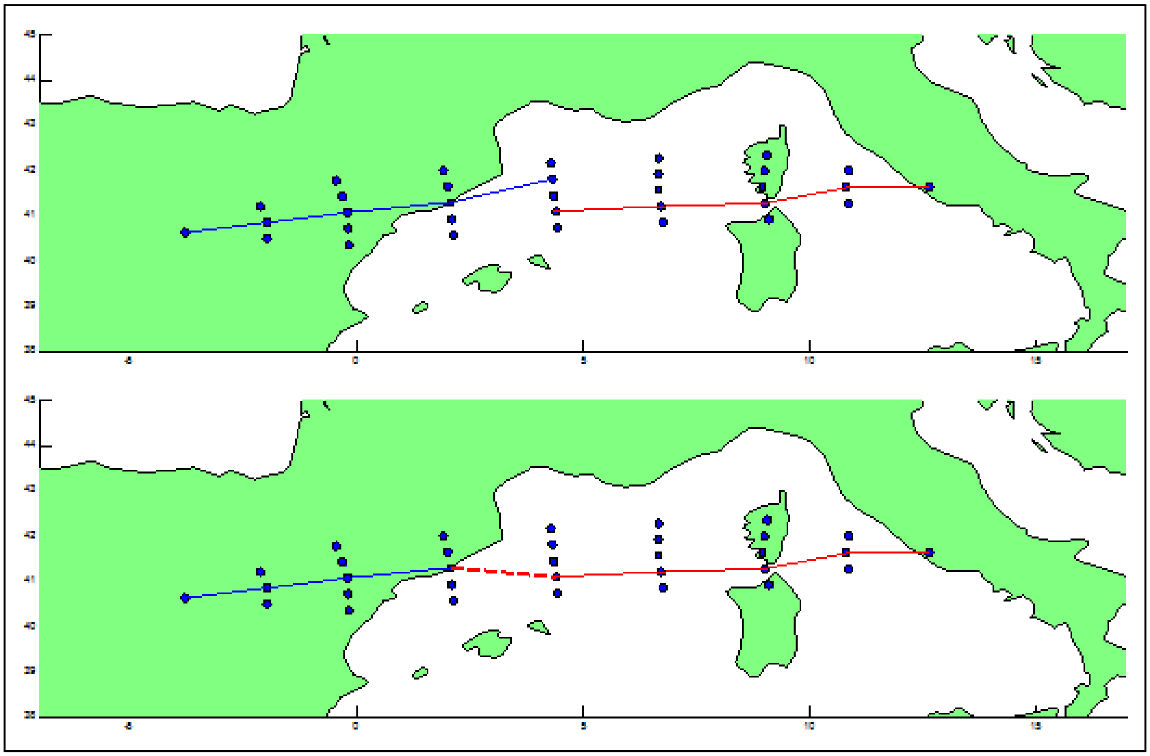

Figure 4.9 Example of an adaptation to an invalid trajectory

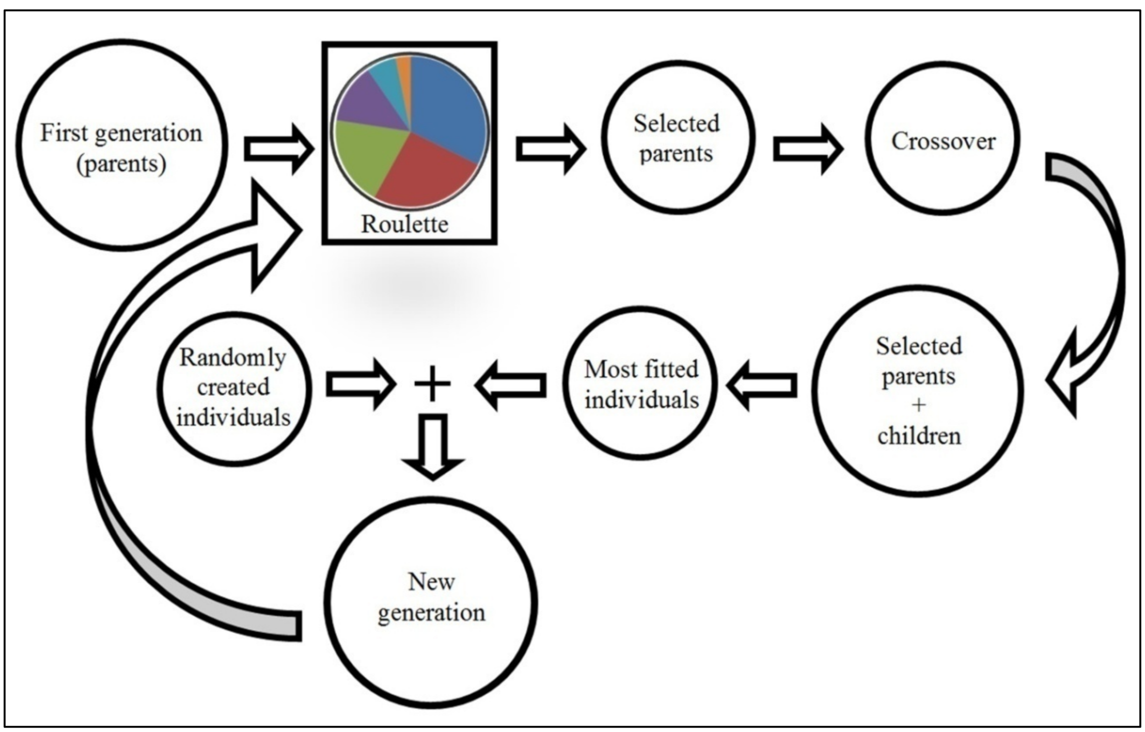

Figure 4.10 GA diagram

4.3 Results

The GA used in this trajectory optimization problem includes a certain amount of randomness. The genetic optimization algorithm, due to its nature, could give a suboptimal trajectory as a result. Also, the wind speeds and directions vary every minute, every day, so it is possible that for some specific cases, no optimization could be made. Each alternative trajectory is compared with the great circle; if the optimal trajectory is found in the great circle, no optimization would be obtained. This algorithm analyzes the possibility of flying alternative routes in order to reduce the flight cost.

In order to obtain the most accurate and realistic results, a “Ceteris Paribus” (Latin for “other things being equal”), methodology has been implemented. The “Ceteris Paribus” methodology is utilized to explain the effect of a single variable, without having to worry about the effect of other variables which are held constant. In this case, the flight trajectory, flight time, fuel flow, altitude and speed are held constant, while the only variable element is the day of the flight.

The behavior of commercial airlines is simulated, in that the time of departure does not (normally) change from day to day for the same trajectory. This approach should thus present the real optimization capabilities of the algorithm, since it provides the results by only modifying the wind’s speed and direction.

A total of 25 different days were tested. The weather for these days was obtained from Environment Canada (2013). The tests were repeated 100 times for each day, to present the percentage of occurrence of the GA in which the optimal trajectory was provided. The CI is left at zero, since we are looking at the flight time optimization; however, as can be seen in Equation (4.3), as the fuel flow remains constant, the global cost increases linearly with time. Table 4.3 presents the results obtained with the algorithm.

The values of the inputs for the tests shown on Table 4.3 are:

- Departure: Montreal
- Arrival: Paris
- Distance: 3,000 nm
- Altitude: 38,000 ft.
- Speed: 0.80 Mach.
- Initial time: 1 UTC.
- Separation angle: 15º.
- Number of WPs per trajectory: 9.
- Number of individuals per generation: 50.
- Number of generations: 50.
- CI: 0.
- Fuel flow: 3800 kg/hr.

The observations that can be made from Table 4.3 are the following:

- Even if minimal, a flight time reduction can be obtained most of the time. It is possible, however, that for a specific day, flight time and trajectory, the great circle is the optimal trajectory, and therefore the optimization percentage obtained is zero. This occurred with the tests for the 2nd and the 9th of May.
- The GA provided the optimal solution, in average, 90% of the time.
- The flight cost could be reduced with an average of 0.51%.
- In those tests where the optimal solution is found with less frequency, is due to the nonlinearity of the wind's model, which may give as a result two or more trajectories with similar optimization results, making it more difficult for the algorithm to identify the true optimal trajectory (instead of a suboptimal solution).

The flight cost reduction represents an improved trajectory with respect to the great circle. The algorithm optimizes flight cost only when an alternative trajectory to the great circle is found to reduce flight time.

Table 4.3 Flight tests from real weather data obtained from Environment Canada with 9 WPs and 5% of initial population

Date	Optimal time flight time (minutes)	Great circle flight time (minutes)	Flight time optimization	Optimal solution frequency given by the GA	Calculation time per optimal trajectory (s)
April 1st, 2012	392.9	395.8	0.72%	94%	2.86
April 3rd, 2012	394.3	394.8	0.13%	95%	2.33
April 4th, 2012	395.2	399.7	1.14%	78%	2.44
April 5th, 2012	394.0	394.3	0.07%	38%	2.33
April 6th, 2012	389.5	389.8	0.09%	98%	2.34
April 9th, 2012	415.4	417.5	0.50%	91%	2.47
April 10th, 2012	409.2	409.6	0.09%	86%	2.62
April 11th, 2012	406.5	406.6	0.03%	93%	2.51
May 1st, 2012	399.6	400.3	0.17%	99%	2.55
May 2nd, 2012	391.4	391.4	0.00%	100%	2.64
May 9th, 2012	411.6	411.6	0.00%	100%	2.43
May 14th, 2012	412.7	415.4	0.64%	100%	2.5
June 5th, 2012	392.6	393.2	0.16%	76%	2.36
June 7th, 2012	397.0	397.1	0.04%	88%	2.41
October 18th, 2012	423.1	426.3	0.76%	100%	2.39
October 29th, 2012	394.7	401.9	1.80%	95%	2.62
October 30th, 2012	386.5	396.6	2.54%	100%	2.54
March 10th, 2013	408.5	411.4	0.69%	100%	2.34
March 21st, 2013	401.2	401.8	0.14%	74%	2.43
May 21th, 2013	402.0	403.9	0.46%	100%	2.34
May 29th, 2013	410.8	411.3	0.13%	60%	2.31
June 4th, 2013	413.2	415.9	0.66%	91%	2.49
June 10th, 2013	403.0	406.2	0.78%	100%	2.39
June 11th, 2013	407.7	410.7	0.72%	99%	2.53
June 13th, 2013	401.7	402.9	0.30%	96%	2.39
Average			**0.51%**	**90.04%**	**2.46**

It is important to mention that only the flight time has been analyzed for flight cost reduction, with the assumption that the aircraft's Mach number will not change during the flight. The aircraft's speed variation depends only on the speed and direction of the wind. The temperature has a direct influence on the fuel flow, which is held constant in this case. The speed of sound, which has a direct influence on the TAS, is calculated using the ISA parameters and thus, remains constant. A CI of zero has been utilized, for maximal fuel savings. Equation (4.3) shows this time dependency for flight cost calculation with a constant CI and fuel flow.

Flight time optimization values of from 0% to 2.54% are indicated in Table 4.3. These results show flight cost reduction for the horizontal profile of the flight trajectories. Only the wind speeds and directions are varied according to the weather.

To analyze the influence of the number of individuals per generation used in the GA on the calculation time, the 25 tests presented in Table 4.3 were repeated, this time, for a different number of individuals per generation. Table 4.4 presents the results for 26 individuals per generation and for 100 individuals per generation. The selection of the initial populations of 26, 50 and 100 individuals in Table 4.3 and Table 4.4, respectively, represent 2.5%, 4.8% and 9.6% of the entire population.

As expected, the reduction of the number of individuals per generations reduces the calculation time, but also the frequency at which the optimal solution is found. When the number of individuals per generations is increased, the calculation time and the frequency of obtaining the optimal solution are both increased.

The selection of the number of individuals should be made in terms of the complete number of solutions. In the previous test cases, for 9 WPs, there are a total of 1,035 possible trajectories. Table 4.5 shows the different times required to calculate the optimal trajectory for different numbers of WPs and the initial population's percentage. The percentage of the initial population represents the number of individuals analyzed with respect to the total number of possible trajectories. The precision of the algorithm could be increased with the number of WPs, allowing a higher discretization of the grid and the wind information, but it is penalized with computation time, as shown in Table 4.5.

Table 4.4 Flight tests for the variation of the initial population, with a separation angle of 15º and 9 WPs

Date	Flight time optimization	Optimal solution given by the GA for 26 individuals	Optimal solution given by the GA for 100 individuals	Calculation time per optimal trajectory for 26 individuals (s)	Calculation time per optimal trajectory for 100 individuals (s)
April 1st, 2012	0.72%	87%	99%	0.69	4.87
April 3rd, 2012	0.13%	84%	97%	0.66	4.99
April 4th, 2012	1.14%	74%	94%	0.66	5.32
April 5th, 2012	0.07%	35%	39%	0.62	5.19
April 6th, 2012	0.09%	92%	99%	0.63	4.9
April 9th, 2012	0.50%	80%	100%	0.62	4.92
April 10th, 2012	0.09%	78%	92%	0.66	5.08
April 11th, 2012	0.03%	76%	100%	0.65	4.91
May 1st, 2012	0.17%	87%	100%	0.65	4.73
May 2nd, 2012	0.00%	100%	100%	0.65	5.2
May 9th, 2012	0.00%	100%	100%	0.65	4.96
May 14th, 2012	0.64%	96%	100%	0.65	4.94
June 5th, 2012	0.16%	55%	72%	0.62	5.21
June 7th, 2012	0.04%	72%	94%	0.64	4.97
October 18th, 2012	0.76%	99%	100%	0.64	5.2
October 29th, 2012	1.80%	84%	98%	0.63	5.25
October 30th, 2012	2.54%	100%	100%	0.63	5.12
March 10th, 2013	0.14%	95%	98%	0.62	4.78
March 21st, 2013	0.69%	67%	80%	0.62	4.78
May 21th, 2013	0.46%	90%	100%	0.65	4.9
May 29th, 2013	0.13%	54%	85%	0.61	5.17
June 4th, 2013	0.66%	80%	89%	0.64	5.11
June 10th, 2013	0.78%	100%	100%	0.63	5.17
June 11th, 2013	0.72%	100%	100%	0.63	5.23
June 13th, 2013	0.30%	87%	100%	0.64	5.18
Average	**0.51%**	**82.88%**	**93.44%**	**0.64**	**5.04**

Table 4.5 Calculation times for different number of WPs and initial population with a separation angle of 15º and 9 WPs

Number of WPs	Number of possible trajectories	Calculation time for a 2.5% initial population	Calculation time for a 5% initial population	Calculation time for a 7.5% initial population	Calculation time for a 10% initial population
7	139	0.025s	0.061s	0.1s	0.188s
8	379	0.11s	0.39s	0.77s	1.25s
9	1,035	0.64s	2.52s	4.43s	5.42s
10	2,827	3.86s	7.49s	11.62s	15.13s
11	7,723	12.36s	22.17s	36.39s	51.26s
12	21,099	33.7s	69.35s	110.63s	145.77s
13	57,642	126.32s	228.26s	344.3s	488.1s

At 18 WPs, the number of possible trajectories is 1,563,803, which would result in a very high calculation time, even at an initial population of only 2.5%. Future work will focus on further reducing the calculation time of the algorithm in order to obtain an optimal solution with a high number of WPs.

Table 4.6 represents the test for the same parameters as in Table 4.3, but for 12 WPs and an initial population of 5%.

The average optimization found at 12 WPs was for a savings of 0.46%. The reason for the difference from the 9 WP optimization is that the wind speed and direction was analyzed at coordinates different than those in the Table 4.3 tests. Some of the trajectories obtain a higher optimization, and some a lower optimization. The optimization capabilities of the algorithm are independent of the number of WPs selected for each trajectory.

Table 4.6 Flight tests with 12 WPs and a 5% initial population

Date	Optimal flight time (minutes)	Great circle flight time (minutes)	Flight time optimization	Calculation time per optimal trajectory (s)
April 1st, 2012	393.4	395.7	0.58%	65.41
April 3rd, 2012	396.5	397.0	0.13%	68.92
April 4th, 2012	397.7	400.8	0.77%	66.9
April 5th, 2012	394.6	396.0	0.35%	68.16
April 6th, 2012	388.6	388.6	0.00%	74.15
April 9th, 2012	417.2	418.8	0.38%	69.38
April 10th, 2012	409.4	410.3	0.21%	65.67
April 11th, 2012	407.8	407.8	0.00%	67.15
May 1st, 2012	396.6	397.3	0.19%	69.83
May 2nd, 2012	390.6	390.7	0.02%	69.82
May 9th, 2012	411.0	411.0	0.00%	71.05
May 14th, 2012	412.8	415.7	0.70%	70.77
June 5th, 2012	394.1	395.3	0.31%	68.95
June 7th, 2012	397.6	397.8	0.04%	74.54
October 18th, 2012	422.1	424.2	0.48%	72.27
October 29th, 2012	394.3	401.6	1.81%	74.84
October 30th, 2012	389.6	396.6	1.76%	80.98
March 10th, 2013	409.0	411.8	0.67%	65.63
March 21st, 2013	402.0	402.0	0.01%	65.78
May 21th, 2013	402.1	404.2	0.53%	69.45
May 29th, 2013	410.0	410.5	0.11%	66.59
June 4th, 2013	412.8	415.0	0.53%	65.29
June 10th, 2013	401.6	404.9	0.83%	66.72
June 11th, 2013	405.0	408.2	0.77%	66.88
June 13th, 2013	402.1	403.8	0.42%	68.56
Average			**0.46%**	**69.35**

The variation of the separation angle, however, can influence the optimization percentage. If the separation angle is small, the alternative trajectories created are closer to the great circle. When the separation angle is large, the alternative trajectories are longer, so the winds would have to be significant for an aircraft to reduce its flight time while flying a longer trajectory. Tests for 5º, 10º, 15º and 20º separation angles are presented in Table 4.7.

Table 4.7 Flight tests for different separation angles

Date	Flight time optimization (5º separation angle)	Flight time optimization (10º separation angle)	Flight time optimization (15º separation angle)	Flight time optimization (20º separation angle)
April 1st, 2012	0.21%	0.52%	0.72%	0.45%
April 3rd, 2012	0.03%	0.10%	0.13%	0.05%
April 4th, 2012	1.21%	1.48%	1.14%	1.09%
April 5th, 2012	0.07%	0.11%	0.07%	0.06%
April 6th, 2012	0.11%	0.10%	0.09%	0.12%
April 9th, 2012	0.43%	0.64%	0.50%	0.66%
April 10th, 2012	0.15%	0.20%	0.09%	0.00%
April 11th, 2012	0.02%	0.02%	0.03%	0.00%
May 1st, 2012	0.09%	0.11%	0.17%	0.16%
May 2nd, 2012	0.07%	0.07%	0.00%	0.00%
May 9th, 2012	0.11%	0.03%	0.00%	0.10%
May 14th, 2012	0.20%	0.51%	0.64%	0.84%
June 5th, 2012	0.13%	0.17%	0.16%	0.16%
June 7th, 2012	0.18%	0.13%	0.04%	0.00%
October 18th, 2012	0.46%	0.67%	0.76%	1.02%
October 29th, 2012	0.06%	1.49%	1.80%	1.48%
October 30th, 2012	0.54%	2.52%	2.54%	2.38%
March 10th, 2013	0.40%	0.73%	0.69%	0.80%
March 21st, 2013	0.20%	0.20%	0.14%	0.00%
May 21th, 2013	0.20%	0.37%	0.46%	0.46%
May 29th, 2013	0.15%	0.15%	0.13%	0.12%
June 4th, 2013	0.27%	1.48%	0.66%	0.55%
June 10th, 2013	0.47%	0.70%	0.78%	0.80%
June 11th, 2013	0.48%	0.69%	0.72%	0.75%
June 13th, 2013	0.22%	0.27%	0.30%	0.34%
Average	**0.26%**	**0.54%**	**0.51%**	**0.50%**

The optimal reduction for the four separation angles was found between 10º and 15º, where the length of the trajectories compared to the great circle is larger, but where the wind magnitudes can help to reduce flight time.

The algorithm presented by Félix Patrón, Botez and Labour (2013) optimized the vertical flight profile by 2.57% with the analysis of the optimum altitudes and speeds. If this vertical profile algorithm and the horizontal algorithm presented here were coupled, a flight cost optimization of around 3% would be expected. Future work will focus on coupling the vertical and the horizontal flight profile.

4.4 Conclusion

It is important to perform an analysis of the alternative trajectories on the horizontal flight profile to reduce flight costs. The algorithm presented here improves the creation of flight trajectories for aircraft in the presence of winds. The GA implemented to reduce the number of calculations was shown to be stable, obtaining the optimal trajectory 90% of the time in average, depending on the selection of the initial population.

This algorithm allows us to vary parameters such as the number of WPs and the deviation angle, which influence the creation of alternative trajectories and the precision of the wind matrix. However, increasing the number of WPs also increases the calculation time.

While the algorithm presented by Félix Patrón, Botez and Labour (2013) improves the vertical flight profile on a commercial FMS with a 2.57% flight cost reduction in the absence of the winds, the methodology defined here reduces the flight cost on the horizontal flight profile for an average of 0.54% compared to the great circle. If the VNAV and LNAV algorithms were coupled together and implemented, a reduction of around 3% would be expected if the initial great circle route was under the influence of unfavorable winds with respect to the alternative trajectories. The implementation of this horizontal algorithm, however, still depends on the availability of the proposed flight trajectories to the ATC.

The coupling of both, the vertical and the horizontal algorithm are considered as future work. The 3% overall potential flight cost optimization represents an important improvement in the creation of trajectories by current FMS platforms, and would be an important step in the use of green aircraft procedures to reduce the aviation footprint on the environment.

CHAPTER 5

ARTICLE 3: NEW METHODS OF OPTIMIZATION OF THE FLIGHT PROFILES FOR PERFORMANCE DATABASE-MODELED AIRCRAFT

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Résumé

Les chercheurs dans le domaine de l'aéronautique ont longuement essayé de réduire la consommation de carburant des avions et ainsi minimiser les émissions provenant du domaine de l'aviation dans l'atmosphère. Cet article présente un algorithme qui améliore les trajectoires créées pour un système de gestion de vol commercial. Une analyse complète de la montée, de la croisière et de la descente a été effectuée et un algorithme génétique a été développé pour évaluer les effets des changements possibles en vitesse et en altitude des avions, ainsi que l'influence du vecteur vent dans les profils latéral et vertical de vol, afin d'obtenir la trajectoire qui réduit la consommation globale du vol.

Abstract

Researchers have been attempting to reduce aircraft fuel consumption for decades to minimize aviation's emissions to the atmosphere. This article presents an algorithm which improves the trajectories created by a commercial FMS. A complete analysis of the climb, cruise, and descent was performed and a GA has been implemented to evaluate the effects of the possible changes to aircraft speeds and altitudes, as well as the influence of the wind vector on the lateral and vertical profiles, all to obtain the flight trajectory that most reduces the global flight fuel consumption.

5.1 Introduction

As the impacts of global warming and climate change have become more severe, many researchers have been trying to further reduce aircraft fuel consumption. The total CO_2 emissions due to aircraft traffic represents between 2.0% and 2.5% of all anthropogenic CO_2 emissions to the atmosphere (ICAO, 2010). In 2011, more than 676 million tons of CO_2 were emitted. The goal for the aviation industry proposed by International Air Transport Association (IATA) and International Civil Aviation Organization (ICAO) is to reduce the CO_2 production of 2005 by 50% in 2050 (ATAG, 2014).

Various approaches have been used to reduce the environmental impact of aviation: the use of biofuels to improve aircraft environmental performance (Hendricks, Bushnell and Shouse, 2011; Sandquist and Guell, 2012), the development of more efficient engines to decrease emissions and to reduce noise (Panovsky J, 2000; Salvat, Batailly and Legrand, 2013; Williams and Starke, 2003), improvements to aircraft frames and wings (Freitag and Schulze, 2009; Nguyen et al., 2013), and the optimization of flight trajectories.

The optimization of flight trajectories has been used by researchers to reduce aircraft fuel consumption for several years now. The FMS is a device used in all current aircrafts to assist the pilot with several tasks, such as navigation, guidance, trajectory prediction and flight path planning. There are three phases during a flight that can be improved: climb, cruise and descent. However, it is during the cruise phase where 80% of the CO_2 emissions from aviation are produced (ATAG, 2014), and thus, many researchers have been studying strategies to improve this phase. Lovegren (2011) analyzed how the fuel burn could be reduced during cruise if the appropriate speed and altitude are selected, or if SCs are performed. Jensen et al. (2013) presented a speed optimization method for cruises with fixed lateral movement by analyzing radar information from the United States FAA's ETMS (Palacios and Hansman, 2013). Their results showed that most flights in the USA do not fly at an optimal speed, which increases their fuel consumption. Dancila, Botez and Labour

(2012; 2013) studied a new method to estimate the fuel burn from aircraft to improve the precision in flight trajectory calculations.

The influence of weather on aircraft flight has been considered as part of strategies to take advantage of winds to reduce flight time and/or to avoid headwinds that could increase global flight costs. Campbell (2010) studied the influence of weather conditions, such as thunderstorms and contrails, and modeled them as obstacles in order to create a trajectory to avoid it, to reduce air pollution and fuel burn. Filippone (2010) analyzed the influence of the cruise altitude on the creation of contrails and its influence on the flight cost. Miyazawa et al. (2013) studied an optimal flight trajectory using dynamic programming including a model of wind patterns from the Japan Meteorological Agency. They modeled the aircraft's performance using BADA (Base of Aircraft Data), which is an open-source database of aircraft models. They minimized fuel consumption while respecting arrival time constraints and the vertical distance safety separation from other aircraft. Murrieta et al. (2013) presented an algorithm which optimized the vertical and horizontal trajectories, taking into account the wind forces and patterns as well as the variation of the CI. Gagné et al. (2013) found the optimal vertical profile by performing an exhaustive search of all the available speeds and altitudes. Bonami et al. (2013) studied a trajectory optimization method capable of guiding aircraft through different WPs considering the wind factors and reducing fuel burn, utilizing a multiphase mixed-integer control. Franco and Rivas (2011) analyzed the minimal fuel consumption for a cruise at a fixed altitude, using a variable arrival-error cost that penalizes both late and early arrivals. They showed that the minimal cost is obtained when the arrival-error cost is null, and found that different optimal cruise altitudes could achieve the goal of minimal cost and fuel consumption with a fixed estimated arrival time.

An alternative method, arranging aircraft in formation, was analyzed by Kent and Richards (2013). Formation flights were used to reduce drag, thereby reducing fuel burn. Kent and Richards used two different methods: an extension to the Fermat-Torricelli problem allowing them to find optimal formations for many routes, and a geometric method to be able to apply the influence of the wind. Nangia and Palmer (2007) reduced overall drag of the order of 15-20% for commercial aircraft flying in formation.

Other research groups have focused specifically on the descent phase, where the goal is to reduce pollution close to air terminals in terms of both noise pollution and fuel burn emissions. Clarke et al. (2004) introduced the CDA method to reduce noise, which consists of the deceleration and descent of an aircraft at its own vertical profile from the TOD. He then presented the design and implementation of an optimized profile descent in high-traffic conditions, such as at the Los Angeles International Airport (LAX), which increased operational efficiency from traffic management and reduced fuel, emissions and noise (Clarke et al., 2013). Dancila, Botez and Ford (2013) created an analysis tool to estimate the fuel and emissions cost produced by aircraft during a missed approach. Reynolds, Ren and Clarke (2007) concluded that the CDA effectively reduced fuel burn and noise near airports simply by keeping the aircraft at the highest possible altitude before creating the descent. Adding together both cruise and descent flight cost reduction strategies would increase the impact of flight trajectory analysis.

Air traffic management has increased significantly. By 2030, an estimated number of 5.9 billion passengers are expected, doubling the amount from 2010 (ATAG, 2014). Over the past few years, this growth has influenced many researchers to include increasing levels of air traffic as a part of the trajectory optimization process. This has also opened a research domain in conflict detection algorithms to increase air security (Gariel, Kunzi and Hansman, 2011; Kuenz, Mollwitz and Korn, 2007; Visintini et al., 2006). Delgado and Prats (2013) worked on the concept of aircraft speed reduction with the objective of selectively causing in-flight delays to avoid traffic congestion near airports. This research was performed so as to delay an aircraft during flight, but with no extra fuel consumption compared to the initially-planned flight, and considering the possible uncertainties due to the weather. Margellos and Lygeros (2013) examined a new concept of target windows, with 4D-imposed constraints at different locations along the flight trajectory, aiming to increase safety by avoiding conflicts with improved prediction. De Smedt and Berz (2007) studied the characteristics of different

FMSs' performance to determine the accuracy of their time constraints calculations and the influence it could have on ATC. Friberg's (2007) study showed that promising results in terms of the environment benefits could be achieved by establishing a proper communication between the FMS and ATC. Fays and Botez (2013) developed a 4D algorithm treating meteorological conditions or air traffic restrictions in a specified air space, defining them as obstacles, to improve the FMS's trajectory creation capabilities. Air traffic conditions have also been identified as the cause of missed approaches.

Since the objective of this trajectory optimization algorithm is to be implemented in a FMS, computation time has to be reduced. GAs have been widely used in the aviation sector to obtain optimal solutions at low computation times (Kanury and Song, 2006; Kouba, 2010; Li et al., 1997; Turgut and Rosen, 2012; Yokoyama and Suzuki, 2001).

At LARCASE, various algorithms have been developed to improve a FMS platform, using the PDB from different types of commercial aircraft as the numerical model. These methods define VNAV optimization in the absence of external perturbations such as wind. More recently, an adaptation to include wind factors was developed, and the LNAV (Lateral NAVigation) profile analyzed. Different techniques have been implemented to reduce the algorithms' calculation time, such as new interpolation methods and time optimization techniques, like the golden section search and GAs.

While in the literature different optimization algorithms have been applied to optimize flight trajectories, an important fact to be considered about the presented optimization approach is that it is applied to database-modeled aircraft, instead of the usual model by equations of motion.

This article describes an algorithm to be implemented in an FMS to create optimal flight trajectories and reduce fuel burn by analyzing the three phases of a flight, and the wind factors, to obtain the maximum flight cost reduction, but not considering any restrictions that may be imposed by air traffic management.

The optimization algorithm described in this article analyzes the climb, cruise and descent, all together, to obtain the highest possible flight cost optimization. A complete wind model is used to calculate a more accurate assessment of the aircraft fuel burn, as well as to analyze the influence of the winds during a flight. During the cruise phase, alternative horizontal trajectories for the LNAV profile, as well as SCs during the VNAV profile are considered to reduce flight cost. A GA has been implemented to analyze the maximal number of possible trajectories while keeping the calculation time low.

This work for article was conducted under the project “Optimized Descents and Cruise”, in collaboration with the Canadian Green Aviation Research and Development Network (GARDN).

5.2 Methodology

The methodology begins with an introduction of the PDBs’ structure, which represents the numeric model of each aircraft. Next, a wind model is developed to calculate the wind’s influence during a flight, including its influence on the flight cost equation. The optimal climb to the TOC is then calculated. The cruise is analyzed from the TOC until the estimated TOD, including an analysis of the influences of different altitudes and lateral trajectories using GAs. Finally, the descent is calculated to obtain the complete flight trajectory.

5.2.1 Aircraft model – Performance database

The algorithms presented below were developed in Matlab®, using the PDB for commercial aircraft. The PDB is a database of over 30,000 lines containing information on the actual performance of the numerical model of the aircraft used for this study. The PDB includes the aircraft weight, altitude, speed, center of gravity and air temperature as inputs; the outputs are the distance traveled and the fuel burn. The travel time is calculated from the aircraft’s TAS, and the wind influence is calculated with a wind triangle methodology which is explained in the next section. The PDB contains a large quantity of very detailed aircraft information;

however, there are five main tables that are used in this program. The inputs and outputs contained in these databases are described in Table 5.1. This information gives the performance (outputs) of each aircraft for different parameters (inputs), at each phase of the flight.

Table 5.1 Inputs and outputs of the PDB

Type of table	Inputs	Outputs
Climb	Center of gravity Speed Gross weight ISA deviation Altitude	Fuel burn (kg) Horizontal distance (nm)
Climb acceleration	Gross weight Initial Speed Initial Altitude Delta speed	Fuel burn (kg) Horizontal distance (nm) Delta altitude (ft)
Cruise	Speed Gross weight ISA deviation Altitude	Fuel flow (kg/hr)
Descent deceleration	Vertical speed Gross weight Initial speed Final altitude Delta speed	Fuel burn (kg) Horizontal distance (nm) Delta altitude (ft)
Descent	Speed Gross weight Standard deviation Altitude	Fuel burn (kg) Horizontal distance (nm)

An example of the data provided by the PDB is shown in Figure 5.1. The framed value shows the fuel consumption for the cruise with a center of gravity of 28% of the mean aerodynamic chord, flying at Mach 0.8 with a total gross weight of 100 tons, at an altitude of 30,000 ft and at a standard deviation temperature of -10°C.

The PDB’s information is used to calculate the fuel burn and the distance traveled by the aircraft at each phase of the flight.

```
MODE CRUISE_PROFILE_MACH
! * * * * * * * * * * * * * * * * *
! Last Update     : 2011/01/23 8h2
! Aircraft        : A310 304 Basic
! Reference CG    : 28
! * * * * * * * * * * * * * * * * *
  SPEED  0.8
     GROSS_WEIGHT  100000
        ISA_DEV  -10
           !Altitude  FuelFlow
             25000     5355
             26000     5146
             27000     4948
             28000     4756
             29000     4574
             30000     4402
             31000     4243
             32000     4091
             33000     3946
             34000     3809
             35000     3685
             36000     3574
             37000     3495
             38000     3441
             39000     3405
             40000     3378
```

Figure 5.1 Example of the PDB

To obtain the performance information from the database, the Lagrange linear interpolation method is applied, as in Equation (5.1).

$$x = \frac{y - y_1}{y_0 - y_1} * \mathrm{f}_0 + \frac{y - y_0}{y_1 - y_0} * f_1 \tag{5.1}$$

With this information a complete flight trajectory can be calculated precisely, in terms of flight time, distance and fuel burn.

5.2.2 Wind model and flight cost equation

5.2.2.1 Wind model

The wind data used in this algorithm is extracted from Environment Canada (2013). The information is presented under a GDPS format. The GDPS model provides a 601×301 latitude-longitude grid with a resolution of 0.6×0.6 degrees. At each point of this grid, information such as the wind direction, speed, temperature, and the pressure can be obtained for different altitudes, in 3-hour time blocks.

Wind directly affects the horizontal distance traveled with respect to ground level, and indirectly affects the fuel consumption. The ground speed is calculated so that it can be considered in the horizontal distance calculation. The speeds below are expressed in knots<kt>.

$$\overrightarrow{\text{Ground speed}} = \overrightarrow{\text{Airspeed}} + \overrightarrow{\text{Effective wind speed}} \tag{5.2}$$

The air speed is an aircraft's speed relative to the air mass, and the wind is the horizontal movement of this air mass relative to the ground. Here, the effective wind is the wind's component of the aircraft's trajectory, and the crosswind is that component perpendicular to the effective wind speed. These are illustrated in Figure 5.2.

$$\overrightarrow{\text{Effective wind speed}} = \overrightarrow{\text{Real wind}} - \overrightarrow{\text{Crosswind}} \tag{5.3}$$

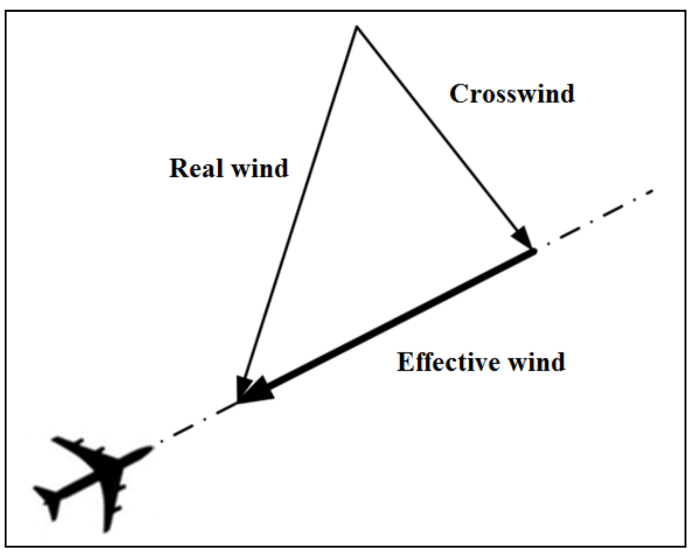

Figure 5.2 Airspeed, crosswind and effective wind

As the aircraft flies on a straight path, the wind affects the aircraft's speed. Depending on the direction and speed of the wind, a distance factor is calculated. As the distance traveled by the aircraft will either be reduced or increased in a particular time segment. The horizontal distance traveled at the ground level is the norm of the ground speed vector. Figure 5.3 shows the influence of the wind of a mass moving from WPT(n) to WPT(n+1).

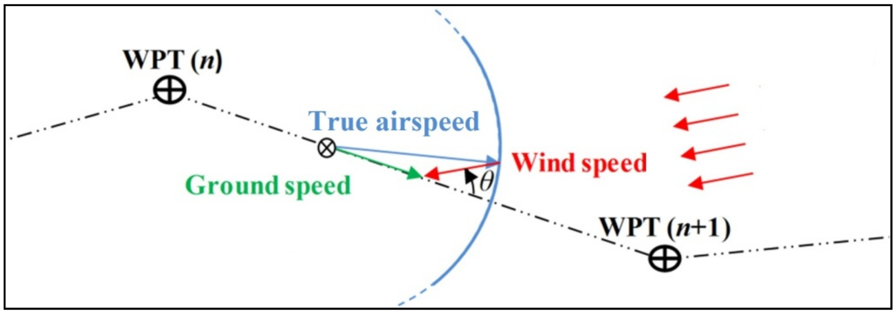

Figure 5.3 Wind factor calculation (Langlet, 2011)

The distance factor is calculated by the wind factor in the following way (Langlet, 2011):

$$Wind\_factor = \cos\left[\arcsin\left(\frac{\sin(\theta) * \|\overrightarrow{Wind_{speed}}\|}{\|\overrightarrow{Air_{speed}}\|}\right)\right] - \frac{\|\overrightarrow{Wind_{speed}}\|}{\|\overrightarrow{Air_{speed}}\|} * \cos(\theta) \tag{5.4}$$

The ground speed is obtained from the ratio between the TAS and the wind speed. The ground distance can be calculated from the ground speed.

The wind data is interpolated in the optimization algorithm at each required geographical position between two consecutive WPs. At each WP between the departure and arrival airports, the altitude, flight time, latitude and longitude are used as inputs to obtain outputs such as the wind speed, wind direction and air temperature from Environment Canada's database. This interpolation is used at each phase of the flight (climb, cruise, descent). For the vertical interpolation, the wind vectors are analyzed according to the Earth's Northern and Eastern axes (selected arbitrarily as a reference parameter) for two different altitudes. Afterwards, an interpolation is made between these axes at the required altitude to obtain the wind vector (speed and direction). The horizontal interpolation is obtained between consecutive WPs. This process is sketched in Figure 5.4.

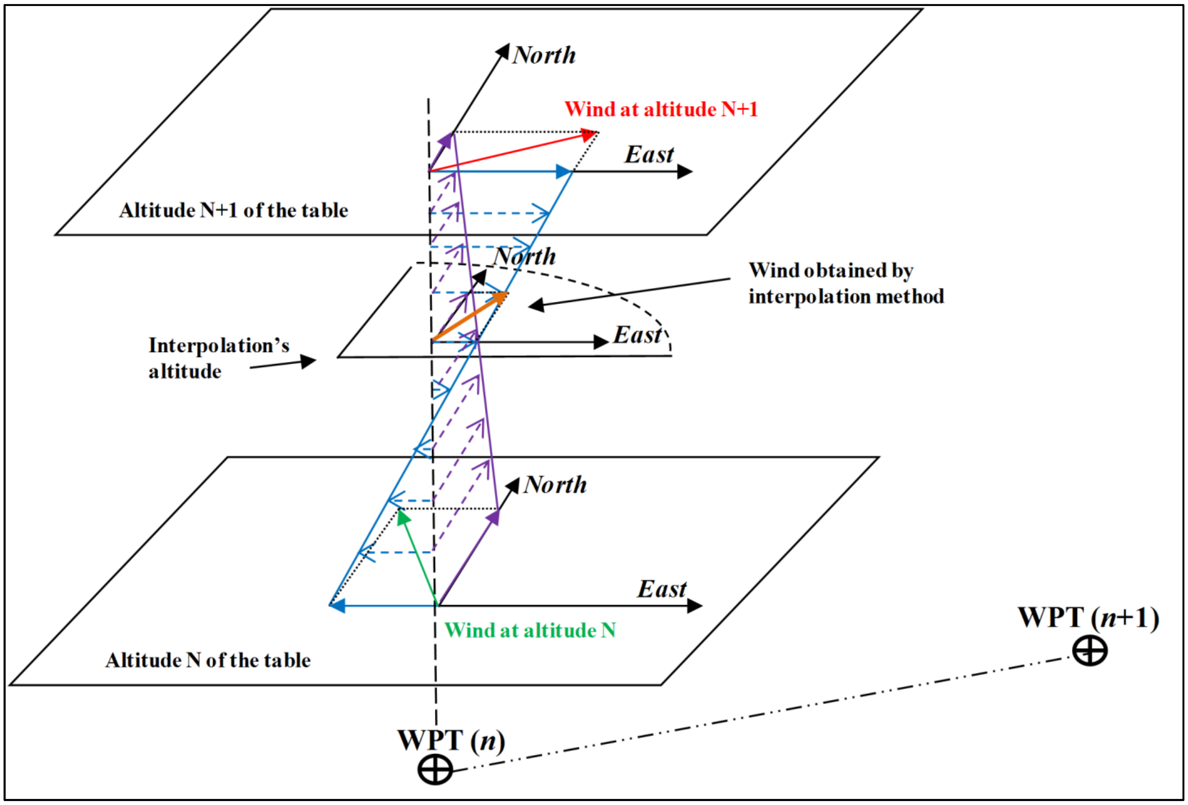

Figure 5.4 Wind interpolation method (Langlet, 2011)

In the flight cost optimization program, the wind's influence is calculated dynamically, i.e., it is updated as the aircraft advances in space and time.

5.2.2.2 Flight cost

In aviation, fuel consumption is not the only information considered for aircraft trajectory planning. In this algorithm, it is the global flight cost that is calculated, and not only the fuel burned. The CI is a variable that influences the global cost of a flight; it is a term used by airlines to calculate their flight operation costs. The CI in this paper is defined as in the commercial FMS used for this study.

The global cost is defined by:

$$\text{Global cost} = \sum \text{Fuel Burn} + \text{CI} * \text{Flight Time} \tag{5.5}$$

Where the Fuel Burn is expressed in <kg>, the CI in <kg/hr> and the Flight Time in <hr>; therefore, the Global Cost is given in <kg>. Since the fuel price (in <$/kg>) changes continuously, in this article the global cost is given in <kg> of fuel. The global cost in money <$> can be obtained by multiplying it by the price of one kg of fuel.

The global cost can be expressed as:

$$\text{Global cost} = \overline{\text{Fuel Flow}} * \text{ Flight Time } + \text{ CI } * \text{ Flight Time} \tag{5.6}$$

Where Fuel Flow is given in <kg/hr> and can be obtained directly from the PDB. Therefore, Equation (5.6) can be further written as follows:

$$\text{Global cost} = \text{Flight Time} * (\overline{\text{Fuel Flow}} + \text{ CI}) \tag{5.7}$$

The optimization of the algorithm is expressed according to the global cost of the flight.

5.2.3 Climb

Before describing the climb, it is important to define the crossover altitude. The PDB divides the TAS values into two different types of speeds: IAS and Mach number. The TAS varies with the altitude. For the IAS, the TAS increases with the altitude, while the Mach decreases with altitude. The altitude at which the TAS due to IAS is equal to the TAS due to Mach is called the crossover altitude.

The initial climb is calculated at a constant IAS speed of 250kt, from 2000ft to 10,000ft, since the information about the take-off procedure is not provided in the aircraft's numeric model. The climb starts at 10,000ft, where the algorithm accelerates from 250kt to all the available IAS in the PDB. It climbs at all the available IAS up to the crossover altitude. At each crossover altitude (it varies for each IAS/Mach speed schedule), a constant Mach climb is calculated at each 1,000ft, up to the maximal climb altitude (40,000ft). A different TOC is obtained for each IAS/Mach/Altitude combination. All the possible IAS/Mach/Altitude combinations are evaluated during this phase. An example of a climb trajectory is shown in Figure 5.5.

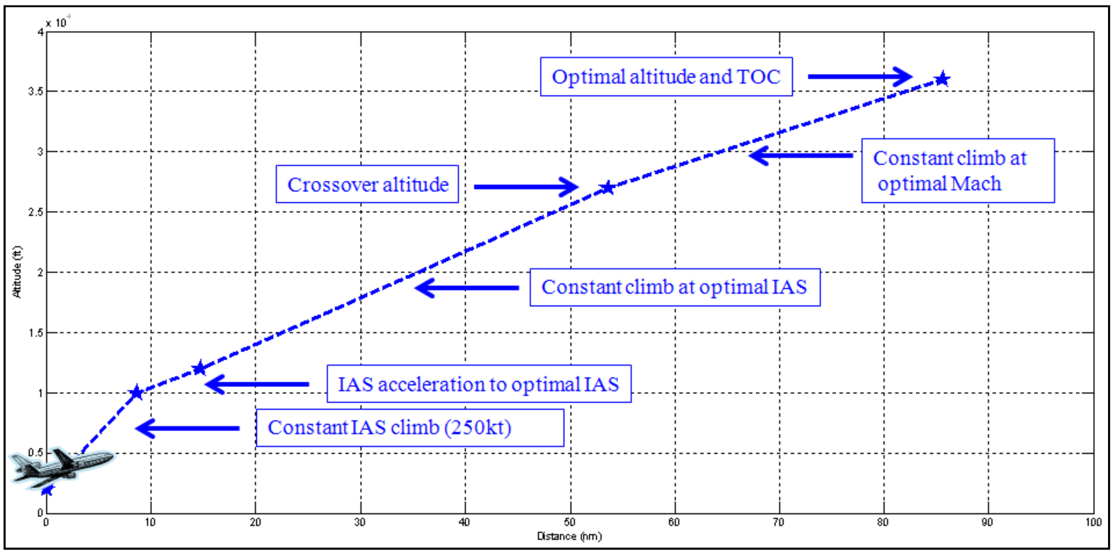

Figure 5.5 Climb trajectory example

5.2.4 Cruise

The cruise phase starts at the end of the climb. At this point, the known cruise parameters are:

- Position of the aircraft in latitude and longitude
- Altitude
- Mach speed
- Aircraft updated weight after climb
- Flight time

The cruise is divided into WPs, where the first WP is the TOC, and the last WP is the estimated TOD.

5.2.4.1 LNAV

In order to perform a complete analysis of the wind, an LNAV optimization is made. At the TOC obtained after the climb, four alternative trajectories are introduced, two on each side of the original trajectory. These trajectories are separated at a variable distance (15 nautical miles for the tests in this paper).

Figure 5.6 shows an example of a real trajectory and its alternatives. Real flight information was downloaded from the website FlightAware (2013), a website that allows users to download flight information such as real coordinates, altitudes, speeds, flight time, airlines and aircraft type (at no charge). The flight shown in Figure 5.6 is from Paris to Montreal, on October 21st, 2013 at 12:25 pm UTC. The original and the alternative trajectories are presented.

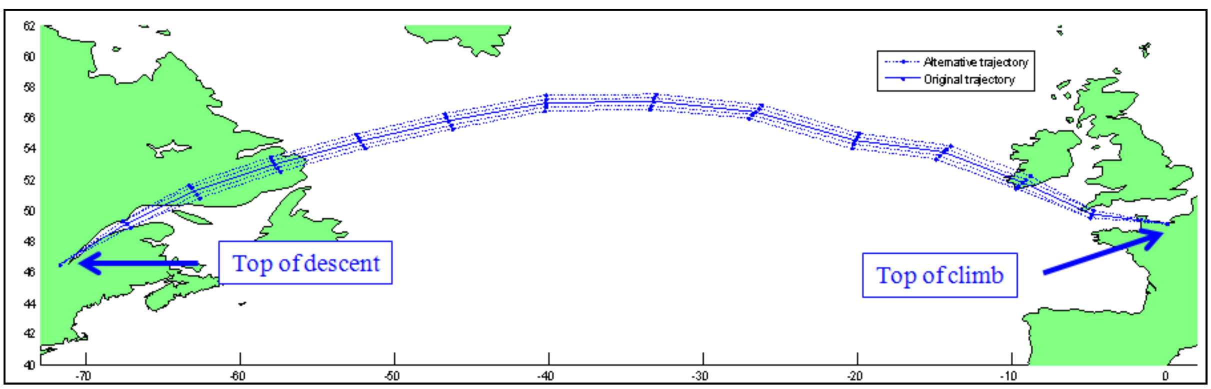

Figure 5.6 In-cruise grid example of a Paris to Montreal flight

Each trajectory is divided into n WPs. The more n increases, the more precisely the trajectories will be calculated, but the longer the calculation time. The algorithm analyzes the possible deviations to find the one that most reduces fuel consumption. The first WP corresponds to the TOC, while the last WP to the TOD.

The grid is represented by an m x n matrix for the latitudes and altitudes, where n is the number of WPs and m the total number of possible routes, which is fixed at five. Adding additional alternative trajectories would increase the algorithm's optimization performance, but it would also increase the calculation time.

Each possible trajectory is represented by a vector containing the specific number of one of the five possible routes, at each WP. For eastbound flights, the trajectory's numbering starts at one from the northern trajectory and ends at five for the southern trajectory. For westbound flights, the trajectories are defined contrariwise. The real trajectory is always defined as number three.

Figure 5.7 shows an example of a westbound flight from Paris to Montreal, represented by the vector (3 2 2 1 1 1 2 3 3 3 2 3 3 3).

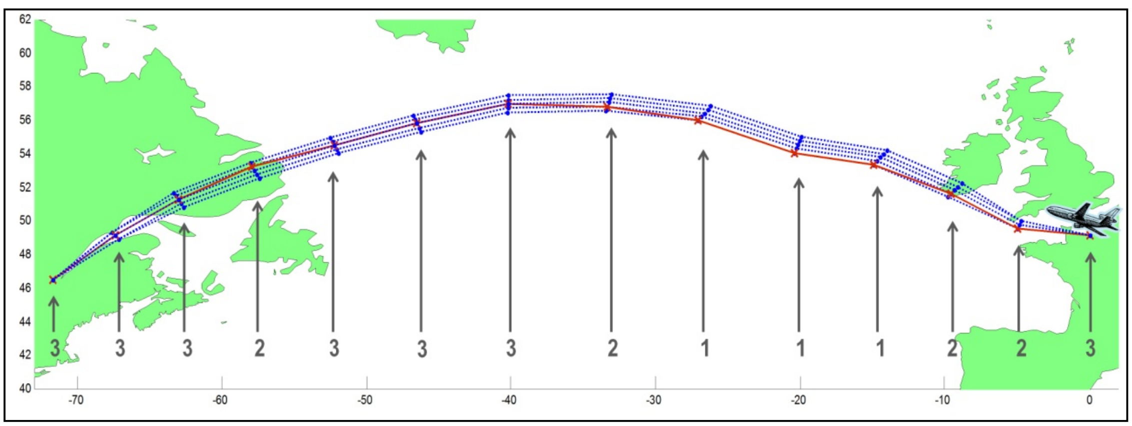

Figure 5.7 Grid numbering example for a westbound flight

If all the possible trajectory combinations were calculated, the algorithm's calculation time would be very large. Due to the non-linear nature of the wind, a GA has been used to calculate the optimal trajectory in a reasonable calculation time. The meteorological forecast is used in the calculation process in order to take advantage of tailwinds and avoid headwinds.

In this section, the term optimization is applied to the calculation time reduction, whereas in the rest of the document it refers to flight cost reduction.

GAs are based on Darwin's theory of evolution, where the fittest survive. The calculation of the optimal horizontal trajectory is performed in four steps:

First, the GA creates **individuals**, defined like random trajectories as in Figure 5.7. These individuals can only be created within the confines of the grid, and must respect two important constraints: the aircraft can only fly to an adjacent WP, and the initial and final WPs have to be the TOC and the TOD, respectively.

Second, the **evaluation** process consists in calculate the cost of the flight for each individual with Equation (5.7) using the following information:

- Distance: Obtained directly from the flight trajectory.
- Aircraft TAS: Calculated from the aircraft Mach number and the flying altitude.
- Wind speed: Obtained from the weather model.
- Flight time: Calculated using the distance and the global aircraft speed, which is given by the aircraft TAS and the wind speed.
- Fuel burned: Calculated with the fuel flow and the flight time.

Third, a set of individuals, those that will reproduce, are obtained by a **selection** process among the total individuals, by means of a selection by roulette. This method consists of assigning a piece of the roulette depending on that individual's cost. The better the cost is in terms of optimization, the larger the piece assigned, and so the more chances it will have to be selected. However, the randomness of the roulette gives even the 'poorest' individuals a chance to be selected. This method allows for diversity in each generation, which is helpful to avoid a quick convergence into suboptimal solutions. A roulette wheel selection example could be seen in Figure 5.8.

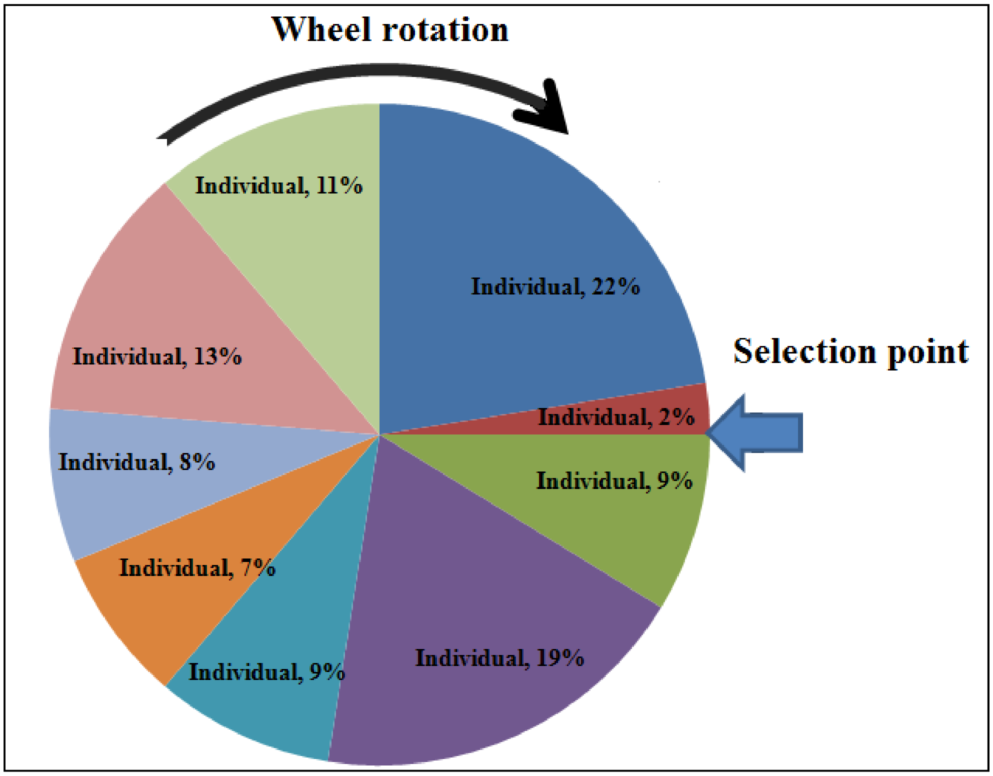

Figure 5.8 Roulette wheel selection example

Finally, the selected individuals will perform a **reproduction** to create a new generation. A crossover method has been selected. This reproduction method crosses the first part of an individual with the second part of another individual. Since each individual (trajectory) is represented by a vector, the crossover takes place at the middle number of each vector. An example of two random individual's crossover is shown in Table 5.2.

Each new individual after the crossover is evaluated as well, obtaining a new individual with a new flight cost.

Table 5.2 Example of individual's crossover

Individual 1	$WP_{1,1}$	$WP_{1,2}$	$WP_{1,3}$	$WP_{1,4}$	$WP_{1,5}$	$WP_{1,6}$	$WP_{1,7}$	$WP_{1,8}$
Individual 2	$WP_{2,1}$	$WP_{2,2}$	$WP_{2,3}$	$WP_{2,4}$	$WP_{2,5}$	$WP_{2,6}$	$WP_{2,7}$	$WP_{2,8}$
Individual obtained	$WP_{1,1}$	$WP_{1,2}$	$WP_{1,3}$	$WP_{1,4}$	$WP_{2,5}$	$WP_{2,6}$	$WP_{2,7}$	$WP_{2,8}$

The process is repeated for a specific number of generations. At the end of the optimization algorithm, the optimal trajectory is obtained, represented by a set of coordinates that the aircraft should follow to reduce fuel burn. This optimal trajectory is defined as the trajectory that best uses the wind to reduce flight time and fuel burn. The LNAV optimization algorithm finds an optimal trajectory which profits from tailwinds, and avoids headwinds as much as possible.

5.2.4.2 VNAV

After the LNAV optimization algorithm has run, the dynamic wind information for the flight has been analyzed, and the optimal horizontal trajectory in terms of flight cost has been found. The VNAV optimization during the cruise is the next step.

The optimal altitude changes as an aircraft burns fuel. The VNAV optimization functions by determining the cruise's optimal altitude. At each cruise WP, the algorithm analyzes if the optimal altitude is the current aircraft altitude, or if a 1,000ft or 2,000ft SC would reduce the global flight cost.

To obtain a more accurate calculation, the algorithm calculates the cost of the entire cruise at the selected altitude, in order to take into account the costs caused by the in-cruise climbs. Figure 5.9 shows an example trajectory that performed three in-cruise SCs to reduce the global flight cost. In this example, 40,000ft is the maximal climb altitude.

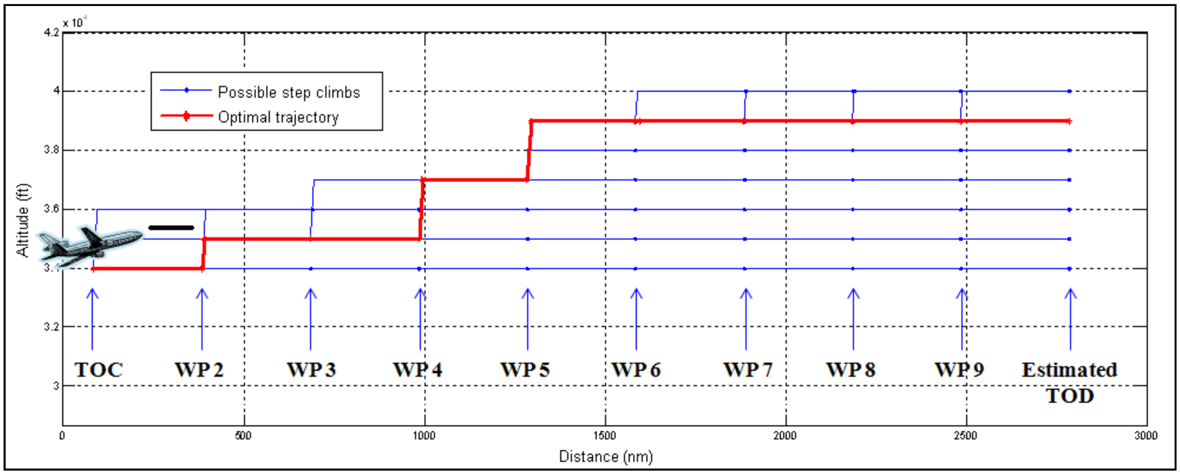

Figure 5.9 In-cruise SCs

In addition to the analysis of the possible SCs, the algorithm analyzes all the available cruise speeds at each WP to determine if the Mach speed can be modified to reduce fuel consumption.

The estimated TOD WP is set 200nm before the destination airport, since the final descent depends on the altitude at which the aircraft is placed. Further explanation of the descent is presented in the next section.

5.2.5 Descent

At the estimated TOD, the updated aircraft weight, the current altitude and the speed are known. Since the TOD is only an estimated value, an iteration process is implemented to accurately calculate the descent. At the current aircraft weight, the optimal descent is calculated to estimate the total horizontal distance necessary to perform the descent. Once this distance is estimated, the remaining cruise required to arrive at the updated TOD is performed, and the descent is recalculated with the new aircraft weight (after the cruise) from the new TOD. Since the estimated horizontal distance due to the descent was calculated with a different weight, this horizontal distance will change, and it is possible that the aircraft will not arrive exactly at the destination airport without further adjustment. The difference in the horizontal distance between the aircraft and the airport is removed from the cruise, and the descent is recalculated. This process is repeated until the aircraft arrives precisely at the destination airport. This process can be seen in Figure 5.10.

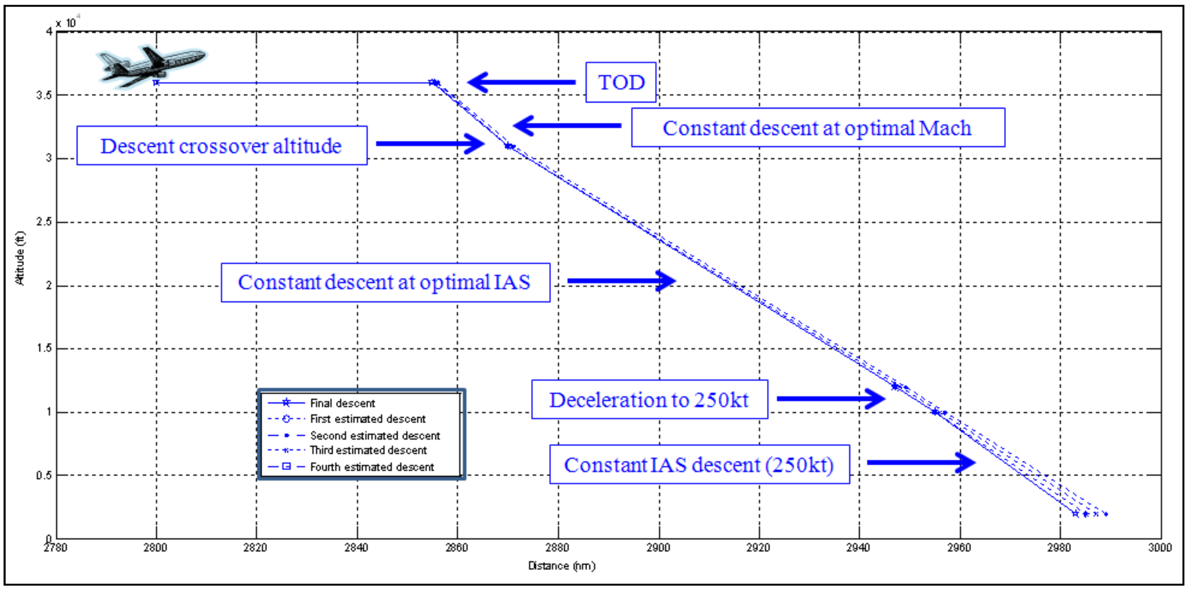

Figure 5.10 Descent trajectory example

All the possible Mach/IAS speed schedules are calculated to obtain the lowest-cost descent. The descent is calculated as follows:

The aircraft descends from its current altitude at constant Mach speed until it reaches the crossover altitude. From the crossover altitude, the aircraft descends at constant IAS until it is time to decelerate, since it needs to arrive at 10,000ft at 250kt. At 10,000ft, the aircraft descends at a constant speed of 250kt until it reaches 2,000ft. The aircraft flight model stops at this altitude, since the PDB does not include information about the landing procedure. This process can also be seen in Figure 5.10.

The global optimization algorithm is explained in the flow chart shown in Figure 5.11.

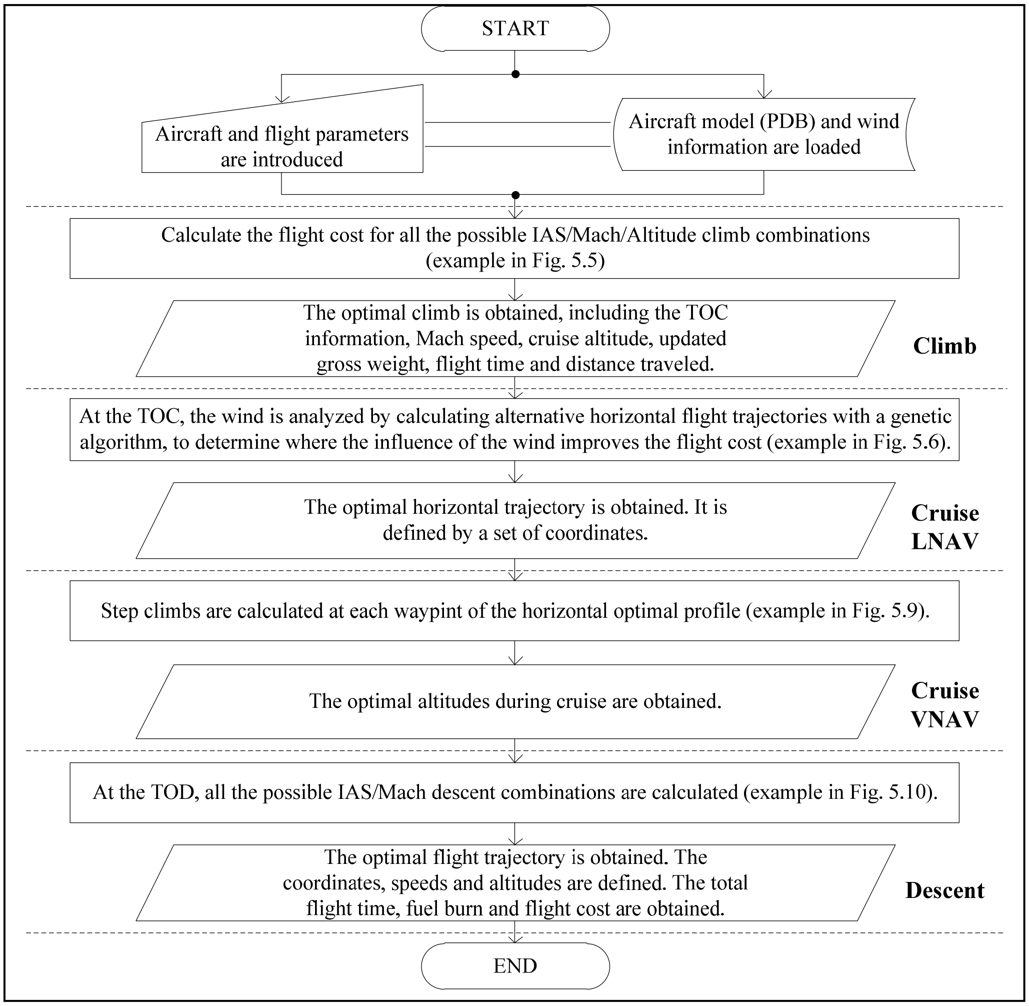

Figure 5.11 Optimization algorithm diagram

5.3 Results

This section presents the results of tests implemented to verify the algorithm's optimization capabilities at reducing fuel consumption.

These tests were performed using real flight information obtained from FlightAware. FlightAware is a website that makes real time and historical information for commercial flights freely available for download, including precise information indicating the flight coordinates, altitude, speed, date and time of flights. To demonstrate the flight cost reduction as a precise percentage, the real flight costs and the calculated optimal trajectory flight costs are compared. The real flight data has been recreated using the information obtained from FlightAware through the PDB, which represents the numerical model for this specific aircraft. The optimal and the real trajectory are calculated using the same aircraft model and fuel burn performance.

Using the same initial flight parameters as those for the real flight, as well as the same wind profile obtained from Environment Canada, the algorithm is executed to obtain the optimal flight trajectory in terms of altitude, speed, and SCs, along with a possible alternative horizontal trajectory determined by considering the wind influence to reduce the flight time and the overall flight cost. Since FlightAware does not provide information about the aircraft weight or the fuel burn, this parameter is defined by the standard aircraft weight as defined in the PDB, and the initial fuel provided will be calculated so the aircraft can complete the entire flight. It is possible that the real trajectory was optimized to a different weight that in the following tests, but this cannot be said certainly. The real trajectory is taken as reference, and for the same weight, the algorithm reduced the fuel consumption. The CI is not likely to influence negatively the optimization results, since the flight time is also optimized.

The optimization algorithm calculates the possibility of performing 1,000 ft or 2,000 ft SCs at each WP. FlightAware provides the information for each flight; the aircraft's location and speed sampling occurs at approximately each minute, providing an extensive flight profile. In order to be able to recreate the real flight in the algorithm, a total number of 13 WPs were selected. These WPs are equally distributed along the trajectory so the flight can be recreated as precisely as possible. The choice of 13 WPs is a number low enough to keep the calculation time acceptable for its application on the FMS platform while still making it possible to precisely recreate the real trajectory. At each WP, the effects of speed change and of a possible horizontal deviation were analyzed. The meteorological conditions of both flight trajectories were analyzed using the same date and time, with the information obtained from Environment Canada.

To show the differences between a real flight and the calculated optimal trajectory, a flight from Lisbon to Toronto, from October 4th, 2013, departing at 15:10 UTC, has been analyzed as an example. Figure 5.12 shows the difference between the flight altitudes and speeds of the real and the optimal trajectories, and Figure 5.13 shows the difference between the horizontal cruise trajectories. The flight information can be found in Table 5.3. A total of 1902 kg of fuel has been saved with the optimization, representing a 6.86% flight cost reduction. The CI was set to zero, indicating that the only parameter to consider was the fuel consumed and not the flight time. In this case, the flight time was reduced by 1.12%.

Table 5.3 Fuel burnt and flight time for a Lisbon to Toronto flight

	Fuel burnt (kg)	**Flight time (hr)**
Real trajectory	27,709.9	6.95
Optimal trajectory	25,807.8	6.87
Optimization	**6.86%**	**1.12%**

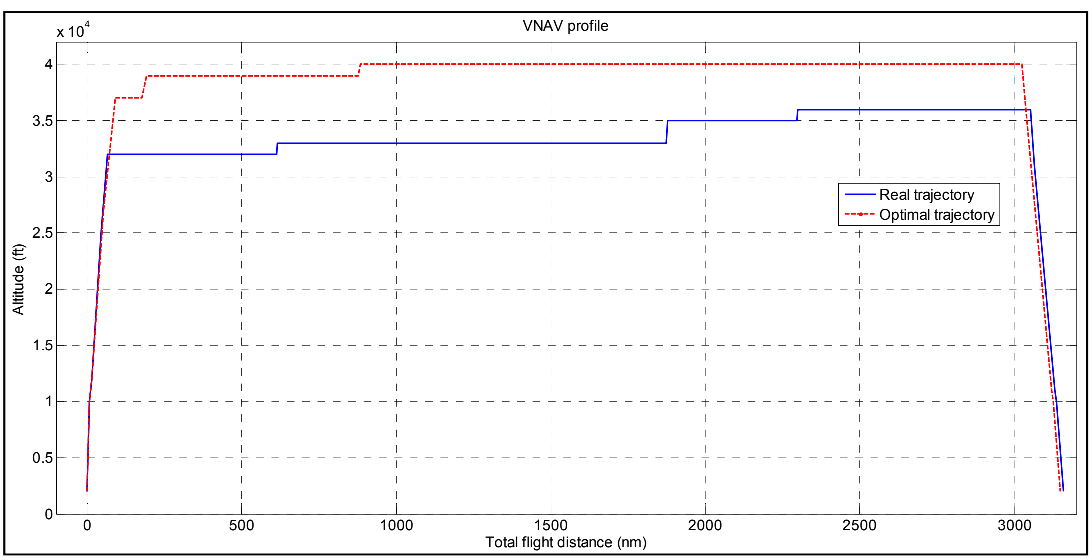

Figure 5.12 VNAV flight from Lisbon to Toronto

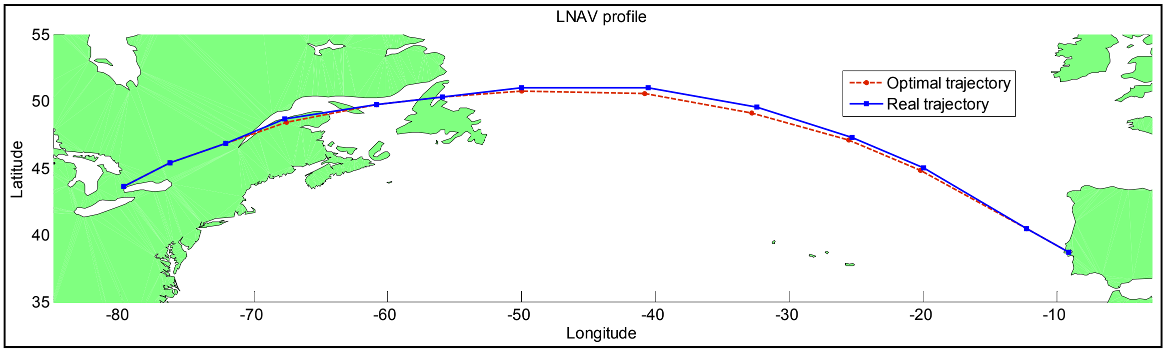

Figure 5.13 LNAV flight from Lisbon to Toronto

It is important to mention that the optimization algorithm does not take into account any restrictions imposed by air traffic management, and it only proposes a trajectory in terms of altitude and coordinates and the flight speed at which the global flight cost is the lowest.

Another example is presented, in which a higher flight cost reduction was obtained. The algorithm was compared with a real trajectory for a flight from London to Toronto, on October 4th 2013, departing at 9:19 UTC. For this flight, the real flight aircraft remained at the same altitude through the entire (cruise) flight (30,000ft), increasing the flight cost significantly. In this case, most of the flight cost reduction was obtained by improving the vertical flight profile, as indicated in Figure 5.14. The reason for the aircraft to remain at a constant altitude could have been an ATS constraint, therefore is important to remember that the optimization algorithm does not include ATC constraints for its calculations. The optimal trajectory reduces the flight cost by 15.25%, presenting a 3.11% flight time penalty for maximal fuel consumption reduction (CI set at zero). The fuel burnt and the flight times are presented in Table 5.4.

Table 5.4 Fuel burnt and flight time for a London to Toronto flight

	Fuel burnt (kg)	**Flight time (hr)**
Real trajectory	31,239.1	6.71
Optimal trajectory	26,475.9	6.82
Optimization	**15.25%**	**-3.11%**

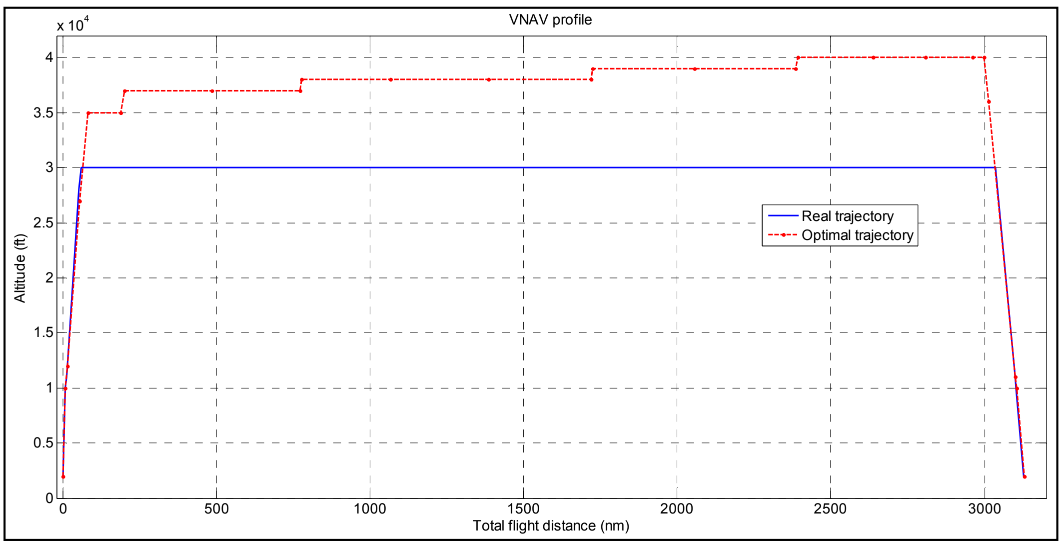

Figure 5.14 VNAV flight from London to Toronto

Table 5.5 shows the results for the optimization of 10 different flights with commercial aircraft.

Table 5.5 Optimization results from the proposed algorithm

Origin - Destination	Flight date	Fuel burnt real flight (kg)	Flight time real flight (hr)	Fuel burnt optimal flight (kg)	Flight time optimal flight (hr)	Fuel burn optimization	Flight time optimization
London - Toronto	23/09/13	28,179	7.14	26,485	7.04	6.01%	1.38%
Ponta Delgada - Boston	01/10/13	22,760	5.85	21,920	5.72	3.69%	2.19%
Paris – Toronto	04/10/13	28,464	7.30	26,703	7.17	6.19%	1.74%
Glasgow - Toronto	04/10/13	26,231	6.47	24,138	6.40	7.98%	1.15%
Lisbon - Toronto	04/10/13	27,710	6.95	25,808	6.87	6.86%	1.12%
London - Toronto	04/10/13	31,239	6.71	26,476	6.82	15.25%	-1.70%
Paris - Montreal	11/11/13	25,283	6.86	24,543	6.41	2.93%	6.51%
Paris - Montreal	11/11/13	25,664	7.02	25,085	6.62	2.26%	5.72%
London - Toronto	11/11/13	26,895	7.41	26,373	6.96	1.94%	6.14%
Lisbon - Cancun	11/11/13	38,054	9.28	35,747	9.15	6.06%	1.44%
Average						**5.92%**	**2.57%**

In all ten of the optimizing tests presented above, the CI was set to zero to obtain the maximal fuel consumption optimization. This setting usually results in a penalty in flight time, but as can be seen in Table 5.4, here it only once resulted in a flight time penalty. For the ten flights evaluated, the optimization resulted in an average fuel burn reduction of 5.92% and a flight time reduction of 2.57%.

5.4 Conclusions

Trajectory planning is one of the essential elements of an efficient flight analysis. The work presented here shows a complete trajectory's optimization, from the climb to the descent, in the presence of winds. During the cruise, both the LNAV and the VNAV profiles are analyzed to obtain the maximal optimization possible during this phase, which is the most fuel-consuming phase of a flight.

This algorithm analyzes real flight information, using real weather forecast data, and calculates an alternative trajectory to those used by commercial airlines. Different altitudes, speeds and alternative WPs are proposed by the algorithm to optimize the flight trajectory. The results from the tests performed have shown an average flight cost reduction of 5.92%, and an average flight time reduction of 2.57%. These results do not consider the restrictions imposed by air traffic management. This algorithm does, however, allow the possibility of imposing WP, speed and altitude restrictions.

CHAPTER 6

ARTICLE 4: FLIGHT TRAJECTORY OPTIMIZATION THROUGH GENETIC ALGORITHMS COUPLING VERTICAL AND LATERAL PROFILES

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Résumé

Pour les vols longs, la croisière est la phase la plus longue où la plus grande proportion de carburant est utilisée. Une nouvelle méthode de calcul des trajectoires de vol utilisant des algorithmes génétiques a été proposée dans cet article. Les profils latéral et vertical de navigation ont été analysés pour obtenir la trajectoire de croisière optimale en termes de consommation de carburant. Avec une analyse complète des courants des vents, un maillage 3D a été créé pour toute la phase de croisière, considérant latitudes, longitudes et altitudes. Différentes trajectoires de vol ont été calculées avec des algorithmes génétiques. Afin d’améliorer la précision et le temps de calcul, ces trajectoires de vol ont été calculées en utilisant une base de données de performances d’avions commerciaux, actuellement utilisées dans des systèmes de gestion de vol. Des économies de coût de carburant allant jusqu’à 5.6% pourraient être obtenues.

Abstract

For long flights, the cruise is the longest phase in which the largest proportion of fuel is consumed. A new flight trajectory calculation method utilizing GAs is proposed here. The LNAV and VNAV profiles are analyzed to obtain the optimal cruise trajectory in terms of fuel consumption. Using a complete analysis of the wind currents, a 3D grid is created for all along the cruise phase, including latitudes, longitudes and altitudes. Different flight trajectories were calculated using GAs. To improve calculation time and precision, the flight trajectories were calculated using the PDBs for commercial aircraft, databases which are used in actual FMSs platforms. This optimization process indicated that fuel cost savings of up to 5.6% can be achieved.

6.1 Introduction

The global aviation industry produced 689 million tons of CO_2 in 2012, which represents around 2% of the total emissions produced worldwide (ATAG, 2013). CO_2 emissions contribute to global warming, one of the biggest environmental problems encountered today. Multiple solutions to reduce aircraft emissions have been put forward. These can be divided in three major categories, aircraft technology improvement, the use of alternative fuels, and improvements in air traffic management and airline operation (Pan, Huang and Wang, 2014). Each of these categories could increase aircraft efficiency and thereby reduce fuel burn and emissions.

One of the research areas in the aircraft technology improvement category is focused on increasing engine efficiency through lighter designs (Williams and Starke, 2003), increased compression rates (Salvat, Batailly and Legrand, 2013) or optimized aerodynamic patterns (Panovsky J, 2000), to name a few. Airlines have been constantly reducing aircraft weight by changing to lighter seats (AirTransat, 2014). Techniques to install more efficient electrical wiring have also been studied (Wattar et al., 2013). Design studies to reduce drag through wing elasticity improvements (Nguyen et al., 2013) or by increasing the aircraft efficiency through the addition of winglets (Freitag and Schulze, 2009) have also contributed to this area.

To reduce its impact on climate change, the aviation industry has been studying sustainable biofuels to provide a cleaner source of fuel (Sandquist and Guell, 2012). Today, the aviation sector uses petroleum-derived liquid fuels, which is not only a limited fuel resource, it also contributes to CO_2 emissions. Hendricks, Bushnell and Shouse (2011) performed a study on biofuels in which they conclude that there is not only a large productive capacity for biofuels, but also the potential for carbon emission neutrality and reasonable costs. Airline companies, such as Porter, already use a 50:50 biofuel/Jet A1 fuel blend to perform a complete flight, which shows that biofuels are an important option for a greener aviation sector (Porter-Airlines, 2012).

Air traffic management and airline operation improvement would also reduce aviation's environment footprint. ATC is in charge of assigning the trajectories to airlines; once in-flight, authorization from ATC is required to perform a trajectory deviation. The FMS is an in-flight device, and can be used to identify optimal trajectories to propose to ATC.

Increased air traffic has opened a research domain in conflict detection algorithms to increase air security (Gariel, Kunzi and Hansman, 2011; Kuenz, Mollwitz and Korn, 2007; Visintini et al., 2006).

Many studies have focused specifically on the descent phase, where the goal is to reduce pollution close to air terminals in terms of both noise and emissions. Clarke et al. (2004) studied the CDA to reduce noise, which consists of the deceleration and descent of an aircraft at its own vertical profile from the TOD. They subsequently presented the design and implementation of an optimized profile descent in high-traffic airports, such as at the Los Angeles International Airport (LAX), which increased operational efficiency from traffic management and reduced fuel, emissions and noise (Clarke et al., 2013).

For long flights, however, the cruise is the phase where the most significant reduction can be obtained. In fact, 80% of the CO_2 emissions produced by aviation come from long flights (more than 1,500 km or 810 nm), and the cruise is where most of the fuel is consumed (ATAG, 2014). To improve the VNAV profile, Lidén (1992) studied the variation of the optimal altitude as fuel is burned during the flight. Lovegren (2011) analyzed how the fuel burn could be reduced during the cruise phase by choosing the appropriate cruise altitudes and speeds and performing SCs. Jensen et al. (2013) presented a speed optimization method for cruises with fixed lateral movement by analyzing radar information from the United States FAA’s ETMS (Palacios and Hansman, 2013). Their results show that most flights in the United States do not fly at an optimal speed, which increases their fuel consumption. Dancila, Botez and Labour (2012; 2013)studied a new method to estimate the fuel burn from aircraft to improve the precision in flight trajectory calculations.

The influence of weather on aircraft flight has been considered as part of strategies to take advantage of winds to reduce flight time and/or to avoid headwinds that could increase global flight costs. Murrieta (2013) presented an algorithm which optimized the vertical and horizontal trajectories, taking into account the wind forces and patterns as well as the variation of the CI. Filippone (2010)analyzed the influence of the cruise altitude on the creation of contrails and its influence on the flight cost. Gagné et al. (2013) performed an exhaustive research of all possible speeds and altitudes to obtain the optimal trajectory and reduce fuel burn. Bonami et al. (2013) studied a trajectory optimization method capable of guiding aircraft through different WPs considering the wind factors and reducing fuel burn, utilizing a multiphase mixed-integer control. Franco and Rivas (2011) analyzed the minimal fuel consumption for a cruise at a fixed altitude, using a variable arrival-error cost that penalizes both late and early arrivals. They showed that the minimal cost is obtained when the arrival-error cost is null, and found that different optimal cruise altitudes could achieve the goal of minimal cost/lowest fuel consumption with a fixed estimated arrival time.

However, in order to achieve maximal optimization from all these proposed techniques, an improved method of communication between the FMS and the ATC must be established.

Mayer (2006) studied the benefits of an integrated aviation modeling and evaluation platform, in which ATC and the FMS could be coupled to obtain better flight path planning.

For both the NGATS in the USA, and the SESAR in Europe, the implementation of RTA as a part of the FMS and ATC was an important step towards better air traffic control. De Smedt and Berz (2007) studied the characteristics of different FMSs' performance to determinate the accuracy of their RTA and the influence it could have on ATC. Friberg's (2007) study showed that promising results in terms of the environment could be achieved by establishing communication between the FMS' RTA function and ATC. Fays and Botez (2013) developed a 4D algorithm treating meteorological conditions or air traffic restrictions in a specified air space, defining them as obstacles, to improve the FMS's trajectory-creation capabilities. Air traffic conditions have also been identified as the cause of missed approaches (Murrieta Mendoza, Botez and Ford, 2014). Dancila, Botez and Ford (2013) created an analysis tool to estimate the fuel cost and the emissions produced by aircraft during a missed approach.

Since the goal is to implement these trajectories' optimization algorithms into the FMS, calculation time constraints have to be considered. GAs have been used widely in aviation research to reduce calculation time. Turgut and Rosen (2012) used GAs to obtain the optimal descent in terms of the fuel flow values and altitudes to reduce the global descent cost. These algorithms are useful when searching for a solution involving multiple imposed restrictions. Kouba (2010) studied GAs as a means to incorporate several constraints into a trajectory optimization problem, where the objective was to find the shortest route while considering different restrictions.

More recently, various LNAV and VNAV profile optimization algorithms were developed at the Research Laboratory in Active Controls, Avionics and Aeroservoelasticity (LARCASE) (Félix Patrón, Botez and Labour, 2012; 2013; 2013).

The algorithm proposed in this article presents a combination of LNAV and VNAV optimization during the cruise phase. Once an aircraft is in cruise, with a predefined Mach speed and altitude, this algorithm creates alternative trajectories and analyzes the possibility of making a deviation according to its potential to reduce fuel burn. In addition, at each WP of these alternative trajectories, it analyses the possibility of performing a SC to improve fuel consumption. A grid is compiled, which contains all the possible alternative latitudes, longitudes and altitudes, including dynamic weather information such as the air temperature, wind speed and wind direction. A complete analysis of the weather was obtained from Environment Canada (2013). Since there are a great number of possible trajectories, especially as the number of WPs of a flight increases, a GA has been implemented to reduce the calculation time. The flight cost of each trajectory is calculated using the flight PDB for commercial aircraft. This PDB was created from real flight aircraft performance information, and improves the precision of the aircraft model, compared to conventional methods that analyze an aircraft's equations of motion.

6.2 Methodology

To reduce the flight cost, the proposed trajectories' optimization algorithm analyzes alternative trajectories which consider the influence of the wind, the outside air temperature and the variation of the optimal altitudes as the fuel is burnt during flight.

Typically, flight trajectories are planned before each flight by large ground-based computers, which consider the restrictions imposed by ATC. These trajectories incorporate the current traffic, weather conditions, the aircraft's weight and the airline's operation costs. However, due to changing weather, current traffic conditions and the variation of the aircraft's weight during the flight, these trajectories may not be optimal in terms of flight costs.

To reduce flight costs, a 3D grid is created around the original flight trajectory, planned before the flight, which allows the analysis of possible SCs to reach the optimal altitude, as well as of horizontal deviations to profit from the tailwinds or avoid the headwinds. Since this algorithm is conceived in order to be implemented in a FMS, calculation time is an important factor. To reduce the number of possible trajectories and thus the calculation time, the aircraft's speed

will remain constant during the entire cruise, and a genetic optimization algorithm is applied to calculate the optimal trajectory without calculating all the possibilities within the grid.
The trajectories' optimization algorithm analyzes the cruise once the aircraft is situated at the TOC, and the flight parameters are known, such as the aircraft's weight, speed, the initial cruise altitude and the flight time.

The methodology is structured as follows: First, the model of the aircraft and the calculation of each trajectory are defined. Next, a dynamic weather model is described, and the grid where the possible alternative trajectories are analyzed is explained after. Finally, the GA applied to reduce the calculation time is defined.

6.2.1 Aircraft model – performance database

The aircraft's model was obtained from a PDB for a commercial aircraft. This PDB includes precise information on the main phases of the flight: climb, cruise and descent. The PDB, as it considers real flight performance information obtained from actual tests, increases the calculation precision compared to conventional methods that apply an aircraft's equations of motions.

The trajectories' optimization algorithm presented in this article analyzes the cruise phase for long flights, with the possibility of performing SCs to reduce flight cost. The structure of the cruise and climb tables provided in the PDB is defined in this section. Table 6.1 presents the inputs and outputs of these tables.

The cruise trajectory is divided into segments called WPs. At each WP, the algorithm analyzes the possibility of performing a SC or a flight to an adjacent horizontal WP. The flight cost for each possible segment is calculated.

Table 6.1 Inputs and outputs for a commercial aircraft's PDB

Type of table	Inputs	Outputs
Climb	Speed (Mach number) Gross weight (kg) ISA deviation (°C) Altitude (ft)	Fuel burn (kg) Horizontal distance (nm)
Cruise	Speed (Mach number) Gross weight (kg) ISA deviation (°C) Altitude (ft)	Fuel flow (kg/nm)

To calculate the cost of each possible climb, the fuel burn and the horizontal values have to be obtained from the PDB. Figure 6.1 represents the interpolations required to obtain these values for the climb phase.

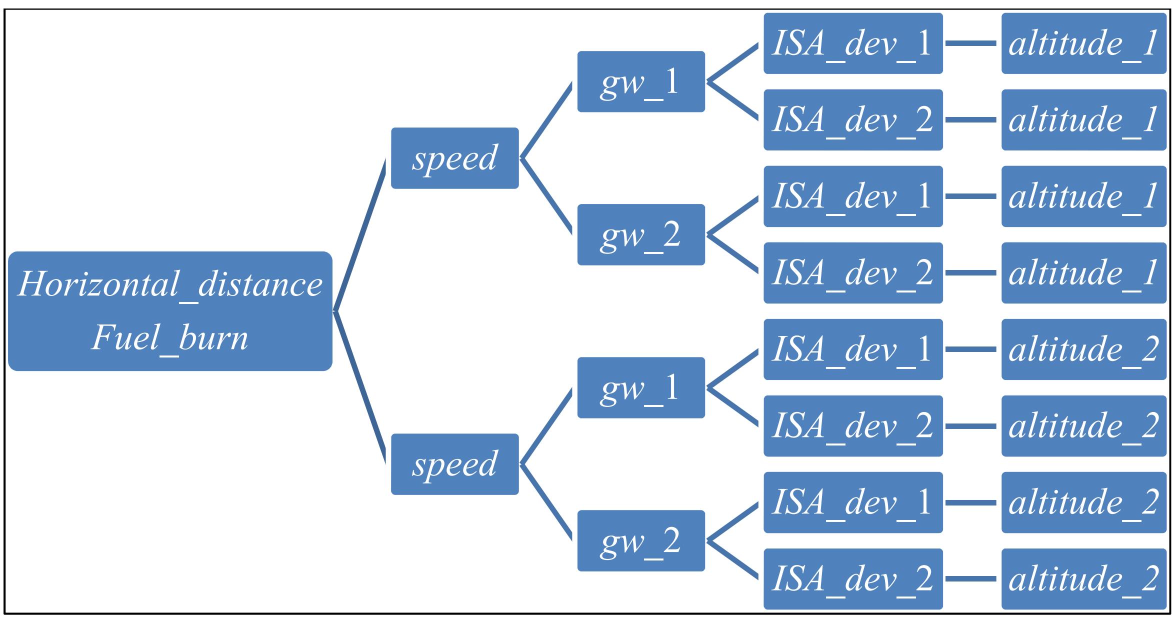

Figure 6.1 Interpolations to obtain the aircraft's flight performance during climb from the PDB

The speed remains constant and it is not interpolated. This means that only one speed is calculated at a time, and only the speed values found in the PDB can be analyzed. The Mach number ranges from 0.6 to 0.84. Variables *gw_1* and *gw_2* define the interval boundaries of the aircraft's gross weight in kg, and *ISA_dev_1* and *ISA_dev_2* are the interval boundaries on the ISA deviation input values in °C.

The altitude is not interpolated, and only the values found in the PDB are analyzed. The cruise altitudes in the PDB are defined in steps of 1,000ft, and it varies from 20,000 to 40,000ft. The PDB sums the cost of each climb for each 1,000ft; thus, in order to calculate the cost in terms of *Horizontal_distance* and *Fuel_burn*, the interpolation results for *altitude_1*, which represent the current aircraft altitude, have to be subtracted from the results obtained for *altitude_2*, which refer to the desired climb altitude.

Possible SCs of 2,000ft are analyzed. Even if real aircraft do sometimes perform 1,000ft SCs, by convention, 2,000ft SCs should be performed to avoid aircraft flying in the opposite direction (eastbound and westbound flights) and to respect the flight levels predefined by ATC (Ojha, 1995).

The Lagrange linear interpolation function is applied to perform the interpolations:

$$x = \frac{y - y_1}{y_0 - y_1} * f_0 + \frac{y - y_0}{y_1 - y_0} * f_1 \tag{6.1}$$

The interpolations required to obtain the fuel flow during cruise are presented in Figure 6.2.

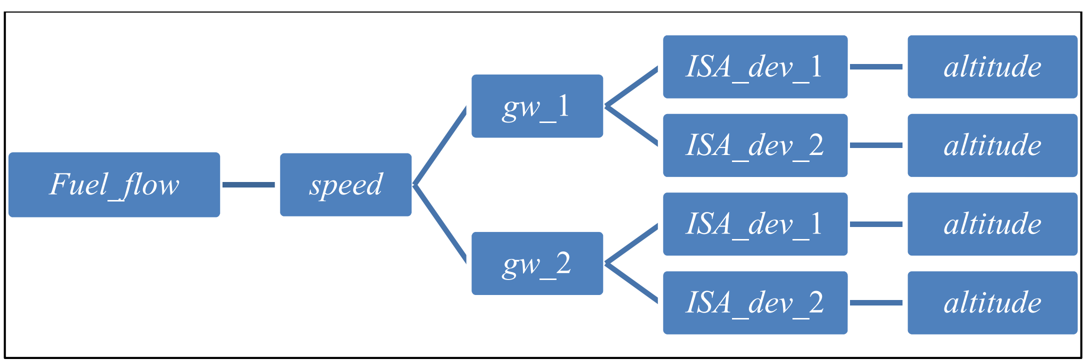

Figure 6.2 Interpolations to obtain the aircraft's flight performance during cruise from the PDB

The fuel flow is updated to calculate the fuel burn at each segment. The distance for each segment is calculated using the coordinates of the grid and Vincenty's method (Vincenty, 1975). In the PDB, the fuel flow is given in terms of kg of fuel burned per nautical mile. In order to calculate the fuel burnt during the cruise, it suffices to multiply both values, as in Equation (6.2).

$$Fuel\_burn\_cruise = Fuel\_flow * Horizontal\_distance \tag{6.2}$$

Where the *Fuel_burn_cruise* is given in <kg>, the *Fuel_flow* in <kg/nm> and the *Horizontal_distance* in <nm>.

6.2.2 Dynamic wind model

The wind data used in this algorithm is extracted from Environment Canada. The information is presented under a GDPS format. The GDPS model provides a 601×301 latitude-longitude grid with a resolution of 0.6×0.6 degrees. At each point of this grid, information such as the wind direction, speed, temperature, and the pressure can be obtained for different altitudes, in 3-hour time blocks. This database is updated every 12 hours, and it is indicated in UTC.

The wind is defined as dynamic. As the aircraft advances in time, the weather is updated to match the current position of the aircraft.

Wind directly affects both the horizontal distance traveled with respect to ground level and the fuel consumption. The ground speed is calculated so that it can be considered in the horizontal distance calculation. The speeds in Equation (6.3) are expressed in knots<kt>.

$$\text{Ground speed} = \text{Airspeed} + \text{Effective wind speed} \tag{6.3}$$

The airspeed is an aircraft's speed relative to the air mass, and the wind is the horizontal movement of this air mass relative to the ground. Here, the effective wind is the wind's component of the aircraft's trajectory, and the crosswind is that component perpendicular to the effective wind speed given in Equation (6.4) (Du Puy de Goyne, 2009). These are illustrated in Figure 6.3

$$\text{Effective wind speed} = \text{Airspeed} - \text{Crosswind} \tag{6.4}$$

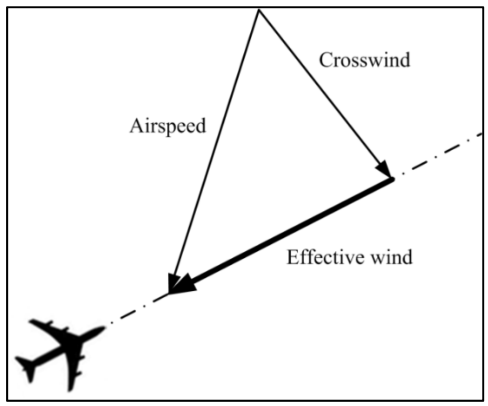

Figure 6.3 Airspeed, crosswind and effective wind

As the aircraft flies on a straight path, the wind affects the aircraft's speed. Depending on the direction and speed of the wind, the distance traveled by the aircraft will either be reduced or increased in a particular time segment. The horizontal distance traveled at the ground level is the norm of the ground speed vector. Figure 6.4 shows the influence of the wind of a mass moving from WPT(n) to WPT(n+1) (Langlet, 2011; Langlet et al., 2011).

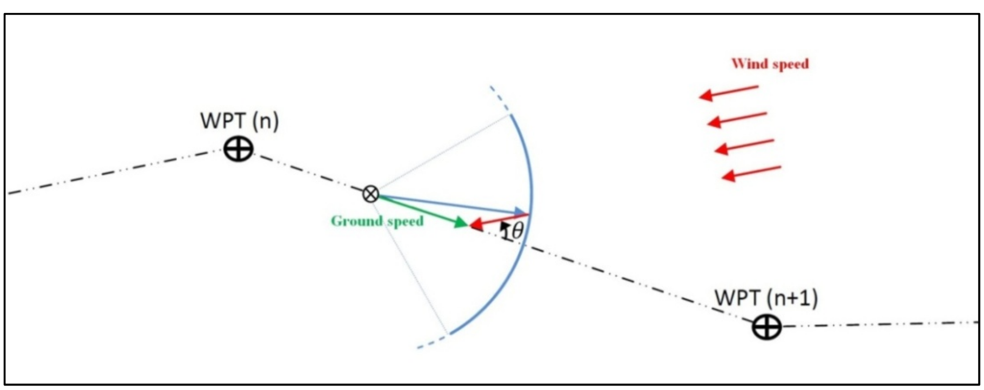

Figure 6.4 Wind factor calculation

The wind factor can be calculated in the following way:

$$Wind\_factor = \cos\left[\arcsin\left(\frac{\sin(\theta)*\|\overrightarrow{Wind_{speed}}\|}{\|\overrightarrow{Air_{speed}}\|}\right)\right] - \frac{\|\overrightarrow{Wind_{speed}}\|}{\|\overrightarrow{Air_{speed}}\|}*\cos(\theta) \qquad (6.5)$$

The wind data is interpolated in the optimization algorithm at each segment. For the vertical interpolation, the wind vectors are analyzed according to the Earth's Northern and Eastern axes (selected arbitrarily as a reference parameter) for two different altitudes. Afterwards, an interpolation is made between these two axes at the required altitude to obtain the wind vector (speed and direction). The horizontal interpolation is obtained between consecutive WPs. This process is sketched in Figure 6.5.

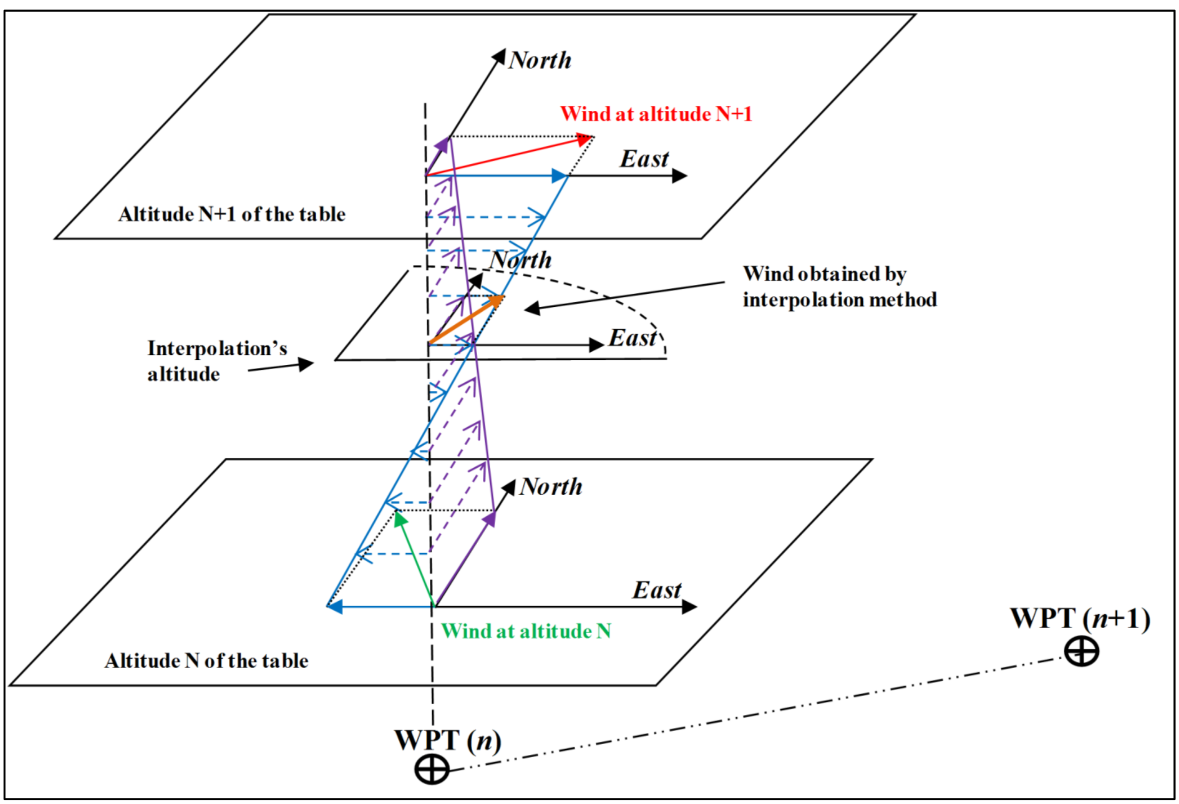

Figure 6.5 Wind interpolation method

6.2.3 The grid

As mentioned before, the flight trajectories are planned before the flight by ground-based computers, respecting the restrictions imposed by ATC. This proposed trajectories' optimization algorithm, however, analyzes alternative trajectories once the aircraft is already in the cruise phase, and finds those which, if approved by ATC, would reduce the global flight cost.

A 3D grid is created around the original trajectory planned pre-flight. Horizontally, two parallel trajectories are added to each side of the original trajectory. A total of five trajectories on the horizontal profile are analyzed. The distance between horizontal trajectories is variable, but predefined at 15nm for test purposes. The cruise, from the TOC to the estimated TOD, is divided into an n number of WPs. Vertically, the profile is divided in m sections at each 1,000ft from the initial cruise altitude up to the maximal cruise altitude (defined by the PDB for each aircraft). The size of the grid is 5 x n x m.

Within the grid, the aircraft can only fly to adjacent horizontal WPs, and can perform 2,000ft SCs.

Each WP in the grid is represented by a latitude, a longitude and an altitude. Each WP is identified as a specific point in the grid. Each trajectory consists of n WPs.
Trajectories are randomly created to be introduced in the GA explained in the next section.

6.2.4 The genetic algorithm

The objective of this trajectories' optimization algorithm is to be implemented on board an actual FMS. The FMS does not have the same processing capabilities as the ground-based computers that plan the trajectories before a flight. This means that the calculation time has to be reduced as much as possible.

Within the grid, the number of possible alternative trajectories increases exponentially as *n* increases. Calculating all the possible alternative trajectories is not only impractical, it is also very time-consuming. Therefore, a GA has been used to reduce calculation time.

GAs were selected because they have proved their ability to obtain optimal solutions where nonlinear data is analyzed in a short calculation time (Félix Patrón et al., 2013; Félix Patrón et al., 2013). These algorithms are based on Darwin's evolution theory, where the fittest individuals in a population survive to reproduce.

GAs mimic the natural evolution process. Starting with an initial population, a group of individuals selected by their fitness will reproduce, creating a second generation of individuals. Once again, the fittest individuals will be selected to create a third, and so on. This process is repeated for a predefined number of generations or until the optimal solution is repeated for a predefined number of times.

GAs comprise the following steps: the definition of the individuals and the creation of the initial population, the evaluation of individuals, the selection of the individuals most-fitted to create the next generation, the reproduction and the process termination conditions; each of which are explained in the following sections.

6.2.4.1 Individuals and initial population

Each individual is defined as a randomly-generated alternative trajectory. These trajectories consist of a set of WPs, defined by latitude, longitude and altitude. These alternative trajectories are created respecting the previously defined grid, and all latitudes, longitudes and altitudes must be within the grid. Table 6.2 describes the individuals created.

Table 6.2 Individuals parameters for the GA

	WP 1	WP 2	…	WP *n*
Individual 1	$Lat_{1,1}$ $Lon_{1,1}$ $Alt_{1,1}$	$Lat_{1,2}$ $Lon_{1,2}$ $Alt_{1,2}$	… … …	$Lat_{1,n}$ $Lon_{1,n}$ $Alt_{1,n}$
Individual 2	$Lat_{2,1}$ $Lon_{2,1}$ $Alt_{2,1}$	$Lat_{2,2}$ $Lon_{2,2}$ $Alt_{2,2}$	… … …	$Lat_{2,n}$ $Lon_{2,n}$ $Alt_{2,n}$
…	…	…	…	…
Individual *m*	$Lat_{m,1}$ $Lon_{m,1}$ $Alt_{m,1}$	$Lat_{m,2}$ $Lon_{m,2}$ $Alt_{m,2}$	… … …	$Lat_{m,n}$ $Lon_{m,n}$ $Alt_{m,n}$

Depending on the size of the grid, there could be thousands or millions of possible trajectories. The initial population should represent a small percentage of all the possible solutions. The size of the initial population is defined according to the size of the entire number of possible solutions.

6.2.4.2 Evaluation

The evaluation process consists of calculating the flight cost of each trajectory using the PDB.

After its evaluation, each individual is represented by the following:

- Coordinates (latitudes and longitudes);
- Altitudes at each WP (if SCs were performed);
- Aircraft speed (which remains constant throughout the entire cruise to save calculation time);
- Aircraft gross weight (updated dynamically as the aircraft advances);
- Partial fuel burnt and flight time (at each WP);

- Wind speeds, directions and outside air temperature (calculated dynamically as the aircraft advances); and
- Fuel burnt and flight time.

The fittest individual is defined as the flight trajectory that minimizes the global flight cost.

6.2.4.3 Selection

According to Darwin's theory of evolution, the best-fitted individuals are those that are more likely to survive and have more chances to reproduce and preserve their genetic heritage. This does not mean that less-fitted individuals do not have the right to be part of the upcoming generations; they bring diversity to the population. This aspect makes it possible to avoid local optimums more efficiently and reach the global optimum.

There are different methods for selecting the individuals to reproduce. Some examples are uniform selection, rank selection, proportional selection (or the roulette wheel) and selection by tournament.

In the uniform selection method, all the individuals are allowed to reproduce, independently of their cost. This method is usually inefficient in terms of calculation time, since the population is too diversified and the convergence to the optimal solution is slow.

The rank selection method has been applied to a VNAV optimization algorithm (Félix Patrón et al., 2013). This method sorts individuals according to their cost, and only the most-fitted individuals are selected to reproduce. This method benefits from a quick convergence towards a solution; however, depending on the complexity of the problem, it may lead to a quick convergence to a suboptimal solution.

A selection by roulette wheel was already implemented on an LNAV optimization algorithm (Félix Patrón et al., 2013), in which the non-linearity of the wind was added to the problem.

This method consists of assigning a piece of roulette to each individual depending on their cost. The more-fitted individuals are represented by a bigger piece, while the less-fitted individuals are represented by a proportionally smaller piece. The selection is performed randomly, as in a roulette wheel. The more-fitted individuals thus have more chances of being selected than the less-fitted individuals, who nonetheless still have a chance. This allows for a more diversified population than the rank selection method, but one that is less diversified than that created by uniform selection. This method is usually slower to converge than rank selection, but the roulette wheel is more efficient at avoiding local optima.

In the GA proposed in this paper, a selection by tournament was carried out. This selection method makes the individuals compete against each other and preserves the strongest one. Along with the roulette wheel selection, this method allows a diversified population. However, the less-fitted individuals have a lower likelihood of reproducing, allowing a quicker convergence towards the global optimal solution. After the first round of the tournament, only half of the individuals survive to reproduce and create the next generation.

6.2.4.4 Reproduction

After the tournament, the strongest individuals survived and half the population was eliminated. The surviving individuals reproduce to create a new set of individuals; filling the places made vacant after the first round. Since the strongest individuals are reproducing with each other, more-fitted individuals are expected at each round or generation.

Each trajectory, as mentioned before, is represented by a set of WPs defined by latitude, longitude and altitude. A crossover method was used to create a new individual. This method consists of taking one half of one individual, and combining it with a half from another individual. The order of the individuals to reproduce after the tournament is defined randomly.

After the new individuals are created, they will be evaluated to obtain their cost. A new generation is thus obtained, consisting of old and new individuals in a 50/50 proportion.
The new generation will be sorted, and the most-fitted individual will be defined as the optimal solution for that generation. In order to increase the diversity of the new generation, the poorest-fit individuals will be eliminated automatically and replaced by a set of new randomly-created individuals.

The process is repeated until a predefined number of generations are reached, or until the optimal solution repeats itself for a predefined number of generations.

6.3 Results

This section is divided into two parts. First, the performance of the GA is described in terms of calculation time and performance optimization. The second part covers the proposed algorithm’s ability to reduce flight costs.

6.3.1 The genetic algorithm

A GA has been implemented in order to reduce the calculation time. A reduced percentage of the total number of possibilities is calculated. In order to define this small percentage, first it is necessary to know the number of total possibilities.

The size of the grid is variable, since the number of WPs can be modified to better adapt it to the original trajectory, and the number of possible SCs is defined by the initial cruise altitude and the maximal climb altitude. The diagram presented in Figure 6.6 represents how the total number of trajectories can be calculated in a 2D grid. The total number of possible trajectories is defined by a sum of the trajectories arriving at each WP from the precedent WPs. As mentioned in the previous section, the aircraft can only fly to adjacent WPs.

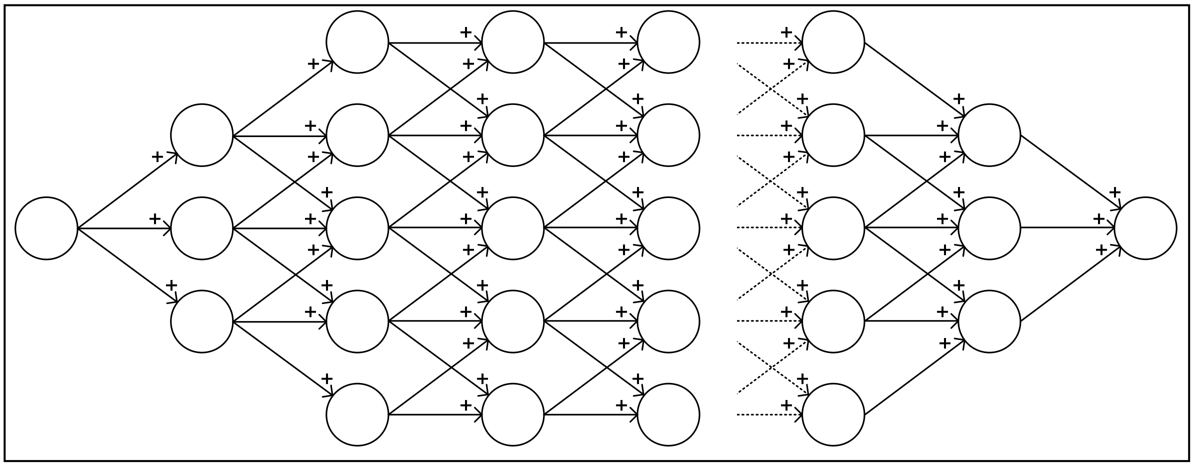

Figure 6.6 Dynamic diagram to calculate the total number of possibilities in a 2D grid

An example of the calculation in 2D (with no SCs) for six WPs is presented in Figure 6.7.

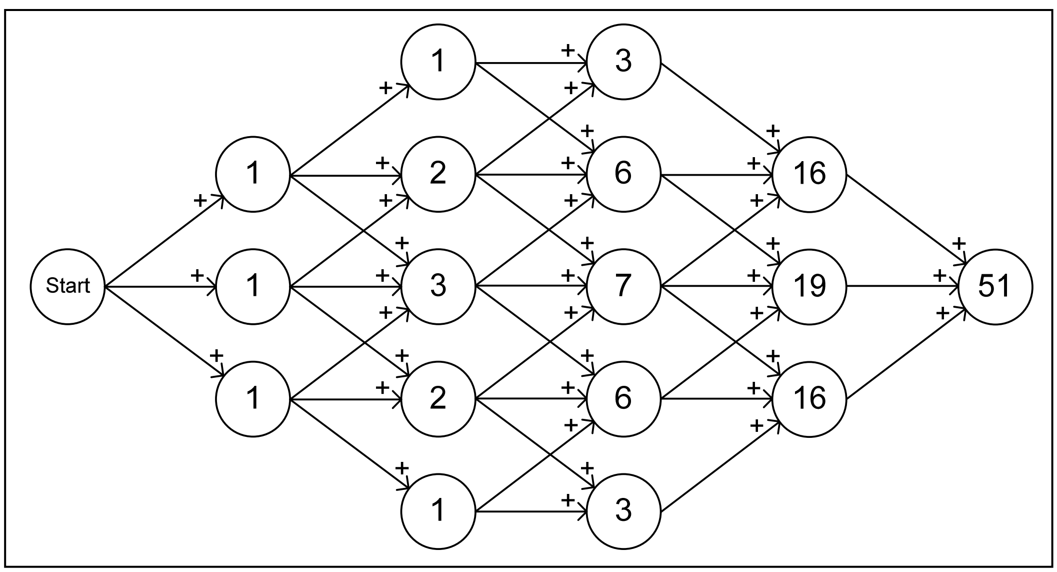

Figure 6.7 Dynamic diagram example to calculate the total number of possibilities in a 2D grid for 6 WPs

The diagram presented in Figure 6.7 can calculate the total number of possibilities if no SCs are performed. In order to include SCs in the calculation of total possible trajectories, at each WP, the total sum of trajectories has to be multiplied by two, in order to include the possibility of the aircraft remaining at the same altitude, or of performing a SC. Table 6.3 shows the total number of possible trajectories that can be produced by varying the number of WPs and the number of SCs during the entire cruise.

Table 6.3 makes it clear that as the number of WPs increases, the number of possible trajectories grows exponentially. Adding a SC to each trajectory doubles the number of possibilities.

Table 6.3 Number of possible trajectories within the 3D grid varying the number of SCs and WPs

WP \ SC	0	1	2	3	4	Unlimited
6	51	102	204	408	816	1,632
7	139	278	556	1,112	2,224	8,896
8	379	758	1,516	3,032	6,064	48,512
9	1,035	2,070	4,140	8,280	16,560	264,960
10	2,827	5,654	11,308	22,616	45,232	1,447,424
11	7,723	15,446	30,892	61,784	123,568	7,908,352
12	21,099	42,198	84,396	168,792	337,584	43,210,752

The results for trajectories divided into nine WPs and allowing four SCs during the cruise are presented next.

To evaluate the performance of the GA, a single flight was tested and repeated 100 times, thereby revealing the percentage of times the optimal trajectory was achieved.

The real flight information was taken from FlightAware (2013). This website provides real flight information in the form of a database that contains flight coordinates, altitudes, time and speeds of aircraft for each flight. These parameters are used as a reference, and the flight trajectories optimization algorithm creates the 3D grid around the original flight trajectory proposed by FlightAware. The costs of each flight, both the real and the optimal one, are calculated using the PDB for a commercial aircraft.

The FlightAware website, however, does not provide information about an aircraft's weight. In order to perform the flight test, the aircraft's standard weight is obtained from the PDB, and the amount of fuel used is the minimal amount required to complete the flight.

The selected trajectory is a flight from Lisbon to Toronto, which departed at 16:30 UTC, on November 15th, 2013. The inputs are the following:

- Aircraft type: Medium to long range commercial aircraft
- Aircraft weight: 100 ton
- Aircraft fuel: 28 ton
- Maximal altitude: 40,000 ft
- Altitude at TOC from the real flight: 31,000
- Distance between alternative horizontal trajectories: 15nm
- Aircraft speed: 0.70 Mach
- For the GA
- Individuals per generation: 50 individuals
- Number of generations: 300
- Maximal number of repetitions for the optimal solution: 50

The real flight is compared with the optimal flight in terms of latitudes, longitudes and altitudes in Figure 6.8. A flight cost reduction of 5.71% of was obtained. The optimization algorithm calculates the optimal trajectory at a constant speed in order to reduce calculation time.

Table 6.4 represents the performance of the GA. The optimal solution, which reduces fuel consumption by 5.71%, was obtained 45% of the time, while calculating only an average of 23.2% of the total possible trajectories. However, the suboptimal solutions still reduce the fuel consumption by an average of 5.60%. If the number of individuals per generation or the number of generations was increased, the optimal solution would be found more often; however, that would substantially increase the number of calculations.

The purpose of this algorithm is to reduce the calculation time as much as possible so that the proposed algorithm can be implemented in a FMS device.

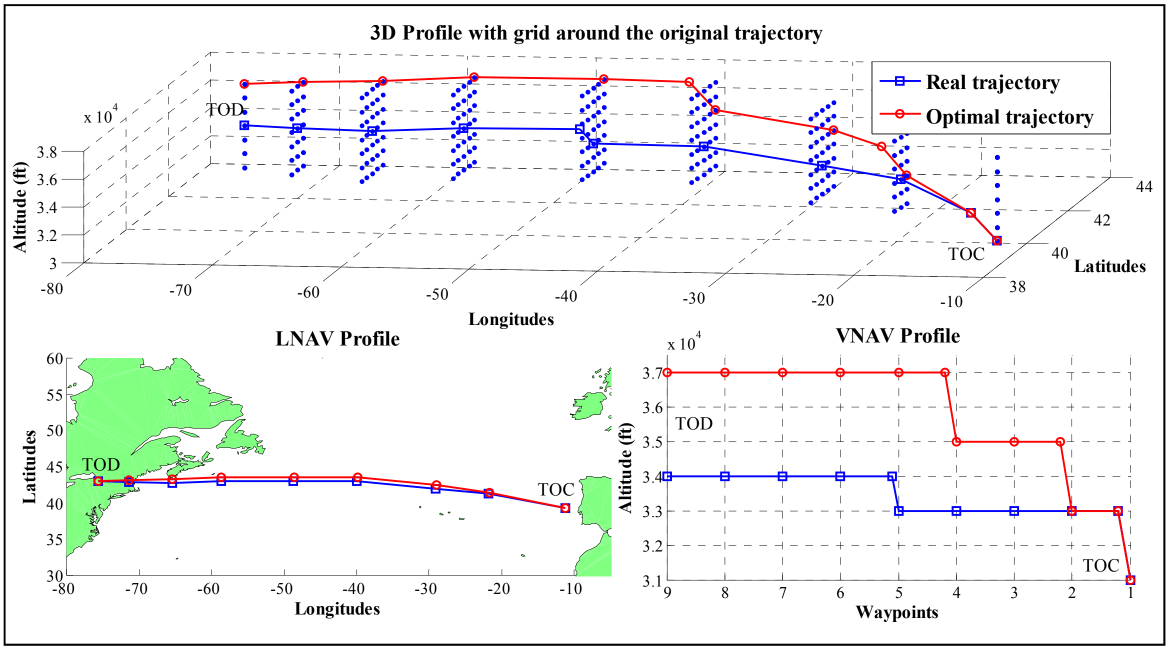

Figure 6.8 Real flight trajectory compared with its optimal flight trajectory

Table 6.4 GA optimization results

Original flight cost (kg)	Optimal trajectory cost (kg)	Average cost of suboptimal trajectories (kg)	Optimal trajectory found by the GA	Average number of trajectories calculated by the algorithm
26,462	24,950 (5.71% cost reduction)	24,979 (5.60% cost reduction)	45%	23.2%

6.3.2 Fuel cost reduction

A total of 20 real flights were compared by the trajectories optimization algorithm to reduce the global flight cost. The results are given in Table 6.5.

The inputs considered for the grid creation and the GA are the following:

- Number of WPs: 9; and
- Maximal number of SCs: 4.

Table 6.5 shows that the trajectories' optimization algorithm reduces the global flight cost by 5.60%. These reductions were obtained by a better selection of the flight altitudes and a better choice of the horizontal trajectory, which improves the fuel consumption by avoiding headwinds and/or taking advantage of tailwinds.

Table 6.5 Flight cost reduction results

Departure	**Arrival**	**Date (2013)**	**Optimal trajectory cost (kg)**	**Original trajectory cost (kg)**	**Cost reduction (%)**
London	Toronto	Sep-23	26341.9	27112.6	2.84
P. Delgada	Boston	Oct-01	17650.7	17818.2	0.93
Paris	Toronto	Oct-04	26349.2	28039.4	6.02
Glasgow	Toronto	Oct-04	23899.7	25916.8	7.78
Lisbon	Toronto	Oct-04	25044.5	26692.4	6.17
Paris	Montreal	Nov-11	24410.9	26107.3	6.49
Manchester	Toronto	Nov-11	24372.3	26657.2	8.57
London	Toronto	Nov-11	24726.6	26424.9	6.42
Lisbon	Cancun	Nov-11	36066.1	37974.5	5.02
Lisbon	Toronto	Nov-15	24950.7	26462.9	5.71
Average flight cost reduction			**5.60%**		

The proposed algorithm analyzed the entire flight by using the mode of the real speed vector, which represents the Mach number used for most of the flight. This speed remained constant for the entire analysis of the optimal flight trajectory in order to reduce calculation time. Varying the speed would result in a significant increase of the optimization algorithm's calculation time.

The influence of the wind was not considered in these tests, since that information is not provided by FlightAware. However, the flight cost as a function of the time could be obtained with this algorithm

As future work, the trajectories' optimization algorithm could consider varying the Mach number during a flight to increase the cost savings. Varying the speed would also allow the implementation of RTA restrictions at each WP, and thus, a 4D flight trajectory analysis.

6.4 Conclusions

The flight cost analysis was performed using a PDB for a commercial aircraft. This allows a better precision and lower calculation time than conventional methods using an aircraft's equations of motion. By applying a GA to reduce calculation time, the flight trajectories' optimization algorithm reduces the flight cost by 5.60%. This was accomplished by analyzing real flight trajectories from FlightAware, which are trajectories that real commercial airlines are flying today. It can be seen that these actual trajectories are not optimal. Airlines may be flying those trajectories because of ATC's restrictions, their own flight policies or due to poor analysis of the optimal performance of the aircraft and the winds. Although the proposed trajectories' optimization algorithm does not consider ATC restrictions, it could serve as an in-flight method of calculating an alternative trajectory which could then be requested of ATC to reduce flight cost, via the FMS device. This algorithm could also serve as a pre-flight analysis. However, pre-flight analyses are usually performed by powerful ground-based computers; in which case, the number of calculations could be increased, and so a more complete flight analysis performed. The main objective of the present research, however, is for its implementation in a small processing device such as the FMS, and therefore, parameters such as the aircraft speed are held constant to reduce calculation time. This algorithm analyzes the behavior of the winds, and uses them as a way to reduce fuel burn. At the same time, SCs are analyzed at each WP to improve aircraft performance as the fuel is consumed and the total aircraft weight reduced. The 5.60% reduction, even if ATC restrictions are not considered, could be an important way to reduce aircraft fuel consumption and pollutant emissions from aviation.

DISCUSSION OF RESULTS

The results obtained for the different phases of the algorithm have been presented in the previous chapters. However, this section presents a summary of these results.

In the first research paper, the results of the aircraft model used by the flight trajectory optimization algorithm presented in this thesis were compared with the FlightSIM® results, since this software considers a complete aircraft aerodynamic model for its simulations, giving accurate results and very close to reality. Then, the PTT results were compared with the results obtained with FlightSIM® in order to see if an improvement in the calculation precision was obtained. After the analysis of the differences between the results obtained with the algorithm and the PTT, it was found that the flight trajectory optimization algorithm improved calculation precision in both, the flight time and the fuel burn.

When the fuel burn calculated with the algorithm was compared with the fuel burn calculated with FlightSIM®, the optimization algorithm gave a difference of **1.75%**, against **2.55%** obtained by the PTT. When the flight time calculation was compared with FlightSIM®, the optimization algorithm had a difference of **0.49%**, against **1.03%** obtained by the PTT. In both cases, the new algorithm improved the precision on the fuel burn calculation.

The algorithm compared the flight cost optimization results with the results obtained by the PTT. When only speed was optimized, **0.15%** of flight cost reduction was obtained. But when both, altitude and speed were optimized, a **2.57%** flight cost reduction was obtained.

In the second research paper, the LNAV profile was optimized. Different results were obtained when the size of the grid was modified, but a maximal flight cost optimization of **0.54%** was found. These results, however, depend on the weather influence. The maximal optimization result of 2.54% was obtained for the tests performed on October 30th, 2012. In this article a GA was also studied, which was proved to be stable by obtaining the optimal trajectory over **90%** of the time.

In the third research paper, the optimized trajectory results where compared with real flight trajectories results. By a comparison of complete flight trajectories, including climb, cruise and descent, an average fuel burn reduction of **5.92%** was found, as well as a **2.57%** flight time improvement. These results show the actual fuel burn reduction capabilities of this project.

In the fourth and final research paper, a fuel burn reduction of **5.60%** was obtained, while the cruise phase was calculated by comparing the optimization algorithm results to real flight trajectories.

CONCLUSIONS AND RECOMMENDATIONS

Different versions of a flight trajectory optimization algorithm have been presented in this manuscript-based thesis. Several conclusions could be made addressing different issues, such as: the advantages of working with a numerical aircraft model, the precision of the weather, the VNAV and LNAV profiles design, the influence of ATC to the flight reduction results and the GAs performance.

One of the important factors about these algorithms is that they have been applied using a PDB as the model of the aircraft. Most of the research done in the flight trajectory optimization problem includes aircraft models represented by equations of motion. As these models can be generally adapted to all kinds of aircraft just by varying their parameters, the precision of the PDB gives the opportunity to create a more precise algorithm in terms of fuel burn calculation. The algorithms could be adapted to any type of aircraft using a PDB in a text format, since most of these PDBs include the same parameters. These PDBs are used in FMSs that are currently in service, allowing a faster implementation of these algorithms in the new technologies.

The weather database used to create the wind models includes very detailed information from around the world. This database has been modeled to include a dynamic weather model to calculate the flight cost. However, the size of the database increases significantly the calculation time, reason for which the optimization algorithms have been adapted to preload the weather database and to implement the data at each WP. This data could be easily replaced by the pilot, in-flight, with the information that they receive for the flight.

It was found in this thesis, that the VNAV profile has a more important relevance in terms of flight cost reduction. The flight altitudes and speeds represent the better way to reduce the fuel burn and thus, the emissions to the atmosphere. The LNAV, however, could sporadically represent an important optimization, when the winds around the original trajectory are non-optimal. The most significant reduction can be obtained during cruise for long flights, where as it can be seen on the results in Chapter 6, commercial airlines do not fly at the optimal cruise speeds, altitudes and directions.

The flight trajectory optimization algorithm could work as a pre-flight planner, even if the initial objective was to be implemented in-flight. However, the ATC has an immense influence on the selection of these trajectories. The results, showing a flight cost reduction over 5% when compared with real flights were calculated assuming that the aircraft could fly freely around the airspace. This algorithm proposes the optimal trajectory to the pilot through the FMS, but if the pilot does not obtain authorization from the ATC, no optimization could be made. The next generation of FMSs would reduce the global aviation fuel consumption if these functions are adapted to air traffic.

Different versions of time-optimization algorithms have been tested in this thesis. GAs were used in this project because of their nature, since they could be conveniently adapted to the problem. It was trivial to separate the trajectories using WPs, and GAs allowed the imposition of restrictions to the calculation of trajectories, which were used at each phase of the optimization process. The different versions of the GAs gave the optimal (or suboptimal) results while reducing calculation time, and they would be more convenient if the size of the problem would be increased. Even if currently, avionics' companies try to avoid the implementation or non-deterministic algorithms for certification purposes, in the short term, these type of calculation-time reduction methods would significantly improve the processing capabilities of current FMS technologies.

Here are a few recommendations to improve in the future the presented optimization algorithms:

A RTA method should be added. RTA is an important function on the FMS that facilitates the task of the ATC to analyze air traffic. With a precise RTA calculation, algorithms to improve the calculation of trajectories through traffic would be developed more efficiently.

ATC should be included as an additional parameter to calculate the optimal flight trajectory. Many researchers have been studying the presence of other aircraft as obstacles, creating no-fly zones to avoid. These zones could also be used to avoid military zones restrictions and weather phenomenon such as thunderstorms.

A function should be included in the algorithm to calculate the minimal fuel required to complete a flight, while considering unexpected circumstances such as missed approach situations.

Even if the GAs improved the calculation time, and were selected over other existing methods because of their practicality, new calculation time-optimization methods have been surging, such as ants or bees' colony optimization algorithms, the branch and bound method, etc. These methods could possibly improve the performance of the proposed trajectory optimization algorithm.

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