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Nila ABOLFATHI NOBARI

BAYESIAN UPDATING OF HYDROELECTRIC TURBINE FATIGUE RELIABILITY

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Mechanical Department at École de technologie supérieure

Mr. Martin Gagnon, ing. Ph.D., Thesis Co-supervisor
Institut de rehcerche d'Hydro-Québec (IREQ)

Mr. Jean-Pierre Kenné, ing. Ph.D., Chair, Board of Examiners
Mechanical Department at École de technologie supérieure

Mr. Michel Rioux, ing. Ph.D., Member of the jury
Automated Manufacturing Engineering Department at École de technologie supérieure

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BAYESIAN UPDATING OF HYDROELECTRIC TURBINE FATIGUE RELIABILITY

Nila.A.Nobari

ABSTRACT

In fatigue design, uncertainties that exist in material, environment, and loading could arise due to manufacturing processes and changing with environment condition. Therefore because of the lack of information and cost of inspection, updating the fatigue model variables to decrease the uncertainties is necessary. In this study, Paris model is used to model the crack growth rate for hydroelectric turbine runner. We applied the Bayesian method to construct the posterior distribution. After constructing the posterior distribution, we update it by Bayesian updating approach. This method is one of the useful methods to decrease the uncertainty of variables at each loading cycle to construct precise prior distribution. The results of updating applied to Kitagawa-Takahashi limit state diagram. After modeling the proper limit state, we apply First Order Reliability Method (FORM) and Monte-Carlo Simulation (MCS) method to calculate the reliability index. In This study all of the procedures that mentioned are described, also we could see the results of effects of prior knowledge and select the distribution to analysis of reliability index. This study follows the (Gagnon, Tahan et al. 2013) research with aim of updating the fatigue reliability amount on hydroelectric turbine runner by Bayesian method.

Key-word: Bayesian methods, reliability, fatigue, hydroelectric runners.

MISE À JOUR BAYÉSIENNE DU MODÈLE DE FIABILITÉ EN FATIGUE DES ROUES HYDROÉLECTRIQUES

Nila.A.NOBARI

RÉSUMÉ

Dans une démarche de conception pour la fatigue, les incertitudes qui existent dans le matériel, l'environnement et le chargement pourraient survenir lors du processus de fabrication et de l'exploitation ce qui a pour effet une incertitude sur la vie résiduelle en fatigue. Par conséquent, en raison du manque d'informations et de coût de l'inspection, la mise à jour des variables d'un modèle de la fatigue est justifiée et nécessaire pour diminuer les incertitudes. Dans ce projet, le modèle de Paris est utilisé pour modéliser le taux de croissance de la fissure pour la roue d'une turbine hydroélectrique. Nous avons appliqué la méthode bayésienne pour construire la distribution postérieure. Après la construction de la distribution postérieure, nous mettons à jour le modèle. Cette méthode est utile pour diminuer l'influence de l'incertitude des variables à chaque cycle de chargement, ce qui permet de construire une distribution plus précise pour modéliser le comportement aléatoire des variables entrants dans le modèle de fatigue. Les résultats de la mise à jour sont appliqués à modèle d'état limite basé sur le diagramme de Kitagawa-Takahashi. Après modélisation de l'état limite approprié, nous appliquons les méthodes FORM (*First Order Reliability Method*) et Monte-Carlo pour calculer l'indice de fiabilité. Dans cette étude, toutes les procédures mentionnées sont décrites, aussi nous avons pu voir les résultats sur les effets des connaissances préalables sur l'indice de fiabilité. Cette étude suit la recherche démarrée par Gagnon *et al.* (2013) avec pour but d'actualiser l'estimation de la fiabilité par la méthode bayésienne.

Mots-clés : Méthodes bayésiennes, fiabilité, fatigue, roues hydroélectriques.

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LIST OF ABREVIATIONS

MCS	Mont Carlo simulation
FORM	First order reliability method
SORM	Second order reliability method
LEFM	Linear elastic fracture mechanic
PDF	Probability Density Function
EVT	Extreme Value Theory
HCF	High Cycle Fatigue

LIST OF SYMBOLS AND UNITS OF MEASUREMENTS

$\frac{da}{dN}$	Crack growth rate by passing cycles
ΔK	Stress intensity factor
K_{Ic}	Stress intensity factor of the LEFM critical in mode I
K_{th}	Stress intensity factor of the LEFM threshold
ΔK_{onset}	Stress intensity factor of the HCF onset
a	Crack length
a_0	Crack length at which the fatigue limit and the LEFM threshold cross
N_f	The final cycle that failure happen
γ	Stress intensity correction factor for crack geometry parameter
$\Delta\sigma$	Stress cycle range
$\Delta\sigma_{th}$	Stress range of the threshold
R	Stress ratio
m, C	Material parameter
$p(\theta\|X)$	Posterior distribution function
$p(X\|\theta)$	Likelihood distribution function
$p(X)$	Normalized distribution function
$\Phi(Z)$	Standard normal cumulative distribution function
$F_{\Delta\sigma}$	Cumulative distribution function of stress cycle range
$f(X)$	Joint probability density function
$g(X)$	Limit state
β	Reliability index
β_t	Target reliability index
$\{Z_1^*, Z_2^*, ..., Z_n^*\}$	Design point at standard space
X	Variable

INTRODUCTION

The industrial problem

In 2014, the production division of Hydro-Québec owned a fleet of 60 hydropower plants, including 347 generating units. This represented a net asset value of 26.6 billion dollars in December 2013 and an annual investment for maintenance and care operations around $400 million dollars (average from 2009-2014). With an approximate value of 7.4 billion dollars, the generator-turbine units represent 28% of these assets. The hydroelectric turbine's modes of operation, age, start and stop and numbering the maintenance have a profound effect on a turbine's lifespan. In this context, the maintenance of hydroelectric facilities is a significant challenge to producers because they need to produce more electricity without decrease in availability and productivity. The consequence of over-used facilities will increase the risk of failure.

One of the factors that limit the life of hydroelectric turbines is material fatigue which causes cracking that decreases system reliability. Some methods based on visual inspection or Non-Destructive Testing (NDT) techniques such as ultrasound can detect and monitor cracks, but these are not more useful because of their high costs (Haapalainen and Leskela 2012), (Goranson 1997).

With cyclic load, material fatigue is characterized by the presence of defects and their propagation to form with each passing cycle crack in the structure. Therefore, an accurate prediction of fatigue life is an important part of the maintenance scheduling. An efficient maintenance policy should also include an inspection schedule, planning repair and a replacement policy. Therefore, several studies are interested in understanding the process of crack growth rate (Kumar and Prashant 2009), (Acar, Solanki et al. 2010), (Thibault, Bocher et al. 2011). Their studies demonstrate that the frequency and quality of inspection, material properties, blade shape and loading strongly impact the cracking process and reliability. Consequently, several models and approaches have been proposed to estimate crack growth rates and fatigue reliability (Castillo, Fernández-Canteli et al. 2008), (Gagnon, Tahan et al. 2013), (Kumar and Prashant 2009).

Generally, deterministic models are used to estimate fatigue reliability. However, when using these types of models uncertainty cannot be considered. According to the case study related to turbine blades, uncertainty is the main subject that is considered in the reliability analyses of this study. Many sources of uncertainties which exist in the parameters can affect the fatigue process and their influence on the propagation of blade cracks in the turbine (Huth 2005) , (Pattabhiraman, Levesque et al. 2010). Thus without considering model uncertainty, errors in estimating crack size, crack growth rate and fatigue reliability are generated. Therefore we need to study models using a probabilistic approach. In this way uncertainties can be characterized and/or estimated. With additional information and data from observations, the uncertainty range can be reduced. This work showcases new sources of information which can be exploited to update model variables used a prior to update uncertainties behaviors.

To illustrate the problem, the case study on the blades of Francis runners from the hydroelectric plants of Hydro-Quebec are used to estimate fatigue reliability. Hydro- Québec and Andritz Hydro (a hydroelectric turbine manufacturer) have collaborated on the issues of identifying variables, parameters and models to account for uncertainties related to fatigue and blade cracking.

More recently (Gagnon, Tahan et al. 2013) proposed a probabilistic fatigue model for life prediction. But in this study, we considered a few settings as appropriate and not all factors affecting the cracking process. In their model, reliability is distinct when the crack does not pass a threshold above which a high cycle fatigue contributes rapidly to crack propagation. If the crack length exceeds a given critical length, the structure needs to be repaired. This defect propagation to form cracks occurs even for stress levels much lower than typical allowable design stresses.

In this study, we want to study cracks that tend to initiate and propagate near the welded joint between the blade and the rest of the structure as shown Figure 0.1. Typically, the critical zone is near the outflow edge of the blade. In this particular case, the critical zone studied is inside a stress relief cut-out region.

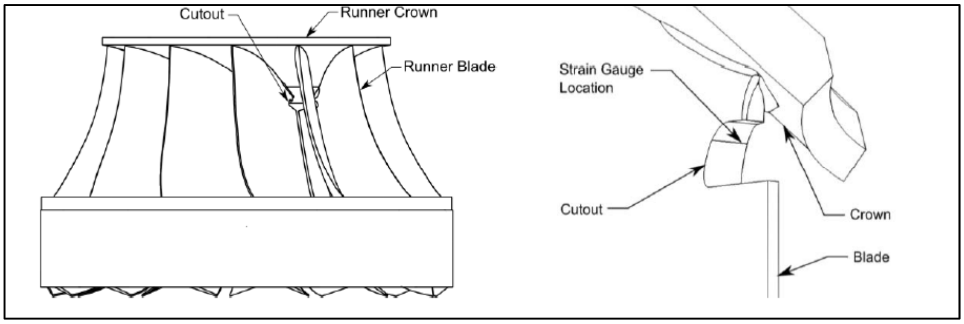


Figure 0.1: Schematic of Francis runner diagram (Gagnon, Tahan et al. 2013)

Objectives

This research aims to update fatigue model behaviour of variables and parameters that predict the results of a reliability index with a Bayesian inference method. The accuracy of a reliability index depends on data quality that is gathered through inspection, expert opinion and laboratory testing. Most of the data from the history and expert opinion contains a lot of uncertainties. These uncertainties affect the reliability analysis. We want the Bayesian method to decrease the uncertainties that exist in parameters and variables by using new information from field (e.g. inspection, measurements,..). Using a Bayesian method means that uncertainties are updated when new information becomes available (Box and Tiao 2011). By updating the probabilistic model parameters and variables in prior distributions, uncertainties related to fatigue life can decrease and the predicted reliability more precise. Therefore, as a first step, we need to recognize important variables that describe fatigue reliability models and construct the limit state. The fatigue reliability model proposed in this study is based on the classical limit state $g(\mathbf{x})$ that (Gagnon, Tahan et al. 2013) used to determine fatigue reliability models. This limit state is named as a Kitagawa –Takahashi diagram. Figure 0.2 shows the Kitagawa –Takahashi limit state.

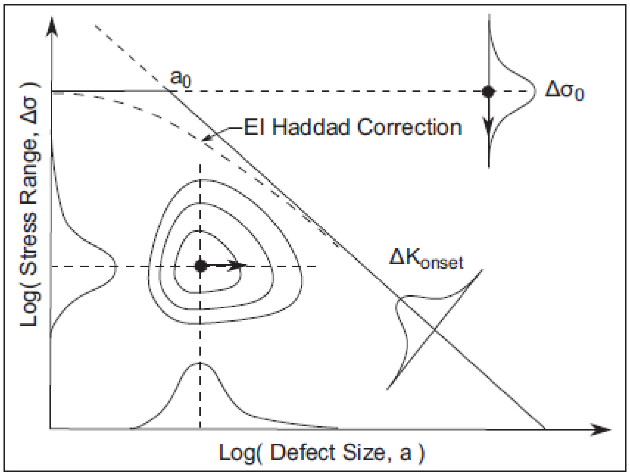


Figure 0.2: Probabilistic model that introduces the Kitagawa-Takahashi limit state (Gagnon, Tahan et al. 2013)

We propose a methodology to integrate new information about the state of variables with *prior* knowledge to obtain the *posterior* distribution of the unknown variable parameters: crack size of the defect a and high cycle fatigue stress range $\Delta\sigma$. Then we want to develop one approach to assess the fatigue reliability of hydroelectric turbine blades with structural reliability methods and update them with Bayesian methods to minimize inspection costs. Therefore, for better maintenance planning, we need to increase the accuracy of predictions for crack size and loads on the hydroelectric runners. To achieve this goal, the Bayesian method is a useful method and we propose three steps:

- Develop a probabilistic fatigue model based on uncertainty techniques followed (Gagnon et al. 2013).
- Construct a *prior* and *likelihood distribution* related to parameters and variables of fatigue model by Bayesian inference method and *updating* them with new data.
- Develop the methodology for model validation.

Thesis structure

The content of this thesis consists of 3 chapters that cover the updating parameters of fatigue models by using the Bayesian inference method and calculating the new reliability index for hydroelectric turbine blades. Following the introduction we will delve into previous research in CHAPTER 1. CHAPTER 2 relates to updating variables and parameters of fatigue models. In CHAPTER 2, analytical modeling for crack size, loading variables and updates using the Bayesian method are highlighted. In CHAPTER 3 a fatigue reliability model adapted to hydroelectric turbine blades is provided and improved by the Bayesian theory. The limit state which allows the calculation of the hydroelectric turbine reliability is defined therein. Some recommendations for this study are included. Finally, in the conclusion, we compare results following updates. We want to answer the following questions in the Abstract:

1. How can we update our prior knowledge in light of new information gathered to obtain a posterior? (CHAPTER 2)
2. Can we estimate and decrease the uncertainty about variables and parameters that exist in fatigue models? (CHAPTER 2)
3. How can we, given this new information, assess the validity of the reliability model used? (CHAPTER 3)

These are legitimate questions that form the basis of the current study. We believe that by using Bayesian statistics, these fundamental problems may be addressed.

CHAPTER 1

STATE OF ART

1.1 Fatigue propagation

Operating hydroelectric turbines causes fatigue and increases the risk of failure (Hadavinia, Kinloch et al. 2003). This process depends on the size of the initial crack, loading, material properties, aging and modes of operation (Liu, Luo et al. 2014), (Pirondi and Moroni 2010). Material properties need to be considered in all analysis of fatigue. In many cases, the total cost associated with material fracture and failure can be high (Rau Jr and Besuner 1980). Most of the researchers consider constant material properties (Castillo, Fernández-Canteli et al. 2008), (Trudel, Sabourin et al. 2014). Therefore, the uncertainties that exist in material property is often overlooked in many analyses.

The prediction of crack due to fatigue is based on two main methods: The first method (safe life) is the S-N curve damage method. Using of this method might be safe when the safe margin is selected as large (Kruzic and Ritchie 2006). The number of fatigue cycles could be determined with this method. This method is very straightforward, but to obtain safe reliability, we need to consider a large safety margin. Therefore, a lot of uncertainties are missed (Castillo and Fernández-Canteli 2009). The second method is based on crack propagation. This method which uses Linear Elastic Fracture Mechanics (LEFM) can predict fatigue and crack growth rates (Gagnon, Tahan et al. 2013).

After finding a suitable fatigue model, the fatigue reliability can be estimated. During the last decade, an increasing number of studies have been published using material and structure fatigue reliability. The basis of fatigue reliability calculation was in the late 60s and early 70s. At that time, the lack of data and capacity to perform numerical calculations affected the probabilistic fatigue results (Tong 2001) , (Manuel, Veers et al. 2001). There are very few recent studies in the literature on the issue of the fatigue reliability of hydroelectric turbines. The following authors, (Gagnon, Tahan et al. 2013); (Karandikar, Kim et al. 2012), (Chan, Enright et al. 2014); (Dong, Gao et al. 2008) study fatigue and crack growth rates based on the LEFM method. Their work shows that fatigue reliability is used for a wide range of applications of areas similar to hydroelectric turbines, aerospace panels, offshore turbines, etc. In the abstract, some factors influence the process of crack formations. The factors are initial crack size, loading, material properties, aging and operating systems.

1.2.1 Initial crack size

Initial crack size that occurs in the material of the structure needs to be investigated. Industrial crack size can be estimated with periodic inspection. Some analyses are focused on finding a way to estimate the initial crack size (Anderson 2005). For example, the location and shape of initial cracks have an effect on the speed of crack propagation (Trudel, Sabourin et al. 2014). Therefore, we need to investigate the following questions before a crack is analyzed:

1. What is the shape of the initial crack?
2. What is the size of the initial crack?
3. How can we model the crack propagation?
4. What direction do we need to find for crack propagating (planar, non-planar)?

1.2.2 Loading

Cyclic loadings and numbers of stress cycles are the main reasons for fatigue. The crack growth rate $\mathrm{d}a/\mathrm{dN}$ is defined as crack extension per cycle. This amount corresponds to the speed of propagation of a crack length $a[mm]$ with a pass the number of cycles N. The fatigue crack growth rate could be explained with the nonlinear functional relationship that is given by equation (1.1).

$$\frac{da}{dN} = f(\Delta K) \tag{1.1}$$

In this equation, $\Delta K \left[MPa\sqrt{m} \right]$ is the stress intensity factor. The stress intensity factor can predict stress intensity when the structure is under load or has the residual stress near the edge of the crack. A typical plot of log $\frac{da}{dN}$ versus log ΔK could help analyzer to estimate fatigue. Figure 1.1 shows plot of log $\frac{da}{dN}$ versus log ΔK.

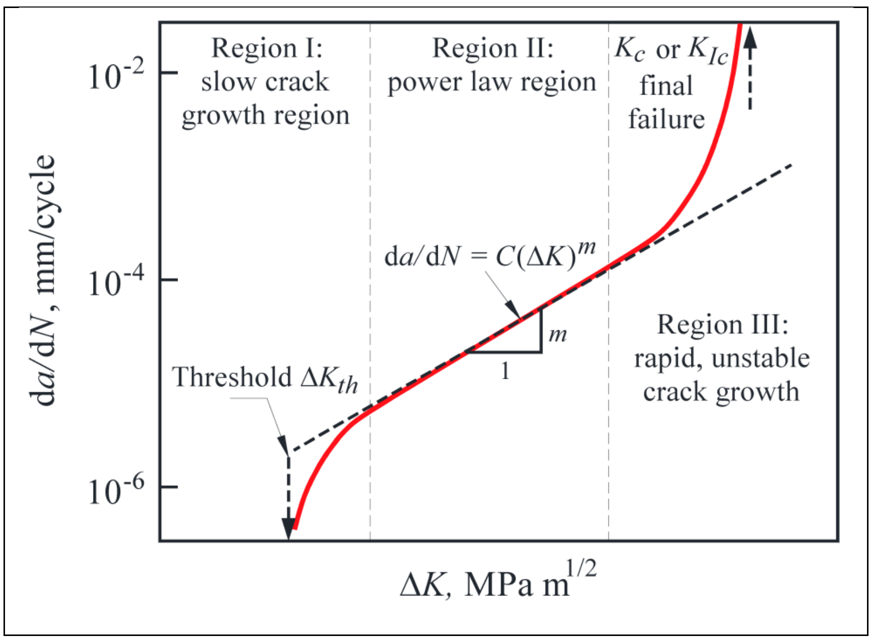


Figure 1.1: Fatigue crack growth rate curve for metals (Ambriz, 2014)

As seen in Figure 1.1, basically the propagation of crack can be divided into three regions:

Region I, The propagating of crack is extremely slow. We have the threshold stress intensity value K_{th} at this region. Below this amount there is no fatigue crack growth rate, or the rate of crack growth is too small to measure. In this project, blades work under Low Cycle Fatigue (LCF) and High Cycle Fatigue (HCF) loads. Existing micro cracks could be motivated by loading types (Huth 2005), HCF affects more to propagate of the crack rather than LCF (Trudel, Sabourin et al. 2014) , (Gagnon, Tahan et al. 2013). Therefore, under these conditions the crack reached becomes a critical size very soon. This can lead to a large crack in a very short time, compared to the life provided in the design (Gagnon, Tahan et al. 2013). It is therefore necessary to study the crack propagating in this project at the threshold point in Region I.

Region II, We can see the linear slope that could fit to the data. Most studies relate to this region. One of the popular models that fit in this region is the Paris model (Raju and O'Brien 2008). The Paris model was used by many researchers. The Paris model for fatigue relation is given in equation (1.2) for $\Delta K \geq K_{th}$.

$$\frac{da}{dN} = C.\Delta K^{m} \tag{1.2}$$

The equation (1.2) shows that the curve of crack growth rate in region II is a function of material parameters C and m, and stress intensity factors ΔK. We find material property amounts from (BS 7910, 2000) that sets guidelines for these parameters.

Region III, The rate of growing crack is very high and little fatigue life is involved. Region III is characterized by rapid, unstable crack growth.

To sum up, all these factors contribute to the turbine's fatigue and accelerate their damage which makes the estimation of their actual life expectancy difficult.

1.2 Reliability assessment

The main objective of reliability assessment is to support decision making, because each action under, or over, the threshold point could affect estimating the reliability and cost of the system. But these decisions are always accompanied by uncertainty (Heyman, Alaszewski et al. 2013) (Liu, Luo et al. 2014) and whether the estimation of the parameters needs to absorb more costs. In an attempt to find system reliability, the use of reliability structural methods led to move precise information on the structure's performance (turbine blades) (Ditlevsen and Madsen 1996). A structural reliability method requires the definition of a model of reliability (taking into account the parameters on which the system is operated) and a threshold of acceptability of the estimated reliability. More generally, this structure could also include a formulation of the criteria for failure modes that identified failures (Ronold, Wedel-Heinen et al. 1999), (Moriarty, Holley et al. 2004) (Ditlevsen and Madsen 1996). What makes the task of analysis difficult is judgment which takes into account parameters that affect the results (Liu, Luo et al. 2014) (Toft and Sørensen 2011). For this reason, the Bayesian method is a good technique to account for parameters and consider the related uncertainties.

CHAPTER 2

UPDATING PARAMETERS WITH BAYESIAN THEORY

2.1 Introduction

The purpose of CHAPTER 2 is to present a Bayesian technique to update the data and information necessary to reduce uncertainty related to fatigue life reliability. The aviation industry, particularly in the military field, is at the forefront of scientific developments in the field of reliability fatigue (Kappas 2002). However, the main difficulties in calculating reliability is model choice and methodology calculation (Cross, Makeev et al. 2006). The classic probability methods used with available information may determine reliability. This method, without updating the information, yields results. Therefore, the uncertainty associated with the initial parameters is very large, because the information initially available is limited. But in the new probability methods, one possible solution is to use observations to update priori estimated values (Wang 2008). This approach is named the Bayesian method. The Bayesian approach can use the information when it becomes available to combine with initial hypotheses and prior information to validate the model. The Bayesian theory is a method for the quantification of uncertainties issues. This method consists of evaluating Probability Density Function (PDF) for variables and model parameters. (Coppe, Haftka et al. 2010) used Bayesian inference to reduce the uncertainty of the parameters of a Paris model in fatigue issues. (Guerin and Hambli 2009) proposed that the Bayesian method is a possible way to reduce the scatter of fatigue distribution.

In this project, periodic inspections are the main source for gathering information to assist in the update of model variables for turbine blades. In CHAPTER 2, the Bayesian update method is used when additional test data are available to decrease the uncertainty of variables. This leads to the improvement of design optimization and system performance. The methodology that we used for updating a fatigue model with a Bayesian method is given in Figure 2.1.

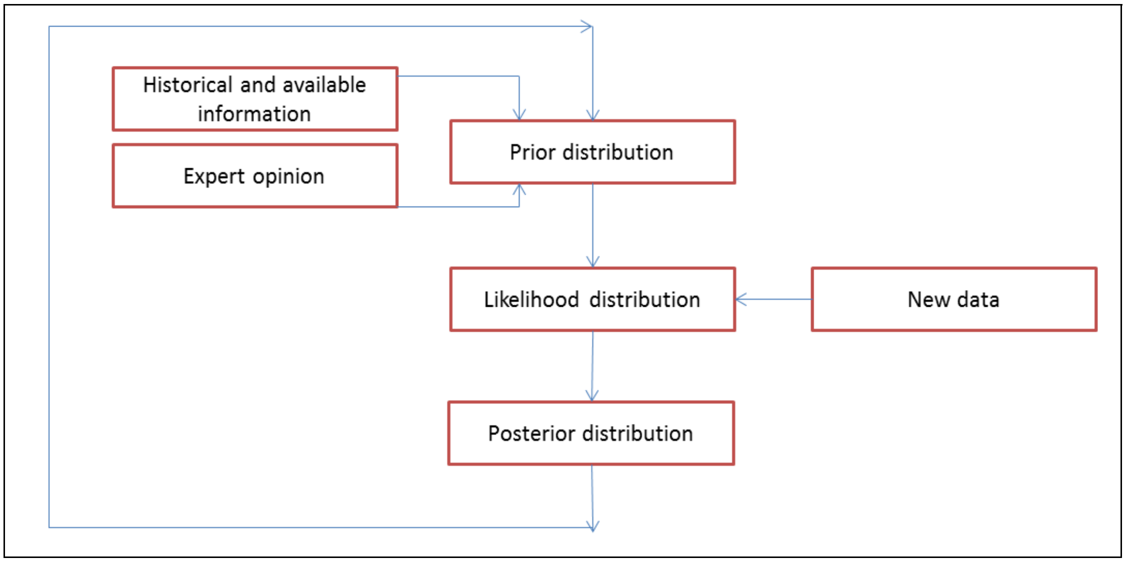


Figure 2.1: Methodology that introduces Bayesian updating method to decrease the uncertainty of parameters and variables.

Figure 2.1 shows that the Bayesian theory helps to update prior knowledge in order to obtain a suitable prediction of fatigue life. We use MCS draw samples from the given distribution. With the use of Bayesian theory, the uncertainties characterized are reduced. The results of updated posterior distribution will be used in the reliability analysis found in CHAPTER 3. We will then compare results in terms of reliability.

2.2 Data uncertainty

Uncertainties come from human errors, model errors, testing methods and measurements. Although the data are supposed to give us a picture of reality, in truth, because of the existence of uncertainty, accurately calculating the degree of truth for a given variable cannot be done (Vorobyev).

According to the difference sources of uncertainties in this project, we first identify a set of variables and related parameters to be used in this project. Therefore, with the use of the Bayesian method, which is a type of probabilistic method, the uncertainties associated with fatigue could decrease in this project.

2.3 Hypothesis

It is important to be familiar with hypothesis for more convenience. In general, two types of material defects are investigated for big structures such as the hydroelectric Francis runner: surface cracks and near surface cracks (Gagnon, Tahan et al. 2013). To decrease the number of parameters related to crack geometry, we study the circular cracks located on the surface in this project. Therefore, we have just one parameter that shows the size of a crack that was able to grow on a two – dimensional diagram. Figure 2.2 shows the surface, and the near the surface, crack. We also consider that the crack grows only in one direction where the ΔK is the maximum amount.

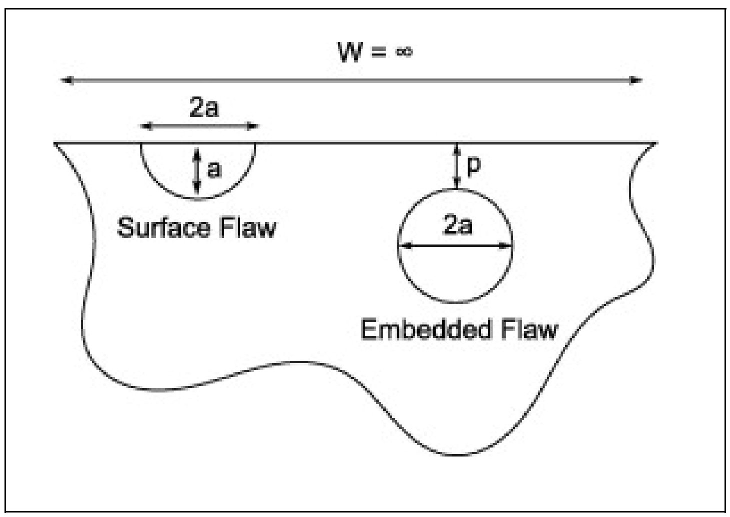


Figure 2.2: Surface crack and near the surface crack (Gagnon, Tahan et al. 2013)

Because of a lack of information about the variables, we only study the stress range and crack size that are more effective on the fatigue problem and consider other variables that affect the crack growth rate as constant and deterministic. For example, the stress intensity factor (ΔK_{th}) is first assumed to be constant and well known. This variable is related to geometry and crack location. The value of (ΔK_{th}) and stress intensity correction factor are taken from the British standard BS7910 (BS 2000). From this standard, the amount of ΔK_{th} is close to 2 [MPa $m^{1/2}$].

About the fatigue variables (crack size and stress range), we have prior knowledge that shows these variables follow the normal distribution (Pattabhiraman and Kim 2009). We want to update the variables when we add the data to our prior knowledge with the Bayesian method. Therefore if we add more data, the values of updated distributions could be precise and reduced. As our prior knowledge about the data follow Normal distribution (Pattabhiraman, 2009), therefore choosing 95% confidence interval for our prior distribution is more confident about the upper and lower bounds of distribution. This confidence interval could be a proper measure in our analytical prediction value. Although when we do not have any idea about the distribution, we could consider the variables following the uniform distribution (An, Choi et al. 2011). Table 2.1 shows the amount of fatigue problems for the hydroelectric Francis runner.

Table 2.1: Amount of parameters

	Location	Upper/Lower band	Distribution
a **[mm]**	μ =1.5, σ =0.5	[0.2, 2.48]	Normal
$\Delta\sigma$ **[MPa]**	μ =28, σ =3	[22.12, 33.88]	Normal
ΔK_{th} **[MPa m$^{1/2}$]**	2		

2.4 Bayesian update for fatigue variables

The essential work for using Bayesian statistical analysis is obtaining and estimating the posterior distribution for variables and model parameters. The posterior is an average distribution before variables are observed (*prior distribution*) (Pattabhiraman, 2009). We then make note of the variables and analyze the information that we observed (*likelihood distribution*). In this project we have analytical results (prior) and test variability (likelihood) that will be used to update posterior distribution to predict fatigue failure and reduce uncertainty in fatigue issues. This method is good way to decrease uncertainty and provide a conservative distribution that covers the error of prior distribution and the variability of likelihood distribution. The relation between the likelihood and prior distribution is shows in equation (2.1) (Pattabhiraman, 2009).

$$p(\text{analytic}|\text{test}) = \frac{p(\text{test}|\text{analitic}) \times p\ (\text{analytic})}{p(\text{test})} \propto p(\text{test}|\text{analitic}) \times p(\text{analytic}) \tag{2.1}$$

In this equation $p(\text{analytic}|\text{test})$ is a posterior distribution and $p(\text{test}|\text{analitic})$ is called the likelihood that introduces the probability of data that achieved from the test given the value of analytic results. The prior distribution is shown by $p\ (\text{analytic})$. The expert opinions are affected in prior distributions. According to the raw data that is used for Bayesian updates, therefore the normalizing of data and all distributions is necessary. Therefore, the dominator of equation (2.1) is brought in to ensure that the posterior PDF integrates to 1. For updating variables (crack size and stress range) we used equation (2.2) (Pattabhiraman, 2009).

$$p^{\,i}(X) = \frac{p_{1,\text{test}}(X)\ .\ p^{ini}\ (X)}{\int_0^{+\infty} p_{1,\text{test}}(X)\ .\ p^{ini}\ (X)dX} \tag{2.2}$$

X in equation (2.2) replaces defect size and stress range. This equation shows that $p^{ini}\ (X)$ is the initial distribution of variables. With iterating *i* times, we could achieve a proper prior distribution and it is very close to a posterior distribution that could be fit with the data. For using the Bayesian theory, we follow these steps to obtain precise results:

1) Decide on a prior distribution, with considering the uncertainty in unknown model parameters before the data observed.

2) Observe the new data and create the likelihood distribution based on the data.

3) Calculate the posterior distribution with a multiplication of prior distribution and likelihood distribution with simulation.

4) Update the posterior distribution.

2.5 Results of updating variables with Bayesian theory

In this section, the posterior distribution with analytical results (prior distribution) and test data (likelihood distribution) according to Table 2.2 is constructed.

Table 2.2: Parameter specification for crack size and stress range

Distribution	a [mm]	Δσ [MPa]
Prior (analytical result)	Normal (μ =1.5, σ =0.5)	Normal (μ =28, σ =3)
Likelihood (test data)	Gumbel (μ =1.80, σ =0.65)	Gumbel (μ =30.4, σ =5)

We choose a proper a prior probability distribution that fits to variables of fatigue models. This is the **first step**. With historical data, the variables (defect size and stress range) follow normal distribution. For estimating the value of the unobservable parameter, we use a confidence interval of 95% that could cover variables. Figure 2.3 show the normal distribution for variables with a 95% confidence interval.

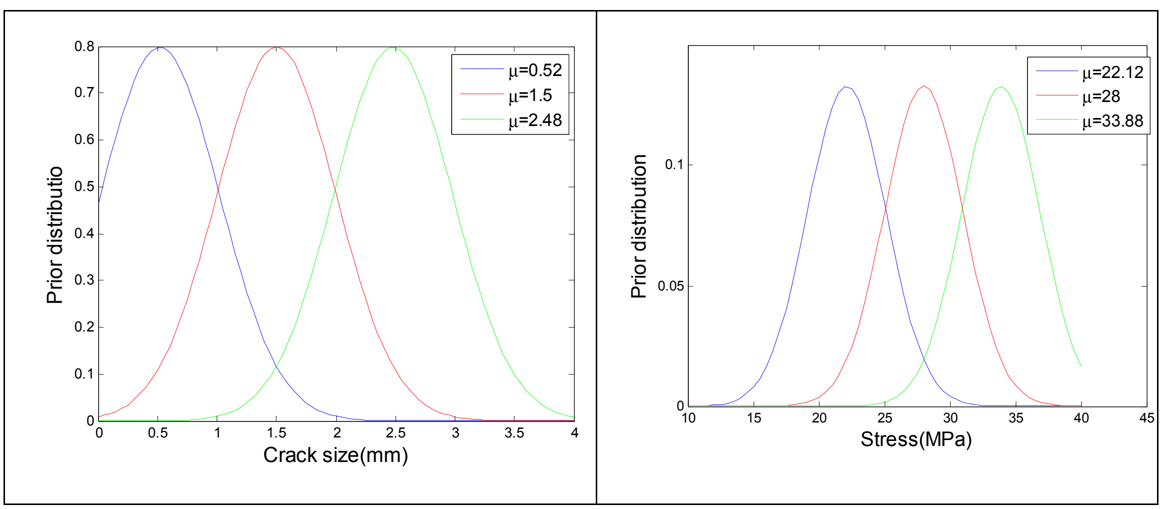


Figure 2.3: Prior distribution with 95% confidence interval

Figure 2.3 shows that the mean of the crack size changes between 0.52 and 2.48 and mean of stress range varies between 22.12 and 33.88.

The **second step** is choosing the likelihood distribution for variables that may be often more problematic rather than to choose prior distribution (Pattabhiraman and Kim 2009).

Therefore, in standard Bayesian methods likelihood distributions could be determined precisely by knowing the sample data. Once the data has been observed, the likelihood function is constructed. Sometimes in special cases, the prior and likelihood could merge together suitably (analytically) so that there is no need to compute the normalization factor that exists in a denominator Bayesian method. Some conjugate pairs for prior and likelihood distributions are given in Table 2.3. In choosing these conjugate pairs, applying Bayesian method could be simplified. In this project we use the industry data and then construct the likelihood distribution. The results demonstrate a crack size and stress range following the Gumbel distribution with specific parameters that is shown in Figure 2.4.

Table 2.3: Conjugate pairs for prior and likelihood distribution

Likelihood Distribution	**Prior Distribution**	**Posterior Distribution**
Normal	Normal	Normal
Exponential	Gamma	Gamma
Normal	Gamma	Gamma
Exponential	Inverse gamma	Inverse gamma

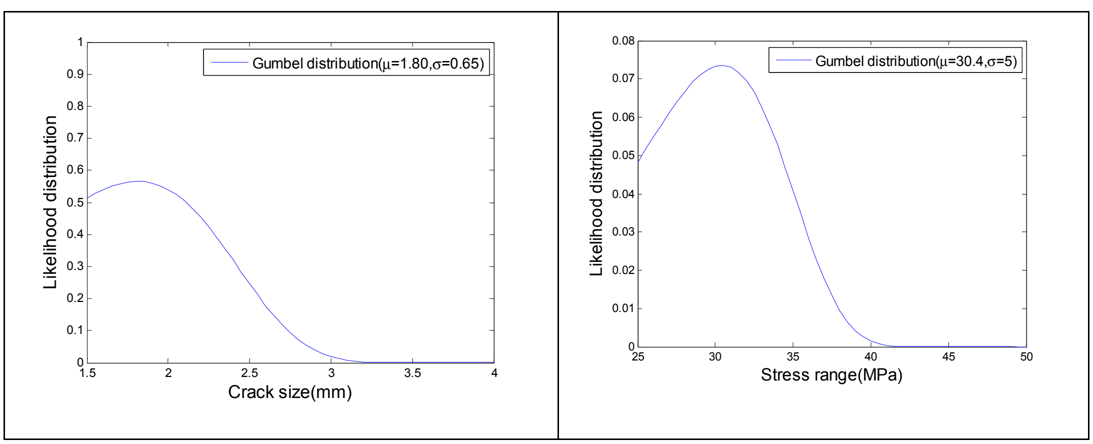


Figure 2.4: Likelihood distribution

According to Figure 2.4, it looks much like a normal distribution. We plot the normal distribution that has a same mean and standard deviation with previous distribution. It can be seen in Figure 2.5. We do this work when we want to obtain the suitable distribution between Normal and Gumbel to cover most values. Also, Figure 2.5 shows the product distribution that is an average of them.

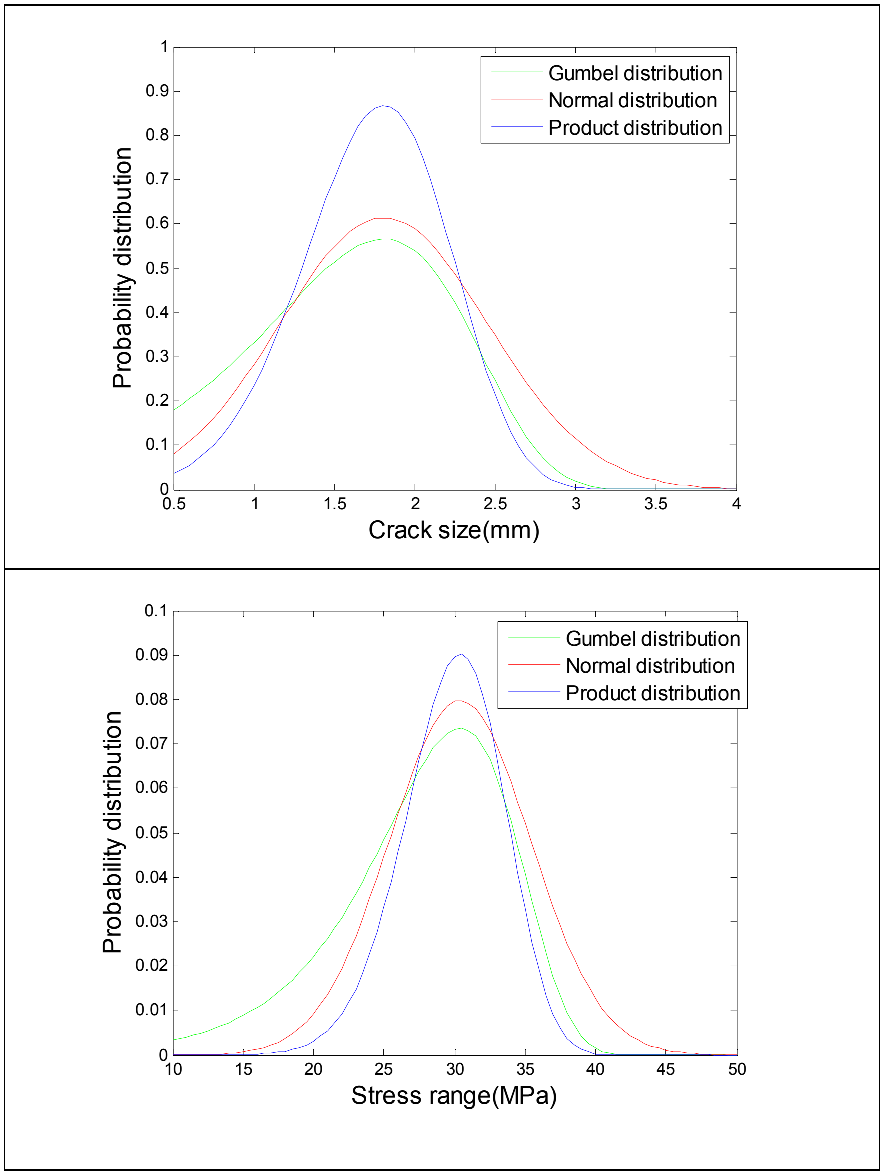


Figure 2.5: Likelihood distribution and product distribution

Figure 2.5 shows the product distribution that it used for updating variables. Figure 2.6 also shows the prior and likelihood distribution with each other.

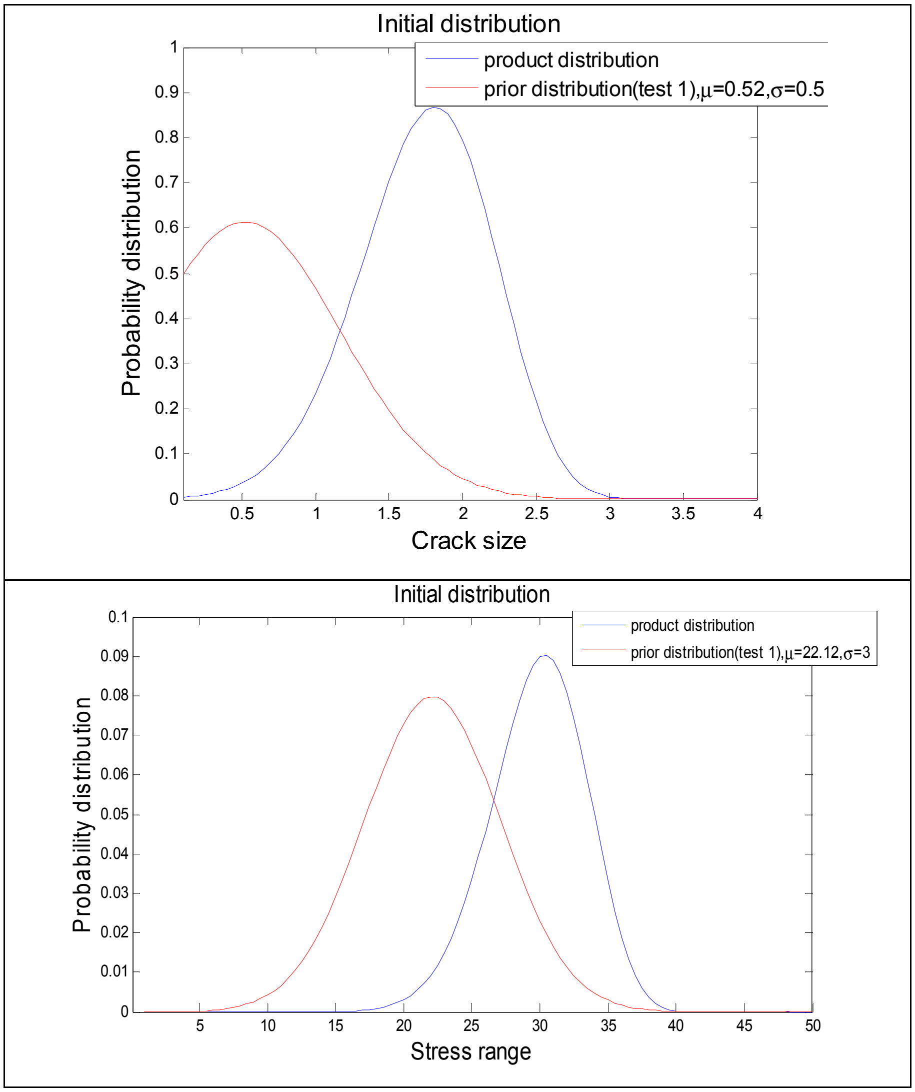


Figure 2.6: Prior and likelihood distribution

Having the prior and likelihood distribution, we could estimate the posterior distribution. We also update posterior distribution with a confidence interval of 95% to not miss the data. Figure 2.7 and Figure 2.8 show variables updated with the Bayesian method.

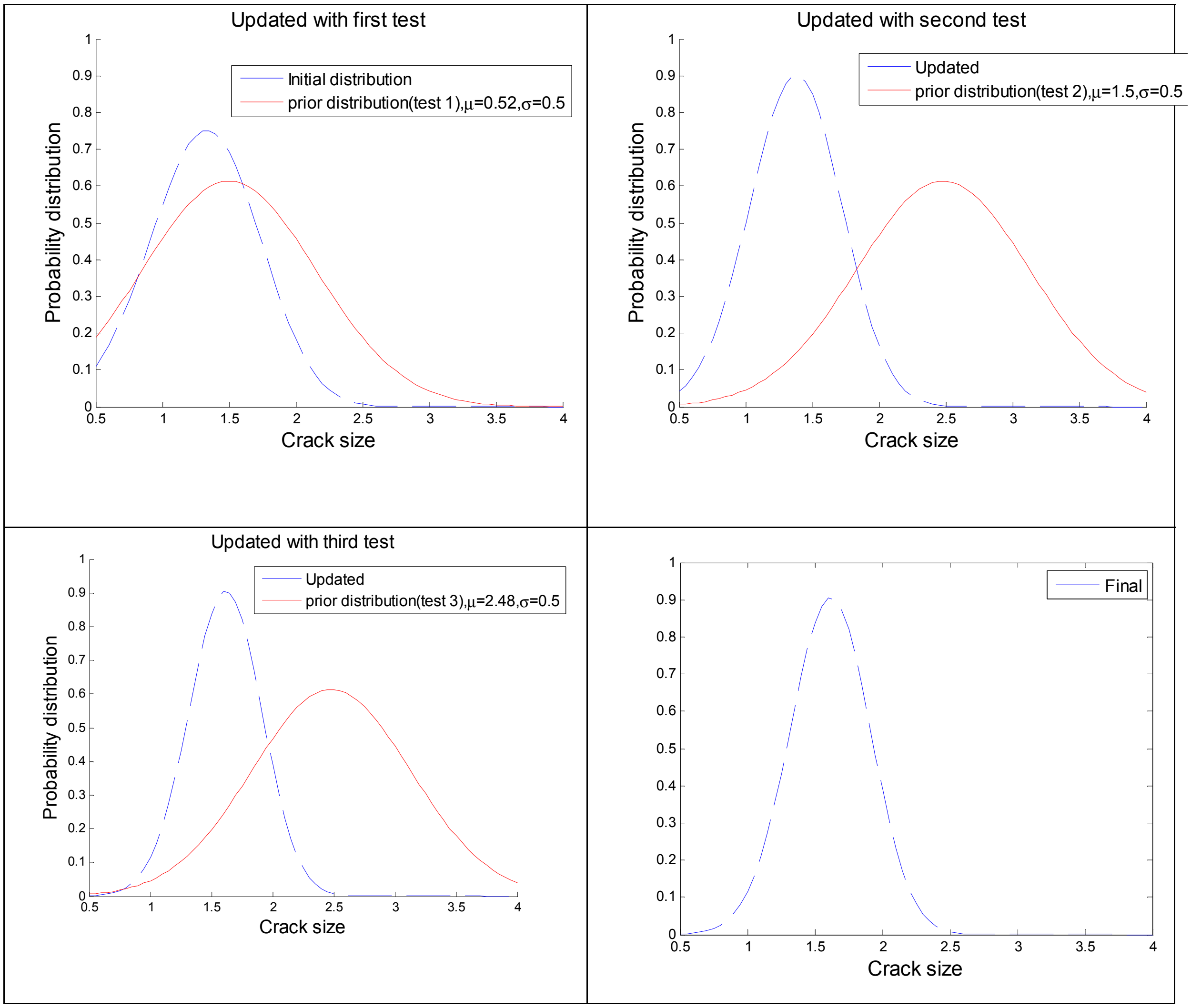


Figure 2.7: Bayesian update for crack size

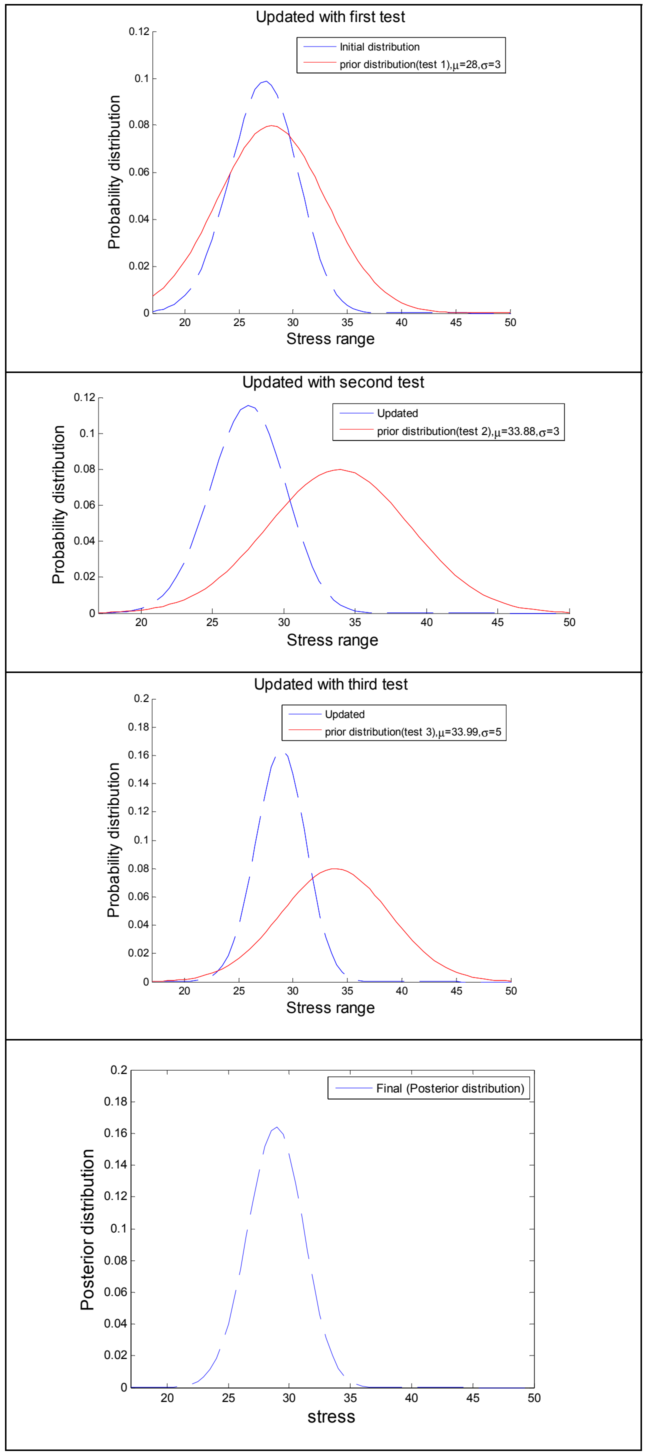


Figure 2.8: Bayesian update for stress range

According to Figure 2.7 and Figure 2.8, the new mean value and standard deviation for new distributions after three updates brings in Table 2.4.

Table 2.4: Mean value and standard deviation for updated distribution

Distribution	a [mm]	$\Delta\sigma$ [MPa]
Prior (analytical result)	Normal (μ =1.5, σ =0.5)	Normal (μ =28, σ =3)
Likelihood (test data)	Gumbel (μ =1.80, σ =0.65)	Gumbel (μ =30.4, σ =5)
Posterior	μ =1.6, σ =0.5	μ =29.1, σ =0.75

Note that the amount of standard deviations for updated posterior distributions is decreased by 33% for crack size and 83% for stress range. Therefore updating the likelihood distribution with three test data sets reduces the uncertainty of fatigue variables.

2.6 Results of updating parameters with Bayesian theory

In the previous section, the variables of fatigue problems were updated with the Bayesian method. However, in predicting fatigue life, updating variable parameters is also recommended (Pattabhiraman, 2009). In this section, we want to update the parameters mean value and standard deviation (μ , σ) of variables using the Bayesian method to decrease the additional uncertainties that exist in fatigue issues. As mentioned earlier, choosing a proper probability distribution as a *prior* for parameters is the first step in the Bayesian method. From the results shown in Table 2.4, we are interested in choosing a probability distribution function for the data with this amount of posterior distribution.

As we know, the parameter amounts affect to the skew and median of distribution, therefore it is important to obtain precise amount parameters because to construct limit states, we need to use these parameters amounts. Therefore, with accurate limit states we can estimate a proper reliability index. So we should model these parameters and study them to decrease the uncertainties that exist in parameters by using new information.

For this study, we consider that μ follows noninformative (uniform distribution) with domain $[b,c]$ (equation (2.3)). This kind of distribution is very common to use when you do not have an idea of the parameters (An, Choi et al. 2011).

$$p(\mu)=\begin{cases}0 & \mu<b\\ \dfrac{1}{b-c} & b\le\mu\le c\\ 0 & \mu>c\end{cases} \tag{2.3}$$

To decrease the uncertainty of σ, we notice that it follows the normal distribution. The equation (2.3) shows the normal probability distribution. The second step is constructing the likelihood distribution from the data that was given in Table 2.4. Figure 2.9 to Figure 2.12 are a prior and likelihood probability distribution of a variation of parameters for stress range and crack size. The third step is the derivation of *posterior* distribution using Bayesian theory. Figure 2.14 and Figure 2.14 show the 3D of the *posterior* distribution for stress ranges and crack size. The result of posterior distribution is shown in Table 2.5.

Table 2.5: Amount of parameters related to $\Delta\sigma$ and a with prior and likelihood distribution

	Prior	Likelihood	Posterior
$\Delta\sigma$	μ is uniform with domain [10 - 60]. $\sigma \sim N(1,1.5)$	Gumbel (μ =29.1, σ =0.75)	μ =28.41 σ =0.495
a	μ is uniform with domain [0.1 , 3.1]. $\sigma \sim N(0.55,0.2)$	Gumbel(μ =1.6, σ =0.5)	μ =1.577 σ =0.435

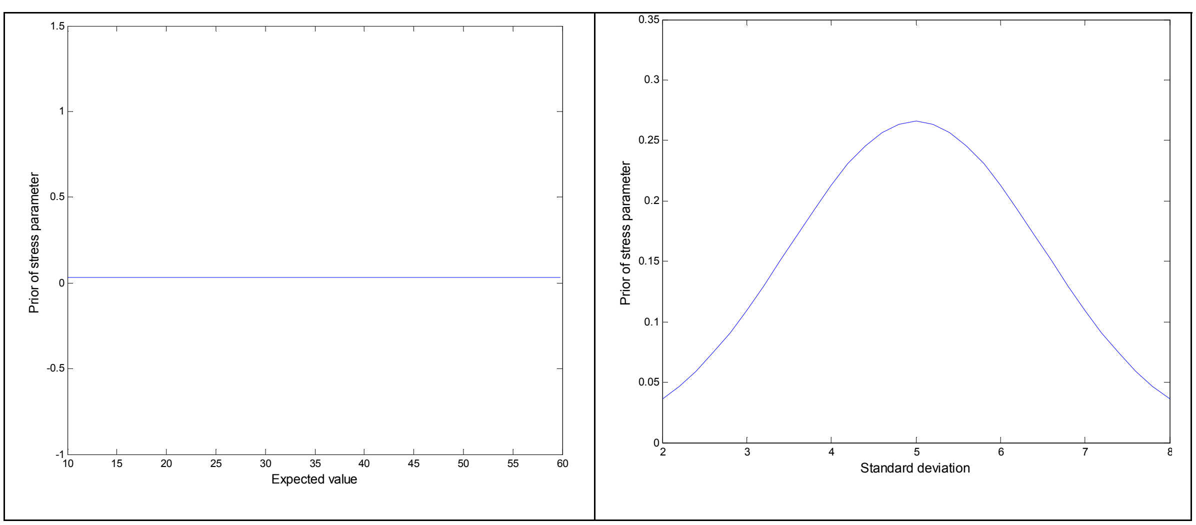


Figure 2.9 : Prior probability distribution for stress range

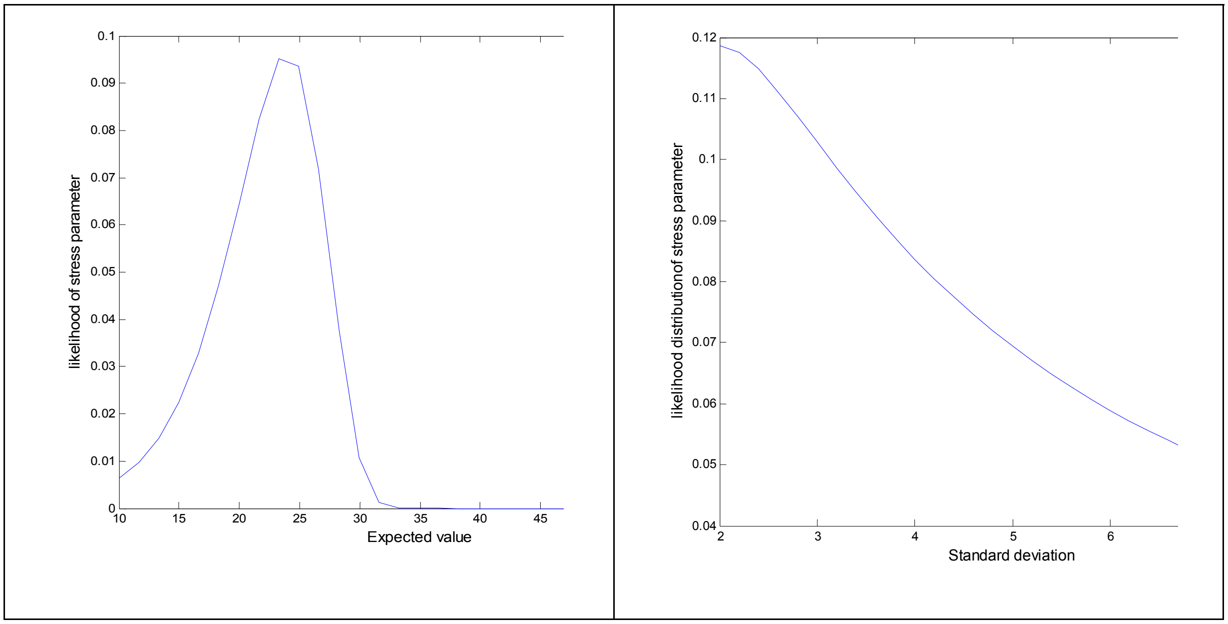


Figure 2.10: Likelihood probability distribution for stress range

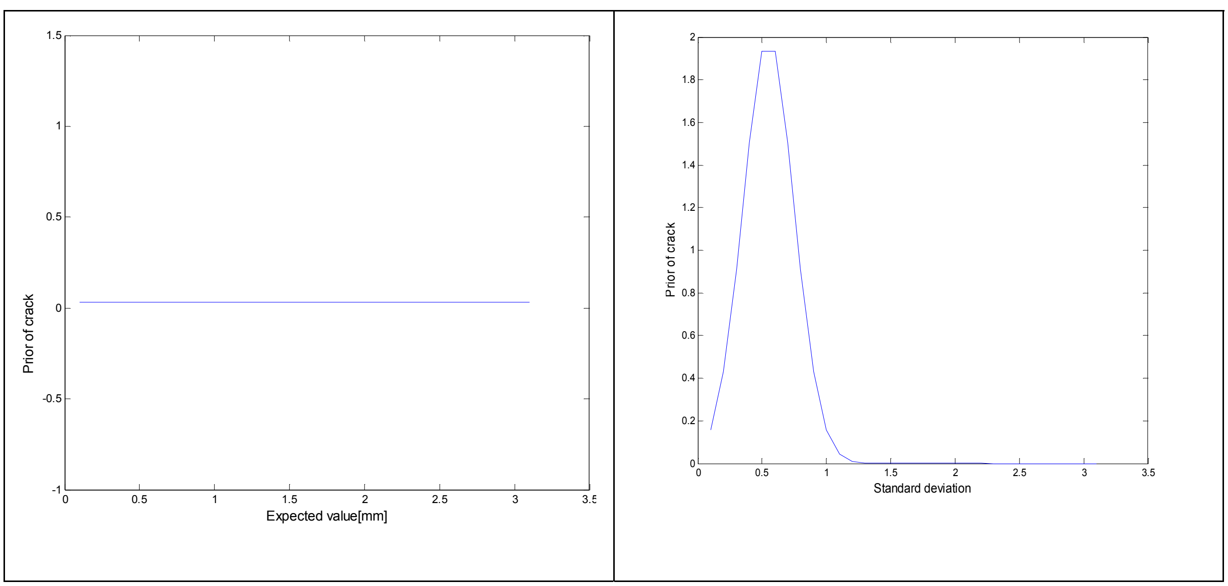


Figure 2.11: Prior probability distribution for crack size

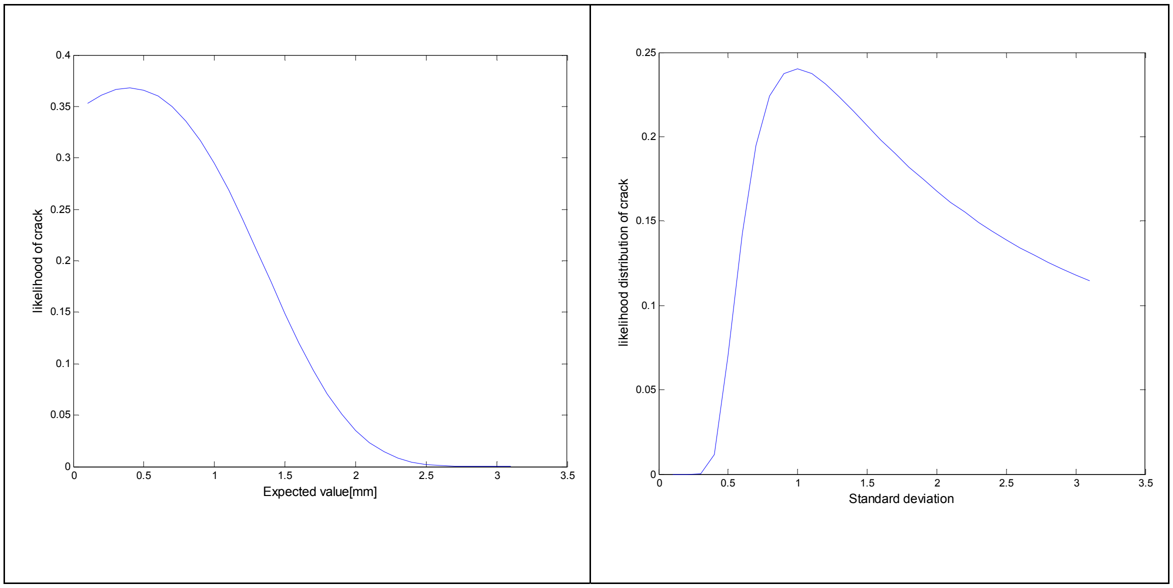


Figure 2.12: Likelihood distribution for crack size

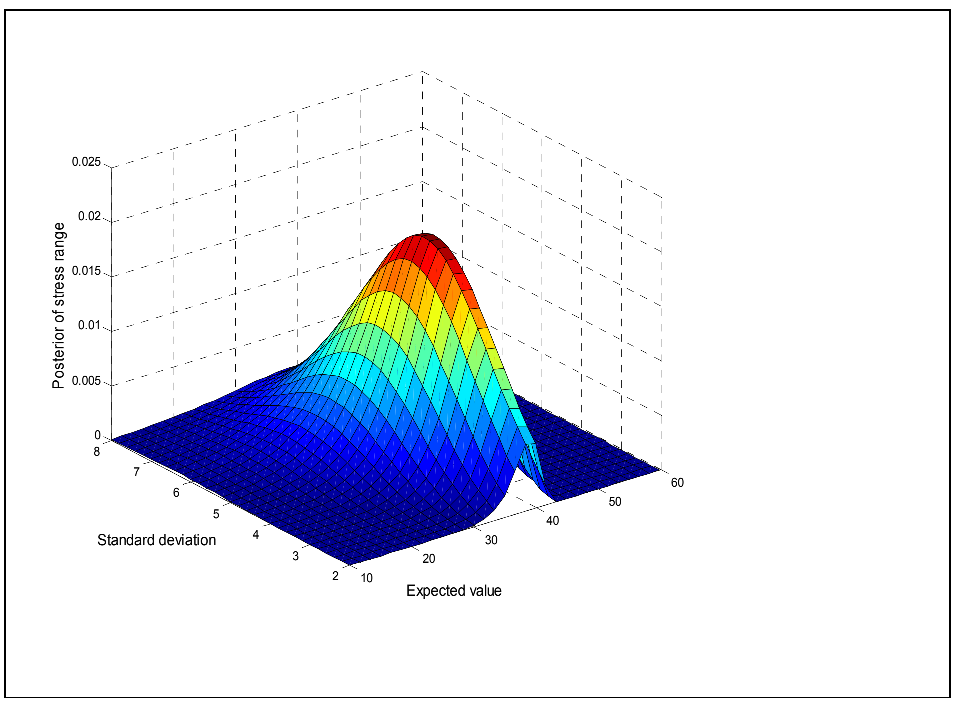


Figure 2.13: 3D of the Posterior distribution for stress range

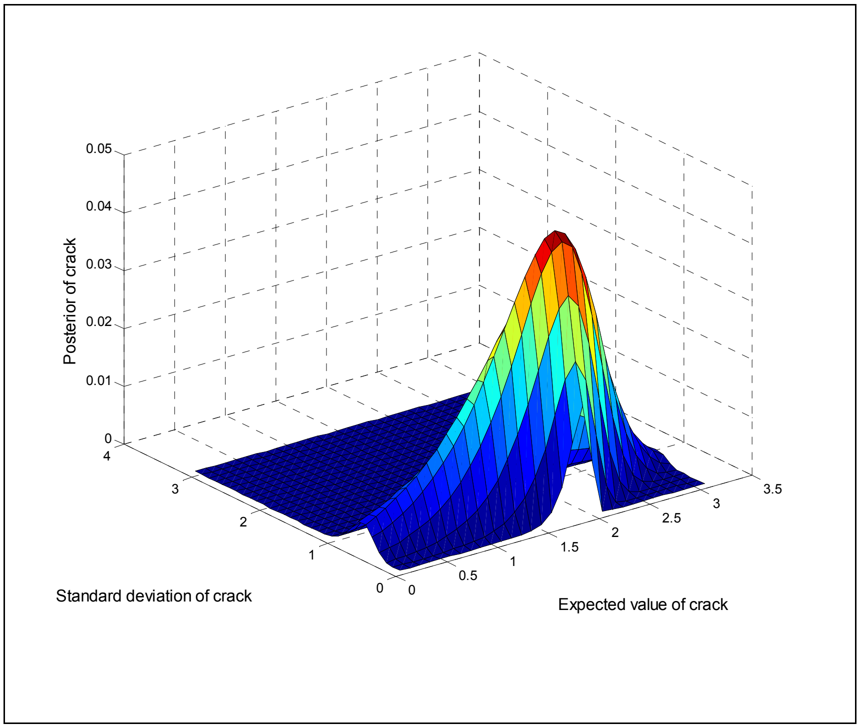


Figure 2.14: 3D of the Posterior distribution for crack size

2.7 Conclusion:

We used an update of the Bayesian method to estimate the amount of variables in section 2.5. The results show that the Bayesian method is a good way to reduce uncertainty in fatigue issues. We see that the standard deviation for crack size and stress range is reduced by more than 30% and 80%. If more data are gathered, the values of posterior distribution could be updated; therefore the credible interval could be decreased. For this reason, in section 2.6, we update the parameters of variables. We thus achieve a conservative estimate of the variables. So with Bayesian theory we did a proper distribution with minimum uncertainties in parameters. In CHAPTER 3, we use the results of CHAPTER 2 to estimate the fatigue reliability index.

CHAPTER 3

UPDATING FATIGUE RELIABILITY MODELS WITH THE BAYESIAN METHOD

3.1 Introduction of the structural reliability method

Reliability is defined as the ability (probability) of the system to do its tasks adequately under determined condition for a definite and specific time (Ebeling 2004). In general, the amount of reliability is defined according to the type of industry and its mission to pursue an index. Probabilistic methods is the major approach to estimate the system's reliability (Cullen and Frey 1999). In a non-probabilistic approach, the determination of reliability can be based on the historical analysis of the frequency of events supported by expert opinion. In the probabilistic method, the reliability is estimated according to statistical-probabilistic methods. One of the useful methods to find reliability is the reliability index used in the structural reliability theory. (Madsen, Krenk et al. 2006) present a formulation of the reliability index β based on the expected value (mean) and standard deviation of each variable analyzed with the subject of a structural reliability method. First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and Monte Carlo Simulation (MCS) are all methods to estimate the reliability index in this theory. The context of this theory, and related formulations, offers a proper framework to quantify uncertainties (Madsen, Krenk et al. 2006).

In this project, the reliability index, noted as β , is defined when the length of crack does not pass a threshold amount of HCF. The Kitagawa-Takahashi limit state is chosen at a threshold amount of loading HCF to estimate fatigue reliability. In CHAPTER 3, the numerical value of the reliability index is estimated by MCS and will be compared with FORM to find the accuracy approximation of reliability index for *prior* and *updated distribution* that is proposed in CHAPTER 2. Therefore for estimating the reliability index with structural reliability method, we need to do following process, described below.

3.2 Step 1: Identify the significant failure modes of hydroelectric turbine blades

As mentioned earlier, operating mode, maintenance strategy, quality of repairs, initial size of the crack by the manufacturer, location and shape of crack, along with stress loading are all parameters that influence the reliability index of fatigue (Raju and O'Brien 2008), (Gagnon, Tahan et al. 2013). CHAPTER 2 identifies the main variables in our model (e.g. crack size and stress range) which lead to the cracking of the hydroelectric turbine's blades that cause the degradation of the system's reliability. Therefore the fatigue reliability in hydroelectric turbines depends on the probabilistic model of a crack length that does not increase after passing a number of cycles under specific loading.

3.3 Step 2: Define probability of failure for turbine blades

Roughly, we could separate variables which affect the system in two groups (VĂCĂREANU, 2007). One of them shows the resistance (strength) of system **R** versus of loading (stress) **S** that disturbs the system. Failure will happen when **R** is less than load **S**. Each of these variables follows a specific probability density function ($f_S()$ and $f_R()$), it is important to study the joint distribution of each to find the probability of failure. In this specific case, **R** and **S** are said in the same units (e.g. MPa). Figure 3.1 shows an example for distributions of resistance and load variables when their joint distribution may lead to the failure of the system.

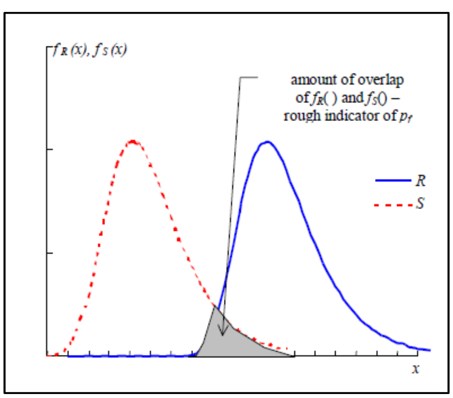


Figure 3.1: Basic failure problem
(VĂCĂREANU, ALDEA et al. 2007)

The gray area in Figure 3.1 shows that some probability for loads in this area surpass resistance behavior. Therefore the probability of failure $\mathrm{Pr_F}$ in this area needs to be estimated. With a structural reliability method, the probability failure $\mathrm{Pr_F}$ could be obtained easily, with new variables. This relation can be stated in equation (3.1) (Melchers 1999).

$$\mathrm{Pr_F} = \Pr(Z \le 0) = \Phi(\frac{0-\mu_z}{\sigma_z}) = \Phi(-\beta) \tag{3.1}$$

In this equation Φ is the standard normal cumulative distribution function, and β is defined as the reliability index. All of the variables exist in a normal distribution form. As we see in equation (3.1) when the standard deviation σ_z is increased, the probability of failure will increase. But in most cases, this simple equation is not appropriate to solve the problem. More general formulation is required. With the theory of the structural reliability method this problem is solved whether it defines the equation that is a safe boundary and in unsafe mode. This equation is named a limit state and shows with $\mathrm{g}(\mathbf{X})$. The $\mathbf{X}$ is the vector of all relevant basic variables. In general, the limit state equation is derived from the physics of the problem. A failure in the structural reliability method is functional of the limit state when the limit state is less than zero ($\mathrm{g}(\mathbf{X}) \le 0$). The probability of failure is evaluated as equation (3.2) and could be written as equation (3.3).

$$\mathrm{Pr_F} = \Pr\{\mathrm{g}(\mathbf{X}) \le 0\} \tag{3.2}$$

$$\mathrm{Pr_F} = \int_{\mathrm{g}(\mathbf{X}) \le 0} f(\mathbf{X})\, d\mathbf{x} \tag{3.3}$$

When the limit state is less than zero it shows the unsafe state. Various methods for solutions of the integral in equation (3.3) have been proposed. Some limit states are linear and an analytical solution is easy to obtain. If limit state functions are nonlinear, we can obtain an approximate solution by linearizing the function using a Taylor series development. FORM, SORM and MCS are frequently used to calculate a reliability index when we have a nonlinear limit state. In all these methods the equation of limit state is equal to zero ($\mathrm{g}(\mathbf{X}) = 0$) and the reliability index (β) is definite as the shortest distance from the origin of standards and normalized variables to the limit state in the same iso-probabilistic space. This definition, which is introduced by (Hasofer and Lind 1974) is seen in Figure 3.2.

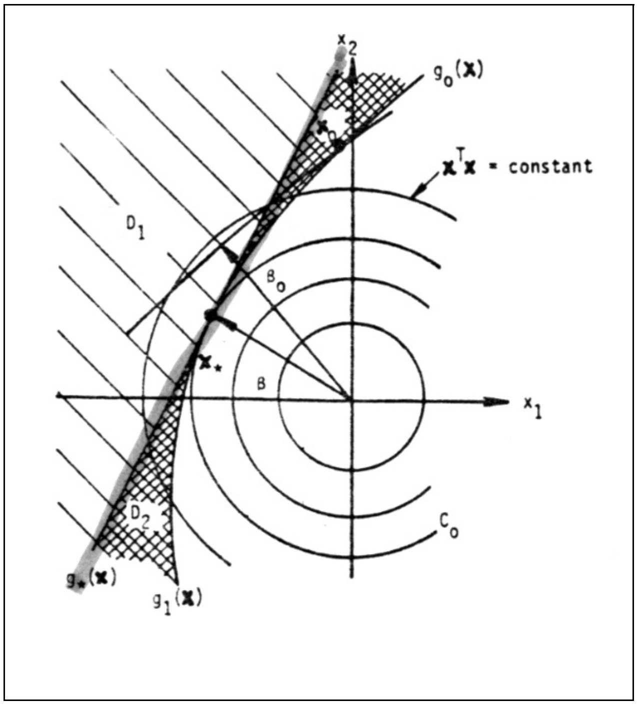


Figure 3.2: Reliability index on nonlinear limit state (Hasofer and Lind 1974)

Figure 3.2 shows the desire point to have a reliability index on the nonlinear limit state. It could be estimated with an iteration relation.

3.4 Step 3: Construct Kitagawa-Takahashi limit state for fatigue reliability

It must be considered in fatigue failure; the Kitagawa-Takahashi limit state is more used (Kruzic and Ritchie 2006). For constructing the Kitagawa-Takahashi limit, data from S-N approach and LEFM approach are used. (Gagnon, Tahan et al. 2013) used the Kitagawa-Takahashi limit state for the HCF onset to estimate the fatigue reliability for turbine blades. As mentioned in CHAPTER 1, no propagation occurs in Region I below a threshold stress intensity factor. Equation (3.4) is used to determine the stress range at HCF with the LEFM method. This equation shows the relation between the HCF stress range at threshold point $\Delta\sigma_{th}$ and stress intensity factor at threshold point ΔK_{th} .

$$\Delta\sigma_{th} = \frac{\Delta K_{th}}{\sqrt{\pi a}\gamma(a)} \qquad (3.4)$$

With equation (3.4) and data from the S-N curve, the Kitagawa-Takahashi limit state could be evaluated in 2D space by equation (3.5).

$$g(a, \Delta\sigma) = \Delta\sigma - \frac{\Delta K_{th}}{\sqrt{\pi a}\gamma(a)} \qquad (3.5)$$

In this equation, g () is the limit state and function of variables (defect size a [mm] and stress range $\Delta\sigma$ [MPa]). Later (El Haddad, Topper et al. 1979) proposed some corrections to the limit state. Figure 3.3 shows a schematic of the Kitagawa –Takahashi limit state with an El Haddad correction.

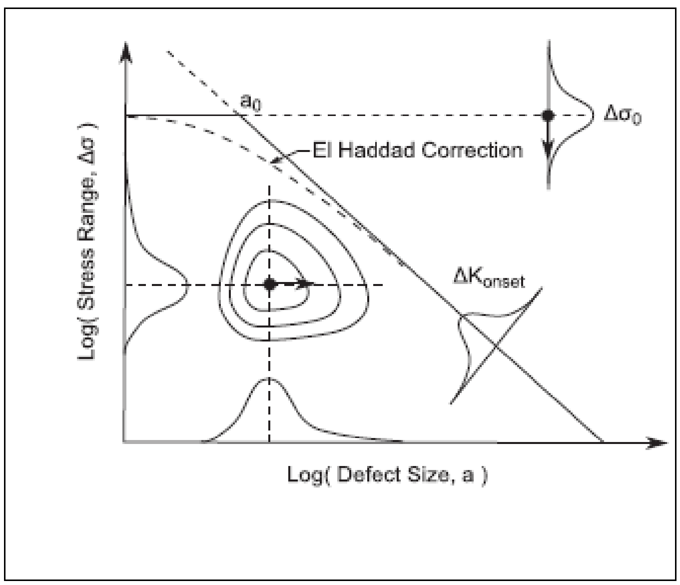


Figure 3.3: Schematic of Kitagawa -Takahashi limit state with El Haddad correction (Gagnon, Tahan et al. 2013)

Figure 3.3 shows the Kitagawa-Takahashi limit state with Log-Log scales. One line is the fatigue limit and represents the limit of the material's resistance and is determined with an S-N approach. In this study, it corresponds to 10^7 cycles when the crack is in the surface and has a circle shape. The second line is represented by the stress range at a threshold point that is obtained by the LEFM approach (equation (3.4)).

3.4.1 Estimated reliability index for the standard normal variables

The standard normal distribution is used to estimate the reliability index β . It is easy to analyze and at this form the variable does not have dimensional consistency. The relation (3.6) shows a standard normal form (Z) for variables. As we mentioned in this project the stress range and crack defect are the main variables.

$$Z_{X_i} = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (3.6)$$

Equation (3.6) reduced all of the normal variables to the standard form. According to the real problem, the variables follow non-normal distribution; we need to transfer the variables from the non-normal space to a standard normal space (iso-probabilistic space). One of the transformation techniques that could be used is a Rosenblatt transformation. In this project we use a Rosenblatt transformation to reduced variables in a standard normal space.

3.4.2 Rosenblatt transformation

When the variables are non-normal, the Rosenblatt transformation is applicable and shows up in equation (3.7).

$$Z = \Phi^{-1}(F_X(X)) \quad (3.7)$$

Where $F_X(X)$ is the cumulative distribution (CDF) of X, Φ is the standard normal cumulative distribution. For generating the new expected value and the standard deviation that relates to equation (3.7), we need to define the relation that is described in (3.8) to (3.9).

$$\Phi(\frac{X_i - \mu_i}{\sigma_i}) = F_{X_i}(X_i) \quad (3.8)$$

$$\frac{1}{\sigma_i}\varphi(\frac{X_i - \mu_i}{\sigma_i}) = f_{X_i}(X_i) \quad (3.9)$$

Where $f_{X_i}(X_i)$ is the probability density function (PDF) for μ,σ. The new μ,σ could be obtained by (3.10) to (3.11) are:

$$\sigma_i = \frac{\varphi(\Phi^{-1}(F_{X_i}(X_i))}{f_{X_i}(X_i)} \tag{3.10}$$

$$\mu_i = X_i - \sigma_i \Phi^{-1}(F_{X_i}(X_i)) \tag{3.11}$$

3.4.3 Estimating reliability index with FORM

As mentioned, the reliability index represents the shortest distance from the origin to the point in a limit state when all of the variables are in the standard normal form. When the limit state is nonlinear, we can obtain an approximate answer by linearizing the function using a Taylor series. Equation (3.12) is used to linearize the limit state.

$$g(X_1, X_2, \ldots, X_n) \approx g(x_1^*, x_1^*, .., x_n^*) + \sum_{i=1}^{n} (X_i - x_i^*) \frac{\partial g}{\partial X_i}\Big|_{evaluated\ at\ design\ point} \tag{3.12}$$

The design point x_i^* is a point on the limit state when the limit state is equal to zero. Since this design point is generally not known, an iterative technique must be used to solve the equation. Equations (3.13a) to (3.13c) show the iteration that is needed to find the reliability index. Calculating this equation needs additional time in order to find the location of a design point. For calculating the design point, we need reduced variables. Therefore all of the variables need to be transferred to a standard normal variable.

$$g(Z_1^*, Z_2^*, ..., Z_n^*) = 0\ ;\ \ Z_i^* = \beta\alpha_i \tag{3.13a}$$

$$\frac{\partial g}{\partial Z_i} = \frac{\partial g}{\partial X_i} \frac{\partial X_i}{\partial Z_i} \tag{3.13b}$$

$$\alpha_i = \frac{\frac{\partial g}{\partial Z_i}\Big|_{\substack{evaluated\\ at\\ design\\ point}}}{\sqrt{\sum_{K=1}^{n} \left(\frac{\partial g}{\partial Z_K}\Big|_{\substack{evaluated\\ at\\ design\\ point}}\right)^2}} \tag{3.13c}$$

The probability failure is estimated directly from the reliability index and is given by equation (3.14).

$$\Pr_F(X<0)=\Phi\left(\frac{0-\mu_X}{\sigma_X}\right)=\Phi(-\beta) \tag{3.14}$$

3.5 Estimate reliability index for prior distribution

As mentioned before, in order to estimate the reliability index, the first step is constructing the limit state with variables that affect the failure. With the use of information in Table 3.1, the Kitagawa-Takahashi limit state constructed.

Table 3.1: Detailed results for a [mm] (Normal (μ =1.5, σ =0.5)) and $\Delta\sigma$ [MPa] (Normal (μ =28 σ =3))

Description for *Prior* distribution	**Values**
Physical space design point (mm, MPa)	(1.5, 28)
Standard space design point	(1.79, 1.88)
MCS reliability index (10^5 simulations)	2.60
FORM reliability index	1.88
MCS probability failure	0.004
FORM probability failure	0.029

After constructing the limit state, we could estimate the reliability index when the variables are reduced to the normal standard space. The prior distribution in this study follows normal; therefore we do not need to use transformation technique. After updating the results (updated to *posterior* distribution) we will use a Rosenblatt transformation to transfer variables to a 2D standard normal space. Figure 3.4 shows the limit state and β when all of the variables are in standard form. The value of reliability index is calculated by MCS method and FORM.

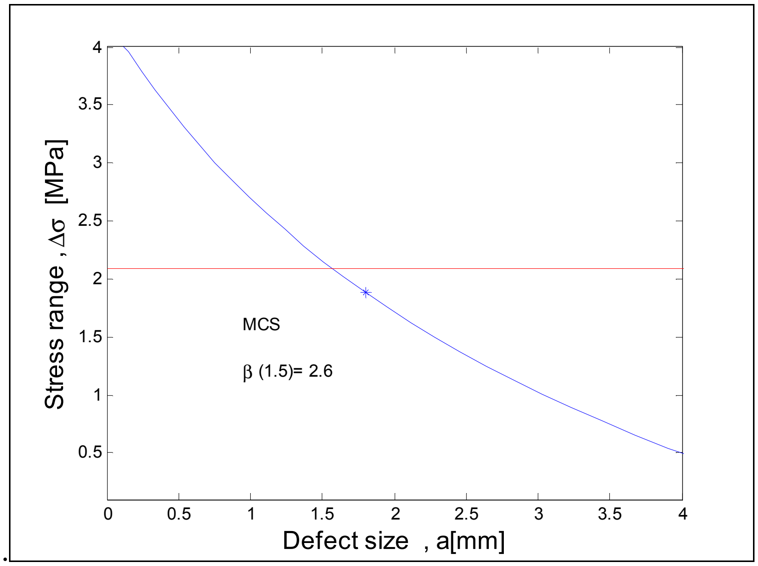


Figure 3.4: Reliability index amount

Figure 3.4 shows the result of reliability index for the crack that obtained by MCS after 10^5 simulations. We generate 10^5 data because the amount of reliability is between 2 and 4. In general for large amount of reliability index we need to simulate more than 10^6 data. This amount has abilities to cover distribution that need to investigate. According to in this project, reliability index is close to 3, selecting the number of 10^5 simulations is reasonable. Figure 3.5 shows the reliability index vs crack size and Figure 3.6 displays probability failure versus crack size when we use FORM and MCS method.

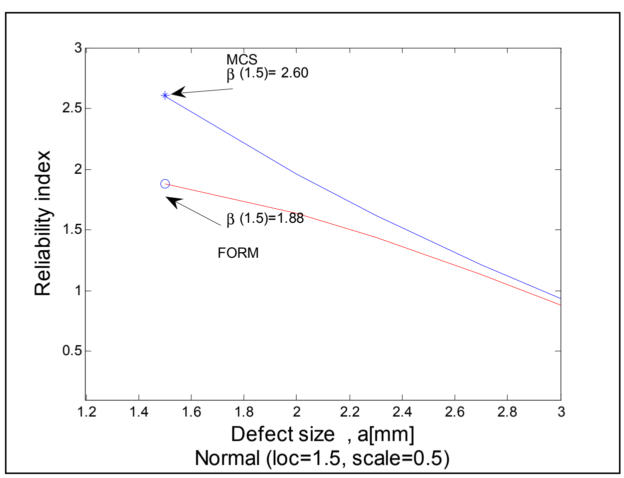


Figure 3.5: Reliability index vs of crack size
with MCS and FORM

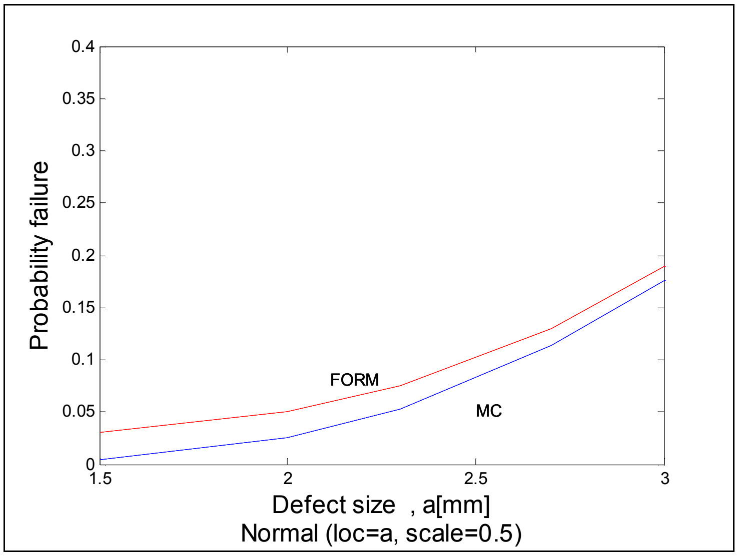


Figure 3.6: Probability of failure vs crack size with MCS and FORM

Figure 3.5 and Figure 3.6 show the large difference between MCS and FORM. However, in both cases, the results follow the same trend, but the amounts are different from each other. One of the reasons is because of existing of large standard deviation of variables specially related to stress range. As we see the equation in (3.13), the impact of the standard deviation in the FORM method is very high. But after updating variables and decreasing uncertainties we will see the curves of FORM and MCS converge to each other. Although we need to consider that the results for FORM are obtained with only 16 iterations as compared to MCS those 10^5 simulations which draws from the distribution.

3.6 Estimate reliability index for *posterior* distribution

After estimating the reliability index for *prior* distribution, we need to update the results to use *posterior* distribution to achieve precise fatigue reliability. As mentioned in CHAPTER 2, the product of *prior* and *likelihood* is *posterior* distribution. The amount of parameters for *posterior* distribution are taken from Table 2.4 to construct the limit state. Afterwards, the use of Rosenblatt transformation and the variables transfer to the 2D standard normal space to estimate the reliability index. Figure 3.7 shows the reliability index point that is obtained by MCS for *posterior* distribution and is equal to 2.62. The amount of the reliability index that is estimated by FORM is 2.11.

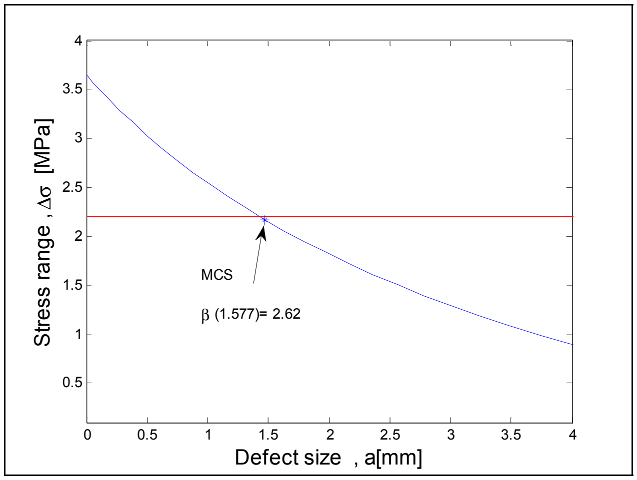


Figure 3.7: Reliability index for posterior distribution with MCS

3.6.1 Updating the posterior distribution to find the precise fatigue reliability

In this section, we want to update the results of previous reliability index to achieve a precise estimation. We have two test data that could replace the *likelihood* distribution and we update the *posterior* distribution with adding data. In order to update the posterior distribution, we use the latest posterior distribution as a prior and add test data that are replaced to the likelihood distribution. The amount of test data is shown in Table 3.2. Figure 3.8 and Figure 3.9 show the updated posterior distribution for two test data for crack size a [mm] and stress range $\Delta\sigma$ [MPa]. Table 3.3 shows the amount of parameter specifications with added data and updates.

Table 3.2: Test data (Likelihood distribution)

Likelihood	a [mm]	$\Delta\sigma$ [MPa]
Test Data 1	Gumbel (μ =1.80, σ =0.65)	Gumbel (μ =30.4, σ =5)
Test Data 2	Gumbel (μ =1.74, σ =0.4)	Gumbel (μ =29.7, σ =3.3)
Test Data 3	Gumbel (μ =1.67, σ =0.33)	Gumbel (μ =30.4, σ =5)

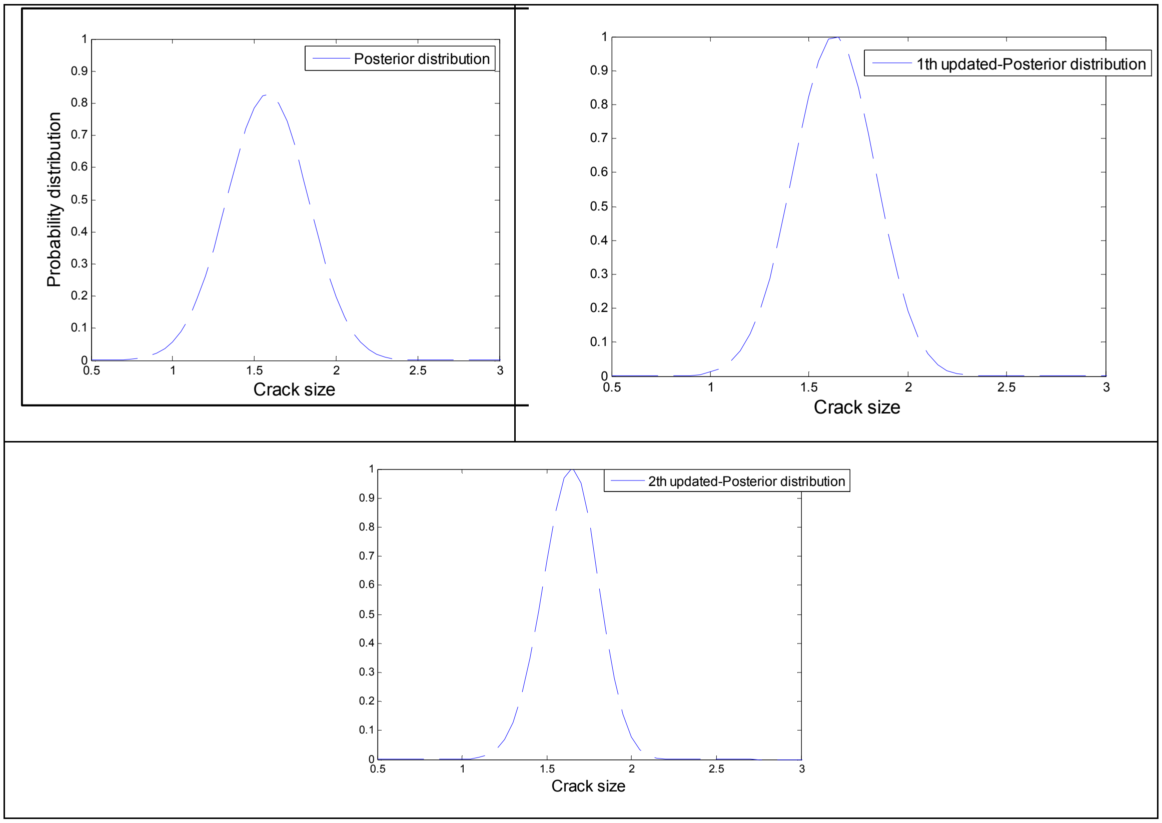
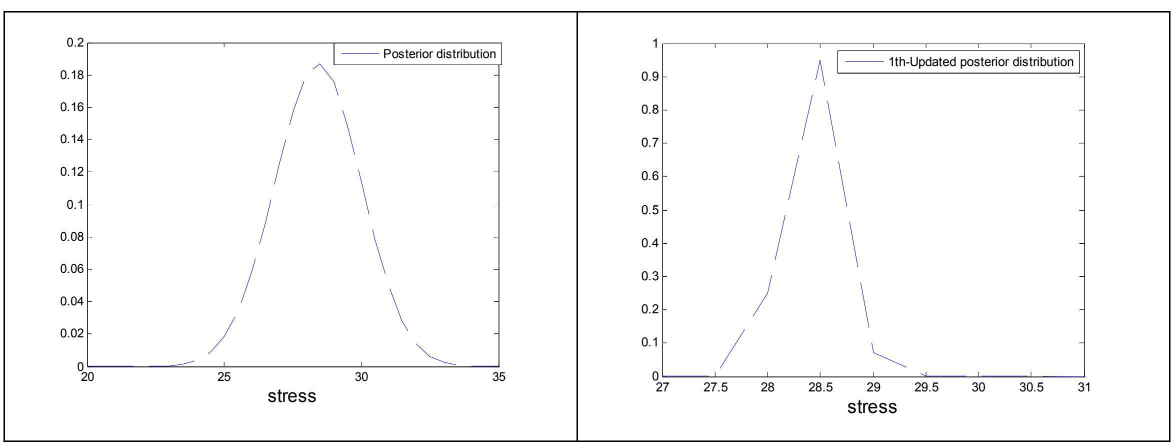


Figure 3.8: Two updated posterior distribution
for crack size



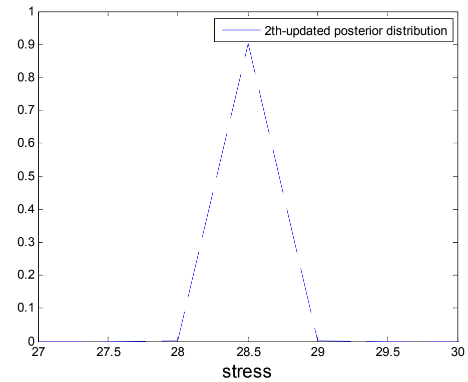


Figure 3.9: Two updated posterior distribution for stress range

Table 3.3: Amount of prior, likelihood and updated posterior distribution

No	Distribution	**Prior** (analytical result)	**Likelihood** (test data)	**Posterior**
1	a [mm]	Normal (μ =1.5, σ =0.5)	Gumbel (μ =1.80, σ =0.65)	μ =1.577, σ =0.435
		μ =1.577, σ =0.435	Gumbel (μ =1.74, σ =0.4)	1th-Update μ =1.624, σ =0.396
		μ =1.624, σ =0.396	Gumbel (μ =1.67, σ =0.33)	2th-Update μ =1.636, σ =0.282
2	$\Delta\sigma$ [MPa]	Normal (μ =28, σ =3)	Gumbel (μ =30.4, σ =5)	μ =28.41, σ =0.49
		μ =28.41, σ =0.49	Gumbel (μ =29.7, σ =3.3)	1th-Update μ =28.42, σ =0.133
		μ =28.42, σ =0.133	Gumbel (μ =30.4, σ =5)	2th-Update μ =28.5, σ =0.03

With the updated posterior distribution results shown in Table 3.3 the reliability index and probability failure could estimate. Table 3.4 shows the reliability index amount and probability failure for posterior distribution and two updated distributions with MCS and FORM. Figure 3.10 shows the probability of failure for the 2^{th} updated posterior distribution.

Table 3.4: Reliability index and probability failure for updated distribution

No	Distribution	β MCS	β FORM	Pr_F MCS	Pr_F FORM
1	**Prior**	2.60	1.88	0.004	0.029
2	**Posterior**	2.62	2.11	0.004	0.017
3	**1th-Update**	2.06	2.08	0.019	0.018
4	**2th-Update**	1.75	1.99	0.039	0.023

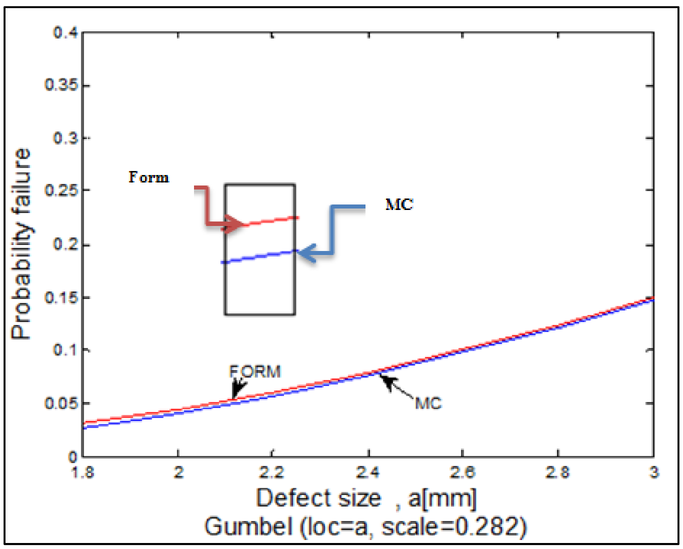


Figure 3.10: Evolution of the probability of failure vs crack size

Figure 3.10 shows that probability failure increases when the crack size grows. Also, we see that the difference of FORM and MCS is near to zero after the 2th updater of posterior distribution which causes a decrease in the uncertainty of parameters. We can see and compare the probability of failure for the prior and posterior distribution with FORM and MCS methods as shown in Figure 3.11 and Figure 3.12.

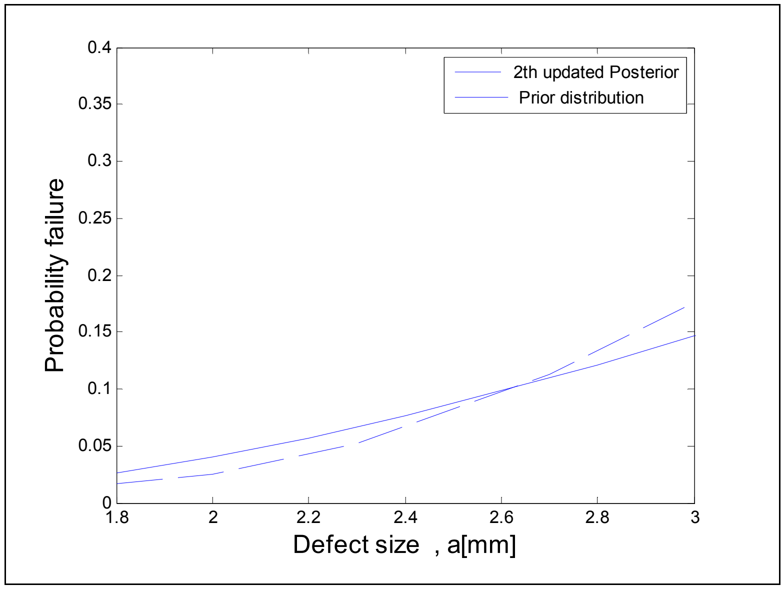


Figure 3.11: Probability of failure for prior and 2th updated posterior distribution with FORM method

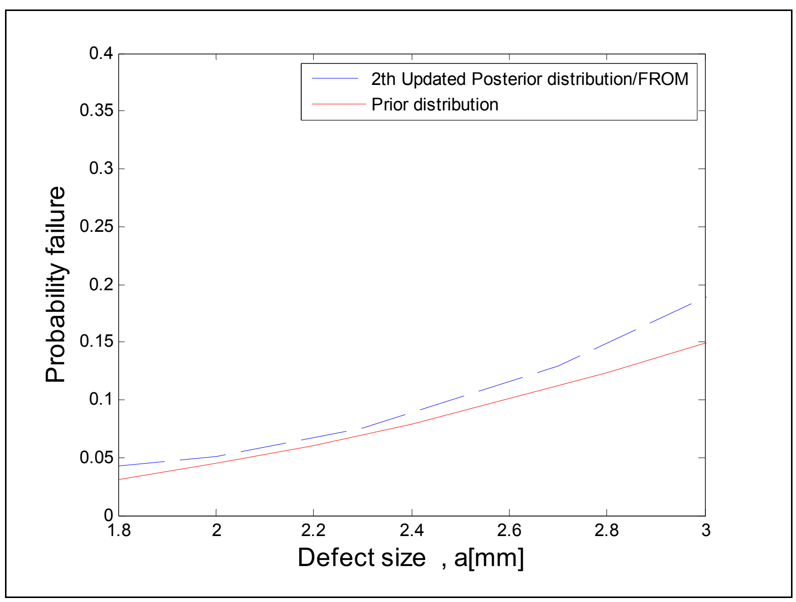


Figure 3.12: Probability of failure for prior and 2th updated posterior distribution with MCS method

As we see in Figure 3.11 and Figure 3.12, the amount of probability failure for posterior distribution is less than the prior distribution. Therefore, the failure occurs sooner than we expected. .

3.7 Select target reliability index

The target reliability index levels related to consequence failures are mainly planned for structures to compare the results with this target. The amount of this target is explained in some standards. Table 3.5 shows some target reliability levels that is suggested by international codes for design and assessment. It varies with the consequences of failure and the reference periods.

Table 3.5: Target reliability index

Codes	Consequences of reliability index			
EN 1990 IS0 9324 JCSS	Small	Low Some Minor	Normal Moderate Moderate	High Great Large
EN 1990-- 50 years IS0 9324--life time JCSS--50 years	- 1.3 -	3.3 2.3 2.5	3.8 3.1 3.2	4.2 3.8 3.5
EN 1990--1 year IS0 9324--1 year JCSS--1 year	- 2.9 -	4.2 3.5 3.7	4.7 4.1 4.2	5.2 4.7 4.4

3.8 Conclusion

According to the previous review, it appears that for non-linear systems, methods of FORM or Monte Carlo are appropriate for the estimation of the reliability index. For time-dependent systems, Monte Carlo simulations are effective (Guo, Watson et al. 2009).

The value of the reliability index must be accompanied by a probability of failure based on the updated prediction of crack growth rates. For calculating the reliability index, a Rosenblatt transformation technique is used to obtain a representation in iso-probabilistic space. After constructing the limit state with variables in standard form, we first estimate the reliability index for prior distribution with FORM and MCS methods. The accuracy of the FORM is compared with the MCS that was shown in Figure 3.10. We understand that when the uncertainties are large, the differences of MCS and FORM are considerable. After updating the distribution with Bayesian methodology, results show that the standard deviation of a stress range is changed (better estimation) and the differences of reliability index with FORM and MCS are smaller than before. As shown, the reliability index is very dependent on the type of distribution of variables and the amount of parameters. Therefore, determining the precise distribution affects estimating the reliability index. With an updated reliability index and a probability of failure, we could better-placed to predict turbine failure. Figure **3.13** shows the methodology that we used in CHAPTER 3 for estimating the reliability index.

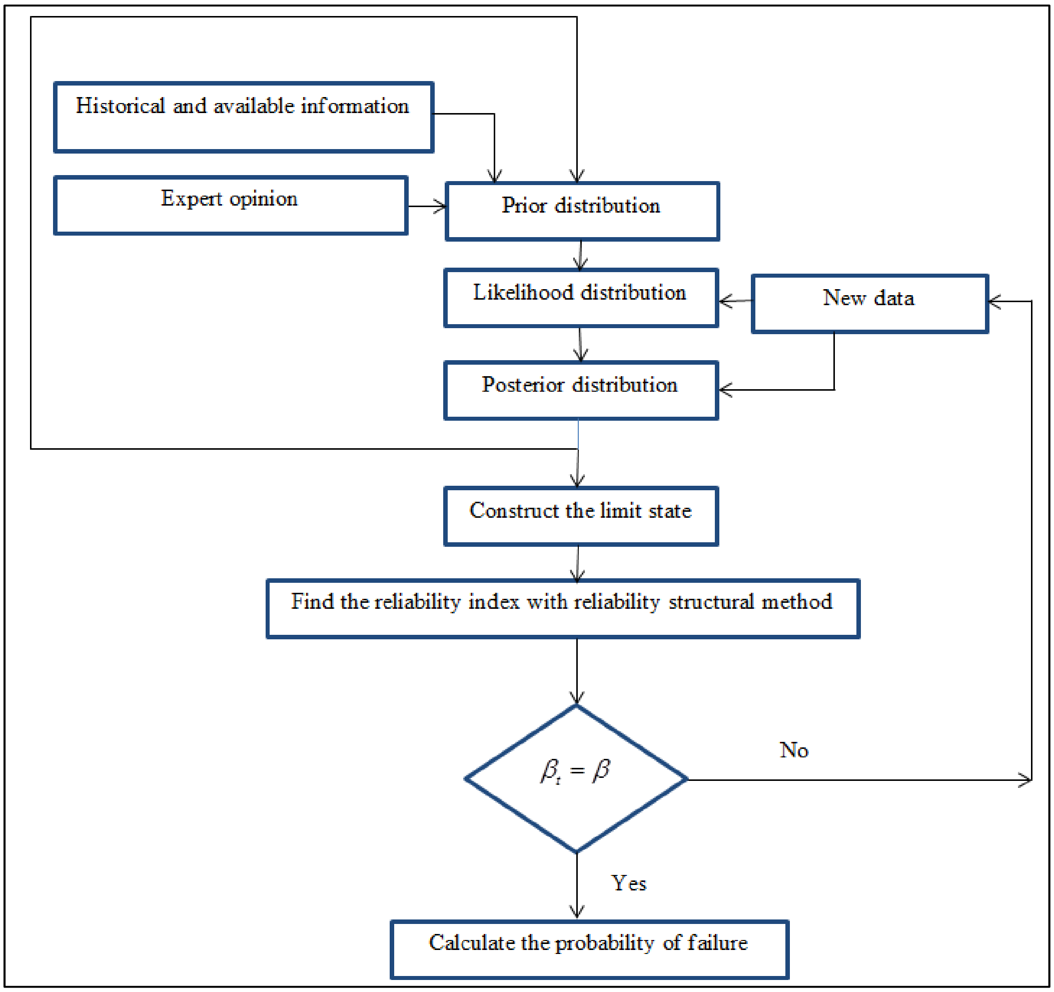


Figure 3.13: Methodology to update reliability index

CONCLUSION

In this thesis, we have described the concept of a fatigue reliability model and updated the variables of models for hydroelectric turbines to obtain a precise reliability index. We answered the following three questions:

- CHAPTER 2: How can we update our prior knowledge in light of new information gathered to obtain a posterior?
- CHAPTER 2: Can we estimate and decrease the uncertainty about the variables and parameters that exist in fatigue models?
- CHAPTER 3: How can we, given this new information, assess the validity of the reliability model used?

In CHAPTER 2, a Bayesian update is used to reduce the uncertainties that exist in variables that are related to a Paris formula. The distribution of variables is estimated by considering an initial uncertainty with normal distribution and a 95% confidence interval. In order to decrease the confidence interval band, we updated the initial distribution three times by using the data as likelihood distribution. Therefore, we found that the Bayesian method could reduce the uncertainty of variables when reducing the scatter data and standard deviation by almost 40%. Moreover, a Bayesian update has been applied to update the parameters of variables to find the precise amount of variables. The final results of posterior distribution after updating the variables and parameters are used to determine the fatigue reliability index. Therefore, the proposed method could account for uncertainties, as well as the presence of confidence intervals or error bands.

In CHAPTER 3 most of the issues that contribute to the phenomenon of cracking and hydraulic turbine fatigue are studied. In this study using the Bayesian method, due to existing complexity and more computational programming, certain consumed variables are constant and just examine two variables: crack size and stress range. The Kitagawa-Takahashi limit state is a suitable limit state for estimating the reliability of fatigue that constructs with two S-N curves and a LEFM method. The limit state is a boundary to determine the component's safe and failure mode. Because of a scarcity of information about variables, variable uncertainty is increased. Therefore, we have an interval limit state.

We demonstrated that that the Bayesian method is a suitable candidate for fatigue reliability modeling of turbine runners where prior information is scarce and highly subjective. With the Bayesian method and updating approach, we may be able to decrease the uncertainty of limit state.

After constructing the limit state, a transformation technique that transfers the variables to 2D standard forms is used. In this study, the Rosenblatt transformation is performed. The reliability index using the FORM and MCS method is estimated in this study. The accuracy of the FORM method is compared with the MCS. As demonstrated, the reliability index is very dependent on the type of distribution of variables, amount of parameters, and limit state function. We have shown that when the amount of parameters changed is very smooth, the result of reliability index is changed significantly. Therefore, each individual source of uncertainty needs to be identified and characterized to allow for a precise reliability index and decrease the risk of structural component failure.

RECOMENDATIONS

The following recommendations are offered as possible ways to improve this study.

- We choose normal distribution as our prior distribution, given that the precision of posterior distribution is very close to prior distribution and since it is recommended to choose several applicant distributions to find the best model to fit the data.

- We consider that ΔK_{th} is constant, but in reality this amount follows a specific distribution. It is therefore advised to find its precise distribution and updated it using by Bayesian method. Afterwards we could construct the limit state with three updated distributions that are highly significant in order to obtain a precise reliability index.

- It is recommended to perform a sensitivity analysis to find out how the output of a model changes with input variations. By doing so we could understand which parameters have more weight to estimate the reliability index. As we observed in CHAPTER 3, a little change in the amount of parameters could affect failure probability and the reliability index.

APPENDIX I

MATLAB CODE

GUMBEL DISTRIBUTION

```
% crack length

x = -5:.01:5;
plot(x,evpdf(x,1.81,0.65),'-', ...
     x,evpdf(x,1.66,0.33),':', ...
     x,evpdf(x,1.75,0.55),'-.');
legend({'mu = 1.81, sigma = 0.65', ...
        'mu = 1.66, sigma = 0.33', ...
        'mu = 1.74, sigma = 0.49'}, ...
       'Location','NW')
xlabel('crack size')
ylabel('f(crack size|mean value,s.deviation)')

%%

% stress range

x = 10:.01:40;
plot(x,evpdf(x,30.4,5),'-', ...
     x,evpdf(x,29.7,3.3),':', ...
     x,evpdf(x,26.6,1.3),'-.');
legend({'mu = 30.4, sigma = 5', ...
        'mu = 29.7, sigma = 3.3', ...
        'mu = 26.6, sigma = 1.3'}, ...
       'Location','NW')
xlabel('Stress range')
ylabel('f(Stress range|mean value,s.deviation)')
```

UPDATING VARIABLES BY BAYESIAN METHOD

```
 % STRESS UPDATED

clear all
clc

% Likelihood Distribution

stress=[1:0.5:50];
mu=30.4;
sigma=5;

% the test shows the result follow the gumbel distributin

gumbel_stress = evpdf(stress,mu,sigma);
normal_stress = normpdf(stress,mu,sigma);
product_stress=gumbel_stress.*normal_stress;

% Normalizing

int_stress(1)=2*10^(-5);
for i=1:98
int_stress(i+1)=product_stress(i+1)*0.5+int_stress(i);
end
int_infinit=0.065;
product_stress_normalized=product_stress/int_infinit;

figure(1)
plot(stress, int_stress,'*')

figure(11)
plot(stress, gumbel_stress)
legend({'Gumbel distribution(\mu=30.4,\sigma=5)'})
set(legend,'FontSize',12)
set(gcf, 'Color', [1,1,1]);
 xlabel('Stress range(MPa)','fontsize',14')
ylabel('Likelihood distribution','fontsize',14')
axis([25 50 0 0.08])

figure(22)
plot(stress, gumbel_stress,'g');
hold on
plot(stress, normal_stress,'r')
plot(stress,product_stress_normalized)
% plot(stress, int_stress,'*')
legend({'Gumbel distribution', ...
        'Normal distribution', 'Product distribution'})
set(legend,'FontSize',12)
set(gcf, 'Color', [1,1,1]);
Xlabel('Stress range(MPa)','fontsize',14')
ylabel('Probability distribution','fontsize',14')
axis([17 50 0 0.1])
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Construct prior distribution
% Tests: (confidence interval 95)%

mu_t1=22.12;
mu_t2=28;
mu_t3=33.88;
normal_stress_test1 = normpdf(stress,mu_t1,sigma);
normal_stress_test2 = normpdf(stress,mu_t2,sigma);
normal_stress_test3 = normpdf(stress,mu_t3,sigma);


% Test 1

figure(33)
plot(stress,product_stress_normalized);
hold on
plot(stress, normal_stress_test1,'r')
legend({'product distribution', ...
        'prior distribution(test 1),\mu=22.12,\sigma=3'})
set(gcf, 'Color', [1,1,1]);

xlabel('Stress range','fontsize',14')
ylabel('Probability distribution','fontsize',14')

title('Initial distribution','fontsize',14')
axis([17 50 0 0.1])

% UPDATE
update_1=product_stress_normalized.*normal_stress_test1;

int_update_1(1)=2*10^(-5);
for i=1:98
int_update_1(i+1)=update_1(i+1)*0.5+int_update_1(i);
end
figure(44)
% plot(stress, int_update_1,'*')
hold on
int_update_1=0.03;
update_1_normalized=update_1/int_update_1;
plot(stress,update_1_normalized,'--')
plot(stress, normal_stress_test2,'r')
 legend({'Initial distribution', ...
        'prior distribution(test 1),\mu=28,\sigma=3'})
set(gcf, 'Color', [1,1,1]);

xlabel('Stress range','fontsize',14')
ylabel('Probability distribution','fontsize',14')
 title('Updated with first test','fontsize',14')
axis([17 50 0 0.12])
```

```
 % Test2
 update_2=update_1_normalized.*normal_stress_test2;

int_update_2(1)=2*10^(-5);
for i=1:98
int_update_2(i+1)=update_2(i+1)*0.5+int_update_2(i);
end

figure(55)
% plot(stress, int_update_2,'*')
hold on
int_update_2=0.068;
update_2_normalized=update_2./int_update_2;
plot(stress,update_2_normalized,'--')
plot(stress, normal_stress_tes      t3,'r')

legend({'Updated', ...
        'prior distribution(test 2),\mu=33.88,\sigma=3'})

set(gcf, 'Color', [1,1,1]);

xlabel('Stress range','fontsize',14')
ylabel('Probability distribution','fontsize',14')
 title('Updated with second test','fontsize',14')
axis([17 50 0 0.12])

 %%%%%%%%%%%%%%%%%%%%%%%%%%%
 % Test3
 update_3=update_2_normalized.*normal_stress_test3;

int_update_3(1)=2*10^(-5);
for i=1:98
int_update_3(i+1)=update_3(i+1)*0.5+int_update_3(i);
end
figure(66)
% plot(stress, int_update_3,'*')
hold on
int_update_3=0.03;
update_3_normalized=update_3./int_update_3;
plot(stress,update_3_normalized,'--')
plot(stress, normal_stress_test3,'r')
 legend({'Updated', ...
        'prior distribution(test 3),\mu=33.99,\sigma=5'})
set(gcf, 'Color', [1,1,1]);


xlabel('Stress range','fontsize',14')
ylabel('Probability distribution','fontsize',14')
title('Updated with third test','fontsize',14')
axis([17 50 0 0.2])
```

```
figure (77)
plot(stress,update_3_normalized,'--')
legend({'Final (Posterior distribution)'})
set(gcf, 'Color', [1,1,1]);
xlabel('stress','fontsize',14')
ylabel('Posterior distribution','fontsize',14')
axis([17 50 0 0.2])


 % Mean Value

n=0;
d=0;
for i=1:99
    n=(update_3_normalized(i)*stress(i))+n;
    d=update_3_normalized(i)+d;
end
mean=n/d


 % Standard Deviation
sq=0;
for i=1:99
    sq=(update_3_normalized(i)*(stress(i)-mean))^2+sq;
end
std=sq^0.5
update_3_normalized_stress=update_3_normalized;

% *******************************************
% ***********
```

CRACK UPDATED

```
clear all
clc

% Likelihood Distribution
step=.05
crack=[-0.9:step:4];
mu=1.81;
sigma=0.65;

% the test shows the result follow the gumbel distributin

gumbel_crack = evpdf(crack,mu,sigma);
normal_crack = normpdf(crack,mu,sigma);
product_crack=gumbel_crack.*normal_crack;


% Normalizing
int_crack(1)=0.003;
for i=1:98
int_crack(i+1)=product_crack(i+1)*step+int_crack(i);
end
int_infinit=0.4;
product_crack_normalized=product_crack/int_infinit;

figure(1)
plot(crack, gumbel_crack)
legend({'Gumbel distribution(\mu=1.81,\sigma=0.65)'})
set(legend,'FontSize',12)
set(gcf, 'Color', [1,1,1]);

xlabel('Crack size(mm)','fontsize',14')
ylabel('Likelihood distribution','fontsize',14')
axis([0.5 4 0 1])

% ********************
figure(2)
plot(crack, gumbel_crack,'g');
hold on
plot(crack, normal_crack,'r')
plot(crack,product_crack_normalized)
plot(crack, int_crack,'*')
legend({'Gumbel distribution', ...
        'Normal distribution', 'Product distribution'})
set(legend,'FontSize',12)
set(gcf, 'Color', [1,1,1]);

xlabel('Crack size(mm)','fontsize',14')
ylabel('Probability distribution','fontsize',14')
axis([0.5 4 0 1])
% ****************************
% ****************************
```

```
% prior distributiin
% Tests: (confidence interval 95%
mu_t1=0.52;
mu_t2=1.5;
mu_t3=2.48;
sigma_t=0.5
normal_crack_test1 = normpdf(crack,mu_t1,sigma_t);
normal_crack_test2 = normpdf(crack,mu_t2,sigma_t);
normal_crack_test3 = normpdf(crack,mu_t3,sigma_t);

% Test 1
 figure(3)
plot(crack,product_crack_normalized);
hold on
plot(crack, normal_crack_test1,'r')
 legend({'product distribution', ...
        'prior distribution(test 1),\mu=0.52,\sigma=0.5'})
set(legend,'FontSize',12)
set(gcf, 'Color', [1,1,1]);
 xlabel('Crack size','fontsize',14')
ylabel('Probability distribution','fontsize',14')
 title('Initial distribution','fontsize',14')
axis([0.1 4 0 1])

% UPDATE
 update_1=product_crack_normalized.*normal_crack_test1;
 int_update_1(1)=2*10^(-5);
for i=1:98
int_update_1(i+1)=update_1(i+1)*step+int_update_1(i);
end

figure(4)
plot(crack, int_update_1,'*')
hold on
int_update_1=0.15;
update_1_normalized=update_1/int_update_1;
plot(crack,update_1_normalized,'--')
plot(crack, normal_crack_test2,'r')
 legend({'Initial distribution', ...
        'prior distribution(test 1),\mu=0.52,\sigma=0.5'})
set(legend,'FontSize',12)
set(gcf, 'Color', [1,1,1]);
xlabel('Crack size','fontsize',14')
ylabel('Probability distribution','fontsize',14')
 title('Updated with first test','fontsize',14')
 axis([0.5 4 0 1])

 % Test2
 update_2=update_1_normalized.*normal_crack_test2;
 int_update_2(1)=2*10^(-5);
for i=1:98
int_update_2(i+1)=update_2(i+1)*step+int_update_2(i);
end
```

```
figure(5)
plot(crack, int_update_2,'*')
hold on
int_update_2=0.65;
update_2_normalized=update_2./int_update_2;
plot(crack,update_2_normalized,'--' )
plot(crack, normal_crack_test3,'r')
 legend({'Updated', ...
        'prior distribution(test 2),\mu=1.5,\sigma=0.5'})
set(legend,'FontSize',12)
set(gcf, 'Color', [1,1,1]);
 xlabel('Crack size','fontsize',14')
ylabel('Probability distribution','fontsize',14')
 title('Updated with second test','fontsize',14')
axis([0.5 4 0 1])

% ****************************
% Test3
update_3=update_2_normalized.*normal_crack_test3;
int_update_3(1)=2*10^(-5);
for i=1:98
int_update_3(i+1)=update_3(i+1)*step+int_update_3(i);
end

figure(6)
plot(crack, int_update_3,'*')
hold on
int_update_3=0.1;
update_3_normalized=update_3./int_update_3;
plot(crack,update_3_normalized,'--')
plot(crack, normal_crack_test3,'r')
legend({'Updated', ...
        'prior distribution(test 3),\mu=2.48,\sigma=0.5'})
set(legend,'FontSize',12)
set(gcf, 'Color', [1,1,1]);
 xlabel('Crack size','fontsize',14')
ylabel('Probability distribution','fontsize',14')

title('Updated with third test','fontsize',14')
axis([0.5 4 0 1])

figure (7)
plot(crack,update_3_normalized,'--')
legend({'Final'})
set(legend,'FontSize',12)
set(gcf, 'Color', [1,1,1]);
xlabel('Crack size','fontsize',14')
ylabel('Probability distribution','fontsize',14')
axis([0.5 4 0 1])

% ****************************
% ****************************
% ****************************
```

```
% MEAN value and STANDARD DEVIATION
n=0;
d=0;
for i=1:99
    n=(update_3_normalized(i)*crack(i))+n;
    d=update_3_normalized(i)+d;
end
mean=n/d

sq=0;
for i=1:99
    sq=(update_3_normalized(i)*(crack(i)-mean))^2+sq;
end
std=sq^0.5
update_3_normalized_crack=update_3_normalized;
```

UPDATING PARAMETERS BY BAYESIAN METHOD

```
clear

% % CODE IS AVAILBLE FOR CRACK SIZE
% this a code for calculating posterior distribution and updating of
% parameters
% the prior is uniform
% the likelihood is gumbel distribution

mu=1.75;
sigma=0.55;

beta_mean=(sigma/1.2)^0.5;
alpha_mean=mu-0.57772*beta_mean;

x=[0:0.1:3];
mu_stress=[0.1:0.1:3.1];
standard_deviation=[0.1:0.1:3.1];
sigbet=0.2;
mubet=sigma;
mu_stress_interval=30;

for k=1:length(standard_deviation)
    for j=1:length (mu_stress)
        for i=1:length(x)

                        likelihood(k,j,i)=1/standard_deviation(k)*exp(-
(x(i)-mu_stress(j))/standard_deviation(k))*exp(-exp(-(x(i)-
mu_stress(j))/standard_deviation(k)));



%             perior
            pbeta(k)=1/(sigbet*(2*pi)^0.5)*exp(-(standard_deviation(k)-
mubet)^2/2/sigbet^2);
            palpha(j)=1/mu_stress_interval;

%             Posterior
            post(k,j,i)=pbeta(k)*palpha(j)*likelihood(k,j,i);


        end
    end
end


plot(mu_stress,likelihood(:,1,1),'-', ...
     mu_stress,pbeta(:),':',...
       mu_stress,post(:,1,1),'-.');

legend({'likelihood', ...
```

```
        'prior', ...
        'posterior'}, ...
       'Location','NE')

xlabel('a[mm]')
ylabel('Posterior of crack lenght')


figure(2)
plot(mu_stress,likelihood(:,1,1),'-', ...
     mu_stress,pbeta(:),':',...
       mu_stress,post(:,1,1),'-.');

legend({'likelihood', ...
        'prior', ...
        'posterior'}, ...
       'Location','NE')

xlabel('a[mm]')
ylabel('Probability distribution of crack lenght')

figure(3)
plot(mu_stress,likelihood(10,:,5))
xlabel('Expected value[mm]')
ylabel('likelihood of crack')


figure(4)
plot(standard_deviation,likelihood(:,8,1))
xlabel('Standard deviation')
ylabel('likelihood distribution of crack')


figure(5)
plot(standard_deviation,pbeta(:))
xlabel('Standard deviation')
ylabel('Prior of crack')

figure(6)
plot(mu_stress,palpha(:))
xlabel('Expected value[mm]')
ylabel('Prior of crack')

figure(7)
surf(mu_stress,standard_deviation,post(:,:,20))
xlabel('Expected value of crack')
ylabel('Standard deviation of crack')
zlabel('Posterior of crack')
clear
```

```
% % CODE IS AVAILBLE FOR STRESS RANGE
% this a code for calculating posterior distribution
% the prior is uniform
% the likelihood is gumbel distribution

 mu=30.4;
sigma=5;
x=[25:1:50];
mu_stress=[10:1.66:60];
standard_deviation=[2:0.2:8];
sigbet=1.5;
mubet=sigma;
mu_stress_interval=30;

for k=1:length(standard_deviation)
    for j=1:length (mu_stress)
        for i=1:length(x)

                        likelihood(k,j,i)=1/standard_deviation(k)*exp(-
(x(i)-mu_stress(j))/standard_deviation(k))*exp(-exp(-(x(i)-
mu_stress(j))/standard_deviation(k)));


%             perior
            pbeta(k)=1/(sigbet*(2*pi)^0.5)*exp(-(standard_deviation(k)-
mubet)^2/2/sigbet^2);
            palpha(j)=1/mu_stress_interval;

%             Posterior
            post(k,j,i)=pbeta(k)*likelihood(k,j,i);



        end
    end
end

 figure(1)

plot(mu_stress,likelihood(:,1,1),'-', ...
     mu_stress,pbeta(:),':',...
       mu_stress,post(:,1,1),'-.');

legend({'likelihood', ...
        'prior', ...
        'posterior'}, ...
       'Location','NE')

xlabel('Stress range')
ylabel('Posterior of stress
range','fontsize',16,'fontweight','b','color','b')
```

```
figure(2)
plot(mu_stress,likelihood(10,:,5))
plot(mu_stress,palpha(:))
plot(mu_stress,post(:,3,20))

xlabel('\Delta\sigma[Mpa]')
ylabel('Prior of \Delta\sigma','fontsize',14)


figure(3)
plot(mu_stress,likelihood(10,:,5))
xlabel('\mu')
ylabel('L(\Delta\sigma|\mu)','fontsize',14)


figure(4)
plot(standard_deviation,likelihood(:,8,1))
xlabel('\sigma')
ylabel('L(\Delta\sigma|\sigma)','fontsize',14)

figure(5)
plot(standard_deviation,pbeta(:))
xlabel('\sigma')
ylabel('Prior of \Delta\sigma','fontsize',14)

figure(6)
plot(mu_stress,palpha(:))
xlabel('\mu')
ylabel('Prior of \Delta\sigma','fontsize',14)

figure(7)
surf(mu_stress,standard_deviation,post(:,:,20))
xlabel('\mu')
ylabel('\sigma')
zlabel('Posterior(\mu,\sigma|\Delta\sigma)','fontsize',14)
```

RELIABILITY INDEX WITH FORM METHOD

```
clear all

mu_s=28.42;
sigma_s=0.7;
mu_c=1.624;
sigma_c=0.396;

stress_g_original=[1:0.5:50];
step=.05
crack=[-0.9:step:4];
crack_g_original=sort(crack);


% ROSENBLATH TRANSFORMATION
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

a=pi/(6^0.5*sigma_s);
b=mu_s-0.5772/a;
x=stress_g_original;
CDF_gum_s = evcdf(x,mu_s,sigma_s)
a=pi/(6^0.5*sigma_c);
b=mu_c-0.5772/a;
x=crack_g_original;
CDF_gum_c = evcdf(x,mu_c,sigma_c)
figure(1001)
plot(crack_g_original,CDF_gum_c)
xlabel('Crack')
ylabel ('CDF of Posterior function');

%phie-1 CDF_gum_s
 z_stress = norminv(CDF_gum_s,0,1);

j=1;
for i=1:length(z_stress)
    if z_stress(i)<8 & z_stress(i)>-8
        z_stress_corect(j)= z_stress(i);
        j=j+1;
    end
end

 %phie-1 CDF_gum_c
z_crack = norminv(CDF_gum_c,0,1);

j=1;
for i=1:length(z_crack)
    if z_crack(i)<6 & z_crack(i)>-6
        z_crack_corect(j)= z_crack(i);
        j=j+1;
    end
end
 %normal distribution pdf
```

```
norm_stress = normpdf(z_stress_corect,0,1);
norm_crack = normpdf(z_crack_corect,0,1);


%Gumbel distribution pdf
stress_g = stress_g_original;
crack_g = crack_g_original;

pdf_gum_s = evpdf(stress_g,mu_s,sigma_s);
pdf_gum_c = evpdf(crack_g,mu_c,sigma_c);


%finding new sigma and mu


for i=1:length(norm_stress)
    sigma_stress_nor(i)=norm_stress(i)/pdf_gum_s(i);
    mu_stress_nor(i)=stress_g(i)-sigma_stress_nor(i)*z_stress_corect(i)
end

for i=1:length(norm_crack)
    sigma_crack_nor(i)=norm_crack(i)/pdf_gum_c(i);
    mu_crack_nor(i)=crack_g(i)-sigma_crack_nor(i)*z_crack_corect(i);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% finding beta and alphas
b(1)=6;  %initial value for beta
a1(1)=50;     %initial value for alpha 1
a2(1)=1;    %initial value for alpha 2

j=1;
for i=1:length(mu_crack_nor)
    if abs(mu_crack_nor(i))<2*mu_c
    MU_crack_nor(j)=mu_crack_nor(i);
    SIGMA_crack_nor(j)=sigma_crack_nor(i);
    j=j+1;
    end

end

j=1;
for i=1:length(mu_stress_nor)
    if abs(mu_stress_nor(i))<2*mu_s
    MU_stress_nor(j)=mu_stress_nor(i);
    SIGMA_stress_nor(j)=sigma_stress_nor(i);
    j=j+1;
    end

end

mu_crack= mean(MU_crack_nor);
```

```
sigma_crack=mean(SIGMA_crack_nor);


mu_sigma= mean(MU_stress_nor);
sigma_sigma=mean(SIGMA_stress_nor);

 % Finding the limit state

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% countour stress range and crack size
x_norm=sort(z_stress_corect);
xx_norm=sort(z_crack_corect);

 %for g=0, we have the following:
Kth=2;
Ya=1;
i=1;
q=75;
s1=sigma_crack;
m1=mu_crack;
s2=sigma_sigma;
m2=mu_sigma;
% mu_sigma=0;
m2=mu_sigma;

for j=1:length(xx_norm)
    if xx_norm(j)>-m1
        norm_crack_Calculated(i)=xx_norm(j);
        norm_stress_reduced_value(i)= (q/(xx_norm(j)*s1+m1)^(0.5)-m2)/s2;
%         norm_stress_reduced_value(i)=Kth/(pi*xx_norm(j))^0.5/Ya;



        i=i+1;
    end

end
mu_cr=mu_crack;
sigma_cr=sigma_crack;
mu_s=mu_sigma;
sigma_s=sigma_sigma;

mu_exp_stress=45;

stress_Experimental=mu_exp_stress*ones(length(norm_crack_Calculated),1);
crack_Calculated=sigma_cr*norm_crack_Calculated+mu_cr;
stress_reduced_value=sigma_s*norm_stress_reduced_value+mu_s;




%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% axes transfer

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


% Tr_x=(b(length(a2))*a2(length(a2)));
% Tr_y=(b(length(a2))*a1(length(a1)));

Tr_x=0;
Tr_y=0;

crack_Tr=norm_crack_Calculated-Tr_x;
stress_Tr=norm_stress_reduced_value-Tr_y;
stress_exp_Tr=(stress_Experimental-mu_sigma)/sigma_sigma-Tr_y;
length_size=min(length(crack_Tr),length(stress_Tr))
for i=1:length_size
    crack_Tr_plot(i)=crack_Tr(i);
    stress_Tr_plot(i)=stress_Tr(i);
end

figure(1)
plot(crack_Calculated,stress_reduced_value,'b')
hold on
plot(crack_Calculated,stress_Experimental,'r')
hold on
plot(mu_c,mu_s,'*')
hold on
xlabel({'Defect size , a[mm]',...
'Normal (loc=1.5, scale=0.5)'},'fontsize',14')
ylabel({'Stress range , \Delta\sigma [MPa]',...
'Normal (loc=28, scale=3)'},'fontsize',14')
set(gcf, 'Color', [1,1,1]);
axis([1.1 5 0.1 50])

figure(2)
plot(crack_Tr_plot,stress_Tr_plot,'b')
hold on
plot(crack_Tr,stress_exp_Tr,'r')
% legend({'Threshold stress', ...
%  'endurance stress'})
% set(legend,'FontSize',12)

set(gcf, 'Color', [1,1,1]);
xlabel('Defect size  , a[mm]','fontsize',14')
ylabel('Stress range , \Delta\sigma  [MPa]','fontsize',14')
set(gcf, 'Color', [1,1,1]);
axis([0 4 0.1 4])
text(0.869,1.21,'  \beta (1.577)= 2.62')
text(0.869,1.6,'  MCS')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% finding minimum beta
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
BETA=1000;

for i=1:length(crack_Tr_plot)
    beta(i)=((crack_Tr(i))^2+(stress_Tr(i))^2)^0.5

    if beta(i)< BETA
        BETA=beta(i);
        indx=i;
    end


end
% betamin=min(beta)


if stress_Tr(indx)> stress_exp_Tr(1)
    BETA=stress_exp_Tr(1);
    figure(2)
    plot(crack_Tr_plot(1),BETA,'*')
else
    figure(2)
    plot(crack_Tr_plot(indx),stress_Tr_plot(indx),'*')
end

hold on

crack_real = sigma_cr*crack_Tr_plot(indx)+mu_cr;
stress_real = sigma_s*stress_Tr_plot(indx)+mu_s;

mu_fixed_stress = mu_exp_stress;
stress_fixed=mu_fixed_stress*ones(length(norm_crack_Calculated),1);
stress_fixed_Tr=(stress_fixed-mu_s)/sigma_s-Tr_y;


BETA_fixed=[2.6 1.96 1.62 1.21 0.93 0.97 1.01];
BETA_fixed_form=[1.88 1.64 1.44 1.13 0.88 0.93 0.96 ]

crack_Tr_plot=[ 1.5  2 2.3 2.7  3 3.5 4]

 prob_beta=normcdf(-BETA_fixed);
prob_beta_form=normcdf(-BETA_fixed_form);

 figure(3)
plot(crack_Tr_plot,prob_beta,'b')
xlabel({'Defect size  , a[mm]',...
'Gumbel (loc=1.5, scale=0.5)'},'fontsize',14')
ylabel('Probability failure','fontsize',14')
set(gcf, 'Color', [1,1,1]);
axis([1.5 3 0.01 0.4])
```

```
figure(33)
plot(crack_Tr_plot,prob_beta,'b')
hold on
plot(crack_Tr_plot,prob_beta_form,'r')
 xlabel({'Defect size  , a[mm]',...
'Normal (loc=a, scale=0.5)'},'fontsize',14')
ylabel('Probability failure','fontsize',14')
set(gcf, 'Color', [1,1,1]);
axis([1.5 3 0.0 0.4])
text(2.5,0.05,'MC')
text(2.1,0.08,' FORM')


% finding beta and alphas
 const=m1^0.5;
 fix=3*q/2/m1;
mul=q/2/m1^1.5;
z1(1)=1;
z2(1)=-1;

% linear limit state
mean_value=-1.5*q*m1^-.5+m2+q/2*m1^-1.5*m1;
standard_linear=(s2^2+(q/2*m1^-1.5*s1)^2)^0.5;
beta_mean=-mean_value/standard_linear;

figure(4)
 plot(crack_Tr_plot,BETA_fixed,'b')
hold on
plot(mu_c,BETA,'*')
hold on
plot(crack_Tr_plot,BETA_fixed_form,'r')
hold on
plot(mu_c,beta_mean,'o')
 set(gcf, 'Color', [1,1,1]);
xlabel({'Defect size  , a[mm]',...
'Normal (loc=1.5, scale=0.5)'},'fontsize',14')
ylabel('Reliability index','fontsize',14')
set(gcf, 'Color', [1,1,1]);
 text(1.7,2.8,'  \beta (1.5)= 2.60')
text(1.7,1.6,'  \beta (1.5)=1.88')
text(1.7,2.9,'  MCS')
text(1.7,1.3,'FORM')
axis([1.2 3 0.1 3])
 BETA_fixed = -norminv(prob_beta);
prob_mcs=normcdf(-BETA);
prob_form=normcdf(-beta_mean);

BETA
indx
 beta_mean
 prob_mcs
```

RELIABILITY INDEX WITH MCS

```
clear all
 c0  =   2.51517;
 c1  =   0.802853;
 c2  =   0.01038;
 d1  =   1.432788;
 d2  =   0.189269;
 d3  =   0.001308;

  %U1

mu1     =   1;
u1  =   exp(-rand(1,10)*5);
x1  =   -1/mu1*(log(u1/mu1));

%U2

sigma2  =   10;
mu2     =   50;
u2_general  =   normrnd(0,1,[1 50]);

u2  =  abs( u2_general/norm(u2_general));

for i=1:length(u2)
    if u2(i)<0.5

        t2  =   sqrt(-log(u2(i).^2));

        z2  =   -t2+(c0+c1*t2+c2*t2.^2)/(1+d1*t2+d2*t2.^2+d3*t2.^3);

    else

        u2_star     =   1-u2(i);

        t2  =   sqrt(-log(u2_star.^2));

        z2  =   -(-t2+(c0+c1*t2+c2*t2.^2)/(1+d1*t2+d2*t2.^2+d3*t2.^3));

    end
    x2 (i)  =   mu2+z2*sigma2;
end

%U3
 sigma3  =   10;
 mu3     =   60;
 u3_general  =   normrnd(0,1,[1 50]);
 u3  =   abs(u3_general/norm(u3_general));

 for i=1:length(u3)
    if u3 (i)<0.5
```

```
        t3  =   sqrt(-log(u3(i).^2));

        z3  =   -t3+(c0+c1*t3+c2*t3.^2)/(1+d1*t3+d2*t3.^2+d3*t3.^3);

    else

        u3_star     =   1-u3(i);

        t3  =   sqrt(-log(u3_star.^2));

        z3  =   -(-t3+(c0+c1*t3+c2*t3.^2)/(1+d1*t3+d2*t3.^2+d3*t3.^3));

    end

    x3(i)  =   mu3+z3*sigma3;

end

% U4
 sigma4  =   0.01;
 mu4     =   14.4;
 u4_general  =   normrnd(0,1,[1 50]);
 u4  =   abs(u4_general/norm(u4_general));

for i=1:length(u4)
    if u4(i)<0.5

        t4  =   sqrt(-log(u4(i).^2));

        z4  =   -t4+(c0+c1*t4+c2*t4.^2)/(1+d1*t4+d2*t4.^2+d3*t4.^3);

    else

        u4_star     =   1-u4(i);

        t4  =   sqrt(-log(u4_star.^2));

        z4  =   -(-t4+(c0+c1*t4+c2*t4.^2)/(1+d1*t4+d2*t4.^2+d3*t4.^3));

    end
        x4 (i) =   mu4+z4*sigma4;
end

% U5

sigma5  =   0.3;
 mu5     =   -29.9;

u5_general  =   normrnd(0,1,[1 50]);
u5  = abs( u5_general/norm(u5_general));
```

```
for i=1:length(u5)
    if u5(i)<0.5

        t5  =   sqrt(-log(u5(i).^2));

        z5  =   -t5+(c0+c1*t5+c2*t5.^2)/(1+d1*t5+d2*t5.^2+d3*t5.^3);

    else

        u5_star     =   1-u5(i);

        t5  =   sqrt(-log(u5_star.^2));

        z5  =   -(-t5+(c0+c1*t5+c2*t5.^2)/(1+d1*t5+d2*t5.^2+d3*t5.^3));

    end

    x5(i)  =   mu5+z5*sigma5;
end

 muA     =   8;

u_A_rep  =   exp(-rand(1,50)*5);

A_rep  =   -1/muA*(log(u_A_rep/muA));


% all xi should be in the interval of interest.
 % varies from 0 to 1

count_principal     =   0;

count_failure       =   0;

ss  =   0;

 for N=0:0.1:.9

    for i=1:length(u1)
        for j=1:length(u2)

            j

            for k=1:length(u3)


                for l=1:length(u4)

                    for m=1:length(u5)
```

```
                    if x1(i)>0 & x1(i)<8 & x2(j)<100 & x2(j)>10 &
x3(k)<100 & x3(k)>10 & x4(l)<16 & x4(l)>13 & x5(m)<-25 & x5(m)>-35


                        Mn  =   u2(j)-((1-
1/2*u4(l))*u5(m)*(sqrt(pi)*u3(k)).^u4(l)*N+u1(i).^(1-.5*u4(l))).^((1-
.5*u4(l)).^-1);


                        for o=1:length(u_A_rep)
                            count_principal     =   count_principal+1;

                            u_X_rep=u2(j)-Mn;

                            H_rep   =   u_A_rep(o)-u_X_rep;


                            ss=ss+1;

                            if H_rep<0


                                if Mn   <   0

                                    count_failure=count_failure+1;

                                end

                            end

                        end

                    end

                end

            end

        end

    end

    CP(round(N*10+1))   =   count_principal;

    CF(round(N*10+1))   =   count_failure;

    P(round(N*10+1))    =   count_failure/count_principal;
```

```
        count_principal     =   0;

        count_failure       =   0;

        p_crack(i,round(N*10+1))     =   P(round(N*10+1));
    end

end
  beta= [ 4.5   4.1   3.4   3   2.8  2.6  2.4  2.2  2.1  2   ]

  N_rep=2*10^5;

N_original=[0:0.1:.9]*(10^7-10^5)+10^5+N_rep;
   figure(1)
 semilogx(N_original,beta)
 xlabel('N')
 ylabel ('\beta')
  x1  =  8*u1;
```

LIST OF REFERENCES

Acar, E., K. N. Solanki, et al. (2010). "Stochastic uncertainty analysis of damage evolution computed through microstructure–property relations." *Probabilistic Engineering Mechanics* **25**(2): 198-205.

An, D., J.-H. Choi, et al. (2011). "Fatigue life prediction based on Bayesian approach to incorporate field data into probability model." Structural engineering & mechanics **37**(4): 427.

Anderson, T. L. (2005). Fracture mechanics: fundamentals and applications, CRC press.

Box, G. E. and G. C. Tiao (2011). Bayesian inference in statistical analysis, John Wiley & Sons.

BS, B. S. (2000). "Guidance on methods for assessing the acceptability of flaws in metallic structures." British Standards Institution.

Castillo, E. and A. Fernández-Canteli (2009). A unified statistical methodology for modeling fatigue damage, Springer.

Castillo, E., A. Fernández-Canteli, et al. (2008). "A general model for fatigue damage due to any stress history." International Journal of Fatigue **30**(1): 150-164.

Chan, K. S., M. P. Enright, et al. (2014). "Development of a probabilistic methodology for predicting hot corrosion fatigue crack growth life of gas turbine engine disks." Journal of Engineering for Gas Turbines and Power **136**(2): 022505.

Coppe, A., R. T. Haftka, et al. (2010). "Uncertainty reduction of damage growth properties using structural health monitoring." Journal of Aircraft **47**(6): 2030-2038.

Cross, R. J., A. Makeev, et al. (2006). "A comparison of predictions from probabilistic crack growth models inferred from Virkler's data." Journal of ASTM International **3**(10): 1-11.

Cullen, A. C. and H. C. Frey (1999). Probabilistic techniques in exposure assessment: a handbook for dealing with variability and uncertainty in models and inputs, Springer Science & Business Media.

Ditlevsen, O. and H. O. Madsen (1996). Structural reliability methods, Wiley New York.

Dong, W., Z. Gao, et al. (2008). "Fatigue reliability analysis of Jacket-type offshore wind turbine considering inspection and repair." Planning **130**.

Ebeling, C. E. (2004). An introduction to reliability and maintainability engineering, Tata McGraw-Hill Education.

El Haddad, M., T. Topper, et al. (1979). "Prediction of non propagating cracks." Engineering Fracture Mechanics **11**(3): 573-584.

Farahmand, B., G. Bockrath, et al. (1997). Fatigue and fracture mechanics of high risk parts: application of LEFM & FMDM theory, Chapman & Hall.

Gagnon, M., A. Tahan, et al. (2013). "On the Fatigue Reliability of Hydroelectric Francis Runners." Procedia Engineering **66**: 565-574.

Gagnon, M., A. Tahan, et al. (2013). "A probabilistic model for the onset of High Cycle Fatigue (HCF) crack propagation: Application to hydroelectric turbine runner." International Journal of Fatigue **47**: 300-307.

Goranson, U. G. (1997). "Fatigue issues in aircraft maintenance and repairs." International Journal of Fatigue **19**(93): 3-21.

Guerin, F. and R. Hambli (2009). "Bayesian method approach for fatigue life distribution estimation of rubber components." International Journal of Product Development **7**(3): 199-217.

Guo, H., S. Watson, et al. (2009). "Reliability analysis for wind turbines with incomplete failure data collected from after the date of initial installation." Reliability Engineering & System Safety **94**(6): 1057-1063.

Haapalainen, J. and E. Leskela (2012). Probability of detection simulations for ultrasonic pulse-echo testing. 18th World Conference on Nondestructive Testing.

Hadavinia, H., A. Kinloch, et al. (2003). "The prediction of crack growth in bonded joints under cyclic-fatigue loading II. Analytical and finite element studies." International journal of adhesion and adhesives **23**(6): 463-471.

Hasofer, A. M. and N. C. Lind (1974). "Exact and invariant second-moment code format." Journal of the Engineering Mechanics division **100**(1): 111-121.

Heyman, B., A. Alaszewski, et al. (2013). "Probabilistic thinking and health risks: an editorial." Health, risk & society **15**(1): 1-11.

Huth, H.-J. (2005). "Fatigue design of hydraulic turbine runners."

Kappas, J. (2002). Review of risk and reliability methods for aircraft gas turbine engines, DTIC Document.

Karandikar, J. M., N. H. Kim, et al. (2012). "Prediction of remaining useful life for fatigue-damaged structures using Bayesian inference." Engineering Fracture Mechanics **96**: 588-605.

Kruzic, J. and R. Ritchie (2006). "Kitagawa-Takahashi diagrams define the limiting conditions for cyclic fatigue failure in human dentin." Journal of Biomedical Materials Research Part A **79**(3): 747-751.

Kumar, P. and K. Prashant (2009). Elements of fracture mechanics, Tata McGraw-Hill Education.

Liu, X., Y. Luo, et al. (2014). Fatigue Analysis of the Piston Rod in a Kaplan Turbine Based on Crack Propagation under Unsteady Hydraulic Loads. IOP Conference Series: Earth and Environmental Science, IOP Publishing.

Madsen, H. O., S. Krenk, et al. (2006). Methods of structural safety, Courier Dover Publications.

Manuel, L., P. S. Veers, et al. (2001). "Parametric models for estimating wind turbine fatigue loads for design." Journal of solar energy engineering **123**(4): 346-355.

Melchers, R. E. (1999). "Structural reliability analysis and predictionJohn Wiley & Sons." New York.

Moriarty, P. J., W. Holley, et al. (2004). Extrapolation of extreme and fatigue loads using probabilistic methods, Citeseer.

Pattabhiraman, S. and N. H. Kim (2009). Bayesian Technique for Reducing Uncertainty in Fatigue Failure Model, SAE Technical Paper.

Pattabhiraman, S., G. Levesque, et al. (2010). "Uncertainty analysis for rolling contact fatigue failure probability of silicon nitride ball bearings." International Journal of Solids and Structures **47**(18): 2543-2553.

Pirondi, A. and F. Moroni (2010). "A progressive damage model for the prediction of fatigue crack growth in bonded joints." The Journal of Adhesion **86**(5-6): 501-521.

Raju, I. and T. O'Brien (2008). "Fracture mechanics concepts, stress fields, strain energy release rates, delamination initiation and growth criteria." Delamination behaviour of composites.

Rau Jr, C. and P. Besuner (1980). "Risk analysis by probabilistic fracture mechanics." Design Feature, Product Engrg.(October 1979).

Ritchie, R. (1977). "Influence of microstructure on near-threshold fatigue-crack propagation in ultra-high strength steel." Metal Science **11**(8-9): 368-381.

Ronold, K. O., J. Wedel-Heinen, et al. (1999). "Reliability-based fatigue design of wind-turbine rotor blades." Engineering Structures **21**(12): 1101-1114.

Thibault, D., P. Bocher, et al. (2011). "Reformed austenite transformation during fatigue crack propagation of 13% Cr–4% Ni stainless steel." Materials Science and Engineering: A **528**(21): 6519-6526.

Toft, H. S. and J. D. Sørensen (2011). "Reliability-based design of wind turbine blades." Structural Safety **33**(6): 333-342.

Tong, Y. C. (2001). Literature review on aircraft structural risk and reliability analysis, DTIC Document.

Trudel, A., M. Sabourin, et al. (2014). "Fatigue crack growth in the heat affected zone of a hydraulic turbine runner weld." International Journal of Fatigue **66**: 39-46.

VĂCĂREANU, R., A. ALDEA, et al. (2007). Structural Reliability and Risk Analysis, UTCB.

Vorobyev, O. Y. Contemporary Uncertainty Theories: an Eventological View. Труды VIII международной ФАМ'2009 конференции. Ч. 1./под ред. ОЮ Воробьева. Красноярск: Сиб. фед. ун-т, 2009. 241 с.

Wang, L. (2008). Stochastic modeling and simulation of transient events, ProQuest.