Distributed Radio Resource Allocation in Wireless Heterogeneous Networks

by

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ALLOCATION DISTRIBUÉE DES RESSOURCES DANS LES RÉSEAUX SANS FIL HÉTÉROGÈNE

Mathew Pradeep GOONEWARDENA

RÉSUMÉ

Cette thèse étudie le problème d'allocation des ressources dans la partie d'accès radio des réseaux hétérogènes à petites cellules (en anglais *Heteregeneous and Small-cell Networks*, HetSNets). Un HetSNet est construit en introduisant des petites cellules, dans une zone géographique desservie par un réseau macro-cellulaire bien structuré. Les petites cellules utilisent les mêmes bandes de fréquence que celui du réseau macro-cellulaire et opèrent ainsi dans un régime limité en interférence. Par la suite, une allocation complexe des ressources radio est nécessaire afin de bien gérer l'interférence et améliorer l'efficacité spectrale du réseau. Afin de résoudre ce problème, plusieurs approches centralisées ou distribuées ont été proposées dans la littérature. Cette thèse se concentre sur l'approche distribuée basée sur le paradigme des réseaux auto-organisés. Plus précisement, elle développe des modèles et des algorithmes d'allocation de ressources en faisant appel à la théorie des jeux et à la théorie d'apprentissage. Bien que cette approche distribuée du paradigme des réseaux auto-organisés peut donner des résultats sous optimaux par rapport à l'approche centralisée, elle est hautement évolutive et tolère les pannes.

Le problème d'allocation des ressources comporte plusieurs facette qui varient selon l'application, la méthodologie de la solution et le type des ressources. Par conséquent, cette thèse se concentre sur quatre sous-problèmes qui ont été choisis en raison de leur importance. La théorie des jeux ainsi que la théorie des mécanismes d'incitation sont les principaux outils utilisés dans cette thèse parce qu'ils fournissent un environnement riche pour modéliser le problème dans le cas du paradigme des réseaux auto-organisés. Premièrement, le problème de l'accès du canal orthogonal sur la liaison montante est considéré. Deux variantes de ce problème sont modélisées comme des jeux bayésiens non coopératifs et l'existence d'équilibre symétrique pure bayésien de Nash est démontré pour chacun. Deuxièmement, cette thèse considère les jeux en forme de satisfaction et étudie leurs équilibres généralisés (en anglais *Generalized Satisfaction Equilibrium*, GSE). Chaque joueur (ou utilisateur sans fil) a une contrainte à satisfaire et le GSE présente un profil de stratégies mixtes à partir duquel aucun joueur insatisfait ne peut unilitéralement dévier à la satisfaction. L'objectif dans ce cas est de développer un équilibre alternatif pour modéliser les utilisateurs sans fil. L'existence du GSE, sa complexité, et sa performance par rapport à l'équilibre de Nash sont discutés. Troisèmement, la thèse introduit des mécanismes de vérification afin de garantir une auto-organisation dynamique dans les HetSNets. L'objectif principal est de remplacer les techniques de transfert monétaire utilisées dans la littérature actuelle. Dans un réseau sans fil, certaines informations privées des utilisateurs, telles que le taux d'erreur par bloc et la classe d'application, peuvent être vérifiées aux niveaux des petites cellules. Cette vérification peut être utilisée pour menacer les faux rapports avec étranglement backhaul. Par conséquent, les utilisateurs apprennent l'équilibre véridique au fil du temps en observant les récompenses et les punitions. Enfin, la thèse modélise le problème de

contrôle d'admission avec des contraintes sur le débit des utilisateurs comme un jeu bayésien dans le cas d'un canal d'interférence à accès mutiple. Ce problème est démontré d'avoir au moins un équilibre bayésien de Nash.

Les résultats obtenus dans cette thèse démontrent que l'auto-organisation peut être utilisée d'une manière efficace dans les HetSNets. Toutefois, ces derniers doivent faire appel à des mécanismes d'incitations, de punitions et d'équilibres spécialemment adaptés à l'environnement sans fil. Afin d'élargir ces résultats, des futurs problématiques de recherche sont identifés à la fin de ce document.

Mots clés: Petites cellules, Théorie des jeux, Auto-organisation, Réseaux sans fil hétérogène, Théorie des mécanismes d'incitation

DISTRIBUTED RADIO RESOURCE ALLOCATION IN WIRELESS HETEROGENEOUS NETWORKS

Mathew Pradeep GOONEWARDENA

ABSTRACT

This dissertation studies the problem of resource allocation in the radio access network of heterogeneous small-cell networks (HetSNets). A HetSNet is constructed by introducing small-cells (SCs) to a geographical area that is served by a well-structured macrocell network. These SCs reuse the frequency bands of the macro-network and operate in the interference-limited region. Thus, complex radio resource allocation schemes are required to manage interference and improve spectral efficiency. Both centralized and distributed approaches have been suggested by researchers to solve this problem. This dissertation follows the distributed approach under the self-organizing networks (SONs) paradigm. In particular, it develops game-theoretic and learning-theoretic modeling, analysis, and algorithms. Even though SONs may perform subpar to a centralized optimal controller, they are highly scalable and fault-tolerant.

There are many facets to the problem of wireless resource allocation. They vary by the application, solution, methodology, and resource type. Therefore, this thesis restricts the treatment to four subproblems that were chosen due to their significant impact on network performance and suitability to our interests and expertise. Game theory and mechanism design are the main tools used since they provide a sufficiently rich environment to model the SON problem. Firstly, this thesis takes into consideration the problem of uplink orthogonal channel access in a dense cluster of SCs that is deployed in a macrocell service area. Two variations of this problem are modeled as noncooperative Bayesian games and the existence of pure-Bayesian Nash symmetric equilibria are demonstrated. Secondly, this thesis presents the *generalized satisfaction equilibrium* (GSE) for games in satisfaction-form. Each wireless agent has a constraint to satisfy and the GSE is a mixed-strategy profile from which no unsatisfied agent can unilaterally deviate to satisfaction. The objective of the GSE is to propose an alternative equilibrium that is designed specifically to model wireless users. The existence of the GSE, its computational complexity, and its performance compared to the Nash equilibrium are discussed.

Thirdly, this thesis introduces *verification mechanisms* for dynamic self-organization of wireless access networks. The main focus of verification mechanisms is to replace monetary transfers that are prevalent in current research. In the wireless environment particular private information of the wireless agents, such as block error rate and application class, can be verified at the access points. This verification capability can be used to threaten false reports with backhaul throttling. The agents then learn the truthful equilibrium over time by observing the rewards and punishments. Finally, the problem of admission control in the interfering-multiple-access channel with rate constraints is addressed. In the incomplete information setting, with compact convex channel power gains, the resulting Bayesian game possesses at least one pure-Bayesian Nash equilibrium in on-off threshold strategies.

The above-summarized results of this thesis demonstrate that the HetSNets are amenable to self-organization, albeit with adapted incentives and equilibria to fit the wireless environment. Further research problems to expand these results are identified at the end of this document.

Keywords: Heterogeneous networks, Game Theory, Self-organization, Mechanism design, Generalized Satisfaction Equilibrium, Verification Mechanisms

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2G	2^{nd} Generation Mobile Networks
3G	3^{rd} Generation Mobile Networks
3GPP	3^{rd} Generation Partnership Project
4G	4^{th} Generation Mobile Networks
5G	5^{th} Generation Mobile Networks
ABS	Almost Blank Subframe
AP	Access Point
ARQ	Automatic Repeat Request
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BLER	Block Error Rate
BNSE	Bayesian Nash Symmetric Equilibrium
BS	Base Station
CA	Carrier Aggregation
CCN	Common Core Network
CE	Correlated Equilibrium
CoMP	Coordinated Multipoint
CQI	Channel Quality Indicator
CSG	Closed Subscriber Group

CSI	Channel State Information
CSIT	Channel State Information at the Transmitter
CSMA	Carrier Sense Multiple Access
CSP	Constraint Satisfaction Problem
DPI	Deep Packet Inspection
DSL	Digital Subscriber Line
EHN	Energy Harvesting Nodes
eICIC	Enhanced Intercell Interference Cancellation
eNB	Enhanced Node B
ETSI	European Telecommunications Standards Institute
E-UTRAN	Evolved UMTS Terrestrial Radio Access Network
FAP	Femto Access Point
FBS	Femto Base Station
FUE	Femtocell User Equipment
GNE	Generalized Nash Equilibrium
GSE	Generalized Satisfaction Equilibrium
GSM	Global System for Mobile Communications
HARQ	Hybrid Automatic Repeat Request
HeNB	Home Enhanced Node B
HetNet	Heterogeneous Network

HetSNet	Heterogeneous Small Cell Network
HO	Handover
ICIC	Intercell Interference Coordination
IEEE	Institute of Electrical and Electronics Engineers
IoT	Internet of Things
ISM	Industrial Scientific and Medical
LTE-A	Long Term Evolution - Advanced
MAC	Medium Access Control
MBS	Macro Base Station
MCC	Minimum Collisions Coloring
MIMO	Multiple Input Multiple Output
MPE	Markov Perfect Equilibrium
MUE	Macrocell User Equipment
MUT	Multiservice User Terminal
NE	Nash Equilibrium
NTU	Non-Transferable Utility
OFDM	Orthogonal Frequency Devision Multiplexing
OFDMA	Orthogonal Frequency Devision Multiple Access
PCI	Physical Cell Identifier
PoA	Price of Anarchy

PoS	Price of Stability
PPP	Poisson Point Process
QoS	Quality of Service
RAN	Radio Access Network
RB	Resource Block
RN	Relay Node
RRM	Radio Resource Management
RRO	Radio Resource Optimization
SAP	Small-cell Access Point
SBS	Small-cell base station
SC	Small Cell
SDR	Software Defined Radio
SE	Satisfaction Equilibrium
SFR	Soft Frequency Reuse
SIMO	Single Input Multiple Output
SINR	Signal to Interference plus Noise Ratio
SLA	Service Level Agreement
SNR	Signal to Noise Ratio
SON	Self-organizing Network
SUE	Small-cell User Equipment

TU	Transferable Utility
UDTS	Uniformly Distributed Threshold Strategies
UE	User Equipment
UMTS	Universal Mobile Telecommunications System
VCG	Vickrey–Clarke–Groves

LISTE OF SYMBOLS AND UNITS OF MEASUREMENTS

Symbols Common to Chapters

$\mathscr{N}$	set of agents (UEs)
$\mathscr{M}$	set of BSs
$\mathscr{K}$	set of subchannels
$\mathscr{P}_i$	set of power levels of agent i
Θ_i	type set of agent i
$\emptyset$	empty set
$\mathbb{R}$	set of real numbers
u_i	utility of agent i
σ^2	AWGN variance
$\mathbb{E}$, Pr, Var, $\mathbb{P}$	expectation, probability, variance, and power set operators

Symbols of Chapter 2

$\mathscr{N}_m$, $\mathscr{N}_{-m}$	set of SUEs of SAP m and set of SUEs not of SAP m
$\mathscr{N}_{mk}$	set of SUEs of SAP m that transmits on channel k
$\mathscr{N}_{-mk}$	set of SUEs not of SAP m that transmits on channel k
$\mathscr{N}'_{-mk}$	set of SUEs not of SAP m that does not transmit on channel k
$\mathscr{A}_i$, $\mathscr{A}_{-i}$,$\mathscr{A}$	action set of SUE i, product set $\prod_{j\in\mathscr{N}\setminus i}\mathscr{A}_j$, and $\prod_{j\in\mathscr{N}}\mathscr{A}_j$
Θ_i, Θ_{-i}, Θ	of game $\mathbf{G}_1$- type set of SUE i, product set $\prod_{j\in\mathscr{N}\setminus i}\Theta_j$, and $\prod_{j\in\mathscr{N}}\Theta_j$
Ω_i, Ω_{-i}, Ω	of game $\mathbf{G}_2$- type set of SUE i, product set $\prod_{j\in\mathscr{N}\setminus i}\Omega_j$, and $\prod_{j\in\mathscr{N}}\Omega_j$

α_i	symbol availability at SUE i
h_i^k	channel power gain of SUE i to home-SAP on channel k
g_{jm}^k	interference power gain of a UE j to SAP m on channel k
ζ_{km}	sum of noise power of MUEs at SAP m on channel k
$f_{h_i^k}, f_{g_{im}^k}, f_{\alpha_i}, f_\zeta$	probability density functions of h_i^k, g_{im}^k, α_i, and ζ_{km}
$\bar{f}_{\theta_{-i}}, \bar{f}_{\omega_{-i}}$	belief densities of SUE i over set Θ_{-i} and Ω_{-i}
u_i	utility of player i
s_i, r_i	pure and mixed strategy of SUE i
$s_{-i}, \mathbf{r}_{-i}$	pure and mixed strategy vectors of SUEs except i
$h_{i\text{th}}, \lambda_{i\text{th}}$	threshold of SUE i in game $\mathbf{G}_1$ and $\mathbf{G}_2$
$\mathrm{q}_1, \mathrm{q}_2$	probability that an SUE transmits on channel k in game $\mathbf{G}_1$ and $\mathbf{G}_2$
$\bar{\mathrm{p}}_l\left(\mathscr{N}_{b_ik}\right), \bar{\mathrm{p}}_l\left(\mathscr{N}'_{-b_ik}\right)$	of game $\mathbf{G}_l$ ($l \in \{1,2\}$) probability that SUEs $\mathscr{N}_{b_ik}$ and $\mathscr{N}_{-b_ik}$ do not transmit on channel k
$\mathrm{p}_l\left(\mathscr{N}_{-b_ik}\right)$	of game $\mathbf{G}_l$ ($l \in \{1,2\}$) probability that SUEs $\mathscr{N}_{-b_ik}$ transmit on channel k
$\mathscr{B}_l\left(N, \mathrm{p}\right)$	of game $\mathbf{G}_l$ ($l \in \{1,2\}$) binomial distribution with N trials and success probability p

Symbols of Chapter 3

$\mathscr{N}_\mathrm{s}$	set of satisfied agents
$\mathscr{N}_\mathrm{u}$	set of unsatisfied agents
$\mathscr{A}_i$	pure-strategy set of agent i
$\mathscr{A}$	set product $\times_{i\in\mathscr{N}} \mathscr{A}_i$

$\mathscr{A}_{-i}$	set product $\times_{j\in\mathscr{N}\setminus i}\mathscr{A}_j$
Π_i	mixed strategy set of agent i
Π	set product $\times_{i\in\mathscr{N}}\Pi_i$
Π_{-i}	set product $\times_{j\in\mathscr{N}\setminus i}\Pi_j$
a_i	element of $\mathscr{A}_i$
π_i	element of Π_i
a_{-i}	element of $\mathscr{A}_{-i}$
π_{-i}	element of Π_{-i}
a	element of $\mathscr{A}$
π	element of Π
g_i	correspondence of agent i
	Symbols of Chapter 4
$\mathscr{O}$	set of outcomes
v_i	valuation of agent i
s_i	pure-strategy of agent i
F	state transition probability
$\hat{\theta}_i$	report of agent i
a	allocation rule for single period mechanism
π	allocation policy for infinite horizon mechanism
T	penalty duration

β	discount factor
α	learning rate
$\mathrm{err}_{\mathrm{I}}$	type I error probability
$\mathrm{err}_{\mathrm{II}}$	type II error probability
V_i	value function of agent i
	Symbols of Chapter 5
$\mathscr{O}$	set of outcomes
v_i	valuation of agent i
s_i	pure-strategy of agent i
F	state transition probability
$\hat{\theta}_i$	type report of agent i

INTRODUCTION

The demand for mobile data has been on the rise and the leading players in the industry expect it to grow at even higher rates (CISCO, 2014). Moreover, surveys show that there is higher demand from indoor environments, such as commercial buildings and households, as much as 50% of mobile calls and 70% of mobile data usage (Andrews *et al.*, 2012; Gilbert, 2012). In addition, the number of communication-enabled user appliances is steadily growing, especially with the impact from the concept of Internet of things (IoT) (Whitmore *et al.*, 2015). Presently, most of the indoor data demand is met by Wi-Fi networks operating in the free industrial scientific and medical (ISM) band. If the mobile standards are to remain competitive in the vast indoor data market, they must provide a seamless extension of the mobile standards to the indoor user with data rates comparable to or higher than Wi-Fi (Andrews *et al.*, 2012). Research is being conducted under the larger banner of 5G networks as the next step to the Long-Term Evolution - Advanced (LTE-A) standard. These proposals in the access network include, but are not limited to, densification of the network, full-duplex communication, massive multiple-input multiple-output (MIMO), cloud radio access network (Cloud RAN) (Demestichas *et al.*, 2013; Hossain & Hasan, 2015). The focus of this dissertation is the small-cells technology, which falls under network densification. Small-cells are low-power base stations (BSs). They are often deployed indoors and operate within the frequency range of the macro-BSs. While shorter radio links increase spectral efficiency, frequency reuse creates intercell interference. This interference is detrimental to the network performance if resource allocation is not properly coordinated. Traditional cellular networks that have been augmented by small-cells are called heterogeneous small-cell networks or HetSNets. There are two main competing paradigms for resource allocation in HetSNets, namely centralized vs. distributed. This dissertation is based on the distributed approach and focuses on self-organization of the network, whereby nodes make decisions based on locally available information with minimum centralized coordination (Peng *et al.*, 2013; Fehske *et al.*, 2014). In particular, this dissertation will consider game-theoretic and learning-theoretic modeling, analysis, and algorithms for self-organization in small-cell networks (Bennis *et al.*, 2013).

The organization of this dissertation is as follows. The body consists of 5 chapters. Chapter 1 introduces the problem domain, the state of the art, the objectives, and the methodology. Each remaining chapter presents an article that has been accepted or under revision in a peer-reviewed journal. Chapter 2 presents the first article, which considers the problem of uplink channel access in a dense cluster of closed-access small-cells that are deployed in a macrocell service area. In this network all small-cell users have access to a common set of channels. This leads to intercell interference. Each channel forms a separate collision domain in each cell. Thus, in a given cell one channel can be successfully used only by one user of that particular cell. This article proposes two noncooperative Bayesian games that are played by the small-cell users. The first game assumes the availability of channel state information at the transmitters, while the second game only assumes the availability of the distribution of the channel state information. Each small-cell user can choose to either transmit over one of the channels or not to transmit. The article considers symmetric threshold strategies. In these strategies the Nash equilibrium is fully determined by a single parameter (Lee *et al.*, 2009). The existence of a Bayesian Nash symmetric equilibrium in threshold strategies is shown. Numerical results presented in this dissertation corroborate the theoretical findings.

Chapter 3 contains the second article, which presents the generalized satisfaction equilibrium (GSE) for games in satisfaction-form. In these games each wireless user (also called an agent) has a constraint to satisfy and the GSE is a strategy profile from which no unsatisfied agent can unilaterally deviate to satisfaction. This new equilibrium is particularly designed to model problems of service-level provisioning, where satisfying all agents is infeasible. The GSE forms a more flexible framework for studying self-configuring networks than the previously defined satisfaction equilibrium (Perlaza *et al.*, 2012b). The existence of the GSE in mixed strategies is shown for the case in which the constraint of a user is defined by a lower limit on its expected utility. The article also demonstrates that the pure-strategy GSE problem is closely related to the constraint satisfaction problem and that finding a pure-strategy GSE with a given number of satisfied agents is NP-hard. For certain games in satisfaction-form, it is shown that the satisfaction response dynamics converge to a GSE. Next, the Bayesian GSE is introduced for games with incomplete information. Finally, this article presents a series of wireless applications that demonstrate the superiority of the GSE over the classical equilibria in solving problems of service-level provisioning.

Chapter 4 contains the third article, which introduces the *verification mechanisms* for dynamic self-organization of wireless access networks. Current research on mechanism design for networks relies on quasi-linear monetary transfers theory of economic mechanisms, such as auctions (Khaledi & Abouzeid, 2015). Pricing results in the loss of separation between network control and business processes. Allocation policies that can be truthfully implemented through pricing are limited. In sharp contrast to auctions of items such as arts, in the wireless environment certain private information of the users, such as block error rate and application class, can be verified at the access points after the allocation by observation of the control messages, channel sensing, or by deep-packet inspection. This verification capability can be used to threaten false reports with backhaul throttling. The proposed mechanisms are designed with the realistic assumption of imperfect verification with non-zero probability of error. The users learn truthful strategy over time by observing the rewards. An experiment is set up to implement verification mechanisms for widely used policies such as proportional-fair, round-robin, weighted sum-rate, and random scheduling. The results demonstrate that the verification mechanism achieves a *high probability of truthfulness* for these scheduling policies.

Chapter 5 presents the fourth article, which considers the problem of admission and discrete power control in the interfering-multiple-access channel with rate constraints on admitted links.

This problem is formulated as a normal-form noncooperative game. The utility function models inelastic demand. An example demonstrates that in some networks in the fading channel, a pure-strategy equilibrium does not exist with strictly positive probability. Hence, the probability of the existence of an equilibrium is analyzed and bounds are computed. To this end, the problem of finding the equilibria is transformed into a constraint satisfaction problem. Next, the article considers the game in the incomplete information setting with compact convex channel power gains. The resulting Bayesian game is shown to possess at least one pure Bayesian Nash equilibrium in on-off threshold strategies. Numerical results are presented to corroborate the findings.

CHAPTER 1

RESEARCH PROBLEM

In order to rise to the challenge of higher demand for throughput in mobile communications, the wireless research community has identified a few promising technologies. These are deduced through information-theoretic properties of the wireless communication process. Let us consider the following simplified equation of the rate (bits/s) of a cellular user (Bhushan *et al.*, 2014)

$$R = \frac{m}{n} W \log_2 \left(1 + \frac{S}{I + \sigma^2}\right), \tag{1.1}$$

where $\frac{m}{n}$ is the ratio of MIMO multiplexing gain to the number of users, W is the bandwidth, $\frac{S}{I+\sigma^2}$ is the signal to interference plus AWGN noise power ratio. Increasing the $\frac{m}{n}$ ratio requires multiple antennas and fewer users per cell. A shorter link distance can improve useful signal power, S. SC technology is aimed at lowering n and improving S and introduces a larger number of low-power access nodes with partial to full *frequency reuse*. Thus, SCs share the user load among a larger number of access nodes. The path loss exponent may increase above 4 when the receiver is obstructed by a building. For instance, halving the distance in this scenario can increase the received power by a factor of $\left(\frac{1}{2}\right)^{-4}$, which is an increase of an order of magnitude (Rappaport, 2009). Thus, SCs provide a power gain, offloading gain, and frequency reuse gain. On the other hand, the received power increases linearly with the number of receiver antennas in a single input multiple output (SIMO) link. Even though the power gain in SIMO is linear (compared to the order of magnitude increase with shorter link distance) it provides a diversity gain. Moreover, if multiple antennas are introduced at the transmitter as well, then a multiplexing gain (transmitting multiple symbols simultaneously) is achieved (Tse & Viswanath, 2005). In addition, full-duplex attempts to reuse time, frequency, and special dimensions simultaneously for transmission and reception. Thus, effectively doubling the spectral efficiency. While the above analysis is simplified, it hints that these radio technologies play complimentary roles in the quest to increase wireless network capacity (Chandrasekhar & Andrews, 2009; Landström *et al.*, 2011). The focus of this dissertation is on the SCs. The next section takes a closer look at the concept of heterogeneity in a modern network and the role of SCs within it.

1.1 Heterogeneous Networks

The term heterogeneous network (HetNet) is employed to identify various technological concepts, therefore a clear disambiguation is required in order to firmly ground this dissertation. In the broadest definition, a HetNet provides seamless communication to a user over wireless networks of varying standards, such as disparate physical layer technologies that are operating over distinct frequency bands (Wu *et al.*, 2002). The emphasis in this context is on the inter-working between different standards in the protocol stack through software defined radio (SDR) based multiservice user terminals (MUTs), protocol translation, and/or a common core network (CCN), which handles the network functionality independent of the access technologies. However, with regards to 3rd Generation Partnership Project (3GPP) access network, the term HetNet is used somewhat more restrictively to describe a wireless network of varying cell sizes, from large macrocells to SCs and relay nodes (RN). Most of the smallest cells are deployed unplanned with semi-autonomous resource allocation and possibly with restricted access (Damnjanovic *et al.*, 2011; Bennis & Saad, 2012). In this context, the heterogeneity is in cell sizes and operation rather than access technology. Thus, the key research emphasis for 3GPP HetNets is primarily on interference between cells, resource allocation, self-organization, and self-optimization, not on interoperability (Guvenc *et al.*, 2013a,b).

Cell miniaturization is not a recent concept. Cell splitting was introduced as a solution to offload traffic from the larger macrocell in densely populated environments, thus increasing the number of users served per unit of area (Dehghan & Steele, 1997). This led to microcells in city centers and picocells inside large buildings. In addition to increasing offloading gains, picocells also formed a solution for the wall penetration loss. These smaller cells were still centralized in their deployment, configuration, and management and their numbers were modest (Madfors *et al.*, 1997). The frequency assignment was pre-planned in order not to have interference between the umbrella macrocell and the overlayed micro- and picocells.

The new trend seen with SCs is fundamentally different to cell splitting. The SC-enabled networks are not only minimizing the cell size further but also altering the deployment and operation of the network. The smallest member of the SC family that is considered in this dissertation is the femtocell. A femtocell has lower transmission power than a picocell, usually in the order of 10 mW and coverage is intended to be in the range of a few tens of meters (Davies, 2007). A large number of SCs are expected to be deployed and most SCs will be deployed by end users. Thus, the configuration and operation of SCs in the network is expected to have minimum human involvement leading to the concept of self-organizing networks (SONs) (Hoydis *et al.*, 2011; Peng *et al.*, 2013). Therefore, frequency pre-planning is no longer possible and overlapping frequency reuse among cells is a design goal. In order to distinguish this new vision of a multitier network (Dhillon *et al.*, 2012) from previously discussed types of HetNets, the term heterogeneous small cell network (HetSNet) has appeared in recent literature (Shakir *et al.*, 2013). Thus, this dissertation chooses to employ the term HetSNet to refer to a 3GPP wireless network that consists of operator deployed macro-, micro-, and pico- cells, and user- or operator-deployed femtocells, which has SON capabilities and provides seamless service to the user equipments (UEs). The term SC is reserved to identify cells of coverage that are smaller than the macrocells.

Fig. 1.1 depicts a 3GPP HetSNet similar to that envisioned in (Damnjanovic *et al.*, 2011). An enhanced node B (eNB) is a base station (BS) that is operator deployed and managed, such as macro- micro- picocell base stations, a home enhanced node B (HeNB) is an SC base station (SBS) of the smallest order that can be deployed by the operator or the user. An HeNB in some literature is identified as a femtocell base station (FBS) or a femtocell access point (FAP). In literature, the term mobile terminal (MT) or UE is used to identify the handset. A SC, specifically a user deployed HeNB, can offer three access modes, namely closed access, open access, and hybrid access (Roche *et al.*, 2010).

Closed access: is also called restricted access and in this mode only the UEs that belong to the network operator of the SC or that are allowed by the private owner of the SC can obtain service. The control is managed by a access control list in the HeNB. This privileged group is called the Closed Subscriber Group (CSG).

Open access: all UEs that are in the network of the operator can access the SC.

Hybrid access: the CSG has guaranteed access while the other UEs of the operator have limited access.

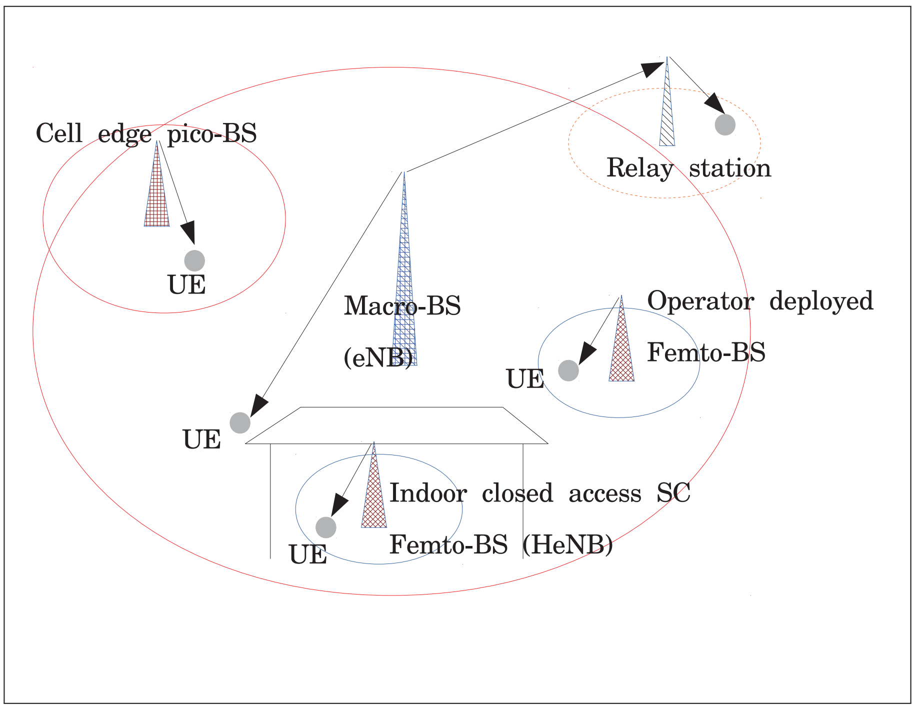

Figure 1.1 HetSNet in 3GPP wireless networks

1.2 Motivation & Impact

SCs are introduced to improve throughput and coverage. However, the reuse of frequency, overlapping cell coverage areas, and closed access cells lead to a multitude of optimization problems in the RAN of a HetSNet. The control variables of these problems are the location of the SCs, the number of SCs that are on, and allocation of channels, power, time slot, and UE. There are two main solution paradigms, centralized vs. distributed. In line with the challenges, controls, and solution paradigms, literature has identified several key research domains for HetSNets. (Guvenc *et al.*, 2013a,b). One key problem due to the introduction of SCs is the interference created due to overlapping cell deployment with frequency reuse. Fig. 1.2 shows how the macrocell interferes the two SCs, which are deployed in its coverage area. In literature, the interfering cell is called the *aggressor* while the interfered cell is called the *victim*. Interference can be in both uplink and downlink. Similarly, the SCs that are closer to the macrocell BS can victimize the macrocell (Saquib *et al.*, 2012). The SC_1 of Fig. 1.2 shows one proposed solution, called the cell range expansion, where SC_1 prevents uplink interference to itself by accepting the macrocell UE (MUE) to the SC network. Cell range expansion and other techniques of interference mitigation are detailed in Section 1.3. Since SCs are expected to

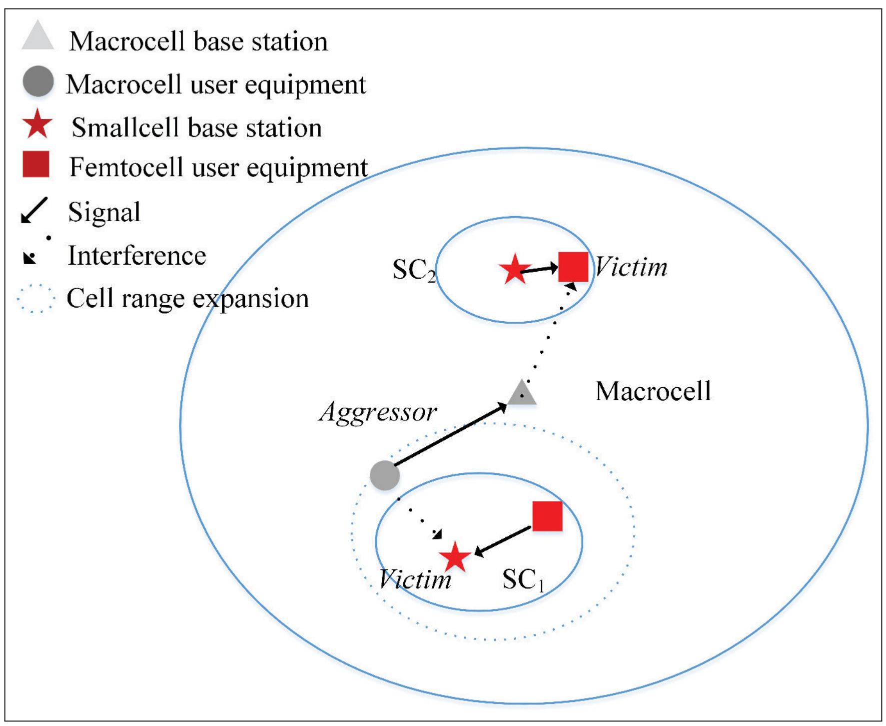

Figure 1.2 The MUE creates interference to SC_1 BS in uplink and macrocell BS creates downlink interference at SC_2 UE.

be deployed in substantial numbers by network operators as well as users, the infeasibility of preplanned radio resource allocation is a forgone conclusion. Instead, HetSNets are expected to follow the new paradigm of self-configuration and self-optimization, which is jointly defined under SON (Peng *et al.*, 2013) and the related concept of automated SC site planning for operator deployed SCs (Guo *et al.*, 2013b; Guo & O'Farrell, 2013). The underlying fundamental issues that both SON and automated deployment attempt to solve are the handling of inter-cell interference and providing improved coverage and throughput where it is in demand in a scalable and dynamic manner. Another research domain is load balancing or traffic steering, among overlapping cells for network performance optimization (Munoz *et al.*, 2013b). Traffic steering may happen while the UE is idle or active. The latter is known as handover (HO). In an overlapping multitier cell environment, HO decision is more complicated than a conventional cell network (Guvenc, 2011). The HO decision now involves not only the received signal to interference plus noise ratio (SINR) of the considered UE but also interference from the UE to nearby cells and also the need of avoiding excessive HOs among cells in order to reduce signaling and minimize call drop rate (Munoz *et al.*, 2013a; Pedersen *et al.*, 2013a). Group HO where a group of UEs is handed over simultaneously, e.g., in a moving vehicle, is also considered an important challenge in HetSNets (Sui *et al.*, 2013).

In order to optimize offloading gain, SCs has to be discovered by the UEs. Optimization of cell discovery, with minimum signaling and minimum delay is an active research domain (3GPP, 2012; Prasad *et al.*, 2013). Energy efficiency is another challenge the research community is currently working on and this topic is usually addressed under the topic of green mobile networks (Xu *et al.*, 2013b; Shakir *et al.*, 2013). Researchers are exploring methods to save energy in various functions of the network from the deployment to operation (rae Cho & Choi, 2013). Network modeling is another challenge in HetSNets (Hwang *et al.*, 2013). As discussed in Section 1.3, classical Wyner interference model is no longer a valid approximation of a HetSNet. Due to a large number of unplanned SCs that are expected to be deployed, centralized solutions for the above-discussed challenges are not scalable. Therefore, the network intelligence has to be distributed among the cells. Especially real-time processes such as interference coordination must be handled through distributed or decentralized resource allocation in the RAN (ElSawy *et al.*, 2013). Such decentralized radio resource optimization (RRO) schemes in HetSNets are still in their infancy and much space is available to make substantial research contributions. We would like to draw the attention to one more issue that is studied by the HetSNet community and that is the limitation of the backhaul. While larger cells are backhauled through fiber links or high-speed dedicated microwave links SCs, especially those deployed in residences, are expected to be backhauled through the existing digital subscriber line (DSL) and home cable networks. DSL was designed for Internet access and hence not optimized for quality of service (QoS). Managing UE in a HetSNet to optimize QoS over backhaul links of SCs has been identified as a key problem (Samarakoon *et al.*, 2013). Also, wireless backhauling SCs to places without wired infrastructure is also studied (Liu & Shen, 2014).

Wireless communications as a field has evolved at a quite unimaginable pace since the first radio transmission by Guglielmo Marconi. The path ahead is just as interesting. The industry is debating the 5G network model. HetSNets and SON have been identified as key technologies that will define a 5G network (Demestichas *et al.*, 2013). Therefore, this research could not possibly be more timely. We share a passion for wireless communication research, which is the key reason to propose to explore RRO in HetSNets that will have an impact on the future wireless networks. Finally, as identified above, the introduction of SCs has given rise to a plethora of new challenges in wireless resource optimization at the RAN, which translates into research opportunities and which gives us the chance to make substantial and valuable research contribution for the industry and the advancement of humanity.

1.3 State of the Art and Their Limitations

This section discusses the state of the art in research and their limitations as related to the thesis. The bulk of the works surveyed in this section are the state of the art as it was at the time of writing the research proposal for this thesis (mid 2014). A few more recent articles were later included. The Chapters 2,3,4, and 5 that present the constituent articles of this thesis have their own discussion on the state of the art with respect to the problems discussed in each of those chapters.

The problem of resource allocation in HetSNets has been the focus of several research works in the past. One of the main problems addressed is intercell interference. Cell range expansion is an attempt to this end. In conventional cellular networks, the UE is handed over to the cell with the highest received referenced signal strength, which works well for cells of comparable transmission power. But in HetSNets there is an imbalance in the transmission powers of macrocells and SCs, therefore the UE may be closer to the SC but may still receive relatively higher power from the macrocell. Therefore, in the downlink, the UE prefers to be served by the macrocell. However, in the uplink the SC receives a stronger signal from the UE than the macrocell due to proximity. This uplink signal from the UE may interfere the transmission of the UEs of the SC. Cell range expansion is a solution that was proposed by 3GPP (see Fig. 1.2) to solve this uplink-downlink imbalance (Lopez-Perez *et al.*, 2011). In cell range expansion, a positive bias is introduced to the received power of the SC signal, thus the UE performs the HO to the SC while receiving a lower SINR compared to the macrocell BS. The equation (1.2) shows the operation of bias where the UE selects the cell m^* which has the highest received power P_m plus bias β_m among the set of cells $\mathscr{M}$. HO frees up the macrocell to serve another UE and also eliminates the uplink interference that was received at the SC. Due to the lower downlink SINR at the UE in the expanded cell, research has shown an overall reduction in sum rate of the network (Guvenc, 2011). Researchers have proposed optimizing the bias value to enhance the performance of the UE in the expanded cell. In (Kudo & Ohtsuki, 2013), Q-learning is used where each UE learns the optimal bias value from past performance. The simulation results depict a reduction in the number of UEs in outage compared to a fixed bias scheme.

$$m^* = \underset{\mathrm{m} \in \mathscr{M}}{\arg\max}\ (P_m + \beta_m) \tag{1.2}$$

3GPP has also proposed two coordination schemes between eNBs to improve the performance of UEs in the expanded area: intercell interference coordination (ICIC) and enhanced ICIC (eICIC). ICIC in 3GPP release 8 is a frequency domain technique, whereas eICIC in release 10 combines both frequency and time domain techniques. In the frequency domain, the interfering BSs of a HetSNet can coordinate to dynamically freeing frequency bands to alleviate an interference condition. The key motivation to move to eICIC is that ICIC is unable to combat interference at the control channel, which is distributed across the full system bandwidth

(Lindbom *et al.*, 2011). In eICIC, frequency domain coordination is extended to carrier aggregation (CA), which is fully compatible with the earlier LTE release 8 standard but also allows control signals and data to be sent over different component carriers (Pedersen *et al.*, 2011). Therefore, two interfering cells in a HetSNet can employ complementary component carriers for signaling, thereby mitigating interference in crucial signaling channels. The eICIC standard also defines time domain coordination. In the time domain, the aggressor allocates certain subframes where only signaling is carried out and no data is sent. These subframes are called almost blank subframes (ABS). Time domain techniques require subframe level time synchronization between the involved BSs, giving rise to yet another challenge, cell synchronization in HetSNets.

While the 3GPP standard defines the messages between the network entities, it is up to the research community to optimize the message passing in eICIC. In (Deb *et al.*, 2013) the problem of UE and BS association and ABS duration optimization is formulated as a mixed integer program, which is then proved to be NP-hard and solved by relaxed heuristic methods. When frequency is negotiated in real-time it is called soft frequency reuse (SFR). In contrast, hard frequency reuse techniques are used in 2G networks, such as GSM. In (Jeong *et al.*, 2010; Oh *et al.*, 2010) SFR schemes are proposed, which divide the cell into inner and outer regions as shown in Fig. 1.3. In the outer regions, the BSs employ non-overlapping frequencies and less power is transmitted over the common frequency bands that serve inner cell UEs.

A related problem for frequency and time-slot allocation is the *Minimum Collisions Coloring* (MCC) problem, which is a variation of the graph coloring problem: a set of colors is assigned so that the number of adjacent vertices in a graph with the same color is minimized. While MCC resembles a variation of graph coloring, the objectives of the two optimization problems are considerably different. In classical graph coloring optimization, the objective is to minimize the number of colors used, always having sufficient colors to achieve zero collisions (Panagopoulou & Spirakis, 2008). On the other hand, in MCC, the number of colors is constant and the objective is to minimize the number of collisions. In a HetSNet, the nodes on the

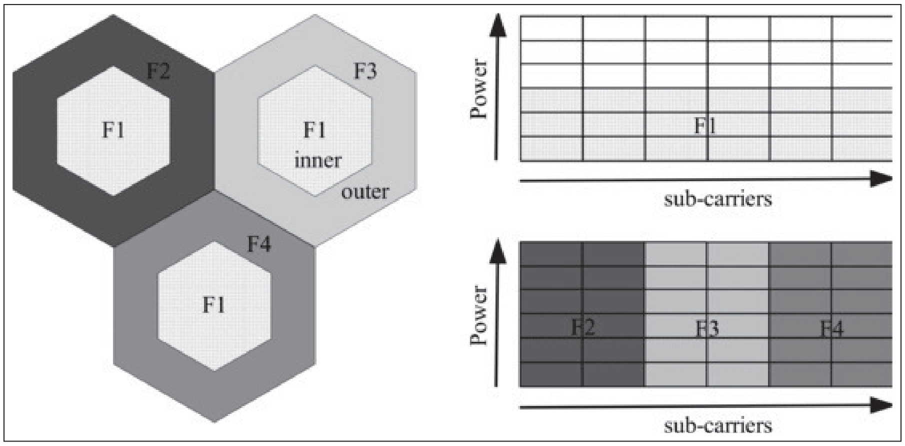

Figure 1.3 Soft frequency reuse (Jeong *et al.*, 2010) © 2010 IEEE

graph are the eNBs, the edges are the interference relations between the eNBs, and the colors are resources, such as frequency bands, time slots, and physical cell identifiers (PCIs).

MCC decision problem is NP-complete. The complexity of MCC requires heuristic solutions. Current research primarily focuses on the classical graph coloring problem and not on MCC. Graph coloring heuristics have been extensively used in wireless networks (Riihijarvi *et al.*, 2005). In (Panagopoulou & Spirakis, 2008) the proper vertex coloring problem is modeled by a game played by the vertices. The actions are depicted by a set of colors. Through potential function and *Best Response* (BR) dynamics, the game is shown to converge to a Nash equilibrium (NE) of proper coloring. In (Chatzigiannakis *et al.*, 2010), the above work is extended to a distributed and parallel implementation. In (Escoffier *et al.*, 2012), the bound for the worst case number of colors presented in (Panagopoulou & Spirakis, 2008) is improved. In (Chaudhuri *et al.*, 2008), a proper coloring game is designed for the case when number of colors k is at least $\Delta(G)+2$, where $\Delta(G)$ is the maximum degree of the graph. Any graph, G can be proper $k-$colored in polynomial time and with knowledge of only the neighbors, if $k \geq \Delta(G)+1$. The novelty of (Chaudhuri *et al.*, 2008) is that the players follow a randomized strategy. In (Halldorsson *et al.*, 2010), the problem of assigning a fixed number of subchannels, k, among access points, is modeled by the graph coloring problem of finding the weighted maximum induced $k-$colorable subgraph (*weighted-Max-k-CIS*).

Mobility management in HetSNet beyond 3GPP Release 11 is considered in (Pedersen *et al.*, 2013b), which explains the mobility problems identified in Release 11 and proposes a hybrid model. The HOs for larger cells are managed by the network while the HOs for SCs are managed by UEs. This scheme reduces the signaling between the UE and the core network during the HO process in a dense SC network while also lowering the probability of HO failure. Mobility management is again the focus of (Guo *et al.*, 2013a); the authors propose to preserve the established data path between the current serving SC (the anchor) and the core network. The new serving SC, after the HO, forwards the data to the anchor SC over the X2 interface between SCs, which is proposed in 3GPP Release 10. The proposed scheme is analyzed against the conventional scheme via Markov chain modeling.

In (Rangan & Madan, 2012), the use of belief propagation algorithm to minimize mutual interference in HetSNets is demonstrated. There, network optimization problems with linear signal and interference model and nonlinear utilities are considered. The standard belief propagation algorithm is shown to be heavy in message passing. Therefore, two approximation algorithms, Gaussian and first order, are derived, which are implementable under the provisions in the current standards. In (Chen *et al.*, 2013) the downlink resource block allocation and power allocation in a HetSNet are formulated as a mixed integer nonlinear optimization problem. Due to the complexity of solving a centralized resource allocation problem, the authors propose a distributed message passing belief propagation algorithm.

Game theory has been employed to analyze the equilibrium outcome of HetSNets. The players can be the BSs, the UE or as (Scutari *et al.*, 2008) shows, the links. Game-theoretical literature can be broadly divided into two groups, noncooperative and cooperative. In noncooperative games, the stability of the system (in this case HetSNet) is usually considered with respect to a single player, whereas in cooperative games the stability is more complicated and must be stable for all feasible subgroups among players. The main stability criterion in noncooperative games is the famous NE. A strategy profile $(s_1,\ldots,s_i,\ldots,s_N)$ is a tuple with one strategy for each player i where there are N players. A strategy profile is an NE of the game if no player can change their strategy and obtain a strictly better outcome. Correlated equilibria (CE) is another stability criteria, which is more general than the NE, i.e., all NEs are CEs but not all CEs are NEs (Aumann, 1987; Shoham & Leyton-Brown, 2009). The recently developed satisfaction-form and the satisfaction equilibrium (SE) (Perlaza *et al.*, 2012b) have been specially designed for wireless networks. In an SE a user only needs to fulfill a constraint and need not maximize the utility as in the NE. This new equilibrium is employed to model the problem of spectrum access in Ren *et al.* (2015); Ellingsæter (2014). In Goonewardena & Ajib (2016) it is shown that the normal-form games discussed in Southwell *et al.* (2014), have satisfaction-form representations.

While NE and CE theories provide existence results it is seldom the case that a player can individually identify the equilibrium point. The reasons being, lack of knowledge of the strategies of the other players and the existence of multiple equilibria. Driving the system into an equilibrium requires the players to learn their utilities and each other's utilities over time. Reinforcement learning is one such algorithm. In (Bennis *et al.*, 2013), downlink power allocation of a HetSNet with a minimum SINR guarantee for MUE is modeled as a noncooperative game. The SCs learn their equilibrium mixed strategy probability distribution over the discrete set of power levels over time through reinforcement learning. The learning algorithm is distributed in nature and it converges to an $\varepsilon-$NE.

A Stackelberg game is a two-stage game with a leader and a follower (Nisan *et al.*, 2007). In (Kang *et al.*, 2012a) a Stackelberg game is devised for the uplink and it is played by the MBS and the femtocell UE (FUE). The MBS protects itself from the FUE by pricing the interference. Two pricing schemes are proposed, uniform and nonuniform. In uniform pricing, the price per received unit interference is identical for all FUEs, whereas in nonuniform pricing each FUE has a unique unit price. The results show the existence of the Stackelberg equilibria under both pricing schemes. Moreover, a centralized algorithm and a distributed algorithm is proposed for nonuniform and uniform pricing models, respectively. It is shown that non-uniform pricing maximizes the MBS revenue and that uniform pricing maximizes the FUE sum rate.

Potential games are a special class of noncooperative games characterized by the existence of a potential function, such that sequential best response dynamics may only increase its value (Nisan *et al.*, 2007). In (Buzzi *et al.*, 2012) joint power and subcarrier allocation in the uplink of an SC network is designed as a potential game. The utility of each UE is a function of both SINR and the energy efficiency measured in bits/Joule. The potential function is shown to be exact.

In (Huang & Krishnamurthy, 2011), the resource block allocation problem in a cognitive FBS network is analyzed for its CE. One resource block consists of a predefined subset of the sub-channels for a predefined constant time duration. The system model assumes constant transmission power. The global objective is to maximize the minimum-rate-to-demand ratio. To achieve this global objective, a local objective consisting of three sections, rate to demand ratio, transmission power, and excess rate over demand is proposed. In (Hart & Mas-Colell, 2000), a regret matching algorithm is proposed to compute a CE in a decentralized manner. Regret matching is a learning rule. The proposed algorithm in (Huang & Krishnamurthy, 2011) utilizes the regret matching procedure to converge to a CE of the constant power resource block allocation problem. In (Bennis *et al.*, 2012), a similar approach is employed to find the ε-CE in the downlink of a HetSNet consisting of one macrocell and a number of underlayed femtocells. The objective is to make the FBSs operate at a CE while guaranteeing the MUE a predefined SINR. Regret matching is again utilized for the FBSs to learn the CE in a decentralized way. In (Lai *et al.*, 2013), the uplink problem is considered. The overlay network model is considered in which the FUEs can only access the resource blocks not used by MUE. The overlay model requires cognitive FUEs in order to sense inactive RBs. Miscalculation of occupied RBs as unoccupied is discouraged by a penalty function. Again, regret matching is used to propose a decentralized algorithm to achieve a CE.

Threshold-based strategies in noncooperative games have been used in the random resource allocation problem in SCs. The channel access model in (Cho & Tobagi, 2008; Cho *et al.*, 2008) is based on ALOHA and carrier sense multiple access (CSMA) with a single access point (AP). Therefore intercell interference is not considered. In (Lee *et al.*, 2009), intercell interference is introduced through a Bayesian game but the authors do not consider multiple subchannels or the possibility of collision within a cell since collision avoidance is assumed. In (Guan *et al.*, 2013), a threshold strategy based game in a multichannel environment is presented for managing queue length of the transmitters in an ad-hoc network. Instead of collisions, interference between transmitter-receiver pairs are considered.

Network formation is another form of noncooperative game used in HetSNet research, mostly in relation to relaying (Samarakoon *et al.*, 2012; Zhou *et al.*, 2014). In addition, mean-field models are being employed to study ultradense networks (Samarakoon *et al.*, 2016).

The other main class of games is cooperative games. Cooperative games can be divided into two main groups, Nash bargaining and coalition formation games. There are two main classes of coalition formation games. Transferable utility (TU) games can arbitrarily transfer the value of the coalition between players. Nontransferable utility (NTU) games have constraints regarding how the value can be divided within the coalition members (Saad *et al.*, 2009). Coalition formation games with externalities are used to group the femtocells to mitigate collisions and reduce interference in (Pantisano *et al.*, 2011). In games with externalities, the grand coalition is unlikely to be stable and the players may form stable subgroups. Due to wall penetration losses and distance, SC to macrocell interference is disregarded. Neighboring SCs must form disjoint groups to mitigate interference and the groups pool their frequency resources. The cost of forming a group is computed as the sum of the maximum power spent by a coalition member to reach another member. The solution method employed in (Pantisano *et al.*, 2011) is the recursive core (Huang & Sjostrom, 2006). In (Zhang *et al.*, 2013), an overlapping coalition formation game is developed between SCs. Constant transmission power is assumed and a sequential algorithm which converges to a locally stable solution is proposed. In (Pantisano *et al.*, 2012), a coalition game together with the solution concept of *recursive core* are used to model the cooperative interaction between MUE and FUE. By forming disjoint coalitions the rates of both MUE and FUE increase. In (Ma *et al.*, 2013), a coalition formation game is employed to partition a dense network of femtocells to minimize interference and a polynomial time group formation algorithm is introduced. While the algorithm converges, it is not guaranteed to be stable. In (Mathur *et al.*, 2008), both TU and NTU coalition formation games are used for co-operation of receivers and transmitters in an interference environment. A more recent work on coalition formation games is (Yuan *et al.*, 2017). This work develops an overlapping coalition formation game between the FUEs for the subchannel allocation problem.

In order to design games with attractive equilibria, the theory of mechanism design is employed. Most research in mechanism design for HetSNets uses utility transfer through payments (Wu *et al.*, 2016). The players pay the marginal contribution as in the VCG auction theory (Khaledi & Abouzeid, 2015). These payment based mechanisms do not fit the HetSNet environment well. In a wireless network, utilities are measured in units of data rate, error rate, and/or delay and not in terms of currencies. Also, flat pricing is preferred by operators (Mcqueen, 2009). HetSNets do not possess a versatile medium of utility transfer similar to money in economic networks (Hartline & Roughgarden, 2008). Rate throttling can replace payments as a punishment. These are called *money burning* mechanisms (Hartline & Roughgarden, 2008). Replacing money with a commonly available resource is discussed in (Cavallo, 2014). Verification after allocation can be employed in some cases (Fotakis & Zampetakis, 2015).

A key issue related to interference coordination is distributed estimation of interference. Cognitive features are employed to estimate interference (Wang *et al.*, 2013). The authors propose a scheme to estimate the interference from an FUE to an MUE in a non-synchronous HetSNet based on MBS-MUE distance, MUE transmission power, and MUE interference at the FUE. The paper also proposes a scheme to employ a cognitive relay node whose objective is to maximize the rate of FUE and minimize the outage probability of MUE.

Coordinated multipoint (CoMP) transmission is when a UE is being served by more than one BS simultaneously. CoMP reception, on the other hand, is joint processing of the received signal of a UE by more than one BS (Sawahashi *et al.*, 2010). CoMP requires synchronization between BSs at the symbol level. In (Sun *et al.*, 2013), the problem of selecting a subset of BSs to form the CoMP group and jointly optimize the transmit beamformers is formulated as a two-stage stochastic optimization problem. The solution is obtained through a weighted minimum mean squared error algorithm. Due to the fine synchronization requirement, CoMP is mostly considered with Cloud-RAN (Ha *et al.*, 2016) rather than small-cells.

Classical network modeling employs the Wyner model where interference from a neighboring cell is modeled by a lumped known value that is fixed over the cell area. In (Dhillon *et al.*, 2012), the locations of the BSs and the UEs in a multitier HetSNet are assumed to follow a Poisson point process (PPP). A UE is covered under a given BS if the SINR of the UE is higher than a given threshold Γ, i.e., SINR $> \Gamma$. The coverage probability in the downlink is calculated under Rayleigh distribution. In (Novlan *et al.*, 2013), a similar analysis is carried out for the uplink. In (Hoydis *et al.*, 2011), a random matrix theory (RMT) is employed to quantify mutual information and ergodic mutual information of a randomly deployed set of SCs.

The above discussion on the state of the art encompassed a broad range of the current research on HetSNets. As mentioned in the beginning of this section, the detailed state of the art specific to each problem that is considered in this thesis is presented in their respective chapters.

1.4 Research Domain, Objectives & Methodology

The networks addressed in this dissertation are 3GPP HetSNets similar to scenarios in (ETSI, 2014). The term HetSNets was defined in Section 1.1 and is concisely stated here for ease of reference. A HetSNet is a wireless network that consists of a large number of overlapping cells of identical radio access interfaces but of varying coverage size including macro- pico- and femtocells where femtocells can be deployed by end users (Damnjanovic *et al.*, 2011; Dhillon *et al.*, 2012). The problem this research attempts to answer is: how to optimize the *radio resources* of BSs and UEs in a HetSNet in a *scalable* and *dynamic* manner to achieve a network-wide *common objective*?

This question requires the definition of the terms radio resources, scalable, dynamic and common objective. The radio resources in a wireless network are the air interface resources and include transmission power, frequency bandwidth, time slots, the transmitters, receivers themselves and their antennas (Kulkarni *et al.*, 2010). These radio resources must be used to maximize the network performance with respect to some objective function. In 3GPP LTE-A the term radio resource management (RRM) is used to identify a specific functionality of the air interface (E-UTRAN) which includes radio admission control, radio mobility control, scheduling and dynamic allocation of resources to UEs (Sesia *et al.*, 2009). While the problem of RRO includes these tasks of RRM, this research is not limited to optimizing the solution methods employed in existing standards and must be taken as an attempt to propose novel solutions to the RRM problem beyond 4G (Nokia, 2011).

The solutions sought in this research must be inherently scalable and dynamic. SCs are expected to be deployed densely in large numbers (Qualcomm, 2012). Centralized solutions to optimize and respond to real-time radio conditions can be less scalable due to the signaling load and could suffer from slow response time (Hatoum *et al.*, 2011). Moreover, static preplanning such as the frequency planning in conventional macrocells is not the type of solutions the HetSNets demand. Therefore, this research explores decentralized or distributed real time control solutions. Decentralized in this context means a system of controllers that do not share information with any other controller or a central controlling unit. On the other hand, distributed controllers can share information with a subset of the other controllers and/or a central controller. The central controller may act as a relay for the information or may include processing as well (Tarau *et al.*, 2009). The goal is to achieve real-time decision making capability with minimum information exchange between the cells of the HetSNet.

In a wireless network the key measurement parameter of performance is area spectral efficiency measured in $\mathrm{bit/s/Hz/m^2}$ (Alouini & Goldsmith, 1999). Area spectral efficiency measure the rate of the system per Hertz per square area. In HetSNets, the objective of maximizing spectral efficiency must be approached cautiously. Area spectral efficiency does not guarantee fairness or game-theoretic stability of the system. As mentioned in Section 1.1, SCs can be user deployed and naturally the user wants to maximize his/her rate. The solutions sought in this research must consider fairness as well as stability. As discussed in Section 1.3, there are multiple definitions of fairness and stability and the solutions must justify the chosen definitions in the context of HetSNets (Shoham & Leyton-Brown, 2009; Akkarajitsakul *et al.*, 2011). Therefore, measurements such as minimum rate achieved by a UE and coverage probability of a UE has to be considered in trade-off with area spectral efficiency (Ghosh *et al.*, 2012). Also there is an interest in energy efficiency in order to provide longer battery life for the UE and also to economize transmit energy at the eNBs to reduce operation cost and to be environmentally responsible (Shakir *et al.*, 2013; Soh *et al.*, 2013).

1.4.1 Objectives of the Research

The key to distributed game-theoretic resource allocation in HetSNets is coordination among the competing BSs and UEs. Coordination in a HetSNet is not a trivial task due to the number of cells involved, the real-time nature of the problem, and the limitations in communication among nodes. Thus, there are trade-offs to be made by any proposed solution. To address this complex resource allocation problem in the air interface of HetSNets, this thesis establishes the following two main objectives:

- Modeling & Analysis: representation of the problem in a game-theoretic formulation, explore existence of game-theoretic equilibria, and quantify the equilibrium performance.
- Implementation: develop distributed algorithms to reach favorable equilibria.

The goal of the modeling and analysis is to transform the resource allocation problem into a well defined game form in order to quantitatively measure their operation. In addition it also enables to explore and justify the expectations that are placed on distributed organization of HetSNets by the industry and the research community. The game-theoretic equilibria are discussed in Section 1.4.2. The outcome of the implementation objective is to present algorithms that are tailored to the properties and control variables of HetSNets. These algorithms are capable of governing the network to the desirable operating conditions identified in the analysis. The results of the modeling & analysis and implementation should manifest the following characteristics:

- The modeling must seek to represent natural objectives of communicating user and the analysis should find out fair and stable operating conditions.
- When possible the algorithms should lead the HetSNet to a stable state and ideally to the *best* stable state.
- The algorithms have to be simulated in realistic wireless communication environments employing standard channel models.
- The algorithms must be analyzed for trade-offs between optimality and complexity of computation and message load between cells.

The above two main objectives are applied to a chosen set of problems. Each problem is addressed in a publication and presented in a unique chapter. The novelty of the analysis and solutions of this thesis are established under each publication in comparison to the state of the art. The simulation parameters depend on the problem and is specified under each chapter.

1.4.2 Methodology

This thesis employs game theory, in order to analyze distributed allocation of radio resources in HetSNets. Game theory is a tool to measure the stability of the final system state after a set of rational agents, also called players, interact with each other (Nisan *et al.*, 2007). There are two main subdivisions of games, namely noncooperative games and cooperative games. The normal-form of a noncooperative game is identified by a finite set of players, the finite set of actions available to each player, and their utility functions. Incomplete information availability can be introduced through player *types* and a common prior over the types, and such games are known as Bayesian games. In noncooperative games the players act to maximize individual utility. There are a few definitions of stability for a noncooperative game, the most common being the NE. While a mixed strategy NE exists in every finite normal-form game (Nash *et al.*, 1950) there is no known efficient algorithm to calculate one (Daskalakis *et al.*, 2006). Therefore, heuristic algorithms must be sought that achieve approximate equilibria such as the ε-NE. As discussed in the previous section, there exists a more general equilibrium than the NE called the CE (Shoham & Leyton-Brown, 2009). It has been shown that the CE can be obtained by solving a linear program (Aumann, 1987). Linear programs are known to be efficiently solvable (Arora & Barak, 2009). Besides there is the theorem that all NE are just special cases of CE (but not the other way around) (Aumann, 1987). Thus, in this research the equilibrium has to be judiciously decided. There are also other forms of equilibria such as quantal response equilibrium (McKelvey & Palfrey, 2007) and trembling hand perfect equilibrium (Shoham & Leyton-Brown, 2009), for situations where the players are expected to make mistakes in playing the rational action or in calculating their utility.

The other main class of games is cooperative games and the solution concept to these games are different from noncooperative games. The two main subclasses of cooperative games are Nash bargaining and coalition formation games. In coalition formation games a group of agents optimize the value of the group, then share the generated value among themselves (Nisan *et al.*, 2007; Saad *et al.*, 2009).

Mechanism design also knowns as reverse game theory is another theoretical tool that is being employed in this research. Mechanism design addresses the problem of construction of a game to achieve a required system state as the equilibrium solution of that game (Nisan *et al.*, 2007; Shoham & Leyton-Brown, 2009). Payments are widely used as incentives to coerce players to a preferred equilibrium, as is the case in the famous VCG mechanism (Nisan *et al.*, 2007). However, this research looks into mechanisms without payments due to their preferred value in wireless networks as discussed in Chapter 4.

While game theory provides the means to model and establishes stability criteria in multiagent systems, the core of this thesis is HetSNets. The link between game theory and HetSNets is established through the action sets available to the players and their utility functions. The utilities can be characterized by the achievable rate, coverage probability, interference to neighboring cells, and bit error rate (Dhillon *et al.*, 2012). The problem of optimizing the player's utility can be modeled and solved as an optimization problem. To optimize its utility the wireless agent requires the knowledge of the actions, utilities and the system variables of other players. However, a complete information model is not practical in a HetSNet. Therefore, learning rules such as fictitious play (Monderer & Shapley, 1996) and multiagent Q-learning (Littman, 2001) have to be employed to build distributed algorithms to drive the players to an equilibrium.

This research users Matlab® and Python as the main simulation software. These programming environments have been used by the wireless research community with great success in modeling and simulating wireless network conditions.

1.5 Summary of Publications

In addition to the articles presented in the four ensuing chapters, this research has produced the following publications.

1.5.1 Competition vs. Cooperation: A Game-Theoretic Decision Analysis for MIMO HetNets

This paper was published in the proceedings of IEEE International Conference on Communications (ICC), 2014 (Goonewardena *et al.*, 2014c).

This paper addresses the problem of competition vs. cooperation in the downlink between BSs, of a MIMO HetSNet. It considers a scenario where a MBS and a cochannel FBS each simultaneously serving their own UE, has to choose to act as individual systems or to cooperate in CoMP. The paper employs both the theories of noncooperative and cooperative games in a unified procedure to analyze the decision making process. The BSs of the competing system are assumed to operate at the maximum expected sum rate CE, which is compared against the value of CoMP to establish the stability of the coalition. It is proven that there exists a threshold distance between the macrocell UE and FBS, under which the region of coordination is non-empty. Theoretical results are verified through simulations.

1.5.2 Pairwise Nash and Refereeing for Resource Allocation in Self-Organizing Networks

This paper was published in the proceedings of IEEE Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), 2014 (Goonewardena *et al.*, 2014a).

This paper considers the allocation of frequency and time resources in a heterogeneous network, in a self-organizing manner. The general problem is to assign a resource set, so as to minimize the number of pairs of adjacent base stations that obtain the same resource. This can be modeled by MCC on an undirected graph, where the colors are the resources, the vertices are the wireless nodes and the edges represent interference relations between nodes. The MCC decision problem is NP-complete. This paper develops a game-theoretic model for the MCC problem. The players of this game are a set of colored agents, which in practice could be software robots. The game is proven to possess multiple pure-strategy NEs. Then a swapping mechanism is developed to improve the NE performance and the resulting coloring is shown to be pairwise-Nash stable. Further refinement is proposed by making use of an external referee. All theoretical results are corroborated through simulations.

1.5.3 On Minimum-Collisions Assignment in Heterogeneous Self-Organizing Networks

This paper was published in the proceedings of IEEE Global Communications Conference (GLOBECOM), 2014 (Goonewardena *et al.*, 2014b).

Minimum-collisions assignment, in a wireless network, is the distribution of a finite resource set, such that the number of neighbor cells which receive common elements is minimized. In classical operator deployed networks, resources are assigned centrally. Heterogeneous networks contain user deployed cells, therefore centralized assignment is problematic. Minimum-collisions assignment includes orthogonal frequency bands, time slots, and physical cell identity (PCI) allocation. Minimum-collisions assignment is NP-complete, therefore a potential-game-theoretic model is proposed as a distributed solution. The players of the game are the cells, actions are the set of PCIs and the cost of a cell is the number of neighbor cells in collision. The price of anarchy and price of stability are derived. Moreover, the paper adapts a randomized-distributed-synchronous-update algorithm, for the case, when the number of PCIs is higher than the maximum degree of the neighbor relations graph. It is proven that the algorithm converges to a optimal pure-strategy Nash equilibrium in finite time and it is robust to node addition. Simulation results demonstrate that the algorithm is sub-linear in the size of the input graph, thus outperforms best response dynamics.

1.5.4 Self-Optimization of Uplink Power and Decoding Order in Heterogeneous Networks

This paper was published in the proceedings of IEEE International Conference on Communications Workshop (ICCW), 2015 (Goonewardena *et al.*, 2015b).

This work demonstrates the distributed joint selfoptimization of power and decoding order, to alleviate uplink intercell interference in a heterogeneous network, with signal to interference ratio constraints. This problem can be formulated as a potential game of coupled action space among the base stations However, best response dynamics of this game require global channel knowledge at all cells. This is unlike when power is the only optimization variable, in which case the users can autonomously reach the unique sum power minimizing equilibrium with locally measurable information at its base station. Thus, in order to propose a distributed scheme, by exploiting the properties of small-cells, this paper designs a perturbation to the original action space and a messaging scheme among neighborhoods of base stations. The resulting perturbed game has best response dynamics that converge with much less information than the original game, under three proposed sufficient constraints. Monte Carlo simulations demonstrate that the perturbed game has performance close to that of the original game.

1.5.5 Generalized Satisfaction Equilibrium: A Model for Service-Level Provisioning in Networks

This paper was published in the proceedings of European Wireless (EW), 2016 (Goonewardena *et al.*, 2016).

This paper presents a generalization of the existing notion of SE for games in satisfaction-form. The new equilibrium, which is referred to as the GSE, is particularly adapted for modeling problems such as service-level provisioning in decentralized self-configuring networks. Existence theorems for GSEs are provided for particular classes of games in satisfaction-form and the problem of finding a pure-strategy GSEs with a given number of satisfied players is shown to be NP-hard. Interestingly, for certain games there exist a dynamic, analogous to the best response of games in normal-form, that is shown to efficiently converge to a pure-strategy GSE under the given sufficient conditions. These contributions form a more flexible framework for studying self-configuring networks than the existing SE framework. This paper is concluded by a set of examples in wireless communications in which classical equilibrium concepts are shown to be not sufficiently adapted to model service-level provisioning. This reveals the relevance of the new solution concept of GSE.

1.5.6 Fair Scheduling for Energy Harvesting Nodes

This article was published in Wireless Commun. Lett. in June, 2015 (Goonewardena *et al.*, 2015c).

This letter considers the problem of scheduling in the MIMO multiple-access wireless channel, where the transmitters are energy harvesting nodes (EHNs) that are powered by renewable energy sources. In this letter the conventional scheduling objective of maximizing rate is augmented by two other objectives, regulating fairness, and stabilization of the stored energy processes of the EHNs. This problem is formulated as a network of energy queues, which represent the batteries. Considering the stochastic nature of the wireless channel and the energy harvesting processes, this letter employs Lyapunov drift plus penalty technique to develop a cross-layer scheduler that operates in a slotted-time and distributed manner. At each epoch it selects an EHN for transmission and computes the transmission power. As an added advantage, the power control algorithm still retains the optimal water-filling solution. Through simulations, the proposed solution is compared against a conventional max-rate scheduler and is shown to better enforce fairness, stabilize the battery levels, and minimize the required battery capacity.

CHAPTER 2

OPPORTUNISTIC DISTRIBUTED CHANNEL ACCESS FOR A DENSE WIRELESS SMALL-CELL ZONE

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This article was published in Wireless Commun. and Mobile Comput. in October 2015 (Goonewardena *et al.*, 2015a).

2.1 Abstract

This paper considers the uplink access of a zone of closed-access small-cells (SCs) that is deployed in a macrocell service area. All user equipments (UEs) have access to a common set of orthogonal channels, leading to intercell interference. Moreover, each channel forms a separate collision domain in each SC, hence can be successfully used only by one SUE that belongs to that SC. This paper proposes two noncooperative Bayesian games, $\mathbf{G}_1$ and $\mathbf{G}_2$, that are played among the small-cell UEs. $\mathbf{G}_1$ assumes the availability of channel state information (CSI) at the transmitters while $\mathbf{G}_2$ only assumes the availability of the distribution of CSI. Each SUE can choose to transmit over one of the channels or not to transmit. The emphasis of the paper is on the set of symmetric threshold strategies where the Nash equilibrium is fully determined by a single parameter. The existence and uniqueness of pure Bayesian Nash symmetric equilibrium (BNSE) of $\mathbf{G}_1$ in threshold strategies and mixed BNSE of $\mathbf{G}_2$ in uniformly distributed threshold strategies are proven. Numerical results corroborate the theoretical findings and benchmark against another decentralized scheme.

2.2 Introduction

Small-cells (SCs) are introduced to improve the coverage and meet the increasing demand for throughput of indoor users. They are low-power, cost-effective, and short-range radio access networks. The throughput is improved due to the reduced link distance between the indoor located user equipments (UEs) and the small-cell access point (SAP). SCs include microcells, picocells, and femtocells (Andrews *et al.*, 2014). A UE connected to an SAP is called a small-cell user equipment (SUE). In an underlay deployment, SCs have access to the same spectrum as the existing macrocell. A wireless network with co-existing macrocells and SCs is called a heterogeneous small-cell network (HetSNet). In a HetSNet, intercell interference is a key limiting factor (Lopez-Perez *et al.*, 2009). Centralized coordinated scheduling of SUEs among multiple SAPs is optimal for interference mitigation, but such a scheme requires extensive global channel state information (CSI) of all the UEs. Acquisition of time-varying global CSI involves a large signaling overhead, which renders centralized schemes less practical. Therefore, distributed interference management in HetSNets with light signaling is a true challenge (Xu *et al.*, 2014a). *Noncooperative game theory* is one of the tools employed in this domain (Nisan *et al.*, 2007).

Recently numerous schemes, which use game theoretic tools, are developed to mitigate interference. A potential-game-theoretic solution for intracell interference mitigation in a cognitive radio network through combined power allocation and base station association is considered in (Hong *et al.*, 2011). Another potential-game is introduced in (Buzzi *et al.*, 2012) for the problem of uplink channel and power allocation in a multicell environment. The convergence time to equilibrium in a potential game can be large. In (Han *et al.*, 2007), a concave game is proposed for an interfering set of orthogonal frequency division multiplex (OFDM) transmit-receive pairs. In a concave game, the action set of each player is compact and convex, the utility of a player is continuous in the action profile and is quasi-concave in its own action (Lasaulce & Tembine, 2011). The strategy space of the transmitters is the rate assignment over the set of OFDM channels. Finding the Nash equilibrium (NE) in a concave game requires complete information at each player.

Multi-user channel access games are also addressed in relation to medium access control (MAC) protocol design. This approach is taken in random access networks, such as ad-hoc, ALOHA and carrier sense multiple access (CSMA). A game theory inspired MAC protocol for transmit power allocation is designed in (Wang *et al.*, 2006) for an interference channel. In (Inaltekin & Wicker, 2008), an uplink channel access game is proposed for a set of co-located transmit-receive pairs over a single channel. It considers a collision-domain approach instead of interference and presents a mixed-strategy nonsymmetric equilibria. Likewise authors in (Chen *et al.*, 2010) assume a collision-domain and model the access probability design as a continuous-action-space game. In the ALOHA setting, a noncooperative game is designed in (Hultell *et al.*, 2011), where the strategy space is the probability of transmission. In (Hanawal *et al.*, 2012), a mobile ad-hoc network where nodes follow slotted ALOHA protocol is considered and they develop a game for the channel access probability at the symmetric NE.

2.2.1 Related Work

Application of game theory for channel access in multichannel case appears in orthogonal frequency division multiple access (OFDMA) networks. In (La *et al.*, 2009), subchannel allocation in an OFDMA based network is considered as a potential game. The utility of a player is a function of interference. An uplink power allocation game among UEs is analyzed in (Al-Zahrani & Yu, 2011). Interference is controlled through a quadratic cost function of the transmitted power. In (Zheng *et al.*, 2012), uplink channel allocation in an OFDMA multicell system is formulated and solved for the correlated equilibrium (CE). In (Gao *et al.*, 2011), a distributed cell selection and resource allocation scheme that is performed by UEs is presented. It is a two-stage game. A UE first selects the cell and then selects the radio resource. In (Hong & Garcia, 2012), resource allocation problem of the OFDMA downlink is addressed in the context of mechanism design. The authors demonstrate that the problem is NP-hard and provide an α-optimal solution. In a multiple femtocell scenario an OFDMA based downlink power allocation with interference constraints is considered, in (Galindo-Serrano *et al.*, 2012), as a generalized NE problem. In (Guan *et al.*, 2013), a threshold strategy based game in a multichannel environment is presented. A threshold strategy of a player is defined by a single parameter.

The above-discussed research consider games that require complete CSI. Therefore, they have limited practical applicability. The incomplete CSI case is taken into consideration through Bayesian games in (Adlakha *et al.*, 2007). Therein, power allocation for transmit-receive pairs over a multichannel system with interference is considered. The authors prove that spreading power equally among the flat fading channels is a pure-strategy NE. The Bayesian symmetric games in (Cho & Tobagi, 2008; Cho *et al.*, 2008; Lee *et al.*, 2009) consider threshold strategies in a single channel wireless network. The channel access model in (Cho & Tobagi, 2008; Cho *et al.*, 2008) is a single collision domain and a single access point (AP). In (Lee *et al.*, 2009), the work of (Cho & Tobagi, 2008) is extended to multiple APs with intercell interference.

A key limitation of applying game theory to design distributed solutions is the complexity in finding the NE. It has been proven, that even for a two player game finding the NE is PPAD (Polynomial Parity Arguments on Directed graphs) complete (Chen *et al.*, 2009). Therefore, we motivate a symmetric game which possesses NE that is computable individually by each player as the unique root of a function (Cho & Tobagi, 2008). In a symmetric game the utility of a player, given the action profile of other players, is independent of the player (Nisan *et al.*, 2007). This article considers the orthogonal multichannel uplink transmission in a *dense zone* of SCs. The qualifier *dense*, in the context of this article, means that the coverage areas of the SCs in consideration overlap with each other. That is as opposed to sparse deployment, where SCs are deployed far apart in order not interfere with each other (ping Yeh *et al.*, 2008). The qualifier *zone*, in the context of this article, means that the set of SCs in consideration is confined to a localized area that is small relative to the macrocell coverage area. Examples of dense zones of SCs are convention centers, hotel lobbies or shopping malls, where more than one telecommunication service providers maintain picocells, each for their own users. Thus, it is important to this research that a *dense zone of SCs* is not understood as a large number of SCs, rather it is an overlapping localized deployment. Fig. 2.1 depicts a dense zone of SCs. In such a dense deployment, cochannel interference between SCs is a key limitation and opportunistic channel access has been proposed as a solution (Zahir *et al.*, 2013). In opportunistic channel access, an SUE exploits the fading nature of the channels to judiciously access, while its own channel gain is relatively higher and the channel gains of the interferes are lower (Tse & Viswanath, 2005).

2.2.2 Contributions

The objective of this article is to propose low complex, decentralized-opportunistic channel access schemes based on Bayesian games. To that end, the article considers symmetric threshold strategies; those that are defined by a single parameter (Cho & Tobagi, 2008; Guan *et al.*, 2013). Two uplink channel access games are discussed. The first game $\mathbf{G}_1$, considers the case where each SUE knows its CSI. This situation is identified in literature as channel state information available at the transmitter (CSIT). The second game $\mathbf{G}_2$, considers the case where each SUE has statistical knowledge of its CSI (statistical-CSIT). In this article:

- we bring together in a game model; multiple channels, intracell per-channel collision domains, inter-SC and SC-macrocell interference, and random symbol availability at the SUEs. These are well identified resource allocation constraints in an underlay dense SC deployment.
- we prove the unique pure-strategy Bayesian Nash symmetric equilibrium (BNSE) in threshold strategies for game $\mathbf{G}_1$ with CSIT.
- we prove the unique mixed-strategy BNSE in uniformly distributed threshold strategies for game $\mathbf{G}_2$ with statistical-CSIT.
- we corroborate the theoretical results through numerical simulations and compare against a scheme where each SC schedules the SUE with highest channel gain.

The pure-strategy BNSE proved in $\mathbf{G}_1$ is an extension of the single channel result of (Lee *et al.*, 2009) to a multichannel case. However, the extension to the multichannel case is nontrivial and the proof method is novel, in that we employ stochastic coupling theory (Thorisson, 2000). The mixed-strategy BNSE proved in $\mathbf{G}_2$ for statistical-CSIT is new in SC research to the best of our knowledge. The advantage of BNSE in threshold strategies is that each player is independently able to find the equilibrium without message passing. However, its applicability is limited to symmetric situations among players, which is the case in a dense SC zone as justified in the following sections.

The remainder of this article is organized as follows. The assumptions and system model are detailed in Section 2.3. Development of game $\mathbf{G}_1$ for CSIT is presented in Section 2.4, and Section 2.5 solves it. Development of game $\mathbf{G}_2$ for statistical-CSIT and its solution is presented in Section 2.6. Numerical results are discussed in Section 2.7 and Section 2.8, concludes the article.

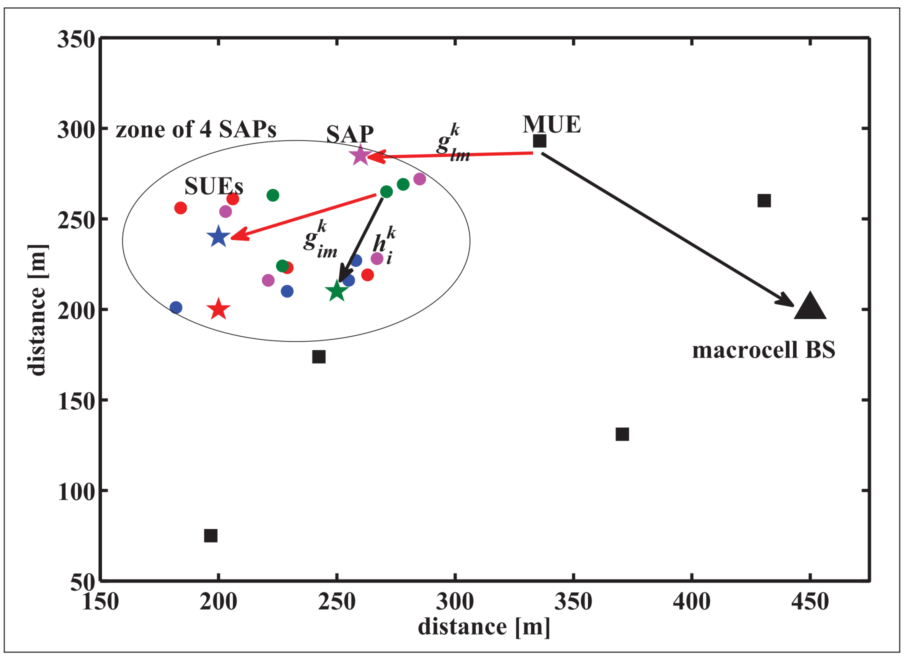


Figure 2.1 A transmission scenario illustrating a dense zone of SCs where 4 operators serve a commercial building. A ⋆ denotes an SAP, ● denotes an SUE, ▲ denotes the macrocell base station (BS), ■ denotes an MUE. The colors match the SUEs to their respective home-SAPs

2.3 System Model

This article considers the uplink access in a dense zone of SCs that is underlayed in a single macrocell coverage area. Each SAP forms a single SC and hence SAP and SC are synonymous.

As was defined in Section 3.2, SCs that are in a dense zone, have overlapping coverage areas and are deployed in a confined area. For example, a convention center where multiple operators maintain picocells and each picocell only serves the customers of the one operator to which it belongs. Fig. 2.1 depicts an example dense zone of SCs. The set of SAPs is $\mathscr{M}$. Let $\mathscr{N}$ denote the set of SUEs and $\mathscr{K}$ the set of orthogonal channels whose cardinalities are given by N and K, respectively. The set of macrocell user equipments (MUEs) is denoted by $\mathfrak{N}$. Each SAP $m \in \mathscr{M}$ operates in *closed-access* mode and hence is accessible only by SUEs that are in its access list and they are called the home-SUEs of that SAP (Sesia *et al.*, 2011). We assume that an SUE i can only be in the access list of only one SAP, which is called its home-SAP and denoted by $b_i \in \mathscr{M}$. The set of home-SUEs of SAP m is denoted by $\mathscr{N}_m$ and its complement is denoted by $\mathscr{N}_{-m}$. Narrow-band single tap Rayleigh fading channels are assumed. Then the baseband equivalent received signal y_m^k, at SAP m on channel $k \in \mathscr{K}$ is,

$$y_m^k = \sum_{i \in \mathscr{N}_m} \bar{h}_i^k x_i^k + \sum_{j \in \mathscr{N}_{-m}} \bar{g}_{jm}^k x_j^k + \sum_{l \in \mathfrak{N}} \bar{g}_{lm}^k x_l^k + n_m^k, \tag{2.1}$$

where $\bar{h}_i^k$ is the complex channel gain from SUE i to its home-SAP m on channel k. The complex interference gain from a UE (can be an SUE or an MUE) $j \in \mathscr{N}_{-m} \cup \mathfrak{N}$ to SAP m on channel k is denoted by $\bar{g}_{jm}^k$ (Tse & Viswanath, 2005). The corresponding power gains are denoted by $h_i^k \triangleq \mid \bar{h}_i^k \mid^2$, $g_{im}^k \triangleq \mid \bar{g}_{jm}^k \mid^2$ and they follow exponential distribution. The complex valued transmit symbols of a UE $j \in \mathscr{N} \cup \mathfrak{N}$ on channel k is denoted by x_j^k. The circular symmetric additive white Gaussian noise (AWGN) is denoted by n_m^k. The second right hand term of (2.1) is the sum interference from the SUEs that belongs to SAPs other than m. The third term is the sum interference of the MUEs. Note that although we sum over all UEs, if a UE j does not transmit, the respective symbol x_j^k is 0.

This article assumes that the SCs are *identically populated*, i.e., the number of home-SUEs in each SC is independent and identically distributed (i.i.d.). In practice, this corresponds to a situation where the SAP operators have equal market share. Then, without loss of generality, the number of home-SUEs in each SC is set to a constant with probability one. Unlike in macrocells, in SCs all home-SUEs lie sufficiently close to the home-SAP so that no power control is needed and therefore SUEs transmit at constant normalized unit power (Novlan *et al.*, 2013).

The following development considers that the load of the MUEs is balanced in distribution over the set of channels $\mathscr{K}$, which is realistic as there are a relatively larger number of MUEs associated with the macrocell cell than the number of channels. From the above assumption and since the SAPs in the zone are close to each other with respect to the coverage area of the macrocell, the sum of MUE interference received at each SAP can be modeled by identically distributed random variables (Chandrasekhar & Andrews, 2009). In other words, when $\zeta_{km} \triangleq \sum_{l \in \mathfrak{N}} g^k_{lm}$ with the probability density $f_{\zeta_{km}}$, we have $f_{\zeta_{km}} = f_{\zeta_{k'm'}} = f_\zeta$, $\forall\ k, k' \in \mathscr{K}$. The distribution f_ζ can be estimated through cognitive features in SCs and for the sake of clarity, the errors associated with estimation is disregarded in this research (Yucek & Arslan, 2009).

The symbol availability at an SUE i is denoted by the Boolean random variable α_i. If a symbol is available for transmission then $\alpha_i = 1$ otherwise 0. The above discussed system model closely represents a mall or a convention center, where a number of service providers operate cochannel SAPs for their own customers.

2.4 Design of $\mathbf{G}_1$: A Game with CSIT

This section defines the components of the Bayesian game with CSIT. The set of players are the SUEs. By the definition of CSIT, each SUE i posses perfect information of $\left(h^k_i, k \in \mathscr{K}\right)$. SUE i also knows its α_i. The actions available for an SUE are: transmit on a channel k denoted by $\mathtt{T}_k$ and the action of *"do not transmit"* denoted by $\mathtt{X}$. Then the action set of SUE i is $\mathscr{A}_i \triangleq \{\mathtt{X}, \mathtt{T}_1, \ldots, \mathtt{T}_K\}$. We define the joint action spaces by $\mathscr{A} \triangleq \prod_{i \in \mathscr{N}} \mathscr{A}_i$ such that $a \in \mathscr{A}$ and $\mathscr{A}_{-i} \triangleq \prod_{j \in \mathscr{N} \setminus i} \mathscr{A}_j$ such that $a_{-i} \in \mathscr{A}_{-i}$. The MUEs do not take part in the game, but exogenously affect the outcome through interference.

2.4.1 Symmetric-Independent Types

Type of a player in a Bayesian game is the private information of that player (Nisan *et al.*, 2007). In our system model, the private information available at SUE i is its channel power gains to the home-SAP and its symbol availability. Since an SUE has a multitude of private information, we define a single private information vector θ_i, called the *type vector*, containing all private information of an SUE, $\theta_i \triangleq \left(h_i^k\right)_{k\in K}, \left(g_{im}^k\right)_{k\in\mathscr{K}, m\in\mathscr{M}\setminus\{b_i\}}, \alpha_i$. The *type set* is denoted by Θ_i such that $\theta_i \in \Theta_i$. We also define the type set product $\Theta \triangleq \prod_{i\in\mathscr{N}} \Theta_i$ such that $\theta \in \Theta$ and $\Theta_{-i} \triangleq \prod_{j\in\mathscr{N}\setminus i} \Theta_j$ such that $\theta_{-i} \in \Theta_{-i}$.

Let the probability densities of h_i^k, g_{im}^k, and α_i, be $f_{h_i^k}$, $f_{g_{im}^k}$, and f_{α_i}, respectively. Then the belief that SUE i holds about the types of other players θ_{-i} is given by the density function $\bar{f}_{\theta_{-i}} = \prod_{\substack{k\in\mathscr{K} \\ j\in\mathscr{N}\setminus i}} \left(f_{h_j^k} \cdot f_{g_{jm}^k}\right) \cdot \prod_{j\in\mathscr{N}\setminus i} f_{\alpha_j}$.

It is customary to model i.i.d symbol availability among SUEs and hence $f_{\alpha_i} = f_\alpha, \forall\, i \in \mathscr{N}$. Recall from Section 2.3 that the SCs lie in a confined zone. Therefore, all the SUEs lie close to the SAPs and experience a similar scattering environment. As such, we assume that all players hold independent and identical beliefs about each other's channels, which in Bayesian games is called *symmetric-independent types* (Nisan *et al.*, 2007). From symmetric-independent types, $f_{h_i^k} = f_h$ and $f_{g_{im}^k} = f_g \;\forall\, i \in \mathscr{N}, m \in \mathscr{M}$, and $k \in \mathscr{K}$.

2.4.2 Utility Design

The utility of player i is a function $u_i : \mathscr{A} \times \Theta \to \mathbb{R}$. When $\alpha_i = 0$, SUE i does not possess a symbol and hence the utility is zero. This article models each channel on each SAP as a separate *collision domain*. It implies that if more than one home-SUE of a given SAP transmits simultaneously on the same channel, then all those home-SUEs (of that cell) obtain a zero utility due to a collision. It is important to emphasize that the SUEs from different SAPs may transmit simultaneously over the same channel. In addition, more than one home-SUE of a given SAP may transmit simultaneously as long as they employ different channels.

However, to obtain a positive rate, avoiding a collision is not sufficient. The SINR needs to be above a detectable threshold Γ_{th} as well (Zahir *et al.*, 2013). Let $\mathscr{N}_{b_ik}$ denote the set of the home-SUEs of SAP b_i except the SUE i , that transmits on channel k. Let $\mathscr{N}_{-b_ik}$ denote the set of SUEs of SAPs $\mathscr{M} \setminus b_i$ that transmits on channel k. Their cardinalities are denoted by N_{b_ik} and N_{-b_ik} respectively. According to the above discussion, we define the utility $u_i\left(a_i, a_{-i}, \theta\right)$, as follows. If SUE i does not transmit

$$u_i\left(\mathtt{X}, a_{-i}, \theta\right) \triangleq \begin{cases} \rho & \text{if } \alpha_i = 1, \\ 0 & \text{if } \alpha_i = 0, \end{cases} \tag{2.2}$$

where the modeling parameter $\rho \in \mathbb{R}$ is an incentive given to the player.

If SUE i has $\alpha_i = 1$, and transmits successfully on channel k, i.e., obtains SINR $\geq \Gamma_{\text{th}}$, and $N_{b_ik} = 0$, then

$$u_i(\mathtt{T}_k, a_{-i}, \theta) \triangleq \log_2\left(1 + \frac{h_i^k}{\sum\limits_{j \in \mathscr{N}_{-b_ik}} g_j^k + \zeta_k + \sigma^2}\right). \tag{2.3}$$

Otherwise, if SUE i has $\alpha_i = 1$, and transmits unsuccessfully on channel k, i.e., obtains SINR $<$ Γ_{th} or $N_{b_ik} \neq 0$, then

$$u_i\left(\mathtt{T}_k, a_{-i}, \theta\right) \triangleq 0. \tag{2.4}$$

For the notational convenience, as the SAP in discussion is clear, subscript m is omitted and g_{jm}^k simplifies to g_j^k, while ζ_{km} simplifies to ζ_k.

2.4.3 Definition of Game G_1

A game in normal-form is defined by the set of players, action set of each player and utility of each player. In addition, when the game is Bayesian, we need to specify the type set and belief of each player. Finally, the system state that is given by the external random variables

Table 2.1 Definition of Game $\mathbf{G}_1$

Players	$\mathscr{N}$
Action	$a_i \in \mathscr{A}_i$
Type	$\theta_i \in \Theta_i$
Belief	$\bar{f}_{\theta_{-i}}$ over Θ_{-i}
System state	$\zeta_{km}\ \forall\ k \in \mathscr{K}, m \in \mathscr{M}$
Payoff	$u_i(a,\theta)$, $a \in \mathscr{A}$ and $\theta \in \Theta$

that affect the utilities. In this article the pairs of terms "player"-"SUE" and "payoff"-"utility" are synonymously used.

2.5 Symmetric-Threshold Equilibrium of $\mathbf{G}_1$

The *ex interim* expected utility of player $i \in \mathscr{N}$ is defined as, $\mathbb{E}_{\theta_{-i}|\theta_i} u_i(\cdot)$ (Shoham & Leyton-Brown, 2009). From the independence of random variables in symmetric-independent types, the conditional expectation $\mathbb{E}_{\theta_{-i}|\theta_i}$ simplifies to $\mathbb{E}_{\theta_{-i}}$. For brevity, the rest of the article refers to ex interim expected utility as the expected utility.

Next we introduce the definitions of pure strategies, best response (BR) strategy, and Bayesian Nash equilibria (Nisan *et al.*, 2007; Shoham & Leyton-Brown, 2009).

Definition 1. In a Bayesian game a pure-strategy of a player i is a relation $s_i : \Theta_i \to \mathscr{A}_i$.

Following standard game-theoretic notation, the strategy vector of all players except SUE i is denoted by $s_{-i} \triangleq \left(s_j, j \in \mathscr{N} \setminus i\right)$ and the strategy profile of all players is denoted by $s \triangleq \left(s_i, i \in \mathscr{N}\right)$.

Definition 2. Given the strategy vector s_{-i}, a BR strategy of player i, denoted by $\tilde{s}_i$, is given by

$$\tilde{s}_i(\theta_i) \in \underset{s_i \in \mathscr{S}_i}{\arg\max} \left\{\mathbb{E}_{\theta_{-i}} u_i\left(s_i(\theta_i), s_{-i}, \theta\right)\right\} \forall \theta_i \in \Theta_i.$$

Definition 3. The strategy profile $\tilde{s} \triangleq (\tilde{s}_i, i \in \mathscr{N})$ is a Bayesian Nash equilibrium if $\tilde{s}_i$ is a BR strategy for $\tilde{s}_{-i}\ \forall\ i \in \mathscr{N}$.

2.5.1 Threshold Strategies

This article considers *threshold strategies* similar to those used in (Cho & Tobagi, 2008; Cho *et al.*, 2008; Lee *et al.*, 2009). Such strategies form a subset in the feasible strategy space of the game. Threshold strategies are attractive since the user only needs to compare its channel gain against a threshold and is every efficient to implement. Let us define the threshold strategy of SUE i as follows:

$$s_{i\text{th}}(\theta_i) \triangleq \begin{cases} \mathtt{T}_k & \text{if } \alpha_i = 1, h_i^k = \max\limits_{k' \in \mathscr{K}} \{h_i^{k'}\} \geq h_{i\text{th}}, \\ \mathtt{X} & \text{otherwise}, \end{cases} \tag{2.5}$$

where $h_{i\text{th}}$ is a non-negative real valued parameter. The threshold-strategy definition (2.5) states that a player transmits on channel k if the channel power gain h_i^k is the largest among all the channels and h_i^k is greater than a threshold $h_{i\text{th}}$. As a consequence of the special form of threshold strategies, we can denote the strategy profile of the players by simply specifying their threshold vector $s_{\text{th}} \triangleq (h_{i\text{th}}, i \in \mathscr{N})$. Similarly, the threshold strategy vector of all players except i is denoted by $s_{-i\text{th}} = (h_{j\text{th}}, j \in \mathscr{N} \setminus i)$. If the threshold is *symmetric*, i.e., common to all players, then we denote the strategy profile by $s_{\text{th}}^{\text{sym}} \triangleq (h_{\text{th}})$. Since we search for a unique BNSE in threshold strategies, according to Definition 3, our goal is to demonstrate that there is a unique threshold $h_{i\text{th}} = \tilde{h}_{\text{th}}\ \forall\, i \in \mathscr{N}$ such that $s_{\text{th}}^{\text{sym}} = (\tilde{h}_{\text{th}})$ is a mutual BR strategy profile.

When SUE i plays the threshold strategy defined by (2.5), the probability that it transmits on a channel k is

$$\mathrm{q}_{ik}(h_{i\text{th}}) = \Pr\left(\alpha_j = 1, h_i^k = \max_{k' \in \mathscr{K}} \{h_i^{k'}\}, h_i^k \geq h_{i\text{th}}\right). \tag{2.6}$$

It is observed that the probability $\mathrm{q}_{ik}(h_{i\text{th}})$ is increasing in h_i^k and decreasing in $h_{i\text{th}}$. From independence in symmetric-independent types, the probability that all the SUEs in $\mathscr{X}_k \subset \mathscr{N}$ transmit on channel k is $\mathrm{p}_1(\mathscr{X}_k) \triangleq \prod_{j \in \mathscr{X}_k} \mathrm{q}_{jk}(h_{j\text{th}})$, and the probability that none of the SUEs in $\mathscr{X}_k$ transmit on k is $\bar{\mathrm{p}}_1(\mathscr{X}_k) \triangleq \prod_{j \in \mathscr{X}_k} \left(1 - \mathrm{q}_{jk}(h_{j\text{th}})\right)$. Let $\mathscr{N}'_{-b_ik}$ denote the set of SUEs that belong to SAPs other than b_i and that *does not* transmit on channel k. Then it follows that $\bar{\mathrm{p}}_1(\mathscr{N}_{b_ik})$ is the probability that i does not encounter a collision on k. The probability that the set $\mathcal{N}_{b_ik}$ transmits is $\mathrm{p}_1\left(\mathcal{N}_{-b_ik}\right)$ and the probability that the set $\mathcal{N}'_{-b_ik}$ does not transmit is $\bar{\mathrm{p}}_1(\mathcal{N}'_{-b_ik})$. These probabilities are used to define the expected utility.

Now the expected utility of player i when $\alpha_i = 1$ and $a_i = \mathtt{T}_k$ is denoted by $\mathbb{E}_{\theta_{-i}} u_i\left(h_i^k, s_{-i\text{th}}, \theta\right)$ and is given by (2.7). The power set of $\mathcal{N}_{-b_i}$ is denoted by $\mathbb{P}\left(\mathcal{N}_{-b_i}\right)$. The integration region $\mathcal{D}$ is $\{(g_{jb_i}^k, j \in \mathcal{N}_{-b_ik}), \zeta_k : \frac{h_i^k}{\sum_{j\in\mathcal{N}_{-b_ik}} g_j^k+\zeta_k+\sigma^2} \geq \Gamma_{\text{th}}\}$ and $f_{\mathbf{g}}$ is the probability density of the random variable vector $(g_j^k, j \in \mathcal{N}_{-b_ik})$.

$$\mathbb{E}_{\theta_{-i}} u_i(h_i^k, s_{-i\text{th}}, \theta) =$$

$$\bar{\mathrm{p}}_1\left(\mathcal{N}_{b_ik}\right) \sum_{\mathcal{N}_{-b_ik}\in\mathbb{P}\left(\mathcal{N}_{-b_i}\right)} \mathrm{p}_1\left(\mathcal{N}_{-b_ik}\right) \bar{\mathrm{p}}_1\left(\mathcal{N}'_{-b_ik}\right) \int_{\mathcal{D}} f_{\mathbf{g}} f_{\zeta} \log_2\left(1 + \frac{h_i^k}{\sum_{j\in\mathcal{N}_{-b_ik}} g_j^k + \zeta_k + \sigma^2}\right) d\mathbf{g} d\zeta. \tag{2.7}$$

$$\mathbb{E}_{\theta_{-i}} u_i(h_i^k, s_{-i\text{th}}^{\text{sym}}, \theta) = \bar{\mathrm{p}}_1^{\text{sym}}\left(\mathcal{N}_{b_ik}\right) \mathbb{E}_X \int_{\mathcal{D}} f_{\mathbf{g}} f_{\zeta} \log_2\left(1 + \frac{h_i^k}{\sum_{j\in X} g_j^k + \zeta_k + \sigma^2}\right) d\mathbf{g} d\zeta. \tag{2.8}$$

Claim 1. When $\alpha_i = 1$, for symmetric-independent types and strategy vector $s_{-i\text{th}}$, it holds that

$$\arg\max_{k\in\mathcal{K}} \{\mathbb{E}_{\theta_{-i}} u_i(h_i^k, s_{-i\text{th}}, \theta)\} = \arg\max_{k\in\mathcal{K}} \{h_i^k\}.$$

Proof. By symmetric-independent types, we have that interference channel gains of a player j to SAP b_i, given by $g_{jb_i}^k$ are i.i.d. $\forall\, k \in \mathcal{K}$. From (2.6) we have that $\mathrm{p}_{ik}\left(h_{i\text{th}}\right) = \mathrm{p}_{ik'}\left(h_{i\text{th}}\right)$ for $k, k' \in \mathcal{K}$. From (2.7) it follows that $h_i^k \geq h_i^{k'}$ implies $\mathbb{E}_{\theta_{-i}} u_i\left(h_i^k, s_{-i\text{th}}, \theta\right) \geq \mathbb{E}_{\theta_{-i}} u_i(h_i^{k'}, s_{-i\text{th}}, \theta)$. Therefore, selecting the channel with best expected payoff is equivalent to selecting the channel with the highest channel gain. □

Claim 1 essentially says that channel with maximum power gain *dominates* all the other channels. Consequently, when $\alpha_i = 1$, it brings down the choices of actions from set $\mathcal{A}_i$, to just 2 actions, namely arg max $\left\{h_i^1, \ldots, h_i^K\right\}$ and $\mathtt{X}$. Without loss of generality we suppose that $k = \arg\max\left\{h_i^1, \ldots, h_i^K\right\}$. Then to select the BR between $\mathtt{T}_k$ and $\mathtt{X}$, player i tests for the condition $\mathbb{E}_{\theta_{-i}} u_i(h_i^k, s_{-i\text{th}}, \theta) \geq \mathbb{E}_{\theta_{-i}} u_i\left(\mathtt{X}, s_{-i\text{th}}, \theta\right)$. If the condition is true then it chooses $\mathtt{T}_k$ otherwise $\mathtt{X}$. The threshold $h_{i\text{th}}$, that h_i^k must exceed in order to meet the above condition, is the solution to the following equation

$$\mathbb{E}_{\theta_{-i}} u_i(h_i^k, s_{-i\text{th}}, \theta) = \mathbb{E}_{\theta_{-i}} u_i\left(\mathtt{X}, s_{-i\text{th}}, \theta\right). \tag{2.9}$$

By (2.2), we observe that the right hand side of (2.9) is equal to ρ. The solution $h_i^k = h_{i\text{th}}$ defines the BR of player i in the set of threshold strategies defined by (2.5).

In the case of symmetric-independent types and strategy profile $s_{\text{th}}^{\text{sym}} = (h_{\text{th}})$, the event that player i transmits on k and the event that player j transmits on k are independent and have equal probabilities given by (2.6). Therefore, let us define the unique probability that any player transmits on any channel by

$$\mathrm{q}_1\left(h_{\text{th}}\right) = \mathrm{q}_{ik}\left(h_{\text{th}}\right) \forall i \in \mathscr{N}, k \in \mathscr{K}. \tag{2.10}$$

From symmetric-independent types and strategy profile $s_{\text{th}}^{\text{sym}} = (h_{\text{th}})$, the probability that player i experiences no collisions on channel k is given by

$$\bar{\mathrm{p}}_1^{\text{sym}}\left(\mathscr{N}_{b_i k}\right) = \left(1 - \mathrm{q}_1\left(h_{\text{th}}\right)\right)^{N_{b_i} - 1}, \tag{2.11}$$

where N_{b_i} is the cardinality of $\mathscr{N}_{b_i}$. Moreover, for symmetric-independent types and strategy profile $s_{\text{th}}^{\text{sym}} = (h_{\text{th}})$, the probability that the subset of SUEs $\mathscr{N}_{-b_i k} \subseteq \mathscr{N}_{-b_i}$ takes action $\mathtt{T}_k$ follows the binomial distribution of $N_{-b_i k}$ successes in a sequence of N_{-b_i} independent binary trials with success probability of one trial given by (2.10). Here $N_{-b_i k}$ and N_{-b_i} are the cardinalities of $\mathscr{N}_{-b_i k}$ and $\mathscr{N}_{-b_i}$ respectively. We denote this binomial distribution by $\mathscr{B}_1\left(N_{-b_i}, \mathrm{q}_1\left(h_{\text{th}}\right)\right)$ and the probability of $N_{-b_i k}$ number of successes is denoted by $\mathrm{p}_{\mathscr{B}_1}\left(N_{-b_i k}\right)$. Due to symmetric-independent types the interference gains g_j^k are i.i.d and therefore the density $f_{\mathbf{g}}$ in (2.7) only depends on the cardinality of the set $\mathscr{N}_{-b_i k}$ (not on the exact SUEs in $\mathscr{N}_{-b_i k}$). Next we use these observations to simplify (2.7). Let us define the binomial random variable

$X \sim \mathscr{B}_1\left(N_{-b_i}, \mathrm{q}_1\left(h_{\mathrm{th}}\right)\right)$, then the expected utility of player i for action $\mathtt{T}_k$ and $s_{-i\mathrm{th}}^{\mathrm{sym}} = \left(h_{\mathrm{th}}\right)$ is given by (2.8), where $\sum_{j \in X} g_j^k$ is the sum of X number of i.i.d random variables g_j^k.

In order to find the symmetric BR strategy for player i, we need to find the unique threshold $\tilde{h}_{\mathrm{th}}$ such that $h_i^k = \tilde{h}_{\mathrm{th}}$ and $s_{-i\mathrm{th}}^{\mathrm{sym}} = \left(\tilde{h}_{\mathrm{th}}\right)$ solves equation (2.9). That is to say that $\mathbb{E}_{\theta_{-i}} u_i\left(\tilde{h}_{\mathrm{th}}, s_{-i\mathrm{th}}^{\mathrm{sym}}, \theta\right) = \rho$. Then note that all players follow the common threshold $s_{\mathrm{th}}^{\mathrm{sym}} = \left(\tilde{h}_{\mathrm{th}}\right)$. Therefore, as player i is arbitrary, this threshold defines the symmetric BR strategy for all players and by Definition 3 it is a unique BNSE.

Theorem 1. For symmetric-independent types and identically populated cells, game $\mathbf{G}_1$ has a unique threshold $h_{i\mathrm{th}} = \tilde{h}_{\mathrm{th}}\ \forall\, i \in \mathscr{N}$, such that the BNSE is given by the profile $s_{\mathrm{th}}^{\mathrm{sym}} = \left(\tilde{h}_{\mathrm{th}}\right)$.

Proof. See *Appendix 1*. □

At the BNSE, all players follow the threshold strategy defined by the symmetric-threshold profile $s_{\mathrm{th}}^{\mathrm{sym}} = \left(\tilde{h}_{\mathrm{th}}\right)$. When $\rho \leq 0$, we have $\tilde{h}_{\mathrm{th}} = 0$, hence, at all times each SUE with a symbol available, transmits over the channel on which it has the highest gain. On the other hand when $\rho > 0$ we have $\tilde{h}_{\mathrm{th}} > 0$, therefore an SUE may not transmit, even if a symbol is available if its maximum channel gain is below the threshold. The proof in *Appendix 1* shows the existence of a unique threshold. In practice the threshold can be computed by solving $\mathbb{E}_{\theta_{-i}} u_i\left(\tilde{h}_{\mathrm{th}}, s_{-i\mathrm{th}}^{\mathrm{sym}}, \theta\right) = \rho$ using the bisection method. We draw the attention of the reader to the similarity of our mechanism to that of backoff probability in CSMA-collision detection (CD). In CSMA-CD the backoff decision is a result of a previous collision, whereas in our scheme the CSI determines the backoff probability. Thus, by increasing $\rho > 0$ we increase the equilibrium threshold $\tilde{h}_{\mathrm{th}}$, which leads to a lowering of the SUEs that simultaneously transmit. Thus, using our scheme the network administration is able to control the number of simultaneous transmissions to match the level of congestion in the network in order to avoid excessive collisions.

2.6 Design of $\mathbf{G}_2$: A Game with Statistical CSIT

In previous sections we developed the BNSE in threshold strategies for game $\mathbf{G}_1$. In $\mathbf{G}_1$ mixed-BNSEs do not exist for threshold strategies of the form (2.5), with probability 1. The reason for the above observation is the fundamental result in game theory which states that in a mixed-strategy NE all the actions that are played with non zero probability must yield the same payoff (Nisan *et al.*, 2007). In $\mathbf{G}_1$, two actions $\mathtt{T}_k$ and $\mathtt{T}_{k'}$, $k, k \in \mathscr{K}$ may yield the same expected utility if and only if (iff) $h_i^k = h_i^{k'}$, which has $\Pr(h_i^k = h_i^{k'}) = 0$. Similarly a player obtains equal expected utilities for $\mathtt{T}_k$ and $\mathtt{X}$ iff $h_i^k = h_{i\text{th}}$, which has zero probability as well.

This section considers the situation where an SUE possesses statistical-CSIT. From (2.1), the channels $\bar{h}_i^k \; \forall \; k \in \mathscr{K}$, from SUE i to its home-SAP, are i.i.d. single tap Rayleigh. Then the statistical knowledge an SUE i must posses is the mean λ_i of the i.i.d. exponential power gains $h_i^k \sim \text{Exp}(\frac{1}{\lambda_i})$, $k \in \mathscr{K}$. Now the type vector of SUE i is $\omega_i \triangleq \left(\left(\lambda_i, g_{im}^k \alpha_i\right), k \in \mathscr{K}, m \in \mathscr{M}\right)$ and the type set is denoted by Ω_i such that $\omega_i \in \Omega_i$. We also define the set products $\Omega \triangleq \prod_{i \in \mathscr{N}} \Omega_i$ such that $\omega \in \Omega$ and $\Omega_{-i} \triangleq \prod_{j \in \mathscr{N} \setminus i} \Omega_j$ such that $\omega_{-i} \in \Omega_{-i}$. In the Bayesian setting the mean power gain λ_i is known only to player i. The other players hold a belief of λ_i that we denote by the probability density f_{λ_i}. Then, the belief player i holds about the types of other players ω_{-i} is given by the density function $\bar{f}_{\omega_{-i}} = \prod_{\substack{k \in \mathscr{K} \\ j \in \mathscr{N} \setminus i}} \left(f_{\lambda_j} \cdot f_{g_{jm}^k}\right) \cdot \prod_{j \in \mathscr{N} \setminus i} f_{\alpha_j}$. Due to the change in types and the beliefs of the players from those of $\mathbf{G}_1$ we introduce the new game $\mathbf{G}_2$ as follows.

Table 2.2 Definition of Game $\mathbf{G}_2$

Players	$\mathscr{N}$
Action	$a_i \in \mathscr{A}_i$
Type	$\omega_i \in \Omega_i$
Belief	$\bar{f}_{\omega_{-i}}$ over Ω_{-i}
System state	$\zeta_{km} \; \forall \; k \in \mathscr{K}, m \in \mathscr{M}$
Payoff	$u_i(a, \omega)$, $a \in \mathscr{A}$ and $\omega \in \Omega$

Game $\mathbf{G}_2$ also follows symmetric-independent types model that was discussed in Section *2.4.1*. Hence, $\lambda_i \ \forall \ i \in \mathscr{N}$ are i.i.d. Following analysis is valid for any distribution f_{λ_i} with finite first and second order moments.

2.6.1 Mixed Threshold Strategies

The definition of a mixed strategy follows (Shoham & Leyton-Brown, 2009).

Definition 4. In $\mathbf{G}_2$, a mixed strategy of player $i \in \mathscr{N}$ is defined as $r_i : \Omega_i \to \triangle^K$ where r_i is a probability distribution over $\mathscr{A}_i$ and $\triangle^K$ is the standard $(K-1)$-simplex in $\mathbb{R}^K_{\geq 0}$.

Following the standard game theoretic notation the mixed-strategy vector of all players except SUE i is denoted by , $\mathbf{r}_{-i} \triangleq \left(r_j, j \in \mathscr{N} \setminus i\right)$, and the strategy profile of all players is denoted by $\mathbf{r} \triangleq (r_i, i \in \mathscr{N})$.

Our interest is in a special subset of the feasible strategy space. We call this sub-strategy space *uniformly distributed threshold strategies* (UDTSs). It consists of strategies of the following form:

$$r_{i\text{th}}(\omega_i) \triangleq \begin{cases} \Pr(\mathtt{T}_k) = \dfrac{1}{K}, \Pr(\mathtt{X}) = 0 & \text{if}\, \alpha_i = 1, \lambda_i \geq \lambda_{i\text{th}} \\ \Pr(\mathtt{T}_k) = 0, \Pr(\mathtt{X}) = 1 & \text{otherwise,} \end{cases} \tag{2.12}$$

where $\lambda_{i\text{th}}$ is a non-negative threshold parameter. The strategy (2.12) essentially means that an SUE i picks a channel uniformly at random, if it has a symbol available and if its private knowledge of λ_i is greater than a parameter $\lambda_{i\text{th}}$. Otherwise it does not transmit. Thus, a UDTS of a player i is completely characterized by the threshold $\lambda_{i\text{th}}$. Thus, in order to specify the strategy profile of the players, it is sufficient to provide the threshold vector. Let us define $\mathbf{r}_{\text{th}} \triangleq (\lambda_{i\text{th}}, i \in \mathscr{N})$ and $\mathbf{r}_{-i\text{th}} \triangleq \left(\lambda_{j\text{th}}, j \in \mathscr{N} \setminus i\right)$.

By (2.12), the probability that player i transmits on channel k is

$$\mathrm{q}_{ik}(\lambda_{i\text{th}}) = \frac{1}{K} \Pr(\lambda_i \geq \lambda_{i\text{th}}, \alpha_i = 1). \tag{2.13}$$

It is observed that $\mathrm{q}_{ik}\left(\lambda_{i\mathrm{th}}\right)$ is increasing in λ_i and decreasing in $\lambda_{i\mathrm{th}}$. Due to independence of types, the probability that a subset of SUEs $\mathscr{X}_k \subset \mathscr{N}$ takes action $\mathtt{T}_k$ is given by $\mathrm{p}_2\left(\mathscr{X}_k\right) = \prod_{j\in\mathscr{X}_k} \mathrm{q}_{jk}\left(\lambda_{j\mathrm{th}}\right)$ and similarly the probability that non of the SUEs in $\mathscr{X}_k$ takes action $\mathtt{T}_k$ is $\bar{\mathrm{p}}_2\left(\mathscr{X}_k\right) = \prod_{j\in\mathscr{X}_k}\left(1-\mathrm{q}_{jk}\left(\lambda_{j\mathrm{th}}\right)\right)$. These probabilities are next used to define the expected utility.

Now the expected utility of i when $\alpha_i = 1$ and $a_i = \mathtt{T}_k$ is denoted by $\mathbb{E}_{h_i^k,\omega_{-i}} u_i\left(\mathtt{T}_k, \mathbf{r}_{-i\mathrm{th}}, \omega\right)$ and is given by (2.14).

$$\begin{aligned}
&\mathbb{E}_{h_i^k,\omega_{-i}} u_i\left(\mathtt{T}_k, \mathbf{r}_{-i\mathrm{th}}, \omega\right) = \\
&\bar{\mathrm{p}}_2\left(\mathscr{N}_{b_ik}\right) \sum_{\mathscr{N}_{-b_ik}\in\mathbb{P}\left(\mathscr{N}_{-b_i}\right)} \mathrm{p}_2\left(\mathscr{N}_{-b_ik}\right) \bar{\mathrm{p}}_2\left(\mathscr{N}'_{-b_ik}\right) \int_{\mathscr{D}} f_{\mathbf{g}} f_\zeta \mathbb{E}_{h_i^k} \log\left(1 + \frac{h_i^k}{\sum\limits_{j\in\mathscr{N}_{-b_ik}} g_j^k + \zeta_k + \sigma^2}\right) d\mathbf{g} d\zeta.
\end{aligned} \tag{2.14}$$

$$\mathbb{E}_{h_i^k,\omega_{-i}} u_i\left(\mathtt{T}_k, \mathbf{r}_{-i\mathrm{th}}^{\mathrm{sym}}, \omega\right) = \bar{\mathrm{p}}_2^{\mathrm{sym}}\left(\mathscr{N}_{b_ik}\right) \mathbb{E}_X \int_{\mathscr{D}} f_{\mathbf{g}} f_\zeta \mathbb{E}_{h_i^k} \log\left(1 + \frac{h_i^k}{\sum\limits_{j\in X} g_j^k + \zeta_k + \sigma^2}\right) d\mathbf{g} d\zeta. \tag{2.15}$$

2.6.2 Best Response Strategies

The UDTSs in (2.12) only forms a strict subset of all possible strategies for a player i. Hence, it is important to demonstrate that this subset is sufficient to contain a BR. In this section we demonstrate that when the set of SUEs $\mathscr{N} \setminus \{i\}$ is playing UDTSs, the SUE i can find a BR strategy also within the subset of UDTSs.

Claim 2. For symmetric-independent types, when $\alpha_i = 1$, the expected utility given in (2.14), is increasing in λ_i almost surely.

Proof. Let us consider two exponential random variables $h_i^{k1} \sim \mathrm{Exp}\left(\lambda_{i1}\right)$ and $h_i^{k2} \sim \mathrm{Exp}\left(\lambda_{i2}\right)$, such that $\lambda_{i1} < \lambda_{i2}$. In order to compare (2.14) for $h_i^k = h_i^{k1}$ and $h_i^k = h_i^{k2}$, we use the theory of coupling (Thorisson, 2000). Let U be an exponentially distributed random variable with mean 1. Then let $h_i^{k1} = \lambda_{i1}U$, $h_i^{k2} = \lambda_{i2}U$. Then $\lambda_{i1}U < \lambda_{i2}U$ almost surely, hence $h_i^{k1} < h_i^{k2}$ almost surely Now let us consider the expected utility conditioned on h_i^k, i.e., $\mathbb{E}_{\omega_{-i}|h_{ik}} u_i(h_i^k, \mathbf{r}_{-i\text{th}}, \omega)$ and observe from (2.14) that it is increasing in h_i^k. Thus, the following inequality holds almost surely.

$$\mathbb{E}_{\omega_{-i}|h_i^{k1}} u_i\left(\lambda_{i1} U, \mathbf{r}_{-i\text{th}}, \omega\right) < \mathbb{E}_{\omega_{-i}|h_i^{k2}} u_i\left(\lambda_{i2} U, \mathbf{r}_{-i\text{th}}, \omega\right). \tag{2.16}$$

Moreover, from properties of expectation, the inequality (2.16) is preserved when expectation is taken over U, and by the law of total expectation, $\mathbb{E}_U \mathbb{E}_{\omega_{-i}|h_i^{k1}} u_i\left(\lambda_{i1} U, \mathbf{r}_{-i\text{th}}, \omega\right) = \mathbb{E}_{h_i^k, \omega_{-i}} u_i\left(\mathtt{T}_k, \mathbf{r}_{-i\text{th}}, \omega\right)$. Therefore, (2.14) is increasing in λ_i almost surely. □

The expected utility of SUE i when $\alpha_i = 1$ and $a_i = \mathtt{X}$ is $\mathbb{E}_{\omega_{-i}} u_i\left(\mathtt{X}, \mathbf{r}_{-i\text{th}}, \omega\right) = \rho$. Next we demonstrate the form of the BR strategy of SUE i when all other SUEs are playing UDTS $\mathbf{r}_{-i\text{th}}$.

Lemma 1. For $\mathbf{r}_{-i\text{th}}$ and $\alpha_i = 1$, a BR mixed strategy of player i, denoted by $\tilde{r}_i$, is given by

$$\tilde{r}_i = \begin{cases} \begin{cases} \Pr\left(\mathtt{T}_k \mid \alpha_i = 1\right) = p_k \\ \Pr\left(\mathtt{X} \mid \alpha_i = 1\right) = 0 \end{cases} & \text{if } \lambda_i \geq \lambda_{i\text{th}}, \\ \begin{cases} \Pr\left(\mathtt{T}_k \mid \alpha_i = 1\right) = 0 \\ \Pr\left(\mathtt{X} \mid \alpha_i = 1\right) = 1 \end{cases} & \text{if } \lambda_i < \lambda_{i\text{th}}, \end{cases}$$

where $0 \leq p_k \leq 1$ are probabilities s.t., $\sum_{k \in \mathscr{K}} p_k = 1$.

Proof. Let U be a exponential random variable with mean 1. Since (2.14) is increasing in λ_i as proved in Claim 2, there exists a $\lambda_{i\text{th}}$ such that for channel k

$$\mathbb{E}_{h_i^k, \omega_{-i}} u_i\left(\lambda_{i\text{th}} U, \mathbf{r}_{-i\text{th}}, \omega\right) = \rho. \tag{2.17}$$

Accordingly, the SUE i transmits iff $\lambda_i \geq \lambda_{i\text{th}}$. Since the exponential distribution is completely characterized by the mean, for symmetric-independent types the expected payoff for two actions $a_i = \mathtt{T}_k$ and $a_i = \mathtt{T}_{k'}$ $k, k' \in \mathscr{K}$, are equal. Consequently, if $\lambda_i \geq \lambda_{i\text{th}}$ at the BR, the player i may play *any* probability distribution $\left(p_k, k \in \mathscr{K}\right)$ and obtains the same expected payoff. □

Lemma 1 essentially says that when other SUEs play (2.12), the BR of SUE i is *any distribution* (not necessarily uniform) over the set of channels, provided that the mean of the power gain is above a threshold. Therefore, player i may as well play the UDTS $r_{i\text{th}} = \lambda_{i\text{th}}$. Thus, we have demonstrated that the subset of UDTSs is sufficiently large to hold a BR.

A symmetric threshold is one that is common to all players and we denote a symmetric UDTS by $\mathbf{r}_{\text{th}}^{\text{sym}} = (\lambda_{\text{th}})$.

For symmetric-independent types and UDTS profile $\mathbf{r}_{\text{th}}^{\text{sym}} = (\lambda_{\text{th}})$, the event that player i transmits on channel k and the event that player j transmits on channel k are independent and have equal probabilities and hence is denoted by

$$\mathrm{q}_2(\lambda_{\text{th}}) = \mathrm{q}_{ik}(\lambda_{\text{th}})\,\forall i \in \mathscr{N}, k \in \mathscr{K}.$$

Moreover, analogous to (2.11), the probability that no collision is encountered by player i on channel k is given by $\bar{\mathrm{p}}_2^{\text{sym}}(\mathscr{N}_{b_i k}) = (1 - \mathrm{q}_2(\lambda_{\text{th}}))^{N_{b_i}-1}$. Also consequently, the probability that the subset of SUEs $\mathscr{N}_{-b_i k} \subseteq \mathscr{N}_{-b_i}$ takes action $\mathtt{T}_k$ follows the binomial distribution $\mathscr{B}_2(N_{-b_i}, \mathrm{q}_2(\lambda_{\text{th}}))$.

Similar to the discussion in Section 2.5.1, when $X \sim \mathscr{B}_2(N_{-b_i}, \mathrm{q}_2(\lambda_{\text{th}}))$, the expected utility of SUE i, for action $a_i = \mathtt{T}_k$ and $\mathbf{r}_{-i\text{th}}^{\text{sym}} = (\lambda_{\text{th}})$ is given by (2.15), where $\sum_{j \in X} g_j^k$ is the sum of X number of i.i.d random variables g_j^k.

If a BNSE in UDTSs exists, then there must be a unique $\lambda_{i\text{th}} = \tilde{\lambda}_{\text{th}}\ \forall\, i \in \mathscr{N}$ that defines a mutual BR $\mathbf{r}_{\text{th}}^{\text{sym}} = (\tilde{\lambda}_{\text{th}})$. That is to say that $\mathbf{r}_{-i\text{th}}^{\text{sym}} = (\tilde{\lambda}_{\text{th}})$ and $\lambda_i = \tilde{\lambda}_{\text{th}}$ solves the following equation:

$$\mathbb{E}_{h_i^k, \omega_{-i}} u_i(\mathtt{T}_k, \mathbf{r}_{-i\text{th}}^{\text{sym}}, \omega) = \rho. \tag{2.18}$$

Theorem 2. For symmetric-independent types and identically populated cells, $\mathbf{G}_2$ has a unique threshold $\lambda_{i\text{th}} = \tilde{\lambda}_{\text{th}}\ \forall\, i \in \mathscr{N}$, such that the BNSE in UDTSs is given by the profile $\mathbf{r}_{\text{th}}^{\text{sym}} = (\tilde{\lambda}_{\text{th}})$.

Proof. The method is similar to the proof of Theorem 1 and hence it is omitted. □

Theorem 2 proves that when only the mean of the CSI is known, the channel access game can still achieve a BNSE in mixed strategies. The equilibrium strategy of an SUE i is to pick a channel uniformly at random if and only if it posses a symbol and its mean channel gain, λ_i, is above the threshold $\tilde{\lambda}_{\text{th}}$. This equilibrium strategy is extremely efficient to implement and each player is able to compute the common threshold $\tilde{\lambda}_{\text{th}}$ without interaction. Such a scheme is ideal for dense SC zones. Furthermore, as in $\mathbf{G}_1$, in $\mathbf{G}_2$ as well, the network administrator may control the number of SUEs that simultaneously transmit, and thus control collisions, by manipulating the parameter ρ. The higher the ρ is, the higher the threshold $\tilde{\lambda}_{\text{th}}$ and therefore the lesser the probability that an SUE transmits.

2.6.3 Limitations of Games $\mathbf{G}_1$ and $\mathbf{G}_2$

The key limitation in both games $\mathbf{G}_1$ and $\mathbf{G}_2$ is the assumptions that were necessary to render the games symmetric. The symmetric assumption holds for a localized overlapping set of SCs, which was justified throughout the above development. We identified this cases as a dense SC zone as depicted in Fig. 2.1. However, once the SCs are no longer clustered in localized zones and instead are dispersed in a larger area, such as the home SCs in a residential area, the assumption of symmetric independent types in Section 2.4.1 does not hold. That situation gives rise to a non-symmetric game.

The following discussion on non-symmetric case is carried out with respect to $\mathbf{G}_1$. Two levels of asymmetry can be observed. Firstly asymmetry among UEs and secondly asymmetry among channels of a given UE. Let us first consider asymmetry only among UEs. Thus, we assume that the channel gains are i.i.d. from a given UE to a given BS, i.e., $\forall\, k \in \mathscr{K}$, $g^k_{jm} = g_{jm}$, but a UE may have different channel distributions to different BSs. Then at the equilibrium of $\mathbf{G}_1$, from (2.9), there must be thresholds $\forall\, i \in \mathscr{N}$, $h_{i\text{th}} = \tilde{h}_{i\text{th}}$, which are the solution to the system of nonlinear equations:

$$\mathbb{E}_{\theta_{-i}} u_i(h_{i\text{th}}, s_{-i\text{th}}, \theta) = \rho, \qquad \forall\, i \in \mathscr{N}, \tag{2.19}$$

where $s_{-i\text{th}} = \left(h_{j\text{th}}, j \in \mathcal{N} \setminus i\right)$. In the symmetric case this system condensed to a single equation $\mathbb{E}_{\theta_{-i}} u_i\left(h_{\text{th}}, s^{\text{sym}}_{-i\text{th}}, \theta\right) = \rho$. The asymmetric game has a equilibrium in threshold strategies iff the system (2.19) has a solution.

Now if we consider asymmetry among UEs together with asymmetry among channels, so that the MUE load need not be balanced among the channels and the channel power gain from a UE to a BS may depend on the channel index, then for an equilibrium to exist we are looking for $K \times N$ threshold values $\forall\, i \in \mathcal{N}$, $h^k_{i\text{th}} = \tilde{h}^k_{i\text{th}}$, which are the solution to the system of nonlinear equations:

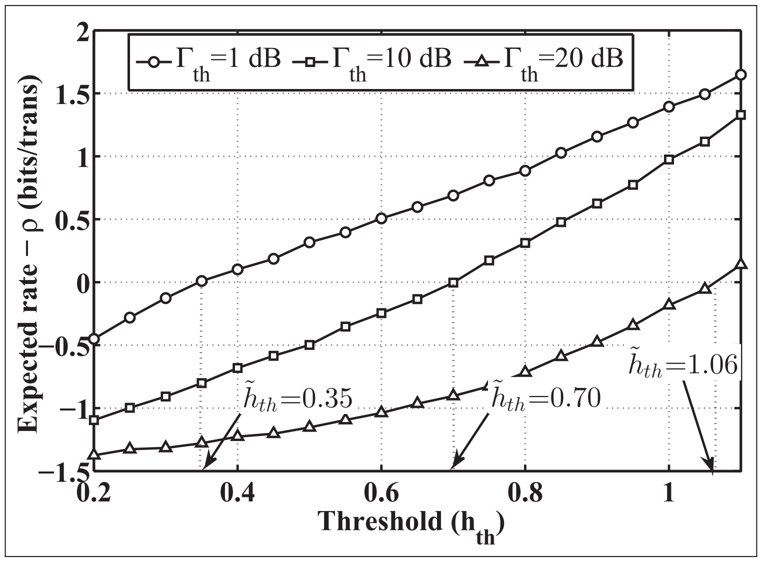
$$\mathbb{E}_{\theta_{-i}} u^k_i(h^k_{i\text{th}}, s_{-i\text{th}}, \theta) = \rho, \qquad \forall\, i \in \mathcal{N}, k \in \mathcal{K}. \tag{2.20}$$

where $s_{-i\text{th}} = (h^k_{j\text{th}}, j \in \mathcal{N} \setminus i, k \in \mathcal{K})$. UE i will transmit on the channel with the highest $\mathbb{E}_{\theta_{-i}} u^k_i(\cdot)$ that has a channel gain above $h^k_{i\text{th}}$.

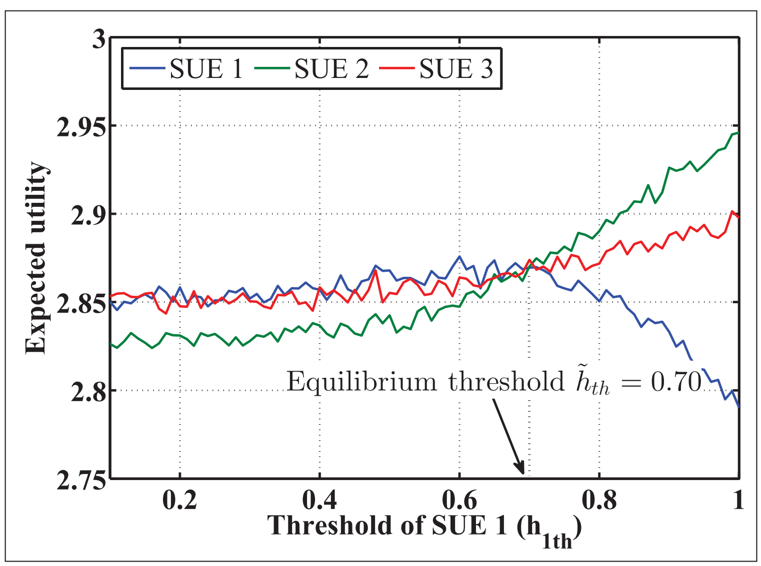
The symmetric assumptions were necessary to establish a single equilibrium threshold value for all UEs. A general existence result of threshold based NEs for the asymmetric case requires to establish the existence of a solution to systems of equations given by (2.19) or (2.20), which is not considered in this article.

2.7 Numerical Results and Discussion

Let us consider a scenario where, in a hotel lobby, 4 service providers have deployed 1 SAP each. This scenario is depicted in Fig. 2.1. There are $K = 8$ channels in the uplink and it is noted when the number changes. The rest of the variables are as follows: $\Pr(\alpha_i = 1) = 0.9$, SINR threshold $\Gamma_{\text{th}} = 20$ dB, power and interference gains h^k_i and g^k_{im} follow exponential distribution with means $\mathbb{E}\left(h^k_i\right) = 0.5$, $\mathbb{E}\left(g^k_{im}\right) = 0.05$ respectively $\forall\, i \in \mathcal{N}$, $\forall\, k \in \mathcal{K}$, and $m \in \mathcal{M}$, and parameters $\rho = 2^{\text{bits}}/_{\text{trans}}$ and $\sigma^2 = 0.001^2$. A constant MUE interference is considered in the simulations. That is the distribution of the MUE interference ζ is $f_\zeta = \delta(\zeta - 0.001)$, where $\delta(\cdot)$ is the impulse function. The MUEs do not take part in the games, therefore this assumption does not introduce any limitations and is a means to simplify the Monte Carlo simulations. Recall that the symmetric model assumed i.i.d. number of SUEs among service providers. For simplicity and with no loss of generality, the simulation consider 5 SUEs for each service provider with probability one. In the discussion the SUEs 1 to 5 belong to SC_1 and SUEs 6 to 10 belong to SC_2 and so forth.

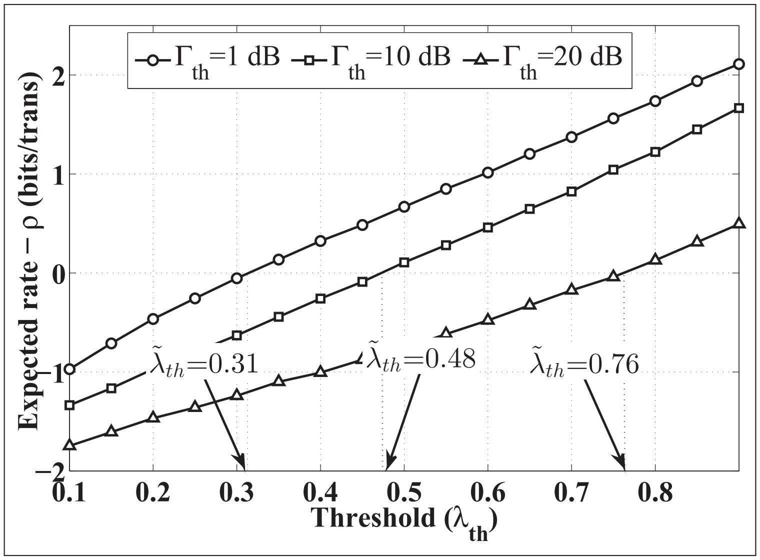


a) Equilibrium threshold computation of pure-strategy BNSE.

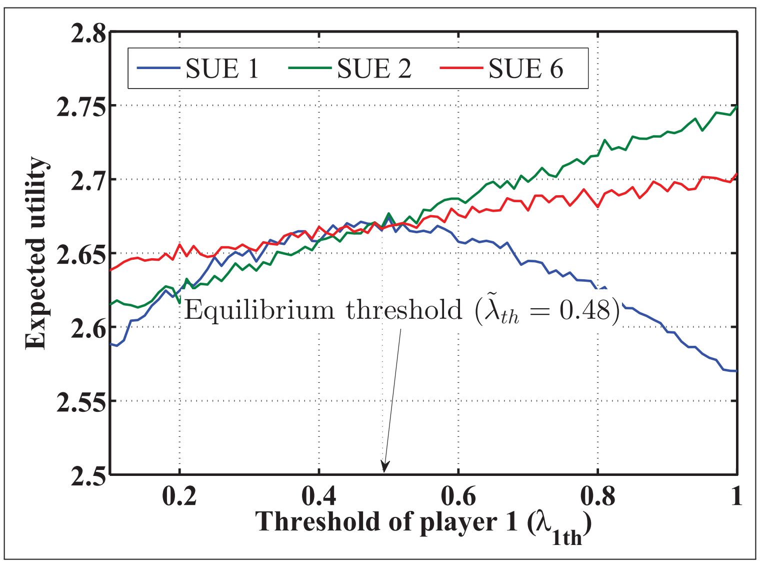


b) The effect of deviation of SUE 1 from equilibrium, when $\Gamma_{\text{th}} = 10$ dB

Figure 2.2 The existence and uniqueness of pure-strategy BNSE of $\mathbf{G}_1$.



a) Equilibrium threshold computation of mixed-strategy BNSE



b) The effect of deviation of SUE 1 from equilibrium, when $\Gamma_{\text{th}} = 10$ dB

Figure 2.3 The existence and uniqueness of mixed-strategy BNSE of $\mathbf{G}_2$.

Fig. 2.2a depicts (expected rate -ρ) vs. symmetric threshold h_{th} for SUE 1. The expected rate of an SUE in $\mathbf{G}_1$ is given by (2.8). The value of h_{th} for which the expected rate is equal to ρ, is the solution of (A I-1) and defines the unique equilibrium $s_{\text{th}}^{\text{sym}} = \left(\tilde{h}_{\text{th}}\right)$ of $\mathbf{G}_1$ according to Theorem 1. Three SINR thresholds $\Gamma_{\text{th}} \in \{5, 10, 20\}$ dB are considered. As Γ_{th} increases, the expected rate (2.8) decreases since the probability of violation of SINR threshold increases and hence the channel power gain required to achieve a rate of ρ increases leading to a higher equilibrium threshold.

Fig. 2.2b demonstrates that the root obtained in the Fig 2.2a is indeed the equilibrium point. To this end, we let SUE 1 deviate from the symmetric equilibrium threshold while all other SUEs follow the symmetric equilibrium threshold strategy $s_{-i\text{th}}^{\text{sym}} = \left(\tilde{h}_{\text{th}}\right)$. As can be seen, SUE 1 is unable to achieve strictly better performance by unilateral deviation i.e., $h_{1\text{th}} \gtrless \tilde{h}_{\text{th}} = 0.7$.

Next let us consider $\mathbf{G}_2$. The simulation setup assumes that λ_i is uniformly distributed in the interval $(0,2)$. The rest of the simulation parameters are kept the same as in the previous section. The expected rate of an SUE in $\mathbf{G}_2$ is given by (2.15). Fig. 2.3a depicts the (expected rate$-\rho$) vs. symmetric threshold λ_{th} for SUE 1. It demonstrates the existence and uniqueness of solution to (2.18), which defines the symmetric threshold $\tilde{\lambda}_{\text{th}}$ of the mixed BNSE. Observe that as Γ_{th} increases, the mean channel power gain required to achieve an expected rate of ρ increases according to (2.18), resulting in a higher $\tilde{\lambda}_{\text{th}}$ at equilibrium.

Fig. 2.3b demonstrates the payoffs of SUEs as SUE 1 deviates from the equilibrium threshold value while all other SUEs follow the equilibrium strategy $\mathbf{r}_{-i\text{th}}^{\text{sym}} = (\tilde{\lambda}_{\text{th}})$. For clarity utilities of only three SUEs are depicted. As expected, SUE 1 is unable to achieve strictly better performance by unilateral deviation, therefore $\mathbf{r}_{\text{th}}^{\text{sym}} = (\tilde{\lambda}_{\text{th}})$ defines the NE.

The configurable network parameter available to the administrator is ρ. It is proven under Theorem 1 that as ρ is increased the collisions in the network must reduce. Fig. 2.4 demonstrates this fact numerically. To emphasize on collisions, a reduced channel number of $K = 4$ is used in this simulation.

2.7.1 Fairness and Benchmark

As a result of the symmetric-independent types, all SUEs achieve equal expected utility in both games $\mathbf{G}_1$ and $\mathbf{G}_2$ at the equilibrium. Hence, fairness among the SUEs is ensured by both

Games. This can be observed in Fig. 2.2b and Fig. 2.3b. When the thresholds of all SUEs coincide at the equilibrium threshold, they all achieve equal expected utility.

In order to compare the system throughput, this article implements the benchmark decentralized scheduling scheme where each SAP schedules its SUE who has a symbol and has the highest channel power gain. Thus, this scheme requires a message to be sent to the selected SUE of each SC in each time slot. Then there are no intracell collisions but only intercell interference. When the scheduled SUEs of the SCs satisfy the SINR threshold Γ_{th}, they achieve the rates given by (2.3). We present the results for CSIT case of the benchmark and the related CSIT game $\mathbf{G}_1$ in Fig. 2.5. The rate distribution of $\mathbf{G}_1$ performs close to the benchmark. Therefore, for a dense SC zone as in Fig. 2.1, employing the proposed game models, rather than the benchmark is reasonable, as the proposed games have the added advantage of being fully distributed, once the parameter ρ has been braodcaseted to the SUEs.

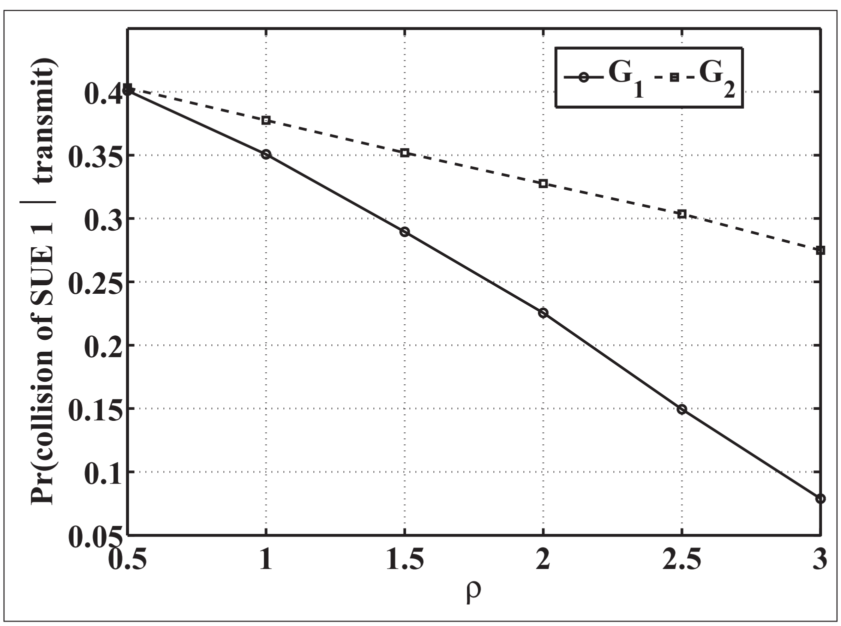


Figure 2.4 Empirical probability of a collision experienced by SUE 1, given it transmits

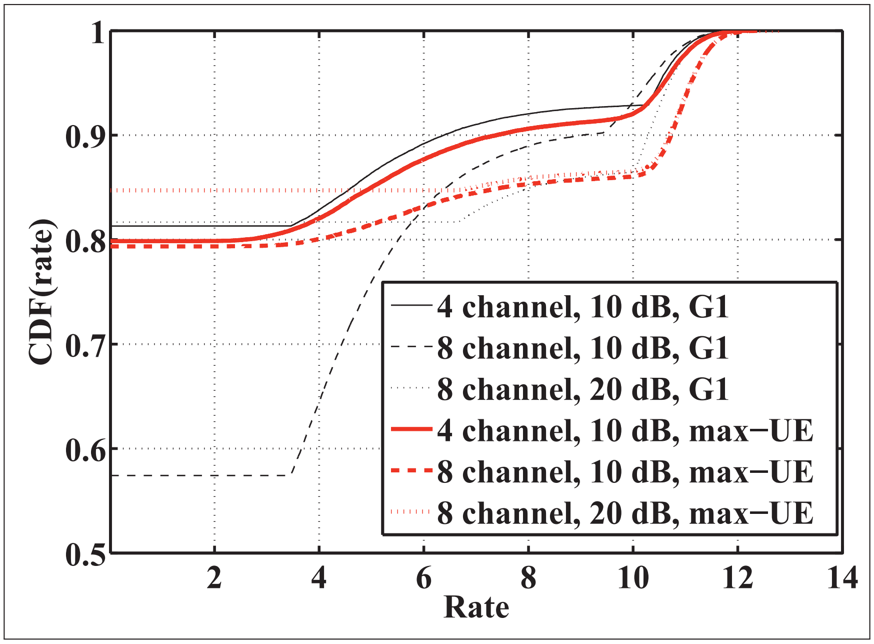


Figure 2.5 Empirical CDF for rate (bits/trans) comparison

2.8 Conclusion

This article analyzed the distributed uplink channel access problem of a cluster of dense underlay SCs. The analysis was carried out using the theory of Bayesian games. The system model was chosen to be sufficiently general and it includes multiple cells and channels, intercell interference, intracell collisions and random symbol availability, which are important parameters in modeling picocells, femtocells, and wireless local area networks. Two CSI availability models are used resulting in two games. The first game, $\mathbf{G}_1$ assumes CSIT and we solve it for pure-strategy symmetric equilibrium. At the equilibrium each SUE transmits on the highest gain channel if that gain is above a threshold. The second game, $\mathbf{G}_2$, only assumes statistical CSIT. $\mathbf{G}_2$ is proved to posses an interesting symmetric mixed-strategy equilibrium where an SUE uniformly distributes channel access if mean channel gains is above a threshold. The two pure- and mixed-strategy equilibria, are particularly interesting for distributed systems as at the equilibrium, the best response strategy is defined by a single threshold parameter and both equilibria can be achieved without interaction among the SUEs. The key extension that remains is to explore nonsymmetric equilibria.

CHAPTER 3

GENERALIZED SATISFACTION EQUILIBRIUM FOR SERVICE-LEVEL PROVISIONING IN WIRELESS NETWORKS

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3.1 Abstract

This paper presents the generalized satisfaction equilibrium (GSE) for games in satisfaction-form. Each wireless agent has a constraint to satisfy and the GSE is a strategy profile from which no unsatisfied agent can unilaterally deviate to satisfaction. This new equilibrium is particularly adapted to model problems of service-level provisioning when satisfying all agents is infeasible. The GSE forms a more flexible framework for studying self-configuring networks than the previously defined satisfaction equilibrium and the generalized Nash equilibrium. The existence of the GSE in mixed strategies is proven for the case in which the constraints are defined by a lower limit on the expected utility. The paper demonstrates that the pure-strategy GSE problem is closely related to the constraint satisfaction problem and that finding a pure-strategy GSE with a given number of satisfied agents is NP-hard. For certain games in satisfaction-form, it is shown that the satisfaction response dynamics converge to a GSE. Next, the Bayesian GSE is introduced for games with incomplete information. Finally, this paper presents a series of wireless applications that demonstrate the superiority of the GSE over the classical equilibria in solving problems of service-level provisioning.

3.2 Introduction

Game theory plays a fundamental role in the analysis of decentralized self-configuring wireless networks, e.g., sensor networks, body area networks, SCs (Han, 2012; Alpcan *et al.*, 2013). In a self-configuring network the transceivers coordinate the resource allocation among themselves without the control of a central authority. Therefore, radio devices (also called agents) must autonomously tune their own strategies to meet a required quality-of-service (QoS). The underlying difficulty of this task is that meeting a given level of QoS depends on the transmit-receive configuration adopted by all other agents. The object of central attention within this context is the equilibrium. The notion of Nash equilibrium (NE) (Nash *et al.*, 1950; Nisan *et al.*, 2007) is probably the most popular solution to normal-form noncooperative games. When the agents operate at an NE, no one is able to unilaterally deviate from that NE to improve its performance. Thus, the relevance of the equilibrium is that it defines the operating states under which a self-configuring network can be considered stable. Aside from the NE, there are other notions of equilibria particularly adapted to self-configuring networks. Each solution concept has advantages and disadvantages, as described in (Perlaza & Lasaulce, 2014).

A major disadvantage that is common to most equilibrium concepts, including the NE, is that the stability depends on whether each agent achieves the highest possible performance. The NE was originally designed for economic markets in which *risk-neutral* and fully *rational* agents maximize their expected profits over the mixed strategies (Nisan *et al.*, 2007). In contrast most widely used applications in wireless networks do not require the agents to operate at their maximum achievable QoS that is measured by signal to interference and noise ratio (SINR), delay, or bit error rate (BER). These include applications that generate inelastic traffic such as voice or video calls, streaming video or music, social networking, messaging and live broadcasts. In order to function, these applications only require a specific level of QoS. Thus, the utility maximization as in markets does not meet the model for wireless resource allocation for these applications (Meshkati *et al.*, 2009). To overcome this constraint, a new solution concept called the satisfaction equilibrium (SE) was suggested in (Ross & Chaib-draa, 2006) and formally introduced in the realm of wireless communications in (Perlaza *et al.*, 2010, 2012b).

The SE is a state in which all agents satisfy their QoS constraints. Thus, the pure-strategy SE of (Perlaza *et al.*, 2012b) is an NE of a normal form game with binary utilities where all agents receive a utility of one. From this perspective, radio devices are no longer modeled by agents that maximize their individual benefit, but by agents that aim to satisfy their individual constraint. This new approach was adopted to model the problem of dynamic spectrum access in (Ren *et al.*, 2015; Ellingsæter, 2014) and SCs in (Perlaza *et al.*, 2012a). Other applications of SE are reported for instance in the case of collaborative filtering in (Xu *et al.*, 2014b). In (Goonewardena & Ajib, 2016) it is shown that the normal-form games discussed in (Southwell *et al.*, 2014), where the agent has a dormant action, have satisfaction-form representations, such that their pure-strategy NEs coincides with the SEs. However, this equilibrium notion of SE as introduced in (Perlaza *et al.*, 2012b) presents several limitations. As pointed out in (Southwell *et al.*, 2014), the notion of SE is too restrictive. Simultaneously satisfying the QoS constraints of all agents might not be always feasible. Hence, the existence of an SE is highly constrained, which limits its application to wireless networks. This same limitation is observed if the generalized NE (GNE) is employed to solve the problem and in fact (Perlaza *et al.*, 2012b) demonstrates that the GNE is more restrictive than their proposed SE. The GNE too cannot exist even if one agent cannot satisfy its constraint (Scutari *et al.*, 2010).

3.2.1 Contributions

This article generalizes the notion of SE presented in (Perlaza *et al.*, 2012b) to the case in which only a subset of the radio devices satisfy their QoS constraints in mixed strategies. This new notion of equilibrium is referred to as the *generalized satisfaction equilibrium (GSE)*. At a GSE strategy profile, there are two groups of agents: satisfied and unsatisfied. The former is the set of agents that meet their QoS conditions and the latter set contains those that do not meet their constraints. The key point is that at a GSE none of the unsatisfied agents can unilaterally deviate to achieve their QoS requirement.

This article studies the existence of GSEs in games in satisfaction form and presents general existence results that apply to a wide range of wireless resource allocation problems. These existence conditions are less restrictive than those observed for the case of SE in (Perlaza *et al.*, 2012b). Specifically, a GSE always exists in a finite game where the individual satisfaction constraint is defined by a lower bound on the expected utility. Nonetheless, for constraints of other forms, the existence of a GSE is shown to be not ensured in general even in the case of mixed strategies. This contrasts with the normal-form, for which there always exists an NE in mixed strategies (Nash *et al.*, 1950). It is shown that there is a relation between the $\{0,1\}$ normal-form game and the pure-strategy GSE. However, this relation does not hold for mixed strategies. The price of stability (PoS) and price of anarchy (PoA) for the number of satisfied agents are also studied and bounds are derived.

The relation between pure-strategy GSE and a class of problems known as the constrained satisfaction problems (CSPs) is exploited to show that the problem of finding a pure-strategy GSE with a given number of satisfied agents is NP-hard. The satisfaction response dynamics, where agents take turns playing a strategy that satisfies their constraint, is studied for both pure- and mixed-strategy spaces and sufficient conditions for convergence to a GSE are derived.

In the incomplete information case *Bayesian games in satisfaction form* are introduced. This class of games builds upon the definition of Bayesian games (Harsanyi, 1967-1968; Nisan *et al.*, 2007) to model satisfaction in which the agents have probabilistic knowledge of the *types* of the other agents. The corresponding solution concept of Bayesian-GSE is defined and the existence of the Bayesian-GSE is proven for constraints of expected utility realization. Sufficient conditions for the convergence of the Bayesian satisfaction-response dynamics are provided.

Finally the relevance of the GSE in the realm of wireless communications is highlighted by several examples that compare the performance of the GSE against the NE solution in pure and mixed strategies. These applications are energy efficiency, power control, admission control, and orthogonal channel allocation.

The rest of the article is organized as follows. Sec. 3.3 introduces games in satisfaction-form and defines the GSE. Sec. 3.4 studies the complexity of the problem of finding a pure-strategy GSE of a finite game in satisfaction-form. Sec. 3.4.2 introduces the satisfaction-response dynamics and sufficient conditions for convergence. Sec. 3.5 introduces Bayesian games in satisfaction-form, the Bayesian-GSE and Bayesian satisfaction-response dynamics. Sec. 3.6 discusses applications of GSE in wireless networks and comparative numerical results are presented. Finally, Sec. 5.7 concludes the article with a discussion on future directions.

3.3 satisfaction-form and Generalized Satisfaction Equilibrium

This section introduces the satisfaction-form representation of games and generalizes the notion of the equilibrium presented in (Perlaza *et al.*, 2012b). Unless otherwise stated, this article considers finite games in which there are finitely many agents and pure strategies.

3.3.1 Games in satisfaction-form

A game $\mathtt{G}_{\mathrm{SF}}$ in satisfaction-form is defined by the triplet

$$\mathtt{G}_{\mathrm{SF}} \triangleq \left(\mathscr{N}, \{\mathscr{A}_i\}_{i\in\mathscr{N}}, \{g_i\}_{i\in\mathscr{N}}\right), \tag{3.1}$$

where $\mathscr{N}$ is the finite index set of the agents and $\mathscr{A}_i$ is the finite set of pure strategies (*actions*) of agent $i \in \mathscr{N}$. Let Π_i denote the set of all probability distributions over $\mathscr{A}_i$. The set valued *correspondence* $g_i : \Pi_{-i} \to \mathbb{P}(\Pi_i)$ determines the set of strategies that satisfy the individual constraint of agent i for a given strategy profile of other agents $\pi_{-i} \in \Pi_{-i}$. Then, in a profile $(\pi_i, \pi_{-i}) \in \Pi$, agent i is said to be satisfied if $\pi_i \in g_i(\pi_{-i})$.

The correspondence g_i should not be confused to a constraint on feasible strategies, as in the case of games with coupled strategies (Scutari *et al.*, 2012). The agent i can choose any $\pi_i \in \Pi_i$ as a response to $\pi_{-i} \in \Pi_{-i}$, however, only the strategies in $g_i(\pi_{-i}) \subseteq \Pi_i$ satisfy its individual constraint. When only pure strategies are considered, with a slight abuse of notation, the correspondence in pure strategies is denoted by $g_i : \mathscr{A}_{-i} \to \mathbb{P}(\mathscr{A}_i)$. Then, for a given $a_{-i} \in \mathscr{A}_{-i}$, the subset $g_i(a_{-i}) \subseteq \mathscr{A}_i$ denotes the set of pure strategies that satisfy the individual constraint of agent i.

3.3.2 Generalized Satisfaction Equilibrium

A strategy profile $\pi \in \Pi$ of the game (3.1) induces a partition $\{\mathscr{N}_{\mathrm{s}}, \mathscr{N}_{\mathrm{u}}\}$ over the set $\mathscr{N}$ of agents. It is possible that one of the two sets $\mathscr{N}_{\mathrm{s}}, \mathscr{N}_{\mathrm{u}}$ is empty. The agents in the set $\mathscr{N}_{\mathrm{s}}$ are satisfied, that is, $\forall i \in \mathscr{N}_{\mathrm{s}}$, $\pi_i \in g_i(\pi_{-i})$. The agents in the set $\mathscr{N}_{\mathrm{u}}$ are unsatisfied, that is, $\forall j \in \mathscr{N}_{\mathrm{u}}$, $\pi_j \in \Pi_j \setminus g_j(\pi_{-j})$. Since an agent in $\mathscr{N}_{\mathrm{s}}$ is satisfied, it has no interest in changing its current strategy. Then, in order to guarantee an equilibrium, it must hold that none of the unsatisfied agents in $\mathscr{N}_{\mathrm{u}}$ are able satisfy their individual constraints by unilateral deviation. This notion of equilibrium, namely the generalized satisfaction equilibrium, is introduced by the following definition.

Definition 5. *Generalized Satisfaction Equilibrium (GSE): $\pi \in \Pi$ is a GSE of the game in (3.1) if there exists a partition $\{\mathscr{N}_{\mathrm{s}}, \mathscr{N}_{\mathrm{u}}\}$ of $\mathscr{N}$ such that $\forall i \in \mathscr{N}_{\mathrm{s}}$, $\pi_i \in g_i(\pi_{-i})$ and $\forall j \in \mathscr{N}_{\mathrm{u}}$, $g_j(\pi_{-j}) = \emptyset$.*

At a GSE strategy profile $\pi \in \Pi$, either an agent i satisfies its constraint or it is unable to satisfy its constraint, since $g_i(\pi_{-i}) = \emptyset$. From Def. 5 it follows that a pure-strategy GSE of (3.1) is an action profile $a \in \mathscr{A}$, where $\forall i \in \mathscr{N}_{\mathrm{s}}$, $a_i \in g_i(a_{-i})$ and $\forall j \in \mathscr{N}_{\mathrm{u}}$, $g_j(a_{-j}) = \emptyset$. This equilibrium notion generalizes previously proposed solution concepts to games in satisfaction-form. For instance, the SE as introduced in (Perlaza *et al.*, 2012b), is a pure-strategy profile that satisfies all agents. This definition comes as a special case of the pure-strategy GSE of Def. 5 when $\mathscr{N}_{\mathrm{u}} = \emptyset$. An ε-SE, of (Perlaza *et al.*, 2012b), is a GSE in which $\forall i \in \mathscr{N}$, $g_i(\pi_{-i}) = \{\pi_i \in \Pi_i : \mathbb{E}_\pi \mathbb{1}_{g_i(a_{-i})}(a_i) = 1 - \varepsilon\}$ and $\mathscr{N}_{\mathrm{u}} = \emptyset$. The expectation $\mathbb{E}_\pi$ is taken over the mixed-strategy profile. Finally when $\varepsilon = 0$, the mixed-strategy SE of (Perlaza *et al.*, 2012b) also follows as a special case of the GSE.

$$\mathtt{G}_{\mathrm{NE}} \triangleq \left(\mathscr{N}, \{\mathscr{A}_i\}_{i \in \mathscr{N}}, \{u_i\}_{i \in \mathscr{N}}\right). \tag{3.2}$$

Define the normal-form game (3.2), in which $u_i : \mathscr{A} \to \mathbb{R}$ is the utility function of agent i (Nisan *et al.*, 2007). The expected utility over a mixed-strategy profile is denoted by (3.3).

$$u_i(\pi) \triangleq \mathbb{E}_\pi u_i(a). \tag{3.3}$$

If the correspondence of the game (3.1) is defined as $g_i(\pi_{-i}) = \{\arg\max_{\pi_i \in \Pi_i} u_i(\pi_i, \pi_{-i})\}$, which is the best response correspondence of (3.2), then the GSEs of (3.1) are identical to the NEs of (3.2). Thus, the NE problem is in the class of GSE problems. The satisfaction form in (3.1) is capable of representing more general correspondences than the normal-form. For instance the GSE allows the modeling of risk-averse agents. Let the risk of a strategy π be measured by the variance of the utility denoted by $\text{Var}(u_i)$. Then, a risk-averse agent i has the correspondence $g_i(\pi_{-i}) = \{\pi_i \in \Pi_i : u_i(\pi_i, \pi_{-i}) \geq \tau_i, \text{Var}(u_i) \leq \rho\}$, where ρ is an upper limit on the variance. Even though the GSE allows the modeling of risk-averse agents, the rest of the article focuses on risk-neutral agents. Risk-neutrality is used to derive existence results for both complete and incomplete information games. Moreover, it also allows a fair comparison of the GSE against the NE, which too is defined for risk-neutral agents.

The GSEs of a game can be categorized by the number of agents that are satisfied. An N_s-GSE denotes a GSE in which $N_\text{s} \leq N$ agents are satisfied. An N-GSE satisfies all agents and thus, it is referred to as an SE in this article. The qualifiers mixed- and pure- may be omitted when the meaning is clear from the context.

3.3.3 Existence of Generalized Satisfaction Equilibria

The existence of a GSE in (3.1) depends on the properties of the correspondences $g_1, \ldots, g_N$. Consider the set valued function $g : \Pi \to \mathbb{P}(\Pi)$ given by (3.4).

$$g(\pi) \triangleq g_1(\pi_{-1}) \times \ldots \times g_N(\pi_{-N}). \tag{3.4}$$

Then an SE of (3.1) is a fixed point of g, i.e., $\pi \in g(\pi)$,and thus, the tools of fixed-point theory (Border, 1985) can be used explore the existence of SEs. However, this is not the case for GSEs. Note that at a GSE profile $\pi \in \Pi$, where $N_\text{s} < N$ there exists $j \in \mathcal{N}_\text{u}$ for which $g_j(\pi_{-j}) = \emptyset$ and thus, a fixed point of g is not properly defined in the set Π. This observation highlights the difficulty of providing a general existence result for a GSE. This is also a point of difference between the GSE and the mixed-strategy NE. The best response correspondence is nonempty for finite games (Nash *et al.*, 1950).

Existence results for GSEs can be given for particular classes of correspondences. For instance, consider a game in which agent are risk-neutral, and an agent i obtains an expected utility (3.3) and it is satisfied if the expected utility is higher than a given threshold $\tau_i \in \mathbb{R}$. That is, the set of mixed strategies that meet the satisfaction constraint of i is given by (3.5).

$$g_i(\pi_{-i}) = \{\pi_i \in \Pi_i : u_i(\pi) \geq \tau_i\}. \tag{3.5}$$

Examples of games in satisfaction-form that follow this correspondence are used in (Perlaza *et al.*, 2012b) to describe several dynamic spectrum access problems. In fact many wireless communication resource allocation problems fall under this class. In this case, the game in satisfaction form possesses at least one GSE. This result is formalized in the following proposition.

Proposition 1. *The finite game in satisfaction-form of (3.1) in which $\forall i \in \mathcal{N}$ risk-neutral agents, the correspondence is (3.5) possesses at least one GSE.*

Proof. The following proof of Prop. 1 argues that every NE of the normal-form game (3.2) coincides with some GSE of the game in (3.1) in which $\forall i \in \mathcal{N}$ g_i is (3.5). From the assumption of finite sets of actions and finite set of agents, it follows from (Nash *et al.*, 1950) that the game in (3.2) possesses at least one NE. At an NE, none of the agents can unilaterally choose another strategy and improve its individual reward. Therefore, at any NE there always exists a partition $\mathcal{N}_\mathrm{s}$ and $\mathcal{N}_\mathrm{u}$ of the set of agents such that $\forall i \in \mathcal{N}_\mathrm{s}$ $u_i(\pi) \geq \tau_i$ and $\forall j \in \mathcal{N}_\mathrm{u}$ $u_j(\pi) < \tau_j$ thus, $g_j(\pi_{-j}) = \emptyset$. This implies that an NE of (3.2) is a GSE of (3.1) in which agents possess the correspondence (3.5). $\square$

It is stressed that to apply Prop. 1, all agents of the game in (3.1) must follow the correspondence in (3.5). Prop. 1 does not hold if the correspondence is modified, for instance $g_i(\pi_{-i}) = \{\pi_i \in \Pi_i : \underline{\tau}_i \leq u_i(\pi) \leq \overline{\tau}_i\}$, with $\underline{\tau}_i$ and $\overline{\tau}_i$, any two reals. This is because, an NE strategy profile $\tilde{\pi} \in \Pi$ of (3.2) only ensures that an agent i may not increase its expected utility, however, it does not prevent the agent from deviating to reduce its utility if $u_i(\tilde{\pi}) \geq \overline{\tau}_i$.

Remark 1. The proof of Prop. 1 only requires that the game (3.2) has an NE. Thus, Prop. 1 extends to noncooperative games of infinite action spaces conditioned that they possess an NE.

The proof of Prop. 1 states that every NE of (3.2) is a GSE of a game in satisfaction-form in which the agent correspondences are of the form (3.5). However, the converse is not always true, i.e. the set of GSEs of the game in (3.1) can be larger than the set of NEs of (3.2). At a GSE, an agent might still unilaterally deviate and achieve a higher expected utility (not above the required threshold if it is in $\mathscr{N}_{\mathrm{u}}$), which contradicts the definition of the NE. The ε-NEs of (3.2) are not necessarily GSEs of the correspondence (3.5). An ε-NE is a profile from which no agent can unilaterally deviate and increase its expected utility more than $\varepsilon \geq 0$ (Nisan *et al.*, 2007). Given an ε-NE there can be an unsatisfied agent that can achieve satisfaction by increasing its expected utility by less than ε. The correspondence (3.5) describes agents with *bounded rationality* (Shoham & Leyton-Brown, 2009). That is the agents assign nonzero probability to actions that are non-optimal for expected utility maximization. Other equilibria with bounded rationality included the logit equilibrium (Chen *et al.*, 1997; Bennis *et al.*, 2013) and ε-NE.

Appendix 1 provides an example of a finite game that does not possess a GSE in mixed-strategies. Thus, the general existence of a GSE is not guaranteed. However, the satisfaction of the form in (3.5) is one of the most common problems in wireless networks. Thus, the existence of a GSE for (3.5), as proven in Prop. 1, is an encouraging result.

3.3.4 Comparison with normal-form

The problem of finding a pure-strategy GSE can be formulated as an NE problem of a particular game, called the $\{0,1\}$ normal-form game (Perlaza *et al.*, 2012b). Given an action profile $a \in \mathscr{A}$, if agent i is satisfied it receives a utility of 1, otherwise 0. This normal-form game is given in (3.6).

$$\mathtt{G}_{\mathrm{NF}} \triangleq \left(\mathcal{N}, \{\mathcal{A}_i\}_{i\in\mathcal{N}}, \{\mathbb{1}_{g_i(a_{-i})}(a_i)\}_{i\in\mathcal{N}}\right). \tag{3.6}$$

The pure-strategy GSEs of the game in (3.1) are identical to the pure-strategy NEs of the game in (3.6). All those who achieve a utility of 1 at an NE are satisfied, while the others are unsatisfied. For instance, in (Perlaza *et al.*, 2012b) an SE is defined as an NE of (3.6), where all agents achieve a utility of 1. However, the mixed-strategy GSEs of (3.1) are not necessarily mixed-strategy NEs of (3.6) and vice versa. This can be observed with respect to the correspondence (3.5). The satisfaction of i depends on the value of $u_i(\pi)$, whereas in (3.6) the agent maximizes $\mathbb{E}_\pi \mathbb{1}_{g_i(a_{-i})}(a_i)$. In addition, if restricted to only pure strategies Prop. 1 does not hold and the satisfaction-form game with correspondence (3.5) may not have a GSE solution. The existence result of Prop. 1 is valid only in the complete joint mixed-strategy space Π.

3.3.5 Efficiency of GSEs

The efficiency of a GSE is defined as the number of satisfied agents at that GSE. In order to compare the GSE performance against the optimal strategy that maximizes the number of satisfied agents, the price of stability (PoS) and price of anarchy (PoA) are defined as follows.

Definition 6. The PoS (resp. PoA) is the ratio of the maximum number of satisfiable agents to the maximum (resp. minimum) number of satisfiable agents at an equilibrium.

While a GSE profile is able to uniquely identify the indices of the satisfied players, when computing these prices the GSEs that satisfy equal number of players are considered to form an equivalence class. The following result upper bounds the PoS of the game with correspondence (3.5) by the PoS of the normal-form game in (3.2).

Corollary 1. *The PoS of a game in satisfaction-form in (3.1), in which the correspondence is (3.5) is upper bounded by the PoS of the normal-form game in (3.2).*

Proof. From Prop. 1 the set of NEs of the normal-form game (3.2) is a subset of the set of GSEs of the game (3.1) with correspondence (3.5). Thus, the maximum number of satisfied agents at a GSE of this satisfaction-form game is lower bounded by the maximum number of satisfied agents at an NE of the normal-form game (3.2). Then, the result follows by taking the ratio with the optimal number of satisfied agents. □

Similarly it can be seen that the PoA of the GSE of a game in satisfaction form with correspondence (3.5) is lower bounded by the PoA of the NE of the normal-form game (3.2). Thus, guiding the agents to an efficient GSE is paramount to the performance of the network.

3.4 Computation of Generalized Satisfaction Equilibria

This section demonstrates that a CSP can be represented as a pure-strategy GSE problem. The converse, mapping a pure-strategy GSE problem to a CSP is also possible and it enables the use of CSP algorithms to solve for pure-strategy GSEs. This section also introduces the satisfaction response algorithms, both in pure and mixed strategies. Sufficient conditions for their convergence are provided.

The *pure-strategy SE search problem* is as follows: given the game in satisfaction-form in (3.1) if there is a pure-strategy SE find it, otherwise, indicate that it does not exist. The following proposition asserts its complexity.

Proposition 2. *The pure-strategy SE search problem is NP-hard.*

In order to prove Prop. 2, the CSP is reduced in polynomial time to the pure-strategy SE search problem. This is called the Karp reduction (Arora & Barak, 2009). The CSP is NP-complete and it is concisely introduced at the beginning of Appendix 2 (Bulatov, 2011; Kumar, 1992).

Proof. The proof of Prop. 2 is given in Appendix 2. □

The *pure-strategy N_s-GSE search problem* is: given the game in satisfaction-form in (3.1) and a natural number $0 \leq N_s \leq N$ if there is an N_s-GSE or higher in pure strategies find it, otherwise indicate that it does not exist.

Corollary 2. *The pure-strategy N_s-GSE search problem is NP-hard.*

Proof. Given a routine to solve the N_s-GSE search problem, the SE search problem can be solved by setting $N_s = N$. Therefore, the N_s-GSE search problem is at least as hard as the SE search problem. □

Finding the complexity of the mixed-strategy GSE search problem is left as an open problem. However, the following result follows from Prop. 1.

Corollary 3. *The problem of finding a mixed-strategy GSE of a game (3.1) in which the correspondence is (3.5) is no harder than finding a mixed-strategy NE of the game (3.2).*

Proof. By Prop. 1, every NE of the game (3.2) is a GSE of a game (3.1) in which the correspondence is (3.5). Moreover, by the theory of NE (Nash *et al.*, 1950), the game (3.2) has at least one NE. Thus, any algorithm that finds an NE of (3.2) finds a GSE of a game (3.1) in which the correspondence is (3.5). □

3.4.1 Mapping the pure-strategy GSE to the CSP

The problem of finding a pure-strategy GSE can be formulated as a CSP. The variables of the CSP are the pure strategies $\{a_i, \dots, a_N\}$. If for $a_{-i} \in \mathscr{A}_{-i}$, $g_i(a_{-i}) \neq \emptyset$, then include a tuple (a_i', a_{-i}), for each $a_i' \in g_i(a_{-i})$, in the $N-$ary relation $\mathscr{R}_i$ of constraint c_i. Otherwise agent i may choose any action and thus there is some flexibility in deciding which tuples to place in $\mathscr{R}_i$. One possibility is to include a tuple (a_i', a_{-i}) for each $a_i' \in \mathscr{A}_i$. Another possibility is to include a single tuple (a_i'', a_{-i}) where $a_i'' \in \mathscr{A}_i$ is the only action the agent wants to take when it cannot achieve satisfaction. For example, in admission control, a_i'' is the zero power action. Repeat these steps $\forall a_{-i} \in \mathscr{A}_{-i}$ and $\forall i \in \mathscr{N}$. The resulting CSP is $(\{a_i\}_{i \in \mathscr{N}}, \{\mathscr{A}_i\}_{i \in \mathscr{N}}, \{c_i\}_{i \in \mathscr{N}})$. By the above construction of the relations $\mathscr{R}_1,\ldots,\mathscr{R}_N$, at a solution $a \in \mathscr{A}$ of this CSP, agent i has either $a_i \in g_i(a_{-i})$ or $g_i(a_{-i}) = \emptyset$. Therefore, any solution of the above constructed CSP is a pure-strategy GSE. Thus, algorithms for CSPs can be employed to solve for pure-strategy GSEs (Yokoo, 2012). For this reason, the normal-form representation of (3.6) and the NE algorithms are not required to solve for pure-strategy GSEs. A CSP is not guaranteed to have a solution or else it may have multiple solutions.

3.4.2 Satisfaction Response Algorithm in Pure Strategies

In the game (3.1), for $a \in \mathscr{A}$, if $a_i \notin g_i(a_{-i})$ and $g_i(a_{-i}) \neq \emptyset$, then there exists an $a_i' \in g_i(a_{-i})$ that agent i can deviate to satisfy its individual constraints. This deviation a_i' is called a satisfaction response and is denoted by $\mathrm{SR}_i(a_{-i}) \in g_i(a_{-i})$. Let $\mathscr{N}_\mathrm{u}' \subseteq \mathscr{N}$ be the subset of unsatisfied agents with nonempty correspondence, i.e., $i \in \mathscr{N}_\mathrm{u}'$, if $a_i \notin g_i(a_{-i})$ and $g_i(a_{-i}) \neq \emptyset$. Then, consider the discrete time update sequence in which at each instance a subset $\mathscr{N}_\mathrm{u}^\star \subseteq \mathscr{N}_\mathrm{u}'$ performs satisfaction response. In *asynchronous* mode, a strict subset $\mathscr{N}_\mathrm{u}^\star \subset \mathscr{N}_\mathrm{u}'$ performs the response and it includes the *sequential* mode in which only one agent at a time performs the response. In *synchronous* mode, all the agents in $\mathscr{N}_\mathrm{u}'$ perform the response. Algorithm 3.1 provides the pseudo-code for satisfaction response and Prop. 3 states its convergence properties. The convergence point of this algorithm depends on the agent selection and on the satisfaction response of those agents.

Algorithm 3.1: Asynchronous Satisfaction Response in Pure Strategies

Initialize $a = \underline{a}$
While $\mathscr{N}_\mathrm{u}'$ is not empty:
  Select $\mathscr{N}_\mathrm{u}^\star \subseteq \mathscr{N}_\mathrm{u}'$
  $a \triangleq \left((\mathrm{SR}_i(a_{-i}))_{i\in\mathscr{N}_\mathrm{u}^\star}, (a_j)_{j\in\mathscr{N}\setminus\mathscr{N}_\mathrm{u}^\star}\right)$

Prop. 2 and Cor. 2 demonstrate that solving for a pure-strategy GSE of the game in (3.1) is a hard problem in general. However, it is possible to identify subclasses of games that have a special structure in their correspondence that allows to efficiently find a pure-strategy GSE by

Algorithm 3.1. Suppose $\mathscr{Y}$ is a totally ordered set so that $\forall y, y' \in \mathscr{Y}$ either $y \leq y'$ or $y' < y$. Define the finite pure-strategy (action) spaces $\mathscr{A}_i \subset \mathscr{Y}, \forall i \in \mathscr{N}$, so that $\mathscr{A}_i$ is totally ordered as well. For all pairs $(a, a') \in \mathscr{A}^2$, the relation $a \leq a'$ holds if $\forall i \in \mathscr{N}$, $a_i \leq a'_i$. Alternatively, the relation $a < a'$ holds if $\forall i \in \mathscr{N}$ $a_i \leq a'_i$ and for at least one $j \in \mathscr{N}$ $a_j < a'_j$. The smallest and largest elements of $\mathscr{A}_i$ are denoted by $\underline{a}_i$ and $\overline{a}_i$ respectively and define the following vectors, $\underline{a} \triangleq (\underline{a}_1, \ldots, \underline{a}_N)$ and $\overline{a} \triangleq (\overline{a}_1, \ldots, \overline{a}_N)$. Consider the following mappings:

$$\underline{\phi}_i : \mathscr{A}_{-i} \to \mathscr{Y} \text{ and} \tag{3.7}$$

$$\overline{\phi}_i : \mathscr{A}_{-i} \to \mathscr{Y}. \tag{3.8}$$

Given the condition $a_{-i} \leq a'_{-i}$, the mapping $\overline{\phi}_i$ is called order-preserving if

$$\overline{\phi}_i (a_{-i}) \leq \overline{\phi}_i \left(a'_{-i}\right) \tag{3.9}$$

and is called order-reversing if

$$\overline{\phi}_i (a_{-i}) \geq \overline{\phi}_i \left(a'_{-i}\right). \tag{3.10}$$

Then define (3.11) in which both $\underline{\phi}_i$ and $\overline{\phi}_i$ are order-preserving.

$$g_i (a_{-i}) = \left\{a_i \in \mathscr{A}_i : \underline{\phi}_i (a_{-i}) \leq a_i \leq \bar{\phi}_i (a_{-i})\right\}. \tag{3.11}$$

Proposition 3. *Considering the game in satisfaction-form* (3.1) *in which* $\forall i \in \mathscr{N}$ *the correspondence is given by* (3.11)*, Algorithm 3.1 converges to a pure-strategy GSE.*

Proof. All agents are initialized at $\underline{a} \in \mathscr{A}$. Then there are two cases. Case one is, at $\underline{a}$ all agents are satisfied and then, $\mathscr{N}'_{\mathrm{u}}$ is empty and Algorithm 3.1 terminates. Case two is, there is at least one unsatisfied agent with a nonempty correspondence and Algorithm 3.1 proceeds. After a finite number of iterations of Algorithm 3.1, suppose that the current strategy profile is a. Consider an agent $i \in \mathscr{N}'_{\mathrm{u}}$ so that $g_i(a_{-i}) \neq \emptyset$. Then, in the current profile a, the action a_i is such that $a_i < \underline{\phi}_i(a_{-i})$ and it cannot be that $a_i > \overline{\phi}_i(a_{-i})$, since the algorithm started at $\underline{a}$.

Now if i performs the satisfaction response, then $a_i < \mathrm{SR}_i(a_{-i}) \leq \overline{\phi}_i(a_{-i})$. This implies that at each satisfaction response, the agents in $\mathscr{N}_\mathrm{u}^\star$ advance at least one action in their ordered action spaces. Since the number of agents and the action spaces are finite Algorithm 3.1 terminates in finite time. When Algorithm 3.1 terminates it is either $\mathscr{N}_\mathrm{u} = \emptyset$ or $\forall i \in \mathscr{N}_\mathrm{u}\ g_i(a_{-i}) = \emptyset$, which by Def. 5 is a pure-strategy GSE. $\square$

There is the implicit assumption that every agent that finds itself in $\mathscr{N}_\mathrm{u}'$ is given the chance to perform the satisfaction response within a finite number of future steps. The worst case iterations for sequential satisfaction response is $O(N \max_{i\in\mathscr{N}}\{|\ \mathscr{A}_i\ |\})$. This worst case occurs when all agents are initially in $\mathscr{N}_\mathrm{u}'$ and each agent advances to $\mathscr{N}_\mathrm{s}$ with $\mathrm{SR}_i(a_{-i}) = \underline{\phi}_i(a_{-i})$ only to be found back in $\mathscr{N}_\mathrm{u}'$ at the beginning of its next chance to respond. Algorithm 3.1 applies to infinite action spaces that are closed intervals in the real line. However, in that case the convergence time depends on the step size. Sequential satisfaction response up to a fixed number of iterations is discussed in (Ross & Chaib-draa, 2006); however, the conditions for convergence are not identified.

3.4.3 Satisfaction Response in Mixed Strategies

The satisfaction response algorithm extends to mixed strategies. To this end, a partial ordering of mixed strategies is required. Recall that the probability assigned by the strategy $\pi_i \in \Pi_i$ to action a_i is denoted by $\pi_i(a_i)$. Two mixed strategies $(\pi_i, \pi_i') \in \Pi_i^2$ of i are ordered $\pi_i \leq \pi_i'$, if $\exists\ a_i' \in \mathscr{A}_i$ such that $\forall a_i < a_i'$, $\pi_i(a_i) \geq \pi_i'(a_i)$ and $\forall a_i \geq a_i'$, $\pi_i(a_i) \leq \pi_i'(a_i)$. The action a_i' acts as a pivot. The profile π_i' must have probabilities no less than the probabilities given by π_i for each action above a_i' and probabilities no greater than the probabilities given by π_i for each action below a_i'. Define $\underline{\pi}_i$ such that $\underline{\pi}_i(\underline{a}_i) = 1$ and $\forall a_i > \underline{a}_i$, $\underline{\pi}_i(a_i) = 0$. Also define $\overline{\pi}_i$ such that, $\overline{\pi}_i(\overline{a}_i) = 1$ and $\forall a_i < \overline{a}_i$, $\overline{\pi}_i(a_i) = 0$. Then, $\underline{\pi} \triangleq (\underline{\pi}_i)_{i\in\mathscr{N}}$ and $\overline{\pi} \triangleq (\overline{\pi}_i)_{i\in\mathscr{N}}$. In addition, the following assumptions are made about the agent utility function. The utility of $i \in \mathscr{N}$ is such that if $a_{-i} \leq a_{-i}'$, then $u_i(a_i, a_{-i}) \geq u_i(a_i, a_{-i}')$, with strict inequality if $a_{-i} < a_{-i}'$. If $a_i \leq a_i'$, then $u_i(a_i, a_{-i}) \leq u_i(a_i', a_{-i})$, again with strict inequality if $a_i < a_i'$.

Recall that $u_i(\pi)$ is the expected utility over mixed-strategy profile $\pi \in \Pi$. The correspondence of i is defined as the set of mixed strategies that achieve an expected utility between a given range and it is denoted by (3.12).

$$g_i(\pi_{-i}) = \{\pi_i \in \Pi_i : \underline{\tau}_i \leq u_i(\pi_i, \pi_{-i}) \leq \bar{\tau}_i\}. \tag{3.12}$$

The agents start off at $\underline{\pi}$. Then, in each iteration, given the current profile, an agent $i \in \mathcal{N}_{\mathrm{u}}^{\star}$ chooses a higher order mixed strategy, according to the above ordering, such that it achieves satisfaction. Thus, the probability distribution transitions from a positive skew (longer right tail) to a negative skew (longer left tail). This process continues till no unsatisfied agent is able to achieve satisfaction. The pseudo-code is given in Algorithm 3.2.

Algorithm 3.2: Satisfaction Response in Mixed Strategies

Initialize $\pi = \underline{\pi}$
While $\mathcal{N}_{\mathrm{u}}'$ is not empty:
  Select $\mathcal{N}_{\mathrm{u}}^{\star} \subseteq \mathcal{N}_{\mathrm{u}}'$
  $\forall i \in \mathcal{N}_{\mathrm{u}}^{\star}$, $\mathrm{SR}_i(\pi_{-i}) > \pi_i$
  $\pi \triangleq ((\mathrm{SR}_i(\pi_{-i}))_{i \in \mathcal{N}_{\mathrm{u}}^{\star}}, (\pi_j)_{j \in \mathcal{N} \setminus \mathcal{N}_{\mathrm{u}}^{\star}})$

Considering the game in satisfaction-form (3.1) in which $\forall i \in \mathcal{N}$ g_i is (3.12), Algorithm 3.2 converges to a mixed-strategy GSE. This convergence can be explored as follows. All agents are initialized at $\underline{\pi} \in \Pi$. There are two cases to consider. Case one: at $\underline{\pi}$ all agents are satisfied. Then, $\mathcal{N}_{\mathrm{u}}'$ is empty and Algorithm 3.2 terminates. Case two: there is at least one unsatisfied agent with a nonempty correspondence. Then Algorithm 3.2 proceeds. Let π^t be the profile after an arbitrary finite t number of iterations. If $i \in \mathcal{N}_{\mathrm{u}}'$, then $\exists$ $\mathrm{SR}_i(\pi^t_{-i}) \in g_i(\pi^t_{-i})$ such that $\mathrm{SR}_i(\pi^t_{-i}) > \pi^t_i$. This argument follows from the properties of the utility function u_i of (3.12) and from the fact that $\forall \pi_i \in \Pi_i \setminus \{\overline{\pi}_i\}, \pi_i < \overline{\pi}_i$. The ordered mixed-strategy space of i is upper bounded by $\overline{\pi}_i$ and $\pi^t_i < \mathrm{SR}_i(\pi^t_{-i}) < \overline{\pi}_i$. Thus, for $t' > t$, $\mathrm{SR}_i(\pi^t_{-i}) < \mathrm{SR}_i(\pi^{t'}_{-i}) < \overline{\pi}_i$. Therefore, Algorithm 3.2 converges and at convergence all unsatisfied agents have empty correspondences; hence, it converges to a mixed-strategy GSE by Def. 5.

The convergence rate of Algorithm 3.2 depends on the manner in which the players advance in the ordered mixed-strategy space, which in turn depends on the utility functions. Both Algorithm 3.1 and Algorithm 3.2 can converge to inefficient GSEs in terms of the number of satisfied agents.

3.5 Bayesian Games in satisfaction-form

In many wireless network problems global channel state information (CSI) is not common knowledge among transceivers. Thus, they can be modeled as Bayesian games (Harsanyi, 1967-1968). In a Bayesian game, an agent possesses private information, called its *type*. The type set of agent i is denoted by $\mathscr{X}_i$ and x_i is a random variable over $\mathscr{X}_i$. All agents share common knowledge of the joint distribution F_x of the random type vector x. A pure-strategy of i is a mapping $s_i : \mathscr{X}_i \to \mathscr{A}_i$ that assigns an action to each type in $\mathscr{X}_i$ (Shoham & Leyton-Brown, 2009). The set of pure strategies of i is denoted by $\mathscr{S}_i$. The mixed-strategy set Π_i of a Bayesian game is the set of all probability distributions over $\mathscr{S}_i$ (Shoham & Leyton-Brown, 2009). Given a type realization $x_i \in \mathscr{X}_i$, the probability that π_i assigns to $a_i \in \mathscr{A}_i$ is denoted by $\pi_i(a_i \mid x_i)$. The correspondence is defined as $g_i : \Pi_{-i} \times \mathscr{X}_i \to \mathbb{P}(\Pi_i)$. Then, a Bayesian game in satisfaction-form is defined by the tuple (3.13).

$$\mathtt{G}_{\mathrm{BSF}} \triangleq \left(\mathscr{N}, \{\mathscr{A}_i\}_{i\in\mathscr{N}}, \{\mathscr{X}_i\}_{i\in\mathscr{N}}, \{g_i\}_{i\in\mathscr{N}}, F_x\right). \tag{3.13}$$

Having a correspondence for each type in $\mathscr{X}_i$ comes useful, for instance, in modeling a minimum rate requirement that depends on a queue length or a minimum SINR based on the channel gain. For a strategy profile $(\pi_i, \pi_{-i}) \in \Pi$, agent i is said to be unsatisfied if $\pi_i \notin g_i(\pi_{-i}, x_i)$ for at least one realization $x_i \in \mathscr{X}_i$ and conversely i is satisfied if $\forall x_i \in \mathscr{X}_i$, $\pi_i \in g_i(\pi_{-i}, x_i)$. Then the Bayesian-GSE is defined as follows.

Definition 7. *Bayesian Generalized Satisfaction Equilibrium (Bayesian-GSE): The profile $\pi \in \Pi$ is a Bayesian-GSE of (3.13) if there exists a partition $\{\mathscr{N}_{\mathrm{s}}, \mathscr{N}_{\mathrm{u}}\}$ of $\mathscr{N}$ such that $\forall i \in \mathscr{N}_{\mathrm{s}}$, $\forall x_i \in \mathscr{X}_i$, $\pi_i \in g_i(\pi_{-i}, x_i)$ and $\forall j \in \mathscr{N}_{\mathrm{u}}$, if for any $x'_j \in \mathscr{X}_i$ $\pi_j \notin g_j(\pi_{-j}, x'_j)$, then $g_j(\pi_{-j}, x'_j) = \emptyset$.*

Def. 7 essentially states that at a Bayesian-GSE, agents in $\mathscr{N}_{\mathrm{u}}$ are unable to deviate and achieve satisfaction for the types in which they are unsatisfied. This equilibrium is Bayesian in the sense that $g_i(\pi_{-i}, x_i)$ can be defined as the achievement of a performance level in expectation over the posterior distribution $F_{x|x_i}$. As in the complete information case, with a slight abuse of notation, the pure-strategy correspondence is denoted by $g_i : \mathscr{S}_{-i} \times \mathscr{X}_i \to \mathbb{P}(\mathscr{S}_i)$.

For $\pi \in \Pi$, let $\mathbb{E}_{x|x_i} u_i(\pi, x)$ denote the *ex interim* expected utilities of i (Shoham & Leyton-Brown, 2009) and $\tau_i(x_i) \in \mathbb{R}$ a threshold, which can take different values for each type $x_i \in \mathscr{X}_i$. The expectation is over the mixed strategies and the posterior $F_{x|x_i}$. A Bayesian game is finite when the sets of agents, actions, and types are all finite. Then, Prop. 4 is the Bayesian counterpart to Prop. 1.

$$g_i(\pi_{-i}, x_i) = \{\pi_i \in \Pi_i : \mathbb{E}_{x|x_i} u_i(\pi, x) \geq \tau_i(x_i)\}. \tag{3.14}$$

Proposition 4. *The finite Bayesian game in satisfaction form in (3.13), where $\forall i \in \mathscr{N}$ risk-neutral agents, the Bayesian correspondence is (3.14), has at least one Bayesian-GSE.*

Proof. The proof follows similar to that of Prop. 1. Given the above Bayesian satisfaction-form game, construct the Bayesian normal-form game as follows

$$\mathtt{G}_{\mathrm{BNE}} \triangleq \left(\mathscr{N}, \{\mathscr{A}_i\}_{i\in\mathscr{N}}, \{\mathscr{X}_i\}_{i\in\mathscr{N}}, \{u_i\}_{i\in\mathscr{N}}, F_x\right), \tag{3.15}$$

where u_i is the utility of $i \in \mathscr{N}$. At a Bayesian Nash equilibrium $\pi \in \Pi$ of (3.15), $\forall i \in \mathscr{N}$ the ex interim expected utility $\mathbb{E}_{x|x_i} u_i(\pi, x)$ is a maximum $\forall x_i \in \mathscr{X}_i$. Thus, if agent i for type x_i has $\mathbb{E}_{x|x_i} u_i(\pi, x) < \tau_i(x_i)$, then i cannot deviate and improve $\mathbb{E}_{x|x_i} u_i(\pi, x)$. Hence, for any unsatisfied type x_i of i, $g_i(\pi_{-i}, x_i) = \emptyset$, which by Def. 7 is a Bayesian-GSE. $\square$

The Bayesian versions of Cor. 1 and Cor. 3 follows from Prop. 4. It is possible to identify *Bayesian* games that converge with satisfaction response, similar to the discussion of Sections 3.4.2 and 3.4.3. The case of pure strategies is developed here. Recall the totally ordered action spaces from Section 3.4.2, where $\forall i \in \mathscr{N}$, $\mathscr{A}_i \subset \mathscr{Y}$. Let $\underline{\phi}_i : \mathscr{S}_{-i} \times \mathscr{X}_i \to \mathscr{Y}$ and $\overline{\phi}_i$: $\mathscr{S}_{-i} \times \mathscr{X}_i \to \mathscr{Y}$. The mapping $\underline{\phi}_i$ is called order-preserving, if $\forall x_{-i} \in \mathscr{X}_{-i}$, $s_{-i}(x_{-i}) \leq s'_{-i}(x_{-i})$, then $\forall x_i \in \mathscr{X}_i$ $\underline{\phi}_i(s_{-i}, x_i) \leq \underline{\phi}_i(s'_{-i}, x_i)$. Let us define $\forall x_i \in \mathscr{X}_i$, $\underline{s}_i(x_i) \triangleq \underline{a}_i$, and $\underline{s} \triangleq (\underline{s}_i)_{i \in \mathscr{N}}$. In the game in (3.13) let the correspondence g_i be

$$g_i(s_{-i}, x_i) = \{s_i \in \mathscr{S}_i : \underline{\phi}_i(s_{-i}, x_i) \leq s_i(x_i) \leq \bar{\phi}_i(s_{-i}, x_i)\}, \tag{3.16}$$

where $\underline{\phi}_i, \bar{\phi}_i$ are order-preserving. The Bayesian counterpart of Prop. 3 is given by Prop. 5. When Algorithm 3.1 is applied to the Bayesian game, the action profile a is replaced by the pure-strategy profile s.

Proposition 5. *Consider a Bayesian game in satisfaction-form in* (3.13)*, in which* $\forall i \in \mathscr{N}$ g_i *is* (3.16)*. Then, Algorithm 3.1 converges to a pure-strategy Bayesian-GSE.*

Proof. The proof is similar to that of Prop. 3, except each type has to be considered. Initialized at $\underline{s}$, if $i \in \mathscr{N}_\mathrm{u}'$ performs satisfaction response at the current profile s, then $\forall x_i \in \mathscr{X}_i$ where $g_i(s_{-i}, x_i) \neq \emptyset$, $s_i(x_i) \leq \mathrm{SR}_i(s_{-i}, x_i) \leq \bar{\phi}_i(s_{-i}, x_i)$ and for at least one x_i (for which i was unsatisfied) $s_i(x_i) < \mathrm{SR}_i(s_{-i}, x_i) \leq \bar{\phi}_i(s_{-i}, x_i)$. Therefore, for each unsatisfied type, the strategies monotonically advance in the ordered action space. Since the number of agents, actions, and types are finite the algorithm terminates when either $\mathscr{N}_\mathrm{u} = \emptyset$ or $\forall i \in \mathscr{N}_\mathrm{u}$ for all unsatisfied types $x_i \in \mathscr{X}_i$, $g_i(s_{-i}, x_i) = \emptyset$. □

3.6 Applications of GSEs and Simulation Results

This section applies the novel GSE framework to several problems in wireless networks and compares the performance against the NE. The first application is energy efficiency in an orthogonal frequency division multiple access (OFDMA) heterogeneous network (HetNet). The second application is power control in the HetNet with rate constraints. The third is orthogonal channel allocation in device-to-device (D2D) communication. The fourth is admission control. Finally, this section presents an application of the Bayesian-GSE to the power control problem.

Since the SE of (Perlaza *et al.*, 2012b) is a special case of the GSE, the applications considered in (Perlaza *et al.*, 2012b; Mérriaux *et al.*, 2012; Ren *et al.*, 2015; Rose *et al.*, 2012; Ellingsæter, 2014), which employ the SE, can also be solved for their GSEs. Moreover, by Prop. 1 and Rem. 1 noncooperative games of utility maximization that possess NEs can be solved for GSEs of correspondence (3.5) and these encompass a vast array of literature (Altman & Altman, 2003; Scutari & Palomar, 2010; Scutari *et al.*, 2012; Samarakoon *et al.*, 2013).

3.6.1 Energy Efficiency in HetNets

Similar to (Buzzi *et al.*, 2012), the energy efficiency is defined as the number of error free information bits per Joule of transmit energy. The user i transmits coded frames of length L bits of which D bits are information at a transmission rate of R bits/s. The channel power gain from small cell user equipment (SUE) $i \in \mathscr{N}$ to the base station (BS) $m \in \mathscr{M}$ on subchannel $k \in \mathscr{K}$ is $\mid h_{im}^{k} \mid^2$ and noise power at the receiver is σ^2. The utility of energy efficiency is given by (3.17). The efficiency function $f(\gamma_i) = (1 - e^{-\gamma_i})^L$ is determined by the SINR $\gamma_i = \frac{p_i |h_{im}^k|^2}{\sum_{j \in \mathscr{N} \setminus \{i\}} p_j |h_{jm}^k|^2 + \sigma^2}$ of i at the home BS m. SUE i transmits at power $p_i \in [0, p_{\max}]$. It has been shown that with per-sub-carrier power constraints the normal-form game $\mathtt{G}_{\text{EFF}-\text{NE}}$ in (3.18) has a unique NE (Buzzi *et al.*, 2012).

$$u_i(p) = R \frac{D}{L} \frac{f(\gamma_i)}{p_i}. \tag{3.17}$$

$$\mathtt{G}_{\text{EFF}-\text{NE}} \triangleq \left(\mathscr{N}, \{\mathscr{P}_i\}_{i \in \mathscr{N}}, \{u_i\}_{i \in \mathscr{N}} \right). \tag{3.18}$$

The satisfaction-form game $\mathtt{G}_{\text{EFF}-\text{GSE}}$ in (3.19) has the correspondence $g_i(p_{-i}) = \{p_i \in [0, p_{\max}] : u_i(p) \geq \underline{\tau}\}$, in which $\underline{\tau}$ is the minimum level of energy efficiency required by the SUE.

$$\mathtt{G}_{\text{EFF}-\text{GSE}} \triangleq \left(\mathscr{N}, \{\mathscr{P}_i\}_{i \in \mathscr{N}}, \{g_i\}_{i \in \mathscr{N}} \right). \tag{3.19}$$

The simulations compare the performance of the GSEs of $\mathtt{G}_{\text{EFF}-\text{GSE}}$ to the NEs of $\mathtt{G}_{\text{EFF}-\text{NE}}$ for different path-loss models that represent different interference scenarios. The simulation setup is an OFDMA HetNet that consists of an urban microcell of radius 200 m and 8 small-cell BSs (SBSs) each with a serving radius of 15 m and each serving 4 SUEs. The network has 4 subchannels which are reused among the cells and each active user receives one subchannel. For simplicity one sub-carrier per subchannel is considered. Within a cell the OFDMA subchannels are assigned to the users in a non-overlapping manner similar to the LTE uplink (3GPP, 2010). Hence, the SUEs of a cell only experience intercell interference from SUEs of other SCs and the microcell users. The microcell users are considered to transmit at their maximum transmission power. Table 3.1 contains the simulation parameters. Fig. 3.1 depicts the results. It is seen that the GSE outperforms the NE to satisfy more users at each threshold level. Fig. 3.2 shows the probability mass function for the same experiment.

Table 3.1 Simulation Parameters for Energy Efficiency

Parameter	Value
Maximum UE transmission power $p_{\max}$	21 dBm
Noise power spectral density	-174 dBm/Hz
Path-loss exponent α	3.76
Small scale fading	Rayleigh($\frac{1}{\sqrt{2}}$)
Carrier frequency	2 GHz
Sub-carrier Spacing	15 kHz
Number of SBSs	8
Number of users per SBS	4
R	100 kb/s
D/L	800/1024

3.6.2 Uplink Power Control for Minimum SINR

The problem of power control under per-user rate requirements has been well studied for its feasible region and Pareto optimal solutions (Hande *et al.*, 2008). The infeasible case in which a subset of the transmitters cannot be satisfied has received less attention (Monemi *et al.*, 2013). The GSE framework provides a well defined solution that applies to both the feasible and infeasible cases. The utility of transmitter i is the spectral efficiency $u_i(p) = \log_2(1+\gamma_i)$ bits/s/Hz. The transmission power is $p_i \in \mathscr{P}_i$, where $\mathscr{P}_i$ is the set of finite power levels. The game in

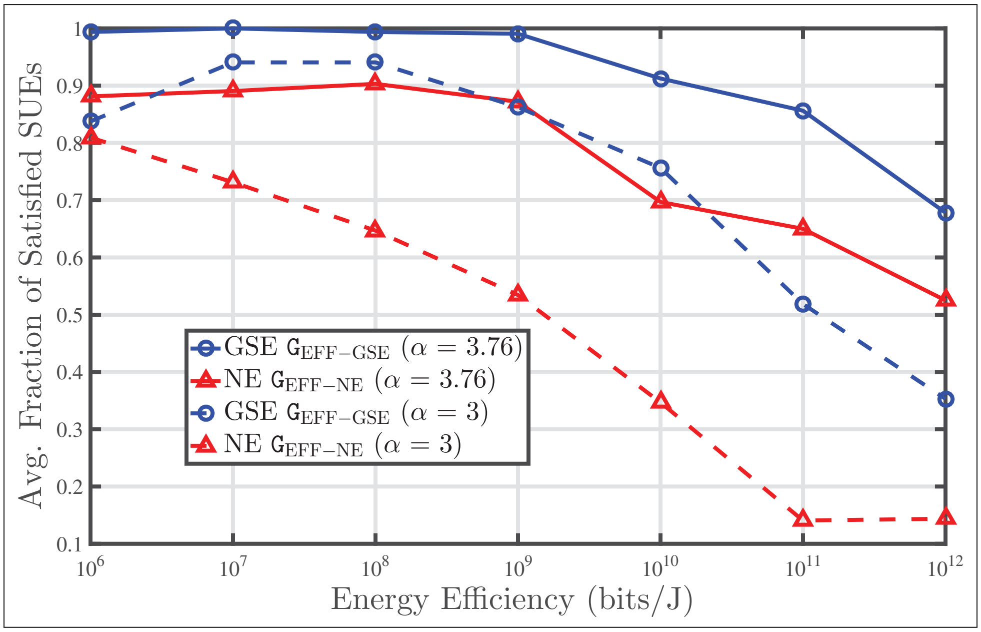

Figure 3.1 The empirical average of the number of satisfied SUEs under varying channel conditions for $\mathtt{G}_{\mathrm{EFF-GSE}}$ and $\mathtt{G}_{\mathrm{EFF-NE}}$. Here α is the path-loss exponent.

satisfaction-form played by the SUEs is

$$\mathtt{G}_{\mathrm{PC-GSE}} \triangleq \left(\mathscr{N}, \{\mathscr{P}_i\}_{i\in\mathscr{N}}, \{g_i\}_{i\in\mathscr{N}}\right), \tag{3.20}$$

in which $\forall i \in \mathscr{N}$, $g_i(\pi_{-i}) = \{\pi_i \in \Pi_i : \underline{\tau} \leq u_i(\pi)\}$, where $0 \leq \underline{\tau}$. It can be verified that the game $\mathtt{G}_{\mathrm{PC-GSE}}$ satisfies the sufficient conditions for convergence of both Algorithm 3.1 and Algorithm 3.2. The power control game can be formulated as a noncooperative game to minimize the transmit power with per-user rate constraints and it is a generalized NE (GNE) problem (Scutari *et al.*, 2010). However, for a GNE to exist it is necessary (but not sufficient) that all the rate constraints can be simultaneously met and thus, a GNE solution may not exist if the problem is over constrained (Scutari *et al.*, 2010). Also note that if a GNE exists, then the satisfaction-form problem has an SE. On the other hand by Prop. 1 the game in (3.20) always has a GSE.

$$\mathtt{G}_{\mathrm{PC-NE}} \triangleq (\mathscr{N}, \{\mathscr{P}_i\}_{i\in\mathscr{N}}, \{u_i\}_{i\in\mathscr{N}}). \tag{3.21}$$

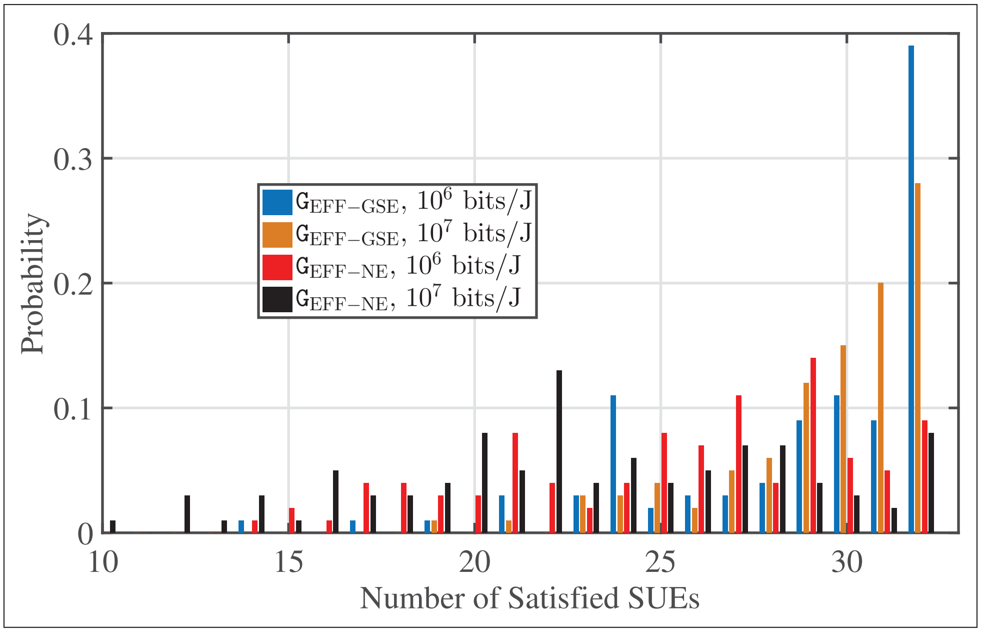

Figure 3.2 The distribution of the number of satisfied SUEs in $G_{EFF-GSE}$ and G_{EFF-NE} at energy efficiency of 10^6 bits/J and 10^7 bits/J.

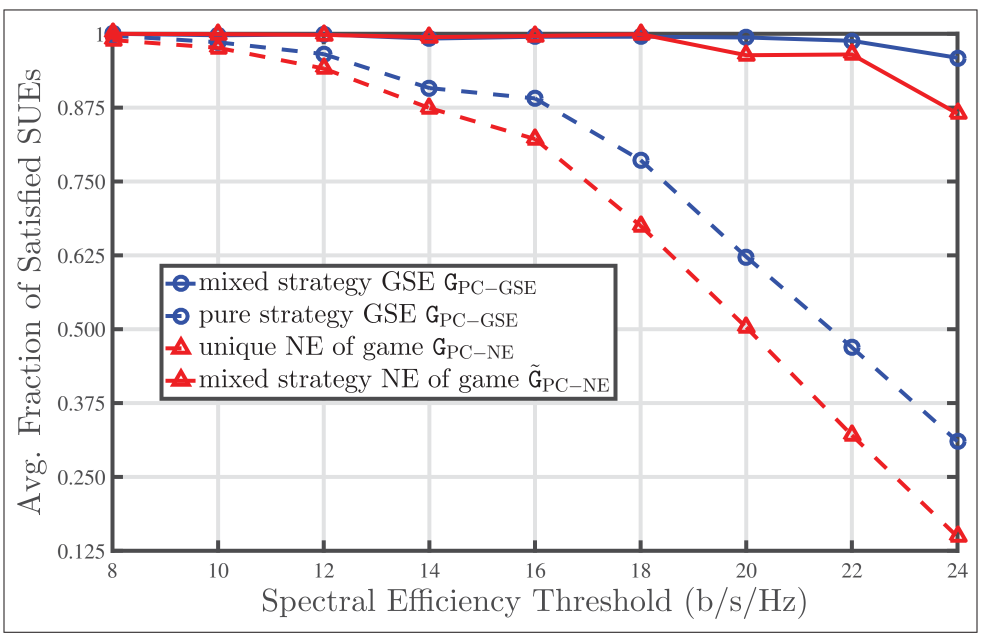

Figure 3.3 The empirical average of the number of satisfied SUEs for G_{PC-GSE}, G_{PC-NE}, and $\tilde{G}_{PC-NE}$.

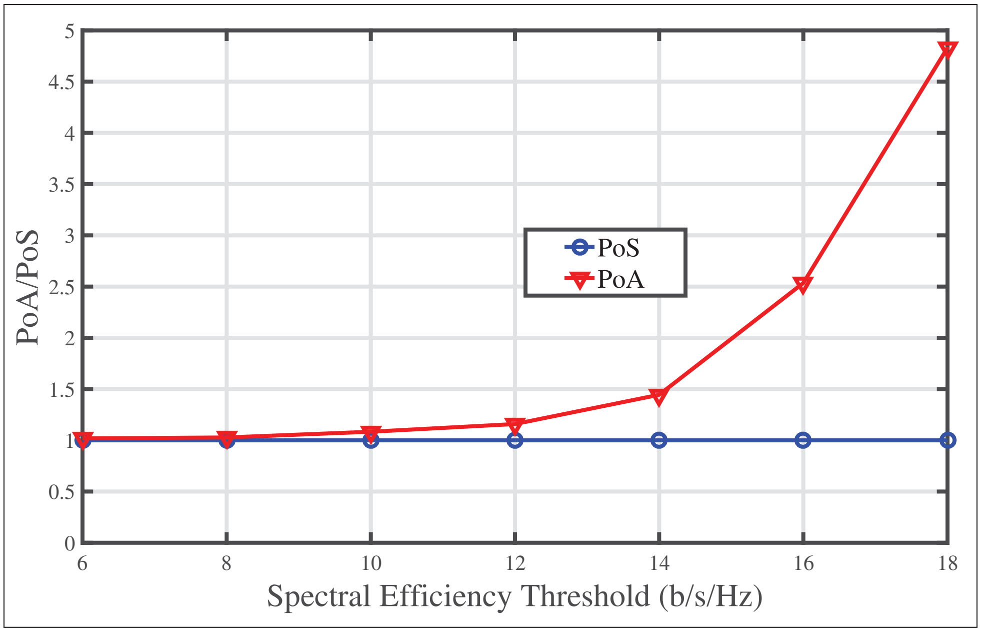

Figure 3.4 PoA and PoS of $\mathtt{G}_{\mathrm{PC-GSE}}$ for pure strategies.

$$\tilde{\mathtt{G}}_{\mathrm{PC-NE}} \triangleq (\mathcal{N}, \{\mathscr{P}_i\}_{i\in\mathcal{N}}, \{\tilde{u}_i\}_{i\in\mathcal{N}}). \tag{3.22}$$

Two normal-form games are used for comparison purposes. The first one is $\mathtt{G}_{\mathrm{PC-NE}}$ in (3.21). From the monotonicity of u_i in p_i, $\mathtt{G}_{\mathrm{PC-NE}}$ has a unique NE where an SUE i transmits at its maximum power. The second one is $\tilde{\mathtt{G}}_{\mathrm{PC-NE}}$ in (3.22). The utility of $\tilde{\mathtt{G}}_{\mathrm{PC-NE}}$ is defined as $\tilde{u}_i = u_i$ if $u_i < \underline{\tau}$ else $\tilde{u}_i = \underline{\tau}$. Thus, for an SUE in $\tilde{\mathtt{G}}_{\mathrm{PC-NE}}$ its utility increases with the spectral efficiency till it reaches the threshold and then its utility value does not change with further increase of the spectral efficiency. The Fictitious Play algorithm is used to compute the mixed-strategy equilibria (Lasaulce & Tembine, 2011). The simulation HetNet has a total of 8 SUEs in 2 OFDMA small cells. Each agent has 3 power levels $\{0, \frac{p_{\max}}{2}, p_{\max}\}$. The other relevant simulation parameters are similar to that of Table 3.1. Fig. 3.3 depicts the simulation results for the fraction of satisfied SUEs for different thresholds. The PoA and PoS of the GSE is given in Fig. 3.4.

3.6.3 Admission Control

At a pure-strategy GSE $p \in \mathscr{P}$ of (3.20), an unsatisfied agent $i \in \mathscr{N}_{\mathrm{u}}$, obtains $u_i(p) < \underline{\tau}_i$, but may have $p_i > \underline{p}_i$. If an agent in $\mathscr{N}_{\mathrm{u}}$ lowers its power, then it is possible that another in $\mathscr{N}_{\mathrm{u}}$ can deviate to satisfaction and thus disrupt the equilibrium. In admission control applications where $\forall i \in \mathscr{N}\ \underline{p}_i = 0$, it is desirable that at a GSE $\forall i \in \mathscr{N}_{\mathrm{u}},\ p_i = 0$. Such GSEs do not necessarily exist. However, unlike traditional admission control schemes (Halldórsson & Mitra, 2012), the GSE admission is stable, i.e., the agents who do not transmit are aware that they cannot achieve satisfaction even at the maximum power. The mapping outlined in Section 3.4.1 can be used to solve for GSE admission control by solving a CSP.

3.6.4 Orthogonal Channel Allocation in D2D Communication

Consider the problem of allocating a finite set $\mathscr{K}$ of orthogonal channels among $\mathscr{N}$ interfering wireless D2D links (Etkin & Ordentlich, 2009). Each link consists of a unique transmitter and a receiver. The transmitter of link $i \in \mathscr{N}$ has the action set $\mathscr{K}_i \subseteq \mathscr{K}$. A strict subset $\mathscr{K}_i \subset \mathscr{K}$ is a situation where the transmitter does not have access to all the channels of $\mathscr{K}$. The transmit power remains constant. Transmitter i is said to be satisfied if the SINR at its receiver is above a threshold $\underline{\tau}$. The pure-strategy game in satisfaction-form is:

$$\mathtt{G}_{\mathrm{CH-GSE}} \triangleq \left(\mathscr{N}, \{\mathscr{K}_i\}_{i\in\mathscr{N}}, \{g_i\}_{i\in\mathscr{N}}\right), \tag{3.23}$$

where $\forall k_{-i} \in \mathscr{K}_{-i}\ g_i(k_{-i}) = \{k_i \in \mathscr{K}_i : \gamma_i(k) \geq \tau,\}$. Since the transmitter is unique to each link, with a slight abuse of notation, the link set $\mathscr{N}$ is used to represent the transmitter set in (3.23).

From Prop. 1 it follows that the game (3.23) has at least one GSE in mixed strategies. Prop. 6 shows that searching for a pure-strategy SE of (3.23) is NP-hard.

Proposition 6. *The pure-strategy SE search problem of (3.23) is NP-hard.*

Proof. The proof is given in Appendix 3. □

From Corollary 2, if an efficient algorithm exists to solve the N_s-GSE search problem then that algorithm can efficiently solve the SE search problem of the same game. Therefore, since the SE search problem is NP-hard by Prop. 6, finding an N_s-GSE of (3.23) is NP-hard as well.

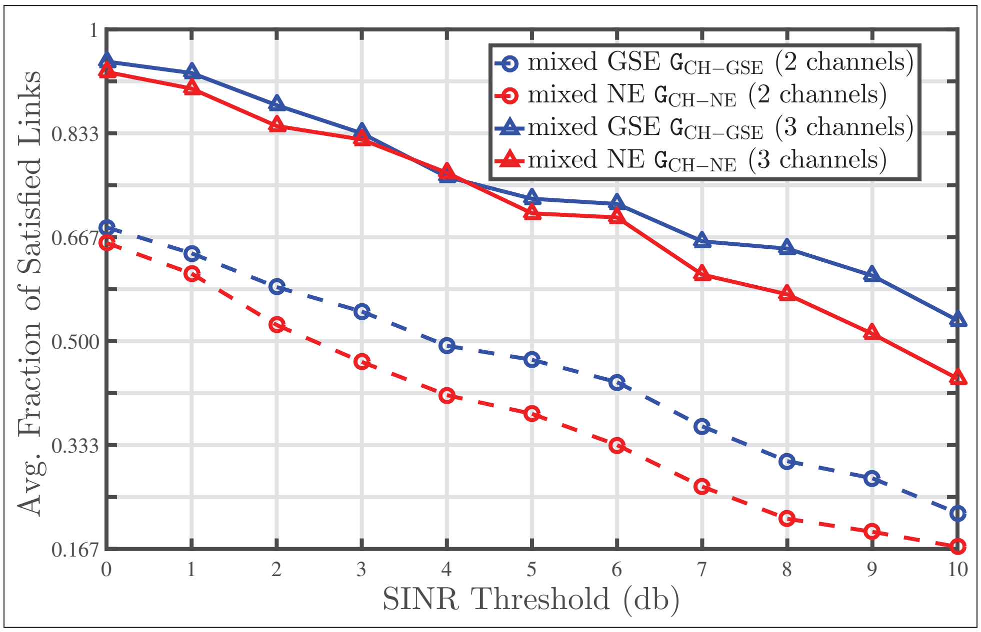

Figure 3.5 The empirical average of the number of satisfied links for $\mathtt{G}_{\text{CH-GSE}}$ and $\mathtt{G}_{\text{CH-NE}}$ in mixed strategies.

Next, consider the game in mixed strategies. Then the correspondence is $\forall \pi_{-i} \in \Pi_{-i}\; g_i(\pi_{-i}) = \{\pi_i \in \Pi_i : \gamma_i(\pi) \geq \tau, \}$. The NE game is given in (3.24). The simulation setup consists of 6 D2D links that are uniformly distributed in a room of radius 10 m. These can be links between smart appliances. The channel parameters are as in Table. 1. Fig. 3.5 depicts the empirical average number of satisfied links for different number of channels. It shows that the number of satisfied users is higher at the GSE than at the NE.

$$\mathtt{G}_{\text{CH-NE}} \triangleq \left(\mathscr{N}, \{\mathscr{K}_i\}_{i\in\mathscr{N}}, \{\gamma_i\}_{i\in\mathscr{N}}\right). \tag{3.24}$$

3.6.5 Bayesian Power Control

This section considers the problem of Section 3.6.2 in the incomplete information case. The private information of SUE i is its direct channel to the home BS, which can be obtained through feedback. Then, let us define the vector of channels (direct and interference) $h = (h_{im}^k)_{i\in\mathscr{N},m\in\mathscr{M},k\in\mathscr{K}}$. A pure-strategy $s_i(h_{im}^k)$ of a user i depends on the channel $h_{im}^k \in \mathscr{X}_i$ between i and its associated home BS. The resulting Bayesian power control game in satisfaction form is

$$\mathtt{G}_{\mathrm{BPC}} \triangleq \left(\mathscr{N}, \{[0,p_{\max}]\}_{i\in\mathscr{N}}, \{\mathscr{X}_i\}_{i\in\mathscr{N}}, \{g_i\}_{i\in\mathscr{N}}, F_h\right), \tag{3.25}$$

where $g_i(s_{-i},h_{im}^k) = \{s_i \in \mathscr{S}_i : \underline{\tau} \leq \mathbb{E}_{h_{-i}|h_{im}^k} u_i(s,h)\}$. Independence of types is assumed. The correspondence can be restated as $g_i(s_{-i},h_{im}^k) = \{s_i \in \mathscr{S}_i : s_i(h_{im}^k) \in [0,p_{\max}], \underline{\phi}_i(s_{-i},h_{im}^k) \leq s_i(h_{im}^k) \leq p_{\max}\}$, where $\underline{\phi}_i(s_{-i},h_{im}^k) = \min_{p_i\in[0,p_{\max}]} \{p_i : \underline{\tau} \leq \mathbb{E}_{h_{-i}|h_{im}^k} u_i(s,h)\}$. Then for any realization h, and $s_{-i}(h_{-i}) \leq s'_{-i}(h_{-i})$ implies $\underline{\phi}_i(s_{-i},h_{im}^k) \leq \underline{\phi}_i(s'_{-i},h_{im}^k)$. Thus, by Prop. 5, Algorithm 3.1 converges to a Bayesian-GSE of $\mathtt{G}_{\mathrm{BPC}}$. The simulation network is similar to that of Section 3.6.2. Since a Bayesian strategy s_i must dictate an action for each type a for numerical tractability a discrete channel model is considered. The channel power gains are equiprobably distributed in two levels $\{0.25, 0.75\}$. Assuming indoor deployment wall penetration loss (WPL) is considered. The convergence of expected utility as Algorithm 3.1 converges to a pure-strategy Bayesian-GSE is shown in Fig. 3.6.

3.7 Conclusion

This article presents the novel generalized satisfaction equilibrium (GSE) for games in satisfaction-form. In a satisfaction-form game the agents attempt to satisfy a required service level rather than maximize their utility and thus, they behave as bounded rational agents. A GSE is a strategy profile from which the unsatisfied agents are unable to unilaterally deviate to achieve satisfaction. An important GSE is when the unsatisfied agents pose the least resistance to the satisfied agents and this is called an admission control problem. The article presents the relation of the GSE to the Nash equilibrium (NE). It also presents results for the existence of

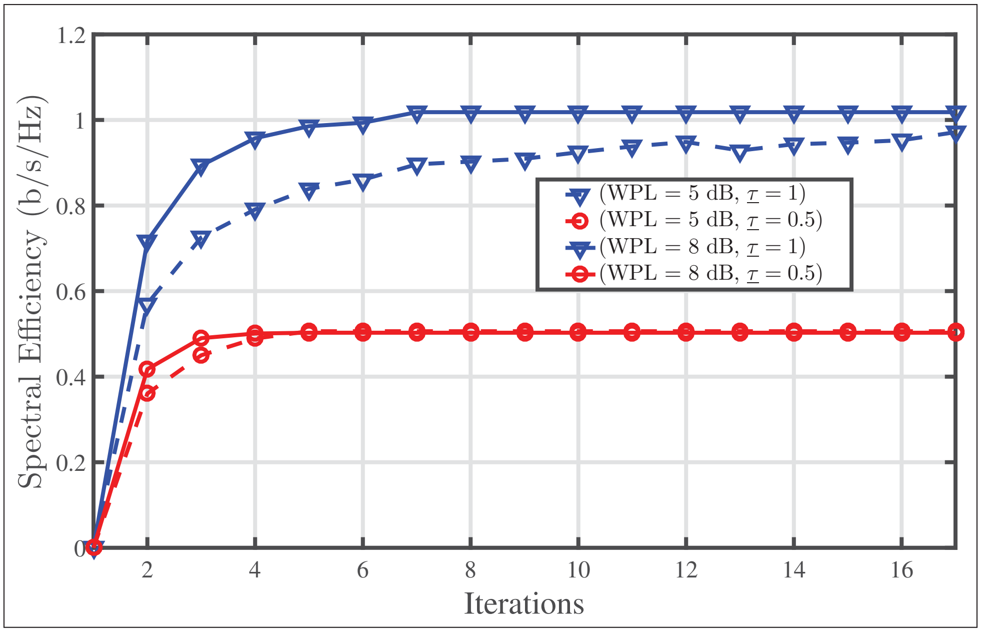

Figure 3.6 The behavior of expected utility of a single agent as Algorithm 1 converges to an equilibrium power level in $\mathtt{G}_{\mathrm{BPC}}$ for different thresholds and WPLs.

GSEs for special classes of games and offers counterexamples in the general case. The GSE bridges the constraint satisfaction problems and the games in satisfaction-form as it is shown that the two problems can be transformed to each other. Finding a pure-strategy GSE is shown to be NP-hard. Sufficient conditions for the convergence of the satisfaction-response dynamics are derived. The incomplete information case is considered under Bayesian-GSEs. To demonstrate the applicability of the GSE, many standard wireless problems are solved and compared in performance against the NE. It is our understanding that GSEs possess immense potential for self-organization in heterogeneous networks.

CHAPTER 4

VERIFICATION MECHANISMS FOR SELF-ORGANIZATION OF HETEROGENEOUS NETWORKS

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4.1 Abstract

This paper introduces *verification mechanisms* for dynamic self-organization of wireless access networks. Current mechanisms in these networks mostly rely on quasi-linear utility transfer through monetary exchanges, as in VCG auctions. In tying-up pricing to resource allocation, the operator can no longer provide flexible pricing schemes, e.g., flat rates, to the clients. Moreover, these mechanisms require additional signaling to exchange prices and it has been shown that the allocation policies that can be truthfully implemented are limited. In contrast to an auction of *objets d'art*, the wireless environment provides the opportunity to verify certain private information (*types*), such as error rate, location, and application class, by observation of the control messages, channel sensing, or by deep-packet inspection. This verification capability can be used to threaten false reports with backhaul throttling. By exploiting these peculiarities, this paper proposes a novel mechanism design framework that also accounts for the possibility of errors in the verification. In addition, the paper also looks into the problem of the feasibility of incentive compatibility constraints and proposes a relaxed implementation of resource allocation policies. In the proposed dynamic mechanism, the agents follow a Q-learning algorithm and learn the truthful strategy over time. Implementations of popular scheduling algorithms in verification mechanisms are demonstrated. By removing monetary exchanges and adapting

the penalties to exploit the wireless environment, this paper demonstrates the feasibility and the necessity of a new theory of mechanism design for wireless access networks.

4.2 Introduction

Future mobile networks are expected to contain a large number of small-cells. The deployment and availability, at least partially, of these cells are at the discretion of the users. Therefore, scalable, dynamic, and distributed resource allocation algorithms that employ the knowledge of the local environment of the nodes are required. Aforesaid algorithms are studied under the domain of self-organizing networks (SONs) (Hwang *et al.*, 2013; Andrews *et al.*, 2014; Xu *et al.*, 2015). One tool in this trade is *mechanism design,* also known as reverse game theory. In the mechanism design problem, each agent possesses private information, called its *type* and the mechanism has a resource allocation rule, called the *social choice* function, that depends on these types. The agents strategically report their types in order to obtain a preferred *allocation* (also called an *outcome*), thus behaving as in a noncooperative game. The Gibbard-Satterthwaite theorem (Reny, 2001), demonstrates that when agents report their preference orders, under mild conditions, only dictatorial social choice functions can be implemented in truthful *dominant strategies*. For agents with real-valued utility functions, this problem can be circumvented through utility transfer by means of monetary exchanges between the agents and the mechanism (Nisan *et al.*, 2007). Then, the utility of an agent is the difference between its *valuation* of the allocation in monetary units and the amount of money paid to the mechanism. These utility functions are known as *quasi-linear preferences* (Nisan *et al.*, 2007). The key problem is to set the prices so that it is an equilibrium for the agents to report the true types. The Vickrey–Clarke–Groves (VCG) (Vickrey, 1961; Clarke, 1971; Groves, 1973) mechanisms compute the payments that maximize *social welfare,* which is the aggregate of valuations of the agents. Money is extremely versatile incentive or a punishment. It can be transferred to-and-from and independently among agents. Moreover, in economic settings, an agent's valuation of an allocation is also in units of money e.g., treasury bill auctions. However, transfer of money is not always possible in other settings, such as elections, in which money transfer is tantamount to bribery (Faliszewski *et al.*, 2009), or allocating resources among internal teams of a company (Cole *et al.*, 2013), or yet again, allocating the internet bandwidth (Dhangwatnotai, 2012).

4.2.1 State of the Art

The majority of game theoretic and mechanism design research for wireless mobile networks assume the possibility of unrestricted monetary transfer. Thus, these works directly appropriate the setting of the economic networks with quasi-linear preferences. In these game-theoretic solutions, the mobile agent, also called the user equipment (UE), pays for the transmitted power and interference. In the mechanism design solutions they pay the marginal contribution as in the VCG auction theory (Saraydar *et al.*, 2002; Huang *et al.*, 2008; Kang *et al.*, 2012b; Xu *et al.*, 2013a; Zhu *et al.*, 2014; Khaledi & Abouzeid, 2015). Appropriation of economic mechanisms into wireless mobile networks enhances the research only if it is clearly confirmed that the underlying assumptions of those mechanisms hold in these networks as well. Auction theory has been successfully used in initial spectrum allocation to operators in many countries (Fox & Bajari, 2013; Cramton, 2013). In this case, since the operators are engaged in a game of generating monetary profits, to them the spectrum blocks have clear monetary values. However, the question remains if mechanisms with payments are the appropriate solution to the distributed dynamic resource allocation problem in wireless networks. This paper argues that they are not and proposes a more realistic alternative that makes use of the physical properties of wireless networks. In a general wireless network setting, the valuation of an allocation is measured in units of data rate, error rate, and/or delay (Tse & Viswanath, 2005). Our first observation is that these units do not possess agreed-upon conversion coefficients into monetary units or vice versa. This leads to the use of arbitrary conversion coefficients (Huang *et al.*, 2008; Khaledi & Abouzeid, 2015). The second observation is that wireless standards decouple pricing from real-time network control. This decoupling is fundamental to the layered architecture of the network design. It also separates short-term resource allocation from the long-term marketing and business processes. This separation allows the operators to offer flexible and simplified pricing schemes that are independent of the dynamic nature of the network. As a result flat pricing is often observed, which is considered one of the key contributors to the popularity of mobile data services (Mcqueen, 2009). Third, the popular VCG pricing mechanisms cannot implement general social choice functions. Specifically, the Roberts' theorem (Nisan *et al.*, 2007), under mild conditions, restricts the implementable social choice functions to affine combinations of agent valuations. This explains why most past works are limited to maximizing the social welfare (Xu *et al.*, 2013a; Khaledi & Abouzeid, 2015). In contrast, resource allocation in wireless networks requires to implement an array of allocation policies varying from simple round-robin or random allocation to more complicated fairness policies and service level agreements (SLAs).

Wireless access networks do not naturally possess a versatile medium of utility transfer similar to money in economic networks (Hartline & Roughgarden, 2008). Limited utility transfer between adjacent agents is possible through relaying radio signals. However, relaying between arbitrary agents in multihop transmission systems is a complex problem and it is difficult to implement and enforce (Xie & Kumar, 2004; Yang *et al.*, 2016). In infrastructure based networks (as opposed to ad-hoc networks), rate throttling in the backhaul can replace payments as a punishment. These are called *money burning* mechanisms where the name alludes to mechanisms that can ruin a portion of the agents' money (which corresponds to rate throttling in our case) but cannot collect nor transfer among the agents. These money burning mechanisms manage to maintain the quasi-linearity, where the utility is the difference between the transmitted rate and the throttled rate (Hartline & Roughgarden, 2008). The major disadvantages are that the throttled rate does not add value to the network operator (unlike collecting payments) nor to the other agents. In addition, social welfare maximization (which corresponds to the maximization of transmitted sum rate) is not achievable. Instead, these rate throttling mechanisms maximize the sum residual rate (the difference between the transmit and throttled rates) (Hartline & Roughgarden, 2008). A mechanism without money that allocates a fixed amount of rate is proposed in (Ko & Wei, 2011). It implements several fairness properties. However, this mechanism is single stage and the proposed setting is not sufficiently rich to consider other allocation rules. Replacing money with a commonly available identical value resource is discussed in (Cavallo, 2014). However, such a common resource has not yet been proposed for wireless networks. In (Angel *et al.*, 2012), a truthful single-stage algorithm without monetary transfer is proposed for the problem of makespan. Yet, it is not possible to know the task duration in most of wireless applications before the end of resources utilization e.g., voice calls. Moreover, information such as channel quality, which *cannot be derived* from task duration, can be more important to the resource allocation decision and thus cannot be modeled by this mechanism.

The mechanism design problem is to know the true type profile of the agents. Monetary transfer and money burning are means to provide incentives/punishments to the agents so that they reveal their true types. However, in these mechanisms, the prices and the resource allocation are computed simultaneously in one shot. Thus, they completely ignore the information revealed by the *environment* during the resource usage. Observing the environment after the allocation can help to verify the truthfulness of reported types (Nisan & Ronen, 1999). Then, in turn, this information can be used to punish false types. This is a two stage process. The punishment can be a hindrance to use the allocated resource or a retraction of the resource. Else it can even be a payment, though this is not the interest of this paper. Thus, the agents know that the mechanism has the capability to verify and punish and this knowledge acts as an incentive to reveal the true types in the allocation stage. In (Nisan & Ronen, 1999), verification with payments is used in the scheduling problem. In (Ben-Porath *et al.*, 2014), verification with retraction is considered for single good allocation without payments. A concise survey of verification mechanisms can be found in (Fotakis & Zampetakis, 2015). Verification mechanisms are different from those based on reputation, which have no capability to directly verify and instead rely on feedback information that is obtained from other agents (Jurca, 2007). This paper employs the terms agent, user, and UE interchangeably.

4.2.2 Contributions

The infrastructure nodes in wireless networks, such as base stations (BSs) and routers, are capable of performing one or more tasks among channel sensing, error detection, localization, and traffic analysis. Thus, after the access network resources are allocated and during the utilization by the UEs the truthfulness of certain types can be verified by probing various properties of the channel, protocol headers, and traffic. Some examples follow. First let us consider a time slotted and frequency orthogonal downlink, such as the LTE-A standard, which employs orthogonal frequency division multiple access (OFDMA). Each UE reports its channel quality indicator (CQI) to the BS. The CQI is based on the signal to interference plus noise ratio (SINR) and it indicates to the BS which modulation and coding schemes to use in order to achieve a predetermined block error rate (BLER) (Kawser *et al.*, 2012; Lopez-Perez *et al.*, 2014). The BS performs resource allocation based on the CQIs of the serving UEs. In LTE-A the CQI reporting is standardized and the UEs passively comply. However, in a self-organizing network, which is the domain of this paper, a UE acts as a rational agent and reports its type to maximize the expected utility. If a user provides a higher CQI than the actual, then the BLER can be estimated at the BS by the ACK/NACK error control messages of the hybrid automatic repeat request (HARQ) process and the false report is thus exposed. As another example of verification, let us suppose the mechanism allocates resources based on the application types of the UEs. During transmission, deep packet inspection (DPI) can be used to verify the reported application type (Deri *et al.*, 2014). Yet another example is the location, where the truthfulness of the location report of a UE can be verified through triangulation (Li *et al.*, 2015). Finally, databases that store SLAs can be accessed to verify the reported quality of service (QoS) demands against the agreements.

Verification alone cannot incentivize the agents to report truthfully. The mechanism requires the capability to punish if a false type is detected. Without punishments, verification cannot enforce truthfulness. Due to reasons presented in the previous section, this paper does not consider payments as a means of punishment. Instead, it considers punishing the agent by blocking its backhaul rate. It is important to note that the blocking in proposed here is a result of failing a test for truthfulness. Thus, it is entirely different from money burning mechanisms, where rate throttling is required *even at the truthful equilibrium* (Hartline & Roughgarden, 2008). That is, in money burning mechanisms rate throttling simply replaces positive payments made by the agent. The combination of the verification procedure and the punishment procedure is called the *verification mechanism.* These it is seen that these mechanisms model the capabilities available in a wireless network environment better than the mechanisms with payments such as auctions.

Verification procedures are prone to errors. Therefore, any realistic verification mechanism has to consider the possibility of an imperfect verification procedure, where a true type may be verified as false or a false type verified as true. In mechanism design, a direct-reporting mechanism is said to implement an allocation policy (social choice) if truthful reporting is an equilibrium of the game induced by the mechanism (Nisan *et al.*, 2007). Such mechanisms are called direct truthful or *incentive compatible* (IC). However, due to imperfect verification, certain resource allocation policies may not be truthfully implemented by a given verification procedure. Therefore, this paper takes a more practical approach and considers the implementation of policies with a high probability of truthfulness. The main contributions of this paper can be summarized as follows:

a. A novel mechanism design framework for wireless networks is proposed based on imperfect verification of agent types and threat of backhaul throttling.

b. To accommodate erroneous verification, the paper proposes the heuristic implementation of policies with a high probability of truthfulness at an equilibrium.

c. The novel oblivious learning equilibrium is proposed for the dynamic verification mechanisms. The agents learn the equilibrium strategy through observing the local rewards.

d. Numerical results are presented for the implementation of widely used resource scheduling policies such as proportional-fair, round-robin, and sum-rate maximization.

It is also demonstrated that the implementability of an allocation policy in a verification mechanism has a direct relation to fairness afforded to the agents by the resource allocation rule. It is our hope that these results would encourage a shift from mechanisms with payments, such as variations of VCG, to verification mechanisms as the basis for distributed protocol design in infrastructure-based wireless networks such as the upcoming 5G standard. In addition, since the allocation rules implementable by weighted-VCG are constrained to affine combinations of utilities (Nisan *et al.*, 2007), it is important to highlight that the proposed verification mechanisms can implement a wider range of allocation rules required by a large scale wireless access network with a high probability of truthfulness.

The rest of the paper is organized as follows. Section 4.3 presents the system model. Section 4.4 and Section 4.5 develop the theory of the single stage and the dynamic verification mechanisms respectively. Section 4.6 presents numerical results from Monte Carlo experiments, and Section 5.7 concludes the paper.

4.2.3 Key Notation

The cardinality of a finite set $\mathscr{N}$ is denoted by the corresponding uppercase letter e.g., $|\mathscr{N}| = N$. For any class of sets $\{\mathscr{S}_i : \forall i \in \mathscr{N}\}$, where $\mathscr{N}$ is a finite index set, the Cartesian product is denoted by $\mathscr{S} \triangleq \times_{i\in\mathscr{N}} \mathscr{S}_i$, the Cartesian product except $\mathscr{S}_i$, by $\mathscr{S}_{-i} \triangleq \mathscr{S}_1 \times \cdots \times \mathscr{S}_{i-1} \times \mathscr{S}_{i+1} \times \ldots \mathscr{S}_N$, and their elements by $s \in \mathscr{S}$ and $s_{-i} \in \mathscr{S}_{-i}$ respectively. Other notations are introduced when they are first encountered.

4.3 System Model

The mechanism design problem of this paper is considered in the context of a heterogeneous small-cell network (HetSNet) that consists of a set of BSs $\mathscr{M}$ that are randomly deployed in a densely populated area serving a set $\mathscr{N}$ of UEs (Hwang *et al.*, 2013). This network model is depicted in Fig. 1.1. The downlink multiple access scheme at a BS is frequency division, similar to OFDMA downlink of the LTE-A, and all BSs share the subchannels with a reuse factor of unity. Let $\mathscr{K} \triangleq \{1, \ldots, K\}$ denote the finite set of subchannels. It is assumed that the BS association problem has been solved, thus presently each active agent (UE) is served by one BS, which is called its home BS. The UEs associated with BS $b \in \mathscr{M}$ are denoted by the set $\mathscr{N}_b$. In addition, uniform power allocation over all subcarriers is assumed (Lopez-Perez *et al.*, 2014). Thus, the key remaining problem is the scheduling of subchannels to the agents. These assumptions are made in order to simplify the notation and also to keep the emphasis on the novel mechanism design framework that is developed. Later in the paper the power allocation assumption is relaxed and it is shown that the proposed mechanisms can solve the larger problem of joint subchannel and power allocation. The UEs possess private information called types. The finite type set of UE $i \in \mathscr{N}$ is denoted by Θ_i. The joint type space of all UEs is denoted by Θ, which is defined as; $\Theta \triangleq \times_{i\in\mathscr{N}}\Theta_i$. The joint type distribution over Θ is denoted by F. A single-stage mechanism defines two components. It defines a message set for each agent. Then it defines an allocation rule, denoted by a. The rule a takes as its independent variable a vector of messages sent by the agents and outputs a particular resource allocation. In defining these two elements the mechanism induces a noncooperative Bayesian game among the agents, where the pure strategies, also called actions, are the messages (Nisan *et al.*, 2007).

At the beginning, each agent observes its private type realization according to F and then chooses a message that it reports to the central mechanism. The mechanism observes the messages of the agents and decides the outcome according to the allocation rule a. The set of all possible outcomes is denoted by $\mathscr{O}$. A mechanism is called direct when the message set of each agent is identical to its type set, and then the allocation rule is a mapping; $a : \Theta \to \mathscr{O}$. (Nisan *et al.*, 2007). A direct mechanism is said to be truthful if reporting the true type is an equilibrium of the induced game. The true type of agent i is denoted by $\theta_i \in \Theta_i$ and the reported type by $\hat{\theta}_i \in \Theta_i$. With a slight abuse of notation the reporting strategy is also denoted by the same notation; $\hat{\theta}_i : \Theta_i \to \Theta_i$. Then, the utility function of agent i is given by $u_i(a(\hat{\theta}), \theta_i)$, where $\hat{\theta} \in \Theta$ is the profile of reported types of all agents. In order to implement a given allocation rule a, the problem is to set the right incentives such that all agents find it mutually optimal to report their true private information while the central mechanism follows the rule a. This mutual optimality is defined by an equilibrium so that no agent can deviate from their reporting strategy and obtain strictly better utility. It is customary to consider a single global allocation rule a. However, since this paper is interested in a self-organizing solution, each BS can possess its own allocation rule, which could be a distributed implementation of a global rule or simply a cell-specific rule selected by the owner of the small-cell. When the outcomes and the allocation rule are specific to each BS they are denoted by $\mathscr{O}_b$ and a_b respectively, where $b \in \mathscr{M}$. Thus, the allocation rule at BS b is; $a_b : \times_{i\in\mathscr{N}_b}\Theta_i \to \mathscr{O}_b$. An agent $i \in \mathscr{N}$ assigns a value $v_i(o,\theta_i)$ for the outcome $o \in \mathscr{O}$. It is assumed that, $v_i(o,\theta_i) \geq 0$ and that it is bounded $\forall o_b \in \mathscr{O}_b$, $\forall \theta_i \in \Theta_i$. The assumption of non-negative values is without a loss of generality, since negative values can be shifted to positive values without affecting the equilibrium strategy (Nisan *et al.*, 2007).

Table 4.1 Notation of the System M odel

BSs	$\mathscr{M} \triangleq \{1,\ldots,b,\ldots,M\}$
Agents of BS $b \in \mathscr{M}$	$\mathscr{N}_b \triangleq \{1,\ldots,N_b\}$
Set of all agents	$\mathscr{N} \triangleq \{\mathscr{N}_1,\ldots\mathscr{N}_M\}$
SCs	$\mathscr{K} \triangleq \{1,\ldots,k,\ldots,K\}$
Type set of agent $i \in \mathscr{N}$	$\Theta_i \ni \theta_i$
Set of outcomes	$\mathscr{O}$
Allocation rule	a
Valuation of $i \in \mathscr{N}$	$v_i(a,\theta_i) \in \mathbb{R}_{\geq 0}$
Blocked state of $i \in \mathscr{N}$	d_i
Utility of agent $i \in \mathscr{N}$	$u_i(a,\theta_i,d_i) \in \mathbb{R}_{\geq 0}$

Data traffic to and from all agents passes through their respective home BSs. Therefore, a BS has full control over the achievable rates of the agents served by it. If the backhaul is blocked for a given agent, then that agent obtains a zero rate. It is considered that a zero backhaul rate has a utility that is identical to the lowest valuation, which is zero. Since a blocked backhaul is equivalent to not obtaining a subchannel, this assumption is justified for non-malicious agents. This assumption is emphasized below.

Assumption 1: For any agent $i \in \mathscr{N}$, a zero backhaul rate provides a utility equal to the lowest valuation v_i of that agent over any type or outcome.

The blocked state of the backhaul of agent i is denoted by the Boolean variable d_i, which takes the value zero when blocked and one otherwise. By the above assumptions, the utility function of agent i is given by (5.1).

$$u_i(a,\theta_i,d_i) = \begin{cases} v_i(a,\theta_i), & d_i = 1 \\ 0, & d_i = 0. \end{cases} \tag{4.1}$$

This system model is summarized in Table 4.1.

4.4 Single Stage Verification Mechanism

The single-stage verification mechanism is concerned with one-time allocation of the resources. The scheduling problem in wireless access networks is dynamic multiperiod in its nature and it is considered in the next section. Therefore, the single-stage mechanism presented in this section is mostly intended as a springboard to the infinite horizon mechanisms of the following section. The agents report types to their home BSs. The BSs perform the subchannel allocation according to the rule, with the reported types as the inputs and then starts transmission to those agents who received a subchannel. During the transmission, the BSs execute the verification procedure to estimate the veracity of the reported types of their associated agents. Notice that verification is applicable only to the UEs that received a subchannel. The verification procedure depends on the type being verified. For instance, if the type represents the application class, then DPI may be used. On the other hand, if the type is the SINR class or CQI, then the ACK/NACK messages can be employed to estimate the BLER, which relates to the SINR or CQI.

In order to model a more realistic network scenario, the verification procedure is assumed to be imperfect. To be more precise, an imperfect verification procedure can be modeled as a hypothesis test in which the two hypothesis are as stated below.

***Null hypothesis*:** the agent is truthful, i.e., $\hat{\theta}_i = \theta_i$

***Alternative hypothesis*:** the agent is not truthful, i.e., $\hat{\theta}_i \neq \theta_i$

The imperfect verification procedure defined by the above two hypothesis has two kinds of error probabilities. Let $\mathrm{err}_{\mathrm{I}}$ denote the probability of a *type I error*, which is the rejection of the null hypothesis when it is true. And $\mathrm{err}_{\mathrm{II}}$ denotes *type II error* probability, which is the acceptance of the null hypothesis when it is false. These probabilities can possibly depend on the agent and the reported type, but for simplicity, it is assumed that these error probabilities are constant for a given verification procedure. Thus, the imperfect verification may mistakenly block a truthful agent with probability $\mathrm{err}_{\mathrm{I}}$ and may fail to block a non-truthful agent with probability $\mathrm{err}_{\mathrm{II}}$. In practice, a viable verification procedure must have low $\mathrm{err}_{\mathrm{I}}$ and $\mathrm{err}_{\mathrm{II}}$. The verification procedure is said to be perfect when these error probabilities are 0. The verification mechanism is denoted by the tuple;

$$\mathtt{M} = < a, \mathrm{err}_{\mathrm{I}}, \mathrm{err}_{\mathrm{II}} > . \tag{4.2}$$

If the verification procedure is perfect, then the design of a truthful mechanism is trivial. This is stated in the below remark.

Remark 2. By Assumption 1, the verification mechanism with $\mathrm{err}_{\mathrm{I}} = 0$ and $\mathrm{err}_{\mathrm{II}} = 0$ is dominant strategy IC for any given allocation rule a. The reason being, with zero verification error probability, it is a weakly dominant strategy to report the true type. Any false report is caught with probability one and thus, results in a zero utility, which is the lowest.

A strong form of implementing an allocation rule a by a mechanism is when it is a dominant strategy for agents to report truthfully regardless of the reporting strategy of others (Nisan *et al.*, 2007). Thus, dominant strategy equilibria are said to be strategy free, i.e., the truth is a best response whichever the strategies employed by other players (Nisan *et al.*, 2007). A slightly weaker implementation is Bayesian Nash equilibrium. In a truthful Bayesian Nash equilibrium, revealing the true type is a best response only if other players also reveal their true types (Nisan *et al.*, 2007). An imperfect verification mechanism is *dominant-strategy incentive compatible*, i.e., reporting the true type is a dominant strategy, if $\forall i \in \mathcal{N}$, $\forall \theta_i \in \Theta_i$, and $\forall \hat{\theta} \in \Theta$,

$$(1-\mathrm{err}_{\mathrm{I}})v_i(a(\theta_i,\hat{\theta}_{-i}),\theta_i) \geq \mathrm{err}_{\mathrm{II}}v_i(a(\hat{\theta}_i,\hat{\theta}_{-i}),\theta_i). \tag{4.3}$$

Notice that in (4.3) expectations are not taken over the types of other agents, since dominant strategy IC mechanism requires that truthfulness is a best response no matter what the reporting strategy of the other players are. On the other hand an imperfect verification mechanism is *Bayesian Nash incentive compatible*, if $\forall i \in \mathscr{N}$, $\forall \hat{\theta}_i \in \Theta_i$,

$$(1-\mathrm{err}_{\mathrm{I}})\mathbb{E}_{\theta_{-i}}v_i(a(\theta_i,\theta_{-i}),\theta_i) \geq \mathrm{err}_{\mathrm{II}}\mathbb{E}_{\theta_{-i}}v_i(a(\hat{\theta}_i,\theta_{-i}),\theta_i). \tag{4.4}$$

A mechanism is said to be *individually rational* if no agent is worst off by taking part in the game. The verification mechanism can achieve individual rationality (Nisan *et al.*, 2007), since not taking part in the game, i.e., not reporting a type, can be countered by setting backhaul rate to zero. Then, those agents that do not report a type obtain zero utilities and thus, voluntary participation is achieved.

At a truthful equilibrium, defined by either (4.3) or (4.4), the verification mechanism causes a loss of traffic of truthful agents due to type I errors. However, as pointed out in the following proposition this loss is only due to the imperfections of the verification procedure.

Proposition 7. *In the verification mechanism (4.2), if type I error probability is zero, then at a truthful equilibrium no traffic is lost.*

Proof. At the truthful equilibrium all agents report the true type. Then, an agent is blocked with probability $\mathrm{err}_{\mathrm{I}}$. If this type I error probability is zero, then the mechanism does not block traffic of truthful reports with probability 1. Thus, with probability 1 all backhaul traffic passes through. □

The implication of Prop. 7 is that a verification mechanism is not wasteful of bandwidth at the truthful equilibrium if the verification procedure can achieve low type I error probability.

That is, as the type I error probability approaches zero, the loss of traffic of truthful reports vanishes. This is in contrast to money burning mechanisms, where the burned rate is nonzero at the truthful equilibrium and rate loss essentially replaces payments (Hartline & Roughgarden, 2008).

4.4.1 Implementability of Social Choice

As discussed earlier all allocation rules can be implemented in dominant strategies if the verification procedure is perfect. However, when the verification procedure is imperfect with nonzero $\mathrm{err}_{\mathrm{I}}$ and $\mathrm{err}_{\mathrm{II}}$, certain allocation rules cannot satisfy the constraints of (4.3). This implies that a dominant strategy IC verification mechanism does not exist for those rules. The following is a simple example scenario. Consider the problem of allocating a single channel at a BS which serves two agents. One agent is near the BS and has line of sight and the other is a cell edge agent. The edge agent has two SINR states, identified as medium and bad each with 0.5 probability. The near agent has the two states good and medium also with 0.5 probability. The BS wants to maximize the rate and thus, the allocation rule is to assign the channel to the agent with best SINR, breaking ties with a fair coin toss. When the far agent is in bad state its expected utility of truthful strategy is 0. However, due to nonzero $\mathrm{err}_{\mathrm{II}}$, and the fair coin toss, its expected utility of falsely reporting medium when in fact it is the bad state is higher than zero. Thus, the verification mechanism with the maximum rate allocation rule is not dominant strategy IC under an imperfect verification procedure. When a mechanism is infeasible, it is customary to relax the equilibrium, i.e., replace the dominant strategy IC constraints with the less strict Bayesian Nash IC constraints of (4.4) (Nisan *et al.*, 2007). However, this relaxation does not always ensure the existence of an incentive compatible mechanism under the relaxed equilibrium. For instance, it can be verified that in the above example the far agent cannot truthfully report the bad state even in a Bayesian Nash equilibrium.

For any given Bayesian Nash equilibrium $\hat{\theta} \in \Theta$, of a mechanism $\mathtt{M}$ (possibly a non-truthful equilibrium), the probability of truthful reports is given by;

$$\Pr\{\texttt{M} \text{ is truthful}\} = \mathbb{E}_\theta \nVdash_{\{\theta\}}(\hat{\theta}(\theta)). \tag{4.5}$$

Here $\hat{\theta}(\theta)$ is the reported type profile of all agents such that $\hat{\theta}_i(\theta_i) \in \Theta_i$ is the reported type of agent i. Notice that if the equilibrium $\hat{\theta}$ is truthful, then (4.5) evaluates to 1. This paper takes a more practical approach and proposes to consider mechanisms where the probability (4.5) is high but not one. Thus, the mechanisms are no longer bounded by the IC constraints (4.4). A temporal interpretation of the probability (4.5) is that in a repeated game, the mechanism would implement the allocation rule a at $\Pr\{\texttt{M} \text{ is truthful}\}$ fraction of times. When achieving Bayesian Nash incentive compatibility is infeasible this method provides a heuristic implementation of the desired rule.

4.4.2 Mechanisms with Optimizable Verification Error

Thus far the allocation rule a was assumed to be given beforehand. Now suppose the network operator wants to choose the allocation rule a to maximize the expected value of a given objective function $f : \mathscr{O} \to \mathbb{R}$. The design of the verification mechanism can then be written as an optimization program. Given the error probabilities of the verification process, there exists a dominant strategy IC verification mechanism that maximize $\mathbb{E}_\theta f(a(\theta))$ if the problem in (4.6) has a solution.

$$\begin{aligned} &\underset{a}{\text{maximize}} : \mathbb{E}_\theta f(a(\theta)), \\ &\text{subject to} : \text{IC constraints } (4.4). \end{aligned} \tag{4.6}$$

Another assumption that was followed so far in this paper is that the verification procedure is fixed. That is, err_{I} and err_{II} are fixed properties of the mechanism and the only optimizable parameter of (4.6) is the allocation rule a. However, some verification procedures can be optimized. That is the errors err_{I} and err_{II} may be reduced, albeit with the extra cost of implementation and operation. When this is the case, the operator is interested in finding the lowest cost mechanism to implement a given allocation rule. For simplicity let us assume a linear cost model for improving the accuracy of the verification procedure, which is given by $c_1(1-\mathrm{err_I})+c_2(1-\mathrm{err_{II}})$, where $c_1, c_2 \in \mathbb{R}_{>0}$ are the marginal costs. Then a solution to problem (4.7) gives the minimum cost mechanism that implements a given allocation rule a.

$$\begin{aligned} &\underset{\mathrm{err_I},\mathrm{err_{II}}}{\mathrm{minimize}}\, c_1(1-\mathrm{err_I})+c_2(1-\mathrm{err_{II}}), \\ &\mathrm{subject\,to}\, \text{IC constraints (4.4)}, \\ &\qquad\qquad 0 \le \mathrm{err_I}, \mathrm{err_{II}} \le 1. \end{aligned} \tag{4.7}$$

Above problem (4.7) is a linear program. Note that the feasibility of this problem is ensured by Rem. 2: if $\mathrm{err_I}$ and $\mathrm{err_{II}}$ are zero, then for any allocation rule a the mechanism is dominant strategy IC. The number of IC constraints can be fairly large even in a moderately sized network. In some applications given the true type θ_i the agent may obtain a higher utility only if the reported type $\hat{\theta}_i$ satisfies the inequality $\hat{\theta}_i \ge \theta_i$. This structure can be employed to reduce the number of IC constraints. The curse of the number of IC constraints is a well-known limitation in solving for a mechanism as an optimization problem. Ways of exploiting special structure to reduce the number of constraints are studied in (Ben-Porath *et al.*, 2014).

4.5 Dynamic Verification Mechanism

Mechanisms of Section 4.4 address the single-stage allocation problem. This section extends the verification mechanism to the infinite horizon stochastic dynamic setting. A stationary resource allocation policy is denoted by π, which is the dynamic counterpart of the allocation rule a of the single stage problem. The time is divided into equal duration periods similar to the dynamic programming setting (Puterman, 1994). It is assumed that the joint agent types evolve in a Markov fashion, where the transfer probability from type $\theta \in \Theta$ to $\theta' \in \Theta$ is given by $F(\theta', \theta, s)$. At the beginning of a period t, each agent $i \in \mathscr{N}$ is revealed its private true type

θ_{it}. Then, each agent reports a type according to its reporting strategy. In the dynamic setting, the stationary reporting strategy of agent i is denoted by s_i. If it is verified that an agent's reported type is non-truthful, then its backhaul is blocked for $T > 0$ future time periods. These steps are repeated in each period. If the verification procedure requires the complete period to assess the truthfulness, then the T blocking periods may not include the present period. The single period error probabilities are time independent and they are given by $\mathrm{err_I}$ and $\mathrm{err_{II}}$, similar to those of the single stage case. The dynamic verification mechanism is denoted by;

$$\mathtt{M_d} =< \pi, \mathrm{err_I}, \mathrm{err_{II}}, T > . \tag{4.8}$$

In the above-identified stochastic dynamic setting, the natural choice of equilibrium is the Markov perfect equilibrium (MPE) (Shoham & Leyton-Brown, 2009). An MPE is a subgame perfect equilibrium in which the players are restricted to Markov strategies. Strategies that depend only on the current state and ignore the history are called Markov strategies (Shoham & Leyton-Brown, 2009). Let $\mathscr{S}_i$ denote the Markov strategy space of agent $i \in \mathscr{N}$. The dynamic verification mechanism $\mathtt{M_d}$ is said to implement a scheduling policy π in an MPE, if truthful reporting is an MPE of the stochastic game induced by the mechanism. Given that the other players follow the stationary profile s_{-i}, the value function $V_i(\theta : s_{-i})$ of i is given in the recursive form by (4.9), where β is the discount factor and $u_i(\pi(s_i, s_{-i}), \theta_i)$ is the stage payoff/reward.

$$V_i(\theta : s_{-i}) = \max_{s_i \in \mathscr{S}} \big(u_i(\pi(s_i, s_{-i}), \theta_i) + \beta \sum_{\theta'} F(\theta', \theta, s_i, s_{-i}) V_i(\theta' : s_{-i})\big). \tag{4.9}$$

An MPE is defined with respect to the current network state θ, and the global state transition probabilities F. In large networks such information requirements are rarely achievable by individual agents. In addition, the global state space size grows exponentially with the number of players. In order to overcome these information and dimensionality limitations, the *oblivious equilibrium* is proposed in (Weintraub *et al.*, 2010). The oblivious equilibrium takes a mean-field approach by assuming that as the number of agents grows, the perceived system state by a single agent remains constant over time. In the oblivious equilibrium an agent i is restricted to the sub-strategy space $\mathscr{S}_i' \subset \mathscr{S}_i$ where a member strategy $s_i \in \mathscr{S}_i'$ depends only on the current local type θ_i of the agent and a summary statistic of the global state. These are called *oblivious strategies* and the agents who follow those are called *oblivious agents*. In wireless networks, where global state and global state transition probabilities are not common knowledge, the oblivious strategies, in fact, represent the reality. This paper defines a novel equilibrium in terms of oblivious agents. These agents learn their optimal strategies by a multiagent Q-learning algorithm and we call the convergent point of the algorithm the *oblivious learning equilibrium.* These agents do not possess the knowledge of state transition probabilities nor any knowledge of the probabilities $\mathrm{err_I}$ and $\mathrm{err_{II}}$. In order to learn the best strategy, they rely only on the local state, the local reward, and the knowledge of its blocked state.

In a dynamic verification mechanism $\mathtt{M_d}$ with $T > 1$, an agent can experience a 0 reward at time t due to a previous type reporting that it did, more than one period back, in the past. In order to incorporate this past memory into the learning process, this paper presents a slight modification to the single-step Q-learning algorithm by way of a timer. The learning process of the agents is as follows. An oblivious agent that is not blocked, observes its current state θ_i, sends the oblivious report $s_i(\theta_i) \in \Theta_i$, obtains a reward u_i, and updates the value $Q_i(s_i(\theta_i), \theta_i)$ according to (4.10). At any state θ_i, if the agent is blocked it obtains a reward of 0. However, an agent could receive a 0 reward without being blocked, for instance, due to not receiving a resource. Therefore, in order to disambiguate, if the verification procedure blocks an agent, the mechanism informs the blocking to the agent. Then, the agent starts a timer to count from 1 to T. During the periods 1 to $T-1$ the agent continues to update the Q value of the state and action that resulted in the blocking, with a reward of zero. This update rule is given in (4.11). During the learning period, the agents have to both explore and exploit the oblivious strategy space. A number of ways to select strategies have been suggested. This paper considers an ε- greedy method where the agent selects the optimal action $\max_{s_i \in \mathscr{S}'} Q_i(s_i(\theta_i)), \theta_i)$ with probability ε

and selects a random strategy with probability $1-\varepsilon$. This iterative learning procedure is stated in Algorithm 4.1.

$$Q_i(s_{it},\theta_{it}) \leftarrow Q_i(s_{it},\theta_{it}) + \alpha\Big(u_{i,t+1} + \beta \max_{s_i' \in \mathscr{S}'} Q_i(s_i',\theta_{it+1}) - Q_i(s_{it},\theta_{it})\Big). \tag{4.10}$$

$$Q_i(s_{it},\theta_{it}) \leftarrow Q_i(s_{it},\theta_{it}) + \alpha\Big(\beta Q_i(s_{it},\theta_{it}) - Q_i(s_{it},\theta_{it})\Big). \tag{4.11}$$

At the convergence of Algorithm 4.1, the oblivious reporting policy of agent i in state θ_i is given by $s_i^*(\theta_i) = \arg\max_{\theta_i' \in \Theta_i} Q_i(\theta_i',\theta_i)$. The oblivious learning equilibrium is defined as $s^* = (s_i^*)_{i\in\mathscr{N}}$. The mechanism $\mathtt{M}_\mathrm{d} =< \pi, \mathrm{err}_\mathrm{I}, \mathrm{err}_\mathrm{II}, T >$ is said to implement the policy π in an oblivious learning equilibrium, if $\forall i \in \mathscr{N}$ and $\forall \theta_i \in \Theta_i$, $s_i^*(\theta_i) = \theta_i$. That is at the convergence of Algorithm 4.1 all players report truthfully. This is defined in Def. 8. At the convergence of Algorithm 4.1 define the oblivious value function of agent i at state θ_i as $V_i(\theta_i) = \max_{\theta_i' \in \Theta_i} Q_i(\theta_i',\theta_i)$. Then, for truth to be an oblivious equilibrium, $\forall i \in \mathscr{N}$ and $\forall \theta_i \in \Theta_i$, $Q_i(\theta_i,\theta_i) = V_i(\theta_i)$. If this is satisfied the mechanism is said to be incentive compatible with respect to the allocation policy π. An arbitrary policy π cannot necessarily satisfy these condition for all players and types.

Definition 8. A dynamic verification mechanism $\mathtt{M}_\mathrm{d}$ implements the scheduling policy π if truthful reporting is an oblivious learning equilibrium.

Algorithm 4.1: Dynamic Learning Algorithm

Initialize $t = 0$, $Q_i(s_{i,0}, \theta_{i,0})$, $\theta_{i,0}$, $\theta_{i,1}$, and unblock all agents

Do:

Unblocked agents take action $s_{i,t}$

Mechanism allocates $\pi(s_t)$

Agents with a channel transmit

Mechanism verifies the agents' types

Agents observe their individual rewards $u_{i,t}$

Unblocked agents update (4.10)

Blocked agents update (4.11)

While: convergence criteria is not met

Proposition 8. *In the dynamic verification mechanism (4.8), if type I error probability is zero, then at a truthful oblivious learning equilibrium no traffic is lost.*

Proof. This result is the dynamic counterpart of Prop. 7. At the truthful oblivious equilibrium, all players report the true type. If type I error probability is zero, then the mechanism does not block traffic of truthful reports with probability 1. Thus, all traffic of agents who receive subchannels passes through. □

Similar to the single state mechanism, at the convergence of Algorithm 4.1, one can observe the probability of truthful reporting. The following section presents numerical results for truthfulness for a variety of allocation policies. $\mathscr{A}_i$s

4.6 Numerical Results

This section presents numerical results for the dynamic verification mechanisms of Section 4.5. Let us consider the downlink of a wireless OFDMA HetSNet that consists of a microcell and a number of underlayed small-cells similar to the network depicted in Fig. 1.1. Each cell is served by a single BS (Lopez-Perez *et al.*, 2014). The network nodes are synchronized and the time is divided into frames. Here one frame corresponds to one period of the dynamic mechanism. A full-buffer traffic model is assumed, so that the BSs always have data to transmit to the associated UEs. The full-buffer assumption is only for modeling purposes and can be relaxed if the distributions of the traffic arrival processes are known. The agents are preassigned to the BSs following a user association algorithm. Appropriate cell biasing may be used during the user association to offload the UEs from congested cells to neighboring cells. The private information of an agent is derived from its received SINR. First, the SINR is estimated by pilot symbols placed on the subchannels and then the SINR is discretized into intervals, which form the private information. The agent reports the SINR interval number to the associated BS. In the LTE and LTE-A systems, it is achieved by mapping the SINR class into a CQI value, such that higher CQI corresponds to better received SINR (Kawser *et al.*, 2012). At the BS the downlink modulation and coding is chosen to match the CQI of the agent such that a certain required average BLER is achieved. For LTE-A the average BLER requirement varies from 2% to 10% (Kawser *et al.*, 2012). The private information is assumed to stay constant during one period due to a block fading channel model. Between periods the channel realizations are independent and identically distributed (i.i.d.). Following a satisfaction model (Goonewardena *et al.*, 2017), if an agent receives a subchannel its value is 1 else the value is 0. At most one subchannel is assigned to an agent.

The verification process is achieved by monitoring the HARQ process at the BSs. In HARQ, when the UE fails to decode a block it informs the BS through a NAK message and the BS retransmits that block (Kawser *et al.*, 2012). Thus, the BS has information of how many blocks were retransmitted during the period, which can be used to estimate the realized BLER of that period. The imperfection of the verification procedure arises from the SINR to CQI mapping. The mapping is designed to ensure the BLER in average. However, during each frame duration the realized BLER is different from the average due to the continuous nature of the stochastic channel processes. Thus, there is a nonzero probability of type I and type II errors. The BS allocates the subchannels according to the scheduling policy assigned to it by the operator.

The goal of the verification mechanism is to implement arbitrary policies. This experiment considers the following allocation policies;

a. random allocation;

b. greedy sum value maximization;

c. weighted greedy sum value maximization;

d. proportional fair allocation;

e. round robin.

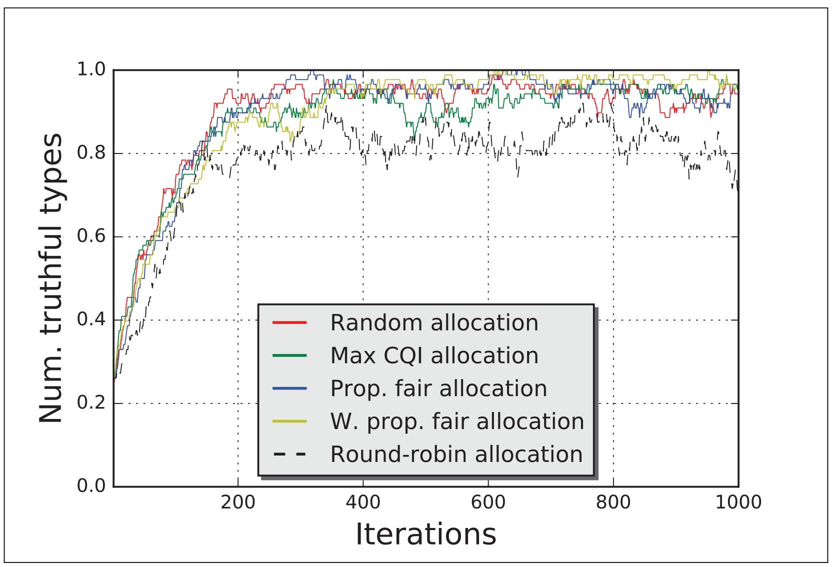

Figure 4.1 Fraction of truthful reports vs. Iteration. Convergence of the verification mechanism to near truthfulness for various scheduling policies.

Let ρ_i represent the average number of subchannels assigned to agent $i \in \mathcal{N}$ in the past. Proportional fair allocates the K subchannels to the first K agents with highest $\frac{1}{\rho_i}$.

The HetSNet of the experiment consists of 1 urban macrocell of radius 500 m and 4 picocells each with a serving radius of $20m$ that are deployed uniformly at random in the same

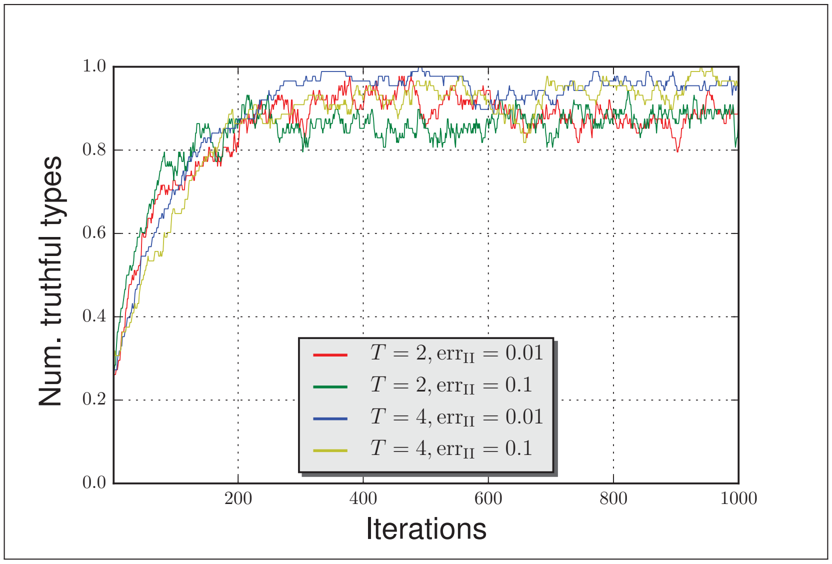

Figure 4.2 Fraction of truthful reports vs. Iteration. The joint influence of blocking time T and $\mathrm{err_{II}}$ on convergence.

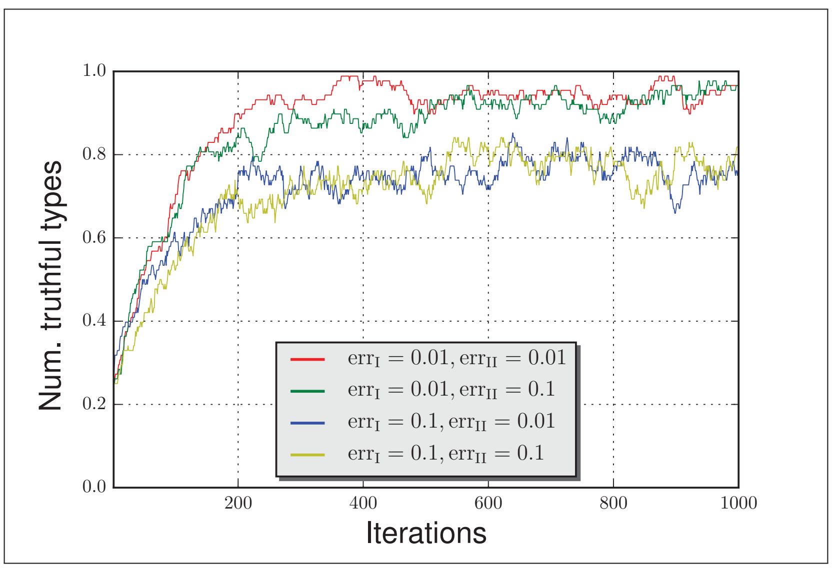

Figure 4.3 Fraction of truthful reports vs. Iteration. The joint influence of $\mathrm{err_{I}}$ and $\mathrm{err_{II}}$ on convergence.

coverage area as the macrocell, thus forming a two-tier network. The SINR range of an agent is discretized into 4 CQIs. The effect of verification probability and penalty duration is explored in the following numerical results. Unless otherwise stated the default values are

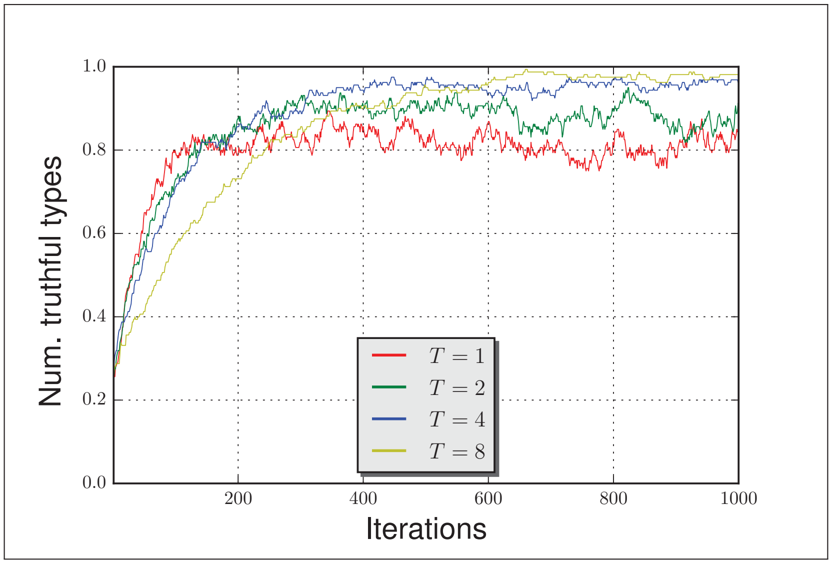

Figure 4.4 Fraction of truthful reports vs. Iteration. The influence of blocking time T on convergence.

$\mathrm{err}_{\mathrm{I}} = \mathrm{err}_{\mathrm{II}} = 0.01$, $T = 4$, $\alpha = 0.6$, $\beta = 0.6$. The experiments record the fraction of truthful types as the learning algorithm proceeds.

The policies are implemented per BS. That is each BS $b \in \mathscr{M}$ acts independently to implement the policy a_b. Fig. 4.1 shows the convergence of the reporting strategies for the above-mentioned scheduling policies. It is observed that many of these policies achieve a high fraction of truthfulness as the Algorithm 4.1 converges. Fig. 4.2 shows the convergence of the fraction of truthful reports for different values of blocking duration T and $\mathrm{err}_{\mathrm{II}}$. As T increases and $\mathrm{err}_{\mathrm{II}}$ reduces a larger fraction of reports are truthful as the learning process converges. The blue and yellow curves can be seen very close to 1, which is the truthful oblivious learning equilibrium. Fig. 4.3 shows the convergence of the fraction of truthful reports for different values $\mathrm{err}_{\mathrm{I}}$ and $\mathrm{err}_{\mathrm{II}}$. As expected, lower error probabilities provide better truthfulness. Fig. 4.4 shows the influence of T on truthfulness as other parameters are at their default values. Notice that larger T values result in better performance in terms of truthfulness. However, the initial learning rate is slower. This is expected, since a larger blocking duration decreases the opportunities to explore the strategy space and thus slows down the learning process.

Thus far this paper only considered the problem of subchannel allocation, assuming that power allocation is uniform over the subchannels. Here we briefly look at how to design a verification mechanism to solve the joint subchannel and power allocation problem when user types are given by CQI. Discrete finite set of power levels are considered $\mathscr{P}$. In the joint problem, the set of outcomes $\mathscr{O}$ contains all possible channel and power allocations over which the scheduling policy operates. The main difference is in the type reports by the agents. The received SINR at the agent is a function of the transmit power, hence so is the CQI. One possibility is that the agents report the CQI for each transmit power level in $\mathscr{P}$. However, this generates $\mid \mathscr{P} \mid$ times more information exchanges than in the uniform power case. One way around this is to report the CQI with respect to a base transmit power level. Then, during verification at the BS, the CQIs related to the actual allocated transmit power level can be derived from a table look up (Kawser *et al.*, 2012). In this way, the signaling load between the BSs and agents remains similar to that of fixed power subchannel assignment problem.

4.7 Conclusion

A closer examination of mechanisms with money transfer urgently validates that the reality of the economic networks, for which these mechanisms were designed, do not directly translate into the wireless network environment. Wireless infrastructure based networks are capable of verifying certain user types during operation. This paper proposes and analyzes mechanisms that employ verification and threat of backhaul throttling to implement resource allocation policies. While under mild assumptions perfect verification can truthfully implement any allocation policy, the mechanisms proposed in this article work with imperfect verification and are shown to implement policies with a high probability of truthfulness. For dynamic networks, this paper proposes the *oblivious learning equilibrium* and demonstrates the implementation of scheduling policies with this equilibrium. The main objective of this paper is to demonstrate that verification mechanisms are a promising and more natural alternative to money transfer for distributed self-organization of future wireless networks. Much work remains to be done in designing efficient and low-error verification procedures for different types that are encountered in wireless agents. In addition, theoretical questions on implementable policies and bounds on optimality must be explored.

CHAPTER 5

EXISTENCE OF EQUILIBRIA IN JOINT ADMISSION AND POWER CONTROL FOR INELASTIC TRAFFIC

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5.1 Abstract

This letter considers the problem of admission and discrete power control in the interfering-multiple-access channel, with rate constraints on admitted links. This problem is formulated as a normal-form noncooperative game. The utility function models inelastic demand. An example demonstrates that in the fading channel, in some networks, a pure-strategy equilibrium does not exist with strictly positive probability. Hence, the probability of existence of an equilibrium is analyzed and bounds are computed. To this end the problem of finding the equilibria is transformed into a constraint satisfaction problem. Next the letter considers the game in the incomplete information setting, with compact convex channel power gains. The resulting Bayesian game is proven to possess at least one pure Bayesian Nash equilibrium in on-off threshold strategies. Numerical results are presented to corroborate the findings.

5.2 Introduction

This letter expounds the problem of distributed admission and power control in a game theoretic setting. The admitted links must satisfy a minimum signal-to-interference plus noise ratio (SINR) requirement. For compact and convex power domains, past works have explored algorithms to solve the feasible as well as the over constrained system, by power allocation, admission control, and/or adjustment of the required SINR level (Rasti & Sharafat, 2011; Monemi & Rasti, 2015). However, the discrete power control problem has seen less results, even if in practice most wireless networks standards follow the discrete model. In (Andersin *et al.*, 1998; Wu & Bertsekas, 2001) it is demonstrated that the continuous power control algorithms can lead to oscillations if applied to the discrete problem. A popular subproblem in the discrete model is *on-off* control.

More specifically, this work considers the *discrete power model* for *inelastic traffic* that requires a specific rate. In (Andrews & Dinitz, 2009) a network of transmitters with strict SINR requirements is analyzed for the path-loss SINR model (without small scale fading) with continuous power control. The channel selection game for inelastic traffic in (Southwell *et al.*, 2014) uses the congestion model. On the other hand this letter follows the SINR model with small scale fading. The problem is formulated as a normal-form game (Shoham & Leyton-Brown, 2009). Throughout this letter only *pure strategies* are considered. In the complete information case the game possesses the important feature that at a Nash equilibrium (NE (Shoham & Leyton-Brown, 2009)) the unsatisfied transmitters have zero power, thus the NEs function as solutions to an admission control scheme. In (Perlaza *et al.*, 2012b) a novel representation called the *satisfaction-form* is introduced for noncooperative games in which players only need to achieve a target performance constraint. The solution of a satisfaction-form game is the *satisfaction equilibrium*. It is demonstrated that the normal-form admission and power control game of this letter has a satisfaction-form representation.

This letter makes two major contributions to the noncooperative game of admission and discrete power control for inelastic traffic. As a first contribution the probability of existence of pure strategy Nash equilibria in complete information case is computed for a general fading channel. Results are presented for both interference channel (IC) and interfering-multiple-access channel (IMAC). In the IMAC, each transmitter is assigned to a single receiver and more than one transmitter may have the same receiver whereas in the IC it is a one-to-one assignment (Hong & Luo, 2013). The second contribution is in the incomplete information game, where the existence of Bayesian Nash equilibria in on-off threshold strategies is proven for compact convex channel power gains.

The rest of the letter is organized as follows. Section 5.3 presents the problem formulation along with transformation to CSP. Section 5.4 analyzes the probability of existence of NEs. Section 5.5 solves the Bayesian game. Section 5.6 presents numerical results. Section 5.7 concludes the letter.

5.3 Problem Formulation

Consider the IMAC with flat fading, single antenna nodes, and synchronized transmission. The finite set of transmitters $\mathscr{N}$ has cardinality N. The transmission power of $i \in \mathscr{N}$ is $p_i \in \mathscr{P}_i$, where $\mathscr{P}_i$ is a finite set of power levels *including* 0 and the *maximum power* $\overline{p}_i$. The channel power gain between $j \in \mathscr{N}$ and the destination of i is h_{ji}. The variance of the additive white Gaussian noise (AWGN) is σ^2. Interference power from external sources is r_i, e.g., the interference from overlaying macrocells that are not in the considered system. The channels fading, interference, and noise are independent. Power profile is $p = (p_i)_{i\in\mathscr{N}}$ and the channel vector is $h = (h_{ij})_{i,j\in\mathscr{N}}$. For single user detection the SINR at the receiver output of the destination of i is $\gamma_i(p,h) = \frac{h_{ii}p_i}{\sum_{j\neq i} h_{ji}p_j + r_i + \sigma^2}$. In this Gaussian IMAC the rate requirement is identical to a lower bound on the SINR. The utility of i is (5.1), where $\tau_i > 0$.

$$u_i(p,h) = \begin{cases} 1, & \gamma_i(p,h) \geq \tau_i, \\ 0, & p_i = 0, \\ -1, & \text{otherwise}. \end{cases} \tag{5.1}$$

The resulting finite noncooperative game in normal-form is;

$$\mathtt{G} = \left(\mathscr{N}, \{\mathscr{P}_i\}_{i\in\mathscr{N}}, \{u_i\}_{i\in\mathscr{N}}\right). \tag{5.2}$$

Remark 3. At an NE of (5.2) a player does not obtain a utility -1. If $u_i(p_i, p_{-i}) = -1$, then $p_i = 0$ is a better response.

By Remark 3 the set of NEs of (5.2) forms a subset in the solution space of the problem of selecting a subset of transmitters that satisfy the SINR requirement. The advantage of NEs over other solutions is that the NEs are stable, i.e., the unadmitted transmitters know that they cannot achieve the required threshold even at maximum transmission power. The best response correspondence of i is a set valued mapping $q_i : \mathscr{P}_{-i} \rightrightarrows \mathscr{P}_i$, where $\mathscr{P}_{-i} \triangleq \mathscr{P}_1 \times \cdots \times \mathscr{P}_{i-1} \times \mathscr{P}_{i+1} \times \cdots \times \mathscr{P}_N$. Given $p_{-i} \in \mathscr{P}_{-i}$, $q_i(p_{-i}) \subseteq \mathscr{P}_i$ is the set of strategies that maximizes u_i. Define $q'_i(p_{-i}) \triangleq \{p_i \in \mathscr{P}_i : \gamma_i(p,h) \geq \tau_i\}$. Then from Remark 3 it follows that;

$$q_i(p_{-i}) = \begin{cases} q'_i(p_{-i}) & \text{if } q'_i(p_{-i}) \neq \emptyset, \\ \{0\} & \text{otherwise.} \end{cases} \tag{5.3}$$

Then the problem of finding an NE of (5.2) is identical to the problem of finding a fixed point of $q(p) \triangleq (q_i)_{i \in \mathscr{N}}$ in the lattice $\mathscr{P} \triangleq \mathscr{P}_1 \times \cdots \times \mathscr{P}_N$. This fixed point problem can be solved as a constraint satisfaction problem (CSP). For details of the CSP the reader is referred to (Soni *et al.*, 2007; Shoham & Leyton-Brown, 2009) and references therein. For the purpose of this letter the CSP is defined by $(\{p_i\}_{i \in \mathscr{N}}, \{\mathscr{P}_i\}_{i \in \mathscr{N}}, \{\mathscr{C}_i\}_{i \in \mathscr{N}})$, where $\{p_i\}_{i \in \mathscr{N}}$ is the set of *variables*, $\mathscr{P}_i$ is the finite *domain of variable* p_i, and $\{\mathscr{C}_i\}_{i \in \mathscr{N}}$ is a collection of N *constraints*. Constraint $\mathscr{C}_i$ is an N*-ary relation* on $\mathscr{P}$. An assignment $a \triangleq (p_i, d_i)_{j \in \mathscr{N}}$, is a value $d_i \in \mathscr{P}_i$ given to each variable. Assignment a is said to solve the CSP if $(d_i)_{i \in \mathscr{N}}$ is a tuple in $\mathscr{C}_i$ $\forall i \in \mathscr{N}$. Algorithm 5.1 constructs a CSP from (5.2). By Algorithm 5.1, every solution of the CSP is an NE of (5.2) and vice versa. Let us define a player as satisfied if it achieves the SINR requirement whenever possible or else if it switches off. Then clearly the correspondence of i in the satisfaction game is also $q_i(p_{-i})$ and the satisfaction equilibria (Perlaza *et al.*, 2012b) coincide with the NE of (5.2).

5.4 Existence of Stable Solutions

Fig. 5.1 illustrates an example that does not possess an NE. For a continuous channel state distributions consider the following counter example. If in Fig. 5.1 $h_{ij} = 1$ then the considered region is $[h_{ij}, h_{ij} - \varepsilon]$ else if $h_{ij} = 0$ then the region is $[h_{ij}, h_{ij} + \varepsilon]$, where $0 \leq \varepsilon < 0.1$, and $\forall i \in \mathscr{N}, \tau_i = 0.8$. Payoff matrices of Fig. 5.1 hold throughout this channel region, thus has a strictly positive probability of not having an NE.

To compute the probability of existence of NEs in the narrowband fading channel, let us define the random variable $y(h,r)$, which evaluates to 1 iff there is at least one solution to the CSP of Algorithm 5.1 and 0 otherwise. Thus, $y(h,r) = \min(1, \sum_{p\in\mathscr{P}} \prod_{i\in\mathscr{N}} \mathbb{1}_{\mathscr{C}_i(h,r)}(p))$, where $\mathbb{1}_{\mathscr{C}_i(h,r)}$ is the indicator function and $\mathscr{C}_i(h,r)$ explicates that the constraint depends on the random variables. The probability of existence of at least one NE is $\mathbb{E}_{hr}(y(h,r)) = \Pr(\sum_{p\in\mathscr{P}} \prod_{i\in\mathscr{N}} \mathbb{1}_{\mathscr{C}_i(h)}(p) \geq 1)$.

For $(p_i, p_{-i}) \in \mathscr{P}$, where $p_i \neq 0, \mathbb{1}_{\mathscr{C}_i(h,r)}(p) = 1$ iff $h_{ii}\frac{p_i}{\tau_i} \geq \sum_{j\neq i} h_{ji}p_j + r_i + \sigma^2$. Else if $p_i = 0$ then $\mathbb{1}_{\mathscr{C}_i(h,r)}(p) = 1$ iff $h_{ii}\frac{\overline{p}_i}{\tau_i} < \sum_{j\neq i} h_{ji}p_j + r_i + \sigma^2$. From independence of channels $\Pr(h_{ii}\frac{p_i}{\tau_i} \geq \sum_{j\neq i} h_{ji}p_j + r_i + \sigma^2) = \mathbb{E}_{hr}(1 - F_{h_{ii}}(\sum_{j\neq i} h_{ji}p_j + r_i + \sigma^2))$ and $\Pr(h_{ii}\frac{\overline{p}_i}{\tau_i} < \sum_{j\neq i} h_{ji}p_j + r_i + \sigma^2) = \mathbb{E}_{hr}(\overline{F}_{h_{ii}}(\sum_{j\neq i} h_{ji}p_j + r_i + \sigma^2))$, in which $F_{h_{ii}}$ and $\overline{F}_{h_{ii}}$ are the CDFs of random variables $h_{ii}\frac{p_i}{\tau_i}$ and $h_{ii}\frac{\overline{p}_i}{\tau_i}$ respectively. This development is independent of the distributions. Let the event $\mathbb{1}_{\mathscr{C}_i(h,r)}(p) = 1$ be denoted by $\mathscr{A}_i(p)$. In the IC if $i \neq j$, then $\mathscr{A}_i(p)$ and $\mathscr{A}_j(p)$ are independent for a given p. Therefore, the joint probability of the set of events $\mathscr{A}(p) \triangleq \{\mathscr{A}_i(p) : i \in \mathscr{N}\}$ is $\Pr(\mathscr{A}(p)) = \prod_{i\in\mathscr{N}} \Pr(\mathscr{A}_i(p))$ and $\mathbb{E}_{hr}(y(h,r)) = \Pr(\cup_{p\in\mathscr{P}} \mathscr{A}(p))$. Let P be the cardinality of $\mathscr{P}$ and index the elements of $\mathscr{P}$ as $p_l \in \mathscr{P}$, $1 \leq l \leq P$ (the indexing is arbitrary). By the inclusion-exclusion principle for a finite set of events, probability of existence of an NE in the IC is given by (5.4).

$$\mathbb{E}_{hr}(y(h,r)) = \sum_{k=1}^{P} (-1)^{k+1} \sum_{\substack{p_{i_1},\dots,p_{i_k} \\ 1\leq i_1 \cdots < i_k \leq P}} \Pr\Big(\bigcap_{l=1}^{k} \mathscr{A}(p_{i_l})\Big). \tag{5.4}$$

As $\mathscr{P}$ grows, evaluation of (5.4) is computationally costly. Let us define $\underline{\Pr}(\cup_{p\in\mathscr{P}}\mathscr{A}(p)) = \max_{1\leq l\leq P}\{\Pr(\mathscr{A}(p_l))\}$ and $\overline{\Pr}(\cup_{p\in\mathscr{P}}\mathscr{A}(p)) = \min(\sum_{k=1}^{P}\Pr(\mathscr{A}(p_k)),1)$. Then the Fréchet bounds (Ferson *et al.*, 2004) are;

$$\underline{\Pr}(\cup_{p\in\mathscr{P}}\mathscr{A}(p)) \leq \mathbb{E}_{hr}(y(h,r)) \leq \overline{\Pr}(\cup_{p\in\mathscr{P}}\mathscr{A}(p)). \tag{5.5}$$

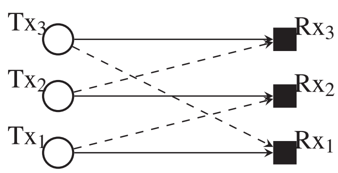
In the IMAC, $\mathscr{A}_i(p)$ and $\mathscr{A}_j(p)$ can be dependent (if players are of the same cell) $i,j\in\mathscr{N}$. Therefore, exact computation of $\Pr(\mathscr{A}(p))$ requires the application of Bayes' rule in a network topology specific manner. For topology independent bounds, let $\underline{\Pr}(\mathscr{A}(p)) \leq \Pr(\mathscr{A}(p)) \leq \overline{\Pr}(\mathscr{A}(p))$, where the two probability bounds are $\underline{\Pr}(\mathscr{A}(p)) = \max(\sum_{i=1}^{N}\Pr(\mathscr{A}_i(p)) - (N-1),0)$ and $\overline{\Pr}(\mathscr{A}(p)) = \min_{1\leq i\leq N}\{\Pr(\mathscr{A}_i(p))\}$. Then p-box (probability-box) is given by (5.6), where $(\underline{\Pr}_1,\overline{\Pr}_1) = \max_{1\leq l\leq P}\left\{\left(\underline{\Pr}(\mathscr{A}(p_l)),\overline{\Pr}(\mathscr{A}(p_l))\right)\right\}$ and $(\underline{\Pr}_2,\overline{\Pr}_2) = \min\left\{\sum_{1\leq l\leq P}\left(\underline{\Pr}(\mathscr{A}(p_l)),\overline{\Pr}(\mathscr{A}(p_l))\right),(1,1)\right\}$.

$$\min\{\underline{\Pr}_1,\underline{\Pr}_2\} \leq \mathbb{E}_{hr}(y(h,r)) \leq \max\left\{\overline{\Pr}_1,\overline{\Pr}_2\right\}. \tag{5.6}$$

Algorithm 5.1: Construction of CSP from (5.2)

variables $\{p_i\}_{i\in\mathscr{N}}$, where $p_i\in\mathscr{P}_i$
for $i\in\mathscr{N}$:
  for $p_{-i}\in\mathscr{P}_{-i}$:
   if $q_i'(p_{-i})\neq\emptyset$:
    $\forall p_i'\in q_i'(p_{-i})$, include (p_i',p_{-i}) in $\mathscr{C}_i$
   else:
    include $(0,p_{-i})$ in $\mathscr{C}_i$

Note that since game (5.2) played by a single player trivially has a pure equilibrium, it is guaranteed that a subset $\mathscr{N}'\subseteq\mathscr{N}$ of users can always be found such that when played by $\mathscr{N}'$, game (5.2) has an equilibrium.



$p_3 = 1$

$p_1 \backslash p_2$	1	0
1	$-1, -1, -1$	$-1, 0, 1$
0	$0, 1, -1$	$0, 0, 1$

$p_3 = 0$

$p_1 \backslash p_2$	1	0
1	$1, -1, 0$	$1, 0, 0$
0	$0, 1, 0$	$0, 0, 0$

Figure 5.1 A counter example: 3 user cyclic Z-interference channel, $\forall i \in \mathscr{N}, \mathscr{P}_i = \{0,1\}, \tau_i = 1$, and $\sigma^2 = 1$, has no stable admission control. An arrow (solid or dashed) indicates a channel gain of 1 and lack of an arrow 0.

5.5 Bayesian Game in Compact Convex Channels

Consider the IMAC with private CSI and $\forall i \in \mathscr{N}$ $[0, \overline{h}_{ii}] \ni h_{ii}$, $0 < \overline{h}_{ii} < +\infty$. The resulting Bayesian game (Shoham & Leyton-Brown, 2009) is:

$$\mathtt{G}_\mathrm{B} \triangleq (\mathscr{N}, \{\mathscr{P}_i\}_{i\in\mathscr{N}}, \{u_i\}_{i\in\mathscr{N}}, \{h_{ij}\}_{i,j\in\mathscr{N}}, F_h), \tag{5.7}$$

where F_h is the joint distribution of the *type* vector $(h_{ij})_{i,j\in\mathscr{N}}$. Let h_{-i} denote the types of all except i. A pure-strategy is a mapping $s_i : [0, \overline{h}_{ii}] \to \mathscr{P}_i$. The strategy profile of all is denoted by s, the strategies of all except i by s_{-i}, and $\mathbb{E}_{-i|h_{ii}}$ denotes the expectation over h, r_i given h_{ii}. The *ex interim* expected utility (Shoham & Leyton-Brown, 2009), when $h_{ii} = h_{i\text{th}}$, is $\mathbb{E}_{-i|h_{i\text{th}}} u_i(p_i, s_{-i}, h_{-i}) = \Pr(\gamma_i(h_{i\text{th}}) \geq \tau_i \mid s_{-i}) - \Pr(\gamma_i(h_{i\text{th}}) < \tau_i \mid s_{-i}) = 2\Pr(\gamma_i(h_{i\text{th}}) \geq \tau_i \mid s_{-i}) - 1$. For $0 < p_i < \overline{p}_i$, $\mathbb{E}_{-i|h_{i\text{th}}} u_i(\overline{p}_i, \cdot) > \mathbb{E}_{-i|h_{i\text{th}}} u_i(p_i, \cdot)$, hence without loss of generality let $\mathscr{P}_i = \{0, \overline{p}_i\}$. Suppose that i's best response to $h_{ii} = h_{i\text{th}}$ is $p_i = 0$. Then from monotonicity, for all $h_{ii} \leq h_{i\text{th}}$ the best response remains 0. This observation allows us to focus on pure threshold strategies of the form

$$s_i(h_i) \triangleq \begin{cases} 0 & \text{if } h_{ii} \leq h_{i\text{th}}, \\ \overline{p}_i & \text{otherwise.} \end{cases} \tag{5.8}$$

Then the strategy profile is fully represented by the thresholds $s = (h_{i\text{th}})_{i\in\mathscr{N}}$. Threshold equilibria for the on-off game with elastic utilities have been proven to exist in symmetric networks and asymmetric case has been numerically solved (Lee *et al.*, 2009; Goonewardena *et al.*, 2015a). At an equilibrium stratgy $\tilde{s} = (\tilde{h}_{\text{ith}})_{i\in\mathscr{N}}$, a player satisfies $\tilde{h}_{\text{ith}} \overset{(a)}{\in} \{h_{ii} \in [0,\overline{h}_{ii}] : \mathbb{E}_{-i|h_{ii}} u_i(\overline{p}_i, \tilde{s}_{-i}, h_{-i}) = 0\}$, else if $\{h_{ii} \in [0,\overline{h}_{ii}] : \mathbb{E}_{-i|h_{ii}} u_i(\overline{p}_i, \tilde{s}_{-i}, h_{-i}) = 0\} = \emptyset$ then $\tilde{h}_{\text{ith}} \overset{(b)}{=} \overline{h}_{ii}$. When $h_{\text{ith}} = \overline{h}_{ii}$, i does not transmit.

Define function $f_i : \mathscr{D} \to [-1,1]$ as follows. The domain $\mathscr{D} \triangleq \mathscr{D}_1 \times \cdots \times \mathscr{D}_N$, where $\mathscr{D}_i \triangleq [0,\overline{h}_{ii}]$. Function f_i is continuous in $\mathscr{D}$, $\forall x_{-i} \in \mathscr{D}_{-i}$, $f_i(0,x_{-i}) < 0$, and $f_i(\cdot,x_{-i})$ is strictly increasing in the region $\{x_i \in \mathscr{D}_i : f_i(x_i,x_{-i}) > -1\}$. Also define $g_i : \mathscr{D}_{-i} \to \mathscr{D}_i$, as follows. For $x_{-i} \in \mathscr{D}_{-i}$, if $\exists\, x'_i \in \mathscr{D}_i$ such that $f_i(x'_i,x_{-i}) = 0$, then $g_i(x_{-i}) = x'_i$, else $g_i(x_{-i}) = \overline{h}_{ii}$. Let $g : \mathscr{D} \to \mathscr{D}$ be $g \triangleq (g_i)_{i\in\mathscr{N}}$.

Claim 3. Function g_i is continuous in $\mathscr{D}_{-i}$.

Proof. If for $x_{-i} \in \mathscr{D}_{-i}$, $\exists\, x_i \leq \overline{h}_{ii}$ such that $f_i(x_i,x_{-i}) = 0$, then from strict monotonicity, x_i is unique $\therefore$ $g_i(x_{-i})$ is *unique*. If $f_i(x_i,x_{-i}) = 0$ and $x_i < \overline{h}_{ii}$ then from continuity of f_i, $\exists$ a neighborhood $\mathscr{U} \subset \mathscr{D}_{-i}$ of x_{-i}, $\forall x'_{-i} \in \mathscr{U}$ $\exists\, x'_i < \overline{h}_{ii}$ such that $f_i\left(x'_i, x'_{-i}\right) = 0$.

If $f_i\left(\overline{h}_{ii}, x_{-i}\right) = 0$, then in a neighborhood $\mathscr{U}$ of x_{-i}, $x'_{-i} \in \mathscr{U}$ either $\exists\, x'_i < \overline{h}_{ii}$ such that $f_i\left(x'_i, x'_{-i}\right) = 0$ or $\exists\, x'_i < \overline{h}_{ii}$ such that $f_i\left(x'_i, x'_{-i}\right) < 0$ and $g_i(x'_{-i}) = \overline{h}_{ii}$. Therefore, g_i is continuous. □

Claim 4. Function g has a fixed point.

Proof. The set $\mathscr{D} \subset \mathbb{R}^N$ is compact and convex. Since g_is are continuous by Claim 3, g is continuous in $\mathscr{D}$. Therefore, by Brouwer fixed-point theorem for compact convex sets (Shoham & Leyton-Brown, 2009) there exits $x \in \mathscr{D}$ such that $g(x) = x$. □

Let $f_{h_{ii}}$ be the PDF of h_{ii}, then the probability that i transmits is $\mathrm{p}'(\{i\}) = \Pr(h_{ii} > h_{\text{ith}}) = \int_{h_{\text{ith}}}^{\overline{h}_{ii}} f_{h_{ii}}(t)dt$. From the independence of channels, the probability that $\mathscr{N}' \subseteq \mathscr{N}$ transmits is $\mathrm{p}'(\mathscr{N}') = \prod_{i\in\mathscr{N}'} \mathrm{p}'(\{i\})$ and the probability that $\mathscr{N}' \subseteq \mathscr{N}$ does not transmit is $\mathrm{p}''(\mathscr{N}') = \prod_{i\in\mathscr{N}'}(1 - \mathrm{p}'(\{i\}))$. The $\Pr(\gamma_i(h_{\text{ith}}) \geq \tau_i \mid s_{-i}) = \sum_{\mathscr{N}'\in\mathbb{P}(\mathscr{N}\setminus\{i\})} \mathrm{p}'(\mathscr{N}')\ \mathrm{p}''(\mathscr{N}\setminus\{i\}\setminus\mathscr{N}') \Pr(\gamma_i(h_{\text{ith}}) \geq \tau_i \mid$

$\mathcal{N}', s_{-i})$, where $\mathbb{P}$ denotes the power set. For any integrable $f_{h_{ii}}$ the function $\int_{h_{i\text{th}}}^{\overline{h}_{ii}} f_{h_{ii}}(x)dx$ is continuous in $h_{i\text{th}}$. Since well defined PDFs are integrable $\Pr(\gamma_i(h_{i\text{th}}) \geq \tau_i \mid \mathcal{N}', s_{-i}) = \Pr(h_{i\text{th}} \frac{\overline{p}_i}{\tau_i} \geq \sum_{j \in \mathcal{N}'} h_{ji}\overline{p}_j + r_i + \sigma^2 \mid h_{jj} > h_{j\text{th}}, j \in \mathcal{N}')$ is continuous in $(h_{i\text{th}}, h_{j\text{th}})_{j \in \mathcal{N}'}$. The external randomness r_i helps to maintain continuity of $\Pr(\gamma_i(h_{i\text{th}}) \geq \tau_i \mid \mathcal{N}', s_{-i})$ when others do not transmit, i.e. $\forall j \in \mathcal{N}'$, $h_{j\text{th}} = \overline{h}_{jj}$ (without the expectation over r_i it would be a step function for this case). Therefore, $\mathbb{E}_{-i|\cdot} u_i(\overline{p}_i, \cdot, h_{-i})$ is continuous in $\mathcal{D}$.

Theorem 3. *The game (5.7) has at least one pure Bayesian Nash equilibrium in threshold strategies of (5.8).*

Proof. $\mathbb{E}_{-i|\cdot} u_i(\overline{p}_i, \cdot, h_{-i})$ satisfies the properties of $f_i(\cdot,\cdot)$. By construction, g_i satisfies aforementioned conditions (a) and (b) for an equilibrium and (a), when satisfied, has a unique solution. Therefore, the fixed point of Claim 4 satisfies conditions (a) and (b) $\forall i \in \mathcal{N}$ g_i. $\square$

Theorem (3) does not assume a channel distribution and only needs the existence of well defined PDFs for channels and external interference. Generally, continuity of the PDFs is sufficient. The single tap Rayleigh channel has exponentially distributed gain, thus the simulations consider truncated exponential distribution. The PDF of a right truncated exponential random variable is $f_{h_{ii}}(x) = \frac{\frac{1}{\lambda}e^{-\frac{x}{\lambda}}}{1-e^{-\frac{\overline{h}_{ii}}{\lambda}}}$. When $0 \leq a < \overline{h}_{ii}$, we have $f_{h_{ii}}(x \mid h_{ii} > a) = \frac{f_{h_{ii}}(x)}{1-F_{h_{ii}}(a)}$, where $F_{h_{ii}}$ is the CDF of h_{ii}. When $a = \overline{h}_{ii}$ player does not transmit and $f_{h_{ii}}(x \mid h_{ii} > a)$ is undefined.

5.6 Numerical Results

The simulation network for game (5.2) consists of 3 home deployed SCs with 2 users in each. The fading channel has unit mean power gain to home access point. Due to wall penetration losses, inter-small cell interference power gain has a mean of 0.25. Scaled noise power is $10^{-3}mW$ and maximum transmission power is 1 mW. External interference r_i $\forall i \in \mathcal{N}$ is ignored in this case for the purpose of simplicity. Fig. 5.2 shows the probability of existence of an NE and p-box of (5.6) for different SINR requirements (that are common to all players) and for 2 and 3 power levels per player. Fig. 5.3 shows price of anarchy (PoA) and price of stability

(PoS) (Shoham & Leyton-Brown, 2009) for the number of satisfied transmitters. PoA is the ratio of maximum number of satisfiable transmitters to minimum number of satisfied transmitters at an equilibrium. PoS is the ratio of maximum number of satisfiable transmitters to maximum number of satisfied transmitters at an equilibrium. Results in Fig. 5.3 are averaged over channel distributions conditioned on existence of equilibria.

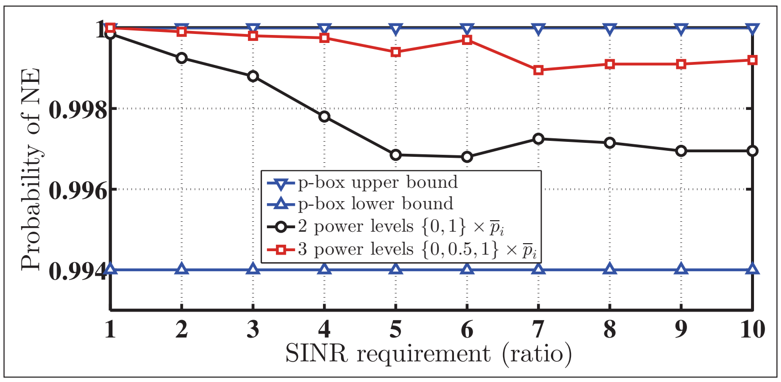


Figure 5.2 Probability of existence of at least one pure-strategy NE

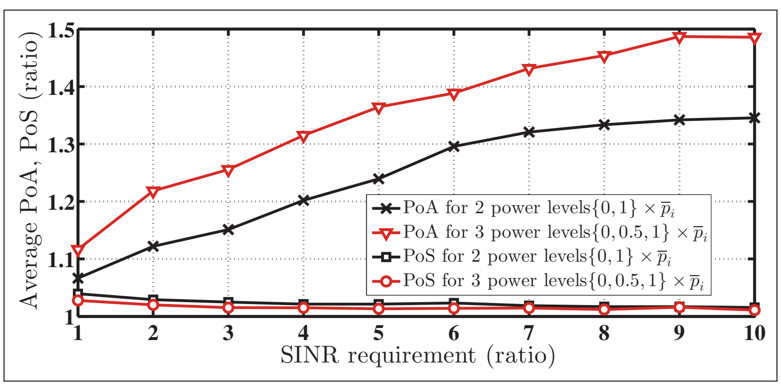


Figure 5.3 PoA/PoS vs. SINR requirement. The closer the lines to 1 the better.

Theorem 3 utilizes the Brouwer's fixed point theorem for compact convex sets, which is hard to construct. Therefore, to compute an equilibrium the iterative sequential update algorithm is followed. Players start at an initial threshold. Then each player, in its turn, updates its threshold knowing the current thresholds of other players. It is not proven that this algorithm

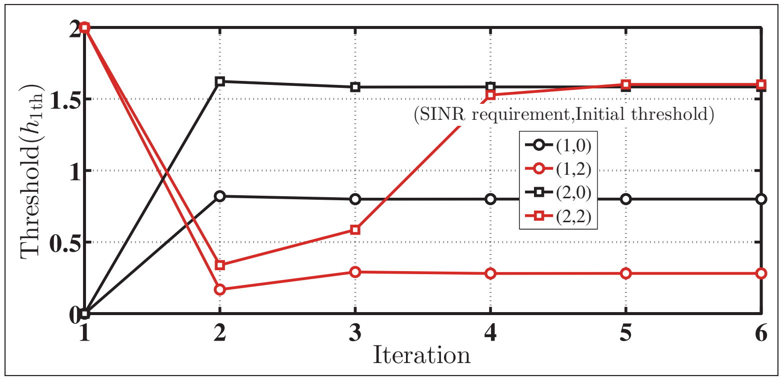


Figure 5.4 Convergence of sequential update for player 1 of cell 1.

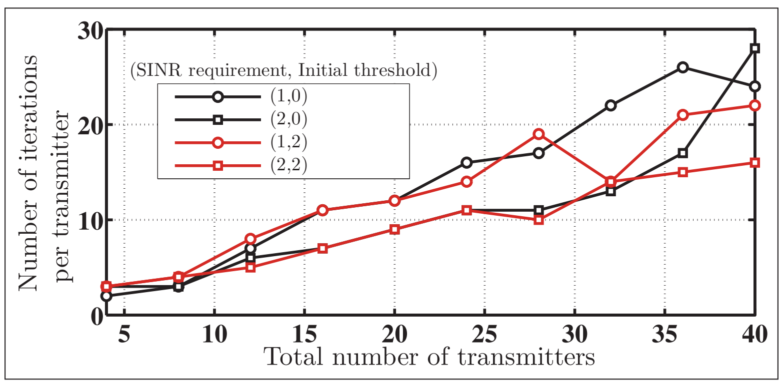


Figure 5.5 Time complexity to convergence of sequential threshold update.

should always converges, however if the algorithm does converge, then by definition it is a fixed point. In simulations it converged in every trial. The right cut-off point of the truncated exponential distribution is 2. Means of exponential inter-small-cell interference power gain and external interference r_i are 0.01. Fig. 5.4 shows the convergence of the threshold of a player for different initial values and SINR requirements where the network consists of 4 SCs with 2 users in each. Fig. 5.5 shows that the time to converge per transmitter grows linearly with the number of transmitters.

5.7 Conclusion

This letter considers distributed admission and power control as a noncooperative game for discrete finite power levels and inelastic traffic utility. In the full information setting, it is shown that a pure NE may not exist in some fading networks, with positive probability and the probability of existence is analytically derived. In the Bayesian setting with compact convex channel power gains, the existence of at least one Bayesian-NE in threshold on-off strategies is proven.

CONCLUSION AND RECOMMENDATIONS

This dissertation treats the problem of radio resource allocation in the heterogeneous small-cell networks (HetSNets). The industry and the academic research community have identified this problem as one of the key challenges that must be overcome, in order to augment spectral efficiency in future wireless networks. Many related subproblems of this resource allocation problem are computationally hard. In addition, the information required to fully define the problem for a practical network is fairly large and highly dynamic. These observations necessitate looking into local self-organization of the resources as a scalable heuristic solution. This dissertation is by and large an attempt to demonstrate the applicability of game-theoretic distributed solutions to this problem. The applicability of game-theoretic solutions into real-time resource allocation relies on the existence of efficient and easily computable equilibria. Chapters 2 and 5 of this thesis demonstrate the existence of threshold-based Bayesian Nash equilibria in the multicell frequency division multiple access problems. Threshold equilibria provide to the users an easy comparison rule to make channel access decisions. The bulk of the theory of games comes from an economic networks perspective. Thus, certain assumptions and results require adaptation to fit into the radio resource allocation setup. For instance, at a Nash equilibrium each player is operating at a local maximum of its expected utility, which is not necessary for many wireless applications. Chapter 3 of this thesis presents a novel alternative equilibrium that has been carefully designed to represent the requirements of wireless users. This equilibrium is called the generalized satisfaction equilibrium (GSE). The GSE does not compel the players to operate at a local maximum. Instead, the players attempt to achieve a certain minimum expected utility as required by their applications. It is demonstrated that such GSEs exist and that they can satisfy more users, for a given amount of resources, compared to the mixed-strategy Nash equilibrium. Finally, Chapter 4 of this thesis presents a novel mechanism design framework for wireless access networks. This chapter addresses the problem of monetary transfer that is popularly used in many research works. Monetary transfer comes from an economic perspective and we believe that such transfers do not fit the HetSNet environment. It is shown that by using verification and threat of blocking, which are more natural for wireless networks, the users can be coerced to a high level of truthfulness. In summary, the results of this thesis demonstrate that the wireless network environment is amenable to game-theoretic analysis and mechanism design. While direct application of classical results is certainly possible, the differences between economic vs. wireless networks justify innovations and adaptations of game theory to the wireless network environment. Therefore, future research has to focus on a theory of games and mechanisms specifically designed with the properties of wireless networks in mind.

There are multiple paths of research that can advance the results of this thesis. Especially, the following two paths are identified based on the results of Chapters 3 and 4. The GSE of Chapter 3 has shown to be promising, due to the strong existence results and the performance in the Monte Carlo experiments. However, the applications that are considered in this thesis are limited. Use cases such as cell range expansion and coordinated multipoint transmission, which are peculiar to HetSNets have to be considered in future works. In addition, distributed learning algorithms and their convergence to GSEs have to be analyzed. One very important future problem would be to explore the existence of GSEs, in which unsatisfied users have the least impact on the users who are satisfied. For example, in the power control problem, we can identify an admission control GSE where all unsatisfied agents are switched-off. It is not clear what conditions are required for these special GSEs. Learning such particular GSEs in a distributed manner is also an interesting problem. Another important path is to extend the GSE to the stochastic dynamic setting. This thesis only considers single stage games with respect to the GSE. The other main path of future research should be focused on verification mechanisms of Chapter 4. Monte Carlo experiments show that verification mechanisms can implement many classical scheduling algorithms with a high probability of truthfulness. However, theoretical results such as the conditions for incentive compatibility and bounds on how close to truthfulness can the verification mechanisms arrive remain to be explored. Finally, combining the GSE and the verification mechanisms to implement scheduling policies in truthful Bayesian GSEs is an interesting problem. From the results of this thesis, we believe that verification mechanisms combined with the Bayesian GSE are well adapted to model the resource allocation problem in a wireless network than the classical mechanisms based on payments and Bayesian Nash equilibria.

APPENDIX I

APPENDIX FOR CHAPTER 2

1. Proof of Theorem 1

Proof. Consider the strategy profile $s_{\text{th}}^{\text{sym}} = (h_{\text{th}})$. Let us define the random variable $X(h_{\text{th}}) \sim \mathscr{B}_1(N_{-b_i}, \mathrm{q}_1(h_{\text{th}}))$ and let

$$z(X(h_{\text{th}}), h_{\text{th}}) = \int_{\mathscr{D}} f_{\mathbf{g}} f_{\gamma} \log_2\left(1 + \frac{h_{\text{th}}}{\sum_{j \in X(h_{\text{th}})} g_j^k + \gamma_k + \sigma^2}\right) d\mathbf{g}\gamma.$$

For the common threshold $s_{\text{th}}^{\text{sym}} = (h_{\text{th}})$, the expected payoff in (2.8) is equal to $\bar{\mathrm{p}}_1^{\text{sym}}(\mathscr{N}_{b_i k})$ $\mathbb{E}_{X(h_{\text{th}})} z(X(h_{\text{th}}), h_{\text{th}})$. Note that $z(X(h_{\text{th}}), h_{\text{th}})$ is increasing in h_{th} (as the $\log(\cdot)$ and integration region $\mathscr{D}$ both grows with h_{th}). Also observe that $z(X(h_{\text{th}}), h_{\text{th}})$ is decreasing in $X(h_{\text{th}})$ (the number of interfering SUEs grows as $X(h_{\text{th}})$ increases).

We can also observe by (2.6) and (2.10) that $\mathrm{q}_1(h_{\text{th}})$ is decreasing in h_{th}. Therefore, if $h_{\text{th}}^1 < h_{\text{th}}^2$, then $\mathrm{q}_1\left(h_{\text{th}}^2\right) < \mathrm{q}_1\left(h_{\text{th}}^1\right)$. By the stochastic coupling theory (Thorisson, 2000) $X\left(h_{\text{th}}^2\right) < X\left(h_{\text{th}}^1\right)$ almost surely (a.s.). Therefore,

$$z(X(h_{\text{th}}^1), h_{\text{th}}^1) < z(X(h_{\text{th}}^1), h_{\text{th}}^2) < z(X(h_{\text{th}}^2), h_{\text{th}}^2) \text{ a.s.}$$

Taking expectations yields $\mathbb{E}_{X(h_{\text{th}}^1)} z(X(h_{\text{th}}^1), h_{\text{th}}^1) < \mathbb{E}_{X(h_{\text{th}}^2)} z(X(h_{\text{th}}^2), h_{\text{th}}^2)$ a.s. From (2.11) and (2.6) we observe that the probability of no collision $\bar{\mathrm{p}}_1^{\text{sym}}(\mathscr{N}_{b_i k})$ is also increasing in h_{th}. Consequently the expected payoff $\bar{\mathrm{p}}_1^{\text{sym}}(\mathscr{N}_{b_i k}) \mathbb{E}_{X(h_{\text{th}})} z(X(h_{\text{th}}), h_{\text{th}})$ is increasing in h_{th}. Thus, we have that the expected payoff $\mathbb{E}_{\theta_{-i}} u_i(h_{\text{th}}, s_{-i\text{th}}^{\text{sym}}, \theta)$ is increasing in h_{th}. Hence there exists unique $\tilde{h}_{\text{th}}$ such that

$$\mathbb{E}_{\theta_{-i}} u_i\left(\tilde{h}_{\text{th}}, s_{-i\text{th}}^{\text{sym}}, \theta\right) = \rho. \tag{A I-1}$$

□

APPENDIX II

APPENDIX FOR CHAPTER 3

1. An Example with no GSE

In the following example, a game in satisfaction-form that does not possess a GSE in mixed strategies is presented. Define a two agent game in which each agent i has two actions $\{a_i^1, a_i^2\}$, $i \in \{1,2\}$. The probability that the strategy of agent i assigns to action a_i^j is $\pi_i(a_i^j)$, $j \in \{1,2\}$. The correspondence of agent 1 is

$$g_1(\pi_2) = \begin{cases} \{\pi_1 \in \Pi_1 : \pi_1(a_1^1) < \pi_1(a_1^2)\} & \text{if } \pi_2(a_2^1) \geq \pi_2(a_2^2) \\ \{\pi_1 \in \Pi_1 : \pi_1(a_1^1) \geq \pi_1(a_1^2)\} & \text{othewise} \end{cases}. \tag{A II-1}$$

and the correspondence of agent 2 is

$$g_2(\pi_1) = \begin{cases} \{\pi_2 \in \Pi_2 : \pi_2(a_2^1) < \pi_2(a_2^2)\} & \text{if } \pi_1(a_1^1) < \pi_1(a_1^2) \\ \{\pi_2 \in \Pi_2 : \pi_2(a_2^1) \geq \pi_2(a_2^2)\} & \text{othewise} \end{cases}. \tag{A II-2}$$

These correspondences generates a response cycle in the mixed-strategy space for any given mixed-strategy profile.

2. The CSP and the Proof of Prop. 2

The CSP is briefly introduced here and a comprehensive description can be found in (Bulatov, 2011; Kumar, 1992) and references therein. In a finite domain $\mathscr{D}$, a $q-$ary relation is a *set* of length q tuples of the form $(d_1, \ldots d_q)$, where the elements are from $\mathscr{D}$. An instance of CSP is defined by $(\mathscr{V}, \mathscr{D}, \mathscr{C})$, where $\mathscr{V} = \{v_1, \ldots, v_V\}$ is the set of variables, $\mathscr{D}$ is the finite domain of the variables, and $\mathscr{C} = \{c_1, \ldots, c_C\}$ is a collection of constraints. Constraint c_i is a pair $(v_{q_i}, \mathscr{R}_i)$, where the list $v_{q_i} = (v_{i1}, \ldots, v_{iq_i})$, $1 \leq q_i \leq V$, $v_{i1}, \ldots, v_{iq_i} \in \mathscr{V}$ and $\mathscr{R}_i$ is a q_i-ary relation on $\mathscr{D}$. An assignment $a = (v_j, d_j)_{j \in \mathscr{V}}$, is a single value $d_j \in \mathscr{D}$ given to each variable $v_j \in \mathscr{V}$. Assignment a is said to solve the CSP if $\forall c_i \in \mathscr{C}$, the v_{q_i} component of a is a tuple in the relation $\mathscr{R}_i$.

In complexity analysis, the representation of the problems are important as they are compared with respect to the input size. Here it is considered that $\forall i \in \mathscr{N}$, g_i is provided in tabular form with two columns a_{-i} and $g_i(a_{-i})$. That is, for each $a_{-i} \in \mathscr{A}_{-i}$ for which $g_i(a_{-i})$ is nonempty there is an entry/row in the table. For a_{-i} with no entry in the table $g_i(a_{-i})$ is empty.

Proof. The proof of Prop. 2 is as follows. The CSP is given by $(\mathscr{V}, \mathscr{D}, \mathscr{C})$. If $C < V$, then introduce $V - C$ number of dummy unary constraints c_j, $C < j \leq V$ of the form $(v_j, \mathscr{R}_j)$ where $\mathscr{R}_j$ has a unary tuple for each element of $\mathscr{D}$. These constraints are dummy as they are satisfied by any assignment to v_j. If $V < C$, then introduce $C - V$ dummy variables. Let this derived, either adding constraints or variables, CSP be $(\bar{\mathscr{V}}, \mathscr{D}, \bar{\mathscr{C}})$. Observe that an assignment is a solution to $(\bar{\mathscr{V}}, \mathscr{D}, \bar{\mathscr{C}})$ iff it solves $(\mathscr{V}, \mathscr{D}, \mathscr{C})$. Define a game in satisfaction-form with $\max\{V, C\}$ agents and set $\mathscr{A}_i = \mathscr{D}$. Assign $v_i \in \bar{\mathscr{V}}$ and $c_i \in \bar{\mathscr{C}}$ to agent i. The strategy of agent i is to assign a value $a_i(v_i) \in \mathscr{A}_i$, to v_i and it is satisfied if c_i is satisfied.

If the list v_{q_i} of c_i contains the v_i, then construct table g_i as follows. Each tuple in $\mathscr{R}_i$ can be considered as values assigned to the variables in v_{q_i} by the respective agents who own each variable i.e., $(a_{i1}(v_{i1}), \ldots a_i(v_i), \ldots, a_{iq_i}(v_{iq_i})) \in \mathscr{R}_i$, where $a_i(v_i)$ is assigned by agent i itself. The idea is to add values of variables of other agents on the left column of g_i and put the corresponding $a_i(v_i)$ on the right column but keeping in mind that more than one tuple in $\mathscr{R}_i$ can have the same value assignment to other variables but with different values to v_i. Take a tuple from $\mathscr{R}_i$, if the values of variables except v_i is not already in the left column of the table add it in a new row and corresponding $a_i(v_i)$ on the right column of that row. If on the other hand that exact value combination of other variables is already in the left column, then append to the right column (to the existing values) the new $a_i(v_i)$. If list v_{q_i} does not contain v_i, then construct g_i with one row for each tuple in $\mathscr{R}_i$ on the left column and the entire set $\mathscr{D}$ on each row on the right column, i.e., the satisfaction of i does not depend on its action but only on the actions of others. These mappings are polynomial time in size of $(\bar{\mathscr{V}}, \mathscr{D}, \bar{\mathscr{C}})$. Construction of game (3.1) is now complete. If an assignment a is an SE, then that assignment is found in $g_i \forall i \in \mathscr{N}$ and construction implies that the assignment is in $\mathscr{R}_i$ $\forall i \in \mathscr{N}$, hence solves $(\bar{\mathscr{V}}, \mathscr{D}, \bar{\mathscr{C}})$. Conversely, if a solves $(\bar{\mathscr{V}}, \mathscr{D}, \bar{\mathscr{C}})$, then it is in $\mathscr{R}_i$ $\forall i \in \mathscr{N}$, then by construction that assignment is in $g_i \forall i \in \mathscr{N}$, hence an SE. It was already established that a solution to $(\bar{\mathscr{V}}, \mathscr{D}, \bar{\mathscr{C}})$ solves $(\mathscr{V}, \mathscr{D}, \mathscr{C})$. Therefore, the SE search problem is NP-hard. $\square$

3. Proof of Prop. 6

Proof. The NP-hardness is proven by a polynomial time reduction from the *set partition problem* to (3.23). Set partition is a known NP-complete problem (Garey & Johnson, 1979). It is defined by an input set $\mathscr{P} = \{p_1, \dots p_P\}$ of positive integers and the problem is to decide if there is a partition of $\mathscr{P}$ into two subsets $\mathscr{P}_1$ and $\mathscr{P}_2$ such that the sum of elements of the two sets are equal. Let us denote by τ the value of sum of each partition so that the total sum of elements of $\mathscr{P}$ is 2τ. Let $N = P+2$ be the number of agents and $\overline{\mathscr{K}} = \left\{k^1, k^2\right\}$, 2 channels. Let $\sigma_i^k = \sigma$, $\forall 1 \leq i \leq P+2$ and $\forall k \in \mathscr{K}$. The transmission powers of the first P agents are the numbers $p_i \in \mathscr{P}$, $1 \leq i \leq P$. For those P agents $1 \leq i \leq P$, $\forall k \in \overline{\mathscr{K}}$ let $\mid h_{ij}^k \mid^2 = 1$, where $1 \leq j \leq P+2$. The last two agents $i \in \{P+1, P+2\}$ have power $p_{P+1} = p_{P+2} = 1$ and they do not interfere the first P agents, i.e., $\forall 1 \leq j \leq P$ and $\forall k \in \overline{\mathscr{K}}$ $\mid h_{ij}^k \mid^2 = 0$, but they interfere each other $\forall ij \in \{P+1, P+2\}$ $\mid h_{ij}^k \mid^2 = 1$.

In summary, the first P agents have identical unit gain channels to all receivers, the last two agents have zero gain channels to the receivers of the first P agents while having unit gain channels to the receivers of those two. Let $\forall i \in \mathscr{N}$ $\tau_i = \frac{p_i}{\tau + \sigma}$. Let $\mathscr{K}_i = \overline{\mathscr{K}}$ $\forall 1 \leq i \leq P$ and $\mathscr{K}_{P+1} = k^1$ and $\mathscr{K}_{P+2} = k^2$. Then for agents $P+1$ and $P+2$ to be satisfied, the sum of received interference powers on each channel due to the first P agents has to be less than or equal to τ, but since $\sum_{1 \leq i \leq P} p_i = 2\tau$, they necessarily have to be equal to τ. Observe that from the construction, if $P+1$ and $P+2$ are satisfied, then all $1 \leq i \leq P$ are satisfied as well. Thus, a pure-strategy channel allocation is an SE of the constructed game iff it is a valid set partition of $\mathscr{P}$. $\square$

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