Production Planning of Perishable Products in an Unreliable Manufacturing System

by

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THESIS PRESENTED TO ÉCOLE DE TECHNOLOGIE SUPÉRIEURE
IN PARTIAL FULFILLMENT FOR THE MASTER'S DEGREE
WITH THESIS IN SYSTEMS ENGINEERING
M.A.Sc.

MONTRÉAL, JUNE 8, 2021

ÉCOLE DE TECHNOLOGIE SUPÉRIEURE
UNIVERSITÉ DU QUÉBEC

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ACKNOWLEDGMENTS

Foremost, I would like to express my deepest appreciation to my parents for supporting me spiritually throughout my life. This thesis stands as a testament to their unconditional love and encouragement.

I would like to take this opportunity to express my gratitude to my research supervisors. Without their valuable insights and dedicated involvement in every step throughout the project, this study would have never been accomplished.

A special thanks to my colleagues at Chinook Trading Canada for all their supports and cooperation during these uncertain difficult times. I am privileged to work with this amazing, professional team.

Last but not least, for my fellows at *Laboratoire de conception et contrôle des systèmes de production* (C2SP), and for my friends living here in Canada and throughout the world, I cannot overlook their unequivocal companionship and for all the fun we have had during these years!

Planification de la production des produits périssables dans un système de fabrication non fiable

Seyedpedram PISHVAEIAN

RÉSUMÉ

Cette étude tente de fournir une solution optimale au problème de la planification de la production de produits périssables dans un système de fabrication sujet aux défaillances. Le système à l'étude consiste en une seule machine de fabrication produisant un type de produit, dans lequel la machine est sujette à des pannes et des réparations aléatoires et les produits sont périssables. Malgré le fait que la demande du client doit être satisfaite, les pénuries sont autorisées. Les produits ne sont plus utilisables après un certain temps et doivent être jetés après leur date d'expiration.

L'objectif de cette étude est de proposer des politiques de contrôle optimales minimisant le coût total, y compris les coûts de stockage, de pénurie et de périssabilité. Les variables de décision et d'état pour ce modèle sont respectivement le taux de production et le niveau de stock des produits périssables finis. Une approche basée sur la simulation en combinaison avec la conception de plans d'expérience et la méthodologie de surface de réponse nous donne la possibilité d'obtenir la politique optimale pour les paramètres à l'étude.

A travers ces modèles, trois scénarios majeurs pour étudier le comportement du système seront examinés. Premièrement, la situation dans laquelle le taux de la demande des clients et la durée de vie des produits sont constantes et prédéterminées sera étudiée. Pour le deuxième cas, on suppose que les produits peuvent périr à différents moments; cependant, le taux de demande suit un schéma invariable. Dans le dernier cas, un caractère aléatoire sera ajouté au taux de la demande des clients, tandis que la durée de conservation des produits reste inchangée et constante. Il a été suggéré que dans tous les cas, les politiques proposées sont valides, vérifiées par les analyses de sensibilité approfondies effectuées et fonctionnent bien.

Mots clés: produits périssables, système de fabrication non fiable, simulation, plan expérimental, politique de contrôle stochastique.

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ABSTRACT

The current study tries to provide an optimal solution to the problem of production planning of perishable products in a failure-prone manufacturing system. The system under study has consisted of a single manufacturing machine producing one type of product, in which the machine is subject to random failures and repairs and the products are perishable. Even though the customer demand must be satisfied, shortages are allowed. Products are no longer usable after a certain course of time and must be discarded from the inventories on hand after their expiration.

The objective of this study is to propose optimal control policies minimizing the total cost imposed on the system, including holding, shortage, and perishability costs. The decision and state variables for this model are the production rate and stock level of finished perishable products, respectively. A simulation-based approach in combination with the experimental design and the response surface methodology provides us with the opportunity to obtain the optimal values for the parameters under study.

Through these models, three major scenarios to study the behavior of the system will be scrutinized. First, the situation in which both customer demand rate, and products' lifetime are constant and predetermined will be studied. For the second case, it is assumed that products may perish at different times; however, the demand rate follows an unvarying pattern. In the last case, the randomness will be added to customer demand rate, while the products' shelf life remains unchanged and constant. It has been suggested that in all cases, the proposed policies are valid, verified by the extensive sensitivity analyses performed and work well.

Keywords: control policy, hedging point policy, failure prone manufacturing system, perishable products, simulation optimization

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LIST OF ABBREVIATIONS

ANOVA	Analysis of Variance
DOE	Design of Experiments
EPQ	Economic Production Quantity
FIFO	First-Input First-Output
FPMS	Failure Prone Manufacturing System
HPP	Hedging Point Policy
M1P1	One Manufacturing Machine Producing One Type of Product
MTTF	Mean Time to Failure
MTTR	Mean Time to Repair
RA	Regression Analysis
RSM	Response Surface Methodology
SRLT	Shortest Remaining Lifetime

LIST OF SYMBOLS

U^{max}	Maximum production rate for the manufacturing machine (product/time unit)
$U(t)$	Production rate for the manufacturing machine at time t (product/time unit)
u^t	Total production during period t (product)
$D(t)$	Demand rate at time t (product/time unit)
d^t	Total demand during period t (product)
$x(t)$	Inventory/backlog level at time t (product)
x_0	Initial inventory level (product)
$x^p(t)$	Perishability of products at time t (product)
x_0^p	Initial perished items (product)
$Pr.$	Perishability rate at time t (product/time unit)
$av.$	Machine availability
$\alpha(t)$	The operational state of the machine at time t
q_{ab}	Transition Rate of Stochastic Process from Mode “a” to “b”
c^+	Inventory cost for each product time unit ($/product/time unit)
c^-	Backlog cost for each product per unit of time ($/product/time unit)
c^p	Perishability cost for each product per unit of time ($/product/time unit)
Z	Threshold level (product)
δ	Coequality level – the level at which production rate is changed to average demand rate (product)
T	Product’s shelf life – the maximum period within a product is usable
$J(.)$	Total cost function
N	Simulation run time (time unit)

INTRODUCTION

Globalization has changed and evolved the nature of the supply chain systems. Today, goods are fabricated and/or assembled from components sourced from locations throughout the globe and are shipped to end users hundreds of miles away. Amongst these products, many of them and, hence, their supply chains are time-sensitive – from dairy products and foodstuff to blood banks and pharmaceuticals whose quality and effectiveness deteriorate over time and must be consumed on schedule and in good condition, and from fruits and flowers that may pass over the international borders under special circumstances to high-tech products and apparel that may be subject to consumer taste change, being replaced as brand-new products or new styles come up and end-user demands for the latest (Nagurney, Yu, Masoumi, & Nagurney, 2013).

Global markets, on the other hand, provide manufacturers with great opportunities to produce and sell more of these so-called perishable products. Firms' profitability can be improved by expanding their sales networks in geographically dispersed markets (He, Wang, & Lai, 2010). On the contrary, for such companies, inventory management of these products raises particular challenges, since they may require temperature-controlled storage conditions, higher quality of service or increased utilization of available production resources (Vrat, Gupta, Bhatnagar, Pathak, & Fulzele, 2018). Moreover, goods that exceed their finite lifetimes become no longer usable or desirable and can impose high costs owing to inventory loss or other unintended consequences, e.g., market loss or becoming life-threatening in exceptional cases. This crucial constraint forces manufacturers to vigilantly plan their production in collaborative efforts with their supply chain partners and stakeholders.

From a holistic view in supply chain management, inventories can act as an intermediary between the supply and the demand. Considering the fact that a match between supply and demand in an efficient manner is not straightforward and seldom occurs, as well as other random phenomena take place in the systems, possessing a buffer seems to be a rational practice to hedge the shortage effects (Lee & Kim, 2014). Nevertheless, the perishability of these products should not be overlooked. In this research, we lean toward considering the problem of production planning of perishable products in a failure-prone manufacturing system and answering the following questions:

- How can the companies with FPMSs reduce extra costs incurred to their systems because of products' perishability by determining the optimal inventory level?
- What is the optimal inventory threshold (safety stock) in terms of the total cost incurred to the system?

When it comes to the production planning of perishable products in unreliable manufacturing systems, some gaps can be found in the field, and in this study the intention is to take some of them into consideration in a tender to provide an applicable solution for the real-world challenges. Therefore, control policies have been proposed for a failure-prone manufacturing system with perishable items in presence of randomness in machine failures and repairs, customer demand rate and products' lifetimes based on Hedging Point Policy and then, their performances in terms of total costs they incur to the system have been evaluated.

This thesis has been organized into five chapters. In the first chapter, a comprehensive literature review is conducted to scrutinize the existing works and specify the research gaps, and to introduce the problem, methodology and contribution toward solving the problem. The second chapter proposes a simulation-based optimization model based on HPP to find the optimal solution where the product lifetime is known a priori to a specified number of periods and customer demand rate is constant. In the third chapter, the randomness of the products' shelf life is studied while demand rate remains constant. In this chapter, two priority rules and their effectiveness are studied. In fourth chapter, a modified HPP is proposed to consider the effect of variable demand rate on the system while the shelf life is deterministic and constant. Finally, in the last section, concluding remarks are presented. This work has been the subject of the article "*Optimal Production Control Policies in a Failure-Prone Manufacturing System with Perishable Products Subject to Demand and Lifetime Variability*", which has been submitted to the Journal of Computers and Industrial Engineering.

CHAPTER 1

LITERATURE REVIEW

1.1 Introduction

Over the past decades, supply chain systems have evolved while emerging challenges have been appearing. One of those eye-catching challenges, which has had a substantial influence on downstream businesses such as production and logistics systems, is managing the perishable finished products efficiently as time passes. Accordingly, there have been ongoing demands for understanding the systems including goods with finite lifetimes to explore the impact of the perishability on the production and inventory systems.

In this chapter, the intention is to provide the structure of such manufacturing systems and address one of the most important features associated with the nature of the outputs of these systems; that is the perishability of the products. In this regard, some pertinent concepts existing in the literature and the various methodologies utilized in previous works to tackle similar issues will be discussed in turn. Having reviewed the existing works and found the research gap, the research problem and its objectives will be introduced, and thereafter, the methodology proposed to solve it will be proposed.

1.2 Manufacturing systems

To better realize the manufacturing systems which will be discussed later, it would be helpful to address the terminology used in the literature. According to Ouaret (2012), manufacturing systems are classified into three major categories in terms of their manufacturing process and material flow, namely discrete, continuous and flow process. He also introduced three major production management approaches, which are make-to-stock (push policy), make-to-order (pull policy) and the hybrid production management system. For further details, readers are referred to Ben Salem (2014), Rached Hlioui (2015), and Samir Ouaret (2018). In this thesis, a kind of make-to-stock (push) approach in a continuous-process manufacturing system is studied.

1.3 Key factors in manufacturing systems

Surprisingly, not only the number of works on perishable products has remarkably augmented, but also the multitude of used key topics in one paper has significantly increased over the years (Janssen, Claus, & Sauer, 2016). Indeed, the joint topics in research works and their wide spectrum bring us about to conduct a comprehensive study on them and present them as briefly as possible. Amongst all the topics and parameters affecting every single manufacturing system, a few of them will be mentioned from a different perspective presented in Figure 1.1.

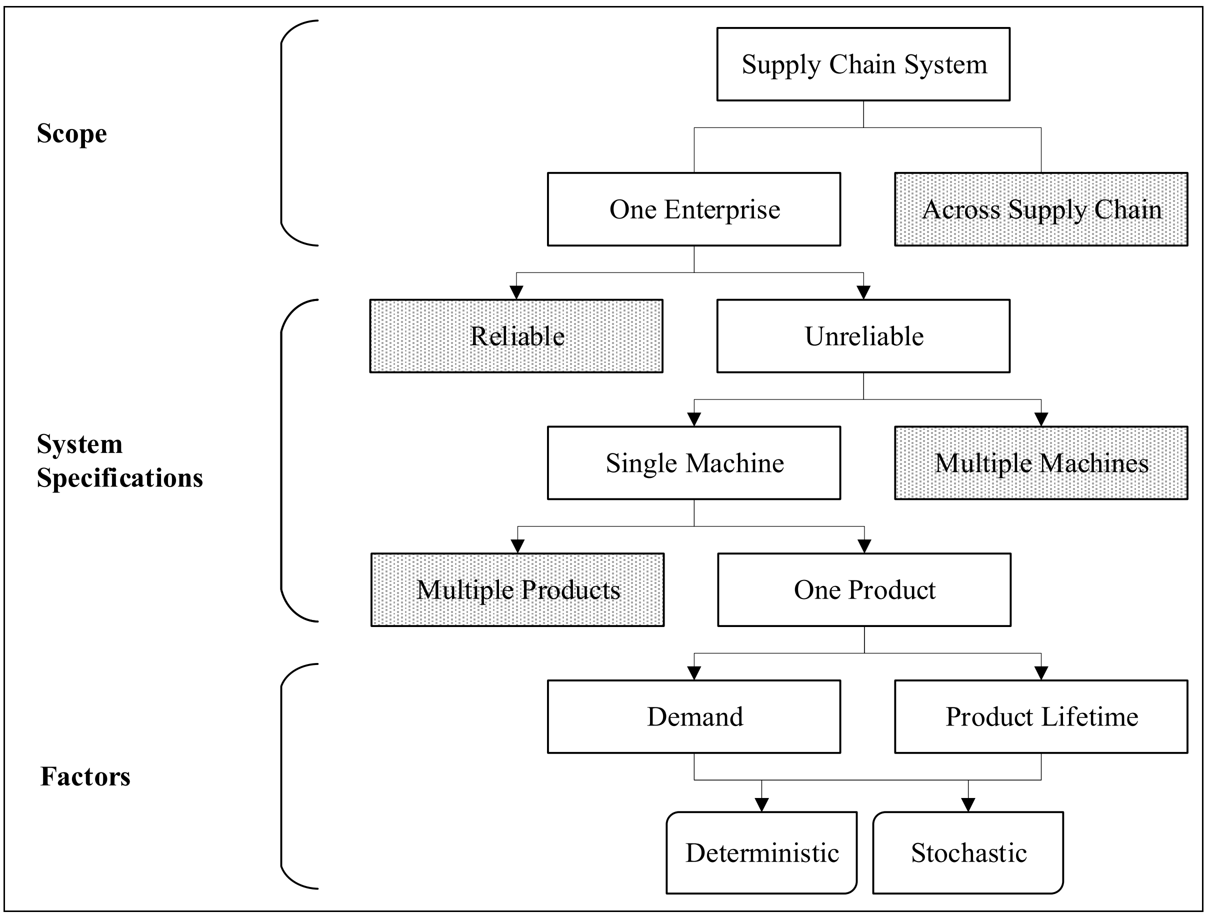


Figure 1.1 Structure of the literature review

From the perspective of scope, one manufacturing company is considered inside a supply chain system and other stages of the supply chain system, such as suppliers, distributors, etc., as well as internal and/or external logistics will not be taken into consideration. In terms of system specifications, a single failure-prone manufacturing machine producing a single type of perfect quality perishable product is studied. At the factor level, the customer demand rate, and the product lifetime are taken into consideration as the two most important parameters studied in previous works, which may be either deterministic or stochastic. By integrating different factors in different system characteristics, different models and consequently, different production-inventory policies may be established.

1.3.1 Reliable vs. failure prone manufacturing systems

In real-world problems, it is hardly possible to find a manufacturing system that never fails. Thus, a system is considered "*unreliable*" if it is to fail at a venture. Since the majority of works in the literature study failure prone systems, like this research, the reliable system will be omitted in this study. To mention a few works carried out on unreliable systems, Boukas and Al-Sunni (1999) proposed a model to study a manufacturing system subject to random failures and repairs, where a fraction of the finished items perish with a constant rate depending on the stochastic process describing the demand rate. The control problem was formulated as a jump-linear quadratic control problem, and the optimal policy was obtained by solving a set of coupled Riccati equations.

In another effort, Chelbi, Ait-Kadi, and Radhoui (2008) introduced an optimal design and operation model for production planning of perishable products in a failure-prone system with a finite-capacity buffer to extenuate the effect of probable dearth of finished products caused by breakdowns and by planned maintenance or overhaul actions occurring at predetermined periods. Products in buffer may degrade their quality with time, and a certain portion of them will be no longer usable after certain time and should be discarded from the inventory. More works on FPMSs with perishable items will follow later.

1.3.2 Perishability of products

In academia, several terms and definitions have been suggested for the items under study. Deteriorating items may refer to those of products becoming decayed, damaged, expired, or undesirable over time (H.-M. Wee, 1993). An important point is inherent in this definition; the deteriorating items can be classified into two major categories. The first category refers to the goods becoming decayed or expired thorough time, such as foodstuff or medicine; the other one encompasses the items losing their perceived value over time due of technology improvement or change in customers' taste, like electronic devices or fashion. Both categories have a similar attribute, *short life cycle*. For the former category, items have a short *natural* life cycle and after a certain point of time, their natural attributes change and lose their desirable value, while for the later one, they possess a short *market* life cycle, which means after a popularity period, they lose the marginal value due to changes in customer preferences.

From decades ago, there have been remarkable efforts in integrating product deterioration into the mathematical models in the operations research literature, such as Ghare (1963), Dave (1979), Nahmias (1982), Raafat (1991), Chakrabarty, Giri, and Chaudhuri (1998), Gupta and Agarwal (2000). Pahl and Voss (2014) explained that deterioration is a process of decay and can be characterized regarding different aspects, namely physical degradation, functionality loss or perceived value loss. This process may occur at once after a certain course of time or gradually. In Figure 1.2, three functional relationships have been illustrated. While the leftmost graph presents the perishability of products once at a time after a certain point in time, the other two represent gradually discrete-in time and continuous deterioration or degradation. To read more, the readers are referred to the preceding survey and its references. Earlier in the literature, researchers used to consider constant deteriorating rate in their models as in Li, Lan, an Mawhinney (2010), while most studies have considered a relationship between time and deteriorating rate, such as a linearly increasing function of time like in Mukhopadhyay, Mukherjee, and Chaudhuri (2004), or a Weibull distribution rate like in Mahapatra and Maiti (2005).

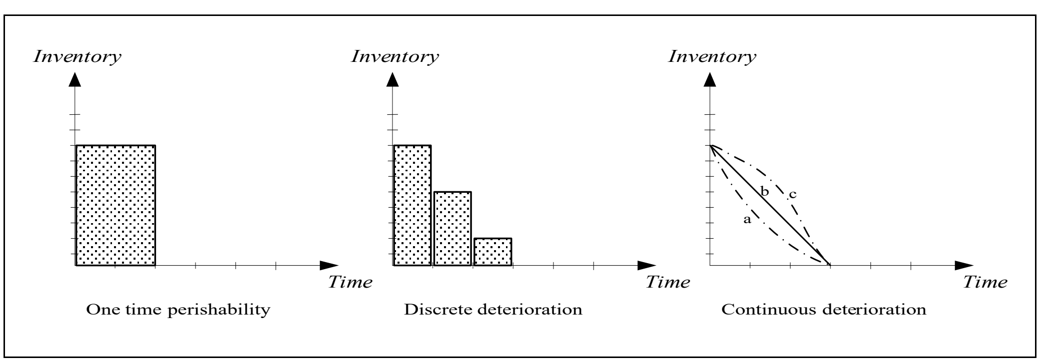


Figure 1.2 Sorts of products’ perishability

In a pioneer and insightful work, Nahmias (1982) provided two major classifications for products' perishability, fixed lifetime as well as random lifetime with an exponential decay as a special case. From the outlook of Bakker, Riezebos, and Teunter (2012), researchers have divided the models for deteriorating items into three different categories according to the products' lifetime; firstly, fixed lifetime, i.e., predetermined deterministic lifetime of e.g., a fixed three-period shelf life; secondly, age-dependent deterioration rate, inferring a known probabilistic distributed or an arbitrary distributed lifetime like Erlang distribution, as in Kouki and Jouini (2015); and thirdly, inventory-dependent deterioration rate, implying a constant deterioration rate per stocked products, as in Pandey (2017). In this research, fixed predetermined lifetimes as well as variable lifetimes based on an arbitrary probability distribution will be studied.

In this regard, it is remarkable to mention that single-period models for perishable products, usually known as "news vendor-type models" have been overlooked despite their valuable insights, since they are unable to provide credible answers to more complicated supply chain problems, especially in case of presence of other uncertainties in the problem (Karaesmen, Scheller–Wolf, & Deniz, 2011). Costs associated with inventory management of perishable products compose a major part of the operational cost of every production-inventory system. This would be more pivotal in cases with higher deteriorating rates.

For various sorts of companies, different perishable product inventory policies should be studied. From the seller and/or distributor point of view, the focus can be on optimal inventory strategies for retailing or distributing or can be on optimal inventory policy under a two-warehouse system, including both supplier and seller and/or distributor warehouses (Li et al., 2010). Lee and Kim (2014) studied this problem under a single-seller single-buyer system, or Perlman and Levner (2014) studied it in under a multi-echelon, multi-supplier inventory system. From the manufacturer standpoint, the emphasis is on developing an optimal production-inventory management system with consideration of the perishability of products, which will be followed in this study.

1.3.3 Customer demand

Reviewing existing literature reveals that there have been done a plethora of efforts to study distinct customer ordering patterns for perishable products because of their undeniable impacts on the production-inventory system. Further reviews suggest that customers may behave differently as these products are approaching their expiry dates. It arises from their perception of taking lower quality or desirability into consideration for them. In comparison with perishability, two major sorts of customer demand commonly studied in the literature are deterministic and stochastic demands. In the case of deterministic demands, the following categorization has been proposed: (Panda, Senapati, & Basu, 2008); (Bakker et al., 2012)

1. Uniform demand rate, e.g., constant rate or a fixed number of items,
2. Stock-dependent demand rate, as in Haijema (2013),
3. Time-varying demand rate,
    a. Linear positive/negative,
    b. Exponentially increasing/decreasing,
    c. Quadratic or parabolic,
    d. Ramp-type, as in Jain and Kumar (2007),
4. Price-dependent demand rate, as in Azadi, et. al (2019).

To read more, the readers are referred to Jain and Kumar (2007) and Panda et al. (2008). From a real-life viewpoint, stochastic demands, either in the form of a specific type of probability distribution or in the form of an arbitrary one, are more plausible (Li et al., 2010). As a prime instance, demands for blood products can be considered stochastic and taking serious measures to fulfill them are extremely important. This is because blood donors are scarce, and donations must be taken place under certain conditions (Belien & Force, 2012). In this regard, the system may face with impatient customers in need of blood transfusion. In cases of blood shortage, customers may decide to leave the system and balk their demands or renege it after a certain waiting time (Perry & Stadje, 1999); (Ioannidis, Jouini, Economopoulos, & Kouikoglou, 2012). Researchers considered "customer reneging" and "perishability of products" a similar phenomenon, which may cause some fluctuations in blood demand forecasting and fulfillment.

Another interesting case, He et al. (2010) studied multiple demands of perishable products in geographically dispersed markets.

As a desultory and complementary point, it should point out that from a holistic view on supply systems, demands in manufacturing systems are those of orders issued by the customer for any intention. Thus, the rate of demand depends on the customer's optimal ordering policy, which may be in any above-mentioned form.

1.4 Existing modeling methods and solution approaches

Decision makers in supply chain systems are interested in exploiting the most reliable and the least risky decision support systems to take proper measures based on the knowledge provided by these systems. For that purpose, different production-inventory strategies have been proposed and the basic idea of all of them is to attain credible answers to ‘‘what if’’ questions. To address the stochastic, dynamic nature of the manufacturing systems, different production-inventory strategies have been proposed. Amongst these strategies, Economic Production Quantity (EPQ) models and feedback control policies based on Hedging Point Policy (HPP) are the most known ones and have always been interesting options for the researchers.

For production and inventory control of perishable products in unreliable manufacturing systems, the EPQ models are commonly used to determine the optimal lot sizing by minimizing the incurred costs. To mention a few of them, Rahim and Ben-Daya (2001) performed a study on an EPQ model with perishable products in an uncalibrated manufacturing system. They assumed that machine failure and perishability of products follow arbitrary probability distributions. An important point here which should not be overlooked is that perishability is related to the nature of the products, while quality-related issues mainly result from the uncalibrated manufacturing machine or out-of-control manufacturing process. Moreover, Pandey (2017) developed an EPQ model for products perishing with a constant rate and the rate of demand depends on the stock level. Huang et al. (2017) studied an EPQ model for deteriorating items with permitted shortages and price-dependent demand rate. They tried to follow two purposes simultaneously: provide a profit maximization model and find the optimal production run time when shortage is allowed. Rout et al. (2019) studied an uncalibrated manufacturing system in which both the serviceable and reworkable items are assumed to deteriorate with time and shortages are allowed.

In the context of EPQ models in unreliable FPMSs, Lin and Gong (2006) tried to study the impact of random machine breakdowns on the EPQ model for the batches of products subject to exponential decay and under a no-resumption (NR) inventory control policy to meet a constant demand rate over an infinite planning horizon. Under the NR policy, the production run is immediately aborted for a fixed, predetermined period in case of failure, and the new run will not be started until all available inventories are depleted. The objective was to determine the optimal production uptime that minimizes the expected total cost per unit of time. A near optimal production uptime was derived under conditions of continuous review, deterministic and fixed demand rate, and no shortages. In another work, Chung, Widyadana, and Wee (2011) made an attempt to develop models with two kinds of stochastic machine failure distributions, lost sales and commodity deficiency. It was as an extension for Abboud, Jaber, and Noueihed (2000), where they introduced an EPQ model for imperishable items with stochastic machine unavailability. Widyadana and Wee (2012) added preventive maintenance and immediate corrective actions into their model. In another work, H. M. Wee and Widyadana (2013) reviewed an unreliable system for perishable products with consideration of rework operation for defective imperfect products using the first-in first-out (FIFO) priority rule and stochastic preventive maintenance.

Several authors, on the other hand, have adopted feedback control policies to consider the unreliable manufacturing systems. In the context, a plethora of studies based on Hedging Point Policy (HPP), as one of the widely used feedback control policies, can be easily found. This policy includes building and maintaining an optimal level of inventory on-hand to meet the customer demands during machine failures. Kimemia and Gershwin (1983) were one of the pioneers in studying the unreliable manufacturing systems. They suggested that Hedging Point Policy (HPP) is an optimal policy for such systems with the logic that the production surplus needs to be stored to compensate the probable future shortages resulted from the stochastic machine failures and repairs. After that, HPP became one of the most effective strategies to address the failure-prone manufacturing systems. This policy aims to control the production rate in terms of the inventory level considering the operational status of the system to build the safety stock (threshold level) for the situations that machine failures occur. For continuous-time failure-prone manufacturing systems, it has been shown that the Hedging Point Policy (HPP) is optimal as presented by Akella and Kumar (1986) and Bielecki and Kumar (1988).

They explicated that this feedback control policy can be simply characterized by so-called Optimal Inventory Level, i.e., should the current inventory level be lower than that of optimal threshold, one requires to produce at the maximum production rate; while if equal, the production rate needs to be exactly same as demand rate; otherwise, production must be stopped. This policy has been formulated as follows:

$$U\big(x(t),\alpha(t)\big) = \begin{cases} \alpha(t)\cdot\ U^{max} & if\ x(t) < z_{B.K}{}^{*} \\ \alpha(t)\cdot\ D & if\ x(t) = z_{B.K}{}^{*} \\ 0 & o.w. \end{cases} \quad (1.1)$$

In which, $U\big(x(t),\alpha(t)\big)$ is the stock-dependent and machine state-dependent production rate where $x(t)$ is stock level and $\alpha(t) \in \begin{cases} 0 & if\ machine\ is\ under\ repair \\ 1 & if\ machine\ is\ operational \end{cases}$.

Moreover, they argued that since there always are uncertainties in the manufacturing systems, either in supply or in demand, zero-inventory policy – like JIT production systems – cannot be possible, thus positive inventories are considered a buffer to hedge or compensate the possible backlogs.

Bielecki and Kumar (1988) then provided an analytical solution of an unreliable manufacturing system with one machine producing one type of imperishable product with exponentially distributed failure and repair times and constant demand rate. The optimal inventory level, $Z_{B.K}{}^{*}$, can be explicitly given by:

$$Z_{B.K}{}^{*} = \left(1 \Big/ \left(\frac{1}{D \cdot MTTF} - \frac{1}{(U^{max} - D) \cdot MTTR} \right) \right) * \ln \left[\frac{U^{max} \cdot MTTR \cdot (C^{+} + C^{-})}{C^{+} \cdot (U^{max} - D) \cdot (MTTR + MTTF)} \right] \quad (1.2)$$

The average inventory cost corresponding to this policy can be accordingly computed as the expected cost with respect to the steady-state distribution, which has been presented as follows:

$$J(Z_{B.K}{}^{*}) = \frac{c^{+} \cdot D \cdot MTTR \cdot MTTF}{(MTTR + MTTF)} + \left(c^{+} \Big/ \left(\frac{1}{D \cdot MTTF} - \frac{1}{(U^{max} - D) \cdot MTTR} \right) \right) * \ln \left[\frac{U^{max} \cdot MTTR \cdot (C^{+} + C^{-})}{C^{+} \cdot (U^{max} - D) \cdot (MTTR + MTTF)} \right] \quad (1.3)$$

These equations will be reviewed, utilized, and discussed in the following sections.

In addition, this policy has been implemented by some other researchers as is evident in Kenne and Gharbi (2000) in considering general distributions for failure and repair times and random demand rates, or in Perkins (2004) in modeling the unreliable manufacturing systems with exponential machine failure times and fixed repair times, and/or in Khoury (2016) with exponentially distributed failure and generally distributed repair times. Moreover, Chan, Wang, and Zhang (2007) developed a two-level HPP to address a time delay in unreliable manufacturing system with uncertain demand to minimize the total cost. Mourani, Hennequin, and Xie (2008) studied a failure-prone manufacturing system with continuous production process with transportation delay. In general, this policy has been widely implemented by researchers as is evident in Kenne and Gharbi (2004), Bouslah, Gharbi, and Pellerin (2013), Rivera-Gomez, Gharbi, and Kenne (2013), Hlioui, Gharbi, and Hajji (2017), Ouaret, Kenne, and Gharbi (2018), Polotski, Kenne, and Gharbi (2019), Entezaminia, Gharbi, and Ouhimmou (2019), etc., all for imperishable products.

In the context of HPP-based control policies in FPMSs, few studies on perishable items have been carried out. Malekpour, Sajadi, and Vahdani (2016) studied a failure-prone network of manufacturing machines with perishable products to meet the fixed incoming demand rate, where the shelf life of the finished products was also constant and predetermined. Similarly, Hatami-Marbini, Sajadi, and Malekpour (2020) considered same assumptions with the previous work. The only difference between these two studies is the optimization approach: in the former one, the researchers utilized Taguchi method to find the optimal values of decision variable, while in the later, researchers used a combination of simulated annealing metaheuristic, simulation, and Taguchi experimental design.

Given the dynamics and complexity of the unreliable production-inventory systems with perishable products because of interactions between products' perishability and randomness in demand rate, products' lifetimes and machine failures, and the high stochastic nature of the problem under study, it is difficult to reach an analytical solution over an infinite planning horizon. Most of EPQ models take some unrealistic assumptions which may make them incapable of providing realistic and credible solutions in real world problems, such as constant production rate or unavailability of the produced items in incomplete lots while the machine is down and under repair process. In addition, EPQ model are usually utilized in determining the optimal lot sizing problems in the context of batch processing systems. There is an underlying assumption in EPQ models stating that a lot with the size Q will not be usable until it is fulfilled with the items, even if a machine failure occurs during the production time. Almost all EPQ models with perishable items assume that perishability has a constant rate or follows a probability distribution, mostly exponential decay, happening in a single period. To the best of our knowledge, none of them have considered products' lifetimes more than one period. These assumptions will be more unrealistic when it comes to studying the perishable items with random shelf life of more than one period in an unreliable production-inventory system.

Notwithstanding its popularity and its implementation in wide range of problems, EPQ is not an appropriate way of modeling for this type of problem because of the abovementioned limitations. Due to constant production rate in reviewed EPQ models, the average inventory level held in the system under EPQ is greater than the case of HPP. This is because in cases of machine failure, the lot in use is seized until the end of the repair process, and amount produced in the lot is not usable. Thus, there should be more products in the system to meet the customer's demand. On the other hand, seizing the lots is an assumption putting a constraint on EPQ models and will result in a lower level of serviceable inventory and a higher risk of backlog. Perishability, in case of occurrence, is higher in EPQ than HPP. This is due to the fact the age of items produced before and after the repair process are identical, and there is no lot including items with different ages, which is an unrealistic assumption in multi-period EPQ models. Since the fulfillment of a lot may take more time, e.g., several units of time, in case of random machine failures and repairs, EPQ model results a bit higher product perishability.

However, under proposed HPP, items can be immediately consumed after production. Moreover, the system under EPQ can produce at the maximum rate, if operational, while the system under HPP can produce at flexible rates, either at the maximum rate or at the average demand rate, depending on the total stock level. Provided the limitations of EPQ models in practice, HPP is adopted in solving the problem under study. As already mentioned, the implementation of HPP in FPMSs with perishable products has been limited to some few cases as mentioned earlier, but in none of them, neither the randomness in demand rate nor in products' lifetimes has been studied.

To solve the problem, several methods in tackling the problem of production planning of perishable products have been proposed, namely simulation, queuing models, optimization methods such as Stochastic Dynamic Programming, Integer Programming or (Meta) heuristics, what-if scenario analysis, statistical analyses, and so forth (Belien & Force, 2012).

As an effort to provide a solution for problems in which HPP has been adopted, Kenne, Gharbi, and Boukas (1997) tried to determine whether the improvement of near-optimal conditions can be obtained by using analytical results as input for simulation models. This was motivated by the fact that a mathematical model could not take all details about the structure and the dynamic of an unreliable manufacturing system into account. Thus, a simulation-based approach was used to predict the system performance and evaluate several production and maintenance strategies, algorithms, rules, etc. Following that, Kenne and Gharbi (1999) described a combinational approach to control the production and preventive maintenance rates in a manufacturing system using computer simulation experiments. The idea of combining analytical approaches and simulation-based optimization methods has been successfully carried out in the literature dealing with similar problems. Earlier, Gupta and Erol (1993) applied this approach to determine the optimum inventory policy which minimizes the average cost per unit of time using simulation output. The combination of simulation and experimental design techniques is also used by Spedding, De Souza, Lee, and Lee (1998) to find optimum buffer size and the number of pallets that maximize keyboard assembly cell throughput.

Computer simulation modeling is one of the most powerful, cost-effective and ease of use techniques (Haijema, van Dijk, van der Wal, & Smit Sibinga, 2009). It can be utilized to analyze multidimensional and chaotic systems and make their dynamics more comprehensible for the practitioners to help them find ways of responding to changes in their structures or assumptions. This confirms the complexity of the problems at hand since simulation -which does not suffer from major computational issues- is often practical in cases where it is rather difficult to find an optimal policy. The main disadvantage of this methodology is being unable to provide any guarantee of optimality and its dependency on the assumptions (Belien & Force, 2012). It is not always clear to what extent the results can be generalized.

The simulation optimization concept is associated with the techniques of optimizing stochastic simulations, including the search for specific settings of input parameters to a stochastic simulation so that the function of the simulation output is minimized without loss of generality (Amaran, Sahinidis, Sharda, & Bury, 2014). In this regard, simulation optimization methods have been categorized in for main approaches, consisting of Stochastic Approximation (gradient-based approaches), Response Surface Methodology, Random Search, and Ranking and Selection (Fu, 2015). Since in this research, Response Surface Methodology is utilized, we will review it shortly.

In Response Surface Methodology (RSM), the objective is to find the relationship between the input variables and the response variables. The process commences with an attempt to fit a linear regression model. If it is not satisfactory to attain the P-value lower than the significance level, a higher degree polynomial regression, usually quadratic, will be implemented. The process of finding a good relationship between input and response variable will be done for each simulation test. However, RSM does not concentrate on characterizing the objective function in the whole solution space but focus on the local area that the search is currently exploring. Nonetheless, RSM is limited in the number of variables designed to handle. The main limitation of this technique is that RSM is a ‘black box’ approach which does not provide gradient information, and particularly cannot make any assumption on the analytic form of objective function (Cox & Baybutt, 1981).

The simulation-based optimization can be used in different fields such as in supply chain management as Rached Hlioui, Gharbi, and Hajji (2015), in manufacturing systems as Rivera-Gómez et al. (2016), and in inventory control as Alrefaei and Diabat (2009).

As an instance, Sajadi, Seyed Esfahani, and Sörensen (2011) concentrated on a single-product failure-prone manufacturing network with fixed, predetermined demand rate in order to minimize the total cost incurred to the system by means of a combination of simulation modeling and the response surface methodology. Another endeavor in applying approximation policies amongst lots of works in this domain to overcome the problem complexities was done by Haijema (2013), where he applied a combination of dynamic programming and simulation to reduce the state space and provide “near-optimal” order-up-to-level inventory policies.

It is also noteworthy to mention that when it comes to the simulation-based optimization, it is significant to make a simulation model which is fast and robust enough. It should be fast to be able to make a wide range of replications and it should be robust enough to deal with the uncertainty and randomness involved in the system (Ait El Cadi, Gharbi, & Artiba, 2016).

1.5 Research problem and motivation

Having accomplished the review, some gaps existing in the literature still exist. Even though HPP has been pulled off in various problems of production planning of imperishable products in unreliable systems, to the best of our knowledge, it has been touched and utilized for perishable items in some few cases as mentioned earlier, but in none of them, neither the randomness in demand rate nor in products' lifetimes has been studied.

The purpose of this research is to propose modified control policies for a failure-prone manufacturing system with perishable items in presence of randomness in machine failures and repairs, demand rate and products' lifetimes based on Hedging Point Policy and then to evaluate the system performance in terms of its total cost, including inventory holding cost, backlog cost and disposal cost of the perished items. The sub-problems addressed in this research are therefore introduced as follows:

Table 1.1 Research problem characteristics

	Problem I	**Problem II**	**Problem III**
Demand Rate	Constant	Constant	Variable
Product Lifetime	Constant	Variable	Constant
Machine Failure and Repair Times	Stochastic	Stochastic	Stochastic
Issuing Priority Rule	FIFO	FIFO / SRLT	FIFO

The manufacturing system under study (Figure 1.3) consists of an unreliable manufacturing machine producing a single type of perishable item. In this problem, it is assumed that customer demand gets through the system at a constant and/or variable rate and must be immediately met. Since the backlog is allowed and may occur, thus a make-to-stock inventory system to avoid shortages would be a desirable measure. It is necessary to trace the age of units in stock to control the inventory accordingly.

To do so, it is assumed that products are labelled at the end of each working period, a portion of them is simultaneously consumed in that period and the remaining will be consumed in the following periods. As an accepted rule, products cannot be infinitely stored in the buffer. Every single product has a lifetime before expiration, which can be constant or variable, but known and the "age" of the products is traceable as time passes. The age of products will be periodically checked at the beginning of each period in this regard.

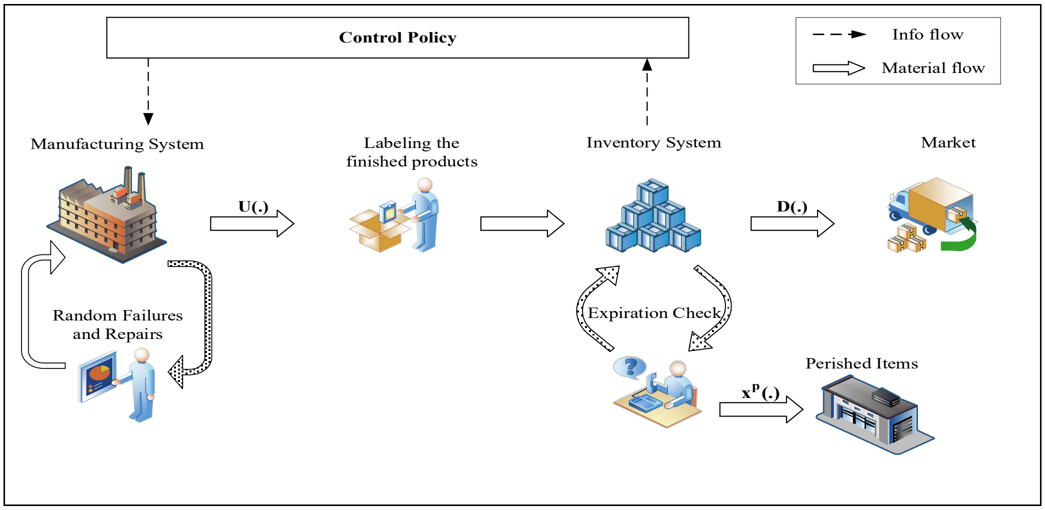


Figure 1.3 The failure-prone manufacturing system under study

The main assumptions considered in this work are as follows:

1. Backlog is allowed and the customers' demand should be immediately met;
2. Products remaining in inventory after their shelf life (T) are no longer usable or desirable and should be immediately discarded from the inventory on-hand;
3. The age of stored products is checked at the beginning of each period;
4. There is no return, recovery, or rectification allowed for the perished items;
5. The planning horizon is infinite.

In this study, the aim is to respond to the following research questions:

- How can the companies with failure-prone manufacturing systems reduce extra costs incurred to their system such as the cost of perishability by determining the optimal control policy? What is the most cost-effective policy?
- What measures can be taken or what technologies are required for companies to monitor perishable goods and to reduce the number of perished ones?
- Which strategies may be practical for companies if it is to reduce the number of perished items? And how much will they cost?

Therefore, our contribution concerning this problem can be summarized as follows:

- Production planning for unreliable manufacturing systems in a dynamic and stochastic context considering the perishability of products while randomness such as machine failures and repairs, variable demand rates, and variable product lifetimes exist in the system.
- Developing optimal, cost-effective control policies for such systems with perishable items to be sure that they can improve their production-inventory operations while keeping their business profitable.

1.6 Solution methodology

Customer satisfaction can be negatively affected by shortages, which may result in balking or reneging the demands in extreme situations. Should a higher level of inventory be maintained to avoid backlogs, the system may face with augmented costs of holding inventory or with an increasing risk of the perishability of items. Although a trade-off between customer satisfaction and cost minimization is reachable through appropriate inventory management policies, the stochastic phenomena existing in the system make such rules ineffective and inapplicable.

Given the complexity of the system dynamics described by HPP equations, because of interactions between products' perishability and randomness in demand rate, products' lifetimes and machine failures, and the high stochastic nature of the problem under study, it is almost impossible to find an analytical solution.

To solve the problem and estimate the optimal values of control parameters, an experimental approach including simulation modeling, Design of Experiments (DOE) and Response Surface Methodology (RSM) has been utilized as indicated in Entezaminia et al. (2019). The main steps of the approach are as follows (Figure 1.4):

Step1: Control policy description

The structure of near-optimal control policies proposed

Step 2: Simulation model

The simulation model is developed to reflect the dynamic system behavior based on the proposed control policies (as the model input) to conduct the experiments and to evaluate the total cost as the system performance criterion.

Step 3: Design of experiment and response surface methodology

In DOE and RSM, the number of experiments and the levels of the considered factor(s) are defined. To specify the main and quadratic effects of the factor on the total cost and the interaction between independent variables (if applicable), the analysis of variance (ANOVA) is conducted. Finally, a response surface methodology is utilized to find out the relationship between the independent variable(s) (i.e., Threshold(s)) and the dependent variable (i.e., total cost), and as a result, to optimize it by finding the best setting of control parameter(s).

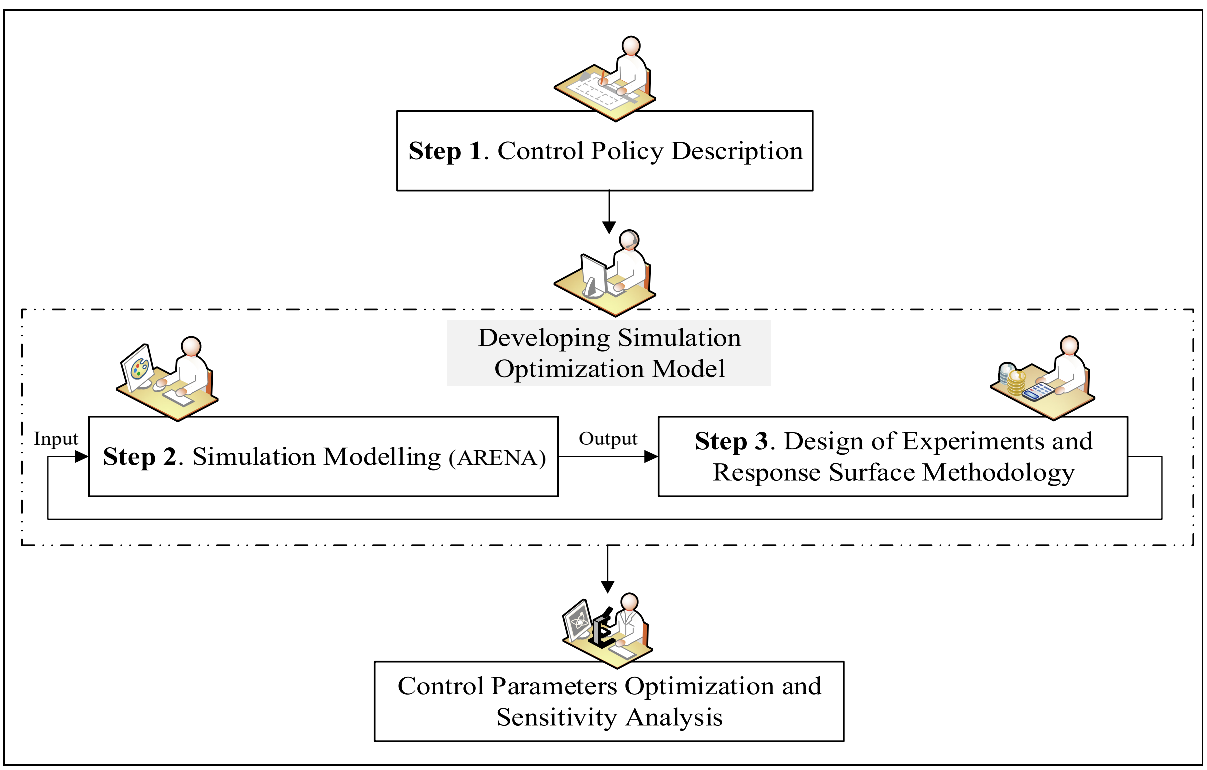


Figure 1.4 Proposed methodology

In this study, having developed the mathematical model based on the problem statement, ARENA software is used to simulate the described manufacturing system by continuous/discrete models, which provide us with the opportunity of having realistic models of the stochastic and dynamic behavior of the system. The continuous modelling enables us making the simulation model faster, while discrete modelling provides us with opportunity to trace the age of items continuously produced and consumed in different working periods. Continuously produced items such as miscible liquids are indistinguishable, and no one can differentiate between what has been produced early in the working period and what produced on late, but the discretization is an appropriate way to trace the age of products based on the period they have been produced in. To estimate the optimal values of cost and control parameters, an experimental approach including a simulation technique, Design of Experiments (DOE) and Response Surface Methodology (RSM) is included. A single/multi-factor statistical analysis of the simulated data is carried out by ANOVA to show the effects of the design factor(s), their interaction (in multi-factor cases) and their main and quadratic effects on the response variable (i.e., the total cost). Henceforth, the main significant factors and their interactions are considered as the input of an RA, used in conjunction with RSM to fit the relationship between the cost and the input factors. RSM is a collection of mathematical and statistical techniques useful for modeling and analyzing problems where the objective is to optimize the response of interest influenced by several variables (Montgomery, 2017).

1.7 Simulation model and validation

In control theory, the optimal solutions are sometimes difficult to reach from the traditional optimization methods and are often approximated by numerical methods under specific conditions. Considering the limitations imposed by the numerical approaches which meet the structure of the control policy, a combinational approach including simulation, DOE and RSM will be utilized as in Gharbi, Kenné, and Hajji (2006). This approach enables us to calculate the total cost of the system under different scenarios and let us find an experimental-based near-optimal threshold for the production-inventory system with consideration of the perishability of products.

Benefiting from the simulation model, various scenarios can be examined with different assumptions, and comprehensive sensitivity analyses can be conducted to verify the accuracy of the developed model. In this study, «ARENA» software is utilized to develop a combined discrete/continuous simulation model for the proposed control policies.

This approach provides us with the possibility of developing a realistic model of the stochastic and dynamic behavior of the system. The continuous modelling enables us making the simulation model faster, while discrete modelling provides us with opportunity to trace the age of items continuously produced and consumed in different working periods. Continuously produced items such as miscible liquids are indistinguishable, and no one can differentiate between what has been produced early in the working period and what produced on late, but the discretization is an appropriate way to trace the age of products based on the period they have been produced in. Figure 1.5 represents the procedure of the simulation model under the proposed control policies.

The model is initialized by defining the parameters required for simulation, e.g., U^{max}., D (.), $MTTF$, etc. (Block 1). Then, the manufacturing system (Block 2) starts producing items according to the state of the system (Block 3) and production policy (Block 4) to satisfy the customers' demands (Block 5). Inventory dynamics of the system is described by (Block 6). The simulation time advances until the end of predetermined time (period length) (Block 7). The age of products is checked and then, perished items are discarded from the system (Block 8), if any, after their shelf life, the total serviceable inventory level x(t) and the age of remaining products are updated (Block 9), which results in updating the control policy (Block 4) affecting the production rate (Block 2). Just upon finishing the simulation run time, the average total cost incurred to the system, including holding, backlog, and perishability, is calculated, and documented by (Block 10).

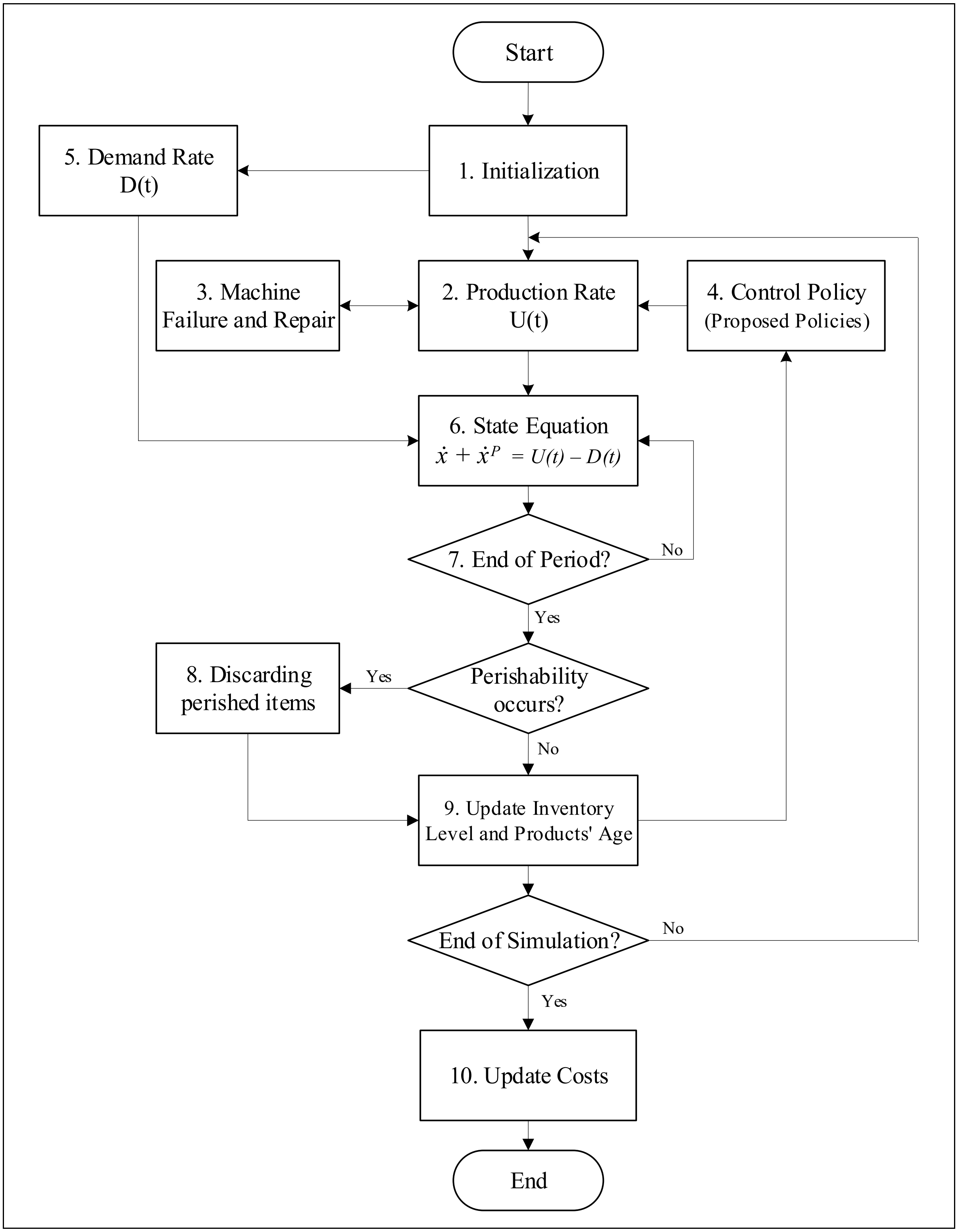


Figure 1.5 Diagram of the simulation model

1.8 Conclusion

Integration of theoretical and applied perspectives is crucial in academic contexts. It gives us consistency in terms of conceptualization, modeling, analysis, and applications. It may also lead to consequent advances and develop novel insights and ideas since a powerful, influential commencement has been established and can be further expanded upon.

In this chapter, we initially took a quick look at manufacturing systems, especially those that are failure-prone and producing items that are subject to perishability and presented a general structure for the study which is pursued in the following chapters. In the next step, an attempt was done in conducting an in-depth literature review of previous works in this domain with respect to the problem under study and its contributing parameters. Having accomplished the review, some gaps existing in the literature are found, i.e., even though HPP has been pulled off in various problems of production planning of imperishable products in unreliable systems, it has only been studied in few cases with perishable items.

We thereupon commenced addressing the problem and adopting an experimental design approach toward solving that. It was pointed out that we would be able to develop an optimal control policy for the production planning of perishable products in the presence of stochastic phenomena, such as random machine failures and repairs, demand rate, and products' lifetime, through this methodology, allowing us to demonstrate the importance of the interaction of different decisions to ensure better performance of the inventory system.

CHAPTER 2

AN OPTIMAL PRODUCTION CONTROL POLICY FOR THE PERISHABLE PRODUCTS WITH CONSTANT DEMAND RATE AND CONSTANT LIFETIME

2.1 Introduction

In this chapter, we are interested in dealing with the problem of producing perishable items in a failure-prone manufacturing system in which product lifetime is deterministic and known and customer demand follows a constant-rate pattern. The principal objective of this chapter is to find an optimal control policy to minimize the total cost incurred to the production-inventory system. Having described the problem and its characteristics, a mathematical model will be suggested, followed by proposing a simulation-based approach to solve it. Afterward, a numerical instance will be examined, and a thorough sensitivity analysis will be performed to check the effectiveness of the proposed policy.

2.2 Problem description

In this problem, it is assumed that customer demand gets through the system at a constant rate and must be immediately met. Since the backlog may occur, thus a make-to-stock approach is required to compensate probable shortages due to the machine unavailability. It is also necessary to trace the age of units in stock to discard the perished items from the system accordingly. In this problem, it is assumed that every single product has a known, constant shelf life before expiration, and the "age" of them is checked at the beginning of each period.

2.3 Problem formulation

For the manufacturing system under study, $U(t, \alpha)$ denotes the production rate of the failure-prone machine. For any time t, the state variables of this discrete/continuous system, $\big(x(t), \alpha(t)\big)$, describe the dynamics of the production-inventory system. In the manufacturing system, discrete section, $\{\alpha(t),\ t > 0\} \in B\,\{0, 1\}$ describes the different states of the manufacturing machine, which means if the machine is operational, it takes the value 1, otherwise, it will be zero. Figure 2.1 depicts the diagram of the state transitions in the system.

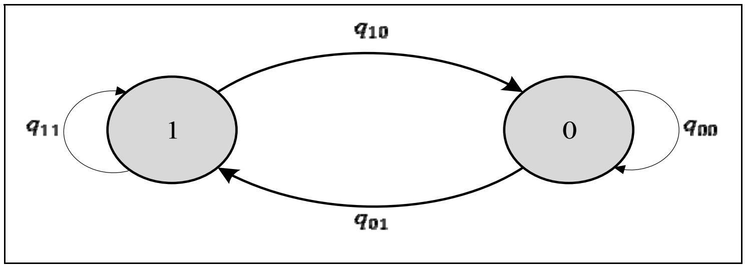


Figure 2.1 State transition diagram of the considered manufacturing machine

The sequence of possible machine states can be described by a continuous-time discrete-state Markov Chain with a transition rate matrix $Q = q_{ab}$ as defined by Gershwin (1994):

$$P[\alpha(t+\delta t) = b \mid \alpha(t) = a] = \begin{cases} q_{ab}\delta t + o(\delta t) & if\ a \neq b \\ 1 + q_{ab}\delta t + o(\delta t) & if\ a = b \end{cases} \tag{2.1}$$

$$q_{ab} \geq 0\,;\ \forall\, a \neq b \quad \& \quad a, b \in B\,\{0,1\}$$

$$q_{aa} = -\sum_{a \neq b} q_{ab}$$

$$Q = \begin{bmatrix} q_{11} & q_{10} \\ q_{01} & q_{00} \end{bmatrix} = \begin{bmatrix} -q_{10} & q_{10} \\ q_{01} & -q_{01} \end{bmatrix} \tag{2.2}$$

Where $P[\alpha(t+\delta t) = b \mid \alpha(t) = a]$ denotes that the conditional probability that the state of machine is b at the time $t + \delta t$, if the state of machine be a at the time t. Given $\pi_0\ and\ \pi_1$, representing the limiting probabilities of modes 0 and 1 respectively, are the solutions of the following equations:

$$\sum_{j=0}^{1} \pi_j \cdot q_{ij} = 0 \quad ; \forall\, i = \{0,1\}$$
$$\sum_{j=0}^{1} \pi_j = 1 \tag{2.3}$$

Having solved these equations, we will have:

$$\pi_0 = \frac{q_{10}}{q_{01} + q_{10}} \qquad\qquad \pi_1 = \frac{q_{01}}{q_{01} + q_{10}}$$

If failure and repair times are exponentially distributed, where $MTTF = \frac{1}{q_{10}}$ and $MTTR = \frac{1}{q_{01}}$, the availability of machine is calculated as follows:

$$Av. = (MTTF)/(MTTF + MTTR) \tag{2.4}$$

Some feedbacks are sent to the system once the inventory level reaches or exceeds the threshold level or the machine fails. In the continuous section, $x(t)$ and $x^p(t)$ describe the cumulative usable inventory surplus level and cumulative perished items before being discarded from the total inventory, respectively. The dynamics of total inventory level including serviceable and perished items is presented by equation (2.5):

$$\dot{x} + \dot{x}^p = U(t) - D(t) \mid x(0) = x_0, x^p(0) = x_0^p \tag{2.5}$$

x_0 and x_0^p indicate the initial inventory level and the initial perished items in the system, respectively. Since the age of products is checked at the beginning of each period, let us say t, $x^p(t^-)$ shows the number of perished items at the end of the previous period, or (t^-), and during the working period, $x^p(t^-)$ will return zero. The dynamics of the system has been explained in section 1.7. The production rate, at any time, must satisfy the capacity constraint of the manufacturing system given by the equation (2.6):

$$\Gamma(.) = \{U \in \mathbb{R} \mid 0 \leq U(.) \leq U^{max}\} \tag{2.6}$$

To ensure that the system is feasible, the following constraint should be checked and met:

$$U^{max} \cdot Av. \geq \overline{D} + Pr. \tag{2.7}$$

Where $\overline{D}$ is the average demand that has been considered in equation (2.7) to make sure the system remains feasible in the long term in case of variable demand rate. If the demand rate is constant, the average demand rate will be equal to the actual rate *D*. Nonetheless, it is difficult to consider the rate of perishability ($Pr.$) in ensuring the system feasibility and provide an analytical solution since this rate highly depends on stochastic parameters of the system, their types, and their interactions, e.g., demand rate, product's lifetime, and failure and repair times. The total cost, as the system performance criterion, consists of the inventory holding, backlog and disposal costs of the perished items. The disposal cost is calculated only in cases that expiry dates have passed the shelf life. Thus, these products must be discarded from the system. The inventory, backlog and perishability costs can be calculated as follows:

$$g(x(t)) = c^+x^+ + c^-x^- \quad | \quad x(0) = x_0 \tag{2.8}$$

$$h(x(t)) = c^p \cdot \int_0^\infty x^p(t)\,dt \quad | \quad x^p(0) = x_0^p \tag{2.9}$$

where $x^+ = max(0, x)$, $x^- = max(-x, 0)$, and c^+, c^- and c^p are inventory, backlog, and perishability costs, respectively. Since the perishability of products does not happen all the time, an average cost of perishability has been considered in calculations. Using the equations (2.8) and (2.9), the average expected total cost can be achieved as:

$$J(x(t), \alpha(t)) = E\left\{\int_0^\infty g(x(t)) + h(x(t))\,dt \,\middle|\, x(0) = x_0, \alpha(0) = \alpha\right\} \mid \forall\, \alpha \in \{0,1\} \tag{2.10}$$

The optimal control policy attempts to minimize $J(.)$ subject to constraints (2.4) to (2.7) to determine the production rate of the machine under study as a function of $x(t)$ and $\alpha(t)$.

2.4 Proposed control policy

The problem under study consists of a failure-prone manufacturing system producing one type of perishable product. In case of constant demand rate and imperishable products, Bielecki and Kumar (1988) analytically proved that the HPP, with only one optimal threshold level $Z_{B.K}{}^*$ as in equation (1.2), is the optimal solution of the control policy presented in equation (2.11) for a similar unreliable system with a Markovian process and imperishable products:

$$U(x(t), \alpha(t)) = \begin{cases} \alpha(t) \cdot U^{max} & if\ x(t) < Z \\ \alpha(t) \cdot D & if\ x(t) = Z \\ 0 & o.w. \end{cases} \tag{2.11}$$

Perishability adds an extra dimension to the production-inventory system; inventory managers do not intend to have excessive quantities of perishable products in their stock; thus, they are likely to produce fewer products than they would in the case of non-perishable products. This is because these products will not be usable or desirable after their shelf life. Given that there are enough inventories available in the system, the amount required to meet the customer constant demand rate (D) within the products' shelf life (T) will be ($D \cdot T$). In other words, in case of existing the amount $D \cdot T$ in the inventory system, they are fully consumed with the constant rate D over the course of T units of time.

Let us assume the inventory level $x(t)$ at the time t exceeds $D \cdot T$. As products have already been in stock, their age must be greater than zero. Now, let us advance the time by T and consider the system at the time $t + T$. As mentioned above, the amount $D \cdot T$ will be consumed and the remaining will either store or perish. Technically, no leftover inventory can remain in the system having passed the course of T. This is because the age of the leftover items has already exceeded their shelf life T (their initial age was greater than zero at the time t). Therefore, any amount higher than $D \cdot T$, i.e., $x(t) - D \cdot T$, will perish and must be disposed.

On the other hand, it was mentioned that ${Z_{B.K}}^*$ can be calculated when products are imperishable, and the failures and repairs are based on a Markovian process. Knowing that if the inventory capacity is up to $D \cdot T$, or the so-called "no-perishability" threshold, all perishable items will be fully consumed within their shelf life. Thus, up to this level, these perishable items can be treated as imperishable ones and ${Z_{B.K}}^*$ can be utilized in determining the optimal inventory threshold and it is guaranteed that it results in the minimum total cost incurred to the system. Nonetheless, if the system parameters return a value for ${Z_{B.K}}^*$ greater than $D \cdot T$, all amount of inventory exceeding $D \cdot T$ will perish, resulting in a cost of perishability add to the total cost of the system.

The equation (2.12) is an extension added to the classical HPP to consider the case in which products are subject to *perishability*.

$$\begin{cases} Z = Z_{B.K}{}^{*} & if \quad Z_{B.K}{}^{*} \leq D \cdot T \\ Z = D \cdot T & if \quad Z_{B.K}{}^{*} > D \cdot T \end{cases} \tag{2.12}$$

To sum up, the $min\ \{Z_{B.K}{}^{*}, D \cdot T\}$ is chosen by the model, which result in the minimum total cost incurred to the system. Under this policy, no perishability happens, and all products are consumed before their expiration.

2.5 Resolution approach

In order to solve the problem under study and estimate the optimal values for the total cost and the control parameter, a combined experimental resolution approach of the simulation modeling, the design of experiments and the response surface methodology has been utilized as described in section 1.6 based on Entezaminia et al. (2019).

2.5.1 Simulation model and validation of the case with constant demand rate and constant lifetime

In control theory, the optimal solutions are sometimes difficult to reach from the traditional optimization methods and are often approximated by numerical methods under specific conditions. Considering the limitations imposed by the numerical approaches which meet the structure of the control policy, a combinational approach including simulation, DOE and RSM will be utilized as in Gharbi et al. (2006). This approach enables us to calculate the total cost of the system under different scenarios and let us find an experimental-based near-optimal threshold for the production-inventory system with consideration of the perishability of products.

In order to verify the accuracy of the simulation model based on the diagram presented in Figure 1.5, a dynamic testing is performed to evaluate the results and behavior of the system in the situation that the demand rate and product life time are constant and predetermined, and the policy represented in equations (2.11) and (2.12) are applied. Figure 2.2 demonstrates the results obtained when the parameters are set as $T = 24$ periods, $D = 60$, $U^{max} = 65$, $TTF \sim EXP\ (12)$ and $TTR \sim EXP\ (0.9)$ based on time units and $Z = 1500$:

- $t < 3170$ units of time; according to HPP, since $x(t) < Z$ (Figure 2.2.b), thus system produces items at its maximum production rate, i.e., $U(t) = U^{max}$ (Figure 2.2.a)
- $t = 3170$ units of time; at ①, the machine adjusts its production rate to the demand rate, i.e., $U(t) = D$ (Figure 2.2.a) since $x(t) = Z$ (Figure 2.2.b).

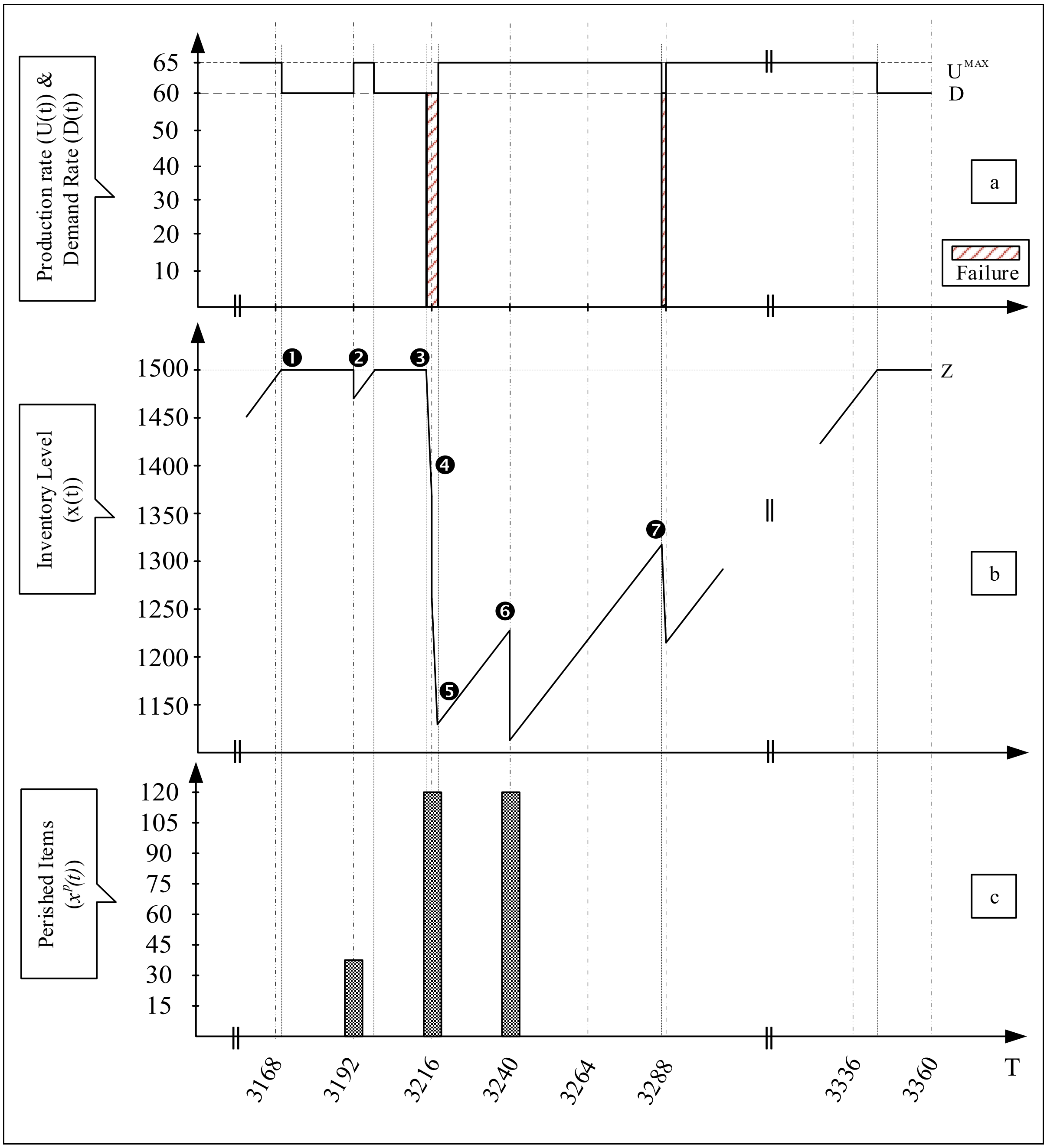


Figure 2.2 System dynamics with constant demand rate and applied equations (2.11) and (2.12)

- $t \leq 3192$ units of time; the machine has been producing with demand rate until ②, where items with age more than 24 units of time perish and must be discarded from inventory (Figure 2.2.c). Hence, at this point, the inventory level decreases by the level of perished items, resulting in $x(t) < Z$ (Figure 2.2.b)and as a result, the system should start producing with its maximum production rate, i.e., $U(t) = U^{max}$ (Figure 2.2.a).
- $3200 \leq t \leq 3212$ units of time; the inventory has already reached at Z and the system has been producing with the demand rate (Figure 2.2.a). At ③, a machine failure occurs, and the production process discontinues. From this point on, demands are met with the serviceable inventory on hand aged less than 24 units of time.
- $t = 3216$ units of time; the machine is still down and under repair. In the meanwhile, at ④, some products with the age exceeding 24 units of time perish and are removed from the system (Figure 2.2.c) and inventory level will further decrease (Figure 2.2.b).
- $3216 < t < 3240$ units of time; Machine is not operational until ⑤. At this point, repair process terminates, and the machine begins working with maximum rate (Figure 2.2.a) since the inventory level is lower than the threshold Z (Figure 2.2.b).
- $t = 3240$ units of time; While the machine is working with a maximum rate and inventory level is lower than the threshold, another production lot perishes (Figure 2.2.b) and causes a drop in the inventory system at ⑥ (Figure 2.2.b).
- $3240 < t \leq 3283$ units of time; System is working with U^{max} (Figure 2.2.a) until another failure takes place at ⑦, making us consume the inventory on hand to meet the customer demand (Figure 2.2.b).

Having examined, verified, and internally validated the simulation model and obtained results, it can be mentioned that the developed simulation model satisfactorily describes the dynamics of the production-inventory system under study.

2.5.2 Design of experiment and response surface methodology for the case of constant demand rate and constant lifetime

It was discussed that in case of constant demand rate, the classical HPP in equation (2.11) and the extension presented in equation (2.12) can be adopted for the problem under study regardless of the type of randomness in product's shelf life.

Now, some experiments are conducted for a range of the factor under study (i.e., Z) to observe the behavior of the response variable (i.e., $J(z)$) under different assumptions and to estimate the relationship between the factor and the response variable. Hereof, the experimental design is concerned with one factor for the simulation model, and with setting the levels for the considered factor and making the decisions on the conditions of the study, such as the length of simulation runs, the number of replications and system parameters.

Table 2.1 System parameters for model with constant demand rate and constant shelf life

Parameter	c^{+}	c^{-}	c^{p}	TTF	TTR	U^{max}	D	T
Value	1	10	15	Exp(12)	Exp(0.9)	65	60	24

To consider the uncertainties existing in model as much as possible and to have better precision for each level, 25 replications for different values of Z leading to 175 runs in total have been conducted. The simulation is performed by ARENA simulation software and each run takes $N = 1{,}500{,}000$ units of time to ensure that the steady state has been reached. The experiments are completely randomized. Then, the statistical software STATGRAPHICS is utilized to conduct a one-factor analysis of variance (ANOVA) and obtain the effect of independent variable (i.e., Z) and its quadratic effect on the dependent variable (i.e., total cost). After that, the model's overall performance is evaluated. This is addressed by the coefficient of determination R-squared and the adjusted R-squared which represent the proportion of total variation in the model explained by the second-order regression model (Myers, Montgomery, & Anderson-Cook, 2016).

The ANOVA report for the constant demand rate and constant product's shelf life has been summarized in Table 2.2, indicating that the main factor and the quadratic effect are significant at a level of 95% significance level. The obtained $R^2 = 99.9\%$ indicates that the RSM model comprises almost all the variability in the expected total cost.

The corresponding response function is given as follows:

$$J(z) = 3429.57 - 3.49348 * Z + 0.00143609 * Z^2 + \varepsilon \quad (2.13)$$

Table 2.2 ANOVA for models with constant demand rate and constant lifetime

Source	*Sum of Squares*	*D.F*	*Mean Square*	*F-Ratio*	*P-Value*
A: Z	1.7061E6	1	1.7061E6	97430.97	0.0000
AA	1.6221E6	1	1.6221E6	92632.95	0.0000
Total error	3011.85	172	17.51		
Total (corr.)	3.3312E6	174			
$R^2 = 99.9\%$					

The main and quadratic effects of the independent variable (i.e., Z) on the response variable (i.e., $J(z)$) are presented in Figure 2.3.

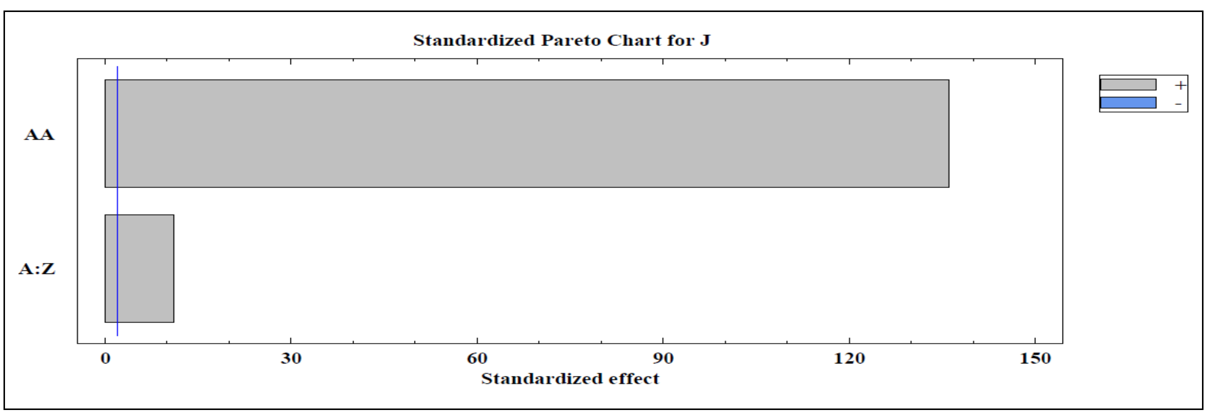


Figure 2.3 Standardized Pareto Chart for total cost with constant demand rate and constant lifetime

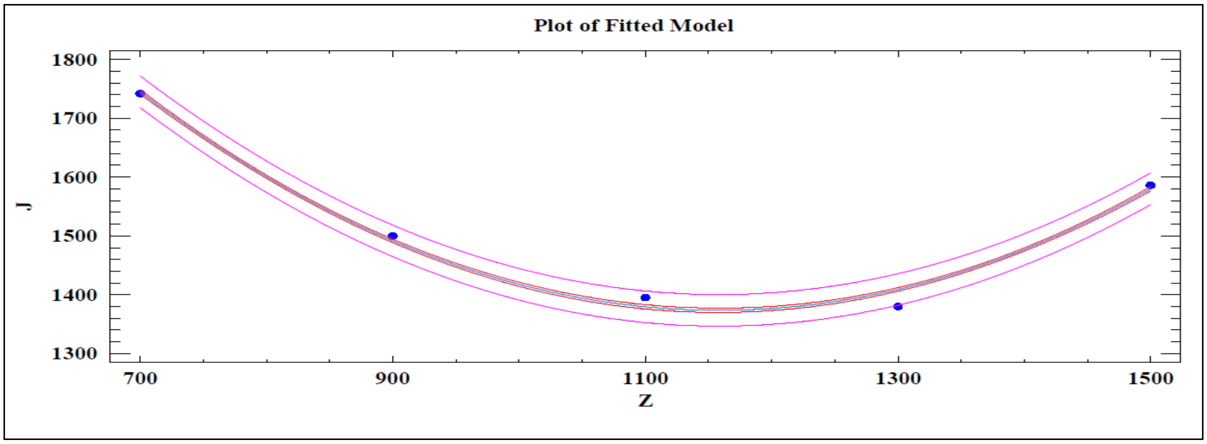


Figure 2.4 Plot of the fitted model with constant demand rate and constant lifetime

The optimum threshold level obtained through the process is $Z^* = 1216$, with $J(Z^*) = 1305$.

To check the accuracy and suitability of the second-order regression model in the local region of the optimal solution, the three steps will be as follows (Noguera & Watson, 2006), (Lavoie, Gharbi, & Kenné, 2010), (Myers et al., 2016):

1. The model's overall performance is evaluated. This is addressed by the coefficient of multiple determination R-squared and the adjusted R-squared which represent the proportion of total variation explained by the second-order regression model. The values of these two coefficients should be close to 1.0.
2. A complete residual analysis is done to check the normality assumption and the homogeneity of residuals.
3. Having performed the optimization, the optimum value should be cross-checked to ensure its validity. To do so, Z^* is considered the input for the simulation model to verify that the optimal cost $J(Z^*)$ falls in the following confidence interval:

$$\hat{J}(n) \pm t_{n-1,1-\alpha/2} \cdot S_n / \sqrt{n} \tag{2.14}$$

that is obtained after n replications of the simulation. $\hat{J}(n)$ is the average cost obtained after n replications; $t_{n-1,1-\alpha/2}$ is the student coefficient function of n and α with (1-α) confidence interval; and S_n is the sample standard deviation.

Table 2.2 contains the required information for the steps one and two. R-squared indicates that the model as fitted explains almost all the variability in the expected total cost $J(z)$. From 50 additional replications of simulation, the solution was validated by verifying that the estimated optimal cost $J(Z^*) = 1305$ falls within the 95% confidence interval [1280, 1313].

2.6 Comparison between the proposed policy and EPQ model with constant demand rate and constant Lifetime

Another production policy widely used by researchers to study the perishability of products is EPQ. To compare the system performance under proposed HPP and classical EPQ in terms of the total cost they impose on the system, a model based on EPQ was developed.

Under the EPQ, the system should have produced the optimal lot size (Q^*) before start consuming products and when the inventory level of each lot reaches at the safety level (s^*), a new production run will start. Q^* and s^* are the decision variables, which should be optimized.

Sometimes, more than one period will be required to complete Q^*. Thus, the products in each lot may have different ages, but in reality, it is difficult to trace the age of products in each lot when it varies from product to product. To simplify the model, it is assumed that the age of products in each lot is the same, however it is not a realistic assumption. Since in this study, the shelf life of products is greater than one period, the EPQ model can still be developed.

However, under proposed HPP, items can be immediately consumed after production. Moreover, the system under EPQ can produce at the maximum rate, if operational, while the system under HPP can produce either at the maximum capacity or at the demand rate depending on the total stock level. Knowing that, if the system parameters in Table 1 are applied to both developed models, the optimal control parameters under HPP and EPQ models will be summarized in Table 2.3.

Table 2.3 Performance comparison between proposed HPP and EPQ with constant demand rate and constant lifetime

	HPP	***EPQ***
Decision Variable(s)	$Z^* = 1216$	$(Q^* = 65, s^* = 5)$
Total Cost	$J(Z^*) = 1305$	$J(Q^*, s^*) = 1{,}619$
95% confidence interval	$[1280, 1313]$	$[1598, 1631]$
Average Inventory Cost	$c^+x^+ = 684$	$c^+x^+ = 739$
Average Backlog Cost	$c^-x^- = 621$	$c^-x^- = 695$
Average Perishability Cost	$c^px^p = 0$	$c^px^p = 185$

The higher total cost of the system under EPQ results from the operational constraints of the system, e.g., the fixed production rate, or inability to consume the stored items if Q^* has not been reached at. Due to the fixed, higher production rate in EPQ than HPP, the average inventory level in EPQ is higher than HPP. Moreover, when failures occur, the incomplete lots are not usable but are counted as inventory on-hand and holding cost should be paid for them.

In some cases, both holding and backlog costs should be paid if machine is down with a lot in progress and serviceable inventory reaching at zero. Backlogs often take place in these situations. Because of holding more inventory under EPQ to compensate backlogs, the chance of perishability of products under EPQ is higher and more products perish after their lifetime.

2.7 Comparison between the proposed policy and classical Bielecki and Kumar's model

It is already mentioned that researchers provided an analytical solution of a similar unreliable system with constant demand rate of imperishable products and exponentially distributed machine failure and repair times, represented by equations (1.2) and (1.3). In case of constant demand rate, if product's lifetime is also constant, the Bielecki and Kumar's policy will be applicable to find out the optimal threshold up to $D \cdot T$ level, which is the maximum level of inventory at which products will not perish based on their shelf life.

To present the point numerically, if ${Z_{B.K}}^*$ is calculated based on the system parameters of the Table 2.1, the outcome will be ${Z_{B.K}}^* = 1242$ and $J({Z_{B.K}}^*) = 1292$ with the 95% confidence interval [1274, 1301]. Having compared the results with the outcomes of the equation (2.13), it reveals that there is no significant difference between the proposed policy and Bielecki and Kumar's policy since the confidence intervals overlap. Moreover, since there is no perishability at this level of ${Z_{B.K}}^*$, no conclusion can be reached on the effect of perishability on the total cost incurred to the system. In this case, if other values of Z are tested, the obtained results of total cost will be higher than $J({Z_{B.K}}^*)$. Figure 2.5 illustrates the abovementioned point when systems parameters are set as in Table 2.1

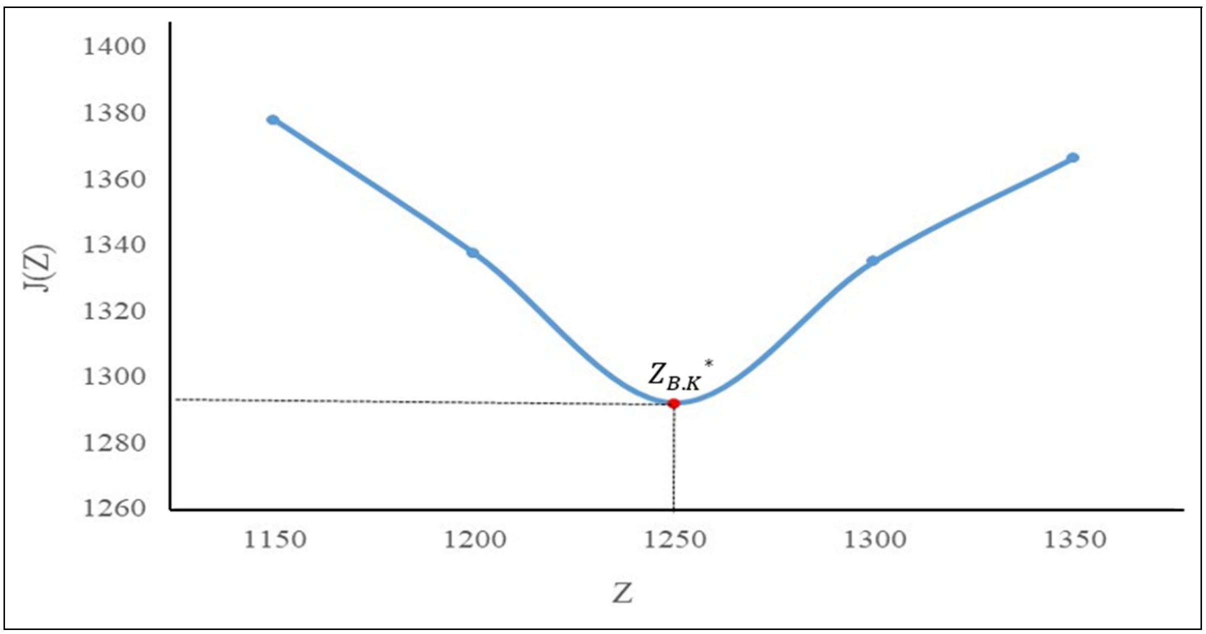


Figure 2.5 Total cost incurred to the system when Table 2.1 with different values of Z

${Z_{B.K}}^*$ is calculated based on $U^{max}, D, c^+, c^-, TTF$ and TTR. Thus, for instance, if the TTR is set on 0.92, it will result in ${Z_{B.K}}^* = 1601$ and $J({Z_{B.K}}^*) = 1653$ with the 95% confidence interval [1648, 1666]. Now, if new experiments based on Table 2.1 and with the new MTTR are run, it is observed that the optimal threshold level obtained from the response function will be equal to $Z^* = 1431.34$, which is almost equal to $D \cdot T$, and $J(Z^*) = 1612$ with the 95% confidence interval [1603, 1621]. Since there is no overlap between these two intervals ($J({Z_{B.K}}^*) > J(Z^*)$), it means there is a significant difference between ${Z_{B.K}}^*$ and Z^* and since

${Z_{B.K}}^* > D \cdot T = 1440$, a portion of stored items will perish and must be discarded from inventory system.

As is evident from Figure 2.6, for $Z^* < D \cdot T$, the different costs of perishability (c^p) have no impact on the total cost incurred to the system, since there is no perishability at these inventory levels. However, the higher threshold, the higher inventory holding (c^+x^+) but lower backlog cost (c^-x^-), and as a result, the lower total cost (note that $c^- > c^+$). This trend is valid up to the point where products start perishing ($D \cdot T$). From this point on, the higher thresholds may reduce the backlog cost, but they result in higher inventory holding cost and disposal cost of the perished items ($c^p x^p$). Now, the higher cost of perishability, the higher total cost of system. The findings reveal that the system will face with the lowest total cost if no product's perishability occurs. That means the system should store the perishable items up to a threshold at which there is no perished items ($x^p = 0$), which is $D \cdot T = 1440$.

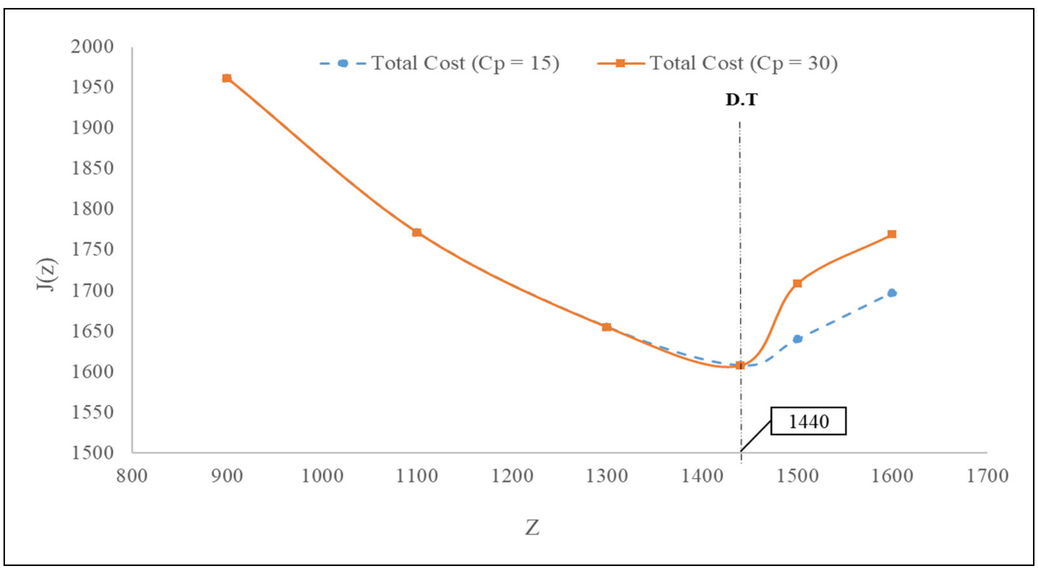


Figure 2.6 The effect of c^p on the total cost when $TTR \sim exp\ (0.92)$

As already mentioned, when ${Z_{B.K}}^* > D \cdot T$, minimum total cost will be incurred to the system if $Z^* = D \cdot T$ and where ${Z_{B.K}}^* < D \cdot T$, the $Z^* = {Z_{B.K}}^*$ will results in the minimum total cost.

2.8 Conclusion

The purpose of this chapter was to adapt the classical hedging point policy for the problem of production planning of perishable products in a failure-prone manufacturing system under stochastic failure and process, constant product lifetimes, and constant demand rate in order to minimize the total cost incurred to the production-inventory system, including inventory holding cost, backlog cost and disposal cost of perished items.

Based on the simulation-based optimization approach, optimal values of control parameters were determined using an experimental approach and response surface methodology. The results show that for the situation with constant demand rate, the classical hedging point policy works well. If the product's lifetime is also constant, the optimal threshold should not exceed the limit at which products start perishing. In the situations that Bielecki and Kumar's policy returns the optimal threshold less than or equal to the "no perishability" level, there is no significant discrepancy between results from this formula and results from the simulation-based approach. However, if it returns an optimal value greater than "no perishability" level, the system should refrain from storing more items to avoid product's perishability as it becomes a matter of importance to save on the perishability cost.

For a managerial point of view, if the machine is operational, when the optimal threshold based on Bielecki and Kumar's policy returns a value lower than $D \cdot T = 1440$, here ${Z_{B.K}}^{*} = 1216$, it is set as the optimal threshold, and the inventory level is compared to this level. If it is lower, then the machine should work on its maximum capacity, or $U^{max} = 65$ products per unit of time. If inventory level reaches at the threshold ${Z_{B.K}}^{*}$, the production rate should be set on average demand rate, or $D = 60$ products per unit of time. Otherwise, the machine should stop working. On the other hand, if Bielecki and Kumar's policy returns a value more than 1440, then $D \cdot T = 1440$ is set as the optimal threshold, and the inventory level is compared to that. Now, if inventory level is lower than 1440, the rate of production will be set on maximum, i.e. $U^{max} = 65$ products per unit of time. If it is equal, the machine will produce at average demand rate, $D = 60$ products per unit of time. Otherwise, the machine should stop working. The summary has been shown in Figure 2.7.

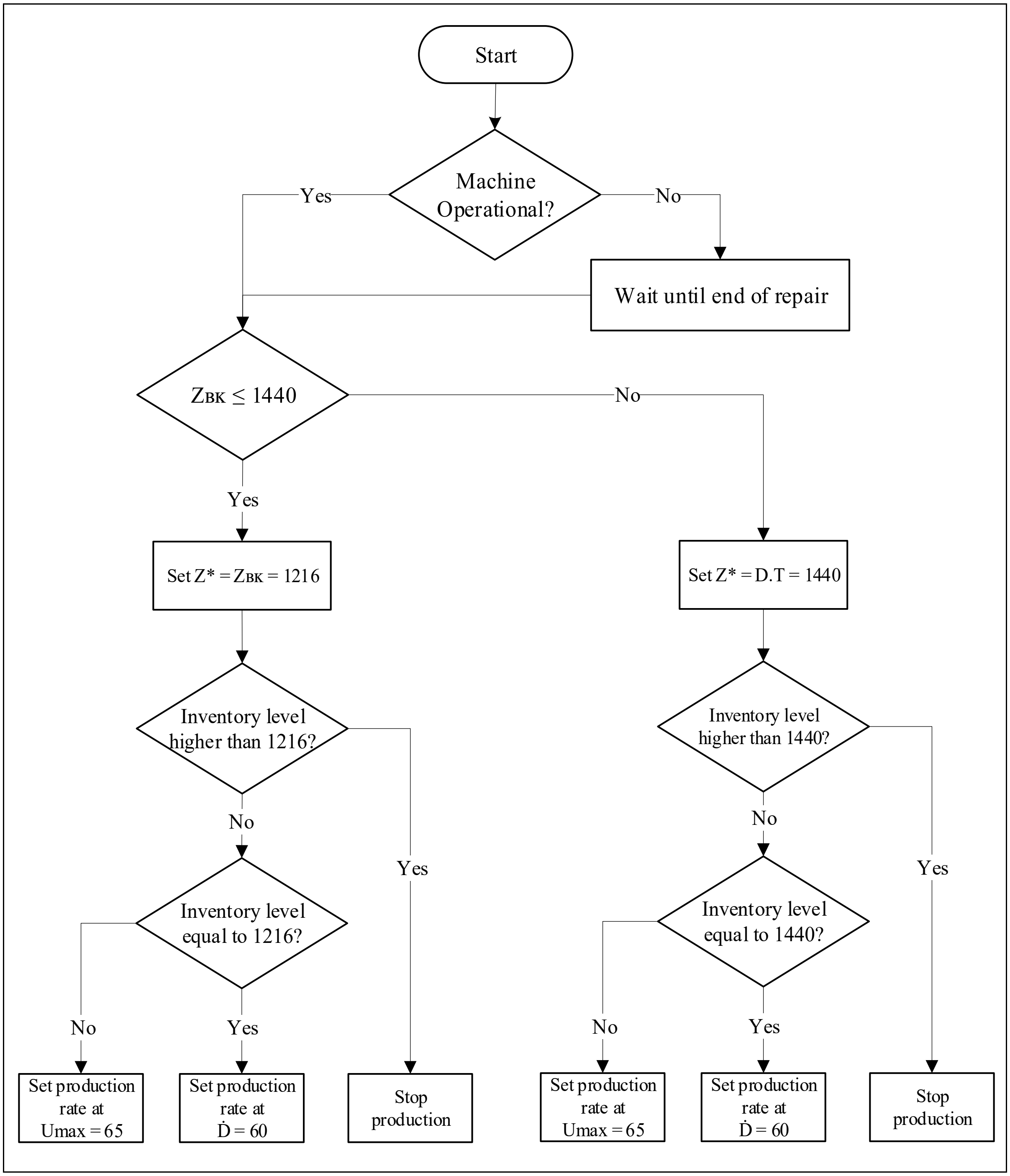


Figure 2.7 Logic chart for implementing the proposed policy (2.11) and (2.12)

CHAPTER 3

AN OPTIMAL PRODUCTION CONTROL POLICY FOR THE PERISHABLE PRODUCTS WITH CONSTANT DEMAND RATE AND VARIABLE LIFETIME

3.1 Introduction

In this chapter, the problem of perishable inventory management will be addressed where the product's lifetime is modeled as a random variable, following an arbitrary distribution. This is a rich modeling allowing us to cover various cases of lifetime variability in practice. The focus will be on the study of this variability and its impact on the system performance, i.e., to what extent may such lifetime variability affect the inventory system operating costs?

To figure it out, a simulation-based study is conducted to examine the important effect of lifetime variability mixed with the cost parameters on the total operating cost.

3.2 Problem description

It was discussed that in case of no randomness in the product lifetime and constant demand rate, the optimal inventory level should not exceed the no-perishability level, i.e., $D \cdot T$. However, the perishability of products will happen at this level if every single product has a random lifetime before expiration. These lifetimes may vary from an item to an item and assumed to be identically randomly distributed.

On account of the periodical age control, the lifetimes are required to be discretized by rounding them down. The reason behind discretizing the lifetimes is that sometimes products remain in stock until their expiration date while they have not been fully consumed. Thus, the remaining should be consumed in the following period. In those cases, there is no idea whether products are perished or usable, because as assumed, they have been continuously produced, and are indistinguishable, such as miscible liquids.

As a case in point, it is assumed that products' lifetimes are normally distributed with mean 24 as in Table 3.1. Based on the empirical rule, or 3-sigma rule, almost all observed data will fall within three standard deviations (denoted by σ) of the mean (denoted by μ) if data follows Normal Distribution, i.e., they vary from $24 - 3\sigma$ to $24 + 3\sigma$. For the illustrative purpose, it is assumed that products' lifetimes are normally distributed with the mean of 24 and standard deviation of 2 periods, i.e., $T \sim N(24, 2)$, and other system parameters remain unchanged.

Table 3.1 Possible normally distributed products' lifetimes after discretization

μ	σ	3σ (per unit of time)	Possible lifetimes after discretization
24	2	6	18, 19, 20, 21, 22 ,23, 24, 25, 26, 27, 28, 29, 30

3.3 Problem formulation and proposed control policy

Since the variable lifetimes only affect the products and their expiry dates, or in other words, only the product's characteristics, the problem is formulated as presented in section 2.3 and the combination of classical HPP, in equation (2.11), and proposed policy extension in equation (2.12) can be still applied and the decision variable (Z) remains unchanged.

Further to provide an efficient turnover of the inventory with random lifetimes, inventory decision-makers try to forward products with different priority rules. To evaluate different priority rules to tackle with the variable lifetimes, First-In First-Out (FIFO) and Shortest Remaining Lifetime (SRLT) priority rules will be compared in terms of their total costs as the priority rules frequently used in practice.

3.3.1 First-In First-Out (FIFO)

For items with no printed expiration date, FIFO issuing rule is widely used in practice, since it is not convenient for customers to identify the products' age or their expiration date. FIFO removes the oldest stored items first, and can be a fair rule, where there is no reliable way to track the age and the expiry date of the items in case of variable lifetimes, and it prioritizes the earliest produced items. In that case, the FIFO rule issues the "oldest" item in stock, resulting in having "younger" items with longer expected remaining shelf life on hand, which allows us to keep the items in stock with longer expected remaining times. Given that the system parameters are still set as in Table 2.1 and Table 3.1 and the decision-makers use FIFO priority rule for issuing their available inventory, the optimum threshold level obtained through the second-order expression of the response surface function, presented in equation (3.1), will be ${Z_{FIFO}}^* = 977$ with $J_{FIFO}(Z^*) = 1803$ and the 95% confidence interval [1791, 1823].

$$J_{FIFO}(z) = 4167.93 - 5.08905 * \mathrm{Z} + 0.00233135 * Z^2 + \varepsilon \quad (3.1)$$

3.3.2 Shortest Remaining Lifetime (SRLT)

There is no doubt that if there is reliable information regarding the expiration date, the FIFO the FIFO may require to be replaced with another issuing rule which release the items having the shortest remaining shelf life (Ioannidis et al., 2012). The use of state-of-the-art technologies are required to obtain the information on residual lifetimes, which enable us to continually update the remaining lifetime of the products (Piramuthu & Zhou, 2013), (Grunow & Piramuthu, 2013). Real-time inventory monitoring and shelf-life prediction can provide a more effective decision-support tool for managers in the supply chain system. Given that the required technologies for tracking the products' remaining lifetime or predicting their expiration dates are accessible, applying the Shortest Remaining Lifetime (SRLT) priority rule would be an interesting option to release available inventories with the shortest remaining lifetime (Spedding et al., 1998). SRLT issues the stored items based on the shortest remaining lifetime, which is not necessarily the "oldest" stored item. optimum threshold level obtained from the equation (3.2) representing the response surface function is ${Z_{SRLT}}^* = 1032$ with $J_{SRLT}(Z^*) = 1647$ and the 95% confidence interval [1631, 1659].

$$J_{SRLT}(z) = 3747.69 - 4.0998 * \mathrm{Z} + 0.00176991 * \mathrm{Z}^2 + \varepsilon \quad (3.2)$$

3.4 Discussion on the degree of the lifetime's variability

The randomness in products' lifetimes adds an extra dimension to the main problem together with the perishability of products. In fact, this variability of the lifetime has a direct impact on the level of perished items. By applying different degrees of variability leads to the following results in terms of the inventory issuing priority rules applied.

Table 3.2 Study on lifetime variability when FIFO is applied

Scenarios	σ	c^+	c^-	c^p	TTF	TTR	T	Z^*	Note
Variation of lifetime (σ) - FIFO									
Base	**2**	**1**	**10**	**15**	**Exp (12)**	**Exp (0.9)**	**24**	**977**	**-**
1	1	1	10	15	Exp (12)	Exp (0.9)	24	1,032	Z↑
2	3	1	10	15	Exp (12)	Exp (0.9)	24	948	Z↓

Table 3.3 Study on lifetime variability when SRLT is applied

Scenarios	σ	c^+	c^-	c^p	TTF	TTR	T	Z^*	Note
Variation of lifetime (σ) - SRLT									
Base	**2**	**1**	**10**	**15**	**Exp (12)**	**Exp (0.9)**	**24**	**1032**	**-**
1	1	1	10	15	Exp (12)	Exp (0.9)	24	1,125	Z↑
2	3	1	10	15	Exp (12)	Exp (0.9)	24	976	Z↓

In both cases, results suggest that in case variable shelf life, the optimal threshold will be lower than the cases without variability in shelf life (${Z_0}^* = 1216$) because of augmented perishability cost incurred in such situations. It is probable that the expiry date for different products varies in terms of the conditions that products are stored in.

As is evident, by increasing the lifetime's variability, more perishability happens in the system, resulting in lower serviceable inventory, and higher shortages. Due to higher perishability cost, the system starts storing fewer products to protect itself against increasing perishability of products. In the meantime, by decreasing the threshold, the backlog cost will go up to the extent at which saving the perishability cost ($c^p x^p$) is no longer beneficial for the system in presence of the augmented cost of backlog ($c^- x^-$).

The experiments show that the tipping points for both FIFO and SRLT rules when the system parameters are set based on Table 1, would be when products' lifetimes have variabilities more than 3 periods and less than 4 periods, or $\sigma \in [3, 4]$ (Figure 3.1).

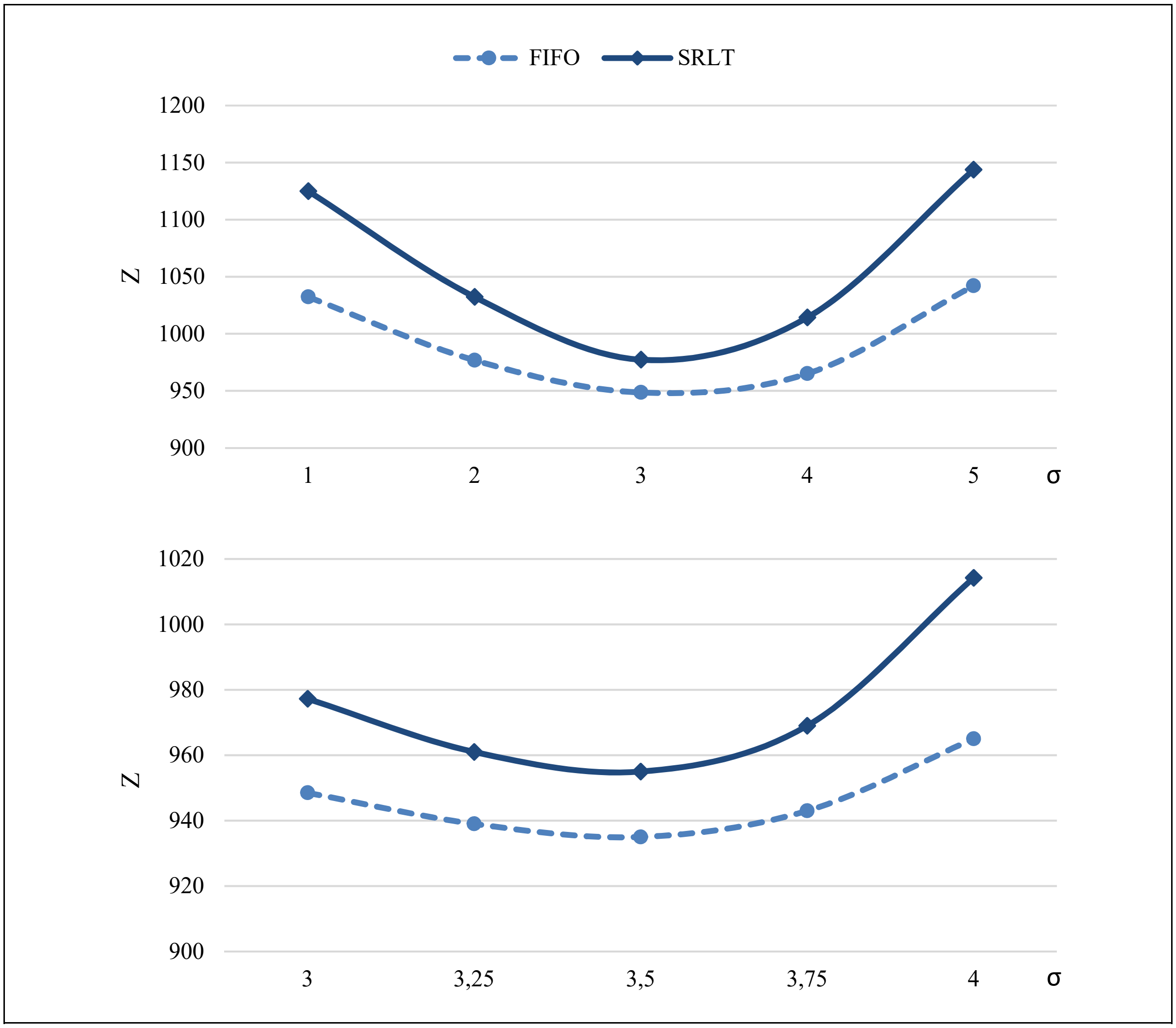


Figure 3.1 Comparing FIFO vs. SLRT rules in terms of Z^* when lifetime variability changes

To find out the cause of this behavior, it should be mentioned that the lifetime is an attribute of the products and when it becomes a variable, the higher degrees of lifetime variability will lead to more perishability of products. As a result, the system tends to store less products (i.e., lower Z) to avoid perishability as much as possible. In the meantime, as products perish more and inventory level decreases as well, backlog of products will happen, and backlog cost will become considerable. Now, the system starts storing more products to avoid further backlog in the system, while the perishability of products will increase again.

The initial decreasing trend is valid up to a point, here until $\sigma \cong 3.5$, where the cost of backlog starts playing a more important role in determining the optimal threshold. Due to continuously increasing cost of backlog and to reduce its impact, the system starts storing more products. Thus, the optimal threshold increases to avoid further backlog cost, while now system will have to pay more for storing more products and perishability of them, literally a trade-off between backlog cost, in one hand, and sum of inventory holding and perishability costs, on the other hand.

To find out the tipping point, the ratios $(c^+x^+ + c^p x^p)/TC$ and c^-x^-/TC were compared under both priority rules. The reason behind this choice was that both inventory holding and perishability costs affect the trend of Z adversely, while there is a direct relationship between backlog cost and the trend of Z.

As is evident from simultaneous consideration of Figure 3.1 and Figure 3.2, the trend of Z changes (Figure 3.1) when $c^+x^+ + c^p x^p)/TC \cong c^-x^-/TC$ (Figure 3.2). Under FIFO, the tipping point happens slightly later, due to the fact perishability under FIFO is higher than SRLT. This higher perishability cost will result in a further decrease in optimal threshold Z before its increase.

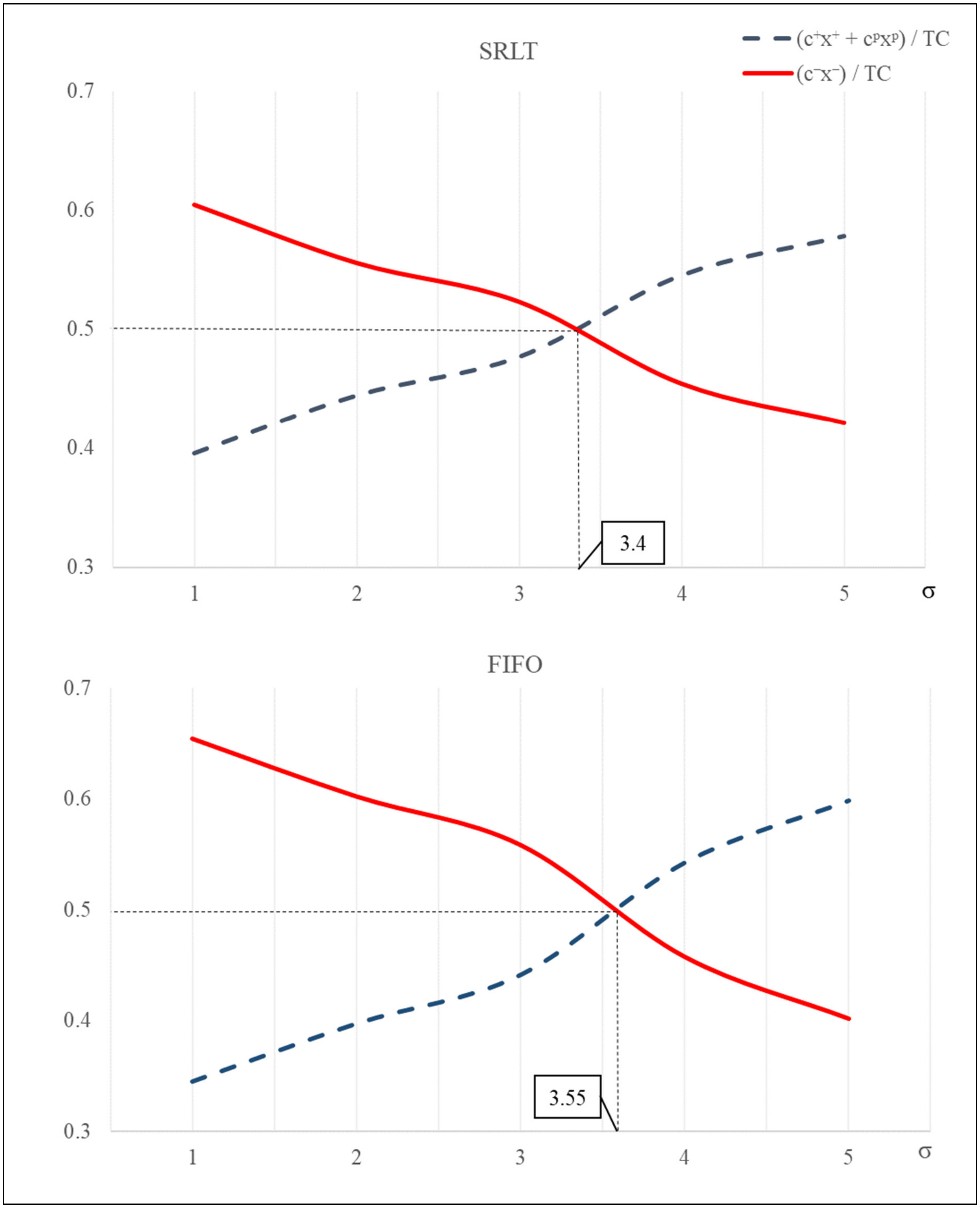


Figure 3.2 Trade-off between the effect of inventory and perishability costs vs. backlog cost – variable lifetime

Generally, by increasing the lifetime variability, the number of perished items and its related cost are continuously increasing, but it comes to studying different priority rules, FIFO always results in a greater number of perished items than SRLT. Similarly, in terms of the total cost incurred to the system as the performance criterion, SRLT results in a better performance than FIFO. As is evident from the results, the higher variability in the system, the wider gap between FIFO and SRLT in terms of their effectiveness in tackling with the perishability (Figure 3.3).

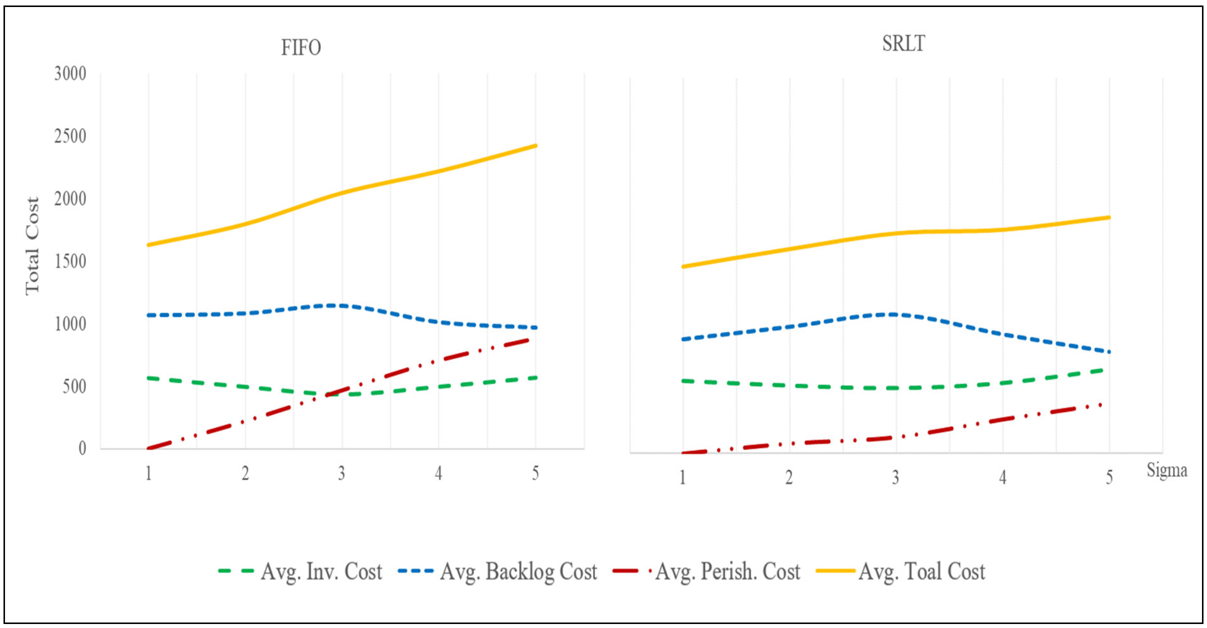


Figure 3.3 Comparing FIFO / SLRT rules in terms of TC^* when lifetime variability changes

When SRLT is applied, the system maintains a higher level of finished products in stock with lower total cost. This is because the perishability of products is lower when SRLT rule is applied, resulting in more serviceable inventory on hand and consequently, lower backlog level. Thus, the inventory system can possess higher threshold level with lower total cost (Figure 3.4).

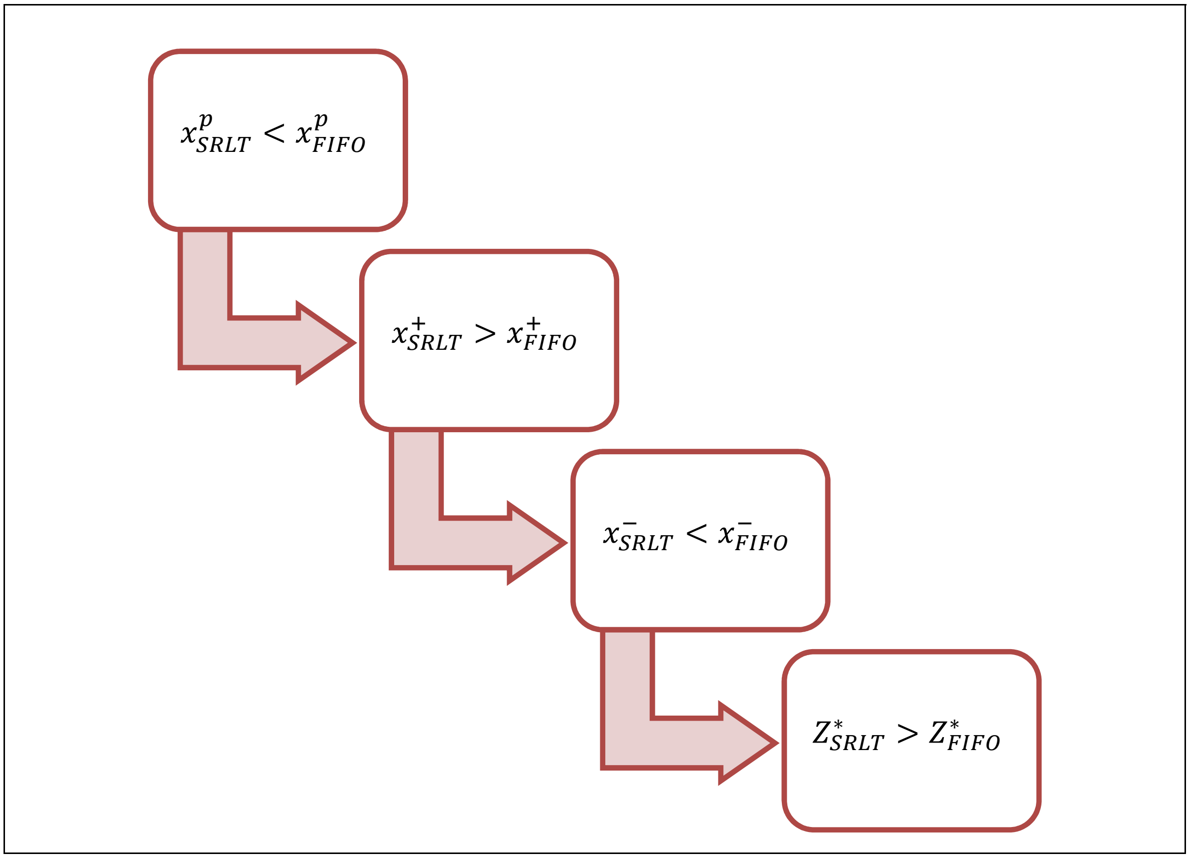


Figure 3.4 Comparison of FIFO vs. SRLT in terms of their performance

3.5 Comparison between the proposed policy and EPQ model with constant demand rate and variable lifetime

Considering the EPQ model presented in section 2.6, an experiment is conducted with developed EPQ model, but with taking variable shelf life into account, to evaluate and compare its performance with classical HPP in terms of the total costs both models incur to the system.

If the system parameters in Table 2.1 are applied to both developed models where demand rate follows Normal Distribution with the mean of 24 and standard deviation of 2 periods, i.e., $T \sim N(24, 2)$, the optimal control parameters under HPP and EPQ models with variable lifetime will be as presented in Table 3.4.

Table 3.4 Performance comparison between proposed HPP and EPQ with constant demand rate and variable lifetime

	HPP	***EPQ***
Decision Variable(s)	$Z^* = 977$	$(Q^* = 57.4, s^* = 3.7)$
Total Cost	$J(Z^*) = 1803$	$J(Q^*, s^*) = 2{,}436$
95% confidence interval	$[1791, 1823]$	$[2418, 2449]$
Average Inventory Cost	$c^+x^+ = 476$	$c^+x^+ = 647$
Average Backlog Cost	$c^-x^- = 1046$	$c^-x^- = 1379$
Average Perishability Cost	$c^px^p = 281$	$c^px^p = 410$

Similar to section 7.4, due to operational limitations and constraints of EPQ model, the total cost under EPQ is higher than HPP when demand rate is constant, and lifetimes are variable.

3.6 Conclusion

The purpose of this chapter was to study the problem of production planning of perishable products in an unreliable manufacturing system under stochastic failure and process, variable product lifetimes, and constant customer demand rate to minimize the total cost incurred to the production-inventory system.

Based on the simulation-based approach, optimal values of control parameters were determined. The results suggest that for the situation with constant demand rate and variable lifetime, the classical hedging point policy works as well. If the product's lifetime is variable, perishability occurs even at the level of "no perishability" level. Thus, applying a proper issuing priority rule will be required to reduce the loss of inventory due to perishability.

Results also show that because of the augmented cost of perishability, the production-inventory system tends to possess fewer inventories on hand as the lifetime's variability increases. However, when it came to evaluate different priority rules to tackle with the variable shelf life, SRLT issuing rule leads to a much more cost-effective performance than FIFO in all scenarios.

For a managerial insight, if the machine is operational and SRLT priority rule is applied in case of variable products' shelf life, when the level of inventory is below the $Z^* = 1032$, the rate of production will be set on maximum, i.e., $U^{max} = 65$ products per unit of time. If it is equal to this threshold, the machine will produce at average demand rate, i.e., $D = 60$ products per unit of time. Otherwise, the production should be stopped. The summary of this insight has been presented in Figure 3.5.

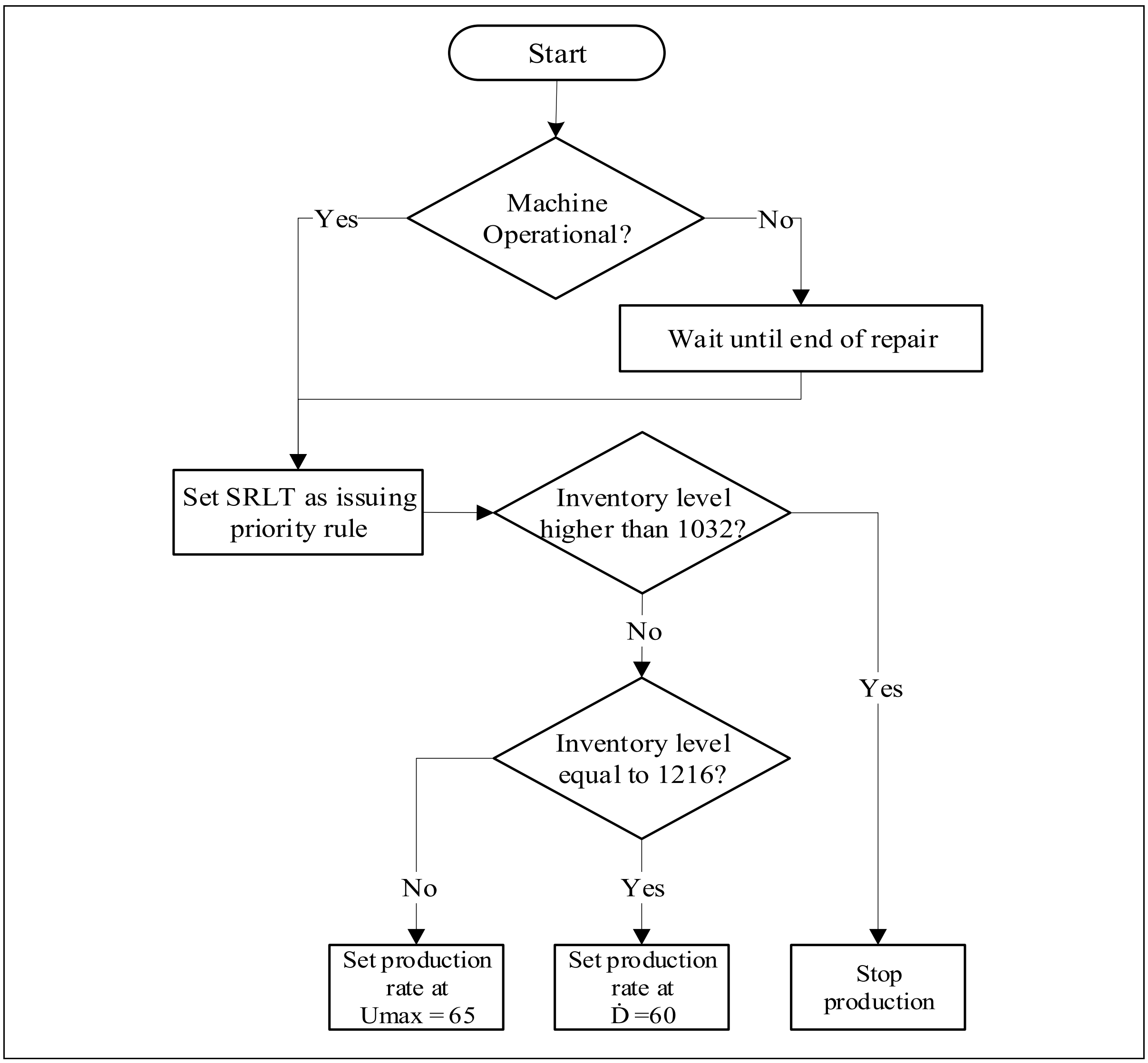


Figure 3.5 Logic chart for implementing the proposed policy with SRLT priority rule

CHAPTER 4

AN OPTIMAL PRODUCTION CONTROL POLICY FOR THE PERISHABLE PRODUCTS WITH VARIABLE DEMAND RATE AND CONSTANT LIFETIME

4.1 Introduction

As presented in section 1.3.3, customer demand may follow different patterns. Amongst them, the stochastic demands are more plausible in studying real world problems. In the current chapter, variable demand rate in the form of an arbitrary distribution and changes coming about in the inventory system and the associated costs will be studied. A modified hedging point policy to consider the variable demand rate will be scrutinized in turn.

4.2 Problem description

It was discussed that in case of no randomness in the product lifetime and constant demand rate, the optimal inventory level never exceeds the no-perishability level, i.e., $D \cdot T$. However, the perishability of products will happen at this level if customer demand rate varies from period to period.

The randomness in demand rate will substantially affect the machine production rate, especially in cases that the inventory level reaches at the threshold. As explained in equation (2.6), the production rate must satisfy the capacity constraint of the manufacturing system. In one hand, if the inventory level reaches at its threshold and demand rate be greater than the maximum production rate, the machine will not be able to produce at the demand rate to comply with the classical HPP as in the form proposed by Bielecki and Kumar in equation (2.11). On the hand, if the demand rate is lower than the maximum production rate, the machine will be required to adjust its production rate to the demand rate in each working period, which technically means the machine should constantly switch its production rate in each period within the interval $[0, U^{\max}]$ depending on the demand rate of the period. Such action based on classical HPP does not seem to be practical and possible.

The interval $[0, U^{\max}]$ comes from the fact that demand rate can be zero or any positive value, and the production rate of the machine cannot exceed its maximum capacity ($U^{\max}$) to meet the customer demand, even if the real demand rate is greater than the capacity.

Given that amount of u^t have been already produced in the period t, and they expire after the constant, predetermined period (T), the following three scenarios may occur at the time $t+T$ while the real total customer demand in the period $t+T$, denoted by d^{t+T}, may be variable due to variable demand rate $D(t)$:

$$if \begin{cases} a. \quad u^t > d^{t+T} & \begin{cases} x^p(t+T) = u^t - d^{t+T} \\ x^-(t+T) = 0 \end{cases} \\ b. \quad u^t = d^{t+T} & \begin{cases} x^p(t+T) = 0 \\ x^-(t+T) = 0 \end{cases} \\ c. \quad u^t < d^{t+T} & \begin{cases} x^p(t+T) = 0 \\ x^-(t+T) = u^t - d^{t+T} \end{cases} \end{cases} \tag{4.1}$$

As equation (4.1.a), if the total demand in the period of perishability $(t+T)$ be lower than the production of period t ($u^t > d^{t+T}$), the leftover products will perish. Equation (4.1.b) explains that in case of a match between production in period t and consumption in $t+T$, there will be no backlog or perishability. On the other hand, equation (4.1.c) states that if the demand in the period of perishability $(t+T)$ be more than the production of period t, there will be a backlog in the system. Kenne and Gharbi (2000) showed that the higher degree of randomness in demand rates results in more backlog in the system. In the current study, the more ups and downs in demand rate will increase not only the backlog, but also the perishability of products.

On the other hand, classical HPP presented in equation (2.11) states that if the inventory level reaches at optimal threshold Z, the system should switch its production rate to demand rate. Now that the demand rate fluctuates over time, the policy represented in equation (1.2) may no longer be applicable. To comply with classical HPP, the real demand rate can be replaced with the average demand rate to reduce variability in customer demand. Moreover, the system should maintain enough inventory around the optimal threshold to meet customer demand.

However, maintaining inventories around optimal threshold Z would be difficult since average demand and real demand rates at the time of reaching at Z may be different and this difference leads to backlog or perishability of products (as explained in equations (4.1.a) and (4.1.c)).

4.3 Proposed control policy

To mitigate the effect of demand uncertainty on the system, an interval can be substituted with the single threshold, where δ, called the coequality threshold, is defined to control the production rate in compliance with the classical HPP. According to this proposition, if the total stock level varies within the interval $[Z - \delta, Z]$, the system only produces items at the average demand rate, instead of having variable production rate based on variable demand rate. The proposed modified policy is formulated as follows:

$$U\big(x(t), \alpha(t)\big) = \begin{cases} \alpha(t) \cdot U^{max} & if\ x(t) < Z - \delta \\ \alpha(t) \cdot \bar{D} & if\ Z - \delta \leq x(t) \leq Z \\ 0 & o.w. \end{cases} \tag{4.2}$$

Our experiments suggest that the policy represented in equation (4.2) with an interval results in lower variability in system and lower incurred total cost comparison with applying the combination of equations (2.11)-(2.12), with only one threshold. By considering only one threshold and the fluctuating demand rate, the system should adjust its production rate in different periods accordingly. When the demand rate is variable, these production rate adjustments lead to more backlogs, and higher thresholds to avoid further backlogs in the system. Higher thresholds then lead to more perishability and consequently, higher total cost.

Since the policy in equation (4.2) considers a fixed, predetermined rate of average demand rate ($\bar{D}$) when the total stock level fluctuates within the coequality interval, it considerably reduces the effect of demand rate variability on the system, and lowers the backlog and perishability, and the total cost of the system. To be more precise, when $x(t) \in [Z - \delta, Z]$ and $D(t)$ is replaced with $\bar{D}$, then the production rate is switched to the average demand rate ($\bar{D}$), the randomness in the system decreases which results in lower backlog (according to equation (4.1.a)) or lower perishability of products (according to equation (4.1.c)).

To mention a similar effort but in the context of lot sizing, Bouslah et al. (2013) proposed an HPP policy with two thresholds, where they adopted a modified HPP to control the production rate, and to decide to produce or not produce a new lot with size Q based on the inventory position at the end of each production cycle to keep the inventory around the optimal threshold Z. They could successfully reduce the variability in the system by proposing an interval, which led to lower total cost incurred to the system. There, the lot size Q was considered a decision variable, which should be optimized, and they tried to maintain the inventory level between $Z - Q$ and Z while the system is operational. In fact, the lot size Q in their policy plays a similar role to δ in the policy represented in equation (4.2).

4.4 Resolution approach

Given the complexity of the system dynamic described by equation (4.2) arising from the stochastic demand rate, the solution methodology proposed in section 1.6 will be utilized.

4.4.1 Simulation model validation for the case with variable demand rate and constant lifetime

To validate the simulation model, a test is performed to evaluate the results and the behavior of the system in the situation that the demand rate is random while shelf life is constant, and the proposed modified policy represented in equation (4.2) is applied. In this regard, system parameters for this test are $T = 5$ periods, $D \sim N(60\,,12)$, $U^{max} = 70$, $TTF \sim EXP\ (135)$ and $TTR \sim EXP\ (20)$ based on time units and $(Z\,,\delta\,) = (20000\,,2500)$. Sample results of this setting are shown in Figure 4.1. Since all the events have not been occurred at the same time, a depiction of the inventory evolution in different periods has been presented.

For ease of reading the graph, the symbol “Y. ®” is used to describe the event shown in the part R of the sub-Figure 4.1. Y. For instance, b.① in Figure 4.1 refers to the event 1 occurred in sub-Figure 4.1.b. This symbol will used with the same sense for the following events. Moreover, u^t and d^t represent the total production and the total demand during the period t.

As can be seen in sub-figures a. and b., as long as the inventory level $x(t)$ fluctuates below the coequality's threshold $Z - \delta$ (b.), the system works on its maximum production capacity while operational (a.). Upon touching the $Z - \delta$ level (b.①), the production rate should be adjusted, being set on it on the average demand rate (a.②). The system continues evolving until the beginning of period 1004, where $\mathrm{D}(1004) < \bar{D}(1004)$ (a.③). Since consumption rate is more production rate, the inventory level starts decreasing until it touches $Z - \delta$ level (b.④). At this point, the system adjusts its production rate and sets it on the maximum rate (a.).

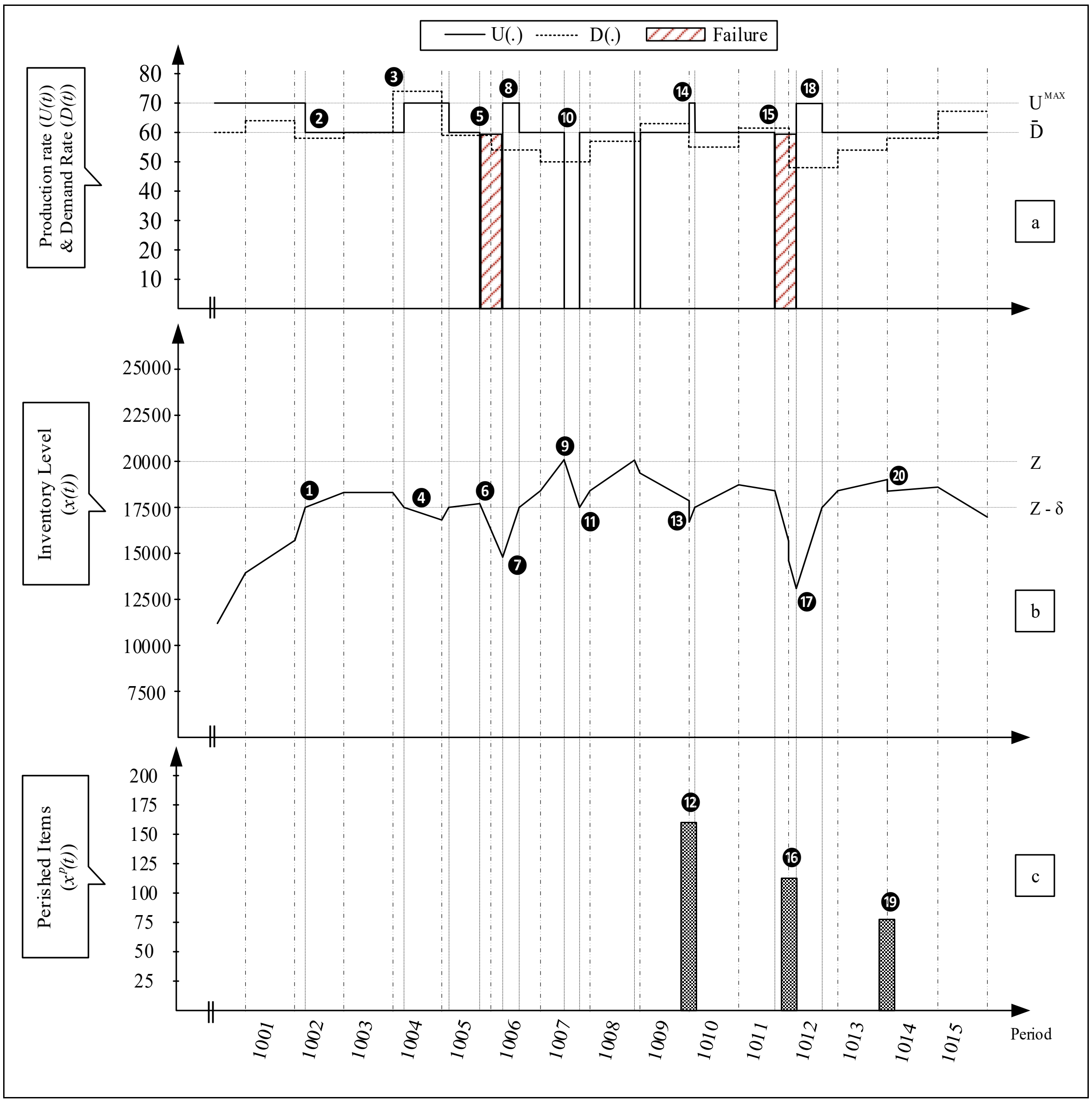


Figure 4.1 Evolution of $x(t)$, $U(t)$ and $x^p(t)$ over time with modified HPP presented in equation (4.2)

When a failure occurs, the machine is no longer able to produce, while the customer demand exists (a.⑤). The inventory level, therefore, starts and continues dropping (b.⑥) until the end of the repair process. Then, since $x(t) < Z - \delta$ (b.⑦), the machine will produce at the maximum rate (a.⑧). The inventory-building process continues until $x(t) = Z$ (b.⑨). At this point, since $\mathrm{D}(1007) < \bar{D}(1007)$ (a.), if the system continues producing the items, the inventory level surpasses the threshold Z. To avoid this situation, the machine temporarily stops working while operational (a.⑩) to let the inventory be consumed until it reaches at $x(t) = Z - \delta$ (b.⑪). Since the machine state is still operational, and $x(1007) \in [Z - \delta\,, Z]$, the machine starts working with the production rate equal to the average demand rate.

Some items being produced and stored in previous periods perish at their expiry date (c.⑫) and must be discarded from the inventory system, resulting a drop in the inventory level (b.⑬). Generally, as mentioned in the equation (4.1), the number of perished items may vary from period to period ($x^p(t) = u^{t-T} - d^t$) depending on the state of the machine, and as a result the amount of production during the period of production (u^{t-T}), and total real demand in the period of the expiry date (d^t). As a case in point for (c.⑫), the machine worked lower than its maximum capacity during the period $t = 1004$ and produced 1618 products, and the real demand during the period of expiry date ($t = 1009$) was 1464 products. Therefore, $x^p(1009) = 154$ products. Having discarded the perished items from the inventory (b.⑬), since $x(t) < Z - \delta$, the system adjusts the production rate to the maximum rate (a.⑭).

The system evolves until the next machine failure (a.⑮), where the next repair process will start and the demands are fulfilled by the current inventory on-hand, resulting a decrease in the level of inventory (b.). In the meantime, products' perishability occurs (c.⑯), resulting in a significant two-aspect decline in the inventory level (b.⑰) until the end of repair process ($x^p(1011) = 118$). Once more, as the production rate and real demand rate may vary in different periods, the amount of products' perishability is variable in different periods (compare c. ⑫/⑯). Since at the end of repair process (b.⑰), $x(t) < Z - \delta$, the system will start working with the maximum production rate (a. ⑱). At the point (c. ⑲), perishability occurs one more time ($x^p(1013) = 72$) but having removed the perished items from the inventory on-hand, the inventory level is still within the coequality interval $[Z - \delta , Z]$ (b.⑳), the system continues producing at the average demand rate (a.).

Having examined, verified, and internally validated the simulation model and obtained results, it can be mentioned that the developed simulation model satisfactorily describes the dynamics of the production-inventory system under study.

4.4.2 Design of experiment and response surface methodology for the case of variable demand rate and constant lifetime

To analyze how the input parameters affect the response variable and then, to estimate the relationship between the total cost and the significant factors, we aim to compute the optimal values of the estimated factors. To show that our simulation optimization model covers a wide range of system configurations under different assumptions, the system parameters are set to the values presented in Table 5 in case of normally distributed demand rate.

Table 4.1 System parameters for model with variable demand rate and constant shelf life

Parameter	c^+	c^-	c^p	TTF	TTR	U^{max}	$\bar{D}$	σ_D	T
Value	1.5	15	30	Exp(135)	Exp(20)	70	60	12	336

In this case, $\bar{D}$ and σ_D are the average demand rate and the demand standard deviation, respectively. A similar approach to what presented in section 2.5.2 has been adopted, except in the number of design factors (i.e., Z and δ). Hence, it is required to utilize a two-factor three-level experimental design (3^2), with 5 replications for each combination of the factors, resulting in 45 simulation runs, to find the admissible region including the optimal control parameter values and to trace the behavior of response variable (i.e., $J(Z, \delta)$) (Bouslah, Gharbi, Pellerin, et al., 2013).

To do so, a multi-factor ANOVA is utilized to find out the effects of independent variables, their interactions, and their quadratic effects on the dependent variable (Entezaminia et al., 2019). The ANOVA report in case of random demand rates is summarized in Table 4.2, showing that all main factors, their quadratic effects, and their interactions are significant at the level of 95%, suggesting that the R-square explains almost 96 percent of the variability in expected total cost of the system. The corresponding response surface function, the equivalent response surface to this function and the main effects plot for the total cost are given as presented in equation (4.3).

$$J(Z,\delta) = 115559 + 5.69872 * \delta - 5.32282 * Z + 0.000552493 * \delta^2 - 0.000292639 * \delta * Z + 0.000123898 * Z^2 + \varepsilon \quad (4.3)$$

Table 4.2 ANOVA for models with variable demand rate and constant product lifetimes

Source	*Sum of Squares*	*D.F*	*Mean Square*	*F-Ratio*	*P-Value*
A: δ	7.05908E7	1	7.05908E7	56.83	0.0000
B: Z	4.0265E8	1	4.0265E8	324.16	0.0000
AA	7.45203E6	1	7.45203E6	6.00	0.0195
AB	6.69046E7	1	6.69046E7	53.86	0.0000
BB	9.59415E7	1	9.59415E7	77.24	0.0000
blocks	5.55515E8	4	1.38879E8	111.81	0.0000
Total error	4.34746E7	35	1.24213E6		
Total (corr.)	1.24253E9	44			
R2 = 95.6%					

The main effects, interactions, and quadratic effects of the independent variables (i.e., Z and δ) on the response variable (i.e., $J(Z,\delta)$) are presented in Figure 4.2.

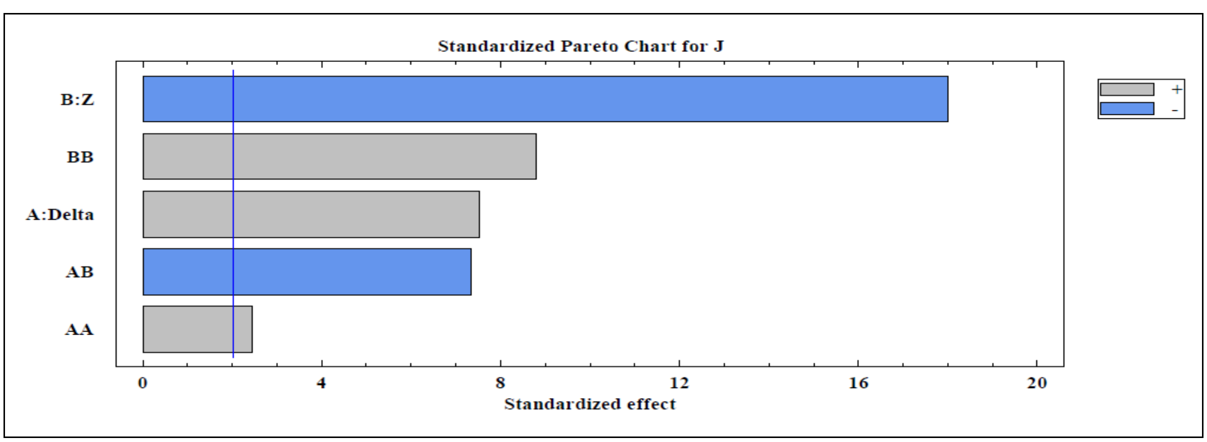


Figure 4.2 Standardized Pareto Chart for the total cost with applied HPP in equation (4.2)

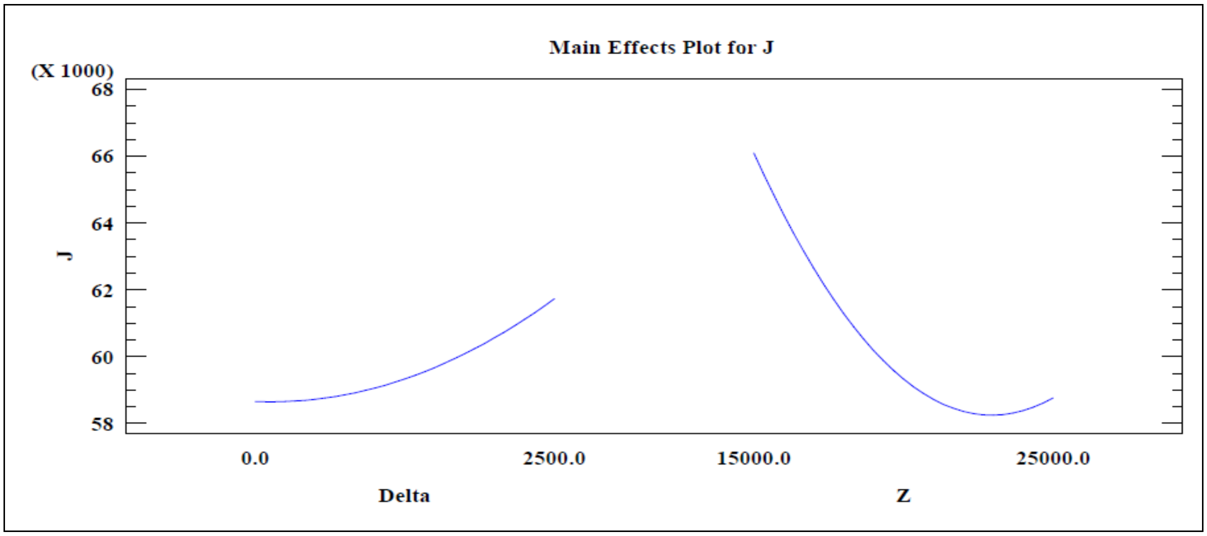


Figure 4.3 Main effects plot with variable demand rate and applied HPP as equation (4.2)

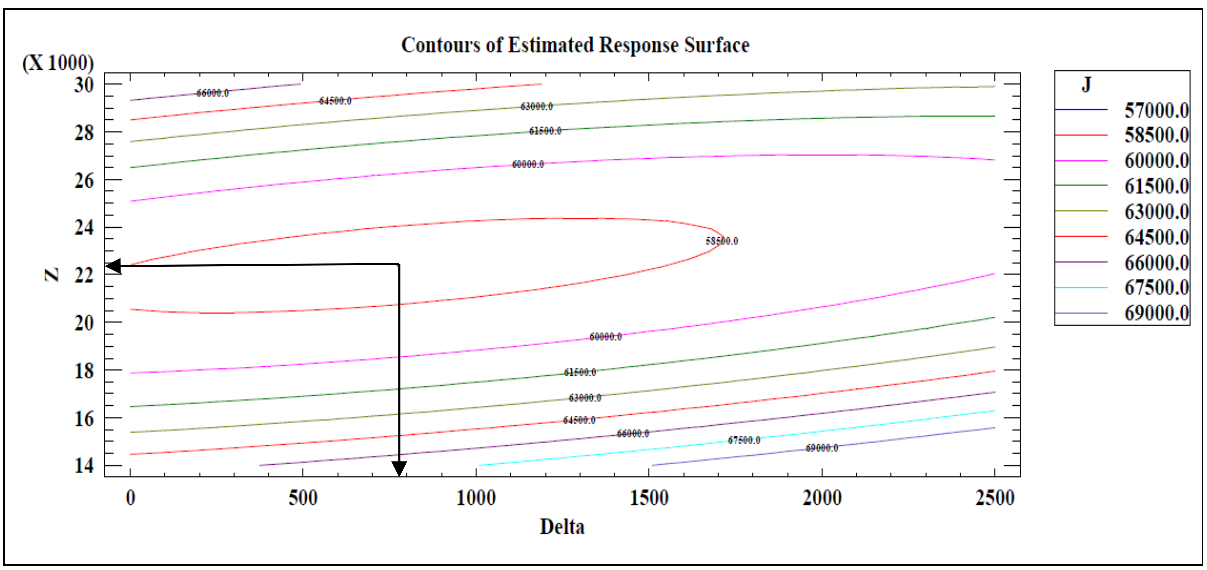


Figure 4.4 Plot of cost response surface on two-dimensional planes with variable demand rate

The results based on the selected setting suggests the optimal values of independent and dependent variables as follows: $Z^* = 22{,}394, \delta^* = 774$ and $J(Z^*, \delta^*) = 58{,}162$ with the 95% confidence interval [58122, 58216] based on 30 additional replications.

4.5 Comparison among the proposed policy, classical HPP and EPQ model with variable demand rate and constant lifetime

The findings show that for the case of random demand rate, applying the proposed HPP with an interval, i.e., equation (4.2), is more cost effective. If we would apply the HPP in its classical form as in equation (2.11) - with only one threshold - for the cases with average demand rate ($\overline{D}$) or with variable demand rate ($D(.)$), the optimal value of the decision variable would be higher as well as higher total cost would be incurred to the system as presented in Table 4.3.

Table 4.3 Comparison among proposed HPP, classical HPP and EPQ with variable demand rate and constant lifetime

	Proposed Policy in equation (4.2)	***Classical Policy in equation (1.1) with $\overline{D}$***	***Classical Policy in equation (1.1) with $D(.)$***	***EPQ***
Decision Variable(s)	$Z^* = 22{,}394$ $\delta^* = 774$	$Z^* = 22{,}881$	$Z^* = 23{,}006$	$Q^* = 69.4$ $s^* = 5.6$
Total Cost	$J(Z^*, \delta^*) = 58{,}162$	$J(Z^*) = 62{,}614$	$J(Z^*) = 64{,}785$	$J(Q^*, s^*) = 69{,}117$
95% confidence interval	$[58122, 58216]$	$[62531, 62649]$	$[64701, 64863]$	$[69021, 69244]$
Average Inventory Cost	$c^+x^+ = 14{,}449$	$c^+x^+ = 15{,}108$	$c^+x^+ = 15{,}520$	$c^+x^+ = 16{,}877$
Average Backlog Cost	$c^-x^- = 36{,}481$	$c^-x^- = 39{,}294$	$c^-x^- = 40{,}572$	$c^-x^- = 42{,}326$
Average Perishability Cost	$c^px^p = 7{,}232$	$c^px^p = 8{,}212$	$c^px^p = 8{,}693$	$c^px^p = 9{,}914$

When only one threshold is applied (as in equation (2.11)), the system should adjust its production rate in different periods accordingly - where the demand rate is either fixed or variable. These adjustments lead to higher variability, more backlog, and more perishability in the system (as explained in section 4.2). To avoid further backlogs, the higher optimal thresholds will be required, whereas these higher thresholds lead to higher product's perishability and consequently, higher total cost of system. Since real demand rate $D(.)$ in classical HPP is more variable than average demand rate $\overline{D}$, the Z^* and total cost incurred to the system in this case will be higher than the case $\overline{D}$ is applied. EPQ model, as already mentioned, will result in higher total cost than either proposed HPP or classical one due to its operational constraints.

4.6 Discussion on the degree of the demand rate randomness

In point of fact, by increasing the randomness in demand rate, the system will face higher products' perishability, which results in lower serviceable inventory, leading to higher product shortages, and an increase in threshold level to avoid higher backlog cost. However, it is obvious that perishability occurs more in case of possessing more inventory on-hand, which results in higher perishability cost leading to an adverse effect on the growing trend. If system parameters in Table 4.1 remain unchanged, by increasing the randomness in demand rate, the optimal inventory threshold and as a result, the total cost will rise (Figure 4.5). As is also evident from this figure, δ will also increase. This is because the more variability in demand rate, the higher backlog and perishability in the system (based on equation (4.1)). Thus, a wider coequality interval is required to mitigate the effect of this randomness on the system.

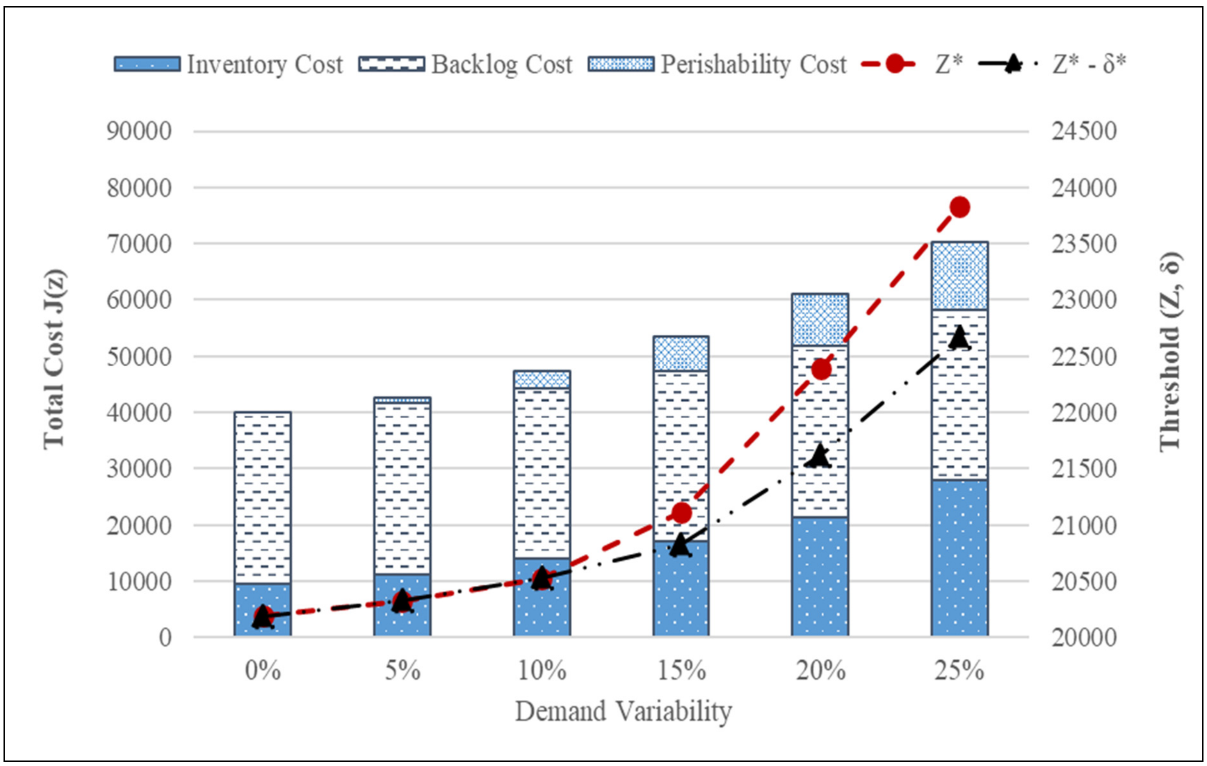


Figure 4.5 The demand rate variability effect on (Z^*, δ^*) and TC^* under Normal Distribution

If there was no randomness in demand rate, the policy based on the equations (2.11)-(2.12) could be applied, without using the coequality threshold δ, and the optimal threshold would be equal to $D \cdot T = 20{,}160$ and not equal to ${Z_{B.K}}^{*} = 24{,}798$ (based on equation (1.2)). It was explained that $min\ \{{Z_{B.K}}^{*}, D \cdot T\}$ will result in minimum total cost incurred to the system.

Now, the question is "why is Z increasing?". To respond this question, an imaginary experiment including 3 levels for the threshold, namely, 15000, 20000 and 25000 and three values for demand rate variability, i.e., σ, which are 15%, 20%, and 25% has been designed. The output of the simulation model is as follows:

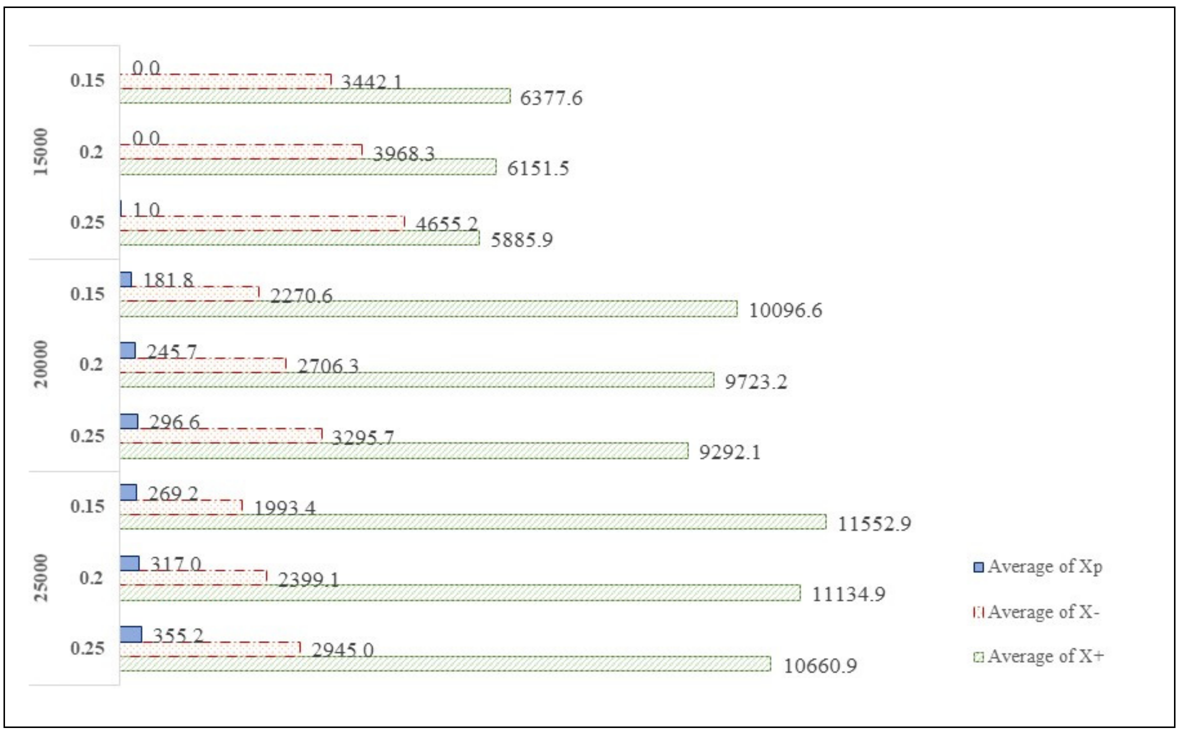


Figure 4.6 Results of the exemplary experiment

If it is assumed that at $\sigma = 0.15$, the optimal threshold is $Z^{*} = 20{,}000$, the decision maker should decide to decrease or increase the inventory threshold as the decision variable if σ increases and reaches at 0.2. The summary of the results is as follows:

Table 4.4 Summary of the results of Figure 4.6

	Z = 15,000 $\boldsymbol{\sigma = 0.2}$	**Z = 20,000** $\boldsymbol{\sigma = 0.2}$	**Z = 25,000** $\boldsymbol{\sigma = 0.2}$
$\boldsymbol{x^+}$	6,151	9,723	11,135
$\boldsymbol{x^-}$	3,968	2,706	2,399
$\boldsymbol{x^p}$	0	245	317

In the *former* case (decrease in Z), the system can save on costs of inventory holding and costs associated with the disposal of perished items, while the total cost related to the backlog will soar. Mathematically speaking,

Cost item	*Calculations*			*Description*
x^+	(6,151 – 9,723)	×1.5	(5,358)	Saving
x^-	(3,968 - 2,706)	×15	18,930	Spending
x^p	(0 - 245)	×30	(7,350)	Saving
Sum			**6,222**	**Spending**

Since by decreasing the level of threshold we will have to pay more, this strategy will not rational and cost-effective. Now, we will try the *later* strategy, increasing the threshold. Mathematically speaking,

Cost item	*Calculations*			*Description*
x^+	(11,135 – 9,723)	×1.5	2,118	Spending
x^-	(2,399 - 2,706)	×15	(4,605)	Saving
x^p	(317 - 245)	×30	2,160	Spending
Sum			**(327)**	**Saving**

As is evident from the results, although the system pays more for inventory holding as well as perishability of the products, the saving due to reducing the level of backlog outweighs the incurred expenses.

Despite the classical HPP in which by adding and/or increasing the demand rate variability for imperishable items, the optimal threshold would increase (Kenne & Gharbi, 2000), the upward or downward trend of Z is now extremely sensitive to the costs the system pays for backlogs and perishability of products. To better illustrate, by increasing the threshold, the system compensates backlogs by paying more for losing perished items and vice vera. In fact, the ratio c^{-}/c^{p} plays a game-changing role in deciding whether Z should increase – i.e., by changing the values c^{-} or c^{p}, this conclusion may be no longer valid, given that c^{+} remains unchanged. To attest the validity of this claim, the values c^{-} and c^{p} were changed to result in different ratios and to let us compare the results with the ratio $c^{-}/c^{p} = 0.5$ and $c^{+} = 1.5$ as the base scenario. The comparison of the scenarios has been suggested in Figure 4.7.

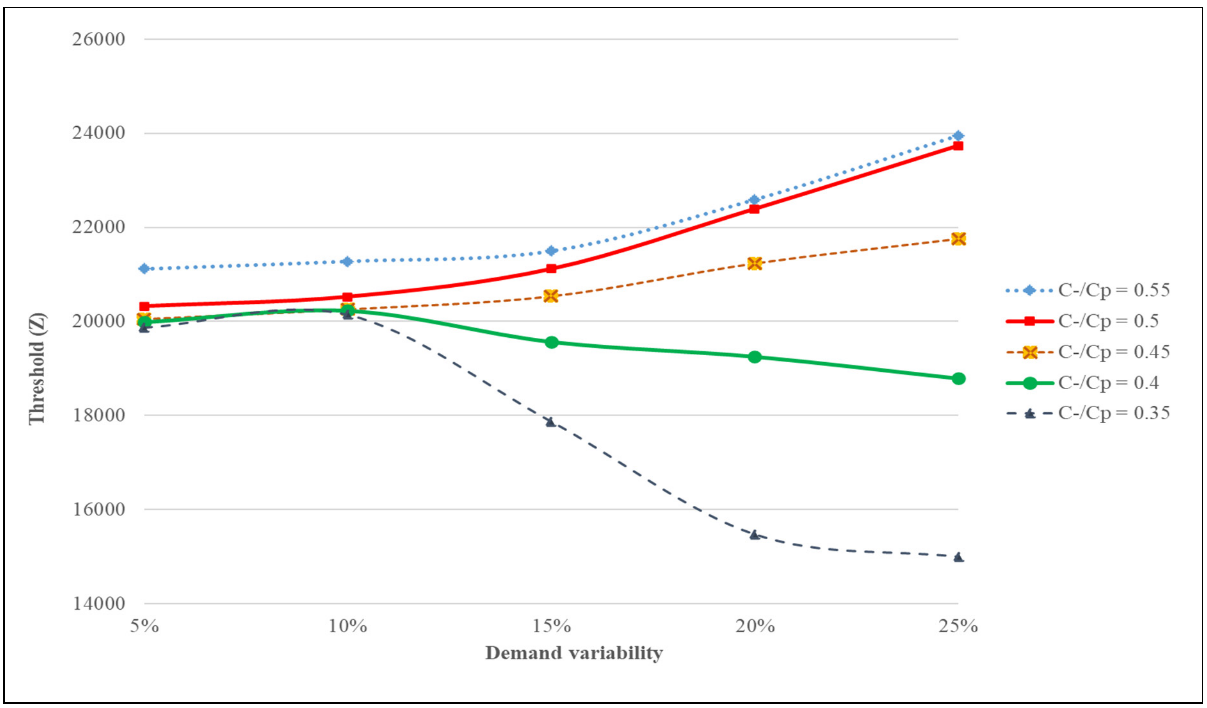


Figure 4.7 The effect of changing c^{-}/c^{p} on the level of optimal threshold when $c^{+} = 1.5$

For some cases like when the cost of shortage is too high, it would be more cost effective to increase the inventory threshold, although the system must pay more for inventory holding and perishability of the products. On the other hand, if perishability cost is too high, it will lead us to decrease the optimal threshold in order not to pay a lot for the disposal of perished items, although the shortages impose an additional cost on the system.

Apparently, by increasing the ratio, either by a decrease in c^p or by an increase in c^- or both, a similar trend is observed. By decreasing the ratio, this conclusion is still valid to the extent in which ${c^-}/{c^p} \in [0.4, 0.45]$. After that, it means the perishability cost is too high or the cost of backlog is too low, and in both cases, it is more cost-effective to decrease the optimal threshold. Thus, the trend of Z will be decreasing where demand rate variability increases (Figure 4.7).

However, c^+ may change the pattern itself given that ${c^-}/{c^p}$ remains unchanged. By changing the inventory holding cost of c^+, a different behavior in the system is observed in terms of the level of optimal threshold where ${c^-}/{c^p} = 0.45$ (Figure 4.8).

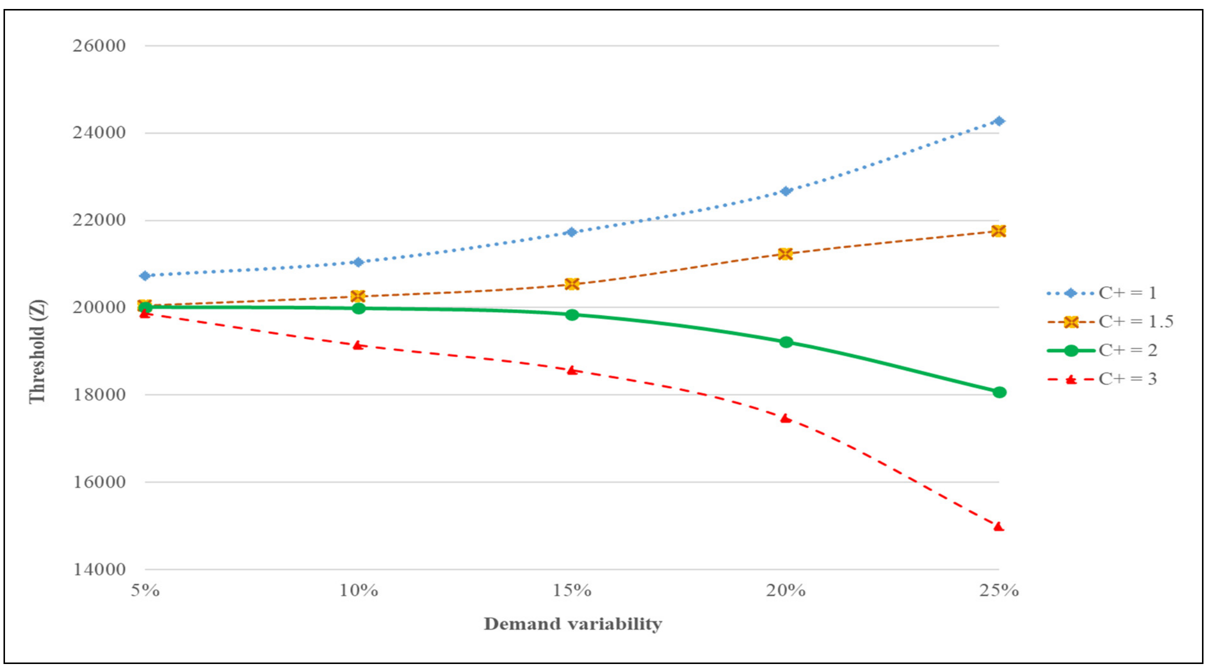


Figure 4.8 The effect of changing c^+ on the level of optimal threshold when ${c^-}/{c^p} = 0.45$

As expected, by increasing the randomness of demand rate, an increase in the threshold would be expected, but at the same time, by growing only inventory holding cost, the threshold level is going up with a gentler slope to the extent in which the decreasing effect of the higher holding cost outweighs the increasing effect of the higher randomness in demand rate, where the slope of threshold trend becomes almost linear at $c^+ \in [1.5, 2]$. For higher holding cost, the decreasing trend will be more significant, even in presence of more variability in demand rate. Generally, by increasing the inventory holding cost (c^+) and/or perishability cost (c^p) independently or simultaneously, the system will require less inventory on-hand, while when backlog cost (c^-) increases, the system will be storing more inventories. It is notable that it would be so difficult to find a single tipping point or ratio including all costs, but there should be a ratio including all costs which determines the direction of changes of the threshold trend.

4.7 Even higher degrees of variability?

Based on the empirical rule, or 3-sigma rule, almost all observed data will fall within three standard deviations (denoted by σ) of the mean (denoted by $\overline{D}$) if data follows Normal Distribution. It means if the standard deviation is equal to one-third of the mean, 99.7% of data observed falls between zero and 2µ. For higher standard deviation, some negative data may be observed. Since demand rate must be always positive or zero, if the demand rate follows the Normal Distribution, negative data must be overlooked. This truncated Normal Distribution will result in a new distribution with different mean and standard deviation than the initial parameters. Thus, Normal Distribution may not be an appropriate fit for modeling higher variabilities in demand rate. To study the system behavior under more demand rate variability, Log-normal Distribution may be a better fit since it always generates positive values and data truncation will no longer be required.

If system parameters in Table 4.1 are applied where variable demand rate follows Log-normal Distribution, the trend of Z in terms of different ratios of c^-/c^p will be as follows (Figure 4.9):

Figure 4.9 Variability effect on Z^* in terms of different ratios ${}^{c^-}/_{c^p}$ under Log-normal Distribution

As the degree of demand rate variability increases, it will lead to more perishability of products. Thus, the system tends to lower the safety stock level (i.e., lower Z) to reduce perishability as much as possible. In the meantime, as products perish more and inventory level decreases as well, backlog of products will go up, and similar to the case of variable product lifetime, backlog cost will become considerable. Now, the system starts storing more products to hedge further backlog in the system, while the perishability of products will increase again. In terms of higher ratios of ${c^-}/{c^p}$, the tipping point happens slightly later, since higher ratios of ${c^-}/{c^p}$ happens when c^- increases, or c^p decreases, or both at the same time. Thus, the safety stock level goes up. As a result, perishability of products will augment. This higher perishability cost will result in a further decrease in optimal threshold Z before its increase. As a case in point for ${c^-}/{c^p} = 0.8$, the trend of Z changes when $c^+x^+ + c^px^p)/TC \cong c^-x^-/TC$ (Figure 4.10).

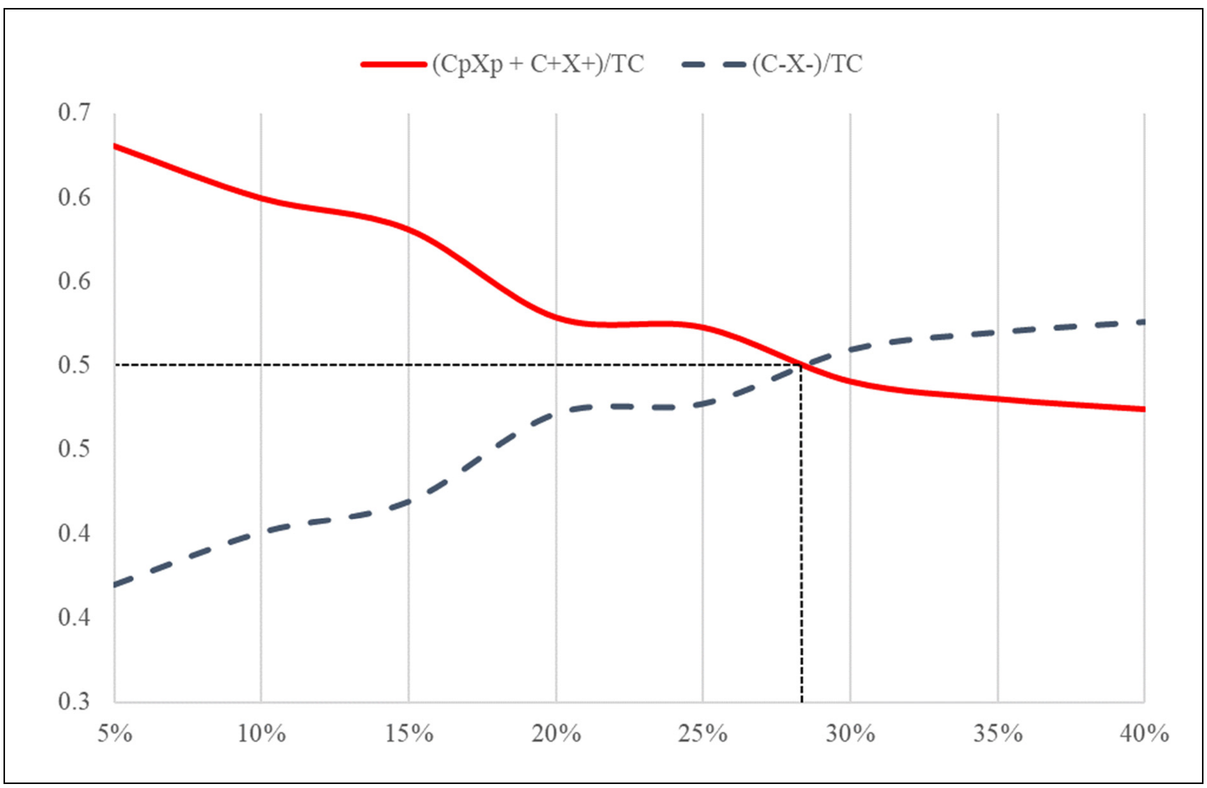


Figure 4.10 Trade-off between the effect of inventory and perishability costs vs. backlog cost – variable demand rate

4.8 Conclusion

The purpose of this chapter was to propose a modified hedging point policy for the problem of production planning of perishable products with variable demand rate in order to minimize the total cost incurred to the production-inventory system.

It was suggested it in case of variable demand rate, applying the classical HPP would result in more total cost incurred to the system in addition to its practical limitations. Thus, a modification to the classical HPP was proposed to mitigate the effect of demand uncertainties on the inventory system by controlling the production rate. A similar experimental methodology was utilized to provide a near-optimal solution to the problem under study.

As discussed, by changing the degree of the demand rate variability, the vector of costs plays a game-changing role in finding the optimal threshold. In fact, when demand rate randomness increases, for the higher ratios of backlog cost to perishability cost, the backlog cost plays a more important role, resulting in an increase in the optimal safety stock, and for the lower values, the perishability affects the optimal threshold negatively. Similarly, when cost of inventory holding goes up, the optimal threshold is likely to be decreasing.

Similar to the case of random lifetimes, by increasing the variability, perishability of products increases, and system initially lowers its safety stock to hedge against the perishability cost, while backlog cost increases. When the backlog cost will become so high, system has no option, but to increase the safety stock level to hedge against the increased backlog cost.

From the decision-makers' viewpoint, if the machine is operational and demand rate is variable, when the level of inventory is lower than the coequality threshold $Z^* - \delta^* = 21{,}260$, the rate of production will be set on maximum, i.e., $U^{max} = 70$ products per unit of time. If the inventory level varies within the coequality interval and lower than $Z^* = 22{,}394$, the machine will produce at average demand rate, i.e., $\overline{D} = 60$ products per unit of time. Otherwise, the production should be stopped. The summary of this insight has been presented in Figure 4.11.

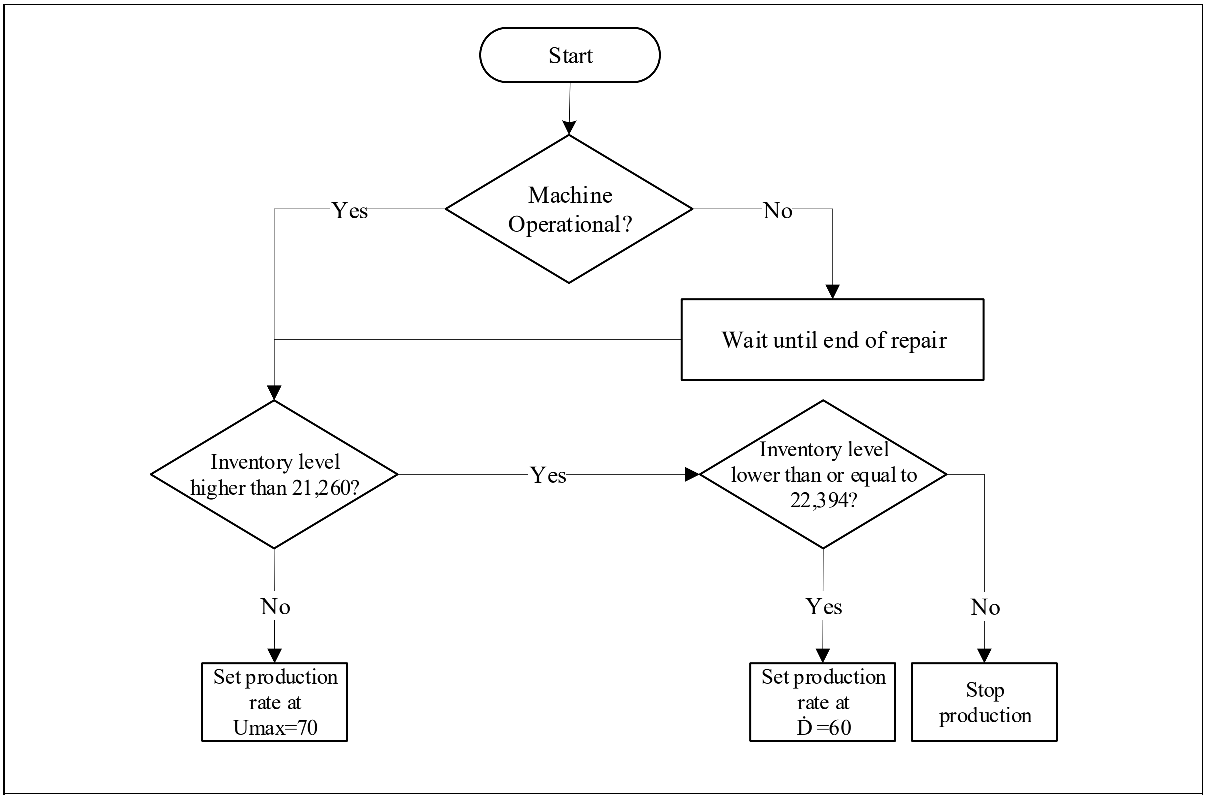


Figure 4.11 Logic chart for implementing the proposed policy with coequality interval

CONCLUSION

In recent decades, the perishability of products throughout the supply chain systems has grabbed extensive attention amongst the practitioners and academia. There have been growing demands for understanding the systems including goods with finite lifetimes and exploring the impact of their perishability on the production and inventory policies.

Since the application of our concern – those of perishable items – arise in daily routine, the methodological framework should be adequately general to capture the scope and complexity of the underlying supply chains. Thus, it is essential to appropriately formulate the perishability of products over space and time. It should also be able to handle different behavioral concepts, including cost optimization, the productivity of available resources, and be sufficiently general to respect and be applicable to the different supply chain structures which are pertinent to the spectrum of perishable products.

In this study, a simulation-based optimization model was developed to describe the dynamic production planning problem of perishable products in a failure-prone manufacturing system by adopting the hedging point policy under specific assumptions for failure process, product lifetimes, and demand rate to minimize the total cost incurred to the production-inventory system, including inventory holding cost, backlog cost and disposal cost of perished items.

Then, an experimental design was used to investigate the effects of specified factors on the cost incurred during the production horizon. The proposed approach combines the simulation method and the statistical method to provide the estimation of the cost function related to the considered control problem. A regression analysis was used to perform this function in terms of significant main factors and interactions given by the experimental design approach. From the estimation of the cost function, the best values of control parameters were easily computed.

An extended hedging point policy was proposed in terms of the type of demand rate. The results show that for the situation with constant demand rate, the classical hedging point policy works well. If the product's lifetime is also constant, the optimal threshold should not exceed the limit at which products start perishing. However, in cases with variable lifetime, the cost of perishability plays an important role, resulting a decrease in the level of optimal threshold. This decrease will eventually lead to more backlogs, resulting in an upward trend for threshold. The tipping point will occur when sum of inventory holding and perishability costs becomes equal to cost of backlog.

To evaluate different priority rules to tackle with the random shelf life, the priority rules widely used in practice, i.e., FIFO, and another rule based on the "Shortest Remaining Lifetime" were compared. It was suggested that in all cases that the perishability of products occurs, SRLT issuing rule leads to a much more cost-effective performance. However, applying priority rules like SRLT may require utilizing advanced technologies for real-time monitoring, like RFID, and will be subject to their availability and accessibility, but in general, these tools are different from industry to industry. For instance, for agricultural products, applying real-time monitoring systems for tracking remaining lifetimes will not be that cost-effective, even if hardly possible. This is because these products are usually cheap and easy to reproduce and replace, and the cost of perishability is not that high. While for some pharmaceutical products, both costs of backlog and perishability are too high, and it is reasonable to invest on advanced technologies for real-time monitoring of these valued inventories and to make their turnover efficient. It should note that in some cases, cost of replacement is also too high, and it makes the efficient inventory turnover a necessity.

Then, a modified hedging point policy was proposed for the case of variable demand rates. It was discussed that in such cases, the cost vector plays a game-changing role in determining the optimal threshold. In fact, for the higher ratios of the backlog cost to the perishability cost, the backlog cost will have a more tangible effect on the system, resulting in an increase in the optimal threshold to avoid the high backlog cost, and for the lower ratios, meaning the perishability affects the system significantly, the optimal threshold goes down to avoid the perishability of products and costs associated with that. Like the case of random lifetimes, by increasing the variability, perishability of products increases, and system initially lowers its safety stock to hedge against the perishability cost, while backlog cost increases. From a managerial insight, when the ratio of the backlog cost to the perishability cost becomes so high, inventory managers have no option, but to increase the safety stock level to hedge against the increased backlog cost, while at the same time, they have to pay low amount as cost of perishability. Like the previous case, the tipping point will occur when sum of inventory holding and perishability costs becomes equal to cost of backlog.

The developed HPP-based simulation-optimization models can be easily implemented and utilized by decision-makers. For the simulation part, they can add, remove, or modify constraints to their models, and test them under different assumptions and system parameters. For the optimization part, they can conduct extensive sensitivity analyses on response variables, usually cost function, to figure out what happens if a change occurs in the system. The tools required to apply these developed policies are ARENA (for discrete/continuous simulation modeling) and STATGRAPHICS (for design of experiments and response surface methodology).

This research made it possible to deal with the problem of production planning of perishable products in an unreliable manufacturing system under different assumptions for customers' demand rate and lifetimes of the products. It was suggested that the HPP can be pulled off in various problems of production planning of perishable products in FPMSs. However, there are lots of areas remaining to be covered in the future works as an extension for this study. Some examples will be provided in the Recommendation section.

RECOMMENDATIONS

Here, some recommendations are proposed for the future studies:

- Changeable Optimal Threshold especially in cases of seasonal demands

There are lots of perishable products having seasonal demands. Fruits are just a case in point. In order to have a more realistic model in this regard, having the possibility of changing the optimal threshold would be an interesting idea. That means, on some occasions or seasons, the demand increases for a specific perishable product. In some other times, a drop in the demand rate may be observed. In the current research, the aim was to determine a fixed optimal threshold minimizing the total cost incurred to the system. Nonetheless, if there are seasonal demands, the system needs to pay a high cost of shortage when total demand is at its peak or needs to pay a high cost of disposal for the perished items when total demand is at its trough.

- Considering pricing policies such as providing a discount for products being about to perish or other incentives to sell such products sooner, like a priority rule.

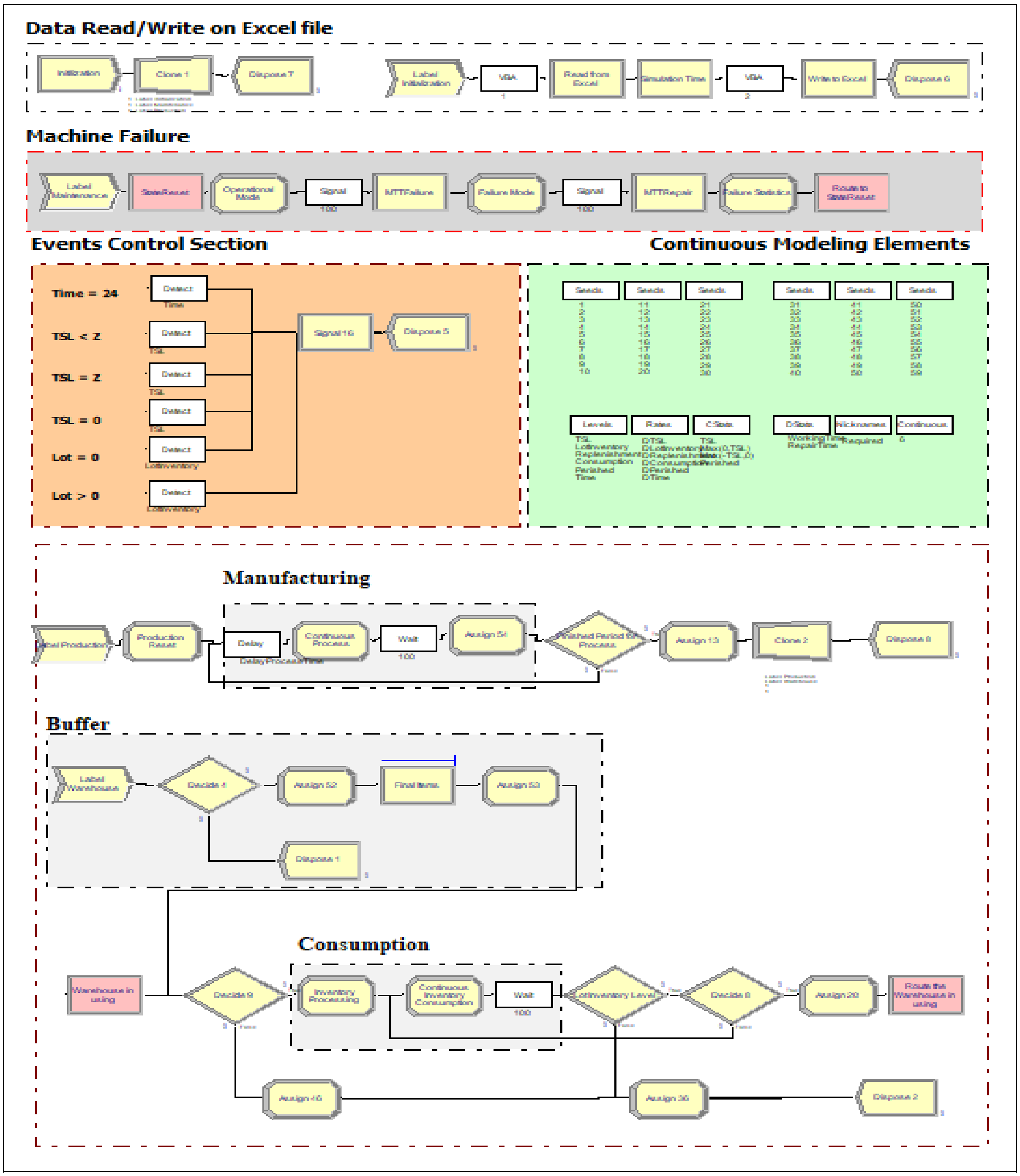
This may result in a change in the customer's behavior and as a result, in the demand pattern. An interdisciplinary study may be required to figure out how customers behave when it comes to purchase discounted perishable items with taking the expiration dates into account.

- Considering different delays in consumption of the perishable products

Perishable items may be produced and consumed in geographically dispersed areas and markets. Most of the time, there are discrepancies between the expected time of arrival and the actual one. Thus, the shipping time adds another random phenomenon to the system which needs to be considered in real-world problems.

ANNEX I

SIMULATION MODEL OF THE PROBLEM UNDER STUDY IN ARENA



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