Simultaneous Control of the Production, Maintenance, and Inspection Strategies for a Failure-Prone Manufacturing System with Quality-Based Financial Penalties/Incentives

by

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FOREWORD

The following manuscript is dedicated to providing an improved optimization model that could minimize the total cost of the manufacturing system. The optimal policies consist of production planning, maintenance policies, and inspection strategies that are affected by the system's condition and parameters. Due to the complexity of the problem, numerical methods have been used to obtain simultaneous optimal policies. The main concentration of this thesis is on the inspection policy by defining a financial incentive and penalty for the production of parts with high and low qualities, respectively.

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Contrôle simultané des stratégies de production, de maintenance et d'inspection pour un système de fabrication sujet aux défaillances avec des pénalités/incitations financières basées sur la qualité

Seyyedmohsen MOUSAVI

RÉSUMÉ

Cette thèse étudie les stratégies simultanées de planification de la production, de maintenance, de remplacement et de contrôle d'inspection d'un système dans trois scénarios différents. Dans le scénario général, le système consiste en une machine de fabrication peu fiable capable de produire un type de pièce pour satisfaire la demande. Dans le deuxième scénario, pour augmenter l'efficacité du système, une machine de rectification est ajoutée au système pour récupérer les parties rectifiables des produits défectueux. Dans le troisième scénario, le plus complet, une machine de reconditionnement aide le système à récupérer les pièces non rectifiables ainsi que les produits renvoyés du marché. De plus, une salle de stock de récupération est ajoutée au système pour stocker les pièces défectueuses pour les processus de reconditionnement.

Le processus de vieillissement et les activités de réparation imparfaites conduisent à la détérioration de la machine de fabrication. Cette détérioration affecte la fiabilité de la machine de fabrication et la qualité des pièces produites. Pour assurer la satisfaction continue de la demande et pallier les effets indésirables de la détérioration, plusieurs activités de maintenance sont appliquées. Les activités de maintenance consistent en des options de maintenance préventive et de commutation de réparation/remplacement. Une étape de contrôle qualité est également effectuée pour conduire le processus d'inspection dans lequel les pièces défectueuses détectées sont envoyées pour être récupérées soit par la machine de rectification, soit par la machine de reconditionnement.

L'objectif principal de cette étude est de déterminer conjointement la politique de production à coûts optimisés pour les machines de fabrication et de reconditionnement, la planification de la maintenance en termes d'activités de commutation PM et réparation/remplacement, et la stratégie d'inspection sur un horizon de planification fini.

Le modèle proposé est soumis à une contrainte de qualité sur la limite de qualité moyenne sortante (AOQL) qui est spécifiée par le marché, où une pénalité financière ou une incitation est envisagée pour fournir des produits d'une qualité inférieure ou supérieure à l'AOQL, respectivement.

De plus, des méthodes numériques sont adoptées pour trouver la solution de contrôle optimale. Enfin, des exemples numériques sont fournis ainsi que l'analyse de sensibilité pour vérifier l'exactitude du modèle proposé.

Mots-clés: Détérioration du système de fabrication, Politiques de fabrication et de remise à neuf, Pénalité/incitation financière basée sur la qualité, Stratégie d'inspection, PM et planification des réparations/remplacements

Simultaneous Control of the Production, Maintenance, and Inspection Strategies for a Failure-Prone Manufacturing System with Quality-Based Financial Penalties/Incentives

Seyyedmohsen MOUSAVI

ABSTRACT

This thesis investigates the simultaneous production planning, maintenance, replacement, and inspection control strategies for a system in three different scenarios. In the general scenario, the system consists of an unreliable manufacturing machine capable of producing one type of part to satisfy the demand. In the second scenario, to increase the efficiency of the system, a rectifying machine is added to the system to recover the rectifiable parts of defective products. In the third and the most complete scenario, a remanufacturing machine helps the system to recover the non-rectifiable parts along with the products returned from the market. Also, a recovery stock room is added to the system to store the defective parts for remanufacturing processes.

The aging process and imperfect repair activities lead to the deterioration of the manufacturing machine. This deterioration affects the reliability of the manufacturing machine and the quality of produced parts. To ensure the continuous fulfillment of the demand and to palliate the undesirable effects of deterioration, several maintenance activities are applied. The maintenance activities consist of preventive maintenance and repair/replacement switching options. A quality control step is also carried out to conduct the inspection process in which the detected defective parts are sent to be recovered either by the rectifying machine or the remanufacturing machine.

The main objective of this study is to jointly determine the cost-optimized production policy for the manufacturing and remanufacturing machines, maintenance planning in terms of PM and repair/replacement switching activities, and inspection strategy over a finite planning horizon.

The proposed model is subject to a quality constraint on the average outgoing quality limit (AOQL) which is specified by the market, where a financial penalty or incentive is considered for providing products with a quality lower or higher than AOQL, respectively.

Furthermore, numerical methods are adopted to find the optimal control solution. Finally, numerical examples are provided as well as the sensitivity analysis to verify the accuracy of the proposed model.

Keywords: Deteriorating manufacturing system, Manufacturing and remanufacturing policies, Financial quality-based penalty/incentive, Inspection strategy, PM and Repair/replacement planning

TABLE OF CONTENTS

Page

INTRODUCTION 1

CHAPTER 1 LITERATURE REVIEW AND RESEARCH'S OBJECTIVES 3
1.1 Introduction of the production systems 3
1.2 Production policy and quality control 5
1.3 Deterioration in manufacturing systems 6
1.4 Maintenance in manufacturing systems 7
1.4.1 Effect of maintenance on reliability of the system 8
1.4.2 Effect of maintenance on quality of the products 9
1.4.3 Effect of maintenance on the reliability of the system and quality of the products 10
1.5 Inspection 11
1.6 Critique of the literature review 13
1.7 Research problematic 13
1.8 Research objectives 14
1.9 Proposed methodology 15

CHAPTER 2 OPTIMIZATION OF THE PRODUCTION, PREVENTIVE MAINTENANCE, REPLACEMENT, AND INSPECTION POLICIES IN MANUFACTURING SYSTEMS 17
2.1 Introduction 17
2.2 System description 18
2.3 Problem statement 19
2.4 Formulation of the control problem 21
2.5 Optimal control policy 35
2.5.1 Numerical example 36
2.5.2 Production policy 37
2.5.2.1 Inspection policy 39
2.5.3 Maintenance Policy 43
2.6 Sensitivity analysis 47
2.6.1 Variation of the backlog costs 47
2.6.2 Variation of the inventory costs 50
2.6.3 Variation of inspection cost 50
2.6.4 Variation of the PM costs 52
2.6.5 Variation of the repair costs 53
2.6.6 Variation of the low-quality financial penalty coefficient 55
2.6.7 Variation of the high-quality financial incentive coefficient 55
2.7 Conclusion 57

CHAPTER 3 PRODUCTION CONTROL OF A FAILURE-PRONE MANUFACTURING SYSTEM WITH RECTIFYING MACHINE 59
3.1 Introduction 59
3.2 System description 59
3.3 Control problem formulation 60
3.4 Production policy 66
3.4.1 Inspection policy 67
3.4.2 Maintenance Policy 73
3.5 Sensitivity Analysis 77
3.5.1 Variation of the backlog cost 77
3.5.2 Variation of the inspection cost 79
3.5.3 Variation of PM cost 79
3.5.4 Variation of the rectifying cost 81
3.5.5 Variation of the low-quality financial penalty coefficient 83
3.5.6 Variation of the high-quality financial incentive coefficient 83
3.6 Conclusion 85

CHAPTER 4 SIMULTANEOUS PRODUCTION PLANNING AND MAINTENANCE CONTROL PROBLEM FOR AN UNRELIABLE HYBRID SYSTEM IN CLOSED-LOOP REVERSE LOGISTICS 87
4.1 Introduction 87
4.2 System description 88
4.3 Formulation of the dynamic of the system 89
4.3.1 Formulation of the control problem 91
4.3.2 Optimal control policy 103
4.4 Numerical example 105
4.4.1 Production policy 106
4.4.2 Inspection policy 111
4.4.3 Maintenance policy 116
4.5 Sensitivity analysis 121
4.5.1 Variation of the backlog costs 122
4.5.2 Variation of the remanufacturing rate 122
4.5.3 Variation of the inspection costs 125
4.5.4 Variation of AOQL 125
4.6 Conclusion 127

CONCLUSION AND RECOMMENDATIONS 129

BIBLIOGRAPHIE 133

LIST OF TABLES

Page

Table 2.1 Parameters of the numerical example 37

Table 4.1 Transition modes $\zeta(t)$ of the hybrid system 93

Table 4.2 Parameters of the numerical example 106

LIST OF FIGURES

Page

Figure 2.1 Transition diagram of proposed model 20

Figure 2.2 Block diagram of proposed model 23

Figure 2.3 Failure rate and Mean time to failure of the machine when $\theta_n = 2$ 26

Figure 2.4 Failure rate and Mean time to failure of the machine 27

Figure 2.5 Production policy 38

Figure 2.6 Production policy 40

Figure 2.7 Nominal inspection policy for $AOQL$=0.05 41

Figure 2.8 Actual and nominal inspection policy at $n = 4$ 42

Figure 2.9 Inspection policy (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$, (e) $n = 5$, and (f) $n = 6$. 44

Figure 2.10 Maintenance policy 45

Figure 2.11 Preventive maintenance for each number of failure n 46

Figure 2.12 Maintenance policies for each number of failure n 48

Figure 2.13 Maintenance policies for different backlog cost at $n = 3$ 49

Figure 2.14 Inspection policies for different backlog cost at $n = 3$ 50

Figure 2.15 Maintenance policies for different inventory cost at $n = 3$ 51

Figure 2.16 Inspection policies for different inventory cost at $n = 3$ 51

Figure 2.17 Maintenance policies for different inspection cost at $n = 3$ 52

Figure 2.18 Inspection policies for different inspection cost at $n = 3$ 53

Figure 2.19 Maintenance policies for different PM cost at $n = 1$ 54

Figure 2.20 Maintenance policies for different PM cost at $n = 3$ 54

Figure 2.21 Maintenance policies for different repair cost at $n = 1$ 55

Figure 2.22 Inspection policies for different bad quality penalty at $n = 3$ 56
Figure 2.23 Inspection policies for different quality incentives at $n = 3$ 57
Figure 3.1 Transition diagram of proposed model 61
Figure 3.2 Block diagram 62
Figure 3.3 Production policy 68
Figure 3.4 Nominal inspection policy for *AOQL*=0.05 69
Figure 3.5 Nominal and actual inspection policies at $n = 4$ 70
Figure 3.6 Inspection policy 72
Figure 3.7 Replacement and preventive maintenance policies 73
Figure 3.8 Maintenance zone 74
Figure 3.9 Maintenance policies 76
Figure 3.10 Maintenance policies for different backlog cost at $n = 3$ 78
Figure 3.11 Inspection policies for different backlog cost at $n = 3$ 78
Figure 3.12 Maintenance policies for different inspection cost at $n = 3$ 80
Figure 3.13 Inspection policies for different inspection cost at $n = 3$ 80
Figure 3.14 Maintenance policies for different PM cost at $n = 3$ 81
Figure 3.15 Maintenance policies for different PM cost at $n = 1$ 82
Figure 3.16 Inspection policies for different PM cost at $n = 1$ 82
Figure 3.17 Inspection policies for different rectifying cost at $n = 4$ 83
Figure 3.18 Inspection policies for different low quality penalty at $n = 3$ 84
Figure 3.19 Inspection policies for different high quality incentive at $n = 3$ 85
Figure 4.1 Transition diagram of hybrid model 90
Figure 4.2 Transition diagram of manufacturing and remanufacturing machines 92
Figure 4.3 Transition diagram of hybrid model 92

Figure 4.4 Production policy for M_1 while M_2 is available 107

Figure 4.5 Production policy for M_1 while M_2 is unavailable 108

Figure 4.6 Production policy for M_2 while M_1 is available 110

Figure 4.7 Actual and nominal inspection policies for $n = 4$ 113

Figure 4.8 Inspection policy for all number of failure when machine M_2 is operating 114

Figure 4.9 Inspection policy for the system while M_2 is in operational state 115

Figure 4.10 Inspection policy of the system while M_2 is in failure state 116

Figure 4.11 PM and replacement policies for all number of failure when machine M_2 is operating 117

Figure 4.12 PM and replacement policies for all number of failure when machine M_2 is operating 119

Figure 4.13 Maintenance policy for different recovery stock level x_2 while M_2 is available 120

Figure 4.14 Maintenance policies for different recovery stock level x_2 while M_2 is unavailable 121

Figure 4.15 Maintenance policies for different backlog costs at $n = 4$ 123

Figure 4.16 Inspection policies for different backlog costs at $n = 4$ 123

Figure 4.17 Maintenance policies for different remanufacturing rate at $n = 4$ 124

Figure 4.18 Inspection analysis for different remanufacturing rate at $n = 4$ 125

Figure 4.19 Inspection policies for different inspection costs at $n = 4$ 126

Figure 4.20 Inspection policies for different AOQL 127

LIST ABBREVIATIONS

x_1	inventory stock level
x_2	recovery stock level
n	number of failures
a	age of the machine after any maintenance activities
u_1	production rate of the manufacturing machine
u_{max}	maximum production rate of the machine
u_2	Production rate of remanufacturing machine
d	demand rate
$\zeta(.)$	mode of the machine
$Q(.)$	transition matrix
$q_{\alpha\beta}$	transition rate from mode α to mode β
$\beta(.)$	rate of defectives
ρ	discount rate
$J(.)$	expected discount cost function
$v(.)$	value function
c_1^+	inventory holding cost
c_2^+	recovery holding cost
c^-	backlog cost
c_{ovr}	replacement cost

c_{pm}	preventive maintenance cost
C_{ins}	inspection cost
c_{rec}	rectifying cost
c_{dis}	disposal cost

INTRODUCTION

To survive in today's competitive market, manufacturers need to be able to produce products with sustainable high quality. This requires a thorough analysis of the production system conditions to find a situation in which the system can produce high-quality products. Moreover, it is necessary to control all aspects of the manufacturing system to produce sustainable products that can continuously satisfy the demand. The concept of controlling the manufacturing system has attracted a lot of attention in recent years judging from the many studies dedicated to investigating the idea. However, given the stochastic nature of the manufacturing system in the presence of unforeseen events such as machine deterioration, break-down, or imperfect repair, etc., it is impossible to exactly predict the behavior of the system.

It is known that in order to improve the control policy, all of the disorder factors should be managed properly. The dual effect of deterioration on the availability of production system and quality of produced parts can make it quite challenging to meet the demand. To palliate the effect of deterioration and improve the performance of the system, various maintenance activities can be carried out. In addition, inspection processes can be performed to enhance the quality of outgoing products and to detect imperfect parts.

The interaction between the quality and maintenance activities and production policy is irrefutable. While most research has focused on the integration of only two of these elements, in recent years the researchers have tried to develop models considering the interactions between all three of them.

CHAPTER 1

LITERATURE REVIEW AND RESEARCH'S OBJECTIVES

1.1 Introduction of the production systems

Many studies have tried to address the issue of production planning for unreliable manufacturing systems. In 1983, Kimemia and Gershwin (Kimemia & Gershwin, 1983) proposed a control policy called a hedging point policy that could determine the production level of a failure-prone manufacturing system. Later in 1986 Akella and Kumar (Akella & Kumar, 1986) formulated an analytical solution for a simple production system where one machine produced a single type of product. Other studies used a solution of the Hamilton-Jacobi-Bellman equations to obtain an optimal control policy to solve a stochastic control problem (Kenné & Boukas, 2003), (Kenné, 1997), and (Gershwin, 1994). Following that, various extensions of the production policy were investigated. For instance, a model was suggested by Sharifinia (Sharifnia, 1988) for multiple manufacturing machines producing one type of product. Next, Mok and Porter (Mok & Porter, 2006) proposed a stochastic model to control the production rate of a manufacturing system with multiple machines producing single or multiple types of products. Another stochastic production model was also suggested by Sajadi et al (Sajadi, Esfahani & Sörensen, 2011). In 2012, Kenné et. al (Kenné, Dejax & Gharbi, 2012) developed a model with a remanufacturing machine for reverse logistic systems. In 2014, Iwao and Kusukawa (Iwao & Kusukawa, 2014) were able to obtain a production strategy for a remanufacturing system using a mathematical model.

There has been an increase in the application of remanufacturing systems in the last decade. Such systems have been shown to significantly reduce the production cost and precious raw material waste as well as negative environmental impacts. However, compared to manufacturing systems, remanufacturing systems are not extensively studied. Such lack of in-depth knowledge can be attributed to factors such as uncertainties and different conditions of the return products that complicate the production planning of remanufacturing systems.

The uncertainty in the quality of received products as well as the output quality, issues with disassembling the returned products, balancing between demand and production, and reverse logistics are among the important aspects of the remanufacturing systems that should be considered (Guide Jr, 2000).

The efforts for the extension of the production system to reverse logistics were triggered by the work of Fleischmann et al(Fleischmann, Bloemhof-Ruwaard, Dekker, Van der Laan, Van Nunen & Van Wassenhove, 1997). Kiesmüller and Scherer (Kiesmüller & Scherer, 2003) worked on a system with manufacturing and remanufacturing machines that produced products of the same quality. Such systems required separate inventories for raw materials and the returned products. The authors were able to determine the optimal control plan for the cases of the stochastic rate of demand and returned products. Kenné et al.(Kenné *et al.*, 2012) extended this work by developing a model with three different stocks for the products of manufacturing and remanufacturing machines as well as the returned products. The authors' goal was to meet the demands by minimizing the total costs through controlling the production rate of the manufacturing and remanufacturing machines.

To describe the reverse logistic system, Polotski et al.(Polotski, Kenne & Gharbi, 2015) considered the case of a hybrid system capable of switching between manufacturing and remanufacturing states where the machine could use either raw material or the returned products.

Francie et al. (Francie, Jean-Pierre, Pierre, Victor & Vladimir, 2015) investigated a hybrid system deteriorating due to the machine's failure, imperfect repair processes, and natural gradual deterioration. In 2018 Ourat et al. (Ouaret, Kenné & Gharbi, 2018a), further developed this work by adding another recovery option to the remanufacturing system. The authors justified that since the remanufacturing machine is always exposed to the different qualities of return products as inputs, a "replacement state" should be added to the system.

Furthermore, Huang et al.(Huang, Chen & Fang, 2013) argued that the effects of deterioration could be alleviated by imposing maintenance activities. However, in this study the effect of deterioration on the product quality was considered as a constant rate of defective products whereas more realistic assumptions such as the work of Kim and Gershwin (Kim & Gershwin, 2005), the effect of deterioration is not always considered constant rather a random phenomenon.

Another major impact of deterioration as indicated by the studies of Colledani and Tolio (Colledani & Tolio, 2011) and Jiang et al. (Jiang, Zhou, Zhang, Wang, Cao & Tian, 2016) is on the availability of the system where the more a machine deteriorates, the more often it will fail.

1.2 Production policy and quality control

By emphasizing the importance of the inter-relation between quality issues and production systems, Inman et al.(Inman, Blumenfeld, Huang & Li, 2003) paved the way for future research on the analysis of the performance of production systems. Later, Kim and Gershwin (Kim & Gershwin, 2005) (Kim & Gershwin, 2008) proposed an analytical method for evaluating the performance of production systems that integrated quality and production issues. Colledani and Tolio (Colledani & Tolio, 2011) employed this approach to analyze the performance of a manufacturing system where the changes of the machine's behavior were tracked by statistical control charts. In their proposed model, the effect of quality control activities on the flow of the product parts was considered.

The relationship between the production policy and quality attracted a lot of attention. Mhada et al. (Mhada, Hajji, Malhame, Gharbi & Pellerin, 2011) presented an optimal production threshold policy based on cost optimization for deteriorating manufacturing systems where a fraction of the output parts were defective. Njike et al. (Njike, Pellerin & Kenne, 2012) employed several operational states based on the condition of the system which was indicated by the number of defective parts. Dhouib et al. (Dhouib, Gharbi & Aziza, 2012) applied a preventive maintenance policy based on the age of the system that helped to maintain the acceptable state of production planning. Bouslah et al. (Bouslah, Gharbi & Pellerin, 2013) used an acceptance sampling plan to monitor the fraction of the defective products. The same plan was utilized in another study by Bouslah et al. (Bouslah, Gharbi & Pellerin, 2016) for controlling the product quality while evaluating the production plan of a manufacturing system subject to reliability and quality deterioration.

1.3 Deterioration in manufacturing systems

An important issue is that the effect of deterioration is not limited to increasing the failure rate or decreasing production intensity, deterioration also affects the quality of the products. In the past decade, many efforts have been made to propose a more realistic model of the deterioration process in production systems that illustrates its effect on the quality of the produced parts as well as the performance and availability of the system. Rosenblatt and Lee (Rosenblatt & Lee, 1986) derived the optimal production cycle for a manufacturing system based on different dynamic models of deterioration (linear, exponential, multi-state) and compared the results to Economic Production Quantity models.

Lai and Chen (Lai & Chen, 2006) considered the age of the machine as an indicator of its level of deterioration. They studied the replacement planning of a system where the failure rate of each machine increased with its age. In another study, Ouaret et al. (Ouaret, Kenné, Gharbi & Polotski, 2015) proposed that the problem of joint production and quality control of a deteriorating system can be addressed by considering that the age of the system affects both the quality of the single-produced part and the failure rate of the production machine.

Another typical approach used to model the deterioration is based on imperfect maintenance and takes into account the number of machine failures as an index of the level of deterioration of the system. Lam (Lam, 2007) presented a lifetime distribution of a deteriorating system for which the average availability after each failure and repair process decreased. Following that Kouedeu et al. (Kouedeu, Kenne, Dejax, Songmene & Polotski, 2015) investigated the effect of imperfect repair on the production and maintenance policy and proposed the stochastic model for a system where the failure rate of the machine is proportionally related to the number of failures.

A more realistic assumption as such made by Deyahem et al. (Nodem, Kenne & Gharbi, 2011c) combines both approaches to indicate the real effect of deterioration on the system. This study presents a model for a production system where not only the failure rate of the machine intensifies as it ages, but also with the increase in the number of the failures, the repair time after each failure increases as well. Moreover, in the models proposed by Rivera-Gomez et al. (Rivera-Gomez, Gharbi & Kenné, 2013b) and Ouaret et al. (Ouaret, Kenné & Gharbi, 2018b) the natural machine wear out process is considered to have been caused by a combination of both the age of the machine and the imperfect repair.

1.4 Maintenance in manufacturing systems

Nowadays, many industries employ preventative maintenance as a countermeasure against machine failure and production loss. A more realistic perspective considers both reliability and quality deterioration phenomena at the same time. In such cases, preventative maintenance plays an even more essential role in ensuring the system's reliability as well as preventing the high defective rate of the products.

Dhouib et al. (Dhouib *et al.*, 2012) considered the age of the equipment as the deterioration index to formulate a model for establishing production control and preventative maintenance plan. In the case of producing defective products, the acquired maintenance strategy was able to reduce the rate at which deterioration expanded to the out-of-control state.

Given the fact that system deterioration directly affects the availability of the production system and the quality of the produced parts, some studies argue that compared to the age of the machine, the quality of the produced parts is a better measure of the system performance for determining the preventative maintenance decisions. It has even been suggested that maintenance and quality control activities can be integrated.

Several studies have proposed the idea of planning preventative maintenance activities based on quality information feedback. For instance, in a study by Njike et al. (Njike, Pellerin & Kenne, 2011) the optimal production and maintenance plan was derived from feedback based upon the number of defective parts. The authors justified their approach by the fact that system deterioration can result in defective products. In other studies, from a similar viewpoint, Panagiotidou and Tagaras (Panagiotidou & Tagaras, 2010), Pan et al. (Pan, Jin, Wang & Cang, 2012), and Zhang et al. (Zhang, Deng, Zhu & Yin, 2015) the statistical process control policies were combined with the maintenance strategies developed from the quality information.

The suggested preventative maintenance plan in the work of Radhoui et al. (Mehdi, Nidhal & Anis, 2010) was also based upon the amount of non-conforming produced parts. Furthermore, Rivera-Gómez et al. (Rivera-Gomez *et al.*, 2013b) presented a model that rendered the optimal production and maintenance strategies for a system undergoing natural ware out process. The proposed model also defined the deterioration process of the system which had a negative impact on the quality of the parts.

1.4.1 Effect of maintenance on reliability of the system

The most important reason for industries to incorporate maintenance activities is to ensure the sustainability of the production systems by preventing decreased reliability caused by deterioration. (Kenné & Nkeungoue, 2008) utilized numerical methods to develop a stochastic model that generated an optimal control policy for the system by controlling both the production and maintenance rates.

Nodem et al. (Nodem *et al.*, 2011c) worked on developing a cost-optimized model of a manufacturing system for repair/replacement switching decisions after the occurrence of each failure. The authors considered the fact that the repair activities depend on the machine repair history. According to the study, the machine repair history increases the average repair time after each failure and changes the dynamic of the system. This work was followed by the work of Nodem (Nodem, Kenné & Gharbi, 2011b) who added preventive maintenance to the model to increase the availability of the system. This improved model determined the optimal condition of preventive maintenance rate and repair/ replacement switching planning after each failure. Considering the presence of the repair history in this model, the semi-Markov decision process was used in this study. (Nodem, Gharbi & Kenné, 2011a) employed the same model for a similar condition to find the optimal economic policy for a deteriorating manufacturing system. In this work, the availability of the system, as well as the mean time required to repair the machine was affected by its number of failures. Moreover, the efficiency of the preventive maintenance was introduced to the model by a reduction factor. Next, a one-to-one correspondence was conducted between the mean time to repair in the absence and presence of PM. Kang (Kang & Subramaniam, 2018) formulated a model for cost-optimal control planning by considering flexible maintenance activities that can be performed at different levels and also in flexible time intervals based on the system condition.

1.4.2 Effect of maintenance on quality of the products

In 2010, Mehdi et al. (Mehdi *et al.*, 2010) proposed a joint quality control and maintenance model to find the optimal rejection rate of the non-conforming units in a lot. This rate would then be used to decide between performing maintenance activities (preventive or overhaul) or stock building to reach the optimal stock size that minimizes total costs. In this study mathematical models were used to investigate three different scenarios based on whether the stock buffer level is reached or not.

The model suggested by (Njike *et al.*, 2011) indicated that it is more appropriate to merge all the parameters that cause machine failures such as the age of the machine and human errors into a single parameter and also to use the quality of the products as an index of the system status.

Based on the system condition, the optimal joint maintenance and production planning of the deteriorating system was then proposed to overcome the progressive defective rate of the products. Next, (Rivera-Gómez, Gharbi & Kenné, 2013) investigated a deteriorating system where the defective rate of the output products was a function of the number of failures. To palliate the effect of deterioration, a stochastic dynamic model was developed which could simultaneously determine the optimal production planning and overhaul processes while minimizing total costs. This model was obtained using numerical methods and response surface methodology for accurate approximations.

To further illustrate the effect of deterioration on the quality of the parts, (Rivera-Gomez *et al.*, 2013b) proposed a manufacturing system with multiple operational states where each state showed different defective rates that increased with the number of failures. (Rivera-Gómez, Montaño-Arango, Corona-Armenta, Garnica-González, Ortega-Reyes & Anaya-Fuentes, 2019) Extended the traditional threshold policy into the JIT (Just In Time) approach which is based on a zero-inventory policy to keep the machine at an insignificant level of deterioration and also save inventory costs. The authors employed a simulation-optimization approach to determine the optimal solution and find the critical age to perform the overhaul activities.

1.4.3 Effect of maintenance on the reliability of the system and quality of the products

In a study by (Rivera-Gomez, Gharbi & Kenné, 2013a) maintenance activities were considered to mitigate the twofold effect of deterioration on quality and reliability. To formulate the effect of maintenance on the system more accurately, a maintenance efficiency parameter was used. According to the paper, preventive maintenance can reset the age of the system to a portion of its previous age. (Ouaret *et al.*, 2015) investigated a system subjected to quality and availability deterioration. This study also considered imperfect repair activities while determining the simultaneous production and replacement policy to minimize the total costs.

(Lu, Zhou & Li, 2016) aimed to acquire a more comprehensive view of the system variables that affect the quality of products. In this study, the variables were identified, divided based on their adjustability through maintenance activities into adjustable and noise variables, and used in the optimal model. A proportional hazard model was utilized to deal with failure interaction of aspect of the system, where the level of degradation of one part affects another component's failure rate. Given the randomness of the demand for the product as well as its breakdown, repair, and quality deterioration, (Ouaret *et al.*, 2018a) formulated the optimal condition in form of the second order of HJB equations. They were able to determine simultaneous production and maintenance strategies to minimize total costs.

(Ouaret *et al.*, 2018b) Also used second-order Ito form of HJB equation to formulate a stochastic control problem in order to capture the uncertainty in diffusion behavioral components of demand and quality. (Rivera-Gómez, Montaño-Arango, Corona-Armenta, Garnica-González, Hernández-Gress & Barragán-Vite, 2018) Constructed a model with four levels of production speeds and a deterioration rate that was also a function of the production speed.

1.5 Inspection

One of the most important works on monitoring the performance of a production system is inspection. While the existence of inspection is an inevitable part of any manufacturing system, only a limited amount of research has been dedicated to investigating the simultaneous contribution of preventive maintenance, inspection, and production policies. The earliest study was conducted by Lee and Rosenblatt (Lee & Rosenblatt, 1987) who developed a model for maintenance by inspection and combined it with the classic Economic Manufacturing Quantity (EMQ) model. The proposed model could determine the optimal inspection planning as well as the optimal size of the batch by using an approximation of the cost function. Besterfield (Besterfield & Gonzlez, 2009) recommended using statistical quality control tools such as control charts. It was also suggested that in cases where a whole-batch-inspection costs more than having a specific fraction of defectives in a batch, it is more beneficial to perform sampling strategies such as the acceptance sampling plan. Bouslah et al. (Bouslah, Gharbi & Pellerin, 2014) proposed an economical model that was able to simultaneously determine the production policy and acceptance sampling plan. According to this study, the size of the lot and the size of the sample can be controlled using a single acceptance sampling plan, which could minimize the total cost. Moreover, the surface methodology was applied to identify the most optimized approach.

Another study considered a proportion of non-conforming items in the raw materials and determined a control policy for the production, replenishment, and quality activities, after applying lot-by-lot acceptance sampling on the received raw materials. The results of the study showed that it is helpful to use a hybrid decision model, which means deciding whether to return the rejected lot or not based on the inventory level of the finished products (Hlioui, Gharbi & Hajji, 2015).

Several sampling strategies have been recommended that can be adopted by industries based on their requirement. Given the dynamic pattern of defects in the semiconductor industry Lee (Lee, 2002) proposed the application of dynamic sampling strategies to accelerate defect detection. In such strategies, the location and size of sampling are determined dynamically.

Bouslah, Gharbi, and Pellerin (Bouslah *et al.*, 2016) presented an optimization model for a batch production system that could simultaneously determine the optimum lot size, inspection strategy, and inventory threshold. In this model, the quality of the product was controlled by inspection and maintenance activities. According to the study, inspection was performed through an acceptance sampling plan by attribute. Preventive maintenance was performed as part of the set-up activity at the beginning of lot production and overhaul activities were undertaken once the proportion of the defectives exceeded the given threshold considering AOQL constraint. The results showed that more than 20% cost saving could be achieved using the designed acceptance sampling plan.

Lopes (Lopes, 2018) also investigated the impact of the quality inspection policy on a production system, where the inspection is imperfect and error type l and type ll are considered. By taking into account the warranty cost and performing PM after each production cycle, an optimal model was obtained that minimized the total cost while considering an AOQL constraint. Moreover, type l and type ll errors were found to have a strong impact on the production and inspection policies. The impact of a rigorous inspection on the warranty cost was also shown to be significant.

(Bouslah, Gharbi & Pellerin, 2018) extended the previous single-stage studies to a multi-stage production line by investigating a two-machine line system where the machines' reliability was correlated and related to the level of incoming product quality. Using mathematics and simulation-based model, it was indicated that the failure correlation of machines had a significant impact on the optimal control policies. Moreover, the maintenance and quality control activities in the preceding machine were shown to improve the reliability of the subsequent machine. It was also suggested to place the inspection stations mainly in the upstream of machines which are affected by the quality of incoming products.

1.6 Critique of the literature review

The existing literature has not examined the inspection rate as a quality controller that can increase the quality of outgoing products in the continuous flow of products. Sampling methods in the batch production system are employed to deliver products to the customer with a certain level of quality. The focus of previous investigations was mostly put into the sampling method in the batch and there was neither a scenario for manufacturing low-quality products nor an incentive for providing products with high quality. However, in the real world, the fabrication of poor-quality products will have consequences in the form of losing the customer, increasing the return rate of the products, or payment of penalties. Also, for high-quality products, there will be incentives like increasing the demand, decreasing the return rate, or even increasing the price of the products.

1.7 Research problematic

The quality of the product plays an important role in determining its amount of demand and even its price in the market. Overall, it can be said that the performance, growth, and future of a company depend on the quality of the parts it produces. Therefore, managers always look for a guideline to reach the highest level of quality. However, manufacturing high-quality products is not easy, nor cheap. This is why it is an interesting challenge to determine a policy to obtain high-quality products while minimizing the total cost. The outcomes of quality need to be well-addressed in the model to be developed. However, importing all aspects of the quality effects into one model is complicated. There is also no specific model that could capture all of these aspects. Thus, our model shows all of these effects in the form of penalties for low quality or financial incentives for the high quality of products.

There are different ways to control and improve the quality of products. One of the most prevalent methods is maintenance practices that can be performed in the form of preventive maintenance, overhaul, and repair activities. Inspection is another inevitable tool to quality management processes that can be used to improve the quality of the parts and detect defective ones.

This study tries to present an optimization model that can simultaneously optimize the production policy and maintenance and inspection planning while minimizing the total costs. Moreover, we try to extend the initial system which includes only a manufacturing machine and one inventory to a more complicated model consisting of manufacturing, remanufacturing, and rectifying machines with two stock rooms.

1.8 Research objectives

The main objective of this research is to implement an integrated model in which the production, maintenance and inspection policies can be determined at the same time while the total incurred costs are minimized. Moreover, since inspection processes are not thoroughly studied in the existing literature, the desired model should provide useful insights on the inspection strategy. To achieve this goal, the following objectives should be adequately fulfilled:

- To properly determine the dynamic of the system as well as quality and availability degradation model which are affected by the age of the system and imperfect repair processes.
- To approximate and illustrate the effect of product quality level on the total incurred costs.
- To develop a model of various aspects of the maintenance processes (e.g. mean time to repair, cost of the repair and maintenance activities, etc.) as a function of the number of machine failures.

Certain assumptions are made while developing the proposed model. These assumptions are:

- The customer demand is known and constant.
- The manufacturing machine deteriorates by age and imperfect repair processes. This means that the failure rate of the manufacturing machine increases by its age and number of failures which results in an increased rate of defective products as well as a decrease in their quality.

- The machine PM activity restores its state into the condition of the machine after the last repair.
- In the replacement process the machine is replaced by a new identical machine with the same features.
- The repair process resets the age of the machine to zero with another failure rate curve.
- The maximum production rate of the manufacturing and remanufacturing machines are known.
- The failure rate of remanufacturing machine is constant.
- The repair time of the rectifying machine is insignificant and the machine is immediately fixed after break-down .
- The defective products are distributed homogeneously in the flow of production parts.
- The return rate of the products is constant and known.

1.9 Proposed methodology

Based on the proposed model, our approach should be capable of modeling and solving the problems design and optimization of different kinds of integrated production, maintenance, and inspection models. We implement this method by the following steps:

- Defining the objective and initial assumptions of the model: The aim of this step is to understand all aspects of the problem and have a comprehensive view of the model.
- Analytical and mathematical formulation of all defined variables of the problem: The goal of this step is to provide an accurate model of the system dynamic. In other words, in this step, the decision and control variables are identified, defined, and formulated based on the assumptions of the problem. The constraints of the problem should also be formulated at this stage.
- Formulation of the optimization problem and numerical resolution: This step aims to represent the simultaneous production policy, maintenance practice scheduling, and maintenance planning problem by an optimal model. By doing so, the optimal control variables that could

minimize the total incurred cost shall be determined. Numerical methods are applied using MATLAB software to determine and find these optimal control variables.

CHAPTER 2

OPTIMIZATION OF THE PRODUCTION, PREVENTIVE MAINTENANCE, REPLACEMENT, AND INSPECTION POLICIES IN MANUFACTURING SYSTEMS

2.1 Introduction

Given the ever-increasing competition in the markets of the modern industrial era, controlling different aspects of the manufacturing system such as quality and production level can be an attractive area of research. It is known that facing unexpected conditions during the operation is inevitable due to the stochastic nature of the production systems. Moreover, the deterioration phenomena caused by various factors such as the age of the machine, environmental conditions, imperfect repair processes, imperfect raw materials, human errors, etc. can have a negative impact on the quality of the production system and output products. Generally, the effect of the deterioration of production systems reflects in the reliability of the system, production pace, and some unfavorable issues such as increasing the environmental risks that can affect the performance of the other elements. To resolve the undesirable effects of deterioration, and especially to moderate the quality degradation of the products, various types of maintenance practices can be carried out.

Another helpful tool for increasing the output quality is detecting defective products through inspection procedures. The inspection process is an essential part of the manufacturing system which not only helps to detect the defective parts and separate them from the outgoing products but also helps to monitor and analyze the system's condition to make decisions. Quality of products plays a crucial role in surviving and prospering in the market. Several studies have been presented in the literature that reveal and formulate the interrelation between quality and production system design. In this study, the concept of quality is defined by comparing the quality level of products with the AOQL. Providing products with a higher quality level than AOQL will be rewarded whereas producing parts with lower quality than AOQL will be punished financially. Due to the presence of the deterioration effects in the model caused by the age of the machine along with imperfect repair processes, we need to use a method that integrates the discrete and continuous aspects of the problem such as the number of failures and age of the machine. Based on our model and due to the presence of repair history, the problem is formulated in the semi-Markov model. Since the problem can be solved mathematically, numerical methods are applied in the form of HJB equations to find the optimal solution. This chapter aims to simultaneously determine the production, PM, replacement, and inspection policies that are cost-optimized considering the backlog, inventory, production, inspection, PM, repair, replacement, and disposal costs.

2.2 System description

The discussed production system consists of one failure-prone deteriorating manufacturing machine that can produce one type of product. The system is subjected to random events like failures as well as different maintenance activities, such as repairs, replacements, and preventive maintenance practices. The deterioration phenomenon has two kinds of effects. On one hand, it decreases the part's quality by increasing the defective rate of the products and on the other hand, it affects the availability of the system by increasing the failure rate of the system. Moreover, as the number of failures goes up, the mean time required to repair the system increases. Several maintenance activities can be performed to mitigate these undesirable effects. For instance, once the failure occurs, two options are available; the machine can be repaired, or it can be replaced with a new identical machine. Also, the repair activities are imperfect and will place the machine's status between AGAN and ABAO conditions. However, the PM activities reset the machine to its initial condition after the last repair. After each repair, the age of the machine resets to zero. However, the availability of the system depends not only on the age of the machine but also on its number of failures. This means that the probability of failure of the machine at the same age increases by the number of failures.

Because of the quality deterioration of the production machine, there are always some defective parts among the output products, however, the market can tolerate a specific percentage of defective products, whereas there are penalties for an excessive amount of them. Inspection is another way to control the output quality. But it is not cost-efficient to perform 100% inspection. Therefore, based on the condition, one may decide to perform a proportion of the total inspection.

2.3 Problem statement

In this section, we develop a control problem for the production system mentioned previously. This system consists of a manufacturing machine that produces one type of product. The machine is subject to deterioration that decreases the availability of the system and quality of products. Also, the system is faced with random events such as failure and maintenance activities. Here, preventive maintenance is performed to lessen the effect of deterioration on the quality of products and also to decrease the failure probability of the system. After each breakdown of the production machine two options are available; either to replace the machine with a new one or to repair it. To control the output defective rate, an inspection process is conducted in addition to maintenance activities to set aside the defective parts.

The system has five different modes denoted by the stochastic variable $\zeta(t)$. The different modes of the machine are classified into the operational $\zeta(t) = 1$, the repair $\zeta(t) = 2$,the preventive maintenance $\zeta(t) = 3$, the replacement $\zeta(t) = 4$, and the failure $\zeta(t) = 5$ modes. Thus the modes of the machine at time t denotes a continuous-time discrete-state stochastic process $\zeta(t) \in \{1, 2, 3, 4, 5\}$ such that:

$$\zeta(t) = \begin{cases} 1 & Operational \\ 2 & Repair \\ 3 & Preventive\ maintenance \\ 4 & Replacement \\ 5 & Failure \end{cases}$$

The machine may randomly be at any of the five modes over an infinite horizon. The following transition diagram describes all five states of the machine.

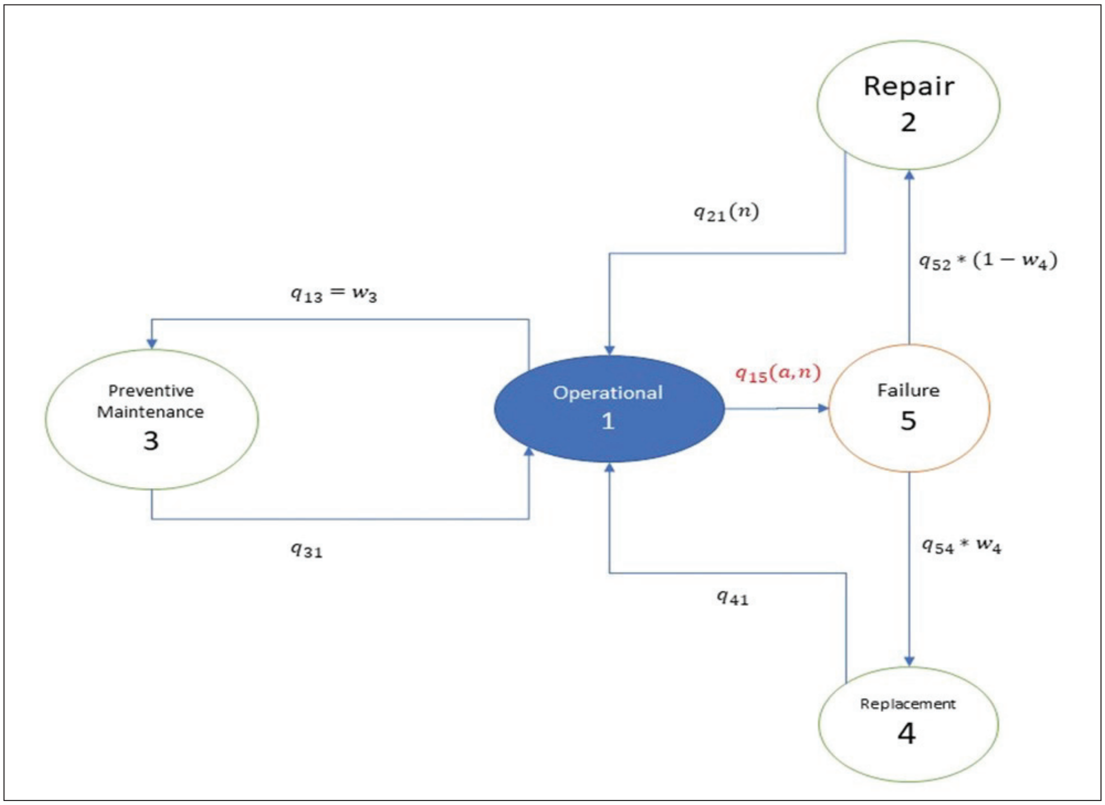


Figure 2.1 Transition diagram of proposed model

The system can be characterized at any given time t,by following these four state variables:

- The machine mode $\zeta(t)$
- The number of failure $n(t)$
- The age of the machine $a(t)$
- The stock level $x(t)$

As it is illustrated in figure 2.1, the state of the manufacturing machine changes from $operational$ mode to PM mode with a transition rate $q_{13} = \omega(.)$ which is one of the control variables. The inverse of the q_{13} points out the average delay time needed to start the preventive maintenance after asking for PM. Considering this delay makes the model more realistic. The state of the machine shifts from PM to $operational$ with q_{31} transition rate. We consider q_{31} as a constant rate because it is assumed that preventive maintenance processes have a routine and standard procedure. Machine goes to the *failure* state from *operational* state at $q_{15}(a,n)$ rate. For a more realistic assumption, we consider that the failure rate of the manufacturing machine is a function of both the age of the machine and its number of breakdowns. After each failure, the state of the machine goes either to *repair* state or to *replacement* state at the rates of $\{q_{52} * (1 - Ind\omega_4)\}$ and $\{q_{54} * Ind\omega_4\}$ respectively. ω_4 is another control variable that indicates whether we should replace the machine or not. The machine goes from *repair* state to *operation* state with $q_{21}(n)$ rate. The repair rate $(q_{12}(n))$ is decreasing as the number of failures increases. This assumption is made because every time the machine fails, repairing it becomes more complicated and more time-consuming. Furthermore, the state of the machine changes from *replacement* state to *operational* state at the constant q_{41} rate. The other transition rates are zeros.

$$Ind(f(.)) = \begin{cases} 1 & \text{if f(.) is realized} \\ 0 & otherwise \end{cases} \tag{2.1}$$

2.4 Formulation of the control problem

In this section, we develop a stochastic model for a deteriorating manufacturing system capable of producing one type of product to satisfy a constant demand. To formulate this problem, random phenomena such as the failure rate of the machine, maintenance activities, and deterioration processes should be considered and defined. Also, the effects of deterioration on the availability of the machine and the quality of products should be taken into account. We suppose that if the manufacturing machine experiences a successive number of breakdowns, it should be replaced with a new one.

We can assume that the maximum number of failures that the machine can undergo before replacement is N. It means if the machine experiences its Nth failure, it is necessary to replace it. Therefore the number of failure is $n(t) \in \{0, 1, 2..., N\}$. A realistic assumption to define the age of the machine $a(t)$, is to consider it is an increasing function of the produced part. In other words, by producing at a higher production rate, the age of the machine increases faster, which also leads to faster deteriorates. The following differential equation defines the relationship between age and production rates since the last maintenance activities.

$$\dot{a} = ku(.), \qquad a(T) = 0 \tag{2.2}$$

k is a given constant coefficient(k>0) and T is defined as the instant at which the last replacement or preventive maintenance activities were performed. We also formulate the age of the machine after nth failure by the following equation:

$$a_n(t) = a(t - t_n) + \alpha_n, \quad t_n \leq t < t_{n+1}, \quad t_0 = 0, \quad \alpha_0 = 0, \tag{2.3}$$

where t_n is the moment at which the nth failure occurs and α_n is the simulated age of the machine after nth repair. The simulated age of the machine is given by:

$$\alpha_n = \phi_a \times a(t_n - t_{n-1}) + \alpha_{n-1}, \quad n \geq 1 \tag{2.4}$$

where ϕ_a is the constant repair age intensity $(0 \leq \phi_a < 1)$. Since the repair activities are not perfect, the condition of the system will not reset to its initial state. This can be attributed to reasons such as imperfect repair processes, imperfect materials or components used to fix the machine, and human errors. Using the number of failures along with the age of the machine to describe the deterioration, enables us to integrate the effect of imperfect repair in our model. The deterioration caused by the imperfect repair affects the age of the machine and its reliability as well as the quality of the products after the repair process. These imperfect repair deterioration which is also known as repair intensity are given by :

$$\phi = \begin{cases} \phi_a & \text{age intensity coefficient} \\ \phi_r & \text{reliability intensity coefficient} \\ \phi_q & \text{quality intensity coefficient} \end{cases}$$

According to the notation, let d be the constant market demand rate that needs to be satisfied, and $u(t)$ the production rate at time t. The production rate of the machine in operation state should always satisfy the constraint: $u(t) \in [0, d, u_{max}]$, where u_{max} is the maximum production rate of the manufacturing machine.

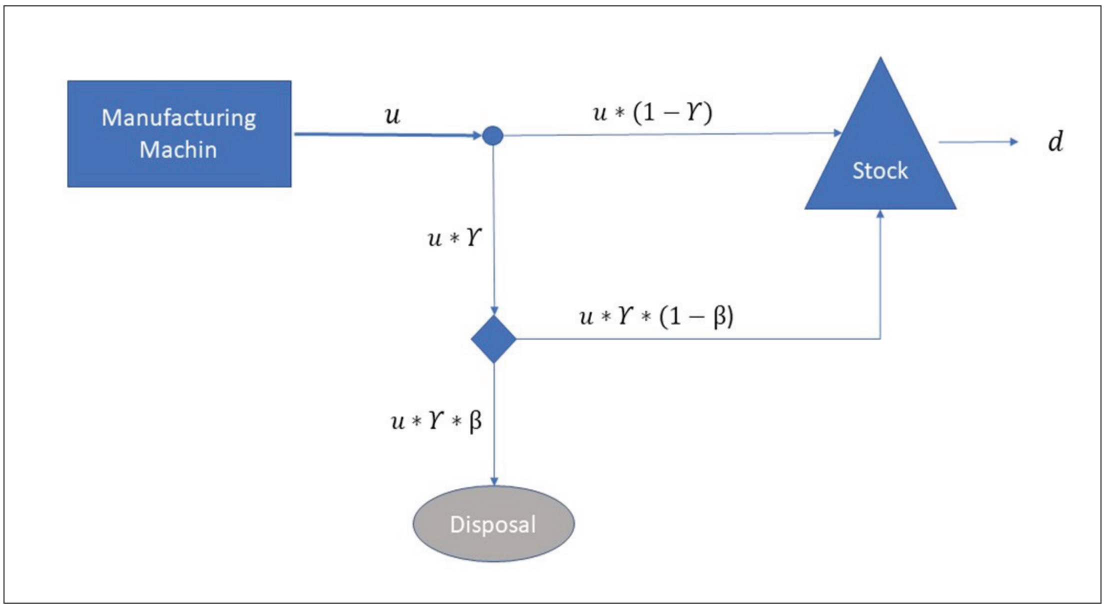


Figure 2.2 Block diagram of proposed model

According to the block diagram of our model in the figure 2.2, the dynamics of the inventory $x(t)$ can be formulated by the following differential equation:

$$\begin{aligned} \dot{x} &= (1-\gamma)u + (1-\beta)\gamma u - d \\ \dot{x} &= (1-\gamma\beta)u - d \quad , \qquad x(0) = x_0 \end{aligned} \tag{2.5}$$

where $\gamma(.)$ is the proportion of inspection that is performed on produced parts before sending them to the inventory. The inspection rate $\gamma(.)$ is one of the control variables that need to be determined at any state of the machine. Also, $\beta(t)$ represents the rate of defectives in this equation which is an increasing function of the age of the machine.

To formulate the effect of deterioration in the production system, several factors such as usage, wear down, imperfect repair, human errors, etc. should be considered. Also, in order to formulate the failure rate, some articles have suggested using the age of the machine as a representative of the deterioration, while others have proposed taking the number of failures as an index of wear down of the machine. In this study, we propose a more comprehensive and realistic model by applying all deterioration-caused factors such as the age of the machine (a), the number of failure (n) and repair reliability factor (ϕ_r) in the failure rate of the machine (q_{15}), which is given by the following expression :

$$q_{15_n} = K_0 + \sum_{i=1}^{n-1} \left(\phi_{ri} \, (\frac{a_i}{A})^{\theta_a} (\frac{i}{N})^{\theta_i} \right) + \left(K_1 - \sum_{i=1}^{n-1} (\phi_{ri} \, (\frac{a_i}{A})^{\theta_a} (\frac{i}{N})^{\theta_i}) \right) \left(1 - e^{(-K_t n^{\theta_{f_1}} a^{\theta_{f_2}})} \right) \quad (2.6)$$

where K_0 is the constant initial and minimum value of failure rate for the new machine and $K_1 + K_0$ is the maximum quality deterioration considered for the manufacturing machine. ϕ_{ri} is the repair quality intensity coefficient for each number of failures. A is the maximum age of the machine in the considered domain. N is the maximum number of failures before replacement processes. a_i is the age of the machine when the ith failure occurs. By using parameters θ_{f_1} and θ_{f_2} we can adjust the failure curve trajectory with different speeds.

The inverse of the failure transition rate q_{15} indicates the meantime to failure $MTTF(a, n)$ which shows the probable average time that the system can operate before breakdown at any number of failures and any age of the machine.

The parameters θ_i and θ_a in Equation indicate the effect of the number of failures and failure age of the machine at each failure on its reliability. The fact that multiple variables are involved in Equation , complicates future graphical illustration and analysis of the problem. Therefore, certain assumptions are made to simplify the equation 2.4. This assumption is given by:

$$\sum_{i=1}^{n-1} \left(\phi_{r_i} \, (\frac{a_i}{A})^{\theta_a} (\frac{i}{N})^{\theta_n} \right) = \acute{\phi_r} \, (\frac{n}{N})^{\theta_n} \qquad ; \qquad \acute{\phi_r} = constant \tag{2.7}$$

Since the repair activities are not perfect, the probable failure age of the machine after repair decreases upon the increase of the number of failures. In the assumption, 2.7, $\theta_n > 1$ specifies that as the number of failures goes up, the capability of repair to reset the reliability decreases, so the increase in failure rate intensifies by the number of failures. $\theta_n < 1$ indicates that by increasing the number of failures, the acceleration in increasing of failure rate decreases by the number of failures. By considering the assumption 2.7 the equation can be simplify to the following form:

$$q_{15_n} = K_0 + \acute{\phi_r} \, (\frac{n}{N})^{\theta_n} + \left(K_1 - \acute{\phi_r} \, (\frac{n}{N})^{\theta_n} \right) \left(1 - e^{(-K_t n^{\theta_{f_1}} a^{\theta_{f_2}})} \right) \tag{2.8}$$

As shown in Figure 2.3(a), by increasing the number of failures, the failure rate of the machine at the same age will increase. For a better illustration, the average time to failure of the machine at each number of failures is presented in Figure 2.3(b) in which the average time to failure decreases by increasing the number of failures.

One of the most important aspects of the production system that are affected by deterioration is the quality of the produced parts. There are different approaches to formulate the relationship between quality and deterioration. Some authors tend to use the age of the machine as an index of the deterioration level and a basis for their maintenance-decision policy. The number of failures is another indicator of the level of the product's quality. Others have mostly emphasized the effect of imperfect repair on the defective rate. In this study, both the age of the machine and the NOF (number of failures) are used to link deterioration with the defective rate of products β. Thus, we can claim that in our model the defective rate ($\beta(a, n)$) is an increasing function of the age of the machine and NOF as given by:

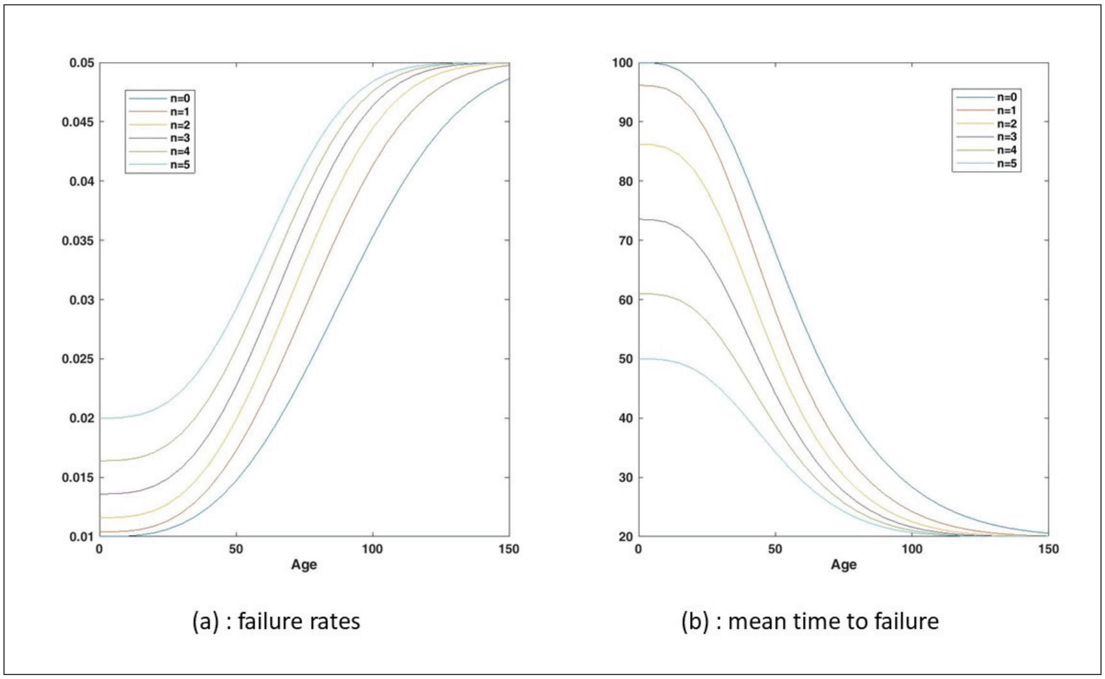


Figure 2.3 Failure rate and Mean time to failure of the machine when $\theta_n = 2$

$$\beta(a,n) = B_0 + \sum_{i=1}^{n-1}\left(\phi_{q_i}\,(\frac{a_i}{A})^3(\frac{n_i}{N})^2\right) + \left(B_1 - \sum_{i=1}^{n-1}(\phi_{q_i}\,(\frac{a_i}{A})^3(\frac{n_i}{N})^2)\right)\left(1 - e^{(-K_b\theta_b a^3)}\right) \quad (2.9)$$

In the equation 4.10 B_0 is the defective rate of the machine at its initial condition and $(B_0 + B_1)$ is the boundary value of the defective rate of the considered machine in its worst condition. k_b is a given constant and θ_b is the adjustment parameter for the defective rate trend.

As the number of failures increases and the machine is subjected to imperfect repairs, the production system's components deteriorate more. In addition, detecting the defects becomes more complicated, and also repairing the machine takes more time. Thereby we can express the meantime to repair the machine with the following equation:

$$MTTR(n) = T_{21}(n) = \frac{A_1}{r^n} + A_2 * n \quad (2.10)$$

where $A_1.A_2$ and r are given constant parameters. The parameter r in equation 2.10 is used to adjust the trace of $MTTR$ and therefore the repair rate trajectory. The parameters A_1,A_2 and r can be obtained through the historical repair time data of the manufacturing machine. The repair rate of the machine can be obtained by:

$$q_{21}(n) = \frac{1}{T_{21}(n)} \tag{2.11}$$

Also we can use following repair rate:

$$q_{21}(n) = R_0 + R_1\left(1 - (\frac{n-1}{N})^r\right) \tag{2.12}$$

As illustrated in Figure 2.4 the repair rate q_{21} decreases with the number of failures and the mean time to repair $MTTR$ increases with the number of failures.

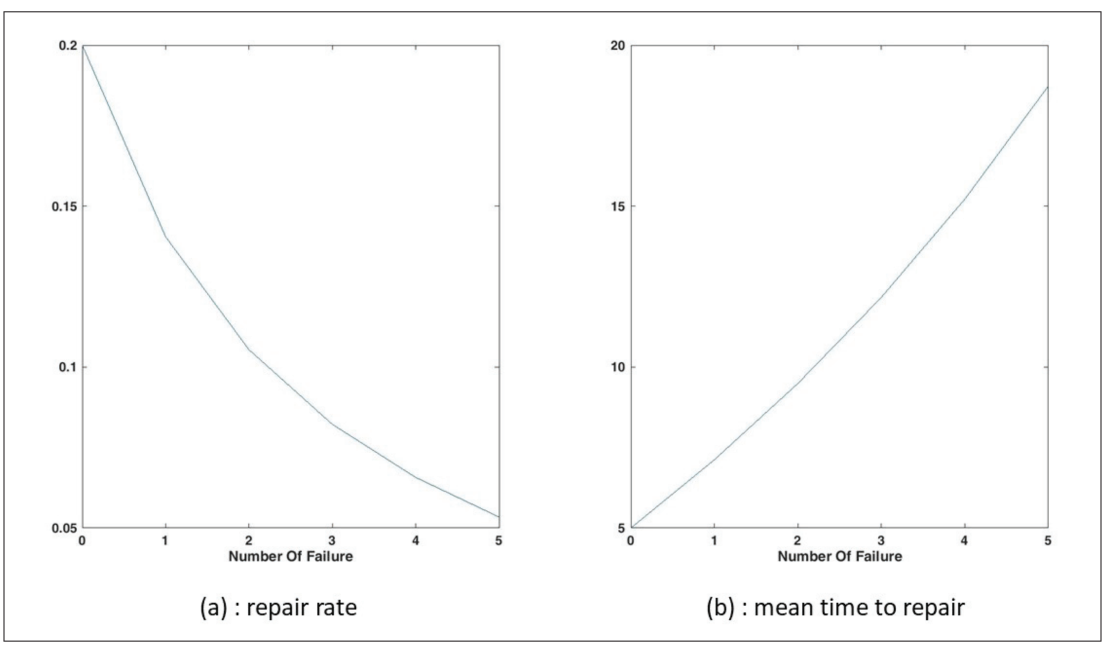


Figure 2.4 Failure rate and Mean time to failure of the machine

Furthermore as indicated in Figure 2.4(a) the repair rate of the manufacturing machine decreases by the number of failures n. In other words, since the complexity of the repair process increases by the number of failures, the mean time required to repair the machine also increases by the number of failures. Figure 2.4(b).

As the production system undergoes more deterioration, the rate of the defective items increases. Moreover, based on the block diagram of the system in Figure 2.2, the defective items that are not detected in the inspection station, will be stored in the stock and delivered to the market. The rate of defective items that are sent to the stock is defined by the *average of outgoing quality*, AOQ is given by the following expression:

$$AOQ = \frac{(1-\gamma)\beta}{1-\gamma\beta} \tag{2.13}$$

where AOQ specifies the proportion of defective products that will reach the customers, γ is the fraction of the inspected products before sending them to the inventory, and β is the fraction of the defective items that the machine produces. Generally, there is always a certain tolerable limit for defectives that is determined by the market and is known as *average outgoing quality limit* $AOQL$. The average outgoing quality limit is the maximum amount of defectives that we are allowed to produce without paying any penalties. $AOQL$ is given by:

$$AOQL = max\{AOQ\} \tag{2.14}$$

By applying the combination of inspection and maintenance activities, the manufacturer can prevent AOQ to surpass the quality limit $AOQL$.

In this model, the inspection rate of control policy that can always ensure that the AOQ is less than $AOQL$, is given by:

$$\gamma \geq \frac{\beta - AOQL}{\beta(1 - AOQL)} \tag{2.15}$$

Furthermore, the minimum amount of inspection that should be performed to keep AOQ less than $AOQL$ is called the nominal inspection rate γ_{nom} and can be defined as:

$$\gamma_{nom} = \begin{cases} 0 & \text{if } \beta < AOQL \\ \frac{\beta - AOQL}{\beta(1-AOQL)} & \text{if } \beta \geq AOQL \end{cases} \tag{2.16}$$

In a more realistic assumption, it is not enough just to define a strict quality constraint to be respected. It is better to also consider an adjustable penalty for violating the quality limit. Generally, the market is more sensitive to the quality of critical products compared to non-critical ones. Therefore, a higher penalty can be established for critical products. There can also be a financial incentive for providing high-quality products. By considering the proportionate penalty for exceeding the quality limit as well as the financial incentives corresponding to the level of good quality, we can handle the market's sensitivity to the quality limit. If the quality of output is considered as the cost of quality can be defined as follows:

$$C_q = \begin{cases} \varepsilon_{q.p}(\frac{(1-\gamma)\beta}{1-\gamma\beta})u & \text{if } AOQ > AOQL \\ 0 & \text{if } AOQ = AOQL \\ \varepsilon_{q.i}(AOQL - \frac{(1-\gamma)\beta}{1-\gamma\beta})u & \text{if } AOQ < AOQL \end{cases} \tag{2.17}$$

where $\varepsilon_{q.p}$ is the penalty coefficient for surpassing the quality limit $AOQL$ required by customer and $\varepsilon_{q.i}$ is the financial incentive coefficient for the production of parts with better a quality than the minimum required level.

Regarding the devised production system, there is an important technical condition that needs to be evaluated to ensure that the production system can fulfill the market's demand. The manufacturing systems can meet the required demand over an infinite, only if the system is feasible. The feasibility condition that needs to be fulfilled is given by:

$$(1 - \gamma\beta)u_{max} \times \pi_1 \geq d \tag{2.18}$$

where π_1 is the limiting probability for the *operational* mode and π_1 can be obtained by the following expressions:

$$\begin{cases} \pi_i * Q(.) = 0 \\ \sum_{i=1}^{m} \pi_i = 1 \qquad i : \textit{modes of the machine} \end{cases} \tag{2.19}$$

where $Q(.)$ is the transition matrix of the different modes of the machine. In order to minimize the total cost and enhance the performance of the production machine, we can control the system using its transition rates between the different states. The transition rate Q is given by:

$$Q = \begin{Vmatrix} q_{11} & 0 & \omega_3 & 0 & q_{15}(a,n) \\ q_{21}(n) & q_{22} & 0 & 0 & 0 \\ q_{31} & 0 & q_{33} & 0 & 0 \\ q_{41} & 0 & 0 & q_{44} & 0 \\ 0 & \omega_2 & 0 & \omega_4 & q_{55} \end{Vmatrix} \tag{2.20}$$

where $\omega_2 = (1 - \acute{\omega}_4)q_{52}$ and $\omega_4 = \acute{\omega}_4 q_{54}$. There are two probabilities at time $t + \delta t$ for the machine in the state of α; the machine either stays in the state α or leaves this state and goes to the state $\acute{\alpha}$. The transition probabilities are given by:

$$P[\zeta(t+\delta t) = \alpha | \zeta(t) = \alpha] = 1 + q_{\alpha\alpha}\delta t + \epsilon \tag{2.21}$$

and

$$P[\zeta(t+\delta t) = \acute{\alpha} | \zeta(t) = \alpha] = q_{\alpha\acute{\alpha}}\delta t + \epsilon \tag{2.22}$$

with

$$\begin{cases} \lim_{\delta t \to 0} \epsilon = 0 \\ q_{\alpha\alpha} = -\sum_{\acute{\alpha} \neq \alpha} q_{\alpha\acute{\alpha}} \\ q_{\alpha\acute{\alpha}} \geq 0 \qquad \forall \alpha, \acute{\alpha} \in E : \acute{\alpha} \neq \alpha \end{cases} \tag{2.23}$$

The control policy of the problem is characterized by the control variables $\{u(.), \omega_3(.), \acute{\omega_4}(.), \gamma(.)\}$. Furthermore, by calculating the equation 2.18 and based on the stochastic process, we have the set of feasibility condition $\Gamma(.)$ as follows:

$$\Gamma(.) = \begin{cases} \{u, \omega_3, \acute{\omega_4}, \gamma\} \in \Re^3 & \begin{cases} 0 \leq u \leq u_{max} \\ \omega_3 \in \{\omega_{3min}, \omega_{3max}\} \\ \acute{\omega_4} \in \{\acute{\omega}_{4min}, \acute{\omega}_{4max}\} \\ 0 \leq \gamma \leq 1 \end{cases} \\ (1-\gamma\beta)u_{max} \times \pi_1 \geq d \\ \pi_1 = \dfrac{1}{1 + \dfrac{\omega_2 q_{15}}{(\omega_2+\omega_4)q_{21}} + \dfrac{\omega_3}{q_{13}} + \dfrac{\omega_4 q_{51}}{(\omega_2+\omega_4)q_{41}} + \dfrac{q_{15}}{(\omega_2+\omega_4)}} \end{cases} \tag{2.24}$$

Due to the deterioration effects caused by the machine age and imperfect maintenance activities, while considering the history of repairs with the number of failures, it is difficult to compute an optimal solution for the semi-Markov model. The proposed mathematical model for solving this semi-Markov problem consists of convergence of numerical methods to ensure that we will have a solution. In the model, the state of the system can be described at any instant time t by the state variables $\big(\zeta(t), x(t), a(t), andn(t)\big)$, with hybrid condition consisting of discrete $\zeta(t)$ and continuous $(x(t), a(t), n(t))$ components. To solve the problem, it is better to define the instantaneous cost function $G(.)$. Let us define $G(.)$ at any state as follows:

$$G(\zeta, x, a, n) = h(x) + c(u) + w(\zeta, n) + z(\text{AOQ}) \tag{2.25}$$

where $h(x) = c^+x^+ + c^-x^-$, that c^+ and c^- are constant incurred costs per unit of product for inventory and backlog, respectively. The cost of backlog is much more that the cost of holding inventories; $c^- >> c^+ > 0$. and let us define x^+ and x^- as follows:

$$\begin{cases} x^{+} = max(0,x) \\ x^{-} = max(0,-x) \end{cases} \tag{2.26}$$

Let us also define $c(u)$ as the cost function of the manufacturing process and $w(\zeta, n)$ as the cost function of maintenance activities such as repair, preventive maintenance, and replacement that can be obtained by following expressions:

$$c(u) = c_m u + c_{ins}\gamma u + c_q u + c_{dep}\gamma\beta u \tag{2.27}$$

$$w(\zeta,n) = c_{over}(Ind\{\zeta(t) = 4\}) + c_{rep}(Ind\{\zeta(t) = 2\}) + c_{pm}(Ind\{\zeta(t) = 3\}) \tag{2.28}$$

and the indicator function Ind is defined as:

$$Ind(f(.)) = \begin{cases} 1 & \text{if f(.) is true} \\ 0 & otherwise \end{cases}$$

Also z(AOQ) is the cost function of the quality of outgoing products and z(AOQ)= C_q.

Our objective is to find the four control decision variables in 2.24, namely, the production rate $u(.)$, preventive control rate ω_3 , replacement rate $\acute{\omega}_4$, and inspection rate γ, that minimize the expected total cost for each initial state condition of the machine (α, x, n). The expected discount total cost (.) can be defined as follows:

$$J(\alpha,x,a,n,u,\omega_3,\acute{\omega}_4) \quad = E\left\{ \int_0^{\infty} e^{-\rho t} G(.)\, dt \mid \zeta(0) = \alpha,\ x(0) = x,\ a(0) = a,\ n(0) = n \right\} \tag{2.29}$$

where ρ denotes the discount rate of the incurred cost and $E\{\bullet|\alpha, x_0, a_0, n_0\}$ is the condition-based expectation symbol. The objective of the model is to determine an optimal control policy $(u^*(.), \gamma^*(.), \omega_3^*, \acute{\omega}_4{}^*)$ that minimizes the total discount cost $J(.)$. The optimal control policies are obtained through the value function v that is defined as follows:

$$v(\alpha, x, a, n) = \inf_{(u(.),\omega_3,\acute{\omega}_4,\gamma\in\Gamma(.))} J(\alpha, x, a, n, u, \omega_3, \acute{\omega}_4, \gamma), \forall \alpha \in \Omega, x \in R, n \in N \qquad (2.30)$$

The value function $v(\alpha, x, a, n)$ refers to the minimum value of the total cost and satisfies the optimality conditions of the problem. Generally, if we consider the $v(., t)$ as a cost-to-go function, and the $t = 0$ as the initial time of the problem, then we can define our optimal function in two parts; The incurred discounted cost in the interval $[0, t]$, and the cost in the interval $[t, \infty)$. therefore, we can break up the cost function in form of the following expression:

$$v(\alpha(.), x(.), n(.), t) = \inf_{(u(.),\omega_3,\acute{\omega}_4,\gamma)_{0\leq s\leq\infty}} E \left\{ \begin{array}{l} \int_0^t e^{-\rho t} g[\alpha, x, a, n, u, \omega] \, ds \\ + \int_t^\infty e^{-\rho t} g[\alpha, x, a, n, u, \omega] \, ds \end{array} |\alpha, x, n \right\} \qquad (2.31)$$

In this equation, we should consider the randomness of and the discount rate ρ. Thus, the conditional expectation operation $\tilde{E}$ can be defined as follows:

$$\tilde{E} H(\alpha(t + \delta t)) = E\{H(\alpha(t + \delta t))|\alpha(t)\} \qquad \forall H(\alpha) \qquad (2.32)$$

By applying the conditional expectation operation $\tilde{E}$ into equation , we can approximate equation 2.31 as follows:

$$v(\alpha(.), x(.), n(.), t) =$$

$$\inf_{(u(.),\omega_3,\acute{\omega}_4,\gamma)\ t\leq s\leq t+\delta t} \tilde{E} \left\{ \begin{array}{l} g[\alpha(t), x(t), a(t), n(t), u(t), \omega(t)]\delta t + \\ + \frac{1}{1+\rho\delta t} v[\alpha(t+\delta t), x(t+\delta t), a(t+\delta t), \\ \qquad n(t+\delta t), u(t+\delta t), \omega(t+\delta t)] \end{array} \right\} + o(\delta t) (2.33)$$

Considering that the value function $v(.)$ is a differentiable function, we can expand the equation 2.4 to:

$$\rho v(\alpha(.), x(.), n(.), t) = \inf_{(u(.),\omega_3,\acute{\omega_4},\gamma)} \left\{ \begin{array}{l} g[\alpha(t), x(t), a(t), n(t), u(t), \omega(t)]\delta t \\ + \dfrac{\partial v}{\partial x}[\alpha(t), x(t), a(t), n(t), u(t), \omega(t)]\delta x(t) \\ + \dfrac{\partial v}{\partial t}[\alpha(t), x(t), a(t), n(t), u(t), \omega(t)]\delta t \\ + \sum_{\acute{\alpha}} v[\acute{\alpha}, x(t), a(t), n(t), t] q_{\acute{\alpha}\alpha}\delta(t) \end{array} \right\} + o(\delta t) \tag{2.34}$$

By considering the equation $\dot{x} = \frac{\delta x(t)}{\delta t}$ and $Q(.) = \{q_{\alpha\acute{\alpha}}\}$ the equation 2.34 can be simplified as:

$$\rho v(\alpha, x, n) = \min_{(u,\omega_3,\acute{\omega_4},\gamma)\in\Gamma(\alpha)} \left\{ \begin{array}{l} g[\alpha, x, n, a, u, \omega_3, \acute{\omega_4}, \gamma] + \dfrac{\partial v}{\partial x}[\alpha, x, n, a]\dot{x} + \\ \quad + \dfrac{\partial v}{\partial t}[\alpha, x, n, a]\dot{a} + Q(.)v[\alpha, x, \varphi(\zeta, n), 0] \end{array} \right\} \tag{2.35}$$

These series of equations are used to obtain the optimal control policy and are known as Hamilton-Jacobi-Bellman equations (HJB). Based on the behavior of the system after maintenance activities, we can define reset function $\varphi(\zeta, n)$ at any jump time τ for the process $\zeta(t)$. After each failure and following the imperfect repair, the machine's failure rate changes with increasing number of failures. However, replacement activities reset the machine to the initial condition of the new machine. The age of the machine after any maintenance activities will reset to zero. The reset function $\varphi(\zeta, n)$ is described using the following expressions:

$$\varphi(\zeta, n) = \begin{cases} n+1 & \text{if} \quad \zeta(\tau^+) = 1 \quad \text{and} \quad \zeta(\tau^-) = 2 \\ 0 & \text{if} \quad \zeta(\tau^+) = 1 \quad \text{and} \quad \zeta(\tau^-) = 4 \\ n & \textit{otherwise} \end{cases}$$

$\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial t}$ are first-order partial derivatives of the value function $v(.)$. As long as the value function $v(.)$ is available, an optimal control policy can be extracted out of the HJB equation 2.35. However, an analytical solution is almost impossible to obtain. BoukasandHauire(1990) performed Kushner's method to solve the problem in the context of production planning. They provided a numerical calculation in the stochastic context to avoid solving the problem analytically.

2.5 Optimal control policy

As mentioned before, the purpose of this study is to determine a control policy for a production system subject to deterioration where maintenance activities along with inspection strategies, help to produce sustainable and acceptable quality parts to meet the constant demand for a single product. To determine the optimal control policy, numerical techniques based on the Kushner approach can be employed to approximate the problem for HJB equations. In this approach, a numerical approximation of the gradient of the value function $v(.)$ is used. Here, in order to approximate the continuous value function v, a discrete function $v_h(.)$ can be applied, and all partial derivatives of the value function can be expressed as a function of v_h given by the following expression:

$$\frac{\partial v}{\partial x}(\alpha, x, a, n) = \begin{cases} \frac{1}{h_x}\Big(v^h(\alpha, x + h_x, a, n) - v^h(\alpha, x, a, n)\Big) & \text{if } \dot{x} \geq 0 \\ \frac{1}{h_x}\Big(v^h(\alpha, x, a, n) - v^h(\alpha, x - h_x, a, n)\Big) & \text{if } \dot{x} < 0 \end{cases} \tag{2.36}$$

$$\frac{\partial v}{\partial t}(\alpha, x, a, n) = \frac{1}{h_a}\Big(v^h(\alpha, x, a + h_a, n) - v^h(\alpha, x, a, n)\Big) \tag{2.37}$$

where h_x and h_a are finite intervals associated with variables x and a, respectively. The application of Kushner discrete equation for every state of the machine in our model is given by:

$$\nu^h(\alpha,x,a,n) = \min_{(u(.),\omega_3,\acute{\omega_4},\gamma\in\Gamma(.))} \left[\left(\rho + |q_{\alpha\alpha}| + \frac{\dot{x}}{h_x} + \frac{\dot{a}}{h_a}\right)^{-1} \left(g(.)+ \begin{Bmatrix} \nu^h(\alpha, x+h_x, a, n)\frac{|\dot{x}|}{h_x} Ind\{\dot{x} \geq 0\} \\ \nu^h(\alpha, x-h_x, a, n)\frac{|\dot{x}|}{h_x} Ind\{\dot{x} < 0\} \end{Bmatrix} + \nu^h(\alpha,x,a+h_a,n)\frac{|\dot{a}|}{h_a} + Q(.)\nu(\alpha,x,\varphi(n,\zeta))\right)\right] \tag{2.38}$$

where

$$\begin{cases} \dot{x} = (1-\gamma\beta)u(.) \ - d \\ \dot{a} = k_a u(.) \qquad\qquad k_a \text{is constant} \end{cases}$$

In general, solving the HJB equation 2.35 analytically is complex and almost impossible. Hence, numerical methods are used to simplify the problems and make them easier to solve. The numerical techniques help us to convert the continuous problem to the discrete semi-Markov process 2.38 within finite grids of states and decision variables. Practically this discrete semi-Markov equation can be solved by policy improvement or value iteration techniques.

2.5.1 Numerical example

In this section we provide an example of the production system discussed in section . By applying the policy improvement method, we try to solve the discrete semi-Markov model2.38. The output discrete approximation $\nu^h(\alpha, x, a, n)$ of the equation 2.38 converges to the continuous function $\nu(\alpha, x, a, n)$ of the equation 2.30. In Table 2.5.1 the value of each parameter required for the application of numerical method is presented. To solve the discrete version of HJB equation by numerical technique, we need to define a finite grid denoted by G^h_{xan} and its computational domain of the state variables (x, a, n) with $h = (h_x, h_a)$ as follows:

$$G^h_{x,a,n} = \{(x,a,n) : -10 \leq x \leq 20, 0 \leq a \leq 150, 1 \leq n \leq 5\} \tag{2.39}$$

Parameter	u_{max}	d	ρ	c^+	c^-
value	1	0.6	0.05	4	400
Parameter	c_{rep}	c_{pm}	c_{ovr}	c_{ins}	c_{dis}
value	50	100	2000	3	2
Parameter	ϕ_{q_i}	$\acute{\phi}_r$	θ_n	θ_{f1}	θ_{f2}
value	0	0.01	2	0.6	3
Parameter	A	N	K_t	K_b	θ_b
value	150	5	$10*10^{-7}$	$18*10^{-7}$	1
Parameter	A_1	A_2	r	$\varepsilon_{q.p}$	$\varepsilon_{q.i}$
value	5	0.1	0.98	10	1
Parameter	N	q_{31}	q_{41}	q_{54}	q_{52}
Value	5	0.3	0.2	0.9	0.9
Parameter	ω_3^{max}	ω_3^{min}	ω_4^{max}	ω_4^{min}	(h_x, h_a)
Value	0.1	0.0001	1	0	(2,5)

Table 2.1 Parameters of the numerical example

2.5.2 Production policy

Based on the proposed model, the optimal production strategy $u^*(\alpha, x, a, n)$ is illustrated in Figure 2.5 which shows the optimal production rate of manufacturing machine at any age of the machine a for each number of failures n and level of the inventory x. As can be seen in Figure 2.5, the space plan (a, x, n) is divided into three regions by production strategies in which the production rate is adjusted to the u_{max}, d and 0 rates. Moreover, we can see the effect of deterioration in the production threshold where the optimal stock level of the system increases by the age and number of failures of the machine, to ensure the fulfillment of the demand. The stock level of the production system is constantly increased before the last failure occurs. Given that the machine should be replaced with a new one once the last failure happens, the stock level should be set to a lower level compared to the previous number of failures to prevent the excessive cost.

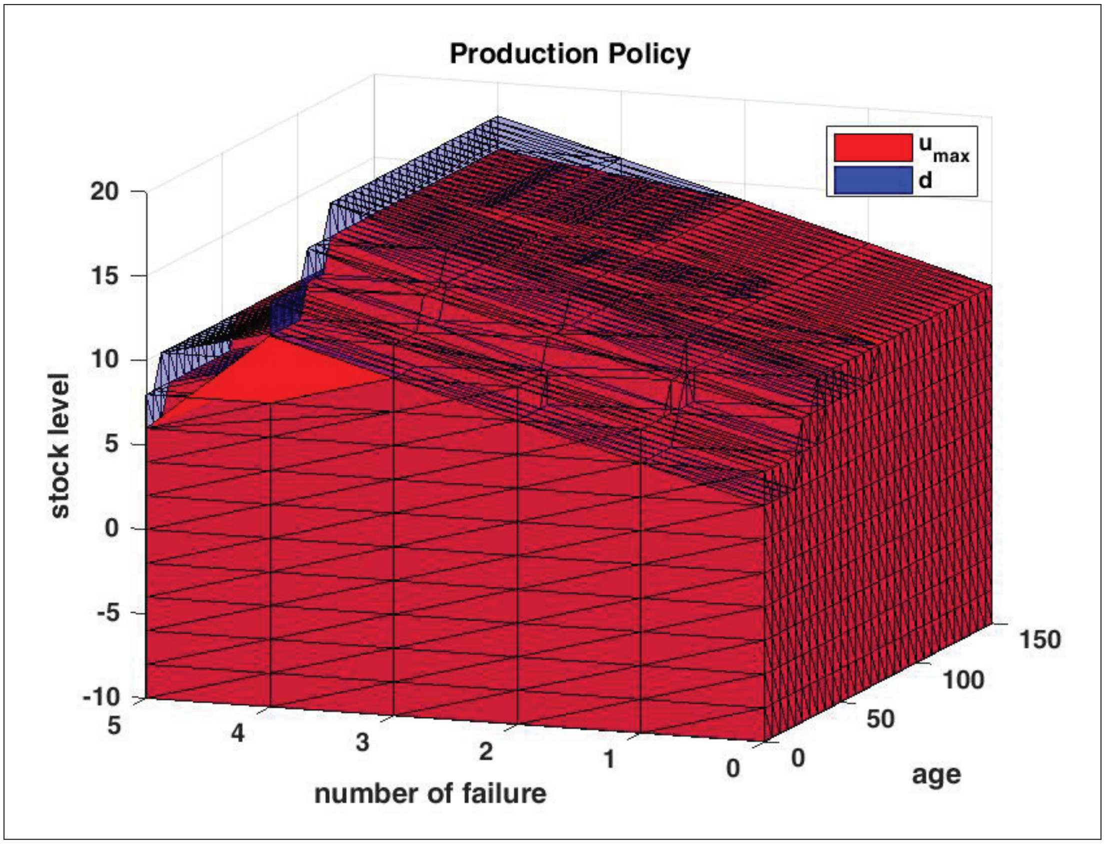


Figure 2.5   Production policy

To better illustrate the production policy, the optimal stock threshold $Z^*_{n_i}$ is shown in Figure 3.5a for each number of failures separately. As can be observed in the plot, the level of stock increases with the number of failures ($Z^*_{n_{i-1}} \leq Z^*_{n_i} \quad \{i = 1, ..., (N-1)\}$), this is given because as the machine gets older and deteriorates more, its availability decreases. In addition, after each failure, technicians need time to repair the machine while the demand should be satisfied, because otherwise we will be penalized for the backlog. Moreover, the mean time needed to repair the manufacturing machine after each failure increases with the number of failures. The reason is that the imperfect machine repair results in more deterioration which in turn further complicates fixing the machine and as a result more time is required. At the last failure number

N, since the machine should be renewed after the next failure, there is no need to keep excessive products in stock, therefore, the optimal production threshold at the last failure number Z_N should be set to a lower level as can be noticed in Figure 3.5b. It is noteworthy that in the case of lower availability of the system, it is essential to maintain a specific amount of inventory in stock to avoid backlog costs.

The optimal production threshold based on the Figures 3.5a, and 3.5b can be defined in the numerical form as the following expression:

$$u^*(1, n_i, .) = \begin{cases} u_{max} & \text{if } x(t) < Z^*_{n_i}(.) \\ d & \text{if } x(t) = Z^*_{n_i}(.) \\ 0 & \text{if } x(t) > Z^*_{n_i}(.) \end{cases} \tag{2.40}$$

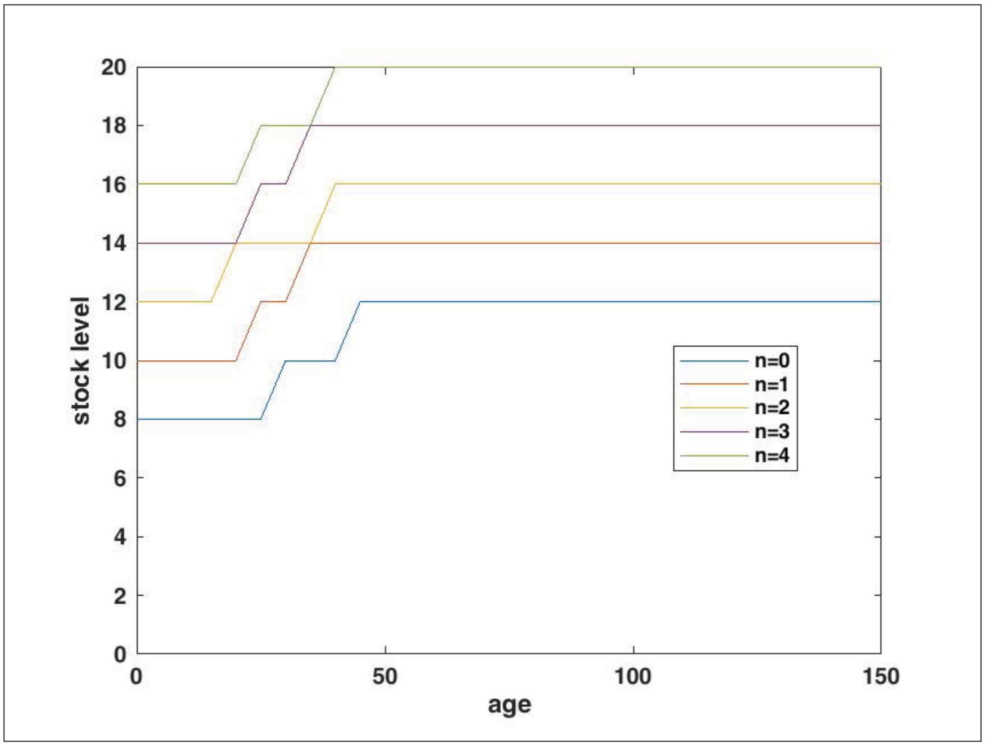
where $Z^*_{n_i}(.)$ is the function that represents the optimal level of production for any given number of failures at any age of the machine as illustrated in Figure 3.3.

2.5.2.1 Inspection policy

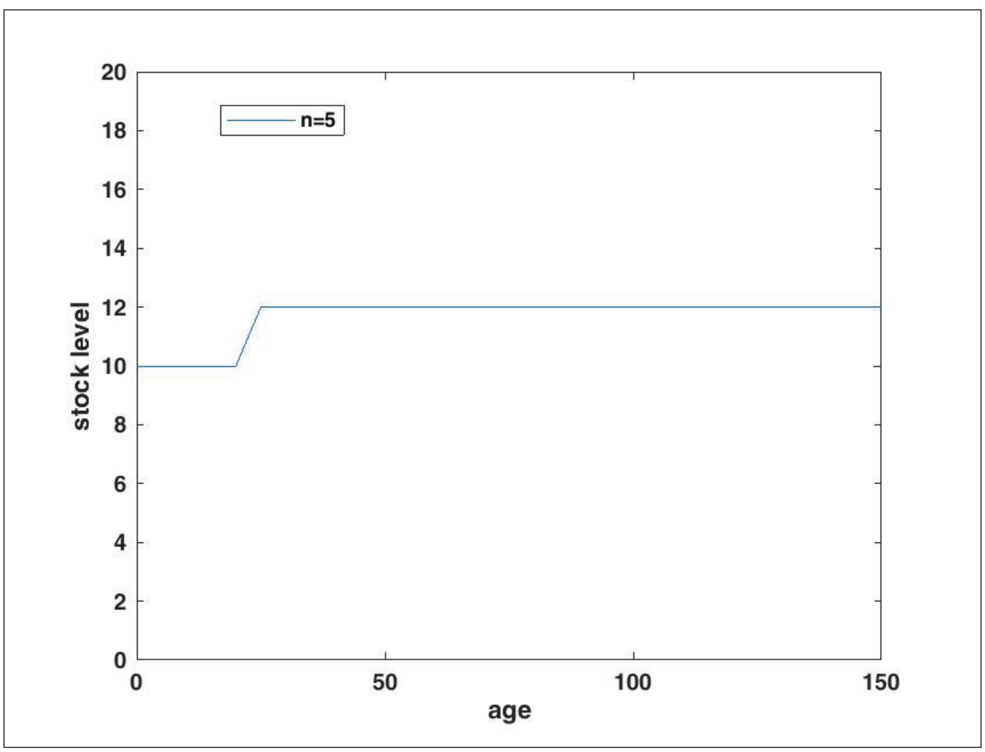
To clarify the issue, we define a nominal inspection level in which an inspection rate γ more than the nominal level causes an output product quality higher than *AOQL* and an inspection rate less than the nominal level results in a quality less than *AOQL*. The nominal inspection level is given as follows:

$$\gamma_{nominal} = \begin{cases} 0 & \text{if } \beta < AOQL \\ \frac{\beta - AOQL}{\beta(1-AOQL)} & \text{if } \beta \geq AOQL \end{cases} \tag{2.41}$$

As shown in Figure 2.7 the inspection rate under the nominal level will result in output quality with higher defective percentage than *AOQL*, and the zone up to the nominal line will lead to output products with higher quality . The obtained inspection policy for the number of failures $n = 4$ is presented in Figure $2.8a$. As can be observed in Figure $2.8a$, the inspection decision variable (γ) can be set to a value between 0 and 1 $\{0 \leq \gamma \leq 1\}$, depending on the age of the machine and the level of the stock at each number of the failures. As the machine gets older and the rate of the defective parts increases, inspection processes should be conducted to detect and



a) production policy at each number of failure



b) production policy at the last number of failure

Figure 2.6 Production policy

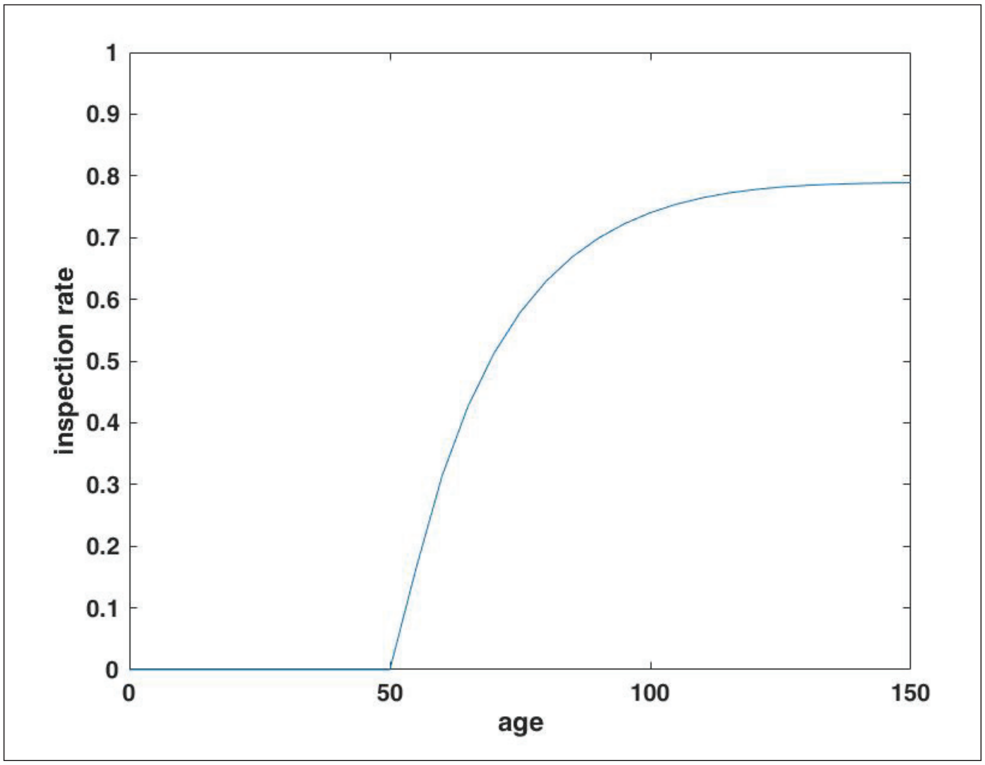
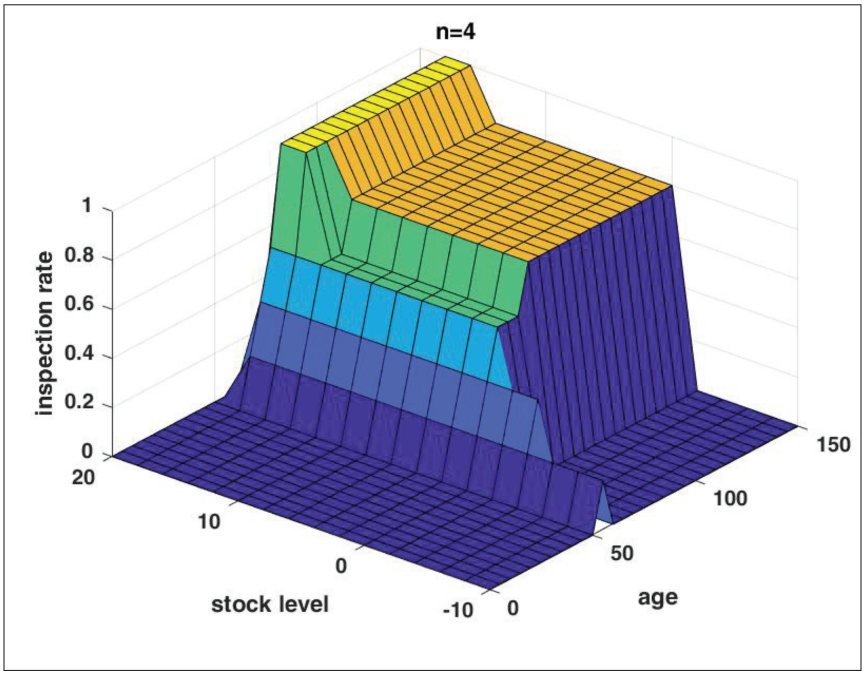


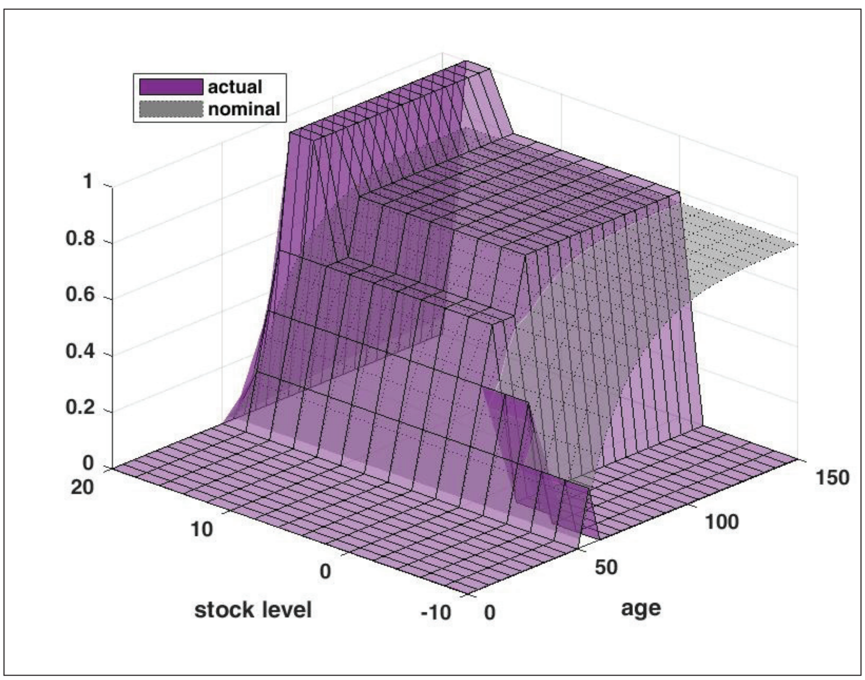
Figure 2.7 Nominal inspection policy for *AOQL*=0.05

eliminate the scrap products and to store products with a certain level of quality.

As it is apparent in Figure 2.7, the nominal inspection rate $\gamma_{nominal}$ is just dependant on the age of the machine. Compared with the production policy with the nominal inspection level provided in Figure 2.8*b*, it can be concluded that contrary to the nominal inspection level, the actual inspection policy also depends on the stock level of the products. Given that financial penalties and incentives are established for low and high qualities respectively, the inspection policy can be flexible. For example, when the machine is old and the stock level is negative (the system is in a backlog situation), the probability of failure and the cost of the backlog are high. Therefore, to avoid the backlog situation, it is cost-efficient to refrain from performing inspection and pay a penalty for producing low-quality parts. However, after a probable failure and during the maintenance activities, if there are enough products in the stock to meet the demand, 100% inspection can be performed. This way we can produce the highest quality level and benefit



a) inspection policy at $n = 4$



b) nominal inspection policy at $n = 4$

Figure 2.8 Actual and nominal inspection policy at $n = 4$

from financial incentives of providing the good quality products. Therefore we can infer from Figure 2.8b that the inspection rate at the negative or low stock level is smaller than the nominal inspection curve and the inspection rate at the higher stock level is greater than the nominal rate. According to Figure 2.8 when the machine is still young performing inspection activities is not recommended. Also, based on the nature of the system's defective rate where the slope of the defective rate curve is almost zero in lower ages of the machine, performing inspection when the machine is young does not change the AOQ considerably, and therefore is not profitable. 100% inspection is mostly performed at higher ages of the machine where the defective rate increases rapidly.

Inspection policy for all numbers of failures is presented in Figure 2.9. The inspection policy $\gamma(.)$ divides the plan (a, x) into six areas. As the number of failures increases the inspection zone inflates. At the last number of failures, because of the mandatory replacement after the next failure, there is no need to have a high stock level, and therefore the area of the inspection decreases.

2.5.3 Maintenance Policy

In this section, we will discuss the optimal maintenance policy that is consisted of repair/replacement and preventive maintenance policies obtained through a numerical example. It can be understood from Figure 2.10 that as the number of failures increases, the area of PM and replacement increases as well. The reason is that the defective rate of the system β and the failure probability q_{15} increase as the machine deteriorates. Moreover, compared to the previous state, the repair rate of the machine $MTTR$ decreases after each failure. Therefore, once the production system wears out progressively, it is necessary to perform the maintenance activities more intensely to increase the system's availability and satisfy the demand, otherwise, there will be great uncertainty about the demand satisfaction. Maintenance activities play an essential role to resolve the deterioration effects.

To better illustrate the maintenance policy, the preventive maintenance policy for each number of failures n is provided in Figure 2.11. As can be noticed, the PM policy divides the plane (a, x) into two regions in a way that based on the age of the machine a and the stock level x for each number of repairs n, the preventive maintenance rate is adjusted

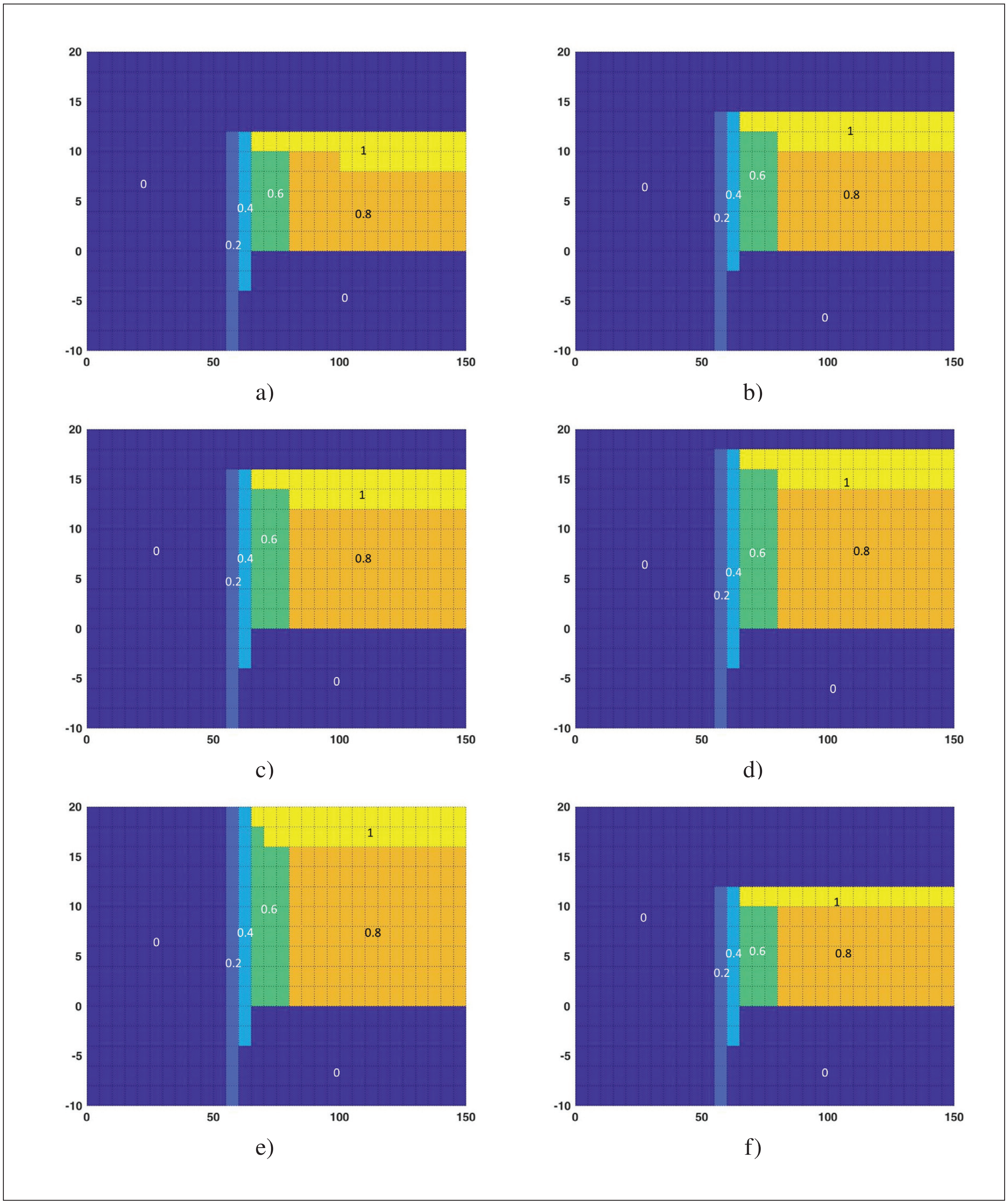


Figure 2.9 Inspection policy (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, (d) $n = 4$, (e) $n = 5$, and (f) $n = 6$.

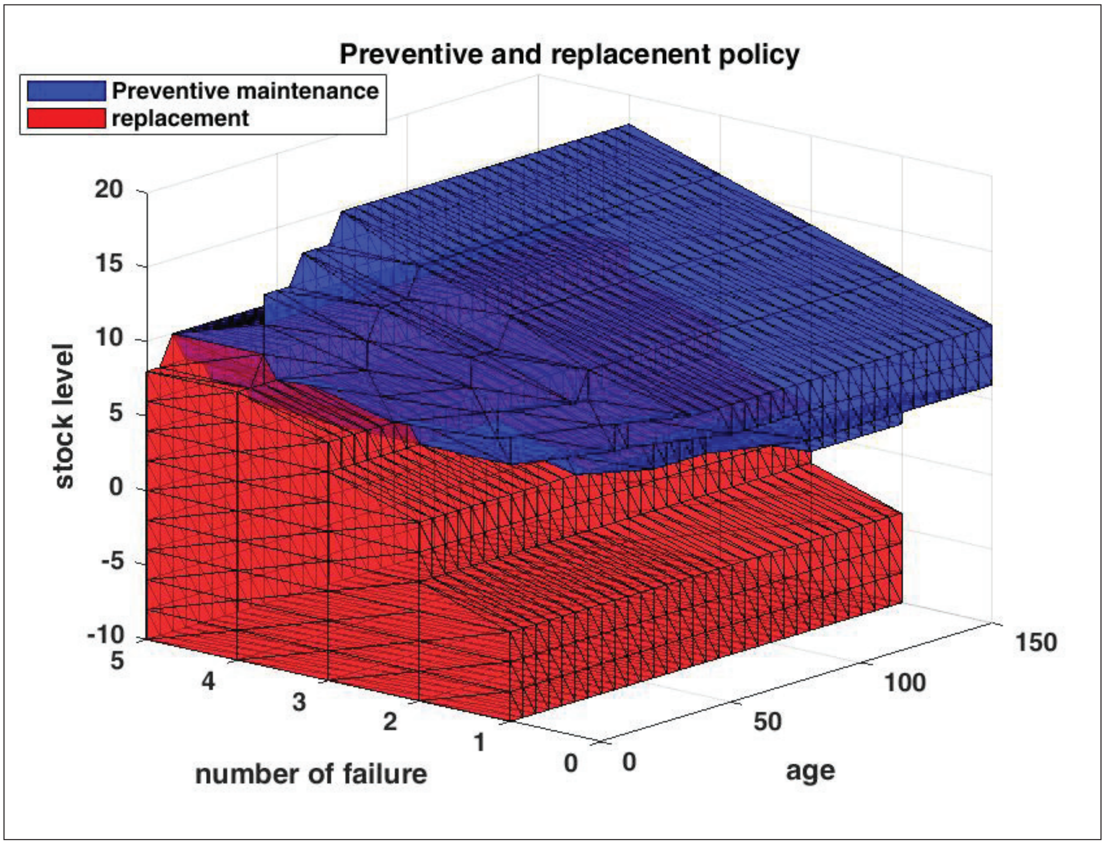


Figure 2.10 Maintenance policy

to the minimum or maximum value $(\omega_3^{min}, \omega_3^{max})$. The description of these two zones is given by:

- Zone A_p: In this zone, since the PM decision variable ω_3 is in its maximum value, it is suggested to send the machine to preventive maintenance mode.
- Zone B_p: The PM decision variable is set to its minimum value, so there is no need in this area to perform PM activities.

Generally, it can be observed that by increasing the number of failure n, the area that PM decision variable is set to maximum ω_3^{max} expands, and also the age in which the PM activities begins, decreases. The optimal preventive maintenance policy is defined by switching function in a given number of failure n based on the age of the machine and level of inventory (a, x) as follows:

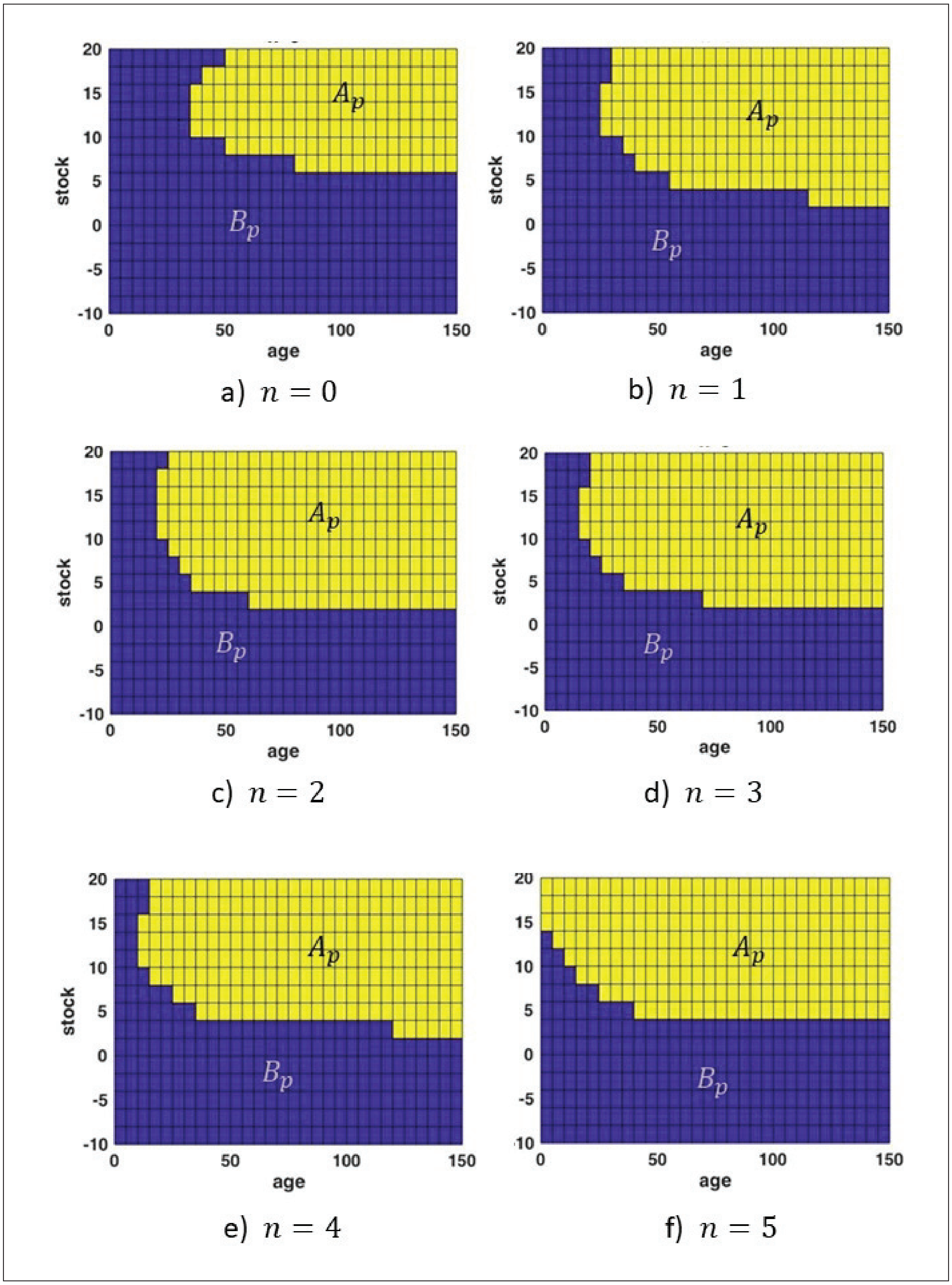


Figure 2.11 Preventive maintenance for each number of failure n

$$\omega_3^*(1,a,x,n) = \begin{cases} \omega_3^{max} & \text{if } a(.) \text{ and } x(.) \in zone\mathrm{A}_p \\ \omega_3^{min} & otherwise \end{cases} \tag{2.42}$$

The last part of this section is about the repair/replacement switching policy. In Figure 2.12 the maintenance activities consisting of PM and replacement policies along with the optimal production threshold level are illustrated. Zone A_r denotes the area where the replacement control variable ω_4 is set to its maximum value. It means that based on the age of the machine a and level of the stock x at a given number of failures n if the production system fails while the system is in the A_r zone, it is recommended to replace the machine. As can be seen in Figure 2.12, the replacement policy is generally set when the machine fails and the system is in the backlog state. This can be attributed to the time-consuming repair process and the fact that the unsatisfied demands are too expensive. Therefore, it is more economical to replace the machine. Also as the machine deteriorates more by imperfect repairs, the optimal replacement zone reaches a higher stock level. The optimal replacement policy function is given by the following expression:

$$\omega_4^*(1,a,x,n) = \begin{cases} \omega_4^{max} & \text{if } a(.) \text{ and } x(.) \in zone\mathrm{A}_r \\ 0 & otherwise \end{cases} \tag{2.43}$$

2.6 Sensitivity analysis

To investigate the effect of variation of the given parameters on the optimal policies, it is essential to perform the sensitivity analysis. In this study, a sensitivity analysis of the production, maintenance, and quality control policies is performed based on the model parameters (such as backlog cost, inventory cost, inspection cost, and ...) by changing their level from the baseline values. First, we focus on the backlog and inventory costs, and then other parameters are investigated.

2.6.1 Variation of the backlog costs

The backlog cost variations c_1^- have a considerable impact on the optimal production threshold policy of the manufacturing system. In Figure 2.13 three different values of the backlog costs

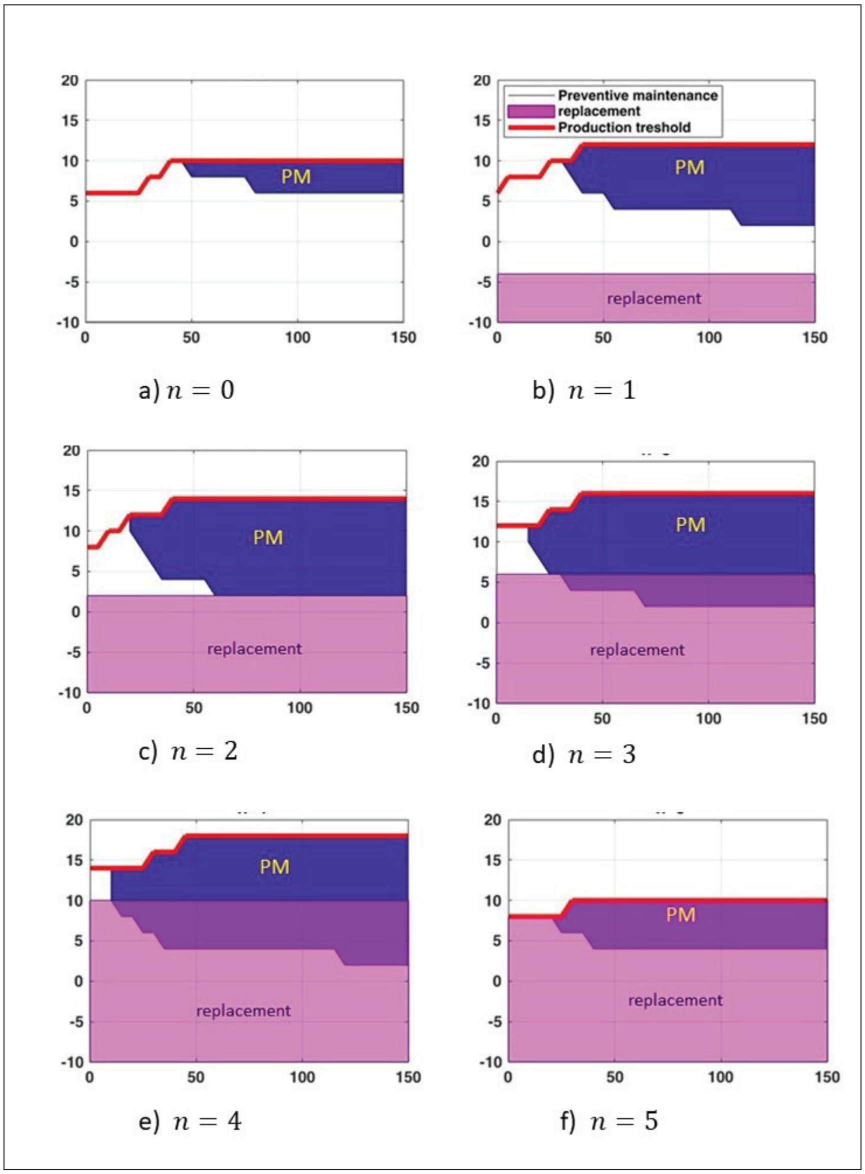


Figure 2.12 Maintenance policies for each number of failure n

are analyzed. It can be understood that the production thresholds have a direct relationship with the backlog cost. When the backlog cost has increased, experiencing a shortage situation will be more expensive for the system. Situations like this might happen during the breakdown of the system, especially as the number of failures increases and the repair process requires a longer time. Therefore, according to the production policy, it is suggested to increase the production threshold in order to be better protected against the backlog.

As can be noticed in Figure 2.13, upon the increase of the backlog cost, replacement should be conducted at a higher level of inventory. This can be justified by the fact that the repair process takes more time after an increased number of failures, therefore, it is more beneficial to perform the replacement at the higher level of stock to avoid facing the backlog situation. Also, for increased backlog costs, PM activities should start earlier to always keep the production machine at low risk of failure. However, in the case of an old machine, it is suggested that in order to protect the system against backlog situations, we should wait until the stock level is higher to perform the PM activities.

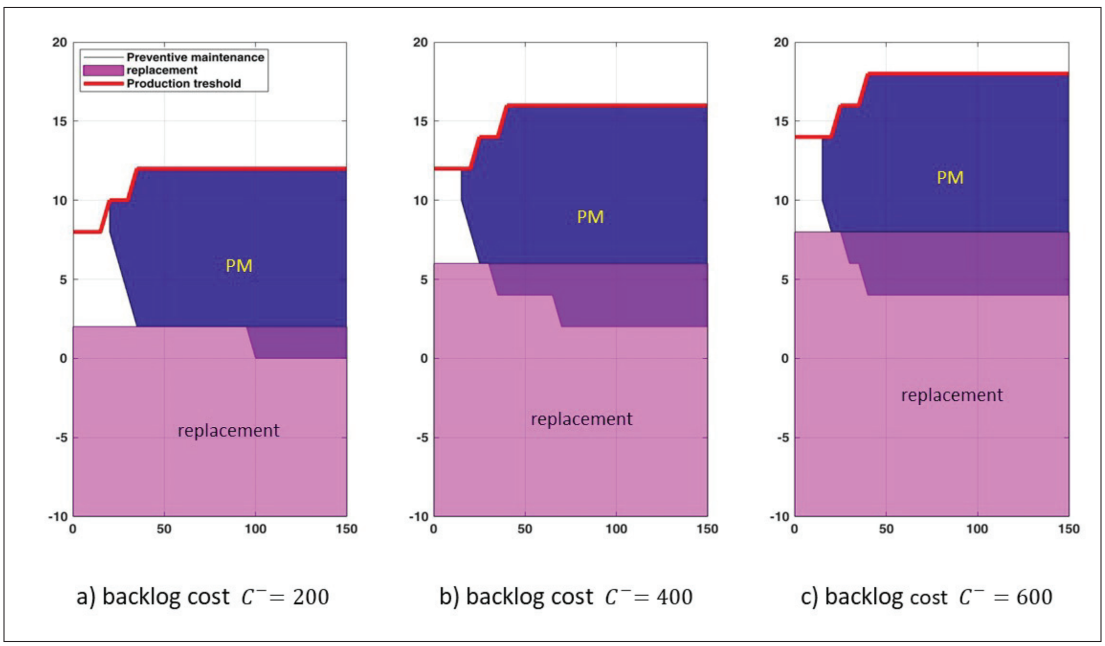


Figure 2.13 Maintenance policies for different backlog cost at $n = 3$

Another aspect of optimal policy that can be affected by the backlog cost variations is the inspection policy. As can be seen in Figure 2.14, as the backlog cost increases, it is not allowed to perform the inspection processes at lower levels of inventory, especially when the system is in a backlog situation and its priority is to escape from this condition even at the expense of paying the quality penalty for providing products with a lower quality than $AOQL$.

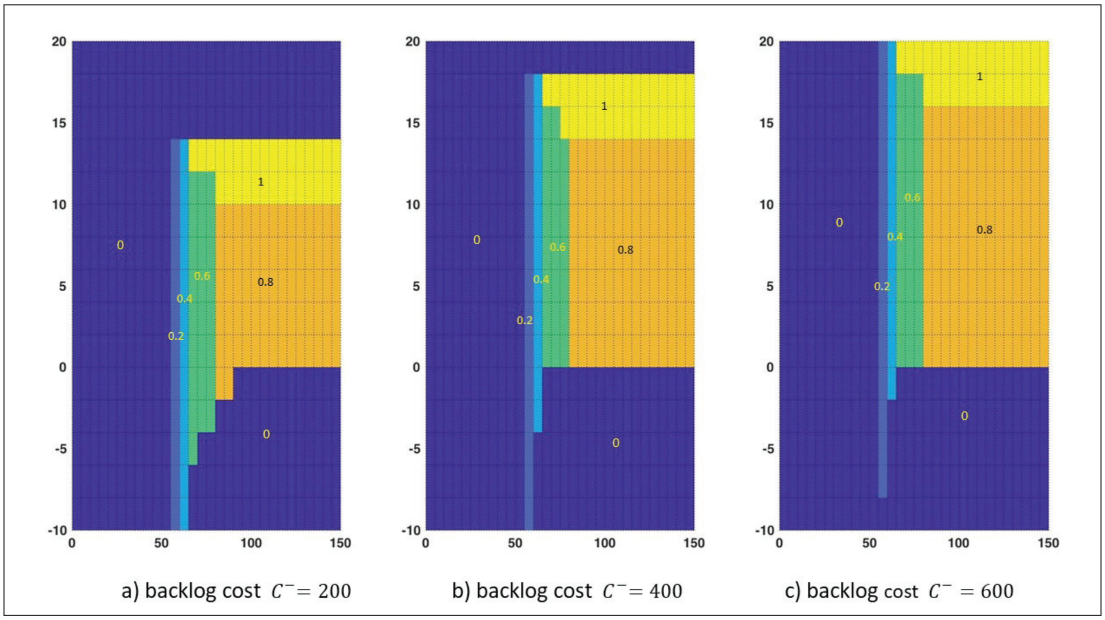


Figure 2.14 Inspection policies for different backlog cost at $n = 3$

2.6.2 Variation of the inventory costs

The effect of variation of the inventory costs c_1^+ is directly reflected in the optimal production threshold which is presented in Figure 2.15. As can be noticed in Figure 2.15, as the inventory cost increases, the production policy suggests reducing the inventory threshold in order to avoid further inventory costs. Consequently, the total inspection area is decreased, but with the same proportionate insensitivity as evidenced by Figure 2.16. It means that 100% can be carried out when we have enough products in inventory and we can benefit from the financial incentives considered for providing products with high quality.

2.6.3 Variation of inspection cost

Following the analysis, three different inspection costs are analyzed. From Figure 2.17, it can be understood that the inspection cost does not affect the production policy, but it has a considerable effect on the inspection policy. On one hand, performing a 100% inspection helps to improve the outgoing product quality and gain the financial benefit of good quality incentives. On the other hand, it makes the production process longer and leads to an excessive cost of inspection and defective part disposal. Therefore, for higher inspection costs, it is no longer economical to

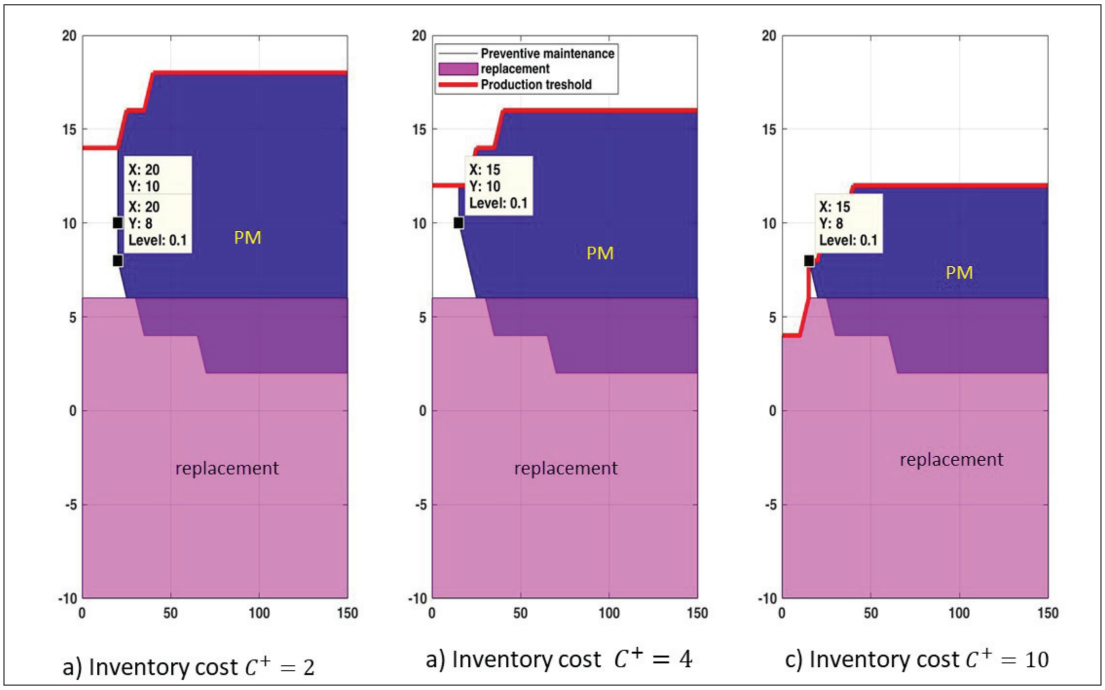


Figure 2.15 Maintenance policies for different inventory cost at $n = 3$

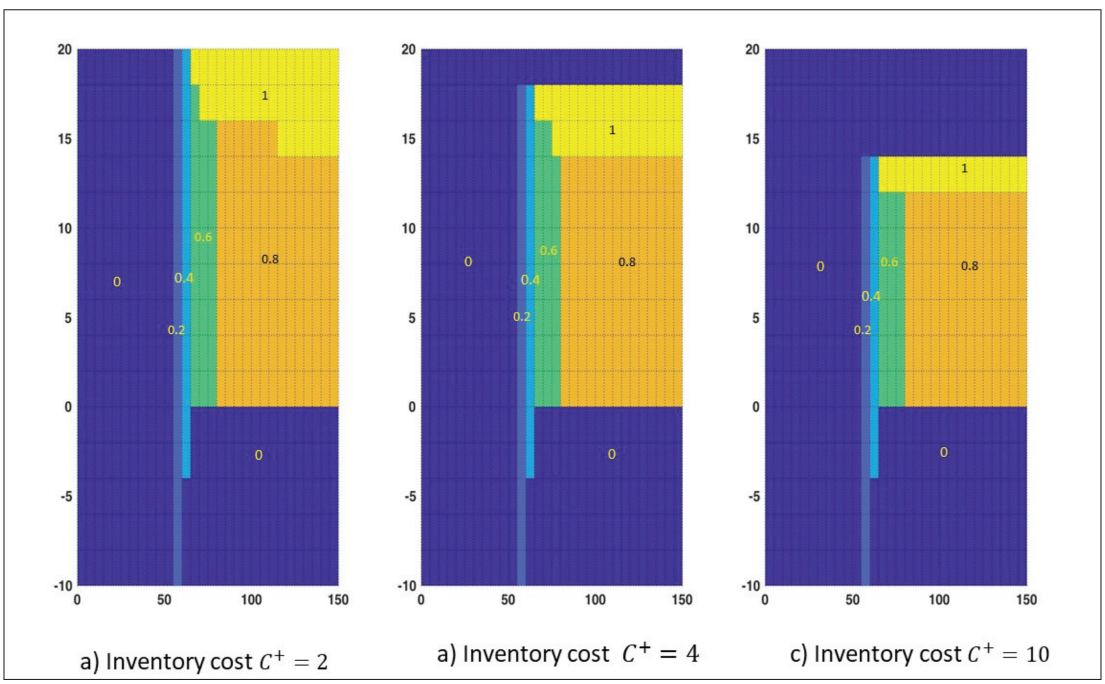


Figure 2.16 Inspection policies for different inventory cost at $n = 3$

perform the 100% inspection. Also, as can be seen in Figure 2.18, the 100% zone decreases considerably for increased inspection costs. However, in the case of higher ages of the machine where the defective rate is high, performing a 100% inspection can have a significant effect on the output quality. Therefore, even if the inspection process is expensive, it is still cost-efficient to perform the 100% inspection.

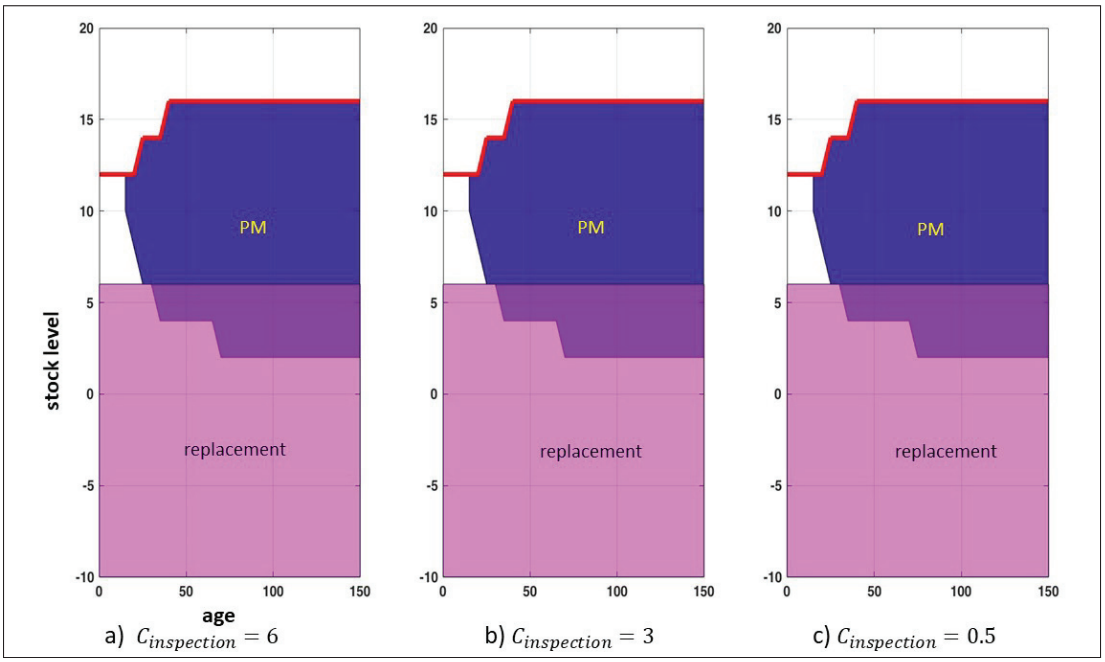


Figure 2.17 Maintenance policies for different inspection cost at $n = 3$

2.6.4 Variation of the PM costs

The preventive maintenance zone will be highlighted under the effect of variation of the PM costs c_{PM}. Figure 2.19 indicates that upon increasing the PM cost, the area of PM decreases considerably, especially when the machine is at its first steps of the number of failures. As the number of failures increases, the cost of repair and the mean time to repair the machine after each failure increases, therefore, for a higher number of failures of the production machine, it is still beneficial to perform the PM activities at an almost similar level even if the PM cost increases (Figure 2.20). In fact, despite the advantages of the PM activities, it is recommended

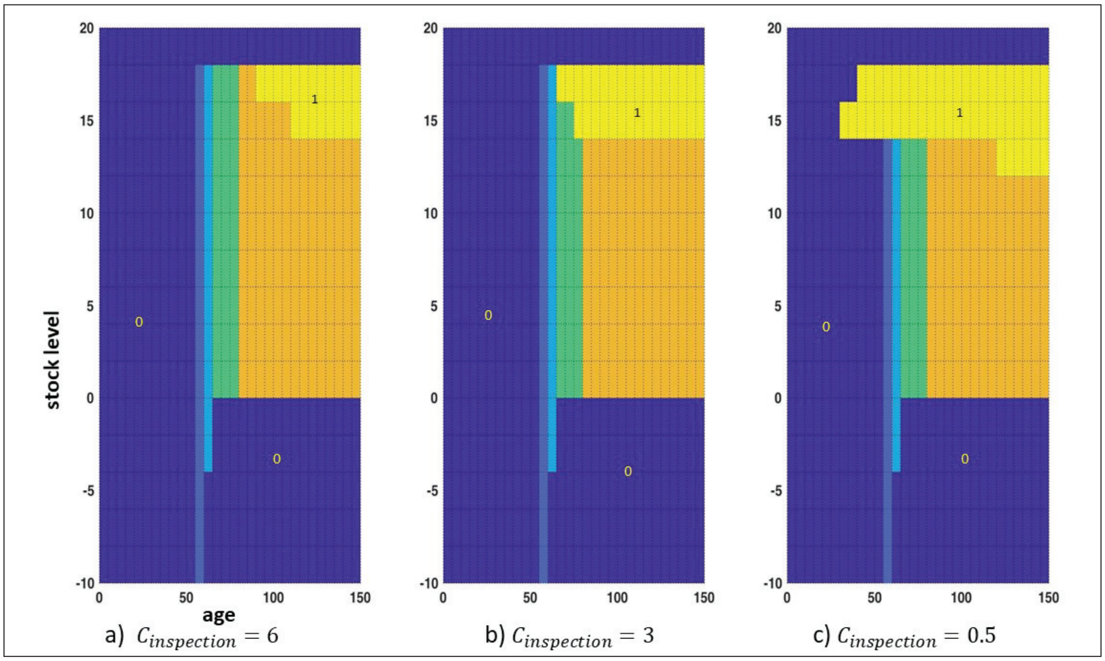


Figure 2.18   Inspection policies for different inspection cost at $n = 3$

to limit them to a lower number of failures considering their higher cost compared to that of repair activities.

Based on Figure 2.19, it can be seen that not only the PM area decreases by increasing the PM cost, but also it is still not recommended to perform PM while the stock level is high even for an old machine. This happens because PM activities are expensive. In other words, when there are enough products in the stock that can cover the demand during the probable failure and repair period, it is not cost-efficient to perform PM, even if the manufacturing machine is too old.

2.6.5 Variation of the repair costs

The repair costs of the manufacturing machine are divided into two parts; the constant repair cost c_{rc} and the variable repair cost c_{rv} which increases by the time required to repair the machine. As illustrated in Figure 2.21, when the repair costs increase, the PM zone expands to avoid probable failure and repair processes that bear costs. It can be concluded that if only the variable part of repair costs increases, it does not affect the PM area significantly for a lower number of failures. However, as the number of failures increases, it affects the PM policy severely and

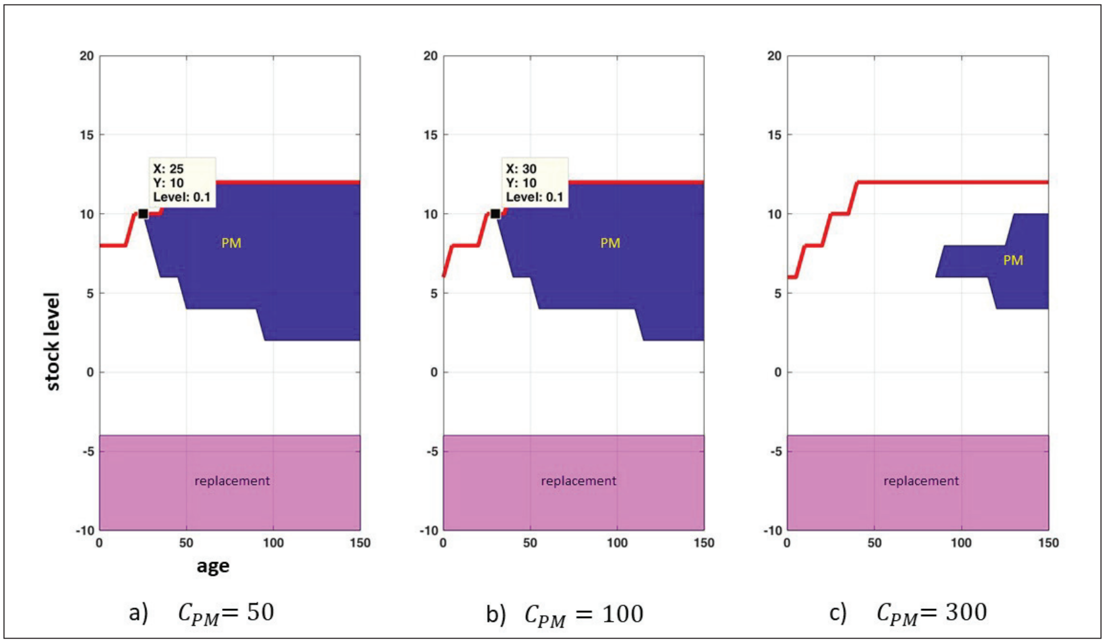


Figure 2.19 Maintenance policies for different PM cost at $n = 1$

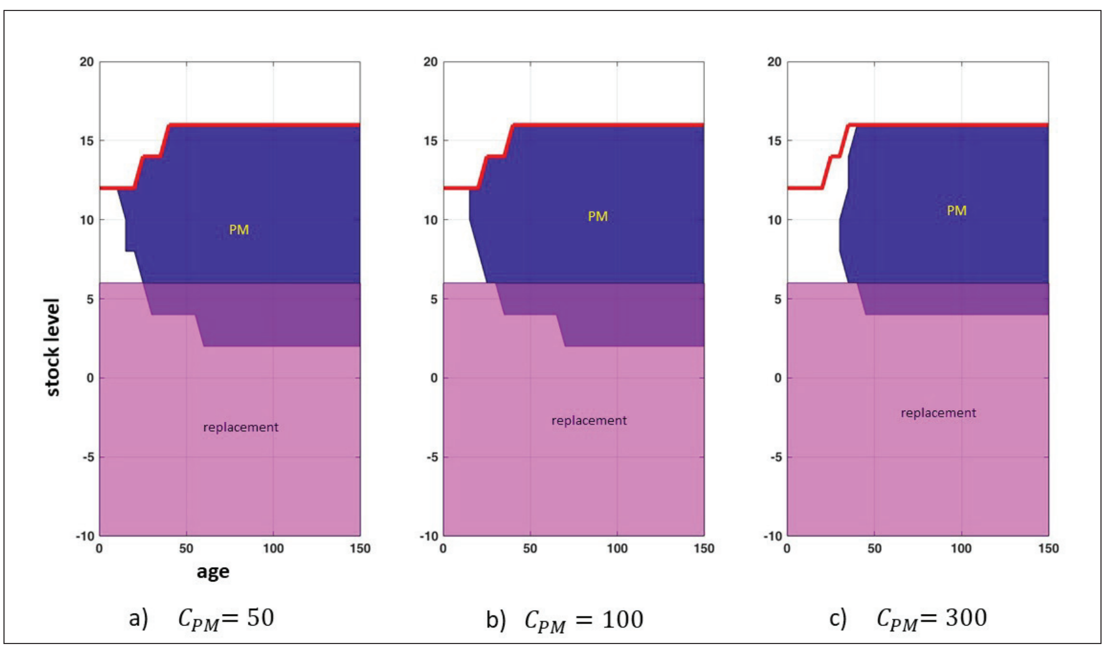


Figure 2.20 Maintenance policies for different PM cost at $n = 3$

increases the effective area of PM. The replacement zone also increases by increasing the repair cost, in order to avoid an expensive repair process after failure.

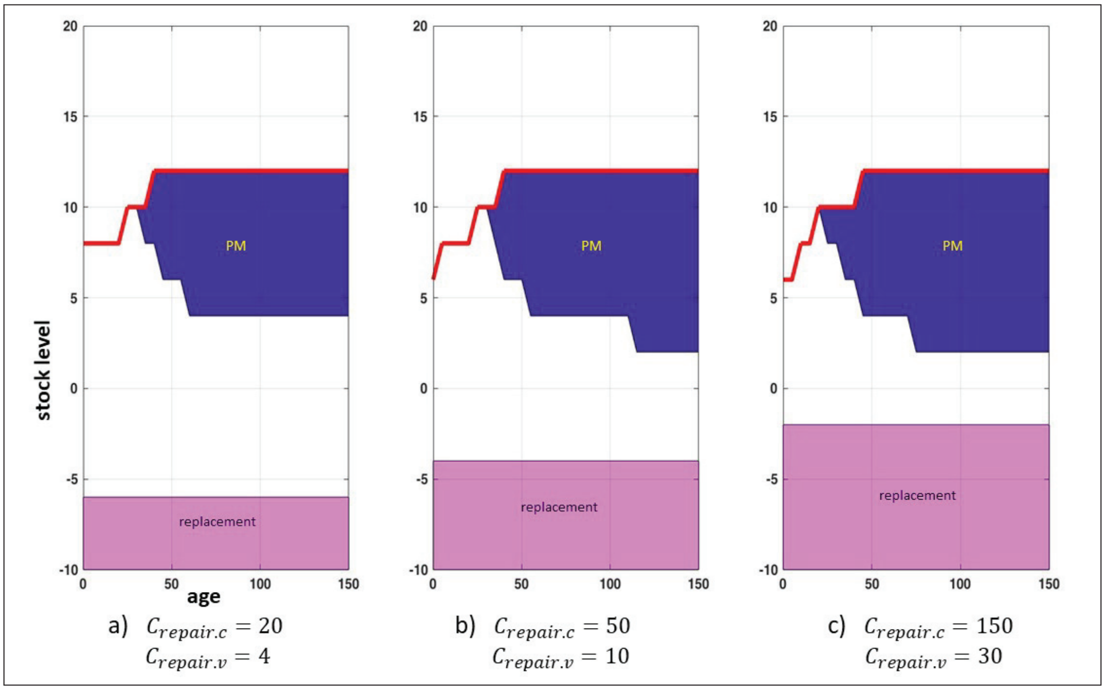


Figure 2.21 Maintenance policies for different repair cost at $n = 1$

2.6.6 Variation of the low-quality financial penalty coefficient

Changing the penalty considered for the manufacturing of products with low quality should have a direct impact on the optimal inspection policy. Figure 2.22, indicates that when the penalty coefficient increases, the area of the inspection zone increases as well. In other words, by increasing the penalty coefficient, the inspection can be started at the lower level of stock to avoid the expensive penalty of exceeding the AOQL. When the penalty considered for low quality is not severe enough, it is not recommended to perform an inspection while the stock is in a backlog situation and the machine is old. However, as the penalty for poor quality increases, conducting the inspection is economically justifiable even in the backlog situation. It is evident in Figure 2.22, that by increasing the value of the penalty, the inspection area increases more.

2.6.7 Variation of the high-quality financial incentive coefficient

Financial incentives are often used to encourage the manufacturer to produce products with better quality. By performing the 100% inspection, the producer can benefit from incentives established for high-grade quality. By increasing the incentive coefficient, as shown by Figure

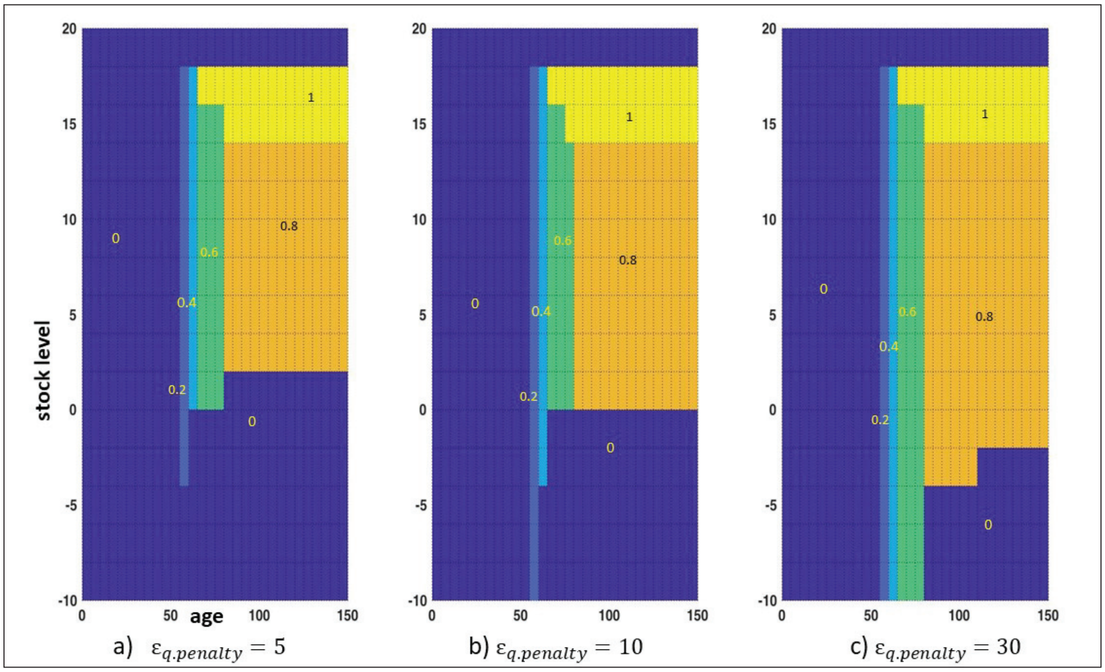


Figure 2.22 Inspection policies for different bad quality penalty at $n = 3$

2.23, both the area of inspection and the area of the 100% inspection increase. Moreover, the initial age of the machine for performing the inspection decreases by increasing the financial incentive. It means that for an increased financial quality incentive, we should start performing the 100% inspection even if the quality of the product is still acceptable and better than $AOQL$, in order to take advantage of the great financial incentives.

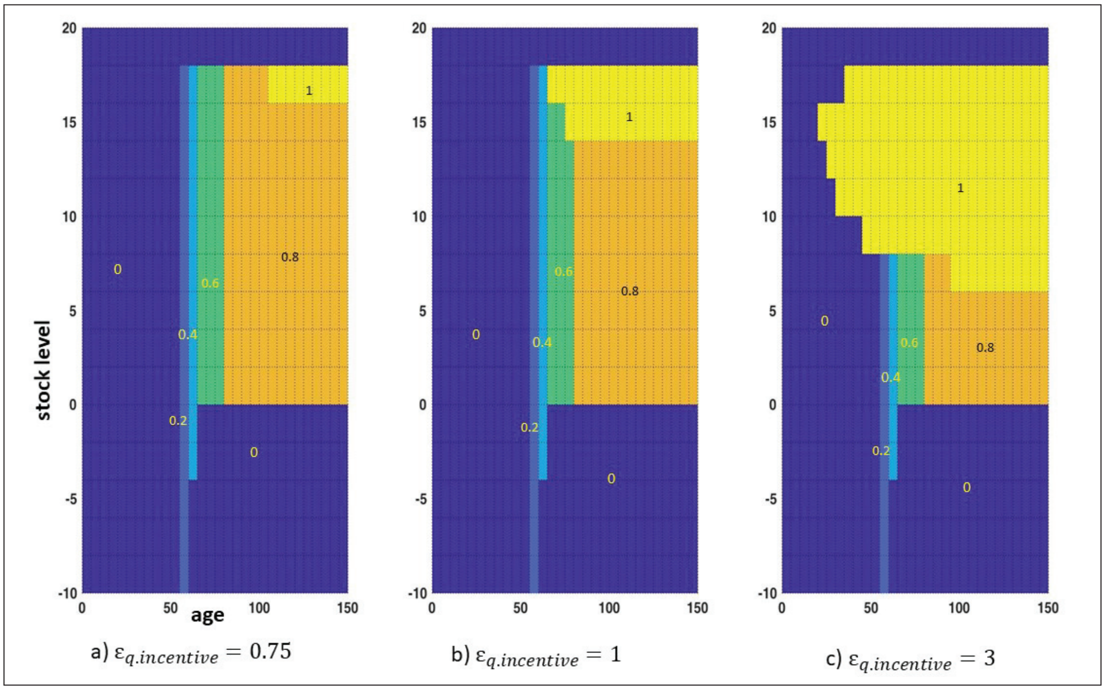


Figure 2.23 Inspection policies for different quality incentives at $n = 3$

2.7 Conclusion

This chapter discusses a simultaneous production planning, inspection policy, and maintenance control problem consisting of PM and repair/replacement switching policies for a manufacturing system under deterioration. The proposed model of the problem consists of four decision variables *(production, Pm, corrective maintenance, and inspection rates)* and three state variables *(age of the machine, number of failures, and level of stock)*. Deterioration of the machine caused by its age and imperfect repair actions has a significant influence on its availability and the quality of the products.

By using dynamic programming in the form of a semi-Markov problem, the stochastic control problem has been formulated and the optimality condition is implemented in the form of HJB equations. Given the complexity of the problem, numerical techniques were employed instead of a direct analytical solution to obtain the optimal policy. Next, the proposed approach was illustrated and validated using a numerical example and sensitivity analysis. It was shown that the application of this approach for the integrated problem yields acceptable and justifiable results. The proposed model indicated that in order to minimize the incurred costs:

- While the stock level should be increased with the age of the machine or the number of failures, the rate of increasing the stock level should be decreased. The reason is that the failure rate of the system changes with a similar trend.
- As the number of machine failures increases, the recommended preventive maintenance activity must be performed earlier. It can also be set at lower stock levels as the age of the machine increases.
- To maintain the quality constraint, the inspection rate should be increased with the age of the machine. However, once the machine is in a backlogged state the inspection is not recommended. Also, when the stock level is high enough, a 100% inspection can be performed to gain profit from the financial incentives considered for the fabrication of high-quality products.

Furthermore, our model is able to yield an optimal solution for a similar situation where the production process is stochastic.

CHAPTER 3

PRODUCTION CONTROL OF A FAILURE-PRONE MANUFACTURING SYSTEM WITH RECTIFYING MACHINE

3.1 Introduction

Quality and production planning play a critical role in modern companies trying to thrive in competitive markets. Given the stochastic nature of the machines, manufacturers might face several issues and uncertainties such as deterioration and unexpected failures that make it difficult to meet the market demand with the specified quality and without delay. The purpose of production planning is to overcome these disruptions by minimizing the total costs. To control the outgoing quality and palliate the effects of deterioration different forms of maintenance activities can be performed as main practices including PM maintenance and overhaul. In addition, inspection is an essential step to monitor the quality of products and detect defective parts. Another appealing way to increase the quality and reduce the production time is to rectify and repair the defective parts detected in the inspection processes. In fact, part of the defective products with minor damage that are detected in the inspection process can be repaired through the rectifying processes. Many authors have so far contributed to the production planning problem for manufacturing systems with regards to the various maintenance activities, but it appears that there are not enough studies that analyze the interaction between production and quality scheduling in the presence of a recovering system. Therefore, in this study, we aim to propose a cost-optimal policy that can integrate simultaneously the optimal production planning, maintenance strategy, and inspection scheduling while applying a rectifying process after inspection activities.

3.2 System description

In this chapter, we extend our previous case and consider a production system with a failure-prone deteriorating manufacturing machine along with a perfect rectifying machine that is not exposed to the deterioration effects. The age of the machine and imperfect repair after failure deteriorate the production system. Not only does deterioration result in machine degradation by increasing its failure rate, but also it reduces the quality of the produced parts. The manufacturing machine wears out more as its age and the number of failures increase. After each failure and based on the system's situation, the machine either can be repaired or replaced by a new one. Since the repair activities are not perfect, after each repair the system will reset to a condition between $AGAN$ and $ABAO$. On the other hand, the PM activities bring the machine to its initial condition after the last repair.

Inspection procedures are usually aimed to detect the parts that are produced defective due to quality deterioration. It is not always economical to perform a 100% inspection, therefore the optimal inspection schedule is also a part of the optimal policy. The defective parts can be classified into two categories: the rectifiable parts that can be fixed by the rectifying machine, and the non-rectifiable that should be disposed of. After being rectified the defective parts have the same quality as the perfect parts. Therefore, in this chapter, we seek to find the optimal policy for the production planning, maintenance control activities, and inspection scheduling of stochastic manufacturing systems in the presence of rectifying machines.

3.3 Control problem formulation

The system that will be discussed in this chapter has five different modes denoted by the stochastic variable $\zeta(t)$. The different modes of the machine can be classified as operational $\zeta(t) = 1$, at repair $\zeta(t) = 2$,under preventive maintenance $\zeta(t) = 3$, under replacement $\zeta(t) = 4$, at failure $\zeta(t) = 5$. Thus, the modes of the machine at time t denote a continuous-time discrete-state stochastic process $\zeta(t) \in \{1, 2, 3, 4, 5\}$ such that:

$$\zeta(t) = \begin{cases} 1 & Operational \\ 2 & Repair \\ 3 & Preventive\ maintenance \\ 4 & Replacement \\ 5 & Failure \end{cases}$$

The machine may randomly be at any of the five modes over an infinite horizon. The following transition diagram describes all five states of the machine.

The system can be characterized at any given time t, by the following four state variables:

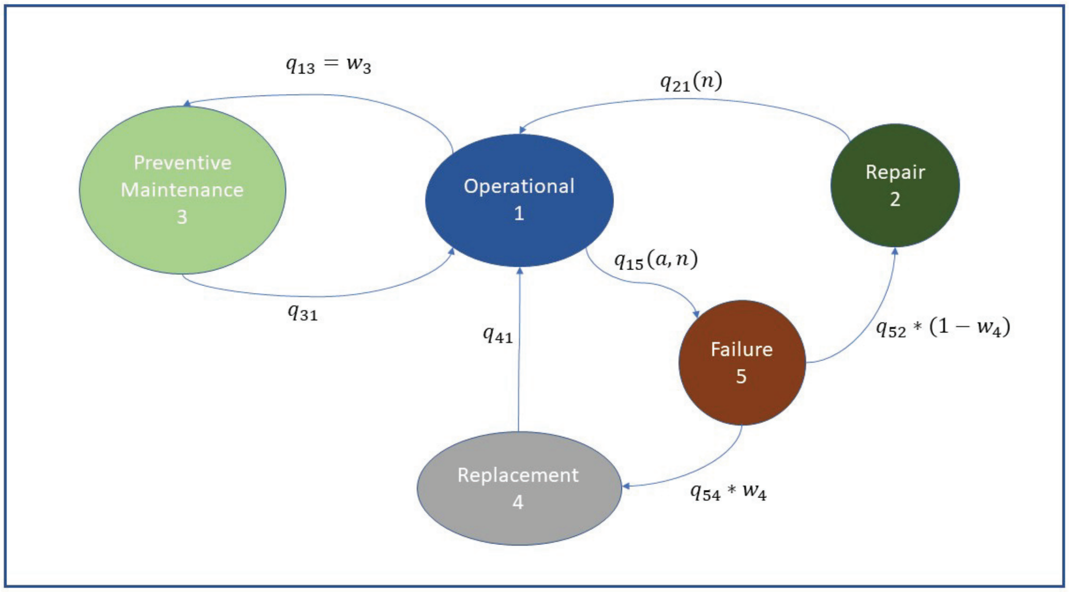


Figure 3.1 Transition diagram of proposed model

- the machine mode $\zeta(t)$
- the number of failure $n(t)$
- the age of the machine $a(t)$
- the stock level $x(t)$

Based on the explanation and to illustrate the model, the block diagram of the proposed model is provided as follows:

Considering the block diagram of the model in the Figure 3.2, the dynamics of the inventory can be expressed as follows:

$$\dot{x} = (1-\gamma)u + (1-\beta)\gamma u + \gamma\beta(1-r_\beta)u - d$$
$$\dot{x} = (1-\gamma\beta r_\beta)u - d \quad , \qquad x(0) = x_0 \tag{3.1}$$

where β is the defective rate of the production machine, $\gamma(.)$ is the inspection rate, and $r_\beta(.)$ is the non-rectifiable rate of the defective parts that need to be disposed of.

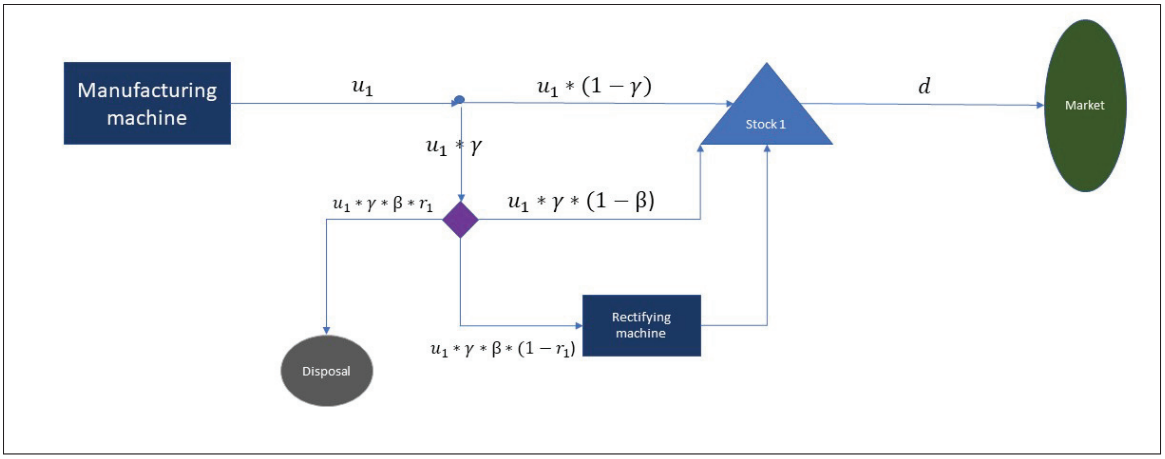


Figure 3.2 Block diagram

As the machine gets older and deteriorates, the quality of the output products decreases whereas the rate of the non-rectifiable defective parts increases. The rate of non-rectifiable parts is given by:

$$r_\beta = R_{b0} + R_{b1}\phi_{qnr}(\frac{n}{N})^{r_2} \tag{3.2}$$

where R_{b0} is the initial non-rectifiable rate of the defectives, R_{b1} is its limit rate and r_2 is a positive adjusting coefficient. n is the number of failures and N is the maximum acceptable number of failures. As can be seen in Equation 3.2, the rate of the non-rectifiable defective parts increases with the number of failures. This is due to the effect of imperfect repair on the quality of the produced parts. The defective parts have more serious damage on them which can not be fixed by the rectifying process, therefore they need to be disposed of.

By performing the rectifying process on the defective parts, some of the parts with minor defects can be fixed, so the quality of outgoing products changes. The AOQ can be defined as follows:

$$AOQ = \frac{(1-\gamma)\beta}{1-\gamma\beta r_\beta} \tag{3.3}$$

where AOQ represents the average quality of the products that will be sent to the inventory. Inspection is a beneficial tools for controlling the AOQ and making decisions based on the defective rate $\beta(.)$ and the designated average outgoing quality limit $AOQL$. The minimum rate of inspection that can assure that the AOQ is less than the *average outgoing quality limit* $AOQL$ is called the nominal inspection rate γ_{nom} and is given by:

$$\gamma_{nom} = \begin{cases} 0 & \text{if } \beta < AOQL \\ \frac{\beta - AOQL}{\beta(1 - AOQL r_\beta)} & \text{if } \beta \geq AOQL \end{cases} \tag{3.4}$$

To define the inspection policy, it is not enough to consider only the $AOQL$ and the defective rate β. There are other parameters that might affect our decision such as the level of inventory, and the value of the penalties and incentives established for products with lower and higher quality than $AOQL$, respectively. From Equation 3.3, the bad quality punishment or the good quality incentive can be defined as:

$$C_q = \begin{cases} \varepsilon_{q.p}(AOQ) & \text{if } AOQ > AOQL \\ 0 & \text{if } AOQ = AOQL \\ -\varepsilon_{q.i}(AOQL - AOQ) & \text{if } AOQ < AOQL \end{cases} \tag{3.5}$$

where($\varepsilon_{q.p}$) is the financial penalty cost coefficient imposed by the customer because of receiving products with low quality and ($\varepsilon_{q.i}$) is the financial incentive coefficient considered for the high quality of products. C_q represents the quality costs that can be positive or negative for low-quality or high-quality products, respectively.

One of the important aspects to be considered in proposing a production model is the feasibility condition. The system can not fulfill the demand on all horizons and it needs to satisfy the feasibility condition that is given by:

$$(1 - \gamma\beta r_\beta)u_{max} \times \pi_1 \geq d \tag{3.6}$$

where π_1 is the limiting probability for the *operational* mode and r_β is the non-rectifiable rate of the defective products. π_1 can also be obtained by the following expressions:

$$\begin{cases} \pi_i * Q(.) = 0 \\ \sum_{i=1}^{m} \pi_i = 1 \qquad i : \textit{modes of the machine} \end{cases} \tag{3.7}$$

where $Q(.)$ is the transition matrix of different modes of the system. Since the machine has the same states as the one discussed in chapter 2, the transition matrix will be the same as Matrix 2.20 that is given by:

$$Q = \begin{Vmatrix} q_{11} & 0 & \omega_3 & 0 & q_{15}(a,n) \\ q_{21}(n) & q_{22} & 0 & 0 & 0 \\ q_{31} & 0 & q_{33} & 0 & 0 \\ q_{41} & 0 & 0 & q_{44} & 0 \\ 0 & \omega_2 & 0 & \omega_4 & q_{55} \end{Vmatrix} \tag{3.8}$$

where $\omega_2 = (1 - \acute{\omega_4})q_{52}$ and $\omega_4 = \acute{\omega_4}q_{54}$. The control variables $\{u(.), \omega_3(.), \acute{\omega_4}(.), \gamma(.)\}$ help us to devise the production planning to minimize the total cost of production. These variables have their limits and work in the finite horizon. A set of feasibility conditions $\Gamma(.)$ for the control variables is provided as follows:

$$\Gamma(.) = \begin{cases} \{u, \omega_3, \acute{\omega_4}, \gamma\} \in \Re^3, & \begin{cases} 0 \le u \le u_{max} \\ \omega_3 \in \{\omega_{3min}, \omega_{3max}\} \\ \acute{\omega_4} \in \{\acute{\omega}_{4min}, \acute{\omega}_{4max}\} \\ 0 \le \gamma \le 1 \end{cases} \\ (1 - \gamma\beta r_\beta)u_{max} \times \pi_1 \ge d \\ \pi_1 = \dfrac{1}{1 + \dfrac{\omega_2 q_{15}}{(\omega_2 + \omega_4)q_21} + \dfrac{\omega_3}{q_{13}} + \dfrac{\omega_4 q_{51}}{(\omega_2 + \omega_4)q_{41}} + \dfrac{q_{15}}{(\omega_2 + \omega_4)}} \end{cases} \tag{3.9}$$

Since it is almost impossible to computationally solve this semi-Markov problem, we use numerical method to solve it. The state of the system at any given time t is defined by variables $\big(\zeta(t), x(t), a(t), n(t)\big)$, that consists of discrete and continuous components.

The instantaneous cost function of the production system $G(.)$ can be defined as follows:

$$G(\zeta, x, a, n) = h(x) + c(u) + w(\zeta, n) + z(\text{AOQ}) \tag{3.10}$$

where $h(x)$ is the inventory cost function, $c(u)$ is the cost function related to the manufacturing process, and $w(\zeta, n)$ is defined as the maintenance activities cost function and can be defined as follows:

$$\begin{cases} h(x) = c^+x^+ + c^-x^- \\ c(u) = c_m u + c_{ins}\gamma u + c_{rec}\gamma\beta(1 - r_\beta)u + c_q u + c_{dis}\gamma\beta r_\beta u \\ w(\zeta, n) = c_{over}(Ind\{\zeta(t) = 4\}) + c_{rep}(Ind\{\zeta(t) = 2\}) + c_{pm}(Ind\{\zeta(t) = 3\}) \\ z(\text{AOQ}) = C_q \end{cases} \tag{3.11}$$

where c^+ and c^- are the costs of inventory and backlog of a single product, respectively. c_m represents the cost of manufacturing a single product, c_{ins} is the cost of inspection per unit of the product, c_q is the quality cost mentioned in Equation 4.16, c_{rec} is the rectifying cost per unit of the product, and c_{dis} is the disposal cost of the non-rectifiable products. c_{over}, c_{rep} and c_{pm} are the costs of replacement, repair, and preventive maintenance, respectively.

In order to obtain the optimal policy that could minimize the total costs, four decision variables namely, the production rate $u(.)$, the preventive control rate ω_3 , the replacement rate $\acute{\omega}_4$, and the inspection rate γ, need to be found. These optimal control variables can minimize the expected total cost for each initial state of the system (x, a, n). The expected discount total cost is given by:

$$J(\alpha, x, a, n, u, \omega_3, \acute{\omega}_4) \quad = E\left\{ \int_0^\infty e^{-\rho t} G(.)\, dt \mid \zeta(0) = \alpha,\ x(0) = x,\ a(0) = a,\ n(0) = n \right\} \tag{3.12}$$

where ρ is the discount rate. We can obtain the optimal control policy of the control variables using the value function v which is is the optimal value of the cost function $J(.)$.

$$v(\alpha, x, a, n) = \inf_{(u(.), \omega_3, \acute{\omega}_4, \gamma \in \Gamma(.))} J(\alpha, x, a, n, u, \omega_3, \acute{\omega}_4, \gamma), \forall \alpha \in \Omega, x \in R, n \in N \tag{3.13}$$

If we consider the value function v as a cost-to-go function for which any point of time can be considered as the initial time, then the problem can be written for two parts:$[0,t]$, and $[t,\infty)$:

$$v(\alpha(.),x(.),n(.),t) = \inf_{(u(.),\omega_3,\acute{\omega_4},\gamma)_{0\leq s\leq\infty}} E\left\{\begin{array}{l}\int_0^t e^{-\rho t} g[\alpha,x,a,n,u,\omega]\,ds \\ +\int_t^\infty e^{-\rho t} g[\alpha,x,a,n,u,\omega]\,ds\end{array} |\alpha,x,n\right\} \quad (3.14)$$

By applying the conditional expectation in equation 3.14 and knowing that the value function is differentiable, we can expand the equation 3.14 to:

$$\begin{aligned}\rho v(\alpha,x,n) = \min_{(u,\omega_3,\acute{\omega_4},\gamma)\in\Gamma(\alpha)} &\Big\{ g\left[\alpha,x,n,a,u,\omega_3,\acute{\omega_4},\gamma\right] + \frac{\partial v}{\partial x}[\alpha,x,n,a]\dot{x} + \\ &+ \frac{\partial v}{\partial t}[\alpha,x,n,a]\dot{a} + Q(.)v[\alpha,x,\varphi(\zeta,n),0]\Big\}\end{aligned} \quad (3.15)$$

where we can define the reset function $\varphi(\zeta,n)$ at any jump time τ as follows:

$$\varphi(\zeta,n) = \begin{cases} n+1 & \text{if} \quad \zeta(\tau^+) = 1 \quad \text{and} \quad \zeta(\tau^-) = 2 \\ 0 & \text{if} \quad \zeta(\tau^+) = 1 \quad \text{and} \quad \zeta(\tau^-) = 4 \\ n & \textit{otherwise} \end{cases}$$

In the series of equations 3.15 that are known as HJB equation, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial t}$ are first-order partial derivatives of the value function $v(.)$. These equations help to find the optimal solution of the model using Kushner's strategy and numerical methods.

3.4 Production policy

The optimal production policy $u^*(\alpha,x,a,n)$ is shown in Figure . The optimal production rate of the manufacturing machines at any state of the machine (x,a,n) is illustrated in Figure. The production policy divides the space plan (a,x,n) into three zones $(U_{max},d,0)$, which means that at any point in the space plan, the production rate should be set to u_{max}, d or zero. Also as can be noticed, as the machine deteriorates more by its age a or the number of failures n, we need to keep more products in the stock to meet the demand. Although the stock level increases with deterioration, at the last number of failures the stock level decreases, that is because of the replacement policy in which after the next failure the machine should be replaced with a new one. Therefore, there is no need to consider a probable repair time after the last failure in the level of stock to fulfill the demand in the repair period.

The production policy for each number of failures is illustrated in Figure 3.3. It can be observed that at each number of failures the production threshold is affected by the age and increases as the age of the machine goes up. Also, the production threshold is increased by the number of failures ($Z^*_{n_{i-1}} \leq Z^*_{n_i} \quad \{i = 1, ..., (N-1)\}$). Moreover, the imperfect repair process has two effects; the failure rate of the machine as well as the mean time required to repair $MTTR$ after each failure increase by the number of failures. This is why it is essential to increase the production threshold by the number of failures. However, at the last number of failures where the machine should be replaced after the next failure, and both the failure rate and $MTTR$ will reset to the initial condition, we can decrease the production threshold to fulfill the demand (Figure 3.5b). According to Figure 3.3, the production threshold is given by:

$$u^*(1, n_i, .) = \begin{cases} u_{max} & \text{if } x(t) < Z^*_{n_i}(.) \\ d & \text{if } x(t) = Z^*_{n_i}(.) \\ 0 & \text{if } x(t) > Z^*_{n_i}(.) \end{cases} \tag{3.16}$$

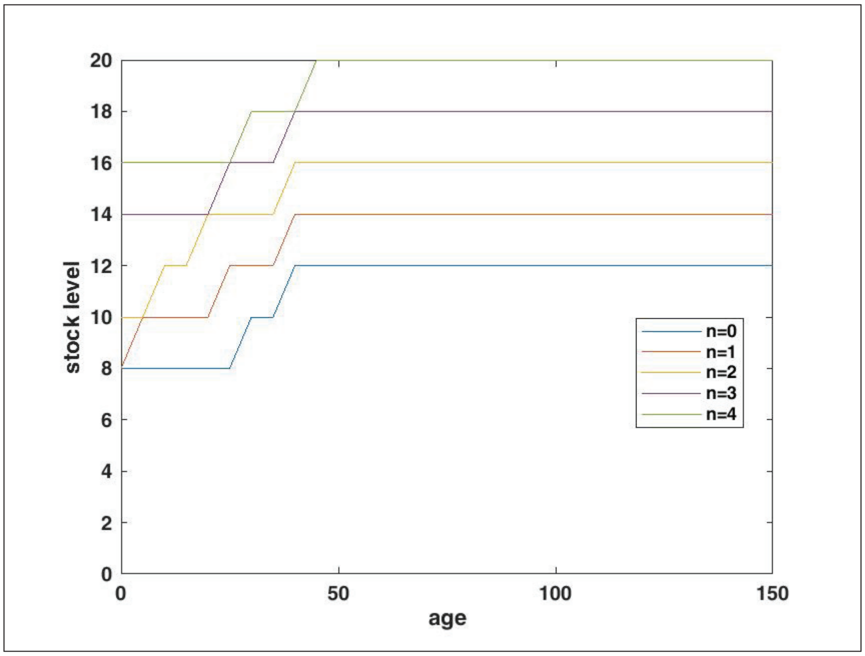
where $Z^*_{n_i}(.)$ is the production threshold at any number of failures.

3.4.1 Inspection policy

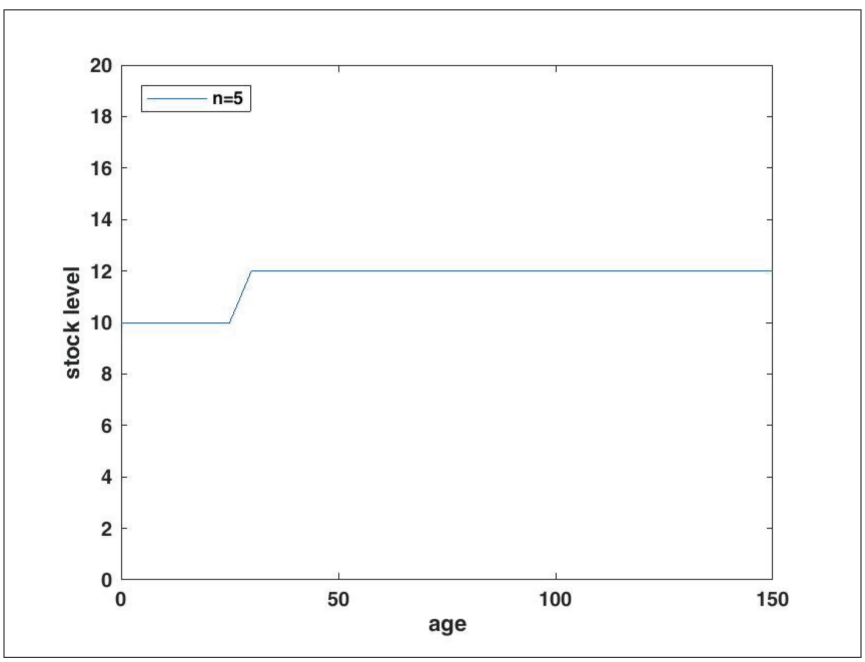
To satisfy the quality limits required by the customer, performing inspection is an essential part of the production process. By defining nominal inspection $\gamma_{nominal}$ as the minimum inspection level that can satisfy product quality limit and comparing it to the inspection policy, we can have an insight of the inspection strategy in our model. The nominal inspection policy is given by:

$$\gamma_{nominal} = \begin{cases} 0 & \text{if } \beta < AOQL \\ \frac{\beta - AOQL}{\beta(1-AOQL)} & \text{if } \beta \geq AOQL \end{cases} \tag{3.17}$$

Based on Figure 3.4, the inspection rates under the threshold (zone A) yield products with a defective rate more than $AOQL$, and conversely, in the zone B where the inspection rate is more than the nominal rate, the defective rate of the output products will be less than $AOQL$. As can



a) production policy at each number of failure



b) production policy at the last number of failures

Figure 3.3 Production policy

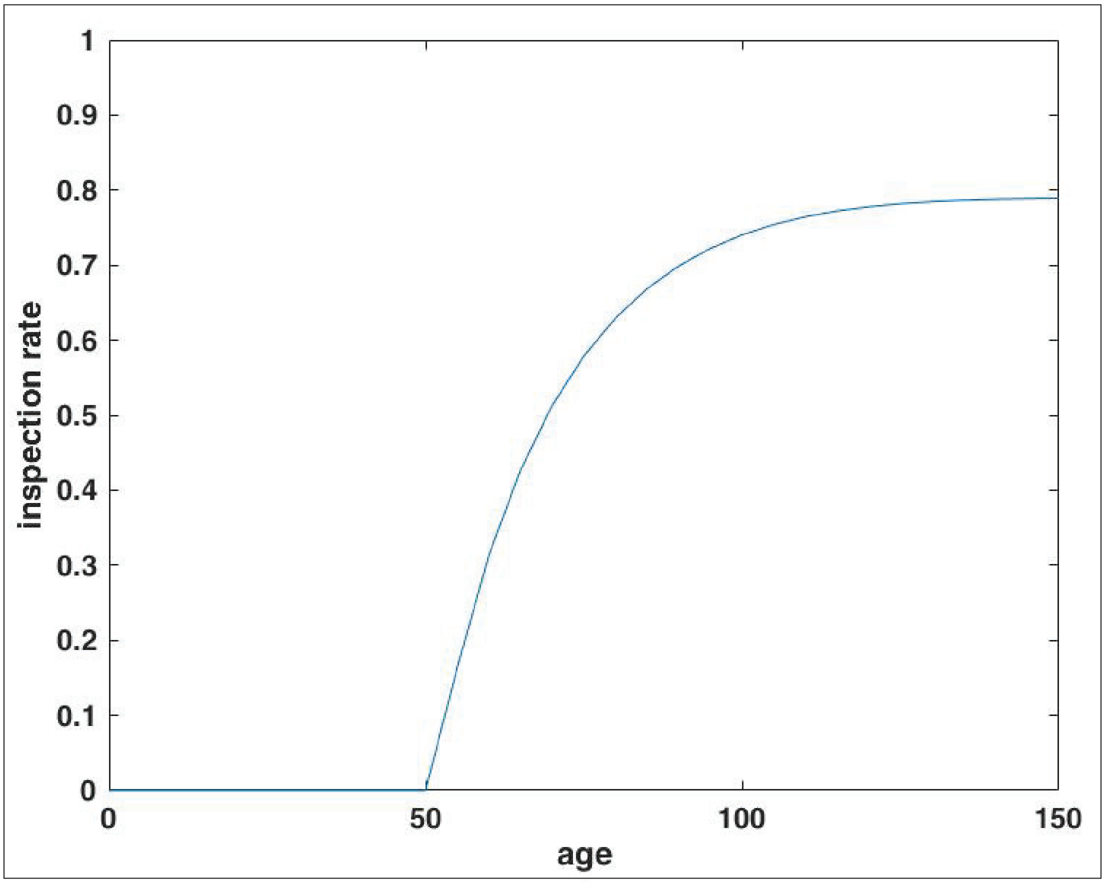
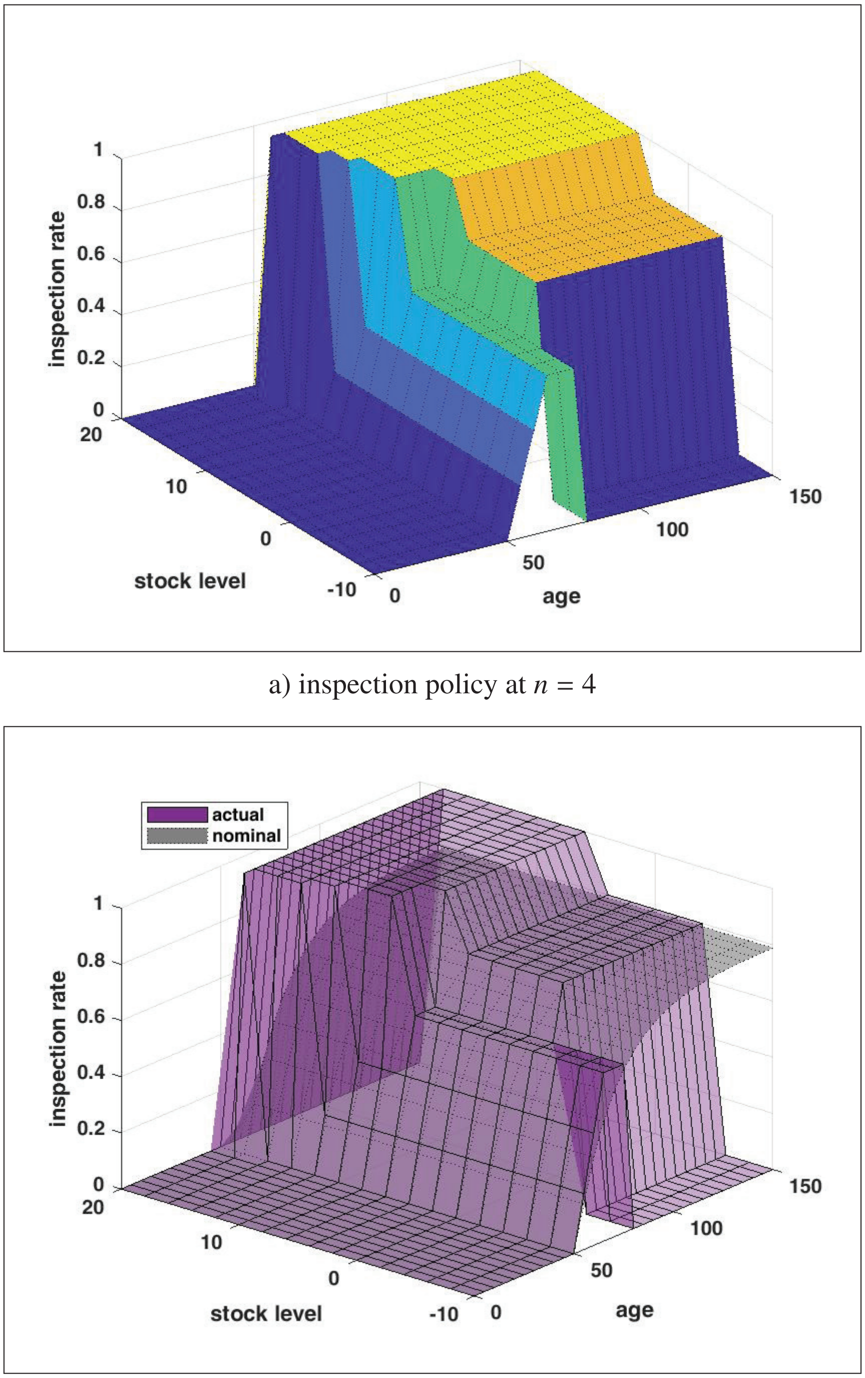


Figure 3.4   Nominal inspection policy for *AOQL*=0.05

be noticed in Figure 3.4, since the defective rate of products only depends on the age of the machine, the nominal inspection rate $\gamma_{nominal}$ depends also only on the age of the machine and not on the number of failures.

The inspection policy for the provided model is illustrated in Figure 3.5a. The inspection policy γ can be interpreted by comparison with the nominal inspection level $\gamma_{nominal}$ in Figure 3.5b. First of all, it can be observed that the inspection rate changes by the stock level for the same machine age. This means that not only the inspection policy depends on the age of the machine but it also relies on the stock level. The punishment for quality lower than $AOQL$ encourages the companies to avoid producing low-quality products. However, when the stock level is in an intense backlog situation and the machine is old, our priority should be fulfilling the demand even without performing any inspection. In other words, since the probability of failure is



a) inspection policy at $n = 4$

b) nominal inspection policy at $n = 4$

Figure 3.5   Nominal and actual inspection policies at $n = 4$

high and staying in the backlog state is expensive, it is more cost-effective to stop performing inspections until a specific level of inventory is reached and then inspection can be started again. However, we should pay the penalty for low quality during the non-inspection periods where the machine is producing products with quality less than $AOQL$. On the other hand, since there is a financial incentive for quality better than $AOQL$, at the specific level of inventory, it is more efficient to perform 100% inspection. Performing 100% inspection helps to achieve products with zero level of defectives. By carrying out the complete inspection we gain the whole amount of incentives for high-quality products.

Figure 3.6 presents the inspection policies γ for each number of failures. The inspection policy divides the (a, x) plan into six areas, including $\gamma = \{0, 0.2, 0.4, 0.6, 0.8, 1\}$. As the number of failures increases, the total inspection zone increases due to an increase in the production threshold. Also, as the number of failures increases the manufacturing machine deteriorates more and produces more non-rectifiable parts that can not be rectified and restored. As a result, it takes more time to build up the stock level and the machine should work even more, which in turn causes more deterioration and increases the failure risk of the machine. Therefore, when the system is too old and in a backlogged state, it is more cost-efficient to pay the penalty for producing the bad quality products rather than risking the machine's failure in the backlog state. (It can be observed in Figure 3.6) that at $n = 4$ and $n = 5$ that when the machine is producing defective more than $AOQL$, there are zones with zero inspection policy $\gamma = 0$).

Another important issue that can be understood from Figure 3.6 is that despite the incentives considered for the high quality, the 100% inspection is only performed when the machine is old, and not when the machine is young. This is due to two reasons: the cost of the inspection process and the nature of the defective rates curve. At earlier ages of the machine the defective rate is small and so is the increase in the amount of the defective rate ($\frac{\partial \beta}{\partial t} \approx 0$), therefore, conducting inspection does not significantly affect the average outgoing quality AOQ, and is not cost-efficient. It is also obvious that the starting stock level for the 100% inspection increases as the number of failures goes up.

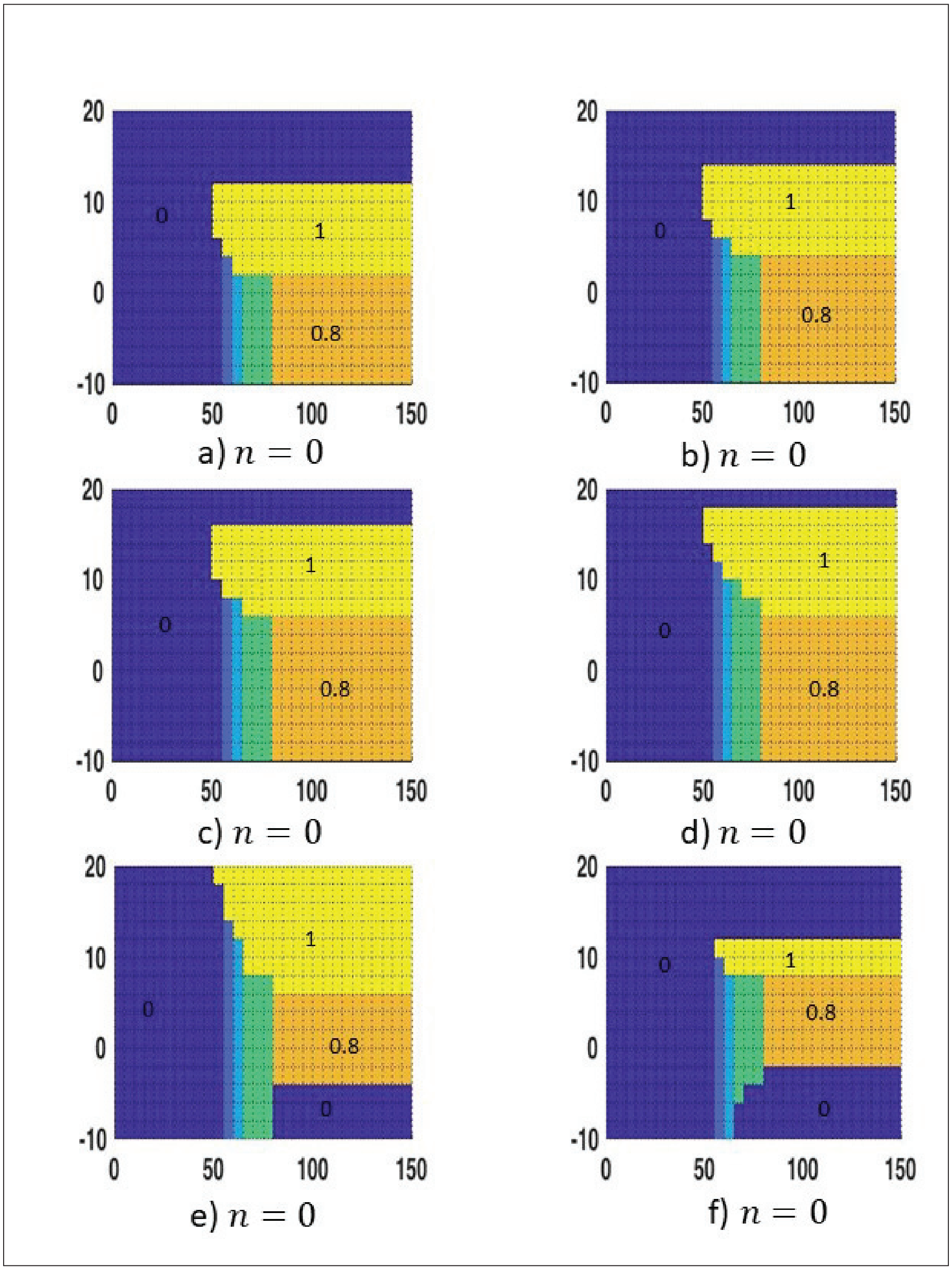


Figure 3.6 Inspection policy

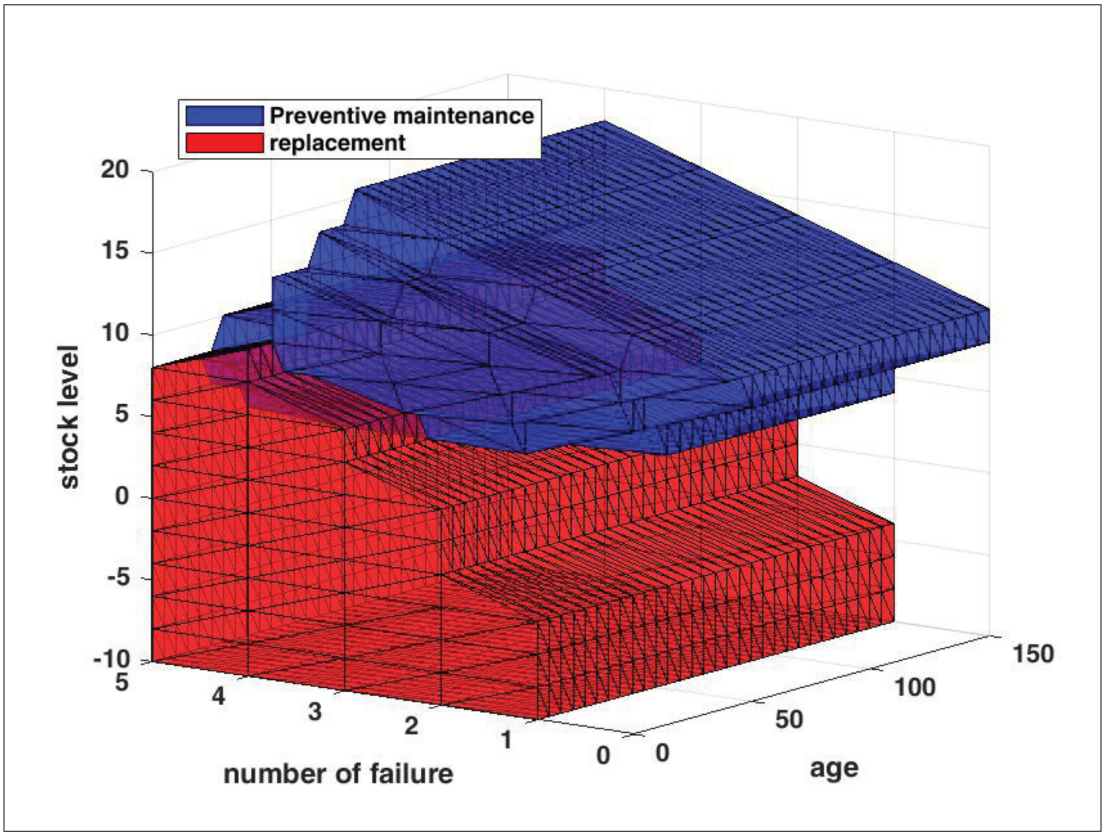
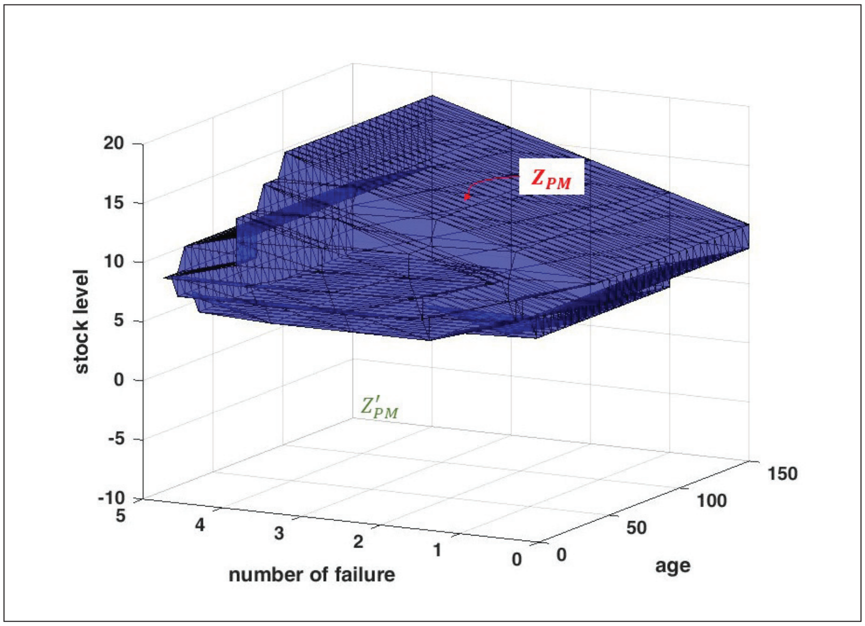


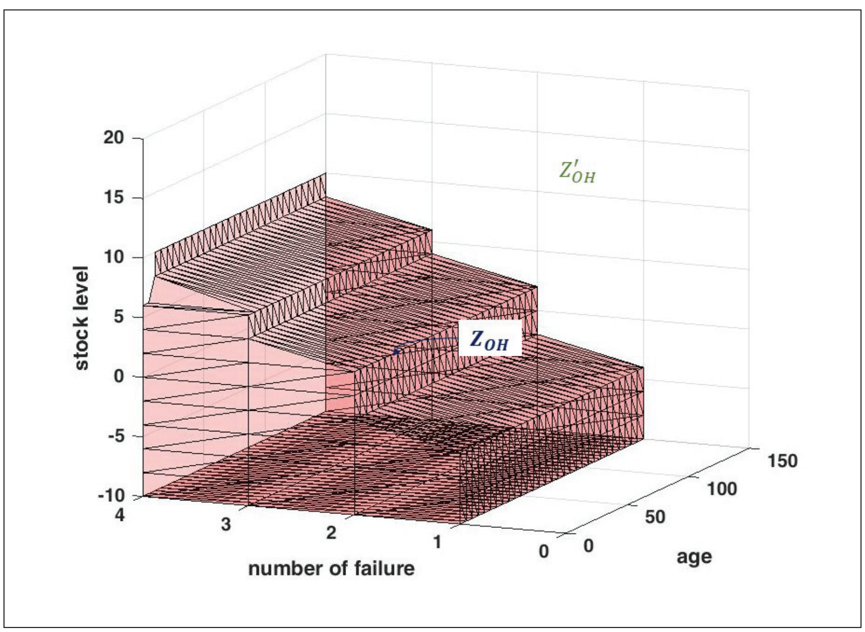
Figure 3.7 Replacement and preventive maintenance policies

3.4.2 Maintenance Policy

In this section, the optimal maintenance policy consisting of PM and repair/replacement switching policies will be discussed. Figure 3.7 represents the maintenance policies in which the areas of PM and replacement activities increase by the number of failures as well as the age of the machine. In order to diminish the effects of deterioration caused by the age of the machine and imperfect repair processes and to guarantee the production of parts with good quality while optimizing the total cost, the areas of maintenance plannings grow by the deterioration. As the manufacturing machine gets older, the defective rate of the system β and failure probability of the machine q_{15} increase. Moreover, as the number of failures n increases, the average time needed to repair the machine $MTTR$ increases as well. Also, after each imperfect repair, the failure rate of the machine goes up, therefore, in order to lessen the effects of deterioration on the machine's availability, the maintenance activities are intensified as its age and number of failures increase.



a) preventive maintenance zone



b) replacement zone

Figure 3.8 Maintenance zone

In Figure 3.8 the preventive maintenance and replacement zones are shown separately. Figures 3.8a and 3.8b indicate that these areas divide the three dimensional plan (a, x, n) into two zones; the PM zone Z_{PM} and the replacement zone Z_{OH}, respectively. These areas can be described as follows:

- Zone Z_{PM}: In Figure 3.8a, in the region Z_{PM}, the PM activity should be set to its maximum value ω_3^{max} considering the increasing deterioration level and in order to have enough products in the inventory to cover the demand during the PM activity time.
- Zone $\acute{Z}_{PM}$: Based on the PM policy shown in Figure 3.8a, in this zone $\acute{Z}_{PM}$ performing the PM activity is not recommended and it should be adjusted to its minimum value ω_3^{min}. The reason is that the machine is new and its level of deterioration is not high, therefore it can continue producing products. In addition, the stock level is not high enough to meet the demand during the PM period.
- Zone Z_{OH}: According to the replacement optimization plan presented in Figure 3.8b, in this area the machine should be replaced after failure. Here, the machine has reached a certain level of deterioration, and more importantly, the inventory stock level x_1 is not enough to satisfy the demand during the repair period. Considering that staying in the backlog state is very expensive and seeing as the average time required to repair increases by the number of failures, it is cost-efficient in this situation to pay the cost of replacement activities.
- Zone $\acute{Z}_{PM}$: It is recommended in Figure 3.8b that if the system is in this zone after failure, repair activities should be performed instead of the replacement activities. If the machine is new or if there are enough products in the inventory that can cover the demand during the repair processes, it is more efficient to avoid performing expensive replacement activities. Therefore, in this zone, the replacement rate should be set to its minimum value ω_4^{min}.

It can be understood from the trend of the maintenance policies in Figure 3.9, that an increase in the number of failures causes the area of ω_3^{max} and ω_4^{max} to increase, and the PM activities start at the earlier ages of the machine. This happens because when the number of failures n goes up, the failure rate of the machine and the mean time to repair $MTTR$ after a probable failure increase.

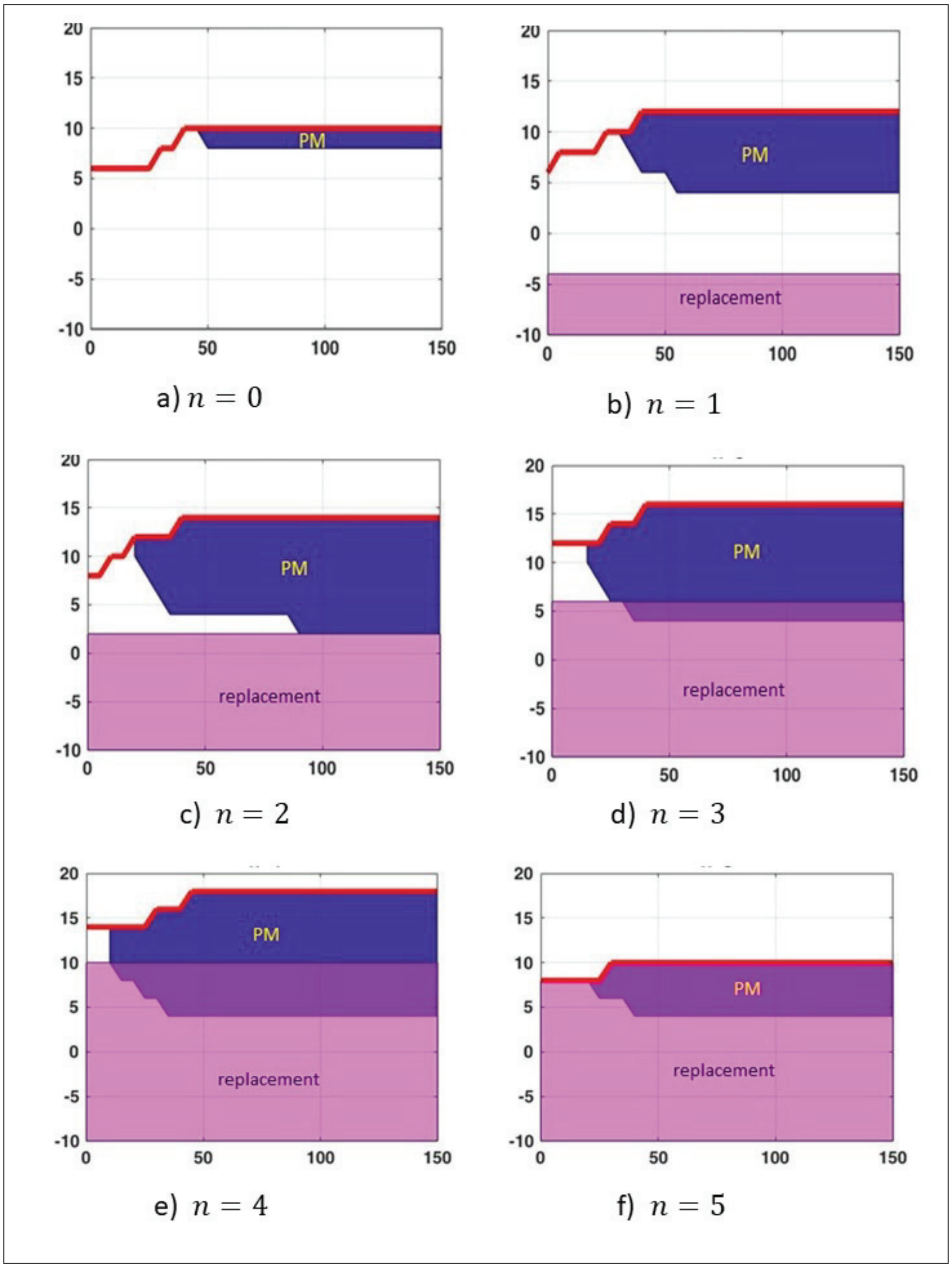


Figure 3.9 Maintenance policies

The optimal preventive maintenance and repair/replacement switching policies can be defined as follows:

$$\omega_3^*(1,a,x,n) = \begin{cases} \omega_3^{max} & \text{if } a(.) \text{ and } x(.) \in zone\mathrm{A}_p \\ \omega_3^{min} & otherwise \end{cases} \tag{3.18}$$

$$\omega_4^*(1,a,x_1,n) = \begin{cases} \omega_4^{max} & \text{if } a(.) \text{ and } x(.) \ \in \text{zone C} \\ \omega_4^{min} & otherwise \end{cases} \tag{3.19}$$

3.5 Sensitivity Analysis

In this section we perform the sensitivity analysis of some of the parameters of the model. We change the value of the parameters involved in the model and examine the change in its behavior.Then, the results are interpreted and compared to the model with the baseline values of the parameters.

3.5.1 Variation of the backlog cost

The optimal production threshold is affected by variations in the backlog costs c_1^-. As can be seen in Figure 3.10, three different values of backlog cost ($c_1^- = 200, 400, 600$) are given and it is clear that by increasing the backlog cost, the inventory level increase. This is due to the fact that it will be a more expensive experience if the system faces a shortage state. Thus, to prevent the system from facing a backlog situation it is suggested to increase the inventory level and the shortage cost. It also can be noticed in Figure 3.10 that by increasing the backlog cost, it is suggested to perform the replacement process at higher levels of inventory. Similarly, performing PM activities are not allowed in the lower inventory levels to avoid going into the backlog state.

Variation in the backlog costs can affect the inspection policy as indicated by Figure 3.11. As can be seen, by increasing the backlog cost, the 100% inspection zone starts from higher stock levels. Also, when the cost is increased to a certain level, the machine is old and the system is in a backlog state, according to Figure 3.11c, it is no longer cost-beneficial to perform inspection even we have to pay penalties for the low quality of products.

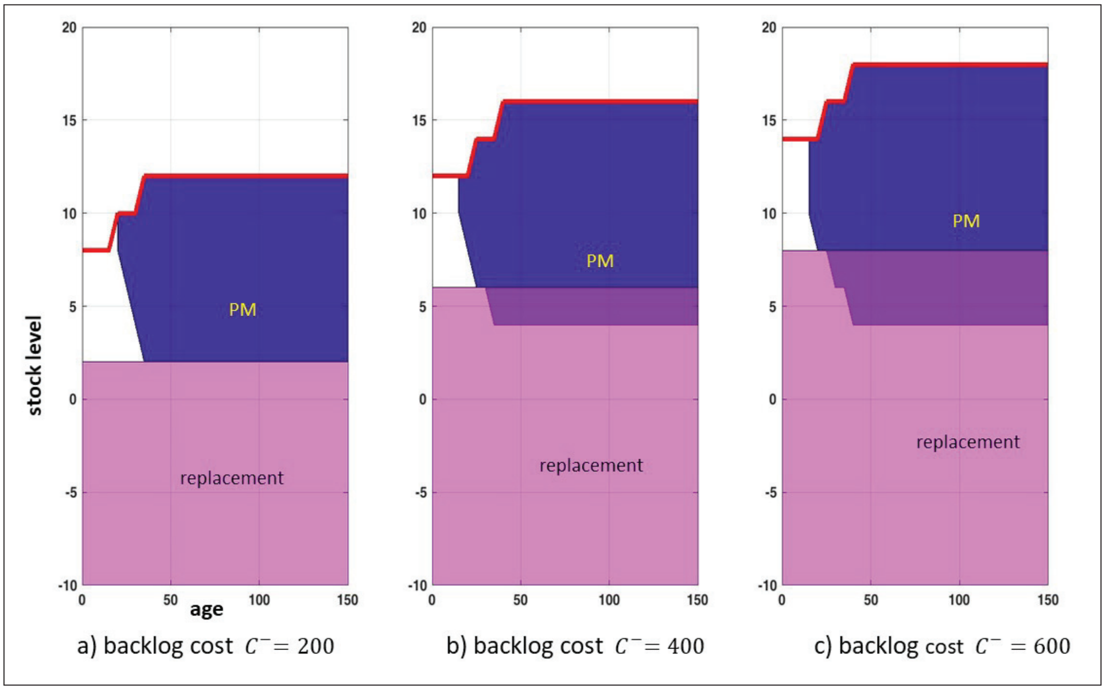


Figure 3.10 Maintenance policies for different backlog cost at $n = 3$

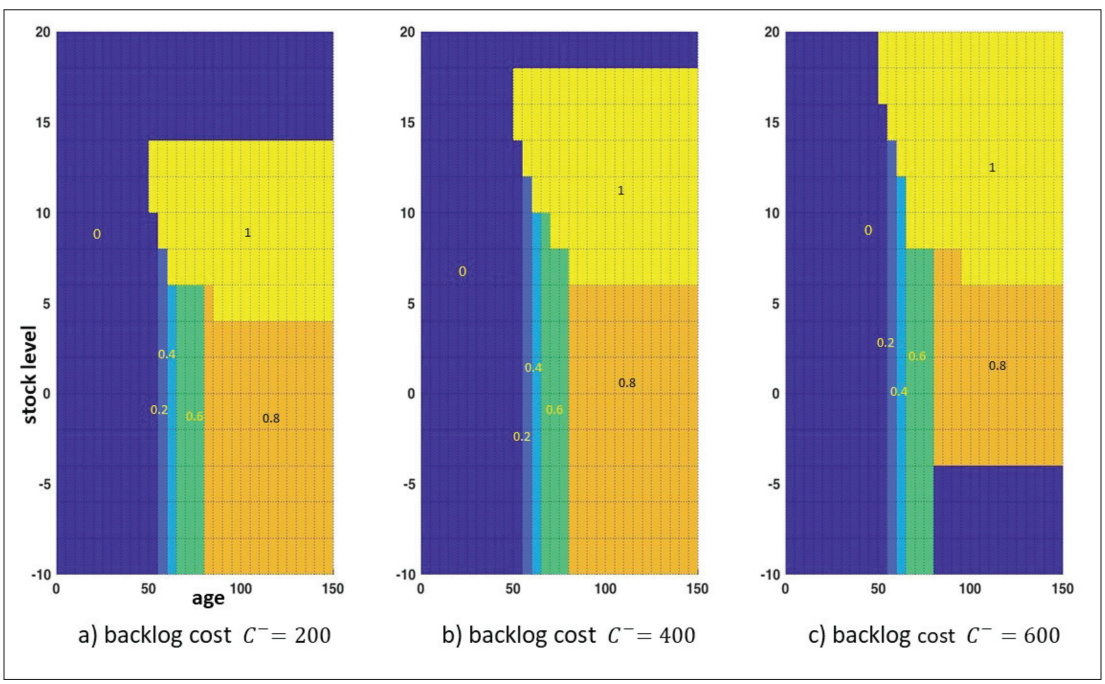


Figure 3.11 Inspection policies for different backlog cost at $n = 3$

3.5.2 Variation of the inspection cost

To study the effect of the variation of the inspection cost on the optimal policy, three different values of inspection costs were analyzed $c_{ins} = 6, 3, and 0.5$. It can be concluded from Figure 3.12, that increasing the value of the inspection costs does not have a considerable effect on the maintenance policy. However, by comparison with Figure 3.12a and Figure 3.12c, it can be understood that when the cost of inspection is decreased, PM can be performed at higher ages of the machine because performing inspection at that age, enables us to benefit from the incentives for providing high-quality products.

Inspection, especially the 100% inspection process, is somehow like a double-edged sword. On one hand, it helps to gain financial incentives by improving the quality of output products. On the other hand, it makes the process longer and delays fulfilling the demand, therefore increasing the inspection cost has a direct effect on the 100% inspection area. As can be seen in Figure 3.13 by decreasing the cost of inspection, the 100% inspection zone increases dramatically. In Figure 3.13c where the inspection cost is low, we can see that it is suggested to perform the inspection process even when the machine is in its earlier ages and the defective rate is very low. Also, in Figure 3.13a where the inspection cost is higher, the machine is old, and the system is in a backlog situation, it is no longer efficient to conduct the inspection.

3.5.3 Variation of PM cost

Following the analysis, Figure 3.14 shows that three different values of PM cost is provided $c_{PM} = 50, 100, and 300$. As the PM cost increases, the PM area decreases and starts at older ages of the machine (Figure 3.14).

The effect of PM cost on the PM policy is more evident for the lower number of failures. Figure 3.15 represents the effect of variation of PM cost when the number of failures is one ($n = 1$). This effect is more clear for lower numbers of failures because as the number of failures goes up, the repair cost and the mean time to repair the machine increases as well, therefore, even though PM activities are expensive, it is still cost-efficient to perform them. However, it can be noticed in Figure 3.15c, that for lower numbers of failures, where PM is expensive and $n = 1$, it is not suggested to perform PM activities.

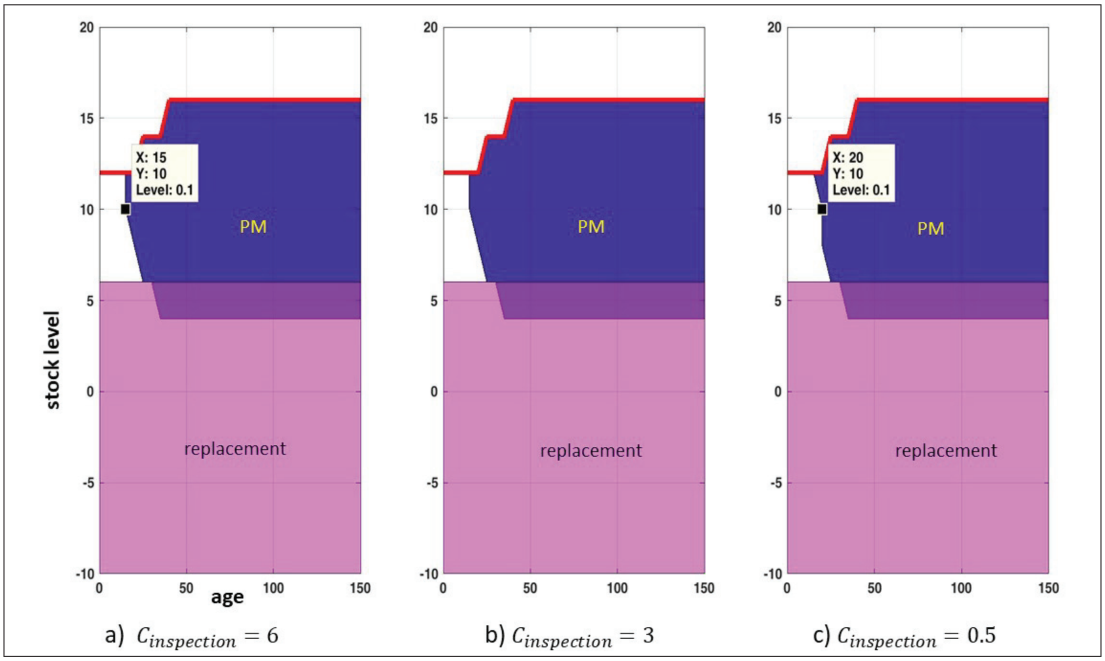


Figure 3.12 Maintenance policies for different inspection cost at $n = 3$

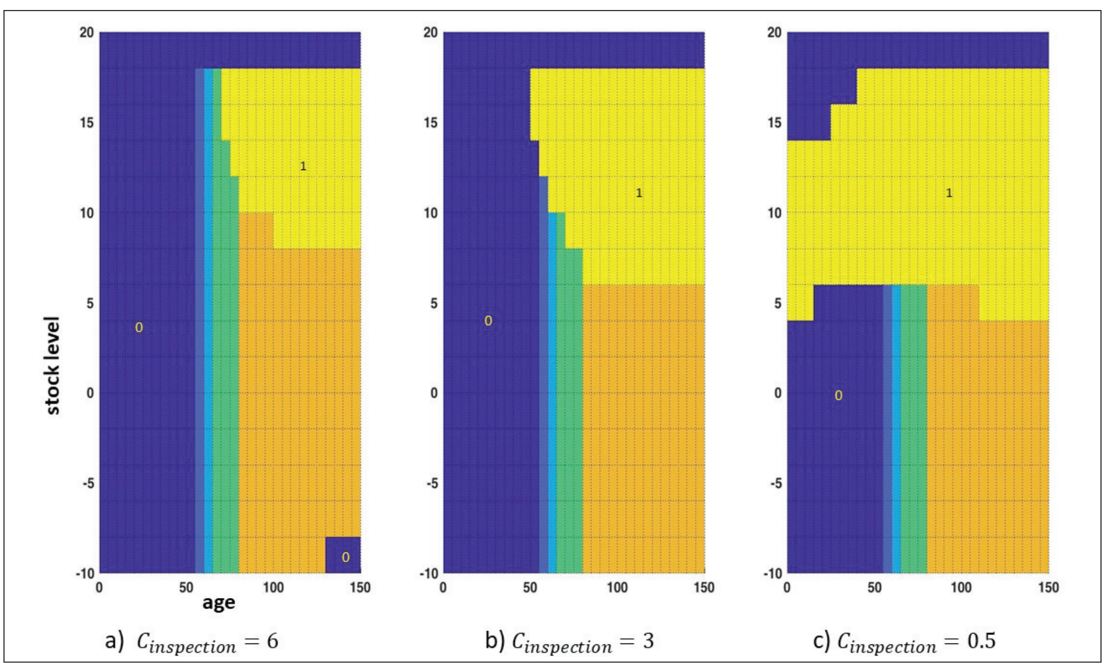


Figure 3.13 Inspection policies for different inspection cost at $n = 3$

Also changing the PM costs does not considerably affect the inspection policy, however, Figure 3.16 indicates that changing the PM cost, results in slight changes in the inspection policy. Here, when the PM cost is increased, the intensity of inspection increases to compensate for the reduction in the PM area.

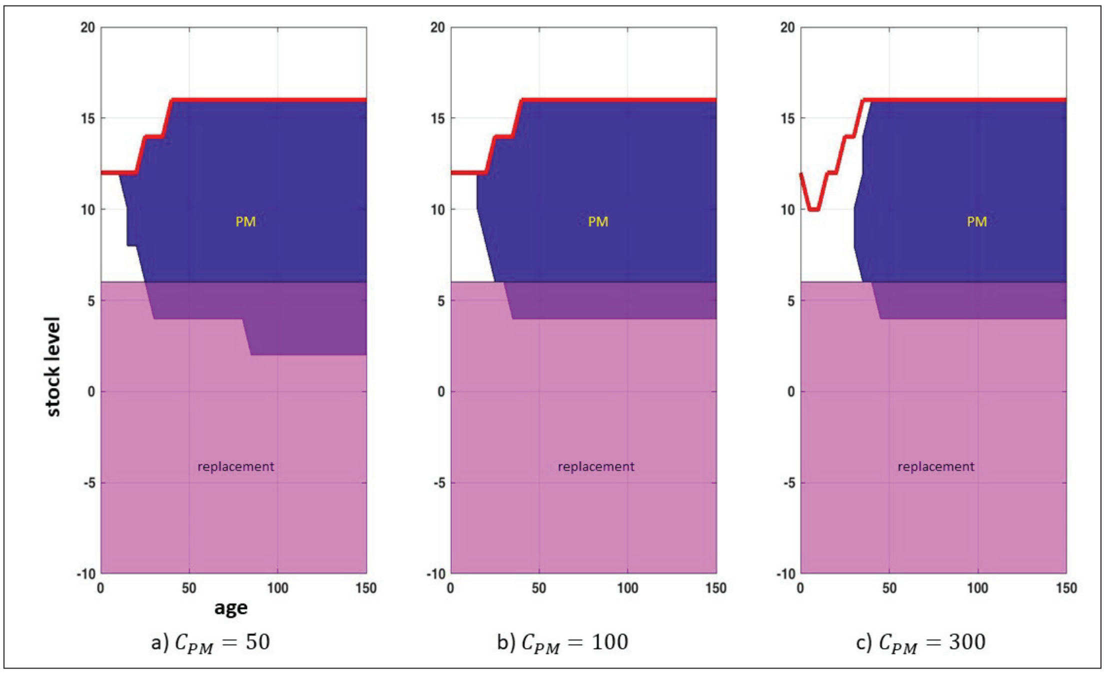


Figure 3.14 Maintenance policies for different PM cost at $n = 3$

3.5.4 Variation of the rectifying cost

The effects of change in rectifying cost are directly reflected in the inspection policy, especially in $zero$ and 100% inspection policy. Three different values of rectifying cost $c_{rec} = 5, 20 and 50$ have been analyzed in Figure 3.17. It can be noticed that by increasing the rectifying cost the area of the 100% inspection decreases and starts from higher levels of stock, also, the $zero$ inspection area grows, and once the machine is old and the system experience the severe backlog situation, inspection is not allowed. Figure 3.17. This happens because when the rectifying cost is high, performing the 100% inspection is less cost-efficient and it is suggested to perform it at a higher stock level. However, when the machine is old, the defective rate is at its maximum value and performing inspection means that more rectifying process is needed. Also, when the

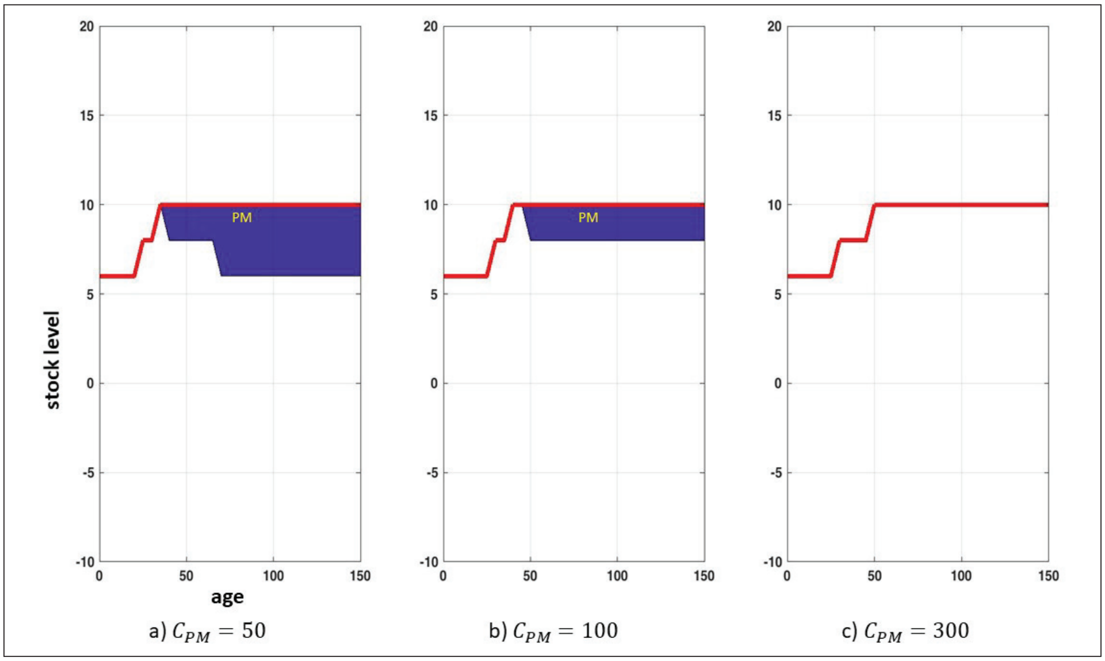


Figure 3.15 Maintenance policies for different PM cost at $n = 1$

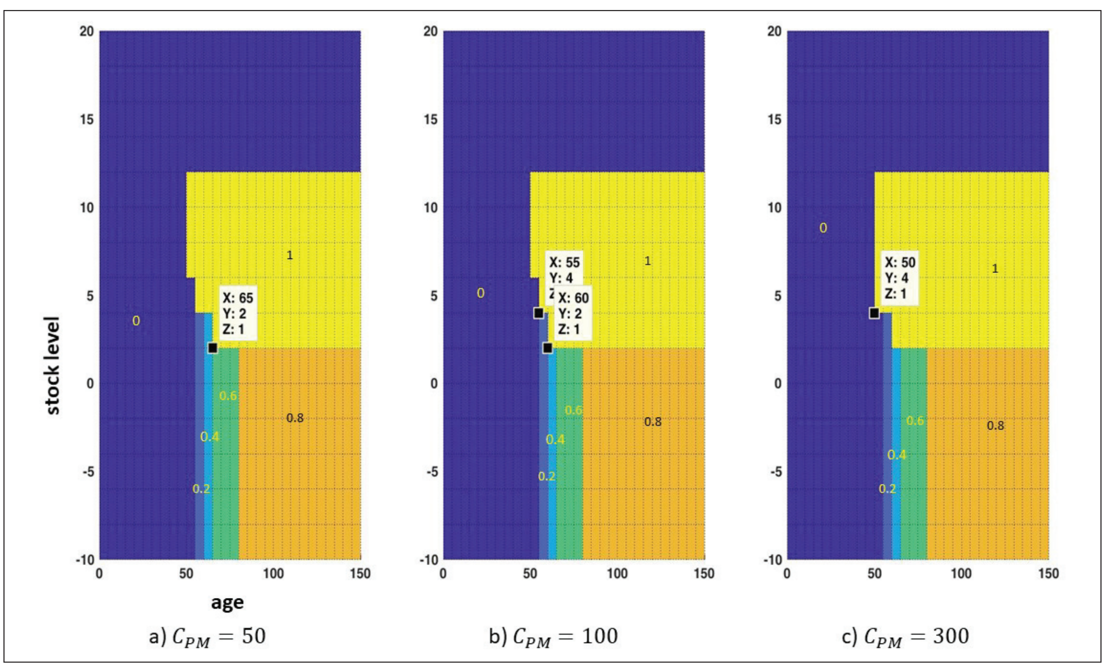


Figure 3.16 Inspection policies for different PM cost at $n = 1$

inspection is carried out it takes more time to fulfill the demand, therefore it is recommended not to perform inspection while the machine is old, the system experiences a severe backlog situation, and the rectifying cost is high. Figure 3.17c.

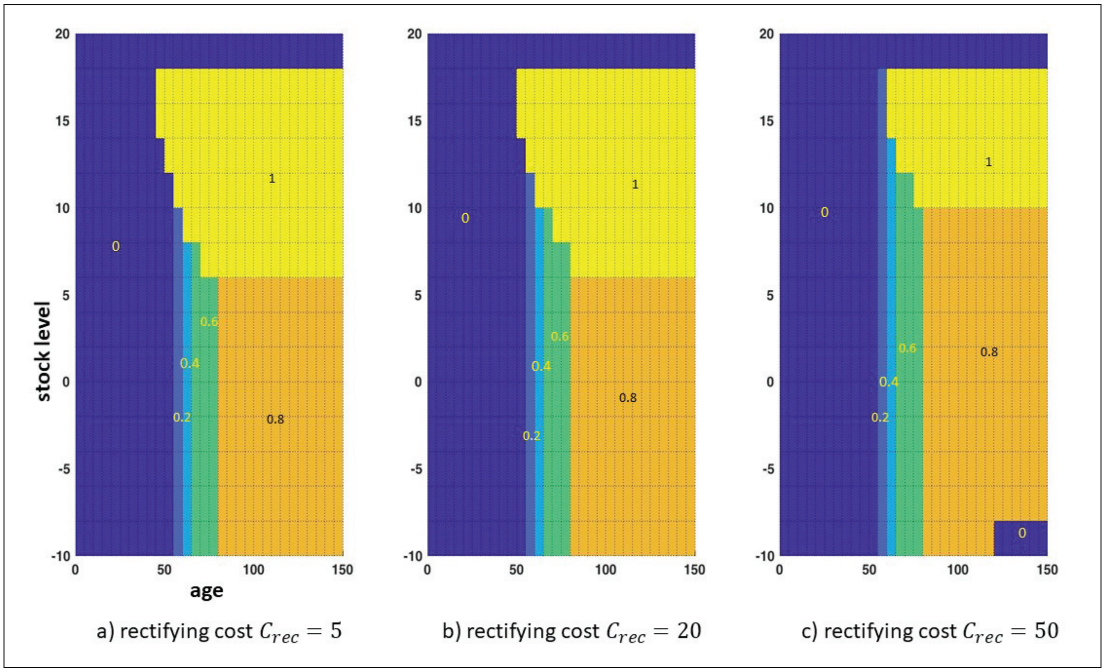


Figure 3.17   Inspection policies for different rectifying cost at $n = 4$

3.5.5   Variation of the low-quality financial penalty coefficient

The effect of changing the penalty for poor quality products will appear in the inspection policy. To illustrate this concept, Three different values are given for the penalty coefficient, $\varepsilon_{q_penalty} = 5, 10, 30$. As can be observed in figure 3.18, when the penalty coefficient increases, the inspection area increases even in the more severe backlog situations. In other words, when the penalty considered for the low quality is not severe enough, performing the inspection while the system is in the backlog state and the machine is old, is not recommended. Increasing the penalty can justify the inspection in the backlog state while the machine is old.

3.5.6   Variation of the high-quality financial incentive coefficient

The effect of variations of the financial incentives considered for high-quality products is shown in Figure 3.19. To analyze the effect of this factor, three different incentive coefficients

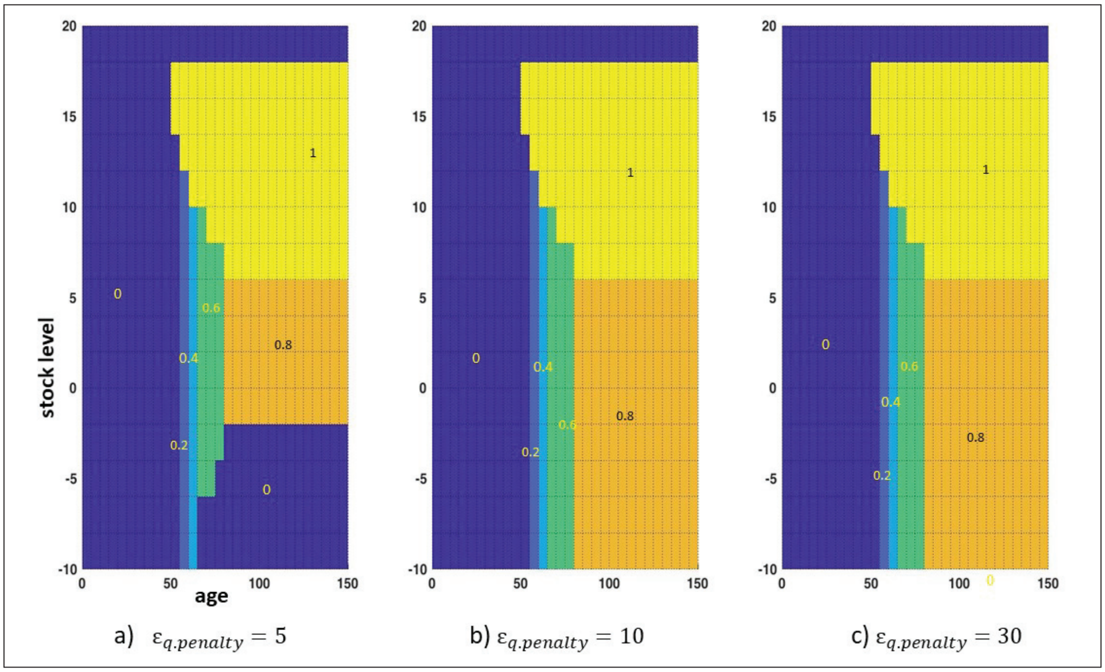


Figure 3.18 Inspection policies for different low quality penalty at $n = 3$

have been considered $\varepsilon_{q_penalty} = 0.75, 1, and 3$. Figure 3.19 indicates that by increasing the incentive, the area of the 100% inspection increases significantly. When the incentive is high, it is recommended to perform the inspection at earlier ages of the machine. The 100% inspection zone can also be carried out at lower stock levels. Performing inspection at earlier ages of the machine even when the products have an acceptable quality can only be justified when the considered incentive is high.

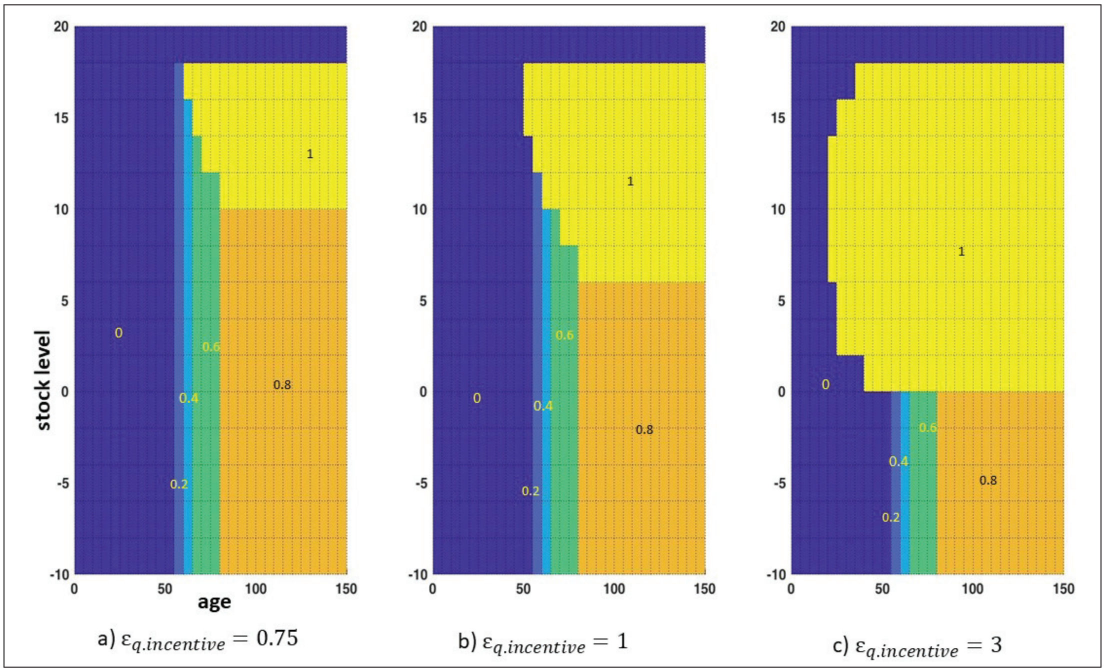


Figure 3.19   Inspection policies for different high quality incentive at $n = 3$

3.6 Conclusion

In this chapter, integrated problems of the interaction between production control, maintenance activities, and quality aspects of products through inspection processes were investigated. The proposed model consists of a single unreliable manufacturing machine that can produce one type of product. This system is supported by a rectifying machine that can recover some of the defective products that are rectifiable. The deterioration has negative effects on the availability of the machine and the quality of the produced parts. The quality of the products decreases by the age of the machine and the rate of non-rectifiable defective products increases by the number of failures. To avoid paying the penalty considered for products with a quality lower than AOQL, and to palliate the negative effects of deterioration, both the maintenance activities and the inspection processes are carried out with the help of the rectifying machine. Moreover, a simulation-optimization approach that can integrate the mathematical formulation and the simulation modeling is presented. This approach helps us to model the structure of the joint production, maintenance, and inspection policy by using numerical techniques. The results show that the production rate depends on the level of deterioration of the machine which increases by its age and number of failures. The maintenance process should also be accelerated as the machine's level of deterioration goes up. The inspection policy is determined by the quality deterioration and the value of the financial penalty or incentive dedicated for the quality level lower or higher than AOQL, respectively. When these quality penalties or incentives are intensified, the inspection process intensifies as well. An experimental design and sensitivity analysis were employed to validate and examine the proposed model over the defined horizon to minimize the total costs. The result obtained from the sensitivity analysis indicates that the proposed optimization approach for the model is accurate and able to yield reliable results.

CHAPTER 4

SIMULTANEOUS PRODUCTION PLANNING AND MAINTENANCE CONTROL PROBLEM FOR AN UNRELIABLE HYBRID SYSTEM IN CLOSED-LOOP REVERSE LOGISTICS

4.1 Introduction

In recent years, the subject of managing defective parts as well as products returned from the market through reusing and major and small recovery operations has received much interest. This growing interest reflects the increasing importance of recovery and remanufacturing issues as people become more conscious of the environmental concerns, sustainable development, and the financial benefits of enhancing remanufacturing processes through technological advancements. There are mainly two ways of recovering the faulty and returned products: recycling (rectifying) and remanufacturing. If there is simply a minor defect in the product that requires only minor repair, the rectifying process can be used. However, if the product has significant damage that needs extensive repair in order to be entirely recovered, the remanufacturing procedure should be employed. The reverse logistic system consists of two separate flows of products; the Forward Flow going from the production system to the customer and the Reverse Flow going back from the customer or the market to the producers. Management and control of this cycle of the flow of products is the main focus of the closed-loop systems studies. One of the interesting topics in production systems research is the hybrid manufacturing-remanufacturing system.

Given the inherent uncertainties in production systems, such as deterioration and random events like unplanned failures and repair processes, developing a comprehensive model that incorporates all of the variables can provide a complete simulation of the real situation. The production policy of such a hybrid stochastic system is investigated in this study in the context of a continuous-discrete state. Along with manufacturing, remanufacturing, PM, and replacement rates, the inspection process is also introduced to the system as a decision variable. In addition, four state variables are taken into account in this model, including inventory and recovery stock levels, as well as the age and number of manufacturing machine breakdowns. We assume a quality limit (AOQL) in our method, with financial incentives and penalties for manufacturing products of higher and lower quality than the limit, respectively. Many research works have been conducted in the area of closed-loop systems, but to our knowledge, there seems to be no similar method like this in the literature. The goal of this research is to develop an optimal model for manufacturing and remanufacturing planning, maintenance scheduling, and inspection strategies that minimize total costs.

4.2 System description

In this section, a simultaneous production planning and maintenance control problem is investigated for an unreliable hybrid system in closed-loop reverse logistics. The proposed system consists of deteriorating manufacturing, re-manufacturing, and rectifying machines to satisfy a constant demand. The system is subjected to random events such as failures and maintenance activities including repair, preventive maintenance, and replacement. The deterioration of the manufacturing machine caused by the aging and imperfect repair processes affects its availability and the quality of the produced parts. The availability of the re-manufacturing machine is randomly affected by the deterioration phenomena. Given that the rectifying machine should mostly work on the defective parts with minimal defects, the deterioration effect on it is not considerable. Therefore, the rectifying machine is assumed to always be available.

The defective products can be classified into two groups based on their quality during the inspection process: rectifiable parts and non-rectifiable parts. The deterioration caused by the imperfect repairs also affects the quality of defective parts and increases the rate of non-rectifiable parts. The rectifiable parts are fixed by the rectifying machine and stored along with the flawless parts in the first inventory room to be delivered to market.

The non-rectifiable parts are stored along with the products returned from the market in the second stock room (recovery stock) to be restored by the re-manufacturing machine. The perfectly restored parts are sent to the first inventory. The system is unable to meet the long-term demands of the products due to the deterioration effects; several maintenance activities can be performed to diminish these unfavorable effects. After each failure, there are two options available to the manufacturing machine: either the machine can be repaired or it can be replaced with a new one.

Since the repair activities are not perfect, they can not restore the machine's state to the initial condition. Therefore, while replacement restores the machine's condition to its initial form, repair takes it to a state between AGAN and ABAO. However, there are PM activities that can be performed during the operational state of the manufacturing machine. The PM processes can restore the machine condition to its initial one after the last repair.

To satisfy the specific quality limit $AOQL$ required by the market and to detect the defective parts produced by the manufacturing machine the inspection processes should be performed. However, because of the inspection processes costs, it is not suggested to perform the 100% inspection in all conditions, and the proportion of the total inspection has to be optimized.

The main goal of this study is to determine simultaneously the optimal production plan, in terms of manufacturing, re-manufacturing, maintenance activities, and inspection strategies, for the hybrid closed-loop reverse system, while minimizing the total costs over an infinite planning horizon.

4.3 Formulation of the dynamic of the system

The hybrid system presented in Figure4.1 consists of manufacturing M_1, re-manufacturing M_2, and rectifying machines M_3 in a closed-loop reverse system and produces one type of product to satisfy the demand. In manufacturing machines, deterioration affects the availability of the machine and the quality of the products. The failure rate of the machine M_1 and the defective rate of the products increase with the age a and the imperfect repair which is represented by the number of failures n of the machine M_1. To mitigate the undesirable effects of deterioration, maintenance activities such as PM, repair, and replacement M_1 are available for the manufacturing machine. The manufactured products are inspected at the rate of $gamma$. The perfect inspected parts are stored along with non-inspected parts in the stock room (x_1) to be sent to the market and satisfy the demand d. During the inspection process, the defective products are classified into rectifiable and non-rectifiable parts. The portion of the defective parts that are non-rectifiable increases progressively with the number of failures n. Rectifiable defective products are fully recovered by the rectifying machine M_3 and then will be sent to the first inventory x_1 whereas the non-rectifiable parts are transferred to the recovery inventory x_2 along with returned products. All of the defective products stored in the recovery inventory x_2 will be sent to the remanufacturing machine to be recovered. Next, the perfect re-manufactured parts will be sent to the first inventory x_1 and the flawed parts will be disposed of. After the failure of the unreliable remanufacturing machine M_2, it can be repaired and be restored to its initial condition.

Considering the severe damages caused by the deterioration, at a certain level, the manufacturing machine M_1 can not fulfill the long-term demand even with remanufacturing machine's support. Therefore, once the failure occurs after a specific number of times which is named the last number of failures N, a mandatory replacement policy is set.

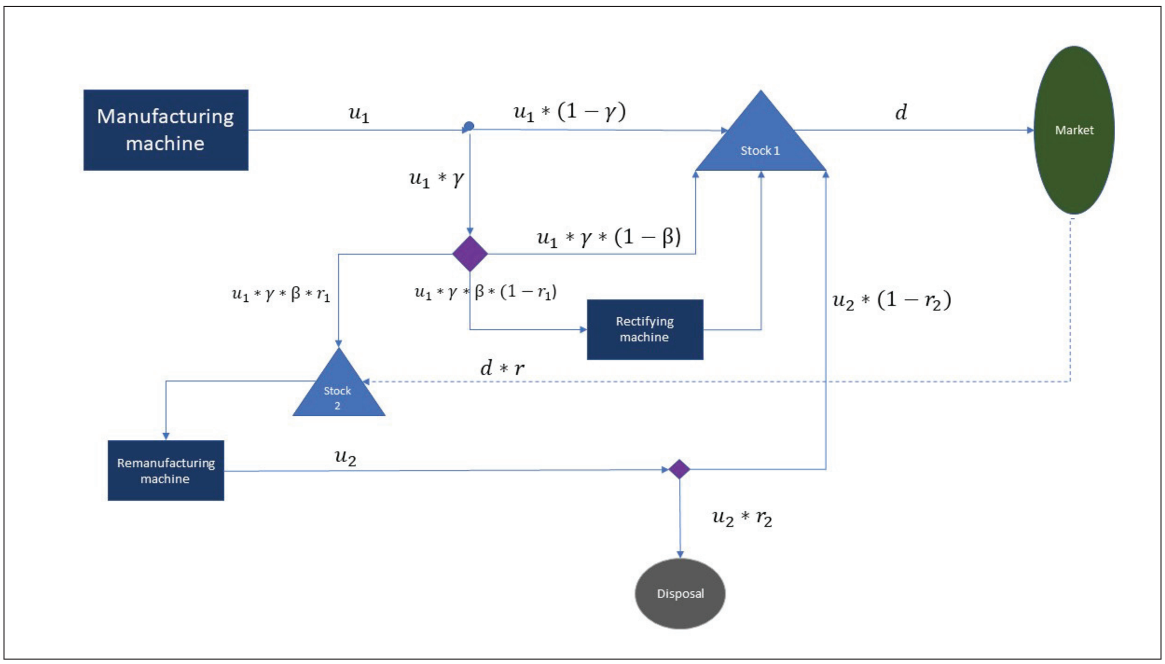


Figure 4.1 Transition diagram of hybrid model

Other features of the hybrid system shown in Figure 4.1 are:

- The demand rate is constant.
- The raw material for the manufacturing machine is always available.
- Finished products produced by manufacturing, remanufacturing, and rectifying machines are identical with the same quality.
- A second remanufacturing process on the same product is unacceptable, therefore, the defective parts from remanufacturing activities are discarded. However, the defective parts

of the manufacturing machine can be recovered either by the rectifying machine or by the remanufacturing machine.

- Since the production is a continuous process, and the rate of inspection and rectifying is higher than the manufacturing rate, the delay time due to inspection and rectifying has not been considered.

Our objective in this section is to determine a joint optimal production policy for the manufacturing u_1 and remanufacturing u_2 machines, the optimal threshold level for the inventory x_1 and recovery x_2, the optimal inspection rate γ, and the optimal maintenance activities, including PM, repair and replacement strategies that minimize the total cost for different scenarios. These costs include: production costs, inventory and backlog costs, inspection costs, rectifying and remanufacturing costs, repair, PM and replacement costs, quality costs such as punishments and incentives, and disposal costs. The optimal solutions should satisfy a number of constraints related to the feasibility conditions.

As already mentioned in the first two chapters, the manufacturing machine has 5 different modes. In the presented hybrid model, the remanufacturing machine M_2 was added to the system with two modes: the *operational* and *failure* modes. After each failure, the machine M_2 will be repaired and reset to its initial condition. Therefore, overall, this hybrid system has 10 different modes. The different modes of manufacturing and remanufacturing machines in which they can be at any mode randomly are depicted in Figure 4.2.

The transition diagram for the whole hybrid system is given by Figure 4.3.

The 10 different modes $\zeta(t)$ of the system shown in figure 4.3, are classified in the table 4.1.

4.3.1 Formulation of the control problem

We started by describing a stochastic model for manufacturing and remanufacturing system under deterioration that is capable of producing one type of product to meet a constant demand. Next, we are going to formulate an optimal hybrid production policy for the manufacturing and remanufacturing machines and determine the best inspection and maintenance strategies to minimize the total costs. In this model, stochastic phenomena such as breakdown and repair activities are considered for both manufacturing and remanufacturing machines. Moreover, PM

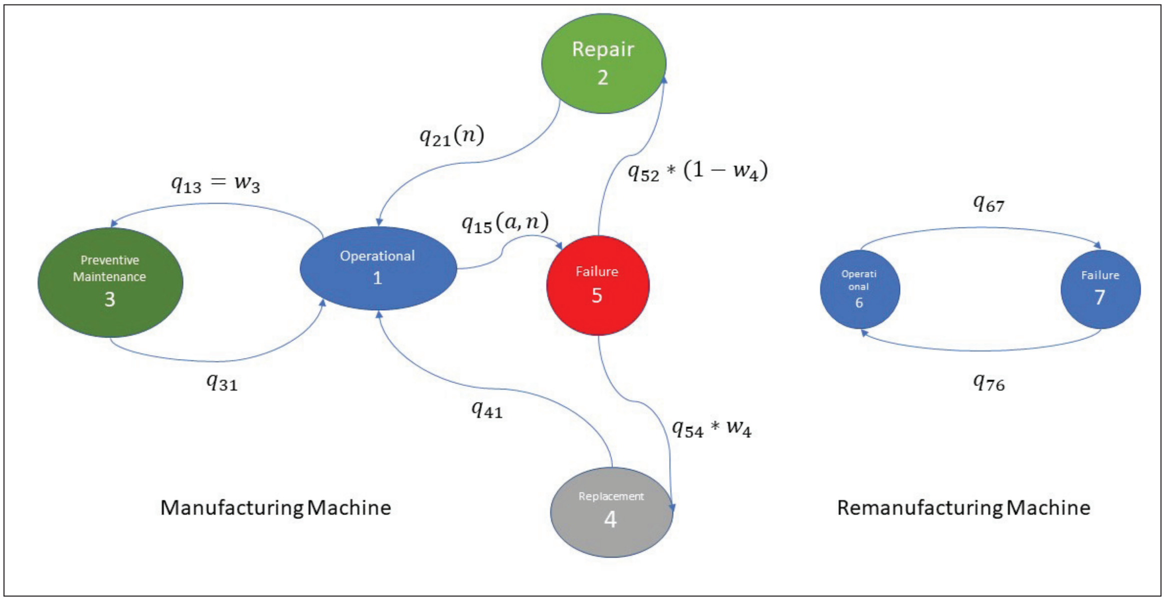
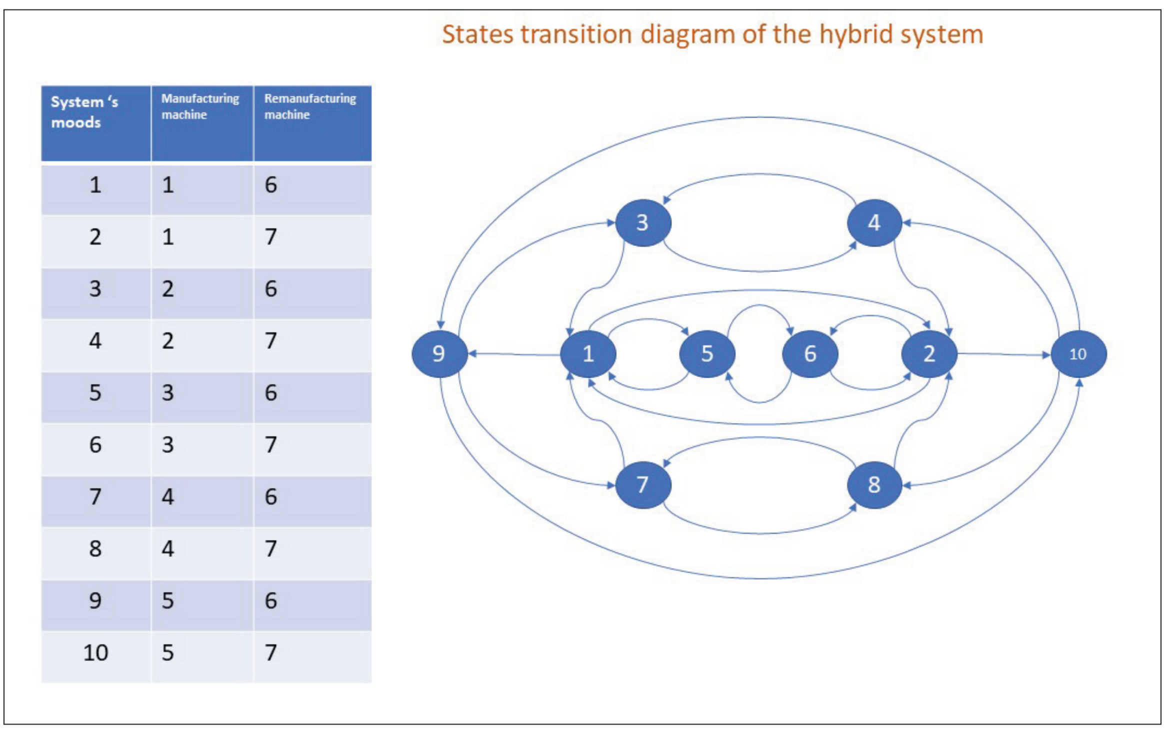


Figure 4.2 Transition diagram of manufacturing and remanufacturing machines



System 's moods	Manufacturing machine	Remanufacturing machine
1	1	6
2	1	7
3	2	6
4	2	7
5	3	6
6	3	7
7	4	6
8	4	7
9	5	6
10	5	7

Figure 4.3 Transition diagram of hybrid model

Table 4.1 Transition modes $\zeta(t)$ of the hybrid system

Hybrid system's mode $\zeta(t)$	Manufacturing machine state	Remanufacturing machine state
1	Operational	Operational
2	Operational	Repair
3	Repair	Operational
4	Repair	Repair
5	PM	Operational
6	PM	Repair
7	Replacement	Operational
8	Replacement	Repair
9	Failure	Operational
10	Failure	Repair

and replacement activities, as well as deterioration effects on the availability of the system and quality of the products, are considered the for manufacturing machine.

We begin the formulation of the control problem by defining the aging process $a(t)$ of the manufacturing machine M_1 as an increasing function of its production rate M_1. This means that the age of the machine M_1 increases only by producing more products. The age of the machine is given by the following differential equation:

$$\dot{a} = ku(.), \qquad a(T) = 0 \tag{4.1}$$

where k is a given constant aging coefficient $(k > 0)$ and T is the instant where the last replacement activity has been performed. The age of the machine after nth failure is given by:

$$a_n(t) = a(t - t_n) + \alpha_n, \quad t_n \leq t < t_{n+1}, \quad t_0 = 0, \quad \alpha_0 = 0, \tag{4.2}$$

where t_n is the instant of nth failure and α_n is the virtual age of the machine after the nth repair which is given by:

$$\alpha_n = \phi_a \times a(t_n - t_{n-1}) + \alpha_{n-1}, \quad n \geq 1 \tag{4.3}$$

where ϕ_a is the constant repair age intensity and $(0 \leq \phi_a < 1)$.

Also, the age of the machine in the event of the nth number of failures when the last maintenance activity was PM, is given by:

$$a_n(t) = a(t - t_{pm}) + \alpha_{pm}^n + \alpha_n, \quad t_{pm} \leq t < t_{n+1}, \tag{4.4}$$

where t_{pm} is the instant of starting PM activities and α_{pm}^n is the virtual age of the machine after PM and in the nth number of failures and is given by:

$$\alpha_{pm}^n = \phi_{pm} \times \big(a(t_{pm} - t_n) + \alpha_n\big), \quad n \geq 1 \tag{4.5}$$

where ϕ_{pm} is the PM intensity coefficient and $(0 \leq \phi_a < 1)$. If the preventive maintenance activities are done perfectly, the coefficient would be zero ϕ_{pm}. Because of the conditions such as human error, imperfect materials that are used during the repair activity, and imperfect repair processes, the repair activities are not perfect and the conditions of the manufacturing machine M_1 after repair, will not reset to its initial state. Therefore, we can use the number of failures and the age of the machine to formulate the deterioration. By using the number of failures, the effect of the imperfect repair can be added to the model. The effects of the imperfect repair can be classified as:

$$\phi = \begin{cases} \phi_a & \text{repair age intensity coefficient} \\ \phi_r & \text{repair reliability coefficient} \\ \phi_q & \text{repair quality coefficient} \\ \phi_{pm} & \text{PM intensity coefficient} \end{cases}$$

As can be seen, not only the imperfect repairs affect the age of the machine but they also can have impacts on its reliability and the quality of the products. The failure rate of the manufacturing machine q_{15} which is a function of its age (a), number of failures n, and the repair reliability factor ϕ_r. The failure rate is given by:

$$q_{15_n}(a) = K_0 + \sum_{i=1}^{n-1}\left(\phi_{r_i}\,(\frac{a_i}{A})^{\theta_a}(\frac{i}{N})^{\theta_i}\right) + \left(K_1 - \sum_{i=1}^{n-1}(\phi_{r_i}\,(\frac{a_i}{A})^{\theta_a}(\frac{i}{N})^{\theta_i})\right)\left(1 - e^{(-K_t n^{\theta_{f_1}} a^{\theta_{f_2}})}\right) \tag{4.6}$$

where K_0 is the constant initial and minimum value of the failure rate for the new machine and $K_1 + K_0$ is the maximum quality deterioration considered for the manufacturing machine. A is the maximum age of the machine M_1 before performing mandatory PM processes and N is the maximum number of failures where the machine should be replaced with a new one after the next failure. ϕ_{r_i} is the repair reliability factor for ith number of failures. n_i and a_i are ith number of failures and the corresponding age where the ith failure occurred. The parameter θ_f can adjust the failure curve trajectory. To simplify the Equation 4.6 following assumption would be helpful:

$$\sum_{i=1}^{n-1}\left(\phi_{r_i}\,(\frac{a_i}{A})^{\theta_a}(\frac{i}{N})^{\theta_i}\right) = \acute{\phi}_r\,(\frac{n}{N})^{\theta_n} \qquad ; \qquad \acute{\phi}_r = constant \tag{4.7}$$

Based on the notation, d is the constant demand that should be satisfied, $u_1(t)$ and $u_2(t)$ are the production rates of manufacturing and remanufacturing machines respectively at time t. According to the block diagram of the model provided in Figure 4.1, the dynamics of the inventory (x_1) and recovery (x_2) are formulated by the following differential equations:

$$inventory\ stock\ dynamic \begin{cases} \dot{x}_1 = (1-\gamma)u_1 + (1-\beta)\gamma u_1 + \gamma\beta(1-r_\beta)u_1 + (1-r_{dis})u_2 - d \\ \dot{x}_1 = (1-\gamma\beta r_\beta)u_1 + (1-r_{dis})u_2 - d \quad , \qquad x_1(0) = x_{10} \end{cases} \tag{4.8}$$

$$recovery\ stock\ dynamic : \dot{x}_2 = (\gamma.\beta.r_\beta)u_1 + r_M.d - u_2 \tag{4.9}$$

where $\gamma(.)$ is the inspection rate of the products produced by the manufacturing machine. β represents the rate of the defective parts produced by the manufacturing machine and r_β is the rate of non-rectifying defective products that increases with the number of failures. r_M is the constant rate of the products that are returned from the market and stored in the recovery stock to be remanufactured. r_{dis} is a constant rate of disposed products that are recovered by remanufacturing machine.

Another disturbing effect of deterioration can be observed in the quality of produced parts by machine M_1. The effect of the repair process on the quality of products is denoted by ϕ_q where $\phi_q = 0$ indicates that the repair processes do not have any effects on the quality of products. The defective rate $\beta(.)$ is given by the following expression:

$$\beta(a,n) = B_0 + \sum_{i=1}^{n-1} \left(\phi_{q_i} (\frac{a_i}{A})^3 (\frac{n_i}{N})^2\right) + \left(B_1 - \sum_{i=1}^{n-1} (\phi_{q_i} (\frac{a_i}{A})^3 (\frac{n_i}{N})^2)\right) \left(1 - e^{(-K_b \theta_b a^3)}\right) \quad (4.10)$$

where B_0 is the initial and minimum defective rate of the machine, and $(B_0 + B_1)$ is the maximum defective rate of the machine in the worst condition. k_b is a given constant and θ_b is the adjustment parameter for defective rate trend.

Another parameter that can be affected by imperfect repair is the quality of the defective parts. A previously mentioned, the defective parts produced by machine M_1 are classified into rectifiable and non-rectifiable parts. The imperfect repair can change the portion of non-rectifiable parts out of the defective parts. The non-rectifiable rate of the defective products r_β is given by:

$$r_\beta = R_{b0} + R_{b1} \phi_{qnr} (\frac{n}{N})^{r_2} \quad (4.11)$$

where R_{b0} , R_{b1} and r_2 are constant parameters and $\phi_q nr$ is the non-rectifiable coefficient. As can be observed in Equation 4.11, as the number of failures go up, the non-rectifiable defective rate increases. In other words, the quality of defective parts decreases by the number of failures. However, when $\phi_q nr$ equals zero, it means that the repair activities do not affect the rectifiable rate.

An increasing number of failures means that the manufacturing machine has been subjected to a couple of imperfect repairs, and after the each failure, it takes longer to detect the reason and repair the machine. Therefore, machine M_1's mean time to repair $MTTR$ increases with the number of failures and is given by the following expression:

$$MTTR(n) = T_{21}(n) = \frac{A_1}{r^n} + A_2 * n \tag{4.12}$$

where $A_1.A_2$ and r are given constant parameters, and n is the number of failures. The repair rate of the manufacturing machine can be obtained by:

$$q_{21}(n) = \frac{1}{MTTR(n)} \tag{4.13}$$

In order to satisfy the quality required by the customer $AOQL$, and because of the quality deterioration caused by age of the machine and imperfect repair activities, maintenance activities should be performed. To reduce the total cost and recover the defective and returned products, rectifying and remanufacturing machines are added to the model. Therefore, the average quality of outgoing products AOQ can be controlled by the inspection process with the help of these recovering machines. According to Figure 4.1, the AOQ can be formulated by:

$$AOQ = \frac{(1-\gamma)\beta.u_1}{(1-\gamma\beta r_\beta).u_1 + (1-r_{dis}).u_2} \tag{4.14}$$

Here, the AOQ represents the average quality of products that are being sent to the inventory. In our model the inspection policy plays an essential role in the cycle of recovering reducing the cost. The nominal inspection level γ_{nom} which is the minimum level of inspection that keeps the AOQ from going beyond the $AOQL$ is provided by:

$$\gamma_{nom} = \begin{cases} 0 & \text{if } \frac{\beta.u_1}{u_1+(1-r_{dis})u_2} < AOQL \\ \frac{(\beta-AOQL).u_1-AOQL(1-r_{dis}).u_2}{\beta(1-AOQLr_\beta).u_1} & \text{if } \frac{\beta.u_1}{u_1+(1-r_{dis})u_2} > AOQL \end{cases} \tag{4.15}$$

The nominal inspection policy γ_{nom} is a strict and solid strategy to keep the AOQ under the $AOQL$. However, it is more beneficial to adopt a more flexible inspection strategy based on the system's situation to minimize the total costs. By establishing a penalty or an incentive,- as the cost of quality - proportionate to the quality level of products AOQ, and taking the $AOQL$ into account as an index level of quality, we can obtain the optimal solution for the inspection strategy. Considering the quality of output products AOQ in Equation 4.14, the cost of quality is formulated as follows:

$$C_q = \begin{cases} \varepsilon_{q.p}(AOQ) & \text{if } AOQ > AOQL \\ 0 & \text{if } AOQ = AOQL \\ \varepsilon_{q.i}(AOQL - AOQ) & \text{if } AOQ < AOQL \end{cases} \tag{4.16}$$

where($\varepsilon_{q.p}$) is the penalty cost coefficient imposed by the customer for receiving products with bad quality and ($\varepsilon_{q.i}$) is a financial incentive coefficient for production of parts with a quality better than $AOQL$. C_q represents the quality cost that can be positive or negative for poor quality or high quality of products, respectively.

To fulfill the market demand over an infinite horizon of workspace, there is an essential technical condition that should be validated over this horizon. Thus, the hybrid system will be able to satisfy the demand if the feasibility condition of the system given by the following expression is verified:

$$\begin{aligned}(\pi_1 + \pi_2)(1 - \gamma\beta r_\beta)u^1_{max} \ + \ (\pi_1 + \pi_3 + \pi_5 + \pi_7 + \pi_9)(1 - r_{dis})\Big(\min \big\{u^2_{max} \ , \\ (r_M.d \ + \ (\pi_1 + \pi_2)(\gamma\beta r_\beta)u^1_{max})\big\}\Big) \ \geq d\end{aligned} \tag{4.17}$$

where π_i , $i = 1, ..., 10$ is the limiting probability of the hybrid system that can be calculated by the following expressions:

$$\begin{cases} \pi(.) * Q(.) = 0 \\ \sum_{i=1}^{m} \pi_i = 1 \qquad i : modes\ of\ the\ machine \end{cases} \tag{4.18}$$

where $\pi(.) = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10})$ and $Q(.)$ is the corresponding $10/times10$ transition matrix of different modes of the hybrid system.

The production rate of remanufacturing machine is limited by u^2_{max} and depends on the level of recovery stockroom x_2. When the stock level in x_2 is positive ($x_2 > 0$), the remanufacturing machine can produce at maximum rate u^2_{max}, and therefore, Equation 4.17 takes the following form:

$$(\pi_1 + \pi_2)(1 - \gamma\beta r_\beta)u^1_{max} \ + \ (\pi_1 + \pi_3 + \pi_5 + \pi_7 + \pi_9)(1 - r_{dis})u^2_{max} \ \ \geq d \tag{4.19}$$

Given the fact that the input rate of the material into the recovery stockroom $\left((1-\gamma\beta r_\beta)u^1_{max} + r_M.d\right)$ is always less than u^2_{max}, if the level of stock in the recovery equals zero ($x_2 = 0$), the remanufacturing machine is not able to produce at a maximum rate and Equation 4.17 can be written as :

$$\begin{aligned}(\pi_1+\pi_2)(1-\gamma\beta r_\beta)u^1_{max} \ + \ (\pi_1+\pi_3+\pi_5+\pi_7+\pi_9)\times \\ (1-r_{dis})(r_M.d \ + \ (\pi_1+\pi_2)(\gamma\beta r_\beta)u^1_{max}) \ \geq d\end{aligned} \tag{4.20}$$

The transition rates $Q(.)$ are given by:

$$Q = \left\| \begin{matrix} q_{11} & q_{67} & 0 & 0 & \omega_3 & 0 & 0 & 0 & q_{15} & 0 \\ q_{76} & q_{22} & 0 & 0 & 0 & \omega_3 & 0 & 0 & 0 & q_{15} \\ q_{21} & 0 & q_{33} & q_{67} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & q_{21} & q_{76} & q_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\ q_{31} & 0 & 0 & 0 & q_{55} & q_{67} & 0 & 0 & 0 & 0 \\ 0 & q_{31} & 0 & 0 & q_{76} & q_{66} & 0 & 0 & 0 & 0 \\ q_{41} & 0 & 0 & 0 & 0 & 0 & q_{77} & q_{67} & 0 & 0 \\ 0 & q_{41} & 0 & 0 & 0 & 0 & q_{76} & q_{88} & 0 & 0 \\ 0 & 0 & q_{52}.(1-\omega_4) & 0 & 0 & 0 & q_{54}.\omega_4 & 0 & q_{99} & q_{67} \\ 0 & 0 & 0 & q_{52}.(1-\omega_4) & 0 & 0 & 0 & q_{54}.\omega_4 & q_{76} & q_{1010} \end{matrix} \right\| \tag{4.21}$$

where ω_3 is the control variable for PM planning and ω_4 is the control variable for the replacement strategy.

The control policy of the hybrid system is characterized by five decision variables including: manufacturing production rate $u_1(.)$, remanufacturing production rate $u_2(.)$, preventive maintenance and replacement policies $(\omega_3(.), \omega_4(.))$, and inspection rate $\gamma(.)$. By considering these decision control variables $(u_1(.), u_2(.), \omega_3(.), \omega_4(.), \gamma(.))$, the stochastic process $\zeta(t)$, and the state constraint, we can provide a set of feasible control policies $\Gamma(.)$ as follows:

$$
\Gamma(.) = \begin{cases} \{u_1(.), u_2(.), \omega_3(.), \omega_4(.), \gamma(.)\} \in \Re^3 \\ 0 \le u_1 \le u_1^{max}\ ,\ 0 \le u_2 \le L_{u_2} \qquad L_{u_2} = \begin{cases} u^2_{max} & if\, x_2 > 0 \\ (1-\gamma\beta r_\beta)u^1_{max}\ +\ r_M.d & if\, x_2 = 0 \end{cases} \\ \omega_3^{min} \le \omega_3 \le \omega_3^{max}\ , \omega_4^{min} \le \omega_4 \le \omega_4^{max} \\ 0 \le \gamma(.) \le 1 \end{cases} \tag{4.22}
$$

Our objective is to determine the optimal control policy $(u_1^*, u_2^*, \omega_{3,4}^{*\,*}, \gamma^*)$ in the feasible control domain $\Gamma(.)$ in Equation 4.22, that is able to minimize the total incurred costs at any initial state condition (α, x_1, x_2, a, n). However, because of the deterioration effects on the quality of products and availability of the system and considering the history of the repair as one of the effective elements in the system, it is almost impossible to find the optimal solution computationally for this semi-Markov model.

To solve this semi-Markov problem, a mathematical method was applied using a numerical approach. In this model, at any given time t, the state of the system can be described by the state variables such as mode of the system $\zeta(t)$, inventory stock level x_1, recovery stock level x_2, age of the manufacturing machine a, and its number of failures n. The instantaneous cost function $G(.)$ of the system is given by:

$$
G(\zeta, x_1, x_2, a, n) = h(x_1, x_2) + c(u_1, u_2) + w(\zeta, n) + z(\text{AOQ}) \tag{4.23}
$$

where $h(x_1, x_2)$ is the inventory cost, $c(u_1, u_2)$ is the manufacturing cost, and (ζ, n) is the maintenance costs given by:

$$
\begin{cases}
h(.) = c_1^+ x_1^+ + c_1^- x_1^- + c_2^+ x_2^+ & \begin{cases} x_1^+ = max(0, x_1) \\ x_1^- = max(0, -x_1) \\ x_2^+ = max(0, x_1) \end{cases} \\
c(.) = c_m u_1 + c_{rm} u_2 + c_{rec} \gamma \beta (1 - r_\beta) u_1 + c_{ins} \gamma u_1 + c_q d + c_{dis} u_2 \\
w(.) = \begin{cases} c_{over}(Ind\{\zeta(t) = 7or8\}) + c_{rep}^1(Ind\{\zeta(t) = 3or4\}) + \\ c_{pm}(Ind\{\zeta(t) = 5or6\}) + c_{rep}^2(Ind\{\zeta(t) = 2or4or6or8or10\}) \end{cases} \\
z(\text{AOQ}) = C_q \\
Ind(f(.)) = \begin{cases} 1 & \text{if f(.) is true} \\ 0 & otherwise \end{cases}
\end{cases}
\tag{4.24}
$$

The expected discount cost $J(.)$ in the model is given by:

$$
\begin{aligned}
&J(\alpha, x_1, x_2, a, n, u_1, u_2, \gamma, \omega_3, \omega_4) = \\
&E\left\{ \int_0^\infty e^{-\rho t} G(.)\, dt \;\middle|\; \zeta(0) = \alpha,\ x_1(0) = x_1,\ x_2(0) = x_2,\ a(0) = a,\ n(0) = n \right\}
\end{aligned}
\tag{4.25}
$$

where ρ denotes the discount rate of the incurred cost and $E\{\bullet|\alpha, x_0^1, x_0^2, a_0, n_0\}$ specifies condition-based expectation operator. We can determine the optimal control policy by calculating the value function $v(.)$ which is given by:

$$
v(\alpha, x_1, x_2, a, n) = \inf_{(u(.), \omega_3, \omega_4, \gamma \in \Gamma(.))} J(\alpha, x_1, x_2, a, n, u, \omega_3, \omega_4, \gamma), \forall \alpha \in \Omega, x_1, x_2 \in R^2, n \in N
\tag{4.26}
$$

The value function $\nu(.)$ is the minimum value of the total incurred costs considering the feasibility condition of the problem. Also, the $\nu(.)$ function can be considered as a cost-to-go problem and hence, the optimal function can be defined in two parts; the occurred cost in the interval $[0, t]$, and the cost in the interval $[t, \infty)$. Moreover, by considering the value function ν as a continuously differentiable function, we can obtain the HJB equation through which the optimality conditions for the stochastic control problem can be extracted. The HJB equation of our hybrid model is given by the following expressions:

$$\rho\nu(\alpha(.), x_1(.), x_2(.), n(.), t) = \inf_{(u(.),\omega_3,\omega_4,\gamma)} \left\{ \begin{array}{l} g[\alpha(t), x(t), a(t), n(t), u(t), \omega(t)]\delta t \\ + \dfrac{\partial \nu}{\partial x_1}[\alpha(t), x(t), a(t), n(t), u(t), \omega(t)]\delta x(t) \\ + \dfrac{\partial \nu}{\partial x_2}[\alpha(t), x(t), a(t), n(t), u(t), \omega(t)]\delta x(t) \\ + \dfrac{\partial \nu}{\partial t}[\alpha(t), x(t), a(t), n(t), u(t), \omega(t)]\delta t \\ + \sum_{\acute{\alpha}} \nu[\acute{\alpha}, x(t), a(t), n(t), t] q_{\acute{\alpha}\alpha}\delta(t) \end{array} \right\} + o(\delta t) \tag{4.27}$$

By considering the equation $\dot{x} = \frac{\delta x(t)}{\delta t}$ and $Q(.) = \{q_{\alpha\acute{\alpha}}\}$ the equation 4.27 can be simplified as:

$$\rho\nu(\alpha, x_1, x_2, a, n) = \min_{(u,\omega_3,omega_4,\gamma)\in\Gamma(\alpha)} \left\{ g(.) + \frac{\partial \nu}{\partial x_1}(.)\dot{x}_1 + \frac{\partial \nu}{\partial x_2}(.)\dot{x}_2 + +\frac{\partial \nu}{\partial t}(.)\dot{a} + Q(.)\nu(\alpha, x_1, x_2, \varphi_n(\zeta, n), \varphi_a(\zeta, n)) \right. \tag{4.28}$$

where

$$\begin{cases} \dot{x}_1 = (1 - \gamma\beta r_\beta)u_1 + (1 - r_{dis})u_2 - d \quad , \qquad x_1(0) = x_1^0 \\ \dot{x}_2 = (\gamma.\beta.r_\beta)u_1 + r_M.d - u_2 \quad , \qquad x_2(0) = x_2^0 \\ \dot{a} = K.u_1 \end{cases} \tag{4.29}$$

Also, $\varphi_n(\zeta, n)$ and , $\varphi_a(\zeta, a)$ are the reset functions for the number of failures and age of the machine at any jump times τ and are given by following expressions:

$$\varphi_n(\zeta, n) = \begin{cases} n+1 & \text{if} \quad \{\zeta(\tau^+) = 3, \zeta(\tau^-) = 9\} \text{ or } \{\zeta(\tau^+) = 4, \zeta(\tau^-) = 10\} \\ 0 & \text{if} \quad \{\zeta(\tau^+) = 1, \zeta(\tau^-) = 7\} \text{ or } \{\zeta(\tau^+) = 2, \zeta(\tau^-) = 8\} \\ n & \textit{otherwise} \end{cases} \tag{4.30}$$

$$\varphi_a(\zeta, a) = \begin{cases} 0 & \text{if} \quad \{\zeta(\tau^+) = 1, \zeta(\tau^-) = 7\} \text{ or } \{\zeta(\tau^+) = 2, \zeta(\tau^-) = 8\} \\ \alpha_n & \text{if} \quad \{\zeta(\tau^+) = 1, \zeta(\tau^-) = 3\} \text{ or } \{\zeta(\tau^+) = 2, \zeta(\tau^-) = 4\} \\ \alpha_{pm}^n & \text{if} \quad \{\zeta(\tau^+) = 1, \zeta(\tau^-) = 5\} \text{ or } \{\zeta(\tau^+) = 2, \zeta(\tau^-) = 6\} \\ a & \textit{otherwise} \end{cases} \tag{4.31}$$

where

$$\begin{cases} \alpha_n = \phi_a.a(t_n - t_{n-1}) + \alpha_{n-1} & \alpha_0 = 0 \\ \alpha_{pm}^n = \phi_{pm}\big(a(t_{pm} - t_n) + \alpha_n\big) + \alpha_n \end{cases} \tag{4.32}$$

In the set of the equations in 4.28 known as Hamilton-Jacobi-Bellman equation (HJB), $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial t}$ are first-order partial derivatives of the value function $v(.)$ and as long as that the derivatives and value function exist, the optimal control policy can be obtained. Based on Equations 4.30 and 4.31, after each failure and successive repair, the failure number increases, and after replacement activities, the number of failures will reset to zero. Moreover, after each repair, the age of the manufacturing machine changes according to the repair intensity. Similarly, based on the PM intensity, the age of the machine will be determined after each PM. It is impossible to solve Equation 4.28 analytically. However, by applying Kushner's method -a numerical method in the stochastic context- the optimal solution can be obtained.

4.3.2 Optimal control policy

The goal of this chapter is to find the optimal control policies for unreliable production systems aiming to meet a constant demand. The production system consists of manufacturing M_1, remanufacturing M_2 and rectifying M_3 machines in which machines M_1 and M_2 are subjected to deterioration. The optimal control policies consist of manufacturing and remanufacturing policies $(u_1,\ u_2)$, preventive maintenance and replacement planning $(\omega_3,\ \omega_4)$, and inspection strategy γ, are used to minimize the total costs. To determine the optimal control policy $(u_1, u_2, \omega_3, \omega_4, \gamma)$ in this model, the problem is formulated into a series of HJB equations. Since these equations are analytically impossible to solve, a numerical approach based on Kushner's method can approximate the solution. To solve the problem with this approach, all gradients of the continuous value function ν should be numerically approximated by the discrete function ν_h. Therefore, we can express all partial derivatives of the value function ν by the function ν_h as follows:

$$\frac{\partial \nu}{\partial x_1}(\alpha, x_1, x_2, a, n) = \begin{cases} \frac{1}{h_{x1}}\Big(\nu^h(\alpha, x_1 + h_{x1}, x_2, a, n) - \nu^h(\alpha, x_1, x_2, a, n)\Big) & \text{if } \dot{x}_1 \geq 0 \\ \frac{1}{h_{x2}}\Big(\nu^h(\alpha, x_1, x_2, a, n) - \nu^h(\alpha, x_1 - h_{x1}, x_2, a, n)\Big) & \text{if } \dot{x}_1 < 0 \end{cases} \tag{4.33}$$

$$\frac{\partial \nu}{\partial x_2}(\alpha, x_1, x_2, a, n) = \begin{cases} \frac{1}{h_{x2}}\Big(\nu^h(\alpha, x_1, x_2 + h_{x2}, a, n) - \nu^h(\alpha, x_1, x_2, a, n)\Big) & \text{if } \dot{x}_2 \geq 0 \\ \frac{1}{h_{x2}}\Big(\nu^h(\alpha, x_1, x_2, a, n) - \nu^h(\alpha, x_1, x_2 - h_{x2}, a, n)\Big) & \text{if } \dot{x}_2 < 0 \end{cases} \tag{4.34}$$

$$\frac{\partial \nu}{\partial t}(\alpha, x_1, x_2, a, n) = \frac{1}{h_a}\Big(\nu^h(\alpha, x_1, x_2, a + h_a, n) - \nu^h(\alpha, x_1, x_2, a, n)\Big) \tag{4.35}$$

where h_{x1}, h_{x2} and h_a are intervals for variables x_1, x_2 and a. By applying the derivatives of value function 4.33, 4.34 and 4.34, the HJB equation 4.28 takes the form of:

$$\begin{aligned} \nu^h(\alpha, x_1, x_2, a, n) = \min_{(u_1, u_2, \omega_3, \omega_4, \gamma \in \Gamma(.))} \Bigg[\Big(\rho + |q_{\alpha\alpha}| + \frac{\dot{x}_1}{h_{x1}} + \frac{\dot{x}_2}{h_{x2}} + \frac{\dot{a}}{h_a}\Big)^{-1} \Big(g(.) + \\ \begin{Bmatrix} \nu^h(\alpha, x_1 + h_{x1}, x_2, a, n)\frac{|\dot{x}_1|}{h_{x1}} Ind\{\dot{x}_1 \geq 0\} \\ \nu^h(\alpha, x_1 - h_{x1}, x_2, a, n)\frac{|\dot{x}_1|}{h_{x1}} Ind\{\dot{x}_1 < 0\} \end{Bmatrix} + \begin{Bmatrix} \nu^h(\alpha, x_1, x_2 + h_{x2}, a, n)\frac{|\dot{x}_2|}{h_{x2}} Ind\{\dot{x}_2 \geq 0\} \\ \nu^h(\alpha, x_1, x_2 - h_{x2}, a, n)\frac{|\dot{x}_2|}{h_{x2}} Ind\{\dot{x}_2 < 0\} \end{Bmatrix} \\ + \nu^h(\alpha, x_1, x_2, a + h_a, n)\frac{|\dot{a}|}{h_a} + Q(.)\nu(\alpha, x_1, x_2, \varphi_n(n, \zeta), \varphi_a(a, \zeta))\Big)\Bigg] \end{aligned} \tag{4.36}$$

The optimal control policy can be determined by solving the discrete semi-Markov problem 4.36 within the finite grids of the state of the system and the decision variables. These kinds of semi-Markov problems can be solved by policy improvement or value iteration methods.

4.4 Numerical example

In this section we extend our previous example to the model discussed in this chapter. The discrete semi-Markov model has been solved by the policy improvement method and the discrete value function $v^h(\alpha, x_1, x_2, a, n)$ in Equation 4.36 converges to the continuous value function in Equation 4.28.

The value for each parameter of the hybrid model that is required to solve the problem, is presented in table 4.4. Also, let us define $G^h_{x_1,x_2,a,n}$ as the finite grid for the computational domain of the state variable with $h = (h_{x1}, h_{x2}, h_a)$:

$$\begin{cases} G^h_{x_1,x_2,a,n} = \{(x_1, x_2, a, n) : -10 \leq x_1 \leq 20, 0 \leq x_1 \leq 10, 0 \leq a \leq 150, 1 \leq n \leq 5\} \\ h = (h_{x1}, h_{x2}, h_a) = (1, 1, 1) \end{cases} \tag{4.37}$$

Parameter	u_1^{max}	u_2^{max}	d	ρ	ω_3^{min}	ω_3^{max}
value	1	0.2	0.6	0.05	0.1	0.0001
Parameter	ω_4^{min}	ω_4^{max}	q_{31}	q_{41}	q_{52}	q_{54}
value	0	1	0.3	0.2	0.9	0.9
Parameter	q_{67}	q_{76}	r_M	K	A	N
value	0.05	0.3	0.1	1	150	5
Parameter	K_0	K_1	k_t	θ_f	B_0	B_1
value	0.01	0.04	10^{-6}	0.6	0.01	0.19
Parameter	K_b	θ_b	A_1	A_2	r	R_0
value	18×10^{-7}	1	5	0.1	0.98	0.1
Parameter	R_1	r_2	ϕ_{qnr}	ϕ_a	$\acute{\phi}_r$	ϕ_q
value	0.4	2	1	0	0.01	0
Parameter	ϕ_{pm}	c_1^+	c_1^-	c_2^+	c_{ovr}	c_{pm}
value	0	4	400	1	2000	100
Parameter	c_{rep}	c_m	c_{rm}	c_{rec}	c_{ins}	$c_{q.p}$
value	50	60	40	20	3	10
Parameter	$c_{q.i}$	c_{dis}	c_{rep}^2	θ_n		
value	1	2	100	2		

Table 4.2 Parameters of the numerical example

4.4.1 Production policy

Based on the proposed model, there are several scenarios for the different states of the system, therefore, the optimal production strategies from the manufacturing and remanufacturing machines can be varied from one state to the other. The production of manufacturing machine M_1 only takes place in the modes $\{\alpha = 1, 2\}$ and there is no production in other modes. The optimal production policy for the manufacturing machine $u^*(\alpha, x_1, x_2, a, n)$ can be shown in two modes. The structure of the production policy for machine M_1 presented in Figure 4.4

indicates the production rate of the manufacturing machine where the remanufacturing machine is available. The production policy is specified for each stock level x_1 at each age a and the number of failures n and for a given recovery stock level x_2. Figure4.5 illustrates the production rate for manufacturing machines at the mode ($\alpha = 2$) in which the remanufacturing machine is unavailable. The production rate for each stock level x_1 at each age a and the number of failures n of the machine can be extracted from the figure. As can be seen in these two figures the space plan (a, x_1) is divided into three zone by the production rates ($u_1 = \{u_1^{max}, d, 0\}$).

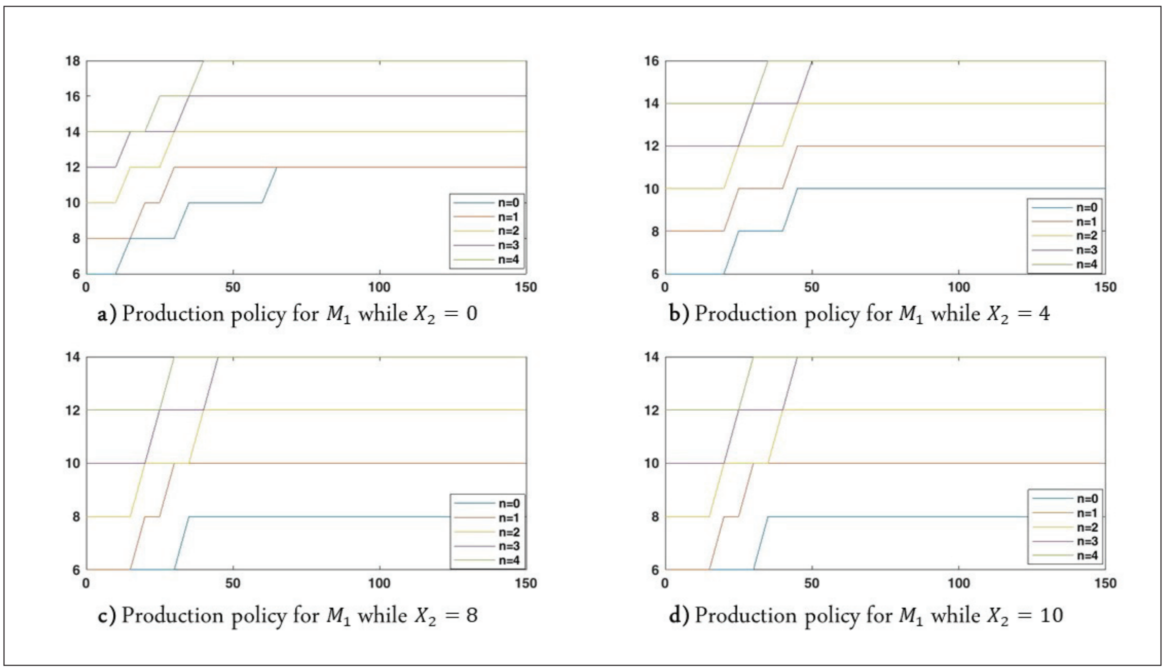


Figure 4.4 Production policy for M_1 while M_2 is available

The production policy for each mode is an extension of the production threshold developed for the hybrid reverse logistics system subjected to deterioration. The threshold levels of machine M_1 are denoted by $Z_1^n(x_2, a)$ and determine the optimal production rate in any state variables of the system. As can be noticed from Figures 4.4 and 4.5, as the machine deteriorates either by age a or by the number of failures n, the thresholds level increases. This is an important feature of the production threshold and is designed to ensure meeting the required demand. As the machine deteriorates by age, its risk of failure increases. Moreover, the increasing number of failures increases the mean time to repair after each failure, therefore, the threshold should be

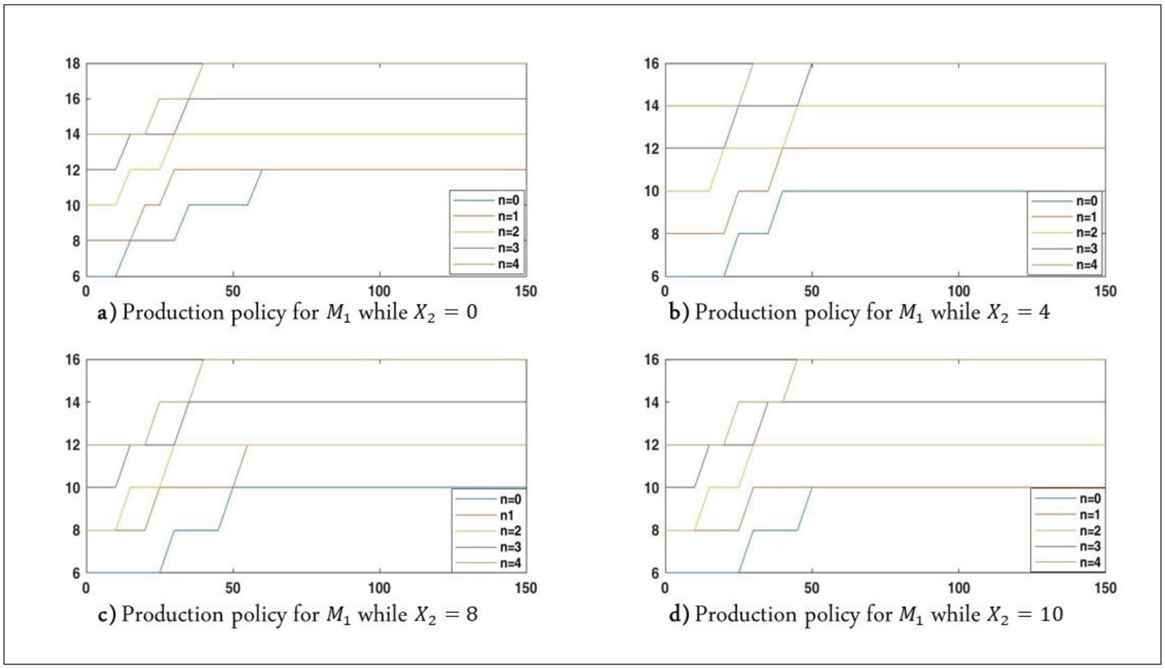


Figure 4.5 Production policy for M_1 while M_2 is unavailable

set at a higher level. It can be noticed in figure 4.4, that the level of recovery stock x_2 affects the production threshold of the manufacturing machine. By increasing the recovery stock level, we can ensure that a part of the demand can be satisfied by the remanufacturing machine for a longer time by maximum capacity. Therefore, the production threshold can be set to a lower level to reduce the total cost and protect the manufacturing machine from further deterioration. For example at the point $(n, a) = (4, 50)$, once the recovery stock level is zero $x_2 = 0$(Figure 4.4a), the production threshold should be set to $Z_1 = 18$, and once the recovery stock level $x_2 = 8$ (Figure 4.4c), the production threshold should be set to $Z_1 = 14$.

The availability of machine M_2 also affects the production policy. By comparing Figures 4.4 and 4.5 we can observe that when the remanufacturing machine is unavailable, the production threshold should be set at a higher level, especially there are enough products in the recovery stock room because the remanufacturing machine can work in its maximum rate. In other words, it is essential to keep more products in the stock x_1 to fulfill the demand when the remanufacturing machine is unavailable.

The production threshold in the remanufacturing machine is presented in Figure 4.6. As can be seen, the production threshold of the remanufacturing machine depends strongly on the recovery stock level x_2. Since one of the main purposes of the remanufacturing machine is to supply the demand once the manufacturing machine is unavailable, when the recovery stock is at the minimum level, it is recommended to operate the remanufacturing machine to a level where the risk of shortage is relatively high. For instance, it can be noticed in Figure 4.6*a*, that once $x_2 = 2$, it is suggested to operate machine M_2 until the stock level reaches the earlier level $x_1 = 2$ and after that, it is better to stop remanufacturing and start building up the recovery stock to protect the system when machine M_1 is unavailable. By increasing the level of x_2, Figure 4.6*b* indicates that once $x_2 = 10$, the remanufacturing machine can operate until higher level of inventory $x_1 = 12$ is reached. Also, as the number of failures goes up, this pattern is valid and is repeated in a higher level of inventory. Figure 4.6*c* and 4.6*d*.

According to the results, the optimal production strategies for machines M_1 and M_2 in the mode α can be defined as follows:

For the manufacturing machine M_1 is given by:

$$u_1^*(\alpha, x_1, x_2, n) = \begin{cases} u_1^{max} & \text{if } \begin{cases} \alpha = 1\ ,\ x_1(t) < Z^1_{\alpha=1,n}(x_2) \\ \alpha\ =\ 2,\ \ x_1(t)\ <\ Z^1_{\alpha=2}(x_2) \end{cases} \\ d & \text{if } \begin{cases} \alpha = 1\ ,\ x_1(t) = Z^1_{\alpha=1,n}(x_2) \\ \alpha\ =\ 2,\ \ x_1(t)\ =\ Z^1_{\alpha=2}(x_2) \end{cases} \\ 0 & \text{if } \begin{cases} \alpha = 1\ ,\ x_1(t) > Z^1_{\alpha=1,n}(x_2) \\ \alpha\ =\ 2,\ \ x_1(t)\ >\ Z^1_{\alpha=2}(x_2) \end{cases} \end{cases} \tag{4.38}$$

and for the remanufacturing machine M_2 is given by :

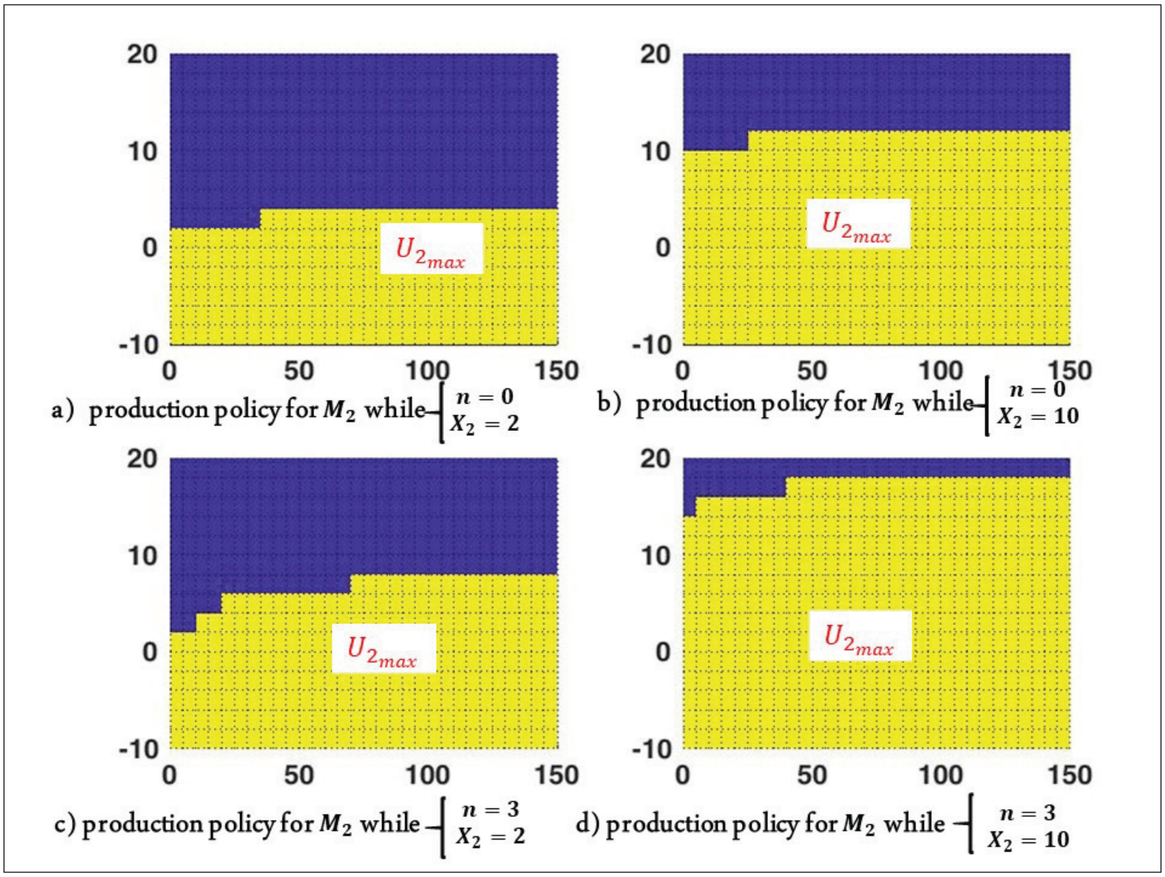


Figure 4.6 Production policy for M_2 while M_1 is available

$$u_2^*(\alpha, x_1, x_2, n) = \begin{cases} u_2^{max} & \text{if } \begin{cases} \alpha = 1 \ , \ x_2 > 0 \ , \ x_1(t) < Z^2_{\alpha=1,n}(x_2) \\ \alpha = \{3,5,7,9\} \ , \ x_2 > 0 \ , \ x_1(t) < Z^2_{\alpha}(x_2) \end{cases} \\ \gamma\beta r_\beta + r_M d & \text{if } \{\alpha = 1 \ , \ x_2 = 0 \ , \ x_1(t) < Z^2_{\alpha=1,n}(x_2)\} \\ r_M d & \text{if } \{\alpha = \{3,5,7,9\} \ , \ x_2 = 0 \ , \ x_1(t) < Z^2_{\alpha}(x_2)\} \\ 0 & \text{if } \begin{cases} \alpha = 1 \ , \ x_1(t) > Z^2_{\alpha=1,n}(x_2) \\ \alpha = \{3,5,7,9\} \ , \ x_1(t) > Z^2_{\alpha}(x_2) \end{cases} \end{cases} \tag{4.39}$$

4.4.2 Inspection policy

Nominal inspection policy (γ_{nom}) can be used as a primary model to be compared with other optimal policies. The nominal inspection policy is the minimum amount of inspection for which the average outgoing quality of products AOQ, can be kept less than $AOQL$. The nominal inspection policy γ_{nom} is a helpful tool to interpret other strategies and is provided by the following expression:

$$\gamma_{nom} = \begin{cases} 0 & \text{if } \frac{\beta.u_1}{u_1+(1-r_{dis})u_2} < AOQL \\ \frac{(\beta-AOQL).u_1-AOQL(1-r_{dis}).u_2}{\beta(1-AOQLr_\beta).u_1} & \text{if } \frac{\beta.u_1}{u_1+(1-r_{dis})u_2} > AOQL \end{cases} \tag{4.40}$$

where the inspection rate less than the nominal inspection level results in the production of parts with quality less than $AOQL$ while the inspection rate higher than the nominal inspection level γ_{nom} leads to the production of parts with higher quality than the specified limit $AOQL$. In Figure 4.7 the optimal inspection policy is represented for the case where the machine M_1 is in its 4th number of failures and the recovery stockroom x_2 is full. As can be observed from Figure 4.7b, unlike the nominal inspection policy γ_{nom}, the real inspection policy depends on the stock level x_1. As the age of the machine increases, the rate of defective parts increases as well, therefore, in order to reduce the defective rate regarding the specified quality limit $AOQL$, the inspection rate should be increased as shown in figure 4.7.

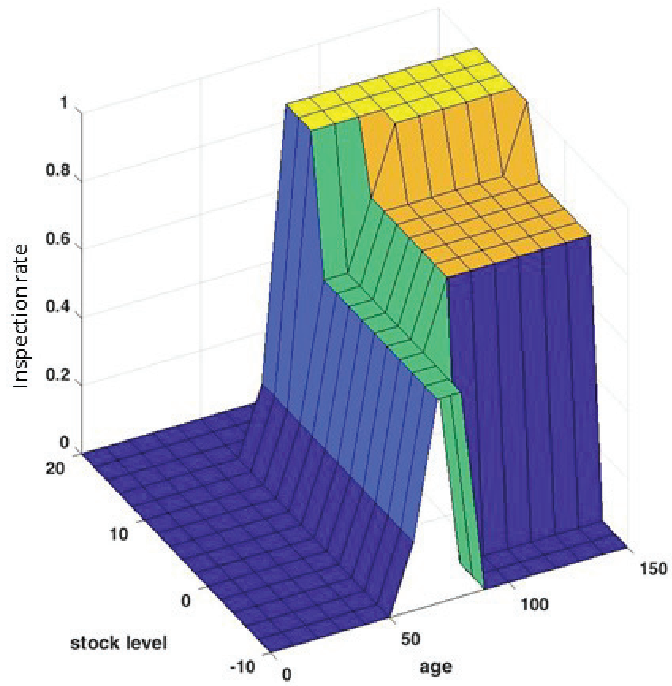
Also, unlike the nominal inspection policy, the actual inspection policy is flexible to the state of the system. Establishing a proportionate financial penalty for low-quality products works as a barrier against the production of parts with a quality lower than $AOQL$. Moreover, allocating a financial incentive for products with high quality encourages the decision-makers to produce high-quality products by increasing the inspection rate. It can also be noticed in Figure 4.7, that when the machine is old and the stock level is negative (backlog state), the optimal inspection level is set to zero, while the nominal inspection is at its standard level. This happens because in this case the failure probability is high and staying in a backlogged state is too expensive, therefore, it is more cost-efficient to stop the inspection process and deliver all the produced products no matter their quality to the customer and pay the penalty. On the other hand, at the same age of the machine, if there are enough products in the inventory stock room to cover the demands after the probable failure of the machine, it is a good opportunity to benefit from the financial incentives considered for high-quality products by performing 100% inspection.

Figure 4.8, presents the inspection policies for all number of failures of the manufacturing machine while the remanufacturing machine is in the operation state. The inspection policy $\gamma(.)$ divides the plan (a, x) into six areas according to the inspection rate $\{0, 0.2, 0.4, 0.6, 0.8, 1\}$. A common feature of all the policies for all number of failures is that there is no inspection when the machine is young. This is due to the nature of the defective rate curve of the products. The defective parts that are produced by the manufacturing machine are increasing by the age of the machine but not with a constant slope (is non-linear). When the machine is young the defective rate curve has a small-slope shape, in other words, at the younger age of the machine M_1 $\dfrac{\partial \beta}{\partial t} \approx 0$. Thus, performing the inspection at earlier ages of the machine will not change the AOQ considerably, and it is cost-efficient to keep it at zero level. As the machine's age is increased, the slope of the defective curve increases faster. This explains why in order to benefit from the financial incentives in place for high-quality parts, the 100% inspection is always performed when the machine is old enough.

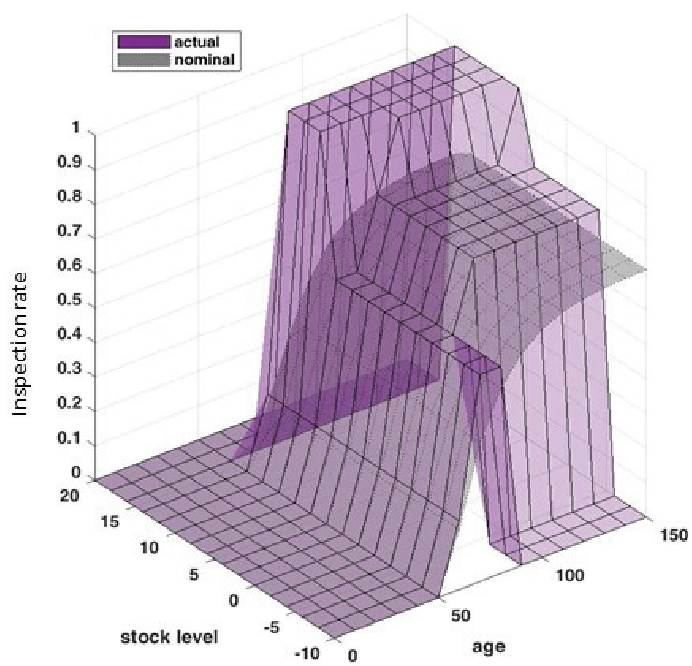
Another interesting point in Figure 4.8 is about the zero-inspection zone when the machine is old and in the backlog state. As can be noticed when the number of failures is low, there is no need to stop performing inspection even when the system is in a backlog state, however, as the number of failures goes up, the zero-inspection zone grows. This happens because of three reasons:

- As the number of failures increases, the machine deteriorates more, therefore, its probable risk of failure increases.
- As the number of failures increases, the mean time to repair the machine after the next failure increases as well, therefore, it is more dangerous to stay any longer in the backlog state.
- As the number of failures increases, the non-rectifiable rate of defective parts goes up. This means that a larger portion of the defective parts should be sent to the remanufacturing machine to be recovered which is more expensive than recovering by the rectifying machine.

Therefore, overall, it can be concluded that as the number of failures goes up, while the machine is old and the system is in a backlog state, the zero-inspection zone increases. In other words, as



a) actual inspection policy



b) nominal inspection policy

Figure 4.7 Actual and nominal inspection policies for $n = 4$

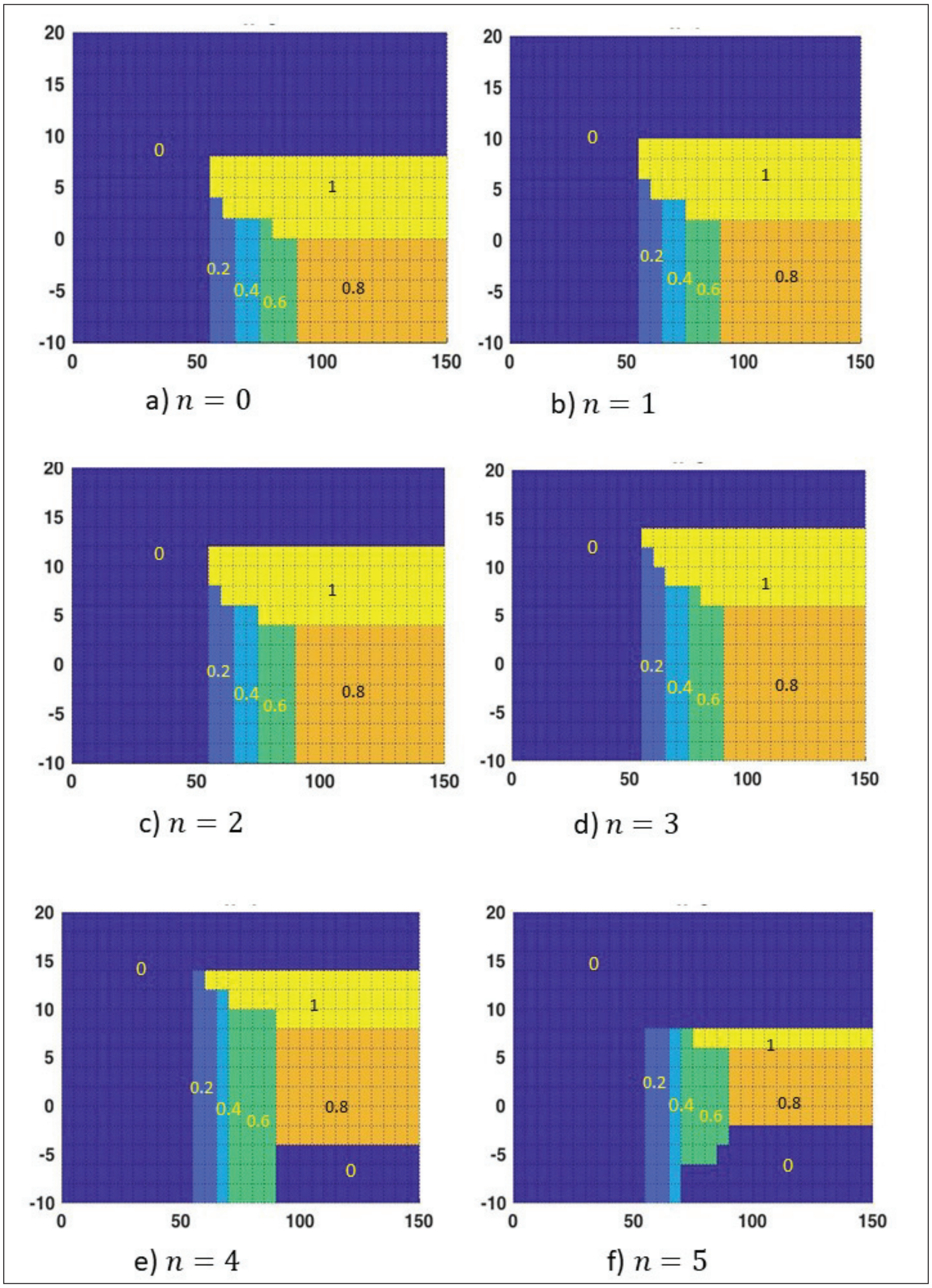


Figure 4.8 Inspection policy for all number of failure when machine M_2 is operating

the number of failures increases, the area for which the inspection rate is less than the nominal inspection policy increases. Obtaining a cost-efficient policy of inspection is complicated as it is interwoven with the state variables of the machine, such as its age a and number of failures n.

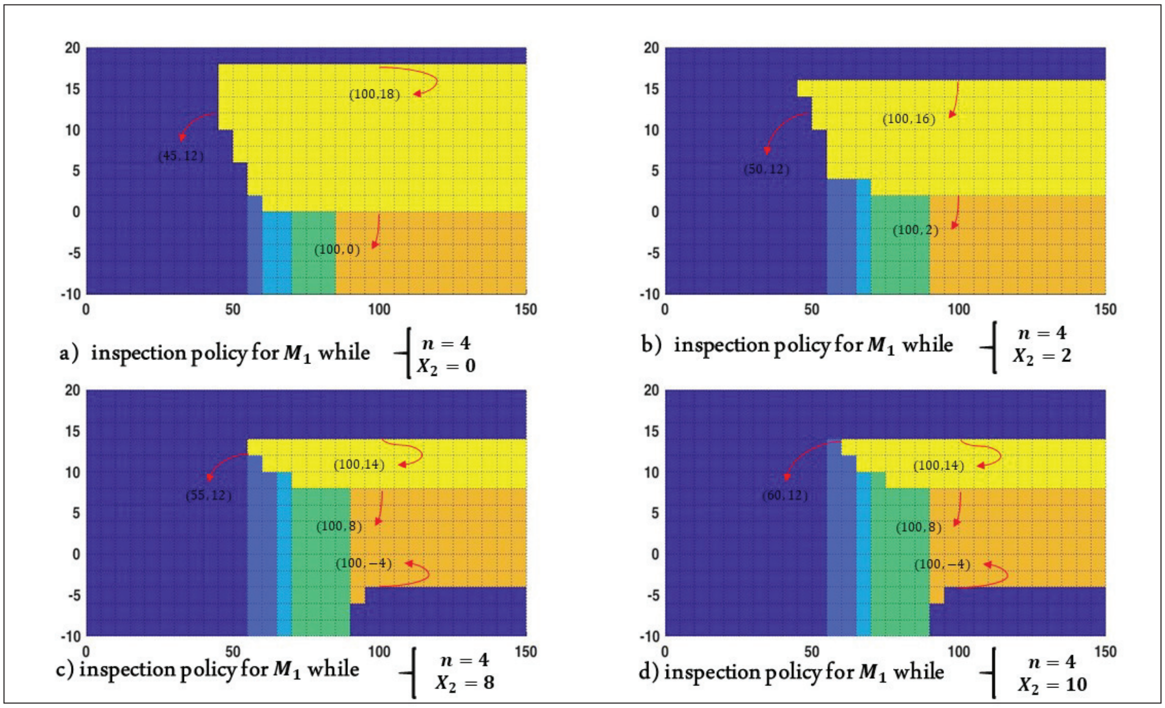


Figure 4.9 Inspection policy for the system while M_2 is in operational state

Figures 4.9 and 4.10 illustrate the effect of the recovery stock level (level of x_2) on the inspection policy in states 1 and 2, respectively. As it is indicated in the figures, by increasing the level of recovery stock room x_2, the intensity of inspection along with the production threshold decrease. One of the additional reasons for performing the inspection is to build up the recovery stock. If there are enough products in the recovery stock room, it is no longer necessary to perform maintenance to accumulate parts for the remanufacturing processes. As the recovery stock level is increased, the intensity of inspection reduces gradually. For instance, according to Figure 4.9, at the point $(n, a) = (4, 100)$, if $x_2 = 0$, the 100% inspection process starts when the inventory level $x_1 = 0$ (Figure 4.9a), and if $x_2 = 10$, the 100% inspection starts when the stock level reaches $x_1 = 8$ (Figure 4.9d). Also, the 100% inspection process starts later with regards to the machine's age. For example, based on Figure 4.9, when the inventory level is $x_1 = 12$, by increasing the recovery stock level from 0 to 10, inspection starts at the age of 45 to 60. Furthermore, by increasing the recovery stock room, there is no longer necessary to perform maintenance while the system is in an intense backlog state and the production machine is old.

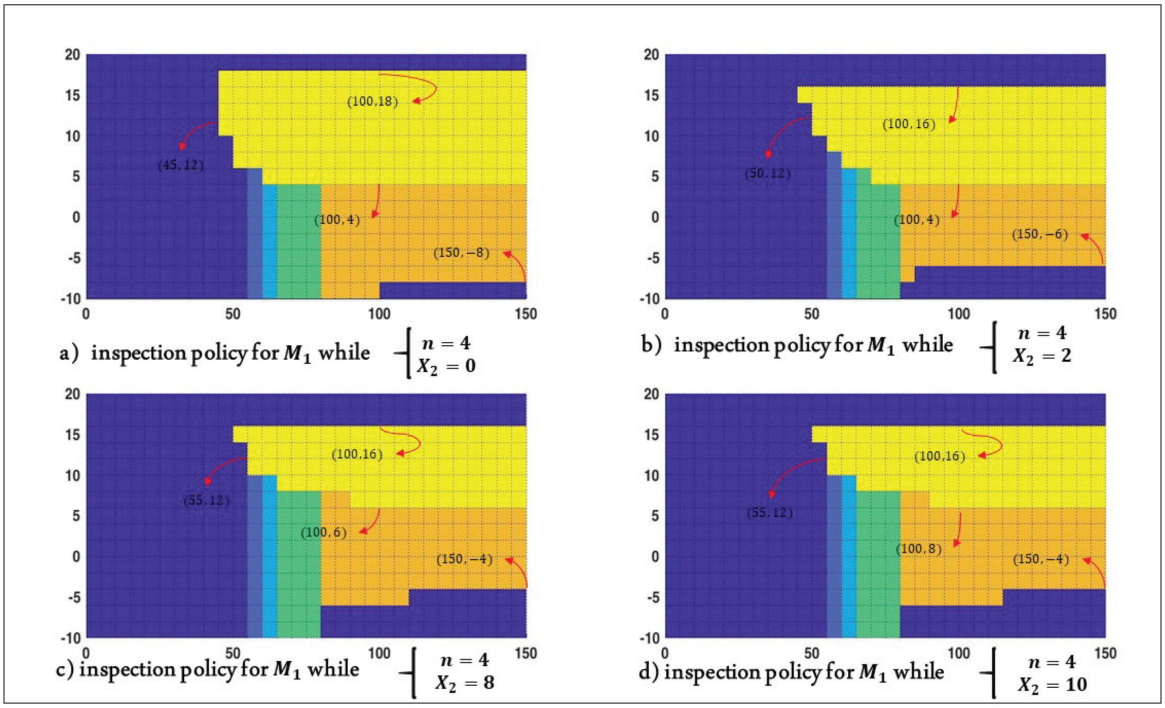


Figure 4.10 Inspection policy of the system while M_2 is in failure state

(Figures 4.9c and d). All of the above explanations also apply to state 2 of the system where the machine M_2 is unavailable. Figure 4.10.

4.4.3 Maintenance policy

Figure 4.11 represents the optimal maintenance policies of the manufacturing machine in mode 1 for every number of failures n of the machine in the plan (x_1, a) at the given recovery stock level x_2. This figure indicates that as the number of failures go up, the replacement zone and and the PM area grow. Also, it can be observed that at each number of failures, once the deterioration of the manufacturing machine reaches a certain level, PM activity should be performed. In addition, if after failure the stock level x_1 is less than a certain amount, replacement activities are required because of the system's incapacity to recover the demand. These maintenance activities help to improve the efficiency of the system and reduce the effects of deterioration. Both of the optimal maintenance policies (replacement and PM) divide the plane (a, n, x_1) at a given level of x_2, into two zones. The PM and replacement activities are assumed to be allowed for all possible values of decision variables ω_3 and ω_4 within $[\omega_3^{min}, \omega_3^{max}]$ and $[\omega_4^{min}, \omega_4^{max}]$,

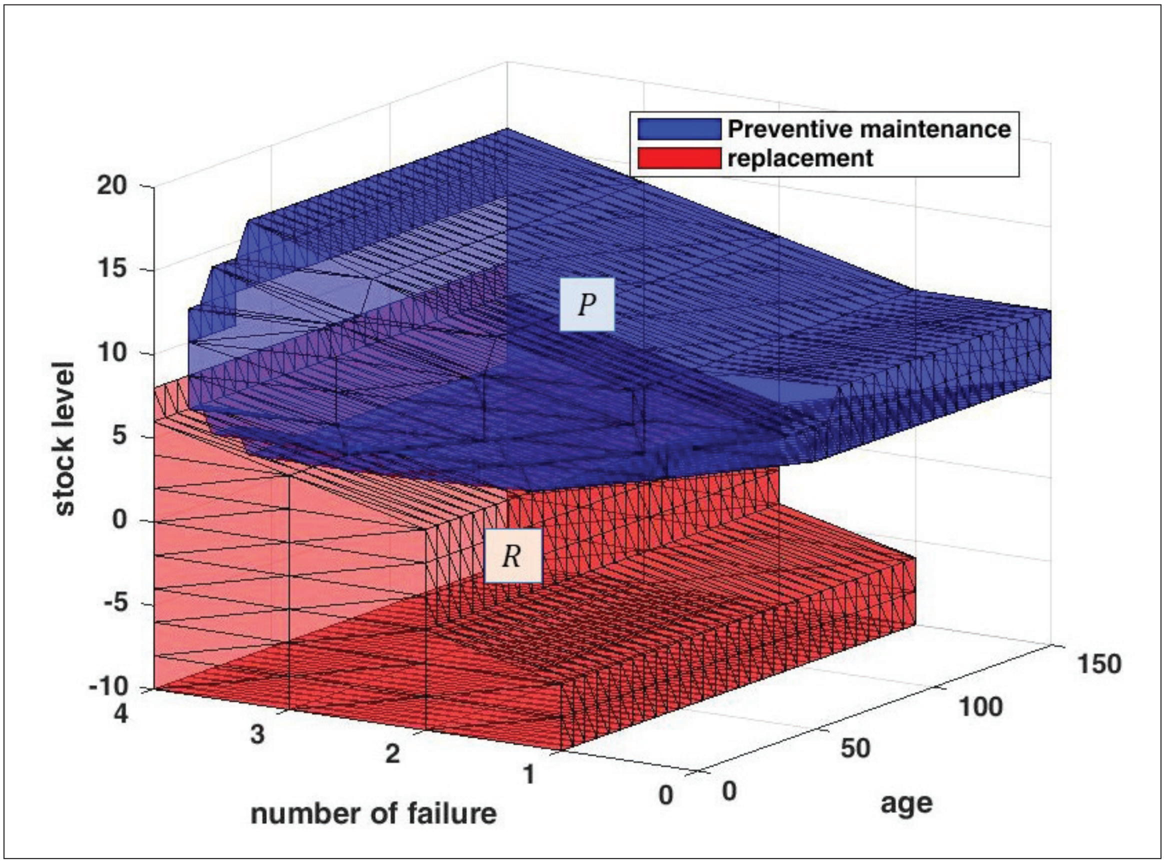


Figure 4.11 PM and replacement policies for all number of failure when machine M_2 is operating

respectively. Generally, at each number of failures of the manufacturing machine, the structure of the maintenance policies alternates between its minimum and maximum values. We can describe these zones as follows:

- Zone P : Here, the deterioration level has reached a point where it is justifiable to perform PM activities. Therefore, the decision variable ω_3 is set to its maximum value.
- Zone $\acute{P}$: In this zone, it is not cost-efficient to perform PM because here the machine is either still new or the stock level is so low that performing PM may cause a backlog of products which would be too expensive.
- Zone R: It is recommended to replace the machine in this zone since it has reached a certain level of deterioration and the stock level x_1 is not enough to support the demand during the

repair process, which is, in turn, increasing by the number of failures. Therefore, the cost of performing replacement can be justified here and the decision variable ω_4 is set to its maximum value.

- Zone $\acute{R}$: In this area, it is not cost-efficient to perform the replacement. One reason is that it is recommended to repair the machine after failure instead of replacing it. Moreover, the manufacturing machine is still new and it is not reasonable to replace it. Even if the machine is old, the stock level x_1 is high and can completely cover the demand during the non-production repair period. Therefore, it is not necessary to pay for an expensive replacement process. The decision variable ω_4 is set to its minimum value and the repair activities are suggested to be performed.

The optimal rate of PM activities ω_3^*, and the optimal rate of replacement ω_4^* at a given recovery stock level x_2 in both operational and failure states of the remanufacturing machine are given by the following equations:

$$\omega_3^{*1}(1, a, x1, x_2, n) = \begin{cases} \omega_3^{max} & \text{if } a(.) \text{ and } x_1(.) \text{ and } n(.) \in zone\mathrm{A}_p \\ \omega_3^{min} & otherwise \end{cases} \tag{4.41}$$

$$\omega_3^{*2}(2, a, x1, x_2, n) = \begin{cases} \omega_3^{max} & \text{if } a(.) \text{ and } x_1(.) \text{ and } n(.) \in zone\mathrm{A}_p \\ \omega_3^{min} & otherwise \end{cases} \tag{4.42}$$

$$\omega_4^{*1}(1, a, x1, x_2, n) = \begin{cases} \omega_4^{max} & \text{if } a(.) \text{ and } x_1(.) \text{ and } n(.) \in zone\mathrm{A}_p \\ \omega_4^{min} & otherwise \end{cases} \tag{4.43}$$

$$\omega_4^{*2}(2, a, x1, x_2, n) = \begin{cases} \omega_4^{max} & \text{if } a(.) \text{ and } x_1(.) \text{ and } n(.) \in zone\mathrm{A}_p \\ \omega_4^{min} & otherwise \end{cases} \tag{4.44}$$

Figure 4.12 shows the joint control policy for maintenance activities including PM and the replacement/repair switching policy for all number of failures at a given level of recovery stock x_2. It can be noticed that as the number of failures increases, preventive maintenance should be performed at an earlier age of the machine because of the effects of deterioration. Also, as the number of failures goes up, the region of ω_3^{max} covers more of the plane (a, x_1) area. Zone R

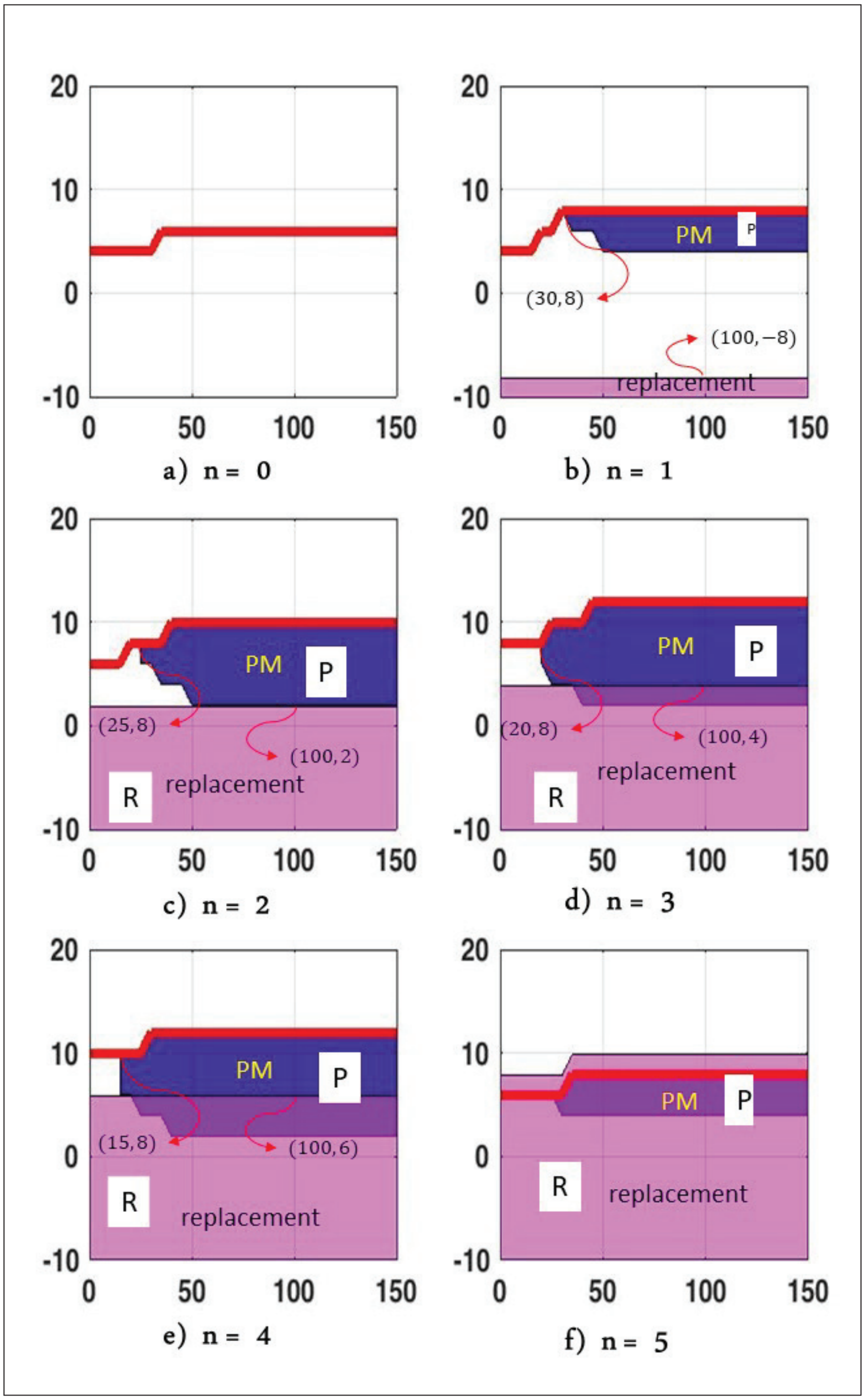


Figure 4.12 PM and replacement policies for all number of failure when machine M_2 is operating

specifies the region in which the replacement variable rate is set to its maximum value ω_4^{max}. Based on the figures, after the failure of the production machine, if the system is in this zone, the machine should be replaced. Also, it can be noticed that the area of ω_4^{max} increases by the number of failures. This happens because after each failure, the mean time to repair the machine increases, and therefore, it is more beneficial to start replacement at a higher level of inventory x_1. For example, in Figure 4.12b to e, it can be noticed that upon the increase of the number of failures from $n = 1ton = 4$, PM activities start from the age of $a = 30toa = 15$. Moreover, replacement are performed from level $x_1 = -8tox_1 = 6$.

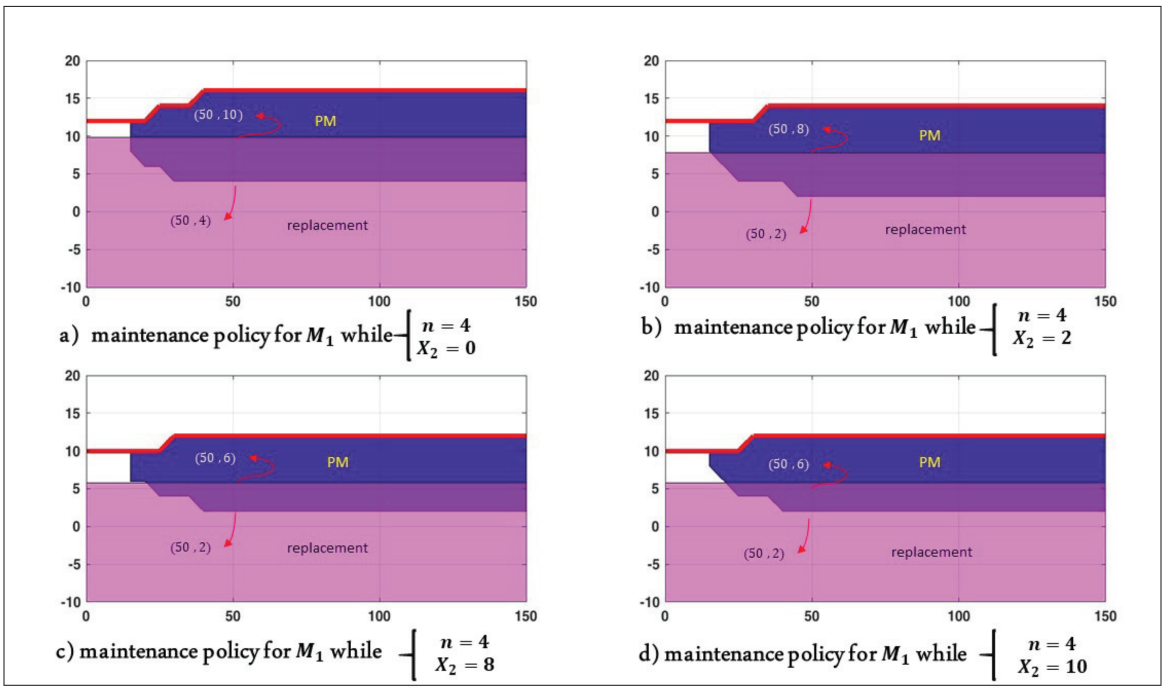


Figure 4.13 Maintenance policy for different recovery stock level x_2 while M_2 is available

In Figures, 4.13 and 4.14, the relationship between maintenance policies and the recovery stock level was investigated for the system at states 1 and 2, respectively. As can be observed in Figure 4.13, by increasing the level of x_2, PM can start from a lower level of inventory x_1. For instance, by increasing the recovery stock level from $x_2 = 0tox_2 = 8$ at the age of $a = 50$, PM activities start from $x_1 = 4ratherthanx_1 = 2$. It means that if there are enough products in the recovery stock room, the system becomes more dependent on the remanufacturing machine to cover the demand, therefore, we can start PM activities earlier. On the other hand, upon increasing the recovery stock level, it is not recommended to carry out the replacement policy, because the

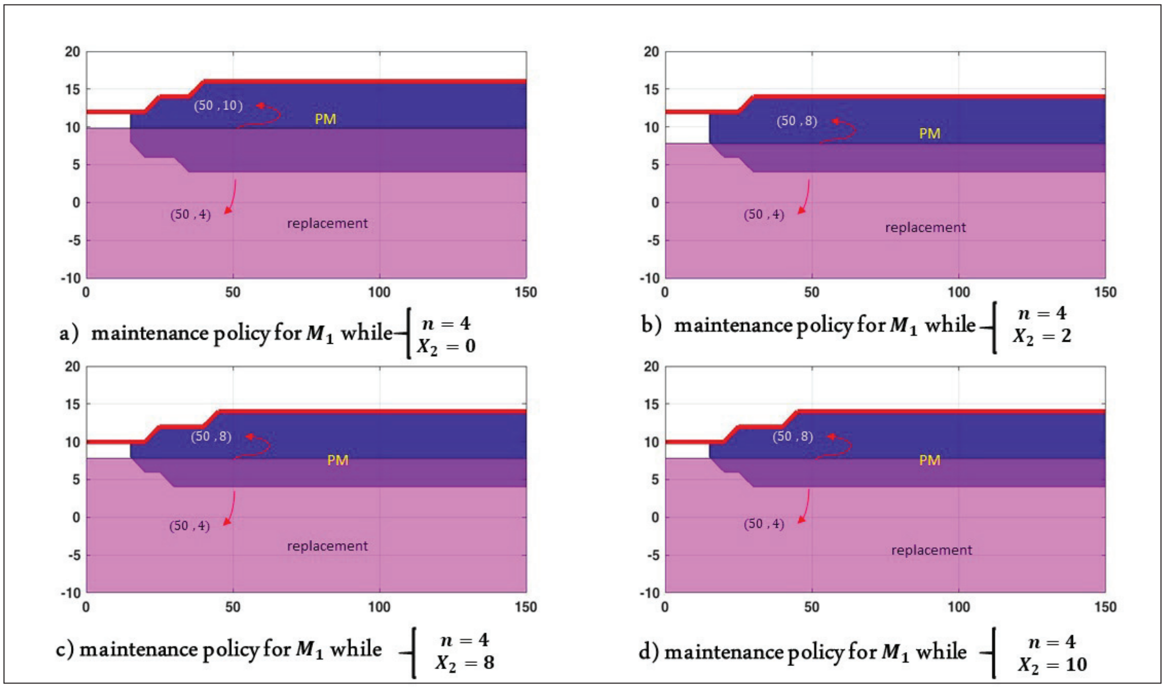


Figure 4.14 Maintenance policies for different recovery stock level x_2 while M_2 is unavailable

remanufacturing machine can cover part of the demand for a longer time. Therefore, if the machine fails at a higher level of inventory, we can repair it. For example, according to Figure 4.13a and 4.13c, by increasing the x_2 from $x_2 = 0$ to $x_2 = 8$ at $(n, a) = (4, 50)$ the maximum required level of inventory for performing replacement decreases from $x_1 = 10$ to $x_1 = 6$. It can also be deducted from Figure 4.14, that the maintenance policies for the production machine at state 2, when the remanufacturing machine is unavailable, are almost independent of the recovery stock level. Only when the recovery level $x_2 = 0$, the replacement activities are performed at a higher level. By increasing the x_2 the maintenance policies remain at the same level given that machine M_2 is unavailable and can not supply the demand upon the failure of the manufacturing machine.

4.5 Sensitivity analysis

The optimal control strategy is related to the system parameters and would change accordingly. To understand how these parameter variations can affect the control policy, a sensitivity analyzes should be performed and compared with the baseline parameter values. First, we focus on the backlog cost. Next, other parameters will be analyzed as well.

4.5.1 Variation of the backlog costs

Variation of the backlog cost c_1^- has a considerable effect on the production threshold. By analyzing the results of the production policy presented in Figure 4.15 for three different backlog values $c_1^- = 200$,$c_1^- = 400$ and $c_1^- = \$600$/missing products/ time unit, it can be concluded that the production threshold increases by increasing the backlog cost. For instance, we can see that at point $(a, n, x_2) = (100, 4, 10)$ of the manufacturing machine the stock level increases from 10 to 14 as the backlog cost increases from 200 to 400. This happens to protect the system from the probability of expensive backlog situations. Therefore, the manufacturing machine should produce more products at its maximum rate. Since deterioration is a function of the production rate, as shown in Figure 4.15, as the backlog cost increases, PM activities should be performed at earlier ages of the machine. Also, when the backlog cost increases from a to b, the starting level of backlog to perform replacement increases from a to b as well.

Backlog costs have also an impact on the inspection policy. In Figure 4.16 it can be noticed that as the backlog cost increases, the areas of 100% inspection that should be performed at higher levels of stock as well as the non-inspection zone increase. Given that by increasing the backlog cost the machine should rapidly produce more products to reach a higher level of stock, it is recommended to start the 100% inspection at a higher level. Furthermore, upon the increase of the backlog cost, staying in a backlog situation costs more, therefore, in order to avoid a severe shortage state, it is recommended not to perform inspection in the larger area. In other words, when the backlog cost has increased and the machine is old, inspection should start at a higher level of stock.

4.5.2 Variation of the remanufacturing rate

The effect of variation of remanufacturing rate u_2 is directly reflected in the stock level of inventory. As it is expected and can be seen in Figure 4.17, by increasing the rate of remanufacturing machines, the optimal recommended production threshold decreases. In Figure 4.17 the optimal production threshold is presented for three different values of remanufacturing rate $0.2, 0.3$ and

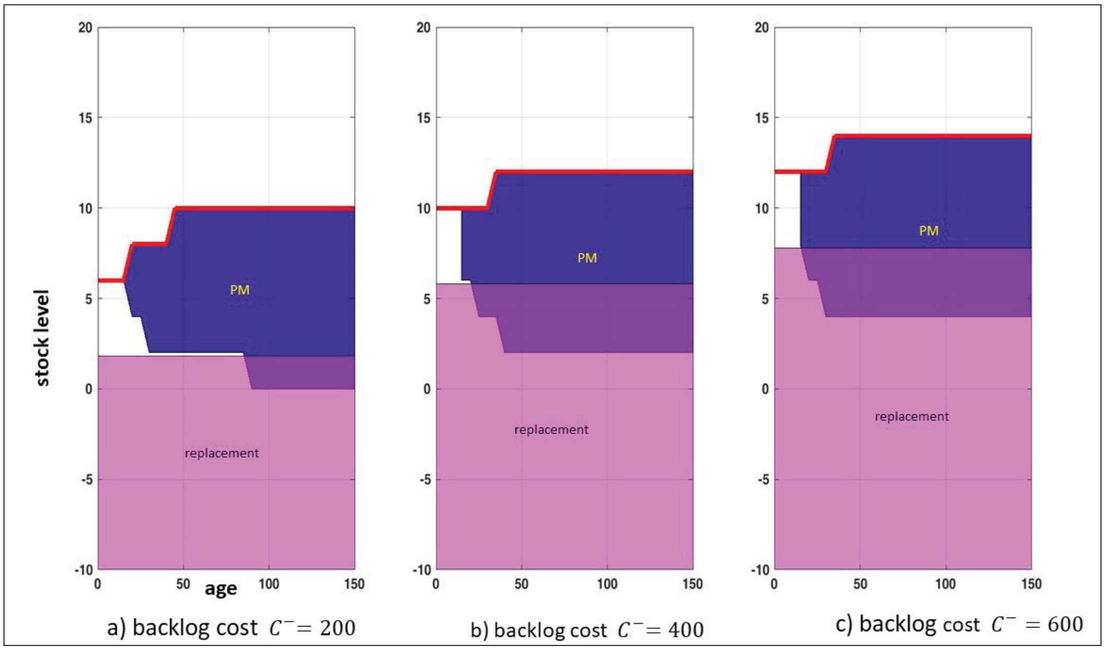


Figure 4.15 Maintenance policies for different backlog costs at $n = 4$

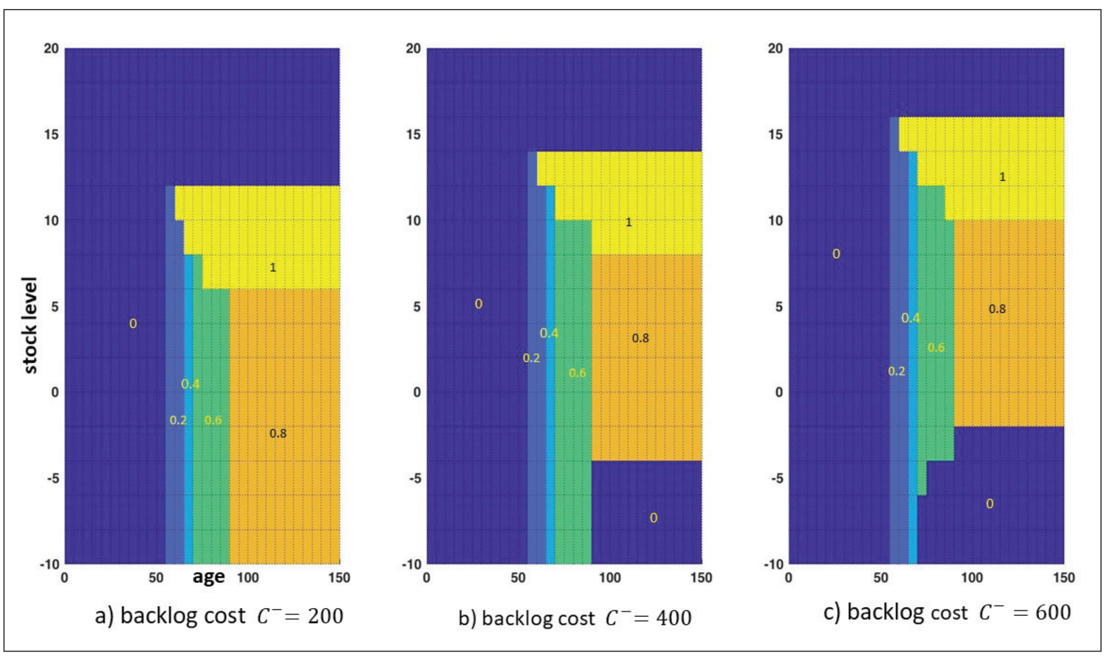


Figure 4.16 Inspection policies for different backlog costs at $n = 4$

0.3 . By increasing the remanufacturing rate u_2 we have a stronger flow of products coming from the recovery stock room which can support the system to meet the demand, we can therefore rely more on the remanufacturing machine to cover the demand. Considering that the failure rate of the remanufacturing machine is constant, it can especially help the system when the manufacturing machine is old and the risk of failure is high. Since the risk of shortage decreases by having a greater flow of remanufacturing products, the production threshold can be set to a lower level to postpone the deterioration of the manufacturing machines. This can be seen in Figure 4.17 where the area of PM activities decreases by increasing the remanufacturing rate. Also, since the remanufacturing machine can work at a higher production rate and cover a bigger part of the demand, performing the replacement procedures can be postponed according to the level of stock. This is also illustrated in Figure 4.17, where the area of replacement decreases by increasing the remanufacturing rate.

Inspection policies also can be affected by the change in the remanufacturing rate. As can be seen in Figure 4.18, by increasing the remanufacturing rate u_2, inspection can be performed at a lower level of the stock. In other words, the inspection zone grows to lower levels of shortage as shown by figure 4.18 where the machine is old.

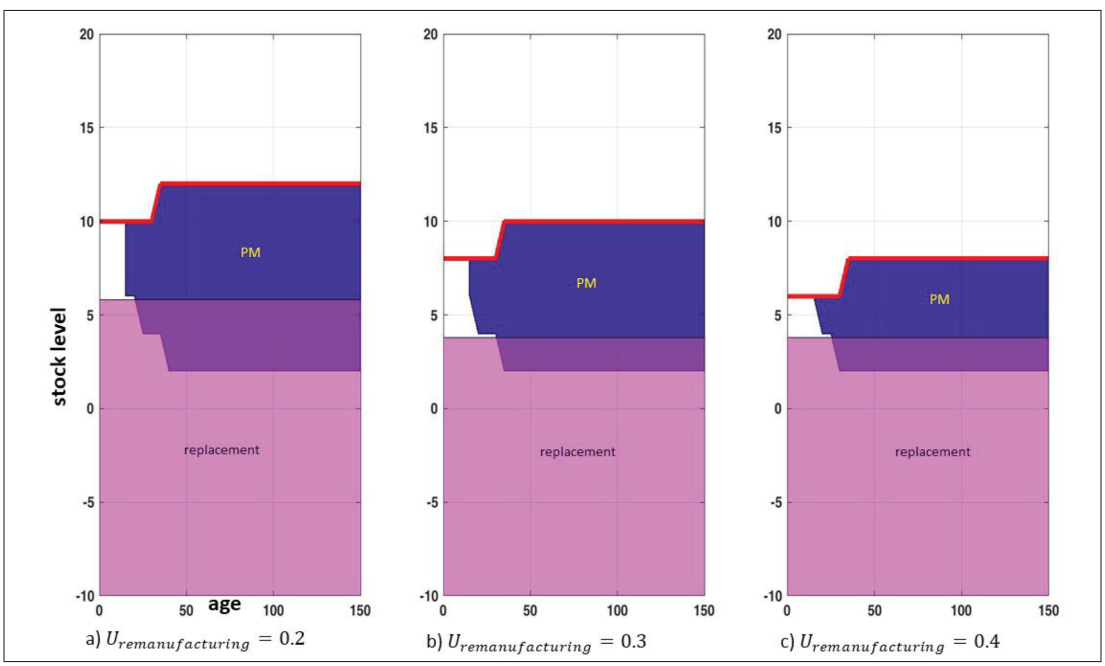


Figure 4.17 Maintenance policies for different remanufacturing rate at $n = 4$

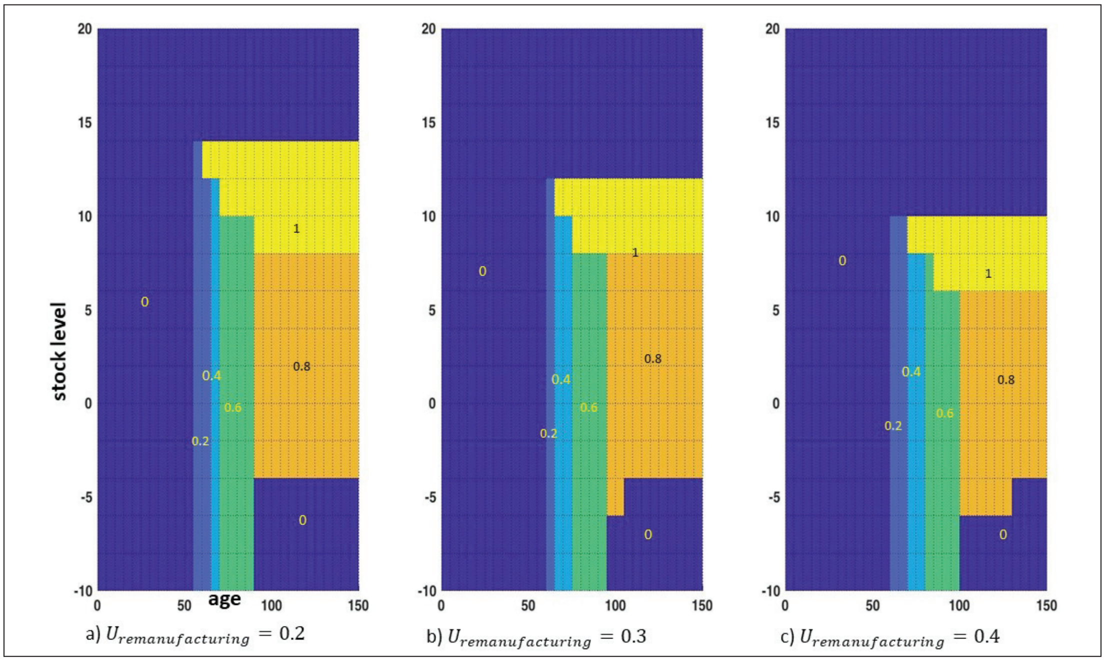


Figure 4.18 Inspection analysis for different remanufacturing rate at $n = 4$

4.5.3 Variation of the inspection costs

It is evident that changing the inspection cost would affect the inspection policy. As can be seen in Figure 4.19, by increasing the inspection cost, the 100% inspection zone decreases dramatically. In fact, setting the inspection rate at a level more than the nominal level and especially at the maximum level (100%) helps to respect the quality limit constraint, improve the quality of the outgoing products, and benefit from the high-quality financial incentives. However, as the inspection cost increases, it is no longer justifiable to perform inspection more than the nominal level. Also, Figure 4.19 indicates that by decreasing the inspection cost, it can be started at earlier ages of the machine, even when the defective rate of the machine is lower than the specified limit for the quality. It is also suggested to perform 100% inspection to gain benefit from the financial incentives established for high-quality products.

4.5.4 Variation of AOQL

The average outgoing quality limit is one of the most important parameters for determining the policies. Following the analysis, it can be noted that the value of $AOQL$ influences the inspection and maintenance policies. Figure 4.20,clearly illustrates the shifts in inspection

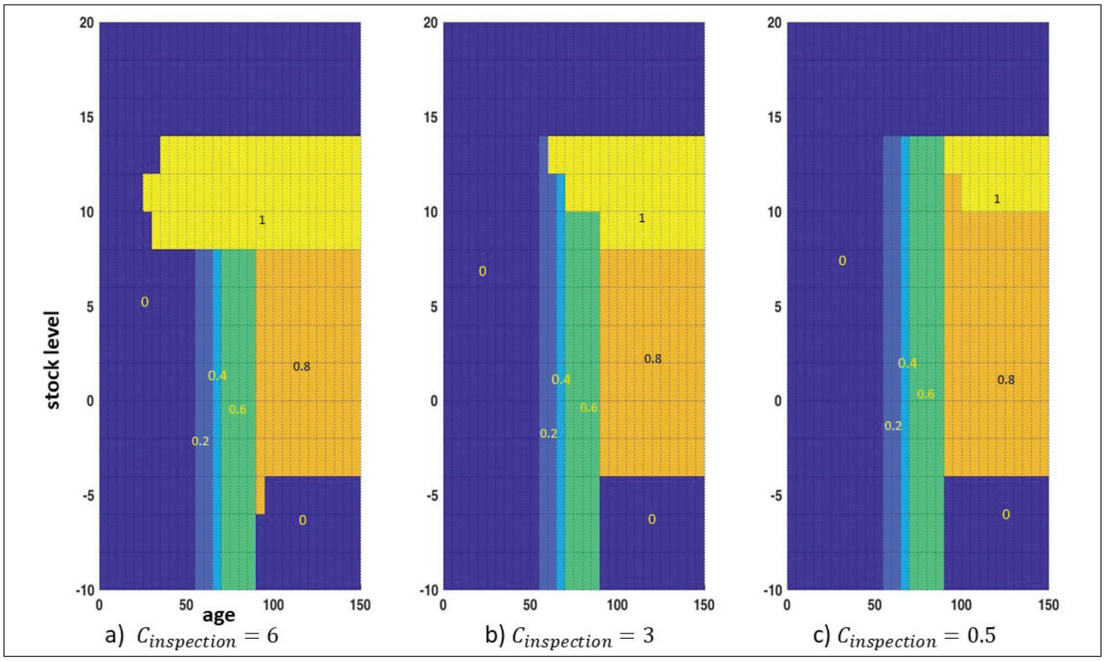


Figure 4.19 Inspection policies for different inspection costs at $n = 4$

planning for three different cases with values $AOQL = 0.1, 0.05, 0.02$. When AOQL decreases, it is necessary to start the inspection and recovering processes earlier to satisfy the required product quality. Also, it can be concluded that the inspection zone increases by decreasing the AOQL. However, when the AOQL is decreased, the zero inspection zone policy -which occurs when the machine is old and deteriorated and the system is in a serious backlog situation- increases. Considering the importance of quality and the specified penalty for low-quality products, it is expected that the zero inspection zone should decrease by decreasing the AOQL, and inspection should be performed even in the worst situation. This can be explained by the fact in the worst situation when the AOQL is decreased, inspection should be performed at a higher rate to satisfy the quality constraint. However, this increases the risk of failure at the backlog state, therefore, it is suggested not to perform the inspection at a lower value of AOQL (when higher quality is required by the costumer). Conversely, as the value of AOQL goes up, since a lower rate of inspection is required to satisfy the quality demand, it is suggested to perform inspection in the backlog state of the machine.

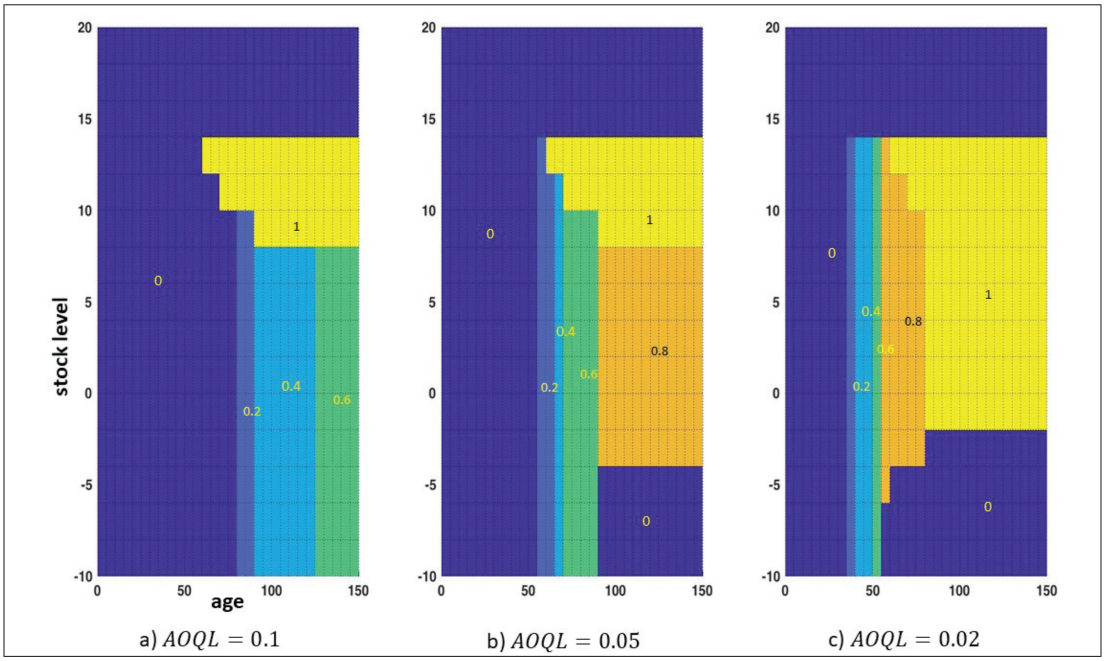


Figure 4.20 Inspection policies for different AOQL

4.6 Conclusion

In this chapter, we developed a stochastic optimization production policy, maintenance planning, and inspection strategy for a deteriorating hybrid closed-loop system consisting of a manufacturing, a remanufacturing, and a rectifying machine with two inventory and recovery stock rooms, that is capable of producing one type of product. Deterioration occurs due to the aging and imperfect repair processes. Since deterioration is partly caused by imperfect repair, we have to use a solution capable of integrating the memory-related issues to model, formulate, and examine this stochastic problem. Therefore, a semi-Markov model was employed to simulate the history of the failures and repairs. The provided model can integrate mathematical formulation and discrete-continuous simulation to represent the complicated inter-relations between the deterioration effects, manufacturing and remanufacturing planning, and maintenance and inspection strategies. In order to study the effect of the input parameters on the decision variables and also to validate the developed model, an experimental example was implemented as well as a sensitivity analysis over the finite horizon and the optimal cost function correlated optimal control policy values were determined.

The results indicated that the stock level can be used for protection against the shortage to counteract the effects of deterioration on limiting the availability of the system. Moreover, to avoid expensive penalties for the production of parts with qualities lower than the AOQL, we require performing combined maintenance and inspection activities. As the manufacturing machine deteriorates, it is recommended to intensify the PM and replacement activities. In addition, it was shown that inspection planning depends not only on the AOQL and the deterioration level of the machine, but also on the inventory stock level. Considering that specific financial penalties and incentives exist for providing low and high-quality parts, the cost-optimized quality of the outgoing products can be determined based on the AOQL, the level of deterioration, and available products in the inventory. The results also suggested that in most areas it is better to keep the outgoing product quality close to the AOQL, however, if the inventory level is high enough, it is recommended to perform a 100% inspection to benefit from incentives considered for high-quality products. Nonetheless, if the machine is old and highly deteriorated and the system is in an intense backlog state, performing the inspection is not cost-efficient. Under such circumstances, it is recommended to only fulfill the market demand even with low-quality products because at a certain point paying the penalty for poor quality products is more cost-efficient than risking staying longer at the backlog state for performing the inspection.

The remanufacturing machine that can recover the returned products, as well as the non-rectifiable defective parts, helps to decrease the production hedging policy level. When the manufacturing machine is deteriorated and we are at a shortage state, the zero inspection area -where we have to keep producing parts even with a low quality- decreases as we increase the remanufacturing rate. To sum up, this closed-loop model and the optimization approach can be applied at an operational level in many industries where the recovery of scrap products is possible.

CONCLUSION AND RECOMMENDATIONS

A growing number of studies have been conducted focusing on manufacturing, remanufacturing, and controlling the production line to obtain maximum efficiency and minimize the total incurred costs. Nevertheless, the existing literature has mainly investigated the forward or reverse flow of products and it appears that there is a lack of research on the relationship between these two flow models. In this study, we tried to integrate the forward and reverse lines of products in a model to obtain the optimal production policy that could minimize the total costs. We expanded the stochastic optimization model of the problem with five decision variables (manufacturing, remanufacturing, preventive maintenance, replacement, and inspection rates) and four state variables (stock level of inventory and recovery products and age and number of failures of manufacturing machine).

The first chapter was devoted to the literature review and investigating the studies related to the subject of controlling the production systems. Several studies on the interrelationship between quality, maintenance, and production were also revisited. Next, the critiques of the literature, the problem statement, objectives of the thesis, and the methodology was presented.

In the second chapter, a deteriorating production system consisting of a machine capable of producing one type of product was analyzed. The inspection process was considered along with the maintenance activities to ensure the production of parts with the specified product quality (AOQL). A finical penalty was assumed to have been established for products with a quality lower than AOQL as well as a financial incentive for providing parts with a quality higher than AOQL.

The third chapter added a rectifying machine - that was able to recover the rectifiable parts of the defective products- to the system. The fourth chapter further expanded the system by considering a remanufacturing machine -which can recover both the non-rectifiable pats of the defective products and the return products.

Numerical methods were employed using HJB equations to solve all three models, and the optimal solutions were presented in the forms of production threshold, PM, and replacement, inspection, and remanufacturing policies. The sensitivity analysis also validated all models. Following conclusions can be drawn by analyzing these three models;

- In the developed models, the manufacturing machine's age was shown to be a function of the machine's usage. One way to evaluate the efficiency of these three models is to compare their rate of outgoing products with the same inspection strategy. These rates can be shown as follows:

$$\frac{\textit{scenario 2}}{\textit{scenario 1}} = 1 + \frac{\gamma\beta(1 - r_\beta)}{1 - \gamma\beta}$$

$$\frac{\textit{scenario 3}}{\textit{scenario 1}} = 1 + \frac{\gamma\beta(1 - r_\beta)}{1 - \gamma\beta} + \frac{u_2(1 - r_{dis})}{u_1(1 - \gamma)}$$

  It can be understood from these equations that the third scenario will perform the task of producing a specified number of products with a specific quality faster and with a lower pressure on the manufacturing machine. For example, for the inspection rate $\gamma = 60\%$, defective rate $\beta = 0.1$ and the non-rectifiable rate of defective products of $r_\beta = 0.3$, the rate of outgoing products for the second model 4% and for third model (considering the recovery stock is not empty) 23% improved compare to the first model. Therefore, in the second and third scenarios, the inventory stock can reach the threshold level faster and the machine can be operational for a longer time.

- One issue of concern for the producers is the system's capacity to store products in the inventory to meet the demand when the production machine is not operating due to either a failure or performing maintenance. However, Keeping excessive products in the inventory increases the inventory cost and jeopardizes the availability of the machine by putting pressure on it to produce extra parts. By comparing the three proposed models, it can be noted that adding the remanufacturing and rectifying machines decreases the number of required products in the inventory for the third model. The is due to the presence of a recovery stock room in this model that can support a part of the demand while the manufacturing machine is unavailable. Also, the continuous flow of the products returned from the market to be recovered by the remanufacturing machine helps to keep the products in the inventory room at a minimum level. However, upon the addition of only the rectifying machine to the primary model, the inventory level does not change (second model). This can be explained by the fact that the products recovered by the rectifying machine are only provided by the manufacturing machine. Therefore, when the production machine is in a non-operational mode, the rectifying machine does not produce any parts. This is the reason the production hedging levels in the first two models are equal.

- In the first model, when the machine is old, and the system is in the backlog state, performing the inspection is not suggested as this region is called the zero inspection policy area. In other words, although the produced parts have a lower quality than AOQL, meeting the market demand and avoiding the expensive backlog state even at the expense of paying the penalty for poor quality products justifies not performing the inspection process. In the next chapter, upon adding the rectifying machine to the model, the inspection area expands to the earlier levels of the backlog state. The 100% inspection area is also recommended to start at a lower level of inventory. Adding a remanufacturing machine to the model in the third scenario further expands the inspection area to the more severe backlog states. Overall, it can be concluded that when the recovering process of defective products is advanced, it is more beneficial to intensify the inspection process. This way, the producer can provide products with higher quality even in shortage situations, and benefit from the financial incentives established for high-quality products. Also, when the penalty for low-quality products is increased, the optimal control model recommends intensifying the inspection process, and the inspection areas extend to earlier ages of the machine.

With regards to suggestions for future studies, the following improvements can be made to the proposed models;

- To increase the accuracy of the model, the presented study can make more realistic assumptions. For instance, considering the demand as a stochastic function of the quality rather than a constant variable can help us to obtain a more realistic optimized model.
- One limitation of our models was assuming that the PM and repair policies are performed with a constant intensity, whereas the intensity of the maintenance practices can vary with the state of the system. Further studies can be conducted by considering different levels of maintenance as a decision variable.
- Another modification to the present model could be considering a variable return rate of products because it can better show the relationship between the quality of provided products for the customer and the rate of returned products. A thorough study of this relationship can be an interesting research avenue.
- Inspection techniques were carried out in the models to control the quality of products. This technique is well suited to b used in the flow of products production systems. Other quality control methods such as process control or acceptance sampling processes can also be applied to the proposed model to gain a more reliable understanding of the problem.
- Another way to expand this study would be to consider a system consisting of a remanufacturing machine, the deterioration rate of which is a function of the level of the deterioration of the manufacturing machine. In general, we can say that any improvement in the proposed models should yield a more realistic simulation of the problem that can be employed in many industrial applications subject to deterioration and unexpected failures.

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