Optimisation des performances des systèmes de manufacturiers en contexte de chaîne d'approvisionnement en boucle fermée

par

Patrick MEGOZE PONGHA

MÉMOIRE PAR ARTICLES PRÉSENTÉ À L'ÉCOLE DE TECHNOLOGIE SUPÉRIEURE COMME EXIGENCE PARTIELLEÀ L'OBTENTION DE LA MAÎTRISE AVEC MÉMOIRE EN GÉNIE MECANIQUE
M. Sc. A.

MONTRÉAL, LE 29 JANVIER 2022

ÉCOLE DE TECHNOLOGIE SUPÉRIEURE
UNIVERSITÉ DU QUÉBEC

PRÉSENTATION DU JURY

CE MÉMOIRE A ÉTÉ ÉVALUÉ

PAR UN JURY COMPOSÉ DE :

M. Jean-Pierre Kenne, directeur de mémoire
Département de génie mécanique à l'École de technologie supérieure

M. Lucas Hof, codirecteur de mémoire
Département de génie mécanique à l'École de technologie supérieure

M. Christian Belleau, président du jury
Département de génie mécanique à l'École de technologie supérieure

M. Yvan Beauregard, membre du jury
Département de génie mécanique à l'École de technologie supérieure

IL A FAIT L'OBJET D'UNE SOUTENANCE DEVANT JURY ET PUBLIC

LE 21 DECEMBRE 2021

À L'ÉCOLE DE TECHNOLOGIE SUPÉRIEURE

REMERCIEMENTS

Le travail présenté dans ce mémoire a été effectué au sein du Laboratoire d'Intégration des Technologies de Production (LITP) du département de Génie Mécanique à l'École de Technologie Supérieure.

J'adresse ma profonde gratitude à mes directeurs de recherche, Professeurs Jean-Pierre Kenne et Lucas Hof pour avoir conduit cette recherche. Je remercie particulièrement leurs disponibilités, leurs conseils, leurs suggestions et toutes les corrections minutieuses qui ont permis de construire ce mémoire. Je ne saurai oublier Edgar De Jésus Garcia, qui a facilité la collecte des données pratiques tout au long de cette recherche. Sans oublier le docteur Guy Richard Kibouka pour les corrections et suggestions apportées dans ce travail.

Je remercie le professeur Christian Belleau qui a bien voulu présider mon jury et aussi le prpfesseur Yvan Beauregard, membre du jury pour avoir accepté d'examiner ce travail.

Mes remerciements vont également à l'endroit de papa Gilbert Nantsa et maman Mireille Nantsa pour les sacrifices qu'ils ont fait pour financer mon projet d'étude à ETS de Montréal. Également à mes Sœurs, Arielle Sonhafouo, Nelly Pongha, Alexa, Tania et Alia Nantsa pour tous leurs encouragements durant ma formation.

Je remercie maman Valérie Dassie et maman Marie Siewé pour leurs conseils, l'encadrement et le soutien durant ma formation. Sans oublier maman Dorette Kenne.

Je tiens à remercier mes chers amis Djionang, Fosso, Embe, Foke, Pamen, Doriane, Brice, Manuella, Elisabeth, Vanessa, qui ont tous facilité mon intégration au Canada. Sans oublier particulièrement, Sylvain, Charnelle, Tchuisseu, Pavel, Cabrel et Nguemo qui m'ont beaucoup aidé pour leurs contributions dans ce travail.

Enfin, j’adresse mes remerciements à mes parents Jérémie et Esther Pongha ainsi que toute ma famille du Cameroun pour leur soutien moral et leur support malgré la distance qui nous sépare

Optimisation des performances des systèmes de manufacturiers en contexte de chaîne d'approvisionnement en boucle fermée

Patrick MEGOZE PONGHA

RÉSUMÉ

La rareté des matières premières, les contraintes environnementales et la mondialisation des marchés poussent les entreprises d'une part à intégrer la réutilisation dans leurs chaînes d'approvisionnement, d'autre part à améliorer la qualité de leurs produits. Ce contexte force les entreprises à se pencher sur le problème d'amélioration de leur système de production en y intégrant des aspects de réutilisation et de contrôle qualité. Ainsi, l'objectif principal de ce travail est de développer des politiques optimales conjointes de production, de réutilisation, de maintenance et de contrôle qualité des systèmes de production hybride.

La détérioration des machines et la qualité des produits sont toutes des nouveaux facteurs incertains pris en compte dans ce travail. Il est donc question à l'issu de cette étude d'atteindre les objectifs suivants : mettre au point une structure de production hybride, élaborer des stratégies de production, déterminer la politique de maintenance et enfin proposer une politique d'inspection qualité des produits. Pour atteindre tous ces objectifs, nous avons divisé le problème principal en 3 sous-problèmes présentés en trois parties comme suit :

La première partie mène une étude d'optimisation d'un système de production constitué d'une unité de production non fiable, qui se dégrade avec le temps. Cette dégradation affecte la qualité des produits. L'objectif de cette partie consiste à proposer des stratégies de production, de maintenance et de contrôle qualité dans le but d'améliorer la qualité des produits et réduire le coût total. Pour atteindre ce but, nous avons utilisé une approche de programmation dynamique stochastique pour développer les conditions d'optimum de type Hamilton-Jacobi-Bellman (HJB). Par la suite, nous avons simulé une application pratique dans une entreprise Québécoise spécialisée dans la production des pompes à incendie. Cette application nous a permis de proposer de nouvelles politiques de production, maintenance et

de contrôle qualité des produits. D'autres entreprises pourront implémenter les politiques proposées. Cependant, dans cette partie, l'influence du taux de production sur la dégradation de la machine n'est pas prise en compte.

Dans la deuxième partie, le taux de production est intégré et l'option de l'ajout d'une seconde machine tel que développée dans kouedeu et Al. (2015) est adoptée. Le système reste non fiable et produit le même type de pièces. Le problème a été modélisé et une approche de programmation dynamique stochastique permet de développer les conditions d'optimum de type HJB. Les résultats obtenus ont permis de proposer des politiques de production, de maintenance et de contrôle qualité. Bien que les résultats obtenus dans les parties 1 et 2 soient très satisfaisants, ils ne répondent pas aux défis auxquels font face les entreprises qui réutilisent les produits en fin de vie.

De ce fait, la troisième partie de ce travail intègre la logistique inverse. La seconde machine s'approvisionnera en produits en fin de vie. Cette partie nous permet de traiter d'un problème de commande optimale pour un système hybride (en boucle fermé). Tous les facteurs aléatoires tels que : les pannes, les réparations et la dégradation des machines sont pris en compte. Une approche de programmation dynamique stochastique permettra de développer les conditions d'optimum de type HJB. Les résultats permettront de développer des politiques optimales conjointes de production, de réutilisation, de maintenance et de contrôle qualité des produits fabriqués et des produits remis à neuf.

Mots-clés: Processus stochastique, détérioration, maintenance, qualité, plan d'échantillonnage, Logistique inverse.

Performance optimization of manufacturing systems in a closed-loop supply chain context

Patrick MEGOZE PONGHA

ABSTRACT

The scarcity of raw materials, environmental constraints and the globalization of markets are pushing companies to integrate reuse in their supply chains and to improve the quality of their products. This context forces companies to address the problem of optimizing their production chain by integrating aspects of reuse and quality control. Thus, the main objective of this project is to develop joint optimal policies for production, reuse, maintenance and quality control of hybrid production systems.

The deterioration of the machines and the quality of the products are all new uncertain factors considered in this work. The objectives of this study are to develop a hybrid production structure, to develop production strategies, to determine the maintenance policy and to propose a product quality inspection policy. To achieve all these objectives, we divided the main problem into 3 sub-problems developed in three parts as follows:

The first part conducts an optimization study of a production system consisting of an unreliable production unit, which degrades over time. This degradation affects the quality of the products. The objective of this part is to propose strategies for production, maintenance and quality control in order to improve the quality of products and reduce the total cost. To achieve this goal, we used a stochastic dynamic programming approach to develop the optimum conditions of Hamilton-Jacobi-Bellman (HJB) type. Then, we simulated a practical application in a Quebec company specialized in the production of fire pumps. This application allowed us to propose new policies for production, maintenance and quality control of products. Other companies could implement the proposed policies. However, in this part, the influence of the production rate on the degradation of the machine is not considered.

In the second part, this production rate is incorporated and the option of adding a second machine as developed in Amie et Al. (2015) is adopted. The System remains unreliable and produces the same type of parts. The problem was modeled, and a stochastic dynamic programming approach is used to develop the HJB type optimum conditions. The results obtained allowed to propose production, maintenance and quality control policies. Although the results obtained in parts 1 and 2 are very satisfactory, they do not address the challenges faced by companies that reuse end-of-life products. Thus, the third part of this work integrates reverse logistics. The second machine will source end-of-life products. This part allows us to conduct an optimal study for a hybrid system (closed loop). All random factors such as: breakdowns, repairs and degradation of the machines are considered. A stochastic dynamic programming approach will be used to develop the optimal conditions of the HJB type. The results will allow the development of joint optimal policies for production, reuse, maintenance and quality control of manufactured and remanufactured products.

Keywords: Stochastic process, deterioration, maintenance, quality, sampling plan, reverse logistics.

TABLE DES MATIÈRES

Page

LISTE DES TABLEAUX

LISTE DES FIGURES

Page

LISTE DES ABRÉVIATIONS, SIGLES ET ACRONYMES

x (t)	Stock of now finished products manufactured t
ξ(t)	Stochastic process of the considered system
Ω	Discrete set of modes of the stochastic process
$q_{\alpha\gamma}$	Transition rate of the stochastic process from mode α to mode γ
M_i	Machine i with $i \in \{1,2\}$
λ_i	Failure rate of the machine I
MTTFMi	Average time between two machine breakdowns Mi
μ_i	Repair rate of the machine I
Q(.)	Matrix of transition rates of the stochastic process
d	Market demand rate
u_i	Production rate of machine M_i
u_1^*	Optimum value of u_i
U_i^{max}	Maximum production rate of machine M_i
u_{insp}	Product inspection rate
U_i^e	Economic production rate of machine M_i
u_i^r	Actual production rate of machine M_i
${}^rU_i^{max}$	Maximum actual production rate of machine M_i
W_i^r	Repair rate of machine M_i
${}^rW_i^{max}$	Maximum repair rate of machine M_i
${}^rW_i^{min}$	Minimum repair rate of machine M_i
W_i^p	Overhaul rate of machine M_i

t_p	Unit production time
t_{insp}	Unit inspection time
C^+	Inventory cost per unit of time and per product
C^-	Cost of out of stock per unit of time and per product
C_{isp}	Inspection cost per unit of time and per product
C_{des}	Destruction cost of non-conforming products
C_m	Preventive maintenance cost
C_{ret}	Cost of parts returned by the customer
C_f	Cost of manufacturing
C_r	cost of remanufacturing
f_i^α	Fraction of products from machine i to be inspected at mode α
f_i^*	Optimum value of f_i
β_i	Defective rate of machine i
β_i^{min}	Minimum defective rate of the machine of machine i
β_i^{max}	Maximum defective rate of the machine of the machine i
AOQ	Average Outgoing quality after inspection
AOQL	Average Outgoing quality limit after inspection
$AOQL_{max}$	Maximal AOQ value
$\mathcal{A}(.)$	Domain of admissibility of admissible contrôl policies
g(.)	Instantaneous cost
J (.)	Incurred total discounted cost
ρ	Discount rate

$v(\alpha, .)$	Value function of the optimization problem at mode α
Z_{α}^{*} (.)	Critical threshold associated with the machine at mode α
HJB	Hamilton-Jacobi-Bellman
EOL	End-Of-Live
MTBF	Mean Time Between Failure

INTRODUCTION

Ces dernières décennies, la mondialisation et la globalisation des marchés, les contraintes climatiques et la rareté des matières premières ont poussé davantage les entreprises à développer de nouvelles techniques d'approvisionnements. Cette situation les confronte d'une part aux problèmes d'optimisation de leurs chaînes de production, d'autre part aux problèmes d'exploitation des produits en fin de vie pour compléter la rareté des matières premières et limiter leurs impacts néfastes sur l'environnement.

Dans ce contexte de fabrication et de réutilisation, il est plus complexe pour une entreprise de maîtriser à la fois des techniques de production et de réutilisation dans sa chaîne d'approvisionnement (Kouedeu et al. 2014). Ceux-ci sont d'autant plus complexe lorsque les produits ne sont pas tous de bonnes qualités (Hliouia et al. 2015). Pour donner suite à cette situation, plusieurs chercheurs se sont penchés sur l'analyse et la modélisation du problème afin de proposer aux entreprises de nouvelles techniques de production et de réutilisation. Cependant, aux vues de la complexité du problème, les modèles proposés traitent séparément les aspects de dégradation, de production, de remise à neuf et de qualité. Or ces facteurs dans une chaine de production interagissent les uns sur les autres. Face à cette interdépendance des facteurs de performance du système de production, il se pose le problème d'optimisation conjointe simultanée de toutes ses caractéristiques (fiabilité, maintenance, qualité, coûts) pour répondre efficacement au besoin du marché.

Fort de ce constat, l'objectif de ce mémoire est de développer conjointement des stratégies de production, de réutilisation et de contrôle qualité. Cette approche vise simultanément à optimiser les coûts totaux de production, à accroître la fiabilité et la disponibilité du système et à améliorer la qualité des produits.

Pour mener à bien ce travail, nous l'avons structuré en 4 chapitres présentés comme suit : le chapitre 1, intitulé revue de littérature, énonce le problème de recherche, les objectifs visés, la méthodologie, les retombées scientifiques et industriels. Le chapitre deux (2) constitue notre premier article de revue intitulé « Optimal joint production, maintenance and product quality control policies for a continuously deteriorating manufacturing system » soumis à « International Journal of Modeling and Simulation ». Dans ce chapitre nous étudions un système de production qui suit un processus markovien homogène stochastique et donc l'inspection qualité est imparfaite. Ensuite le chapitre trois (3) qui présente notre deuxième article intitulé « Production and quality control optimization strategies for deteriorating manufacturing systems under the influence of production rates » soumis à « Journal of Manufacturing Systems ». Dans ce chapitre, l'influence du taux de production négligée dans l'article un (1) est pris en compte. Enfin le chapitre quatre (4) qui présente notre troisième article intitulé « Production planning and quality control of manufacturing and remanufacturing hybrid systems deteriorizing under the influence of production rates » sera soumis à « International Journal of Production Research ». Dans ce chapitre la logistique inverse est intégrée pour étendre les travaux de l'article deux (2) à travers la collecte et la réutilisation des produits.

CHAPITRE 1

REVUE DE LITTÉRATURE

1.1 Introduction

Dans ce chapitre, nous présentons premièrement la problématique de recherche, deuxièmement, nous présentons la structure du système ayant fait l'objet d'étude de ce mémoire, troisièmement une revue de littérature, ensuite la méthodologie adoptée pour résoudre le problème et en fin les retombées scientifiques et industrielles de cette recherche.

1.2 Problématique de recherche

Au cours des dernières décennies, de nombreux chercheurs ont étudié les systèmes de production et de logistique inverse dans le but d'accroitre leur efficacité. Ceci d'accroitre la fiabilité et la disponibilité des composantes industrielles malgré les pannes et les réparations aléatoires auxquelles elles sont soumises. D'autres chercheurs se sont penchés sur la dégradation et ses effets sur la fiabilité et la qualité. Cependant la nature stochastique et la dynamique des processus auxquels sont soumis ses systèmes industriels, rend très complexe l'étude simultanée de ses facteurs (Rivera-Gomez et al. 2020). C'est pour cette raison que certains chercheurs dans leurs études ne prennent pas compte de l'influence du taux de production sur la dégradation. Pourtant cette influence est toujours présente (Kenne and Gharbi, 1999), d'autres négligent l'aspect qualité, réutilisation afin de simplifier et résoudre partiellement le problème. Ainsi le problème que doit résoudre cette recherche est celui de l'optimisation d'un système de production hybride de production et de réutilisation en intégrant simultanément les facteurs de performances tels que la production, la réutilisation, la fiabilité, la disponibilité et la qualité. Ainsi les questions de recherche sont :

- Quelles stratégies optimales de production et de réutilisation sont-elles convenables pour des systèmes de production qui se dégradent en fonction de leur taux d'utilisation, qui fabriquent aussi des pièces non conformes et qui suivent une dynamique stochastique?

- À quel moment produire (ou remettre à neuf) et quelle quantité produire (ou remettre à neuf) afin de réduire le coût total tout en satisfaisant la demande variable du marché?
- Quelles stratégies de maintenance adoptée pour limiter la dégradation et accroitre la fiabilité et la disponibilité?
- Quelles stratégies de contrôle qualité adoptée pour respecter efficacement aux exigences qualité du marché?

Pour répondre à ces questions afin d'analyser, modéliser et optimiser un système industriel en contexte stochastique et en intégrant simultanément tous les aspects de commande optimale; réutilisation; dégradation; maintenance et amélioration qualité, il est nécessaire de définir les mots clés utilisés dans des chaines d'approvisionnements hybrides (production et réutilisation).

1.2.1 Définition des mots clés – Terminologie

Dans cette session, nous allons définir les mots clés qui faciliteront la compréhension du sujet traité. Il s'agit entre autres de définir ce que c'est que : une chaine d'approvisionnement; un processus stochastique; une chaine de Markov (homogène et non homogène); Le processus de détérioration; La qualité la logistique inverse; produits non conformes et en fin remise neuf

1.2.1.1 Chaîne d'approvisionnement

Une chaine d'approvisionnement ou « supply chain», est un ensemble de processus et de flux exécutés de façon cohérente dans le but de transformer une matière première en un produit utile aux clients (J. P. kenne 2006) Elle va de l'approvisionnement auprès du fournisseur de matière première vers les clients en passant par la production (manufacturier). Dans ce travail, nous nous intéressons uniquement au manufacturier et au client.

1.2.1.2 Processus stochastique

Selon Sader et Sorensen (Sader and Sorensen 2003), Un système suit un processus stochastique si l'un ou plusieurs de ses paramètres est/sont aléatoire(s) (pannes, réparations, demande).

1.2.1.3 Chaine de Markov

Selon Kouedeu et al. (Kouedeu et al. 2014), « Une chaîne de Markov est homogène quand ses taux de transition d'un état à un autre sont considérés constants. ». Cependant une chaine de Markov est dite non homogène lorsque le taux de pannes est fonction de l'âge des équipements et de leurs cadences d'utilisation (Boukas and Haurie 1990).

1.2.1.4 Détérioration

La détérioration d'un équipement ou une machine industrielle est l'affaiblissement progressive de ses composantes qui n'arrivent plus à assumer pleinement leur fonction principale. Cette défaillance réduit les performances (rendement, qualité, coût, …) de l'équipement.

1.2.1.5 Maintenance

Selon l'office québécoise de la langue française, la maintenance est : « Ensemble des opérations exécutées par un technicien spécialisé, dans le but de maintenir un système ou une partie du système dans un état de fonctionnement normal »[1].

---

[1] Maintenance (gouv.qc.ca) consulté le 12/10/2021

1.2.1.6 Qualité

La qualité d'un produit industriel se définit comme étant son aptitude à satisfaire aux exigences des clients, tout en respectant les contraintes environnementales, sociales et économiques.

1.2.1.7 Logistique inverse

Fleischmann (Fleischmann 2001) définit la logistique inverse comme étant : « le processus de planification, l'exécution et le contrôle du flux des produits collectés (retour) dans une chaine d'approvisionnement (production) dans le but de les remettre sur le marché. »

1.2.1.8 Produit non conforme

Un produit est dit non conforme lorsqu'après un contrôle qualité, il ne respecte pas toutes les exigences définies au préalable (Baillargeon 2013).

1.2.1.9 Remise à neuf

La remise à neuf d'un produit en contexte manufacturier est l'ensemble des opérations effectuées sur un produit en fin de vie ou un produit non conforme pour l'amener dans un état comme neuf (Samir et al. 2013). Autrement dit, les produits remis à neuf sont considérés identiques aux produits issus de la fabrication directe.

1.3 Structure du système de la chaine d'approvisionnement étudiée

Le système ayant fait l'objet de notre étude est un modèle hybride de fabrication et de remise à neuf tel que défini à la Figure 1.1. Ce système est constitué de deux lignes de production. La première constitue une machine pour produire des pièces à partir de la matière première brute. La deuxième quant a-t-elle produit des pièces à partir des produits en fin de vie (Logistique inverse) ou des produits non conformes issus de la fabrication.

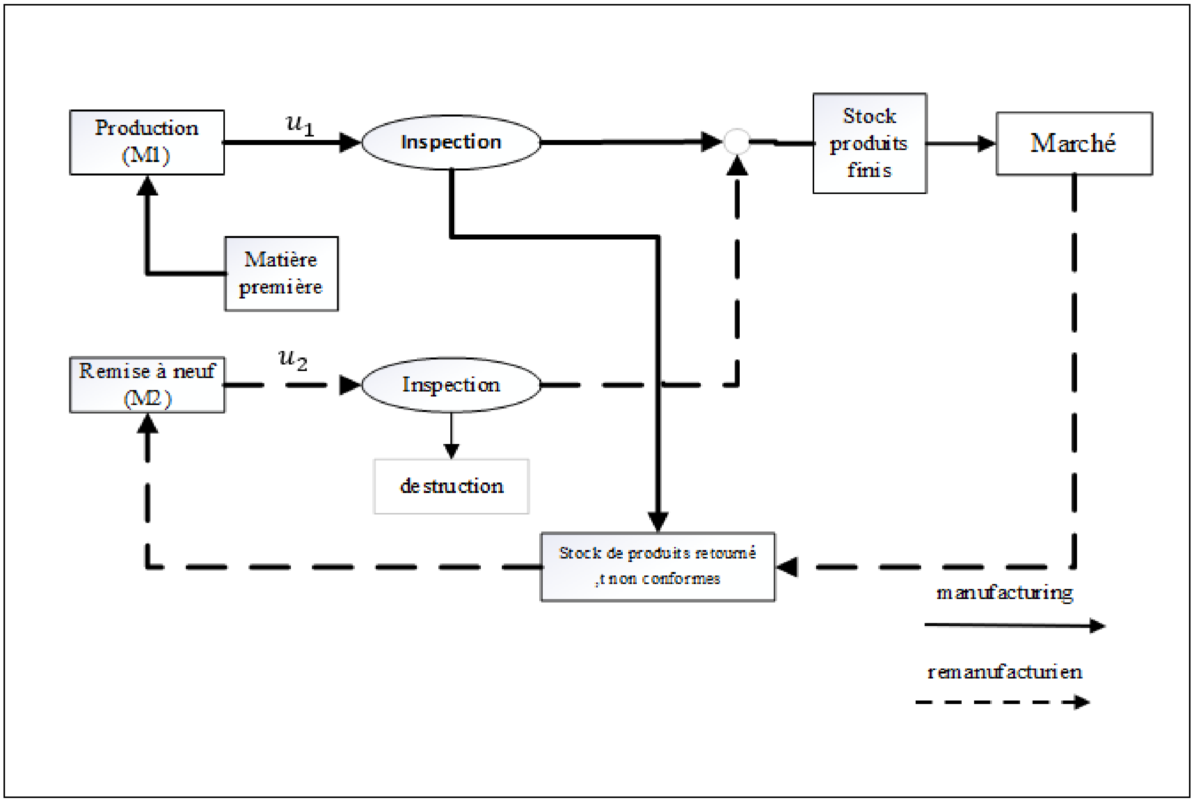


Figure 1.1 Structure du système hybride étudié

Ce système est soumis aux pannes et réparations aléatoires et les composantes se dégradent au fil du temps en fonction de leur utilisation. Les machines de manufacturing /remanufacturing suivent un processus stochastique dynamique modélisé par une Chaine de markov homogène. Ainsi le problème qui se pose est celui de savoir comment nous devons optimiser simultanément la production et la remise à neuf des produits afin de satisfaire en tout temps la demande et à des coûts de production faible, comment planifier les opérations de maintenance afin d'accroitre la disponibilité du système et enfin comment améliorer la qualité des produits distribués. Dans le souci de répondre à ces questions, nous devons conduire cette recherche par certaines hypothèses.

1.3.1 Hypothèses de travail

Les hypothèses qui conduisent ce travail sont les suivantes :

- Les matières premières et les produits en fin de vie sont toujours disponibles ;
- Le taux de demande est connu et est constant ;
- Les produits remis à neuf et les produits fabriqués sont considérés identiques.
- Le taux de production maximal de chaque machine est connu ;
- Les différents coûts : stockage, pénurie, contrôle, maintenance sont constants et connus ;
- Les produits non conformes reçus par les clients sont récupérés et remplacés par l'entreprise, ce qui entraîne des pertes;
- Les machines se dégradent en fonction de leur taux de production ;
- Les pannes et réparations des machines sont aléatoires.
- Chacune des machines produit des pièces donc une certaine proportion est non conforme.
- La proportion moyenne des produits non conformes sur le marché ne doit pas dépasser une certaine limite maximale;
- Les produits non-conformes identifiés sont remis à neuf ;
- Le système de contrôle de la qualité est de type échantillonnage.

Les hypothèses sont mieux détaillées dans les différents chapitres de ce travail.

1.3.2 Objectifs de la recherche

L'objectif principal de ce mémoire est de développer des stratégies (politiques) de production et de réutilisation conjointement aux stratégies de maintenance et de contrôle qualité imparfaite afin de minimiser le coût total de production, limiter l'influence du taux de production sur la dégradation, augmenter la fiabilité et améliorer la qualité des produits. Pour cela nous visons les objectifs spécifiques suivants :

- Proposer de politique d'amélioration qualité au moyen d'une inspection imparfaite (article 1) ;
- Maitriser un système constitué de machines qui se dégradent en fonction de la vitesse d'utilisation (article 2) ;
- Proposer une structure hybride de production et de réutilisation qui intègre à la fois la maintenance et le contrôle qualité (article 3) ;
- Modéliser ce système hybride afin d'étudier et d'analyser ses paramètres (article 3) ;
- Optimiser les performances du système (minimiser le coût, augmenter la disponibilité, réduire la dégradation et améliorer la qualité).

1.4 Revue critique de la littérature

Dans cette session, nous allons mener une revue de littérature sur les principaux concepts de notre recherche. Il s'agit entre autres de :

- Les chaines d'approvisionnements;
- Les stratégies de maintenance industrielle;
- La gestion de la production combinée aux stratégies de maintenance;
- L'étude sur le processus de dégradation des systèmes manufacturiers;
- Les stratégies d'inspection qualité;
- La gestion du système hybride de production et de réutilisation.

1.4.1 Les chaines d'approvisionnements

Selon Nakhla (Nakhla 2009) et Heizer (Heizer 2011), « une chaine d'approvisionnement est un réseau de fournisseurs, fabricants, distributeurs et détaillants qui vise à soutenir la circulation des produits, des informations et des flux financiers depuis la commande des matières premières chez le fournisseur jusqu'à la livraison des produits finis au client final ». Il en ressort que les principaux acteurs d'une chaine d'approvisionnement sont :

- Le fournisseur qui fournit la matière première;
- Le fabricant qui transforme cette matière première en produits utiles pour un client;
- Le distributeur qui se charge d'acheminer les produits fabriqués aux clients.

Les échanges entre ces principaux acteurs sont tenus par la chaine de logistique (c'est l'organe moteur de la chaine d'approvisionnement). Cette chaine logistique est caractérisée par un flux de matière qui va du fournisseur vers le client et un flux d'information qui va du client vers le fournisseur (Wang, Deli, and Guoqing 2010).

Les processus de la chaine d'approvisionnement peuvent se voir selon le but de la production en deux catégories à savoir les processus tirés et les processus poussés. On parle de processus tirés lorsque le manufacturier produit pour satisfaire les commandes enregistrées du client et on parle de processus poussés lorsque ce dernier produit pour satisfaire les prévisions futures des commandes des clients. Selon les cycles nous avons :

- Le cycle d'approvisionnement (fournisseur) : Manufacturier
- Le cycle de fabrication/réutilisation (interne au manufacturier)
- Le cycle de réapprovisionnement (manufacturier) : distributeur
- Le cycle commande clients.

L'optimisation des performances de chacun de ses cycles permet de construire un réseau optimal d'approvisionnement. Pour une bonne optimisation, nous avons besoin de modéliser le problème. De ce fait, il existe quatre approches d'optimisation selon Mim et Zou (Min and Gengui 2002) à savoir : l'approche déterministe pour une chaine d'approvisionnement dont les paramètres sont connus et constants; l'approche stochastique lorsque au moins un des paramètres est aléatoire; l'approche hybride (déterministe et stochastique) et enfin l'approche basée sur les technologies de l'information qui se sert de l' échange mutuelle d'informations entre les différents cycles pour permettre le contrôle en temps réel : c'est l'industrie 4.0 . La figure 1.2 illustre une chaine d'approvisionnement de processus tiré.

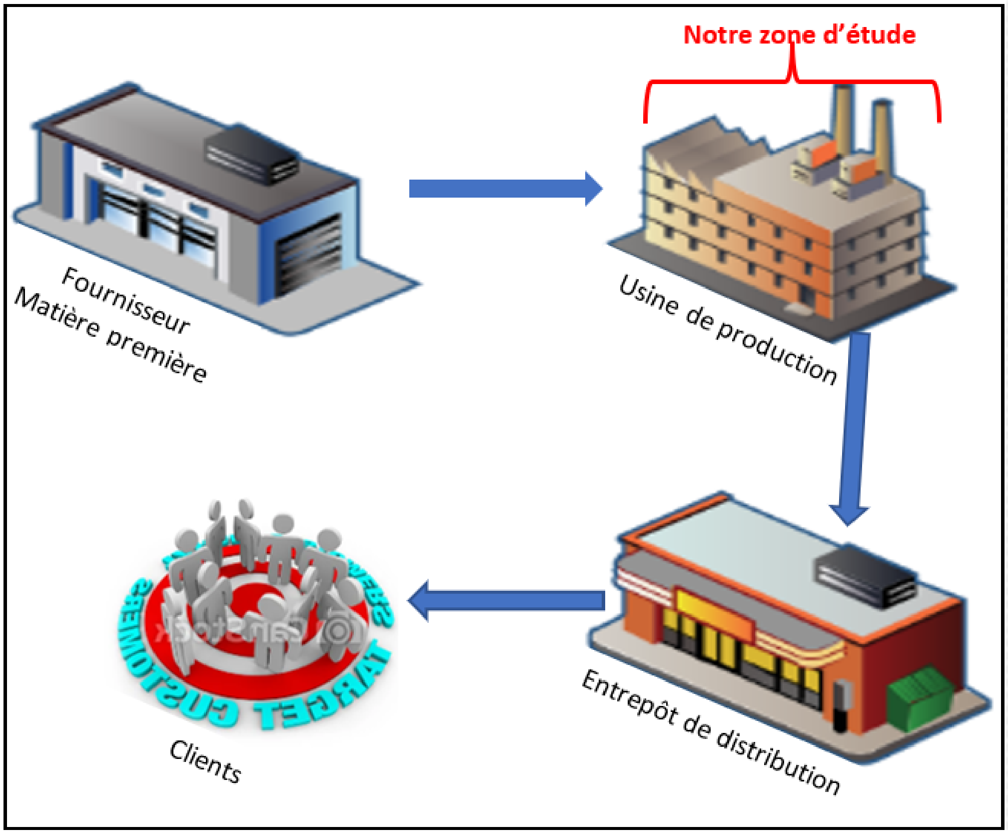


Figure 1.2 Illustration d'une chaine d'approvisionnement à flux tiré

Dans le cadre de cette étude nous allons utiliser une approche stochastique modélisée et optimiser une chaine d'approvisionnement à flux tiré. Les paramètres variables tels que les pannes et réparation pris en compte permettent de mieux se rapprocher de la réalité industrielle (Kouedeu et al. 2014). Alors pour limiter l'influence négative de ces paramètres, il est nécessaire de développer de bonnes stratégies de maintenance.

1.4.2 Maintenance industrielle

Pour augmenter la disponibilité du système de production et fabriquer des produits de bonnes qualités, les entreprises ont besoin de développer de bonnes stratégies de maintenance (Liu et al. 2020). Aussi, les coûts de maintenance varient entre 15% et 70% du coût total de production des entreprises (Ling et al. 2007). Ainsi, appliquer de bonnes stratégies de maintenance peut réduire significativement le coût total de production d'une entreprise. Selon la norme NF X 60-010, il existe deux types d'opérations de maintenance : la maintenance corrective et la maintenance préventive.

La maintenance corrective est l'ensemble des actions menées sur un bien en panne afin de le rétablir dans son état de fonctionnement. Dans ce type de maintenance, trois options sont possibles : Le dépannage, la réparation et le remplacement. En cas de défaillance, « le dépannage est une opération qui permet de rétablir un bien provisoirement en état de fonctionnement; La réparation quant à elle est une opération qui vise à rétablir le bien dans un état de fonctionnement conforme à des exigences prédéfinies. Le remplacement permet de rétablir le composant d'un bien en l'état de panne à l'état neuf »[2].

La maintenance corrective permet à l'entreprise de maintenir disponible son système de production. Cependant, elle favorise la dégradation des équipements et ne réduit pas la fréquence des défaillances (Ling et al. 2007). De ce fait, la maintenance préventive peut être une solution dans ce cas.

La maintenance préventive est une opération menée sur un bien en fonctionnement dans le but de limiter sa dégradation et réduire les défaillances. Cette stratégie de maintenance se fait à travers les inspections, des contrôles et des visites systématiques ou conditionnels (Wang, et al. 2010). De ce fait, on parle de maintenance préventive conditionnelle ou de maintenance préventive systématique. La maintenance préventive dite systématique est celle qui s'effectue systématiquement indépendamment de la dégradation ou de l'âge des équipements tandis que la maintenance préventive conditionnelle est effectuée selon l'état de dégradation observée des équipements. La figure 1.3 présente la structure générale des opérations de maintenance.

---

[2] Maintenance corrective, préventive et prévisionnelle (qualiteonline.com) consulté le 20/10/2021

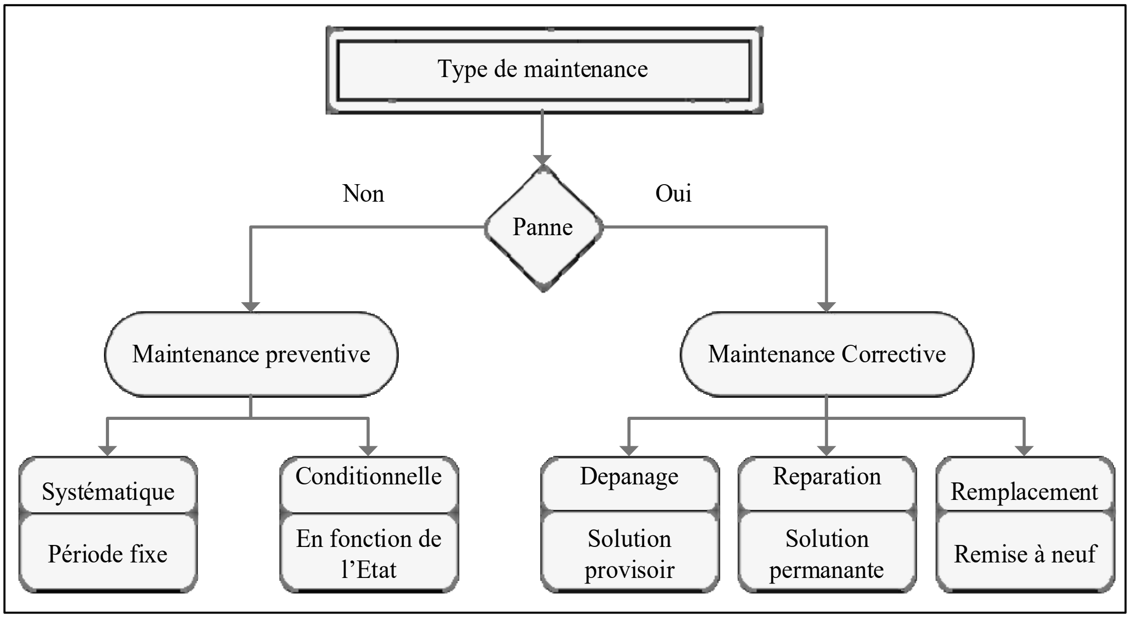


Figure 1.3 Structure générale de maintenance

En somme, la maintenance, préventive apporte des avantages tels que : la réduction des pannes ce qui favorise la production. Aussi les opérations de maintenance préventive contrairement aux opérations de maintenance corrective sont planifiées à l'avance ce qui permet une bonne préparation et optimisation des opérations. Cependant, le caractère stochastique des processus de maintenance (réparation, pannes) ne permet pas de garantir la disponibilité en tout temps des équipements de production. De ce fait, il n'est toujours pas possible de satisfaire la demande. Ce qui nous pousse à mener une réflexion sur l'optimisation dans la production.

1.4.3 Gestion de la production combinée aux stratégies de maintenance

Les systèmes de productions sont soumis aux pannes et réparations aléatoires ce qui perturbe la production malgré l'exécution des bonnes stratégies de maintenance (Kouedeu et al. 2015); c'est dans cette optique que Kenne et al. (Kenne et al 2003) ont publié une étude conjointe de la production et la maintenance d'un système peu fiable. Grâce aux stratégies développées, ils ont augmenté la disponibilité des machines à travers la maintenance corrective et le contrôle de la production.

Ces travaux seront étendus par Kenne et al. (Kenne, Gharbi, and Beit 2007; Kenne et al. 2003) qui proposent une politique de production à seuil critique. Cela leur a permis de déterminer quand effectuer les opérations de maintenance et quel seuil d'inventaire doit-on avoir pour réduire le coût total de production. Cette politique de production a été reprise dans les travaux de Rivera-Gomez et al. (Rivera-Gomez et al. 2020), Kouedeu et al. (Kouedeu et al. 2014) et bien d'autres ; elle consiste à déterminer le seuil critique Z^* qu'il faut en stock pour optimiser le coût total de production et permettre de satisfaire la demande malgré les paramètres (pannes, réparations) aléatoires du système. L'équation (1.1) suivante décrire la politique optimale à seuil critique :

$$u^*(\alpha, x) = \begin{cases} U_{max} & \text{si } x(t) < Z^* \\ d & \text{si } x(t) = Z^* \\ 0 & \text{si } x(t) > Z^* \end{cases} \tag{1.1}$$

Les travaux précédents ont permis d'optimiser la production, augmenter la disponibilité, satisfaire la demande même en temps de pannes et réduire le coût total de production. Cependant, la dégradation de l'unité de production en fonction du temps et en fonction de fréquence d'utilisation n'a pas été prise en compte.

1.4.4 Processus de dégradation du système manufacturier

La politique à seuil critique développé dans la section précédente a été adopté et étendu plusieurs fois et au fil des années. Cette politique a permis de minimiser le coût total de production des systèmes de production suivant un processus Markovien (Kouedeu et al. 2015) et aussi ceux suivant un processus semi-Markoviens (Dehayem N. et al. 2011), (Kazaz and Sloan 2013). Cependant, Kenne et al. (Kenne and Gharbi 1999) ont démontré que le taux de production influence négativement la dégradation des équipements. Aussi Dehayem et al. (Dehayem N. et al. 2011) dans leurs travaux considèrent que le système se dégrade en fonction de l'âge et du taux de pannes.

Cette situation rend plus complexe le problème et présente les limites de la politique optimale de production à seuil critique tel que développé initialement par Berlecki et Kumar (Bielecki and Kumar 1998). Dans ce contexte, la politique à seuil critique n'est plus optimale si le taux de panne n'est pas une fonction linéaire du taux de dégradation (Hu et al. 1994). Pour résoudre ce problème, Kouedeu et al. (Kouedeu et al. 2015) dans une étude d'optimisation d'une chaîne de production qui se dégrade en fonction du taux de pannes, ont proposé l'option d'ajout d'une seconde machine sous-traitante qui assiste la machine principale. Cette stratégie de machine sous-traitante leur a permis de réduire le taux de production de la machine principale de sa valeur maximale à une valeur économique. Grâce à cette technique dans la politique de production à seuil critique, le coût total de production a été minimisé et l'influence du taux de production sur la dégradation de la machine principale a été réduite. Mais dans les travaux précédents l'influence de la dégradation sur la qualité des produits est négligée.

1.4.5 Contrôle qualité

La dégradation des équipements n'affecte pas uniquement la fiabilité telle que développée dans les travaux précédents mais elle influence aussi négativement la qualité des produits (Ait-El-Cadi et al. 2021). Pour se rassurer donc que les produits à distribuer aux clients respectent les exigences qualité, les entreprises doivent contrôler au moyen d'une inspection qualité leurs produits après fabrication. Cette inspection permet d'identifier les produits conformes et les produits non conformes (Baillargeon 2013). Alors pour se rassurer de la conformité des produits distribués, plusieurs chercheurs ont proposé dans la littérature diverse techniques d'inspections à savoir l'inspection 100% et l'inspection en plan d'échantillonnage.

1.4.5.1 L’inspection 100%

Cette technique utilisée dans Rivera-Gomez et al. (Rivera-Gomez et al. 2013) consiste à inspecter tous les produits fabriqués ou tous les produits remis à neuf (Ouaret et al. 2018b). Grace à cette technique, tous les produits non conformes sont retirés de l’inventaire et peuvent être rectifiés afin de devenir conformes, ils peuvent dans certains cas être détruits ou même réutilisés comme matières premières. Cette technique permet de garantir 100 % en théorie la satisfaction des clients. Elle est nécessaire lorsque la présence d’un défaut est susceptible d’occasionner des dommages importants ou des accidents mortels. Cependant cette politique peut être très coûteuse lorsque les coûts unitaires d’inspection sont élevés. Elle peut aussi dans certains cas être impossible au vu de la taille de production (grande série) et du temps d’inspection. C’est aussi le cas lorsque l’inspection est destructive. Pour remédier à cette situation, une autre stratégie d’inspection a vu le jour.

1.4.5.2 Inspection de type échantillonnage

Comme mentionné précédemment, inspecter 100% des produits est quasi impossible pour les entreprises de grande taille avec une grande série de production et dont le temps d’inspection d’un produit est assez long et lorsque l’inspection est très coûteuse. C’est le cas des industries automobiles et les industries agroalimentaires et bien d’autres. Ainsi l’inspection de type échantillonnage a été développée chez Dodge (Dodge 1943b). Cette stratégie consiste à ne pas inspecter tous les produits mais plutôt une certaine proportion choisie aléatoirement. Cette fraction peut être fixe ou variable. Lorsqu’elle est fixe on parle d’inspection de type échantillonnage simple statique et lorsqu’elle varie on parle d’inspection de type échantillonnage multiple dynamique (Hajej, Rezg, and Gharbi 2021).

1.4.6 Gestion des systèmes hybrides de production et de réutilisation

Au regard des enjeux économiques, sociaux et environnementaux, plusieurs opportunités s’offrent aux entreprises afin de leur permettre d’adopter de nouvelles techniques de production et de réutilisation (Colledani and T. Tolio 2012). C’est pour cela qu’au cours de ces dernières années, les systèmes de fabrication hybrides ont connu une expansion fulgurante (Kouedeu et al. 2015).

C'est dans ce contexte que Samir et al. (Samir et al. 2013) ont développé une étude optimale pour un système hybride de fabrication et de réutilisation constitué de deux machines sujets aux pannes et aux réparations aléatoires. Cette étude leur a permis de proposer des politiques de production, de réutilisation et de maintenance. Kouedeu et al. (Kouedeu et al. 2015) vont étendre ces travaux en prenant en compte la dégradation de la machine principale en fonction de son taux d'utilisation. Cette dégradation affecte la disponibilité de la machine mais n'affecte pas la qualité des produits. Pour être plus réaliste, Ouaret et al. (Ouaret et al. 2018b) vont prendre en compte l'influence de la dégradation sur la qualité des produits. Dans cette nouvelle approche, ils proposent l'option des remplacements de la machine comme stratégies de maintenance. L'ajout de cette possibilité de remplacer la machine après une certaine dégradation a permis de réduire l'impact sur la qualité des produits. Ainsi le schéma global d'un système hybride de production et de réutilisation avec contrôle qualité est présenté sur la figure 1.4.

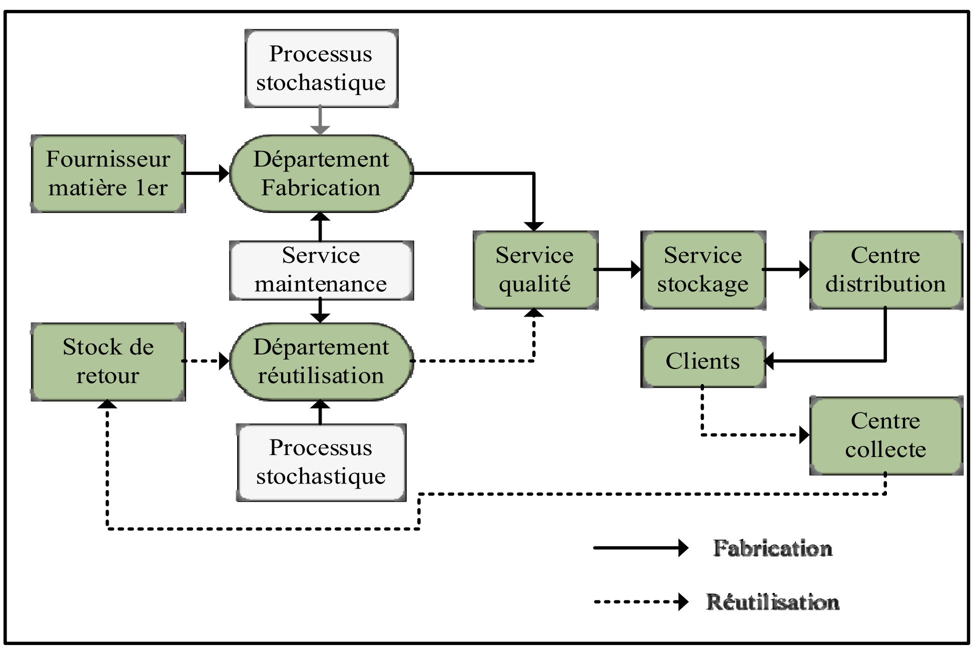


Figure 1.4 Système hybride (production / réutilisation) et contrôle qualité
Reproduite et adaptée avec l'autorisation de Kouedeu et al. (2014, p. 19)

Les deux systèmes de production et de réutilisation sont constitués des machines soumises aux phénomènes aléatoires à savoir : les pannes, les réparations, la dégradation, ainsi que la demande client. L'approvisionnement en matière première se fait auprès des fournisseurs extérieurs. L'approvisionnement en matière réutilisable se fait auprès des centres de collecte de produits en fin de vie. Tous les produits finis issus de la fabrication et de la réutilisation passent par le service qualité avant d'être envoyés en inventaire. Les produits non conformes identifiés par le service qualité sont détruits (Baillargeon 2013). De même les produits en fin de vie collectés mais non conformes aux exigences de réutilisation sont détruits (Van der Laan and Marc 1997). Le service de maintenance mène des opérations de maintenance corrective et préventive dans le but de limiter la dégradation et les défaillances des machines. Au-delà de la complexité des systèmes de fabrication et réutilisation peu d'auteurs ont mené des études conjointes de système hybride et de maintenance (Kouedeu et al. 2015).

1.4.7 Synthèse de la revue de littérature

Le tableau 1.2 suivant résume l'ensemble des domaines parcourus dans cette revue de littérature.

Tableau1.1 Synthèse de la revue de littérature

Revues	Domaine étudiés				
	Contrôle Production	Optimisation maintenance	Dégradation et taux de production	Contrôle Qualité	Logis-tique inverse
Chaînes d'approvisionnement	Étude générale d'une chaine d'approvisionnement à flux tiré et poussé. Présentation des cycles d'approvisionnement, de production, de stockage et de distribution				
Maintenance		✓			
Gestion production et maintenance	✓	✓			

Tableaux 1.1 Synthèse de la revue de littérature (Suite)

Revues	Domaine étudiés				
	Contrôle Production	Optimisation maintenance	Dégradation et taux de production	Control Qualité	Logis-tique inverse
Processus de dégradation	✓	✓	✓		
Contrôle qualité				✓	
Gestion des systèmes hybride	✓	✓		✓	✓
	✓	✓	✓		✓

Le tableau 1.1 montre que les travaux publiés qui traitent la gestion de la production combinée aux stratégies de maintenance ont permis d'optimiser la production, limiter la fiabilité et augmenter la disponibilité mais ne prennent pas en compte la dégradation, le contrôle qualité et la logistique inverse. De même les travaux qui traitent la dégradation et influence du taux de production ont négligé le contrôle qualité. Enfin, parmi les auteurs qui traitent la gestion des systèmes hybrides de production et de réutilisation, d'une part certains ont mené une étude conjointe de production, de réutilisation, de maintenance et de contrôle qualité mais négligent la dégradation et l'influence du taux de production. D'autre part les autres auteurs ont mené une étude conjointe de production, de réutilisation, de maintenance dans un système qui se dégrade en fonction de son utilisation mais négligent le contrôle qualité. L'objectif de notre recherche est de proposer un modèle d'optimisation qui intègre simultanément tous ces aspects du système hybride de production et de réutilisation. En somme cette revue de littérature préliminaire avait pour but de faire un état de l'art général sur la question de recherche qui conduit ce mémoire. Cette revue nous a permis d'une part de prendre connaissance des politiques déjà développées pour l'optimisation des systèmes hybrides de production et de réutilisation. D'autre part d'identifier les facteurs qui ont été négligés afin de les intégrer pour étendre la recherche sur cette thématique.

Pour mener à bien cette étude afin de proposer une politique qui intègre tous les facteurs du tableau 1.2 nous allons suivre la méthodologie présentée à la section suivante.

1.5 Méthodologie proposée

Pour atteindre l'objectif principal de ce mémoire, Nous avons partitionné le problème principal en trois sous problèmes dont la résolution fera l'objet de chacun des trois chapitres qui vont suivre. Par la suite, pour chacun de ses sous – problèmes (articles), la méthodologie que nous avons utilisé pour atteindre le but est la suivante :

- Une revue critique de la littérature. Cette étape consiste à parcourir les travaux déjà publiés sur l'optimisation les systèmes hybrides (manufacturing/ remanufacturing), soumis aux pannes et aux réparations aléatoires qui se dégradent. Cette revue des travaux passés permet de situer chacun de nos sous problèmes par rapport à la littérature et ressortir les contributions de notre recherche.

- Une théorie de commande optimale stochastique. Cette étape consiste à formuler le problème de commande optimale et utiliser une approche de programmation dynamique stochastique pour développer les conditions d'optimum de type Hamilton-Jacobi-Bellman (HJB).

- Une résolution par la méthode numérique. Cette étape consiste à utiliser la méthode numérique pour résoudre les équations de d'HJB

- Une simulation d'un exemple pratique. Cette étape consiste à simuler les modèles précédemment définis sur un cas de système de production et/ou réutilisation réelle afin de proposer de politiques optimales de production, de maintenance et de contrôle qualité.

- Une analyse de sensibilité. Cette étape consiste à faire varier les paramètres du cas pratique simulé, observer le comportement des politiques proposées afin d'assurer leur robustesse et leur validité.

- Une étude comparative. Cette étape consiste à faire une étude comparative de notre politique avec les politiques existantes afin de ressortir les retombées économiques de notre contribution.

La figure 1.5 présente un résumé de notre méthodologie.

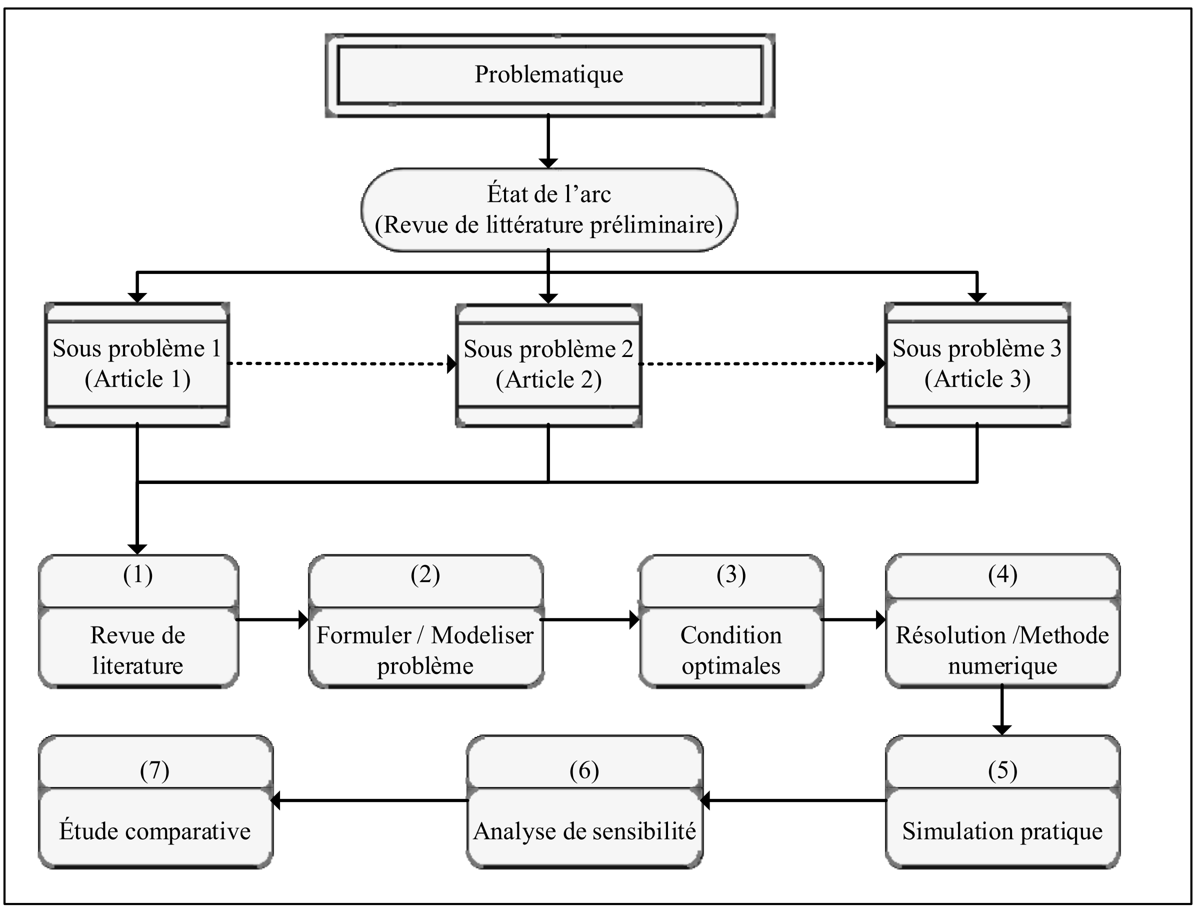


Figure 1.5 Diagramme de la méthodologie proposée

La démarche précédente nous a permis de résoudre les sous problèmes définis (03) et par suite le problème principal. De ce fait, les contributions de cette recherche sont présentées dans la section suivante.

1.6 Contribution et structure du mémoire

Ce mémoire a fait l'objet d'un article de conférence avec comité de lecture (Exposé à la conférence) et de trois (3) articles de revues. Ces articles sont développés dans les 3 prochains chapitres. La figure 1.6 présente un résumé des thèmes développés dans chaque article.

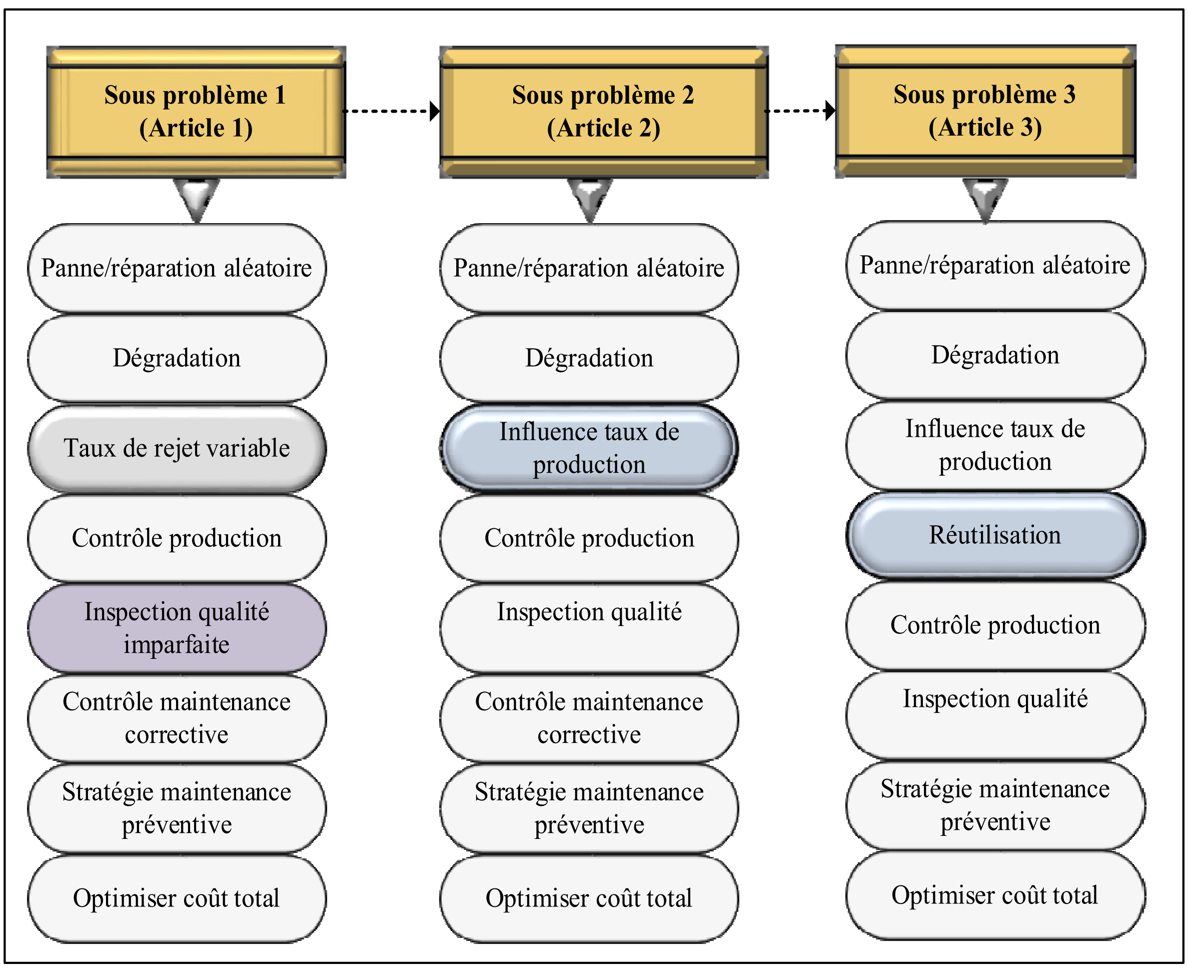


Figure 1.6 Synthèse des thèmes développés et contributions

Notre premier article présenté dans le chapitre deux (2) de ce mémoire, présente une étude d'un système de production, constitué d'une machine qui produit un seul type de pièces, qui se dégrade dans le temps et dont la dégradation affecte la qualité des produits fabriqués.
L'objectif de cette étude est de trouver une politique conjointe de production, de maintenance et de contrôle de qualité imparfaite, afin d'augmenter la disponibilité de la machine, d'améliorer la qualité des produits fabriqués et de minimiser le coût total de production. Les variables d'état sont le niveau d'inventaire et le taux de rejet de la machine. Les variables de décision sont : le taux de production, la fraction de produits à inspecter, les taux de maintenance préventive et corrective. Cet article a été soumis dans la revue International Journal of Modeling and Simulation sous la référence :

Megoze, P. P., J. P. Kenne, Edgar D. J. G and L. Hof. 2021. "Optimal joint production, maintenance and product quality control policies for a continuously deteriorating manufacturing system''. International Journal of Modeling and Simulation. Soumis en février 2021. Confirmation de soumission : JMS-ID: 215354006.

Dans le deuxième article du chapitre trois (3), nous avons ajouté une seconde machine qui assiste la première. Les deux machines produisent le même type de produits, le taux de production des machines impacte sur leur dégradation et elle produise des produits dont une certaine proportion n'est pas conforme. Les variables d'état sont le niveau d'inventaire et le taux de rejet de la machine. Les variables de décision sont : les taux de production, les fractions de produits à inspecter, les taux de maintenance préventive et corrective. Cet article vise donc à développer une politique conjointe de production, de maintenance et de contrôle de la qualité, afin de réduire le coût total de production, améliorer la qualité et limiter l'influence du taux de production sur la dégradation. Cet article a été soumis dans la revue Journal of Manufacturing Systems sous la référence :

Megoze, P. P., J. P. Kenne, Edgar D. J. G G. R. Kabuki and L. Hof. 2021. ''Production and quality control optimization strategies for deteriorating manufacturing systems under the influence of production rates''. Journal of Manufacturing Systems. Soumis en octobre 2021.

Dans le troisième article présenté au chapitre quatre (4), la machine qui assistait la première au chapitre trois (3) ne produit plus à partir de la matière première brute mais elle réutilise plutôt les produits en fin de vie (issu du marché) pour produire. De ce fait notre système d'étude est hybride (manufacturing/remanufacturing). Les deux machines se dégradent en fonction de leurs taux de production et cela affecte non seulement leurs fiabilités mais aussi la qualité des produits. Les variables d'état sont le niveau d'inventaire et les taux de rejet des machines. Les variables de décision sont : le taux de production, le taux de remise à neuf, les fractions de produits à inspecter, les taux de maintenance préventive et corrective. L'objectif principal de ce projet est de développer des politiques conjointes optimales de production, de réutilisation, de maintenance et de contrôle de qualité pour des systèmes de production hybrides en boucle fermée. Cet article a été soumis dans la revue International Journal of Advanced Manufacturing Technology sous la référence :

Megoze, P. P., J. P. Kenne, Edgar D. J. G G. R. Kabuki and L. Hof. 2021. ‘‘Production, maintenance and quality inspection planning of a hybrid manufacturing/remanufacturing system under production rate dependent deterioration’’. International Journal of Advanced Manufacturing Technology. Soumis en Décembre 2021.

1.7 Retombées et impact industriel

Les contributions issues des modèles développés dans cette recherche peuvent permettent d'optimiser la chaine de production des entreprises telle que :

- Les industries de fabrication de l'outillage mécanique : améliorer la qualité des outils, optimiser la production et limiter la dégradation.
- Les chaines industrielles pour l'usinage des pièces mécaniques : Limiter l'usure des outils de coupe en fonction de la fréquence d'usinage, optimiser la maintenance et réduire les couts.
- La ligne de fabrication et d'assemblage des automobiles : Réduire le temps d'inspection, réduire le coût d'inspection et le coût total de production.

- La ligne hydrique pour la fabrication et la réutilisation des Cartouches d'imprimante. (Kouedeu et al. 2015)

En général les modèles proposés permettent d'optimiser sous certaines hypothèses (définies plus haut) toute chaine de production /réutilisation soumise à des pannes et réparations aléatoires, et qui subit une dégradation en fonction de son utilisation et dont certains produits finis sont non conformes.

1.8 Conclusion

En résumé, il était question dans ce chapitre de présenter la question de recherche et la structure de ce mémoire. Pour cela, nous avons tout d'abord présenté le contexte industriel actuel afin de dégager la problématique de recherche, présenté la structure de la chaine de production étudiée, les hypothèses de travail et l'objectif principal de cette recherche. Par la suite, nous avons présenté une revue des concepts issus des travaux antérieurs et lié à notre domaine d'étude. Par la suite nous avons présenté la méthodologie qui sera suivie. Ensuite les contributions scientifiques de nos travaux et en fin les retombées sur le plan industriel.

CHAPITRE 2

OPTIMAL JOINT PRODUCTION, MAINTENANCE AND PRODUCT QUALITY CONTROL POLICIES FOR A CONTINUOUSLY DETERIORATING MANUFACTURING SYSTEM

Patrick Megoze Pongha [a], Jean-Pierre Kenné[b], Edgar De Jesus Garcia[c], Lucas A. Hof[d]

[a, b, d] Département de Génie de mécanique, École de Technologie Supérieure,
1100 rue Notre-Dame Ouest, Montréal, Québec, Canada, H3C 1K3
[c] C.E.T. FIRE PUMPS MFG INC. 75 Hector Street, P.O. Box 90, Pierreville, Québec, Canada

Article submitted for publication, February 2021

Résumé

Cet article présente une étude dans le but d'optimiser un système de production, constitué d'une machine produisant un seul type de pièces, la machine se dégrade dans le temps. Cette dégradation affecte sa disponibilité et augmente le taux de rejet des produits fabriqués. La machine est soumise à des pannes et des réparations aléatoires. Elle a un taux de rejet qui augmente en fonction de la dégradation. Une révision de la machine permet de réduire ce taux de rejet et de le ramener à sa valeur initiale. L'objectif de cette étude est de trouver une politique conjointe de production, de maintenance et de contrôle de qualité, afin d'augmenter la disponibilité de la machine, d'améliorer la qualité des produits fabriqués et de minimiser le coût total de production. Pour atteindre cet objectif, nous avons formulé le problème de recherche et utilisé une approche de programmation stochastique dynamique pour développer les conditions optimales de type Hamilton-Jacobi-Bellman (HJB). Ensuite, nous avons simulé une application pratique hybride pour optimiser la production d'une machine de découpe CNC Router classe R qui transforme des feuilles de polypropylène dans une entreprise québécoise (Canada) spécialisée dans la production de pompes à incendie. Les résultats obtenus nous ont permis de proposer à l'entreprise une politique de production à seuil critique, des stratégies de maintenance corrective et préventive, et un type de contrôle de qualité par échantillonnage. Enfin, nous avons effectué une analyse de sensibilité pour assurer la validation des politiques proposées.

Mots clés : processus stochastique, détérioration, maintenance, fabrication, qualité, politique optimale.

Abstract

This article presents a study with the aim of optimizing a production system, consisting of a machine that produces a single type of parts, that degrades over time. This degradation affects the availability of the deployed machine and increases the defective rate of the manufactured products. The machine is subject to random failures and repairs. It has a defective rate which increases with its degradation. An overhaul of the machine allows to reduce this defective rate and bring it back to its initial value. The objective of this study is to find a joint policy for production, maintenance and quality control, in order to increase the availability of the machine, improve the quality of the manufactured products and minimize the total cost of production. To achieve this goal, we formulated the research problem and used a dynamic stochastic programming approach to develop the Hamilton-Jacobi-Bellman (HJB) type optimum conditions. Then, we simulated a practical hybrid application to optimize the production of a Router class R CNC cutting machine that transforms polypropylene sheets in a Quebec (Canada) company specialized in the production of fire pumps. The obtained results allowed us to propose a critical threshold production policy, corrective and preventive maintenance strategies, and a sampling type of quality control to the company. Finally, we performed a sensitivity analysis to ensure the validation of the proposed policies.

Keywords: stochastic process, deterioration, maintenance, manufacturing, quality, optimal policy.

2.1 Introduction

The process of transforming raw material into a consumable product, goes typically through an imperfect manufacturing system. Such system is subject to random breakdowns and repairs and undergoes deterioration over time. This deterioration affects not only the availability of the system but also the quality of the produced parts. Therefore, maintenance, production control and product quality control strategies are key factors for the control and reliability of manufacturing systems. According to Rivera-Gomez et al. (Rivera-Gomez et al. 2020) the stochastic nature and the interactions between these three factors make the optimization of manufacturing systems that simultaneously integrate these factors a complex task. Faced with this complexity, several studies have been carried out firstly independent on each of these factors and subsequently mutual interactions have been progressively integrated. This is the case of the work of Kim and Gershwin (Kim and Gershwin 2005), who presented the structure of a manufacturing system and proposed processes for analyzing the performance of such a system. The same approach was deployed by Kouedeu et al. (Kouedeu et al. 2015) who have conducted a study to optimize a production system in jointly with maintenance strategies. Following this work, other studies have also emerged, and product quality control has been integrated into these studies. For example, we can refer to the work of Bouslah et al. (Bouslah et al. 2016). It should be noted that the independent optimization of one of these factors in some cases reduces production costs but deteriorates the quality of the products which makes the company less credible to its customers. In other cases, it improves the quality of the products but is very expensive. Hence, this context pushes companies to consider how to optimize their production systems by jointly considering the production, maintenance and quality control strategies of the products on the market. To address this research problem, we have structured this work in eight (8) parts as follows.

Section 2.2 presents a review of relevant literature and developments made in the field. The industrial context is detailed in section 2.3. In section 2.4, we formulated the research problem, defined the hypotheses and the parameters for modeling and simulation. Section 2.5 details the modeling of the problem, and it highlights the developed Hamilton-Jacobi-

Bellman (HJB) equations. Furthermore, section 2.5 entails the simulation of the developed model, and it presents the optimal policies, followed by sensitivity analysis to validate our model, which is discussed in section 2.6. Section 2.7 presents the implementation of the proposed policies, and finally section 2.8, concludes the research study, presenting its limitations and directions for future work.

2.2 Literature review

In the last decade, more and more companies have been looking at optimizing their production system through good maintenance and production strategies, as well as product quality control. This context has given rise to a growing number of scientific publications on the study of manufacturing systems subjected to a stochastic process with quality control (Cheng and Li 2020), (Guoqing Cheng 2020) Hence, the proposed system carries out a joint study on production, maintenance strategies and product quality control. These aspects have been widely studied individually in research in recent decades and very few joint two-by-two studies have been published. Assid et al. (Assid at al. 2015) and Colledani et al. (Colledani et al. 2014) In this work, we will divide our literature review into three (3) main parts, namely: optimization and maintenance strategies; degradation of the system and quality control; production, maintenance, and quality control.

2.2.1 Joint production and maintenance strategies

Most production systems are subject to random breakdowns and repairs. This disrupts production. To remedy this, new policies that consider maintenance strategies have been developed. In this context, Barlow and Hunter (Barlow. and Hunter. 1990) were among the first to conduct a joint production and maintenance. As a result, they have increased the availability of their production system, which led to several other studies. For example, Kenne and Gharbi (Kenne and Gharbi 2004) studied the production and its control strategies in an unreliable system to minimize the total production cost. They increased the availability of the machines through corrective maintenance strategies and demonstrated in their study that the policy of peak coverage is optimal. Research initiated by Barlow and Hunter

(Barlow. and Hunter. 1990) and further developed by Anis (Anis Chelbi 2008) and Roux et al. (Roux et al. 2013) as well as Chouikhi et al. (Chouikhi, Khatab, and Rezg 2011), resulted in the development of preventive maintenance strategies to reduce system degradation. They also proposed corrective maintenance strategies; however, they did not consider the influence of the production rate on system degradation. This effect was considered by, Kenne and Gharbi (Kenne and Gharbi 1999), where they have shown that the degradation of a system is also influenced by its production rate. This influence of the production rate on the degradation of the system is also considered by Dellagi et al. (Dellagi1, Rezg, and Gharbi 2010) and Ayed et al. (Ayed, Sofiene, and Nidhal 2012) in their work. Hence, to satisfy customer demand, they have studied the option of adding a subcontracted machine as a support to maintenance strategies. This is also the case for (Kouedeu et al. 2015) who took the production rate into account in their studies and proposed a joint production and maintenance policy for a system that produces a single type of product and that gradually deteriorates over time. This work was extended by (Tarek, Hajej, and Rezg 2016) for a system composed of several machines. This allowed them to propose corrective maintenance strategies and an economically viable production rate. Integration of preventive machine maintenance was studied by (F.I.Dehayem Nodem 2010) to reduce failure and repair times that increase over time, which was further optimized in their additional research work, (F. I. Dehayem Nodem 2011).. Here, when a failure occurs, in addition to a simple repair, a replacement of the machine is considered. This replacement option allowed the presented new policy to be more efficient than the one proposed in (F.I.Dehayem Nodem 2010). To complement prior work in this field, (Assid, Gharbi, and Hajji 2015) proposed new production policies combined with preventive maintenance. (Polotski, Kenne, and Gharbi 2019) extended the hedging point policy of (Kenne and Gharbi 2004) and proposed the curved hedging strategy. They demonstrated that in the event of a breakdown, it is advantageous to repair at the minimum rate and at low cost when inventory is close to the threshold level of coverage and demand is close to its minimum value. These studies do not consider the effects of system degradation on product quality. However, in an industrial context, it is necessary not only to satisfy customers in terms of quantity but also to distribute good quality products.

2.2.2 Production control with integration of the effects of degradation on product quality

To further improve research on the optimization of production systems, Ouaret et al. (Ouaret, Kenne, and Gharbi 2018a) extended the work of and developed new strategies to optimize a system whose deterioration impacts product quality and system availability. this allowed them to extend the work of Ouaret et al. (Ouaret et al. 2015). Therefore, they developed the option of replacing the machine after a certain period. The addition of this option allowed them to reduce the influence of deterioration on product quality. Nourelfath et al. (Nourelfath, Nahas, and Ben-Daya 2016) and several other researchers, Mehdi et al. (Mehdi, Nidhal, and Anis 2010), (Colledani 2011) and Colledani et Tolio T. (Colledani and T. Tolio 2012), proposed production policies whose decision variables are influenced and adapted depending on the quality of the manufactured products. This is also the case for Rivera-Gomez et al. (Rivera-Gomez, Gharbi, and Kenne 2013) who conducted research on the interactions between production and quality control aspects of produced parts for an unreliable system that deteriorates with age and produces non-conforming parts. They proposed critical threshold production strategies with corrective and imperfect maintenance to increase system availability and a so-called major overhaul to reduce system degradation. Bouslah et al. (Bouslah, Gharbi, and Pellerin 2016), extended this work to a two-machine system and studied the simultaneous interaction of machine degradation on product quality. Researck work conducted by Wu et al. (Wu et al. 2020),Ghaleb et al. (Ghaleb et al. 2020) and Duffuaa et al. (Duffuaa et al. 2020) proved that there is a strong influence of the level of machine deterioration on decision making and planning. Although the presented studies present significant advances in the optimization of production systems, they do most often not integrate an inspection of manufactured products, which is not reducing the quantity of non-compliant products received by customers.

2.2.3 Joint production, maintenance, and quality control strategies

To improve the quality of distributed products, new optimization policies that integrate product inspection have been developed. Hence, a product quality control policy with a constraint on the proportion of non-compliant products that can be distributed was developed by Bouslah et al. (Bouslah, Gharbi, and Pellerin 2016) in addition to proposing optimal production and maintenance strategies. They showed that the quality of manufacturing products influences the flow of decision-making information. However, the inspection parameters remain fixed, and the inspection was considered being perfect. To approach more realistic manufacturing scenarios, Mohammadi et al. (Mohammadi et al. 2015) considered imperfect inspection for a progressively deteriorating manufacturing system in their optimization policy. Nevertheless, this study did not include the development of maintenance strategies. In addition to considering imperfect quality control, Lopes (Lopes 2018) proposed a preventive maintenance policy. This new approach allowed to examine inspection errors and to get closer to a practical industrial context, although he did not consider the quality constraint on the distributed products. Kang and Subramaniam (Kang and Subramaniam 2018) developed policies that, in addition to the results of Lopes (Lopes 2018) included corrective and opportunistic maintenance strategies according to the inventory and the age of the machine. In this new policy, production downtime is exploited for machine overhaul operations. He Liu et al. (He et al. 2019), have developed a quality control policy using a process with a time between events control charts. This new approach allowed them to better study the optimization of the system performance and to control the quality of the products. Nevertheless, as the previous discussed work, He Lieu et al. (He et al. 2019), kept the product inspection parameters static. This approach considers only one state of degradation, which is often not realistic for systems that degrade with age or use. To address this issue, Paraschos et al. (Paraschos et al. 2020) considered several states of deterioration of the system and proposed a general overhaul of the machine to initialize the system. In this new approach, quality inspection is periodic. Another study was conducted by Guoqing Cheng (Guoqing Cheng 2020), which details a system consisting of several machines in parallel and discusses the development of a policy of permanent quality control of all manufactured products. This control system is more efficient but very expensive and sometimes impossible for mass production industries. Thus, Rivera-Gomez et al. (Rivera-Gomez et al. 2020) proposed a policy that jointly integrates preventive maintenance, quality control and production strategies with a constraint on Average Outgoing Quality (AOQ). They consider the quality control system being perfect (i.e., all non-conforming products are identified and rectified during inspection). It also does not propose a corrective maintenance policy, contrary of, industrial inspection systems that are generally not perfect (Zhang et al. 2021).

2.2.4 Summary and conclusion of the reviewed literature

A comparative summary of the previously discussed literature is presented in table 2.1 The rows of the table present the studied research and the columns present for each study the specific aspects developed as well as the considered factors.

Tableau 2.1 Synthesis of the literature

Author \ Research axis	Production control	Optimal stochastic control	Corrective maintenance	Preventive maintenance	Deterioration On availability	Effect of deterioration on quality	Constraint quality	Inspections imperfect	Variable inspection parameters
Studies for optimizing production and maintenance strategies									
(Barlow. and Hunter. 1990)	✓	✓	✓	✓	✓				
(Ayed, et al. 2012)	✓	✓		✓	✓				
(Kouedeu et al. 2015)	✓	✓	✓	✓	✓				
(Tarek, Hajej, and Rezg 2016)	✓	✓	✓	✓	✓				
(Chouikhi, et al. 2011)	✓	✓	✓	✓	✓				
(Assid, Gharbi, and Hajji 2015)	✓	✓	✓	✓	✓				
(F.I.Dehayem Nodem 2010)	✓	✓	✓		✓				
(F. I. Dehayem Nodem 2011)	✓	✓	✓	✓	✓				
(Anis Chelbi 2008)	✓	✓		✓	✓				
(Roux et al. 2013)	✓	✓		✓	✓				
(Dellagi1 et al. Gharbi 2010)	✓	✓		✓	✓				
(Kenne and Gharbi 2004)	✓	✓	✓		✓				
(Polotski et al. 2019)	✓	✓	✓		✓				

Tableau 2.1 Synthesis of the literature (Suite)

Research axis / Author	Production control	Optimal stochastic control	Corrective maintenance	Preventive maintenance	Deterioration On availability	Effect of deterioration on quality	Constraint quality	Inspections imperfect	Variable inspection parameters
Degradation effect of the system on the quality of the product									
(Colledani 2011)	✓	✓				✓			
(Colledani and T. Tolio 2012)	✓	✓		✓	✓	✓			
(Ouaret et al. 2018a)	✓	✓	✓	✓	✓	✓			
(Nourelfath et al. 2016)	✓	✓		✓	✓	✓			
(Rivera-Gomez et al. 2013)	✓	✓	✓	✓	✓	✓			
(Ouaret et al. 2015)	✓	✓		✓	✓	✓			
(Duffuaa et al. 2020))	✓	✓	✓	✓	✓	✓			
production, maintenance and quality control of products									
(Bouslah et al. 2016)	✓	✓		✓	✓	✓	✓		
(Lopes 2018)	✓	✓		✓	✓	✓		✓	
(He et al. 2019)	✓	✓		✓	✓	✓			✓
(Kang and Subramaniam 2018)	✓	✓	✓	✓	✓	✓		✓	
(Guoqing Cheng 2020)	✓	✓	✓	✓	✓	✓			✓
(Rivera-Gomez et al. 2020)	✓	✓		✓	✓	✓ ✓			✓
Proposed solution	✓	✓	✓	✓	✓	✓	✓	✓	✓

It appears from the above table, that many researchers have developed optimization policies integrating simultaneously inspection and preventive maintenance strategies but very few have considered the inspection system as imperfect, and very few integrate a constraint on the quality of the distributed products. In the presented work, we propose a common optimal policy of production, maintenance and quality control for a manufacturing system whose rejection rate varies with its degradation. To approach a realistic industrial scenario, we will consider that the inspection of manufactured products is imperfect as studied by (Lopes 2018). We will also improve this work by proposing corrective maintenance strategies. In summary, this study mainly concentrates on the following challenges:

- Propose a production policy that satisfies the random demand of the market while reducing production costs.
- Establish a quality constraint to limit the proportion of non-compliant distributed products.
- Propose an optimal quality inspection strategy according to the degradation of the system, while considering the errors of the inspection system.
- Propose corrective maintenance strategies to improve the availability of the system, which is not considered in the literature studies on quality inspection.
- Propose preventive maintenance strategies to reduce system degradation.

2.3 Industrial context

« The Quebec company, C.E.T. Fire Pumps Mfg. has been designing and building reliable and high-quality fire pumps, foam systems and portable devices for over 100 years, C.E.T Fire Pumps Mfg dates to 1908. »[3] In addition to this core product line, C.E.T. also offers its huge customer base from plastic water tanks to replacement impellers for pumps. C.E.T.'s tanks are manufactured through several stages including design, nesting, cutting, melting, assembly, water and high voltage leakage testing and tagging.

---

[3] About Us | CET Firepump (fire-pump.com) consult on the 02/12/2020.

Among its stages, we are interested in the cutting process. The conception phase ends once the three-dimensional (3D) model is approved by the client giving way to the nesting process that seeks to minimize the loss of material in the cutting table. A cutting program, optimized by a specialized software (MasterCam), is the input to the cutting process. There is only one cutting board to provide the required material for seven (7) assembly cells, whereby any unexpected delays at this stage will considerably penalize the assembly operation. Cutting is done on a CNC Router class R machine, which transforms the polypropylene sheets into several shapes intended for the manufacture of custom tanks. This machine is subject to random breakdowns and repairs; it degrades over time and this degradation affects its availability and the quality of the products. Therefore, to optimize the company's production at this stage, while considering the product quality problems that the company will encounter after a certain random time of operation, a hybrid simulation of the production of the CNC cutting machine was developed. The obtained result allowed us to: 1) propose a common policy of production, 2) perform quality control and 3) perform maintenance of the machine, which reduces the current production costs of the company and anticipates the improvement of the quality of the products. These strategies can also be used by other companies whose production system is unreliable and where the quality of the manufactured products deteriorates over time. This is the case, for example, for automotive companies, ink cartridge manufacturing or mechanical parts (Kouedeu et al. 2015).

2.4 Assumptions, and problem statement

To model this problem and facilitate its understanding, this section first presents the assumptions that are considered by this moderator and second present the problem.

2.4.1 Assumptions

To model the discussed manufacturing problem, we consider the following assumptions:

- H_1: The raw material is reliable good quality et always available.
- H_2 : The demand rate is known and is constant.

- H_3: Non-conforming products received by customers are returned to the company, which represents a cost of loss for the company.
- H_4: The machine degrades over time, and this negatively influences its availability and the quality of the products manufactured.
- H_5: The defective rate of the machine increases with its degradation.
- H_6: Repairs carried out during breakdowns are imperfect and bring the machine back to as bad as old condition (ABAO) which does not improve its state of degradation.
- H_7: The major repair is perfect and brings the machine back to as good as new condition (AGAN).
- H_8: The quality control system is sampling type, and the inspection is not perfect, therefore some non-conforming products are not identified during inspection.
- H_9: The average of non-conforming products (AOQ) on the market must not exceed a certain maximum limit (AOQLmax).
- H_{10}: Identified non-conforming products are destroyed.

2.4.2 Problem description

The structure we are studying, consists of a machine subject to random breakdowns and repairs, which produces a single type of parts to satisfy a deterministic demand. The system under study is subject to random breakdowns and repairs. This system produces non-conforming parts at a rate that varies depending on its condition. The machine degrades randomly over time and this manifest itself on its availability and production rate of non-conforming products. Repairs that are made following machine breakdowns are minimal and imperfect, which returns the machine to its previous state of degradation. However, a major overhaul (of a preventive nature) returns the machine to its original operating condition. Non-conforming products received by customers are also penalized and to reduce costs, an inspection using a quality-sampling plan is implemented. During this inspection, a fraction of the manufactured products is controlled, and non-compliant products are identified and subsequently destroyed. This inspection limits the quantity of non-conforming products in inventory; it also allows the manufacturer to identify the level of deterioration (function of AOQL based inspection sampling plan) and to initiate a major preventive revision of the system. The decision variables studied in this model are the production rate of the machine, the fraction of products to be inspected (depending on the deterioration of the system), the rate of admissible repairs and the rate of preventive maintenance. The objective here is to minimize the total cost of production, including the costs of shortages, inventory, inspection, repair, major overhaul, and penalty due to the unsatisfactory quality of the products received by the customer. figure 2.1 presents the structure of this developed system.

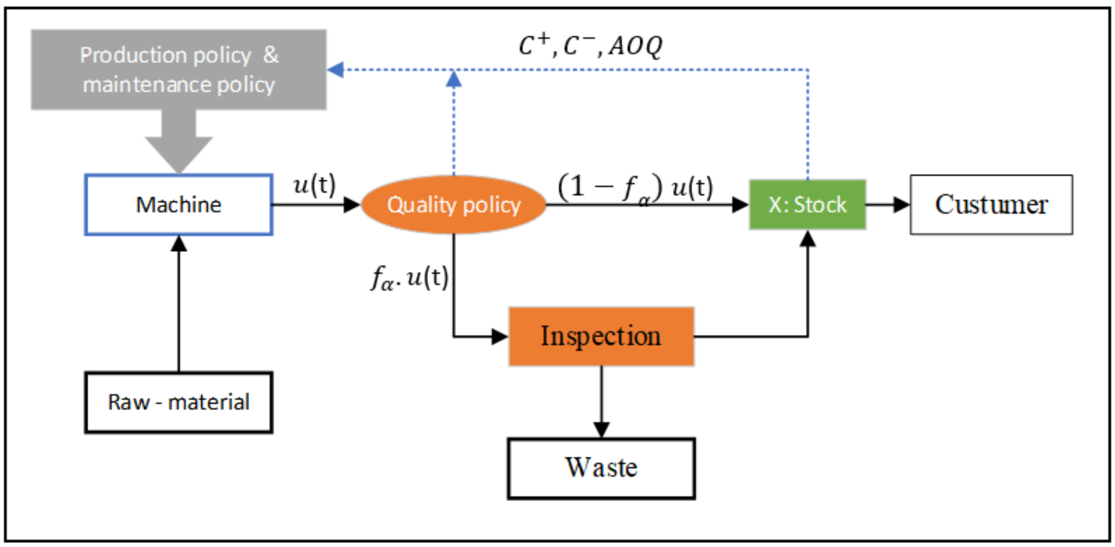


Figure 2.1 Structure of the studied system

2.5 Formulation of the optimal control problem

The machine is imperfect in its new state afterwards it works with its defective rate β1 (mode 1). This defective rate varies with the degradation of the system and after a certain random time, this degradation becomes very significant, and the defective rate noted β2 of the machine increases (mode 2). Let ξ (t) be the stochastic process describing the dynamics of the system at time t. Table 2.2 presents the different modes of ξ (t).

Tableau 2.2 mode of the stochastic process

Mode ξ (t)	Nature
1	Machine in operation and defective rate β1
2	Machine running but degraded (defective rate β2)
3	Faulty machine coming from mode 1 (corrective maintenance)
4	Faulty machine coming from mode 2 (corrective maintenance)
5	Machine in major overhaul (preventive maintenance)

The system can therefore be in the modes $\alpha \in \Omega = \{1,2,3,4,5\}$. Figure 2.2 illustrates the interaction between these different modes.

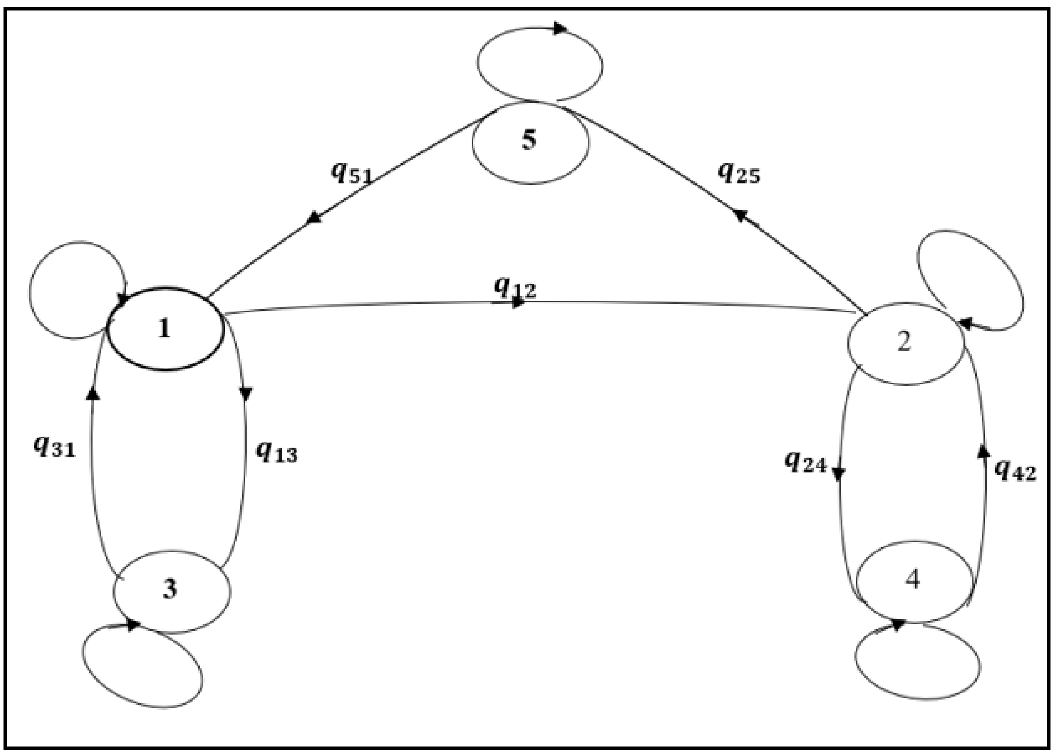


Figure 2.2 State transition diagram

The 100% quality process, which consists in inspecting all the manufactured products, is the best way to guarantee a very good quality of the products distributed, because during this inspection all the nonconforming products are identified, rectified, or destroyed and only the good quality products are received by the customers. This is the case of the policy proposed by (Rivera-Gomez et al. 2013). However, for mass production systems, or products that may take a long time to inspect, this policy is almost inapplicable. Thus, a quality control policy of batch sampling type based on statistical techniques was developed by and used by (Dodge 1943a) and has been used in several works such as (Rivera-Gomez et al. 2020). For this inspection, a fraction f ($0 \leq f \leq 1$) of the production is randomly selected and inspected. Since not all products are inspected, non-compliant products are among the products distributed.

For a production series:

- The control fraction is f_α *(*$0 \leq f_\alpha \leq 1$*)* and the remaining fraction is *(*$1-f_\alpha$*)*.
- Among the uncontrolled fraction, the proportion of non-conforming product is $(1-f_\alpha)\beta_\alpha$.
- The proportion of non-conforming product in the control fraction f is: $f_\alpha.\beta_\alpha$.

The inspection process is not perfect, so the resulting errors cause the non-identification of all non-conforming products. Note *Pei* the probability that a non-conforming product is not identified during the inspection. The proportion of non-conforming product identified is therefore $(1-Pe)f_\alpha.\beta_\alpha$. The proportion $f_\alpha.\beta_\alpha\ .Pe$ of non-compliant products is not identified. Thus, Average quality after control is noted (AOQ(α)) and given by the following relationship

$$\mathrm{AOQ}(f_\alpha,\beta_\alpha) = \frac{(1-f_\alpha)\beta_\alpha + f_\alpha.\beta_\alpha\ .Pe}{(1-(1-Pe)f_\alpha.\beta_\alpha)} \tag{2.1}$$

The control process is explained in Figure 2.3.

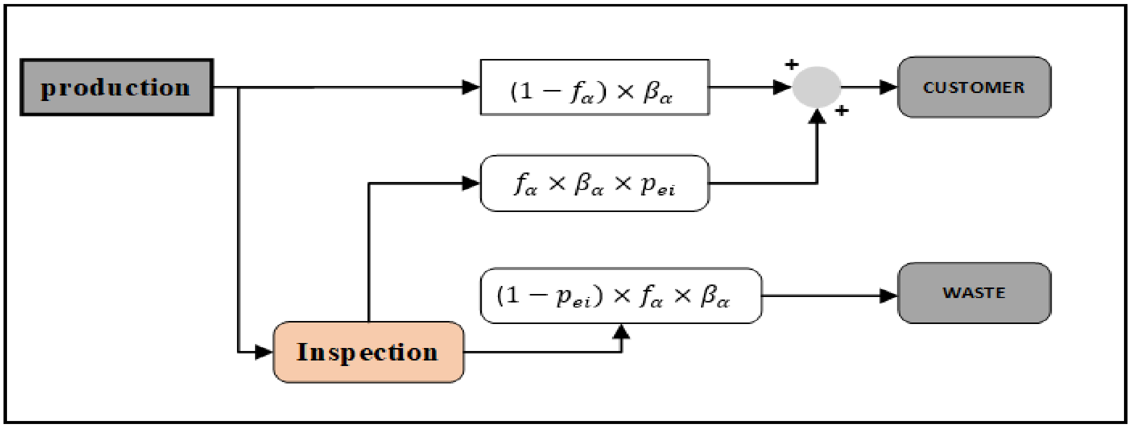


Figure 2.3 Inspection process and the average outgoing quality (AOQ)

The equation shows that the AOQ (f_α, β_α) depends only on f_α and β_α for Pe defined, then its limit value.

$$\text{AOQL} = \max \text{AOQ}\,(f_\alpha, \beta_\alpha) \qquad (2.2)$$
$$\text{With:} \quad 0 \le f_\alpha \le 1 \text{ and } 0 \le \beta_\alpha \le 1$$

The quality control policy that we will propose is to determine for a degradation mode characterized by β_α; the optimal fraction f_α^* ($0 \le f_\alpha^* \le 1$) which minimizes production costs. However, this determined fraction must also satisfy the following quality constraint AOQL ≤ AOQL^{max} as proposed by Rivera-Gomez et al. (Rivera-Gomez et al. 2020). The entire production of the machine is not put in stock because some of the non-conforming products are identified and destroyed upon inspection. Thus, the actual production rate of the system is lower than the production rate of the machine. This actual production rate, expressed in Rivera-Gomez et al (Rivera-Gomez et al. 2020) is given by:

2.5.1 Actual production rate of the machine in the system

The actual production rate, u_s^{α}, of the machine in the system is defined as follows:

$$u_s^{\alpha} = \frac{1}{t_p + t_{insp}} - f_{\alpha} \times \beta_{\alpha}(1 - Pe) \times \mathrm{u_p} = \frac{\mathrm{u_p} \times u_{insp}}{f_{\alpha} \times \mathrm{u_p} + u_{insp}} - f_{\alpha} \times \beta_{\alpha}(1 - Pe) \times \mathrm{u_p}. \tag{2.3}$$

with $\mathrm{t_p} = \frac{1}{\mathrm{u_p}}$; $\mathrm{t_{insp}} = \frac{f_{\alpha}}{u_{insp}}$; and $f_{\alpha} \times \beta_{\alpha}(1 - Pe) \times \mathrm{u_p}$ representing the quantity of products destroyed during the inspection.

The maximum production rate of the system is therefore:

$$^{\alpha}U_s^{max} = \frac{U_p^{max} \times \mathrm{u}_{insp}}{f_{\alpha} \times U_p^{max} + u_{insp}} - f_{\alpha} \times \beta_{\alpha}(1 - Pe) \times U_p^{max} \tag{2.4}$$

The process is modeled by a time-continuous, discrete-state, semi-Markovian chain with a matrix of transition rates $Q = (q_{\alpha j})$. Transition rates are defined as follows:

$$P[\xi(t + dt) = \gamma \,/\, \xi(t) = \alpha] = \begin{cases} q_{\alpha\gamma} * \delta t + o(\delta t)\ si\ \alpha \neq \gamma \\ 1 + q_{\alpha\gamma} * \delta t + o(\delta t)\ si\ \alpha = \gamma \end{cases} \tag{2.5}$$

With, $\lim_{(\delta t - - 0)} o(\delta t) = 0$ et $\alpha, \gamma \in \Omega = \{1, 2, 3, 4\}$

The transition between modes 2 and 4 corresponds to the sending of the machine to preventive maintenance.

Therefore,

$$Q = \begin{bmatrix} q_{11} & q_{12} & q_{13} & 0 & 0 \\ 0 & q_{22} & 0 & q_{24} & q_{25} \\ q_{31} & q_{32} & q_{33} & 0 & 0 \\ 0 & q_{42} & 0 & q_{44} & 0 \\ q_{51} & 0 & 0 & 0 & q_{55} \end{bmatrix} \tag{2.6}$$

$$q_{25}= w_p = \{0, 1\}; q_{31} = \{\ {}^1w_r^{min},\ {}^1w_r^{max}\}; q_{42} = \{\ {}^2w_r^{min},\ {}^2w_r^{max}\} \tag{2.7}$$

$$q_{25=w_p=\{0,1\}}\ q_{31=w_r^1=\{w_{min}^1, w_{max}^1\}}$$

The limits probabilities are given by the following equation:

$$\begin{cases} \sum_{j=1}^{5} \pi_j * q_{ij} = 0 \\ \sum_{i=1}^{4} \pi_i = 1 \end{cases} \tag{2.8}$$

Where π_α is the probability that the machine is in α mode; with $\alpha \epsilon \Omega = \{1, 2, 3, 4,5\}$.
The system is feasible if and only if: the actual maximum overall production of the system, in its production modes 1 & 2, is greater than or equal to the market demand. This translates into the following equation 12:

$$\pi_1 *\ {}^1U_s^{max} + \pi_2 *\ {}^2U_s^{max} \geq \frac{d}{1 - AOQL} \tag{2.9}$$

For each production model of our system (x= {1, 2}) we have: $U_\alpha(t)$ the production rate, $f_\alpha$the fraction to be controlled, w_r^3 the repair rate in mode 3 and w_r^4 the repair rate in mode 4. All production, quality control, corrective and preventive maintenance policies are defined as follows:

$$\Gamma(\alpha) = \begin{Bmatrix} u_\alpha(\cdot), f(\cdot), w_p, w_r^3, w_r^4 / \ 0 \leq u_p \leq U_p^{max}; 0 \leq f_\alpha \leq 1; \\ w_p = \{\ 0,1\ \}; 0.66 \leq w_r^3 \leq 1; 0.66 \leq w_r^4 \leq 1 \end{Bmatrix}, \tag{2.10}$$

For a production rate u_p (α), a defective rate $\beta(\alpha)$ and the fraction to be controlled f_α. The dynamics of the stock is:

$$\begin{cases} \frac{dx(t)}{dt} = u_s^{\alpha}(t) - \frac{d}{1 - AOQ(f_\alpha)} \\ x(0) = x_0 \end{cases} \tag{2.11}$$

With x_0 the initial inventory level and u_s^{α} (t) the production rate of the system at time t. The instantaneous production cost is made up of cost of shortage or stock possession, inspection cost, scrap cost, penalty cost on returned products and finally maintenance costs. Tableau 2.3 defines the expression of each of these costs.

Tableau 2.3 Different factors making up the instantaneous cost

Source	**Costs**	**Notation**
Cost incurred due to inventory and shortage	$C^+x^+ + C^-x^-$ *With* $x^+ = max\ (0, +x)$ *and* $x^- = max\ (-x, 0)$	$g_1(.)$
Cost Control	$C_{ins} \times f\alpha \times u_p$	$g_2(.)$
Destruction cost	$(1\text{-}Pe)\ \beta\alpha \times f\alpha \times u_p \times C_{des}$	$g_3(.)$
Return cost	$AOQ(f\alpha) \times u_p \times C_{ret}$	$g_4(.)$
Maintenance cost	$C_m^c \times w_m * Ind\{\alpha=4\} + C_m^p \times w_m \times Ind\{\alpha=5\}$ With: $Ind\ \{d(.)\} = \begin{cases} 1\ if\ d(.)\ is\ true \\ 0\ otherwise \end{cases}$	$g_5(.)$

The instantaneous cost is therefore:

$$g(\alpha, x) = g_1(.) + g_2(.) + g_3(.) + g_4(.) + g_5(.) \tag{2.12}$$

The discounted cost noted J(.), is defined by the following equation:

$$J(x, \alpha, u(.), f_\alpha) = E\left[\int_0^\infty e^{-\rho t} g(\alpha, x(t))\,.dt \mid x(t) = x, \xi(t) = \alpha\right] \tag{2.13}$$

with ρ the discount rate. The objective of our policy is to minimize the total discounted cost, we need to determine the variables in the eligible domain that minimize J(.). The value function of the problem is given by the following equation:

$$\text{v}(\text{x}, \alpha) = \min_{(u(.), f_\alpha) \epsilon \Gamma(\alpha)} J(x, \alpha, u(.), f_\alpha, .) \tag{2.14}$$

Solving this equation by the analytical method is very complex. Therefore, we will use Kushner's numerical method, which is discussed in section 2.6.

2.6 Optimal conditions and numerical example

The optimum condition of our policy is the one that minimizes the discounted cost of production. Consequently, its variable must satisfy equation E.2 of the value function. Thus, from equation E.2, we have followed the same approach as (Kouedeu et al. 2015) to develop the HJB equations. The general equation of HJB obtained is defined as follows.

$$\rho\vartheta(x, \alpha) = \min_{(u;f)\epsilon\Gamma(\alpha)} \left[g(x, \alpha, u) + \frac{\partial\vartheta(\alpha, x)}{\partial x} * \frac{dx(t)}{dt} + \sum_{j\epsilon\Omega} q_{\alpha j}\vartheta(j, x) \right] \tag{2.15}$$

For example, we will consider the Router class R CNC cutting machine from the Quebec company CET Fire Pumps, which can transform 4' x 8' polypropylene sheets of half an inch thick. They are cut into all kinds of shapes and are used to manufacture custom tanks. The maximum number of sheets that can be cut per day is 7. The average processing time of a sheet is 1.5 hours, the average running time between breakdowns is 110 days and the repair time varies between 1 and 1.5. We will later expand its data to improve our model. Table 2.4 summarizes the data we have collected from CET Firepumps.

Tableau 2.4 Case study data for the CNC Router Class R cutting machine

Variables	U_p^{max}	d	C^+	C^-	C_r	C_m	C_d	C_{reb}	C_i	P_{ei}	q_{12}
Values	7	4	3	50	30	50	150	30	10	0,05	0,002
Variables	q_{13}	q_{24}	q_{25}	q_{31}^{max}	q_{31}^{min}	q_{42}^{max}	q_{42}^{min}	q_{51}	β_1	β_2	U_i
Values	0.01	0,008	0 or 1	1	0.66	0.66	0,04	1	0,1	0,3	10

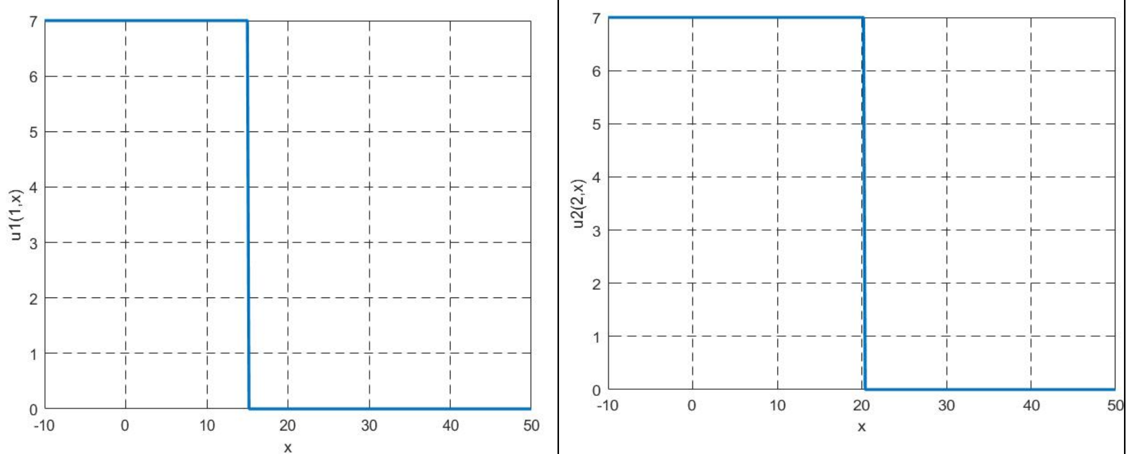
The degradation of the system influences not only the availability but also the quality of the products. This can be continuous and characterized by a high defective rate.

$$B(t) = b_0 + b_1 t \tag{2.16}$$

Where b_0 and b_1 are either given by the machine manufacturer or determined experimentally. The data in table 2.4 are such that the system satisfies the feasibility condition of the equation (2.16). Hence, it can be concluded that the system is feasible.

2.6.1 Production Policy

By applying the production conditions and the values of the parameters given above to the MATLAB resolution program in the appendix, we obtain the results and the figures so we will interpret as follows.



(a): Production rate of machine in mode 1 | (b): Production rate of machine in mode 2

Figure 2.4 Production Policy

- Figure 2.4.a informs us that if the stock level of the product is below the threshold stock $Z_1^* = 15$, the machine must produce its maximum production rate (U^{max}). Then the machine must produce on demand adjusted until it reaches a stock threshold point of $Z_1^{*\prime} = 15.01$, beyond which it will stop producing.
- Figure 2.4.b indicates that the machine must produce in mode 2 at its maximum production rate($Umax$) if the stock level of the products is below the threshold stock $Z_2^{*\prime} = 20.2$. Between Z_2^{*}=20.21, the machine's production rate varies if the machine produces on demand. Beyond $Z_2^{*\prime}$, the machine stops producing.

The critical threshold in mode 2 is higher than the critical threshold in mode 1. This is quite normal because in mode two (02) the machine is degraded afterwards its defective rate and failure rate are higher, so the system protects itself by increasing its protection stock. The policy we propose is at a critical threshold as proposed by Rivera-Gomez et al. (Rivera-Gomez et al. 2020), kouedeu et al. (Kouedeu et al. 2015) and many others in the literature. It will consist in our case for the two modes of production to determine the critical thresholds Z_1* and Z_2* which minimizes the total cost of production and allows to satisfy the demand despite the uncertainties (breakdown and distribution time), which influence the production. The production policy for each mode of operation of the machine is therefore:

$$u_p^*(\alpha, x) = \begin{cases} U_p^{max} & \text{if } x(t) < Z_\alpha^* \\ d/(1 - AOQ(\alpha)) & \text{if } x(t) = Z_\alpha^* \\ 0 & \text{if } x(t) > Z_\alpha^* \end{cases} \quad (2.17)$$

In summary we have:

$$u_P(1, x) = \begin{cases} U_p^{max} & \text{if } x(t) < Z_1^* \\ d/(1 - AOQ(1)) & \text{if } x(t) = Z_1^* \\ 0 & \text{if } x(t) > Z_1^* \end{cases} \quad (2.18)$$

$$u_P(2, x) = \begin{cases} U_p^{max} & \text{if } x(t) < Z_2^* \\ d/(1 - AOQ(2)) & \text{if } x(t) = Z_2^* \\ 0 & \text{if } x(t) > Z_2^* \end{cases} \quad (2.19)$$

2.6.2 Quality control policy

Figure 2.5 show the total cost of production as a function of controlled fractions. For each fixed value of the fraction to be controlled in mode 2, we varied the fraction to be controlled in mode 1. This allowed us to obtain for each fixed value of $f_2$the curve representing the total production cost as a function of the fraction to be controlled f_1. Figure 2.6.a illustrated its different costs. In the same way, we fixed f_1 and varied f_2 (Figure 2.6.b).

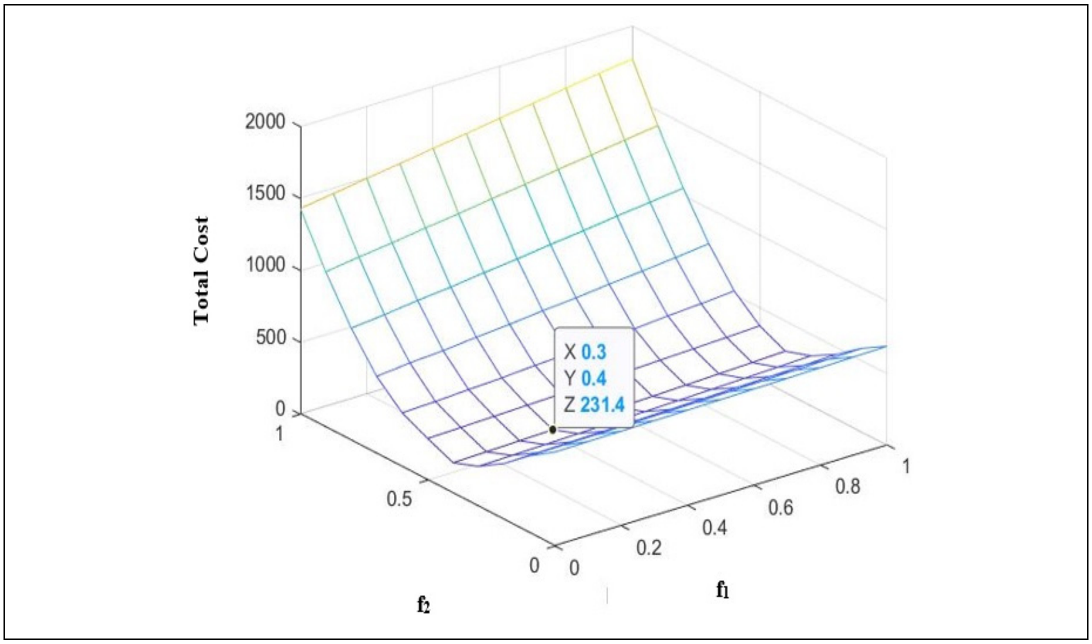


Figure 2.5 Total product cost as a function of controlled product fractions

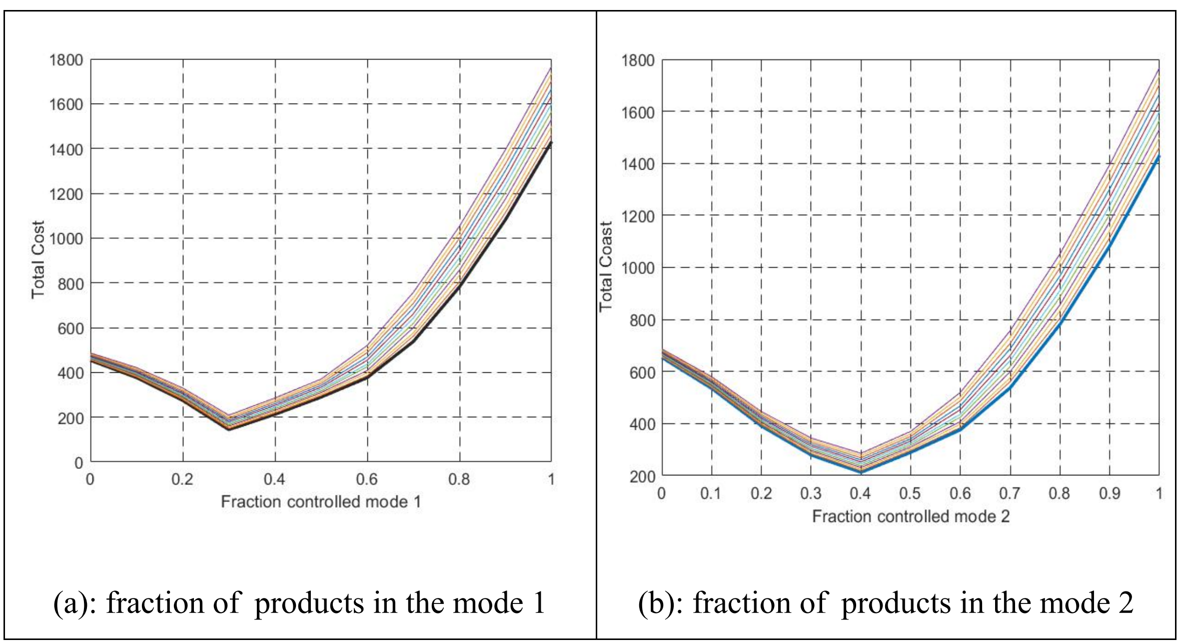


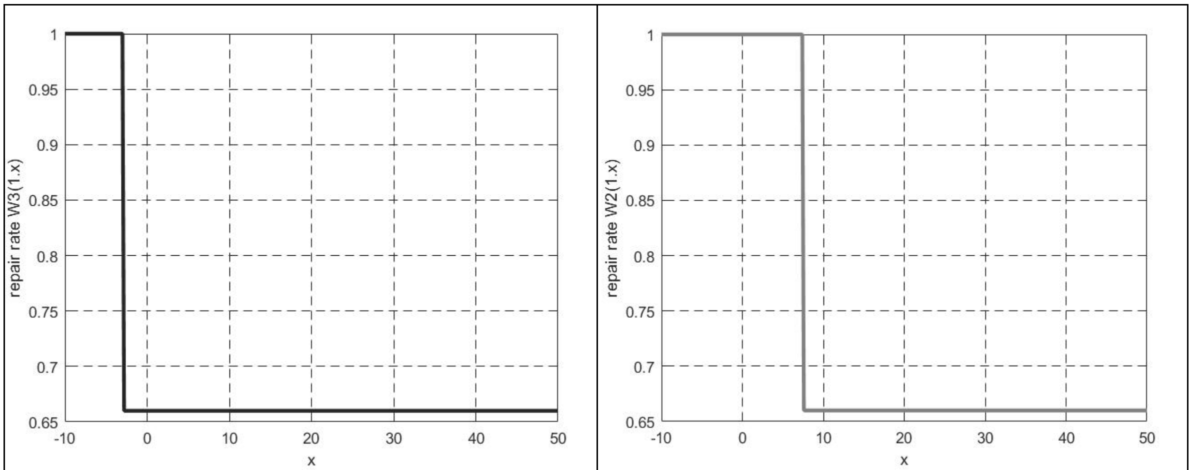
Figure 2.6 Inspection policy in modes 1 and 2

These curves show us that:

- In the initial operating mode 1, f_1 = 30% of the production must be controlled to minimize production costs.
- In mode 2 of degraded operation of the machine, f_2 = 40% of the production must be controlled to minimize production costs.

2.6.3 Corrective maintenance policy

The maintenance policy represented below allows the manufacturer to make decisions when a breakdown occurs. Thus, depending on the stock level, he can call upon an expert to reduce the repair time from its maximum value (${}^{\alpha}w_r^{max}$) to its minimum value (${}^{\alpha}w_r^{min}$). This policy is presented in the following Figure 2.7.a, and Figure 2.7.b



(a): Repaire rate of the machine in mode 3 (b): Repaire rate of the machine in mode 4

Figure 2.7 Corrective maintenance Policy in modes 3 and 4

So, we have:

$$W_r^3 = \begin{cases} {}^{3}w_r^{max} = 0.66 & \text{if } x(t) \le Y_1^* = -3 \\ {}^{3}w_r^{min} = 0.01, & \text{if not} \end{cases} \tag{2.20}$$

$$W_r^4 = \begin{cases} {}^4w_r^{max} = 0.02 & \text{if } x(t) \leq Y_1^* = 7.5 \\ {}^4w_r^{min} = 0.01 & \text{if not} \end{cases} \tag{2.21}$$

2.6.4 Preventive maintenance policy

Preventive maintenance: The machine is deteriorating, and this is reflected in the significant increase in its defective rate from β_1 to β_2. A general overhaul reduces this defective rate and returns the machine to its initial operating mode (Mode 1: β_1). The following Figure 2.8 illustrates this for our example.

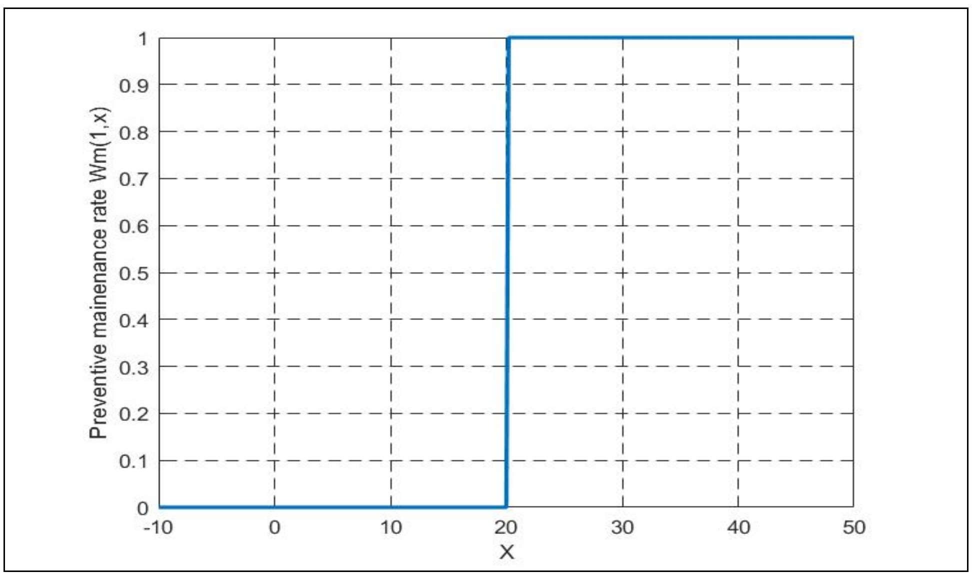


Figure 2.8 Preventive maintenance policy in degraded mode 2

Thus, we have:

$$W_P = \begin{cases} 1 & if\ \ \beta(t) = \beta 2\ \ and\ x(t) = Y_2^* = Z_2^* \\ 0 & if\ not \end{cases} \tag{2.22}$$

The previous practical example allowed us to develop interactive policies for joint optimization of production, maintenance, and quality control. However, we need to do more practical simulations to analyze the sensitivity of our model.

2.7 Sensitivity Analysis

To analyze the sensitivity of the proposed model, we will perform various simulations by varying each time one of the system costs and other parameters of the machine. Then we will analyze the variation of the optimal parameters of the different proposed policies and compare this variation to the expected predictions. In our case, we are interested in six (06) parameters, namely: The change in inventory cost (C^+) for inventory x; the change in the of shortage cost (C^-) for inventory x, the change in the cost of inspections (Cinsp), the change in the cost of penalties on non-compliant products distributed to customers (Cret), change in inspection rate (ui), change in the cost of repairs (Cr) and preventive maintenance (Cm). The varying behavior of these policy parameters is summarized in the table 2.5.

Tableau 2.5 Variation of system parameters

C^+	**3**	**5**	**7**	**13**	**15**	**Variation**
Z_1	15,00	10,71	7,29	6,43	2,14	↓
Z_2	20,02	15,28	12,66	9,95	4,87	↓
F_1	0,3	0,47	0,51	0,6	0,65	↑
F_2	0,4	0,5	0,57	0,66	0,75	↑
C^-	**50**	**60**	**65**	**70**	**75**	
Z_1	15,00	19,29	30,00	32,14	42,86	↑
Z_2	20,02	33,01	40,69	42,75	50,86	↑
F_1	0,3	0,28	0,25	0,17	0,1	↓
F_2	0,4	0,39	0,35	0,32	0,3	↓
Cret	**150**	**200**	**300**	**400**	**450**	
Z_1	15,00	12,86	11,01	8,57	5,5	↓
Z_2	20,02	15,47	13,20	10,50	8,5	↓
F_1	0,3	0,52	0,55	0,6	0,66	↑

Tableau 2.5 Variation of system parameters (Suite)

C_{ret}	150	200	300	400	450	
F_2	0,4	0,6	0,65	0,72	0,82	↑
C_i	**10**	**12**	**14**	**17**	**20**	
Z_1	15,00	20,29	25,71	31,43	33,86	↑
Z_2	20,02	27,05	29,54	35,71	40,04	↑
F_1	0,5	0,49	0,47	0,42	0,35	↓
F_2	0,66	0,6	0,59	0,55	0,49	↓
Ui	**10**	**11**	**12**	**13**	**14**	
Z_1	15,00	13,86	12,07	6,57	5	↓
Z_2	20,02	17,37	16,80	14,50	12.56	↓
F1	0,3	0,48	0,55	0,6	0.66	↑
F2	0,4	0,67	0,75	0,85	0.88	↑

C_r	**30**	**35**	**40**		**C_m**	**50**	**55**	**60**	
Z_1	15,00	12.02	9.01	↓	Z_1	15,00	15,8	17	↑
Z_2	20,02	17.016	16.02	↓	Z_2	20,02	21,01	22.04	↑
F_1	0,3	0,26	0.15	↓	F_1	0.3	0.57	0.65	↑
F_2	0,4	0,37	0.26	↓	F_2	0.4	0.67	0.85	↑

- Inventory Cost (C^+): We see that the critical thresholds (Z_1 *and* Z_2) decrease with the increase of C+ which is normal because the system stocks less to reduce the storage costs. The fractions to be controlled increase to reduce unnecessary costs on poor quality products stored. Figure 2.9.a, and figure 2.9.b shows the behavior of these parameters as a function of the inventory Cost.

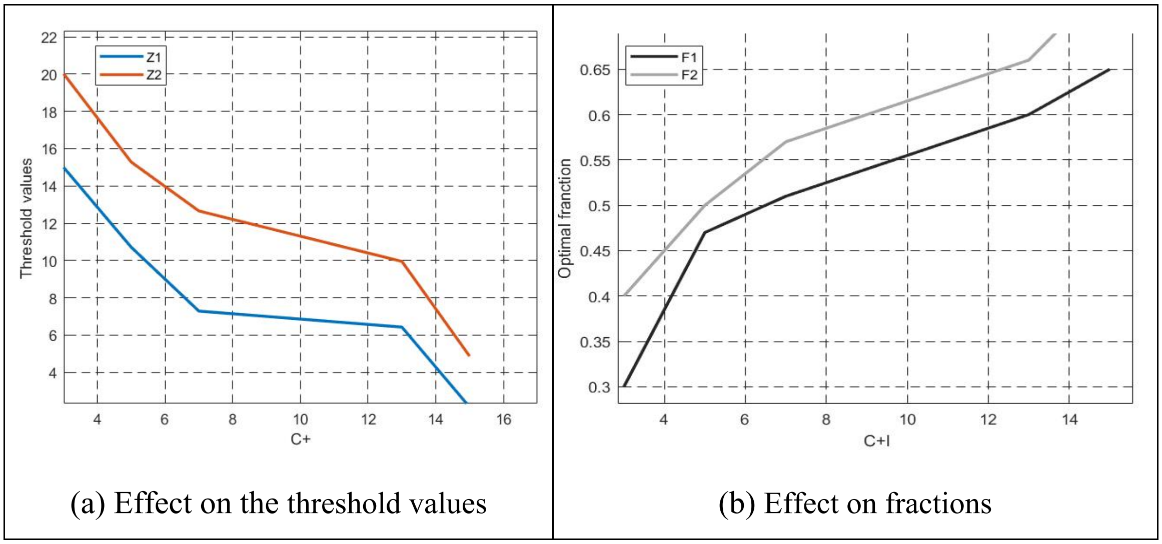


Figure 2.9 Effect of variation of inventory cost

- <u>Cost of shortages (C^-):</u> We see the opposite effect to the previous one because as shortages become more costly, the system stocks more to reduce costs and the fractions to be controlled decrease. Figure 2.10.a, and figure 2.10.b shows the behavior of these parameters as a function of the storage cost.

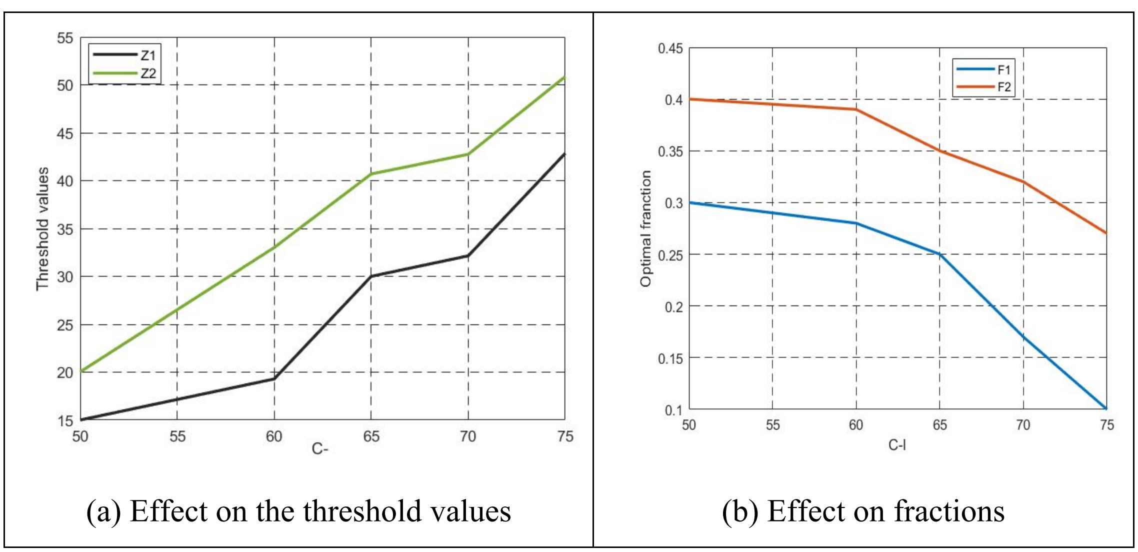
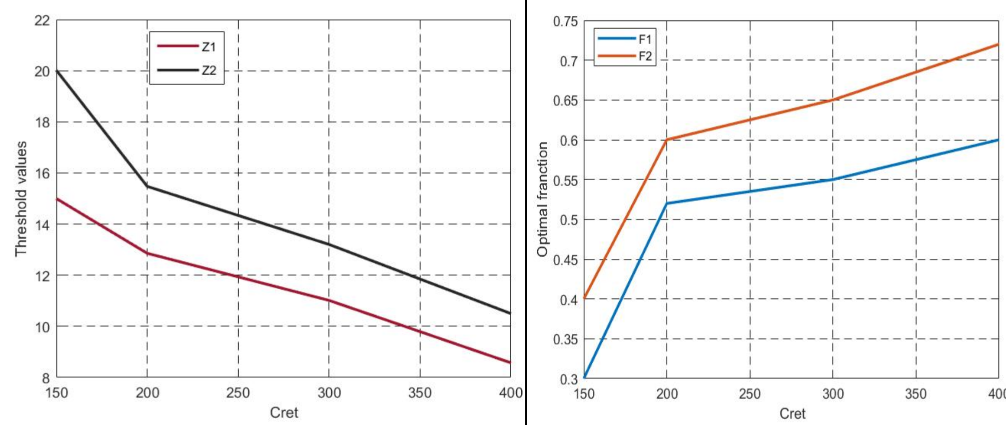


Figure 2.10 Effect of variation of Cost of shortages

- <u>Cost of penalties on the return of non-conforming products (**C**$_{ret}$):</u> (Penalty cost on non-compliant products): The increase in these costs causes an increase in the fractions to be controlled, which is normal as the system reduces the non-compliant products distributed. Figure 2.11.a, and Figure 2.10.b shows the behavior of these parameters as a function of the Cost of penalties.

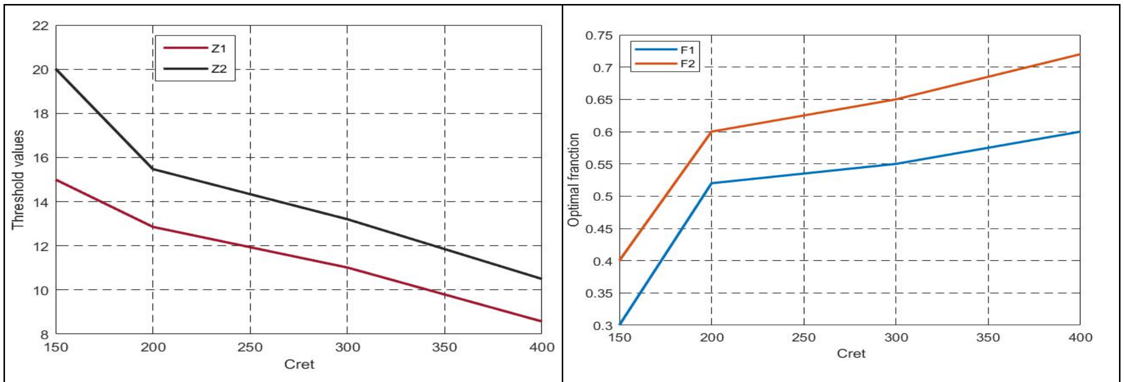


(a) Effect on the threshold values

(b) Effect on fractions

Figure 2.11 Effect of variation of Cost of penalties

- <u>Inspection cost (Ci):</u> We notice that when the inspection cost increases the fractions to be controlled decreases which is normal because the objective is to minimize the total production cost. We also note that the safety stock levels have increased, which is normal because the products are less inspected and therefore there is more non-compliant product in inventory than before. Figure 2.12.a, and Figure 2.12.b shows the behavior of these parameters as a function of the Cost of penalties.



(a) Effect on the threshold values | (b) Effect on fractions

Figure 2.12 Effect of variation of Inspection cost

- Inspection rate (Ui). When the inspection rate increases, the fractions to be inspected increase, which is normal because the inspection time has decreased. It is also noted that the recommended stock threshold has decreased which is also normal because more product is inspected which reduces bad products in inventory.

- Maintenance cost (Cm and Cr). When the cost of repairs increases (repairs carried out by experts), the fractions to be inspected decrease, which is normal because the degradation of the machine is slowed down. It is also noted that the recommended stockpiles have decreased, which is also normal because repairs are faster, and the machine is more available. As far as preventive maintenance costs are concerned, we observe an inverse variation. This is normal because when the cost of preventive maintenance increases, we reduce the sending of the machine to a general overhaul because we must inspect more product and have a higher safety threshold before this overhaul.

2.8 Implementation of the obtained policies

For optimal production with quality control, the procedure to be followed by the manufacturer is as follows:

- Step 1 (S1): identification of the state (operation /failure) of the machine.
  During this step, the manufacturer checks whether the CNC cutting machine is broken or in operation.
- Step 2 (S2): Identify the percentage of non-conforming products manufactured by the machine in operation or just before the failure.

The manufacturer must, after step I, identify the proportion of propylene sheet cut by the machine that does not meet all the standards. If the machine is failure, the manufacturer must consider the proportion that precedes the failure.

Depending on the state of failure or operation of the machine (step I), depending on the defective rate identified (step II), the operator must follow one of the following four procedures.

- If the machine is in operation and this percentage is less than 10%, the operator must inspect 30% of the total production. He must produce at the maximum rate if the sheet already processed and stored is less than 15. Once this quantity is reached in inventory, the manufacturer must reduce the production rate and cut the sheets according to the adjusted demand. Once the inventory level exceeds 15 products, production must be stopped.
- If the machine is in operation and this percentage is greater than 10%, the operator must inspect 40% of the Total production. Produce at the maximum rate of the machine if the inventory level is less than 20 sheets. Once 20 sheets are processed, the production rate must be reduced to the adjusted market demand rate. As soon as the inventory level exceeds 20 products, production must be stopped, and the machine must be sent for preventive maintenance for a general overhaul.

- If the machine is out of order and was rejecting more than 10% of its production before the breakdown, the manufacturer must repair the machine normally if there is a shortage of more than three products. The average repair time allowed is 1.5 days. In case there is a shortage of at least 3 products, the manufacturer must call in experts, if necessary, to repair the machine within 1 day on average.
- If the machine is out of order and it was rejecting more than 10% of its production before the breakdown, the manufacturer must repair the machine normally if the inventory is greater than 7 products. The average repair time allowed is 1.5 days. Otherwise, the manufacturer must call in experts, if necessary, to repair the machine within 1 day on average.

To better illustrate its procedures, we have schematized them on the diagram in Figure 2.13

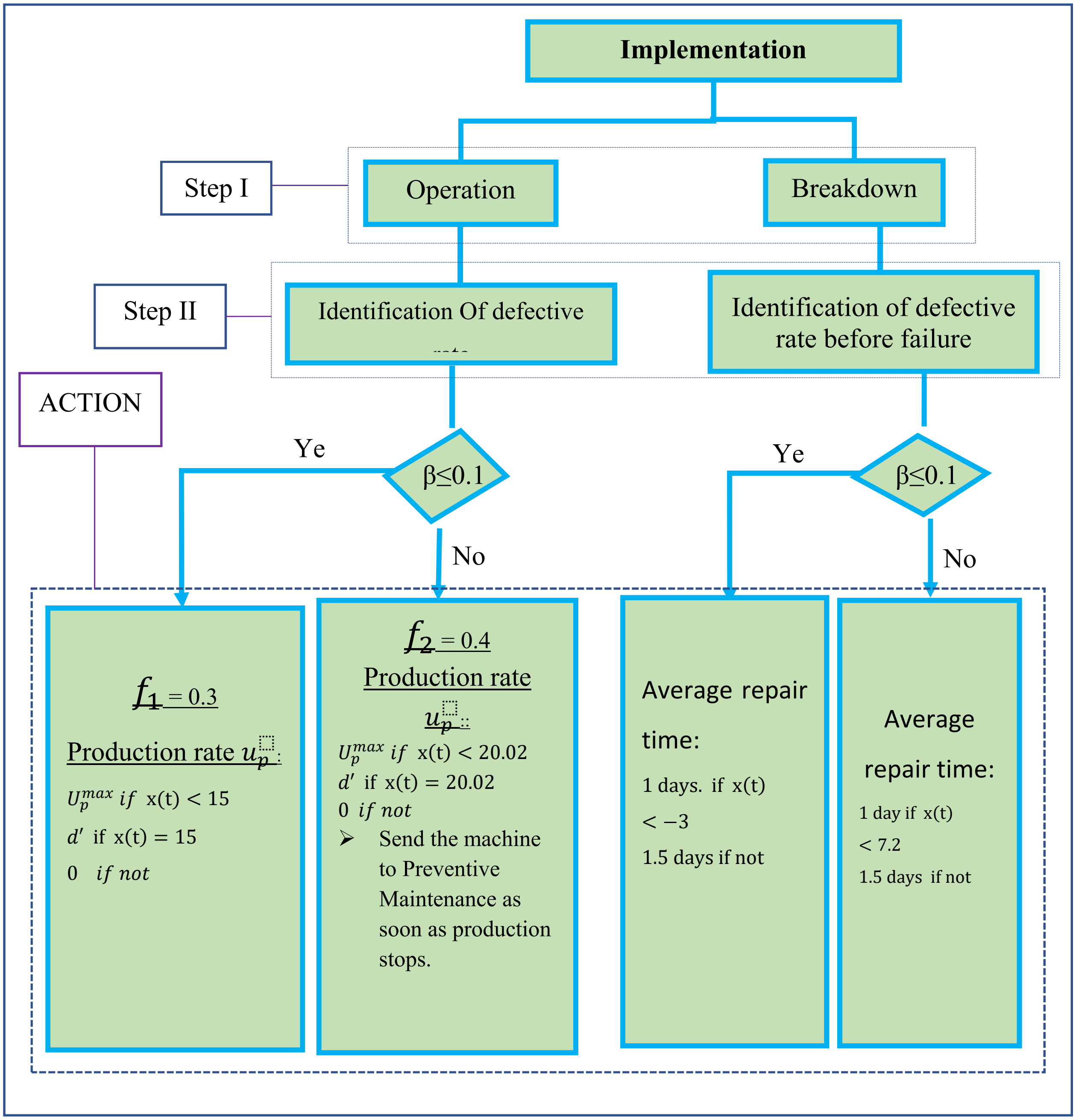


Figure 2.13 Implementation diagram

The obtained results allowed us to propose a: 1) critical threshold production policy, which is to maintain a safety stock of cut sheet to be used to meet demand during unintentional system shutdowns; 2) a sampling type quality control, which consists in controlling a proportion of the cut sheets, identifying the non-conforming ones and destroying them; 3) a preventive maintenance policy, which consists in exploiting the voluntary downtime of the machine when it is degraded to carry out a general overhaul; 4) a corrective maintenance policy, which consists of repairing the machine at its maximum rate (more expensive) or at its minimum rate (less expensive) depending on the stock level. Finally, a sensitivity analysis allowed us to validate the proposed policies.

2.9 Conclusion

This work focused on the study and optimization of a production system in conjunction with maintenance and quality control strategies to increase the availability of the system, improve the quality of the manufactured products and minimize the total cost of production. To achieve this objective, we formulated the problem and developed the dynamics of the system to optimize the decision variables (production rate, repair rate, preventive maintenance dispatch rate and fraction to be inspected). We have by the numerical method through the resolution of the equations H JB, determine the optimal conditions of our decision variables. We then simulated a numerical example of our policy to optimize the production of the Router class R CNC cutting machine of the Quebec company CET Fire Pumps. Our results can be useful for companies that have unreliable machines, that degrade over time and that produce parts that do not comply. This work can be extended in future work for systems with variable demand rates, non-Markovian systems or systems made up of several machines.

CHAPITRE 3

PRODUCTION AND QUALITY CONTROL OF DETERIORATING MANUFACTURING SYSTEMS UNDER THE INFLUENCE OF THE PRODUCTION RATES

Patrick Megoze Pongha [a], Jean-Pierre Kenné[b], Edgar De Jesus Garcia[c],
Guy Richard Kibouka[d], Lucas A. Hof[e]

[a, b, e] Department of Mechanical Engineering, École de Technologie Supérieure,
1100 Rue Notre-Dame Ouest, Montréal, Québec, Canada, H3C 1K3
[c] C.E.T. FIRE PUMPS MFG INC. 75 Hector Street, P.O. Box 90, Pierreville, Québec, Canada
[d] Department of Mechanical Engineering, Omar Bongo University,
Libreville, Gabon

Article submitted for publication, October 2021

Résumé

Cet article présente l'étude d'un système de production composé de deux machines qui produisent un seul type de produit. Les deux machines, fonctionnant en parallèle, ne sont pas identiques, et sont soumises à des pannes et des réparations aléatoires. Elles se dégradent en fonction de leurs cadences de production. Cette dégradation récurrente des systèmes industriels réduit leur disponibilité et affecte négativement la qualité des produits fabriqués. Ainsi, cette recherche vise à développer une politique conjointe de production, de maintenance et de contrôle de la qualité, qui permet de réduire le coût total de production, et de limiter les effets de la dégradation sur la disponibilité et sur la qualité des produits. Pour atteindre cet objectif, dans une première étape, nous avons modélisé le problème de contrôle optimal et nous avons utilisé une approche de programmation dynamique stochastique pour développer les conditions de Hamilton-Jacobi-Bellman (HJB). Dans un deuxième temps, nous avons utilisé une méthode numérique basée sur l'approche de Kushner pour résoudre les équations HJB. Dans un troisième temps, nous avons simulé un exemple pratique où les résultats obtenus nous ont permis de proposer une politique de production de type seuil critique. Cette stratégie consiste à réduire le taux de production à l'approche du seuil critique de stock. De plus, nos résultats nous ont permis de proposer une politique d'inspection de type échantillonnage pour limiter la distribution de produits de mauvaise qualité. Dans un troisième temps, nous avons effectué une analyse de sensibilité pour assurer la validation des politiques proposées. Enfin, nous avons effectué une analyse comparative de notre politique par rapport aux politiques adaptées couramment utilisées dans la littérature. Cette étude a confirmé l'optimalité de la politique conjointe proposée.

Mots-clés : fabrication, processus stochastique, détérioration, maintenance, taux de production, qualité, politique, méthodes numériques.

Abstract

This paper presents the study of a production system consisting of two machines, each producing a single product type. The two machines, operating in parallel, are non-identical, and they are subjected to breakdowns and random repairs. They degrade according to their production rates. This recurrent degradation of industrial systems reduces their availability and negatively affects the quality of manufactured products. Hence, this research aims to develop a joint policy of production, maintenance and quality control, which allows reducing the total cost of production, and limits the effects of the degradation on the availability and on the quality of the products. To achieve this objective, in a first step, we modeled the optimal control problem and we used a stochastic dynamic programming approach to develop the Hamilton-Jacobi-Bellman (HJB) conditions. Secondly, we used a numerical method based on the Kushner approach to solve the HJB equations. In a third step, we

simulated a practical example where the obtained results allowed us to propose a critical-threshold type production policy. This strategy consists in reducing the production rate when approaching the critical inventory threshold. In addition, our results allowed us to propose a sampling-type inspection policy to limit the distribution of poor-quality products. In a third step, we performed a sensitivity analysis to ensure the validation of the proposed policies. Finally, we conducted a comparative analysis of our policy against adapted policies commonly used in the literature. This study confirmed the optimality of the proposed joint policy.

Keywords: manufacturing, stochastic process, deterioration, maintenance, production rate, quality, policy, numerical methods.

3.1 Introduction

In recent decades, many researchers have studied production systems to increase their efficiency, reduce their total production cost and ensure customer satisfaction. However, production systems are subject to random breakdowns and repairs, as well as to the degradation of their components. In this context, it is difficult for companies to continuously satisfy the market demand if they do not stock certain products. This is due to production downtime and the influence of degradation on product quality (Hajej, Rezg, and Gharbi 2021). In addition, the market demand is variable and asking for ever-increasing product personalization (Hof and Wüthrich 2017). According to Hof and Wüthrich (Hof and Wüthrich 2017), this trend of the industrial market will push companies progressively to revolutionize their manufacturing processes and production systems to adapt to the evolving industry 4.0 paradigm. Therefore, to anticipate on this randomly varying demand, some companies, as presented in Yang et al. (Yang al. 2014), He, Liu et al. (He et al. 2019), manufacture and stock their products in advance. The major disadvantages of this practice are increased storage costs, and risks of stock destruction of some products. To solve these problems, other production strategies such as Just-in-Time, are practiced for long time (Kootanaee et al. 2013). This strategy consists in producing only when the customer's need arises, which limits the stock, resulting in cost reduction and limiting the risk of destruction. As this strategy has solved the problems resulting from excess inventory, the stochastic nature of the production process can cause stock-outs when the customer need arises, which is also negatively impacting the company. Kouedeu et al. (Kouedeu et al. 2015) proposed a mixed solution, which consists in determining an optimal threshold stock that the company must store to satisfy the demand when the system is not in operation. Several studies based on this strategy have emerged and they have proposed strategies for production control, maintenance and quality control of products. In this context, we simulated a practical example to optimize the CNC cutting system of a Canadian company (C.E.T. Fire Pumps Mfg Inc.) specialized in the manufacturing of fire pumps, which are a crucial part of most water-based fire protection systems. In this case study, we have considered joint strategies of production, maintenance, and quality control of the products on the market. To achieve this research objective, we have structured this work in nine (9) parts as described in the following.

Section 3.2 presents a review of the literature already done in the field and section 3.3 presents the industrial context and case study. Section 3.4 outlines the parameters, assumptions, and a formulation of the optimal control problem. Section 3.5 defines the optimal conditions, and it demonstrates a numerical example. Section 3.6 reveals the sensitivity analysis, which has been conducted to ensure validation of the proposed policies. Section 3.7 details a comparative study of our developed policy with other existing policies. Section 3.8 presents the implementation procedure, and it finally in section 3.9, it concludes this research, presenting its limitations and perspectives for future work.

3.2 Literature review

During the last decades, several scientific studies on the optimization of production systems have been conducted to propose best strategies for maintenance, production, and improvement of product quality for manufacturing companies. The random nature of its relating parameters (repair, breakdown, demand, production, and quality) makes the simultaneous study of these aspects complex. Hence, researchers have conducted separate studies of each of these aspects and they published gradually their joint studies. In this context, Barlow and Hunter (Barlow. and Hunter. 1990) had published one of the first joint studies of production and corrective maintenance. After this new optimization approach, several other studies were published. In summary, the different contributions listed in this literature review can be divided into four (4) research areas: (i) joint production and maintenance policies with influence of degradation on reliability; (ii) production policy with integration of degradation on product quality; (iii) optimal policy with integration of the influence of the production rate on the degradation; and finally (iv) production control with integration of product quality control strategies. These research fields are discussed in detail in the following sections 2.1, 2.2, 2.3. and 2.4 respectively.

3.2.1 Joint production and maintenance strategies with influence of degradation on reliability

Manufacturing systems comprise machines that are subject to random breakdowns and repairs. In addition, these machines degrade progressively. This degradation reduces the performance of the production system, negatively affecting its reliability. Hence, to improve the production process of a company, researchers must consider the effects of machine degradation on production performance in their studies. Therefore, Kenne and Gharbi (Kenne and Gharbi 2004) have proposed new corrective maintenance strategies to increase the availability of machines. These new policies allowed for increasing productivity, however without considering machine degradation issues. to consider such machine degradation, other preventive maintenance strategies have been developed (Roux et al. 2013); (Chouikhi, Khatab, and Rezg 2011). Another preventive maintenance strategy was proposed by Nodem et al. (Nodem, J.-P.Kenné, and A.Gharbi 2010) to reduce the repair times that increase with machine degradation. In a next step, they proposed the option of replacing the machine after a certain degradation to be more optimal (Nodem, Gharbi, and Kenne 2011). This work was extended by Polotski et al. (Polotski, Kenne, and Gharbi 2019) through their proposed hedge-curve strategy. This strategy involves reducing the cost of corrective maintenance by making repairs at the minimum rate when the inventory can meet the demand. Previous work has mitigated the negative impact of machine degradation on the reliability of the production system, but the influence of the production rate on degradation has been neglected so far. However, according to Kouedeu et al. (Kouedeu et al. 2015) the production rate of a system also influences its degradation.

3.2.2 Optimal policy with integration of the influence of the production rate on the degradation

In their work, Kouedeu et al. (Kouedeu et al. 2015) have shown that the production rate of the machine impacts negatively on the machine and accelerates its degradation. Hence, the degradation of the machines does not depend only on their age but also on their rates of production. To limit this influence, Dellagi et al. (Dellagi1, Rezg, and Gharbi 2010) and Ayed et al. (Ayed, Sofiene, and Nidhal 2012) have proposed the option of adding a subcontracting machine that will assist the main machine in production. The addition of this second machine satisfied the demand and reduced the production time to the maximum rate by the main machine. Similarly, Kouedeu et al. (Kouedeu et al. 2015) proposed a joint production and maintenance policy for a system of two machines that produces a single type of product and gradually degrades over time. By adding the second machine, they were able to limit the influence of the production rate on the degradation of the principal machine. The work of Kouedeu et al. (Kouedeu et al. 2015) was continued and extended to a multi-machine system by Tarek et al. (Tarek, Hajej, and Rezg 2016). The works have proposed very interesting policies to optimize the performance of a production system that degrades according to its utilization rate. This new approach has brought us closer to a practical industrial application. Nevertheless, the effects of the system degradation on the quality of the products was not yet considered. In an industrial context, it is not only necessary to satisfy customer demands, but also to ensure good product quality.

3.2.3 Production policy with integration of degradation on product quality

To enhance product quality, Colledani and Tolio (Colledani and Tolio 2011) established the relationship between production and quality to control the effect of system deterioration on its availability, the product quality and production optimization. However, during this study they do not consider maintenance and quality control strategies (Colledani and Tolio 2011). To remedy this situation, Ouaret et al. (Ouaret, Kenne, and Gharbi 2018a) extended the work of Ouaret et al. (Ouaret et al. 2015) by proposing a maintenance policy including replacement of the machine after a significant degradation. In this context, other researchers, such as Mehdi et al. (Mehdi, Nidhal, and Anis 2010), Colledani and Tolio(Colledani and T. Tolio 2012), and Nourelfath et al.(Nourelfath, Nahas, and Ben-Daya 2016), adapted the decision variables according to the quality of products at inspection. Wu et al. (Wu et al. 2020), Ghaleb et al. (Ghaleb et al. 2020) and Duffuaa et al. (Duffuaa et al. 2020) have adapted decision making and planning of production operations in their work to improve product quality. These studies have significantly improved the optimization of production lines and reduced the effect of degradation on product quality. However, the continuous inspection of products is not considered. As a result, the amount of non-conforming products received by the customers is not reduced.

3.2.4 Production control with integration of product quality control strategies

To compete and retain customers, companies must not only satisfy customer demand but also distribute good quality products. Therefore, products must be of good quality, according to Robotis et al. (Robotis, Boyaci, and Verter 2012). To reduce the number of non-conformities products in inventory, an inspection strategy for manufactured products must be implemented in the production chain. In this context, Rivera-Gomez et al. (Rivera-Gomez, Gharbi, and Kenne 2013) proposed an inspection of all manufactured products in their policies to be more effective (Rivera-Gomez, Gharbi, and Kenne 2013). Bouslah et al. (Bouslah, Gharbi, and Pellerin 2016) applied the work of Rivera-Gomez et al. (Rivera-Gomez, Gharbi, and Kenne 2013) in a system consisting of two machines. This study allowed them to propose a policy that simultaneously integrates product inspection and machine degradation control. A continuous inspection has been developed by Mohammadi et al. (Mohammadi et al. 2015) and Bouslah et al. (Bouslah, Gharbi, and Pellerin 2016) in their work. This allowed them to propose optimal production, maintenance, and quality control strategies. This new approach allows us to effectively address industrial production problems. This work has been extended by Lopes (Lopes 2018) by including the errors of the inspection system, but they do not integrate the maintenance strategies. Kang and Subramaniam (Kang and Subramaniam 2018) and He et al. (He et al. 2019) optimized the work of Lopes (Lopes 2018), adding opportunistic maintenance policies. This novel approach consists of preventive maintenance operations during voluntary non-production times. However, the previous studies do not consider the influence of the progressive degradation of the system on the quality of the products. Consequently, during their studies, they maintained the static inspection parameters. This approach is limited because age and production rates influence the degradation of a production system Paraschos et al. (Paraschos, Koulinas, and Koulouriotis 2020). In their work, Paraschos et al. (Paraschos, Koulinas, and Koulouriotis 2020) proposed a general overhaul as a strategy to rejuvenate the machine and a dynamic product inspection. This is also the case in the work of Guoqing Cheng (Guoqing Cheng 2020) with a continuous inspection of all manufactured products. This new approach has significantly improved the quality of distributed products. However, this policy remains very expensive for large system series or systems with long inspection times. In this context, Rivera-Gomez et al. (Rivera-Gomez et al. 2020) proposed a sampling type of inspection with a constraint on average non-conforming products in inventory to limit the fraction of distributed non-conforming products. In their studies (Rivera-Gomez et al. 2020), they have considered the inspection system as perfect, and they do not consider the influence of the production rate on the degradation of the machine and the product quality. They also neglected the influence of the operation of the machine on its degradation. However, according to Zhang et al. (Zhang, Shen, and Ma 2020), the industrial inspection system is generally imperfect. Similarly, Kenne and Gharbi (Kenne and Gharbi 1999) showed that the degradation of a system is also influenced by its production rate. To extend the work of Rivera-Gomez et al. (Rivera-Gomez et al. 2020), Ait-El-Cadi et al. (Ait-El-Cadi et al. 2021) proposed a dynamic sampling design considering the effects of degradation on product quality and system reliability. In addition, they also studied the influence of operation on degradation but neglected the influence of production rate on degradation and product quality. This influence of production rate is considered in the work of Hajej et al. (Hajej, Rezg, and Gharbi 2021). They proposed a dynamic sampling plan for a production unit consisting of a machine affected by its production rate, nevertheless, they neglected the inspection time as well as the rectification time of the inspected non-conforming products.

3.2.5 Summary and conclusion of the reviewed literature

Table 3.1 outlines a summary of the papers discussed in this literature review. The horizontal rows of the table detail the studied papers and the columns present the developed key aspects of optimal controls well as identifying the elements considered by the authors of each paper.

Tableau 3.1 Literature review synthesis

Research axis / Authors	Production control	Maintenance	Effect of deterioration On reliability	Effect of deterioration On quality	Influence of the production rate	Quality inspection	Quality constraint	Variable inspection	Dynamic sampling	Influence of inspection
(Barlow and Hunter 1990)	✓	✓								
(Kenne and Gharbi 2004)	✓	✓	✓							
(Roux et al. 2013)	✓	✓	✓							
(Chouikhi et al. 2011)	✓	✓	✓							
(F. Dehayem 2010)		✓	✓							
(F. Dehayem 2011)		✓	✓							
(Polotski et al. 2019)		✓	✓							
(Kenne and Gharbi 1999)		✓			✓					
(Sofiene Dellagi1 2010)	✓		✓		✓					
(Ayed et al. 2012)	✓		✓		✓					
(Kouedeu et al. 2015)	✓	✓	✓		✓					
(Tarek et al. 2016)	✓	✓	✓		✓					
(Colledani and Tolio 2011)	✓		✓	✓						

Table 3.1 Literature review synthesis (Suite)

Authors \ Research axis	Production control	Maintenance	Effect of deterioration On reliability	Effect of deterioration On quality	Influence of the production rate	Quality inspection	Quality constraint	Variable inspection	Dynamic sampling	Influence of inspection
(Ouaret et al. 2015)	✓		✓	✓						
(Ouaret et al. 2018a)	✓	✓	✓	✓						
(Nourelfath et al. 2016)	✓		✓	✓						
(Mehdi et al. 2010)	✓	✓		✓						
(Colledani and Tolio 2012)	✓	✓	✓	✓						
(Wu et al. 2020)	✓	✓	✓	✓						
(Ghaleb et al. 2020)	✓		✓	✓						
(Duffuaa et al. 2020)	✓	✓	✓	✓						
(Robotis et al. 2012)	✓	✓	✓	✓		✓				
(Rivera-Gomez et al. 2013)	✓		✓	✓		✓				
(Bouslah et al. 2016)	✓		✓	✓		✓				
(Bouslah et al. 2016)	✓	✓	✓	✓			✓			
(Lopes 2018)	✓		✓	✓		✓	✓			
(Kang and Subramaniam 2018)	✓	✓	✓	✓		✓	✓			
(He et al. 2019)	✓	✓	✓	✓		✓	✓			
(Paraschos et al. 2020)	✓	✓	✓	✓		✓	✓			
(Guoqing Cheng 2020)	✓	✓	✓	✓		✓		✓		
(Zhang et al. 2020)	✓	✓	✓			✓				
(Rivera-Gomez et al. 2020)	✓	✓		✓		✓	✓	✓	✓	✓
(Hajej et al. 2021)	✓	✓	✓	✓	✓	✓	✓		✓	
(Ait-El-Cadi et al. 2021)	✓	✓	✓	✓		✓	✓		✓	
Proposed solution	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

We can see from this summary that several authors have conducted research on production systems and have proposed optimal production, maintenance, and quality inspection policies. However, very few have considered an imperfect inspection system. In addition, the influence of production rate on machine degradation and product quality is not considered. In this work, we will study a system consisting of one principal machine subject to breakdowns and random repairs that manufactures parts with a rate of rejection depending on the rate of production, and a second machine that assists the principal one, which is also subject to repairs and random breakdowns. Thus, we will focus on a joint production, maintenance and quality control policy model that simultaneously incorporates the following specific objectives:

- Satisfy the demand even in times of failure.
- Limit the negative influence of the production rate on the degradation of the machines.
- Reduce the negative influence of the production rate on the quality of the products.
- Propose a quality inspection strategy according to the level of degradation.
- Limit and reduce the distribution of non-conformities products.
- Optimize the total production cost.

3.3 Industrial context

The policies proposed in this paper aim at optimizing standard industrial systems consisting of machines that are subject to random breakdowns and repairs and that degrade with time according to their production rate. Such systems are subject to a degradation of their different components, depends on their frequency of use. This degradation negatively affects the performance of an industrial system (Kouedeu et al. 2015) and reduces the quality of the manufactured product (Ait-El-Cadi et al. 2021). It is therefore important to study the negative effect of this degradation on product quality to limit their undesirable impact. In addition, the simultaneous integration of all aspects in the proposed policy allows the company not only to satisfy customer demand but also to distribute high quality products. To increase practical relevance of the proposed policy development, we used a practical manufacturing environment case study for this work. Therefore, we integrated actual data from a computer numerical control (CNC) cutting machine (EBF, Canada, Class R – CNC router) of the Quebec company C.E.T. Fire Pumps Mfg[4]. in our study. This machine transforms 4 feet by 8 feet polypropylene sheets of half an inch thickness into different shapes for the manufacturing of custom water tanks. The CNC cutting machine is subject to random breakdowns and repairs and degrades over time as a function of its production rate. This degradation influences the reliability of the machine as well as the quality of the obtained products. Therefore, to reduce the consequences of machine degradation according to its operation, we proposed to the company the possibility of adding a second machine to assist the principal machine in the production. We have modeled and simulated the hybrid production system consisting of the CNC cutting machine and assisted by the second machine that supports it. The obtained results allowed us to propose a joint policy to control the machine's production rate, the quality of finished products, as well as the preventive and corrective maintenance of the machines. This proposed model allows reducing the total production cost and limiting the negative influence of the production rate on the machines' reliability and on the quality of the manufactured products. These strategies can also be used by other industries facing a continuous deterioration of their manufacturing system. For example, this is the case in the automotive industry, in the chemical industry, and in the manufacturing of mechanical and electronic parts (Kouedeu et al. 2015).

3.4 Assumptions, and problem formulation

In this section, we firstly define the assumption nomenclature that drive the proposed model and secondly, we present the problem statement of the study.

---

[4] The Flow Starts Here. Powerful, Portable, Pumps | CET Firepump (fire-pump.com) consulted on 25-09-2021

3.4.1 Assumptions

To formulate the optimization problem, we will consider the following assumptions:

- H_1: The main machine M1 has a higher production rate and can satisfy the demand by itself.
- H_2: The second machine M2 has a lower production rate and cannot satisfy the demand by itself.
- H_3: The raw material is of good quality and always available.
- H_4: The demand rate is known and is constant.
- H_5: The maximum production rates of each machine are known.
- H_6: The different costs: storage, shortage, inspection, maintenance are constant and known;
- H_7: The non-conforming products received by the customers are returned and replaced by the company, which causes losses for the company.
- H_8: The machines degrade with time, and this negatively influences their availability.
- H_9: The repairs carried out at the time of the breakdowns of the machines are imperfect and bring back the machine in a state as bad as old (ABAO) which does not improve its state of degradation.
- H_{10}: The preventive maintenance of a machine is perfect and brings it back to as good as news condition (AGAN).
- H_{11}: The quality control system is of the sampling type; so that a proportion of the products manufactured is inspected.
- H_{12}: The average proportion of non-compliant products (AOQ) on the market must not exceed a maximum limit (AOQLmax).
- H_{13}: Identified non-conforming products are destroyed.

Assumptions H_1 and H_2 are a classic feature of this approach, as the second machine is a subcontracting machine that assists the principal one to reduce its degradation (Kouedeu et al. 2015). Assumptions H_3 and H_4 are standard assumptions in supply chain network models (Kenne et al. 2012). Assumptions H_5 and H_6 are common in production planning. Assumptions H_7, H_8 and H_9 are unique to this approach because of the consideration of customer satisfaction and the effect of operation on the frequency of machine failures. Assumption H_{10} and H_{11} are common in maintenance policy. Assumption H_{12} is frequent in dynamic production inspection processes. Assumption H_{13} is common in the unit product production model.

3.4.2 Description of the problem

The studied manufacturing system consists of two non-identical machines in parallel, which are both unreliable as they are subject to random breakdowns and repairs. They produce a single type of parts to satisfy a variable demand. Figure 1 outlines the structure of such system. Each machine produces parts of which a fraction is non-conforming, and their degradation varies with their production rates. A part of the production noted f_i^{α} subjected to a quality control, and the identified non-conforming products are destroyed. The first machine, noted M_1,, is the principal machine, which is assisted by the second machine, noted M_2. A fraction of the manufactured products is inspected during the quality control, and the non-conforming products are identified and destroyed. The studied decision variables in this model are the production rate of the machines M_1, and M_2, the fraction of products to be inspected, corrective rate and preventive maintenance rate. The objective is to minimize the total cost of production including the costs of shortage, inventory, inspection, repair, preventive maintenance, and penalty due to non-satisfaction of the customers having received non-conforming products.

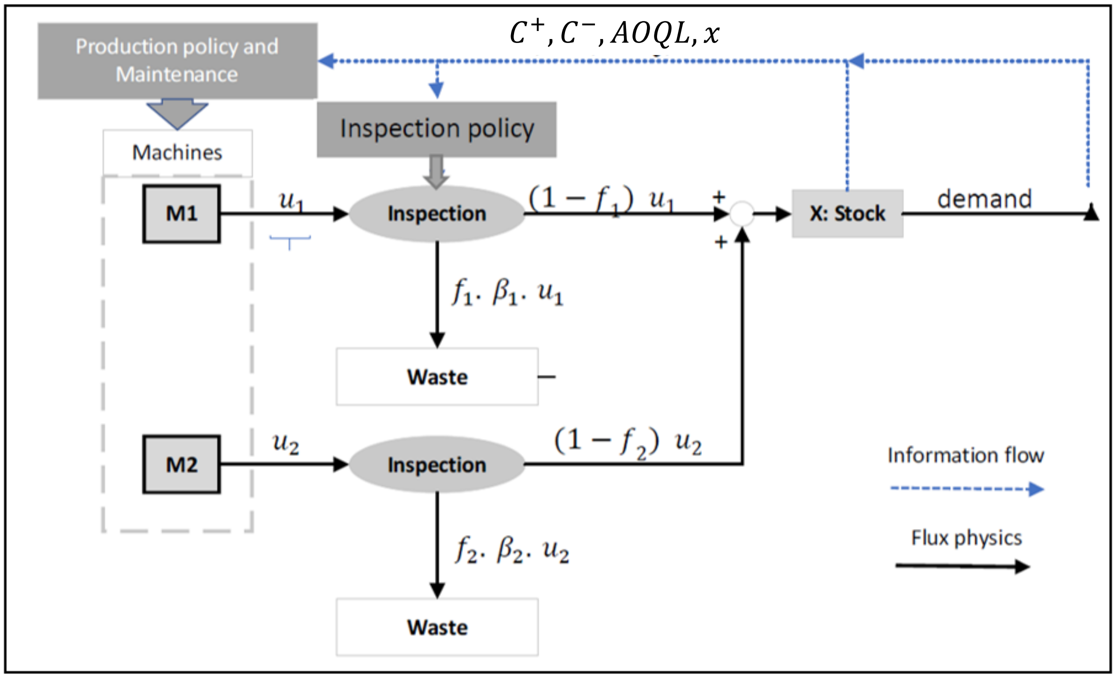


Figure 3.1 General structure of the system studied

3.4.3 Formulation of the optimal control problem

The machines M_1 and M_2 are subject to random failures and repairs. The defective rates of the two machines are $\beta 1$ & $\beta 2$, respectively. Let ξ (t) be the stochastic process describing the system dynamics at time t.

We have:

$$\xi(t) = \begin{cases} 1 \text{ if both machines are in operation} \\ 2 \text{ if the main machine is in operation and the second machine is down} \\ 3 \text{ if the main machine is down and the second machine is in operation} \\ 4 \text{ if both machines are down} \end{cases} \tag{3.1}$$

As well, the system can be in the mode α (with α ϵ Ω = {1, 2, 3, 4,}). Figure 2 details the transition diagram of the system.

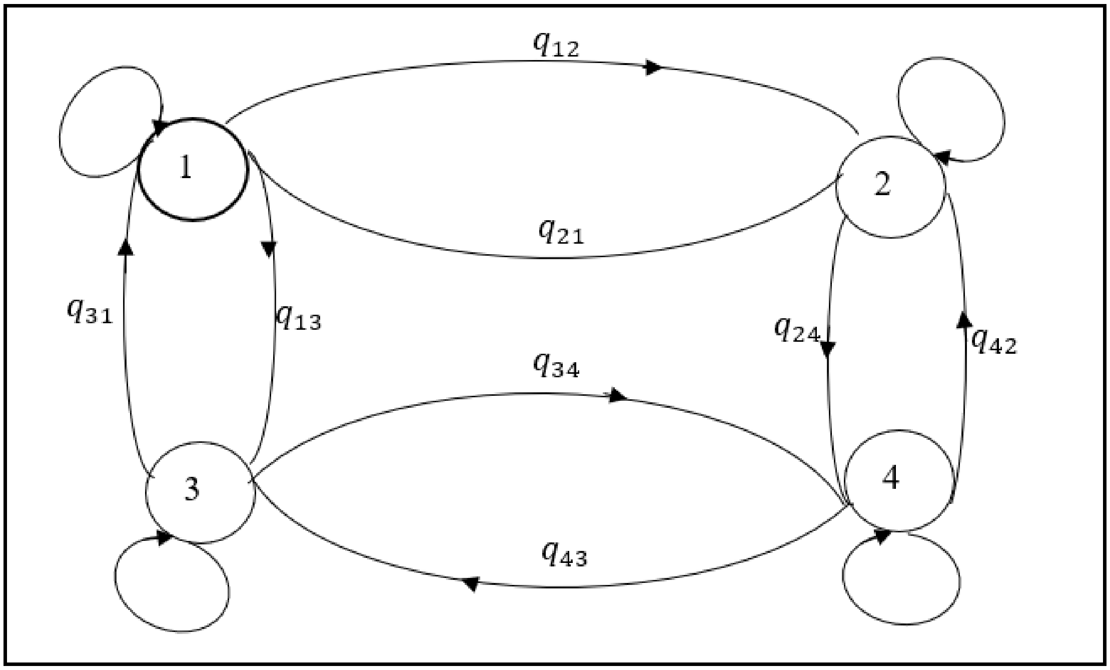


Figure 3.2 State transition diagram of the stochastic process

We have: $q_{12} = \lambda_2$; $q_{13} = \lambda_1$; $q_{21} = \mu_2$; $q_{31} = \mu_1$; $q_{24} = \lambda_1$; $q_{34} = \lambda_2$; $q_{42} = \mu_1$; $q_{43} = \mu_2$; with $\lambda_i = \frac{1}{MTBF^i}$ and $\mu_i = \frac{1}{MTTR^i}$; i ∈ {1, 2} denoting the failure and repair rates of machine M_i, respectively.

3.4.4 Average Outgoing Quality

When a system degrades with use, it affects not only its reliability but also the quality of its products. To be competitive, companies must control the quality of the products they distribute (Robotis et al. 2012). Thus, Rivera-Gomez et al (Rivera-Gomez et al. 2013) in their policy, inspect all products. This strategy had identified all non-conforming products so that only good quality products are distributed to the customer. However, for high volume production systems or systems with low product inspection rates, inspection of the entire production is very difficult or impossible. Hence, a sampling type quality control policy has been used in several works such as (Rivera-Gomez et al. 2020), and (Hajej et al. 2021).

For inspection, a fraction f_i^α ($0 \leq f_i^\alpha \leq 1$) of the production is randomly selected and inspected. Since not all products are inspected, non-conforming products are part of the distributed products. The principal machine produces defective parts with a defective rate β_1 . This rate increases when the machine produces at its maximum rate and could reach β_{1_max}. The machine *M2*, which assists the main machine, has a rejection rate noted β_2 . This rate increases when the machine produces at its maximum rate and could reach β_{2_max}. For a production series, a certain proportion f_i^α ($0 \leq f_i^\alpha \leq 1$) of the products is inspected, so the proportion ($1-f_i^\alpha$) is sent directly to inventory. Among the not inspected products, the proportion of non-compliant products is $\sum_1^2 (1 - f_i^\alpha) u_i \,.\, \beta_i$. Among the inspected products, the proportion of non-compliant products - $\sum_1^2 f_i^\alpha . u_i \,.\, \beta_i$ - is identified and destroyed. Hence, we we find for the average outgoing quality (AOQ):

$$\text{AOQ}(f_1^\alpha, f_2^\alpha) = \frac{(1 - f_1^\alpha) u_1 \,.\, \beta_1 + (1 - f_2^\alpha) u_2 \,.\, \beta_2)}{(u_1 + u_2) - (f_1^\alpha . \beta_1 . u_1 \; + f_2^\alpha \beta_2 . u_2)} \tag{3.2}$$

Figure 3.3 outlines the flow in the considered manufacturing system:

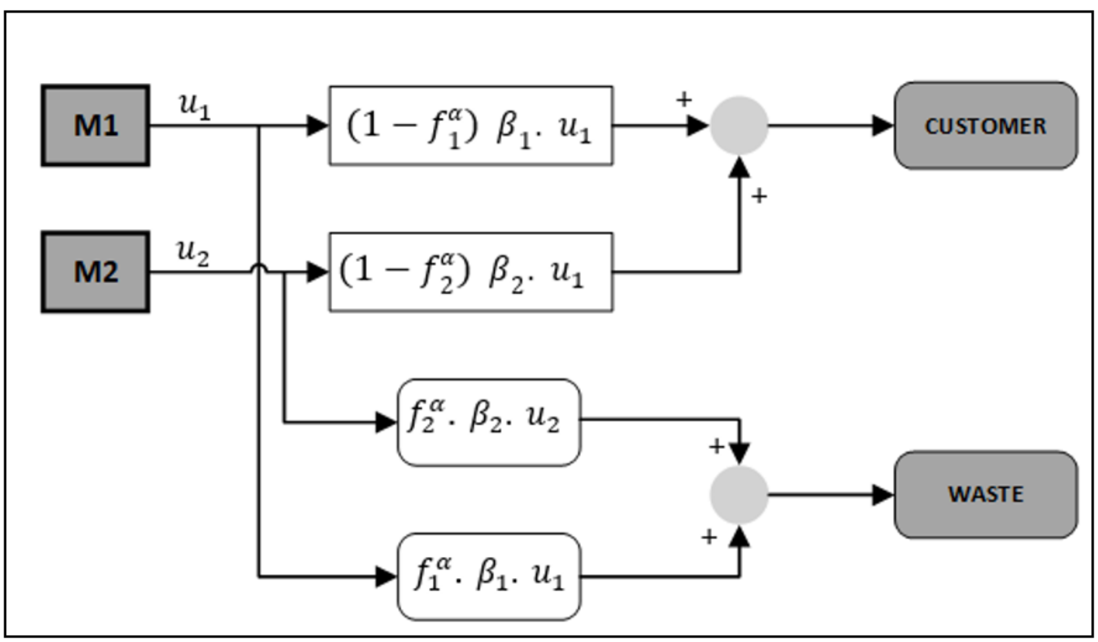


Figure 3.3 Inspection process and the average outgoing quality (AOQ)

Equation (3.2) demonstrates that the AOQ depends only on f_i^{α} and β_i, so its limiting value is as follows,

$$AOQL = maxAOQ(f_i^{\alpha}, \beta_i)$$
$$\text{with } 0 \leq f_i^{\alpha} \leq 1 \text{ and } 0 \leq \beta_i \leq 1. \tag{3.3}$$

Here, the quality objective is to determine, for each production mode, the optimal fraction f_{α}^{*} ($0 \leq f_{\alpha}^{*} \leq 1$) that minimizes the total production cost and limits the proportion of nonconforming products (AOQ). An average of nonconforming products, AOQ ($f_1^{\alpha}; f_2^{\alpha}$) given by equation (2) is distributed to customers before being returned by the customer to the company, so to anticipate the replacement of returned products, Hlioui et al. (Hlioui et al. 2015) proposed in their work an adjustment of the demand rate that becomes: $\frac{d}{1 - AOQL}$.

- Actual production rate of the machine in the system. The actual production rate, u_i, of each machine is defined as follows:

$$u_i^r = \frac{1}{t_{insp} + t_p} - f_i^{\alpha} * \beta_i * u_i = \frac{u_{insp} * u_i}{f_i^{\alpha} * u_i + u_{insp}} - f_i^{\alpha} * \beta_i * u_i \tag{3.4}$$

with $t_p = \frac{1}{u_p}$; $t_{insp} = \frac{f_i^{\alpha}}{u_{insp}}$; and $f_i^{\alpha} * \beta_i * u_i$ representing the quantity of products destroyed during the inspection. Hence, we have:

$$u_1^r = \frac{u_{insp} * u_1}{f_1^{\alpha} * u_1 + u_{insp}} - f_1^{\alpha} * \beta_1 * u_1 \tag{3.5}$$

The maximum real production rate of the main machine M1 is therefore:

$${}^{r}U_1^{max} = \frac{u_{insp} * U_1^{max}}{f_1^{\alpha} * U_1^{max} + u_{insp}} - f_1^{\alpha} * \beta_1^{max} * U_1^{max} \tag{3.6}$$

Similarly, the actual production rate of machine two in inventory is:

$$u_2^r = \frac{u_{insp} * u_2}{f_2^\alpha * u_2 + u_{insp}} - f_2^\alpha * \beta_2 * u_2 \tag{3.7}$$

The actual maximum production rate of my machine M_2 in the system is therefore:

$$^{r}U_2^{max} = \frac{u_{insp} * U_2^{max}}{f_2^\alpha * U_1^{max} + u_{insp}} - f_2^\alpha * \beta_2^{max} * U_2^{max} \tag{3.8}$$

For a production rate $u_{\mathrm{i}}^r(t)$, a rejection rate βi (α) and the fraction to be controlled f_{i}^{α}, the stock dynamics is:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = u_1^r(t) + u_2^r(t) - \frac{d}{1 - AOQ(\alpha)} \; ; \; x(0) = x_0 \tag{3.9}$$

Where x_0 is the initial inventory level. The process is modeled by the semi-Markovian chain ξ(t) at continuous time and discrete state with a transition rate matrix $Q = (q_{\alpha j})$ and transition probabilities defined as follows:

$$P[\xi(t + dt) = \gamma \,/\, \xi(t) = \alpha] = \begin{cases} q_{\alpha\gamma} * dt + o(dt) & if \ \alpha \neq \gamma \\ 1 + q_{\alpha\gamma} * dt + o(dt) & if \ \alpha = \gamma \end{cases} \tag{3.10}$$

Where $\lim_{(\delta t \to 0)} o(\delta t) = 0$ and $\alpha, \gamma \in \Omega = \{1, 2, 3, 4\}$.

As the failure rate and the defective rate of the machines M_1 and M_2 depend on their production rate, we will consider expressions of the following equations developed in (Kouedeu et al. 2015) and which gives the expression of the failure rate as a function of the production rate.

$$\lambda_1(t) = \begin{cases} \lambda_1^{min} & if \ u_1 \in [0, U_1^e] \\ \lambda_1^{max} & if \ u_1 \in [U^1, U_1^{max}] \end{cases} \tag{3.11}$$

$$\lambda_2(t) = \begin{cases} \lambda_2^{max} & if \;\; u_2(t) = \; U_2^{max} \\ \lambda_2^{min} & if \; not \end{cases} \tag{3.12}$$

In the same way, the influence of the production rate on the quality of the product is defined by the following two equations:

$$\beta_1 (t) = \begin{cases} \beta_1^{min} & if \; u_1 \; \in \; [0, U_1^e] \\ \beta_1^{max} & if \; u_1 \; \in \; [U^1, U_1^{max}] \end{cases} \tag{3.13}$$

$$\beta_2(t) = \begin{cases} \beta_1^{max} & if \; u_2(t) = \; U_2^{max} \\ \beta_1^{min} & if \; not \end{cases} \tag{3.14}$$

The transition rate matrix (Q) of the stochastic process is defined as follows:

$$Q= \begin{cases} Q_1 & if \; u_1(t) \; \in \; [0, U_1^e] \text{ and } u_2(t) \; \neq U_2^{max} \\ Q_2 & if \; u_1(t) \; \in \; [U_1^e, U_1^{max}] \text{ and } u_2(t) \; \neq \; U_2^{max} \\ Q_3 & if \; u_1(t) \; \in \; [0, U_1^e] \text{ and } u_2(t) = U_2^{max} \\ Q_4 & if \; u_1(t) \; \in \; [U_1^e, U_1^{max}] \text{ and } u_2(t) = U_2^{max} \end{cases} \tag{3.15}$$

with:

$$Q_1 = \begin{bmatrix} -(\lambda_2^{min} + \lambda_1^{min}) & \lambda_2^{min} & \lambda_1^{min} & 0 \\ \mu_1 & -(\mu_2 + \lambda_1^{min}) & 0 & \lambda_1^{min} \\ \mu_1 & 0 & -(\mu_1 + \lambda_2^{min}) & \lambda_2^{min} \\ 0 & \mu_1 & \mu_1 & -(\mu_1 + \mu_1) \end{bmatrix} \tag{3.16}$$

$$Q_2 = \begin{bmatrix} -(\lambda_2^{min} + \lambda_1^{max}) & \lambda_2^{min} & \lambda_1^{max} & 0 \\ \mu_1 & -(\mu_2 + \lambda_1^{max}) & 0 & \lambda_1^{max} \\ \mu_1 & 0 & -(\mu_1 + \lambda_2^{min}) & \lambda_2^{min} \\ 0 & \mu_1 & \mu_1 & -(\mu_1 + \mu_1) \end{bmatrix} \tag{3.17}$$

$$Q_3 = \begin{bmatrix} -(\lambda_2^{max} + \lambda_1^{min}\ ) & \lambda_2^{max} & \lambda_1^{min} & 0 \\ \mu_1 & -(\mu_2 + \lambda_1^{min}\ ) & 0 & \lambda_1^{min} \\ \mu_1 & 0 & -(\mu_1 + \lambda_2^{max}\ ) & \lambda_2^{max} \\ 0 & \mu_1 & \mu_1 & -(\mu_1 + \mu_1) \end{bmatrix} \tag{3.18}$$

$$Q_4 = \begin{bmatrix} -(\lambda_2^{max} + \lambda_1^{max}) & \lambda_2^{max} & \lambda_1^{max} & 0 \\ \mu_1 & -(\mu_2 + \lambda_1^{max}) & 0 & \lambda_1^{max} \\ \mu_1 & 0 & -(\mu_1 + \lambda_2^{max}) & \lambda_2^{max} \\ 0 & \mu_1 & \mu_1 & -(\mu_1 + \mu_1) \end{bmatrix} \tag{3.19}$$

The limiting probabilities are denoted by equation (3.20) as follows:

$$\begin{cases} \sum_{j=1}^{4} \pi_j * q_{ij} = 0 \\ \sum_{i=1}^{4} \pi_i = 1 \end{cases} \tag{3.20}$$

The system is feasible if and only if the actual maximum aggregate output of the system, in its production modes 1, 2 and 3, is greater than or equal to the market demand. This translates into the following equation:

$$\pi_1(U_1^{max} + U_2^{max}) + \pi_2 * U_1^{max} + \pi_{\mathbf{3}} * U_2^{max} \geq \frac{d'}{1 - AOQL} \tag{3.21}$$

The domain of admissible control policies is defined by equation (3.22) as follows.

$$\Gamma(\alpha) = \begin{Bmatrix} \mathrm{u}_1(\cdot), \mathrm{u}_2(\cdot), f_1(\cdot), f_2(\cdot), \mathrm{w}_1^r, \mathrm{w}_i^p : /\ 0\ \leq u_1 \leq U_{1_{max}}; 0\ \leq u_2\ \leq U_{2_{max}}; 0 \leq f_1^\alpha \leq 1; \\ 0 \leq f_2^\alpha \leq 1;\ \mathrm{w_p} = \{\ 0{,}1\ \};\ {}^rW_1^{min} \leq \mathrm{w}_1^r \leq\ {}^rW_1^{max};\ {}^rW_2^{max} \leq \mathrm{w}_2^r \leq\ {}^rW_2^{max} \end{Bmatrix} \tag{3.22}$$

The instantaneous cost of production is composed of the cost of shortage or possession of inventory, the cost of inspection, the cost of scrap, and the cost of penalty on returned products. Table 2 defines the expression, denoted by $g_i(.)$, for each of these costs.

Tableau 3.2 Summary of instant costs

Source	**Costs**	**Notation**
The cost incurred due to inventory and shortage	$C^+x^+ + C^-x^-$ with x^+= max (0, +x) and x^- = max (-x, 0)	$g_1(.)$
Inspection cost	$C_{insp}\,[f_1^{\alpha}.\,u_1(\alpha) + f_2^{\alpha}.\,u_2(\alpha)]$	$g_2(.)$
Cost of destruction	$C_{dest}[f_1^{\alpha}.\beta_1.u_1 + f_2^{\alpha}.\beta_2.u_2]$	$g_3(.)$
Cost of replacing non-conforming products	$C_r\,[(1-f_1^{\alpha})\,u_1.\beta_1 + (1-f_2^{\alpha})u_2.\beta_2)\,]$	$g_4(.)$
Maintenance cost	$c_m * W_i^m(\alpha) * \text{Ind}\{\alpha = 2{,}3{,}4\}$ with Ind {d(.)} = $\begin{cases} 1 \; if \; d(.) \; is \; true \\ 0 \; if \; not \end{cases}$	$g_5(.)$

The instantaneous cost is therefore:

$$g(\alpha, x_1, x_2) = g_1(.) + g_2(.) + g_3(.) + g_4(.) + g_5(.) \tag{3.23}$$

The discounted cost is defined by the following equation:

$$J(x, \alpha, u(.)) = E\left[\int_0^s e^{-\rho t} g(\alpha, x)\,.\,dt |\, x(0) = 0, \zeta(t) = \alpha\right] \tag{3.24}$$

with ρ the discount rate. The value function of the problem is therefore:

$$\vartheta(\alpha, x) = \min_{[u_1(\cdot),u_2(\cdot),f(\cdot)]\epsilon\Gamma(\alpha)} J(\alpha, x), \qquad \alpha \in \Omega \tag{3.25}$$

3.5 Optimal conditions and numerical example

By applying the stochastic optimal control theory based on dynamic programming we showed that optimality conditions are of the Hamilton- Jacobi- Bellman (HJB) equation as in (Kouedeu et al. 2015) and (Kenne, Dejax, and Gharbi 2012). Hence, the HJB equations are defined by equation (26) as follows:

$$\rho\vartheta(x,\alpha) = \min_{([u_1(\cdot),u_2(\cdot),f(\cdot)]\epsilon\Gamma(\alpha)} \left[g(x,\alpha,u) + \frac{\partial\vartheta(\alpha,x)}{\partial x} * \frac{dx(t)}{dt} + \sum_{j\epsilon\Omega} q_{\alpha j}\vartheta(j,x) \right] \tag{3.26}$$

To find the production and quality control policy, we need to solve this equation in the admissibility domain presented in equation (22). This solution by the analytical method is very complex and almost impossible. We will therefore use numerical methods to solve the optimality conditions. The deployed numerical methods are presented in Appendix A. In the following, we will simulate a numerical example to illustrate the structure of optimal production, quality control and maintenance management policies. To illustrate the above example, let us consider the CNC Router Class R cutting machine as introduced in section 3, which cuts polymer sheets into all sorts of shapes for custom water tank fabrication. The maximum number of sheets that can be cut per day is seven (7). The average processing time of a sheet is 1.5 hours, the average run time between two breakdowns is 110 days and the repair time varies between 1 and 1.5 days. To reduce the effects of degradation of this machine as a function of its production rate, we added a second machine with data from the work of kouedeu et al. (Kouedeu et al. 2015). Table 3.3 summarizes the data used in our practical example.

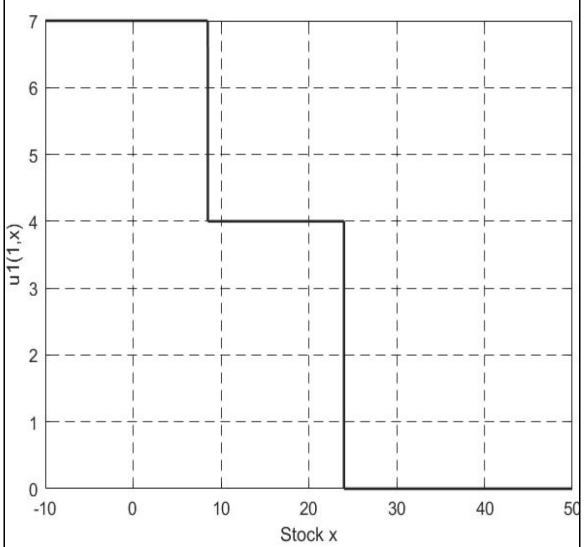
Tableau 3.3 Case study data for the CNC Router Class R cutting machine

Variables	U_1^{max}	U_1^e	U_2^{max}	d	C^+	C^-	C_r	Cm	C_d	C_{ret}	C_i
Values	7	4	3	2.7	3	50	30	50	150	30	10
Variables	λ_1^{min}	λ_1^{max}	λ_2^{min}	λ_2^{max}	$\mu 1$	$\mu 2$	β_1	β_1^{min}	β_2	u_{isp}	
Values	0.66	1	0.01	0.06	0.02	0.04	0.15	0.25	0.1	10	

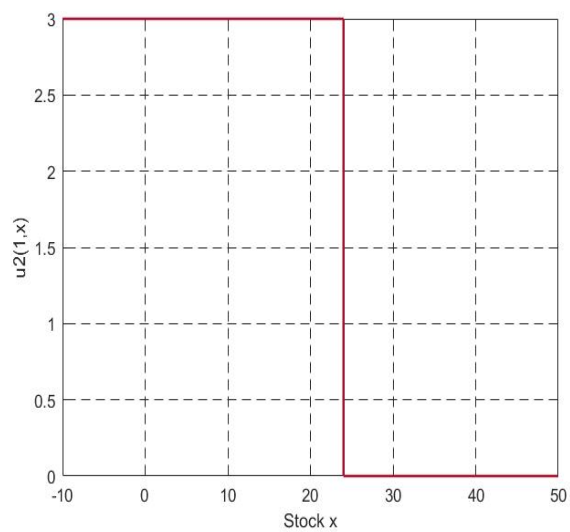
The data in Table 4 are such that the system satisfies the feasibility condition in equation (22). We can therefore conclude that the system is feasible. By applying the conditions of productions and the values of the parameters given in table 4 for the domain D_{h1}=$\{-10 \leq x \leq 50\}$ (Therefore an acceptable shortage of 10 products and a maximum inventory of 50 products), we obtain the results, represented graphically, for the production, quality control, corrective, and preventive maintenance policies, which are detailed respectively in section 3.6.1, section 3.6.2, section 3.6.3, and section 3.6.4.

3.5.1 Production policy

After simulation and solution of equation (3.26), the policies obtained for production in modes 1 and 2 are presented in figure 3.4.a, figure 3.4.b and figure 3.5.



(a) Production rate of machine M_1 in mode 1



(b) Production rate of machine M_2 in mode 1

Figure 3.4 Production policies in modes 1

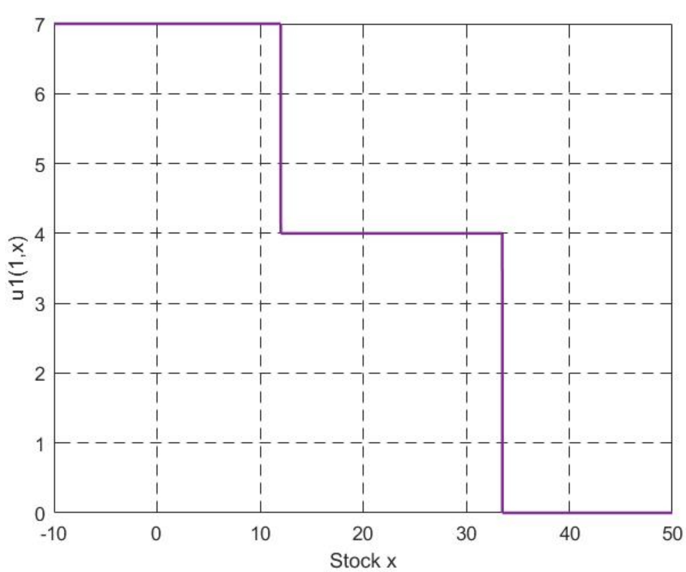


Figure 3.5 Production policies of machine M_1 in modes 2

The graphs above reveal the threshold policy to be adopted for the two production modes:

- Figure 3.4.a shows that if the product's inventory level is below the threshold inventoryZ_1^* = 8.5, the machine should produce at its maximum production rate (U_1^{max}). Then the machine must produce at its economic production rate until it reaches a threshold point in the stock of Z_2^* = 24; thereafter it produces at the adjusted demand rate. When the stock exceeds the second threshold value equal to 24, the machine must stop producing.
- Figure 3.4.b reports that if the product's stock level is below the threshold stock Z_2^* = 24, the machine should produce its maximum production rate (U_2^{max}). When the stock exceeds the second threshold value Z_2^*, the machine should stop producing.
- Figure 3.5.c informs us that if the product's inventory level is below the threshold inventory Z_3^*= 12, the machine should produce at its maximum production rate (U_1^{max}). Then the machine must produce at its economic production rate until it reaches a threshold point in the stock of Z_4^* = 33.5, thereafter it produces at the adjusted demand rate. When the stock exceeds the second threshold value, the machine should stop producing.

The policy that we propose is a critical threshold policy. In our practical example case, it will consist for the two modes of production to determine the critical thresholds Z_1^*, Z_2^*, Z_3^* et Z_4^* that minimize the total production cost and allows to satisfy the demand despite the uncertainties (breakdown and dispatch time), which influence the production.

In summary we have:

$$u_1^*(1, \mathrm{x}) = \begin{cases} U_1^{max} & \text{if } \mathrm{x(t)} < Z_1^* \\ U_1^e & \text{if } Z_1^* < \mathrm{x(t)} < Z_2^* \\ 0 & \text{if } \mathrm{x(t)} > Z_2^* \end{cases} \tag{3.27}$$

$$u_2^*(1, \mathrm{x}) = \begin{cases} U_2^{max} & \text{if } \mathrm{x(t)} < Z_2^* \\ 0 & \text{if } \mathrm{x(t)} > Z_2^* \end{cases} \tag{3.28}$$

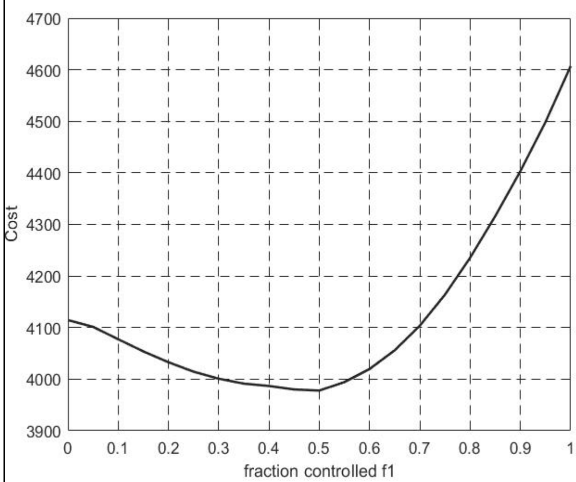
$$u_1^*(2,\mathrm{x}) = \begin{cases} U_1^{max} & \text{if } \mathrm{x(t)} < Z_3^* \\ U_1^e & \text{if } Z_3^* < \mathrm{x(t)} < Z_4^* \\ 0 & \text{if } \mathrm{x(t)} > Z_4^* \end{cases} \tag{3.29}$$

The critical thresholds in mode 2 are higher than the critical thresholds in mode 1, which is common as in mode 2 the second machine M_2 is down. Therefore, to satisfy the demand, machine *M1* produces more at its maximum rate, which accelerates its degradation, decreases its availability, and increases the rejection rate of the manufactured products. The system therefore protects itself by increasing its safety inventory.

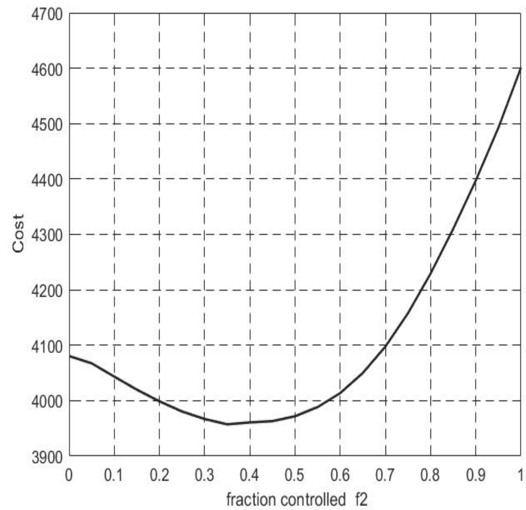
The production policy in mode 3 is not represented because the secondary machine is the only one in operation in this mode. Hence, the feasibility condition is not met, and the system cannot satisfy the demand. Therefore, machine M_2 produces at its maximum rate until the principal machine, M_1, is repaired.

3.5.2 Quality control policy

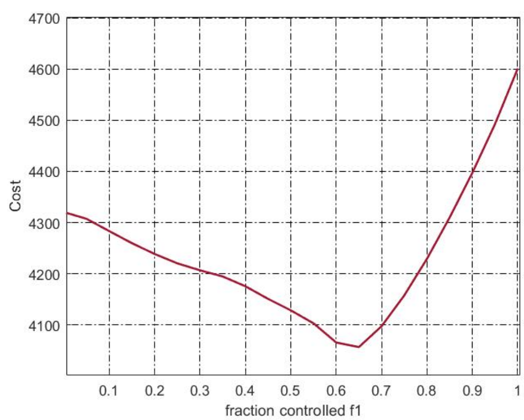
Figures 3.6.a, 3.5.b and 3.5.c show the total production cost according to the fractions controlled in production mode 1 (two machines operational) and production mode 2 (only machine 1 is operational).



(a) fraction of M_1 products in the mode 1



(b) fraction of M_2 products in the mode 1



(c) fraction of M_1 products in the mode 2

Figure 3.6 Inspection policy in modes 1 and 2

When both machines are in operation (mode 1), the quality control policy is to:

- Inspect $f_1^{1*} = 50\%$ (Figure 3.6.a) of the output of the primary machine.
- Inspect $f_2^{1*} = 35\%$ (Figure 3.6.b) of the output of the secondary machine.

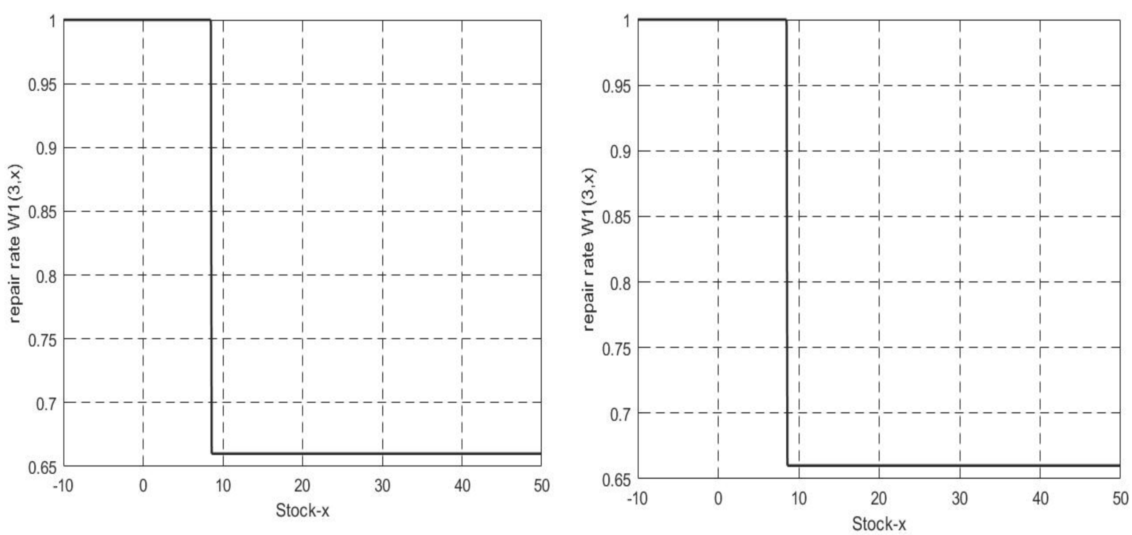
When only the primary machine is in operation (mode 2), the quality control policy is to.

- Inspect $f_1^{2*} = 65\%$ (Figure 3.6.c) of the main machine's output.

We notice that the optimal fraction f_2^{1*} of the products of machine 2 in mode 1 is smaller than the fraction f_1^{1*} of the products of the main machine. This can be explained by the fact that the principal machine degrades more and produces more non-conforming parts. It can also be noticed that in mode 2 the optimal fraction f_1^{2*} of the products to be controlled is higher than the optimal fraction f_1^{1*}, because machine M_1 is no longer assisted by machine M_2. Therefore, it produces more at its maximum rate ($Z_1^* \leq Z_3^*$), which negatively influences the quality of the products. Hence the need to increase the fraction of products to be controlled.

3.5.3 Corrective maintenance policy

Figure 3.7 illustrates the repair rate of the machines as a function of the inventory level to minimize the production cost.



(a)Repaire rate of the machine M_1 in mode 3

(b) Repaire rate of machine M_2 in mode 2

Figure 3.7 Corrective maintenance policy for the machines in modes 1 and 2

The deployed maintenance strategy depicted in Figure 3.7.a and 3.7.b, consists of having an expert to repair the machines at its maximum repair rate (${}^{r}W_1^{max}$ / ${}^{r}W_2^{max}$), representing the reduced repair time, when the main machine fails before the inventory has reached the threshold value Y_1^*= 8.5 for the main machine and Y_2^*= 12 for the second. Once this threshold values are exceeded in inventory, each machine can be repaired at its minimum (${}^{r}W_1^{max}$/ ${}^{r}W_2^{min}$). representing the standard repair time, because calling in an expert will be more expensive. In summary, we have:

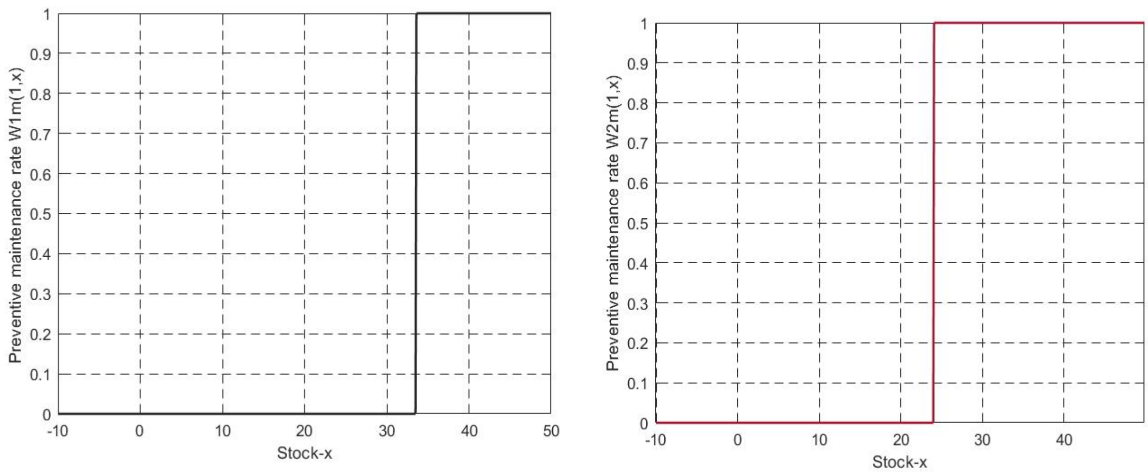
$$W_1^r = \begin{cases} {}^{r}W_1^{min} = 0.66 & \text{if } x(t) \leq Y_1^* \\ {}^{r}W_1^{max} = 1 & \text{if not} \end{cases} \tag{3.30}$$

$$W_1^r = \begin{cases} {}^{r}W_2^{min} = 0.66 & \text{if } x(t) \leq Y_2^* \\ {}^{r}W_2^{max} = 1 & \text{if not} \end{cases} \tag{3.31}$$

We notice that Y_1^* is greater than Y_1^*, which can be explained since the main machine alone can satisfy the demand contrary to machine 2. Hence, we have more risk of not satisfying the demand when machine 1 breaks down before a certain threshold level in inventory.

3.5.4 Preventive maintenance policy

The preventive maintenance policy as proposed in this work is an opportunistic preventive maintenance approach based on developments by Kang and Subramaniam (Kang and Subramaniam 2018), and He et al. (He et al. 2019). This strategy consisted of exploiting the voluntary downtime of the machine to perform general overhaul operations of its components. Figure 3.8 present the frequency of sending each machine for revision according to the in level.



(a): Preventive maintenance of machine M_1 in mode 1.

(b): Preventive maintenance of machine M_2 in mode 1.

Figure 3.8 Preventive maintenance policies of M_1 and M_2 in mode 1

Figure 3.7.a reveals that when the main machine is in production stop in mode 1 (Z_5^*), we must send it to a voluntary stop time exploited to make a preventive revision of the machine components. This is to reduce their degradation because during this mode, the main machine is the only one in operation which accelerates its degradation. Figure 7.b shows that when the inventory level is equal to its threshold value Z_6^*, the secondary machine stops producing. This downtime should be exploited to perform preventive maintenance on this machine.
In summary we find for the preventive maintenance rates:

$$W_1^p = \begin{cases} 1 & \text{if } Z_5^* \leq x(t) \\ 0, & \text{if not} \end{cases} \tag{3.32}$$

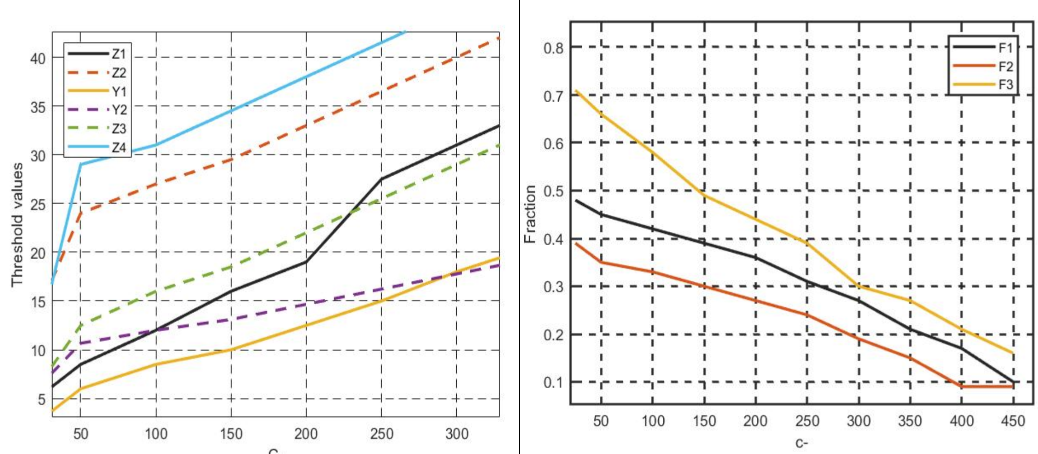
$$W_2^p = \begin{cases} 1 & \text{if } Z_6^* \leq x(t) \\ 0 & \text{if not} \end{cases} \tag{3.33}$$

The previous practical example allowed us to develop interactive policies for joint optimization of production, maintenance, and quality control. However, we need to do more practical simulations to analyze the sensitivity of our model.

3.6 Sensitivity analysis

The robustness of the proposed approach is presented in this section. Multiple simulations of the model were performed to observe the optimal parameters of the policies developed in section 5. The presented sensitivity analysis consists in varying independently the system parameters, such as the cost of setting in inventory C+, cost of shortage C-, the cost of inspections Cisp, and the cost of penalties on the returned products Cd. Based on these varying parameters, the system behaviour is observed as defined by parameters, such as the rate of inspection of products uisp, the failure rate, the defective rate of the main machine (λ1 & β1) and the Average Outgoing Quality (AOQ). The varying behavior of these policy parameters are summarized in Figure 3.9 to Figure 3.15, and they can be explained as follows:

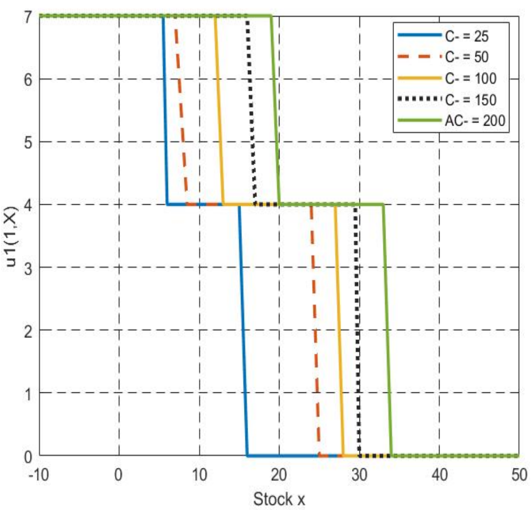
- <u>Variation of shortage cost:</u> The variation of the shortage cost allowed us to observe the variation of the threshold parameters of our policy. Figure 3.9(a-e) shows the behavior of these parameters as a function of the storage cost.



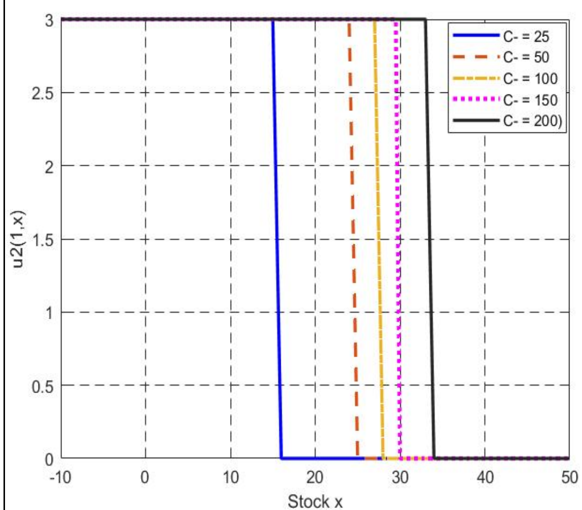
(a) Effect on threshold values

(b) Effect on the threshold fractions

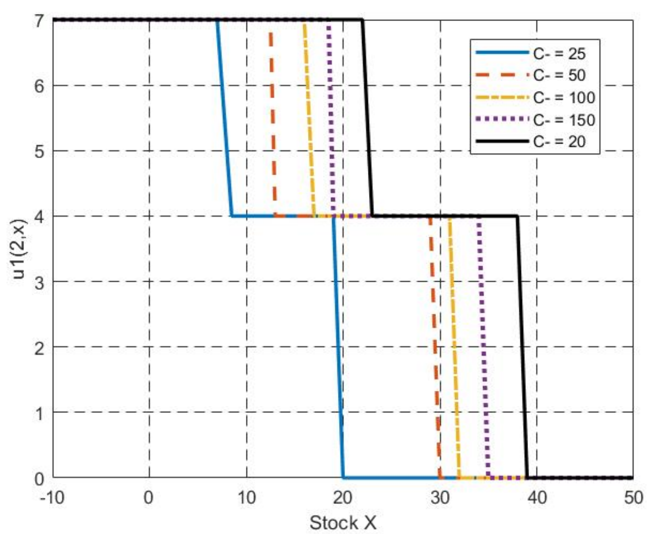
Figure 3.9 Effect of variation of shortage cost



(c) variation of the production rate of M_1 in mode 1



(d) variation of the production rate of M_2 in mode 1

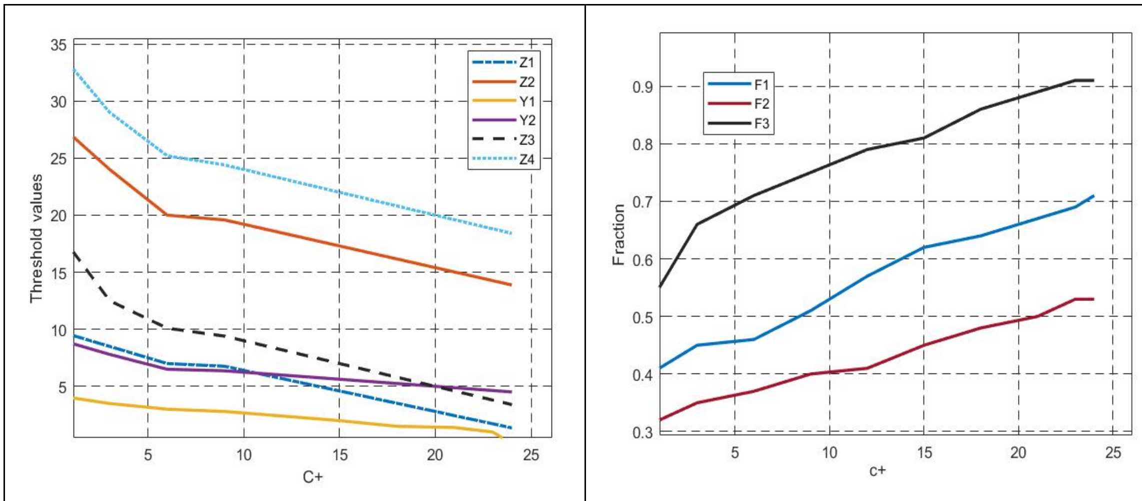


(e) variation of the production rate of M_1 at mode 2

Figure 3.9 Effect of variation of shortage cost (Suite)

We see that the critical thresholds $(Z_1^*, Z_2^*, Z_3^*, Z_4^*, Y_1^*)$ increase. This variation is normal because as shortages become more expensive, the system stores more to reduce costs. We can also see that the fractions to be controlled (f_1^*, f_2^*) are decreasing which is also normal because the increase of the costs of shortage makes it more expensive to miss a product than to have a non-conforming product on the market.

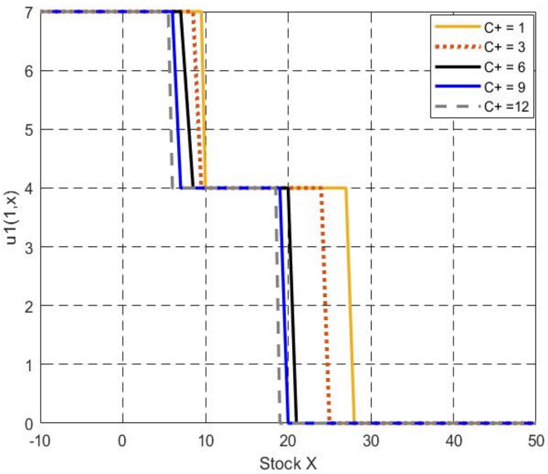
- Variation of inventory cost: The variation of the inventory cost allowed us to observe the variation of the threshold parameters of our policy. Figure 3.10 shows the behavior of these parameters as a function of the storage cost.



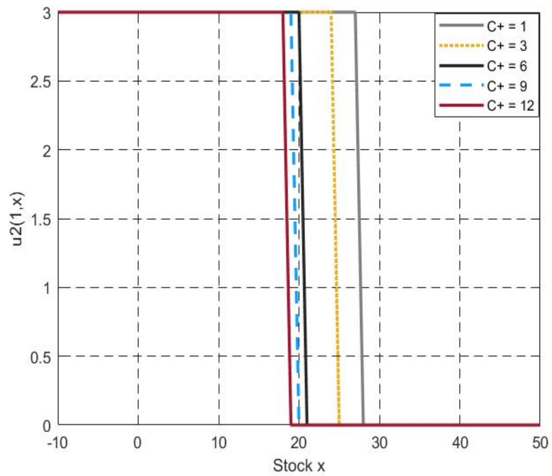
(a) Effect on the threshold values

(b) Effect on fractions

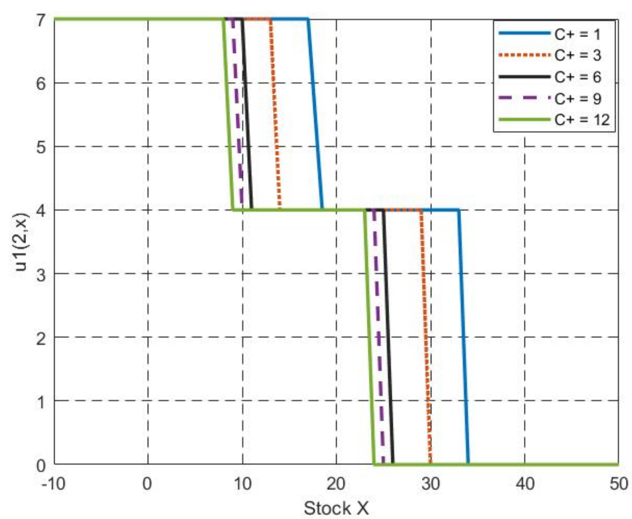
Figure 3.10 Effect of variation of inventory cost



(c) variation of the production rate of M_1 in mode 1



(d) variation of the production rate of M_2 in mode 1

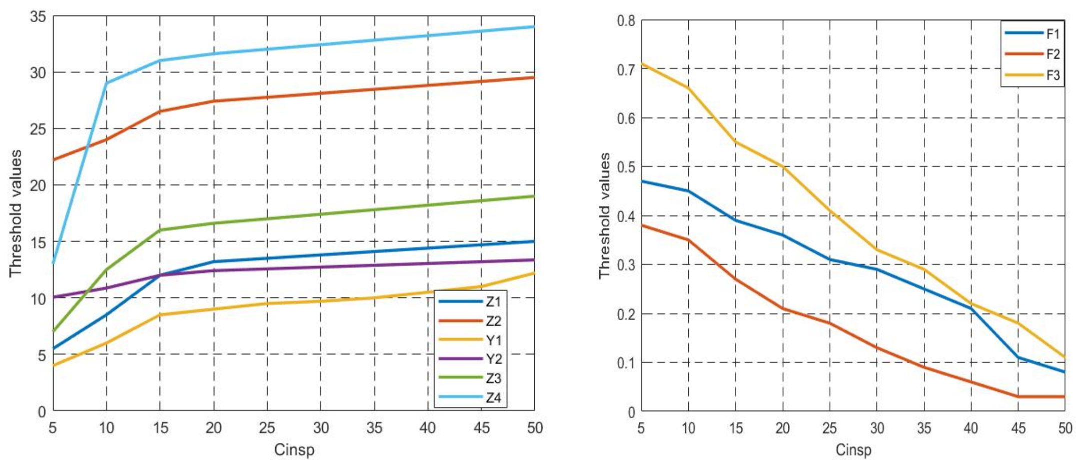


(e) variation of the production rate of M_1 at mode 2

Figure 3.10 Effect of variation of inventory cost (Suite)

When the inventory cost C^+ increases, the opposite effect observed for the increase of the shortage cost is observed. This is since the increase in storage costs leads to a decrease in the inventory threshold levels. Also, the fractions to be controlled increase, because it makes it possible to eliminate the products of bad quality consequently to reduce the level of stock.

- <u>Variation of the inspection cost C_{isp}:</u> when varying the inspection cost C_{isp}, we notice (Figure 3.11.a) that the fractions to be inspected decrease which is normal because it becomes more and more expensive to inspect a product. In the same way, the threshold stocks increase (Figure 3.11.b) because there are more non-conforming products on the market, which increases the number of returned products to replace.

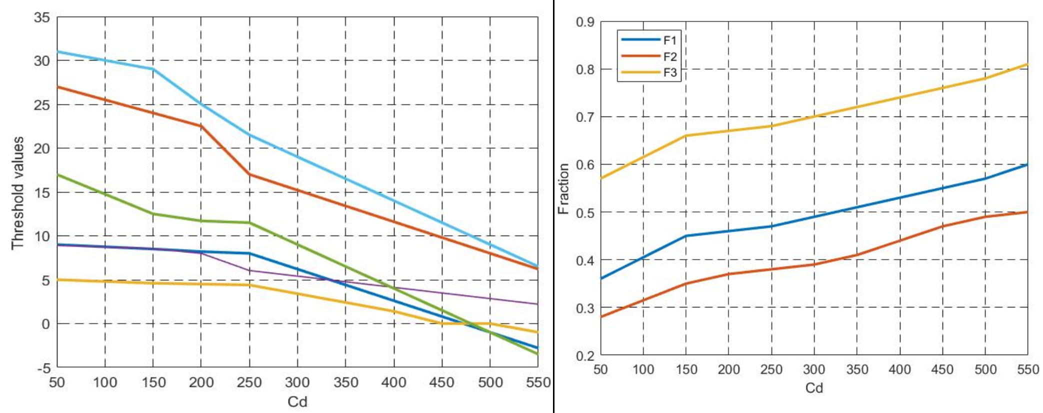


(a) Effect on the threshold values

(b) Effect on fractions

Figure 3.11 Effect of variation of inspection cost

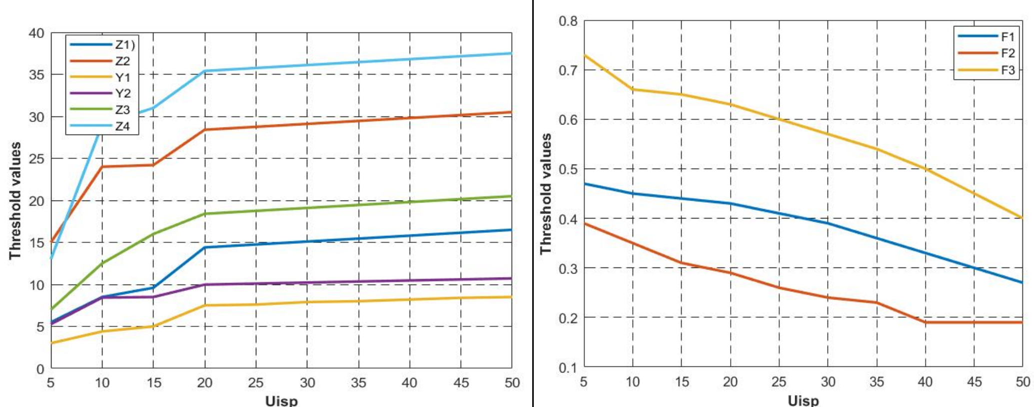
- Variation of penalty cost Cd: As the penalty cost increases, the system must limit the distribution of non-conformities products to reduce the total cost, the fractions of products to be controlled increase (Figure 3.12.b). the critical threshold levels decrease (Figure 3.12.a) because the non-conforming products are reduced from the inventory.



(a) Effect on the threshold values (b) Effect on fractions

Figure 3.12 Effect of variation penalty cost

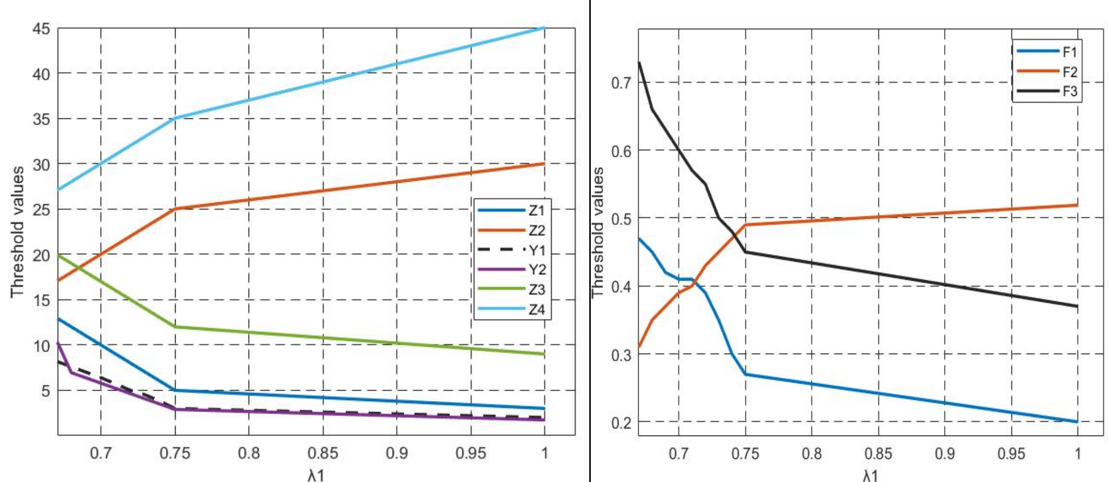
- Variation of the inspection rate u_{isp}: When the inspection rate increases, we notice that the fractions to be inspected decrease, which allows to reduce the real production time (Figure 3.13.b). However, the threshold stocks increase (Figure 3.13.a) because more non-conforming products pass through the system.



(a) Effect on the threshold values (b) Effect on fractions

Figure 3.13 Effect of variation of inspection rate

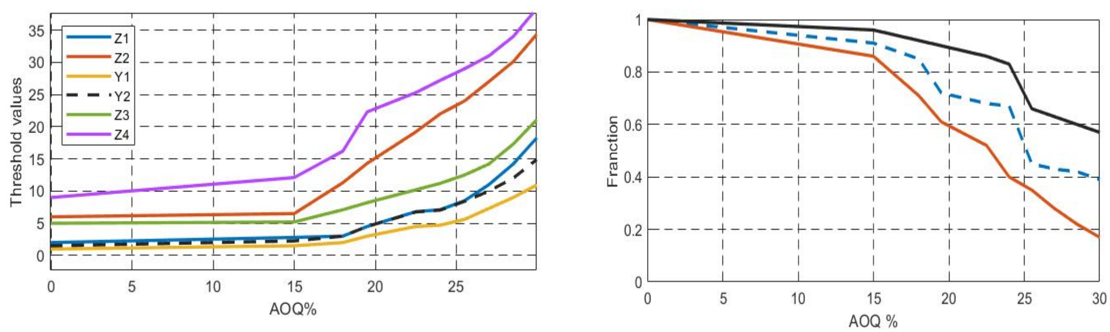
- Variation in the failure rate**:** Figure 3.14.a shows that when the failure rate increases, the thresholds Z_1^* and Z_3^* decrease while the thresholds Z_2^* and Z_4^* increase. this is normal because the main machine produces less at its maximum rate and more at its economic rate. this reduction allows to limit the degradation of the machine. Figure 3.14.b shows that the fraction f_1^{1*} and f_1^{2*} decrease while the disk f_2^{1*} increases. this is because the second machine increases its production to support the main machine which produces more at its maximum rate.



(a) Effect on the threshold values (b) Effect on fractions

Figure 3.14 Effect of variation of failure rate

- Variation in AOQ: Figure 3.15 show respectively that: as the AOQL increases, the optimal thresholds increase and the optimal fractions to inspect decrease. This means that the system reduces the fractions to inspect to reduce the total inspection time. This increases the real production rate of the system to meet the demand.



(a) Effect on the threshold values (b) Effect on fractions

Figure 3.15 Effect of variation of AOQ

3.7 Comparative Study

In this section, we make a comparative study to highlight the economic aspects of our policy as well as the new contribution to our approach. It should be noted that most of the published papers have separately treated the aspects of maintenance, quality improvement and optimal control. Hence, our model, by considering the influence of machine degradation as a function of the production rate, brings more realistic results because in the literature, researchers who have integrated quality control into their policies have neglected the influence of production rate. This is the case of the policies proposed by Rivera-Gomez et al. (Rivera-Gomez et al. 2020), Kang and Subramaniam (Kang and Subramaniam 2018), Zhang et al. (Zhang et al. 2020), Paraschos et al. (Paraschos et al. 2020), Kang and Subramaniam (Kang and Subramaniam 2018). In terms of economics, we will compare our results with four of the policies proposed in the literature review. These policies proposed in Rivera-Gomez et al (Rivera-Gomez, Gharbi, and Kenne 2013), Kouedeu et al (Kouedeu et al. 2015), Zhang et al (Zhang, Shen, and Ma 2020), Paraschos et al (Paraschos et al. 2020) and Rivera-Gomez et al (Rivera-Gómez et al. 2021) are briefly summarized below.

- Policy I (1): This is the policy proposed in this paper. The influence of the production rate on the degradation is considered, the parameters of production (production rate), quality inspection (inspected fraction) and maintenance (repair and general overhaul rate) are dynamic and adjusted according to the degradation level of the system.

- Policy II (2): In this policy, the influence of the production rate on the degradation is not considered. All products are inspected (f equal to 100%) whatever the level of degradation. The maintenance parameters (repair rate) are static (Rivera-Gomez et al. 2013).

- Policy III (3): In this policy, the influence of the production rate is considered, but no product is inspected (f= 0%). All the manufactured products are distributed and only the production rate and the repair rate are adjusted according to the degradation (Kouedeu et al. 2015).

- Policy IV (4): In this policy the influence of the production rate according to the degradation is not considered as in policy I. On the other hand, the fraction to be inspected, the production rate and the maintenance frequency are dynamic as in policy I (HéctorRivera-Gómez et al. 2021) and (HéctorRivera-Gómez et al. 2021)

- Policy V (5): In this policy, the influence of the production rate according to the degradation is considered as in policy I. However, the parameters of quality control are not dynamic but rather static whatever the level of degradation. (Zhang, Shen, and Ma 2020)

Figure 3.16 shows the location of policies (1), (2), (3), (4) and (5) in relation to their total production costs incurred based on the quality inspection.

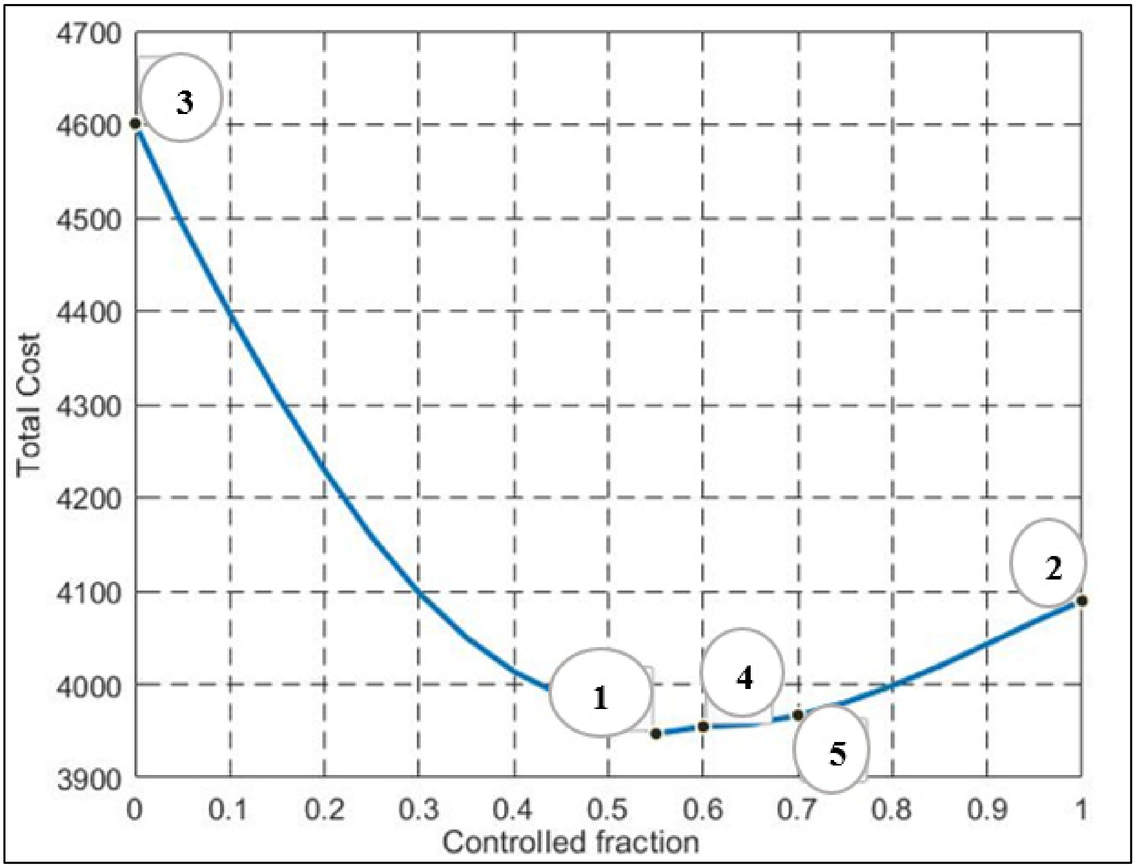


Figure 3.16 Comparative study of policies (1), (2), (3), (4) and (5)

Each of these policies (1-5) are defined as follows:

- *Policy I (1):* This is the policy proposed in this research, for this practical case we must inspect 55% of the production and the total cost of production is 3947$.
- *Policy II (2):* This policy the inspected fraction f =100% because all products are inspected. Thus, the inspected fraction is not optimal because figure 10 shows us that for a rejection rate β the total production cost is 4090$ instead of 3947$ if only 55% of the production had been inspected. This is a reduction of about 3.5% of the total production cost. This situation is since in this policy I, several products of good quality are inspected which costs money. In addition, the machine is not degraded initially, so more good quality products are produced.
- *Policy III (3):* This policy, the influence of the production rate on the system is considered but no inspection is performed on the manufactured products: Thus,

the controlled fraction $f = 0\%$.Thus we observe on figure 3.15 that for a rejection rate β, the total production cost is \$4601 instead of \$3947 if 55% of the production had been inspected. This is a reduction of about 14.2% of the total production cost. This is due to the additional losses caused by the distribution of nonconforming products to customers.

- *Policy IV (4):* In this policy, the influence of the production rate on the system is not considered but the fraction to be inspected is adjusted according to the level of degradation. Thus, we observe on figure 3.15 that for a rejection rate β, the total production cost is 3955$ instead of 3947$ if 55% of the production had been inspected. This is a reduction of about 2% in total production cost. This is due to increased degradation of the machine when producing at its maximum rate.
- *Politique V (5):* In this policy, the influence of the production rate as a function of the degradation is considered as in policy I and the quality control parameters do not change regardless of the level of degradation. Thus, we observe in figure 3.15 that for a beta rejection rate β, the total production cost is $ 3967 instead of $ 3947 if 55%. This means a reduction of about 5% of the total production cost.

To further illustrate our policy, we ran multiple simulations based on the rejection rate of the main machine. In each simulation we compared the total cost of policies (1), (2), (3), (4) and (4) according to our policy. The following table presents the results obtained.

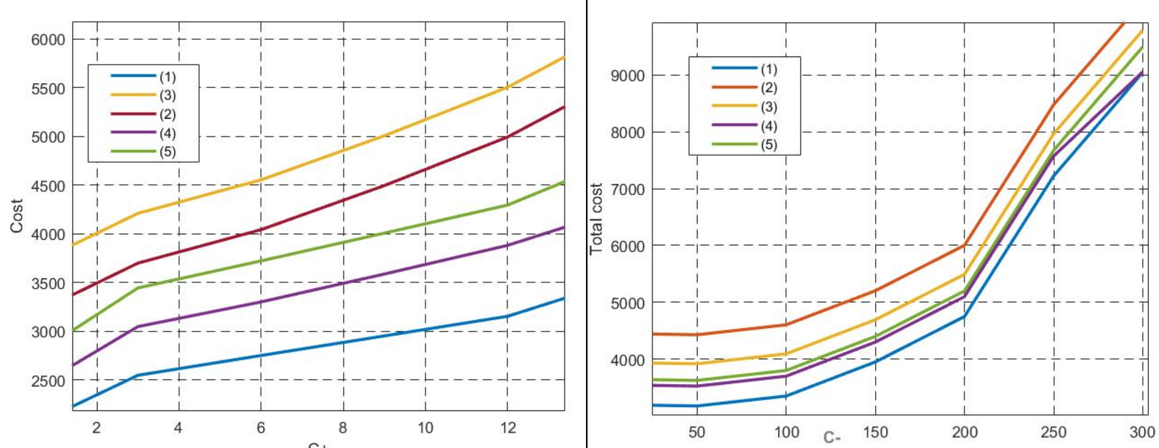
Tableau 3.4 Comparative study of policies (1), (2), (3), (4) and (5)

Product rejection rate (β%)	5%	10%	15%	20%	25%
Policy I (cost $)	2902	3502	3947	4333	4573
Policy II (cost $)	3270	3789	4090	4442	4648
Policy III (cost $)	3167	3951	4601	5175	5594
Policy IV (cost $)	3008	3609	4205	4642	4882
Policy V (cost $)	3117	3720	4217	4655	4896

Tableau 3.4 Comparative study of policies (1), (2), (3), (4) and (5) (Suite)

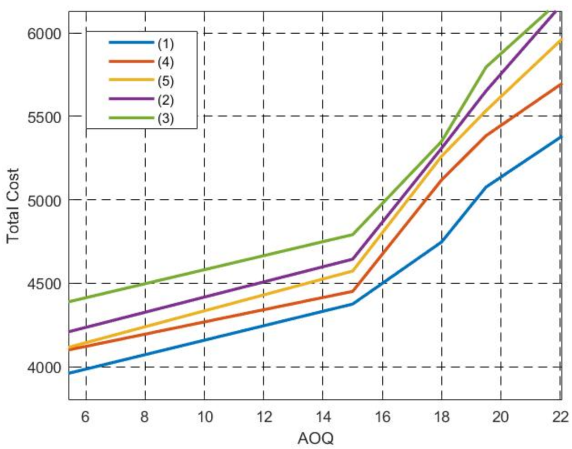
Product rejection rate (β%)		5%	10%	15%	20%	25%
REDUCTION RATE (%)	Policy II (2)	11,26%	7,57%	3,50%	2,46%	1,62%
	Policy III (3)	8,35%	11,36%	14,21%	16,27%	18,26%
	Policy VI (4)	3,52%	2,97%	6,14%	6,65%	6,33%
	Policy V (5)	6,89%	5,85%	6,40%	6,92%	6,60%

To extend this comparative study, we vary other key parameters of the production system to observe the behavior of different policies (1, 2, 3, 4 and 5). Figure 3.17 (a, b, c, d, e) present the obtained results, when varying respectively the inventory cost, the shortage cost, the AOQ, the inspection cost, and the penalty cost.

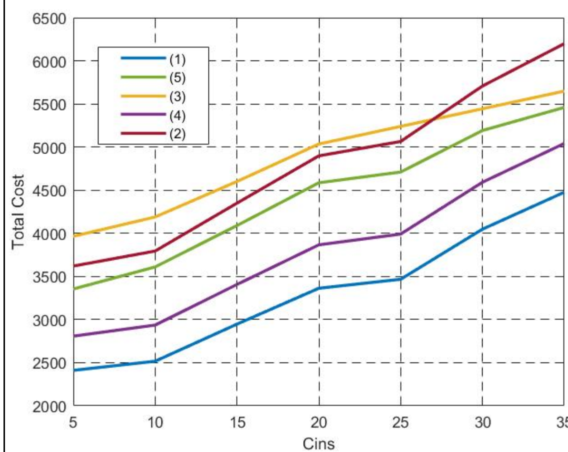


(a) Variation of inventory cost. (b) Variation of shortage cost

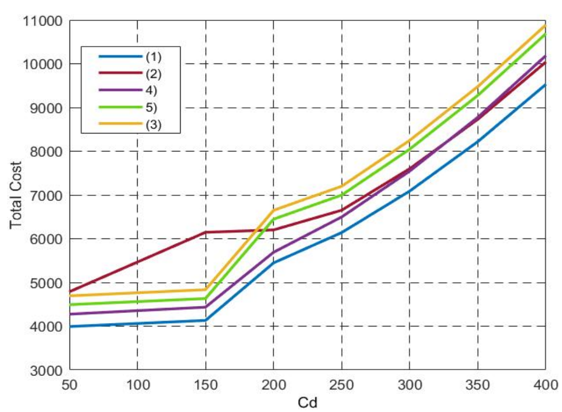
Figure 3.17 Comparative study



(c) Variation of AOQ.



(d) Variation of inspection cost.



(e) Variation of the penalty cost

Figure 3.17 Comparative study (Suite)

Figure 3.17.a shows that as the cost of storage increases, the total cost of each of the five policies increases, however, the cost of the policy proposed in this model remains lower and policy (3) remains the costliest. This is normal because in this policy no product is inspected ($f = 0\%$) and therefore poor-quality products are in inventory which costs money. In Figure 3.17.b our model remains less costly, and policy (2) is the costliest. This is since in this policy, all products are inspected ($f = 100\%$) which increases the inspection time and

reduces the actual production rate of the system. This slow production rate creates shortages. Figure 3.17.c our model is the most economical and policy (3) is the most expensive, this is since in this policy no product is inspected. Figure 3.17.d we see that policy (3) is the most expensive for low inspection costs but for high inspection costs it is rather policy (2). This is since in policy two, the inspection of all products (conforming and non-conforming) costs money. Figure 3.17.e shows that policy (3) is the most expensive for large penalty costs. This is because in this policy all non-conforming products are distributed to the customers which causes penalties for the company. In summary, Figure 3.17 shows the effectiveness of the policy obtained in this research (optimal policy). Indeed, on the one hand this policy allows to improve simultaneously the quality of the products and the reliability of the system. On the other hand, this policy allows to minimize the total production cost even when the different parameters such as, the stock cost, the shortage cost, the inspection cost, the replacement cost of the nonconforming products increase. In conclusion, this strategy finds its effectiveness in the consideration of quality improvement and in the implementation of a dynamic inspection process of sampling type. In addition, the proposed policy integrates an optimal production management strategy, an inspection policy, and a corrective and preventive maintenance management policy.

3.8 Policy implementation and decision support tool

To optimize production and quality control in the case of our numerical example (section 5), the procedure to be followed by the manufacturer is proposed as follows:

- Step 1 (S1): Identify the principal machine and the secondary machine. In this step, the manufacturer identifies the machine with the highest maximum production rate (the principal machine). The machine with the lower rate is the secondary machine.
- Step 2 (S2): Identify the status (operation or failure) of each of the two machines. During this step, the manufacturer verifies whether each of the two machines is broken down or in operation.

Based on the basic case study (data presented in table 3) and depending on the failure or operating status of the machines (S1 and S2), the operator must follow one of the three procedures, which are detailed below. Figure 3.17 illustrates the obtained logical diagram for the implementation of the optimal joint production, maintenance, and quality control process of the proposed policies.

- If both machines are in operation, the operator must inspect 50% of the production of the principal machine and he needs to produce at the maximum rate until the number of products in inventory (x) is equal to 8.5. Once this quantity is reached in stock, he must reduce the production rate and produce at the economic rate until the number of products in stock (x) is equal to 24. Once this inventory level is reached, the manufacturer must stop production and perform a general overhaul of the machine components. For the second machine that assists the first one, the manufacturer must inspect 35% of its production and he must produce at the maximum rate until the number of products in inventory (x) is equal to 24. Once this quantity is reached in stock, the manufacturer must stop the production and perform a preventive revision of the machine's components.

- If the principal machine is in operation and the secondary machine is down, the operator must, for the principal machine, inspect 65% of the production and he needs to produce at the maximum rate until the number of products in inventory (x) is equal to 12.

- Once this quantity is reached in inventory, he must reduce the production rate and produce at the economic rate until the number of products in inventory (x) is equal to 33.5. Once this inventory level is reached, the manufacturer must stop production and perform a preventive overhaul of the machine components.

- If the main machine is down and the secondary machine is in operation, the manufacturer must inspect 35% of the production of this machine. Then, produce at maximum rate until the main machine is operational. Once both machines are up and running, follow the procedure described in action 1.

- If the principal machine, before its failure, had less than 8.5 products in inventory (i.e. x < 8.5), the manufacturer needs to call in an expert to repair the machine.

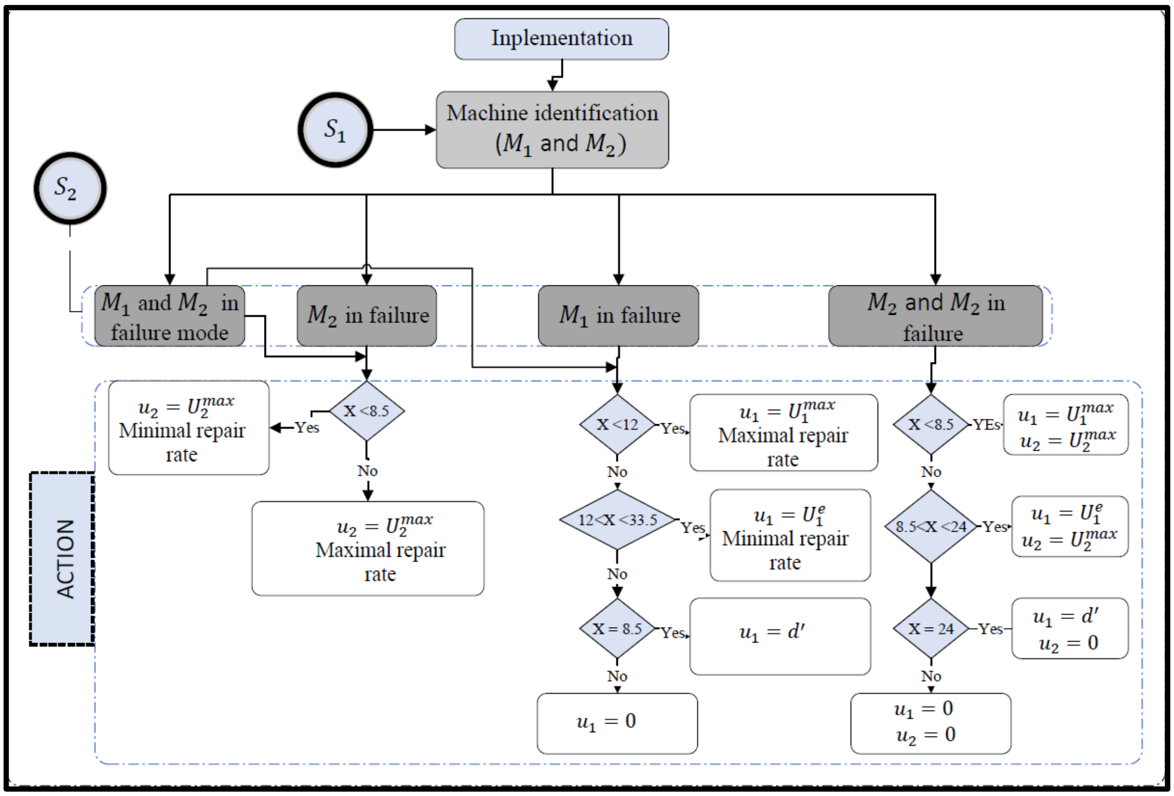


Figure 3.18 Logical diagram for the proposed policies implementation

3.9 Conclusion

In conclusion, this study focused on the optimization of the production of a system made of machines that degrade according to the production rate. The goal was to improve product quality, increase machine availability and reduce production costs. To achieve this goal, we formulated the general problem, and we developed the system dynamics to optimize the decision variables (production rate and fraction rate to be inspected). This optimization study was conducted in a system consisting of two machines that degrade according to their production rate. After having formulated the optimal control problem, we have determined the optimal values of our decision variables by a numerical method through the resolution of the HJB equations. In a second step, a real-world numerical example was simulated using the developed production optimization policy for a Router class R CNC cutting machine of the Canadian company CET Fire Pumps. The obtained results allowed us to propose a joint policy of production, quality control and maintenance depending on the degradation of the manufacturing machines and linked to their production rates. Finally, a sensitivity analysis has shown the robustness of the proposed approach. Hence, to satisfy the demand, even in case of failure, we have proposed a critical threshold production policy. This strategy consists in continuously maintaining an optimal stock of products in inventory before any voluntary stop of the production. To limit the influence of the production rate on the degradation, the proposed strategy consists in reducing the production rate from its maximum value to an intermediate value when the inventory reaches an optimal threshold value. To limit the distribution of poor-quality products, we proposed a sampling and dynamic quality control strategy. An opportunistic preventive maintenance and a corrective maintenance control with the option of using subcontracted experts was proposed to improve maintenance. Thanks to this novel approach and the proposed policies, we have contributed to optimize production lines that degrade according to their use and therefore affecting not only the reliability but also the quality of products. Our results can be useful for companies that have unreliable machines, which degrade over time and produce non-conforming parts. This work can be extended in future works for systems with variable demand rates, non-Markovian systems, or systems with recycling of non-conforming products.

CHAPITRE 4

PRODUCTION, MAINTENANCE AND QUALITY INSPECTION PLANNING OF A HYBRID MANUFACTURING/REMANUFACTURING SYSTEM UNDER PRODUCTION RATE DEPENDENT DETERIORATION

Patrick Megoze Pongha [a], Jean-Pierre Kenné[b], Guy Richard Kibouka[c], Lucas A. Hof[d]

[a, b, d] Département de Génie de mécanique, École de Technologie Supérieure,
1100 Rue Notre-Dame Ouest, Montréal, Québec, Canada, H3C 1K3
[c] Department of Mechanical Engineering, Omar Bongo University,
Libreville, Gabon

Article submitted for publication, December 2021

Résumé

Cet article mène une étude sur un système hybride de fabrication et de remise à neuf et qui se dégrade en fonction de son taux de production. Le système se compose de deux machines sujettes aux pannes et aux réparations. Les deux machines produisent un seul type de produit et se dégradent en fonction de leur taux de production. Cette dégradation affecte la disponibilité et la qualité des produits. L'objectif principal de cette étude est de développer des politiques conjointes optimales de fabrication, de réutilisation, de maintenance et de contrôle de la qualité pour un système de production qui se dégrade. Une approche de programmation dynamique stochastique est utilisée pour développer les conditions d'optimalité de type Hamilton-Jacobi-Bellman (HJB). Par la suite, nous avons utilisé des méthodes numériques pour résoudre les équations HJB obtenues afin de déterminer les seuils optimaux de fabrication et de réutilisation, les fractions optimales de produits à contrôler et les conditions optimales de démarrage des opérations de maintenance préventive. Pour illustrer ce travail, nous avons simulé un exemple numérique d'une ligne de production hybride (fabrication / réutilisation). Les résultats obtenus nous ont permis de développer simultanément une politique de production à seuil critique, une politique d'inspection par échantillonnage, et une politique de maintenance opportuniste. Ensuite, nous avons effectué une analyse de sensibilité de nos modèles pour montrer leur robustesse. Enfin, nous avons comparé notre politique avec des politiques adaptées de la littérature. Cette comparaison nous a permis de mettre en évidence les gains générés par les politiques de contrôle proposées.

Mots-clés : Systèmes de fabrication hybrides, processus stochastique, détérioration du système de fabrication, maintenance, logistique inverse, qualité

Abstract

This paper conducts a study on a hybrid manufacturing and remanufacturing system that degrades according to its production rate. The system consists of two failures and repairs prone machines that produce a single type of product. Both machines degrade according to their production rates, which affects their availability and the quality of the products. The main objective of this study is to develop optimal joint manufacturing, remanufacturing, maintenance, and quality control policies for a deteriorating production system. A stochastic dynamic programming approach is used to develop the Hamilton-Jacobi-Bellman (HJB) type optimality conditions. Subsequently, we used numerical methods to solve its obtained HJB equations in order to determine the optimal manufacturing and remanufacturing thresholds, the optimal fractions of products to be controlled and the optimal conditions to start preventive maintenance operations. To illustrate this work, we have simulated a numerical example of a hybrid production line (manufacturing / remanufacturing). The obtained results allowed us to develop simultaneously a critical threshold production policy, a sampling inspection policy, and an opportunistic maintenance policy. Next, we performed a sensitivity

analysis of our models to show their robustness. Finally, we compared our policy with policies adapted from the literature. This comparison allowed us to highlight the gains generated by the proposed control policies.

Keywords: Hybrid manufacturing systems. Stochastic process, Manufacturing system deterioration, Maintenance, Reverse logistics, Quality

4.1 Introduction

In recent decades, the scarcity of raw materials, environmental constraints and the globalization of markets have pushed companies to improve the quality of their products and to integrate the reuse of EOL products in their supply chains. This context has forced companies to address the problem of optimizing their production chain by integrating reuse and quality control aspects. As a result, many companies today have already integrated strategies to recover and reuse EOL products as raw materials. This new approach has allowed to reduce the negative impact of the industry on the environment as well as to obtain the new finished products at very low prices (Ouaret, Kenne, and Gharbi 2018b). Facing the growing potential of this new production approach, researchers have focused on the study of hybrid production systems, which are defined as joint manufacturing / remanufacturing systems, with the aim of increasing their performance (Huang, Chen, and Fang 2013). However, hybrid systems are subject to failure and random repair on the one hand, and to degradation of their components on the other (Hajej, Rezg, and Gharbi 2021). This degradation affects the quality of the products and reduces the reliability of the system. As such, it is difficult for companies to meet the market demand and it is even more difficult to manage the reverse logistics of the company. Therefore, new production and reuse policies for companies are needed which allow joint optimization of maintenance and product quality control strategies. In this context, we propose the presented study on the optimization of hybrid production and remanufacturing systems. This research study is structured in nine (9) sections as described in the following.

Section 4.2 presents a review of the literature already conducted in the field. Section 4.3 presents the industrial context. In section 4.4, we define the parameters and assumptions of the problem. Subsequently, in section 4.5, we formulate the general problem, define the optimal conditions and finally simulate a numerical example. Section 4.6 presents the sensitivity analysis performed to validate our model. Section 4.7 presents a comparative study of our policy with other existing policies. Section 4.8 presents the implementation procedure and finally section 4.9 concludes this research, presenting its limitations and the prospects for future work.

4.2 Literature review

Numerous publications on studies of production systems have helped developing production policies, maintenance policies, and quality control strategies to support companies to optimize their production, reduce costs and meet customer demand. These systems are subject to several random factors, such as breakdowns and repairs, component degradation and product quality. Therefore, it is very difficult to propose optimal policies that simultaneously integrate all these aspects. Hence, to enhance clarity and structure of this literature review, we have subdivided it into three (3) parts according to the relevant areas addressed by the authors. First, the study of the effect of the production rate on degradation, reliability and quality will be discussed in section 4.2.1. Then, the optimal control with integration of quality control strategies will be reviewed in section 4.2.2. Finally, closed-loop reverse logistics with quality control of manufactured and remanufactured products are discussed in section 4.2.3.

4.2.1 Effect of production rate on degradation, reliability, and quality

Considering the complexity of the simultaneous optimization of production system performance, researchers have focused more on the independent optimization of the different factors that are integrated in a supply chain. This is mainly done through modeling of such systems following the homogeneous Markovian process or the semi-Markovian process. This approach has allowed researchers to get closer to the industrial context and, gradually, joint studies have emerged. For example, Kenne and Gharbi (Kenne and Gharbi 2004) have proposed production strategies in conjunction with maintenance policy to reduce the cost of production and increase the reliability of their study system. However, they have neglected the influence of machine degradation on the system's reliability. Therefore Roux et al., (Roux et al. 2013) and Chouikhi et al. (Chouikhi et al. 2011) proposed joint policies of production and maintenance with a new strategy of preventive maintenance to reduce the degradation of machines. Nodem et al. (Nodem et al. 2010) extended this work by proposing the option of machine replacement as another alternative to preventive maintenance. The addition of this new replacement option reduced the repair times, which increases with the degradation of the system. Later, Kouedeu et al. (Kouedeu al. 2015) conducted a study to optimize a system consisting of two machines, one of which degrades as a function of the production rate and neglected the effects of this degradation on product quality. However, the objective of a manufacturing system is not only to satisfy the customer's demand but also to ensure that the products comply with certain regulations (technical, environmental, health, etc.) (Kim and Gershwin 2005; Kim and Gershwin 2008). Hence, Colledani and Tolio (Colledani and Tolio 2011) proposed to integrate a feedback of production strategies according to the quality aspects to evaluate the performance of the system. Following this work, several other studies integrating the impact of quality on the performance and optimization of production systems have emerged. This is the case of Rached et al. (Rached et al. 2015) who integrated quality control in a broader way. Their strategy was to propose an optimal production policy that considers the constraints and quality control due to the unreliability of raw material suppliers. This work not only allowed to control the production and to optimize the supply of raw material, but it also permitted to increase the quality of the manufactured products. Also in the same context, Mehdi et al. (Mehdi et al. 2010), Colledani (Colledani and T. Tolio 2012), and Nourelfath et al. (Nourelfath et al. 2016) established the relationship between production and product quality. This new approach allows to control the effect of system deterioration on its availability and improve product quality. This work has significantly reduced the impact of machine degradation on the quality of manufactured products, but it did not allow to limit the distribution of non-conforming, already manufactured products.

4.2.2 Optimal control with integration of product quality control strategies

According to Robotis et al. (Robotis et al. 2012), an inspection of manufactured products is necessary to limit the distribution of non-conforming products. Therefore, Rivera-Gomez et al. (Rivera-Gomez et al. 2013) developed production policies in conjunction with inspection strategies for all manufactured products. In the same manner, Mohammadi et al. (Mohammadi et al. 2015) and Bouslah et al. (Bouslah, Gharbi et al. 2016) have developed, a continuous product inspection strategy, simultaneously to the production and maintenance policies, however they neglected the influence of the production rate on machine degradation. Another limitation of the inspection strategy proposed by Rivera Gomez (Rivera-Gomez et al. 2013), Mohammadi et al. (Mohammadi et al. 2015) and Bouslah et al. (Bouslah et al. 2016) is its inapplicability for production systems that include large series and that are unable to inspect all products. In addition, this strategy is expensive and requires more inspection time. For this reason, a dynamic sampling type inspection is more advantageous in some production systems. Hence, Rivera-Gomez et al. (Rivera-Gomez et al. 2020) studied a production system consisting of a machine that progressively degrades with time. They developed joint strategies for production, preventive maintenance, and dynamic inspection of manufactured products. During this study, they considered the effect of degradation on the quality of the products, but they neglected its influence on the reliability. In another study, Ait-El-Cadi et al. (Ait-El-Cadi et al. 2021) integrated the influence of the machine operation on its reliability and on the quality of the products in their policies. This new approach extends the work of Rivera-Gomez et al. (Rivera-Gomez et al. 2020). Hajej et al. (Hajej et al. 2021) have also conducted a study for a system that degrades as a function of its production rate. They proposed a dynamic inspection policy and maintenance strategies to reduce the impact of the production rate on reliability and product quality. The obtained results in the discussed works were satisfactory as they allowed companies to optimize their production, reduce the impact of machine degradation and improve product quality. However, these studies do not address the economic and environmental challenges faced by companies that reuse EOL products.

4.2.3 Reverse logistics in closed loop with quality control and remanufacturing

In view of economic, social and environmental issues, Colledani (Colledani and Tolio 2012) presented new business opportunities and the need for companies to adopt new production and repackaging strategies. Yoo et al. (Yoo et al. 2012) conducted studies to minimize internal and external costs due to poor product quality. They implemented sampling type policies to minimize the total cost, extending previous work that only focused on internal costs and which did not propose maintenance policies. Kenne et al. (Kenne et al. 2012) and Samir et al. (Samir, P et al. 2013) studied a hybrid manufacturing and remanufacturing system consisting of machines subject to random failures and repairs. This study allowed them to propose an optimal policy that simultaneously integrates maintenance, reverse logistics and stochastic optimal control strategies. However, the effect of system degradation wass neglected and no strategy was implemented to control or improve product quality. Kouedeu, et al. (Kouedeu et al. 2014) extended the work of Samir et al. (Samir, P et al. 2013) by proposing an optimal production policy in conjunction with maintenance strategies for a hybrid system consisting of two separate machines manufacturing the same product. Both machines are subject to random failures and repairs. In addition, the machine for remanufacturing degrades according to its production rate and this degradation affects its reliability. The degradation of the remanufacturing machine as well as the impact of the degradation of the manufacturing machine on the quality of the products were neglected. Therefore, Ouaret et al. (Ouaret et al. 2018) proposes a more extensive policy, which include the option of machine replacement as a solution to limit the infinite influence of the system degradation on its reliability and on the quality of the products from manufacturing and remanufacturing. This new policy is more realistic and closer to a real industrial context, but it does not propose a strategy to control and reduce the fraction of nonconform products distributed to customers. Polotskiet al. (Polotski et al. 2019) studied a manufacturing system producing a single type of product from raw material and EOL products. Ouaret et al. (Ouaret et al. 2018) studied a production system consisting of a manufacturing machine and an unreliable recovery machine that produces the same type of parts. The manufacturing machine degrades as it ages, and this degradation affects the reliability and quality of the products. All nonconforming products produced by the principal machine are identified during inspection and sent to reuse inventory. Non-conforming products from the remanufacturing machine are identified at the inspection station and destroyed. Thanks to the addition of this quality control station, Ouaret et al. (Ouaret et al. 2018) have limited the distribution of non-conforming products. However, as in the work presenented by Rivera-Gomez et al. (Rivera-Gomez et al. 2013), this strategy is very costly according to Rivera-Gomez et al. (Rivera-Gomez et al. 2020) and Hajej et al. (Hajej et al. 2021). As well, for large series production or for systems where the product inspection time is long, it challenging to inspect all the products.

4.2.4 Summary and conclusion of the reviewed literature

Table 4.1 outlines a summary of the papers discussed in this literature review. Thus, for each article studied, we show the factors considered to optimize the performance of the system studied by each of the authors.

Tableau 4.1 Literature review synthesis

Research axis / Authors	Maintenance strategies	Deterioration on reliability	Deterioration on quality	Influence production rate manufacturing	Influence production rate remanufacturing	Quality control	Inspection strategy based on degradation	Dynamic sampling	Reverse Logistics	remanufacturing
study of the effect of production rate on degradation, reliability, and quality										
Effect of degradation on reliability										
(Kenne and Gharbi 2004)	✓	✓								
(Roux et al. 2013)	✓	✓								
(Chouikhi et al.2011)	✓	✓								
(F Dehayem 2010)	✓	✓								
(F Dehayem 2011)	✓	✓								
(Polotski et al. 2019)	✓	✓								
Influence of the production rate										
(Kenne and Gharbi 1999)	✓			✓						
(Sofiene Dellagi1 2010)		✓		✓						
(Ayed et al.2012)		✓		✓						
(Kouedeu et al. 2015)	✓	✓		✓						
(Tarek et al. 2016)	✓	✓		✓						
Effect of degradation on quality										
(Colledani and Tolio 2011)		✓	✓							
(Ouaret et al. 2015)		✓	✓							
(Ouaret et al. 2018a)		✓	✓							
(Nourelfath et al. 2016)		✓	✓							
(Colledani and Tolio 2012)	✓	✓	✓							

Table 4.1 Literature review synthesis (Suite)

Research axis / Authors	Maintenance strategies	Deterioration on reliability	Deterioration on quality	Influence production rate manufacturing	Influence production rate remanufacturing	Quality control	Inspection strategy based on degradation	Dynamic sampling	Reverse Logistics	remanufacturing
Optimal control with integrated product quality control strategies										
(Robotis et al. 2012)	✓	✓	✓			✓				
(Rivera-Gomez et al. 2013)		✓				✓				
(Bouslah et al. 2016)	✓	✓	✓			✓				
(Mohammadi et al. 2015)		✓	✓			✓				
(Lopes 2018)	✓	✓	✓			✓				
(He et al. 2019)	✓	✓	✓			✓				
(Paraschos et al. 2020)	✓	✓	✓			✓				
(Guoqing Cheng 2020)	✓	✓	✓			✓	✓			
(Zhang et al. 2020)	✓	✓				✓				
(Rivera-Gomez et al. 2020)	✓		✓			✓	✓	✓		
(Hajej et al. 2021)	✓		✓	✓		✓		✓		
(Ait-El-Cadi et al. 2021)	✓	✓	✓			✓		✓		
reverse logistics in closed loop with quality control and remanufacturing										
(Colledani and Tolio 2012)									✓	
(Kenne et al.i 2012)	✓								✓	✓
(Yoo, Kim, and Park 2012)	✓								✓	✓
(Samir et al. 2013)	✓	✓	✓						✓	
(Kouedeu et al. 2014)									✓	✓
(Ouaret et al.2018b)	✓	✓	✓			✓			✓	✓
Proposed solution	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

We can observe from this literature summary that several works have been published in the field. Nevertheless, very few have simultaneously treated the influence of the degradation on the reliability of the system and on the quality of the products in a context of closed loop production. In fact, none of these works considers simultaneously the influence of the production rate on the reliability and the quality of the products in a system including the reverse logistics in closed loop. In this work, we extend the work of Kouedeu et al (Kouedeu et al. 2014), by considering the influence of the production rate on both reliability and quality of the manufacturing system and its produced parts respectively, in order to better reflect industrial system reality. In addition, we will also extend the work of Ait-El-Cadi et al. (Ait-El-Cadi et al. 2021) and Hajej et al. (Hajej et al. 2021) by integrating recovery and remanufacturing strategies. Hence, to carry out this study we must jointly integrate the following specific objectives:

- Propose dynamic inspection strategies for manufactured products.
- Propose dynamic inspection strategies for remanufactured products.
- Propose an optimal production control strategy for each machine.
- Propose a maintenance policy to limit the degradation of the machine.

4.3 Industrial context

The production, maintenance and quality control strategies developed in this study can be useful for companies that have machines subject to breakdowns and random repairs and that degrade as they are used. In addition, the developed strategies aim to support companies, which manufacture products from raw materials and recycle products at the end of their life for remanufacturing. This is the case, for example, of mechanical machining centers made up of machines that follow stochastic dynamics, and the degradation of their different components depends on their frequency of us (Kouedeu, Kenne et al. 2015). This degradation influences the reliability and quality of manufactured products (Rivera-Gomez, Gharbi et al. 2013) and remanufactured products (Ouaret, Kenne et al. 2018). The joint study of all discussed aspects, such as reliability, production frequency, quality, reverse logistics maintenance and remanufacturing, allows any company that adopts our policy to 1) reduce its total production cost, 2) to improve the quality of the manufactured products, 3) to enhance the quality of the remanufactured products and finally 4) to limit the degradation of the machines by introducing an opportunistic preventive maintenance. These strategies can also be used by other industries that face a continuous deterioration of their components, e.g. the automotive industries, chemical industries and manufacturing industries for mechanical and electronic parts (Kouedeu, Kenne et al. 2015).

4.4 Assumptions, and problem formulation.

This section first presents the parameters used throughout this work, then the assumptions driving this research and finally states the optimal control problem.

4.4.1 Assumptions.

To model this problem, we will consider the following assumptions:

- H_1: Raw material and end-of-life products are always available.
- H_2: The demand rate is known and is constant.
- H_3: The maximum production rate of each machine is known.
- H_4: A known proportion of end-of-life products is returned for remanufacturing.
- H_5: The various costs: storage, shortage, inspection, maintenance are constant and known.
- H_6: The non-conforming products received by the customers are recovered and replaced by the company, which causes losses for the company.
- H_7: The machines degrade according to their production rate and this influences the reliability and quality of the products.
- H_8: The customers demand a certain minimum quality, so the average of non-conforming products (AOQ) on the market must not exceed a certain maximum limit (AOQLmax).
- H_9: The remanufactured products are identical to the directly manufactured products.

- H_{10}: Identified non-conforming products are remanufactured
- H_{11}: The repairs carried out at the time of the breakdowns are imperfect and bring back the machine in a state as good as old (ABAO) which does not improve its state of degradation.
- H_{12}: The preventive revision of its components is perfect and brings the machine to a new state (AGAN).
- H_{13}: The quality control system is of the sampling type (AOQL).

Assumptions H_1 and H_2 are classic assumptions in supply chain network models (Kenne, Dejax, and Gharbi 2012). Assumption H_3 is common in production planning (Kouedeu et al. 2014). Assumption H_4 is specific to this type of model and has been incorporated in the work of (Kenne, Dejax, and Gharbi 2012), (Ouaret, Kenne, and Gharbi 2018b). Assumption H_5 is standard. Assumptions H_6 and H_7 and H_8 are specific to this approach due to the consideration of customer satisfaction and the effect of production rate on reliability and quality. Assumption H_9 is specific to the remanufacturing model including inspection for quality control. Assumption H_{10} is specific to this type of model and has been incorporated in the work of Ouaret et al. (Ouaret, Kenne, and Gharbi 2018b). Assumptions H_{11} and H_{12} are common in maintenance policy. Assumption H_{13} is common in for dynamic production inspection processes.

4.4.2 Description of the problem

The manufacturing structure we are studying consists of two machines that produce a single type of part. The main machine produces parts from raw material and the second machine reuses and refurbishes EOL products and non-conforming products identified at inspection. Both machines are subject to random breakdowns and repairs. A certain fraction of the products do not meet the required quality. Both machines are degrading as a function of their production rates. This degradation affects the reliability and quality of the products, when the production rate of a machine increases, its failure rate and the defective rate increase reciprocally. To avoid stock-outs due to system unavailability or degradation, a safety stock threshold is necessary to reduce the costs of delays and non-distribution. To limit the distribution of non-conforming products, an inspection by means of a quality-sampling plan is established. The studied decision variables in this model are the production rate of each machine and the fraction of products to inspect for each machine. The objective is to minimize the total cost of production including the costs of shortage, inventory, inspection, repair, major overhaul, and penalty due to the non-satisfaction of the quality of products received by the customer. Figure 4.1 presents the structure of this hybrid manufacturing system.

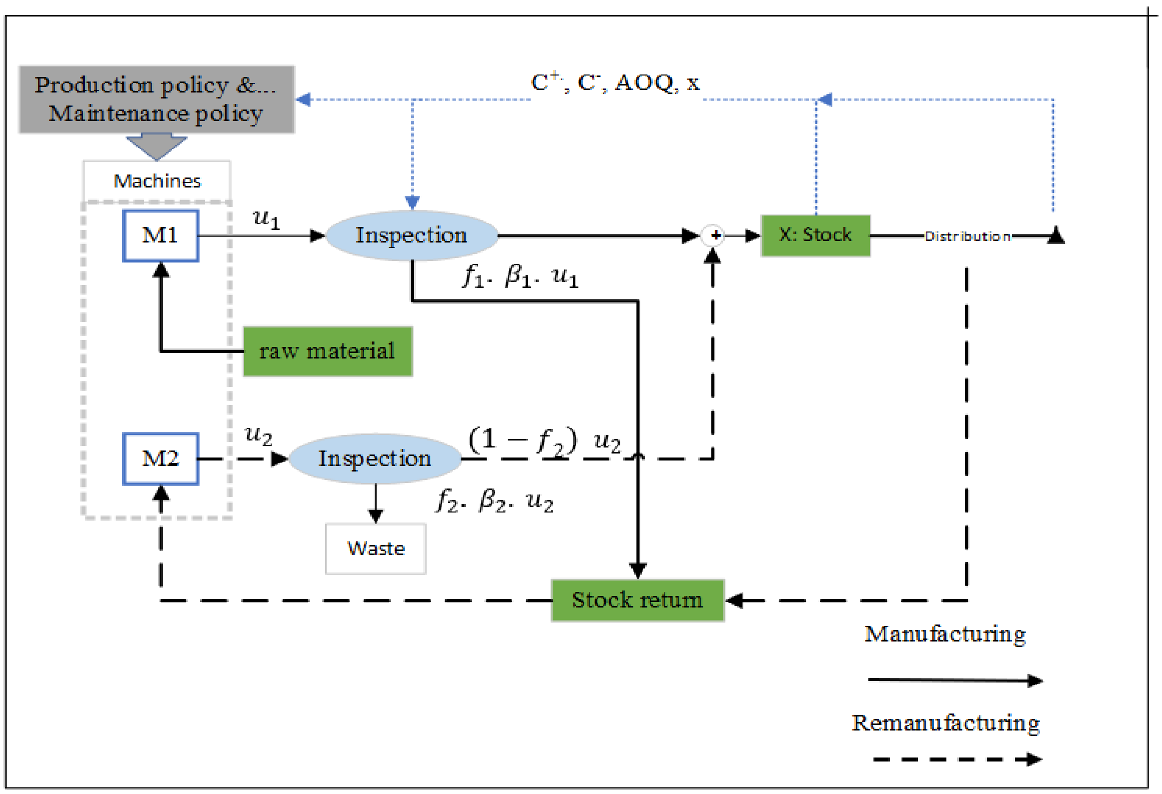


Figure 4.1 Structure of the studied hybrid manufacturing - remanufacturing system

4.4.3 Formulation of the optimal control problem

The machines M_1 and M_2 are subject to random failure and repair. Let ξ (t) be the stochastic process describing the dynamics of the system at time t. Then ξ (t) is defined as follows.

$$\xi(t) = \begin{cases} 1 & if\ \ both\ machines\ are\ in\ operation \\ 2 & if\ the\ main\ machine\ is\ in\ operation\ and\ \text{the second}\ machine\ \ is\ down \\ 3 & if\ the\ main\ machine\ is\ down\ and\ \text{the second}\ machine\ \ is\ in\ operation \\ 4 & if\ both\ machines\ are\ down \end{cases} \quad (4.1)$$

The system can therefore be in the following modes α ϵ Ω = {1, 2, 3, 4,}. The following figure 4.2 shows the transition diagram of the system with $\lambda_i = \frac{1}{MTBF^i}$ and $\mu_i = \frac{1}{MTTR^i}$; i ∈ {1, 2} denoting the failure and repair rates of machine i, respectively

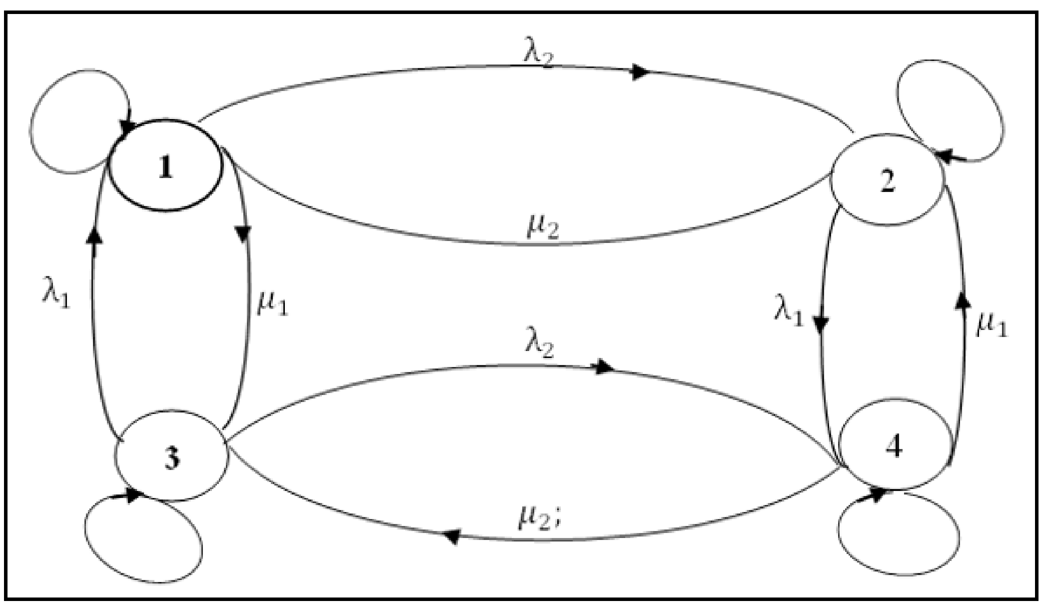


Figure 4.2 State transition diagram of the stochastic process

.

4.4.3.1 Average quality after control (AOQ)

For a production run a certain fraction f_1^α of the manufactured products is inspected and the identified nonconforming products are sent to reuse inventory. Similarly, a certain fraction f_2^α of the remanufactured products is inspected and the identified non-conforming products are destroyed: both machines produce non-conforming products at respective rates of β_1 for machine 1 and β_2 for machine 2. Thus, for a production and remanufacturing run, the inspected fraction of products from machine is f_i^α and the uninspected fraction is *(1-f_i^α).* Since not all products are inspected, a certain proportion of the distributed products are non-compliant. This proportion, denoted as average quality after control (AOQ), is given by the following equation (4.2).

$$\text{AOQ}(f_1^\alpha, f_2^\alpha) = \frac{(1-f_1^\alpha)u_1 . \beta_1 + (1-f_2^\alpha)u_2 . \beta_2)}{(u_1+u_2) - (f_1^\alpha . \beta_1 . u_1 \ + f_2^\alpha \beta_2 . u_2)} \tag{4.2}$$

$$AOQL_i = \max \text{AOQ} \ (f_i^\alpha, \beta_i) \qquad \text{whit} \ \ 0 \le f_i^\alpha \le 1 \ and \ \ 0 \le \beta_i \le 1 \tag{4.3}$$

Figure 4.3 outlines such manufacturing control circuit:

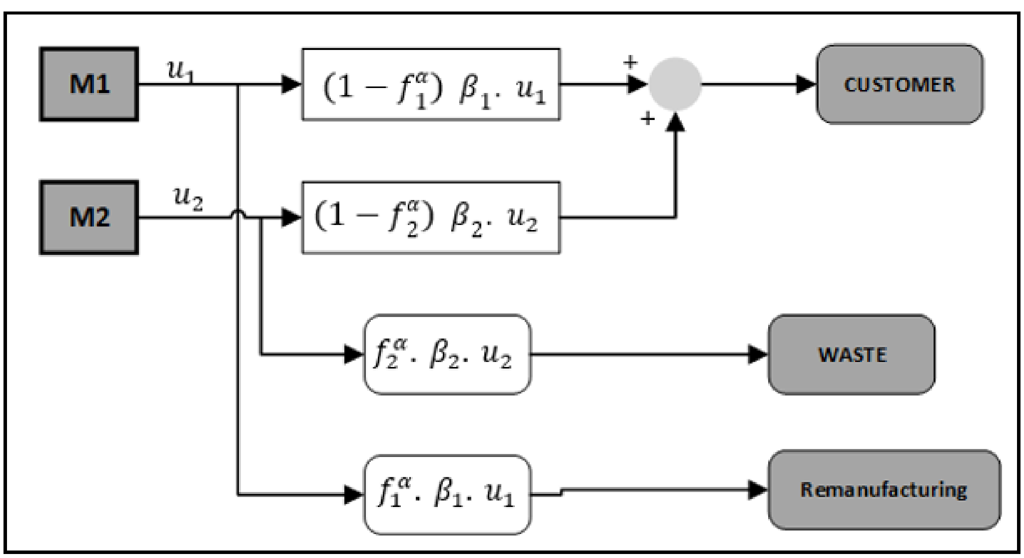


Figure 4.3 Inspection process including average quality after control (AOQ)

To anticipate the replacement of non-conforming products received by customers before being returned to the firm, Hlioui, et al. (Hlioui, Gharbi, and Hajji 2015) proposed an adjustment of the demand rate from its initial value d to the adjusted value d' defined as:

$$d' = \frac{d}{1 - \text{AOQL}} \tag{4.4}$$

Each machine produces at a rate $u_i(t)$ but the product does not enter the inventory at this same rate because the inspection time is considered. Hence, as developed in Rivera-Gomez et al. (Rivera-Gomez et al. 2020), the actual rate $u_i^r(t)$ at which the product enters the inventory for each machine is defined by the equation (4.5).

$$u_i^r = \frac{u_{insp} * u_i}{f * u_i + u_{insp}} - f_i^\alpha * \beta_i * u_i \tag{4.5}$$

where: $f_i^\alpha * \beta_i * u_i$ represents non-conforming products identified during the inspection. The real production rate u_1^r of machine M_1 is defined by the following equation (4.6).

$$u_1^r = \frac{u_{insp} * u_1}{f * u_1 + u_{insp}} - f_1^\alpha * \beta_1 * u_1 \tag{4.6}$$

By replacing $u_1$by its maximum value U_1^{max} , we obtain the real maximum production rate of M_1 defined by equation (4.7).

$${}^{r}U_1^{max} = \frac{u_{insp} * U_1^{max}}{f_1^\alpha * U_1^{max} + u_{insp}} - f_1^\alpha * \beta_1^{max} * U_1^{max} \tag{4.7}$$

Using the same method, we obtain the real production rates u_2^r of M_1 defined by the following equation (4.8) its maximum value defined by the equation (4.9).

$$u_2^r = \frac{u_{insp} * u_2}{f_2^\alpha * u_2 + u_{insp}} - f_2^\alpha * \beta_2 * u_2 \tag{4.8}$$

$$ {}^{r}U_2^{max} = \frac{u_{insp} * U_2^{max}}{f_2^{\alpha} * U_1^{max} + u_{insp}} - f_2^{\alpha} * \beta_2^{max} * U_2^{max} \tag{4.9} $$

Now, we can define the stock of finished manufactured products by the equation (4.10):

$$ \frac{dx(t)}{\mathrm{d}t} = \dot{x}(t) = u_1^r(t) + u_2^r(t) - d' \; ; \; x(0) = x_0 \tag{4.10} $$

Where x_0 is the level of stocks in the initial state $t = 0$.
The presented manufacturing process is modeled by a semi-Markovian chain with continuous time and discrete state with a transition rate matrix $Q = (q_{\alpha j})$. The transition rates are defined as follows:

$$ P[\xi(t+dt) = \gamma \,/\, \xi(t) = \alpha] = \begin{cases} q_{\alpha\gamma} * \delta t + o(\delta t) \; si \; \alpha \neq \gamma \\ 1 + q_{\alpha\gamma} * \delta t + o(\delta t) \; si \; \alpha = \gamma \end{cases} \tag{4.11} $$

$$ \lim_{(\delta t - - 0)} o(\delta t) = 0 \text{ et } \alpha, \gamma \in \Omega = \{1, 2, 3, 4\} $$

Increasing the production rate of each machine increases its degradation. This degradation influences the reliability of the machine by increasing its failure rate as defined in Kouedeu et al. (Kouedeu et al. 2015). Similarly, this degradation influences the quality of the products as developed in Hajej et al. (Hajej, Rezg, and Gharbi 2021). Thus, the failure rate and rejection rate of each of the machines can now be defined as follows:

$$ \lambda_1(t, u_1) = \begin{cases} \lambda_1^{min} \; if \;\; u_1(\mathrm{t}) \in \; [0, U^1] \\ \lambda_1^{max} \; if \;\; u_1(\mathrm{t}) \; \in \; [U^1, U_1^{max}] \end{cases} \tag{4.12} $$

$$ \beta_1(t, u_1) = \begin{cases} \beta_1^{min} \;\; if \;\; u_1(\mathrm{t}) \; \in \; [0, U^1] \\ \beta_1^{max} \; if \;\; u_1(\mathrm{t}) \; \in \; [U^1, U_1^{max}] \end{cases} \tag{4.13} $$

$$\lambda_2(t, u_2) = \begin{cases} \lambda_2^{min} & if \;\; u_2(\mathrm{t}) = U_2^{max} \\ \lambda_2^{max} & if \; not \end{cases} \tag{4.14}$$

$$\beta_2(t, u_2) = \begin{cases} \beta_2^{min} & if \;\; u_2(\mathrm{t}) = U_2^{max} \\ \beta_2^{max} & if \; not \end{cases} \tag{4.15}$$

Hence, the transition rate matrix is defined as follows:

$$\mathrm{Q} \;\; (u_1, u_{2)}$$
$$= \begin{bmatrix} -(\lambda_2(u_2) + \lambda_1(u_1)\;) & \lambda_2(u_2) & \lambda_1(u_1) & 0 \\ \mu_1 & -(\mu_2 + \lambda_1(u_1)\;) & 0 & \lambda_1(u_1) \\ \mu_1 & 0 & -(\mu_1 + \lambda_2(u_2)) & \lambda_2(u_2) \\ 0 & \mu_1 & \mu_1 & -(\mu_1 + \mu_1) \end{bmatrix} \tag{4.16}$$

Let Q_1, $\mathrm{Q}_2, \mathrm{Q}_3, \mathrm{Q}_4$ be the transition matrices that represent $\mathrm{Q} \;\; (u_1, u_{2)}$ according to the values taken by the production rates $u_1$and u_2 and defined by equation (4.17)

$$\mathrm{Q}(u_1, u_{2)} = \begin{cases} \mathrm{Q}_1 & if \; u_1(t) \in [0, U_1^e] \text{ and } u_2(t) \neq U_2^{max} \\ \mathrm{Q}_2 & if \; u_1(t) \in [U_1^e, U_1^{max}] \text{ and } u_2(t) \neq U_2^{max} \\ \mathrm{Q}_3 & if \; u_1(t) \in [0, U_1^e] \text{ and } u_2(t) = U_2^{max} \\ \mathrm{Q}_4 & if \; u_1(t) \in [U_1^e, U_1^{max}] \text{ and } u_2(t) = U_2^{max} \end{cases} \tag{4.17}$$

By replacing $\lambda_2(u_2)$ and $\lambda_1(u_1)$ by their values as defined in equations (4.12) and 4.14), we obtain the values of $\mathrm{Q}_1, \mathrm{Q}_2, \mathrm{Q}_3, \mathrm{Q}_4$ defined by equations (4.18) to (4.21).

$$\mathrm{Q}_1 = \begin{bmatrix} -(\lambda_2^{min} + \lambda_1^{min}\;) & \lambda_2^{min} & \lambda_1^{min} & 0 \\ \mu_1 & -(\mu_2 + \lambda_1^{min}\;) & 0 & \lambda_1^{min} \\ \mu_1 & 0 & -(\mu_1 + \lambda_2^{min}) & \lambda_2^{min} \\ 0 & \mu_1 & \mu_1 & -(\mu_1 + \mu_1) \end{bmatrix} \tag{4.18}$$

$$Q_2 = \begin{bmatrix} -(\lambda_2^{min} + \lambda_1^{max}) & \lambda_2^{min} & \lambda_1^{max} & 0 \\ \mu_1 & -(\mu_2 + \lambda_1^{max}) & 0 & \lambda_1^{max} \\ \mu_1 & 0 & -(\mu_1 + \lambda_2^{min}) & \lambda_2^{min} \\ 0 & \mu_1 & \mu_1 & -(\mu_1 + \mu_1) \end{bmatrix} \tag{4.19}$$

$$Q_3 = \begin{bmatrix} -(\lambda_2^{max} + \lambda_1^{min}\ ) & \lambda_2^{max} & \lambda_1^{min} & 0 \\ \mu_1 & -(\mu_2 + \lambda_1^{min}\ ) & 0 & \lambda_1^{min} \\ \mu_1 & 0 & -(\mu_1 + \lambda_2^{max}\ ) & \lambda_2^{max} \\ 0 & \mu_1 & \mu_1 & -(\mu_1 + \mu_1) \end{bmatrix} \tag{4.20}$$

$$Q_4 = \begin{bmatrix} -(\lambda_2^{max} + \lambda_1^{max}) & \lambda_2^{max} & \lambda_1^{max} & 0 \\ \mu_1 & -(\mu_2 + \lambda_1^{max}) & 0 & \lambda_1^{max} \\ \mu_1 & 0 & -(\mu_1 + \lambda_2^{max}) & \lambda_2^{max} \\ 0 & \mu_1 & \mu_1 & -(\mu_1 + \mu_1) \end{bmatrix} \tag{4.21}$$

The limiting probabilities are defined as follows:

$$\begin{cases} \sum_{j=1}^{4} \pi_j * q_{ij} = 0 \\ \sum_{i=1}^{4} \pi_i = 1 \end{cases} \tag{4.22}$$

The system is feasible if and only if the variables satisfy the following feasibility condition:

$$\pi_1(U_1^{max} + U_2^{max}) + \pi_2 * U_1^{max} + \pi_{\mathbf{3}} * U_2^{max} \geq \frac{d'}{1 - AOQL} \tag{4.23}$$

The domain of admissibility denoted Γ(α) of the control variables is defined as follows:

$$\Gamma(\alpha) = \begin{Bmatrix} \mathrm{u}_1(\cdot), \mathrm{u}_2(\cdot), f_1(\cdot), f_2(\cdot), \mathrm{w}_i^p : /\ 0\ \leq u_1 \leq U_1^{max}; 0\ \leq u_2\ \leq U_2^{max}; 0 \leq f_1^\alpha \leq 1; \\ 0 \leq f_2^\alpha \leq 1;\ \mathrm{w_p} = \{\ 0{,}1\ \}; \end{Bmatrix}, \tag{4.24}$$

Let $g(\cdot)$ be the instantaneous cost, composed of the costs for production, remanufacturing, shortage, storage, inspection, scrap, and penalty cost on returned products. Tableau 4.2 defines the expression of each of these costs.

Tableau 4.2 Summary of instant costs

Source	**Cost**	**Notation**
The cost incurred due to inventory and shortage	$C^{+}x^{+} + C^{-}x^{-}$ with x^{+}= max (0, +x) and x^{-} = max (-x, 0)	$g(1)$
Manufacturing and remanufacturing costs	$u_1 . C_f + u_2 . C_r$	$g(2)$
Inspection cost	$C_{insp}\ [f_1^{\alpha} .\ u_1(\alpha) + f_2^{\alpha} .\ u_2(\alpha)]$	$g(3)$
Cost of destruction	$C_{dest}[f_1^{\alpha} . \beta_1 . u_1 + f_2^{\alpha} . \beta_2 . u_2]$	$g(4)$
Cost of replacing non-compliant products distributed	$C_{ret}\ [(1 - f_1^{\alpha})\ u_1 . \beta_1 + (1 - f_2^{\alpha}) u_2 . \beta_2)\ ]$	$g(5)$
Maintenance.	$C_m \ * W_i^m(\alpha) * \text{Ind}\{\alpha = 2,3,4\}$ with Ind {d(.)} = $\begin{cases} 1\ if\ d(.)\ is\ true \\ 0\ if\ not \end{cases}$	$g(6)$

The instantaneous costs are determined by the following equation:

$$g(\ \alpha, x_1, x_2) = \ g(1) + g(2) + g(3) + g(4) + g(5) + g(6) \tag{4.25}$$

The discounted cost and value function are defined by equations (4.26) and (4.27):

$$J(x, \alpha, u(.)) = E\left[\int_0^s e^{-\rho t} g(\ \alpha, x_1, x_2)\ .\ dt |\ x_1(0) = x_{10}, x_2(0) = x_{20}, \zeta(t) = \alpha\right] \tag{4.26}$$

$$v(\alpha, x_1, x_2) \ = min\, J(\ \alpha, x_1, x_2)/(u_{i\alpha}, f_\alpha\ ) \in \ \Gamma(\alpha) \tag{4.27}$$

Since, $\vartheta(\alpha, x) = \min_{u\epsilon\Gamma(\alpha)} \ J(\ \alpha, x_1, x_2),\ \ \alpha \in \Omega$ The HJB equation is defined as:

$$\rho\vartheta(x, \alpha) = \min_{([u_1(\cdot), u_2(\cdot), f(\cdot)]\epsilon\Gamma(\alpha)} \left[g(x, \alpha, u) + \frac{\partial\vartheta(\alpha, x)}{\partial x} * \frac{dx(t)}{dt} + \sum_{j\epsilon\Omega} q_{\alpha j}\vartheta(j, x) \right] \tag{4.28}$$

Solving this equation by analytical methods is very complex. Therefore, we will use Kushner's numerical method, which is developed in Appendix A. To illustrate the practical use our model, we will simulate a numerical example in section 4.5

4.5 Simulation and numerical example

In this section, we illustrate a numerical example for the hybrid manufacturing and remanufacturing system. Our system describes a 4-state Markov process: Ω = {1, 2, 3, 4,}. For this study we considered the following domains:

$$D_h = \{-10 \leq x \leq 50\} \tag{4.29}$$

In the following, we consider the parameters of the Router (class R) CNC cutting machine of the Quebec company CET Fire Pumps, which transforms 4' x 8' polypropylene sheets of half an inch thickness. They are cut into various geometries, which are used to manufacture custom water-tanks. For remanufacturing, we added a second machine whose data is taken from Kouedeu et al. (Kouedeu et al. 2015). To illustrate the previously defined policy, we will consider an example with the parameter summarized in the Table 4.3.

Tableau 4.3 Parameters of numerical example

Variables	U_1^{max}	U_1^e	U_2^{max}	d	C^+	C^-	C_f	C_r	C_m	C_{ret}	C_{des}	$\boldsymbol{C_{isp}}$
Values	0.7	0.65	0.3	0.37	3	50	1	2	50	150	30	10

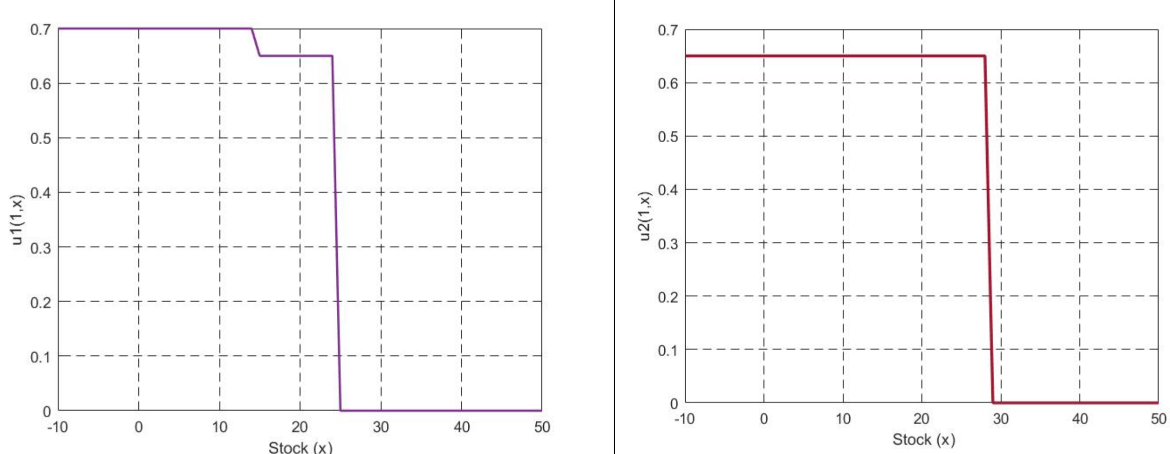
Tableau 4.3 Parameters of numerical example (Suite)

Variables	λ_1^{min}	λ_1^{max}	λ_2^{min}	λ_2^{max}	$\mu1$	$\mu2$	β_1	β_1^{max}	β_2	β_2^{max}	u_{isp}	
Values	0.01	0.06	0.66	1.00	0.02	0.04	0.1	0.15	0.15	0.20	10	

The parameters illustrated in table 3 are such that the admissibility condition of equation (4.24) is satisfied. the following results have been obtained in section 4.5.1, section 4.5.2, section 4.5.3, and section 4.5.4.

4.5.1 Production policy

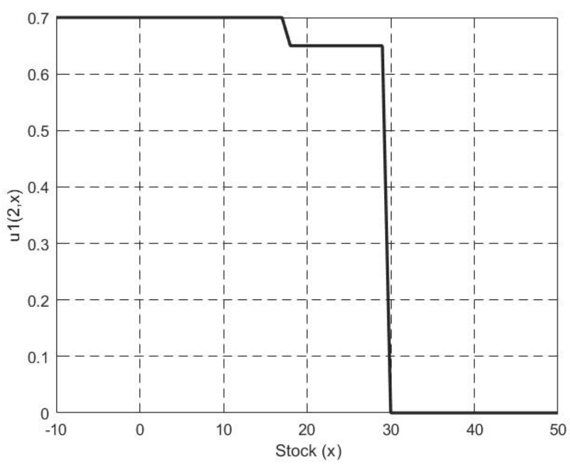
Applying the given production conditions defined by the equation (4.28) and parameter values (table 4.3), we obtained the following results (Figure 4.4)



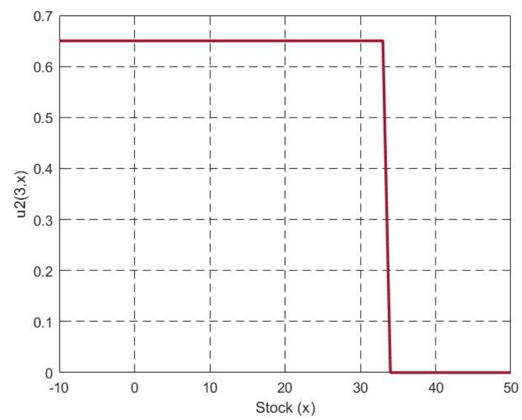
(a) Production rate of the manufacturing machine in mode 1

(b) Production rate of the remanufacturing machine in mode 1

Figure 4.4 Production policy for M_1 and M_2



(c) Production rate of the manufacturing machine in mode 2



(d) Production rate of the remanufacturing machine in mode 3

Figure 4.4 Production policy for M_1 and M_2 (Suite)

Figure 4.4.a presents the production rate of the manufacturing machine M_1 when it is in operation simultaneously with the remanufacturing machine M_2 (referred to as mode 1). Figure 4.4.b presents the production rate of the remanufacturing machine M_2 when it is in operation simultaneously with the manufacturing machine M_1 (mode 1). Figure 4.4.c shows the production rate of the manufacturing machine when it is the only one in operation (mode 2) and Figure 4.4.d present the remanufacturing machine when it is the only one operational.

- Figure 4.4.a shows that when the inventory level of manufactured and remanufactured products is less than $Z_1^*(1) = 14$ we must produce at the maximum rate. Then when this inventory is between $Z_1^*(1) = 14$ et $Z_2^*(1) = 24$, we must reduce the production rate from its maximum value to its economic value. This reduction limits the effect of the production rate on the degradation of the machine (Kouedeu et al. 2014). Then we must stop production as soon as the stock is greater than $Z_2^*(1)$.

- Figure 4.4.b shows that when the inventory level of products in inventory is less than $Z_3^*(1) = 28$ the machine must produce at its maximum rate. When this stock level is exceeded, reduce the production rate to zero.
- Figure 4.4.c shows the production policy of the manufacturing machine when the remanufacturing machine is down (mode 2). We notice that this policy is similar to mode 1 but with higher threshold inventory which are respectively: $Z_4^*(2) = 17$ and $Z_5^*(2) = 29$.
- Figure 4.4.d shows the remanufacturing machine is policy when it is the only machine in operation, is like its policy when both machines are in operation. However, the threshold inventory is higher $Z_6^*(3) = 33$.. This is because the machine is producing alone and is degrading more severely.

In summary we have:

$$u_1^*(1,\mathrm{x}) = \begin{cases} \mathrm{U}_1^{max} & \text{if } \ x(\mathrm{t}) < Z_1^* \\ \mathrm{U}_1^{e} & \text{if } \ Z_1^* < x(\mathrm{t}) < Z_2^* \\ 0 & \text{if } \ x(\mathrm{t}) > Z_2^* \end{cases} \tag{4.30}$$

$$u_2^*(1,\mathrm{x}) = \begin{cases} \mathrm{U}_2^{max} & \text{if } \ x(\mathrm{t}) < Z_3^* \\ 0 & \text{if } \ x(\mathrm{t}) > Z_3^* \end{cases} \tag{4.31}$$

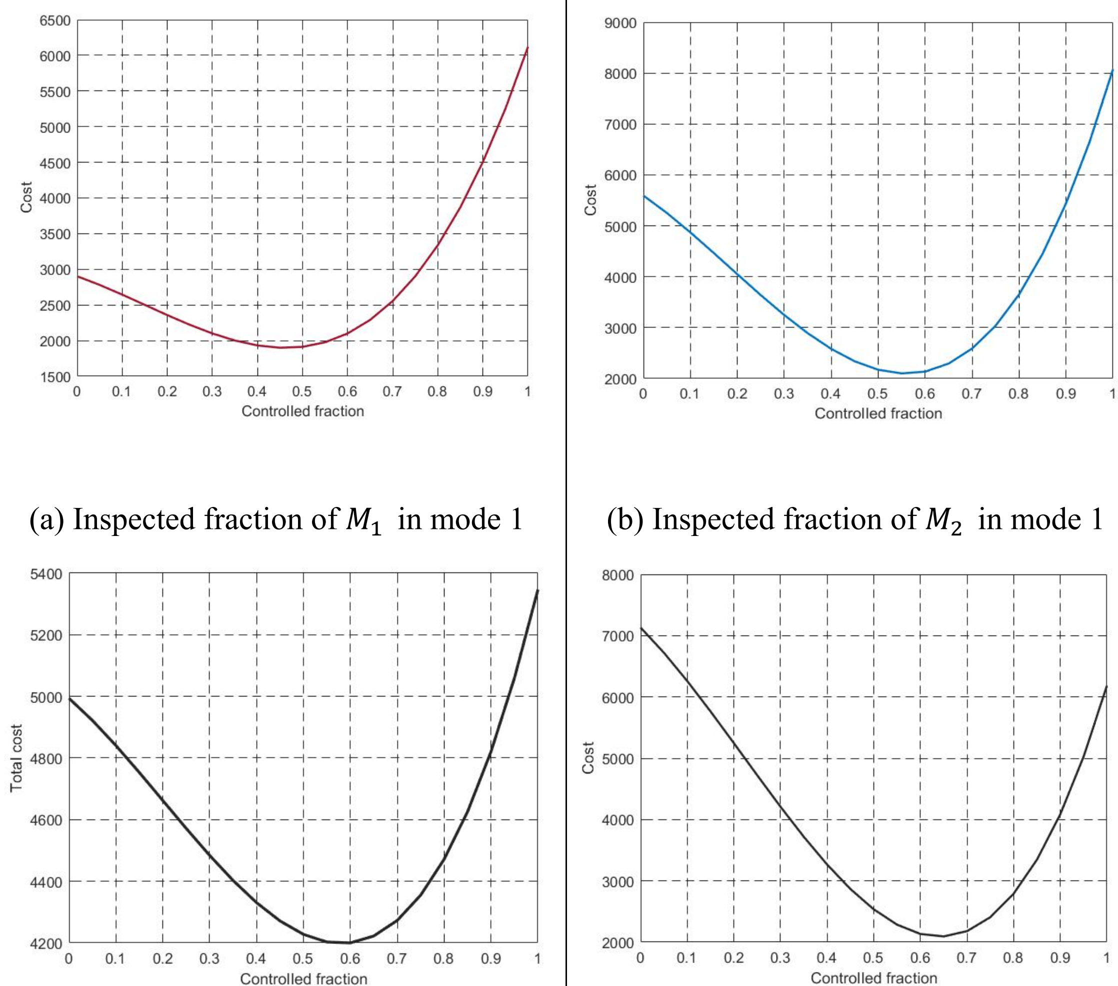
$$u_1^*(2,\mathrm{x}) = \begin{cases} \mathrm{U}_1^{max} & \text{if } \ x(\mathrm{t}) < Z_4^* \\ \mathrm{U}_1^{e} & \text{if } \ Z_4^* < x(\mathrm{t}) < Z_5^* \\ 0 & \text{if } \ x(\mathrm{t}) > Z_5^* \end{cases} \tag{4.32}$$

$$u_2^*(3,\mathrm{x}) = \begin{cases} \mathrm{U}_2^{max} & \text{if } \ x(\mathrm{t}) < Z_6^* \\ d' & \text{if } \ x(\mathrm{t}) = Z_6^* \\ 0 & \text{if } \ x(\mathrm{t}) > Z_6^* \end{cases} \tag{4.33}$$

We also notice that the threshold inventory level of the remanufactured products is larger than that of the manufactured products when both machines are in operation. This can be explained since the considered failure rate of the remanufacturing machine is greater than the failure rate of the manufacturing machine ($\lambda_2 \geq \lambda_1$ or $\mathrm{MTTFM1} > \mathrm{MTTFM2}$). The remanufacturing machine degrades more severely and produces more nonconforming parts than the direct manufacturing machine. We notice that the threshold stock of manufactured products is higher in mode 2 than in mode 1. This is normal because the manufacturing machine is the only one in operation in mode 2. Thus, on one hand it is the only machine to satisfy all the adjusted demand of the market d'. On the other hand, in this mode, the manufacturing machine operates longer at its maximum production rate which accelerates its degradation and increases its failure rate. Therefore, the manufacturing system, to satisfy the demand in case of failure - increases the threshold level to protect itself. We also notice that the threshold stock of remanufactured products in mode 3 is higher than in mode 1. This can be explained because this machine is solely producing in this mode. In mode 4 both machines are down so no production is occurring.

4.5.2 Quality control policy

Figure 5 shows the total production cost as a function of the inspected fractions. Figure 4.5 (a and c) show the total production cost as a function of the inspected proportion of the manufactured products respectively when both machines are in operation and when solely the manufacturing machine is in operation. Figure 4.5.b shows the total production cost as a function of the inspected proportion of remanufactured products.



(a) Inspected fraction of M_1 in mode 1

(b) Inspected fraction of M_2 in mode 1

(c) Inspected fraction of M_1 in mode 2

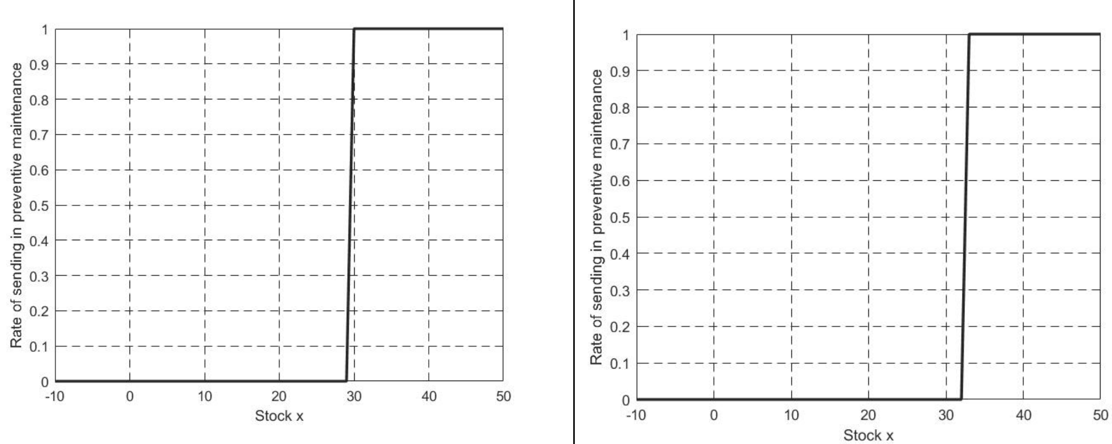
(d) Inspected fraction of M_2 in mode 3

Figure 4.5 Inspection policy for manufacturing /remanufacturing products

The following figures show that: When both machines are in operation, to reduce the production cost while respecting the AOQ constraint, the manufacturer must inspect $f_1^{1*} =$ 45% (Figure 4.5.a) of the manufactured products and $f_2^{1*} = 55\%$ (Figure 4.5.b) of the remanufactured products. However, when only the manufacturing machine is in operation, $f_1^{2*} = 60\%$ (Figure 4.5c) of the manufactured products must be inspected and when solely remanufacturing machine is in operation f_1^{3*} =65 (Figure 4.5.d) of the production must be inspected. The optimal fraction of remanufactured products to be inspected $f_2^{1*}$is larger than the inspected fraction of manufactured products f_1^{1*} because the remanufacturing machine produces more non-conformities products than the manufacturing machine.

4.5.3 Preventive maintenance policy

The preventive maintenance strategy chosen in this study is an opportunistic preventive overhaul of machine components. This strategy was developed and proposed in Kang et al (Kang and Subramaniam 2018) and He et al (He et al. 2019). Here, the components of each of the machines are overhauled during voluntary production downtime especially when the threshold level of each inventory (manufactured products and remanufactured products) is exceeded. In short, when the manufacturing and remanufacturing machines are in voluntary production stoppage in mode 2 and mode 3 respectively, an overhaul is performed on each of the machines. A preventive overhaul is performed on each of the machines.



(a) preventive maintenance from M_1 to mode 2.

(b) preventive maintenance from M_2 to mode 3.

Figure 4.6 Preventive maintenance policy

$$W_1^p(2) = \begin{cases} 1 & if \ Z_5^* < x(t) \\ 0 & if \quad not \end{cases} \tag{4.34}$$

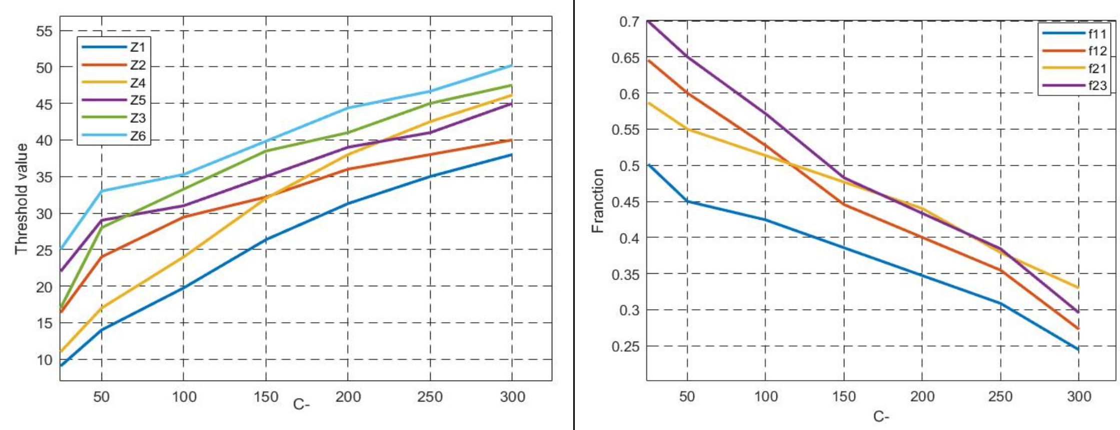
$$W_2^p(3) = \begin{cases} 1 \ if \ Z_6^* < \ x(t) \\ 0, \qquad if \ not \end{cases} \quad (4.35)$$

The policies defined by the equations (4.30) to (4.35) obtained previously allow us to get closer to the industrial context because they simultaneously consider economic issues, customer satisfaction and reverse logistics. However, we need to perform other practical simulations to analyze the sensitivity of our model. This sensitivity analysis aims at validating and illustrating the robustness of the proposed policies.

4.6 Sensitivity analysis

In this section, we perform sensitivity analyses on the cost parameters. For better understanding of the proposed model. The studied parameters include inventory cost C^+shortage cost C^-, manufacturing cost C_f, remanufacturing cost C_f, inspection cost C_{isp}, and the penalty cost on returned products C_{ret}, as well as manufacturing system parameters such as: failure rate, inspection rate and AOQ. The varying behavior of these policy parameters are summarized in Figure 4.7 to Figure 4.16 and they can be explained as follows:

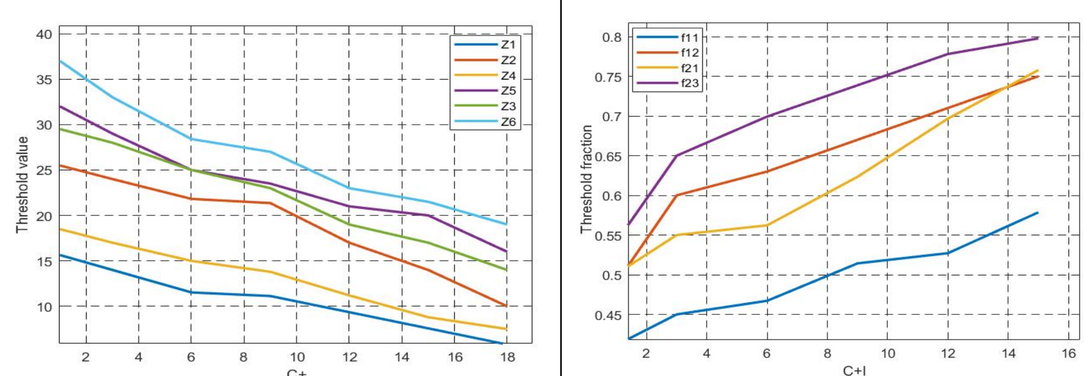
- Variation in shortage cost C^-: When the shortage cost for stock x(t) increases, the critical thresholds ($Z_1^*, Z_2^*, Z_3^*, Z_4^*, , Z_5^*, , Z_6^*$) increase (Figure 4.7.a) while the fractions ($f_1^{1*}, f_2^{1*}, f_1^{2*}, f_2^{3*}$) to be controlled decrease (Figure 4.7.b). This can be explained because the system must store more products to limit shortages. The fractions decrease because with the increase of the shortage costs it becomes preferable to have products even if the quality has not been improved.



(a) Effect on the threshold values (b) Effect on optimal fractions

Figure 4.7 Variation of shortage cost

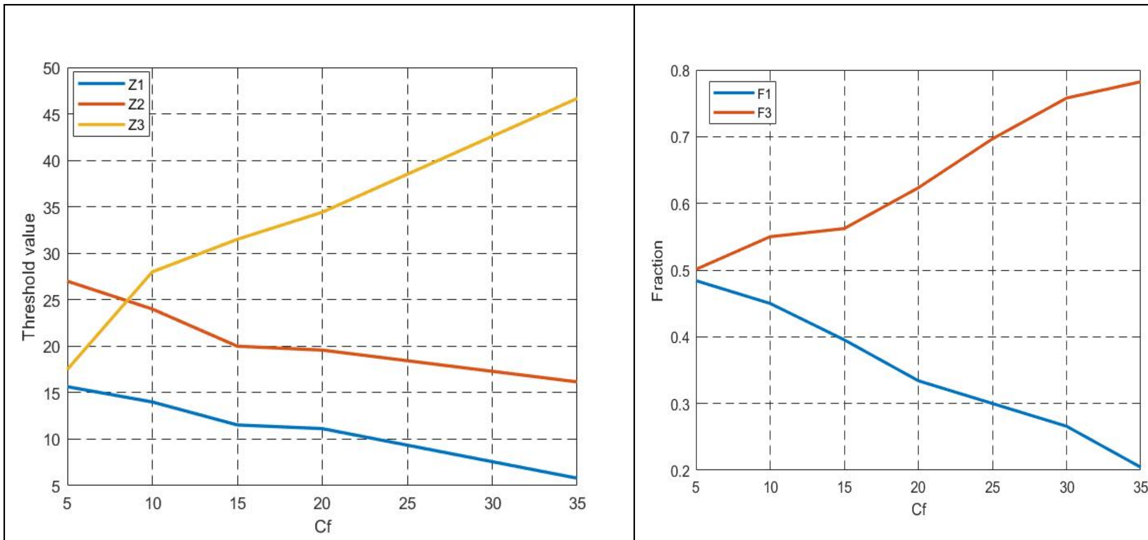
- Variation in inventory cost C^{+}: When the inventory cost for manufactured and remanufactured products increases, the critical thresholds $(Z_1^*, Z_2^*, Z_3^*, Z_4^*,, Z_5^*,, Z_6^*)$ decrease (Figure 4.8.a) and the optimal fractions $(f_1^{1*}, f_2^{1*}, f_1^{2*}, f_2^{3*})$ increase(Figure 4.8.b). This can be explained by the fact that to reduce the costs, the system stocks less products and the fractions to inspect increase to reduce the number of non-conforming products in stock.



(a) Effect on the threshold values (b) Effect on optimal fractions

Figure 4.8 Variation of inventory cost

- Variation of the manufacturing cost C_f of M_1: When the cost of manufacturing products directly increases, the threshold Z_1^* *and* Z_2^* and the fraction $f_1^{1*}$of manufacturing products at mode 1 decreases while the threshold Z_3^* and the fraction f_2^{1*}at this mode increases (Figure 4.9.a and Figure 4.9.b). This can be explained because the system, with the increase in the cost of the manufactured product, will require more remanufactured products to satisfy the demand. The fraction $f_1^{1*}$of the products decreases because the production cost increases while the penalty cost on the nonconforming products is constant.

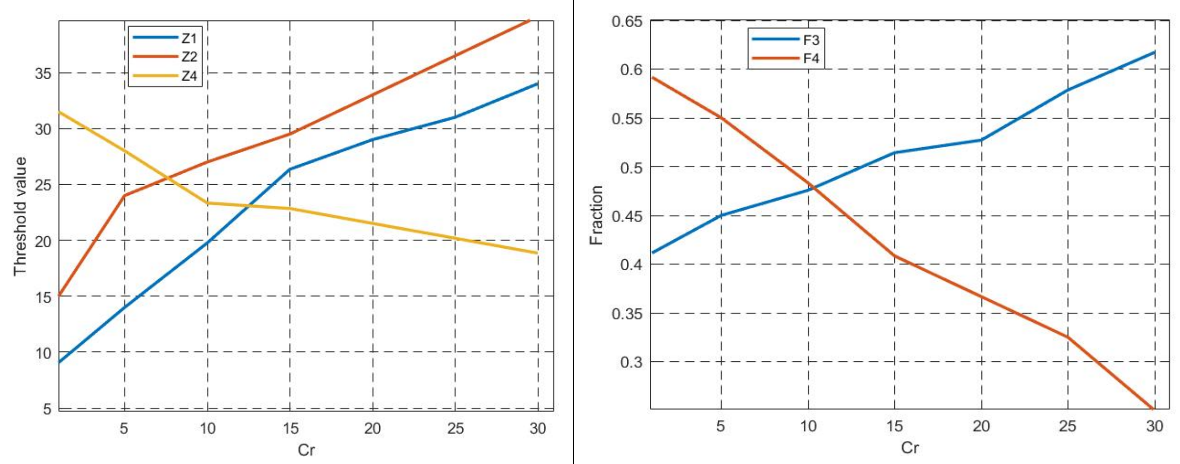


(a) Effect on the threshold values

(b) Effect on optimal fractions

Figure 4.9 Variation of manufacturing cost

- Variation of the remanufacturing cost C_r of M_2: As the remanufacturing cost increases, we see the opposite effect with the increase in direct production cost. The costs (Z_3^* *and* f_2^{1*}) decrease while (Z_1^*, Z_2^* *and* f_1^{1*}) increase (Figure 4.10.a , and Figure 4.10.b),because the system, to optimize the costs, will put more strain on the manufacturing machine.

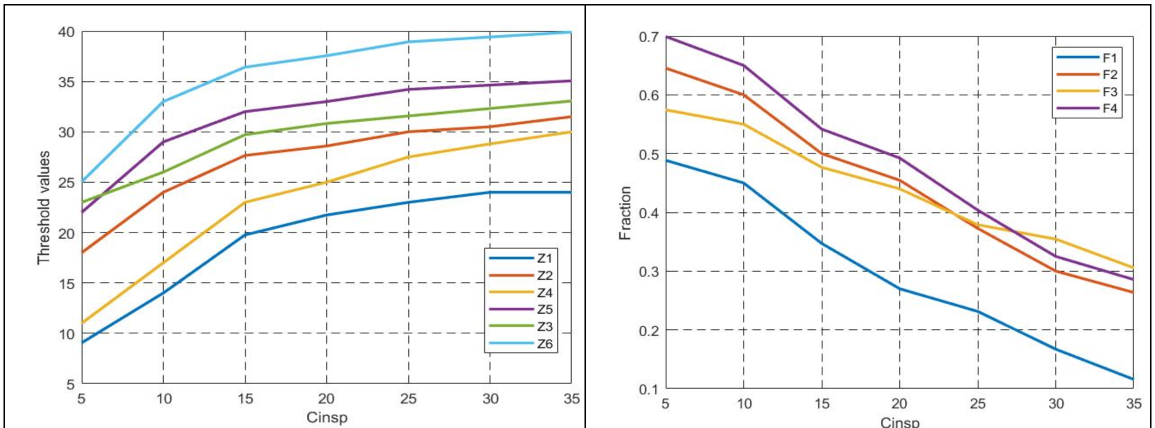


(a) Effect on the threshold values (b) Effect on optimal fractions

Figure 4.10 Variation of remanufacturing cost

The behavior of these policy settings varies and is detailed as follows:

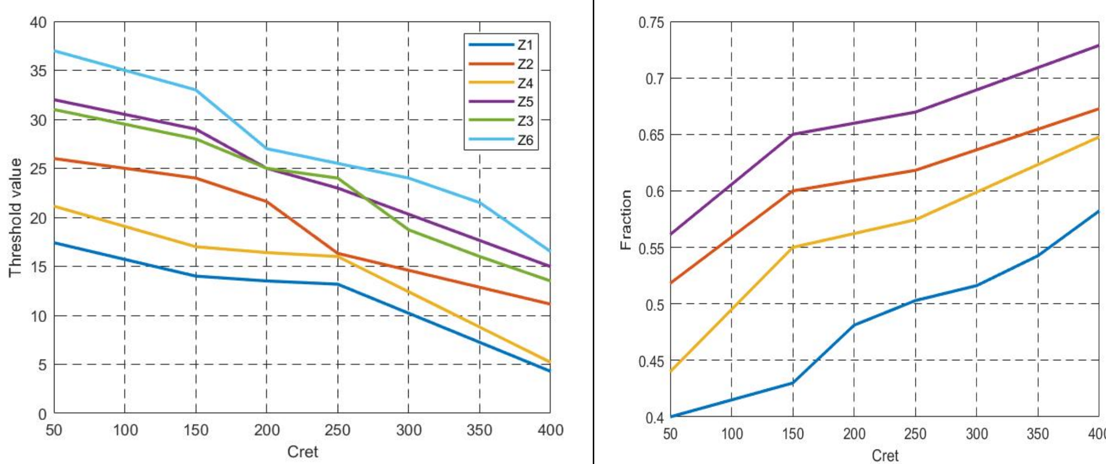
- Variation in inspection cost C_{isp}: When the inspection cost increases, the fractions $(f_1^{1*}, f_2^{1*}, f_1^{2*}, f_2^{3*})$ to be inspected decrease (Figure 4.11.b) to reduce the total production cost. However, the thresholds $(Z_1^*, Z_2^*, Z_3^*, Z_4^*,, Z_5^*,, Z_6^*)$ (Figure 4.11.a) increase because there are more non-conforming product in inventory.



(a) Effect on the threshold values (b) Effect on optimal fractions

Figure 4.11 Variation of inspection cost

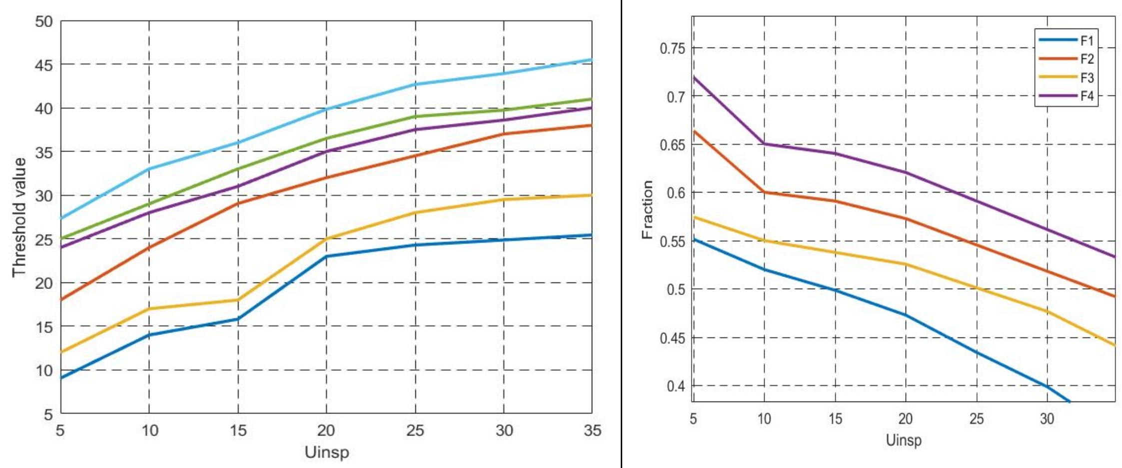
- Variation of the penalty cost on non-conforming products sold C_{ret}: As the cost of replacing non-conforming products returned from the market increases, the fractions to be controlled also increase and the optimal thresholds $(Z_1^*, Z_2^*, Z_3^*, Z_4^*,, Z_5^*,, Z_6^*)$ decrease (Figure 4.12.a and Figure 4.12.b). This is normal because the system limits the distribution of non-conforming products as much as possible.



(a) Effect on the threshold values (b) Effect on optimal fractions

Figure 4.12 Variation of penalty cost

- Variation in inspection rate U_i: Figure 4.13.a shows that as the inspection rate increases, the fractions $(f_1^{1*}, f_2^{1*}, f_1^{2*}, f_2^{3*})$ to be inspected decrease to reduce the overall inspection time. Threshold stocks $(Z_1^*, Z_2^*, Z_3^*, Z_4^*,, Z_5^*,, Z_6^*)$ increase (Figure 4.13.a) to anticipate the increase of non-conforming products in inventory.

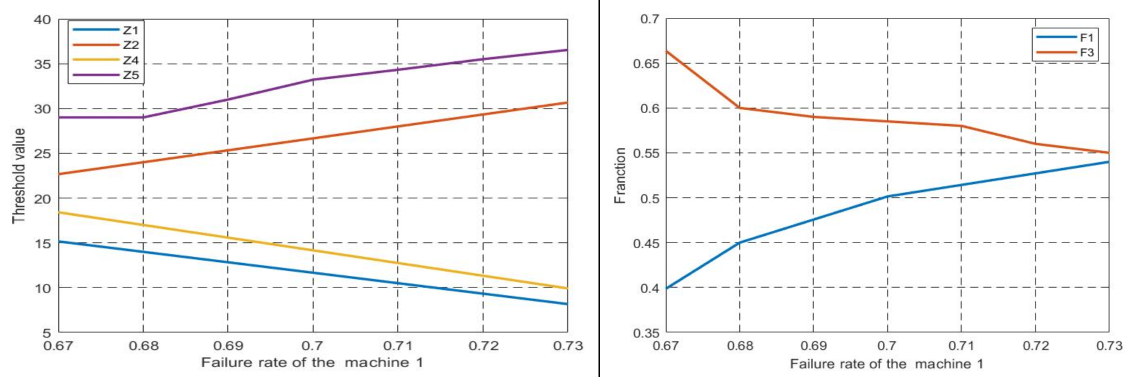


(a) Effect on the threshold values

(b) Effect on optimal fractions

Figure 4.13 Variation of inspection rate

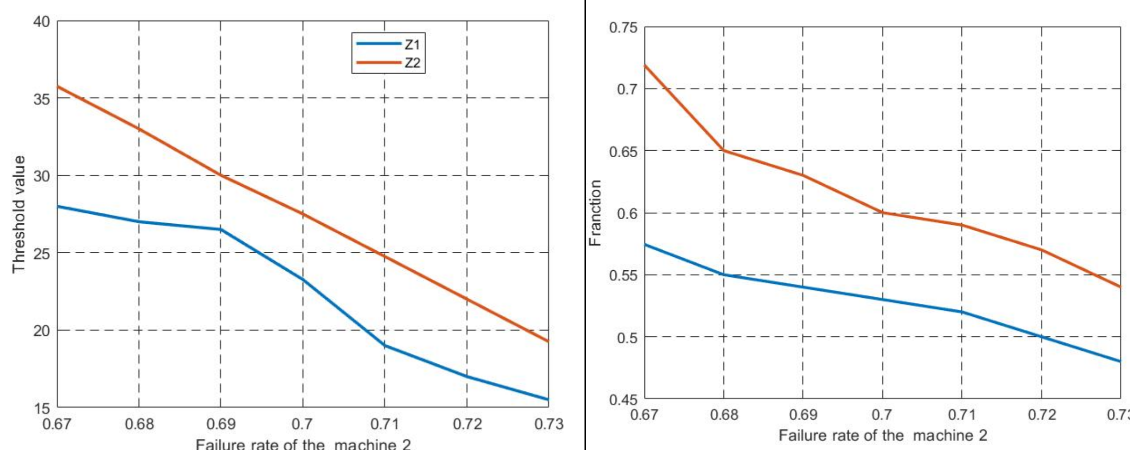
- Variation of the failure rate λ_1 and λ_2: When the failure rate of the manufacturing machine increases (Figure 4.14 and Figure 4.15), the thresholds $Z_1^*$and Z_4^*, decrease while the thresholds Z_2^*,and Z_5^*increase. This behavior is normal because the machine's failure rate is influenced by its production rate. So, the machine produces less at its maximum rate as its failure rate increases. However, when the remanufacturing machine's pass rate increases, its threshold inventory also increases. When the remanufacturing machine's failure rate increases, the Z_3^* and Z_6^* thresholds increase as well, since the system protects itself from production interruption caused by failures.



(a) Effect on the threshold values

(b) Effect on optimal fractions

Figure 4.14 Variation of failure rate λ_1

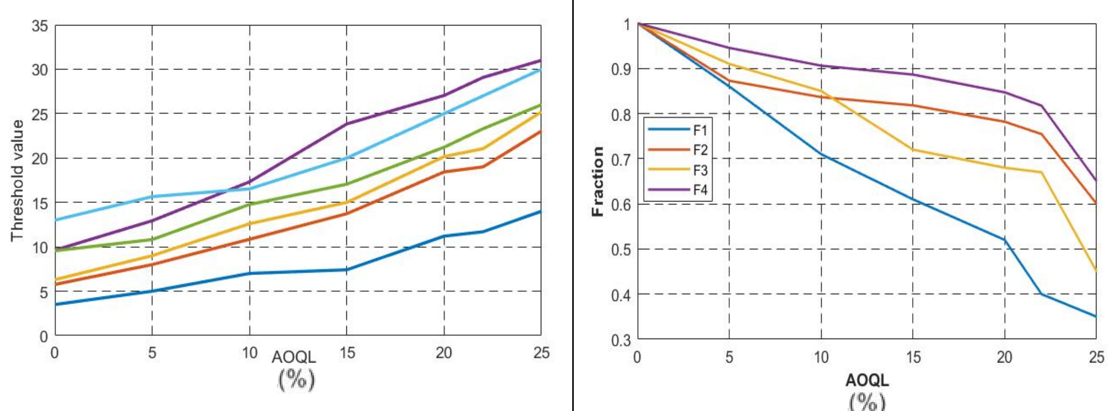


(a) Effect on the threshold values

(b) Effect on optimal fractions

Figure 4.15 Variation of failure rate λ_2

- <u>Variation of AOQL:</u> When the constraint on the AOQL increases, the fractions $(f_1^{1*}, f_2^{1*}, f_1^{2*}, f_2^{3*})$ to be controlled decrease (Figure 4.16.b) and the $(Z_1^*, Z_2^*, Z_3^*, Z_4^*,, Z_5^*,, Z_6^*)$ thresholds increase (Figure 4.16.a). This behavior is normal because more non-conforming products are allowed on the market. Thus, the system to reduce the inspection and scrap costs, reduces the amount of product to be inspected. Once the inspection thresholds are reduced, more non-conforming products go into inventory, which increases the amount of inventory.



(a) Effect on the threshold values | (b) Effect on optimal fractions

Figure 4.16 Variation of *AOQL*

4.7 Comparative Study

This section presents the economic and social merits of the proposed policy compared to the policies proposed in literature. During these decades, several studies have been published in the field of closed-loop production optimization. Nevertheless, very few have addressed jointly, the influence of degradation on reliability and on product quality in a manufacturing and remanufacturing context. Our developed hybrid-manufacturing model, by considering the influence of the production rate of the manufacturing and remanufacturing machines on the systems' reliability and the product quality, brings results that are closer to a real-world industrial environment. As a result, the policy proposed in our study, in addition to considering the effect of the production rate on reliability as in the work of Kouedeu et al. (Kouedeu et al. 2014) also considers the influence of this production rate on the quality of manufactured products and remanufactured products. This policy also differs from the policies proposed in Ait-El-Cadi et al. (Ait-El-Cadi et al. 2021) and Hajej et al. (Hajej, Rezg, and Gharbi 2021) because it proposes optimal recovery and remanufacturing strategies.

As well, our policy considers the influence of production rate on degradation, which was neglected in the policy of Ouaret et al. (Ouaret, Kenne, and Gharbi 2018b) and it proposes a dynamic sampling type inspection strategy. To conclude this part of our study, we will compare our policy, noted policy I (1), with three other policies developed in previous work by assessing them on economical viability.

- Policy I (1). In the policy proposed in this paper, the influence of production rate on reliability and product quality is considered simultaneously and including a dynamic sampling type inspection in a hybrid manufacturing and remanufacturing context.

- Policy II (2). This is the policy proposed in Kouedeu et al (Kouedeu et al. 2014). In this policy the influence of the production rate on reliability is considered but the quality control of the products is neglected, so all manufactured and remanufactured products are distributed. To simulate this policy, we considered that the fraction of product to be inspected is zero ($f_i^{\alpha} = 0$).

- Policy III (3). This is the policy proposed in Ouaret et al. (Ouaret, Kenne, and Gharbi 2018b). In this policy the products are produced and remanufactured as in Policy I(1). However, the influence of production rates on degradation is neglected. The inspection of products is not of the sampling type as in Policy I (1) because all products are inspected. To simulate this Policy III (3), we will consider that the fraction to be inspected is equal to one ($f_i^{\alpha} = 1$).

- <u>Policy IV (4).</u> This is the proposed policy by Ait-El-Cadi et al. Ait-El-Cadi et al. 2021). They take the influence of the production rate on degradation and quality into accont. Also, the inspection strategy is a dynamic sampling type. However, in their work the authors did not consider the reverse logistics. To simulate this policy IV (4), we considered that all products were manufactured directly, i.e. the remanufacturing machine is not considered.

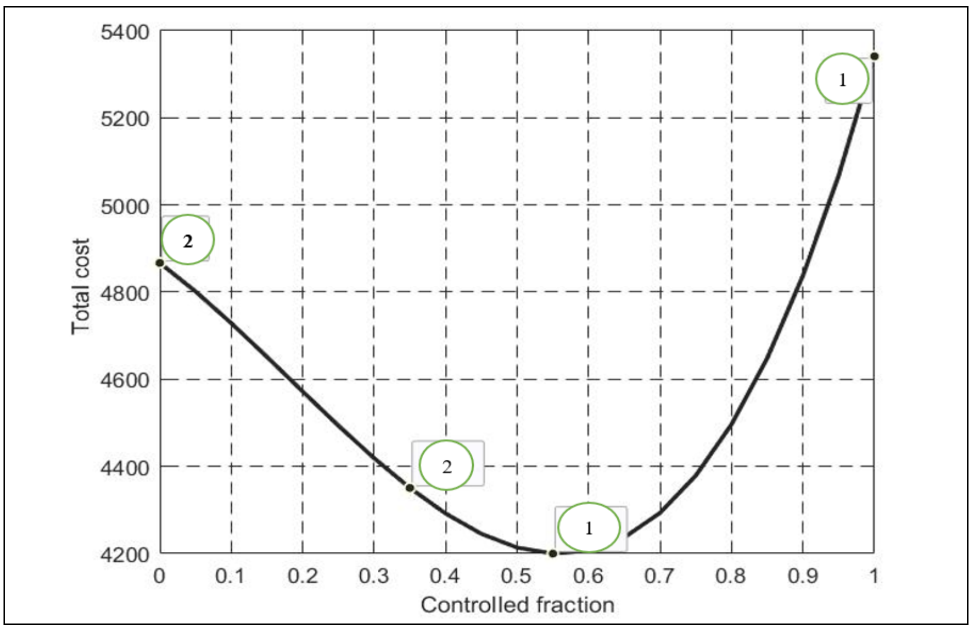


Figure 4.17 Comparative study

Figure 4.17 above shows policies (1), (2), (3) and (4) and their total production costs. Each of these policies is defined as follows

- Policy II (2): The influence of production rate on reliability is considered but neglected on product quality. Also, the products are not inspected ($f = 0\ \%$), hence we observe on Figure 4.17 that for a defective rate $\beta = 15\%$ the total production cost is 4867 \$. However, for a sampling type inspection, this total cost is 4200 \$ for a 55% production faction. This is a reduction of about 13.70 % of the total production cost. This is due to the additional losses caused by the distribution of non-conforming products to customers.

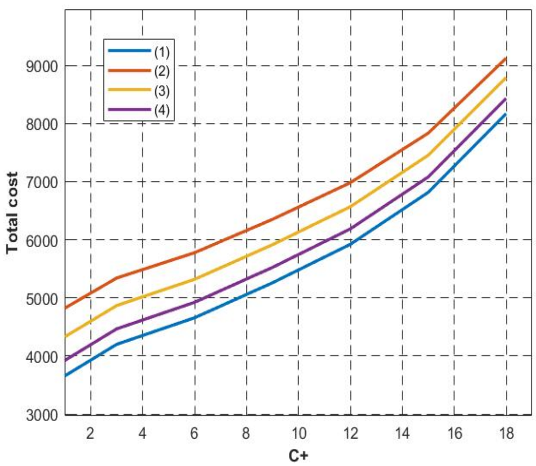
- Policy III (3): The influence of the production rate on degradation is neglected and all products are inspected ($f = 100\%$) For a rejection rate of $\beta = 15\%$,, the total production cost is 5341 $. However, for a sampling type inspection, this total cost is 4200 $ for a 55% faction of the inspected production. This is a reduction of about 21.36% of the total production cost. Policy I (1) has a high cost because all the good quality products (1- β) are inspected which adds to the costs.
- Policy IV (4): The influence of the production rate is considered, but all the products originate directly from manufacturing, as no reverse logistics is considered here. Since the remanufacturing machine is not considered, the fraction of products to be controlled is ($f = 35\%$). Here, the total production cost is 4351 $ compared to 4200 $ in our developed system (Policy I (1)), hence a reduction of about 4% is realized.

To complete the investigation of this comparative study, additional simulations were performed by varying the rejection rate of the manufacturing machine, which are presented in table 4.

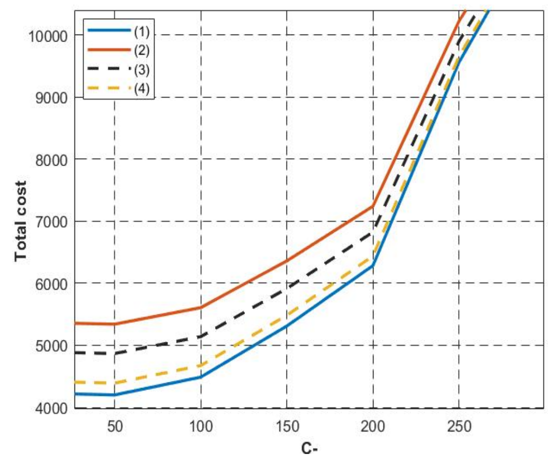
Tableau 4.4 Comparative study according to the defective rate

Product rejection rate (β%)		5%	10%	15%	20%	25%
Policy I (cost $)		3088	3726	4200	4611	4866
Policy II (cost $)		3437	4209	4867	6007	6494
Policy III (cost $)		3996	4786	5341	5486	5531
Policy IV (cost $)		3321	4118	4773	5425	5794
REDUCTION RATE (%)	Policy II (2)	10%	11%	14%	23%	25%
	Policy III (3)	23%	22%	21%	16%	12%
	Policy IV (4)	7%	10%	12%	15%	17%

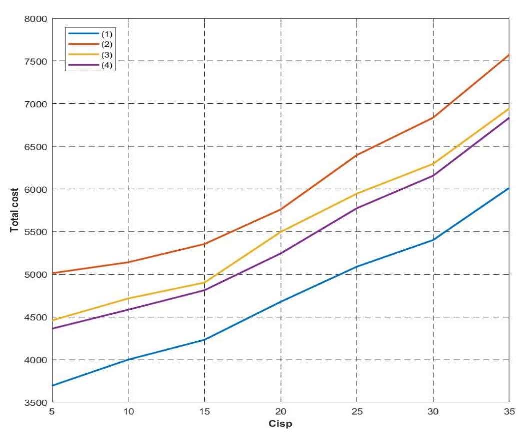
To extend this comparative study, we vary other parameters of the system to observe the behavior of different policies. The Figure 4.18 (a, b, c, d and e) present the obtained results.



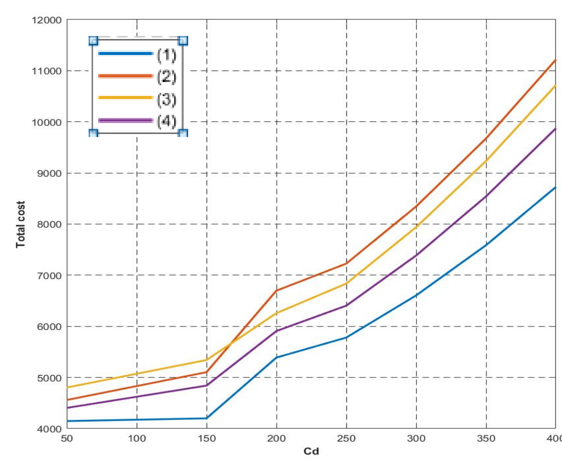
(a) Variation of inventory cost.



(b) Variation of shortage cost



(c) Variation of inspection cost.



(d) Variation of the penalty cost

Figure 4.18 Comparative study

Figure 4.18.a shows that as the cost of storage increases, the costs of all four policies increase. The cost of policy 2 increases more because in this policy no product is inspected. Thus, many of the products in stock are of poor quality and cost the company unnecessarily. However, policy (1) remains the most economical because the inspection is dynamic and varies with the cost of storage. Figure 4.18.b shows that policy (2) remains the most

expensive when the cost of shortage increases because all the non-conforming products are distributed and returned by the customers for replacement which creates the shortage of good quality products. Figure 4.18.c shows that policy 3 is more expensive for large values of inspection cost. This is because in this policy all products are inspected which costs the company money. Figure 4.18.d shows that policy (2) is the most expensive for high penalty costs while for low penalty costs it is policy (3). In sum, figure 11 shows that the proposed policy allows to reduce the production cost while improving the quality of the products. Indeed, on the one hand this policy allows to improve simultaneously the quality of the products and the reliability of the system. On the other hand, this policy allows to minimize the total production cost even when the different parameters such as, the stock cost, the shortage cost, the inspection cost, the replacement cost of the nonconforming products increase.

4.8 Policy implementation and decision support tool

In the case of our numerical example (section 4.5). To optimize production and quality control, the procedure to be followed by the manufacturer is proposed as follows:

- Step 1 (S1): identification; in this first step, the manufacturer must identify the manufacturing machine and the reconditioned machine. the manufacturing machine uses the raw material while the re manufacturing machine uses the returned end-of-life products
- Step 2 (S2): Condition of the machines; the manufacturer shall verify whether each machine is in a state of breakdown or in good working order.

After the identification of the machines (S1) and the identification of the state of each machine (S2), 4 cases are possible: the actions to be carried out by the operator according to the various possible cases and summarized on figure 4.19 are the following ones:

- If both machines are in good working condition. The operator must inspect 45% of the manufactured product and 55% of the remanufactured product produced at the maximum rate on each of the two machines. When the inventory level reaches 14, the operator must reduce the production rate of the manufacturing machine from its maximum value to its economic value and when it reaches the value of 24, the operator must stop the manufacturing machine and continue with the remanufacturing machine always at its maximum rate until the inventory is 28 and then also stop the remanufacturing machine. The operator should perform this procedure repeatedly depending on the inventory level.

- If only the manufacturing machine is in operation. The operator must send the remanufacturing machine for repair, produce at the maximum rate of the manufacturing machine, and inspect 60% of its production. Once the inventory level is 17, the operator should reduce the production rate from its max value to its economic value and then stop producing when the inventory level is 29. Do a preventive overhaul of the machine during this downtime.

- If only the remanufacturing machine is in operation. The operator must send the manufacturing machine for repair, produce at the maximum rate of the remanufacturing machine, and inspect 65% of its production. Once the stock level is equal to 33, the operator should stop production and do a preventive overhaul of the machine.
- Both machines have failures. The operator must send both machines for repair.

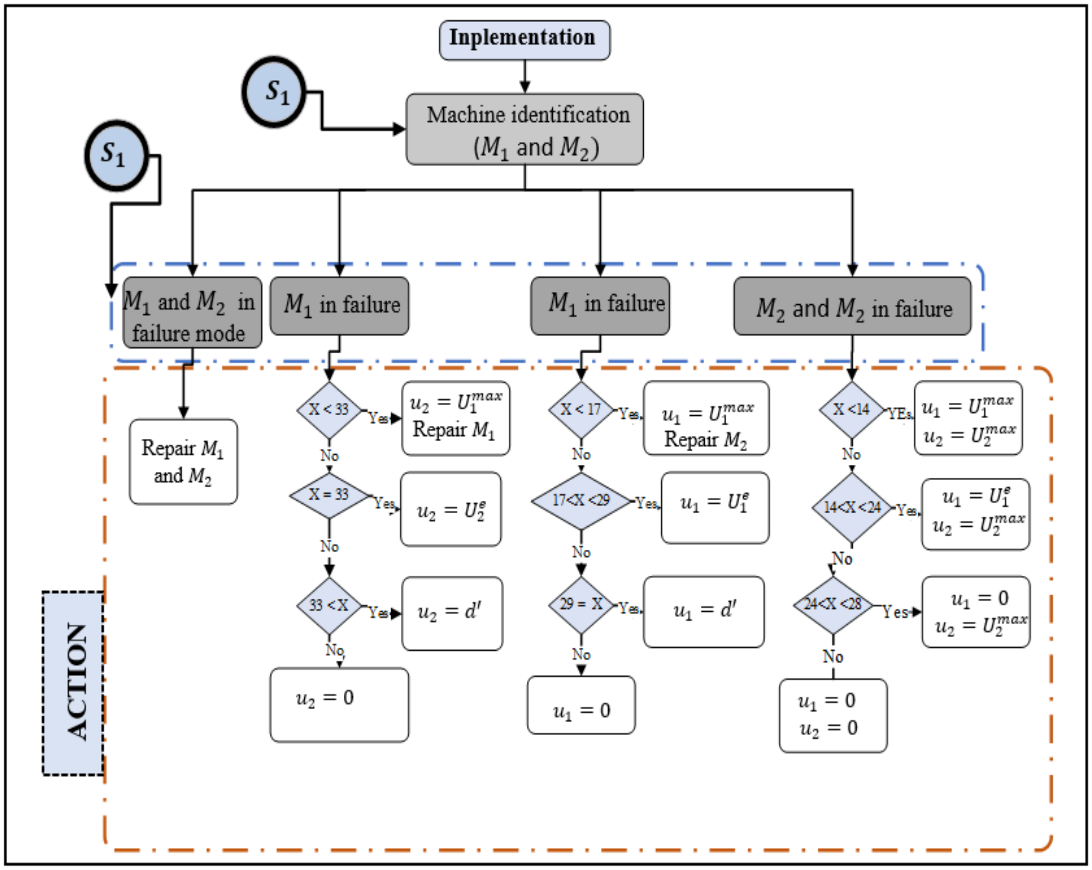


Figure 4.19 Logical diagram for the proposed policies implementation

4.9 Conclusion.

Ultimately, this work focused on the study and optimization of a hybrid production /remanufacturing system in conjunction with maintenance and quality control strategies. The main objective was to increase system reliability, improve product quality and minimize total production cost. To achieve this goal, we formulated the problem and developed the system dynamics to optimize the control variables (i.e., production rates and inspection fractions for each machine). Our study system consists of two non-identical machines (manufacturing machine and remanufacturing machine). After formulating the optimal control problem that describes the study system, we solved HJB equations using the numerical method. This allowed us to determine the optimal conditions for our decision variables. Then, we simulated a numerical example. Through this simulation, we proposed a critical threshold production policy that reduces production costs and satisfies demand. An inspection policy to improve the quality of the distributed products. A maintenance policy to limit the effects of degradation on reliability and quality. Finally, to illustrate the robustness of the proposed approach, we performed a sensitivity analysis. Our results can be useful for companies with unreliable machines that degrade over time and that produce parts from raw materials and recycle end-of-life products. This work can be extended in future work for systems with variable demand rates, systems with raw material control. And a system that degrades as it ages.

CONCLUSION

Il a été question dans ce mémoire d'étudier un système hybride de fabrication et de réutilisation constitué de machines non fiables. Elles sont soumises aux pannes et aux réparations aléatoires. Les machines (manufacturing / remanufacturing) se dégradent en fonction de leurs taux d'utilisation et cela affecte la fiabilité et la qualité des produits. Ainsi, cette étude a été faite dans le but de proposer une politique conjointe de production, réutilisation, de maintenance et de contrôle qualité des systèmes hybrides de fabrication et de réutilisation. Pour mener à bien cette recherche, nous l'avons structuré en 4 chapitres.

Dans le chapitre 1, nous avons décrire le problème principal d'étude. Ensuite nous avons fait un état de l'art à travers une revue des travaux scientifiques publiés dans le domaine des chaines d'approvisionnement de la gestion optimale de la maintenance du contrôle qualité et de la logistique inverse. Cette revue nous a permis de comprendre les acquis et les manquements dans notre domaine d'étude. Pour donner suite à cela, nous avons dans ce chapitre ressortit l'objectif principal, les objectifs spécifiques et les contributions attendues.

Au chapitre 2, nous avons étudié un système constitué d'une seule machine et qui produit un seul type de pièces. La machine se dégrade progressivement et son taux de rejet varie. L'inspection qualité est imparfaite. Grace à cette étude, nous avons développé un modèle stochastique pour optimiser la production conjointement aux politiques de maintenance et de contrôle qualité. Ces politiques ont permis de limiter la dégradation, augmenter la fiabilité, améliorer la qualité des produits et minimiser le coût de production. Cependant, l'influence du taux de production sur la dégradation est négligée.

Dans le chapitre 3, nous avons intégré à notre étude l'influence du taux de production sur la dégradation de la machine. Ainsi le système d'étude est constitué de deux machines qui produisent le même type de pièces. La seconde machine a été ajoute à la machine principale de l'étude fait au chapitre 1. L'ajout de cette machine sous-traitant a permis de réduire le taux de production de la machine principale afin de limiter son influence sur la dégradation.

La politique conjointe de production, maintenance et contrôle qualité développée dans ce chapitre sont utiles pour les systèmes de production qui se dégradent en fonction de leurs vitesses d'utilisation. Mais dans ce chapitre, les enjeux socio-économiques de logistique inverse ne sont pas pris en compte.

Dans le chapitre 4, nous avons intégré la logistique inverse et avons proposé un système hybride de fabrication et réutilisation qui intégré à la fois la maintenance et le contrôle qualité. En effet dans ce chapitre, la seconde machine du chapitre précédant ne produit plus à partir de la matière première mais plutôt à partir des produits en fin de vie collectés auprès des clients. Les deux machines se dégradent en fonction de leur taux d'utilisation. A l'issu de notre étude nous avons une politique optimale conjointe de production, de réutilisation, de maintenance et de contrôle qualité des produits fabriqués et des produits remis à neuf.

En somme, dans ce mémoire, nos contributions ont permis d'étendre les modèles d'optimisation stochastique existant en y intégrant à la fois des stratégies de production, de réutilisation, de maintenance et de contrôle qualité. Aussi la prise en compte des erreurs d'inspection qualité et l'influence négative du taux de production sur la dégradation et sur la qualité des produits, nous a permis de rapprocher au mieux nos politiques au contexte réel des systèmes industriels. Cependant dans cette recherche nous avons considéré la demande variable et linéaire or dans certains cas elle est bien plus complexe. Aussi nous n'avons pas pris en compte la dégradation en fonction de l'âge des machines, nous avons également considérer que la matière première est de qualité parfaite. De ce fait, la résolution de ces limites peut faire l'objet des travaux futurs afin d'étendre cette recherche.

ANNEXE I

NUMERICAL APPROACH FOR ARTICLE 1

The idea of Kushner's approach is to approximate the partial derivative $\frac{\partial \vartheta(\alpha,x)}{\partial x}$ by a finite difference. Let h be the step of discretization of the finite interval of possible values of stock x, the function ϑ (α, x) is approximated by $\vartheta^h(\alpha, x)$. We have:

$$\frac{\partial \vartheta(x,\alpha)}{\partial x} = \begin{cases} \frac{\vartheta^h(x+h,\alpha) - \vartheta^h(x,\alpha)}{h} & \text{if } \frac{dx(t)}{dt} \geq 0 \\ \frac{\vartheta^h(x,\alpha) - \vartheta^h(x-h,\alpha)}{h} & \text{if not} \end{cases} \quad \text{(AI - 1)}$$

By injecting the last equation into HJB, we have:

$$\rho\vartheta^h(x,\alpha) = \min_{(u;f)\epsilon\Gamma(\alpha)} \left[g(.) + q_{\alpha\alpha}\vartheta^h(x,\alpha) + \sum_{j\neq\alpha} q_{\alpha j}\, \vartheta^h(x,j) + \frac{dx(t)}{dt} * \begin{cases} \frac{\vartheta^h(x+h,\alpha) - \vartheta^h(x,\alpha)}{h} \text{ind}(\dot{x}) \geq 0 \\ \frac{\vartheta^h(x,\alpha) - \vartheta^h(x-h,\alpha)}{h} \text{ind}(\dot{x}) < 0 \end{cases} \right] \quad \text{(AI - 2)}$$

$$ind\{P(.)\} = \begin{cases} 1 \; if \; P(.) \; is \; true \\ 0 \;\; Otherwise \end{cases}$$

$$\vartheta^h(x,\alpha) = \min_{u\epsilon\Gamma(\alpha)} \left[\frac{g(.) + \sum_{j\neq\alpha} q_{\alpha j}\, \vartheta^h(x,j) + \frac{\dot{x}}{h} \begin{cases} \vartheta^h(x+h,\alpha)\text{ind}(u-d) \geq 0 \\ \vartheta^h(x-h,\alpha)\text{ind}(u-d) < 0 \end{cases}}{(\rho + \frac{|\dot{x}\,|}{h} + |q_{\alpha\alpha}|)} \right] \quad \text{(AI- 3)}$$

With :

$$\Omega_h^\alpha = \frac{|\dot{x}|}{h} + |q_{\alpha\alpha}| \qquad \qquad P_x^+(\alpha) = \begin{cases} \frac{\dot{x}}{h\Omega_h^\alpha} & if \; \dot{x} > 0 \\ 0 & if \; note \end{cases}$$

$$P_x^-(\alpha) = \begin{cases} \dfrac{\dot{x}}{h\Omega_h^\alpha} & if\ \dot{x} \le 0 \\ 0 & if\ note \end{cases} \qquad P^j(\alpha) = \frac{q_{\alpha j}}{\Omega_h^\alpha}$$

Thus, we have:

$$\vartheta^h(x,\alpha) = \min_{(u,f)\epsilon\Gamma(\alpha)} \left[\frac{g(x,\alpha)}{\Omega_h^\alpha\left(1+\frac{\rho}{\Omega_h^\alpha}\right)} + \frac{1}{1+\frac{\rho}{\Omega_h^\alpha}} (P_x^\pm(\alpha)\ \vartheta^h(x \pm h,\alpha) + \sum_{j\neq\alpha} P^j(\alpha)\ \vartheta^h(x,\alpha) \right] \qquad \text{(AI- 4)}$$

Develop the numerical equations for each mode.

- Mode 1:

The machine is in operation and its rejection rate is β_1, the controlled fraction f_1

$$\vartheta^h(x,1) = \min_{(u,f)\epsilon\Gamma(1)} \left[\frac{g(x,1)}{\Omega_h^1(1+\rho/\Omega_h^1)} + \frac{1}{1+\frac{\rho}{\Omega_h^1}} (P_x^\pm(1)\ \vartheta^h(x \pm h,1) + \frac{q_{12}}{\Omega_h^1}\ \vartheta^h(x,2) + \frac{q_{13}}{\Omega_h^1}\ \vartheta^h(x,3) \right] \qquad \text{(AI- 5)}$$

- And $AOQL \le AOQ_{max}$

- Mode 2:

The machine is in operation and its defective rate is β_2, the control fraction f_2.

$$\vartheta^h(x,2) = \min_{(u,f)\epsilon\Gamma(2)} \left[\frac{g(x,2)}{\Omega_h^2(1+\rho/\Omega_h^2)} + \frac{1}{1+\frac{\rho}{\Omega_h^2}} (P_x^\pm(2)\ \vartheta^h(x \pm h,2) + \frac{q_{24}}{\Omega_h^2}\ \vartheta^h(x,4) + \frac{q_{25}}{\Omega_h^2}\ \vartheta^h(x,5) \right] \qquad \text{(AI- 6)}$$

And $AOQ_L(2) \le AOQ_{max}$

- Mode 3:

The machine is being repaired and before the failure it was in mode 1.

$$\vartheta^{h}(x,3) = \min_{(u,f)\epsilon\Gamma(3)}\left[\frac{g(x,3)}{\Omega_{h}^{3}(1+\rho/\Omega_{h}^{3})}+\frac{1}{1+\frac{\rho}{\Omega_{h}^{3}}}(P_{x}^{-}(3)\,\vartheta^{h}(x-h,3)+\frac{q_{31}}{\Omega_{h}^{3}}\,\vartheta^{h}(x,1)\right] \tag{AI- 7}$$

- Mode 4:

The machine is being repaired and before the failure it was in mode 2.

$$\vartheta^{h}(x,4) = \min_{(u,f)\epsilon\Gamma(4)}\left[\frac{g(x,4)}{\Omega_{h}^{4}(1+\rho/\Omega_{h}^{4})}+\frac{1}{1+\frac{\rho}{\Omega_{h}^{4}}}(P_{x}^{-}(4)\,\vartheta^{h}(x-h,4)+\frac{q_{42}}{\Omega_{h}^{4}}\,\vartheta^{h}(x,2)\right] \tag{AI- 8}$$

- Mode 5:

The machine is in preventive maintenance because it was in mode 2 and can only transit in mode 1.

$$\vartheta^{h}(x,4) = \min_{(u,f)\epsilon\Gamma(5)}\left[\frac{g(x,5)}{\Omega_{h}^{5}(1+\rho/\Omega_{h}^{5})}+\frac{1}{1+\frac{\rho}{\Omega_{h}^{5}}}(P_{x}^{-}(5)\,\vartheta^{h}(x-h,5)+\frac{q_{51}}{\Omega_{h}^{5}}\,\vartheta^{h}(x,1)\right] \tag{AI- 9}$$

ANNEXE II

NUMERICAL SOLUTION BY THE KUSHNER APPROACH FOR ARTICLE 2

The idea of the Kushner approach is to approximate the partial derivative $\frac{\partial\vartheta(\alpha,x)}{\partial \mathrm{x}}$ by a finite difference. Let *h* be the discretization step of the finite interval of possible values of the stock *x*, the function ϑ *(α, x)* is approximated by $\vartheta^h(\alpha, x)$ We have:

$$\frac{\partial\vartheta(x,\alpha)}{\partial x} = \begin{cases} \dfrac{\vartheta^h(x+h,\alpha) - \vartheta^h(x,\alpha)}{h} & \text{if } \dfrac{dx(t)}{dt} \geq 0 \\ \dfrac{\vartheta^h(x,\alpha) - \vartheta^h(x-h,\alpha)}{h} & \text{if not} \end{cases} \qquad \text{(AII- 1)}$$

By injecting the last equation into HJB, we have:

$$\rho\vartheta^h(x,\alpha) = \min_{([u_1(\cdot),u_2(\cdot),f(\cdot)]\epsilon\Gamma(\alpha)} \left[g(.) + q_{\alpha\alpha}\vartheta^h(x,\alpha) + \sum_{j\neq\alpha} q_{\alpha j}\, \vartheta^h(x,j) + \frac{dx(t)}{dt} * \begin{cases} \dfrac{\vartheta^h(x+h,\alpha) - \vartheta^h(x,\alpha)}{h} ind(\dot{x}) \geq 0 \\ \dfrac{\vartheta^h(x,\alpha) - \vartheta^h(x-h,\alpha)}{h} ind(\dot{x}) < 0 \end{cases} \right]$$

(AII- 2)

Isolating all ϑ*(x, α)* members in the left-hand member, we have:

$$\left(\rho + \frac{|\dot{x}|}{h} + |q_{\alpha\alpha}|\right) \vartheta^h(x,\alpha) = \min_{[u_1(\cdot),u_2(\cdot),f(\cdot)]\epsilon\Gamma(\alpha)} \left[g(.) + \sum_{j\neq\alpha} q_{\alpha j}\, \vartheta^h(x,j) + \frac{\dot{x}}{h} \begin{cases} \vartheta^h(x+h,\alpha) ind(u-d) \geq 0 \\ \vartheta^h(x-h,\alpha) ind(u-d) < 0 \end{cases} \right] \qquad \text{(AII- 3)}$$

Hence:

$$\vartheta^h(x,\alpha) = \min_{[u_1(\cdot),u_2(\cdot),f(\cdot)]\epsilon\Gamma(\alpha)} \left[\frac{g(.) + \sum_{j\neq\alpha} q_{\alpha j}\,\vartheta^h(x,j) + \frac{\dot{x}}{h}\begin{cases}\vartheta^h(x+h,\alpha) ind(u-d) \geq 0 \\ \vartheta^h(x-h,\alpha) ind(u-d) < 0\end{cases}}{(\rho + \frac{|\dot{x}\,|}{h} + |q_{\alpha\alpha}|)} \right] \quad \text{(AII- 4)}$$

$$\Omega_h^\alpha = \frac{|\dot{x}|}{h} + |q_{\alpha\alpha}| \qquad \mathrm{P}_x^+(\alpha) = \begin{cases} \frac{\dot{x}}{h\Omega_h^\alpha} & if\ \dot{x} > 0 \\ 0 & if\ not \end{cases} \quad \text{(AII- 5)}$$

$$\mathrm{P}_x^-(\alpha) = \begin{cases} \frac{\dot{x}}{h\Omega_h^\alpha} & \text{if } \dot{x} \leq 0 \\ 0 & if\ not \end{cases} \qquad P^j(\alpha) = \frac{q_{\alpha j}}{\Omega_h^\alpha} \quad \text{(AII- 6)}$$

We have:

$$\vartheta^h(x,\alpha) = \min_{(u,f)\epsilon\Gamma(\alpha)} \left[\frac{g(x,\alpha)}{\Omega_h^\alpha\left(1 + \frac{\rho}{\Omega_h^\alpha}\right)} + \frac{1}{1+\frac{\rho}{\Omega_h^\alpha}} (P_x^\pm(\alpha)\,\vartheta^h(x\pm h,\alpha) + \sum_{j\neq\alpha} P^j(\alpha)\,\vartheta^h(x,\alpha) \right] \quad \text{(AII- 7)}$$

- **Mode 1**:

Both machines are in operation. The rejection rate of the main machine is β, the control fraction is f_1.

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x} = u_{s1}(t) + u_2(t) - \frac{d}{1 - AOQ(x)} \quad \text{(AII- 8)}$$

$$\vartheta^h(x,1) = \min_{(u,f)\epsilon\Gamma(1)} \left[\frac{g(x,1)}{\Omega_h^1(1+\rho/\Omega_h^1)} + \frac{1}{1+\frac{\rho}{\Omega_h^1}} (P_x^\pm(1)\,\vartheta^h(x\pm h,1) + \frac{q_{12}}{\Omega_h^1}\,\vartheta^h(x,2) + \frac{q_{13}}{\Omega_h^1}\,\vartheta^h(x,3) \right] \quad \text{(AII- 9)}$$

And

$$\mathrm{AOQL}(1) \leq \mathrm{AOQ}_{\max}$$

- **Mode 2**:

Only the main machine M1 is in operation. The rejection rate of the main machine is β, the control fraction is f_2.

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x} = u_{s1}(t) - \frac{d}{1 - AOQ(x)} \tag{AII-10}$$

$$\vartheta^h(x,2) = \min_{(u,f)\epsilon\Gamma(2)} \left[\frac{g(x,2)}{\Omega_h^2(1+\rho/\Omega_h^2)} + \frac{1}{1+\frac{\rho}{\Omega_h^2}} (P_x^{\pm}(2)\, \vartheta^h(x \pm h, 2) + \frac{q_{24}}{\Omega_h^2}\, \vartheta^h(x,4) + \frac{q_{25}}{\Omega_h^2}\, \vartheta^h(x,5) \right] \tag{AII-11}$$

And

$$AOQL(2) \leq AOQ_{max}$$

Mode 3: Only the second M2 machine is in operation. No fraction to control in this case.

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x} = u_{s1}(t) - \mathrm{d} \tag{AII-12}$$

because $AOQ(3) = 0$

$$\vartheta^h(x,3) = \min_{f\epsilon\Gamma(1)} \left[\frac{g(x,3)}{\Omega_h^3(1+\rho/\Omega_h^3)} + \frac{1}{1+\frac{\rho}{\Omega_h^3}} (P_x^{-}(3)\, \vartheta^h(x - h, 3) + \frac{q_{31}}{\Omega_h^3}\, \vartheta^h(x,1) \right] \tag{AII-13}$$

And

$\mathrm{AOQL}(1) \leq \mathrm{AOQ}_{\mathrm{max}}$

- **Mode 4**:

Both machines are out of order.

$$\dot{x} = -d \tag{AII-14}$$

$AOQ(4) = 0$

$$\vartheta^h(x,4) = \min_{f \epsilon \Gamma(2)} \left[\frac{g(x,4)}{\Omega_h^4 (1 + \rho/\Omega_h^4)} + \frac{1}{1 + \frac{\rho}{\Omega_h^4}} (P_x^-(4)\, \vartheta^h(x-h,4) + \frac{q_{42}}{\Omega_h^4}\, \vartheta^h(x,2) \right] \tag{AII- 15}$$

and

$$AOQL(2) \leq AOQ_{max}$$

ANNEXE III

ARTICLE DE CONFÉRENCE

Réference

Megoze, P. P., J. P. Kenne, Edgar D. J. G and L. Hof. 2021. “Production and maintenance policies for a deteriorated unreliable manufacturing system’’. CIGI-Qualita21 : Conférence Internationale Génie Industriel QUALITA – Grenoble 5-7 mai 2021 Grenoble (France).

Résumé

Cet article étudie et propose une politique conjointe de production, de maintenance et de contrôle de la qualité des produits pour un système constitué d'une seule machine qui produit un seul type de pièce. Cette machine est soumise à des pannes et des réparations aléatoires. Elle se dégrade au fil du temps, cette dégradation affecte sa disponibilité et son taux de rejet. Ainsi, après un temps aléatoire, le taux de rejet de la machine est significativement plus élevé que sa valeur initiale. La demande du marché est déterministe, et son taux est constant. L'objectif de ce travail est de proposer une politique de maintenance corrective et préventive pour augmenter la disponibilité du système et réduire son niveau de dégradation ; également de proposer une politique de contrôle de la qualité pour réduire la proportion de produits non conformes qui pourraient se retrouver sur le marché. Pour atteindre ces objectifs, nous avons formulé le problème et avons utilisé une approche de programmation dynamique stochastique pour développer les conditions d’optimum de type Hamilton-Jacobi-Bellman (HJB). Par la suite nous avons résolu ces équations par les méthodes numériques. Les résultats obtenus définissent : un contrôle qualité de type échantillonnage, une politique de production avec seuil critique et enfin des politiques de maintenance préventive et corrective.

Mots clés : processus stochastique, détérioration, maintenance, qualité, fabrication.

Abstract

This paper studies and proposes a joint production, maintenance and product quality control policy for a system consisting of a single machine and producing a single type of part. This machine is subject to random breakdowns and repairs, and it degrades over time affecting its availability and defective rate. Hence, after a random time, the machine's defective rate is significantly higher than the initial defective rate. The considered market demand is deterministic, and its rate is constant. The objective of this work is to propose a corrective and preventive maintenance policy to increase the availability of the system and reduce its defective rate. In addition, we propose a quality control policy to reduce the proportion of non-compliant products entering the market. To achieve these objectives, we formulated the problem and used a dynamic stochastic programming approach to develop Hamilton-Jacobi-Bellman (HJB) type optimum conditions. The obtained results define a quality control of sampling type, a production policy with critical threshold and finally a preventive and corrective maintenance policy.

Keywords: stochastic process, deterioration, maintenance, quality, manufacturing.

LISTE DE RÉFÉRENCES BIBLIOGRAPHIQUES

Ait-El-Cadi, A., A. Gharbi, K. Dhouib, and A. Artiba. 2021. 'Integrated production, maintenance and quality control policy for unreliable manufacturing systems under dynamic inspection', *International Journal of Production Economics*, 236.

Anis Chelbi, Daoud Ait-Kadi and Houda Aloui 2008. 'Optimal inspection and preventive maintenance policy for systems with self-announcing and non-self-announcing failures', *Journal of Quality in Maintenance Engineering*, 14: 34 - 45.

Assid, M., A. Gharbi, and A. Hajji. 2015. 'Joint production, setup and preventive maintenance policies of unreliable two-product manufacturing systems', *International Journal of Production Research*, 53: 4668-83.

Ayed, S., D. Sofiene, and R. Nidhal. 2012. 'Joint optimisation of maintenance and production policies considering random demand and variable production rate', *International Journal of Production Research*, 50: 6870-85.

Baillargeon, G. . 2013. 'Plans d'échantillonnage en contrôle de la qualité : contrôle par attributs.', *Trois-Rivières: Éditions SMG, xv,*, 291

Barlow., and Hunter. 1990. 'Manufacturing Flow-Control and Preventive Maintenance - a Stochastic-Control Approach', *Ieee Transactions on Automatic Control*, 35: 1024-31.

Bielecki, T. R. , and P. R. Kumar. 1998. 'Optimality of zero-inventory policies for unreliable manufacturing systems', *Operations Research*, 36: 532-41.

Boukas, E. K., and A. Haurie. 1990. 'Manufacturing flow control and preventive maintenance: A stochastic control approach', *IEEE Transactions on Automatic Control*, 35: 1024-31.

Bouslah, B., A. Gharbi, and R. Pellerin. 2016. 'Integrated production, sampling quality control and maintenance of deteriorating production systems with AOQL constraint', *Omega-International Journal of Management Science*, 61: 110-26.

Cheng, G. Q., and L. Li. 2020. 'Joint optimization of production, quality control and maintenance for serial-parallel multistage production systems', *Reliability Engineering & System Safety*, 204.

Chouikhi, H., A. Khatab, and N. Rezg. 2011. 'A maintenance policy for a production system under environment constraints', *Proceedings of International Conference on Industrial Engineering and Systems Management (Iesm'2011): Innovative Approaches and Technologies for Networked Manufacturing Enterprises Management*: 1060-69.

Colledani, M., and T. Tolio. 2012. 'Integrated quality, production logistics and maintenance analysis of multi-stage asynchronous manufacturing systems with degrading machines', *Cirp Annals-Manufacturing Technology*, 61: 455-58.

Colledani, M., and T. Tolio. 2011. 'Integrated analysis of quality and production logistics performance in manufacturing lines', *International Journal of Production Research*, 49: 485-518.

Colledani, M. and T. Tolio. 2012. 'Integrated quality, production logistics and maintenance analysis of multi-stage asynchronous manufacturing systems with degrading machines', *Cirp Annals-Manufacturing Technology*, 61: 455-458.

Colledani, M., T. Tolio, A. Fischer, B. Iung, G. Lanza, R. Schmitt, and J. Vancza. 2014. 'Design and management of manufacturing systems for production quality', *Cirp Annals-Manufacturing Technology*, 63: 773-96.

Colledani, Marcello. 2011. 'Joint Design of Quality and Production Control in Multi-Stage Asynchronous Manufacturing Systems', *IFAC Proceedings Volumes*, 44: 10408-15.

Dehayem Nodem, F.I., J.-P. Kenne, A. Gharbi, and 2011. 'Simultaneous control ofproduction, repair/replacement and preventive maintenance of deteriorating manufacturing systems ', *International Journal of Production Economics*, 134: 271-82.

Dellagi1, Sofiene, Nidhale Rezg, and Ali Gharbi. 2010. 'Optimal maintenance/production policy for a manufacturing system subjected to random failure and calling upon several subcontractors', *International Journal of Management Science and Engineering Management*, 5: 261 - 67.

Dodge, H. F. 1943a. 'A Sampling Inspection Plan for Continuous Production', *Annals of Mathematical Statistics*, 14: 264 - 79.

Dodge, H. F. . 1943b. 'A sampling inspection plan for continuous production', *The Annals of mathematical statistics*, 14(3): 264-79.

Duffuaa, S., A. Kolus, U. Al-Turki, and A. El-Khalifa. 2020. 'An integrated model of production scheduling, maintenance and quality for a single machine', *Computers & Industrial Engineering*, 142.

F. I. Dehayem Nodem, J.-P. Kenné, A. Gharbi. 2011. 'Simultaneous control of production, repair/replacement and preventive maintenance of deteriorating manufacturing systems', *International Journal of Production Economics*, 134: 271 - 82.

F.I.Dehayem Nodem , J.-P.Kenné and A.Gharbi. 2010. 'Preventive Maintenance and Replacement Policies for Deteriorating Manufacturing Systems', *IFAC Proceedings Volumes* 43: 98 - 103.

Fleischmann. 2001. 'The Impact of Product Recovery on Logistics Network Design ', *Production and Operations Management*, 10: 156-73.

Ghaleb, M., S. Taghipour, M. Sharifi, and H. Zolfagharinia. 2020. 'Integrated production and maintenance scheduling for a single degrading machine with deterioration-based failures', *Computers & Industrial Engineering*, 143.

Guoqing Cheng, Ling LiJoint 2020. 'Joint optimization of production, quality control and maintenance for serial-parallel multistage production systems', *Reliability Engineering & System Safety*, 204.

Hajej, Z., N. Rezg, and A. Gharbi. 2021. 'Joint production preventive maintenance and dynamic inspection for a degrading manufacturing system', *International Journal of Advanced Manufacturing Technology*, 112: 221-39.

He, Y. H., F. D. Liu, J. M. Cui, X. Han, Y. X. Zhao, Z. X. Chen, D. Zhou, and A. Q. Zhang. 2019. 'Reliability-oriented design of integrated model of preventive maintenance and quality control policy with time-between-events control chart', *Computers & Industrial Engineering*, 129: 228-38.

HéctorRivera-Gómez, Ali Gharbi, Jean-Pierre Kenné, Ruth Ortiz-Zarco, and José Ramón Corona-Armen. 2021. 'Joint production, inspection and maintenance control policies for deteriorating system under quality constraint', *Journal of Manufacturing Systems*, 60: 585-607.

Heizer, J. . 2011. 'Operations Management with MyOMLab', *Global Edition. Pearson Education, Limited*.

Hlioui, R., A. Gharbi, and A. Hajji. 2015. 'Replenishment, production and quality control strategies in three-stage supply chain', *International Journal of Production Economics*, 166: 90-102.

Hliouia , A. , Gharbia, and A. Hajjib. 2015. 'Replenishment, production and quality control strategies in three- stage supply chain R', *International Journal of Production Economics*, 166.

Hof, Lucas A., and Rolf Wüthrich. 2017. 'Industry 4.0 – Towards fabrication of mass-personalized parts on glass by Spark Assisted Chemical Engraving (SACE)', *Manufacturing Letters*, 15: 76-80.

Hu, J.-Q., P. Valiki, and G.-X. Yu. 1994. 'Optimality of hedging point policicies in the production control of failure prone manufacturing systems ', *IEEE Transactions on Automatic Control*, 39: 1875-80.

Huang, Y. S., M. W. Chen, and C. C. Fang. 2013. 'A study of the optimal production strategy for hybrid production systems', *International Journal of Production Research*, 51: 5853-65.

Kang, K. C., and V. Subramaniam. 2018. 'Joint control of dynamic maintenance and production in a failure-prone manufacturing system subjected to deterioration', *Computers & Industrial Engineering*, 119: 309-20.

Kazaz, B. , and T.W. Sloan. 2013. 'The impact of process deterioration on production and maintenance policies ', *European Journal of Operational Research*, 227: 88 - 100.

Kenne, J. P., E. K. Boukas, and A. Gharbi. 2003. 'Control of production and corrective maintenance rates in a multiple-machine, multiple-product manufacturing system ', *Mathematical and Computer Modelling*, 38: 351 - 65.

Kenne, J. P., P. Dejax, and A. Gharbi. 2012. 'Production planning of a hybrid manufacturing-remanufacturing system under uncertainty within a closed-loop supply chain', *International Journal of Production Economics*, 135: 81-93.

Kenne, J. P., and A. Gharbi. 1999. 'Experimental design in production and maintenance control problem of a single machine, single product manufacturing system', *International Journal of Production Research*, 37: 621-37.

Kenne, J.P. and A. Gharbi. 2004. 'Stohastic optimal production control problem with corrective maintenance', *Computers & Industrial Engineering*, 46: 865-75.

Kenne, J. P., A. Gharbi, and M. Beit. 2007. 'Age-dependent production planning and maintenance strategies in unreliable manufacturing systems with lost sale ', *European Journal of Operational Research*, 178: 408-20.

Jean-Pierre Kenné (2006). 'Optimisation en production manufacturière' Montréal : *Ecole de Technologie supérieure*, P 224.

Kim, J., and S. B. Gershwin. 2005. 'Integrated quality and quantity modeling of a production line', *Or Spectrum*, 27: 287-314.

Kim, J. Y., and S. B. Gershwin. 2008. 'Analysis of long flow lines with quality and operational failures', *Iie Transactions*, 40: 284-96.

Kootanaee, Akbar Javadian, Dr. K. Nagendra Babu, and Hamidreza Fooladi Talari. 2013. 'Just-in-Time Manufacturing System: From Introduction to Implement ', *International Journal of Economics, Business and Finance*, 1: 07-25.

Kouedeu, A. F., Kenné Jean-Pierre, Pierre Dejax, Victor Songmene, and Vladimir Polotski. 2014. 'Production Planning of a Failure-Prone Manufacturing/Remanufacturing System with Production-Dependent Failure Rates', *Applied Mathematics*, 5: 1557-72.

Kouedeu, A. F., J. P. Kenne, P. Dejax, V. Songmene, and V. Polotski. 2015. 'Production and maintenance planning for a failure-prone deteriorating manufacturing system: a hierarchical control approach', *International Journal of Advanced Manufacturing Technology*, 76: 1607-19.

Ling, Wang, J. Chu, and J. Wu. 2007. 'Selection of optimum maintenance strategies based on a fuzzy analytic hierarchy process ', *International Journal of Production Economics*, 107: 151 -63.

Liu, Bin, Xiujie Zhao, Yiqi Liu, and Phuc Do. 2020. 'Maintenance optimisation for systems with multi-dimensional degradation and imperfect inspections', *International Journal of Production Research*.

Lopes, R. 2018. 'Integrated model of quality inspection, preventive maintenance and buffer stock in an imperfect production system', *Computers & Industrial Engineering*, 126: 650-56.

Mehdi, R., R. Nidhal, and C. Anis. 2010. 'Integrated maintenance and control policy based on quality control', *Computers & Industrial Engineering*, 58: 443-51.

Min, H. , and Z. Gengui. 2002. 'Supply chain modeling: Past, present and future', *Computers and Industrial Engineering*, 43: 231-49.

Mohammadi, B., A. A. Taleizadeh, R. Noorossana, and H. Samimi. 2015. 'Optimizing integrated manufacturing and products inspection policy for deteriorating manufacturing system with imperfect inspection', *Journal of Manufacturing Systems*, 37: 299-315.

Nakhla, M. 2009. 'L'essentiel du management industriel : maîtriser les systèmes : production, logistique, qualité, supply chain', *, 2e éd.. Paris: Dunod/L'Usine nouvelle, xii,*: 419 p.p.

Nodem, F. I. D., A. Gharbi, and J. P. Kenne. 2011. 'Preventive maintenance and replacement policies for deteriorating production systems subject to imperfect repairs', *International Journal of Production Research*, 49: 3543-63.

Nodem, F.I.Dehayem, J.-P.Kenné, and A.Gharbi. 2010. 'Preventive Maintenance and Replacement Policies for Deteriorating Manufacturing Systems', *IFAC Proceedings Volumes* 43: 98 - 103.

Nourelfath, M., N. Nahas, and M. Ben-Daya. 2016. 'Integrated preventive maintenance and production decisions for imperfect processes', *Reliability Engineering & System Safety*, 148: 21-31.

Ouaret, S., J. P. Kenne, and A. Gharbi. 2018a. 'Production and replacement policies for a deteriorating manufacturing system under random demand and quality', *European Journal of Operational Research*, 264: 623-36.

Ouaret, S., J.P. Kenne, and A. Gharbi. 2018b. 'Stochastic optimal control of random quality deteriorating hybrid manufacturing/remanufacturing systems', *Journal of Manufacturing Systems*, 49: 172-85.

Ouaret, S., J. P. Kenne, A. Gharbi, and V. Polotski. 2015. 'Joint Production and Replacement Strategy for a Quality Deteriorating Failure-Prone Manufacturing System', *Ifac Papersonline*, 48: 1198-203.

Paraschos, P. D., G. K. Koulinas, and D. E. Koulouriotis. 2020. 'Reinforcement learning for combined production-maintenance and quality control of a manufacturing system with deterioration failures', *Journal of Manufacturing Systems*, 56: 470-83.

Polotski, V., J. P. Kenne, and A. Gharbi. 2019. 'Optimal production and corrective maintenance in a failure-prone manufacturing system under variable demand', *Flexible Services and Manufacturing Journal*, 31: 894-925.

Rached, M., Z. Bahroun, and J. P. Campagne. 2015. 'Assessing the value of information sharing and its impact on the performance of the various partners in supply chains', *Computers & Industrial Engineering*, 88: 237-53.

Rached HLIOUI, Ali Gharbi, Adnène Hajji, (2015). Stratégies manufacturière intégrant la production, l'approvisionnement et le contrôle de qualité pour des chaines d'approvisionnement non fiables ', *Montréal* : *Ecole de Technologie supérieure*, P 180.

Rivera-Gomez, H., A. Gharbi, and J. P. Kenne. 2013. 'Joint production and major maintenance planning policy of a manufacturing system with deteriorating quality', *International Journal of Production Economics*, 146: 575-87.

Rivera-Gomez, H., A. Gharbi, J. P. Kenne, O. Montano-Arango, and J. R. Corona-Armenta. 2020. 'Joint optimization of production and maintenance strategies considering a dynamic sampling strategy for a deteriorating system', *Computers & Industrial Engineering*, 140.

Robotis, A., T. Boyaci, and V. Verter. 2012. 'Investing in reusability of products of uncertain remanufacturing cost: The role of inspection capabilities', *International Journal of Production Economics*, 140: 385-95.

Roux, O., D. Duvivier, G. Quesnel, and E. Ramat. 2013. 'Optimization of preventive maintenance through a combined maintenance-production simulation model', *International Journal of Production Economics*, 143: 3-12.

Sader, B.H. , and C.D Sorensen. 2003. 'Deterministic and Stochastic Dynamic Modeling of Continuous Manufacturing Systems Using Analogies to Electrical systems ', *In: Proceedings of the 2003 Winter Simulation Conference*, 2: 1134-42.

Samir, O, Vladimir. P, Kenné Jean-Pierre, and G Ali. 2013. 'Optimal Production Control of Hybrid Manufacturing/Remanufacturing Failure-Prone Systems under Diffusion-Type Demand', *Applied Mathematics*, 4: 10.

Sofiene Dellagi1 , Nidhale Rezg, Ali Gharbi. 2010. 'Optimal maintenance/production policy for a manufacturing system subjected to random failure and calling upon several subcontractors', *International Journal of Management Science and Engineering Management*, 5: 261 - 67.

Tarek, A., Z. Hajej, and N. Rezg. 2016. 'Production and Maintenance Optimization for Multi-Machines under Degradation Constraint', *Ifac Papersonline*, 49: 149-54.

Tcheumtchoua S. R. , Kenné, J. P. and Gharbi A. , (2019). ‘Commande des systèmes de production non fiables avec contrôle de la qualité’, *Montréal : Ecole de technologie Supérieure*, P 151.

Van der Laan, E. , and Marc. 1997. 'Production planning and inventory control with remanufacturing and disposal', *European Journal of Operational Research*, 102: 264-78.

Wang, Y., Y. Deli, and Y. Guoqing. 2010. 'Logistics Supply Chain Management Based on Multi-Constrained Combinatorial Optimization and Extended Simulated Annealing', *In International Conference on Logistics Systems and Intelligent Management (ICLSIM 2010)*, 1: 188-92.

Wu, O., G. D. Ave, I. Harjunkoski, A. Bouaswaig, S. M. Schneider, M. Roth, and L. Imsland. 2020. 'Optimal production and maintenance scheduling for a multiproduct batch plant considering degradation', *Computers & Chemical Engineering*, 135.

Yang, D. L., C. J. Lai, and S. J. Yang. 2014. 'Scheduling problems with multiple due windows assignment and controllable processing times on a single machine', *International Journal of Production Economics*, 150: 96-103.

Yoo, S. H., D. Kim, and M. S. Park. 2012. 'Inventory models for imperfect production and inspection processes with various inspection options under one-time and continuous improvement investment', *Computers & Operations Research*, 39: 2001-15.

Zhang, F. X., J. Y. Shen, H. T. Liao, and Y. Z. Ma. 2021. 'Optimal preventive maintenance policy for a system subject to two-phase imperfect inspections', *Reliability Engineering & System Safety*, 205.

Zhang, F. X., J. Y. Shen, and Y. Z. Ma. 2020. 'Optimal maintenance policy considering imperfect repairs and non-constant probabilities of inspection errors', *Reliability Engineering & System Safety*, 193.