Autoregressive Model Updating for Modal Analysis of Non-Stationary Vibration Systems

by

Thi Thuyet BUI

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Mr. Zhaoheng Liu, Thesis Supervisor
Department of Mechanical Engineering, École de technologie supérieure

Mr. Viet Hung Vu, Thesis Co-supervisor
Department of Mechanical Engineering, École de technologie supérieure

Mr. Ambrish Chandra, President of the Board of Examiners
Department of Electrical Engineering, École de technologie supérieure

Mr. Henri Champliaud, Member of the jury
Department of Mechanical Engineering, École de technologie supérieure

Mr. Martin Gagnon, External Evaluator
Research Institute of Hydro-Quebec

THIS THESIS WAS PRESENTED AND DEFENDED

IN THE PRESENCE OF A BOARD OF EXAMINERS AND PUBLIC

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DEDICATION

For my family

Mise à jour du modèle autorégressif pour l'analyse modale des systèmes vibratoires non stationnaires

Thi Thuyet BUI

RÉSUMÉ

L'analyse modale opérationnelle (OMA) est récemment devenue l'une des techniques les plus efficaces pour extraire les paramètres modaux (fréquences naturelles, rapports d'amortissement et formes de mode) des vibrations pratiques dans des conditions de travail. De nombreux bons résultats ont été observés dans de nombreux domaines, tels que la robotique, l'interaction fluide-structure et les ponts. Comme ces applications sont considérées dans des conditions de travail, leurs paramètres modaux changent avec le temps. Par conséquent, l'identification des paramètres modaux opérationnels dans les structures est une procédure complexe sur le plan informatique et qui prend du temps. En outre, les signaux non stationnaires collectés sont généralement mélangés à un bruit important causé par les conditions de fonctionnement ou les environnements changeants. Par conséquent, la résolution des problèmes à l'aide des moindres carrés est déficiente en termes de rang. Pour atténuer ces inconvénients dans l'OMA, il est essentiel de développer des algorithmes de mise à jour.

L'objectif principal de cette étude est de développer de nouveaux algorithmes pour réduire la complexité de calcul, la charge de temps et la singularité de la matrice pour l'identification modale de l'OMA. Les méthodes proposées peuvent être appliquées aux structures vibrantes non stationnaires à variation lente et sont validées par des expériences sur l'interaction fluide-structure. Pour ce faire, la recherche est menée en deux étapes.

L'objectif de la première étape est de présenter une nouvelle méthode de mise à jour des paramètres des modèles autorégressifs et de surveillance du changement des paramètres modaux pour les systèmes de vibrations non stationnaires à variation lente. Cette méthode évite la complexité de calcul et l'analyse modale fastidieuse pour les systèmes de vibrations non stationnaires à variation lente. La technique de la fenêtre glissante est utilisée pour extraire les paramètres modaux des systèmes. Pour ce faire, le complément de Schur est appliqué à la fenêtre glissante pour mettre à jour les paramètres du modèle en temps et en ordre. La simulation numérique et les expériences sur une plaque émergente sont des moyens efficaces pour valider que l'approche proposée améliore les performances en termes de complexité de calcul et de temps d'exécution. En outre, la méthode proposée est utilisée pour surveiller et suivre le changement des paramètres modaux pour les systèmes non stationnaires à variation lente. Les restrictions de l'approche proposée sont également discutées. Les résultats de la première étape ont été acceptés pour publication dans le Journal of Mechanical Systems and Signal Processing (MSSP).

Sur la base des résultats prometteurs concernant la réduction de la complexité de calcul et du temps d'exécution pour l'identification de l'analyse modale, la deuxième étape consiste également à développer un algorithme de mise à jour, qui peut être appliqué aux structures dans des conditions de travail avec uniquement les réponses vibratoires de sortie. En outre, la

méthode proposée permet de faire face à la singularité de la matrice causée par les conditions de fonctionnement pour l'analyse modale opérationnelle. L'idée de base de cette méthode consiste à utiliser une fenêtre glissante à court terme (STSW) pour identifier les paramètres modaux des structures vibratoires non stationnaires à variation lente. Cette méthode utilise la méthode récursive des moindres carrés multivariables avec décomposition en valeurs singulières (SVD) pour trouver les solutions dans un segment de données de chaque fenêtre temporelle. L'identification du modèle de mise à jour est effectuée en mettant à jour la SVD de la matrice de données à travers l'ordre et le temps de la fenêtre de calcul précédente pour contrôler les paramètres modaux des systèmes non stationnaires à variation lente. Des applications potentielles sont trouvées dans l'analyse modale des turbines hydrauliques immergées excitées par des écoulements turbulents pour extraire et surveiller le changement des paramètres modaux. Les résultats de cette étape ont été soumis au Journal of Mechanical Systems and Signal Processing (MSSP) pour publication.

Mots clés: Modèle autorégressif, moindres carrés récursifs, sélection de l'ordre du modèle, identification des paramètres modaux, système variable, fenêtre glissante

Autoregressive model updating for modal analysis of non-stationary vibration systems

Thi Thuyet BUI

ABSTRACT

Operational modal analysis (OMA) has recently become one of the most effective techniques to extract modal parameters (natural frequencies, damping ratios, and mode shapes) for practical vibrations under working conditions. Many good results have been observed in numerous domains, such as robotics, fluid-structure interaction, and bridges. Due to these applications being considered under working conditions, their modal parameters change over time. Hence, identifying the operational modal parameters in structures is a computationally complex and time-consuming procedure. In addition, collected non-stationary signals are usually mixed with heavy noise caused by operating conditions or changing environments. Consequently, problem resolution using least-squares is rank deficient. To mitigate these disadvantages in OMA, it is essential to develop updating algorithms.

The main objective of this study is to develop novel algorithms to reduce the computational complexity, time burden, and matrix singularity for the modal identification of OMA. The proposed methods can be applied to slow-varying non-stationary vibration structures and are validated by experiments on the fluid-structure interaction. To achieve this, the research is conducted in two steps.

The objective of the first step is to present a novel method for updating model parameters of autoregressive models and monitoring the change of modal parameters for slow-varying non-stationary vibration systems. This method avoids computational complexity and time-consuming modal analysis for slow-varying non-stationary vibration systems. The sliding window technique is used to extract modal parameters for systems. For this objective, the Schur complement is applied to the sliding window to update model parameters in time and order. Numerical simulation and experiments on an emerging plate are effective ways to validate that the proposed approach enhances performance in terms of computational complexity and execution time. In addition, the proposed method is used to monitor and track the change of modal parameters for slow-varying non-stationary systems. Restrictions of the proposed approach are also discussed. The results of the first step have been accepted for publication in the Journal of Mechanical Systems and Signal Processing (MSSP).

Based on the promising results regarding reduced computational complexity and execution time for modal analysis identification, the second step is also to develop an updating algorithm, which can be applied to structures under working conditions with only the output vibratory responses. In addition, the proposed method copes with the matrix singularity caused by operating conditions for operational modal analysis. The basic idea of this method consists of using a short-time sliding window (STSW) to identify modal parameters for slow-varying non-stationary vibration structures. This method uses the recursive multivariable least-squares method with singular value decomposition (SVD) to find the solutions in a data segment from

each time window. The updating model identification is conducted by updating the SVD of the data matrix through the order and time from the previous computational window to monitor the modal parameters of the slow-varying non-stationary systems. Prospective applications are found in the modal analysis of the submerged hydraulic turbines excited by turbulent flows to extract and monitor the change of modal parameters. The results of this step have been submitted to the Journal of Mechanical Systems and Signal Processing (MSSP) for publication.

Keywords: Autoregressive model, recursive least squares, model order selection, modal parameter identification, varying system, sliding window

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LIST OF ABBREVIATIONS AND ACRONYMS

AIC	Akaike information criterion
AM	Average modal amplitude
AFS-VTAS	Adaptable functional series vector time-dependent autoregressive
AP	Average modal power
AR	Autoregressive
ARMA	Autoregressive Moving Average
ARX	Autoregressive exogenous excitation
BIC	Bayesian information criterion
DOF	Degree of freedom
DMSN	Damped modal signal-to-noise ratio
EFDD	Enhanced frequency domain decomposition
EMA	Experimental modal analysis
FDD	Frequency domain decomposition
FEA	Finite element analysis
FPE	Final prediction error
FRF	Frequency response function
FS	Functional series
FS-TAR	Functional series time-autoregressive model
FS-TARMA	Functional series time-autoregressive moving average model
ITD	Ibrahim time domain
IV (E)	Instrumental variable (estimate)
LCSE	Least square complex exponential

MDL	Maximum description length
MLE	Maximum likelihood estimate
MSN	Modal signal-to-noise
NOF	Noise-ratio order factor
NSR	Signal to noise ratio
OMA	Operational modal analysis
PDF	Power density function
PEM	Prediction error method
PP	Peak picking
PSD	Power spectral density
QR	Factorization QR
SSI	Stochastic subspace identification
STAR	Short-time autoregressive
STFT	Short-time Fourier transform
SVD	Singular value decomposition
Std	Standard derivation
STSW	Short-time sliding window
TAR	Time autoregressive model
TF	Time-frequency
TV	Time-varying
TVAR	Time-varying autoregressive

LIST OF SYMBOLS

$\mathbf{A}_i$	Matrix of parameters relating the output $\mathbf{y}[t-i]$ to $\mathbf{y}[t]$
p	Model order
n	Dimension or number of sensors
t	Time index
T_s	Sampling period
$\mathbf{I}$	Unity matrix
f	Natural frequency
k	Sample index
$\mathbf{L}$	Complex eigenvectors matrix
$\mathbf{l}$	Complex modal vector
λ	Continuous complex eigenvalue
ξ	Damping ratio
$\mathbf{P}_{np\times n}$	Model parameters matrix
$\mathbf{e}[t]$	Residual vector of all output channels
σ	Real part of the continuous eigenvalue
θ	Imaginary part of the continuous eigenvalue
$\mathbf{h}$	Vector of modal scale factors
π	Pi number
H	Hermitian transpose
$\lvert\ \rvert$	Absolute value

$\mathbf{y}[t-i]$	Output vector with time delay $i \times Ts$
$\mathbf{z}[t]$	Regressor for the output vector $\mathbf{y}[t]$
$\mathbf{K}$	Data matrix
$\mathbf{R}$	Sub-data matrix
N	Number of available data samples
$[k]$	Parameters at time index k
(p)	Order index p
$(p+m)$	Order index $p+m$

INTRODUCTION

Operational modal analysis (OMA) aims to determine the modal parameters (modal frequencies, mode shapes, and mode damping ratios) of vibration systems using only their vibration response signals under working rather than laboratory conditions. Nowadays, OMA has become a powerful technique widely utilized in many fields, such as mechanical, aerospace, and civil engineering.

In practice, operational modal analysis can deal with practical vibration systems whose parameters are time-varying (Dong, Man, Wang, Yu & Zhao, 2018). Such systems are named time-varying non-stationary systems. The identification methods for the time-varying system can be classified into two types: time-domain and frequency-domain. The advantages and disadvantages of these methods have been examined in detail (Poulimenos & Fassois, 2006; Zhou, Heylen & Sas, 2014; Zhou et al., 2018; Li, Zhu, Law & Samali, 2020; Ramnath, 2010).

Generally, the researchers try to improve the quality and efficiency of identifying time-varying vibration systems. The methods for non-stationary vibration are computationally expensive and time-consuming. When the data size is large, the procedure of the inverse matrix always repeats, which means a lot of time and storage space are required to run these algorithms. The complexity of the methods has significant repercussions on their performance in OMA. Due to the disadvantages of the methods for non-stationary systems, developing novel algorithms to overcome these disadvantages is still an open problem.

The methods for non-stationary system identification may be classified as fast and slow non-stationary (fast and slow time-varying). In the fast non-stationary methods, the parameters are explicit functions of time. The slow non-stationary methods, on the other hand, are based on conventional stationary frequencies or time domain system identification and signal segmentation techniques.

There are several difficulties in developing algorithms for identifying fast non-stationary systems. The presence of time-dependent coefficients results in more computational complexity and matrix singularity. In addition, it is very challenging to choose the functions for time-varying parameters in the methods for non-stationary vibration systems.

Slow-varying non-stationary vibration systems

Therefore, the slow non-stationary methods have drawn much attention thanks to their potential application in various practical systems. Under the assumptions of short time-invariance and the theories of "time-freezing" (Zadeh, 1950), the time-varying systems can be regarded as time-invariant systems over short periods. The vibration response signals of slow-varying systems are entirely obtained simultaneously, but they can be received by continuous sampling over time.

Many researchers have developed methods to extract modal parameters for the slow-varying non-stationary vibration systems (Verboven, Cauberghe, Guillaume, Vanlanduit & Parloo, 2004; Cheng Wang et al., 2018; Cheng Wang, Huang, Lai & Chen, 2020; Zong, Chen, Tao, Wang & Xiahou, 2020). Although many problems have been solved, the methods for slow-varying non-stationary vibration remain limited as computational complexity and matrix singularity. Hence, developing the algorithms for slow-varying non-stationary structures to overcome these disadvantages is an open problem. Furthermore, updating the computational model to track and monitor the modal parameters can be developed in conjunction with the identification method to provide a more efficient online modal analysis technique.

The applications of this thesis are aimed at modal analysis in the operation of the fluid-structure interactions. The presence of the fluid around the structure generates a reactive force that causes an added mass and damping for the vibration systems. In submerging vibration systems, natural frequencies can decrease by 3 to 50% depending on the modes considered. In addition, the flow effect is a complex behavior that causes changes in modal parameters for the submerged structures. Theoretical solutions have been developed to predict the outcome of added mass. Besides, the modeling of added damping is still in the laboratory stages. It has not yet been developed in industrial environments (Grüber & Carstens, 2001; Yamamoto, 2008). Hence, identification of the fluid-structure interaction is still open due to the added damping.

Organization of this thesis

This thesis concentrates on developing updating algorithms for slow-varying non-stationary vibration systems. The proposed approaches are then verified by numerical simulation and experiments on the fluid-structure interaction. The thesis is organized into four chapters. Chapter 1 introduces the background and literature review on operation modal analysis. Chapter 2 shows the main objective, specific objectives, and methodology. Chapter 3 and Chapter 4, respectively, present specialized works on developing updating methods to meet the specific objective of this thesis. Numerical simulation and experimental results are presented to validate the effectiveness of the proposed updating algorithms. The conclusion and future avenues of exploration are provided at the end of this thesis.

CHAPTER 1

BACKGROUND AND LITERATURE REVIEW

This chapter presents a background and literature review on operational modal analysis, current methods of non-stationary vibration systems, and modal analysis of fluid-structure interactions. A discussion on the identified problems and knowledge gaps resulting from the literature review are also given in this chapter.

1.1 Operational modal analysis by autoregressive modeling

Modal parameters (natural frequencies, damping ratios, and modal shapes) play significant roles in mechanical systems, such as vibration control, structural health monitoring, and aerospace engineering. The main techniques to extract modal parameters are the theoretical solution, finite element analysis (FEA) method, experimental modal analysis (EMA), and operational modal analysis (OMA). Theoretical solutions may be suitable only for simple structures. The validity of modeling is vital for the FEA method, which is greatly affected by the treatment of boundary conditions. Although modal parameters can also be derived using EMA, the laboratory conditions will be the main priority for obtaining the frequency response functions during the identification process.

Operational modal analysis (OMA) has drawn the attention of a high concentration of researchers due to its series of objective benefits in recent years. Firstly, it is being able to solely measure the output responses of the system eliminates the need for costly excitation instruments. Also, the OMA does not need to measure the excitation on the machines. More significantly, the modal properties extracted by OMA reflect the dynamic characteristics of the processing system in working conditions. The methods of OMA are classified into time-domain and frequency-domain.

Frequency-domain methods

Frequency-domain methods are based on the Fourier transform and have developed in vibration structures. (Brincker & Ventura, 2015; Masjedian & Keshmiri, 2009; Reynders, 2012; Cai, Du, Liu, Gu & Gao, 2011) provide a description of these methods. The peak picking (PP) technique (Bendat, 1993), the frequency domain decomposition (FDD) (Brincker, Zhang & Andersen, 2000), and the enhanced frequency domain decomposition (EFDD) method (Brincker, Ventura & Andersen, 2001) are well-known methods in the frequency-domain.

Frequency-domain methods have received much attention thus far. (Dong, Lian, Wang, Yu & Zhao, 2018) use frequency domain decomposition to reduce structural failure and accidents in offshore wind turbines. Another application of frequency-domain methods is in health monitoring structure, as presented by (Papadimitriou, Fritzen, Kraemer & Ntotsios, 2011). This method is based on the available frequency-domain stochastic fatigue method to obtain the damage accumulation caused by fatigue within an entire metallic structure. Although many studies have been conducted, research in the frequency-domain remains limited due to inadequate frequency resolutions, spectrum distortions, and long data.

Time-domain methods

Time-domain methods show more significant benefits and avoid the difficulties associated with frequency-domain techniques. In time-domain identification in OMA, the information is extracted from the correlation or similar functions, such as the random decrement function. The Ibrahim time-domain (ITD) (Ibrahim, 1983), random decrement (Cole, 1971), and polyreference (Vold, Kundrat, Rocklin & Russell, 1982) techniques are widely utilized in the time-domain methods. The implementation of time-domain identification techniques can be found in many fields, such as suspension bridges (Siringoringo & Fujino, 2008), offshore structures (Bao, Wang & Iglesias, 2021), and wind turbine rotors (Juul, Kovacs, Brincker & Balling, 2016).

Although, time-domain methods have many advantages over frequency-domain methods, they still require a very high signal-to-noise (SNR) ratio from measured signals during modal identification.

1.1.1 Time-series methods

The time-series method, first considered in vibration problems in the 1970s in the book of (Box, George, Jenkins & Gwilym, 1976), is a system identification for both time and frequency-domain approaches. The basic idea of time-series analysis is to determine a system and predict its present and future response based on information about its past input and output signals. The models of the time-series method are established by the model's relationships between input signals, output signals, and model errors.

Recently, the time-series method has been used in many applications, such as flexible robots (Vu, Liu, Thomas, Li & Hazel, 2016), spacecraft (Hai, Wei & Hong, 2008), gear tooth crack detection (Chen, Schmidt, Heyns & Zuo, 2021), and a moving-mass supported beam (Vu, 2008).

Time-series methods offer several potential benefits over alternatives, such as no requirement for physical, complete structural models and effective operation even in the low-frequency range. In the application of vibrations, the input signals are the excitation forces, output signals are the dynamic responses, and the errors are the Gaussian white noise. Some literature on time-series methods is listed in the following:

Autoregressive exogenous excitation (ARX) model

The ARX model, with n sensors, is convenient for obtaining the general relation between input and output signals. The autoregressive exogenous excitation model ARX can be written as follows:

$$\mathbf{y}[t]+\mathbf{y}[t-1]\mathbf{A}_1+\ldots+\mathbf{y}[t-p]\mathbf{A}_p=\mathbf{w}[t]+\mathbf{w}[t-1]\mathbf{C}_1+\ldots+\mathbf{w}[t-q]\mathbf{C}_q+\mathbf{e}[t] \tag{1.1}$$

where $\mathbf{y}[t\text{-}i]\in R^{1\times n}$ are the output signals with delayed time iT_s, $\mathbf{w}[t-i]\in R^{1\times n}$ is the input signals, T_s is the sampling period, p, q are the order of models, and $\mathbf{A}_i\in R^{n\times n}(i=1:p)$, $\mathbf{C}_j\in R^{n\times n}(j=1:q)$ are the parameters of models, $\mathbf{e}[t]\in R^{1\times n}$ is a residual of the model at time t .

The ARX model can be used to identify modal parameters of the vibration systems under operating conditions. (Vu, 2008) presents a method for modal identification of bridge modal parameters under live loading. The modal parameters extracted by the ARX model of real bridge structures with correlated noise exhibits are compared to the finite element method's results to validate this approach's efficiency. Recently, a method proposed by (Kapsalas, Koustoumpardis & Aspragathos, 2016) also uses the ARX model to address the problem of vibration control of a flexible metallic beam. In this algorithm, a simple closed-loop control system is developed to minimize induced force at the wrist of the robot's beam vibration control. During working conditions, the ARX modeling of the robot-beam system is established by the measurements obtained from the force sensor of robots. The purpose of (Kapsalas, Sakellariou, Koustoumpardis & Aspragathos, 2018) is to control the vibrations of flexible beams manipulated by industrial robots.

Autoregressive moving average (ARMA) model

The ARMA is considered a complete model to properly characterize the dynamic behavior of a system or structure as follows.

$$\mathbf{y}[t]+\mathbf{y}[t-1]\mathbf{A}_1+\ldots+\mathbf{y}[t-p]\mathbf{A}_p=\mathbf{e}[t]+\mathbf{e}[t-1]\mathbf{C}_1+\ldots+\mathbf{e}[t-q]\mathbf{C}_q \tag{1.2}$$

where $\mathbf{y}[t\text{-}i] \in R^{1\times n}$ are the output signals with delayed time iT_s, T_s is the sampling period, p, q are the order of models, and $\mathbf{A}_i \in R^{n\times n}(i=1:p), \mathbf{C}_j \in R^{n\times n}(j=1:q)$ are the parameters of models, $\mathbf{e}[t] \in R^{1\times n}$ is a residual of the model at time t .

The ARMA model is a very efficient tool for the modal identification of vibration systems, especially when noisy signals are presented. Regarding the ARMA model applications on vibration systems, (Peter Carden & Brownjohn, 2008) use the ARMA model for structural health monitoring of civil infrastructures to detect a structure in a healthy state and various states of ill-health. Another application, based on the autoregressive moving average vector, (Bodeux & Golinval, 2003) shows the performance of modal identification and damage detection on the Steel-Quake structure to distinguish between the damaged and undamaged conditions of the structures.

Autoregressive (AR) model

In operational modal analysis, excitation cannot be measured directly and is modeled by Gaussian white noise. According to (Vu, Liu, Thomas, Li & Hazel, 2016), the AR model defines the dynamic behavior of a system as follows:

$$\mathbf{y}[t]+\mathbf{y}[t-1]\mathbf{A}_1+\ldots+\mathbf{y}[t-p]\mathbf{A}_p=\mathbf{e}[t] \tag{1.3}$$

Or Eq. (1.3) is rewritten in a regression form

$$\mathbf{y}[t]=\mathbf{z}[t]\mathbf{P}_{np\times n}+\mathbf{e}[t] \tag{1.4}$$

where $\mathbf{z}[t]=\left(\mathbf{y}[t-1],\mathbf{y}[t-2],\ldots,\mathbf{y}[t-p]\right) \in \mathbf{R}^{1\times np}$ is the output data $\mathbf{y}[t]$, $\mathbf{e}[t] \in \mathbf{R}^{1\times n}$ is a residual of the model with zero means, $\mathbf{A}_i \in \mathbf{R}^{n\times n}$ is the AR parameter matrix, and

$\mathbf{P}_{np\times n} = [-\mathbf{A}_1 - \mathbf{A}_2 \ldots - \mathbf{A}_p]$ is the model parameter matrix, T_s is the sampling period, p are the order of models.

The AR model is easy to use for modal identification. Many methods using the AR model have been developed in the past decade. Based on the AR model, (Vu, Liu, Thomas, Li & Hazel, 2016) propose a method for a flexible robot under impact excitation and operational harmonic excitation. The AR model is utilized to obtain the precise signal processing method for the multi-channel measurement to consider the joint and link flexibility of the robot. A method is presented by (Wang & Wong, 2002) for detecting and diagnosing gear faults. This technique is first based on an autoregressive model on the vibration signal of the gear in the healthy state. Then the model is utilized as a linear prediction error filter. The value based on the error between the filtered and unfiltered signals is used to diagnose the health condition of the gear. This method points out that the AR model approach is an effective technique for detecting and analyzing gear faults and may lead to a good solution for the in-flight diagnosis of helicopter transmissions.

In system identification, the existence of input vectors in ARX and ARMA models can perform better than in AR models. In addition, ARX models show several benefits, including accuracy and compactness of representation. However, these models lead to significant, sparse parameter matrices that increase with the number of time points. When the input data is unmeasurable, it is preferable to apply AR models since they only require the output data or structural responses for modeling.

Moreover, (Poulimenos & Fassois, 2006; Tian, Juhola & Grönfors, 1997; Zhang, Xiong, Liu, Zou & Guo, 2010) point out that auto-regressive (AR) models show good performance in terms of high accuracy, good resolution, sharp peak, and their ability to track the varying modal parameters of the structures. This is why AR models are applied in the vibration systems in this thesis.

1.1.2 Model order selection

The precision of the AR model is sensitive to the selection of model order. If the model order is inappropriate, the model parameters will not describe the underlying nature of the process, and the signal's representation will be inaccurate. For the model-based spectral analysis, the choice of model order is too low, resulting in a smoothed spectral estimate. If an optimal model order is too high, it will cause spurious spectral peaks. The model order selection has received much attention in recent years. These methods can be classified in three directions: (1) Estimating model parameters is required. (2) Estimation of model parameters is not necessary. (3) Estimation order and parameters are concurrent.

A trade-off is found in the first collection. The model must know prior parameters to estimate the span of the prediction error variance via a range of model orders. As a result, the model is chosen from the minimum prediction error variance. This method’s computational complexity is a weakness because of the model parameter estimated in each time. The second collection of model order selection does not require a priori knowledge of model parameters to determine the optimal model order. Most techniques are based on the eigenvalues of the data covariance matrix. However, these methods only apply to ARX and ARMA models with high signal-to-noise ratios. The third collection of the model order selection determines the model order and parameters concurrently. However, the cost of increased computational requirements is a limitation of these methods in the system identification process. Well-known criteria for estimating order models are listed in the following:

- Akaike information criterion (AIC) (Akaike, 1974) uses the mean square error to obtain the optimal order of models. The benefit of this criterion is that it is easy to calculate. Although this method has many benefits, AIC fails to consider over-fit. This drawback means the selected order can be greater than the optimal order. Finite samples and asymptotic effects are the main factors that cause the AIC over-fit. These are proven by (Shaarawy & Ali, 2012). The penalize parameter of AIC, for the AR model, is expressed as:

$$AIC(p) = \log \frac{\left(\mathbf{y}[t] - \mathbf{z}[t] \ \ \mathbf{P}_{np\times n}\right)}{N} - p \tag{1.5}$$

- (Schwarz, 1978) presents an approach called the Bayesian information criterion (BIC). This criterion is based on maximizing the likelihood function. The main limitation of BIC is under-fit. Additionally, BIC is only valid for a sample size N much larger than the p order of models (Weakliem, 1999). It cannot handle complex collections of models as in the variable selection. The penalize parameter of BIC, for the AR model, is established in the following:

$$BIC(p) = \log \frac{\left(\mathbf{y}[t] - \mathbf{z}[t] \ \ \mathbf{P}_{np\times n}\right)}{N} - \frac{p}{2}\log N \tag{1.6}$$

- According to (Larbi & Lardies, 1999), the minimum description length criterion (MDL), the criterion, is derived employing non-information prior densities of the model parameter. One can obtain a model that trades goodness-of-fit on the observed data with the "complexity" and "richness" of the model. MDL procedures automatically and inherently prevent over-fitting. However, the main drawback of MDL is prior model parameter estimation. For the AR model, the penalize parameter of MDL is calculated by:

$$MDL(p) = \frac{N}{2}\log \frac{\left(\mathbf{y}[t] - \mathbf{z}[t] \ \ \mathbf{P}_{np\times n}\right)}{N} + \frac{p}{2}\log N \tag{1.7}$$

- (Liang, Wilkes & Cadzow, 1993) present an approach to estimate the optimal model order of the models. This study is based on the MDL criterion and the minimum eigenvalue of a covariance matrix. The main advantage of this method is that it does not require prior estimation of the model parameters. As a result, this seems to be an innovative approach for model order selection. Unfortunately, this method is only applied to ARMA and ARX models with a high signal-to-noise ratio of outputs (Al-Smadi, 2004). For ARMA (p, q), $p1 = \max(p,q)$, $D(p,q)$ is a composite data matrix is defined as:

$$\mathbf{D}(p,q) = \begin{bmatrix} \mathbf{y}[p1] & \mathbf{y}[p1-p] & \mathbf{e}[p1] & \mathbf{e}[p1-q] \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{y}[N-1] & \mathbf{y}[N-1-p] & \mathbf{e}[N-1] & \mathbf{e}[N-1-q] \end{bmatrix} \tag{1.8}$$

The model order chosen is based on the MDL:

$$MDL(p,q) = \frac{N}{2}\log(\lambda_{\min}) + \frac{(p+q)}{2}\log N \tag{1.9}$$

where $\lambda_{\min}$ is the minimum eigenvalue of $\mathbf{D}(p,q)^T \mathbf{D}(p,q)$.

- Many attempts have been made by (Vu, Thomas, Lakis & Marcouiller, 2011) in order to select the optimal model order. Based on the noise-to-signal ratio, this order is considered the smallest order value that can fit the data without discrepancy and can be effectively utilized for modal analysis. The noise-ratio order factor (NOF) is calculated as being the variation of the NSR between two successive orders. The NOF is estimated by:

$$NOF(p) = NSR(p) - NSR(p+1) \tag{1.10}$$

The optimal order is chosen after a significant change in the NOF. The NOF converges to a minimum value after the optimum order is reached. This is presented in detail in (Vu, Thomas, Lakis & Marcouiller, 2011).

Each criterion results in different model orders because of the different penalty functions. Hence, they should be utilized with careful assessment. Among these criteria, AIC and BIC are widely utilized methods. However, they are known to suffer the potential problems regarding overfit and underfit, respectively. This thesis uses MDL to choose the optimal order of models due to its benefits for non-stationary vibrations.

1.1.3 Model parameter identification

Model parameter determination has gained much attention due to its importance for OMA. Various methods for model parameter determination, such as the least-squares method (Lennart, 1999), the maximum likelihood method (Brincker & Ventura, 2015), and the instrumental variable estimation (Söderström & Stoica, 2002), have been presented in the last decade.

The least-squares method uses prediction errors to obtain the model parameters. This method is popular and versatile in the identification process. Another unbiased method for estimating the parameters of models is the maximum likelihood method (MLE) (Lardies & Larbi, 2001; Capecchi, 1989). The method constructs a likelihood function of the noises based on the power density function (PDF) and estimates the parameters by maximizing the logarithm of the likelihood function. The instrumental variable (IV) method is a new method with the advantage of being much less computationally intensive than previous methods. It is recognized that the ordinary least squares method is biased since the data matrix is correlated with the noise. So, it has been proposed that a new matrix be sought, called the instrumental variables matrix, which is correlated with the responses but quite independent of the noise (Söderström & Stoica, 2002).

The computational complexity and time consumption are limitations of the above methods. Several researchers have attempted to present new approaches to diminish the drawbacks. (Vu, Thomas, Lakis & Marcouiller, 2011) introduce an updating algorithm to resolve the least-squares methods by QR factorization concerning model order and time. The technique successfully modeled and identified an output-only modal analysis of a non-stationary system. The improvements of this method are the parameters of the current order models used in the next step of the system identification process. This algorithm is found to be very fast and stable.

In this thesis, we used the least-squares method to obtain the model parameters because the excitation acting on the vibration structures are of a random type considered as white Gaussian noise, which allows for a fast and unbiased use of the least-squares method with good accuracy (See Chapters 3 and 4).

1.2 Non-stationary vibrations systems

In practical operating systems, the dynamic properties of engineering systems change during operational conditions. Such systems are thus known as non-stationary (Priestley, 1988; Roberts & Spanos, 2003). Extracting the modal parameter of non-stationary systems is more complicated than for stationary systems. Non-stationary vibration is modeled and analyzed using two primary methods: parametric and non-parametric.

Non-parametric methods

The non-parametric techniques are based on time-dependent spectral representation (Hammond & White, 1996), and well-known non-parametric methods are listed in the following

- The short-time Fourier transform (STFT) (Hlawatsch & Boudreaux-Bartels, 1992) is the most typical approach for calculating a time-varying (TV) spectrum, which is established on the assumption that the signal can be regarded as being stationary in a short time segment. This method splits the signal into small segments fitting into a sliding window. The Fourier transform of the windowed signal is utilized to receive the energy distribution, and the TV window obtains the desired frequency resolution at different times.

- The wavelet transform method has been widely involved to time-varying structural parameter identification. (Ghanem & Romeo, 2000) presented a discrete wavelet identification for discrete linear models of dynamical systems from noisy data. The procedure is based on representing the governing differential equations on a wavelet basis and formulating an inverse algebraic problem in the associated subspace. Another study by (Hou, Hera & Shinde, 2006) presented a discrete wavelet identification approach for structures subjected to earthquake excitation utilizing the instantaneous modal information.

- If the signal is not windowed, time-frequency (TF) distribution is utilized and renders higher TF resolutions than the STFT (Cohen, 1989). The TF distribution figure illustrates more clearly the temporal localization of a signal's spectral components. However, it is difficult to interpret the energy distribution for signals containing several TF components.

- The evolutionary spectrum is presented to specify a time-varying (TV) spectrum, which diminishes many of the drawbacks of the time-frequency distribution (TV) spectrum (Priestley, 1988). In the evolutionary spectral theory, non-stationary signals are illustrated using sinusoids with slowly varying amplitudes representing the spectrum. The TV amplitudes of a signal are characterized by a set of orthonormal expansion functions in each frequency, so the time-frequency solutions can be exploited by changing the number of expansion functions.

These methods are easy to utilize but are still limited by low representation parsimony, frequency resolution, and the inability to track fast variations in the vibration systems.

Parametric methods

Parametric methods are mainly based on parameterized representations of the time-dependent auto-regressive or related types. Their parameters are time-dependent (Grenier, 1989; Cooper & Worden, 2000; Owen, Eccles, Choo & Woodings, 2001). Compared to their non-parametric methods, parametric methods are known to offer several benefits (Fouskitakis & Fassois, 2002; Conforto & D'Alessio, 1999; Zhan & Jardine, 2005), such as representation parsimony, enhanced accuracy, improved frequency resolution, and improved tracking of the time-dependent dynamics. In parametric methods, the type of structure imposed upon the time-varying model parameters is used to classify into the unstructured, stochastic, and deterministic parameters.

- Unstructured parameter methods impose no "structure" upon the model parameters. The techniques of this type are known as the short-time AR, ARMA, and recursive methods (Cooper & Worden, 2000; Niedzwiecki, 2000; Gersch & Brotherton, 1982; Cooper, 1990; Zhan, Makis & Jardine, 2006; Ljung & Soderstrom, 1983; Bellanger, 1987). These methods may show improvements over non-parametric methods, accuracy, and frequency resolution but still suffer in representation parsimony and tracking.

- Stochastic parameter methods impose stochastic "structure" upon the TV parameters through stochastic smoothness constraints. These methods are utilized to model and analyse earthquake ground motion signals (Kitagawa, 1983; Gersch & Kitagawa, 1985; Kitagawa & Gersch, 1985; Jiang & Kitagawa, 1993). These methods may enhance accuracy and tracking, but parsimony may still be insufficient.

- Deterministic parameter methods impose a deterministic ''structure'' upon TV parameters. These methods are of the functional series TAR and TARMA types and represent model parameters using deterministic functions belonging to specific

> functional subspaces. These approaches are applied to many fields, such as earthquake ground motion, vibration analysis in rotating machinery, and robot vibration (Hall, Oppenheim & Willsky, 1983; Niedźwiecki & Kłaput, 2002; Niedźwiecki & Kłaput, 2003; Grenier, 1983; Fouskitakis & Fassois, 2001).

The authors (Fouskitakis & Fassois, 2002; Conforto & D'Alessio, 1999; Zhan & Jardine, 2005) show that parametric methods are superior to non-parametric methods, such as the short-time Fourier transform (STFT). These methods have many advantages, namely, as high accuracy, good resolution, sharp peak, and fewer spurious components. Several other time-frequency methods such as wavelets, superlets and s-transforms are also widely used in dealing with non-stationary systems. An application of wavelets in the space domain for crack identification in structural elements has also been presented by (Liew & Wang, 1998). Another study of (McFadden, Cook & Forster, 1999) proposed the generalised s-transform to detect the early failure of the gearboxes. In this study, the generalised s-transform and the new window functions are used for the decomposition of experimentally measured gear vibration data. An approach based on the superlets transform was introduced in (Srikanth & Koley, 2021) to identify the type of power system faults occurring in an interconnected power system. This study shows that the superlets technique can lead to quicker classification and early corrective actions when the power system faults involve high-frequency disturbances.

Non-stationary vibration systems are more complex than stationary vibration systems. Since the configurations of parameters for the non-stationary vibration systems are time-dependent functions, it is not easy to estimate the functions of these parameters. As a result, the identification of non-stationary vibration systems is still an open problem.

1.3 Slow non-stationary systems

In operational systems, the systems are often time-varying. Hence, research on time variance is essential, both academically and in practice. The time-varying vibration systems can be separated into fast and slow varying vibration systems (Kopmaz & Anderson, 2001; Li, Zhu, Law & Samali, 2020). The study by (Ramnath, 2010) indicated that the slow varying systems whose variation in coefficient is much less than variation in solution. The identification methods for non-stationary vibration systems are generally divided into fast or slow.

In the fast non-stationary methods, the theory of vibration is induced by the fast variation of the systems and the parameters are explicit functions of time. There are several difficulties in developing algorithms for identifying fast non-stationary structures. The presence of time-dependent coefficients results in more computational complexity and matrix singularity. In addition, it is not easy to choose the functions for time-varying parameters in the methods for non-stationary vibrating structures. To overcome this, these functions need to be simplified. Several authors have written about these methods:

- (Poulimenos & Fassois, 2006) systematically reviewed parametric time-domain methods in terms of model parsimony, model parameters estimation accuracy, and tracking capability of time-varying dynamics. (Spiridonakos & Fassois, 2014) propose the adaptable FS-TARMA model based on the basic functions. The conventional and adaptable FS-TARMA models are compared in parameter estimation, model order selection, and model validation.

- Recently, (Ma et al., 2018) developed a method based on kernel methods in the FS-TARMA models for the recursive identification of TV systems. The algorithm is applied in a laboratory setting, to a time-varying structure, that is, a simply supported beam across which a moving mass slides to track the time-varying modal parameters. The results of this method are compared to Monte Carlo experiments. The comparisons prove this method’s superior accuracy, lower computational complexity, and enhanced online tracking.

- (Li, Vu, Liu, Thomas & Hazel, 2017) present a method for extracting the real modal parameters of time-varying systems using an adaptable functional series vector time-dependent autoregressive (AFS-VTAR) model. The modal identification of a flexible robot in motion is presented as an effective way to validate this algorithm.

Despite the progress that has been achieved so far, there are still many issues, for example, the high demand for computing power or matrix singularity problems.

Slow non-stationary methods are based on conventional stationary frequency or time-domain system identification and signal segmentation. Non-stationary responses of time-varying systems can be divided into the stationary response time series of time-invariant systems in a short interval. The length of each segment is chosen to ensure an approximately stationary behavior in these methods. Consequently, the characterization of a non-stationary signal constitutes a series of locally stationary models. This method presents a tool for monitoring the change of modal parameters to mitigate the weakness of fast non-stationary methods.

- (Wang et al., 2018) present an approach based on the frozen-in coefficient technique. The adaptive identification of the time-varying modal parameters becomes the decomposition of the principal components of stationary response signals. This method can extract modal parameters from the non-stationary random responses for weakly-damped slow time-varying structures.

- An identification method based on the assumption of short-time linear variances for time-varying dynamic systems has been developed by (Vu, Liu, Thomas, Li & Hazel, 2016). In this method, the modal shape coupling in a flexible robot based on the STSW for conventional AR modeling is identified on the stationarity of each data segment. This method uses the AR model for the modal analysis of the flexible robot, first under impact conditions and then under operational excitations.

- To overcome the disadvantages of the fast non-stationary methods, (Ma & Ding, 2019) propose a method in which the short data-based output identification for TV systems is considered to capture their fast varying dynamics via regularization of ill-posed problems.

The identification process of time-invariant systems is repeated many times in each data segment. This results in computational complexity, increased execution time, and matrix singularity for modal analysis. To date, many methods have successfully implemented slow non-stationary vibration systems. However, an approach that avoids complex and time-consuming computing in online monitoring for slow non-stationary vibration systems is still a very open problem.

1.4 Modal analysis of submerged structures

The fluid properties and proximity of surrounding structures impact modal parameters of mechanical systems submerged in a fluid. The fluid surrounding the system gives rise to vibration characteristics that differ from those when surrounded by air, namely, the postponing response, and the increased mass and damping of systems. The main types of fluid-structure interaction are added stiffness, added mass, and added (hydrodynamic) damping (Dehkharqani, Aidanpää, Engström & Cervantes, 2018).

Added stiffness

Added stiffness is a component of the fluid force expressed by the ratio of the fluid force to the structural displacement. The added stiffness of the mechanical systems is usually neglected, as it is considerably less than the added mass and damping effects. Only a few methods have investigated the fluid-added stiffness in submerged vibration systems. A study by (Münch, Ausoni, Braun, Farhat & Avellan, 2010) is presented that predicts fluid-structure coupling by linearizing the hydrodynamic load acting on a rigid, oscillating hydrofoil. According to this study, the fluid-added stiffness is negligible at high frequencies. In addition, the reduced frequencies affect the dimensionless stiffness of the oscillation motion. (Gauthier, Giroux, Etienne & Gosselin, 2017) used the two steady-state simulations to evaluate the fluid-added stiffness. This study showed that the added modal stiffness was approximately 2% of the structural modal stiffness.

Added damping

Submerged vibration systems involve contains material damping, structural damping (friction), and fluid-added damping (hydrodynamic). The material properties relate to the material damping (Trivedi, 2017; Kareem & Gurley, 1996). The fluid-added damping depends on two factors, viscous and fluid pressure. The viscosity increases with increasing submergence levels and could result in an increase in fluid-added damping. The fluid pressure is connected to the mode shape of the structure (Dehkharqani, Aidanpää, Engström & Cervantes, 2018). A set of publication (Seeley, Coutu, Monette, Nennemann & Marmont, 2012; Kammerer & Abhari, 2009) also pointed out that damping ratios increase with the flow velocity for hydrofoil profiles. Generally, the fluid-added damping ratio may not decrease as the submergence level increases. Structural damping depends on the adjacent surface interaction of structures and induced friction (Kareem & Gurkey, 1996).

Based on the literature on fluid surface interaction, the added damping of the vibration system submerged in the fluid is affected by free-stream velocity (Kaminer & Kavitskii, 1976), reduced frequency and excitation frequency (Trivedi & Cervantes, 2017), rotational speed (Kielb & Abhari, 2003), and cavitation and vortex shedding (Benaouicha & Astolfi, 2012).

Added mass

The natural frequency of vibration systems in a fluid is significantly less than in air. Moreover, when the mass of a system increases, its natural frequencies decrease (Rodriguez, Egusquiza, Escaler, Liang & Avcllan, 2006; Lakis & Païdoussis, 1972; Kerboua, Lakis, Thomas & Marcouiller, 2008; Selmane & Lakis, 1997). In the case of plates, their natural frequencies when in water may be 50–70 (%) lower than when in the air (Kramer, Liu & Yin, 2013). As for the partially submerged structure, the added mass changes in conjunction with the submergence levels (De La Torre, Escaler, Egusquiza & Farhat, 2014; Motley, Kramer & Yin, 2013). The added mass factors have been calculated for different cases (Lussier, 1998). (Sinha, Singh & Rao, 2003) published a paper on the added mass and damping effects of perforated and immersed plates. In this study, the mass added in water is assumed to be equal to the mass of water corresponding to the reaction force of the plate. They studied the model by finite elements and performed an experimental validation using an identification method in the spectral domain.

Added mass prediction is necessary to analyze the dynamic response of the submerged structures. Free-stream velocity, reduced frequency and excitation frequency, rotational speed, cavitation, fluid-to-structure density ratio, submergence level, and nearby solid structures are factors (Benaouicha & Astolfi, 2012; De La Torre et al., 2014; Valentín, Presas, Egusquiza & Valero, 2016) to take into consideration for the added mass of vibration systems submerged in a fluid.

The main challenges in numerical and experimental approaches relate to the submerged structure analyses that need to know the proprieties of dimensional flow and the experiments during operational working. Operational modal analysis (OMA) has shown its potential in fluid-structure interaction in the last decade. (Thomas, Abassi & Lakis, 2005) present an experimental modal analysis study of a hydraulic turbine blade-type structure subjected to turbulent flow. The test was conducted in the laboratory in three different cases of vibration, namely: in air, in stagnant water, and water with turbulent flow. The ARMA model was applied as modal analysis technique. The results of the method were compared with different spectral methods. Another work by (Vu, Thomas & Lakis, 2007) was conducted on the effects of added mass on submerged plates. A classical experimental modal analysis was performed to compare the structures in air. The studied structure was placed at different depths and subjected to different turbulent flow rates. This study uses the AR model to extract and monitor the modal parameters.

In this thesis, the study of the added mass and damping effects on a submerged structure were investigated by applying the modal analysis methods developed in Chapter 3 and Chapter 4. The tests were carried out on a submerged plate structure and a hydraulic turbine blade. The impact of submergence levels made it possible to monitor the evolution of the modal parameters over time. This work can be potentially used for investigating the dynamic behavior of hydraulic turbines in operational conditions.

CHAPTER 2

OBJECTIVE AND RESEARCH APPROACH

This chapter outlines the objective of this thesis and the corresponding methodologies based on the literature.

2.1 Objective of thesis

Main objective :

The main objective of this study is to develop an updating algorithm that can be applied to the vibration systems under working conditions using only the output responses. Prospective applications are found in modal analysis of the slow time-varying non-stationary vibration systems, such as the submerged plate and hydraulic turbine, to monitor the modal parameters of these systems. This thesis proposes a study on modal analysis using an autoregressive model. A new method for online monitoring of modal parameters over time is developed to improve the practical performance of OMA in terms of computational complexity and execution time.

Numerical simulations and experiments access the effectiveness of the proposed approach. This work is verified on a fluid-structure interaction, namely, a plate and hydraulic turbine blade submerged at different depths, to extract and monitor the change of modal parameters.

Specific objectives:

The main objective is subdivided in two specific goals that will be gradually verified as described in the following section.

Specific objective 1: This objective aims to develop an updating algorithm to obtain the model parameters of the AR model using the Schur complement (Hager, 1989), in the solution of the least squares method. The short-time sliding window (STSW) technique is utilized on the output signals. This study aims to avoid computational complexity and reduce time-consumption. In addition, this study is used to monitor and track the modal parameters for slow-varying non-stationary vibration structures.

Specific objective 2: This second objective also aims to reduce computational complexity, time-consumption, and matrix singularly for identifying modal analysis of slow-varying non-stationary vibration structures, but it will use the singular value decomposition (SVD) (Bunch & Nielsen, 1978) in the solution of the least squares method. The short-time sliding window (STSW) technique is utilized on the output signals. In addition, this algorithm can monitor the slow-varying modal parameters and obtain more accuracy for the non-stationary signals, which are usually mixed with heavy noise caused by operating conditions or uncertain environments.

2.2 Research approach

For the first specific objective, an updating algorithm is developed to obtain the model parameters of AR models. This method can be updated with respect to the order and time for the AR models. The short-time sliding window technique is used on the signals. The responses of time-varying systems can be divided into the stationary response time series of time-invariant systems in a short interval. In each segment, the Schur complement is applied to the solution of the least-squares method. The model parameters are updated following the order and time. Numerical simulations for the gradual changing system acted on by the random excitations and sinusoidal force are an excellent way to validate the effectiveness of the proposed method. The modal parameters for the multi-channel data measured on an experimental steel plate emerging from the water are identified and compared with those obtained with the analytical method and short-time Fourier transform (STFT), providing a powerful means of validating the proposed method. This approach provides a tool for monitoring the change of modal parameters to mitigate the above weakness of fast non-stationary methods. The first objective is conducted during the first phase of this study. Details of the proposed method are discussed in Chapter 3.

Based on the promising slow non-stationary vibration system results, the second phase is to develop another updating algorithm. In the first phase of this study, the Schur complement (Hager, 1989; Brezinski, 2005) is utilized to update the parameters of the VAR model and monitor the varying modal parameters for the slow non-stationary vibration systems. In terms of computational time, the Schur method is faster because it solves the standard equations of the least squares. However, the main obstacle to vibration signal analysis is that the collected non-stationary signals are usually mixed with heavy noise caused by variable operating or environmental conditions. As a result, the least-squares estimation is rank deficient and must be coped with to overcome this.

Therefore, an updating algorithm for the solution of the least squares method is conducted in this second phase. The short-time sliding window (STSW) technique is used on the signals. This method uses the recursive multivariable least-squares method by singular value decomposition (SVD) (Bunch & Nielsen, 1978; Brand, 2006) to find the solutions within a data segment from each time window. This work has been accessed by the numerical simulations of the gradual changing system acted on by an excitation force and further by an experiment on a hydraulic turbine blade. Details of the proposed method are discussed in Chapter 4.

CHAPTER 3

MODAL ANALYSIS OF SLOW VARYING NON-STATIONARY VIBRATION BY MODAL UPDATING WITH SCHUR COMPLEMENT

The performance of time-varying auto-regressive is limited due to more coefficients estimated in the system identifications. This disadvantage can lead to computational complexity and matrix singularity. This chapter introduces an enhanced algorithm for the online monitoring of slow-varying modal parameters in the vibration systems subjected to unknown excitations to overcome these problems. In this chapter, this method applies an autoregressive (AR) model in a short-time sliding window (STSW) on measured signals. The model parameters are determined and updated through the order and time obtained from the previous computational window. The recursive multivariable least-squares method is utilized with the Schur complement method to find solutions. Numerical simulations of the gradual changing system acted by the random excitations and sinusoidal force are presented. Finally, the modal parameters for the multichannel data measured on an experimental steel plate emerging from the water are identified and compared with those obtained with the analytical method and short-time Fourier transform (STFT), providing a powerful means of verifying the proposed method. This chapter is based on an article titled "*Modal analysis of slow varying non-stationary vibration by model updating with Schur complement,*" published in Mechanical Systems and Signal Processing (MSSP), accepted in May 2021.

3.1 Introduction

This section presents a background and literature review on operational modal analysis and current methods of non-stationary vibrations.

3.1.1 Non-stationary vibration background

In practical cases, the dynamic properties of engineering systems change during operational conditions. Such systems are thus known as non-stationary systems (Poulimenos, Spiridonakos & Fassois, 2006; Chen et al., 2020; Spiridonakos & Fassois, 2014; Yang, Liu, Da Zhou & Ma, 2015; Da Zhou et al., 2018; Zhang, Shan & Li, 2019). Extracting the modal parameter of non-stationary systems is more complicated than for stationary systems. In non-stationary identification, many coefficients need to estimate and change over time.

Numerous approaches are introduced to identify and track the change of modal parameters for non-stationary vibration systems. Although many studies have been conducted, reducing the computational complexity while extracting modal parameters for non-stationary vibration is still open for further research.

There are two primary modelling and analysis directions of the non-stationary vibration (Dziedziech, Czop, Staszewski & Uhl, 2018), namely parametric (Ma et al., 2018) and non-parametric (Ghanem & Romeo, 2000). The non-parametric methods have received much attention thus far. The periodogram estimator, the correlogram estimator, or the Blackman-Tukey (Rao & Swamy, 2018) are well-known non-parametric methods.

In parametric methods, the signal is assumed to satisfy a generating model with a known functional form. Time-varying autoregressive model (TVAR) and time autoregressive moving average (TARMA) (Döhler & Mevel, 2013; Maanan, Dumitrescu & Giurcăneanu, 2017) are parametric methods used in modal parameter identification. These methods are applied to several applications, such as the fluid-structure interaction (Vu, Thomas, Lakis & Marcouiller, 2007) and bridges (Vu, 2008).

Parametric methods used for non-stationary system identification can mainly be categorized as purely non-stationary and locally stationary (fast and slow non-stationary) (Dimitriadis, Fassois, Poulimenos & Shi, 2004).

Functional series (FS) methods (Petsounis & Fassois, 2000; Poulimenos & Fassois, 2003) are examples of purely non-stationary methods. These methods have attracted much attention because of their broad application to many fields. (Spiridonakos & Fassois, 2013) present the stochastic functional series time-dependent auto-regressive (FS-TAR) for effective fault diagnosis in inherently non-stationary structures. That study identified the suitable non-stationary FS-TAR describing the system in each state. It then extracted and used the AR coefficients of a projection parameter vector as the characteristic quantity representing the structural state in each case.

(Zhang, Xiong, Liu, Zou & Guo, 2010) use the time-varying auto-regressive model (TVAR) to extract fault symptoms under non-stationary operating conditions from a rotor test rig during run-up stages. The authors showed that TVAR is superior to some non-parametric methods, such as the short-time Fourier transform (STFT), as it inherits all the advantages of AR in terms of high accuracy, good resolution, sharp peak, and fewer spurious component. However, because many more coefficients are estimated in TVAR than in the conventional AR, using the covariance and correlation similarly to what is done in the conventional AR could lead to computational complexity and matrix singularity.

For this part, slow non-stationary methods are based on conventional stationary frequency or time-domain system identification and signal segmentation (Van Overschee & De Moor, 1996). Each segment is long enough to ensure an approximately stationary behavior in these methods. Consequently, the characterization of a non-stationary signal boils down to a sequence of locally stationary models. This approach provides a tool for monitoring the change of modal parameters to mitigate the above weakness of purely non-stationary methods. (Vu, Liu, Thomas, Li & Hazel, 2016) attempted to identify the modal shape coupling in a flexible robot using the STSW for conventional AR modeling. Based on the stationarity of each data segment, this method used the AR model for the modal analysis of the flexible robot, first under impact conditions and then under operational excitations. An identification approach based on the assumption of “short-time linear varying” for nonlinear time-varying dynamic systems is developed by (Chen et al., 2020). The whole period is divided into a series of shifting windows in that work. The nonlinear time-varying system model can be represented in each window by regression equations, and a least-squares algorithm determines all the coefficients of the models.

In the identification, the least-squares method is repeated many times in terms of the data matrix inversion. This result in computational complexity and time-consuming computing for modal analysis. Although many techniques are successfully implemented on slow non-stationary vibration systems, an approach that avoids complex and time-consuming computing in the online slow varying monitoring of non-stationary cases is still a very open problem.

3.1.2 Proposed approach

As discussed in the previous section, the computational complexity and matrix singularity are the problems of modal analysis. This chapter presents a proposed method to improve practical performance in terms of computational complexity and time-consuming computing. An approximative approach can be used to take advantage of the well-developed related knowledge in the literature. If the non-stationary effect is considered slow varying and modal parameters remain unchanged over a short period, the applicability of the modal analysis can be retained. Hence, the only question will be how to update the model from one window to the next.

The autoregressive model develops as a time-series parametric method in the modal analysis. It can handle short time least-squares (Ma & Ding, 2019; Vu, Thomas, Lakis & Marcouiller, 2011) and direct functional time-varying parameters (Dimitriadis, Fassois, Poulimenos & Shi, 2004; Tian & Zhang, 2020). In modal analysis applications, both the subspace and the time-series methods require a matrix inversion. Therefore, applying the Schur complement in conjunction with the least squares is innovative to take advantage of updating the matrix inversion for time and model order. This will be discussed in this chapter.

The proposed approach introduces some exciting properties, which can be summarized as follows. A new algorithm will be presented to monitor the modal parameters. This method utilizes the STSW for the AR model. The Schur complement is applied to update the matrix inversion in the least-squares algorithm in each segment. This algorithm will allow parameters to be updated via the model order and time. This method avoids complex and time-consuming computing in the online slow varying monitoring of non-stationary cases. Consequently, this method inherits the practicability of the short-time window technique and the accuracy of the autoregressive model, which improves the speed and the versatility of the model updating, allowing better modal analysis and monitoring.

This chapter is organized as follows. Section 3. 2 briefly reviews the autoregressive model description and the identification of operational modal analysis. The use of the Schur complement to update the matrix inversion is discussed in Section 3. 3. Section 3. 4 presents the novel method for updating the model's parameters. The identification of mechanical and vibration systems is illustrated in Sections 3. 5 and Section 3. 6. Section 3.7 presents the summary and the contribution of this work.

3.2 Autoregressive model description and identification

The AR model and the identification of modal parameters from AR models are presented in this section. As mentioned earlier, Gaussian white noise may be modelled as the excitation used in operational modal analysis. Information on the model is acquired during the sampling period T_s. A multivariate autoregressive model of order p and dimension n can be used to fit the data as follows (Vu, Liu, Thomas, Li & Hazel, 2016).

$$\mathbf{y}[t] = \mathbf{z}[t]\mathbf{P}_{np\times n} + \mathbf{e}[t] \tag{3.1}$$

where $\mathbf{z}[t] = (\mathbf{y}[t-1], \mathbf{y}[t-2], \ldots, \mathbf{y}[t-p]) \in \mathbf{R}^{1\times np}$ is the output data $\mathbf{y}[t]$, $\mathbf{e}[t] \in \mathbf{R}^{1\times n}$ is an unobservable innovation sequence with zero means, $\mathbf{A}_i \in \mathbf{R}^{n\times n}$ is the AR parameter matrix, and $\mathbf{P}_{np\times n} = [-\mathbf{A}_1 - \mathbf{A}_2 \ldots - \mathbf{A}_p]$ is the model parameter matrix.

The least-squares estimation can be applied if the data are assumed as a white noise environment. Consider N successive vectors of the output responses from $\mathbf{y}[t]$ to $\mathbf{y}[t+N-1]$. The model parameters matrix $\mathbf{P}_{np\times n}$ can be estimated:

$$\mathbf{P}_{np\times n} = \left(\mathbf{K}[t]^T_{N\times np} \mathbf{K}[t]_{N\times np} \right)^{-1} \mathbf{K}[t]^T_{N\times np} \mathbf{Y}[t]_{N\times n} \tag{3.2}$$

where

$$\mathbf{K}[t]_{N\times np} = \begin{bmatrix} \mathbf{z}[t] \\ \mathbf{z}[t+1] \\ \cdots \\ \mathbf{z}[t+N-1] \end{bmatrix}, \mathbf{Y}[t]_{N\times n} = \begin{bmatrix} \mathbf{y}[t] \\ \mathbf{y}[t+1] \\ \cdots \\ \mathbf{y}[t+N-1] \end{bmatrix} \tag{3.3}$$

Once the model parameters are identified, a state matrix is estimated as follows:

$$\mathbf{A}_{np\times np} = \begin{bmatrix} -\mathbf{A}_1 & -\mathbf{A}_2 & -\mathbf{A}_3 & \ldots & \ldots & -\mathbf{A}_p \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \ldots & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \ldots & \ldots & \mathbf{0} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{I} & \mathbf{0} \end{bmatrix} \tag{3.4}$$

where $\mathbf{I} \in \mathbf{R}^{n \times n}$ is the identity matrix. The eigenvalue decomposition of the state matrix to determine modal parameters of a mechanic system is presented as:

$$\mathbf{A}_{np \times np} = \mathbf{L} \Lambda \mathbf{L}^{-1} = \mathbf{L} \begin{bmatrix} g_1 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \ldots & g_{np} \end{bmatrix} \mathbf{L}^{-1} \tag{3.5}$$

where $g_i, i = 1, 2, \ldots, np$ are discrete eigenvalues and $\mathbf{L} \in \mathbf{R}^{np \times np}$ are eigenvectors of the state matrix. Then, g_i each complex eigenvalue of the discrete system corresponds to one eigenvalue λ_i of the mechanical system: $\lambda_i = \dfrac{\ln(g_i)}{T_s}$. Therefore, natural frequencies f_i and ξ_i damping ratio are computed from complex conjugate pairs λ_i as follows:

$$f_i = \frac{\sqrt{\mathrm{Re}^2(\lambda_i) + \mathrm{Im}^2(\lambda_i)}}{2\pi}, \xi_i = -\frac{\mathrm{Re}(\lambda_i)}{2\pi f_i} \tag{3.6}$$

The eigenvectors can be rewritten as follows:

$$\mathbf{L} = \begin{bmatrix} \lambda_1^{p-1} \mathbf{l}_1 & \lambda_2^{p-2} \mathbf{l}_2 & \ldots & \lambda_{np}^{p-1} \mathbf{l}_{np} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_1 \mathbf{l}_1 & \lambda_2 \mathbf{l}_2 & \ldots & \lambda_{np} \mathbf{l}_{np} \\ \mathbf{l}_1 & \mathbf{l}_2 & \ldots & \mathbf{l}_{np} \end{bmatrix} \tag{3.7}$$

The representation of the AR model and the procedure for extracting modal parameters are shown in Eqs. (3.2)-(3.6) will be used to develop the proposed algorithm in the following sections.

3.2.1 Modal Signal-to-Noise ratio

This section reviews the modal signal-to-noise, the practical technique is used to find the spurious modest and select the modes of interest for the modal identification. The number of eigenvalues and mode shapes of the modal decomposition is large, containing modal features of a system with noises and excitation frequencies.

The modal signal-to-noise ratio (MSN) (Vu, Thomas, Lafleur & Marcouiller, 2007) is used together with previous information on frequencies and damping to classify and identify the modes based on their participation in the following modal decomposition into deterministic and stochastic parts:

$$\mathbf{y}[t]=\sum_{i=1}^{np}\left[\mathbf{l}_i\mathbf{h}\lambda_i^t+\sum_{j=0}^{t-p}\mathbf{l}_i\left(\sum_{r=1}^{n}\mathbf{L}^{-1}(i,r)\{\mathbf{e}(t-j)\}_r\right)\lambda_i^{j+p+1}\right] \tag{3.8}$$

The discrete eigenvalue is transformed into a continuous one by $\lambda_i = e^{(\sigma_i + j\theta_i)T_s}$

The scale factor is derived from the initial repressor:

$$\mathbf{h}_{np\times 1}=[\mathbf{L}]^{-1}_{np\times np}\left\{\mathbf{z}[p+1]\right\}_{np\times 1} \tag{3.9}$$

The estimated covariance matrix of the error:

$$\hat{\mathbf{E}}=\frac{1}{N}\sum_{t=k}^{k+N-1}\left(\mathbf{y}[t]-\mathbf{z}[t]\hat{\mathbf{P}}_{np\times n}\right)\left(\mathbf{y}[t]-\mathbf{z}[t]\hat{\mathbf{P}}_{np\times n}\right)^T \tag{3.10}$$

The continuous average modal power (AP) of each eigenmode in the deterministic signal, is estimated as follows:

$$AP_i = \frac{\mathbf{l}_i^H \mathbf{l}_i \left|\mathbf{h}[i]\right|^2}{\sigma^2} \tag{3.11}$$

The modal variance characterizing the participation of modes in the stochastic part is written by:

$$MV_i = \frac{\mathbf{l}_i^H \mathbf{l}_i [\mathbf{L}^{-1}(i,1:n)][\hat{\mathbf{E}}][\mathbf{L}^{-1}(i,1:n)]^H \left[N - \frac{|\lambda_i|^2 (1-|\lambda_i|^{2N})}{1-|\lambda_i|^2} \right]}{1-|\lambda_i|^2} \tag{3.12}$$

The modal signal-to-noise ratio (MSN) is built for each eigenmode:

$$MSN_i = \frac{AP_i}{MV_i} \tag{3.13}$$

A very low participating factor can point to a physical mode by giving a smaller than AP of a computational mode. This technique allows distinguishing the physical features from the spurious modes, allowing users to select the modes of interest for the modal surveillance.

3.2.2 Model order selection

The selection of the optimal model order is crucial in parametric model-based methods. Several authors have proposed different criteria such as AIC (Kashyap, 1980), BIC (Weakliem, 1999), and MDL (Stine, 2004) for model order selection.

The minimum description length criterion (MDL) is derived from employing non-information prior densities of the model parameter. One can obtain the order that trades the goodness-of-fit on the observed data with the complexity and richness of the model. MDL procedures automatically and inherently protect against over-fitting. Consequently, MDL is a good criterion for short data modelling. MDL is obtained as follow:

$$MDL(p) = \frac{\log\left(\left\|\hat{\mathbf{e}}[t]\right\|\right)}{n} + \log\left(1 + \frac{2np}{N}\log N\right) \tag{3.14}$$

where $\left\|\hat{\mathbf{e}}[t]\right\|$ is a norm of the estimated model error.

Due to the benefits of MDL for short data modelling, the optimal model order estimated by MDL will be presented in the next section.

3.3 Schur complement to update the matrix inversion with added rows and columns

Concerning the mathematical component, the Schur complement is a powerful method for decreasing the number of iterations in the matrix inversion computation (Brezinski, 2005). The Schur complement has been widely applied in matrix analysis, statistics, numerical analysis, and in many other areas of mathematics and its applications.

Regarding operational modal analysis, the Schur complement has been used in several studies of stochastic subspace models (Zarzycki, Wielgus & Libal, 2017; Ma, Ding & Zhou, 2020) to track the modal parameter changes for non-stationary cases. Due to the advantages of the Schur complement in the matrix inversion computation, the Schur complement will be discussed and used in this section.

In this section, a matrix $\mathbf{X}_{r\times r}$ is considered. A matrix $\mathbf{B}_{r\times r}$ can be defined as:

$$\mathbf{B}_{r\times r} = \left(\mathbf{X}_{r\times r}^{T}\,\mathbf{X}_{r\times r}\right)^{-1} \tag{3.15}$$

If columns and rows are appended to the matrix $\mathbf{X}_{r\times r}$, the matrix $\mathbf{B}_{r\times r}$ must be updated.

3.3.1 Matrix inversion updating with added rows

If a row vector $\mathbf{a}$ is added to $\mathbf{X}_{r\times r}$ so that $\tilde{\mathbf{X}} = \begin{bmatrix} \mathbf{X}_{r\times r} \\ \mathbf{a} \end{bmatrix}$, the new inverse matrix can be defined as:

$$\tilde{\mathbf{B}} = \left(\tilde{\mathbf{X}}^T\ \tilde{\mathbf{X}}\right)^{-1} = \left(\begin{bmatrix} \mathbf{X}_{r\times r}^T & \mathbf{a}^T \end{bmatrix} \begin{bmatrix} \mathbf{X}_{r\times r} \\ \mathbf{a} \end{bmatrix}\right)^{-1} = \left(\mathbf{X}_{r\times r}^T \mathbf{X}_{r\times r} + \mathbf{a}^T \mathbf{a}\right)^{-1} \tag{3.16}$$

Based on the inverse matrix modification formula (Hager, 1989), the matrix inversion in Eq. (3.16) is expressed as follows:

$$\tilde{\mathbf{B}} = \left(\mathbf{X}_{r\times r}^T \mathbf{X}_{r\times r} + \mathbf{a}^T \mathbf{a}\right)^{-1} = \left(\mathbf{X}_{r\times r}^T \mathbf{X}_{r\times r}\right)^{-1} - \frac{\left(\mathbf{X}_{r\times r}^T \mathbf{X}_{r\times r}\right)^{-1} \mathbf{a}^T \mathbf{a} \left(\mathbf{X}_{r\times r}^T \mathbf{X}_{r\times r}\right)^{-1}}{\left[1 + \mathbf{a}\left(\mathbf{X}_{r\times r}^T \mathbf{X}_{r\times r}\right)^{-1} \mathbf{a}^T\right]} \tag{3.17}$$

Note that with this Eq. (3.17), it can be concluded that the matrix inversion can be updated when the matrix is appending rows, without the inversion of the full matrix as in Eq. (3.15).

3.3.2 Matrix inversion updating with added columns

If a column vector $\mathbf{b}$ is appended to $\mathbf{X}_{r\times r}$ so that $\tilde{\mathbf{X}} = \begin{bmatrix} \mathbf{X}_{r\times r} & \mathbf{b} \end{bmatrix}$, the new inverse matrix can be defined as:

$$\tilde{\mathbf{B}} = \left(\tilde{\mathbf{X}}^T\ \tilde{\mathbf{X}}\right)^{-1} = \left(\begin{bmatrix} \mathbf{X}_{r\times r}^T \\ \mathbf{b}^T \end{bmatrix} \begin{bmatrix} \mathbf{X}_{r\times r} & \mathbf{b} \end{bmatrix}\right)^{-1} = \begin{bmatrix} \mathbf{X}_{r\times r}^T \mathbf{X}_{r\times r} & \mathbf{X}_{r\times r}^T \mathbf{b} \\ \mathbf{b}^T \mathbf{X}_{r\times r} & \mathbf{b}^T \mathbf{b} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{E} & \mathbf{F} \\ \mathbf{G} & \mathbf{H} \end{bmatrix}^{-1} \tag{3.18}$$

where $\mathbf{E} = \mathbf{X}_{r\times r}^T \mathbf{X}_{r\times r}$, $\mathbf{F} = \mathbf{X}_{r\times r}^T \mathbf{b}, \mathbf{G} = \mathbf{b}^T \mathbf{X}_{r\times r}$, $\mathbf{H} = \mathbf{b}^T \mathbf{b}$.

The Schur complement for the inverse matrix (Hager, 1989) is used to rewrite Eq. (3.18) as:

$$\tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{E}^{-1} + \mathbf{E}^{-1}\mathbf{FSGE}^{-1} & \mathbf{E}^{-1}\mathbf{FS} \\ -\mathbf{SGE}^{-1} & \mathbf{S} \end{bmatrix} \tag{3.19}$$

In this notation, the element $\mathbf{S}$ is given by $\mathbf{S} = \left(\mathbf{H} - \mathbf{GE}^{-1}\mathbf{F}\right)^{-1}$.

3.4 Modal analysis of non-stationary vibration using updating method

In structural health monitoring, modal parameters may change due to many reasons, such as the damage, flow rate, and the presence of cavitation. Therefore, an updating algorithm will be used to track changes in the modal parameters. This section identifies the model parameters by applying the Schur complement and the inverse matrix modification formula to the least-squares algorithm. This algorithm will allow parameter updates with respect to model order and time.

3.4.1 Updating in time for the parameters of AR model

The data matrix $\mathbf{K}_{N\times np}$ and the output vector $\mathbf{Y}_{N\times n}$ at time $t = k$ of the AR model at order p are formed from N successive samples:

$$\mathbf{K}[k]_{N\times np} = \begin{bmatrix} \mathbf{z}[k] \\ \mathbf{z}[k+1] \\ \cdots \\ \mathbf{z}[k+N-1] \end{bmatrix}, \ \mathbf{Y}[k]_{N\times n} = \begin{bmatrix} \mathbf{y}[k] \\ \mathbf{y}[k+1] \\ \cdots \\ \mathbf{y}[k+N-1] \end{bmatrix} \tag{3.20}$$

The model parameters at time $t = k$ are determined by the least-squares method. This leads to the following expression:

$$\mathbf{P}\left[k\right]_{np\times n}=\left(\mathbf{K}\left[k\right]_{N\times np}^{T}\mathbf{K}\left[k\right]_{N\times np}\right)^{-1}\mathbf{K}\left[k\right]_{N\times np}^{T}\mathbf{Y}\left[k\right]_{N\times n} \tag{3.21}$$

Similarly, the data matrix $\mathbf{K}_{N\times np}$ and model parameters at time $t=k+s$ of the AR model can be represented by Eqs. (3.22) and (3.23).

$$\mathbf{K}\left[k+s\right]_{N\times np}=\begin{bmatrix}\mathbf{z}\left[k+s\right]\\ \cdots\\ \mathbf{z}\left[k+N+1\right]\\ \cdots\\ \mathbf{z}\left[k+s+N-1\right]\end{bmatrix},\ \mathbf{Y}[k+s]_{N\times n}=\begin{bmatrix}\mathbf{y}[k+s]\\ \mathbf{y}[k+s+1]\\ \cdots\\ \mathbf{y}[k+s+N-1]\end{bmatrix} \tag{3.22}$$

$$\mathbf{P}\left[k+s\right]_{np\times n}=\left(\mathbf{K}\left[k+s\right]_{N\times np}^{T}\mathbf{K}\left[k+s\right]_{N\times np}\right)^{-1}\mathbf{K}\left[k+s\right]_{N\times np}^{T}\mathbf{Y}\left[k+s\right]_{N\times n} \tag{3.23}$$

From the data matrices of the model at time $t=k$ and $t=k+s$, Eqs. (3.20) and (3.22) can respectively be rewritten as:

$$\mathbf{K}\left[k\right]_{N\times np}=\begin{bmatrix}\mathbf{z}\left[k\right]\\ \mathbf{z}\left[k+1\right]\\ \cdots\\ \mathbf{z}\left[k+N-1\right]\end{bmatrix}=\begin{bmatrix}\mathbf{R}_{1_{s\times np}}\\ \mathbf{R}_{2_{(N-s)\times np}}\end{bmatrix},\ \mathbf{Y}[k]_{N\times n}=\begin{bmatrix}\mathbf{y}[k]\\ \mathbf{y}[k+1]\\ \cdots\\ \mathbf{y}[k+N-1]\end{bmatrix}=\begin{bmatrix}\mathbf{Y}_{1_{s\times n}}\\ \mathbf{Y}_{2_{(N-s)\times n}}\end{bmatrix} \tag{3.24}$$

$$\mathbf{K}\left[k+s\right]_{N\times np} = \begin{bmatrix} \mathbf{z}\left[k+s\right] \\ \dots \\ \mathbf{z}\left[k+N+1\right] \\ \dots \\ \mathbf{z}\left[k+s+N-1\right] \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{2_{(N-s)\times np}} \\ \mathbf{R}_{3_{s\times np}} \end{bmatrix},$$

$$\mathbf{Y}\left[k+s\right]_{N\times n} = \begin{bmatrix} \mathbf{y}\left[k+s\right] \\ \mathbf{y}\left[k+s+1\right] \\ \dots \\ \mathbf{y}\left[k+s+N-1\right] \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{2_{(N-s)\times n}} \\ \mathbf{Y}_{3_{s\times n}} \end{bmatrix} \tag{3.25}$$

where

$$\mathbf{R}_{1_{s\times np}} = \begin{bmatrix} \mathbf{z}\left[k\right] \\ \dots \\ \mathbf{z}\left[k+s-1\right] \end{bmatrix}, \mathbf{R}_{2_{(N-s)\times np}} = \begin{bmatrix} \mathbf{z}\left[k+s\right] \\ \dots \\ \mathbf{z}\left[k+N-1\right] \end{bmatrix} \mathbf{R}_{3_{s\times np}} = \begin{bmatrix} \mathbf{z}[k+N] \\ \dots \\ \mathbf{z}[k+s+N-1] \end{bmatrix} \tag{3.26}$$

$$\mathbf{Y}_{1_{s\times n}} = \begin{bmatrix} \mathbf{y}\left[k\right] \\ \dots \\ \mathbf{y}\left[k+s-1\right] \end{bmatrix}, \mathbf{Y}_{2_{(N-s)\times n}} = \begin{bmatrix} \mathbf{y}\left[k+s\right] \\ \dots \\ \mathbf{y}\left[k+N-1\right] \end{bmatrix}, \mathbf{Y}_{3_{s\times n}} = \begin{bmatrix} \mathbf{y}\left[k+N\right] \\ \dots \\ \mathbf{y}\left[k+s+N-1\right] \end{bmatrix} \tag{3.27}$$

Inserting Eq. (3.24) into Eq. (3.21) leads to:

$$\mathbf{P}\left[k\right]_{np\times n} = \left(\begin{bmatrix} \mathbf{R}_{1_{s\times np}}^{T} & \mathbf{R}_{2_{(N-s)\times np}}^{T} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{1_{s\times np}} \\ \mathbf{R}_{2_{(N-s)\times np}} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{R}_{1_{s\times np}}^{T} & \mathbf{R}_{2_{(N-s)\times np}}^{T} \end{bmatrix} \begin{bmatrix} \mathbf{Y}_{1_{s\times n}} \\ \mathbf{Y}_{2_{(N-s)\times n}} \end{bmatrix}$$

$$=\left(\mathbf{R}_{1_{s\times np}}^{T}\mathbf{R}_{1_{s\times np}}+\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1}\begin{bmatrix}\mathbf{R}_{1_{s\times np}}^{T} & \mathbf{R}_{2_{(N-s)\times np}}^{T}\end{bmatrix}\begin{bmatrix}\mathbf{Y}_{1_{s\times n}}\\ \mathbf{Y}_{2_{(N-s)\times n}}\end{bmatrix}$$

Based on Eq. (3.17), the parameters $\mathbf{P}[k]_{np\times n}$ of the model at time $t=k$ can be obtained:

$$\mathbf{P}[k]_{np\times n}=\left[\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1}-\frac{\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1}\mathbf{R}_{1_{s\times np}}^{T}\mathbf{R}_{1_{s\times np}}\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1}}{\left[1+\mathbf{R}_{1_{s\times np}}\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1}\mathbf{R}_{1_{s\times np}}^{T}\right]}\right]\dots$$

$$\dots\begin{bmatrix}\mathbf{R}_{1_{s\times np}}^{T} & \mathbf{R}_{2_{(N-s)\times np}}^{T}\end{bmatrix}\begin{bmatrix}\mathbf{Y}_{1_{s\times n}}\\ \mathbf{Y}_{2_{(N-s)\times n}}\end{bmatrix} \quad (3.28)$$

After some simple transformation steps, Eq. (3.28) can be rewritten as:

$$\mathbf{P}[k]_{np\times n}=\left(\mathbf{C}-\alpha_1\mathbf{C}\mathbf{R}_{1_{s\times np}}^{T}\mathbf{R}_{1_{s\times np}}\mathbf{C}\right)\mathbf{R}_{1_{s\times np}}^{T}\mathbf{Y}_{1_{s\times n}}+\dots$$

$$\dots+\left(\mathbf{C}-\alpha_1\mathbf{C}\mathbf{R}_{1_{s\times np}}^{T}\mathbf{R}_{1_{s\times np}}\mathbf{C}\right)\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{Y}_{2_{(N-s)\times n}} \quad (3.29)$$

The model parameters of the AR model at the time $t=k+s$ can be calculated by:

$$\mathbf{P}[k+s]_{np\times n}=\left(\begin{bmatrix}\mathbf{R}_{2_{(N-s)\times np}}^{T} & \mathbf{R}_{3_{s\times np}}^{T}\end{bmatrix}\begin{bmatrix}\mathbf{R}_{2_{(N-s)\times np}}\\ \mathbf{R}_{3_{s\times np}}\end{bmatrix}\right)^{-1}\begin{bmatrix}\mathbf{R}_{2_{(N-s)\times np}}^{T} & \mathbf{R}_{3_{s\times np}}^{T}\end{bmatrix}\begin{bmatrix}\mathbf{Y}_{2_{(N-s)\times n}}\\ \mathbf{Y}_{3_{s\times n}}\end{bmatrix}$$

$$=\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}+\mathbf{R}_{3_{s\times np}}^{T}\mathbf{R}_{3_{s\times np}}\right)^{-1}\begin{bmatrix}\mathbf{R}_{2_{(N-s)\times np}}^{T} & \mathbf{R}_{3_{s\times np}}^{T}\end{bmatrix}\begin{bmatrix}\mathbf{Y}_{2_{(N-s)\times n}}\\ \mathbf{Y}_{3_{s\times n}}\end{bmatrix}$$

Based on Eq. (3.17), the parameters $\mathbf{P}[k+s]_{np\times n}$ of the model at time $t=k+s$ can be obtained:

$$=\left[\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1}-\frac{\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1}\mathbf{R}_{3_{s\times np}}^{T}\mathbf{R}_{3_{s\times np}}\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1}}{\left[1+\mathbf{R}_{3_{s\times np}}\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1}\mathbf{R}_{3_{s\times np}}^{T}\right]}\right]\ldots$$

$$\ldots\left[\mathbf{R}_{2_{(N-s)\times np}}^{T}\quad\mathbf{R}_{3_{s\times np}}^{T}\right]\begin{bmatrix}\mathbf{Y}_{2_{(N-s)\times n}}\\ \mathbf{Y}_{3_{s\times n}}\end{bmatrix}$$

$$\begin{aligned}\Rightarrow \mathbf{P}[k+s]_{np\times n}&=\left(\mathbf{C}-\alpha_2\mathbf{C}\mathbf{R}_{3_{s\times np}}^{T}\mathbf{R}_{3_{s\times np}}\mathbf{C}\right)\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{Y}_{2_{(N-s)\times n}}+\ldots\\ &\ldots+\left(\mathbf{C}-\alpha_2\mathbf{C}\mathbf{R}_{3_{s\times np}}^{T}\mathbf{R}_{3_{s\times np}}\mathbf{C}\right)\mathbf{R}_{3_{s\times np}}^{T}\mathbf{Y}_{3_{s\times n}}\end{aligned}\tag{3.30}$$

where

$$\mathbf{C}=\left(\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{R}_{2_{(N-s)\times np}}\right)^{-1},\alpha_1=\frac{1}{\left(1+\mathbf{R}_{1_{s\times np}}\mathbf{C}\mathbf{R}_{1_{s\times np}}^{T}\right)},\alpha_2=\frac{1}{\left(1+\mathbf{R}_{3_{s\times np}}\mathbf{C}\mathbf{R}_{3_{s\times np}}^{T}\right)}$$

Upon carrying out the subtraction of two Eqs. (3.30) and (3.29), it can easily be seen that the model parameters $\mathbf{P}[k+s]_{np\times n}$ lead directly to the following formula:

$$\begin{aligned}\mathbf{P}[k+s]_{np\times n}&=\mathbf{P}[k]_{np\times n}-\left(\mathbf{C}-\alpha_1\mathbf{C}\mathbf{R}_{1_{s\times np}}^{T}\mathbf{R}_{1_{s\times np}}\mathbf{C}\right)\mathbf{R}_{1_{s\times np}}^{T}\mathbf{Y}_{1_{s\times n}}+\left(\mathbf{C}-\alpha_2\mathbf{C}\mathbf{R}_{3_{s\times np}}^{T}\mathbf{R}_{3_{s\times np}}\mathbf{C}\right)\mathbf{R}_{3_{s\times np}}^{T}\mathbf{Y}_{3_{s\times n}}+\ldots\\ &\ldots+\left(\alpha_1\mathbf{C}\mathbf{R}_{1_{s\times np}}^{T}\mathbf{R}_{1_{s\times np}}\mathbf{C}-\alpha_2\mathbf{C}\mathbf{R}_{3_{s\times np}}^{T}\mathbf{R}_{3_{s\times np}}\mathbf{C}\right)\mathbf{R}_{2_{(N-s)\times np}}^{T}\mathbf{Y}_{2_{(N-s)\times np}}\end{aligned}\tag{3.31}$$

From this expression, the parameters of the models at time $t = k + s$ are updated through the model parameters and data sub-matrices at time $t = k$.

3.4.2 Updating the order for parameters of AR model

It is assumed that the data will be measured in a white noise environment. At the sample index t, the data matrix $\mathbf{K}_{N\times np}$ at the order p of the AR model is constructed from N successive samples:

$$\mathbf{K}_{N\times np} = \begin{bmatrix} \mathbf{z}[t] \\ \mathbf{z}[t+1] \\ \dots \\ \mathbf{z}[t+N-1] \end{bmatrix}, \ \mathbf{Y}[t]_{N\times n} = \begin{bmatrix} \mathbf{y}[t] \\ \mathbf{y}[t+1] \\ \dots \\ \mathbf{y}[t+N-1] \end{bmatrix} \tag{3.32}$$

Using the least-square procedure, the modal parameters at order p are determined by:

$$\mathbf{P}_{np\times n} = \left(\mathbf{K}_{N\times np}^{T} \mathbf{K}_{N\times np} \right)^{-1} \mathbf{K}_{N\times np}^{T} \mathbf{Y}[t]_{N\times n} \tag{3.33}$$

At the order $p + m, (m \le p)$, similarly, the data matrix at order $p + m$ can be defined by adding a data sub-matrix as:

$$\mathbf{K}_{N\times n(p+m)} = \left(\mathbf{K}_{N\times np} \quad \mathbf{K}_{1_{N\times nm}} \right) \tag{3.34}$$

where $\mathbf{K}_{1_{N\times nm}} = \begin{bmatrix} \mathbf{y}\left[t-(p+1)\right] & \mathbf{y}\left[t-(p+m)\right] \\ \mathbf{y}\left[t+1-(p+1)\right] & \mathbf{y}\left[t+1-(p+m)\right] \\ \cdots & \cdots \\ \mathbf{y}\left[t+N-1-(p+1)\right] & \mathbf{y}\left[t+N-1-(p+m)\right] \end{bmatrix}$

The model parameters of the model at order $p+m$ can be expressed through the following equation:

$$\mathbf{P}_{n(p+m)\times n} = \left(\mathbf{K}_{N\times n(p+m)}^{T}\mathbf{K}_{N\times n(p+m)}\right)^{-1}\mathbf{K}_{N\times n(p+m)}^{T}\mathbf{Y}\left[t\right]_{N\times n} \tag{3.35}$$

More specifically, the general expression for the parameters of the model at order $p+m$ can be written as

$$\mathbf{P}_{n(p+m)\times n} = \begin{pmatrix} \mathbf{K}_{N\times np}^{T}\mathbf{K}_{N\times np} & \mathbf{K}_{N\times np}^{T}\mathbf{K}_{1_{N\times nm}} \\ \mathbf{K}_{1_{N\times nm}}^{T}\mathbf{K}_{N\times np} & \mathbf{K}_{1_{N\times nm}}^{T}\mathbf{K}_{1_{N\times nm}} \end{pmatrix}^{-1}\begin{pmatrix}\mathbf{K}_{N\times np} & \mathbf{K}_{1_{N\times nm}}\end{pmatrix}^{T}\mathbf{Y}\left[t\right]_{N\times n}$$

$$= \begin{pmatrix} \mathbf{E} & \mathbf{F} \\ \mathbf{G} & \mathbf{H} \end{pmatrix}^{-1}\begin{pmatrix}\mathbf{K}_{N\times np} & \mathbf{K}_{1_{N\times nm}}\end{pmatrix}^{T}\mathbf{Y}\left[t\right]_{N\times n} \tag{3.36}$$

where

$\mathbf{E} = \mathbf{K}_{N\times np}^{T}\mathbf{K}_{N\times np}$, $\mathbf{G} = \mathbf{K}_{1_{N\times nm}}^{T}\mathbf{K}_{N\times np}$, $\mathbf{H} = \mathbf{K}_{1_{N\times nm}}^{T}\mathbf{K}_{1_{N\times nm}}$, $\mathbf{F} = \mathbf{K}_{N\times np}^{T}\mathbf{K}_{1_{N\times nm}}$, $\mathbf{S} = \left(\mathbf{H} - \mathbf{G}\mathbf{E}^{-1}\mathbf{F}\right)^{-1}$.

From the Schur complement in Eq. (3.19), the expression in Eq. (3.36) can be obtained analytically as:

$$\mathbf{P}_{n(p+m)\times n} = \begin{bmatrix} \mathbf{E}^{-1} + \mathbf{E}^{-1}\mathbf{F}\mathbf{S}\mathbf{G}\mathbf{E}^{-1} & -\mathbf{E}^{-1}\mathbf{F}\mathbf{S} \\ -\mathbf{S}\mathbf{G}\mathbf{E}^{-1} & \mathbf{S} \end{bmatrix}\begin{pmatrix}\mathbf{K}_{N\times np} & \mathbf{K}_{1_{N\times nm}}\end{pmatrix}^{T}\mathbf{Y}\left[t\right]_{N\times n}$$

$$=\begin{bmatrix} \left(\mathbf{E}^{-1}+\mathbf{E}^{-1}\mathbf{FSGE}^{-1}\right)\mathbf{K}_{N\times np}^{T} - \mathbf{E}^{-1}\mathbf{FSK}_{1_{N\times nm}}^{T} \\ -\mathbf{SGE}^{-1}\mathbf{K}_{N\times np}^{T} + \mathbf{SK}_{1_{N\times nm}}^{T} \end{bmatrix}\mathbf{Y}[t]_{N\times n}$$

$$=\begin{bmatrix} \mathbf{E}^{-1}\mathbf{K}_{N\times np}^{T}\mathbf{Y}[t]_{N\times n} \\ \mathbf{0}_{nm\times n} \end{bmatrix} + \begin{bmatrix} \mathbf{E}^{-1}\mathbf{FSGE}^{-1}\mathbf{K}_{N\times np}^{T} - \mathbf{E}^{-1}\mathbf{FSK}_{1_{N\times nm}}^{T} \\ -\mathbf{SGE}^{-1}\mathbf{K}_{N\times np}^{T} + \mathbf{SK}_{1_{N\times nm}}^{T} \end{bmatrix}\mathbf{Y}[t]_{N\times n}$$

$$=\begin{bmatrix} \mathbf{P}_{np\times n} \\ \mathbf{0}_{nm\times n} \end{bmatrix} + \begin{bmatrix} \mathbf{E}^{-1}\mathbf{FSGE}^{-1}\mathbf{K}_{N\times np}^{T} - \mathbf{E}^{-1}\mathbf{FSK}_{1_{N\times nm}}^{T} \\ -\mathbf{SGE}^{-1}\mathbf{K}_{N\times np}^{T} + \mathbf{SK}_{1_{N\times nm}}^{T} \end{bmatrix}\mathbf{Y}[t]_{N\times n} \tag{3.37}$$

Or, in a short form:

$$\mathbf{P}_{n(p+m)\times n} = \mathbf{P}_{np\times n} + \Delta\mathbf{P}\left(\mathbf{K}_{N\times np}, \mathbf{K}_{1_{N\times nm}}\right) \tag{3.38}$$

where

$$\Delta\mathbf{P}\left(\mathbf{K}_{N\times np}, \mathbf{K}_{1_{N\times nm}}\right) = \begin{bmatrix} \mathbf{E}^{-1}\mathbf{FSGE}^{-1}\mathbf{K}_{N\times np}^{T} - \mathbf{E}^{-1}\mathbf{FSK}_{1_{N\times nm}}^{T} \\ -\mathbf{SGE}^{-1}\mathbf{K}_{N\times np}^{T} + \mathbf{SK}_{1_{N\times nm}}^{T} \end{bmatrix}\mathbf{Y}[t]_{N\times n} \tag{3.39}$$

$\Delta\mathbf{P}\left(\mathbf{K}_{N\times np}, \mathbf{K}_{1_{N\times nm}}\right)$ is an expression that is a function of order p, data matrix $\mathbf{K}_{N\times np}$, and sub-matrix $\mathbf{K}_{1_{N\times nm}}$ of the model at order $p+m$. It is explicitly seen that the model parameters at higher model orders are identified through the estimated parameters of previous orders and data sub-matrices. This means that modal parameters can be defined with low computational complexity and computational time effectiveness. This technique is preferable to the repetitive Eq. (3.33) for each order value.

3.4.3 Reverse order updating for parameters of AR model

Let us consider that, at the sample index t, the data matrix $\mathbf{K}_{N\times np}$ at order p can be partitioned into the data matrix $\mathbf{K}_{N\times n(p-m)}$ by removing its last $n\times m$ columns $\mathbf{K}_{2_{N\times nm}}$ from the output data:

$$\mathbf{K}_{N\times np} = \begin{pmatrix} \mathbf{K}_{N\times n(p-m)} & \mathbf{K}_{2_{N\times nm}} \end{pmatrix} \tag{3.40}$$

Then, it can readily be seen that the model parameters are:

$$\begin{aligned}
\mathbf{P}_{np\times n} &= \left(\mathbf{K}^{T}_{N\times np}\mathbf{K}_{N\times np} \right)^{-1} \mathbf{K}^{T}_{N\times np}\mathbf{Y}[t]_{N\times n} \\
&= \begin{pmatrix} \mathbf{K}^{T}_{N\times n(p-m)}\mathbf{K}_{N\times n(p-m)} & \mathbf{K}^{T}_{N\times n(p-m)}\mathbf{K}_{2_{N\times nm}} \\ \mathbf{K}^{T}_{2_{N\times nm}}\mathbf{K}_{N\times n(p-m)} & \mathbf{K}^{T}_{2_{N\times nm}}\mathbf{K}_{2_{N\times nm}} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{K}_{N\times n(p-m)} & \mathbf{K}_{2_{N\times nm}} \end{pmatrix}^{T} \mathbf{Y}[t]_{N\times n} \\
&= \begin{pmatrix} \mathbf{E}_1 & \mathbf{F}_1 \\ \mathbf{G}_1 & \mathbf{H}_1 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{K}_{N\times n(p-m)} & \mathbf{K}_{2_{N\times nm}} \end{pmatrix}^{T} \mathbf{Y}[t]_{N\times n}
\end{aligned} \tag{3.41}$$

where

$$\mathbf{E}_1 = \mathbf{K}^{T}_{N\times n(p-m)}\mathbf{K}_{N\times n(p-m)},\ \mathbf{G}_1 = \mathbf{K}^{T}_{2_{N\times nm}}\mathbf{K}_{N\times n(p-m)},\ \mathbf{H}_1 = \mathbf{K}^{T}_{2_{N\times nm}}\mathbf{K}_{2_{N\times nm}}$$
$$\mathbf{F}_1 = \mathbf{K}^{T}_{N\times n(p-m)}\mathbf{K}_{2_{N\times nm}},\ \mathbf{S}_1 = \left(\mathbf{H}_1 - \mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{F}_1\right)^{-1}$$

In this case, the model parameters are identified by using the Schur complements to invert the matrix in Eq. (3.19), which leads to:

$$\mathbf{P}_{np\times n}=\begin{bmatrix}\mathbf{E}_1^{-1}+\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1} & -\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\\ -\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1} & \mathbf{S}_1\end{bmatrix}\begin{pmatrix}\mathbf{K}_{N\times n(p-m)} & \mathbf{K}_{2_{N\times nm}}\end{pmatrix}^T\mathbf{Y}[t]_{N\times n}$$

$$=\begin{bmatrix}\left(\mathbf{E}_1^{-1}+\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\right)\mathbf{K}_{N\times n(p-m)}^T-\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\\ -\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T+\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\end{bmatrix}\mathbf{Y}[t]_{N\times n}$$

$$=\begin{bmatrix}\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T\mathbf{Y}[t]_{N\times n}\\ \mathbf{0}_{nm\times n}\end{bmatrix}+\begin{bmatrix}\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T-\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\\ -\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T+\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\end{bmatrix}\mathbf{Y}[t]_{N\times n}$$

$$=\begin{bmatrix}\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T\mathbf{Y}[t]_{N\times n}\\ \mathbf{0}_{nm\times n}\end{bmatrix}+\begin{bmatrix}\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T-\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\\ -\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T+\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\end{bmatrix}\mathbf{Y}[t]_{N\times n}$$

$$=\begin{bmatrix}\mathbf{P}_{n(p-m)\times n}\\ \mathbf{0}_{nm\times n}\end{bmatrix}+\begin{bmatrix}\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T-\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\\ -\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T+\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\end{bmatrix}\mathbf{Y}[t]_{N\times n} \tag{3.42}$$

From Eq. (3.42), it can be established as follows:

$$\mathbf{P}_{n(p-m)\times n}=\mathbf{P}_{np\times n}-\Delta\mathbf{P}_1\left(\mathbf{K}_{N\times n(p-m)},\mathbf{K}_{2_{N\times nm}}\right) \tag{3.43}$$

In this notation, the element of $\Delta\mathbf{P}_1\left(\mathbf{K}_{N\times n(p-m)},\mathbf{K}_{2_{N\times nm}}\right)$ is given by:

$$\Delta\mathbf{P}_1\left(\mathbf{K}_{N\times n(p-m)},\mathbf{K}_{2_{N\times nm}}\right)=\begin{bmatrix}\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T-\mathbf{E}_1^{-1}\mathbf{F}_1\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\\ -\mathbf{S}_1\mathbf{G}_1\mathbf{E}_1^{-1}\mathbf{K}_{N\times n(p-m)}^T+\mathbf{S}_1\mathbf{K}_{2_{N\times nm}}^T\end{bmatrix}\mathbf{Y}[t]_{N\times n} \tag{3.44}$$

Similarly, Eq. (3.43) shows the explicit updating form for the model parameters at a lower model order from a given model at order p. The set of three equations Eqs. (3.31), (3.38) and (3.43) provide a complete algorithm for updating the autoregressive model with respect to both time and model order.

This updating is beneficial for the operational modal analysis and monitoring of any time-varying system, especially in non-stationary vibrations, as demonstrated in the next section. The efficiency of the developed algorithm will be verified in comparison with ARX method and QR method.

3.5 Application to simulation data

This section applies the proposed method to identify the modal parameters in both stationary and non-stationary cases.

The minimum description length (MDL) criterion is used to select an optimal model order for models. The eigenvalues of the state matrix at any order are decomposed and classified by the MSN index. This technique automatically separates physical features from spurious modes, and lets users select the modes of interest for their modal surveillance.

A two-degrees-of-freedom (2DOF) system, as shown in Figure 3.1, is used to assess the proposed method. The system consists of two related lumped masses constrained by systems of springs and dampers.

The differential governing equation is given as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ f(t) \end{Bmatrix} \quad (3.45)$$

where

$$m_1 = 15, m_2 = 20(kg), k_1 = k_3 = 10000, k_2 = 20000(N/m), c_1 = c_2 = 100, c_3 = 300(N.s/m)$$

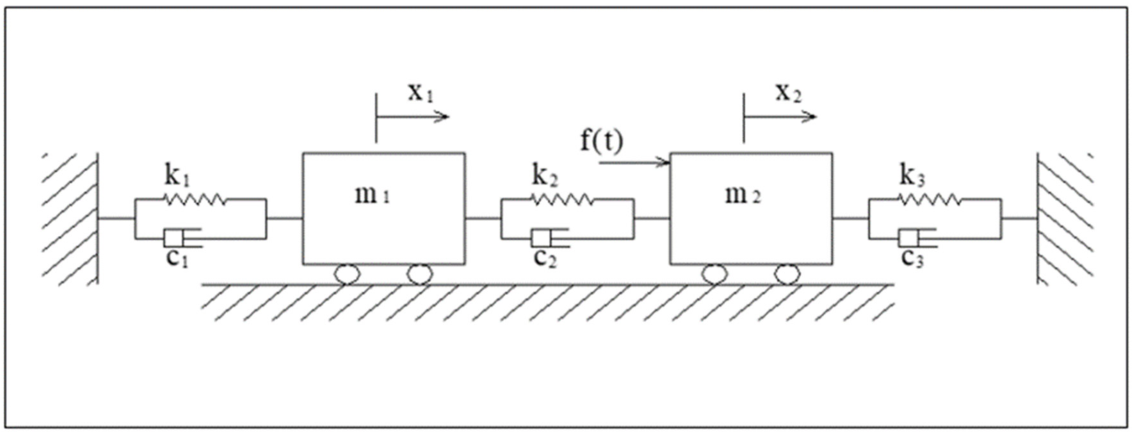

Figure 3.1 Diagram of the simulated system

Based on the analysis method with the given physical parameters, the natural frequencies and damping ratios are calculated as follows:

$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 3.81 \\ 8.58 \end{bmatrix} (Hz); \zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} = \begin{bmatrix} 24.66 \\ 19.96 \end{bmatrix} (\%) \tag{3.46}$$

For the numerical simulation, using the function ODE45 in MATLAB, with a time step of 0.01 (seconds), the system is simulated to give output-only response data from a given excitation force. Each stationary response realization time is 80 (seconds).

3.5.1 Stationary case under harmonic excitation

First, the machines are subjected to harmonic excitations and it can be challenging to separate the modal parameters from the excitation frequencies (Gagnon, Tahan, Coutu & Thomas, 2012).

In this case, a sinusoidal excitation at 20 (Hz) is acted on the system. Using the function ODE 45 in MATLAB, the temporal responses of the system are shown Figure 3.2 (a). The natural frequencies of the systems extracted by the fast Fourier transform, for the one-channel data, are plotted in Figure 3.2 (b).

The stabilization diagrams obtained from the estimated ARX model (MATLAB software) and the proposed method are shown in Figure 3.3 and Figure 3.4.

Table 3.1 presents a comparison of the computation times for the stabilization diagrams of the proposed method with the ARX model (MATLAB software) and MODALAR software (Vu, Thomas, Lakis & Marcouiller, 2010).

In Figure 3.4 Stabilization diagram of the system by the proposed method, the proposed method is applied on the simulated signals. Data signal processing can begin with a model at an arbitrary model order p. This model is updated to the next sample index and then, the order is updated to $(p-1)$, and $(p+1)$. In this case, the stabilization diagrams are used to select the modal parameters from the spurious modes by iteratively calculating the eigenvalues on a range of orders.

Observing the stabilization diagram in Figure 3.4, one can see that the modal parameters start to stabilize from a model order which can be called a necessary order. This order is small from 4 to 10. The necessary order of the damping rate is always higher than that of the frequency for each mode (Vu, Thomas, Lakis & Marcouiller, 2011).

As can be seen from Table 3.1, the estimates of the three methods are quite accurate for all modal parameters, but each has a different computation time. The proposed method performs best in terms of time consumption. It can be observed from these figures that the proposed method is better at eliminating spurious frequencies, which tend to appear with the ARX (MATLAB) method.

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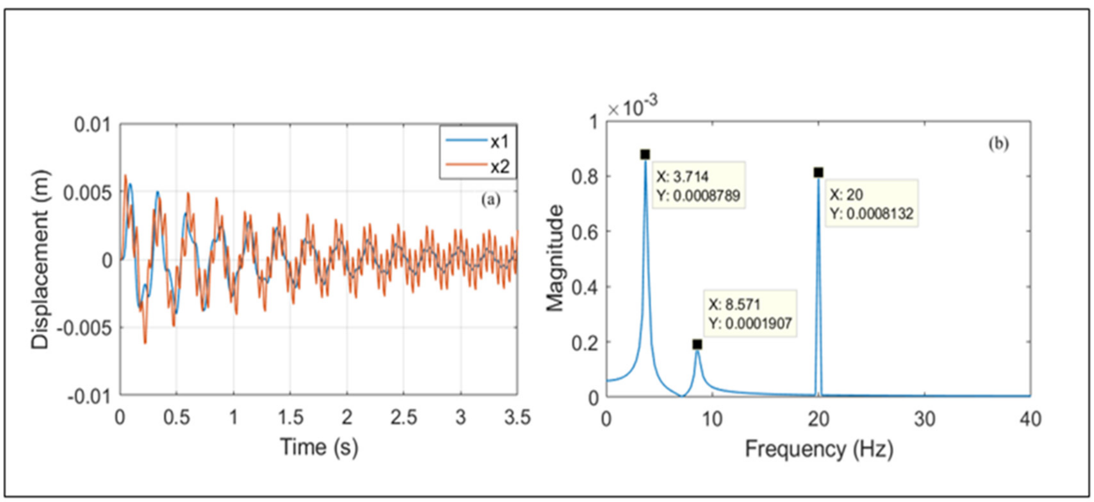


Figure 3.2 (a) The response of the 2DOF system under harmonic excitation,
(b) FFT of the 2DOF system under harmonic excitation

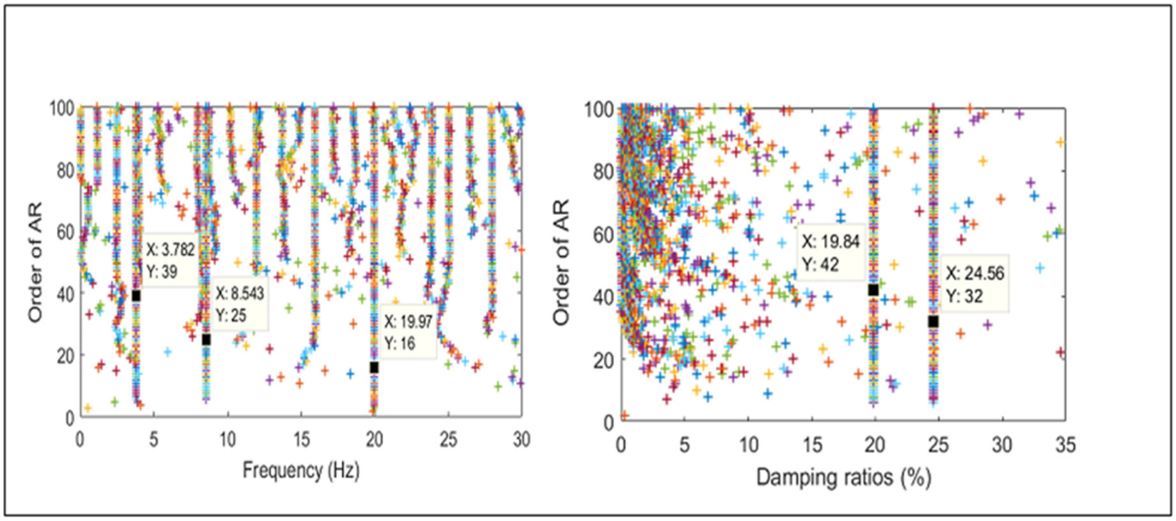


Figure 3.3 Stabilization diagram of the system by ARX model

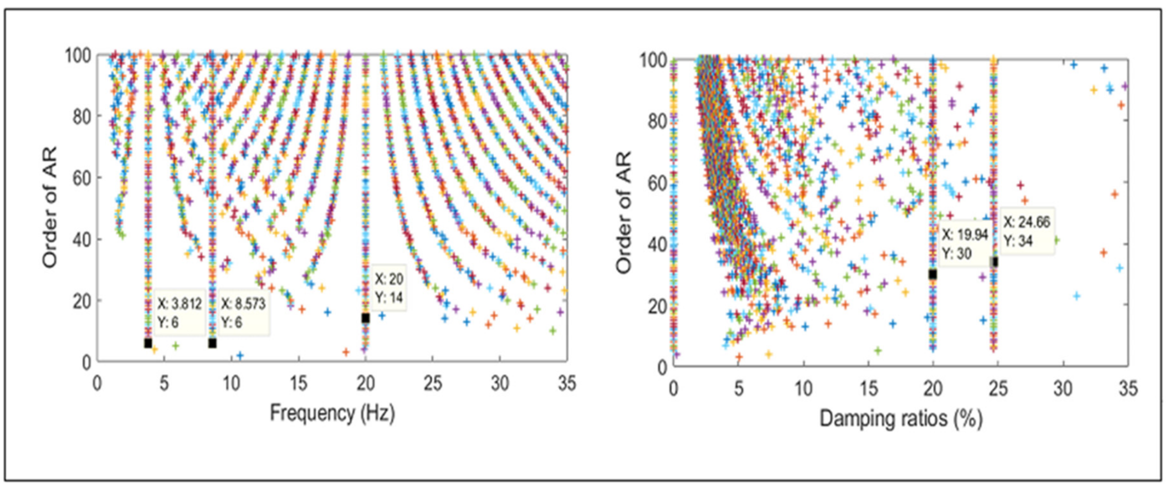


Figure 3.4 Stabilization diagram of the system by the proposed method

Table 3.1 Comparison of the computational times of three methods: ARX, QR-Vu (Vu et al., 2010), and proposed method for the stabilization diagram (two-channels data)

	ARX (MATLAB)				QR-Vu				Proposed method			
	f_1	f_2	ζ_1	ζ_2	f_1	f_2	ζ_1	ζ_2	f_1	f_2	ζ_1	ζ_2
Frequencies	3.78	8.54	24.56	19.84	3.81	8.64	24.66	19.95	3.81	8.57	24.66	19.94
Time (s)	54.8				51.5				48.7			

3.5.2 Non-stationary case

In this case, the masses of the system are assumed to vary as shown in Figure 3.5. The excitation $f(t)$ is a stationary, zero-mean, and uncorrelated signal. The corresponding frozen modal parameters of the system are depicted in Figure 3.7. The first natural frequency varies within the 3.57-3.81 (Hz) range, while the second varies within the 8.00-8.57 (Hz) range.

Given a particular excitation, the system equations are integrated via the ODE45 method in MATLAB by the fixed integration step and are recorded at a frequency $f_s = 100(\mathrm{Hz})$. Each non-stationary response realization time is 80 (seconds). The non-stationary vibration displacement signal is plotted in Figure 3.7 (a). Figure 3.7 (b) presents the short-time Fourier transform of the signal under random excitation.

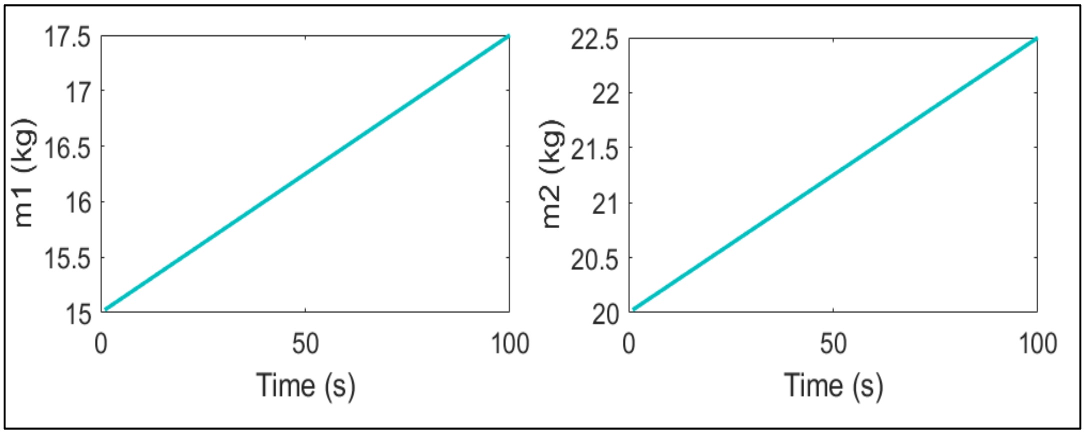


Figure 3.5 Masses changing function

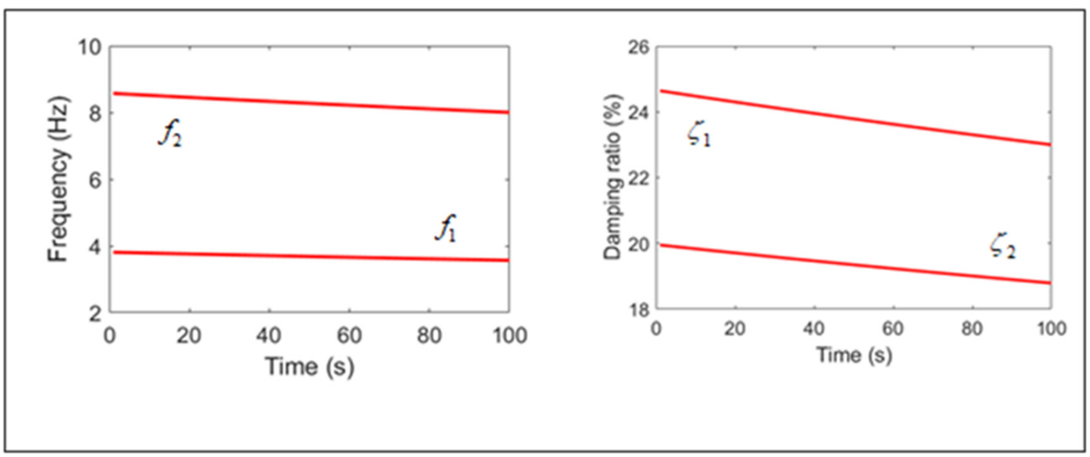


Figure 3.6 Modal parameters of the system

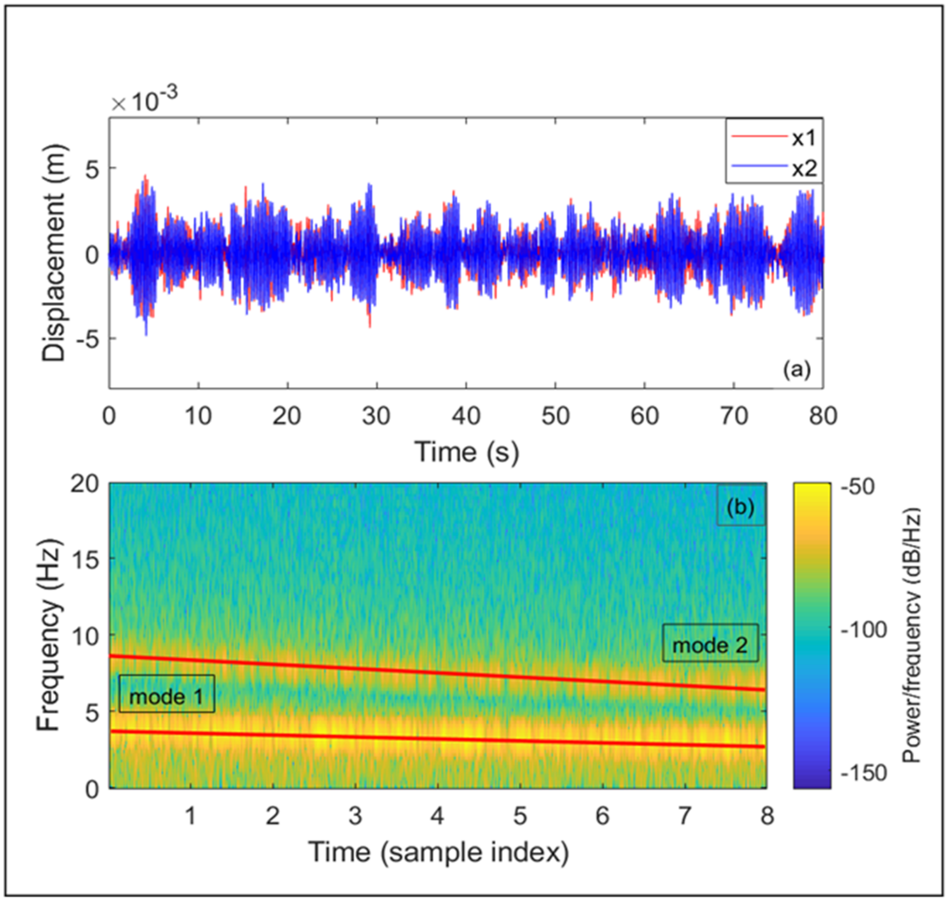


Figure 3.7 (a) Realization of the non-stationary vibration displacement signal, (b) Short-time Fourier transform of the signal

Discussion on data block length

In the operational modal analysis, the dynamic parameters of the system are concealed. As a result, a priori understanding of the model order is unknown. In the proposed method, the data is processed in a block-wise Gabor expansion. In this model, the number of samples in each block N must satisfy $N \rangle np + n$ where p is the computing model order. This number of samples can be changeable in non-stationary vibration.

The block is long enough to exhibit vibratory features and cover the largest vibration period in the signal. Consequently, the block length is adjusted for each sliding window. Figure 3.8 identifies the natural frequencies of the systems at different block sizes.

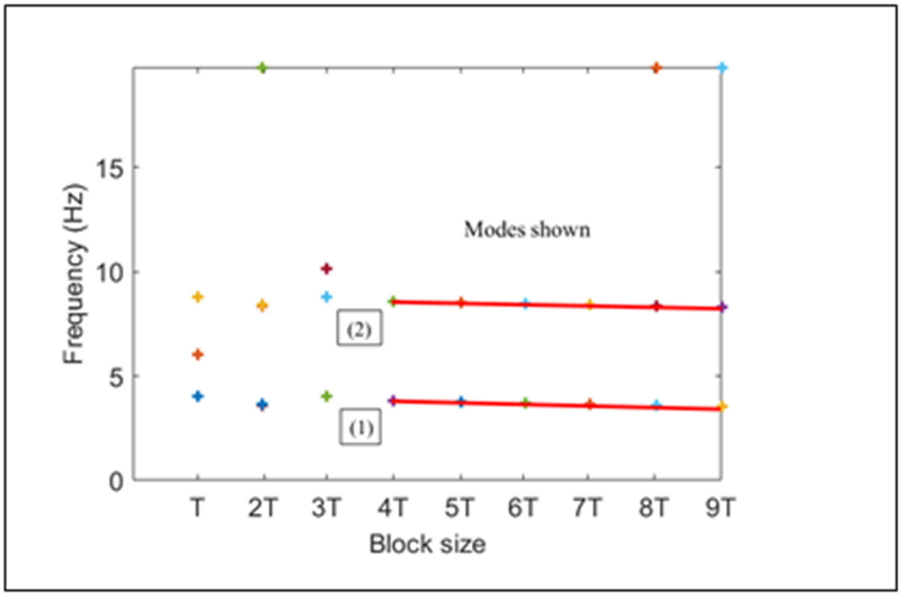


Figure 3.8 Natural frequency identification with the block size

In Figure 3.8, the block size must be chosen to be at least 4 times the period of the lowest frequency. The initial block size is selected to be 4 times for the fundamental period which can be available from a numerical analysis or a measuring bandwidth.

The MDL criterion is used to estimate an optimal model order, which may be updated by increasing or decreasing the order. The modal parameters are extracted at the first sliding window. The signal-to-noise ratio of each eigenvalue is applied to separate spurious and real modes, and to then find the smallest frequency for the next window computation. An overlapping process can also be employed by changing the sliding step, which can vary from only one sample to the whole length of the rectangular block window.

Harmonic excitation

Harmonic excitation is applied to the mechanical system. The identification of modal parameters from the excitation frequencies is a challenging process. A sinusoidal excitation at a frequency 20 (Hz) is added to the system. At this point, the algorithm is used to determine the parameters of the model by increasing or decreasing the order to find the most efficient order for consecutive sliding windows.

Figure 3.9 shows the results of the frequencies and damping ratios with their variations. It can be seen that natural frequencies are accurately identified and tracked. The damping ratios, having a greater uncertainly and being frequency-dependent, are challenging to identify and monitor.

Random excitation

In the case of random excitation, it is apparent that the monitoring of modal parameters is a function of the randomness of the force. In this case, the modal parameters are monitored with standard derivations (s.t.d.) of the random excitation at 15 (N).

The monitoring of modal parameters is plotted in Figure 3.10. It can be observed that under random excitation, frequency identification and monitoring are still very accurate. Damping identification and monitoring are better than in the case of harmonic excitation. This finding is consistent with the current practice, where a random excitation is preferred over a harmonic excitation in modal analysis and vibration testing.

As shown in Figure 3.9 and Figure 3.10, the confidence levels of estimated frequencies are very high while the estimation of damping ratios have larger dispersions.

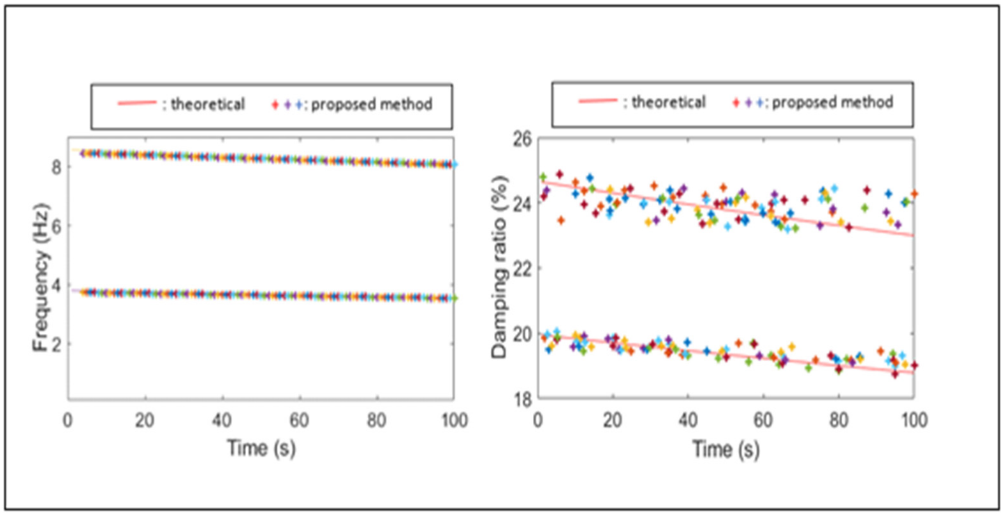


Figure 3.9 Monitoring of modal parameters under harmonic excitation

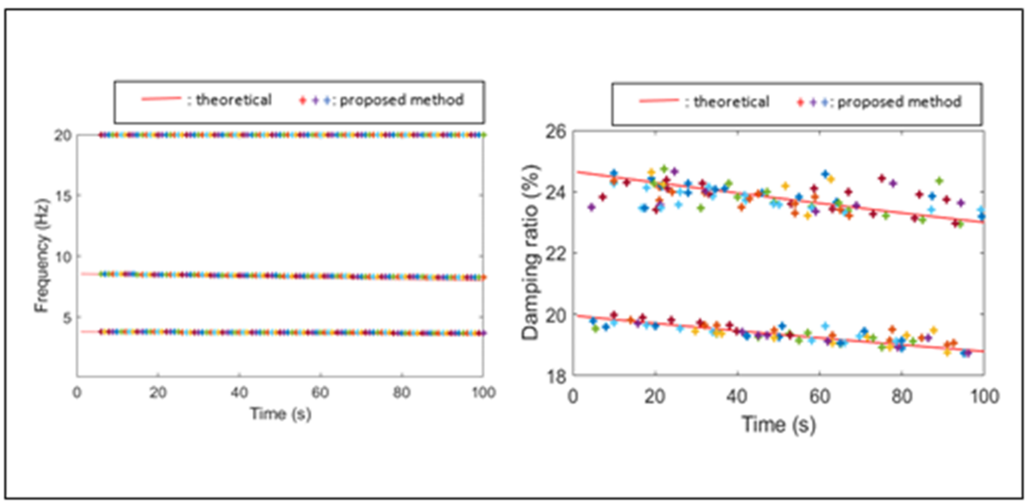


Figure 3.10 Monitoring of modal parameters under random excitation

3.6 Real vibration application

In this section, experiments are performed to access the effectiveness of the proposed method.

3.6.1 Experiment setup

To evaluate the fluid-structure interaction, a hydraulic test bench was designed and assembled at ÉTS in the Dynamo team's laboratory. The dimensions of this test container were 2.769 x 2.515 x 2.210 (m) and its empty weight is 490 kg.

The test bench must meet the following specifications:

- Possibility of performing vibration measurements on a structure in air and water.
- Tests on a submerged structure can be carried out in still water or turbulent flow.
- Several types of structures can be tested, including a thin plate and a turbine blade.
- Submerged depth can be varied.
- Flow rates can be varied according to the desired flow velocities.
- Quiet and safe operation.

The test structure is placed inside a perforated tank, and the flow speed can be regulated by one of the valves and the different outlet nozzles. Therefore, the bench can test structures with a maximum dimension of 0.6 (m). The fastening system was a multi-tasking tool, which allowed mounting a plate in different boundary conditions.

In order to obtain a wide range of flow velocities, several nozzles were manufactured with different diameters. The structure is tested with a maximum speed of 30.95 (m/s).

The PCB330A sensors are used for the vibration measurements. Six sensors were used in all our tests to record the accelerations. Miniature pressure sensors were employed to record dynamic pressures applied on submerged structures due to flow.

The acquisition box called the Vishay System 6000, was used to record pressure sensors during different tests. This model is capable of recording at a maximum speed of 10,000 samples per second per channel. It has 16 carts used for measuring the thermocouples, piezoelectric accelerometers, and stress or pressure.

The software “Strain Smart” is designed to work with the Vishay acquisition box and allows data to be exported and saved under different formats, including .xls and .txt. A PCB impact hammer is acted on the structure in static tests. In order to hit the tested plate at different depths, the hammer is equipped with a steel extension.

The configuration of the experiment is depicted in Figure 3.11. The tested steel plate measures 500 x 200 x 2 (mm). Figure 3.12 presents temporal response data at a sampling frequency 1280 (Hz). The low amplitude portion corresponds to the submerging period, while the high amplitude portion is attributable to the emergence of the plate from the water and into the air.

It is seen that because of the effect of the fluid, the natural frequencies of the plate change with respect to the submerged depth. Before the plate rises, its modal parameters are computed and identified using analytical and experimental methods at different depths, which are illustrated by the depth on plate length ratio (D/L), as shown in Table 3.2.

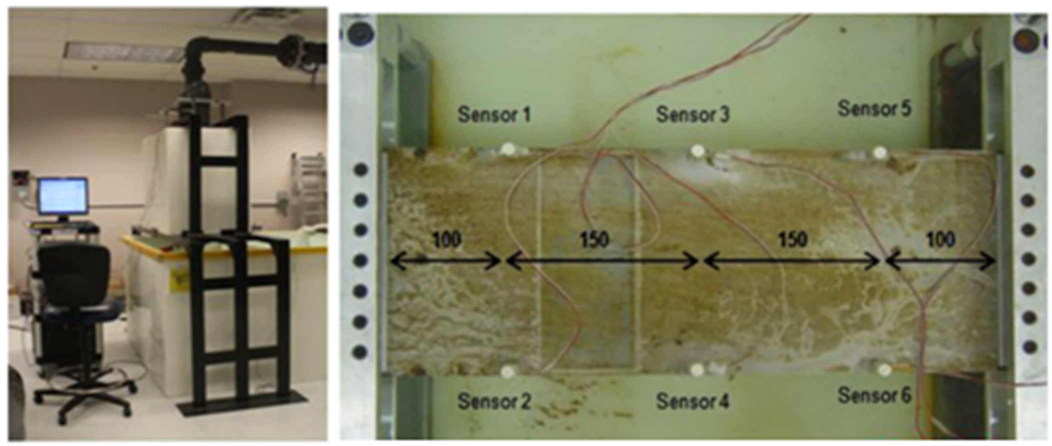


Figure 3.11 Photo images showing the configuration of the experiment

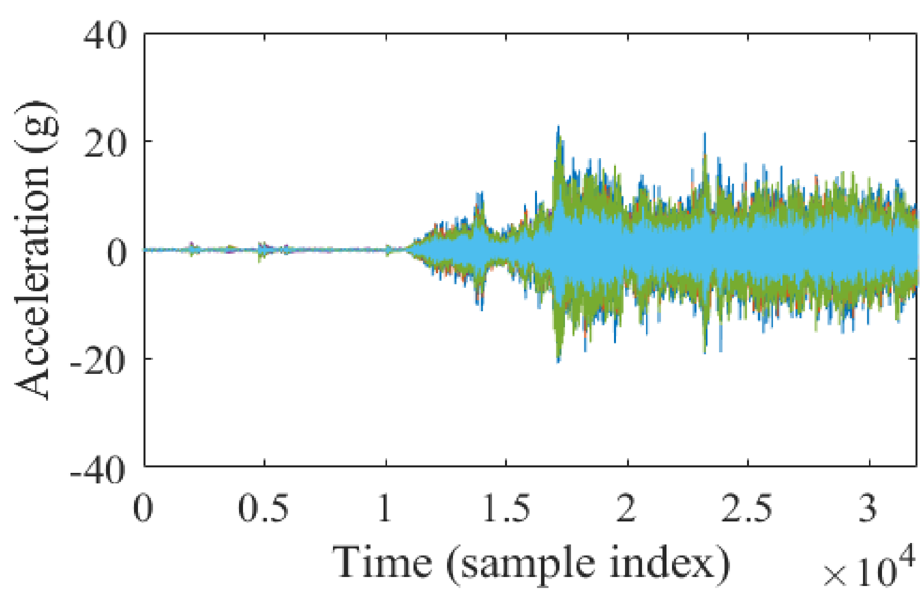


Figure 3.12 The temporal response of the plate

3.6.2 Identifications for the submerged plate

The proposed method is applied to monitor for vibrations in the submerged steel plate. In each data segment, the model parameters are identified by the updating algorithm over time.

MDL and MSN are used to identify the optimal model order and separate the spurious and real modes. The length of the block window is a function of the minimum frequency in the previous data block. The size length of the sliding window varies and is chosen to be at least 4 times that of the longest natural period of the previous block.

Figure 3.13 shows the monitoring frequencies of the plate using the proposed method. The frequency variations match the emergence of the plate, showing the slow change from the time system is still in the water to the abrupt change when it appears at the surface. Even though the detected frequencies have large dispersions, the general trends for the 5 modes appear clearly in the diagram. Note that the dispersion can also be observed in the short-time Fourier transform shown in Figure 3.14, owning to the nature of the system.

It is seen that the proposed method is a powerful technique for the monitoring of natural frequencies. In terms of identification, these frequencies are accurately identified thanks to the autoregressive parametrical method. Moreover, with the proposed updating algorithm, the frequencies are qualitatively and quantitatively tracked and monitored in the varying dynamic systems.

Compared to the calculated values in Table 3.2 and the short-time Fourier transform of the data plotted by the one-channel data with the same data configuration in Figure 3.14. During the rise-up, the plate is in moving condition and the boundary condition is affected by the movement. The clamp and the fixing systems are also in vibration and hence the whole system is a little more flexible than in the modal testing where the condition is perfect. Therefore, the natural frequencies monitored are a little smaller than in modal testing, mostly for higher frequencies and in free air where the boundary condition becomes less damped than in the water.

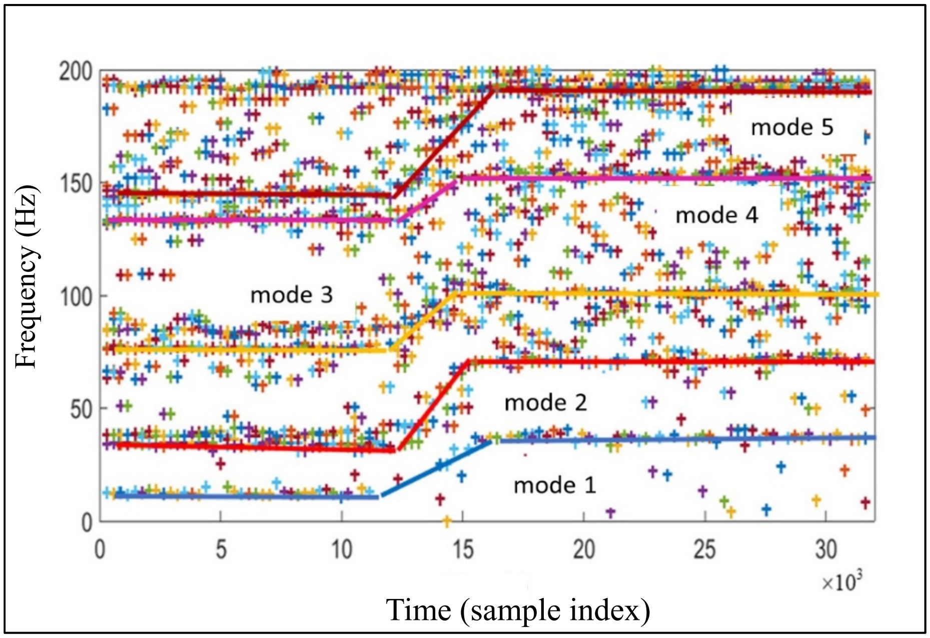


Figure 3.13 Monitoring of plate's natural frequencies by the proposed method

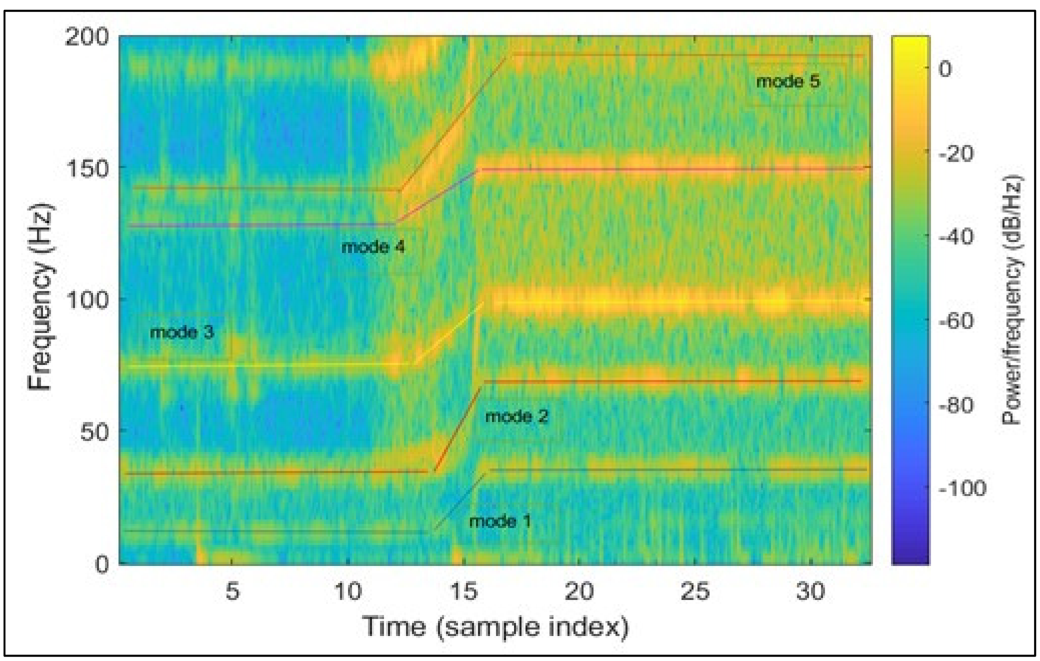


Figure 3.14 Short-time Fourier transform of data

Table 3.2 Modal identification of emerging plate

Mode	Frequencies (Hz) of submerged condition (Depth/plate length)				
	0.6 (totally submerged)	0.4	0.2	0.1	0 (totally in air)
1st	11.9	12.0	12.2	12.7	39.4
2nd	34.1	34.1	34.2	35.0	75.0
3rd	77.7	77.9	78.1	79.5	108.6
4th	135.3	135.4	135.6	137.5	164.0
5th	151.3	151.3	151.1	152.6	210.0

3.7 Summary

This chapter presented the updated method to avoid complex and time-consuming computing for slow-varying non-stationary vibration structures. The main contributions of this study can be listed as follows:

- It presents an updated algorithm of modal identification for the slow-varying non-stationary vibration systems.

- It introduces the sliding window technique, which is used with the multivariate autoregressive model to identify and track changes in modal parameters of modal analysis identification.

- It presents the simulation and experimental studies to verify the proposed method. A plate emerging from the water is presented. Results show that the proposed method dramatically reduces computational time and is a powerful technique for analyzing and monitoring the modal parameters in slow-varying non-stationary vibration systems. Natural frequencies can be accurately extracted, and their variations can be observed correctly in random and harmonic excitation. The identification and monitoring of damping ratios are qualitatively convincing.

CHAPTER 4

UPDATING SINGULAR VALUE DECOMPOSITION FOR MODAL ANALYSIS IN SLOW-VARYING NON-STATIONARY VIBRATION STRUCTURES

This chapter presents a novel method for the modal analysis of slow-vary vibration structures based on vector autoregressive models. The basic idea of this method consists of using a short-time sliding window (STSW) to identify modal parameters for non-stationary vibrations. This method uses the recursive least-squares estimation for multivariable systems with the singular value decomposition (SVD) method to find the solutions within a segment of the data from each time window. Model identification is conducted by updating the SVD of the data matrix using the order and time from the previous computational window to monitor the modal parameters of a slow-varying system. Finally, this work was verified first by numerically simulating a system's gradual changes submitted to an exciting force and further by an experiment on a hydraulic turbine blade. This chapter is based on an article titled: *"Updating singular value decomposition for modal analysis in slow-varying non-stationary vibration structures"* submitted to Mechanical Systems and Signal Processing (MSSP) in September 2022.

4.1 Introduction

As discussed in the previous chapter, in real-life structural systems, the dynamic properties of these systems change under working conditions and are known as non-stationary systems (Chen et al., 2020). Typical structural systems include traffic-excited bridges, earthquake-excited structures, surfaces of any kind, sea vehicles, robotic devices, and rotating machinery (Au et al., 2004; Verboven et al., 2004).

The extraction of modal parameters for non-stationary systems is more complex than for stationary systems whose dynamic properties remain constant over time. Parameter identification methods are generally divided into parametric and non-parametric methods (Petsounis & Fassois, 2001).

Non-parametric methods are based on non-parameterized representations, which may be the impulse response function, the autocovariance function, and describing the signal's power spectral density. Many studies have used the Hilbert-Huang transform (Shi, Law & Xu, 2009), some focusing on the Cohen class of distribution (Lee, Robb & Besant, 2001; Meltzer & Ivanov, 2003; Roshan-Ghias, Shamsollahi, Mobed & Behzad, 2007), and others on the wavelet-based representation (Wang, Ren, Wang & Zhu, 2013).

Based on parameterized representations of the time-dependent series models, parametric methods are advantageous in terms of their improved accuracy, resolution, and tracking of time-varying dynamics. The parametric methods are the functional time-dependent auto-regressive (TAR) series (Spiridonakos & Fassois, 2013), time-dependent auto-regressive moving average (TARMA) series (Petsounis & Fassois, 2000), and specific functional subspaces, and they have drawn much attention because of their broad application to many fields.

- (Ma et al., 2018) presented the parametric output-only identification of time-varying structures using a kernel recursive extended least-squares TARMA approach. More specifically, the study used the TARMA model in kernel Hilbert space to track the time-varying dynamics.

- (Yang et al., 2015) proposed a moving kriging shape function modeling of vector TARMA models for modal identification and then validated the identification algorithm with a moving cantilever beam experiment.

- From another point of view, (Li, Zhou, Liu, Kang & Ma, 2019) presented a Bayesian estimation of operational modal parameters for linear time-varying mechanical systems based on the functional series vector TAR model. This built the analytical expression conjugate prior to the unknown parameters, the spanning AR coefficients, and showed the excellent performance of TAR models based on the Bayesian estimation for the time-varying vibration.

- Another method, employed by (Spiridonakos & Fassois, 2013), was to apply the stochastic functional series time-dependent auto-regressive (FS-TAR) method in each state for effective fault diagnosis in inherently non-stationary structures, after which, the AR coefficients of the projection parameter vector are extracted and utilized as the characteristic quantity representing the structural state in each case.

The time-varying auto-regressive model (TVAR) is similar to the conventional auto-regressive (AR) model. However, TVAR has more time-varying coefficients that could lead to certain disadvantages, such as computational complexity and matrix singularity for the identification. In addition, the time-varying system's ambient excitation is usually difficult to measure under operating conditions.

A powerful technique to mitigate these disadvantages is the slow varying non-stationary method based on the conventional stationary frequency domain or time domain system identification and signal segmentation technics. Many endeavors have been made in this direction. (Ma & Ding, 2019) assumed that the system parameters vary linearly with time in each window. A linear function describes the temporal variation of the parameter in the shifting window. Hence, the time-varying parameters are identified in the different time windows. In another identification approach, (Vu, Thomas, Lakis & Marcouilleret, 2010) use the short-time auto-regressive (STAR) modeling for OMA of a non-stationary mechanical system. Based on the stationary state of each data segment, the modal parameter variations are monitored by autoregressive models for the emerging steel plate.

Repeating the identification process of time-invariant systems in each data segment causes computational complexity and matrix singularity for the slow non-stationary vibration systems. Although, many problems of slow varying non-stationary vibration systems have been reached. They are known to suffer potential problems regarding computational complexity and matrix singularity. Hence, developing the methods for slow-varying non-stationary vibration systems to overcome these disadvantages is an open problem.

4.1.1 Singular value decomposition

The singular value decomposition is a widely used technique to decompose a matrix into several component matrices (Gandhi & Rajgor, 2017; G. W. Stewart, 1990). It has been used in system identification (Brincker, Zhang & Andersen, 2001; Shen & Wai, 2021; Sun, Li, Luo & Li, 2021) to monitor the modal parameters for both cases: stationary and non-stationary vibrations. (Jiang, Wang & Zhong, 2021) have presented a method of damage detection using singular value decomposition (SVD) for beam structures. SVD is applied to decompose the trajectory matrix of the attractor reconstructed from shape data to localize the damage to detect the defects for beam-like systems, simplifying the measurement method and reducing testing work. Another technique, found in (Lobos, Kozina & Koglin, 2001), is to use singular value decomposition (SVD) to estimate the harmonics in signals in the presence of high noise. The method was developed to locate the frequencies in closely spaced sinusoidal signals. The study also presented the superiority of SVD with the standard FFT technique for signals buried in the noise. It concluded that the SVD method is especially suitable for offline analysis of recorded waveforms.

The singular value decomposition of a matrix is a valuable and important method used in the least-squares fitting of data. Many applications, for instance, signal processing, mechanical engineering, or statistics, employ SVD. In many cases, the computing procedure of SVD is repeated. This repetition could lead to high computational costs. Some authors have shared that updating the SVD is a further development that overcomes this drawback. Such an update algorithm, is described in (Bunch & Nielsen, 1978). It is reliable and efficient for a matrix with SVD and is applied when adding or deleting a row or column. (Brand, 2006) recently developed a fast algorithm to update a few dominant singular values of an augmented matrix used to perform background elimination in multiple analysis systems. A thin SVD is calculated through a matrix's column updates and downdates.

4.1.2 Proposed approach

In this chapter, we propose a new algorithm for the online monitoring of slow-varying modal parameters in vibrating structures subjected to unknown excitation. The proposed method applies a vector auto-regressive model (VAR) in a short time sliding window (STSW) on measured signals. The model parameters are determined and updated through the order and time from the previous computational window. The recursive least-squares estimation for multivariable systems is used to find the solutions by the singular value decomposition (SVD). This work aims to avoid the computational complexity of identifying and monitoring modal parameter variations for slow-varying non-stationary vibrations.

In our previous work (Bui, Vu & Liu, 2022), the authors use the Schur complement to update the parameters of the VAR model and monitor the varying modal parameters for a submerged plate. In terms of computational time, the Schur method is faster because it solves the standard equations of the least-squares. However, the main obstacle to vibration signal analysis is that the collected non-stationary signals are usually mixed with heavy noise caused by variable operating or environmental conditions. As a result, the rank deficient in least-squares estimation must be coped with to overcome these problems. The singular value decomposition method is a highly reliable, computationally stable mathematical tool that could obtain more accurate results and help to resolve these problems.

Therefore, this study extends previous works as follows:

- By applying the updating SVD to find the solution for the least-squares estimation, the proposed method is used to extract modal parameters for slow-varying non-stationary structures under working conditions or uncertain environments.

- It provides experimental results on the hydraulic turbine blade.

This chapter is organized as follows. Section 4. 2 briefly introduces the vector autoregressive models and singular value decomposition. The updated approximation for the singular value decomposition of the matrix will be discussed in Section 4. 3. Section 4. 4 presents the proposed method for updating the modal parameters of the VAR model. The identification of the mechanical systems will be presented in Section 4.5. Section 4.6 presents the results on experimental data. The conclusion is summarized in the final Section.

4.2 Vector autoregressive models and singular value decomposition

This section present the autoregressive models for modal analysis identification, and the singular value decomposition is applied to the least-squares methods to obtain the model parameters of autoregressive models.

Considering the general time-invariant recursive process for signal $\mathbf{y}[t] \in \mathbf{R}^{1\times n}$, referred to as a multivariate autoregressive model at p, dimension n and sampling period T_s, that is given by the following equation (Vu, Thomas, Lakis & Marcouiller, 2011):

$$\mathbf{y}[t] + \sum_{i=1}^{p} \mathbf{y}[t]\mathbf{A}_i = \mathbf{e}[t] \tag{4.1}$$

where t designates the normalized discrete time, $\mathbf{e}[t] \in \mathbf{R}^{1\times n}$ is a residual vector with zero means, and $\mathbf{A}_i \in \mathbf{R}^{n\times n}$ the AR parameter matrix. Eq. (4.1) is rewritten into the following linear regression form:

$$\mathbf{y}[t] = \mathbf{z}[t]\mathbf{P}_{np\times n} + \mathbf{e}[t] \tag{4.2}$$

where $\mathbf{z}[t] = (\mathbf{y}[t-1], \mathbf{y}[t-2], \ldots, \mathbf{y}[t-p]) \in \mathbf{R}^{1\times np}$ is the output data $\mathbf{y}[t]$, and $\mathbf{P}_{np\times n} = [-\mathbf{A}_1 - \mathbf{A}_2 \ldots - \mathbf{A}_p]$ is the model parameter matrix.

A least-squares estimation can be applied if the data are assumed to be measured in a white noise environment. Considering N successive vectors of the output responses from $\mathbf{y}[t]$ to $\mathbf{y}[t+N-1]$, the modal parameters matrix $\mathbf{P}_{np\times n}$ can be found in the least-squares method by minimizing the summed squared error between the left and right-hand sides of the equation. The objective function to be minimized may be expressed in the norm-2 vector notation form (Lobos et al., 2001) as:

$$\mathbf{E} = \frac{1}{2}\left\|\mathbf{K}_{N\times np}\mathbf{P}_{np\times n} - \mathbf{Y}[t]_{N\times n}\right\|_2^2 \tag{4.3}$$

where

$$\mathbf{Y}[t]_{N\times n} = \begin{bmatrix} \mathbf{y}[t] \\ \mathbf{y}[t+1] \\ \cdots \\ \mathbf{y}[t+N-1] \end{bmatrix}, \ \mathbf{K}_{N\times np} = \begin{bmatrix} \mathbf{z}[t] \\ \mathbf{z}[t+1] \\ \cdots \\ \mathbf{z}[t+N-1] \end{bmatrix} \tag{4.4}$$

The singular value decomposition of the $\mathbf{K}_{N\times np}$ matrix is used to compute the solution of the least-squares method. There are orthogonal matrices $\mathbf{U}_{N\times np}, \mathbf{V}_{np\times np}$ and a diagonal matrix $\mathbf{D}_{np' \ np}$ such that $\mathbf{K}_{N\times np} = \mathbf{UDV}^T$. Here, $\mathbf{U}_{N\times np}$ and $\mathbf{V}_{np\times np}$ are the left singular vectors and the suitable right singular vectors of $\mathbf{K}_{N\times np}$, respectively, and the diagonal entries $\mathbf{D} = diag(d_1, d_2, \ldots, d_{np})$ are the singular values of $\mathbf{K}_{N\times np}$. The model parameters of the AR model are estimated as follows (Lobos et al., 2001):

$$\mathbf{P}_{np\times n} = \mathbf{V}\mathbf{D}^{-1}\mathbf{U}^{T}\mathbf{Y}\left[t\right]_{N\times n} \tag{4.5}$$

4.3 Updating the singular value decomposition of a matrix

In many least-squares and signal processing applications, one updates a matrix $\mathbf{K}_{N\times p}$ by appending or deleting a row or a column. After each update or downdate, the computing process of the SVD must be repeated for the resulting matrix. This section presents the updating formulation SVD for the matrix when appending and deleting a row or a column (Bunch & Nielsen, 1978).

Consider the singular value decomposition of a given matrix $\mathbf{K}_{N\times p}$ as:

$$\mathbf{K}_{N\times p} = \mathbf{U}\mathbf{D}\mathbf{V}^{T} \tag{4.6}$$

where $\mathbf{U}\in \mathbf{R}^{N\times p}$ and $\mathbf{V}\in \mathbf{R}^{p\times p}$ are orthogonal, and $\mathbf{D}\in \mathbf{R}^{p\times p}$ is zero except on the main diagonal $\mathbf{D} = diag(d_1, d_2, \ldots, d_p)$.

4.3.1 Updating the SVD of a matrix when appending a row

Define a new matrix $\tilde{\mathbf{K}}_{(N+1)\times p}$ that is based on the given matrix $\mathbf{K}_{N\times p}$ when appending a row $\mathbf{a}^{T}$ as follows:

$$\tilde{\mathbf{K}}_{(N+1)\times p} = \begin{pmatrix} \mathbf{K}_{N\times p} \\ \mathbf{a}^{T} \end{pmatrix} = \tilde{\mathbf{U}}\tilde{\mathbf{D}}\tilde{\mathbf{V}}^{T} \tag{4.7}$$

where $\tilde{\mathbf{U}}\in \mathbf{R}^{(N+1)\times p}$, $\tilde{\mathbf{V}}\in \mathbf{R}^{p\times p}$, $\tilde{\mathbf{D}} = diag(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_p)$.

One can compute $\tilde{\mathbf{U}}, \tilde{\mathbf{D}}, \tilde{\mathbf{V}}$ matrices of $\tilde{\mathbf{K}}_{(N+1)\times p}$ by using the provided information of the matrices $\mathbf{U}, \mathbf{D}, \mathbf{V}$ of $\mathbf{K}_{N\times p}$.

with $\mathbf{z} = \mathbf{V}^T\mathbf{a} = \left[z_1 \; z_2 \; \dots \; z_p \right]^T$, a factorized representation, is defined by:

$$\tilde{\mathbf{K}}_{(N+1)\times p}\tilde{\mathbf{K}}_{(N+1)\times p} = \mathbf{K}_{N\times p}^T\mathbf{K}_{N\times p} + \mathbf{a}\mathbf{a}^T = \mathbf{U}\left(\mathbf{D}^2 + \mathbf{z}\mathbf{z}^T\right)\mathbf{V}^T \tag{4.8}$$

From Eq. (4.8), the singular values of $\tilde{\mathbf{K}}_{(N+1)\times p}$ are computed by the eigen decomposition of the matrix $\mathbf{D}^2 + \mathbf{z}\mathbf{z}^T$. The SVD of $\mathbf{D}^2 + \mathbf{z}\mathbf{z}^T$ is expressed by: $\mathbf{D}^2 + \mathbf{z}\mathbf{z}^T = \mathbf{Q}\mathbf{\Omega}^2\mathbf{Q}^T$. Here, $\mathbf{Q} \in \mathbf{R}^{p\times p}$ is the orthogonal and $\tilde{\mathbf{D}} = \mathbf{\Omega} \in \mathbf{R}^{p\times p}$.

The singular values of the new matrix are updated through the singular values of the matrix $\mathbf{K}_{N\times p}$. The singular values of the new matrix can be updated as follows (Bunch & Nielsen, 1978):

$$\tilde{d}_i = d_i + \mu_i, 1 \le i \le p \tag{4.9}$$

where μ_i satisfy the secular equation:

$$1 + \sum_{i=1}^{p} \frac{z_i^2}{\left(d_i + d_j + \mu\right)\left(d_i - d_j - \mu\right)} = 0, 1 \le i \le p \tag{4.10}$$

Instead of computing the singular values directly of the matrix $\tilde{\mathbf{K}}_{(N+1)\times p}$, one can update them through the secular equation Eq. (4.10). Once the singular values of the new matrix have been updated, the right singular vector $\tilde{\mathbf{V}} = \left[\tilde{\mathbf{v}}_1 \; \tilde{\mathbf{v}}_2 \dots \tilde{\mathbf{v}}_p \right]$ is obtained by:

$$\tilde{\mathbf{v}}_i = \frac{\mathbf{V}\mathbf{T}_i^{-1}\mathbf{z}_1}{\left\|\mathbf{T}_i^{-1}\mathbf{z}_1\right\|_2}, \mathbf{T}_i = \mathbf{D}^2 - \tilde{d}_i\mathbf{I}, \mathbf{z}_1 = \frac{\mathbf{z}}{\left\|\mathbf{a}\right\|_2} \tag{4.11}$$

The updated left singular vectors $\tilde{\mathbf{U}} = \left[\tilde{\mathbf{u}}_1\ \tilde{\mathbf{u}}_2 \ldots \tilde{\mathbf{u}}_p\right]$ are related to the updated right singular vectors as:

$$\tilde{\mathbf{u}}_i = \frac{1}{\tilde{d}_i}\begin{pmatrix}\mathbf{K}_{N\times p} \\ \mathbf{a}^T\end{pmatrix}\tilde{\mathbf{v}}_i,\ 1 \le i \le p \tag{4.12}$$

Obviously, the need to repeat SVD of the matrix as Eq. (4.7) is avoided by using the updated formulas from the four equations Eqs. (4.9)-(4.12), which further reduces the computational complexity of computing SVD. Thus, solving the least-squares problem by SVD is beneficial for the identification process.

4.3.2 Updating the SVD of a matrix when deleting a row

A new matrix based on the given matrix $\mathbf{K}_{N\times p}$ when deleting a row $\mathbf{a}^T$ is defined as Eq. (4.13)

$$\mathbf{K}_{N\times p} = \begin{pmatrix}\tilde{\mathbf{K}}_{(N-1)\times p} \\ \mathbf{a}^T\end{pmatrix} = \mathbf{U}\mathbf{S}\mathbf{V}^T \tag{4.13}$$

where $\tilde{\mathbf{K}}_{(N-1)\times p} = \tilde{\mathbf{U}}\tilde{\mathbf{D}}\tilde{\mathbf{V}}^T, \tilde{\mathbf{U}} \in \mathbf{R}^{(N-1)\times p}, \tilde{\mathbf{V}} \in \mathbf{R}^{p\times p}, \tilde{\mathbf{D}} = diag(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_p)$.

Eq. (4.13) implies that $\tilde{\mathbf{K}}_{(N-1)\times p}^T\tilde{\mathbf{K}}_{(N-1)\times p} = \mathbf{K}_{N\times p}^T\mathbf{K}_{N\times p} - \mathbf{a}\mathbf{a}^T = \mathbf{U}\left(\mathbf{D}^2 - \mathbf{z}\mathbf{z}^T\right)\mathbf{V}^T$

where $\mathbf{z} = \mathbf{V}^T \mathbf{a} = \left[z_1 \ z_2 \ \ldots \ z_p \right]^T$. Thus, the singular values of $\tilde{\mathbf{K}}_{(N-1)\times p}$ can be found by computing the eigen decomposition of $\mathbf{D}^2 - \mathbf{z}\mathbf{z}^T = \mathbf{Q}\mathbf{\Omega}^2\mathbf{Q}^T$, where $\mathbf{Q} \in \mathbf{R}^{p\times p}$ is the orthogonal matrix, and $\tilde{\mathbf{D}} = \mathbf{\Omega} \in \mathbf{R}^{p\times p}$ is a non-negative and diagonal matrix.

The singular values decomposition of the matrix $\tilde{\mathbf{K}}_{(N-1)\times p}$ is defined through the SVD of the matrix $\mathbf{K}_{N\times p}$ as follows (Bunch & Nielsen, 1978):

$$\tilde{d}_i = d_i + \mu_i, 1 \le i \le p \tag{4.14}$$

where μ_i satisfy the secular equation:

$$-1 + \sum_{i=1}^{p} \frac{z_i^2}{\left(d_i + d_j + \mu\right)\left(d_i - d_j - \mu\right)} = 0, 1 \le i \le p \tag{4.15}$$

The updated right singular vectors $\tilde{\mathbf{V}} = \left[\tilde{\mathbf{v}}_1 \ \tilde{\mathbf{v}}_2 \ldots \tilde{\mathbf{v}}_p \right]$ and the updated left singular vectors $\tilde{\mathbf{U}} = \left[\tilde{\mathbf{u}}_1 \ \tilde{\mathbf{u}}_2 \ldots \tilde{\mathbf{u}}_p \right]$ are estimated as forms:

$$\tilde{\mathbf{v}}_i = \frac{\mathbf{V}\mathbf{T}_i^{-1}\mathbf{z}}{\left\| \mathbf{T}_i^{-1}\mathbf{z} \right\|_2}, \mathbf{T}_i = \mathbf{D}^2 - \tilde{d}_i \mathbf{I}, 1 \le i \le p \tag{4.16}$$

$$\tilde{\mathbf{u}}_i = \frac{1}{\tilde{d}_i} \tilde{\mathbf{K}}_{(N-1)\times p} \tilde{\mathbf{v}}_i \ 1 \le i \le p \tag{4.17}$$

Thus, the set of four equations Eqs. (4.14)-(4.17) is an effective procedure for updating the SVD of the matrix when deleting a row. The aim of this technique is to reduce the computational complexity when computing the SVD of a matrix.

4.3.3 Updating the SVD of a matrix when appending a column

In this case, a matrix based on the given matrix $\mathbf{K}_{N\times p}$ when adding a column is defined as:

$$\tilde{\mathbf{K}}_{N\times(p+1)} = \left(\mathbf{K}_{N\times p}\ \mathbf{b}\right) = \tilde{\mathbf{U}}\tilde{\mathbf{D}}\tilde{\mathbf{V}}^T \tag{4.18}$$

where $\tilde{\mathbf{U}} \in \mathbf{R}^{N\times(p+1)}, \tilde{\mathbf{V}} \in \mathbf{R}^{(p+1)\times(p+1)}, \tilde{\mathbf{D}} = diag(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_{p+1})$.

Letting $\mathbf{z}' = \mathbf{U}^T b = \left[z_1'\ z_2' \ldots z_p'\right]^T, \alpha = \dfrac{1}{\|\mathbf{b}\|_2}$, the factorized representation can be rewritten as:

$$\tilde{\mathbf{K}}^T_{N\times(p+1)}\tilde{\mathbf{K}}_{N\times(p+1)} = \mathbf{K}^T_{N\times p}\mathbf{K}_{N\times p} - \mathbf{b}\mathbf{b}^T = \mathbf{U}\left(\mathbf{D}^2 + \mathbf{z}'\mathbf{z}'^T\right)\mathbf{V}^T \tag{4.19}$$

The eigen decomposition of $\mathbf{D}^2 + \mathbf{z}'\mathbf{z}'^T$ can be written as $\mathbf{Q}\mathbf{\Omega}^2\mathbf{Q}^T$, where $\mathbf{Q} \in \mathbf{R}^{(p+1)\times(p+1)}$, $\tilde{\mathbf{D}} = \mathbf{\Omega} \in \mathbf{R}^{(p+1)\times(p+1)}$. The singular values of the matrix $\tilde{\mathbf{K}}_{N\times(p+1)}$ are updated via the eigen decomposition of $\mathbf{D}^2 + \mathbf{z}'\mathbf{z}'^T$ as follows (Bunch & Nielsen, 1978):

$$\tilde{d}_i = d_i + \mu_i, 1 \le i \le p+1 \tag{4.20}$$

where μ_i satisfy the secular equation in (Bunch & Nielsen, 1978):

$$1 + \frac{1}{\alpha^2}\sum_{i=1}^{p}\frac{z_i'^2}{\left(d_i + d_j + \mu\right)\left(d_i - d_j - \mu\right)} - \frac{\left(1 - \|\mathbf{z}'\|_2^2\right)^2}{\alpha^2\mu} = 0, 1 \le i \le p \tag{4.21}$$

The updated right singular vector $\tilde{\mathbf{V}} = \left[\tilde{\mathbf{v}}_1\ \tilde{\mathbf{v}}_2 \ldots \tilde{\mathbf{v}}_{p+1}\right]$ is determined through the left singular vector $\mathbf{V} \in \mathbf{R}^{p\times p}$ and the regular values $\tilde{\mathbf{D}} \in \mathbf{R}^{(p+1)\times(p+1)}$ form:

$$\tilde{\mathbf{v}}_i = \eta_i \begin{bmatrix} \mathbf{V}\mathbf{T}_i^{-1}\mathbf{D}\mathbf{z}' \\ -1 \end{bmatrix}, \mathbf{T}_i = \mathbf{D}^2 - \tilde{d}_i \mathbf{I}, \eta_i = \left\| \begin{bmatrix} \mathbf{V}\mathbf{T}_i^{-1}\mathbf{D}\mathbf{z}' \\ -1 \end{bmatrix} \right\|_2^{-1} \tag{4.22}$$

The updated left singular vectors $\tilde{\mathbf{U}} = \left[\tilde{\mathbf{u}}_1 \; \tilde{\mathbf{u}}_2 \ldots \tilde{\mathbf{u}}_{p+1} \right]$ are updated via the right singular vectors and the given matrix by:

$$\tilde{\mathbf{u}}_i = \frac{1}{\tilde{d}_i} \left(\mathbf{K}_{N \times p} \; \mathbf{b} \right) \tilde{\mathbf{v}}_i, 1 \le i \le (p+1) \tag{4.23}$$

Instead of repeating the computing SVD of the matrix, the SVD updating procedure of the matrix when appending a column is presented in the set of four equations Eqs. (4.20)-(4.23).

4.4 Simulation on lumped-mass mechanical model

As presented in the previous section, many studies have used the matrix's singular value decomposition (SVD) in system identifications. Updating the SVD of the matrices by appending or deleting a column and a row has been presented in many algorithms. This section develops a formula for updating model parameters for AR models by updating the SVD of a data matrix through the identification procedure with respect to time and model order. The proposed process for the modal analysis is also presented in this section.

4.4.1 Updating in model order for AR model parameters

Consider that, the data matrix $\mathbf{K}_{N \times np}$ and the output vector $\mathbf{Y}[t]_{N \times n}$ of the AR at order p are formed from N successive samples by:

$$\mathbf{Y}[t]_{N\times n} = \begin{bmatrix} \mathbf{y}[t] \\ \mathbf{y}[t+1] \\ \cdots \\ \mathbf{y}[t+N-1] \end{bmatrix}, \mathbf{K}_{N\times np} = \begin{bmatrix} \mathbf{z}[t] \\ \mathbf{z}[t+1] \\ \cdots \\ \mathbf{z}[t+N-1] \end{bmatrix} \tag{4.24}$$

The goal of the model identification is to determine all the model parameters. Based on Eq. (4.5), solving the least-squares problem for the model parameters of AR models by SVD is defined by:

$$\mathbf{P}_{np\times n} = \mathbf{V}_{(p)}\mathbf{D}_{(p)}^{-1}\mathbf{U}_{(p)}^{T}\mathbf{Y}[t]_{N\times n} \tag{4.25}$$

where $\mathbf{K}_{N\times np} = \mathbf{U}_{(p)}\mathbf{D}_{(p)}\mathbf{V}_{(p)}^{T}$ with $\mathbf{U}_{(p)} \in \mathbf{R}^{N\times np}, \mathbf{V}_{(p)} \in \mathbf{R}^{np\times np}$ are orthogonal matrices, and $\mathbf{D}_{(p)} \in \mathbf{R}^{np\times np}$ is a square diagonal matrix.

The data matrix at order $p+1$ can be built by appending a data sub-matrix into the data matrix $\mathbf{K}_{N\times np}$ as follows:

$$\mathbf{K}_{N\times n(p+1)} = \left(\mathbf{K}_{N\times np}\ \mathbf{K}'_{N\times n}\right), \text{ where } \mathbf{K}'_{N\times n} = \begin{bmatrix} \mathbf{y}[t-(p+1)] \\ \mathbf{y}[t+1(p+1)] \\ \cdots \\ \mathbf{y}[t+N-1-(p+1)] \end{bmatrix}$$

The model parameter estimation of AR models at order $p+1$ is obtained by updating the SVD of the data matrix. The proposed algorithm is summarized as follows:

Input: $\mathbf{K}_{N\times np}, \mathbf{K}'_{N\times n}, \mathbf{Y}[t]_{N\times n}, \mathbf{V}_{(p)}, \mathbf{D}_{(p)}, \mathbf{U}_{(p)}^{T}, \mathbf{z}' = \mathbf{U}^{T}\mathbf{K}'_{N\times n}, \alpha = \left\|\mathbf{K}'_{N\times n}\right\|_2^{-1}$

Step1. Computation of the model parameters at order p.

Using the SVD in Eq. (4.6) of the data matrix to estimate the model parameters at order p in Eq. (4.25).

$$\mathbf{P}_{np\times n} = \mathbf{V}_{(p)}\mathbf{D}_{(p)}^{-1}\mathbf{U}_{(p)}^{T}\mathbf{Y}\left[t\right]_{N\times n} \tag{4.26}$$

Step 2. Update of the singular values of the data matrix at order $p+1$.

Solving the secular equation in Eq. (4.21) to calculate the singular values of $\mathbf{D}_{(p+1)}$ via $\mathbf{z}', \mathbf{D}_{(p)}$ at order p.

Step 3. Update of the right vector $\mathbf{V}_{(p+1)}$ of the data matrix at order $(p+1)$.

Based on $\mathbf{V}_{(p)}, \mathbf{z}', \mathbf{D}_{(p)}, \mathbf{D}_{(p+1)}$, the right vectors $\mathbf{V}_{(p+1)}$ at order $(p+1)$ are updated in Eq. (4.22).

Step 4. Update of the left vectors $\mathbf{U}_{(p+1)}$ of the data matrix at order $(p+1)$.

Once the singular values and the right vectors at order $(p+1)$ are obtained, the left vectors $\mathbf{U}_{(p+1)}$ are updated through $\mathbf{V}_{(p)}, \mathbf{z}', \mathbf{D}_{(p)}, \mathbf{D}_{(p+1)}, \mathbf{K}_{N\times np}, \mathbf{K}'_{N\times n}$ in Eq. (4.23).

Step 5. Computation of the model parameters of AR models at order $(p+1)$.

Because the singular values, the left and right vectors at order $(p+1)$, are calculated via $\mathbf{U}_{(p)}, \mathbf{D}_{(p)}, \mathbf{V}_{(p)}^{T}$ at order p.

The solution of the least square problem $\mathbf{P}_{n(p+1)\times n} = \mathbf{V}_{(p+1)}\mathbf{D}_{(p+1)}^{-1}\mathbf{U}_{(p+1)}^{T}\mathbf{Y}[t]_{N\times n}$ can be directly updated.

The above algorithm identifies the model parameters at higher model orders through the SVD of the data matrix at the previous order. The model parameters are defined with low computational complexity and computational time effectiveness. This technique is preferable to the repetitive approach of Eq. (4.25) for each order value.

4.4.2 Updating in time for AR model parameters

Observation matrix $\mathbf{K}[k]_{N\times np}$ and output vector $\mathbf{Y}[k]_{N\times n}$ at time $t = k$ of the AR at order p are extracted from the measured portion of the global response as follows:

$$\mathbf{K}[k]_{N\times np} = \begin{bmatrix} \mathbf{z}[k] \\ \mathbf{z}[k+1] \\ \cdots \\ \mathbf{z}[k+N-1] \end{bmatrix}, \mathbf{Y}[k]_{N\times n} = \begin{bmatrix} \mathbf{y}[k] \\ \mathbf{y}[k+1] \\ \cdots \\ \mathbf{y}[k+N-1] \end{bmatrix} \tag{4.27}$$

Based on Eq. (4.5) solving the least-squares problem for the model parameters of AR models at $t = k$ by SVD is defined as:

$$\mathbf{P}[k]_{np\times n} = \mathbf{V}[k]\mathbf{D}[k]^{-1}\mathbf{U}[k]^{T}\mathbf{Y}[k]_{N\times n} \tag{4.28}$$

where $\mathbf{K}[k]_{N\times np} = \mathbf{U}[k]\mathbf{D}[k]\mathbf{V}[k]^{T}$. At time $t = k + s$, the data matrix of AR models can be represented by:

$$\mathbf{K}[k+s]_{N\times np}=\begin{bmatrix}\mathbf{z}[k+s]\\ \cdots\\ \mathbf{z}[k+N+1]\\ \cdots\\ \mathbf{z}[k+s+N-1]\end{bmatrix},\ \mathbf{Y}[k+s]_{N\times n}=\begin{bmatrix}\mathbf{y}[k+s]\\ \mathbf{y}[k+s+1]\\ \cdots\\ \mathbf{y}[k+s+N-1]\end{bmatrix} \tag{4.29}$$

From the data matrices of the model at time $t=k$ and $t=k+s$, one can rewrite Eq. (4.27) and Eq. (4.29) , respectively as:

$$\mathbf{K}[k]_{N\times np}=\begin{bmatrix}\mathbf{z}[k]\\ \mathbf{z}[k+1]\\ \cdots\\ \mathbf{z}[k+N-1]\end{bmatrix}=\begin{bmatrix}\mathbf{R}_{1_{s\times np}}\\ \mathbf{R}_{(N-s)\times np}\end{bmatrix},\ \mathbf{K}[k+s]_{N\times np}=\begin{bmatrix}\mathbf{z}[k+s]\\ \cdots\\ \mathbf{z}[k+N+1]\\ \cdots\\ \mathbf{z}[k+s+N-1]\end{bmatrix}=\begin{bmatrix}\mathbf{R}_{(N-s)\times np}\\ \mathbf{R}_{2_{s\times np}}\end{bmatrix} \tag{4.30}$$

where

$$\mathbf{R}_{1_{s\times np}}=\begin{bmatrix}\mathbf{z}[k]\\ \cdots\\ \mathbf{z}[k+s-1]\end{bmatrix},\mathbf{R}_{(N-s)\times np}=\begin{bmatrix}\mathbf{z}[k+s]\\ \cdots\\ \mathbf{z}[k+N-1]\end{bmatrix},\mathbf{R}_{2_{s\times np}}=\begin{bmatrix}\mathbf{z}[k+N]\\ \cdots\\ \mathbf{z}[k+s+N-1]\end{bmatrix} \tag{4.31}$$

Instead of the iterative solution procedure required to calculate the model parameters at time $t=k+s$, the model parameters of the AR model at time $t=k+s$ can be obtained by updating the SVD of the matrix. The algorithm is implemented in five steps:

Input: $\mathbf{K}[k]_{N\times np},\mathbf{Y}[k]_{N\times n},\mathbf{R}_{1_{s\times np}},\mathbf{R}_{2_{s\times np}},\mathbf{R}_{(N-s)\times np},\mathbf{Y}[k+s]_{N\times n},\mathbf{U}[k],\mathbf{D}[k],\mathbf{V}[k]$.

Step 1. Computation the model parameters at time $t=k$.

The model parameters of the AR model at time $t = k$ are calculated using Eq. (4.28).

Step 2. Update of the singular values of the data matrix at time $t = k + s$.

The singular values of the data matrix $\mathbf{K}[k+s]_{N \times np}$ are identified by solving the secular equation in Eq. (4.10) and Eq. (4.15).

Step 3. Update of the right vectors $\mathbf{V}[k+s]$ of the data matrix at time $t = k + s$.

Once, the singular values of the data matrix $\mathbf{K}[k+s]_{N \times np}$ at time $t = k + s$ are obtained in step 2. The right vector are identified by Eq. (4.11) and Eq. (4.16) through:

$$\mathbf{R}_{1_{s \times np}}, \mathbf{R}_{2_{s \times np}}, \mathbf{R}_{(N-s) \times np}, \mathbf{U}[k], \mathbf{D}[k], \mathbf{V}[k]$$

Step 4. Update of the left vectors $\mathbf{U}[k+s]$ of the data matrix at time $t = k + s$.

The left vectors $\mathbf{U}[k+s]$ are computed using Eq. (4.12) and Eq. (4.17) through

$$\mathbf{R}_{1_{s \times np}}, \mathbf{R}_{2_{s \times np}}, \mathbf{R}_{(N-s) \times np}, \mathbf{V}[k+s], \mathbf{D}[k], \mathbf{D}[k+s]$$

Step 5. Computation of the model parameters of the AR models at time $t = k + s$.

The AR model's model parameters are obtained by using the updated singular values, right vectors, and left vectors in steps 2, 3 and 4 as:

$$\mathbf{P}[k+s]_{np \times n} = \mathbf{V}[k+s]\mathbf{D}[k+s]^{-1}\,\mathbf{U}[k+s]^{T}\,\mathbf{Y}[k+s]_{N \times n}$$

From this algorithm, the parameters of models $t = k + s$ are updated through the SVD of the data matrix at the time $t = k$.

A lumped-mass dynamic system and a hydraulic turbine blade experimental setup are presented to validate the proposed method, extract the modal parameters and monitor the systems. Results are detailed in the next section.

4.5 Simulations on lumped-mass mechanical model

A numerical simulation was carried out to produce numerical system input-output data. A mechanical model of the simulated system used for this study is shown in Figure 4.1.

The motion equation of the system can be derived from either a Newton or Lagrange formulation, as follows:

$$\begin{bmatrix} m_1(t) & 0 \\ 0 & m_2(t) \end{bmatrix} \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 + \dot{m}_1(t) & -c_2 \\ -c_2 & c_2 + \dot{m}_2(t) \end{bmatrix} \begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} f(t) \\ 0 \end{Bmatrix} \tag{4.32}$$

where $\dot{m}_1(t) = \dfrac{dm_1(t)}{dt}$, $\dot{m}_2(t) = \dfrac{dm_2(t)}{dt}$.

Because the variation of the masses is small in this example, the derivative of the masses with respect to time is negligible in the damping matrix. Therefore, Eq. (4.32) can be rewritten as follows:

$$\begin{bmatrix} m_1(t) & 0 \\ 0 & m_2(t) \end{bmatrix} \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} f(t) \\ 0 \end{Bmatrix} \tag{4.33}$$

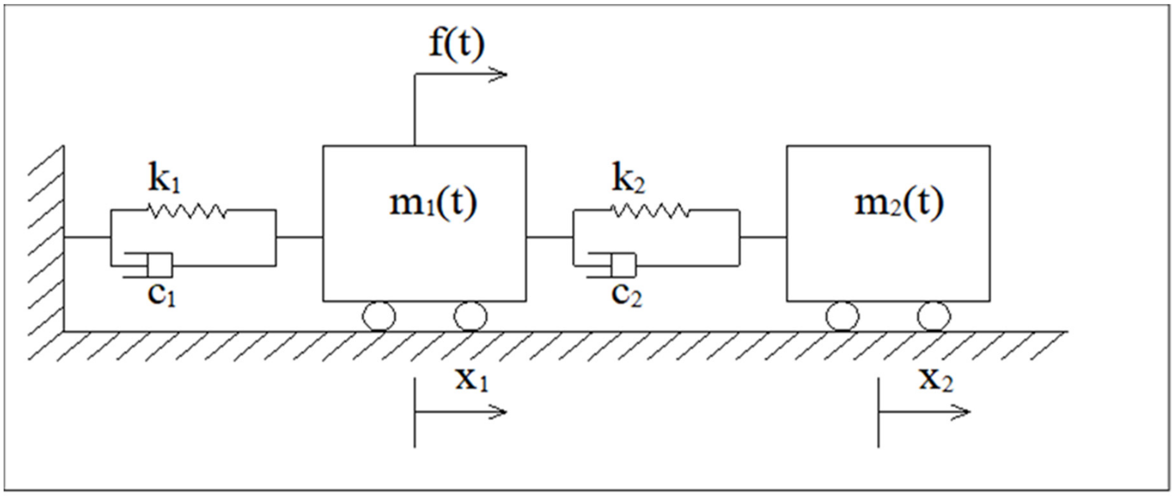


Figure 4.1 Schematic model of 2 DOF time-varying system

An excitation signal with a white-noise shape is applied in this sub-section with the different rate change of system's masses.

Figure 4.2 shows the gradual changing masses for two different rates. The numerical values of the system parameters are given as follows:

$$c_1 = 10(Ns/m), c_2 = 20(Ns/m), k_1 = 10000(N/m), k_2 = 22000(N/m)$$

The system's varying modal parameters are simulated in Figure 4.3 on a theoretical basis for the two different rates, 2.5 (%/s) and 5 (%/s). At the masse's change rate 2.5 (%/s), the first natural frequency varies within the range of (2.32-2.47) (Hz), and the second varies within the range of (7.95-8.51) (Hz). At the masse's change rate 5 (%/s), the two natural frequencies vary within the range of (2.19-2.47) (Hz) and (7.49-8.51) (Hz), respectively.

The displacement responses of the system under a particular excitation were obtained through the ODE45 method in MATLAB using a fixed-integration step and recorded at a sampling frequency of 100 (Hz).

At the mass change rate 2.5 (%/s), Figure 4.4 (a) plots the non-stationary vibration displacement signal. The signal's spectrogram under a random excitation is presented in Figure 4.4 (b).

Figure 4.5 (a) and (b) show the non-stationary vibration displacement signal and the signal's spectrogram under a random excitation at the mass change rate 5 (%/s).

The proposed method uses the AR model to identify the mechanical system's modal parameters in each data segment. The model's parameters at the next segment are updated using the results of previous segments. The length of each window is sufficient to contain the modal parameters, and the length must be at least four times the highest period in order to track the changes of all modes. The model order selection procedure is based on the minimum description length (MDL) (Bui, Vu & Liu, 2022).

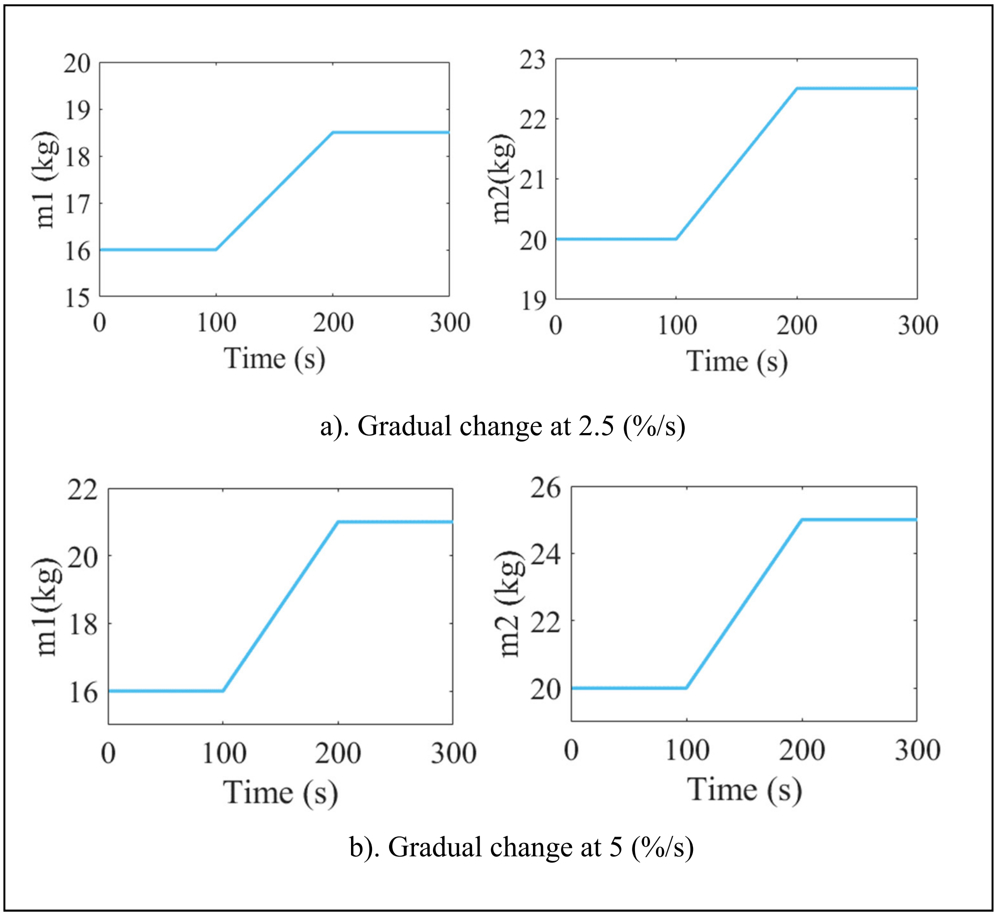
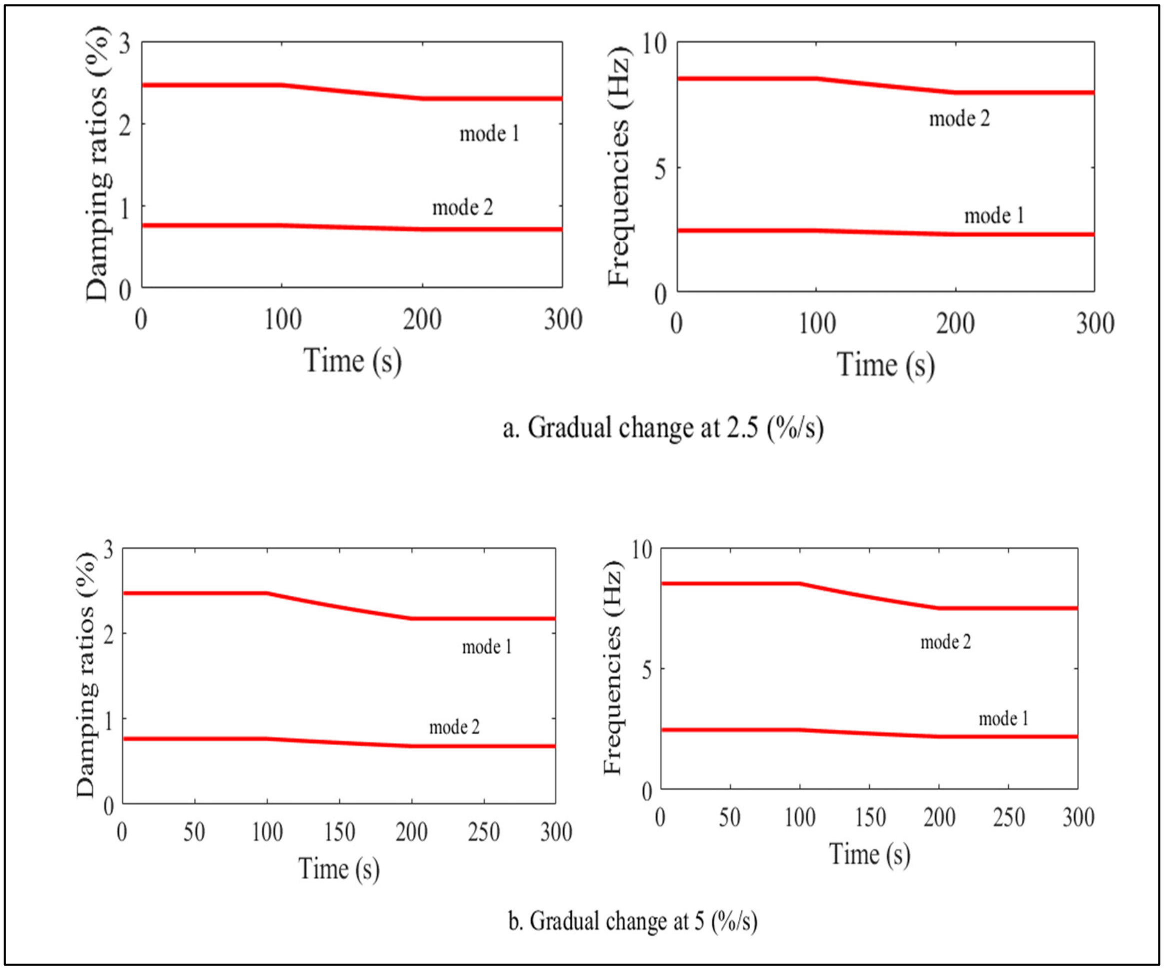


Figure 4.2 Masses changing function



a. Gradual change at 2.5 (%/s)

b. Gradual change at 5 (%/s)

Figure 4.3 Modal parameters of the system

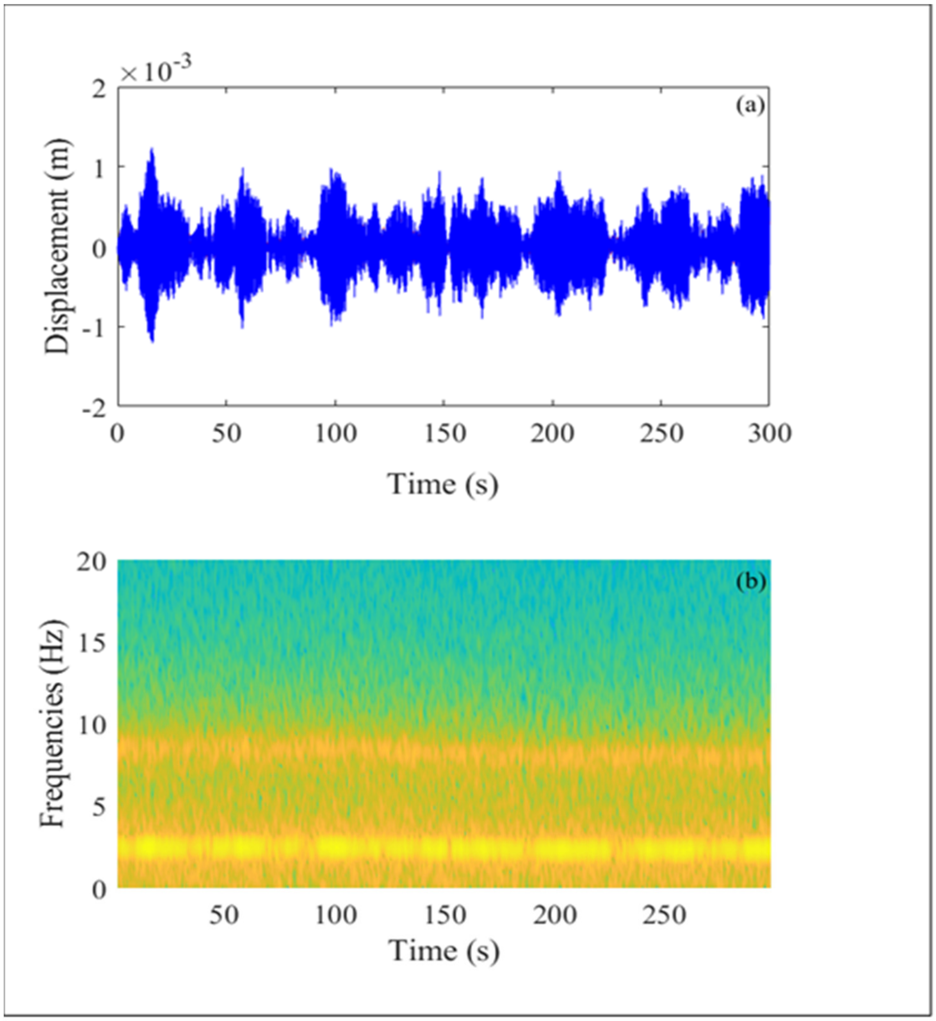


Figure 4.4 (a) Realization of the non-stationary vibration displacement, (b) Short-time Fourier transform of the signal with the gradual change at 2.5 (%/s)

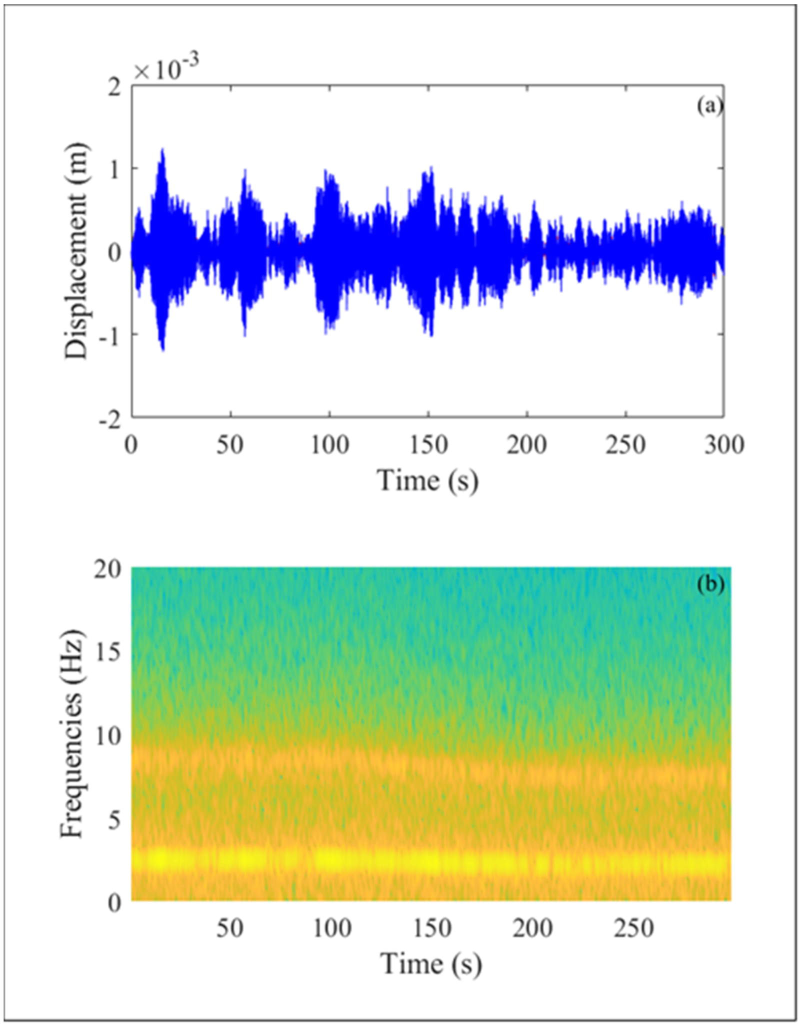


Figure 4.5 (a) Realization of the non-stationary vibration displacement, (b) Short-time Fourier transform of the signal with the gradual change at 5 (%/s)

Figure 4.6 presents the identification of the modal parameters with the proposed method at the mass change rate 2.5 (%/s). Obviously, the natural frequencies are accurately determined and tracked. The results of damping ratios for the first and second modes are also in very good agreement with the calculated values.

The proposed method performs better in terms of accuracy resolution than STFT. It can be observed from these figures that the proposed method matches well with theoretical variations.

At the higher change rate of the masses, the modal parameters of the system using the proposed method are shown in Figure 4.7.

It can be seen that the first mode is still accurately identified and tracked. However, the second mode is largely dispersed, especially its damping ratio, which is known to associate with a greater uncertainty (Vu, Thomas, Lakis & Marcouiller, 2011). Conservatively, the proposed method can identify and monitor the mass change rate at 5 (%/s). This change rate is quite high for real systems.

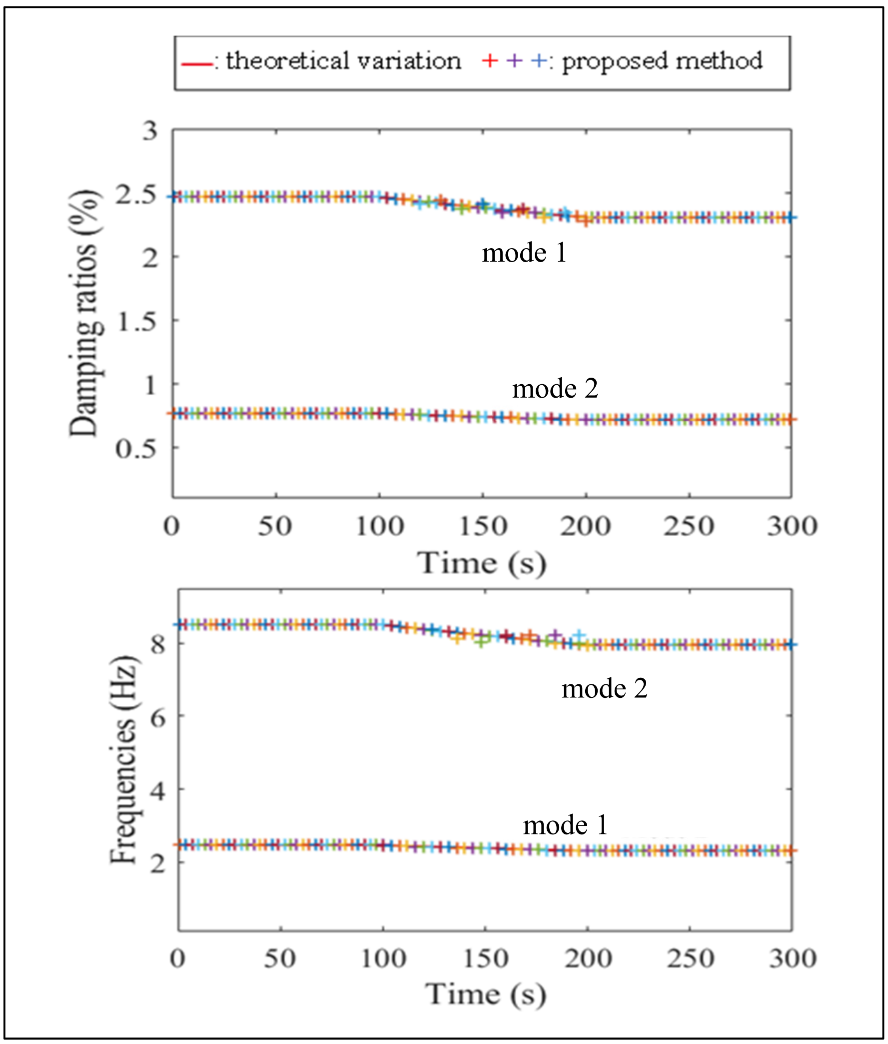


Figure 4.6 Modal parameters of the system with the gradual change at 2.5 (%/s) using the proposed method

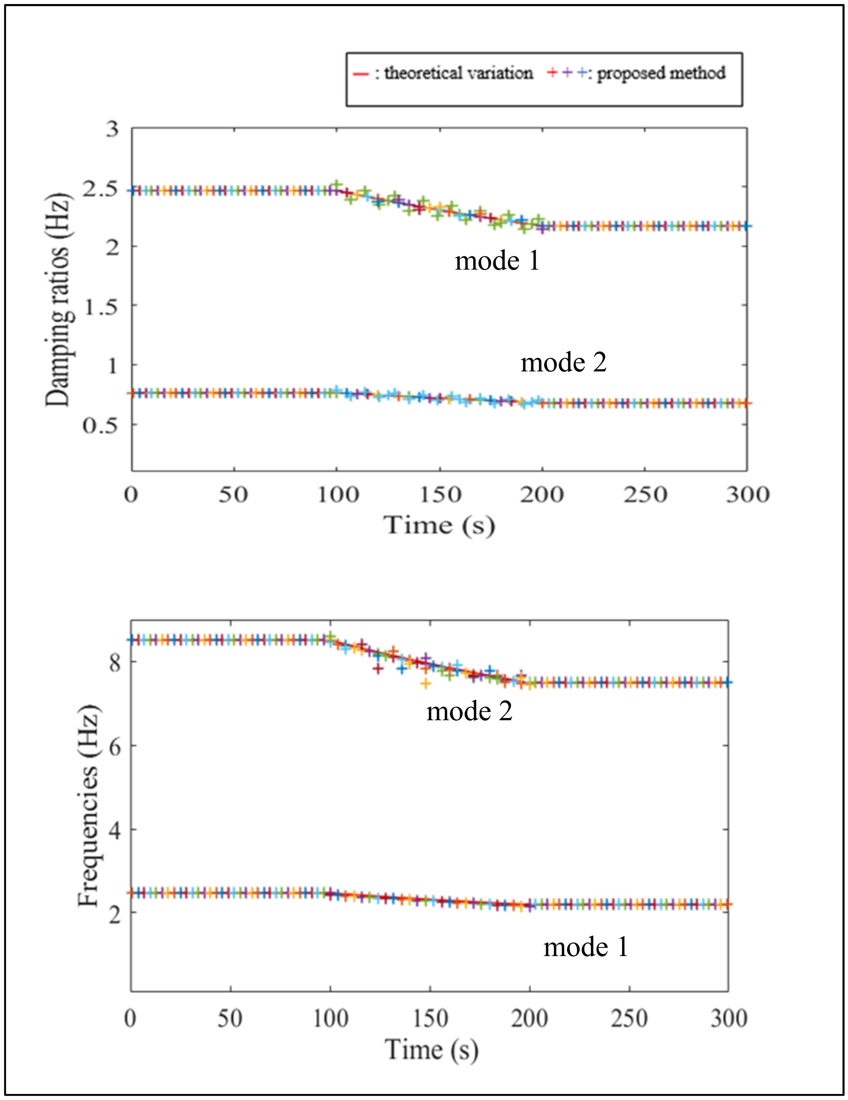


Figure 4.7 Modal parameters of the system with the gradual change at 5 (%/s) using the proposed method

4.6 Experimental data

In this section, the proposed method is applied to a hydraulic turbine blade to monitor the modal parameters. The turbine blade is made of bronze alloy “M”- C92300 corresponding to the standard designation 87Cu- 8Sn- IPb-4Zn.F. Figure 4.8 shows the configuration of the test in which four accelerometers have been mounted to record the accelerations of this blade.

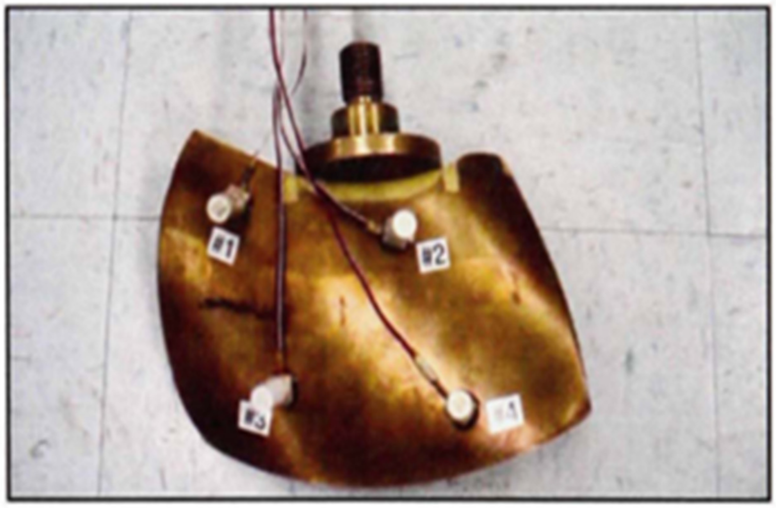


Figure 4.8 Blade with four accelerometers

4.6.1 The turbine blade in air

The test was carried out using an LMS system. A PCB impact hammer with a sampling frequency of 6400 (Hz) acted on the structure in the static test. Figure 4.9 shows the configuration of the test in the air. The natural frequencies of this blade are shown in Table 4.1 with: Ansys, PSD in MATLAB, AR of Vu (Vu, Thomas, Lakis & Marcouilleret, 2010), and the proposed method.

The first frequency (204.7 Hz) was not identified using the finite element method because it was the mounting-structure mode. The system's PSD plot obtained from the spectrogram function in MATLAB is presented in Figure 4.10.

The stabilization diagram obtained from the proposed method is shown in Figure 4.11. As shown in Table 4.1, the identifications of four frequencies are pretty accurate for all modal parameters. The proposed method with order updating was applied to the systems to extract the natural frequencies.

Table 4.1 Modal identification of the blade in the air

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Ansys	Non-identify	332.9	643.16	896.9	963.6	1157.6
PSD in MATLAB	204.7	359.4	582.8	843.8	1050	1295.0
AR of Vu	208.0	368.0	576.0	848.0	1040	1294.5
Proposed method	204.4	360.3	582.5	846.4	1055	1297.0

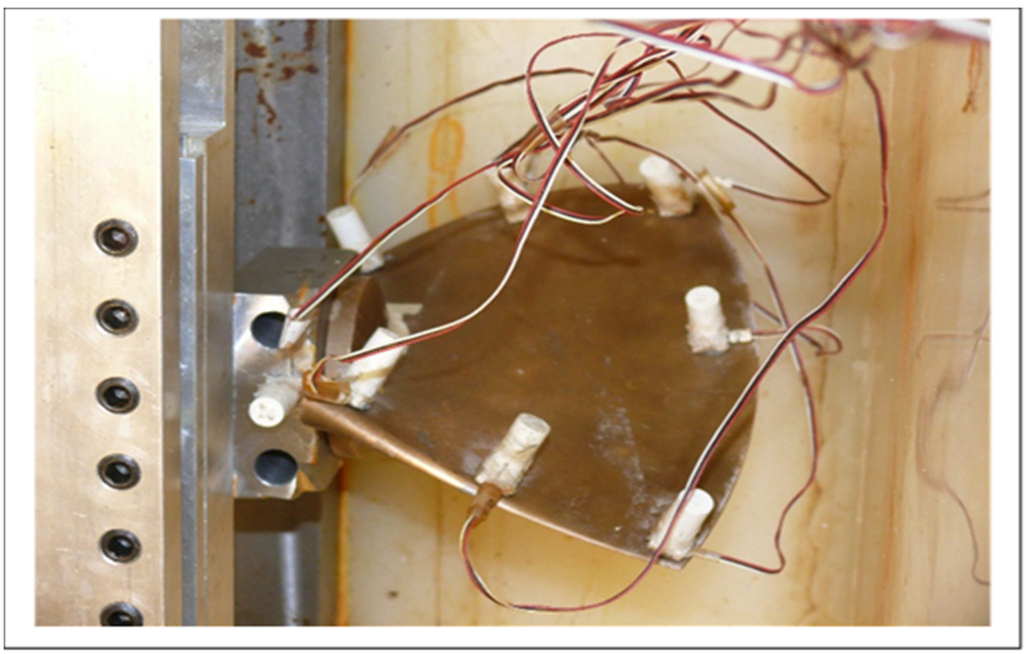


Figure 4.9 Modal test of the blade in the air

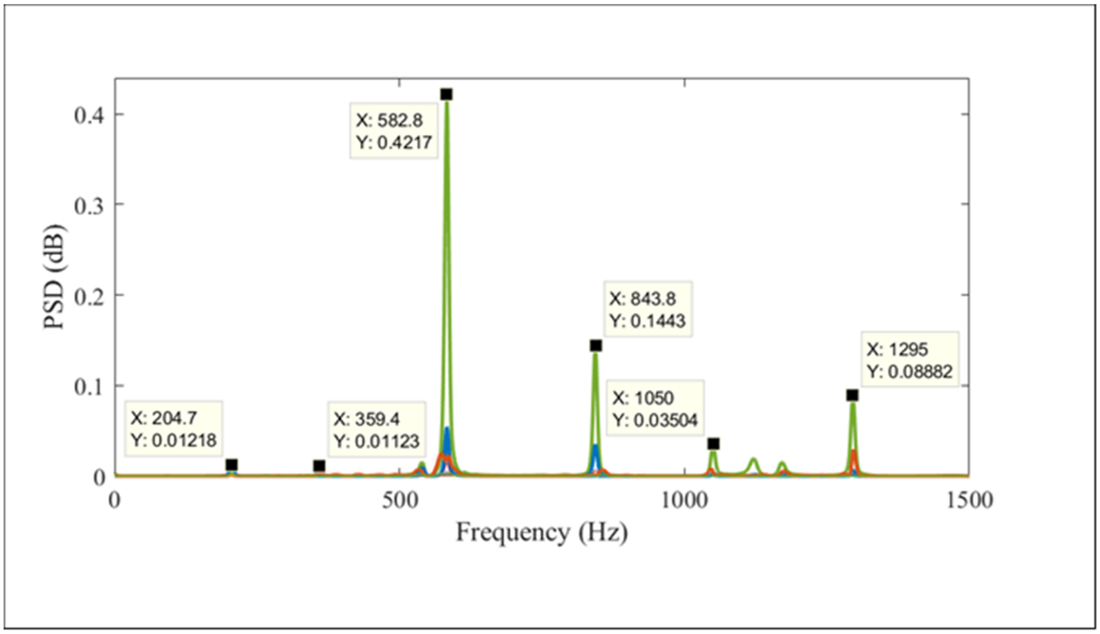


Figure 4.10 Spectra of the blade in the air by spectrogram functions in MATLAB

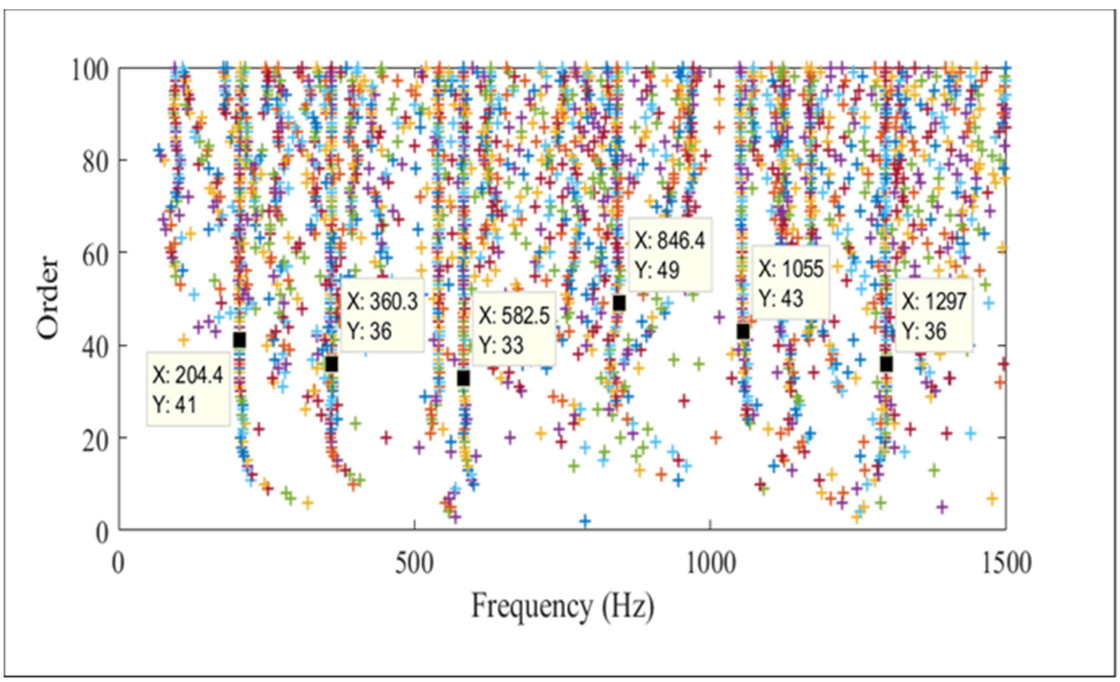


Figure 4.11 Order-updating stabilization diagram of the blade in the air using the proposed method

4.6.2 Turbine blade raised from water

Experiment setup

ÉTS designed a hydraulic test bench in the Dynamo research laboratory to evaluate fluid-structure interactions.

When the blade was placed inside a perforated tank, the vibration of the blade in air and water could be measured. The flow rates can be varied according to the desired flow velocities. The blade was placed inside a perforated tank. The valves and various outlet nozzles are used to control the flow speed. A fastening system is a multi-tasking tool for mounting a blade in variable boundary conditions.

The PCB330A sensors are utilized for vibration measurements. The Vishay System 6000 was used to record pressure sensors during different tests. The data is exported and saved in different formats, such as .xls and .txt, by the software “Strain Smart” designed to work with the Vishay acquisition box. A PCB impact hammer is acted on the structure tests. The hammer is equipped with a steel extension to hit the tested structure at different depths.

The configuration of the experiment is depicted in Figure 4.12. The blade is submerged at different depths, and the depth/length ratio varies from 0.4 (totally submerged) to 0 (in air).

The natural frequencies of the blade change with respect to the submerged depth due to the effect of the fluid. Before the blade rises, its modal parameters was determined by analytical and experimental methods at different depth length ratios (D/L). The result is shown in Table 4.2. The responses of the blade at sampling frequency 8192 (Hz) are depicted in Figure 4.13 (a). Figure 4.13 (b) shows the short-time Fourier transform of the signals.

Modal signal-to-noise ratio

The modal signal-to-noise ratio (MSN) (Vu, Thomas, Lafleur & Marcouiller, 2007) is utilized to separate spurious and real modes. The smallest frequency is then obtained for the next window computation.

The length of the block window

In this section, the proposed method was applied to track modal parameters for the submerged blade. The length of each data segment was chosen to track all the modes of the signals.

The length of the block window is a function of the minimum frequency in the previous data block. The size length of the sliding window varies and is chosen to be at least 4 times that of the longest natural period of the previous block (Bui, Vu & Liu, 2022).

In each window, the minimum description length (MDL) was used to obtain the order of the models. The system's natural frequencies were extracted using the proposed method as shown in Figure 4.14. Compared with the modal testing in Table 4.2 and STTF of the signals shown in Figure 4.13 (b), the identifications of frequencies (147-151.6, 191.2-199, 292.2-303, 435.3-451.4, 579.6-605.2) (Hz) are quite accurate for all modal parameters.

Figure 4.14 shows that the natural frequencies increase slightly when the blade rises from the water to the surface under turbulence. This increase is relevant to the depth ratio of the blade submerged in water. This result agrees well with the conclusion of (Vu, Thomas & Lakis, 2007).

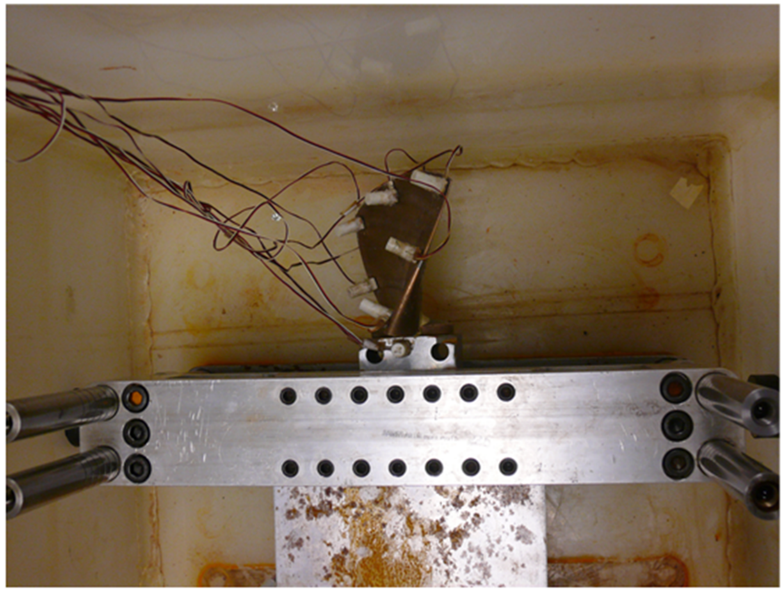


Figure 4.12 Modal test of the blade in water

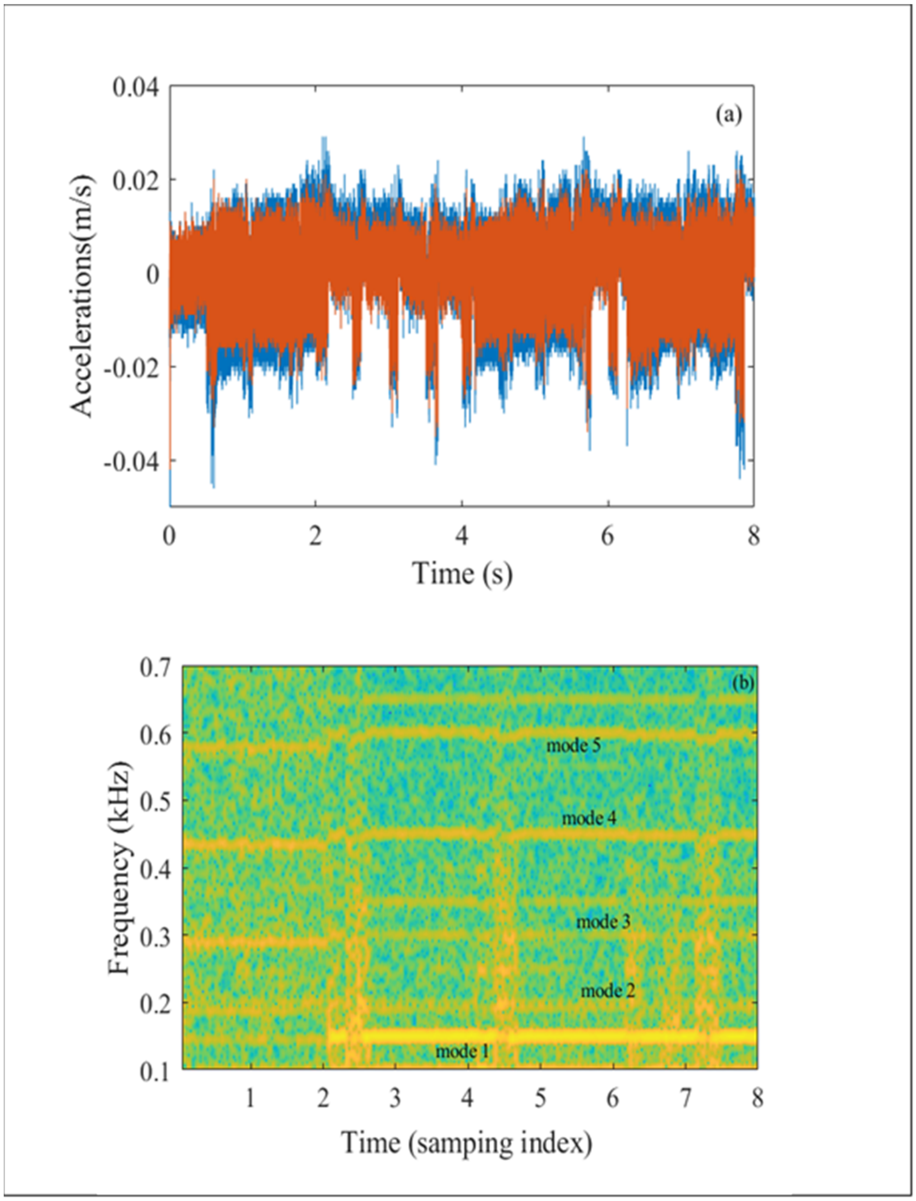


Figure 4.13 a) Response to the blade's acceleration, (b) Short time Fourier transform of the signal

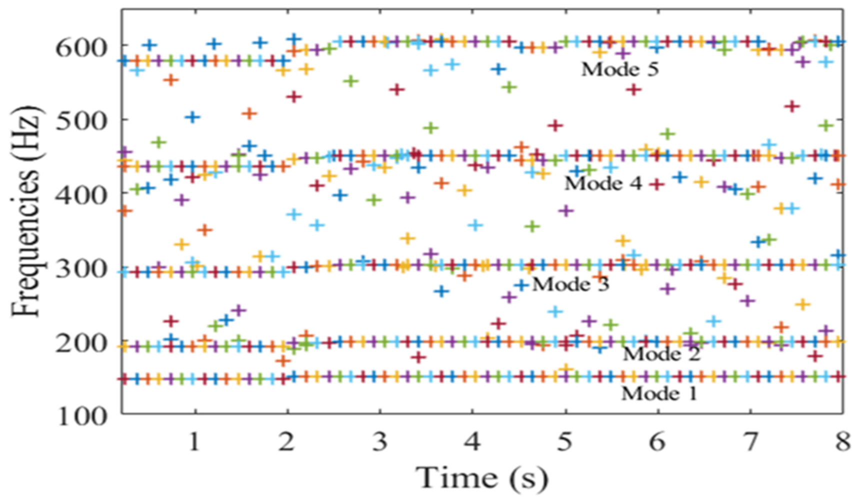


Figure 4.14 Natural frequencies of the blade using the proposed method

Table 4.2 Modal identification of submerged blade

Mode	Frequencies (Hz) of submerged condition (depth/blade length)			
	0.4 (totally submerged)	0.2	0.1	0 (totally in air)
1st	160.8	166.85	170.8	Non-identify
2nd	288.1	288.1	290.0	332.9
3rd	352.7	354.0	358.0	643.2
4th	496.9	498.0	500.0	896.9
5th	640.5	642.8	645.0	963.6

4.7 Summary

This chapter introduces a new method of modal parameter identification and monitoring using a vector autoregressive model for slow-varying non-stationary vibration systems. The main contributions of this study can be listed as follows:

- It presents the updated algorithm for the model parameters of autoregressive models in the modal identification.

- It can monitor and track modal parameters change in slow-varying non-stationary vibration systems under working conditions or in uncertain environments.

- It presents the simulation and experimental studies to access the proposed method. The proposed method was verified first through a numerical simulation of a mechanical system at different rates of masses and then through experiments on a submerged hydraulic turbine blade. Results show that the proposed method is a powerful technique for analysis and monitoring the modal parameters in non-stationary vibration systems under a reasonable varying rate at 5 (%/s).

CONCLUSION

This thesis presents a study on modal identification for non-stationary vibration systems. The objectives of this thesis are divided into two goals which resulted in the development of two algorithms, one based on the Schur complement for matrix inversion and the other based on the singular value decomposition (SVD). The proposed algorithms share the same model performance but differ in effectiveness.

The first step of the study was to develop an updating algorithm for modal identification, and this was presented in Chapter 3. The significant contributions of this study for modal analysis of non-stationary vibration systems are: (i) Introduction of a novel method for updating parameters over time and model order using the least-squares method in conjunction with the Schur complement. This method presents certain advantages, such as updating the inverse matrix based on sub-matrices when the model order increases or decreases. In addition, it can be combined with time updating to provide an effective method for monitoring the modal parameters of a slow-varying system; (ii) Application of the sliding window technique, used with the multivariate autoregressive model, to identify and track the changes in modal parameters; (iii) Experimental studies on a plate emerging from water. The results of this study show that the proposed method dramatically decreases computational time and is a powerful technique for analyzing and monitoring the modal parameters in non-stationary vibration systems.

To reduce the complexity represented by computational time, and matrix singularity, an updating algorithm based on the singular value complement was also developed in the second step, as presented in Chapter 4. This study provided the following critical contributions to modal analysis for slow non-stationary vibrations: (i) A new method for updating parameter estimation AR models; (ii) Reduction of computing complexity for online slow-varying monitoring of non-stationary cases by updating SVD to update AR model parameters through the order and time from the previous computational window; (iii) Experimental validation using the hydraulic turbine blade with interesting industrial findings on added mass of submerged turbine structure.

FUTURE WORKS

The research presented the updating model parameters using the auto-regressive model for operational modal analysis. However, some issues still need to be improved, and these can be considered as directions for future research.

Future research should consider the potential effects of damping rates on selecting a maximum order. In this study, natural frequencies are well identified, even though they vary over time. However, the damping rates have higher uncertainties. Therefore, the order's maximum value is recommended based on a threshold of the uncertainty of the damping rates. Once this order is reached, the damping rates will be identified more precisely, and then monitoring the change in these rates becomes feasible.

In this study, modal tracking is performed using the sliding window technique. The length of the windows was selected to be four times the period of the first natural frequency. This length also varies along with the variation of this frequency. Although this length works well to reveal frequency changes, the results still need to be validated to identify the variation in damping rates. This is desirable for future work.

It will be important that future research could conduct on non-stationary systems with highly varying rates. This study applies the proposed algorithms to slow-varying non-stationary vibration systems. In the future, the proposed method could be used on pure non-stationary vibration systems. These updating algorithms can be used in a functional auto-regressive time series framework or smoothness priors time-dependent auto-regressive for better and more accurate analysis and monitoring of the non-stationary vibration systems.

In this study, the model parameters of AR models are updated over time and in order. Simulation and experiment data results are considered an excellent way to verify the effectiveness of the proposed methods in terms of computational complexity and time consumption. However, updating modal parameters directly over time and order is still a very open problem that needs further study.

In this study, the confidence interval did not estimate during the identification and monitoring of modal parameters. Hence, the uncertainty qualifications in vibration systems should be considered in future research.

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