ÉCOLE DE TECHNOLOGIE SUPÉRIEURE
UNIVERSITÉ DU QUÉBEC

THÈSE PAR ARTICLES PRÉSENTÉE À
L'ÉCOLE DE TECHNOLOGIE SUPÉRIEURE

COMME EXIGENCE PARTIELLE
À L'OBTENTION DU
DOCTORAT EN GÉNIE
Ph.D.

PAR
Fleur Inès DEHAYEM NODEM

COMMANDE OPTIMALE STOCHASTIQUE APPLIQUÉE À LA GESTION DE CAPACITÉ DES SYSTÈMES DYNAMIQUES EN ENVIRONNEMENT MANUFACTURIER

MONTRÉAL, LE 6 MAI 2009

CETTE THÈSE A ÉTÉ ÉVALUÉE

PAR UN JURY COMPOSÉ DE

M. Jean Pierre Kenné, directeur de thèse
Département de génie mécanique à l'École de technologie supérieure

M. Ali Gharbi, codirecteur de thèse
Département de génie de la production automatisée à l'École de technologie supérieure

Mme Sylvie Nadeau, présidente du jury
Département de génie mécanique à l'École de technologie supérieure

M. Robert Pellerin, membre du jury
Département de mathématiques et de génie industriel à l'École polytechnique de Montréal

M. Pierre Dejax, membre du jury, examinateur externe
Département Automatique et Productique à l'École des mines de Nantes

ELLE A FAIT L'OBJET D'UNE SOUTENANCE DEVANT JURY ET PUBLIC

LE 21 AVRIL 2009

À L'ÉCOLE DE TECHNOLOGIE SUPÉRIEURE

REMERCIEMENTS

Les professeurs Jean-Pierre Kenné et Ali Gharbi ont encadré mon travail de thèse. Je leur adresse ma profonde gratitude pour le temps qu'ils ont consacré à examiner les différentes questions soulevées par ce travail, l'engagement dans les discussions, suggestions et contributions qui s'en sont suivies. Merci également pour leur soutien financier grâce auquel j'ai pu me concentrer sur cette thèse.

Mes remerciements vont au professeur Sylvie Nadeau qui m'a fait l'honneur de présider le Jury de la soutenance, aux professeurs Robert Pellerin et Pierre Dejax qui ont accepté de participer au Jury de la soutenance.

Mon père André et ma mère Bibienne Tsayem, mes frères Lezin, Mesmin et mes sœurs Bovary, Alix, Lidwine et Isabelle Dehayem : je vous remercie pour votre irremplaçable et inconditionnel soutien. Merci à Yves Robertson et à toute sa famille : Eugène, Jeanine, Yvan, Jocelyne, Kim, Benoit, Laurence, vous m'avez offert une deuxième famille. Je désire également remercier Michael pour m'avoir incité à retourner sur les bancs d'école, Dorette, Diane, Vital et Honorine pour leur soutien moral lors de la réalisation de ce travail.

Mes remerciements vont à mes collègues de bureau et du laboratoire d'intégration des technologies de production pour leurs diverses contributions. À toi dont le nom n'apparait pas et qui m'aurait aidé d'une manière ou d'une autre, je te dis sincèrement merci.

COMMANDE OPTIMALE STOCHASTIQUE APPLIQUÉE À LA GESTION DE CAPACITÉ DES SYSTÈMES DYNAMIQUES EN ENVIRONNEMENT MANUFACTURIER

DEHAYEM NODEM, Fleur Inès

RÉSUMÉ

Le travail présenté dans cette thèse porte sur l'approche intégrée de gestion optimale de production, de capacité, de remplacement, de maintenance corrective et préventive des ressources d'un système manufacturier. Lesdites ressources sont sujettes à une dégradation progressive dans un environnement caractérisé par des incertitudes. Le travail est développé en quatre (04) phases.

Dans la première phase, une étude est menée sur l'impact de l'introduction des stratégies de maintenance corrective des équipements sur les décisions d'acquisition de capacité et de planification de la production. Le système constitué de plusieurs machines est modélisé par un processus qui dépend de la politique de maintenance corrective. L e problème d'optimisation est ensuite résolu par des méthodes numériques. L'introduction des stratégies de maintenance corrective dans le modèle proposé permet d'améliorer la disponibilité des machines et réduit le coût total encouru, comparé aux modèles existants. Cependant, dans cette première phase, nous ne tenons pas compte de la dégradation de la machine, phénomène pourtant inhérent en contexte manufacturier. En effet, les machines des systèmes de production sont remplacées à long terme, ce qui démontre qu'il y a une dégradation progressive.

La deuxième phase du travail a permis de prendre en compte cette réalité. Pour cela, nous avons travaillé sur des machines pour lesquelles le vieillissement se traduit par l'âge que prend la machine chaque fois qu'une pièce est fabriquée. De plus, le temps de réparation de ces machines croît avec le nombre de pannes. Une approche de recherche simultanée des stratégies de production, de réparation et du remplacement de la machine est utilisée pour déterminer les politiques optimales de réparation, de remplacement et de production. Bien que les temps de réparation deviennent de plus en plus longs au fil des réparations, dans cette phase, nous considérons que les activités de maintenance permettent de remettre l'âge de la machine à zéro, ce qui n'est pas réaliste. D'où l'apport de la prochaine phase.

Dans la troisième phase, les machines après réparation ont un âge non nul, appelé âge virtuel de la machine. Une approche hiérarchique de prise de décision permettant au premier niveau de déterminer la politique de réparation/remplacement de la machine et au second niveau la politique de production est utilisée. Elle montre que sous des hypothèses raisonnables, les décisions de réparation ou de remplacement peuvent être fondées sur l'âge de la machine et le nombre de pannes. Le niveau opérationnel de gestion peut ensuite déterminer un plan de production pour le système en tenant compte de ces décisions.

Les phases deux (02) et trois (03) de notre travail apportent une contribution importante. Elles permettent de montrer que le nombre de pièces à mettre en stock pour se protéger des pénuries durant les périodes de non production n'augmente pas seulement avec l'âge de la machine, mais qu'il augmente également avec le nombre de pannes.

Nous ne pouvions conclure ce travail sans explorer l'impact de l'introduction des stratégies de maintenance préventive. En effet, la maintenance préventive est une des pratiques les plus courantes dans l'industrie manufacturière. Elle permet d'améliorer la disponibilité des équipements lorsque ces derniers subissent une dégradation progressive et nous l'avons intégrée dans la dernière phase.

Dans la quatrième phase de ce travail, nous introduisons la stratégie de maintenance préventive et analysons son effet sur les différentes politiques. Le système est modélisé par un processus qui prend en compte la détérioration et la maintenance préventive. Le modèle est résolu par des méthodes numériques. Des analyses de sensibilité sont élaborées pour montrer la pertinence de l'approche et l'impact de l'introduction des stratégies de maintenance préventive.

Mots clés : Commande optimale, planification de production, gestion de capacité, réparation/remplacement, détérioration, maintenance préventive, systèmes manufacturiers.

OPTIMAL STOCHASTIC CONTROL APPLIED TO THE CAPACITY MANAGEMENT OF DYNAMIC SYSTEMS IN MANUFACTURING ENVIRONMENT

DEHAYEM NODEM, Fleur Ines

ABSTRACT

The work presented in this thesis deals with the integrated optimization of capacity, production, repair, replacement and maintenance planning of stochastic manufacturing systems. The resources of the manufacturing systems are subject to a gradual deterioration in an environment characterized by uncertainties. The work is developed in four (04) phases.

In the first phase, a study is conducted on the impact of the introduction of corrective maintenance strategies of equipment on decisions to acquire capacity and on production planning. The system consists of multiple machines is modeled by a process that depends on the corrective maintenance policy. The optimization problem is solved by numerical methods. The introduction of corrective maintenance strategies in the proposed model improves machine availability and reduces the total incurred cost, compared to existing models. In this first phase, we consider that the machine does not age. In reality however, production systems machinery is replaced in the long term, which proves that there is a gradual deterioration.

The second phase of work permits to take into account this reality. To do this, we worked on machines that age every time a part is manufactured. In addition, the repair time of the machines increases with the number of breakdowns. A simultaneous production, repair and replacement planning of the machine is used to determine optimal repair/replacement and production policies. Although repair times increase with the number of failures in this phase, maintenance activities can reset the age of the machine to zero value, which is not realistic hence the contribution of the third phase.

In the third phase, after repairing the machines, they have a non-zero age value, called virtual age of the machine. A hierarchical decision-making approach allowing the first level to determine the repair / replacement policy of the machine and the second level the production policy is used. It shows that under reasonable assumptions, decisions of repair or replacement can be based on the age of the machine and the number of breakdowns already occurred. The operational level management can then determine a production plan for the system in light of these decisions.

Phases two (02) and three (03) of this thesis provide an important contribution by showing that the number of parts to be stored to hedge against shortages during periods of non-production increases with the age of the machine. The number of parts also increases with the number of breakdowns. Preventive maintenance is one of the most common practices in manufacturing. It improves the availability of equipment when they undergo a gradual deterioration.

In the fourth phase of this work, we introduce preventive maintenance strategy and analyze its effect on various policies. The system is modeled by a process that takes into account the degradation and preventive maintenance. The problem is solved by numerical methods. Sensitivity analyses are developed to show the relevance of the approach and the impact of introducing the preventive strategies.

Keywords: Optimal control, Production planning, Capacity management, repair/replacement, deterioration, Preventive maintenance, Manufacturing systems.

TABLE DES MATIÈRES

LISTE DES TABLEAUX

Page

LISTE DES FIGURES

LISTE DES ABRÉVIATIONS, SIGLES ET ACRONYMES

x_0	Stock initial du produit
x^+	Stock positif du produit
x^-	Stock négatif du produit
$x(t)$	Stock du produit à l'instant t
h_x	Pas de discrétisation suivant l'inventaire du produit
$X_{\min}$	Niveau de stock minimal
$X_{\max}$	Niveau de stock maximal
$a(t)$	Age de la machine à l'instant t
h_a	Pas de discrétisation suivant l'âge de la machine
$u(t)$	Taux de production à l'instant t
$u^*(t)$	Taux de production optimal à l'instant t
d	Taux de demande
$\xi(t)$	Processus de Markov en temps continu et état fini décrivant le système
ρ	Taux d'actualisation
α	Mode de la machine
Ω	Ensemble des modes de la machine
$J(.)$	Coût moyen sur un horizon infini
$q_{\alpha\beta}(.)$	Taux de transition de la machine du mode α au mode β
$q^s{}_{\alpha\beta}(.)$	Taux de transition de la machine du mode α au mode β avec remplacement de la machine au dessus de l'âge s
T	Instant de dernière remise en marche de la machine
N	Nombre maximal de pannes
M	Limite d'âge de réparation de la machine pour l'ensemble des pannes
$\omega(.)$	Taux d'envoi de la machine en maintenance préventive
$\omega_{\max}$	Taux d'envoi maximal de la machine en maintenance préventive

τ	Instant de saut du processus
$Q(.)$	Matrice des taux de transition de la machine
$\bar{\alpha}$	Disponibilité de la machine
$\Gamma(.)$	Ensemble des commandes admissibles
$V(.)$	Fonction valeur
n	Nombre de pannes
$a_n(.)$	Age de la machine avant la $n^{ième}$ panne
A_n	Age virtuel de la machine avant la $n^{ième}$ panne
$G(\cdot)$	Fonction coût instantané
n_p	Nombre de pannes avant remplacement
s	Age de la machine au dessus duquel la machine doit être systématiquement remplacée si une panne survient
N_m	Nombre maximal de pannes avant remplacement systématique de la machine si une autre panne survient.
S_n	Age de la machine au dessus duquel la machine doit être systématiquement remplacée si la panne $(n+1)$ survient
θ	Facteur d'âge virtuel
C_0	Coût de remplacement de la machine
C_m	Coût de maintenance préventive de la machine
C_{rep}	Coût par unité de temps de réparation de la machine
$C_1(\cdot)$	Coût instantané de réparation de la machine
$w(\cdot)$	Fonction coût de réparation et de remplacement
$h(\cdot)$	Fonction coût d'inventaire
C^-	Coût de stockage
C^+	Coût de pénurie

$k(t)$	Processus de Markov en temps continu et état fini décrivant la capacité du système
ε	Facteur de réduction des temps de réparation
HJB	Hamilton-Jacoby-Bellman
SMDP	Processus de décision semi-Markovien
SMDM	Modèle de décision semi-Markovien
HPP	Politique a seuil critique (Hedging point policy)

INTRODUCTION

L'objectif de l'industriel est de satisfaire le client, l'actionnaire et l'employé, tout en faisant un maximum de profits. La réalisation d'un maximum de profits se fait notamment en minimisant les coûts liés à la gestion des ressources de l'entreprise. La gestion optimale des ressources nécessite un dimensionnement convenable desdites ressources. L'entreprise à cet effet ne doit disposer que de la capacité qu'il faut et au moment opportun. Cela suppose la connaissance de la capacité nécessaire, de celle disponible à tout moment dans le système et de tout ce qui peut l'affecter.

En contexte manufacturier, les ressources de production telles que les machines sont sujettes aux phénomènes aléatoires que sont par exemple les activités de production, les pannes, les réparations, les activités de maintenance et les variations de demande. Ces aléas se manifestent à travers les dégradations que les machines subissent et affectent les stratégies de production et de maintenance.

La multiplication des pannes et des réparations associées à l'activité de production accentuent la dégradation des machines et entraine des conséquences telles que la diminution progressive de la limite de la charge de la machine appelée communément capacité ou celle de la disponibilité de la machine. Si cette diminution progressive n'est pas prise en compte lors des choix d'acquisition des ressources, leur gestion ne sera pas réaliste.

Les solutions les plus évidentes en matière d'optimisation de la gestion des ressources sont les solutions telles que la planification optimale de la production, le contrôle optimal de maintenance (amélioration de la fiabilité ou de la disponibilité des équipements). Ces solutions se basent sur le concept selon lequel la capacité disponible dans le système est fixe ou bien varie peu. Or, dans le contexte industriel réel, la capacité change. Ce qui fait que si elle est insuffisante ou trop grande par rapport aux besoins, malgré l'utilisation de ces solutions, la minimisation des coûts ne sera pas réaliste. Une meilleure gestion devrait à cet effet intégrer la planification optimale de capacité à la planification optimale de la production et au contrôle optimal de maintenance afin de faire une minimisation réaliste des coûts.

L'objectif de cette thèse est d'élaborer la structure de la loi de commande qui détermine simultanément les stratégies optimales de gestion de ressources de production, de gestion de production et de contrôle de maintenance des systèmes dynamiques en environnement manufacturier. L'approche utilisée vise à considérer non seulement la dégradation des ressources, mais aussi à tenir compte de la performance des différentes interventions effectuées.

Notre travail apporte une contribution scientifique en reformulant les modèles mathématiques existants pour intégrer le vécu des machines en contexte manufacturier. Ces modèles sont résolus afin d'obtenir la loi de commande qui donne simultanément l'instant optimal d'acquisition ou de réduction de capacité, les politiques optimales de production et de maintenance. Les résultats de nos travaux sont confirmés à travers des études par modélisation, résolution numérique et analyse de sensibilité sur des cas de systèmes manufacturiers. Ces résultats permettront pour l'industrie, d'améliorer la productivité des ressources matérielles et humaines, de diminuer les coûts totaux encourus et par conséquent les coûts de produits finis.

Le prochain chapitre présente la structure des systèmes manufacturiers et une revue de littérature associée à la problématique de la commande optimale stochastique appliquée à la gestion de capacité des systèmes dynamiques en environnement manufacturier.

CHAPITRE 1

STRUCTURE DES SYSTEMES MANUFACTURIERS ET REVUE DE LITTÉRATURE

1.1 Introduction

La première partie de ce chapitre présente la structure des systèmes manufacturiers, domaine dans lequel s'est orientée la présente thèse. Nous présentons dans la deuxième partie du chapitre une revue de littérature sur les systèmes manufacturiers. Le chapitre se termine par la méthodologie, les contributions et la structure de cette thèse.

1.2 Structure des systèmes manufacturiers

Un système se définit comme un ensemble d'éléments ou d'entités qui interagissent entre eux selon un certain nombre de principes ou de règles. La figure 1.1 présente la structure globale du système considéré dans cette thèse avec ses entités, représentées par des rectangles. Dans chaque rectangle est inscrit le nom de l'entité, ce dernier se rapportant à son contenu ou à sa fonction.

Les entités d'un système manufacturier interagissent à l'aide des flux. Le flux est un transfert ou un échange qui va d'une entité à une autre. Cela peut être un échange d'informations, de données, un transfert de matières premières, de produits finis, d'équipements, de services ou de personnel. Sur le graphique de la figure 1.1, le flux est représenté par des flèches. Lorsque le flux est exprimé par quantité par unité de temps, on parle de débit.

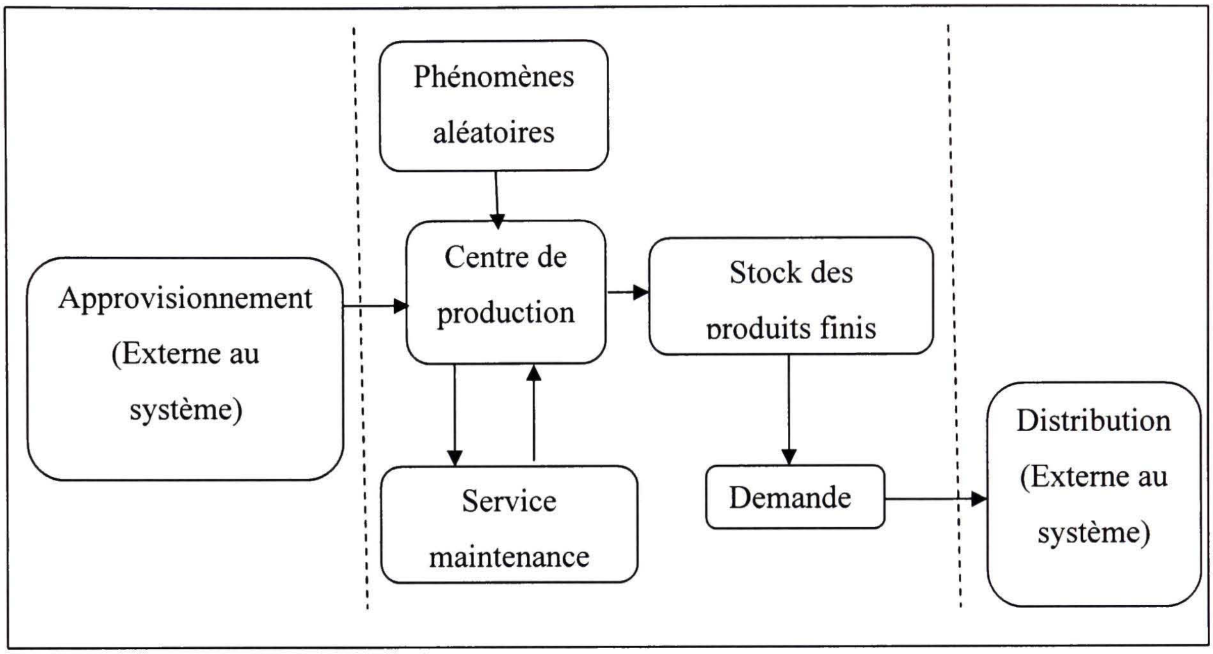


Figure 1.1 Structure d'un système manufacturier.

Le débit maximal ou limite maximale de la charge d'une entité est la capacité de production de cette entité. Si par exemple l'entité est une machine, alors on va parler de capacité de production de la machine.

La capacité de production d'une machine est la quantité maximale de produits que peut fabriquer cette machine par unité de temps. Elle s'exprime en nombre de pièces par unité de temps. La quantité de pièces fabriquée par la machine par unité de temps est le taux de production de la machine. Il est inférieur ou égal à la capacité de production.

Le système manufacturier est un ensemble de machines ou d'équipements, d'éléments de transport, d'unités de stockage, de personnes, d'ordinateurs ou tout autre élément mis ensemble pour la fabrication (Gershwin, 1994).

Le système évolue au cours du temps suivant des éléments tels que les pannes et les réparations des machines, les contraintes environnementales, l'âge de la machine, les fluctuations du stock et de la demande. Les fluctuations reliées aux machines (âge, pannes) affectent la disponibilité des machines ou bien limitent leur performance et engendrent soit un besoin de capacité additionnelle, soit un problème de disponibilité de capacité. Par conséquent, lorsque le besoin de capacité additionnelle se pose, il est possible de procéder à une expansion de capacité. Si par contre le système se retrouve avec un problème de disponibilité de capacité non soluble par les méthodes classiques que sont la maintenance corrective et préventive, on procède à un remplacement de machine. Ces opérations exigent une flexibilité du système manufacturier: flexibilité par rapport aux ressources, à l'expansion, aux produits, au routage des produits et à la quantité de produits fabriquée. L'objectif du système est de fabriquer des produits pour satisfaire les besoins de la clientèle tout en permettant de faire des profits. La nature du système manufacturier est de ce fait fortement dépendante des produits fabriqués. Nous présentons à la prochaine section la classification des systèmes manufacturiers.

1.3 Classification des systèmes manufacturiers

De nombreux critères permettent de faire une classification des systèmes manufacturiers. Ainsi, en se basant sur la nature (mode de circulation) et la dynamique du flux, on classifie les systèmes manufacturiers par leur organisation, leur mode et leur dynamique.

1.4 Organisation de production des systèmes manufacturiers

On distingue trois (03) principaux types d'organisations de production symbolisés par les lettres A, T et V. À ces principaux types s'ajoute le type X. La base de la lettre représente l'entrée des matières premières tandis que le sommet correspond à la sortie des produits finis. Certains auteurs associent les produits au type d'organisation de production et de ce fait identifient quatre (04) types de produits: A, T, V et X (Draghici, 1997).

Les systèmes manufacturiers de type A sont ceux pour lesquels peu de produits finis sont fabriqués à partir de nombreux composants. Les industries d'assemblage par exemple ont une typologie en A. Dans le cas des systèmes manufacturiers de type T, de nombreux produits finis sont assemblés à partir de composants communs. Les industries d'assemblage dans lesquelles des produits fabriqués massivement sont individualisés à la toute fin ont une typologie en T. Pour les systèmes manufacturiers de type V, un nombre restreint de matériaux conduit à un vaste éventail de produits finis. Dans l'industrie métallurgique comme les laminoirs, sont fabriqués à partir du lingot d'acier et aux diamètres variés du fer carré, rond et torsadé. Le système manufacturier de type X correspond à un système manufacturier pour lequel un dérivé des produits de type T est obtenu par une combinaison des produits de type A et V : de nombreux produits finis sont fabriqués à partir de peu de matériaux bruts.

1.5 Mode des systèmes manufacturiers

La circulation du flux permet de distinguer trois (03) types de systèmes manufacturiers : les systèmes manufacturiers discrets, les systèmes manufacturiers continus et les systèmes manufacturiers hybrides.

1.5.1 Systèmes manufacturiers discrets

On parle de système discret lorsque les produits fabriqués sont des pièces distinctes (Elhafsi et Bai, 1996). Le mode discret du système peut également se justifier par une suite d'opérations indépendantes réalisées sur des moyens indépendants, par exemple deux (02) machines distinctes (Gharbi et Kenne, 2003). On parle également de système discret lorsqu'il y a des points de rupture dans le flux sous forme d'en-cours et de stockages intermédiaires (Bironneau, 2000). D'autre part, le fait que chaque produit soit réalisé suivant un processus de production pouvant être fractionné pour permettre la reprise des produits finis confère à un système une nature discrète.

1.5.2 Systèmes manufacturiers continus

Lorsque les flux des matières ne s'interrompent pas entre les postes de travail consécutifs comme c'est le cas lors de la raffinerie, la coulée continue en fonderie, le système est continu. Les systèmes discrets et continus pris individuellement n'étant pas toujours en mesure de satisfaire les exigences d'une entreprise qui se veut compétitive sur le marché, on peut procéder à leur intégration dans un même système, donnant ainsi naissance aux systèmes de fabrication hybrides (Bhattacharya et Coleman, 1994).

1.5.3 Systèmes manufacturiers de nature hybride

Les systèmes de fabrication hybrides en général transforment une matière première continue à travers des équipements qui fonctionnent de façon partiellement discrète. Leur configuration correspond aussi bien à celle des systèmes de type A et T que celle des systèmes de type X. Les systèmes de fabrication hybrides ne sont donc ni continus, ni discrets par ce que les processus de types continus alternent avec des évènements discrets (Lazarescu *et al.*, 1998). Les évènements discrets apparaissent à des instants discrets suite à des évènements ponctuels qui sont par exemple l'arrivée d'un client (Liberatore *et al.*, 1995), l'achèvement d'une tâche (Balduzzi, 2001). Lorsque ces évènements sont assujettis à l'itération d'une fonction d'évolution, le système de fabrication est dynamique (Elhafsi et Bai, 1996). Un système de fabrication dynamique est un système sujet à des changements dans le temps.

1.6 Dynamique des systèmes manufacturiers

En se basant sur leur dynamique, on classifie les systèmes manufacturiers en systèmes à dynamique déterministe et systèmes à dynamique stochastique. Un système manufacturier a une dynamique stochastique si au moins une de ses sorties ou un de ses paramètres est aléatoire. Dans le cas contraire, le système a une dynamique déterministe (Sader et Sorensen, 2003). Des paramètres d'un même système peuvent avoir une dynamique évoluant de façon déterministe (la demande par exemple, lorsqu'elle est à taux constant et connu) et d'autres de façon aléatoire (par exemple l'arrivée des pannes). Lorsqu'il n'y a pas d'interaction entre ces paramètres, alors le système a une dynamique hybride (Cassandras *et al.*, 1999; Savkin, 2003).

Les systèmes de production à dynamique hybride combinent par exemple une dynamique d'évolution du temps et des évènements (Cassandras *et al.*, 1999). C'est une dynamique qui demeure cependant sujette à des perturbations. Les perturbations sont des informations qui contredisent une prévision faite aussi bien sur le fonctionnement du système opérationnel que sur l'évolution des objectifs élaborés à partir de l'évolution de l'environnement (Megartsi, 1996). Ce sont également des actions qui engendrent des effets entravant le respect des objectifs ou tout événement imprévu et susceptible d'être néfaste pour le système de production (BAILLET, 1994).

Dans un environnement industriel où le marché est caractérisé par des incertitudes et des perturbations, les systèmes manufacturiers doivent être conçus de façon à permettre la modification rapide de leur capacité pour satisfaire de façon optimale la demande. La prochaine section présente la gestion de capacité des systèmes manufacturiers.

1.7 Gestion de capacité des systèmes manufacturiers

La gestion optimale de capacité consiste à utiliser de manière rationnelle et judicieuse la capacité. L'utilisation rationnelle des ressources exige au préalable une taille appropriée de ces ressources. En environnement incertain, la capacité disponible peut devenir insuffisante ou au contraire être excessive. Dans les deux (02) situations, il faut envisager soit l'acquisition de capacité additionnelle, soit une réduction de capacité. La gestion de capacité d'une ressource telle qu'une machine va permettre par exemple de savoir :

- Quand fabriquer des pièces avec la machine;
- À quel rythme (débit, taux) fabriquer les pièces;
- Quand arrêter la machine et ne pas produire (même s'il n'y a pas de défectuosité);
- Quand envoyer la machine en maintenance préventive;

- À quelle vitesse effectuer la maintenance corrective en cas de panne, etc.

L'objectif étant de permettre à l'entreprise de satisfaire la demande et faire des profits. En environnement incertain caractérisé également par la dégradation des machines, la capacité diminue en conséquence. À long terme, les vieilles machines sont remplacées par de nouvelles, ce qui implique une augmentation de capacité suite au remplacement. Une bonne gestion de capacité doit tenir compte de ces fluctuations et de ce fait est indissociable des décisions d'expansion ou de la réduction de capacité. Cependant, combiner tous ces éléments rend le problème d'optimisation très complexe et dans certains cas insoluble analytiquement. Dans la section suivante, nous allons présenter une revue de littérature sur la commande optimale stochastique appliquée à la gestion de capacité des systèmes manufacturiers.

1.8 Revue de littérature de la gestion de production, de capacité et de maintenance des systèmes manufacturiers

Bien que la littérature soit riche en travaux sur les systèmes manufacturiers, peu d'entre eux traitent, par une approche combinée, la gestion de production et d'acquisition de capacité en environnement incertain. Dans leur travail sur les décisions hiérarchiques de planification de production et d'expansion de capacité des systèmes manufacturiers en environnement incertain, Sethi *et al.* (1995) répertorient trois (03) approches couramment utilisées. Ces approches utilisent une hiérarchisation à deux (02) niveaux (supérieur et inférieur). La première approche, utilisée par Dempster *et al.* (1981) permet au niveau supérieur de déterminer le nombre optimal de machines à acheter et au niveau inférieur la programmation de tâche à assigner aux machines. La deuxième approche, celle de Schneeweiss et Schröder (1992) développée pour le service maintenance de la Deutsche Lufthansa AG détermine au niveau supérieur le nombre et l'agencement d'outils à mettre à la disposition du service de maintenance. Au niveau inférieur, elle utilise les données du niveau supérieur pour garantir un niveau de service spécifique aux différents composants à réparer, sachant que la demande est stochastique. La troisième approche, proposée par Bitran *et al.* (1986) a été développée pour un système manufacturier en environnement caractérisé par des incertitudes résultant d'une estimation de demande et d'une révision des prévisions. Dans cette dernière approche, le niveau supérieur considère une structure hiérarchique de produits à deux (02) niveaux pour déterminer la séquence de familles de produits et la taille de lot dans chaque famille. Le niveau inférieur détermine ensuite la taille des lots à fabriquer à chaque période pour chaque type de produit.

A la suite des travaux précédents, Sethi *et al.* (1992) traitent le cas d'un système manufacturier devant satisfaire une demande à taux constant et considèrent que sur un horizon infini, l'acquisition de capacité ne peut se faire qu'une seule fois. Ils font ensuite une extension de ce problème aux cas de systèmes manufacturiers faisant face à une demande monotone et croissante, mais sur un horizon fini durant lequel le système peut acquérir un nombre donné et fini de machines (Sethi *et al.*, 1995). Ce travail leur a permis de montrer, pour un système fabricant plusieurs types de produits avec des machines tombant en panne de manière aléatoire, que lorsque les pannes et les réparations sont suffisamment fréquentes, le coût de leur plan d'expansion converge asymptotiquement vers le coût optimal.

La richesse des systèmes manufacturiers dépend de leur aptitude à générer, à s'approprier et à combiner de nombreux éléments de développement et l'intégration des nouvelles technologies de production. A cet effet, Rajagopalan *et al.* (1998) ont introduit dans leur travail le remplacement d'anciennes machines par de nouvelles dans un environnement caractérisé par la prévision d'une série de percées technologiques, mais dont l'ampleur et le calendrier demeurent incertains. Par la suite, en considérant le temps d'acquisition d'équipements comme une variable de décision continue, Çakanyildirm *et al.* (1999) minimisent les coûts de pénurie et de capacité pour une entreprise fabricant un produit pour satisfaire une demande aléatoire. Ils trouvent que la séquence optimale d'acquisition de machines est indépendante de la demande. Cependant, pour un système fabricant plusieurs produits, Zhang *et al.* (2004) montrent que ce n'est pas réaliste dans la mesure où l'attribution de capacité aux produits n'est pas directe. Ces derniers ont fait une extension aux cas de systèmes fabricant plusieurs produits en temps discret en proposant un schéma d'allocation de capacité en mesure de résoudre les problèmes comportant un nombre élevé de périodes de temps, de produits et de machines. En combinant une demande non stationnaire et incertaine au schéma d'allocation de capacité, ils ont obtenu non seulement un modèle traitable, mais incluant encore plus de contraintes.

Pour améliorer la disponibilité de la capacité du système dans le cas des systèmes manufacturiers se dégradant avec l'âge des machines, plusieurs auteurs dont Kenne et Gharbi (1999); Kenne *et al.* (2003); Kenne et Gharbi (2004); Liu *et al.* (2004) et Gharbi et Kenne (2005) ont contrôlé le taux de réparation des machines, sans toutefois envisager une acquisition supplémentaire de capacité. Dans le même ordre d'idée, Boukas et Haurie (1990) contrôlent le taux d'envoi en maintenance préventive de la machine. Kenne et Nkeungoue (2008) en plus du taux de maintenance corrective, contrôlent le taux d'envois des machines en maintenance préventive pour améliorer la capacité du système. Par ailleurs, une machine qui vient d'être installée et qui fonctionne parfaitement est soumise à des événements tels que les pannes, les réparations, l'usure, la fatigue, les fissures, la corrosion et parfois l'érosion. Si les réparations et autres opérations de maintenance ne sont pas parfaites, alors avec le temps et les cycles de production, la machine vieillit et la fréquence de défaillances augmente. À long terme, il est économiquement justifiable de remplacer cette machine. Le problème qui se pose souvent est de savoir à quel moment (âge de la machine, nombre de pannes/réparations ou maintenances préventives) remplacer la machine versus continuer à la réparer. Plusieurs auteurs dont Makis et Jardine (1993) et Love *et al.* (2000) ont déterminé la politique optimale de réparation versus le remplacement d'un système manufacturier subissant des réparations imparfaites.

Une réparation imparfaite remet l'âge de la machine à un niveau qui est plus petit que l'âge réel, rajeunit la machine mais ne la renouvelle pas intégralement (Makis and Jardine, 1991; Love *et al.*, 2000; Doyen and Gaudoin, 2004; Cassady *et al.*, 2005; Shirmohammadi *et al.*, 2007). Makis et Jardine (1993) et Love *et al.* (2000) ont montré que lorsque le coût de réparation ne dépend que de l'âge du système, alors la politique optimale de remplacement tombe dans la classe de politiques appelées politiques-T, examinés antérieurement dans la littérature pour les modèles nécessitant une réparation minimale. Une réparation minimale remet la machine dans son état juste avant panne (Phelps, 1981; Nakagawa et Kowada, 1983; Ohnishi *et al.*, 1994; Aven et Castro, 2008). Phelps (1983) a utilisé un modèle de décision semi-Markovien et a démontré qu'une stratégie dans laquelle les réparations minimales sont effectuées jusqu'à l'âge T, avec un remplacement par une nouvelle machine à la première panne après T, est la stratégie optimale à utiliser sur un horizon infini. Nous référons le lecteur aux travaux de Valdez-Flores et Feldman (1989) pour plus de détails sur la politique de réparation minimale.

En utilisant également un modèle de décision semi-Markovien, Makis et Jardine (1993) et Love *et al.* (2000) ont déterminé la politique de réparation versus le remplacement d'une machine subissant des réparations imparfaites se traduisant par un âge virtuel que prend la machine au fil des réparations.

Une bonne étude conceptuelle prévoyant les possibilités d'expansion ou de réduction de capacité et une mise en place efficace constituent des atouts de modélisation en vue de la commande optimale stochastique des systèmes manufacturiers. Dans la prochaine section, nous présentons la modélisation des systèmes manufacturiers.

1.9 Modélisation des systèmes manufacturiers

La modélisation est la phase de conception du système au cours de laquelle l'utilisateur fait une représentation de l'ensemble des paramètres du système (Kubota *et al.*, 1999). Aussi est-il important de faire au préalable une séparation explicite des fonctions de prise de décision des fonctions d'exécution (Jones et Saleh, 1990). De ce fait, le système doit être décomposé en deux (02) parties complémentaires (George *et al.*, 1998): la partie opérative et la partie commande. La partie opérative est constituée de l'ensemble des machines, du flux matériel et de matière, des opérateurs et des outils de productions. La partie commande par contre est le niveau de développement des stratégies de commande et de prise des décisions à long terme.

On rencontre dans la littérature plusieurs approches de modélisation des systèmes manufacturiers. Megartsi (1996) donne une étude comparative des méthodes d'analyse et de modélisation des systèmes de production. La modélisation doit faire ressortir de façon naturelle la nature discrète, continue, déterministe, stochastique, dynamique ou hybride du système (Liafa, 2003). Cependant, malgré les efforts effectués pour modéliser les systèmes de production, il demeure difficile de définir un modèle qui couvre la totalité des aspects pris en compte dans le développement des systèmes manufacturiers (Magartsi, 1996). Aucun modèle n'est donc parfait et très souvent le jugement d'un expert industriel est requis si les résultats mathématiques ne s'accordent pas avec la réalité (Gershwin, 1994).

Le modèle est indissociable des activités et permet de faire le choix de commande du système que nous présentons dans la section suivante lorsqu'on est dans un environnement incertain.

1.10 Commande optimale stochastique des systèmes manufacturiers

Diverses approches sont utilisées pour la recherche des solutions de problèmes de commande optimale stochastique appliquée à la gestion de production des systèmes manufacturiers. Les plus utilisées sont :

- l'algorithme;
- l'intelligence artificielle (Basnet et Mize (1986); Chiodini (1995));
- l'heuristique (Stecke et Solberg (1981); Williams et Wirth (1996); Thesen (1999));
- la simulation (Kimemia et Gershwin (1983); Manz *et al.* (1989); Haurie *et al.* (1990) Kenne et Gharbi (2004)).

Bien que ces approches présentent de nombreux avantages, il est à noter qu'elles utilisent pour trouver une solution une relaxation plus ou moins importante. Dans le contexte des systèmes manufacturiers, les résultats disponibles ne sont pas entièrement satisfaisants par ce qu'ils ne tiennent pas compte des aléas et autres sources de perturbation du système (pannes, demande) et de ce fait, les solutions obtenues ne sont pas réalistes. Pour cette raison, la théorie de commande stochastique a été mise au point. C'est une approche permettant au modèle développé de tenir compte des aléas et de la dynamique discrète ou continue du système. Elle consiste à trouver des stratégies de commande qui sont soit moins simplificatrices que celles des autres approches, soit elles permettent sur un horizon infini d'aboutir au problème original. Pour y parvenir, la théorie de commande stochastique combine plusieurs autres approches.

Pour des systèmes dont l'état à un instant donné ne dépend que de ses états antérieurs (Processus de markov homogènes), Rishel (1975) a développé les conditions d'optimum (nécessaires et suffisantes) pour obtenir la solution optimale en utilisant la programmation dynamique. Basés sur le formalisme de Rishel (1975), Olsder et Suri (1980) modélisent la commande stochastique de planification de production d'un système manufacturier sujet aux pannes aléatoires suivant un processus markovien homogène. Ce qui leur permet d'obtenir l'équation de programmation dynamique de la politique de commande optimale, mais dont ils n'ont pu obtenir la solution à cause de la complexité du problème.

À la suite des travaux ci-haut cités, Kimemia et Gershwin (1983) modélisent les incertitudes du système par des processus Markoviens homogènes pour déterminer la politique de production dont le taux de production permet de minimiser le coût de stockage et de pénurie. Le processus d'états homogènes suppose que les taux de transition sont constants. Ce n'est pas le cas en réalité dans les systèmes de production puisque des critères tels que l'âge influencent le taux de transition (Boukas et Haurie (1990); Kenne et Boukas (2003)).

Akella et Kumar (1986), Chiodini (1986) montrent que pour les systèmes Markoviens homogènes (Taux de transition constants), la politique permettant de maintenir un stock de sécurité non négatif pendant les périodes d'excès de capacité pour prévenir les futures insuffisances de capacité est une politique de commande optimale. Cette politique est appelée politique à seuil critique ou Hedging point policy ou (HPP) (Liberopoulos et Caramanis, 1994).

Une formulation rigoureuse de ce problème est fournie dans les travaux de Gershwin (1994) pour la modélisation de la dynamique et l'optimisation de la production, la résolution analytique pour le cas d'une machine produisant un seul type de produit dans les travaux d'Akella et Kumar (1986).

Dans le cas des systèmes manufacturiers complexes, on montre que la fonction qui réalise le coût optimal appelée fonction valeur doit satisfaire un ensemble d'équations différentielles appelées équations d'Hamilton-Jacobi-Bellman (HJB). Nous référons le lecteur aux travaux de Boukas et Haurie (1990); Boukas et Kenné (1997); Rishel (1975) pour plus de détails sur les équations d'HJB. Pour les systèmes complexes dont les conditions d'optimum sont décrites par les équations d'HJB, le calcul réel de la commande optimale demeure insurmontable analytiquement et parfois même numériquement dans des cas de systèmes plus larges (Kenne et Gharbi, 2001).

Sachant qu'il n'existe pas de solution analytique connue aux équations d'HJB, Boukas et Haurie (1990) et Yan et Zhang (1997) ont trouvé une solution à ce problème de commande optimale stochastique en utilisant une méthode numérique basée sur l'approche de Kushner (Kushner et Dupuis, 1992) pour un système fabricant plusieurs produits. La politique de commande obtenue, par ce qu'elle est asymptotiquement optimale, on parle de sous-optimalité de la politique de commande.

En se basant sur la méthode de résolution et les résultats de Yan et Zhang (1995) plusieurs auteurs ont pu combiner à l'approche numérique une approche expérimentale basée sur la simulation et étendre le concept aux processus non-markovien. En effet, les pannes et les réparations des machines ne sont pas toujours modélisables par des processus markoviens.

Ce cas de figure a été étudié à partir des solutions obtenues pour les processus markoviens, à travers une extension du concept de la politique à seuil critique aux processus non-markoviens (Kenne et Gharbi, 2000; Gharbi et Kenne, 2003). L'approche de commande utilisée est basée sur les plans d'expérience et les modèles de simulation. Ce qui a permis d'obtenir des solutions pour le cas d'une machine fabricant deux (02) produits, deux (02) machines fabricant cinq (05) produits (Gharbi et Kenne, 2003).

Dans ces travaux, la maintenance corrective est effectuée après détection d'une panne et est destinée à remettre le système de production dans l'état lui permettant de produire avec sa pleine capacité. Cette hypothèse est loin d'être réaliste. Les pannes, les dégradations, les réparations et certaines activités de maintenance préventive sont des phénomènes aléatoires. Dans un tel contexte, la probabilité de panne des machines croit en fonction de l'âge et de ce fait, la distribution des pannes dépend de l'âge de la machine. Cette dépendance affecte le contrôle du flux de production du système et la recherche des politiques de contrôle optimales doit en tenir compte.

Le problème de contrôle de flux optimal avec distribution des pannes de la machine dépendant de l'âge a été traité par Kenne et Gharbi (1999) pour quatre (04) types de distribution de panne. Dans leur travail, il est montré pour une machine fabricant un produit que la politique à seuil critique, sous des hypothèses restrictives, est optimale. Le système est modélisé par des chaines de Markov non homogènes, en termes de dépendance des taux de pannes de la machine de l'âge. A l'aide d'une procédure d'optimisation basée sur l'approche analytique, le modèle de simulation, le plan d'expérience, l'analyse de régression et les surfaces de réponse, ils trouvent expérimentalement les paramètres de la politique à seuil critique modifiée qui donnent la meilleure approximation du coût total encouru. Leur approche permet de représenter les interactions entre les facteurs et leurs effets sur les surfaces de réponse, ce que plusieurs autres approches telles que l'approche numérique ne permettent pas de faire.

Les travaux de Boukas et Haurie (1990), Kenne et Gharbi (1999) prennent en considération une probabilité de pannes des machines croissante en fonction de l'âge. L'âge étant défini comme une fonction de la production totale de la machine depuis sa dernière réparation ou maintenance. Toutefois, ces auteurs procèdent à une restauration de l'âge des machines à zéro après les actions de réparation et de maintenance préventive. Cette procédure annule la considération de la possibilité des pannes destructives et de ce fait ne traduit pas la vie réelle des systèmes manufacturiers.

Nous avons présenté dans cette section les diverses considérations faites et les diverses approches utilisées dans la littérature pour résoudre les problèmes de gestion de capacité et de commande des systèmes manufacturiers en environnement incertain. Au regard de ces considérations et des approches, on note une insuffisance lorsque les ressources sont sujettes à une dégradation progressive. Les machines des systèmes manufacturiers sont remplacées à long terme, étant donné qu'elles subissent une dégradation. Cette dégradation n'est non seulement pas toujours constante, mais peut s'accentuer dans le temps et avec le nombre de fois qu'il y a intervention sur la machine. Malgré toutes les tentatives, aucune approche n'a développé le modèle qui intègre simultanément la gestion optimale de production, de capacité, de remplacement, de maintenance corrective et préventive des ressources du système manufacturier. Plusieurs raisons expliquent ce manque : en contexte de dégradation, la dynamique du système change dans le temps et d'une intervention à une autre. En intégrant la dégradation progressive des machines, on intègre également la notion de mémoire et par conséquent le modèle devient non markovien. Le problème d'optimisation devient de ce fait très complexe et nous proposons à la section suivante la méthodologie proposée pour résoudre ce problème.

1.11 Méthodologie proposée

La méthodologie adoptée pour réaliser le travail consiste à :

- Développer les modèles mathématiques correspondant à la dynamique des systèmes manufacturiers considérés;
- Partir des modèles simples (markoviens) et incorporer progressivement des paramètres qui rendent les modèles plus réalistes (semi-markoviens);
- Élaborer l'approche de résolution des conditions d'optimum;
- Fournir des exemples au moyen de résolutions numériques;
- Faire des analyses de sensibilité pour confirmer les structures des politiques obtenues;
- Tirer les conclusions conséquentes.

Le schéma global est présenté à la figure ci-dessous :

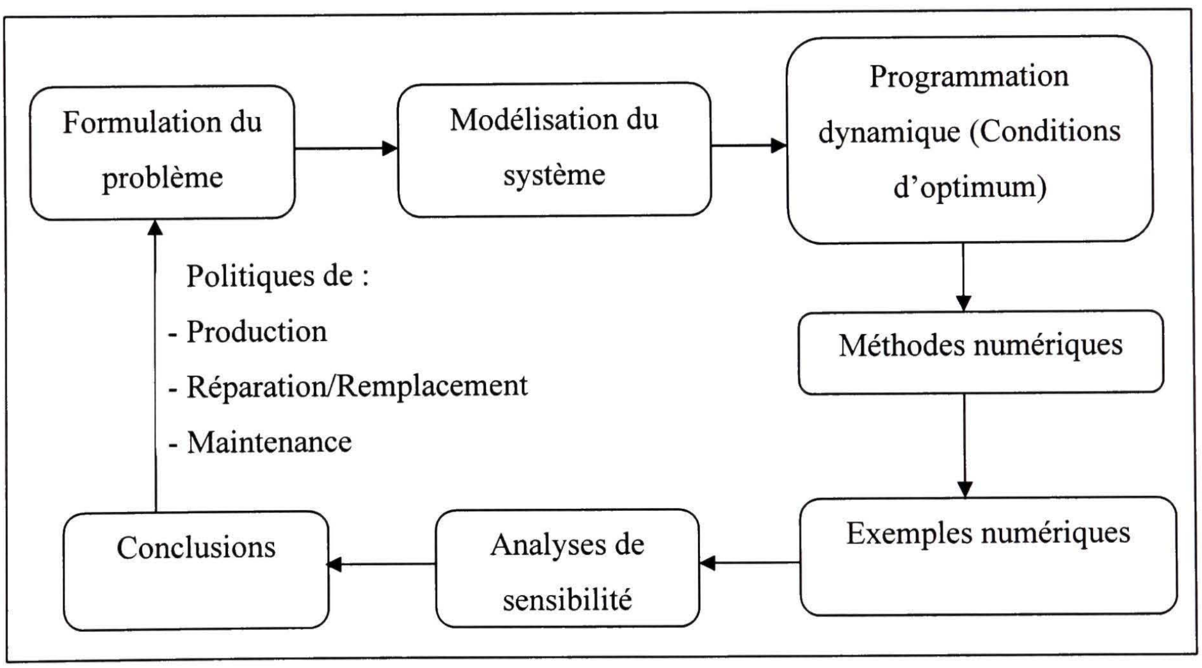


Figure 1.2 Méthodologie de la recherche proposée.

La prochaine section présente les contributions de cette thèse.

1.12 Contributions et structure de la thèse

Cette thèse propose une approche de formulation et de résolution du problème de commande optimale stochastique des systèmes dynamiques en contexte manufacturier. Cette approche permet de trouver la stratégie optimale de gestion de capacité, de gestion de production et de contrôle de maintenance qui tient compte du vécu des machines.

Dans le domaine de la gestion simultanée de production et de capacité au moyen des stratégies de maintenance et de remplacement des machines des systèmes manufacturiers en environnement incertain, cette recherche apporte une contribution par :

- La prise en compte de la dégradation progressive que subit la machine combinée à des réparations imparfaites;
- L'utilisation d'une approche simultanée basée sur un processus de prise de décision semi-Markovien pour déterminer la politique optimale de réparation/remplacement et la politique de production lorsque l'imperfection des réparations se traduit par un temps de réparation croissant avec le nombre de pannes;
- La prise en compte de l'âge virtuel que prend la machine à mesure que le nombre de pannes augmente;
- L'introduction des stratégies de maintenance préventive pour l'amélioration de la disponibilité des machines en contexte de dégradation et de réparations imparfaites;
- L'application de la notion de facteur de réduction d'âge et d'intensité de défaillance à la réduction du temps de réparation au moyen de la maintenance préventive;
- La résolution de ces problèmes d'optimisation par des méthodes numériques;
- La confirmation des résultats et de la structure des politiques obtenus à travers des exemples numériques et des analyses de sensibilité permettant de matérialiser et d'interpréter les politiques obtenues;

La suite de cette thèse comporte cinq (05) chapitres et une conclusion. Pour spécifier le contenu de chacun des chapitres, définissons les différents types de dégradations :

Type 1: Dégradation proportionnelle à la production. L'âge de la machine est une fonction du nombre de pièces fabriquées;

Type 2: Dégradation proportionnelle au temps de production. L'âge de la machine est une fonction du nombre d'unités de temps passé en opération;

Type 3: Temps de réparation croissant avec le nombre de pannes;

Type 4: Accumulation d'âge de la machine après chaque réparation (âge virtuel).

Le tableau ci-dessous récapitule les différentes considérations de chaque chapitre. La considération est symbolisée par la lettre "x".

Tableau 1.1 Considérations de chaque chapitre

	Chapitres	2	3	4	5	6
Considérations de chaque chapitre	Planification de production	x	x	x		x
	Acquisition de capacité additionnelle	x				
	Réparation/Remplacement		x	x	x	x
	Stratégies de maintenance corrective	x				
	Stratégies de maintenance préventive				x	x
	Dégradation de type 1		x	x		x
	Dégradation de type 2		x	x	x	x
	Dégradation de type 3			x		
	Dégradation de type 4				x	
	Stratégies de réduction des temps de réparation au moyen de la maintenance préventive				x	

Les contributions de cette thèse sont obtenues à travers la participation à deux (02) conférences et la rédaction de cinq (05) articles de revues.

Les articles de conférence sont référencés par :

- F. I. Dehayem Nodem, J. P. Kenne and A. Gharbi (2005). "*Capacity expansion, production and maintenance planning in stochastic manufacturing systems.*" International Conference on Numerical Analysis and Applied Mathematics (ICNAAM), pp. 423-427.

- F. I. Dehayem Nodem, J. P. Kenne and A. Gharbi (2008). "*A replacement policy of deteriorating production systems subject to imperfect repairs.*" 3rd IEEE Conference on Industrial Electronics and Applications (ICIEA), pp. 6-11.

Les articles de revues sont présentés dans les cinq (05) prochains chapitres.

L'article du deuxième chapitre étudie l'impact de l'introduction des stratégies de maintenance corrective des équipements sur les décisions d'acquisition de capacité et de planification de la production. Cet article a été publié dans la revue Applied Mathematical Sciences (AMS) sous la référence: F. I. Dehayem Nodem, J. P. Kenne and A. Gharbi (2008). "*Joint optimization of capacity expansion, Production and Maintenance strategies in Stochastic Manufacturing System.*" Applied Mathematical Sciences, 2(5): pp. 195-212.

Dans l'article du troisième chapitre, nous avons introduit la dégradation progressive des équipements. Nous avons de ce fait travaillé sur des machines pour lesquelles le vieillissement se traduit par l'âge que prend la machine chaque fois qu'une pièce est fabriquée. Nous avons également tenu compte du temps de réparation croissant avec le nombre de pannes. Cet article est en révision dans International Journal of Production Economics (IJPE) sous la référence : F. I. Dehayem Nodem, J. P. Kenne and A. Gharbi (2008) "*A replacement policy of production systems subject to imperfect corrective maintenance.*" Submitted on July 29, 2008; Revised on February 11, 2009. Submission Confirmation Ms. Ref. No.: IJPE-D-08-00591R1.

Dans l'article du chapitre quatre, nous avons introduit la notion d'âge virtuel de la machine. C'est une accumulation d'âge que prend la machine chaque fois qu'elle séjourne en opération, qu'elle subit une panne et qu'elle est réparée. Cet article est sous presse dans European Journal of Operational Research (EJOR) sous la référence: F. I. Dehayem Nodem, J. P. Kenne and A. Gharbi (2008). "*Hierarchical Decision Making in Production and Repair/Replacement Planning With Imperfect Repairs Under Uncertainties.*" European Journal of Operational Research, 198(1), pp. 173-189.

Dans l'article du chapitre cinq, nous avons analysé l'impact de l'introduction des stratégies de maintenance préventive sur les politiques de réparation versus remplacement de la machine. Ici la maintenance préventive, en plus de réduire la probabilité de pannes de la machine diminue le temps de réparation à la prochaine panne. L'article est soumis dans Reliability Engineering & System Safety sous la référence: F. I. Dehayem Nodem, J. P. Kenne and A. Gharbi (2008) "*Preventive maintenance and replacement policies for deteriorating production systems subject to imperfect repairs.*" Submitted on December 18, 2008. Ms. Ref. No.: RESS-D-08-00413.

Dans l'article du chapitre six, nous avons, compte tenu de l'interaction entre les activités de maintenance préventive des systèmes manufacturiers et les activités de production, combiné ces deux éléments. Cette combinaison a permis de déterminer les politiques de production, de réparation/remplacement et de maintenance préventive de la machine. L'article a été soumis dans Applied Stochastic Models in Business and Industry sous la référence: F. I. Dehayem Nodem, J. P. Kenne and A. Gharbi (2008) "S*imultaneous control of production, repair/replacement and preventive maintenance of deteriorating manufacturing systems.*" Manuscript number: ASMB-09-68.

Nous finissons la thèse par une conclusion qui récapitule les principales contributions apportées et nos travaux futurs.

CHAPITRE 2

ARTICLE 1: JOINT OPTIMIZATION OF CAPACITY EXPANSION, PRODUCTION AND MAINTENANCE IN STOCHASTIC MANUFACTURING SYSTEMS

F. I. Dehayem Nodem [1], J. P. Kenne[1] and A. Gharbi[2]

[1]Mechanical Engineering Department, Production Technologies Integration Laboratory, University of Quebec,

[2]Automated Production Engineering Department, Production System Design and Control Laboratory, University of Quebec

École de technologie supérieure, 1100 Notre Dame Street West, Montreal, Quebec Canada H3C 1K3

This chapter is published as an article in Applied Mathematical Science. 2008. Vol. 2, no 5-8, pp. 195-212.

Résumé

Cet article présente un modèle de détermination conjointe des politiques optimales de production, d'expansion de capacité et de maintenance corrective d'un système manufacturier sujet aux pannes et réparations aléatoires. Des décisions concernant la production, la maintenance et l'investissement en expansion de capacité doivent être prises afin de minimiser l'ensemble des coûts encourus. Les coûts sont constitués par les coûts d'investissement en capacité, de maintenance corrective, d'inventaires et de pénuries dans un environnement caractérisé par des incertitudes. L'introduction de la stratégie de maintenance corrective dans le modèle proposé permet d'améliorer la disponibilité des équipements et de ce fait réduire le coût total encouru comparé aux modèles existants. Les variables de décision sont le temps d'arrêt aléatoire auquel acquérir une nouvelle machine, le taux de production et le taux de maintenance corrective avant et après acquisition d'une nouvelle machine. L'objectif est de minimiser les coûts d'investissement en capacité, d'inventaire et de pénurie sur un horizon infini. Les conditions d'optimum sont développées à l'aide de la programmation dynamique. Les méthodes numériques sont ensuite utilisées pour résoudre le problème et déterminer les diverses politiques de commande. Un exemple numérique et des analyses de sensibilité sont présentés pour illustrer l'utilité de l'approche proposée.

Abstract

This paper presents an analytical model for the joint determination of optimal production, corrective maintenance and capacity expansion policies for a repairable production system subject to random failures. The production system have to make decisions regarding production and maintenance, as well as investment in capacity expansion in order to minimize costs of investment, production, maintenance, inventories and backlogs in an uncertain environment. The introduction of corrective maintenance strategy in the proposed model improves the availability of the machines and hence reduces the total incurred cost compared to available models. The control variables are the random stopping times at which to purchase a new capacity, the production and corrective maintenance rates before and after capacity purchase. The objective is to minimize production, capacity investment, inventories, backlogs and maintenance costs over an infinite planning horizon. Optimality conditions are given and numerical methods are used to solve them and to determine the control policy. A Numerical example and sensitive analyses are presented to illustrate the usefulness of the proposed approach.

Keywords: Optimal Control, Numerical Methods, Production Planning, Capacity Management, Maintenance Control, Manufacturing Systems

2.1 Introduction

This paper models and illustrates an optimal policy for a manufacturing system with a simultaneous control of capacity expansion, production and corrective maintenance strategies. As demand increases, new machines must be purchased at given epoch (or stopping time) while production and preventive maintenance are well planned before and after the stopping time at which to purchase a new capacity. The system under study, described in Sethi and Zhang (1994), consists of a firm that must satisfy a given demand rate for its product over time to minimize its discounted cost of investment and inventory/shortage. The firm has an existing machine that is failure-prone with given rates of breakdowns and repairs. At a given time, due to demand fluctuations, the demand for the firm's product is higher than the average production capacity of the existing machine. However, the firm has some initial inventory of its products to absorb the excess demand for a few initial periods. Such a firm may have to increase its production capacity at some future time. For this purpose, the firm has an option to purchase a new machine, identical to the existing machine, at a given fixed cost, in order to double its average production capacity. This assumes that the firm has sufficient repair capacity to handle two machines event when they are both broken down during some time interval. The purpose of this paper is to extend the model presented in Sethi and Zhang (1994) by controlling both production with capacity expansion and corrective maintenance in order to reduce the overall incurred cost.

The objective of this work is, for the aforementioned purpose, to find the optimal policy which integrates simultaneously capacity management strategy, production and maintenance control for a flexible manufacturing system with a wafer demand trend approximated in this paper by a multiple steps staircase structure of the demand rate. The demand rate is assumed constant in the proposed model for a given planning horizon. For the considered manufacturing system, this assumption is motivated by the fact that the rate of change in the machines states is much larger than the rate at which the demand rate change significantly. In a more general context, capacity expansion, results of capacity purchase, can be done by machines, manpower and technology acquisitions or by using subcontracting to hedge against sudden demand jumps.

In this work, we focus only on capacity expansion by machine purchase. There have been some attempts to deal with the problem of capacity expansion by machines purchase under uncertainty. An extensive survey of early work of stochastic capacity expansion problems is provided in Luss (1982). Models that appeared after Luss (1982) are presented in Çakanyıldırım *et al.* (1999) where it is noted that when the time is a continuous variable rather than a discrete one, the capacity expansion problem may be formulated as an optimal control problem. For a single product with a stochastic demand process, Davis *et al.* (1987) used an investment rate function as the control variable to regulate the expansion rate. For multiple products with deterministic demand rates, Sethi and Zhang (1994) and Çakanyıldırım *et al.* (1999) studied machine capacity expansion when machines can break down randomly. They proposed a capacity planning using the mean available production capacity for a multiple identical machines system. When breakdowns and repairs happen sufficiently fast, they showed that the cost of their expansion plan asymptotically converges to the optimal cost. Unfortunately, as breakdowns increase (failures occurrence become fast), the system availability decreases. Although, there is a great literature based on the using of maintenance to increase production system availability, we refer the reader to Boukas and Haurie (1990) and Kenne and Boukas (2003) for preventive maintenance and to Boukas (1998), Kenne *et al.* (2003), Gharbi and Kenne (2005) and references therein, for corrective maintenance, none of those studies, based on production and maintenance planning, examined the case in which capacity expansion is considered. They assumed that the average machines capacity is sufficient to satisfy demand of produced parts in the considered horizon. In stochastic environment, while the system average capacity remains the same with possible demand jumps, the backlogs can increase, possibly without bounds, and hence increasing the overall incurred cost. In this paper, we propose to improve the capacity of the manufacturing system by capacity expansion and by controlling the machines repair rates.

An important question that arises is to know if the contribution of the approach proposed herein in terms of total cost reduction is significant compared to a fixed repair rate situation as in Sethi and Zhang (1994). The theory presented in this paper answers this question in the affirmative under reasonable assumptions (demand rates of various products are constants, the machines are completely flexible, etc). This theory is based on the fact that the structure of the control policy (production, capacity expansion and machine repair rates) can be obtained by using the fact that the value function is the unique solution to the associated Hamilton-Jacobi-Bellaman (HJB) equations. We first used a numerical approach to determine an approximate value function, instead of the true value function, to construct the control policy. Under certain appropriate conditions, the control policy constructed is asymptotically optimal as the difference between the true value function goes to zero (see Kenne *et al.* (2003) for details). Finally, we presented a numerical example and a sensitive analysis that illustrate the usefulness of the proposed approach.

The paper is organized as follows: In the next section, we state the model of the problem under consideration. In section 2.3, we present the HJB equations and show in section 2.4 that a numerical scheme can provide an approximation of the value function. Then in section 2.5, we present a numerical example and a sensitive analysis to illustrate the contribution of the paper. The paper is finally concluded in section 2.6.

2.2 Problem statement

The system under study consists of n machines producing one part type. The machines capacities are assumed herein to be described by a finite state Markov chain. After the investment in new capacity, the enhanced capacity process is represented by another finite state Markov process having a larger average capacity. The stochastic nature of the system is due to the machines that are subject to random breakdowns and repairs. At any given time, the system is in state $k_1(t) \in \{0,1,2,\cdots,m_1\}$ before capacity purchase and $k_2(t) \in \{0,1,2,\cdots,m_1+m_2\}$ if there is additional new capacity purchase m_2 at time $t=0$ with m_1 describing the existing maximum capacity. Each of the process $k_1(t)$, before capacity expansion, and $k_2(t)$, after the capacity expansion, is a Markov chain with state at time t describing the number of operational machines, called here capacity process of the system at time t.

Let $\{\mathcal{F}_1(t)\}$ and $\{\mathcal{F}_2(t)\}$ denoted the filtration generated by $k_1(s)$ and $k_2(s)$, $0 \le s < t$, respectively, i.e., $\{\mathcal{F}_1(t)\} = \sigma\{k_1(s) \colon s \le t\}$ and $\{\mathcal{F}_2(t)\} = \sigma\{k_2(s) \colon s \le t\}$, $s \in [0,t]$. We can describe the dynamics of the system by jump processes corresponding to the discrete states of the machines generated by a continuous time and discrete states Markov process

$k_1(t)$ or $k_2(t)$ with values in $M_1 = \{0,1,2,\cdots,m_1\}$ or $M_2 = \{0,1,2,\cdots,m_1+m_2\}$. For any $\mathcal{F}_1(t)-$Markov $\tau \geq 0$, the state of the system can then be described by a new process $k(t)$ as follows:

$$k(t) = \begin{cases} k_1(t) & \text{if } t < \tau \\ k_2(t-\tau) & \text{if } t \geq \tau \end{cases} \quad \text{and} \quad k(\tau) = k_2(0) := k_1(\tau) + m_2 \tag{2.1}$$

where τ is the purchase time of additional capacity at a cost K. Note that $0 \leq \tau \leq \infty$ and $\tau = \infty$ means not to purchase additional capacity.

Each machine in the system is either up or down and the system is either down ($k(t) = 0$) or up ($k(t) \neq 0$) having a maximum of $k(t) \neq 0$ units of capacity available at time t. The total production rate at each instant is limited by the capacity of the operational machines. Hence, at time t, the production rate depends on the machines states and thus is subject to sudden changes due to the dynamics of the stochastic process $k(t)$. The production constraint is then defined by:

$$p \cdot u(t) \leq k(t) \tag{2.2}$$

where $p = (p_1,\dots,p_n)$ is the vector of processing times of the part type on the machine M_i, with $i = 1,\cdots,m_1$ (before capacity expansion) or $i = 1,\cdots,m_1+m_2$ (after capacity expansion); and $u = (u_1,\dots,u_n)$ is the vector of the corresponding production rates ($0 \leq u_k \leq U_k^{\max}$ and $p_k \leq (U_k^{\max})^{-1}$).

To increase the system availability, we considered that the transition rate from a failure mode to operation mode is a control variable, called here $u_r(t)$. Thus, by controlling $u_r(t)$, one acts on the mean time to repair. The system capacity is then described by a finite state Markov chain that depends on the corrective maintenance policy. The transition rates matrix in such a situation is given by:

$$Q_m(u_r) = \{q_{\alpha\beta}^m(u_r)\} \tag{2.3}$$

with

$$q_{\alpha\beta}^{m}(u_r) \geq 0 \text{ if } \alpha \neq \beta \text{ and } q_{\alpha\alpha}^{m}(u_r) = -\Sigma_{\alpha\neq\beta}\, q_{\alpha\beta}^{m}(u_r)$$

where $q_{\alpha\beta}^{m}$ describes the transition rate of the system from mode α to mode β, with $m=1$ before capacity expansion and $m=2$ after capacity expansion.

For a production rate $u(t) \in R^n$, the surplus ($x(t) \in R^n$), of the manufacturing system under consideration (corresponding to inventory if positive or to backlog if negative) is described by the following equation:

$$\dot{x}(t) = u(t) - z, \quad x(0) = x \tag{2.4}$$

where $z \in R^n$ denotes the constant demand rate and x the initial surplus level.

For any capacity $k(t) \in M = M_1$ or M_2, let

$$U_{rk(t)} = \left\{u_{rk}, U_{rk}^{\min} \leq u_{rk} \leq U_{rk}^{\max}\right\},$$
$$U_{k(t)} = \left\{u_{k(t)} = (u_1, u_2, \ldots, u_n) \geq 0,\ p_1 u_1 + p_2 u_2 + \ldots + p_n u_n \leq k(t)\right\},$$

with $u_k \leq U_k^{\max}$.

The set of admissible decisions at state $k(t)$ is defined by:

$$\Gamma_r(k) = \left\{ \begin{array}{l} \left(\tau,\ u_{k(t)},\ u_{rk(t)}\right) = ((\tau, u_1, u_{rk}), (\tau, u_2, u_{rk}), \ldots, (\tau, u_n, u_{rk}))\ /\ u_{k(t)}(t) \in U_{k(t)}, \\ U_{rk}^{\min} \leq u_{rk(t)} \leq U_{rk}^{\max},\ 0 \leq u_{k(t)} \leq U_k^{\max} \end{array} \right\} \tag{2.5}$$

The control policy at state $k(t)$ is $(\tau, u(\cdot), u_r(\cdot))$ and $U_{rk}^{\min}, U_{rk}^{\max}$, $U_k^{\max}$ are minimum repair, maximum repair and maximum production rates respectively. Such a policy states that the decisions variables are production rate $u(\bullet)$, repair rate $u_r(\bullet)$ and time of additional capacity purchase $\tau \geq 0$.

The objective of the control problem is to minimize the following discounted function:

$$J(x,k,\tau,u(\bullet),u_r(\bullet)) = E\left(\int_0^{\infty} e^{-\rho t} G(x(t),u(t),u_r(t))dt + Ke^{-\rho\tau}\right) \tag{2.6}$$

where $x(0)=x$; $k(0)=k$; $G(x,u,u_r)=c_1^+x_1^+ + c_1^-x_1^- + c_r u_r$ is the instantaneous cost, $c_1^+ > 0$, $c_1^- > 0$, $c_r > 0$ are incurred costs per unit produced parts for inventory, backlog and corrective maintenance respectively. In addition $x^+ = \max\{0,x\}$ *and* $x^- = \max\{0,-x\}$.

The considered stochastic optimal control problem is to find an admissible control $(\tau, u(\cdot), u_r(\cdot))$ that minimizes $J(\cdot)$ given by equation (2.6) subject to equations (2.1)-(2.5). This problem is formulated in the next section as a dynamic stochastic optimization problem with a stopping time at which to purchase new capacity, corrective maintenance and production rates over time before and after the acquisition of the new capacity as decision variables.

2.3 Optimality conditions

Let $\upsilon(x,k)$ denote the value function or minimum expected discounted cost if there is no capacity purchase ($\tau = +\infty$):

$$\upsilon(x,k) = \inf_{(\infty,\, u(\bullet),u_r(\bullet)) \in \Gamma_r(k)} J(x,k,\tau,u(\bullet),u_r(\bullet)), \quad k \in M_1 \tag{2.7}$$

Let us define

$$h(x) = c_1^+x_1^+ + c_1^-x_1^-;$$

$$c(u_r) = c_r u_r$$

and

$$J_1(x,k,u(\bullet),u_r(\bullet)) = J(x,k,\infty,u(\bullet),u_r(\bullet)) = E\left(\int_0^\infty e^{-\rho t} G(x(t),u(t),u_r(t))dt\right)$$

Using the previous notation, $J_1(\cdot)$ can be rewritten as follows:

$$J_1(x,k,u(\bullet),u_r(\bullet)) = E\left(\int_0^\infty e^{-\rho t}\left[h(x(t)) + c(u_r)\right]dt\right), \; x(0)=x;\; k(0)=k$$

The value function $v(x,k)$ in the case where there is no need to purchase a new capacity is given by the following equation:

$$\upsilon(x,k)=\min_{u\in U_k,u_r\in U_{rk}} J_1(x,k,u(\cdot),u_r(\bullet))=\inf_{u(\bullet)}\inf_{u_r(\bullet)} J_1(x,k,u(\bullet),u_r(\bullet)) \tag{2.8}$$

It is shown in Boukas (1998) or in Kenne *et al.* (2003) that such a value function satisfies the set of Hamilton-Jacobi-Bellman equations given by the following expression:

$$\rho\upsilon(x,k)=\min_{u\in U_k,u_r\in U_{rk}}\left[(u-z)\upsilon_x(x,k)+Q_1(u_r)\upsilon(x,.)(k)+c(u_r)\right]+h(x),$$

$$\forall x\in\mathbb{R}^n,\ k\in\mathrm{M}_1$$

That is:

$$\rho\upsilon(x,k)=\min_{u\in U_k,u_r\in U_{rk}}\left[(u-z)\upsilon_x(x,k)+Q_1(u_r)\upsilon(x,.)(k)+G(x,u,u_r)\right],$$

$$\forall x\in\mathbb{R}^n,\ k\in\mathrm{M}_1$$

Let $\upsilon_a(x,k)$ denote the value function or minimum expected discounted cost if there is a capacity purchase at initial time ($\tau=0$).

$$\upsilon_a(x,k+m_2)=\inf_{(0,\ u(\bullet),u_r(\bullet))\in\Gamma_r(k)} J(x,k,\tau,u(\bullet),u_r(\bullet)),\quad k\in M_1 \tag{2.9}$$

From the definition of the process $k(t)$, see equation (2.1), it follows that:

$$\upsilon_a(x,k+m_2)\geq\upsilon(x,k),\quad k\in M_1$$

Let $J_{a0}(\bullet)$ be the cost function when there is capacity purchase at cost $K=0$ and define the corresponding value function as follows:

$$\upsilon_{a0}(x,k)=\min_{(\tau,u(\bullet),u_r(\bullet))\in\Gamma_r(k)} J_{a0}(x,k,\tau,u(\bullet),u_r(\bullet))$$

with

$$J_{a0}(x,k,\tau,u(\bullet),u_r(\bullet))=E\left(\int_0^\infty e^{-\rho t}G(x(t),u(t),u_r(t))dt\right)$$

This value function corresponds to the one of the control of production and corrective maintenance rates presented in Kenne *et al.* (2003) in the context of multiple identical machines manufacturing systems. The value function $\upsilon_{a0}(x,k)$ satisfies the set of Hamilton-Jacobi-Bellman equations given by the following expression:

$$\rho \upsilon_{a0}(x,k) = \inf_{u \in U_k, u_r \in U_{rk}} \left[(u-z)(\upsilon_{a0})_x(x,k) + Q_1(u_r)\upsilon_{a0}(x,.)(k) + c(u_r) \right] + h(x),$$

$\forall x \in \mathbb{R}^n,\ k \in M_1$

With $(\upsilon_{a0})_x(x,k)$, the gradient $\frac{\partial}{\partial x}\upsilon_{a0}(x,k)$

From equation (2.9), we have

$$\upsilon_a(x,k+m_2) = \inf_{(0,\ u(\bullet),u_r(\bullet)) \in \Gamma_r(k)} J(x,k,0,u(\bullet),u_r(\bullet)), \quad k \in M_1$$

and

$$\upsilon_a(x,k) = \inf_{(0,\ u(\bullet),u_r(\bullet)) \in \Gamma_r(k)} J(x,k,0,u(\bullet),u_r(\bullet)), \quad k \in M_2$$

$$= \inf_{(0,\ u(\bullet),u_r(\bullet)) \in \Gamma_r(k)} E\left(\int_0^\infty e^{-\rho t} G(x(t),u(t),u_r(t))dt + Ke^{-\rho\tau} \right)$$

Thus,

$$\upsilon_a(x,k) = \inf_{u \in U_k, u_r \in U_{rk}} E\left(\int_0^\infty e^{-\rho t} G(x(t),u(t),u_r(t))dt \right) + K$$

$$= \inf_{u \in U_k, u_r \in U_{rk}} J_{a0}(x,k,0,u(\bullet),u_r(\bullet)) + K$$

Hence,

$$\upsilon_a(x,k) = \upsilon_{a0}(x,k)\Big|_{\tau=0} + K$$

We then have:

$$\rho \upsilon_a(x,k) = \inf_{u \in U_k, u_r \in U_{rk}} \left[(u-z)\upsilon_a(x,k) + Q_1(u_r)\upsilon_a(x,.)(k) + c(u_r) \right] + h(x) + K,$$

$\forall x \in \mathbb{R}^n,\ k \in M_2$

Since we are interested in optimal purchase time, optimal production and corrective maintenance rates, we write the HJB equations as follows:

$$\begin{cases} \min\Big\{ \min_{u \in U_k, u_r \in U_{rk}} \big\{ (u-z).\upsilon_x(x,k) + G(x,u,u_r) + Q_1(u_r)\upsilon(x,k) \\ -\rho\upsilon(x,k), \upsilon_a(x,k+m_2) - \upsilon\ (x,k) \big\} \Big\} = 0 \qquad k \in M_1 \\ \min_{u \in U_k, u_r \in U_{rk}} \left[(u-z).(\upsilon_a)_x(x,k) + G(x,u,u_r) \right] + Q_2(u_r)\upsilon_a(x,.)(k) \\ -\rho\left(\upsilon_a(x,k) - K\right) = 0, \qquad k \in M_2 \end{cases} \tag{2.10}$$

The optimal control policy (τ^*, u^*, u_r^*) denotes a minimizer over U_{rk} of the right hand side of equation (2.10). This policy corresponds to the value function described by equations (2.7) and (2.9). Then, when the value functions are available, an optimal control policy can be obtained as in (2.10). However, an analytical solution of (2.10) is almost impossible to obtain. In the next section, we construct a near optimal control policy through numerical methods. It is by now well known that an approximation of the corresponding control policy or near optimal control policy can be obtained by a small perturbation of the true value function. This can be done by using numerical techniques which provide a close form of the value function under reasonable assumptions.

2.4 Numerical approach

In this section, we develop the numerical method for solving the optimality conditions presented in the previous section. This method is based on the Kushner approach (see Kushner and Dupuis (1992), Boukas and Haurie (1990) and Kenne *et al.* (2003) for details). The main idea behind this approach consists of using an approximation scheme for the gradient of the value functions $v(x,k)$ and $v_a(x,k)$. Let h denotes the length of the finite difference interval of the variable x. Using h, $v(x,\alpha)$ is approximated by $v^h(x,k)$ and $v_x(x,k)$ is approximated as in equation (2.11).

$$v_x(x,k)\times(u-z)=\begin{cases}\frac{1}{h}\left(v^h(x+h,k)-v^h(x,k)\right)\times(u-z) & \text{if } u-z>0\\ \frac{1}{h}\left(v^h(x,k)-v^h(x-h,k)\right)\times(u-z) & \text{otherwise}\end{cases} \tag{2.11}$$

Using h, $(v_a)_x(x,k)$ is approximated as in equation (2.12).

$$(v_a)_x(x,k)\times(u-z)=\begin{cases}\frac{1}{h}\left(v_a^h(x+h,k)-v_a^h(x,k)\right)\times(u-z) & \text{if } u-z>0\\ \frac{1}{h}\left(v_a^h(x,k)-v_a^h(x-h,k)\right)\times(u-z) & \text{otherwise}\end{cases} \tag{2.12}$$

With approximations given by equations (2.11) and (2.12) and after a couple of manipulations, the HJB equations (2.10) can be rewritten as follows:

$$\upsilon^h(x,k) = \min_{u \in U_k^h} \min_{u_r \in U_{rk}^h} \frac{1}{\left(\rho + Q_h^{k1}(u_r)\right)} \Big(\left[\upsilon^h(x+h,k) p_x^k(1) + \upsilon^h(x-h,k) p_x^k(2) \right] + G(x,u,u_r) + \sum_{k' \neq k} q_{kk'}^1(u_r) \upsilon^h(x,k') \Big) \qquad k \in M_1 \tag{2.13}$$

$$\upsilon_a^h(x,k) = \min_{u \in U_k^h, u_r \in U_{rk}^h} \frac{1}{\left(\rho + Q_h^{k2}(u_r)\right)} \Big(\left[\upsilon_a^h(x+h,k) p_x^k(1) + \upsilon_a^h(x-h,k) p_x^k(2) \right] + G(x,u,u_r) + \sum_{k' \neq k} q_{kk'}^2(u_r) \upsilon_a^h(x,k') + \rho K \Big) \qquad k \in M_2 \tag{2.14}$$

where (U_k^h, U_{rk}^h) is the discrete feasible control space or the so-called control grid and the other terms used in equations (2.13) and (2.14) are defined as follows:

$$Q_h^{k_i}(u_r) = \left| \frac{u-z}{h} \right| + \left| q_{k_i k_i}^i(u_r) \right|, \quad i = 1,2$$

$$p_x^k(1) = \begin{cases} \dfrac{u-z}{h} & \text{if} \quad u-z \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$p_x^k(2) = \begin{cases} \dfrac{z-u}{h} & \text{if} \quad u-z < 0 \\ 0 & \text{otherwise} \end{cases}$$

The system of equations (2.13) and (2.14) can be interpreted as the infinite horizon dynamic programming equation of a discrete-time, discrete-state decision process as in Boukas and Haurie (1990), Kenne *et al.* (2003), for capacity expansion, production and maintenance planning problems. The obtained discrete event dynamic programming can be solved using either policy improvement or successive approximation methods.

The next theorem shows that $v^h(x,k)$ and $v_a^h(x,k)$ are approximations to $v(x,k)$ and $v_a(x,k)$ for small step size h.

Theorem

Let $v^h(x,k)$ and $v_a^h(x,k)$ denote a solution to HJB equations (2.13) and (2.14). Assume that there are constants C_g and κ_g, C_g^a and κ_g^a such that

$$0 \le v^h(x,k) \le C_g(1+|x|^{\kappa_g})$$
$$0 \le v_a^h(x,k) \le C_g^a(1+|x|^{\kappa_g^a})$$

Then,

$$\lim_{h\to 0} v^h(x,k) = v(x,k)$$
$$\lim_{h\to 0} v_a^h(x,k) = v_a(x,k)$$

Proof:

The proof of this theorem can be obtained by extending the one presented in Yan and Zhang (1997).

□

In this paper, we use the policy improvement technique, given by the following algorithm, to obtain a solution of the approximating optimization problem.

Step 1: Initialisation

Choose $\delta \in \mathbb{R}^{+}$, set $n := 1,\ (v^h(x,k))^n := 0,\ (v_a^h(x,k))^n := 0,\ \forall k \in M_{1,2}\ \forall x \in G_x^h$

Step 2: Compute

$$(v^h(x,k))^{n-1} := (v^h(x,k))^n, \quad \forall k \in M_1, \quad \forall x \in G_x^h$$
$$(v_a^h(x,k))^{n-1} := (v_a^h(x,k))^n, \quad \forall k \in M_2, \quad \forall x \in G_x^h$$

Step 3: Compute the correspondent value function to obtain the control policy: $(\tau, u(\cdot), u_r(\cdot))$

Step 4: Test the convergence

$$c_1^- = \min_{x\in G_x^h}\left[(v^h(x,k))^n - (v^h(x,k))^{n-1}\right], \quad c_1^+ = \max_{x\in G_x^h}\left[(v^h(x,k))^n - (v^h(x,k))^{n-1}\right]$$

$$c_2^- = \min_{x\in G_x^h}\left[(v_a^h(x,k))^n - (v_a^h(x,k))^{n-1}\right], \quad c_2^+ = \max_{x\in G_x^h}\left[(v_a^h(x,k))^n - (v_a^h(x,k))^{n-1}\right]$$

$$c_1^{\min} = \frac{\rho}{1-\rho}c_1^-, \quad c_1^{\max} = \frac{\rho}{1-\rho}c_1^+$$

$$c_2^{\min} = \frac{\rho}{1-\rho} c_2^-, \quad c_2^{\max} = \frac{\rho}{1-\rho} c_2^+$$

If $\min\left(\left|c_1^{\max} - c_1^{\min}\right|, \left|c_2^{\max} - c_2^{\min}\right|\right) \le \delta$, then stop; else $n := n+1$ and go to step 2

If min $\left(\left|c_1^{\max} - c_1^{\min}\right|, \left|c_2^{\max} - c_2^{\min}\right|\right) = \left|c_1^{\max} - c_1^{\min}\right|$ there is no capacity expansion else there is a capacity expansion.

Where G_x^h is the state space grid related to the surplus x and a given value of h. The boundary conditions presented in Yan and Zhang (1997) are used here with the previous algorithm to solve the optimality conditions given by equations (2.13) and (2.14).

2.5 Numerical example and sensitivity analysis

Let us consider a firm with a two states Markov process $k_1 \in M_1 = \{0,1\}$ describing the capacity of the system before expansion. After capacity expansion, a three states Markov process $k_2 \in M_2 = \{0,1,2\}$ describes the capacity of the system. The discrete dynamic programming equations (2.13) and (2.14) give the equations (2.15) and (2.16) before capacity purchase:

$$\upsilon^h(x,0) = \min_{u_r \in U_{r0}} \left(\frac{1}{\left(\rho + Q_h^{01}(u_r)\right)} \left(\left[\upsilon^h(x+h,0) P_x^0(1) + \upsilon^h(x-h,0) P_x^0(2) \right] + G(x,0,u_r) + q_{01}^1(u_r) \upsilon_a^h(x,1) \right) \right) \tag{2.15}$$

$$\upsilon^h(x,1) = \frac{1}{\left(\rho + Q_h^{11}\right)} \left(\min_{u_1 \in U_1} \left[\upsilon^h(x+h,1) P_x^1(1) + \upsilon^h(x-h,1) P_x^1(2) \right] + G(x,u_1,0) + q_{10}^1 \upsilon_a^h(x,0) \right) \tag{2.16}$$

and equations (2.17) to (2.19) after capacity purchase:

$$\upsilon_a^h(x,0) = \min_{u_r \in U_{r0}} \left(\left(\frac{1}{\left(\rho + Q_h^{02}(u_r)\right)} \right) \left(\left[\upsilon_a^h(x+h,0) P_x^0(1) + \upsilon_a^h(x-h,0) P_x^0(2) \right] + G(x,0,u_r) + q_{01}^2(u_r) \upsilon_a^h(x,1) + q_{02}^2 \upsilon_a^h(x,2) + \rho K \right) \right) \tag{2.17}$$

$$\upsilon_a^h(x,1) = \min_{u_r \in U_{r1}} \left(\left(\frac{1}{\left(\rho + Q_h^{12}(u_r)\right)} \right) \left(\min_{u_1 \in U_1} \left[\upsilon_a^h(x+h,1) P_x^1(1) + \upsilon_a^h(x-h,1) P_x^1(2) \right] + G(x,u_1,u_r) + q_{10}^2 \upsilon_a^h(x,0) + q_{12}^2 \upsilon_a^h(x,2) + \rho K \right) \right) \tag{2.18}$$

$$\upsilon_a^h(x,2)=\frac{1}{\left(\rho+Q_h^{21}\right)}\left(\min_{u_2\in U_2}\left[\upsilon_a^h(x+h,2)P_x^2(1)+\upsilon_a^h(x-h,2)P_x^2(2)\right]+G(x,u_2,0)\right.$$
$$\left.+q_{20}^2\upsilon_a^h(x,0)+q_{21}^2\upsilon_a^h(x,1)+\rho K\right) \tag{2.19}$$

We use the following computational domain:

$$G_x^h=\{x:-5\le x\le 25,\ x(i)=-5+(i-1)\times h,\ i=1,2,\ldots\}$$

Other parameters of the considered manufacturing system are given in table (2.1)

Table 2.1 Parameters of the considered manufacturing system

Parameter	C^+	C^-	C_r	U_2^{max}	U_1^{max}	Z	K	ρ	q_{10}^1	q_{10}^{1min}
Value	1	15	100	0.2	0.4	0.12	50000	0.001	0.05	0.4
Parameter	h	q_{01}^{1max}	q_{01}^{2min}	q_{01}^{2max}	q_{02}^2	q_{10}^2	q_{12}^{2min}	q_{12}^{2max}	q_{20}^2	q_{21}^2
Value	0.1	0.6	0.4	0.6	0	0.05	0.05	0.1	0	0.05

The policy iteration technique is used to solve the optimality conditions related to equations (2.15) to (2.19). Obtained results are presented in figures (2.1) and (2.2).

Figure 2.1 gives simultaneously the optimal purchase time (associated to X_{op}) and the production rate according to the initial inventory level. It is interesting to note from figure 2.1 that if the initial inventory level is less than $X_{op}=-2.10$, then the optimal purchasing time will be at initial production time, meaning that $\tau=0$. Backlog values under X_{op}, corresponding to the initial inventory of the products used for the firm to absorb the excessive demand for a few initial periods, as stated in section 2.1. Thus, the optimal control policy suggests to purchase a new capacity when the initial surplus (i.e., backlog if negative) is under X_{op}.

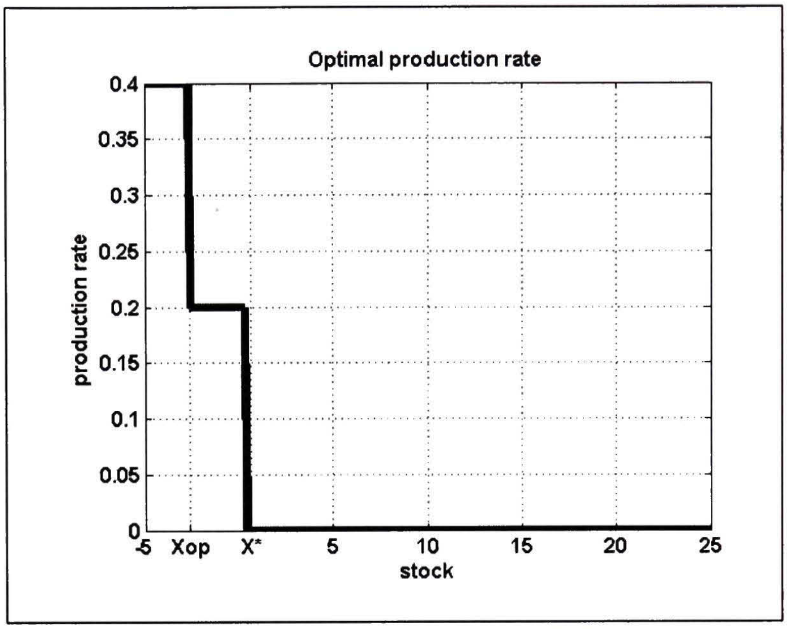


Figure 2.1 Production rate.

After a purchase of a new capacity, the production rate of each machine (the existing and the new machines) is set to its maximal value to satisfy unmet demands (backlog) and to build the safety stock described by the threshold value X^*. The production control policy obtained is an extension to the so-called hedging point policy given that the previous behaviour respect the structure presented in Akella and Kumar (1986) for production without maintenance and capacity expansion. The obtained modified hedging point policy, characterized by the switching trend illustrated by figure 2.1, is given by the following equation:

$$u(t,x,k) = \begin{cases} U_{3\max} & \text{if } x \le X_{op} \\ U_{2\max} & \text{if } X_{op} < x < X^* \\ 0 & \text{if } x \ge X^* \end{cases} \tag{2.20}$$

The corrective maintenance policy, plotted in figure 2.2, divides the computational G_x^h into two regions after capacity expansion (i.e., $\left[-5, X_{r2}\right[$ and $\left[X_{r1}, X_{r2}\right[$) and another two regions before capacity expansion (i.e., $\left[X_{r1}, X_r^*\right[$ and $\left[X_r^*, 25\right]$).

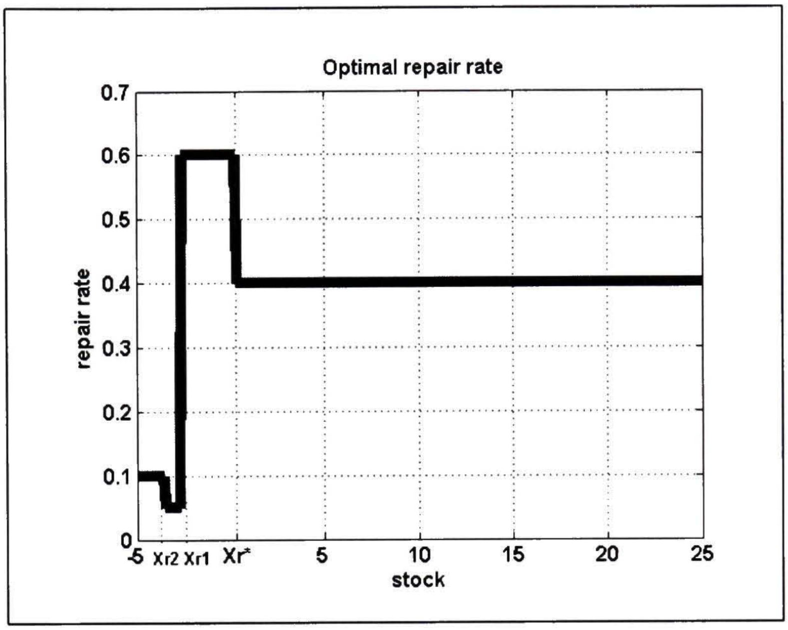


Figure 2.2 Maintenance rate.

With capacity expansion, if the stock level is under X_{r2} and a failure occurs, one have to repair the failed machine at the maximal repair rate $q_{23}^{\max}$ while the minimal repair rate $q_{23}^{\min}$ is applied in $[X_{r1}, X_{r2}[$. Without capacity expansion, if the stock level is under X_r^* and a failure occurs, one have to repair the failed machine at the maximal repair rate $q_{12}^{\max}$ while the minimal repair rate $q_{12}^{\min}$ is applied in $\left[X_r^*, 25\right]$. The corrective maintenance policy, illustrated in figure 2.2, is then summarized as in equation (2.21). For this example, $Xr^* = 0.20$, $X_{r1} = -2.10$ and $X_{r2} = -3.60$

$$u_r(x,k) = \begin{cases} q_{23}^{\max} & \text{if } x \le X_{r2} \\ q_{23}^{\min} & \text{if } X_{r2} < x \le X_{r1} \\ q_{12}^{\max} & \text{if } X_{r1} < x < X_{r*} \\ q_{12}^{\min} & \text{if } x \ge X_{r*} \end{cases} \tag{2.21}$$

The control policy described by equations (2.20)-(2.21) is completely defined for given values of parameters X^*, X_{op}^*, X_r^*, X_{r1}^* and X_{r2}^* in the case of two-identical machine as in the example presented in this section.

Sensitivity analysis and comparative study.

We performed a couple of experiments using the numerical example presented previously. A set of analysis have then been considered to illustrate the sensitivity of the obtained control policy with respect to capacity purchase, inventory, backlog, maintenance costs and machines availability. The results presented in tables 2.2 illustrate four different situations used to show the variation of the production and new capacity purchasing parameters when the purchase cost increase and when the repair rate is controlled or not.

Table 2.2 Impact of the corrective maintenance policy on purchase time and production policy

	K	Cr	C-	Xop	X*
Without control of machine repair rate	5 000	-	15	0	0.7
With control of machine repair rate	5 000	100	15	0.1	0.1
Without control of machine repair rate	80 000	-	15	-3.9	0.7
With control of machine repair rate	80 000	100	15	-4.6	0.4

Table 2.2 shows that controlling machines repair rates (corrective maintenance) reduces the optimal purchase time and production threshold level. The proposed joint optimization of capacity expansion, production and maintenance activities significantly reduces the overall incurred cost compared to separate optimization models as shown in figure 2.3.

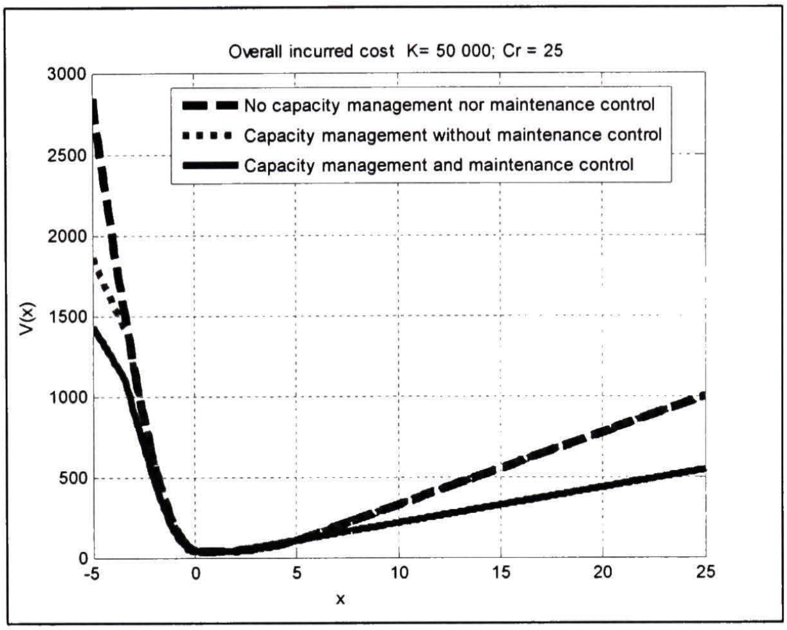


Figure 2.3 Overall incurred cost.

As one can observe from figure 2.3, there is a significant difference between the system performances in three situations. Such situations are:

- Production planning with no capacity expansion and no corrective maintenance as in Akella and Kumar (1986);
- Production planning with capacity expansion and no corrective maintenance as in Sethi and Zhang (1994);
- Production planning with capacity expansion and corrective maintenance as in this paper for the control policy given by equation (2.20)-(2.21).

For a comparative purpose, we used the algorithm presented in section 2.4 to solve the optimality conditions or HJB equations of the above three situations and to obtain results presented in figure 2.3. Through the observations made from figure 2.3, it clearly appears that the proposed approach, based on a simultaneous control of capacity expansion, production and corrective maintenance rates, provides interesting results in the context of manufacturing systems under uncertainties, for given initial inventory levels.

For a m-identical machines manufacturing system producing one part type, the capacity expansion, the production and corrective maintenance policies could be defined by $2(m+1)$ parameters or input factors. The experimental design approach, combined to simulation and analytical models could be used to determine the effects of considered factors on the incurred costs and to determine their optimal values. Details on experimental design and simulation modelling could be find in Gharbi and Kenne (2003) and references therein.

2.6 Conclusion

In this paper, we develop a capacity expansion model for one-product, multiple-machine manufacturing systems with constant demand. We develop an effective solution approach to determine the optimal capacity purchase time, production and maintenance decisions over time. The introduction of the maintenance planning increases the availability of the production system, which guarantees the improvement of the system's productivity. Through a computational study, we show the effectiveness of the proposed models in terms of contribution in the control theory area and investigate the impact of capacity cost, maintenance control and other important parameters on the control policy.

In this work, we have only considered the model with a single capacity purchase, constant demand, equipment purchase and capacity expansion, mainly for simplicity, without loosing the generality of the proposal. An extension of the proposed models could significantly reduce the overall incurred cost if it incorporates models with any finite number of capacity purchases, stochastic demand or capacity reduction. In addition to machine purchase, the use of other resources, define as any part of the system that is not consumed or transformed during the production process, for capacity expansion could be considered.

REFERENCES

Akella R. and Kumar P. R. (1986). "*Optimal Control of Production Rate in a Failure Prone Manufacturing System.*" IEEE Transactions on Automatic Control, 31: pp. 116–126.

Boukas E. K. and Haurie A. (1990). "*Manufacturing Flow Control and Preventive Maintenance : Stochastic Control Approach.*" IEEE Transactions on Automatic Control, 33: pp. 1024–1031.

Boukas E. K. and Kenne J. P. (1997). "*Maintenance and production control of manufacturing systems with setups.*" In Lectures in applied mathematics, 33: pp. 55-70.

Boukas E. K. (1998). "*Hedging point policy improvement.*" Journal of Optimization Theory and Control, 9: pp. 47-70.

Cakanyildirim M., Roundy R. and Wood S.C. (1999). "*Machine purchasing strategies under demand-and technology-driven uncertainties.*" Technical paper no. 1250, SORIE, Cornell University, Ithaca, NY.

Davis M.A.H., Dempster, Sethi S.P. and Vermes D. (1987). "*Optimal capacity expansion under uncertainty.*" Advances in Applied Probability, 19: pp. 156-176.

Gharbi A., Kenne J.P. (2005). "*Maintenance Scheduling and Production Control of Multiple-Machine Manufacturing Systems.*" Computer and Industrial Engineering, 48: pp. 693-707.

Gharbi A. and Kenne J.P. (2003). "*Optimal Manufacturing Systems Control Problem in Stochastic Multiple-Product, Multiple-Machine Manufacturing Systems.*" IIE transactions, 35: pp. 941-952.

Kenne J.P., E. K. Boukas (2003). "*Hierarchical Control of Production and Maintenance Rates in Manufacturing Systems.*" Journal of Quality in Maintenance Engineering, 9(1): pp. 66-82.

Kenne J.P., Boukas E. K. and Gharbi A. (2003). "*Control of Production and Corrective Maintenance Rates in a Multiple-Machine, Multiple-Product Manufacturing System.*" Mathematical and Computer Modelling, 38: pp. 351-365.

Kenne J.P. and Gharbi A. (2000). "*Optimal Production Planning Problem in Stochastic Multiple-Product Multiple-Machine Manufacturing Systems.*" Submitted to IIE transaction, 2000.

Kusner H. J. and Dupuis P. G. (1992). "*Numerical Methods for Stochastic Control Problem in Continuous Time.*" Springer-Verlag, New York.

Luss H. (1982). "*Operations research and capacity expansion problems: a survey.*" Operations Research, 30: pp. 907-947.

Older G. J. and Suri R. (1997). "*A numerical method in optimal production and setup scheduling to stochastic manufacturing systems.*" IEEE Transactions on Automatic Control, 42: pp. 1452-1455.

Sethi S. P., Taksar M. I. and Zhang Q. (1992).. (1995). "*Hierarchical capacity expansion and production planning decisions in stochastic manufacturing systems.*" Journal of Operations Management 12(3-4): pp. 331-352.

Sethi S. P., and Zhang Q. (1994). "*Hierarchical Decision Making in Stochastic Manufacturing Systems.*" Birkhauser.

Yan H. and Zhang Q. (1997). "*Numerical method in optimal production and setup scheduling to stochastic manufacturing systems.*" IEEE Transactions on Automatic Control, 42 : pp. 1452-1455.

CHAPITRE 3

ARTICLE 2: A REPLACEMENT POLICY FOR PRODUCTION SYSTEMS SUBJECT TO IMPERFECT CORRECTIVE MAINTENANCE

F. I. Dehayem Nodem [1], J. P. Kenne[1] and A. Gharbi[2]

[1]Mechanical Engineering Department, Production Technologies Integration Laboratory, University of Quebec,

[2]Automated Production Engineering Department, Production System Design and Control Laboratory, University of Quebec

École de technologie supérieure, 1100 Notre Dame Street West, Montreal, Quebec Canada H3C 1K3

This chapter is being revised as an article for publication in International Journal of Production Economics. Submitted on July 29, 2008; Revised on February 11, 2009. Submission Confirmation Ms. Ref. No.: IJPE-D-08-00591R1

Résumé

Le présent article analyse un système manufacturier dont la machine est sujette à la dégradation, aux pannes et réparations aléatoires et aux activités de réparation ou de remplacement. La machine fabrique un seul type de produit et lorsqu'une panne survient, une option est choisie entre réparer la machine ou bien la remplacer. Si l'option de remplacement est choisie, la machine est remplacée par une autre machine, neuve et identique. En cas de remplacement, aucune action de réparation n'est prise en considération. Les variables de décision du problème de commande considéré sont le taux de production et la politique de basculement de la réparation au remplacement de la machine lorsqu'une panne survient. L'objectif est de trouver les variables de décision qui permettent de réduire les coûts totaux, comprenant les coûts de réparation, de remplacement, d'inventaire et de pénurie sur un horizon infini de planification. Étant donné que les activités de réparation de la machine dépendent de l'historique des pannes et réparations, un processus de décision semi-Markovien est utilisé pour décrire la dynamique du système. Les conditions d'optimum proposées sont développées en utilisant la méthode de programmation dynamique stochastique. Un exemple numérique est donné pour illustrer l'utilité de l'approche proposée et une analyse de sensibilité permet de confirmer la structure obtenue des politiques de commande.

Abstract

In this paper, we investigate a deteriorating production system subject to random machine breakdowns, repair or replacement activities. The machine produces one type of product, and when a breakdown occurs, either a repair or a replacement action is chosen. The machine is replaced with a new one if the replacement option is selected, and no repair action is considered. The decision variables of the control problem are the production rate and the repair/replacement switching policy upon machine failure. The objective is to find the decision variables that minimize overall cost, including costs associated with repair, replacement, inventory holding and backlog over an infinite planning horizon. Because machine repair activities depend on the repair history, a Semi-Markov Decision Process (SMDP) is used to describe the dynamic of the system. The proposed optimality conditions are developed using the stochastic dynamic programming approach. A numerical example is given to illustrate the utility of the proposed approach, and a sensitivity analysis is considered to confirm the structure of the control policy obtained.

Keywords: Replacement policy; Corrective maintenance; Production planning; Numerical method; Manufacturing systems.

3.1 Introduction

In a manufacturing environment, the availability of an operating machine decreases with its age. Maintenance activities are often imperfect, that is, a machine is not new after a corrective or a preventive maintenance is performed. For that reason, the machine must be replaced after a certain number of failures, as it progressively degrades over time, and in the long run, will reach the end of its useful life following rebuilding. Production system availability could be increased by performing appropriate replacements in critical situations characterized by a large number of failures. The literature contains many proposals for optimizing the performance of production systems, based on repair/replacement, maintenance and the production planning problems, with the models considered individually. In this work, we carry out a joint optimization of repair/replacement and production activities in order to come up with more realistic control policies.

Many authors have studied manufacturing systems subject to progressive degradation in order to determine optimal repair versus replacement conditions. Two replacement policies are commonly used. The first one is based on the accumulated working time, or age T of the machine and the second one is based on the accumulated number of failures N of the machine. Phelps (1983) developed solutions covering situations where minimal repairs are performed on the system up to age T, with replacement by a new machine at first failure after T. A minimal repair does not change the failure rate of the system. Kijima *et al.* (1988) introduced a periodic replacement model with general repair, in which a system is replaced only at scheduled intervals and is repaired whenever it fails. They defined a general repair model as one in which the repair upgrades the mode of the machine, and proposed an approximation procedure to be used to find the optimal replacement period. Makis and Jardine (1991) considered a deteriorating system subject to random failures such that at failure, the system can be replaced or can undergo a repair with three possible outcomes. Those outcomes are the following: the machine is as good as new, it is returned to its operating state just prior to failure, or it must be scrapped and replaced at additional replacement cost. Makis and Jardine (1991) assumed that the repair cost increases with the system's age, and that maintenance activities take very little time. They determined the optimal replacement time for the system. Makis and Jardine (1992) later extended this model to consider a cost that depends on time and on the number of repairs in the cycle. They showed by numerical examples that the optimal policy is considerably better than the optimal periodic policy. Yuan (1994) proposed a bivariate replacement policy (T, N) and showed that the optimum of the combined replacement policy is better than the optimum of individual T and N policies. Love *et al.* (1998) and Love *et al.* (2000) considered a system that may undergo a repair which can partially reset the failure intensity of the machine. They introduced the concept of virtual age of the machine and assumed that repairs could at most shift back the failure intensity so as to remove the most recent run-time sojourn. They formulated the decision to repair or replace the machine on failure as a discrete semi-Markov decision process and showed that optimal decisions are of the threshold type. Mathew and Kennedy (2003) examined issues including whether the replacement should occur at fixed time intervals or not, the consequences of a delay beyond the fixed time interval, the existence of a valid optimal fixed time interval for each replacement. They studied whether an optimal fixed time is something inherent to equipment or if factors like depreciation rate, discount factor influence it greatly. They showed that a Delayed Time Interval policy would be more efficient for the replacement of components and subassemblies of equipment. Hariga *et al.* (2006) addressed the problem of determining optimum inspection schedules for a single deteriorating production system with a predetermined replacement cycle. They developed an optimal joint inspection-preventive maintenance policy when the inspections are carried out at discrete points in time. In their model, the time to shift follows a general distribution with increasing failure rate. Zhou *et al.* (2007) integrate sequential imperfect maintenance policy into condition-based predictive maintenance. They proposed a reliability centered predictive maintenance policy for a system subject to degradation due to the imperfect maintenance effect.

Corrective maintenance which is mainly employed in manufacturing industries is usually unplanned and more often than not, it brings production to a standstill (Gabriella *et al.*, 2008). In many industries including automobile and semiconductor, buffer stock are usually used to guarantee the continuous supply of the subsequent production machine during interruptions of service due to corrective or preventive maintenance activities (Chelbi and Ait-Kadi (2004), El-Ferik (2008)). Both research papers mentioned above and many others made the assumption that failures are instantly detected and repaired, or that replacements are instantaneous. This assumption is not realistic for production systems because, more often than not, repairing the machine requires a considerable amount of time (Wang and Zhang, 2006). Moreover, the repair time can increases with the age of the machine or the number of failures it has undergone. Failures are random phenomena, and repair times are non-production times for the system. Since these uncertainties imply disruptions on replenishment lead times, the company must therefore hold buffer stocks in order to be able to ensure delivery when such lead times are uncertain (El-Ferik, 2008). On the other hand, holding stock and backlogs involve costs. Although several researchers have studied the problem of production and repair/replacement planning, most of the researches that have been published are based either on instantaneous repair or replacement or both. Therefore they ignore the interactions that exist between production and repair/replacement planning because repair and replacement activities do not significantly affect production activities.

A great deal of research has been focused on determining the optimal production planning of manufacturing systems subject to failures and repairs (Rishel (1975), Gharbi and Kenne, (2000), Cassidy et Kutanoglu (2005), Pellerin *et al.*, (2007)). Following the work by Rishel (1975) and Kimemia and Gershwin (1983) on production planning for a system affected by jump disturbances, Boukas and Haurie (1990) combined production and preventive maintenance planning when the machine's failure probability increases with its age, and obtained a modified hedging point policy. The hedging point policy introduced by Kimemia and Gershwin (1983) consists in maintaining an optimal inventory level during times of excess capacity in order to hedge against future capacity shortages brought about by machine failures. Boukas and Haurie (1990) determined the production rate and the maintenance policy which minimize the total expected cost of a two-machine system. They adopted a numerical scheme which is computationally difficult to provide the optimal control for large-scale manufacturing systems. To cope with this difficulty, Kenne and Boukas (2003) formulated a singular perturbation-based model for production and preventive maintenance rate planning in a manufacturing system, and obtained a limiting problem that was numerically more tractable. Gharbi *et al.* (2007) investigated the joint implementation of preventive maintenance and safety stocks in a manufacturing environment. For more realistic purpose, they used a stochastic model not restricted to Markovian processes and derived expressions of the overall incurred cost. They used the overall incurred cost as the basis for optimal determination of the jointly production and preventive maintenance policies. Kenne *et al.* (2007) formulated an analytical model for the joint determination of an optimal age-dependent buffer inventory and preventive maintenance policy in a production environment that is subject to random machine breakdowns. They provided optimality conditions for the manufacturing systems considered, and used numerical methods to obtain the corresponding multiple threshold levels hedging point policy. Gharbi *et al.* (2008) studied the multiple repair rate control problem of remanufacturing systems and solved the problem in the case of one product type. Based on control theory, a near-optimal control policy was proposed. The policy, called multiple hedging point policy (MHPP), is described by two thresholds related to the two accelerated repair rates.

Some of the previous works on production planning took into account the fact that the system deteriorates with age, but did not consider the cases in which some parameters, such as repair time, change with the number of failures. One reason for this is that considering the variations in such parameters renders the problem more complex. Wang and Zhang (2006) studied a simple deteriorating system with repair. They considered that the successive operating times of the system after preventive repair formed a stochastically decreasing geometric process, while the consecutive preventive repair times of the system formed a stochastically increasing geometric process. They assumed a negligible replacement time and did not consider a production planning. El-Ferik (2008) considered a joint determination of the economic lot size for an unreliable facility maintained according to age-based maintenance policy with increasing failure rate. In his model, the system is replaced after N production cycles. Although replacement time has an average value Tr, repair times are presumed negligible and production rates are constant.

Our aim is thus to extend these previous models in order to allow simultaneous production and repair versus replacement policies determination when repair times increase with the number of failures. We considered a production system consisting of one failure-prone machine that deteriorates with age. The machine is subject to random breakdowns, repairs and replacements. The failure of the machine means that the production activity must be stopped. When this occurs, the machine should be repaired or replaced, and the repair time increases with the number of failures. When at failure the machine is replaced after it experiences a certain number of failures or it age reaches a predetermined level, then the age level and the number of failure after which the machine will be replaced is almost known by the maintenance team. That allows to order the machine and to prepare the installation kit such that the time taken to replace the machine at the corresponding breakdown is reduced. The average replacement time is constant and low compared to the repair time. This problem fell under the class of machine age dependent replacement models, which has been covered in the literature. However, the specific problem considered in this work is more complex for the four following reasons:

- Production planning (non constant production rate) is combined with a repair versus replacement policy determination in a stochastic environment;
- The machine has different dynamics after each breakdown and repair;
- The machine deteriorates while in operation and the failure rate increases with the age of the machine;
- Corrective maintenance activities are imperfect and repair times increase with the number of failures.

The main contribution of the paper is the simultaneous determination of the production (rate) and repair versus replacement policies under those four considerations, which has not yet been addressed in literature. Because machine repair activities depend on the number of failures the machine has undergone, the simultaneous control approach consists in developing a Semi-Markov Decision Process (SMDP), in order to determine an optimal repair and replacement policy and a production plan for the system. Those policies are determined in order to minimize inventory, backlogs, repairs, and replacement costs over an infinite planning horizon.

The paper is organized as follows. In Section 3.2, we present the problem statement. Optimality conditions are given in Section 3.3. Numerical methods are used in Section 3.4 to solve the optimality conditions obtained in Section 3.3. Numerical examples are presented in Section 3.5, and a sensitivity analysis is performed in Section 3.6 to illustrate the usefulness of the proposed approach. We finally conclude the paper in Section 3.7.

3.2 Problem statement

The machine of the considered system produces one type of product, and at any given time t, the system is characterized by the following four-state variables: the machine mode $\xi(t)$, the number of failures $n(t)$, the age of the machine $a_n(t)$, and the inventory/backlog level $x(t)$. If we define the production rate to be $u(t)$ and the demand rate d, then the dynamic of the stock level can be described by the one-dimensional ordinary differential equation:

$$\dot{x}(t) = u(t) - d, \;\; x(0) = x_0 \tag{3.1}$$

where x_0 is the initial inventory/backlog level. When $x(t) \geq 0$, the system has available inventory, and backlog, otherwise. The mode of the machine at time t is given by the random variable $\xi(t) \in \Omega = \{1,2,3\}$ defined by:

$$\xi(t) = \begin{cases} 1 & \text{if the machine is operationnal} \\ 2 & \text{if the machine is under repair} \\ 3 & \text{if the machine is under replacement} \end{cases}$$

The age of the machine between failure n and failure $(n+1)$ is given by:

$$a_n(t) = a(t - t_n), \;\; t_n < t \leq t_{n+1}, t_0 = 0 \tag{3.2}$$

where t_n is the instant of the n^{th} failure.

The function $a(t)$ is an increasing function of number of produced parts, described by the following differential equation:

$$\frac{da(t)}{dt} = f(u(t))$$

There could be several ways to express the relationship between the age $a(t)$ and the production rate $u(t)$. By referring to the literature (Kenne and Boukas, 2003), we define it linear. That is:

$$\frac{da(t)}{dt} = ku(t) \tag{3.3}$$

where k is a given constant used to define the age of the machine and its increasing rate. Thus, equations (3.2) and (3.3) state that the age of the machine is a linear function of the number of produced parts since the last restart of the machine.

At the initial time ($\xi(t)=1$), the machine is operational, and its number of failures $n(t)=0$. Its age is a_0 and the inventory/backlog level is $x_0(t)$. Thus $(\xi(t), n(t), a_n(t), x(t)) = (1, 0, a_0(t), x_0(t))$. After the first failure, $n(t)=1$. If the machine is to be repaired, then $\xi(t)=2$, otherwise $\xi(t)=3$. Assuming that the repair or replacement does not damage the machine, the age of the machine remains unchanged until the restart time after the repair. At the restart time of the machine after the first failure, the four-state variable is $(1,1,a_1(t),x(t))$ and upon the next breakdown, it can become $(2,2,a_1(t),x(t))$ or $(3,2,a_1(t),x(t))$, depending on the action undertaken. The transition state variable upon the $(n+1)^{th}$ failure will be $(3,n+1,a_n(t),x(t))$ if the machine is replaced, or $(2,n+1,a_n(t),x(t))$, if the machine is repaired. If the transition state variable is $(3,n+1,a_n(t),x(t))$, then it will subsequently be $(1,0,a_0(t),x(t))$, otherwise it will be $(1,n+1,a_n(t),x(t))$.

Let $q_{\alpha\beta}(a_n,n)$ $\alpha,\beta \in \Omega$, be the transition rates for the machine from mode α to mode β at time t. We then have:

$$q_{12}(a_n(t)) = \lim_{\partial t \to 0} \left\{ \frac{P[\xi(t+\partial t) = 2 / \xi(t) = 1]}{\partial t} \right\}$$

$$q_{21}(n(t)) = \lim_{\partial t \to 0} \left\{ \frac{P[\xi(t+\partial t) = 1 / \xi(t) = 2,\ n(t) = n]}{\partial t} \right\}$$

$$q_{13}(a_n(t)) = \lim_{\partial t \to 0} \left\{ \frac{P[\xi(t+\partial t) = 3 / \xi(t) = 1]}{\partial t} \right\}$$

$$q_{31}(t) = \lim_{\partial t \to 0} \left\{ \frac{P[\xi(t+\partial t) = 1 / \xi(t) = 3]}{\partial t} \right\}$$

All other transition rates of the machine from mode α to mode β ($\beta \neq \alpha\,; \alpha, \beta \in \Omega$) equal 0.

Let $s \geq 0$ be a given positive real, representing the age above which if a failure occurs, the machine is replaced.

We define $q_{12}^{s}(a_n) = q_{12}(a_n) * Ind\{a_n \leq s\}$ and $q_{13}^{s}(a_n) = q_{13}(a_n) * Ind\{a_n > s\}$. s is a repair/replacement switching age, and $q_{\alpha\beta}^{s}(.)$ is a transition rate of the machine from mode $\alpha = 1$ to mode $\beta \in \{2,3\}$, given the switching age s. If $Ind\{a_n \leq s\} = 1$, then the age of the machine a_n is equal to or less than s and the machine should be repaired. If not, the machine needs to be replaced.

The mean time to replace the machine (that is the mean time the machine spends in operation before replacement at failure (T_{13})) is longer than the mean time to repair (T_{12}). But the mean repair time, (that is the mean time it takes to replace the machine after a failure (T_{31})) is lower than the time it takes to repair the machine when a failure occurs (T_{21}). Thus in our model, $T_{13} \geq T_{12}$ and $T_{21} \geq T_{31}$. The repair time is an increasing function of the number of failures n. Let $\xi(\cdot) = \{\xi(t) : t \geq 0\}$ denote the stochastic process with value in Ω. $\xi(\cdot)$ is a semi-Markov chain.

Let $Q^s(\cdot)=\left(q^s_{\alpha\beta}(a_n)\right)$ denote a 3×3 matrix such that $q^s_{\alpha\beta}(a_n)\geq 0$ if $\alpha\neq\beta$, $q^s_{13}(\cdot)=q_{13}(\cdot)$; $q^s_{31}(\cdot)=q_{31}(\cdot)$ and $q^s_{\alpha\alpha}(\cdot)=-\sum_{\alpha\neq\beta} q^s_{\alpha\beta}(\cdot)$

We shall refer to $Q^s(\cdot)=\begin{bmatrix} q_{11}(.) & q^s_{12}(.) & q^s_{13}(.) \\ q_{21}(.) & q_{22}(.) & 0 \\ q_{31}(.) & 0 & q_{33}(.) \end{bmatrix}$ as the transition matrix of the semi-Markov chain $\xi(\cdot)$

The replacement cost is fixed at C_0 and the repair cost $C_1(.)$ is a bounded non-decreasing function of repair time.

Let $G(\cdot)$ be the cost rate defined as follows:

$$G(\xi,a_n,x,n,u,s)=h(x)+C_1(.)*Ind\{\xi(t)=2\}+C_0*Ind\{\xi(t)=3\} \tag{3.4}$$

where $h(x)=c_s x^+ + c_p x^-$. The constants c_s and c_p are used to penalize inventory and backlog, respectively. $x^+ = \max(0, x)$ and $x^- = \max(0,-x)$.

The expected discounted cost is given by:

$$J(\alpha,a_n,x,n,u,s)=E\left[\int_0^\infty e^{-\rho t}G(\cdot)dt \,/\, x(0)=x,\ \xi(0)=\alpha,\ a_n(0)=a_n\right] \tag{3.5}$$

where ρ is the discounted rate.

Equation (3.5) is subject to constraints given by equations (3.1)-(3.4), and the capacity constraint is given by:

$$0 \leq u(t) \leq U_m \tag{3.6}$$

where U_m is the maximum production rate of the machine.

Our objective is to find the number of failures N_m after which the machine should be replaced at the next failure; the age S_n before a systematic replacement at the next failure, and the production rate $u^*(t)$ in order to minimize $J(\cdot)$.

Let S be the set of $a_n \geq 0$ such that at the next failure after a_n, a replacement is undertaken, P the set of $n \geq 0$ such that at the next failure after n, a replacement is undertaken, and $\Gamma_u = \{u(t) / 0 \leq u(t) \leq U_m\}$.

Any three-state variable $(s, n_p, u) \in S \times \mathrm{P} \times \Gamma_u$ is admissible, and our problem is to minimize the cost $J(\cdot)$ across all admissible three-state variables.

The control variables are the age s after which the machine should be systematically replaced at the next failure, the number of failures before replacement at the next failure n_p, and the production rate u.

$S_n = \min\{s \geq 0 / s \in S\}$ and $N_m = \min\{n_p \geq 0 / n_p \in P\}$. Since at N_m, $S_n = 0$ $\forall x$, it is sufficient to determine S_n instead of the two variables.

The value function of the problem for a given number of failures n is:

$$V(\alpha, a_n, x, n) = \inf_{s(t) \in S,\, u(t) \in \Gamma_u} J(\alpha, a_n, x, n, u, s) \tag{3.7}$$

In the remainder of this work, we will use a instead of a_n, and the notation $z=(a,x)$. The value function $V(\alpha,z,n)$ satisfies specific properties called optimality conditions, which are presented in the next section.

3.3 Optimality conditions

In this section, we show that under an appropriate assumption, the value function $V(\alpha,z,n)$ satisfies a set of coupled partial derivative equations derived from the application of the dynamic programming approach.

Assumption 3.1

One of the goals of scheduling the production is to meet the demand with minimal costs due to inventory. It's better to have x as close as possible to 0. The costs due to x is generally summarized in a convex function which has its minimum at $x=0$ and which grows as $|x|\to\infty$ (Gershwin, 2002). Thus, we make the following assumptions A.3.1 and A.3.2).

A.3.1) $h(\cdot)$ is a nonnegative convex function with $h(0)=0$. There are positive constants C_g and κ_g, such that: $h(x)\le C_g(1+|x|^{\kappa_g})$ and

$$|h(x_1)-h(x_2)|\le C_g(1+|x_1|^{\kappa_g}+|x_2|^{\kappa_g})|x_1-x_2|$$

Thus, $h(.)$ satisfies the Lipschitz property.

A.3.2) The repair replacement costs $w(n,a)=C_1(.)*Ind\{\xi(t)=2\}+C_0*Ind\{\xi(t)=3\}$ is a bounded non-decreasing function in (n,a).

Lemma 3.1

i) if $G(\xi,z,n,u,s)$ is jointly convex in x, then $V(\alpha,z,n)$ is convex in x for each $(\alpha,a,n)\in\Omega\times[0,\infty]\times P$

ii) if $G(\xi,z,n,u,s)$ is locally Lipschitz, then $V(\alpha,z,n)$ is also locally Lipschitz.

iii) if $G(\xi,z,n,u,s)$ is a bounded increasing function in a, then $V(\alpha,z,n)$ is also a bounded increasing function in a for each $(\alpha,x,n)\in\Omega\times\mathbb{R}\times P$

Proof:

We just need to show that $J(\alpha,.,.,n,.)$ is jointly convex to prove i).

Let x_1 and x_2 be any initial values, $u^1(\cdot)$ and $u^2(\cdot)$ any admissible controls. Let $x^1(t)$ and $x^2(t)$ $t\geq 0$ denote the trajectories corresponding to $\left(x_1,u^1(\cdot)\right)$ and $\left(x_2,u^2(\cdot)\right)$.

For any $\lambda\in[0,1]$,

$$\lambda J\left(\alpha,a,x_1,n,u^1(\cdot),s\right)+(1-\lambda)J\left(\alpha,a,x_2,n,u^2(\cdot),s\right)$$
$$=E\left[\int_0^\infty e^{-\rho t}\left(\lambda G\left(\alpha,a,x_1,n,u^1(\cdot),s\right)+(1-\lambda)G\left(\alpha,a,x_2,n,u^2(\cdot),s\right)\right)dt\right]$$
$$=E\left[\int_0^\infty e^{-\rho t}\left(\lambda h(x_1)+\lambda w(n,a)+(1-\lambda)h(x_2)+(1-\lambda)w(n,a),s\right)dt\right]$$
$$=E\left[\int_0^\infty e^{-\rho t}\left(\lambda h(x_1)+(1-\lambda)h(x_2)\right)dt\right]+E\left[\int_0^\infty e^{-\rho t}\left(w(n,a)\right)dt\right]$$

We used assumptions in section 3 to show that if $G(\xi,z,n,u,s)$ is jointly convex in x, then $V(\alpha,z,n)$ is also convex. Thus with assumptions A.3.1) and A.3.2) we have:

$$\lambda J\left(\alpha,a,x_1,n,u^1(\cdot)\right)+(1-\lambda)J\left(\alpha,a,x_2,n,u^2(\cdot),s\right)\geq E\left[\int_0^\infty e^{-\rho t}G(\alpha,a,x(t),n,u(t),s)dt\right]$$

where $u(t)=\lambda u^1(t)+(1-\lambda)u^2(t)$ and $x(t)$ denotes the trajectory with initial value $x=\lambda x_1+(1-\lambda)x_2$ and control $u(\cdot)$.

Thus:

$$\lambda J\left(\alpha,a,x_1,n,u^1(\cdot)\right)+(1-\lambda)J\left(\alpha,a,x_2,n,u^2(\cdot),s\right)\geq J\left(\alpha,a,\lambda x_1+(1-\lambda)x_2,n,\lambda u_1(\cdot)+(1-\lambda)u_2(\cdot),s\right),$$

which means that $J(\alpha,a,.,n,.s)$ is jointly convex, and consequently, $V(\alpha,a,x,n)$ is convex.

The proof of ii) is similar to the proof of lemma 3.1 of Sethi and Zhang (1994) for a given number of failures value.

To prove iii), it suffices to show that $J(\alpha,.,n,.)$ is a bounded increasing function in a for each $(\alpha,x,n)\in\Omega\times\mathbb{R}\times P$.

□

Lemma 3.2

$V(\alpha,z,n)$ is a unique viscosity solution to the Hamilton-Jacobi-Bellman (HJB) equations:

$$\begin{aligned}\rho V(\alpha,z,n)=\underset{u\in\Gamma}{Min}\,\underset{s\in S}{Min}\{\; & G(\cdot)+\frac{\partial}{\partial a}V(\alpha,z,n)k(u(t))+\frac{\partial}{\partial x}V(\alpha,z,n)\left(u(t)-d\right)\\ & +Q^s(a,n)V(\cdot,\varphi_a(a,\cdot,n),x,\varphi_n(a,\cdot,n))(\alpha)\;\}\end{aligned}\tag{3.8}$$

with

$$\varphi_a(a,\xi,n)=\begin{cases}0 & \text{if } \xi(\tau^+)=1 \text{ and } \xi(\tau^-)=2\\ 0 & \text{if } \xi(\tau^+)=1 \text{ and } \xi(\tau^-)=3\\ a(\tau^-) & \text{otherwise}\end{cases}\qquad \text{where } \xi(t)=\alpha\in\Omega$$

$$\varphi_n(a,\xi,n)=\begin{cases}n & \text{if } \xi(\tau^+)=1 \text{ et } \xi(\tau^-)=2\\ 0 & \text{if } \xi(\tau^+)=1 \text{ et } \xi(\tau^-)=3\\ n+1 & \text{otherwise}\end{cases}$$

$\frac{\partial}{\partial x}V(\alpha,z,n)$ and $\frac{\partial}{\partial a}V(\alpha,z,n)$ are the partial derivatives of the value function $V(\alpha,z,n)$.

Proof:

It suffices to show that $V(\cdot)$ is both a viscosity subsolution and viscosity supersolution of equation (3.8).

Let us consider any fixed $\alpha_0 \in \Omega,\ a_0 \geq 0,\ x_0 \in \mathbb{R}$ and $n_0 \in \mathbb{N}$. Let $\phi(a,x) \in C^1(R^1)$ be such that $V(\alpha_0,a,x,n_0) - \phi(a,x)$ attains its maximum at $(a,x) = (a_0,x_0)$ in a neighborhood $N(a_0,x_0)$. Let τ denote the first jump time of $\xi(t)$. We consider $a(t) < s$ and the control $u(t) = u$ for $0 \leq t \leq \tau$; where u is a constant. Let consider $\theta_d \in (0,\tau]$ be such that $z(t) = (a(t),x(t))$ starts at $z_0 = (a_0,x_0)$ and stay in $N(a_0,x_0)$ for $0 \leq t \leq \theta_d$. Define

$$\psi_d(\alpha,a,x,n) = \begin{cases} \phi(a,x) - V(\alpha_0,a_0,x_0,n_0) - \phi(a_0,x_0) & \text{if } \alpha = \alpha_0 \text{ and } n = n_0 \\ \\ V(\alpha,a,x,n) & \text{if not} \end{cases}$$

For the remainder of the proof, it suffice to use dynkin's formula and the fact that $\xi(\theta_d) = \alpha_0$, $0 \leq t \leq \theta_d$. After a couple of manipulations and by letting $\theta_d \to 0$, we can conclude that

$$\min_{u\in\Gamma}\min_{s\in S}\left[\begin{array}{l} G(\alpha_0,a_0,x_0,n_0) + (u-d)\phi_x(x_0) + (ku)\phi_a(a_0) \\ +Q^s(a,n)V(.,\varphi_a(a,.,n),x,\varphi_n(a,.,n))(\alpha_0) \end{array}\right] - \rho V(\alpha_0,a_0,x_0,n_0) \geq 0$$

Thus, $V(.)$ is a viscosity subsolution.

The basic idea to show that $V(.)$ is a viscosity supersolution is similar to that of Lemma H.2 of Sethi and Zhang (1994) when replacing x by $z = (a,x)$ and $V(x,\alpha)$ by $V(\alpha,z,n)$.

□

Because of the complexity of equations (3.8) according to state variables ξ, n, a and x, obtaining the analytical expression of decision variables s and u is very difficult. In the next section, we develop a numerical method based on the Kushner approach (see Kushner and Dupuis (1992)) to solve the optimal conditions and obtain an approximation of the optimal control policy.

3.4 Numerical approach

In this section, we solve the HJB equations (3.8) by approximating $V(\alpha,a,x,n)$ by a function $V^h(\alpha,a,x,n)$ and the first-order partial derivatives of the value function $\frac{\partial}{\partial x}V(\alpha,a,x,n)$ and $\frac{\partial}{\partial a}V(\alpha,a,x,n)$ by:

$$\frac{\partial}{\partial x}V(\alpha,a,x,n)=\begin{cases}\frac{1}{h_x}\begin{bmatrix}V^h(\alpha,a,x+h_x,n)\\ -V^h(\alpha,a,x,n)\end{bmatrix} & if\ \frac{dx}{dt}\geq 0\\ \frac{1}{h_x}\begin{bmatrix}V^h(\alpha,a,x,n)\\ -V^h(\alpha,a,x-h_x,n)\end{bmatrix} & \text{otherwise}\end{cases}$$

$$\frac{\partial}{\partial a}V(\alpha,a,x,n)=\frac{1}{h_a}\begin{bmatrix}V^h(\alpha,a+h_a,x,n)\\ -V^h(\alpha,a,x,n)\end{bmatrix}$$

where h_x and h_a are discrete increments associated to state variables x and a.

The following three equations are the discrete dynamic programming equations obtained:

$$V^h(1,a,x,n)=\frac{1}{\left(\rho+\frac{1}{h_a}ku(t)+\frac{1}{h_x}|u(t)-d|-q_{11}\right)}*\begin{pmatrix}c^+x^+ +c^-x^- +\frac{1}{h_a}V^h(1,a+h_a,x,n)(ku(t))\\ +\frac{1}{h_x}|u(t)-d|\begin{pmatrix}V^h(1,a,x+h_x,n)Ind\{(u(t)-d)\geq 0\}\\ +V^h(1,a,x-h_x,n)Ind\{(u(t)-d)<0\}\end{pmatrix}\\ +q_{12}^sV^h(2,0,x,n+1)+q_{13}^sV^h(3,0,x,0)\end{pmatrix}$$

(3.9)

$$V^h(2,a,x,n)=\frac{1}{\left(\rho+\dfrac{d}{h_x}-q_{22}\right)}*\begin{pmatrix}c^+x^+ + c^-x^- + C_1(n,a) \\ +\dfrac{d}{h_x}V^h(2,a,x-h_x,n)+q_{21}V^h(1,a,x,n)Ind\{a\le s\}\end{pmatrix} \tag{3.10}$$

$$V^h(3,a,x,n)=\frac{1}{\left(\rho+\dfrac{d}{h_x}-q_{33}\right)}*\begin{pmatrix}c^+x^+ + c^-x^- + C_0 \\ +\dfrac{d}{h_x}V^h(3,a,x-h_x,n)+q_{31}V^h(1,a,x,n)Ind\{a> s\}\end{pmatrix} \tag{3.11}$$

The next theorem shows that the value function $V^h(\alpha,a,x,n)$ is an approximation of $V(\alpha,a,x,n)$ for small size step h.

Theorem 3.1

Let $V^h(\alpha,a,x,n)$ denote a solution to HJB equations (3.9) to (3.11). Assume that there are constants C and K such that: $0\le V^h(\alpha,a,x,n)\le C(1+|x|^K)$, then

$$\lim_{h\to 0} V^h(\alpha,a,x,n)=V(\alpha,a,x,n) \tag{3.12}$$

Proof:

This theorem can be proved in the same manner as in Yan and Zhang (1997).

□

In the next section, we provide a numerical example to illustrate the structure of the control policies.

3.5 Numerical example

The computational domain D is given by:

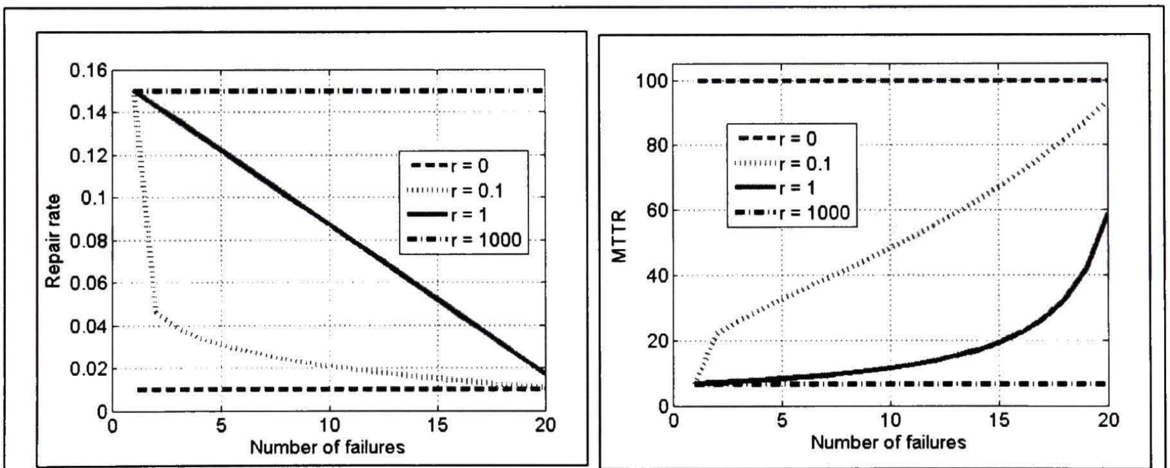
$$D=\{(a,x,n):0\le a\le 300;-5\le x\le 15;0\le n\le 20\}.$$

We define the repair transition rate by:

$$q_{21}(n) = q_0 + q_1\left(1-\left(\frac{n-1}{N}\right)^r\right), \text{ where } q_1, q_0, N \text{ and } r \text{ are given parameters.}$$

The mean time to repair $MTTR(n) = \dfrac{1}{q_{21}(n)}$.

Thus, the repair rate $q_{21}(n)$ decreases with the number of failures and the mean time to repair $MTTR(n)$ increases with the number of failures, as illustrated in Figures 3.1 a. and 3.1.b.



a. $q_{21}(n)$ **b.** $MTTR(n)$

Figure 3.1 Repair rate and mean time to repair for $n = 1, 2, 3, \ldots 20$.

As illustrated in Figure 3.1.a and 3.1.b, we can act on the constant parameter r to modify the trend of the repair rate (according to the number of failures having occurred) of a specific type of machine. If the repair rate is almost constant with a high limit value $q_0 + q_1$, a high value of r (i.e., value 1000 in this example) will be used. Conversely, to represent a low repair rate with a limit value of q_0, we can use low values of r (example $r = 0$ in Figure 3.1.a). In this work, we use a value of $r = 1$ because it lies between the two extreme cases, and allow us to observe the impact of changes on the mean time to repair (see Figure 3.1.b) from one failure to the next. For a specific manufacturing system, the typical value of r can be determined from repair time's historical data.

Given the frequent utilization of the Weibull distribution for semi-Markov process (Love *et al.* 2000, Ouhbi and Limnios, (2003), Zhou *et al.*, (2007)), we assume that the lifetime distribution of a new machine follow a Weibull distribution. Let λ_w be the scale parameter and α_w the shape parameter. The machine failure transition rate or transition rate for the two possible actions, repair ($q_{12}(\cdot)$) and replacement ($q_{13}(\cdot)$) is $\lambda_w \Gamma(1+\frac{1}{\alpha_w})(1-e^{-(\lambda_w a)^{\alpha_w}})$, where $\Gamma(.)$ is the gamma function. Other parameters used in the numerical example are presented in table 1. For a given manufacturing system, they can be determined from the system's historical data.

Table 3.1 Parameters of the numerical example №1

Parameter	U_m	d	h_x	h_a	ρ	Cs	Cp	λ_w
Value	0.4	0.25	0.2	1	10^{-5}	2	100	0.03
Parameter	N	q_0	q_1	q_{31}	r	C_0	c_r	α_w
Value	20	0.01	0.14	10	1	20 000	10	2

The control policies obtained are presented in figures 3.2-3.9. As we can see from figure 3.2, for each surplus level and number of failures, there is a limit on the age of the machine after which if a failure occurs, the machine should be replaced. This limit defines the repair/replacement policy $S_n(x)$ for a given number of failures n. If a failure occurs below $S_n(x)$, a repair is conducted; otherwise, the machine is replaced with a new identical one.

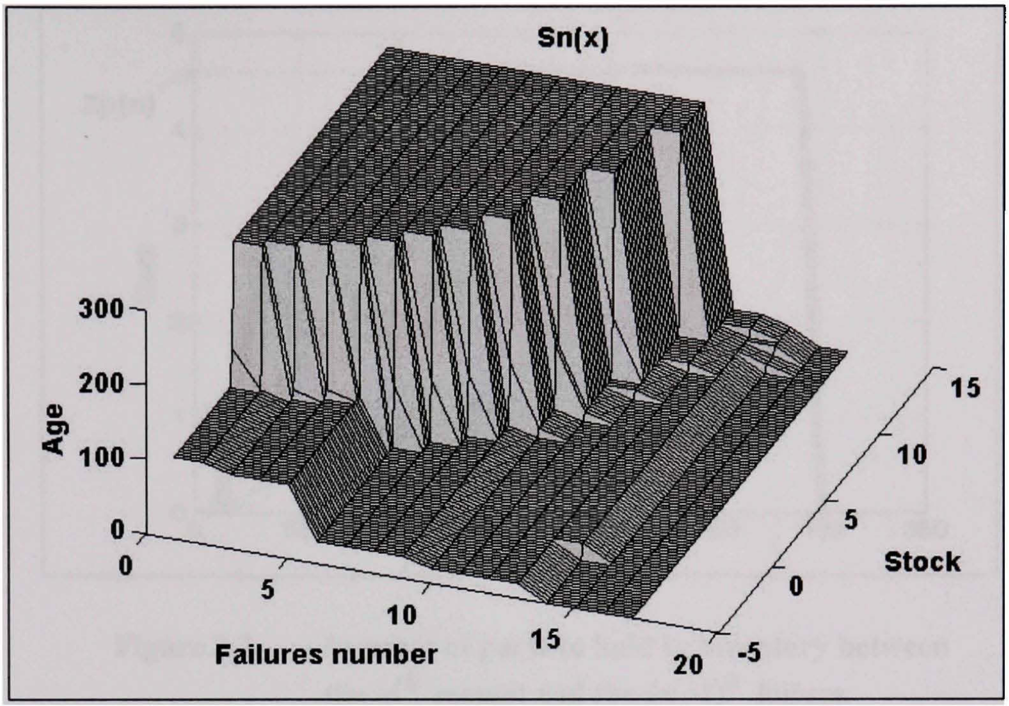


Figure 3.2 Repair/replacement policy $S_n(x)$

It was predictable that the $S_n(x)$ policy would drop as the number of failures value dropped and attain its minimum at zero. $S_{16}(x)=0$, $\forall x$; that is, the machine would have at least $N_m = 16$ failures before a systematic replacement would be required upon a next failure.

The production policy presented in figure 3.3 shows the number of parts $z_n(a)$ to hold in inventory in order to hedge against machine breakdown at each age, for a given number of failures.

If the stock in the system $x(t)$ is less than $z_n(a)$, production should be at a maximum rate; if $x(t)$ is equal to $z_n(a)$, then production should be at demand rate, otherwise, there would be no need to produce parts. As we can see in figures 3.3 and 3.4, the threshold level $z_n(a)$ increases and reaches a certain level $Z_p(n)$, which is the maximum number of parts to hold in inventory before the $(n+1)^{th}$ failure of the machine.

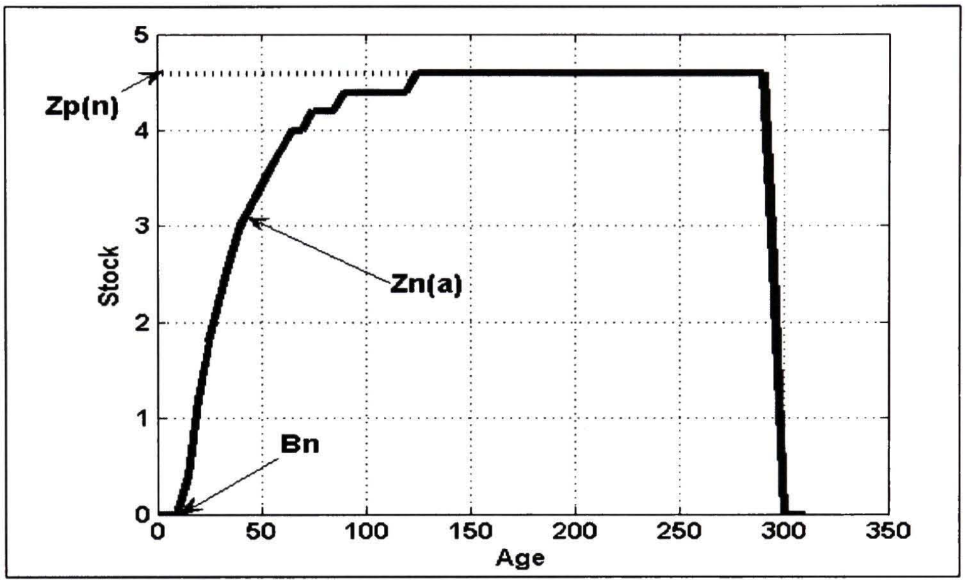


Figure 3.3 **Number of parts to hold in inventory between the n^{th} restart and the $(n+1)^{th}$ failure.**

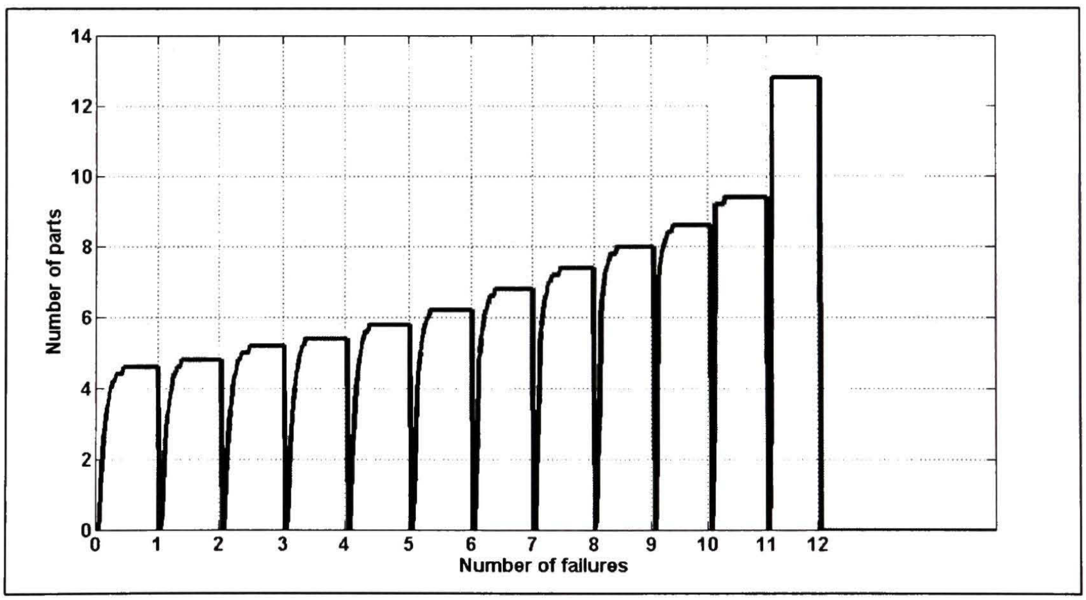


Figure 3.4 **Number of parts to hold in inventory for each failure number.**

Figure 3.5 below presents the maximum number of parts to hold in inventory to hedge again machine breakdown for several failures number.

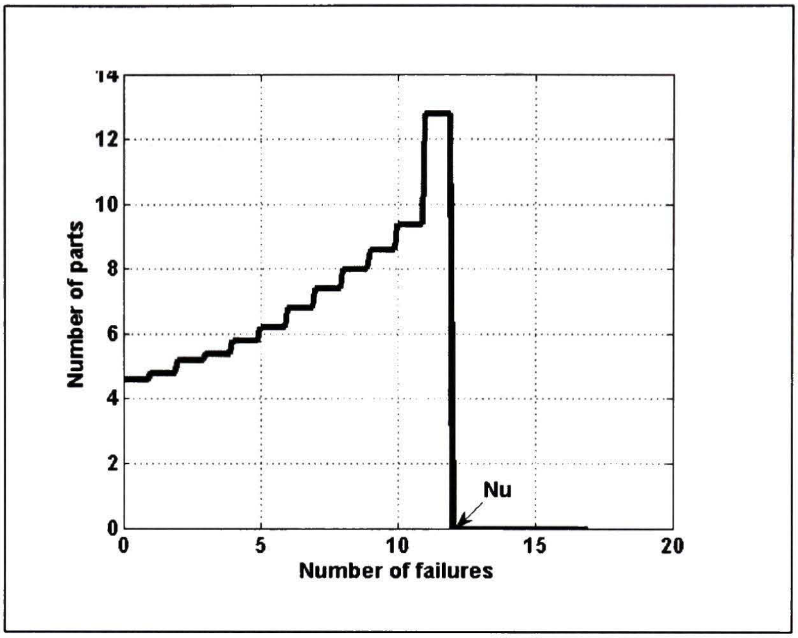


Figure 3.5 Maximum number of parts to hold in inventory for each failure number.

As can be seen in figures 3.4 and 3.5, the number of parts to hold in inventory in order to hedge against failures increases with the number of failures for a certain number of failures N_u. That number of parts becomes null after N_u, that is, just prior to machine replacement. Thus, the repair/replacement policy is:

When the next breakdown of the machine occurs at age $a(t)$,

$$\begin{cases} \text{Repair} & \text{if } a(t) <= S_n(x) \\ \text{Replace} & \text{if } a(t) > S_n(x) \end{cases} \quad \text{with } S_n(x) \text{ given in figure 3.2} \tag{3.13}$$

The above results enable us to clearly illustrate the production control policy (production rate) as follows: before each failure n,

$$u^*(\alpha, a, x, n) = \begin{cases} U_{\mathrm{m}} & \text{if } x(t) < z_n(\mathrm{a}) \\ d & \text{if } x(t) = z_n(\mathrm{a}) \\ 0 & \text{if } x(t) > z_n(\mathrm{a}) \end{cases} \tag{3.14}$$

With $\begin{cases} z_n(a) > 0 \text{ if } B_n < a(t) < S_n(x) \\ z_n(a) = 0 \text{ otherwise} \end{cases}$ and $z_n(a) \le z_p(n)$ and $\alpha = 1$ (Production mode)

The repair/replacement policy obtained is more appropriate for manufacturing systems than those presented in Makis and Jardine (1993), Love *et al.* (1998) and Love *et al.* (2000) . For each stock level, it gives the production rate and the replacement age for a given number of failures, instead of a single replacement age for all system stock levels.

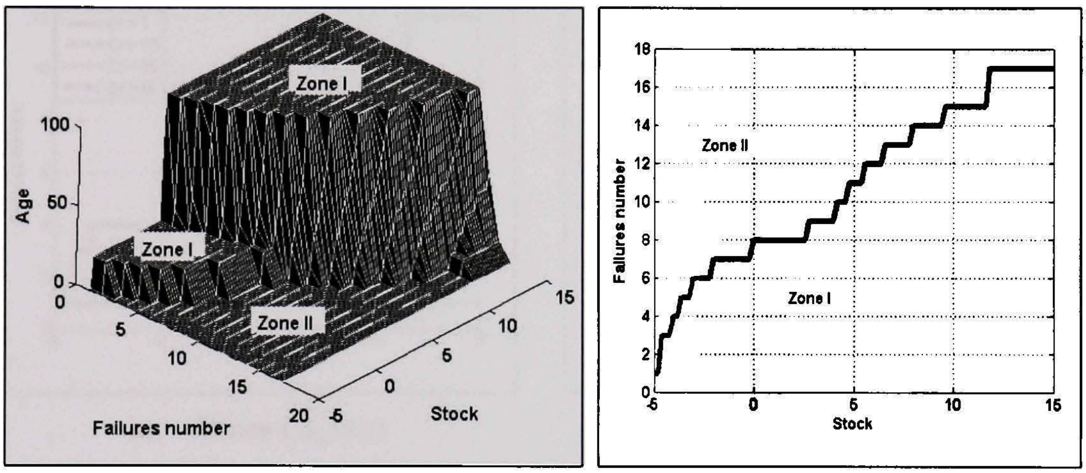
We could have chosen to use the results of Makis and Jardine (1993), Love *et al.* (1998) and Love *et al.* (2000) as the input, and to integrate production control, given the repair/replacement policy. Unfortunately, under these circumstances, there is no evidence to say with certainty that the structure of the policy remains valid when production control is introduced. However, by simultaneously controlling the production and repair versus replacement as we have done in this article, we obtained a directly appropriated repair/replacement policy.

In the next section, we will analyze the sensitivity of the policies obtained according to some parameters of the system.

3.6 Result and sensitivity analysis

It appears from figure 3.5 that the maximum number of parts *Zp*(*n*) to hold in inventory increases with failures until a number N_u of failures is reached; after N_u, the number of parts to hold in inventory in order to hedge against breakdowns is nil. This result is logical because the probability of the machine being replaced at the next breakdown after N_u is almost equal to 1, due to the fact that $S_n(x)$ must then necessarily be exceeded. Furthermore, the replacement takes a negligible amount of time compare to repair time, and after replacement, the machine is new. It is thus normal to avoid holding parts in inventory just prior to replacement.

For other sensitivity analysis, we used $0 \le a \le 100$. The repair/replacement policy in figure 3.6 a. below shows two zones: zone I, where $S_n(x) > 0$ and zone II where $S_n(x) = 0$. Whenever a failure occurs in zone II, the machine is systematically replaced with a new identical one, regardless of age; if it occurs in zone I, then if the age is less than $S_n(x)$, the machine is repaired, otherwise, it is replaced with a new identical one. We will call the boundary of the two zones Trace $(S_n(x))$, presented in figure 3.6 b.



a. $S_n(x)$ **b. Trace** $(S_n(x))$

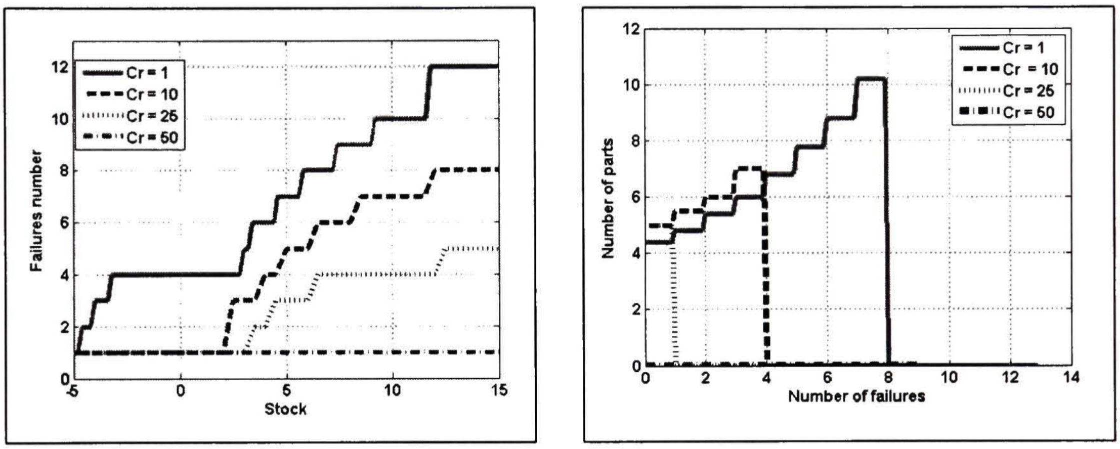
Figure 3.6 Repair/replacement policy $S_n(x)$ and its trace.

From figure 3.6.a, we extract figure 3.6.b in order to carry out a sensitivity analysis of the repair/replacement policy $S_n(x)$ because Trace$(S_n(x))$ just has to change for $S_n(x)$ to change as well.

In figure 3.6.b, we note the requirement for an increasing number of failures before systematic replacement as the stock level in the system increases. For example, when the stock level is at -5, the machine is systematically replaced after the first failure. With a stock level of 5, the machine is systematically replaced after 11 failures, and with a stock level of 15, the machine is systematically replaced after 17 failures, regardless of its age.

We will now analyze the sensitivity of the above results according to the cost of repair per unit of time and the cost of replacement, and see if the graphics in figures 3.5 and 3.6.b maintain their structures as the parameters change.

We will first analyze the sensitivity of the Trace ($S_n(x)$) according to the cost of repair per unit of time. The result obtained for four repair cost values is presented in figure 3.7.a.



a. **Trace ($S_n(x)$)**

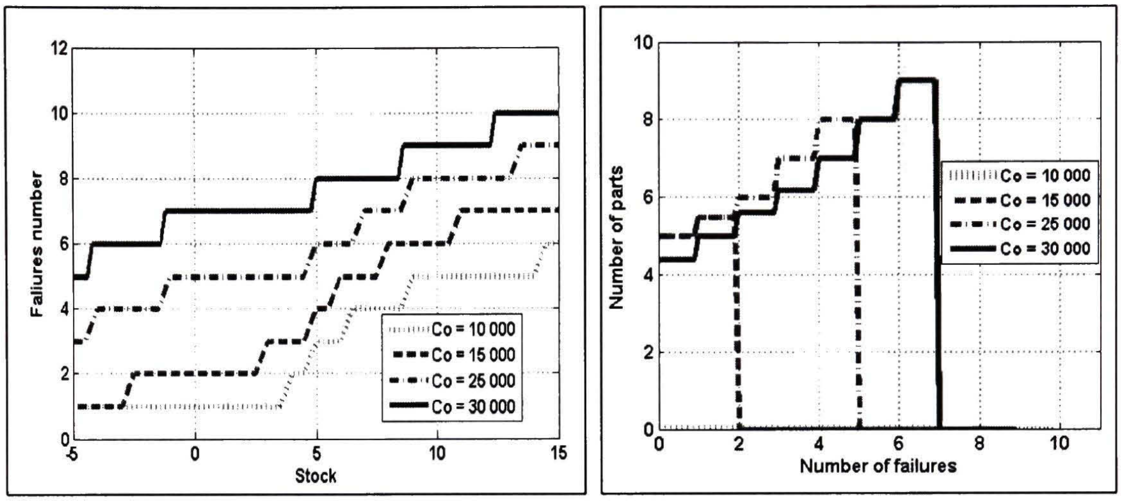
b. **Maximum number of parts $Z_p(n)$ to hold in inventory**

Figure 3.7 Sensitivity of Repair/replacement policy $S_n(x)$ and production policy according to repair cost.

From figure 3.7.a, we conclude that zone 1 decreases as repair costs increase, that is, the machine should be replaced earlier if the repair cost per unit of time increases.

The production policies obtained, which are presented in figure 3.7.b show that, the maximum number of parts to hold in inventory decreases and reaches zero when the repair cost is too high. In this example, 30 is a high value for repair cost per unit of time.

As we can see from figure 3.8.a below, zone 1 also decreases as the machine replacement costs decrease, that is, for a higher value of replacement cost, the number of failures we should expect before systematic replacement for each stock level is higher than the number of failures to be expected before a systematic replacement for low replacement cost values.



a. Trace ($S_n(x)$)

b. Maximum number of parts $Z_p(n)$ to hold in inventory

Figure 3.8 Sensitivity of Repair/replacement policy $S_n(x)$ and production policy according to replacement cost.

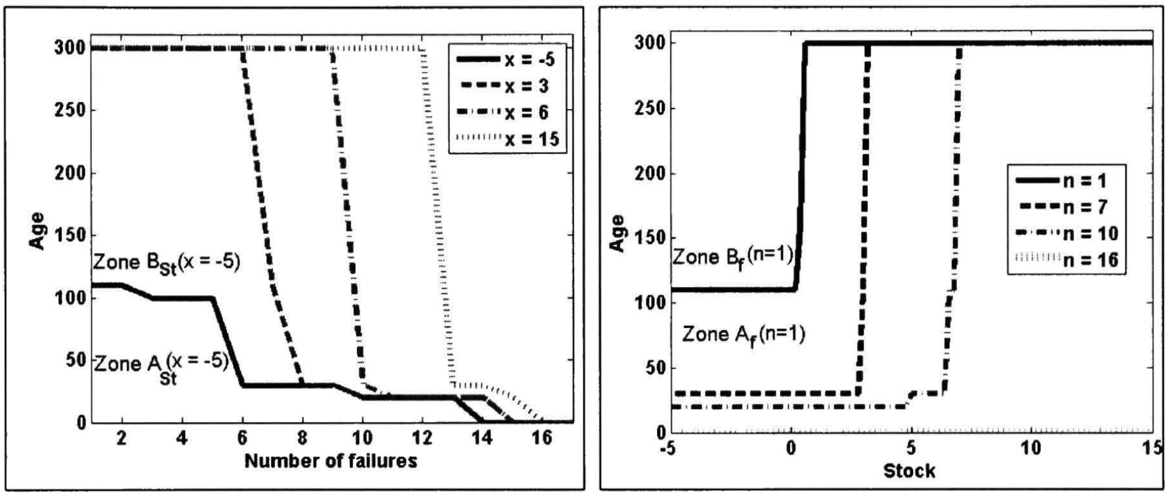
Figure 3.8.b shows that the maximum hedging surplus value increases as the replacement cost increases.

The above results are consistent with our expectations because in reality, for a given replacement cost, if the repair cost increases, the machine will be replaced earlier, and vice versa. In this section, we analyze the results of the control policies obtained, and through various sensitivity analyses, we show that the control policies are robust.

Let us define zone $A_{st}(x)$ as the zone in which if a failure occurs, the machine is repaired and zone $B_{st}(x)$ as the zone in which if a failure occurs, the machine is systematically replaced for a given stock level x. Let us also define $A_f(n)$ as the zone in which if the

n^{th} failure occurs, the machine is repaired and zone $\mathrm{B_f}(n)$ as the zone in which if the n^{th} failure occurs, the machine is systematically replaced.

Figures 3.9.a and 3.9.b below are more appropriated for industrial application when the stock level is known or when the number of failures is known.



a. For a fixed value of stock level x **b. For a fixed number of failures** n

Figure 3.9 Repair/replacement policy $S_n(x)$.

If the stock level in the system is known and constant, we could use the policy given in figure 3.9.a, where the policy is presented for four values of stock level x. As shown in figure 3.9.a, the result of Makis and Jardine (1993), Love *et al.* (1998) and Love *et al.* (2000) appears as a particular case of our result.

Figure 3.9.a shows that zone $\mathrm{A_{st}}(x)$ where the machine is repaired if a failure occurs increases with the stock level in the system while zone $\mathrm{B_{st}}(x)$ where the machine should be replaced if a failure occurs decreases. That is, the more the stock level in the system is high, longer the machine will be kept in the system before replacement after a failure.

Figure 3.9.b illustrates the policy for four number of failures values n. If the number of

failures is known, we could use the policy given by figure 3.9.b, in which the repair/replacement policy is presented for four values of n, according to the number of parts in the system. If a failure occurs in zone $A_f(n)$, the machine is to be repaired and when it occurs in zone $B_f(n)$, the machine is to be replaced.

From figure 3.9.b, we conclude that zone $A_f(n)$ decreases when the number of failures increases. That is, the machine should be replaced earlier after a failure as the number of failures occurred increases. For example, when the number of failures occurred is 16, according to figure 3.9.b, $S_{16}(x) = 0$, for all x. That is, $A_f(16) = 0$ for all x and $B_f(16) = \{a; 0 \leq a \leq 100\}$, meaning that at the 17^{th} failure, the control policy recommend to replace the machine, no matter the age of that machine.

It is important to note that in many situations, the dynamics of several variables change after breakdowns as a result of the machine degradation phenomenon. As shown in the above results, taking into account some degradation with age after a machine failure leads to a policy comprising several critical threshold values, which increase from one breakdown to the next. The production policy for a given number of breakdowns is characterized by three parameters: $Zp(n)$; $B_{fail}(n)$; $S_n(x)$. These parameters are such that at the $(n+1)^{th}$ restart, if the machine age is less than $B_{fail}(n)$, then there is no need to stock parts; when it is situated between $B_{fail}(n)$ and $S_n(x)$, the inventory should be brought up to $Zp(n)$, and maintained at this value. After the machine reaches age $S_n(x)$, it is no longer necessary to have parts in inventory because the machine will be replaced at the next breakdown after $S_n(x)$, where $S_n(x)$ is the repair/replacement policy.

The results presented in this paper indicate that, as expected, the optimal production policy for the considered manufacturing system is characterized by a special structure of the hedging point policy. Such a policy is defined by the aforementioned three parameters for production (i.e. $Zp(n)$; $B_{fail}(n)$; $S_n(x)$) and two parameters for the repair/replacement switching policy (i.e. $S_n(x)$; Nu). The overall control policy given by equations (3.13) and (3.14) is completely defined by values of parameters ($Zp(n)$; $B_{fail}(n)$; $S_n(x)$) and ($S_n(x)$; Nu) both for production and repair/replacement switching policy.

3.7 Conclusion

In this work, we combined repair versus replacement planning with a production control in an age deterioration manufacturing system, and determined the impact of the stock level on the repair/replacement policy. We showed that the stock level required to hedge against breakdowns increases with the number of failures, and that the optimal age at which to replace the machine for manufacturing systems is not constant for each failure number, but rather, depend on the stock level in the system. A numerical example is given to illustrate the utility of the proposed approach, and a sensitivity analysis is considered in order to confirm the control policy structure obtained. Further extensions of the work presented in this paper could involve performing preventive maintenance in order to increase system availability, to decrease operating costs, to prevent the occurrence of system failures, and to avoid failure or unnecessary replacement.

REFERENCES

Boukas E.-K. and Haurie A. (1990). "*Manufacturing flow control and preventive maintenance: A stochastic control approach.*" IEEE Transactions on Automatic Control, 35(9): pp. 1024-1031.

Cassady C. R. and Kutanoglu E. (2005). "*Integrating preventive maintenance planning and production scheduling for a single machine.*" IEEE Transactions on Reliability, 54(2): pp. 304-309.

Chelbi A. and Ait-Kadi D. (2004). "*Analysis of a production/inventory system with randomly failing production unit submitted to regular preventive maintenance.*" European Journal of Operational Research, 2004; 156(3): pp. 712-718.

El-Ferik S. (2008). "*Economic production lot-sizing for an unreliable machine under imperfect age-based maintenance policy.*" European Journal of Operational Research, 186(1): pp. 150-163.

Gabriella B., Rommert D. and Robin P. N. (2008). "*Maintenance and Production: A Review of Planning.*" Complex System Maintenance Handbook. S. London, 2008.

Gharbi A. and Kenne J. P. (2000). "*Production and preventive maintenance rates control for a manufacturing system: An experimental design approach.*" International Journal of Production Economics, 65(3): pp. 275-287.

Gharbi A., Kenne J. P. and Beit M. (2007). "*Optimal safety stocks and preventive maintenance periods in unreliable manufacturing systems.*" International Journal of Production Economics, 107(2): pp. 422-434.

Gharbi A., Pellerin R. and Sadr J. (2008). "*Production Rate Control Policy for Stochastic Remanufacturing Systems.*" International Journal of Production Economics, 112(1): pp. 37-47.

Gershwin, S. B. (2002). "*Manufacturing systems engineering.*" Englewood Cliffs, NJ: prentice-Hall.

Hariga M. A., Azaiez M. N., and Daya M. B. (2000). "*A discounted integrated inspection-maintenance model for single deteriorating production facility.*" International Transactions in Operational Research, 13(4): pp. 353-364.

Kenne J. P. and Boukas E. K. (2003). "*Hierarchical control of production and maintenance rates in manufacturing systems.*" Journal of Quality in Maintenance Engineering, 9(1): pp. 66-82.

Kenne J.P., Gharbi A. and Beit M. (2007). "*Age-Dependent Production Planning and Maintenance Strategies in Unreliable Manufacturing Systems with Lost Sale.*" European Journal of Operational Research, 178(2): pp. 408–420.

Kijima M., Morimura H. and Y. Suzuki. (1988). "*Periodical replacement problem without assuming minimal repair.*" European Journal of Operational Research, 37(2): pp. 194-203

Kimemia J. and Gershwin S. B. (1983). "*Algorithm for the computer control of a flexible manufacturing system.*" Institute of Industrial Engineers Transactions, 15(4): pp. 353-362.

Kushner H. and Dupuis P. G. (1992). "*Numerical methods for stochastic control problems in continuous time.*" Springer-verlag; New York.

Love C. E., Zhang Z. G. Zitron M.A. and Guo R. (1998). "*An SMDP approach to optimal repair/replacement decisions for systems experiencing imperfect repairs.*" Journal of Quality in Maintenance Engineering, 4(2): pp. 131-149.

Love C. E., Zhang Z. G. Zitron M.A. and Guo R. (2000) "*Discrete semi-Markov decision model to determine the optimal repair/replacement policy under general repairs.*" European Journal of Operational Research, 125(2): pp. 398-409.

Makis V. and Jardine A. K. S. (1992). "*Optimal replacement policy for a general model with imperfect repair.*" Journal of the Operational Research Society, 43(2): pp. 111-120.

Makis V. and Jardine A. K. S. (1993). "*Note on optimal replacement policy under general repair.*" European Journal of Operational Research, 69(1): pp. 75-82.

Makis V. and Jardine A. K. S. (1991). "*Optimal replacement of a system with imperfect repair.*" Microelectronics and Reliability, 31(2-3): pp. 381-388.

Mathew S. and Kennedy D. (2003). "*A strategy for optimal equipment replacement.*" Production Planning and Control, 14(6): pp. 571-577.

Ouhbi B. and Limnios N. (2003). "*Nonparametric reliability estimation of semi-Markov processes.*" Journal of Statistical Planning and Inference, 109(1-2): pp. 155-165.

Pellerin R., Boulet J.-F., Gharbi A. and Kenne J.-P. (2007). "*Développement d'une politique intégrée de contrôle des taux de production et de maintenance corrective avec* *diagnostic (Development of an integrated control policy of the rates of production and of corrective maintenance with diagnosis).*" INFOR, 45(4): pp. 197-207.

Phelps A.V. (1983). "*Transport data for the modeling of electrical breakdown and discharges. Electrical Breakdown and Discharges in Gases. Part A: Fundamental Processes and Breakdown.*" Plenum Press, New York, NY, USA, 89(a): pp. 109-132.

Rishel R. (1975). "*Control of systems with jump Markov disturbances.*" IEEE Transactions on Automatic Control, 20(2): pp. 241-244.

Sethi S. P. and Zhang Q. (1994). "*Hierarchical decision making in stochastic manufacturing systems.*" Birkhäuser, 1994.

Wang G. J. and Zhang Y. L. (2006) "*Optimal periodic preventive repair and replacement policy assuming geometric process repair.*" IEEE Transactions on Reliability, 55(1): pp. 118-122.

Xiaojun Z., Lifeng X. and Jay L. (2007). "*Reliability-centered predictive maintenance scheduling for a continuously monitored system subject to degradation.*" Reliability Engineering & System Safety, 92(4): pp. 530-534.

Yan H. and Zhang Q. (1997). "*Numerical method in optimal production and setup scheduling of stochastic manufacturing systems.*" IEEE Transactions on Automatic Control, 42(10): pp. 1452-1455.

Yong J. and Zhou X. Y. (1999) "*Stochastic controls: Hamiltonian systems and HJB equations.*" New York, Springer-Verlag New York, Inc., ISBN 0-387-98723-1.

Yuan Lin, Z. (1994). "*A bivariate optimal replacement policy for a repairable system.*" Journal of Applied Probability, 31(4): pp. 1123-1127.

CHAPITRE 4

ARTICLE 3: HIERARCHICAL DECISION MAKING IN PRODUCTION AND REPAIR/REPLACEMENT PLANNING WITH IMPERFECT REPAIRS UNDER UNCERTAINTIES

F. I. Dehayem Nodem [1], J. P. Kenne[1] and A. Gharbi[2]

[1]Mechanical Engineering Department, Production Technologies Integration Laboratory, University of Quebec,

[2]Automated Production Engineering Department, Production System Design and Control Laboratory, University of Quebec

École de technologie supérieure, 1100 Notre Dame Street West, Montreal, Quebec Canada H3C 1K3

This chapter is published as an article in European Journal of Operational Research. 2008. Vol. 198, no 1, pp. 173-189.

Résumé

Dans cet article, nous analysons les politiques optimales de production, de réparation et de remplacement des machines d'un système manufacturier sujet aux pannes aléatoires. Le système fabrique des pièces et lorsqu'une panne de machine survient, soit une réparation imparfaite est effectuée, soit la machine est remplacée par une autre machine, neuve et identique. Dans le présent article, les machines après réparation n'ont pas un âge nul, mais plutôt plus faible que leur âge au moment de la survenue de la panne. L'âge de la machine ainsi obtenu après réparation est appelé âge virtuel de la machine. Par conséquent, les temps successifs d'opération de la machine avant panne deviennent de plus en plus courts à mesure que le nombre de pannes augmente. La dynamique d'un tel système change après chaque panne et l'historique des pannes et réparations est prise en considération, rendant l'optimisation de la planification de production, des réparations et du remplacement de la machine très complexe. Pour cette raison, nous utilisons une approche hiérarchique de prise de décision basée sur un modèle semi-Markovien. Les variables de décision sont le taux de production et la politique de réparation versus le remplacement de la machine. L'objectif est de trouver les variables de décision qui permettent de réduire les coûts totaux encourus sur un horizon de planification infini. L'approche hiérarchique utilisée permet dans un premier temps d'élaborer la politique de réparation et de remplacement. Ensuite, le taux de production est déterminé, compte tenu de la politique de réparation versus le remplacement. Les conditions d'optimum sont développées et les méthodes numériques utilisées pour les résoudre et déterminer les politiques de commande. Des exemples numériques et des analyses de sensibilité sont présentés pour illustrer l'utilité de l'approche proposée.

Abstract

In this paper, we analyse an optimal production, repair and replacement problem for a manufacturing system subject to random machine breakdowns. The system produces parts, and upon machine breakdown, either an imperfect repair is undertaken or the machine is replaced with a new identical one. The decision variables of the system are the production rate and the repair/replacement policy. The objective of the control problem is to find decision variables that minimize total incurred costs over an infinite planning horizon. Firstly, a hierarchical decision making approach, based on a Semi-Markov Decision Model (SMDM), is used to determine the optimal repair and replacement policy. Secondly, the production rate is determined, given the obtained repair and replacement policy. Optimality conditions are given and numerical methods are used to solve them and to determine the control policy. We show that the number of parts to hold in inventory in order to hedge against breakdowns must be readjusted to a higher level as the number of breakdowns increases or as the machine ages. We go from the traditional policy with only one high threshold level to a policy with several threshold levels, which depend on the number of breakdowns. Numerical examples and sensitivity analyses are presented to illustrate the usefulness of the proposed approach.

Keywords: Manufacturing systems, Numerical methods, Optimal control, Production planning, imperfect repairs, damage failures, replacement.

4.1 Introduction

Consider a new machine that has just been installed and is operating perfectly. Such a machine is subject to events such as breakdowns and repairs, demand fluctuations, wear, fatigue, crack, corrosion and erosion. Overtime, this machine ages, given the presence of these events as well as many other factors. As the machine ages, the frequency of failure increases, and at some point, it may no longer be economically justifiable to keep the machine in the system. The system under consideration consists of a failure prone machine subject to random breakdowns, repairs and replacement. Upon a failure, the machine can be replaced or an imperfect repair undertaken. Such a system was considered by Makis and Jardine (1993) and Love *et al.*(2000) after it experienced its n^{th} failure and was repaired. They determined the optimal replacement policy, requiring the shortest repair and replacement time. Makis and Jardine (1993) showed that if the repair cost depends only on the age of the system, then the optimal replacement policy would fall under a certain class of policies called T-policies, which were considered earlier in the literature for models requiring minimal repair. Phelps (1983) used a semi-Markov decision model to prove that a strategy, in which minimal repairs are performed on a machine up to age T, with a replacement by a new machine at the first failure after T, is the optimal strategy to use over an infinite horizon. Nakagawa and Kijima (1989) applied the periodic replacement with minimal repair at a failure cumulative model. They provided the mean cost-rate, the optimal time, the optimal number of shocks, and the optimal number of damages. They provided those parameters when the unit is replaced at a specific time, at shock or at damage, with minimal repair undertaken between replacements. A minimal repair is the repair that brings the state of a failed machine to a level in which it was prior to failure. We refer the reader to the work of Valdez-Flores and Feldman (1989) for an extensive examination of minimal repair policies.

Using the two-dimensional state space proposed by Makis and Jardine (1993), Love *et al.* (2000) developed a discrete semi-Markov structure to obtain the optimal repair/replacement policies for systems subject to imperfect repairs. An imperfect repair resets the age of the machine at a level that is smaller than the real age, but that does not renew the machine. For a case study of the preventive maintenance, repair and replacement of such a system in the water industry, Ansell *et al.* (2004) showed that repair and replacement should only be performed when the equipment has failed, and describe general conditions under which replacement is appropriate. Those authors obtained the optimal replacement policy in order to minimize the expected average cost. However, in a context of production and demand satisfaction, machine preventive maintenance, repair and replacement management are not sufficient to minimize general cost since inadequate production planning can generate additional storage or shortage costs.

A great amount of research has been focused on the determination of the optimal production planning of manufacturing systems under repair. Following the work by Rishel (1975) and Sworder (1969) on production planning for a system affected by jump disturbances, Boukas and Haurie (1990) combined production and preventive maintenance planning in cases where the machine's failure probability increases with its age, using the hedging point policy concept introduced by Kimemia and Gershwin (1983). For more details on this concept, we refer the reader to the age-dependent hedging point concept presented by Boukas, (1998); Kenne and Gharbi, (2000). They determine production rate and maintenance rule which minimize the total expected cost of a two-machine system. However, with the numerical scheme adopted in their work, it remains computationally difficult to obtain the optimal control of a large scale manufacturing systems. To cope with this difficulty, Kenne and Boukas (2003) formulated the singular perturbation problem of production and preventive maintenance rates planning in an manufacturing systems, and obtained a limiting problem that was numerically more tractable. Gharbi and Kenne (2005) extended this production and maintenance rates control model and determined the control policy for a large case including non-identical machine manufacturing systems. These works on production planning under general repairs have the merit of taking into account the fact that the system deteriorates with age, just like real life systems. However, following repairs or preventive maintenance, the machine is either as good as new (Boukas and Haurie (1990); Kenne *et al.* (2007); Kenne and Boukas (2003)) or is as bad as an old machine. Generally, assuming that the repair does not damage the machine, the repair brings the state of a failed machine to a level somewhere between new and prior to failure. That type of repair is called the *imperfect repair*. The level of repair is known as the *intensity of repair*, and varies from zero, for a perfect repair, to one, for minimal repair. The repair intensity could reflect the impact of the n^{th} repair and be function of the n^{th} repair as described in Love *et al.* (2000) or be stochastic (see Kijima(1989)). It is a function of the quality of intervention performed and depends on the skill level of the maintenance team as well as the number and nature of the components repaired (Shin *et al.* 1996). However, little work has been developed to take into account the case where this factor is stochastic (see Mohafid and Castanier (2006)). In deterministic cases, the repair intensity, used to model the effectiveness of maintenance, is assumed to be known and constant (see Love *et al.* (2000)). Kijima (1989) proposed that upon failure, the repair undertaken could serve to reset the age of the machine only as far back as its age at the start of the last failure, called the virtual age. The literature refers to this repair model as Kijima's Type I imperfect repair model, and it has largely been used in cumulative damage models. Note that the virtual age is equal to or less than the real age, and thus, minimal repair and perfect repair are, by extension, two special cases of the imperfect repair model.

The goal of this paper is to determine the production rate and the repair/replacement policy that minimizes the total expected cost when the system deteriorates with age, and is subject to damage failures. Upon failure, an imperfect repair can be undertaken or the machine replaced by a new identical one. For consistency with previous authors and Type I repairs, the replacement cost is considered constant and the repair cost is a bounded non-decreasing function of real age, virtual age or number of failures. Such a control problem is very complex due to the fact that although the machine is initially new or is new after each replacement, it has clearly different dynamics after each breakdown and repair. One way of coping with this complexity is to develop a hierarchical decision making model for the system.

Considering that human factor is hardly interpreted by analytical methods because of its unpredictable nature, fuzzy logic (see Zimmerman (1990) and Vasant (2004)) is an important tool that could also be used to include the effects of human factors like technician's experience. For production and maintenance cooperate scheduling, Coudert *et al.* (2002) used multi-agent and fuzzy logic and show that fuzzy logic provides interesting facilities for modeling the degrees of freedom of the negotiation in a quite natural way. Labib and Sudiarso (2002) presented an algorithm for transforming maintenance data to shop floor information. These shop floor data are then used via a fuzzy-logic based scheduling algorithm to determine optimal production systems control policies. Yuniarto and Labib (2006) integrated production control, corrective and preventive maintenance by a method based on fuzzy logic.

In studying hierarchical models involving stochastic, multilevel decision processes, Dempster *et al.* (1981) argued that the objective at each level is the minimization of current costs plus the expected value of the lower-level decisions. Sethi and Zhang (1994) provided a review of several different approaches to hierarchical decision making in an uncertain environment. Our hierarchical proposed approach consists in developing a Semi-Markov Decision Model (SMDM) in order to make a lower level determination of the optimal repair and replacement policy for a system that deteriorates with age, and is subject to damage failures. Upon failure, the system can be repaired or replaced by a new identical one. Given that policy, at higher levels, we derive a production plan for the system, which minimizes surpluses, backlogs, repairs, and replacement costs over an infinite planning horizon.

The paper is organized as follows. In Section 4.2, we present the notations and the problem statement, followed in Section 4.3 by optimality conditions. Numerical methods are used in Section 4.4 to solve the Hamilton-Jacobi-Bellman (HJB) equations obtained in Section 4.3. The overall search algorithm for the control policy is also provided in Section 4.4. Numerical examples are presented in Section 4.5. In Section 4.6, results and sensitivity analyses are presented to illustrate the usefulness of the proposed approach. We finally conclude in Section 4.7.

4.2 Notations and Problem statement

The system considered consists of one machine which produces one part type. Before we formulate the problem, let us define in the next sub-section the notations to be use in this paper.

4.3 Notations

The following notations will be used in the rest of the paper.

$x(.)$	inventory level
$u(.)$	production rate of the system
n	number of failures
$a(.)$	machine age
$a_n(.)$	machine age before the $(n)^{th}$ failure
A_n	machine virtual age before the $(n)^{th}$ failure
d	demand rate
$\xi(.)$	random process of the system
$\mathrm{U_m}$	maximum production rate of the system
ρ	discounted rate
$G(\cdot)$	instantaneous cost
$J(\cdot)$	expected discounted cost
$q_{\alpha\beta}(\cdot)$	transition rate from mode α to mode β
$V(\cdot)$	value function
n_p	number of failures before replacement
s	age after which the machine should be replaced
N_m	maximal number of failures before replacement at the next failure

s_n age after which the machine should automatically be replaced at the $(n+1)^{th}$ failure

$\Gamma(\cdot)$ set of $u(\cdot)$ given s_n

τ jump time of $\xi(t)$

θ impact of the repair action

C_0 instantaneous replacement cost

$C_1(\cdot)$ instantaneous repair cost

$w(\cdot)$ repair/replacement cost

$h(\cdot)$ inventory/backlog cost

4.4 Problem formulation

To properly characterize the system, the formulation of the problem requires five state variables: number of failures to date n, real age $a_n(t)$, virtual age $A_n(t)$, inventory/backlog level $x(t)$ and the stochastic process that described the machine mode $\xi(t)$. Figure 4.1 presents the fives state variables of the system, which has already had a n^{th} failure, and shows what could happen at the next breakdown.

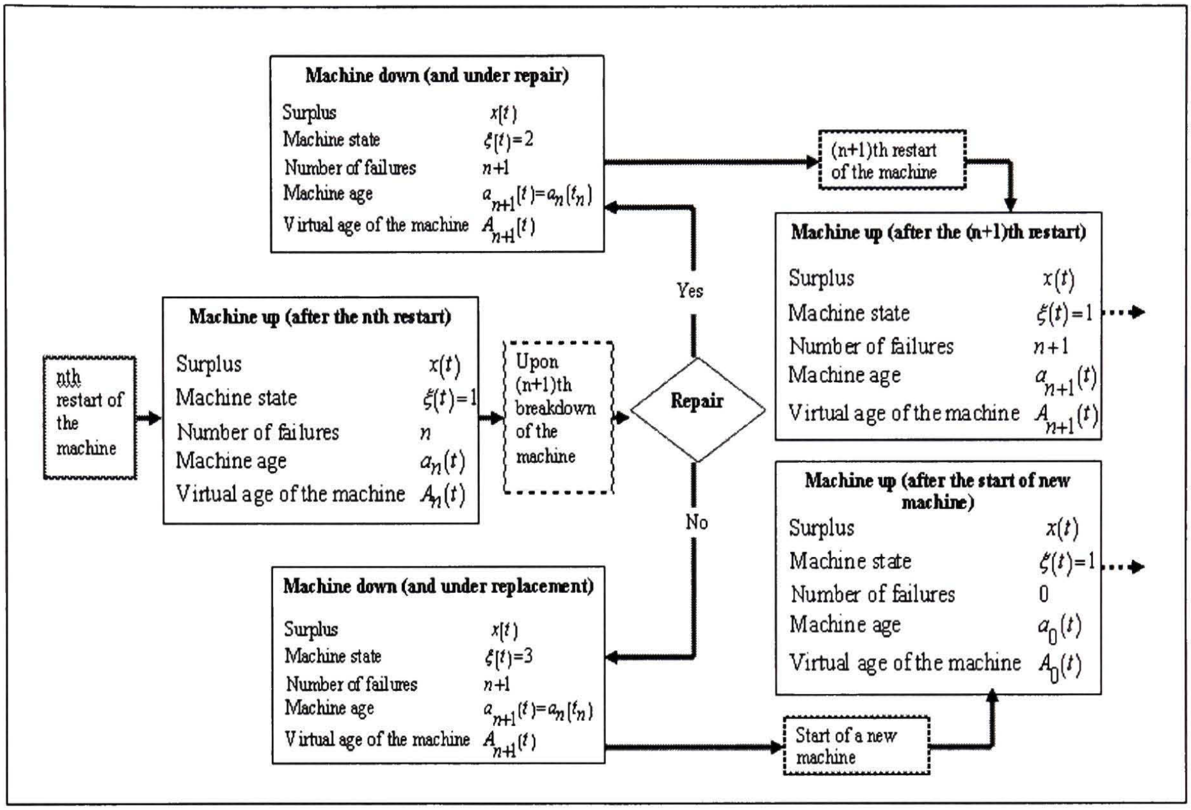


Figure 4.1 State variables of the system.

At any given time t, the number of failures is n, with $n=0,1,2,\ldots$. From the initial instant $t_0=0$, failures occur at instants $t_1,t_2,t_3\ldots$. Thus, the number of failures occurring from the initial instant to a given instant t (if only repair actions are taken at failures) is:

$$n=\max\{i\geq 0/t_i\leq t\}=\sum_{i=1}^{+\infty} Ind\{t_i\leq t\} \tag{4.1}$$

where the function $Ind\{(\cdot)\}$ equals 1 when the condition $(\cdot)$ is satisfied, and is zero otherwise. The equation $n=0$ implies the machine has not had its first failure, but is not necessarily new because its age increases over time.

The three modes of the machine are described by a stochastic process $\xi(t) \in \Omega = \{1,2,3\}$ such that $\xi(t)=1$ when the machine is operational (corresponding to an operational mode in figure 4.1, $\xi(t)=2$ when the machine is under repair, and $\xi(t)=3$ when it is under replacement. The last two correspond to down modes in figure 4.1.

We assume that the distribution of the sojourn time between the $(n-1)^{th}$ and the n^{th} failure depends only on the failure rate function and the imperfect repair model.

Let A_n be the virtual age of the machine after the n^{th} repair. The machine age before the $(n+1)^{th}$ failure is given by:

$$a_n(t) = a(t-t_n) + A_n \ , \ t > t_n \qquad t_0 = 0 \, , \ A_0 = 0 \tag{4.2}$$

We consider that the age of the machine is an increasing function of chronological time when the machine is operational. The machine age function $a(t)$ is then described by the differential equation:

$$\frac{da(t)}{dt} = Ind\{\xi(t) = 1\} \tag{4.3}$$

The virtual age after the n^{th} repair is:

$$A_n = \theta a(t_n - t_{n-1}) + A_{n-1}; \quad n \geq 1 \tag{4.4}$$

where $0 \leq \theta \leq 1$ is the impact of the repair or the repair intensity, assumed to be known and constant. In this case, there is a one-to-one correspondence between real age and post-repair virtual age (see Love *et al.*, (1998) for more details).

If both repair and replacement take a negligible amount of time, $Ind\{\xi(t)=1\}\approx 1,\ \forall\, t\ \geq 0$. The virtual age after the repair is then $A_n = \theta t_n$. Using that form of virtual age, we obtain a semi-Markovian model with a Markovian property (see Love *et al.*, (1998)). The same results are obtained when the repair or replacement takes time, by having two stopwatches installed on the machine. The first moves only when the machine is operational, and indicates the cumulated chronological time of production, while the second moves only when the machine is under repairs, and is reset to zero when the machine becomes operational again. In addition, if the time of repair $t_r(i)$ is not negligible, but is only a function of n $\left(\text{i.e } \sum_{i=1}^{n} t_r(i) = T_n\right)$, then

$$A_n = \theta\left[t_n - T_{n-1}\right] \approx \theta\varphi_n \tag{4.5}$$

with $\varphi_n \leq t_n$.

An imperfect repair cannot shift the virtual age A_n below the virtual age A_{n-1}, as shown in figure 4.2, where A_1 and A_2 are clearly represented. In this work, we use the real-time path with a repair time that is a function of the number of failures and a stopwatch that gives the cumulative production time. Replacement takes a negligible amount of time. The path when both repair and replacement take a negligible amount of time is also presented in figure 4.2.

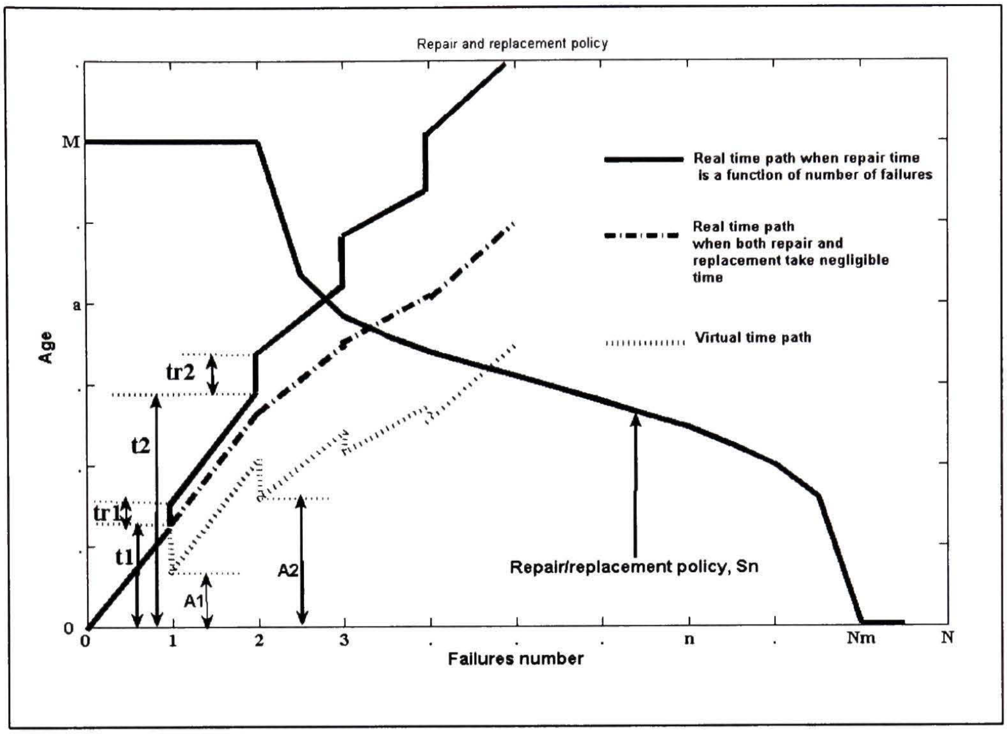


Figure 4.2 **Example of repair and replacement policy** S_n **with virtual and real-time path.**

Figure 4.2 states that the first failure occurs at t_1. It takes t_{r1} units of time to repair the machine, since the failure occurs below S_1, which is equal to M in this example. S_n is the age after which the machine should automatically be replaced at the next failure. Other details on S_n will be provided in the next sections. Upon restart after the first repair, the age of the machine is A_1, and the next failure occurs at t_2. It takes t_{r2} units of time to repair the machine. $A_1 = \theta t_1$; $T_1 = t_{r1}$; $A_2 = \theta(t_2 - T_1)$; $T_2 = t_{r1} + t_{r2}$.

Let $f(a)$ be the probability density function of the first age to failure in a replacement cycle and $F(a)$ the cumulative distribution function. With b_n being the time between the n^{th} repair and the $(n+1)^{th}$ failure, if at age $a_n(t)$ a repair action is taken, the probability density function of b_{n+1}can be writte n as:

$$f_{A_n}(b) = \frac{f(b + A_n)}{1 - F(A_n)} \tag{4.6}$$

The cumulative distribution function of the $(n+1)^{th}$ lifetime b_{n+1}, given that the virtual age $A_n = \Delta$ is:

$$\mathrm{F}_\Delta(b) \equiv P(b_{n+1} \le b / A_n = \Delta) = \frac{(F(b+\Delta) - F(\Delta))}{1 - F(\Delta)} \tag{4.7}$$

The mean time to repair is given by:

$$\tau_1(n, a_n) = \int_0^\infty x f_{\theta a_n}(x)\, dx \tag{4.8}$$

And the mean time to replacement by:

$$\tau_0(n, a_n) = \int_0^\infty x f(x)\, dx \tag{4.9}$$

The replacement cost is fixed at C_0, while the repair cost $C_1(n, a_n)$ is a bounded non-decreasing function of n and a_n. The repair/replacement policy cost $w(n, a_n)$ is defined by:

$$w(n, a_n) = \min\left\{ \left(C_1(n, a_n) + \int_0^{+\infty} w(n+1, \theta a_n + y) \mathrm{F}_{\theta \mathrm{a_n}}(\mathrm{d}y) \right) Ind\{\xi(t) = 2\}, \atop \left(C_0 + \int_0^{+\infty} w(1, y) \mathrm{F}(\mathrm{d}y) \right) Ind\{\xi(t) = 3\} \right\} \tag{4.10}$$

Makis and Jardine (1993) and Love *et al.* (2000)) showed that the optimal replacement policy for this type of system is a threshold-type policy characterized by a set of positive integers $S_1 \ge S_2 \ge \ldots \ge S_N$ and an upper bound M beyond which the machine is automatically replaced. If at the n^{th} failure the machine's age is $a_n(t_n) \ge S_n$, then it should be replaced. Thus, after the n^{th} failure, the machine jumps from operational (up) mode to two possible down mode, and later, from a down mode to operational mode, as shown in figure 4.1 above.

The hierarchical approach proposed in this paper consists of a high level determination of the production plan of the system in operational mode, given the optimal replacement policy S_n obtained at a lower level. The levels are presented in figure 4.3.

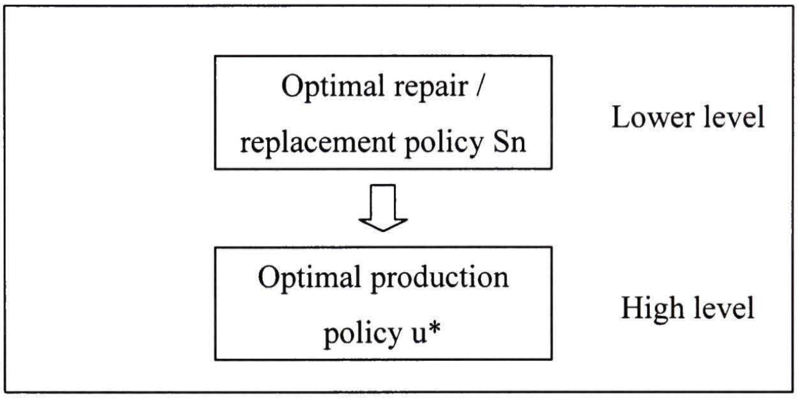


Figure 4.3 Hierarchical approach.

When the machine is operational, it produces parts and when it is under repair or under replacement, it does not produce anything. The surplus may take either a positive value, called an inventory, or a negative value, call a backlog. The state equation of the surplus is given by:

$$\dot{x}(t) = u(t) - d, \quad x(0) = x, \quad t \in \mathbb{R}^+ \tag{4.11}$$

where $d \in \mathbb{R}$ denotes the constant demand rate, and x the initial surplus value.

The transition rates $q_{\alpha\beta}(a_n, n)$ of the machine after the n^{th} failure from mode $\xi(t) = \alpha \in \Omega$ to mode $\xi(t) = \beta \in \Omega$ at instant t are defined by:

$$q_{12}(a_n(t), n) = \lim_{\delta t \to 0} \left\{ \frac{1}{\delta t} \left(\mathrm{P}\left[\xi(t+\delta t) = 2 / \xi(t) = 1 \right] \right) \right\} \mathrm{Ind}\left\{ a_n(t) < S_n \text{ and } \mathrm{n} < \mathrm{N_m} \right\} \tag{4.12}$$

$$q_{13}\left(a_n(t),n\right)=\lim_{\delta t\to 0}\left\{\frac{1}{\delta t}\left(\mathrm{P}\left[\xi(t+\delta t)=3\,/\,\xi(t)=1\right]\right)\right\}\mathrm{Ind}\left\{a_n(t)\geq S_n \text{ or } n\geq \mathrm{N_m}\right\} \quad (4.13)$$

where N_m is the maximal number of breakdowns after which the machine is replaced at the next breakdown. It is important to note that at N_m, $S_n = 0$.

$$q_{21}\left(a_n(t),n\right)=\lim_{\delta t\to 0}\left\{\frac{1}{\delta t}\left(\mathrm{P}\left[\xi(t+\delta t)=1\,/\,\xi(t)=2\right]\right)\right\} \quad (4.14)$$

$$q_{31}\left(a_n(t),n\right)=\lim_{\delta t\to 0}\left\{\frac{1}{\delta t}\left(\mathrm{P}\left[\xi(t+\delta t)=1\,/\,\xi(t)=3\right]\right)\right\} \quad (4.15)$$

All other transition rates are equal to zero.

Let $G(\cdot)$ be the cost rate defined as follows:

$$G(\xi,a,x,n,u)=h(x)+w(n,a_n) \quad (4.16)$$

where $h(x)=c^+x^+ + c^-x^-$. The constants c^+ and c^- are used to penalize inventory and backlog, respectively. $x^+ = \max(0, x)$; $x^- = \max(0, -x)$.

Our objective is then to find the number of failures N_m after which the machine should be replaced at the next failure; the age S_n before automatic replacement at the next failure, and the production rate $u^*(t)$ to minimize the expected discounted cost $J(\cdot)$ given by:

$$J(\alpha,a_n,x,n,u)=E\left[\int_0^{\infty} e^{-\rho t}G(\cdot)dt\,/\; x(t)=x,\ \xi(t)=\alpha,\ a_n(t)=a_n,\ n(t)=n\right] \quad (4.17)$$

where ρ is the discounted rate.

Equation (4.17) is subject to equations (4.11) - (4.15), and the capacity constraint given by:

$$0 \leq u(t) \leq U_m \tag{4.18}$$

where U_m is the maximum production rate of the machine.

Let $\omega(a_n, n) = \begin{cases} 1 & \text{if at the } n^{th} \text{ failure at age } a_n, \text{ a replacement is undertaken} \\ 0 & \text{underwise} \end{cases}$

Let also

$$S = \{a_n \geq 0 / \omega(a_n, n) = 0\}, \ \mathrm{P} = \{n \geq 0 / \omega(a_n, n) = 1\} \text{ and } \Gamma = \{u(t) / 0 \leq u(t) \leq U_m\} \tag{4.19}$$

Any plan $(s, n_p, u) \in S \times \mathrm{P} \times \Gamma$ that satisfies (4.19) is called an admissible plan, and our problem lies in minimizing the costs across all admissible plans.

The control variables are the number of failures before replacement at the next failure n_p, the age s after which the machine should automatically be replaced at the next failure, and the production rate u.

$S_n = \max\{a_n \geq 0 / \omega(a_n, n) = 0\}$ and $N_m = \min\{n \geq 0 / \omega(a_n, n) = 1\}$. Since at N_m, $S_n = 0$, it is sufficient to determine S_n instead of the two variables.

The value function of the problem is:

$$V(\alpha, a_n, x, n) = \inf_{u(t) \in \Gamma(\alpha, S_n)} J(\alpha, a_n, x, n, u, s_n) \tag{4.20}$$

with $\Gamma(\alpha, S_n) = \{u(t) / 0 \leq u(t) \leq U_m, \ \xi(t) = \alpha, \text{ given } S_n\}$

In the remainder of this work, we will use a instead of a_n, and use the notation $z = (a, x)$.

The value function $V(\alpha,z,n)$ satisfies specific properties called optimality conditions, which are presented in the next section.

4.5 Optimality conditions

In this section, we will show that a sufficient condition for optimal control states that the value function $V(\alpha,z,n)$ satisfies a set of coupled partial derivative equations (HJB) derived from the application of the dynamic programming approach. We used the following assumptions, definition and lemmas.

Assumptions 4.1

A.4.1) $h(\cdot)$ is a nonnegative convex function with $h(0)=0$. There are positive constants C_g and κ_g, such that:

$h(x) \le C_g(1+|x|^{\kappa_g})$ and $|h(x_1)-h(x_2)| \le C_g(1+|x_1|^{\kappa_g}+|x_2|^{\kappa_g})|x_1-x_2|$

A.4.2) The function $w(n,a)$ is nonnegative and twice differentiable in both intervals $[0,S_n]$ and $[S_n,\infty]$, with $w(0,0)=0$. Moreover, $w(n,a)$ is either strictly convex or linear in both interval $[0,S_n]$ and $[S_n,\infty]$.

Definition (4.1)

A function $u(z,\alpha,s_n)$ is known as an admissible feedback control or simply a feedback control, if :

a) For any given initial z, the equations

$$\begin{cases} \dfrac{dx(t)}{dt} = u(z(t),\xi(t),s_n)-d, & x(0) = x \\ \dfrac{da(t)}{dt} = Ind\{\xi(t)=1\}, & a(0) = A_n,\ \xi(0) = \alpha, \end{cases}$$

have a single solution $z=(a,x)$, and

b) $u(\cdot)=\{u(t)=u(z(t),\xi(t),s_n),t\geq 0\}\in\Gamma(\alpha,S_n)$

Thus, $u(z,\alpha,s_n)\in\Gamma(\alpha,S_n)$ implies that $u(z,\alpha,s_n)$ is an admissible feedback control.

Lemma 4.1

i) if $G(\xi,z,n,u)$ is jointly convex, then $V(\alpha,z,n)$ is convex in z for each $(\alpha,n)\in\Omega\times N_p$

ii) if $G(\xi,z,n,u)$ is locally Lipschitz, i.e., there exist positive constants C and k, such that:

$$\left|G(\cdot,z_1,\cdot)-G(\cdot,z_2,\cdot)\right|\leq C\left(1+|z_1|^k+|z_2|^k\right)|z_1-z_2|$$

Then $V(\alpha,z,n)$ is also locally Lipschitz, that is:

$$\left|V(\cdot,z_1,\cdot)-V(\cdot,z_2,\cdot)\right|\leq C\left(1+|z_1|^k+|z_2|^k\right)|z_1-z_2|$$

Proof:

We just need to show that $J(\alpha,.,n,.)$ is jointly convex to prove i).

Let $Z_1=(a_1,x_1)$ and $Z_2=(a_2,x_2)$ be any initial values, $u^1(\cdot)$ and $u^2(\cdot)$ any admissible controls, given S_n. Let $Z^1(t)=\left(a^1(t),x^1(t)\right)$ and $Z^2(t)=\left(a^2(t),x^2(t)\right)$ $t\geq 0$ denote the trajectories corresponding to $\left(z_1,u^1(\cdot)\right)$ and $\left(z_2,u^2(\cdot)\right)$.

For any $\lambda\in[0,1]$,

$$\begin{aligned}
&\lambda J\left(\alpha,z_1,n,u^1(\cdot)\right)+(1-\lambda)J\left(\alpha,z_2,n,u^2(\cdot)\right)\\
&=E\left[\int_0^\infty e^{-\rho t}\left(\lambda G\left(\alpha,z_1,n,u^1(\cdot)\right)+(1-\lambda)G\left(\alpha,z_2,n,u^2(\cdot)\right)\right)dt\right]\\
&=E\left[\int_0^\infty e^{-\rho t}\left(\lambda h(x_1)+\lambda w(n,a_1)+(1-\lambda)h(x_2)+(1-\lambda)w(n,a_2)\right)dt\right]\\
&=E\left[\int_0^\infty e^{-\rho t}\left(\lambda h(x_1)+(1-\lambda)h(x_2)\right)dt\right]+E\left[\int_0^\infty e^{-\rho t}\left(\lambda w(n,a_1)+(1-\lambda)w(n,a_2)\right)dt\right]
\end{aligned}$$

Using assumptions 4.1.A1) and 4.1.A2) we have:

$$\lambda J\left(\alpha, z_1, n, u^1(\cdot)\right)+(1-\lambda) J\left(\alpha, z_2, n, u^2(\cdot)\right) \geq E\left[\int_0^{\infty} e^{-\rho t} G(\alpha, z(t), n, u(t)) dt\right]$$

where $u(t)=\lambda u^1(t)+(1-\lambda) u^2(t)$ and $z(t)$ denotes the trajectory with initial value $Z=\lambda z_1+(1-\lambda) z_2=\left(\lambda a_1+(1-\lambda) a_2, \lambda x_1+(1-\lambda) x_2\right)$ and control $u(\cdot)$.

Thus:

$$\lambda J\left(\alpha, z_1, n, u^1(\cdot)\right)+(1-\lambda) J\left(\alpha, z_2, n, u^2(\cdot)\right) \geq J\left(\alpha, \lambda z_1+(1-\lambda) z_2, n, \lambda u_1(\cdot)+(1-\lambda) u_2(\cdot)\right)$$

which means that $J(\alpha,.,n,.)$ is jointly convex, and consequently, $V(\alpha, z, n)$ is convex.

For ii), we consider an admissible control $u(\cdot)$, $Z^1(\cdot)=\left(a^1(\cdot), x^1(\cdot)\right)$ and $Z^2(\cdot)=\left(a^2(\cdot), x^2(\cdot)\right)$ denote the state trajectories under $u(\cdot)$ with initial values $z_1=\left(a_1, x_1\right)$ and $z_2=\left(a_2, x_2\right)$, respectively. We then have:

$$\left|z^1(t)-z^2(t)\right| \leq\left|z_1-z_2\right|, \quad \left|z^1(t)\right| \leq C_1\left(1+\left|z_1\right|\right) \text{ and } \left|z^2(t)\right| \leq C_1\left(1+\left|z_2\right|\right)$$

Assuming a locally Lipschitz condition, there exists a constant C_2 independent of $u(\cdot)$, z_1 and z_2, such that:

$$\left|J\left(\alpha, z_1, n, u(\cdot)\right)-J\left(\alpha, z_2, n, u(\cdot)\right)\right| \leq C_2\left(1+\left|z_1\right|^k+\left|z_2\right|^k\right)\left|z_1-z_2\right|$$

From that:

$$\left|V\left(\alpha, z_1, n\right)-V\left(\alpha, z_2, n\right)\right| \leq \underset{u(\cdot) \in \Gamma\left(\alpha, S_n\right)}{Sup}\left|J\left(\alpha, z_1, n, u(\cdot)\right)-J\left(\alpha, z_2, n, u(\cdot)\right)\right|$$

that is:

$$\left|V\left(\alpha, z_1, n\right)-V\left(\alpha, z_2, n\right)\right| \leq C_2\left(1+\left|z_1\right|^k+\left|z_2\right|^k\right)\left|z_1-z_2\right| \qquad \square$$

Lemma 4.2

$V(\alpha, z, n)$ is the single viscosity solution to the HJB equations:

$$\rho V(\alpha, z, n)=\underset{u \in \Gamma\left(\alpha, S_n\right)}{Min}\left\{\begin{array}{l} G(\cdot)+\frac{\partial}{\partial a} V(\alpha, z, n) Ind\{\alpha=1\}+\frac{\partial}{\partial x} V(\alpha, z, n)(u(t)-d) \\ +Q(a, n) V\left(\cdot, \varphi(a, \cdot, n), x, n'\right)(\alpha) \end{array}\right\}, \quad z=(a, x),\ \xi(t)=\alpha \in \Omega \tag{4.21}$$

with

$$\varphi(a,\xi,n)=\begin{cases} A_n & \text{if } \xi(\tau^+)=1 \text{ et } \xi(\tau^-)=2 \\ 0 & \text{if } \xi(\tau^+)=1 \text{ et } \xi(\tau^-)=3 \\ a(\tau^-) & \text{otherwise} \end{cases}$$

$$n'=\begin{cases} n & \text{if } \xi(\tau^+)=1 \text{ et } \xi(\tau^-)=2 \\ 0 & \text{if } \xi(\tau^+)=1 \text{ et } \xi(\tau^-)=3 \\ n+1 & \text{otherwise} \end{cases}$$

$\frac{\partial}{\partial x}V(\alpha,z,n)$ and $\frac{\partial}{\partial a}V(\alpha,z,n)$ the partial derivatives of the value function $V(\alpha,z,n)$.

Proof:

It suffices to show that $V(\cdot)$ is both a viscosity subsolution and viscosity supersolution of equation (4.21), using the procedure provided by Sethi and Zhang, (1994). For more details on viscosity subsolution and supersolution, see Yong and Zhou (1999).

Let $C^1(\mathbb{R}^+\times\mathbb{R}^1)$ be the class of functions defined on $\mathbb{R}^+\times\mathbb{R}$ that are continuously differentiable on $\mathbb{R}^+\times\mathbb{R}$. Set α and $Z_0=(a_0,x_0)$. Next, let $\phi(\cdot)\in C^1(\mathbb{R}^+\times\mathbb{R}^1)$ be such that $V(\alpha,z,n)-\phi(z)$ attains its maximum at $(a=a_0,x=x_0)$ in a neighborhood $N(a_0,x_0)$.

Let τ denote the first jump time of $\xi(t)$. We consider the control $u(t)=u$ for $0\le t\le\tau$, where $u\in\Gamma(\alpha,S_n)$ is a constant. Moreover, let $\varepsilon\in(0,\tau]$ be such that z starts at z_0 and stays in $N(a_0,x_0)$ for $0\le t\le\varepsilon$. Define

$$\gamma(\xi,z,n)=\begin{cases} \phi(z)+V(\alpha,z_0,n)\text{-}\phi(z_0) & \text{if } \xi(t)=\alpha \\ V(\xi,z,n) & \text{if } \xi(t)\neq\alpha \end{cases} \tag{4.22}$$

Then, by Dynkin's formula and the fact that $\xi(\varepsilon)=\alpha$ and $0\le t\le\varepsilon$,

$$Ee^{-\rho\varepsilon}\gamma(\xi(\varepsilon),z(\varepsilon),n)-V(\alpha,z_0,n)=E\int_0^{\varepsilon}e^{-\rho t}\begin{bmatrix}-\rho\gamma(\alpha,z(t),n)+\phi_x(Z)(u(t)-d)\\+\phi_a(Z)Ind\{\xi(t)=1\}+Q(a,n)\gamma(\cdot,x(t),\cdot)(\alpha)\end{bmatrix}dt \tag{4.23}$$

$V(\alpha,z,n)-\phi(z)$ attains its maximum at $N(z_0)$ and $z(t)\in N(z_0)$ for $0\le t\le\varepsilon$. Thus $V(\alpha,z_0,n)\text{-}\phi(z_0)\ge V(\alpha,z,n)-\phi(z)$, that is:

$$\phi(z)\ge V(\alpha,z,n)-(V(\alpha,z_0,n)\text{-}\phi(z_0)) \qquad \text{for } 0\le t\le\varepsilon \tag{4.24}$$

$V(\alpha,z_0,n)\text{-}\phi(z_0)$ is constant, and when we replace $\gamma(\alpha,z,n)$ in (4.23) by $V(\alpha,z,n)-(V(\alpha,z_0,n)\text{-}\phi(z_0))$, we obtain

$$Ee^{-\rho\varepsilon}V(\alpha,z(\varepsilon),n)-V(\alpha,z_0,n)\le E\int_0^{\varepsilon}e^{-\rho t}\begin{bmatrix}-\rho V(\alpha,z(t),n)+\phi_x(Z)(u(t)-d)\\+\phi_a(Z)Ind\{\xi(t)=1\}+Q(a,n)V(\cdot,x(t),\cdot)(\alpha)\end{bmatrix}dt \tag{4.25}$$

And by the optimality principle,

$$V(\alpha,z_0,n)\le E\left[\int_0^{\varepsilon}e^{-\rho t}G(\alpha,z(t),n)\,dt+e^{-\rho\varepsilon}V(\alpha,z(\varepsilon),n)\right] \tag{4.26}$$

Combining (4.25) and (4.26), we have:

$$0\le E\int_0^{\varepsilon}e^{-\rho t}\left[G(\alpha,z(t),n)+\phi_x(Z)(u(t)-d)+\phi_a(Z)Ind\{\xi(t)=1\}+Q(a,n)V(\cdot,x(t),\cdot)(\alpha)\right]dt$$

By letting $\varepsilon\to 0$, we conclude that:

$$\min_{u\in\Gamma(\alpha,S_n)}\left[G(\alpha,z_0,n)+\phi_x(z_0)(u-d)+\phi_a(z_0)Ind\{\alpha=1\}\right]+Q(a,n)V(\cdot,x,\cdot)(\alpha)-\rho V(\alpha,z_0,n)\ge 0$$

Thus, $V(\cdot)$ is a viscosity subsolution of equation (4.21).

We will show that $V(\cdot)$ is a viscosity supersolution of equation (4.21) by supposing that it is not.

If $V(\cdot)$ is not a viscosity supersolution, it implies that there exist α, z_0 and $\delta_0 > 0$ such that for all $u \in \Gamma(\alpha, S_n)$:

$$G(\alpha,z,n,u)+\phi_x(z)(u-d)+\phi_a(z)Ind\{\alpha=1\}+Q(a,n)V(\cdot,x,\cdot)(\alpha)-\rho V(\alpha,z,n)\geq\delta_0 \tag{4.27}$$

in the neighborhood $N(a_0, x_0)$, where $\phi(\cdot) \in C^1(\mathbb{R}^+ \times \mathbb{R}^1)$, such that $V(\alpha,z,n)-\phi(z)$ attains its maximum at z_0 in the neighborhood $N(a_0, x_0)$.

Then for all $z(t) \in N(a_0, x_0)$:

$$V(\alpha,z,n)\geq\phi(z)+V(\alpha,z_0,n)-\phi(z_0) \tag{4.28}$$

For any $u \in \Gamma(\alpha, S_n)$, let ε_0 denote a small number such that z starts at z_0 and stays in $N_0(a_0, x_0)$ for $0 \leq t \leq \varepsilon_0$. Note that ε_0 depends on the control $u(\cdot)$. However, since $u(t)-d$ is always bounded, there exists a constant $\varepsilon_1 > 0$ such that $\varepsilon_0 \geq \varepsilon_1 > 0$. Let τ denote the first jump time of the process $\xi(\cdot)$. Then for:

$$\begin{aligned}
J(\alpha,z_0,n,u(\cdot)) &\leq E\left[\int_0^{\varepsilon} e^{-\rho t}G(\alpha,z(t),n)dt+e^{-\rho\varepsilon}V(\alpha,z(\varepsilon),n)\right] \\
&\geq E\left\{\begin{array}{l}\int_0^{\varepsilon} e^{-\rho t}\left[\begin{array}{l}\delta_0-\phi_x(z)(u(t)-d)-\phi_a(z)Ind\{\xi(t)=1\}+\rho V(\alpha,z(\varepsilon),n) \\ +Q(a,n)V(\cdot,x(t),\cdot)(\alpha)\end{array}\right]dt \\ +e^{-\rho\varepsilon}V(\alpha,z(\varepsilon),n)\end{array}\right\}
\end{aligned}$$

Now, we can use the differentiability of $\phi(\cdot)$ together with (4.24) to show:

$$V(\alpha,z_0,n) \le E\int_0^{\varepsilon} e^{-\rho t}\begin{bmatrix}\rho V(\alpha,z(t),n)-\phi_x(z)(u(t)-d)-\phi_a(z)Ind\{\xi(t)=1\} \\ +Q(a,n)V(\cdot,x(t),\cdot)(\alpha)\end{bmatrix}dt+e^{-\rho\varepsilon}V(\alpha,z(\varepsilon),n)$$

Thus,

$J(\alpha,z_0,n,u(\cdot)) \ge V(\alpha,z_0,n)+\delta_0 E\int_0^{\varepsilon} e^{-\rho t}dt \ge V(\alpha,z_0,n)+\eta$, which is a contradiction. This shows that $V(\xi,z,n)$ is a viscosity supersolution of equation (4.21).

Thus, $V(\xi,z,n)$ is a viscosity solution to equation (4.21).

□

Theorem 4.1 (Uniqueness Theorem)

Given assumptions A.4.1), A.4.2) and since the function $w(n,a_n)$ is nonnegative and either strictly convex or linear with $w(0,0)=0$, the HJB equations (4.21) has a single viscosity solution.

Proof:

For the proof, see Sethi and Zhang (1994) and Yong and Zhou (1999).

□

The numerical solution of equations (4.21) is developed in the next section.

4.6 Numerical approach

In this section, we develop the numerical approach for solving the optimality conditions presented in the previous section. Since the optimal repair and replacement policy is a threshold type policy, we will search directly for the optimal control limits S_n, using the algorithm provided by (Love *et al.*, 2000), with a few cost definition modifications. Although the cost definition in this present work differs from those used in their work, they nevertheless have the same properties.

4.7 Repair and replacement search algorithm

The repair and replacement search algorithm consists in choosing, right from the beginning, a limit on the number of repairs N, an initial control limit $S_1 \geq S_2 ... \geq S_N$ and a bound on threshold age M, with $M \gg S_1 \geq S_2 ... \geq S_N$. Having $M \gg S_1 \geq S_2 ... \geq S_N$ ensures that the machine's probability of reaching age M without incurring a failure is zero.

Let us discretize the state space of age a when determining S_n with a scale parameter h_i defined as $ih_i \leq a \leq (i+1)h_i$. The repair and replacement cost function becomes:

$$w(n,i) = C_1(n,i) + \sum_{k=i}^{M} P_{(n,i)(n+1,k)}(1) w(n+1,k); \; i \leq S_n - 1, \; 1 \leq n \leq N_m - 1 \tag{4.29}$$

$$w(n,i) = C_0 + \sum_{k=0}^{M} P_{(n,i)(1,k)}(0) w(1,k); \quad i \geq S_n, \; 1 \leq n \leq N_m - 1 \tag{4.30}$$

$$w(N_m,i) = C_0 + \sum_{k=0}^{M} P_{(N_m,i)(1,k)}(0) w(1,k); \quad w(1,0) = 0; \; w(0,i) = 0 \tag{4.31}$$

with

$$P_{(n,i)(n+1,k)}(1) = \int_{k-i}^{k-i+1} f_{\theta i h_i}(x) dx, \; n \leq N_m \text{ -1}, \; i \leq k \leq M \text{ -1} \tag{4.32}$$

$$P_{(n,i)(n+1,M)}(1) = \int_{M-i}^{\infty} f_{\theta i h_i}(x) dx, \; n \leq N_m \text{ -1} \tag{4.33}$$

$$P_{(n,i)(1,k)}(0) = \int_{k}^{k+1} f(x) dx, \quad 0 \leq k \leq M \text{ -1} \tag{4.34}$$

$$P_{(n,i)(1,M)}(0) = \int_{M}^{\infty} f(x) dx \tag{4.35}$$

Solving equations (4.29) to (4.31) gives the value of the initial policy in terms of repair/replacement cost. The optimal control limit policy S_n is obtained by improving this policy.

4.8 HJB equations resolution

We solved the HJB equations (4.21) by approximating $V(\alpha,a,x,n)$ by a function $V^h(\alpha,a,x,n)$ and the first-order partial derivatives of the value function $\frac{\partial}{\partial x}V(\alpha,a,x,n)$ and $\frac{\partial}{\partial a}V(\alpha,a,x,n)$ by:

$$\frac{\partial}{\partial x}V(\alpha,a,x,n)=\begin{cases}\frac{1}{h_x}\left[V^h(\alpha,a,x+h_x,n)-V^h(\alpha,a,x,n)\right] & \text{if } \frac{dx}{dt}\geq 0\\[2ex] \frac{1}{h_x}\left[V^h(\alpha,a,x,n)-V^h(\alpha,a,x-h_x,n)\right] & \text{otherwise}\end{cases}$$

$$\frac{\partial}{\partial a}V(\alpha,a,x,n)=\frac{1}{h_a}\left[V^h(\alpha,a+h_a,x,n)-V^h(\alpha,a,x,n)\right]$$

where h_x and h_a are discrete increments associated to state variables x and a.

The following three equations are the discrete dynamic programming equations obtained:

$$V^h(1,a,x,n,u)=\frac{1}{\left(\rho+\frac{1}{h_a}+\frac{1}{h_x}|u(t)-d|-q_{11}\right)}\left(\begin{array}{l}c^+x^++c^-x^-+\frac{1}{h_a}V^h(1,a+h_a,x,n)\\ +\frac{1}{h_x}|u(t)-d|\left(\begin{array}{l}V^h(1,a,x+h_x,n)Ind\{(u(t)-d)\geq 0\}\\ +V^h(1,a,x-h_x,n)Ind\{(u(t)-d)<0\}\end{array}\right)\\ +q_{12}V^h(2,A_{n+1},x,n+1)+q_{13}V^h(3,0,x,0)\end{array}\right)$$

(4.36)

$$V^h(2,a,x,n,u)=\frac{1}{\left(\rho+\frac{d}{h_x}-q_{22}\right)}\left(\begin{array}{l}c^+x^++c^-x^-+W_1(n,a,n+1)\\ +\frac{d}{h_x}V^h(2,a,x-h_x,n)+q_{21}V^h(1,a,x,n)\end{array}\right) \tag{4.37}$$

$$V^h(3,a,x,n,u)=\frac{1}{\left(\rho+\frac{d}{h_x}-q_{33}\right)}\left(\begin{array}{l}c^+x^++c^-x^-+W_0(n,a,0)\\ +\frac{d}{h_x}V^h(3,a,x-h_x,n)+q_{31}V^h(1,a,x,n)\end{array}\right) \tag{4.38}$$

With $w(n,a)=\begin{cases} W_1(n,a,n+1) & \text{if at the } n^{th} \text{ failure a repair action is undertaken} \\ W_0(n,a,0) & \text{otherwise} \end{cases}$

For details on how to obtain equations (4.36)-(4.38), see appendix 4.A.

The next theorem shows that the value function $V^h(\alpha,a,x,n)$ is an approximation of $V(\alpha,a,x,n)$ for small size h step.

Theorem 4.2

Let $V^h(\alpha,a,x,n)$ denote a solution to HJB equations (4.36)-(4.38). Assume that there are constants C and K such that:

$$0 \le V^h(\alpha,a,x,n) \le C(1+|z|^K)$$

Then

$$\lim_{h\to 0} V^h(\alpha,a,x,n)=V(\alpha,a,x,n) \tag{4.39}$$

Proof:

The proof of this theorem can be obtained by extending the one presented in Yan and Zhang (1997).

□

4.9 Overall search algorithm

The overall search algorithm which provides the production rate $u^*(a,x,n)$ requires that we proceed in several steps, as follows:

Step 0: Determine the repair/replacement control-limit policy S_n

Compute A_n, $\forall n$

Set $n := 0$; if $S_n > 0$

Step 1: For a given control policy u_n, let W_{u_n} and W^* be defined by:

$$W_{u_n}\left(V^h(\alpha,z,n)\right) = V^h(\alpha,z,n)$$

$$W^*\left(V^h(\alpha,z,n)\right) = \min_{u_n \in \Gamma(\alpha,S_n)}\left\{W_{u_n}\left(V^h(\alpha,z,n)\right)\right\}$$

For a given finite difference interval h:

Step 2: Choose $\sigma \in \mathbb{R}^+$ a given precision

Set $i_t := 1$ and $\left(V^h\right)^{i_t}(\alpha,z,n) := 0, \forall \alpha \in \Omega, \forall z$

Step 3: $\left(V^h\right)^{i_t-1}(\alpha,z,n) := \left(V^h\right)^{i_t}(\alpha,z,n),\ \forall \alpha \in \Omega,\ \forall z$

Step 4: Determine the policy $u_n^{i_t}$, such that:

$$W_{u_n^{i_t}}\left(\left(V^h(\alpha,z,n)\right)^{i_t-1}\right) = \left(V^h\right)^{i_t}(\alpha,z,n) = W^*\left(\left(V^h(\alpha,z,n)\right)^{i_t-1}\right),\ \forall \alpha \in \Omega,\ \forall z$$

Step 5: Test 1:

$$e_{\inf} := \min_{\substack{\alpha\in\Omega \\ z}}\left[\left(V^h(\alpha,z,n)\right)^{i_t} - \left(V^h(\alpha,z,n)\right)^{i_t-1}\right]$$

$$e_{\sup} : \max_{\substack{\alpha\in\Omega \\ z}}\left[\left(V^h(\alpha,z,n)\right)^{i_t} - \left(V^h(\alpha,z,n)\right)^{i_t-1}\right]$$

$$e_{\min} := \frac{\rho}{1-\rho} e_{\inf}; \qquad e_{\max} := \frac{\rho}{1-\rho} e_{\sup}$$

if $\left|e_{\max} - e_{\min}\right| \le \sigma$ then stop $u_n^* = u_n^{i_t}$; else operate $\left(V^h\right)^{i_t} := W_{u_n}\left(V^h\right)^{i_t}$ until a σ – fixed point is obtained. Let $i_t = i_t + 1$, and go to step 3.

Step 6: Test 2:

While $S_{n-1} > 0$ we have $N_m > n$

Set $n := n+1$;

Go to step 1

If $n > N_m$ then stop $u^*(z,n) = u_n^*$, $\forall n \le N_m$.

It is not necessary to compute the policy after N_m because if the number of failures ever reached N_m, then the machine would be replaced at failure $N_m + 1$, which is similar to having $n = 0$.

4.10 Numerical Example

The computational domain D is given by: $D = \{(a,x,n): 0 \le a \le 450; -5 \le x \le 75; 0 \le n \le 20\}$. Let us assume that the lifetime distribution of a new machine followed a Weibull distribution, with scale parameter λ and shape parameter α. We use a repair rate given by:

$$q_{21}(n) = q_1\left(q_0 + 1 - \left(\frac{n}{N}\right)^r\right)$$

where q_1, q_0 and r are constant with the value given in table 4.1. The virtual age after the n^{th} repair is $A_n = \theta t_n$, with t_n being the cumulated production time. The instantaneous repair cost is a function of the mean repair time, and is given by:

$$C_1(n,a) = \frac{C_r}{q_{21}(n)} \text{ if } 1 \le n \le N;\ C_1(0,a) = 0$$

where C_r is a constant with value given in table 4.1. We then have $C_1(n+1,a') - C_1(n,a) \ge 0;\ 0 \le a \le a';\ 1 \le n \le N;$ and:

$$\lim_{n \to N} \lim_{a \to \infty} C_1(n,a) = \frac{C_r}{q_1 q_0}$$

Thus, $C_1(n,a)$ is a bounded non-decreasing function of (n,a). Other parameters used in the numerical example are presented in table 4.1:

Table 4.1 Parameters of the numerical example №2

Parameter	θ	U_m	d	h_x	h_a	ρ	c^+	c^-	λ
Value	0.25	0.4	0.25	0.5	1	10^{-5}	5.6	150	2
Parameter	c_0	M	q_0	q_1	q_{31}	r	N	c_r	α
Value	3000	300	0.01	0.14	10	2	20	150	4

We used the overall search algorithm to solve equations (4.29)-(4.31) and (4.36)-(4.38). The control policy obtained is shown from figures 4.4 to 4.6.

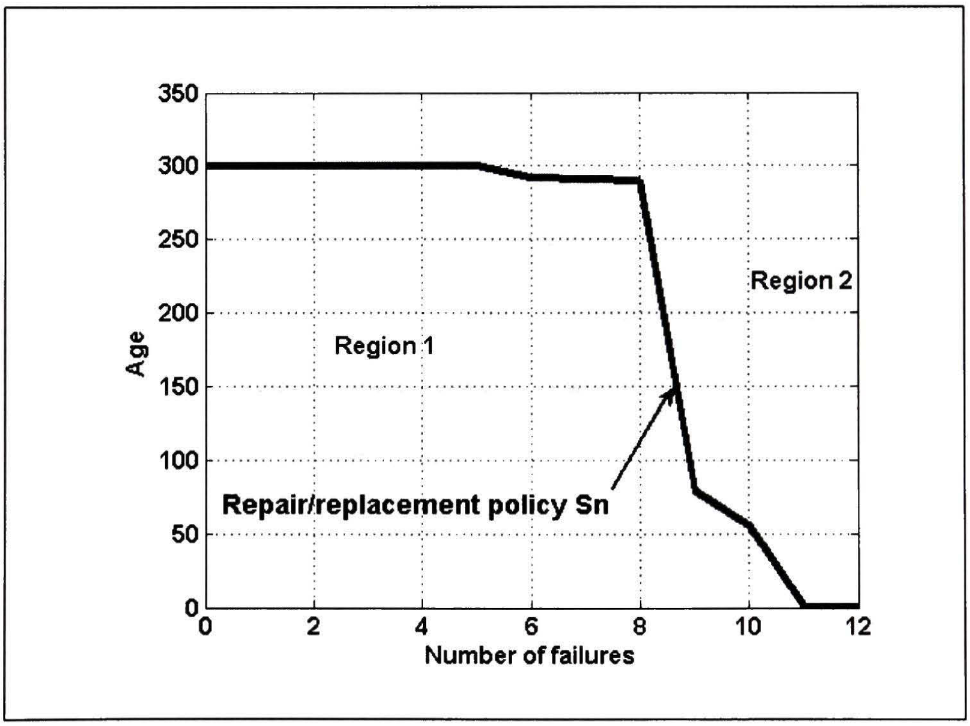


Figure 4.4 Repair/replacement policy S_n.

The repair/replacement policy S_n divides the plan (n,a) into two regions: region 1 denotes the states in which, if a failure occurs, an imperfect repair is conducted, whereas if the failure occurs in region 2, the machine is replaced. If the upper bound on the number of failures

$N_m = 11$ or the age S_n is reached, the machine is replaced at the next failure. Although we used a maximum number $N = 20$ of failures for the implementation of the algorithm, the machine should be replaced at the 12^{th} failure. It means that such a machine would never have 13 failures.

Given the repair/replacement policy, we obtain the plot of the number of parts to hold in inventory between the n^{th} and the $(n+1)^{th}$ failure presented in figure 4.5 as well as the number of parts to hold in inventory before each failure in order to hedge against the breakdowns presented in figure 4.6.

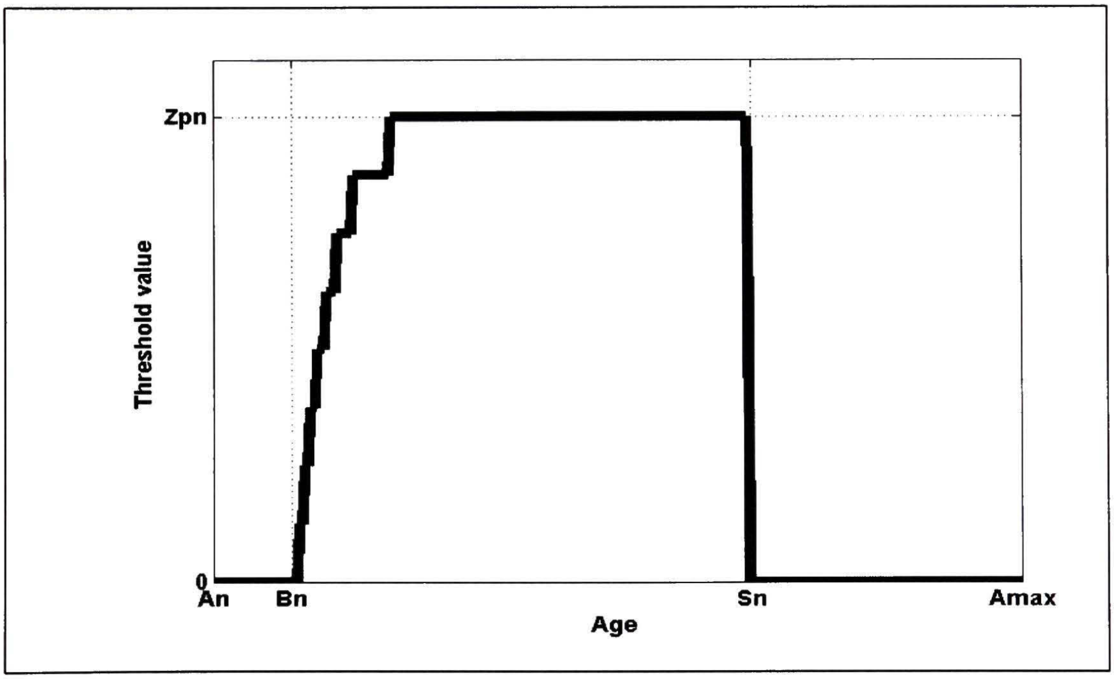


Figure 4.5 Number of parts to hold in inventory between. the n^{th} restart and the $(n+1)^{th}$ failure .

It is shown in figure 4.5 that the number of parts to maintain in inventory between the n^{th} restart and the $(n+1)^{th}$ failure in order to hedge against machine breakdowns increases as the machine age increases, until a maximum number Zpn of parts held in inventory is reached.

After this point, the machine is nearing replacement, and so there is no need to stock parts. B_n is the oldest age at which parts will no longer need to be stocked after the n^{th} restart.

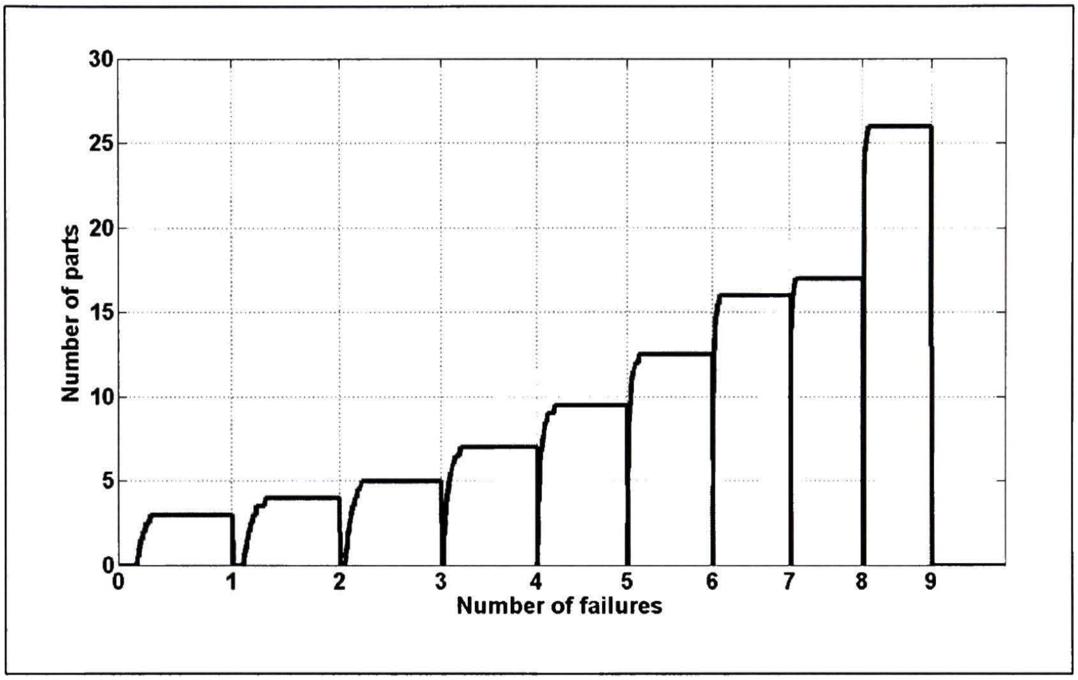


Figure 4.6 Number of parts to hold in inventory for each failure.

Based on the above results, we can obtain a general production control policy (production rate) as follows:

The production control policy $$u^*(\cdot) = \begin{cases} U_{\mathrm{m}} & \text{if } x(t) < Zpn \\ d & \text{if } x(t) = Zpn \\ 0 & \text{if } x(t) > Zpn \end{cases}$$

where the threshold value is $$\begin{cases} Zpn > 0 & \text{if } B_n < a(t) < S_n \\ Zpn = 0 & \textit{otherwise} \end{cases}$$

There is no need to stock parts when the machine age after the n^{th} failure and repair is less than Bn. Similarly, there is no need to stock parts when the machine age is higher than S_n.

4.11 Results and sensitivity analysis

It appears from figures 4.6 and 4.7 that the number of parts *Zpn* to hold in inventory increases with failures until a number N_u of failures is reached; after N_u, the number of parts to hold in inventory in order to hedge against breakdowns is nil. This result is logical because the probability of the machine being replaced at the next breakdown after N_u is almost equal to 1, due to the fact that with the virtual age, S_n must necessarily then be exceeded. Furthermore, the replacement takes a negligible amount of time, and after replacement, the machine is new. It is thus normal for parts not to be held in inventory just prior to replacement.

As also illustrated in figures 4.6 and 4.7, while the maximum stock level is significant for old machines, it is even more so for old machines, which have undergone a high number of failures. That is, until prior to replacement.

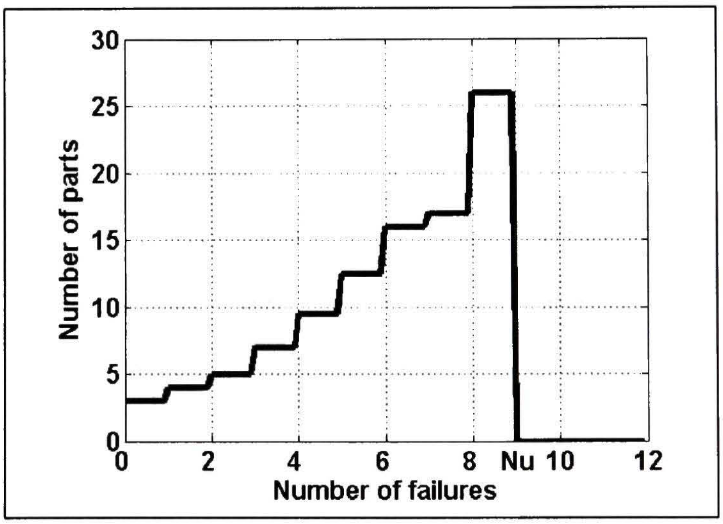


Figure 4.7 Maximum number of parts to hold in inventory for each failure.

We will now analyze the sensitivity of the above results according to the cost of surplus, backlog, repair and the intensity of repair, and see if figures 4.4, 4.6 and 4.7 maintain their structure as the parameters change. We will first analyze the sensitivity of the S_n policy according to the cost of repair. The result obtained for three repair cost values is presented in figure 4.8.

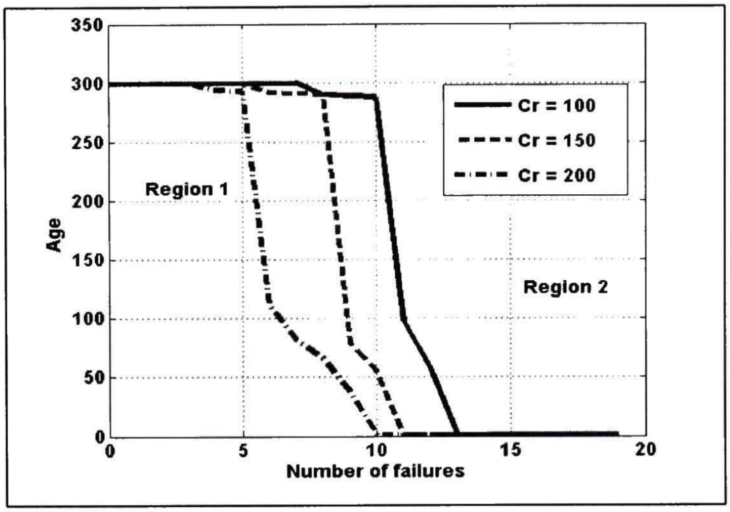


Figure 4.8 Sensitivity of Sn policy according to the cost of repair.

From figure 4.8, we conclude that region 1 decreases as repair costs increase. As we can see from figure 4.9, this region also decreases as the machine replacement costs decrease.

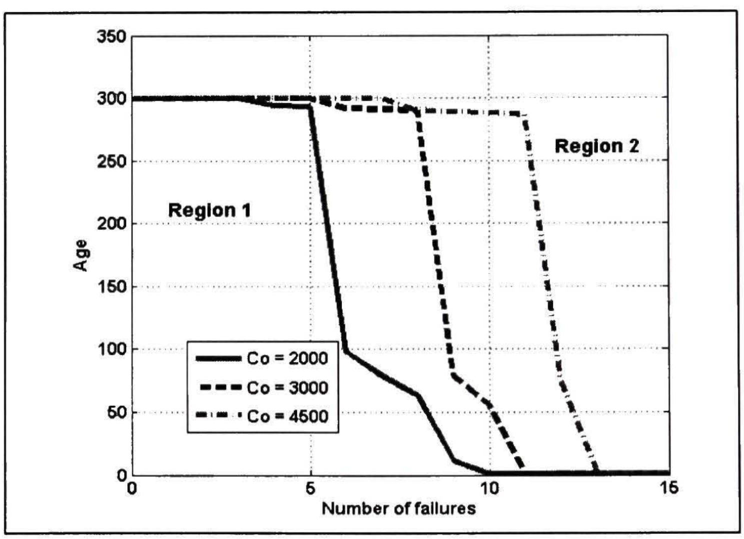


Figure 4.9 Sensitivity of Sn policy according to the cost of replacement.

Let us now analyze the sensitivity of the maximal number of parts to hold for each number of failures for three backlog cost values: 150, 175 and 200. The result, presented in figure 4.10, shows that the hedging inventory value increases as the backlog cost increases. Conversely, as we can see in figures 4.11 and 4.12, it decreases as the inventory cost increases, and takes the three values, 2, 5 and 8.

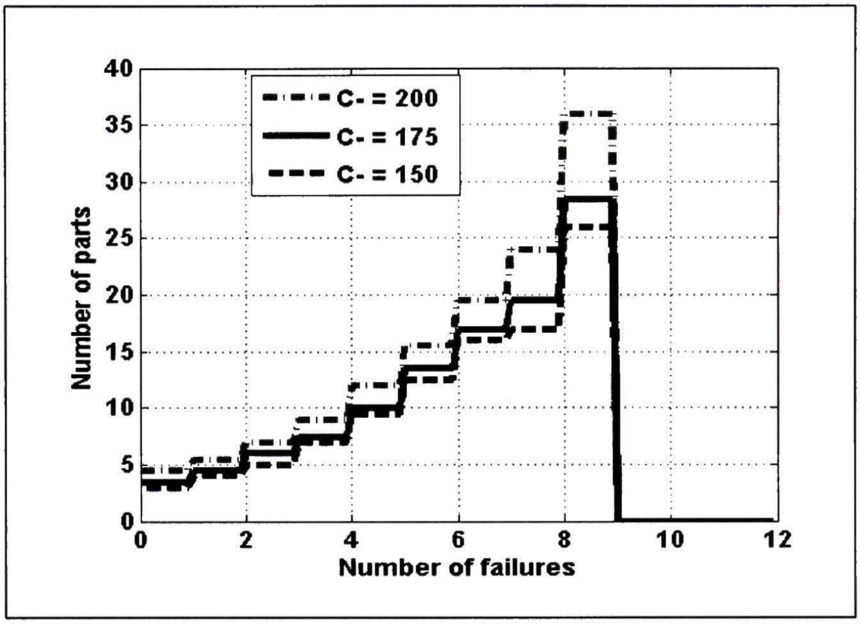


Figure 4.10 Sensitivity of maximum number of parts Zpn to hold in inventory to cost of backlog.

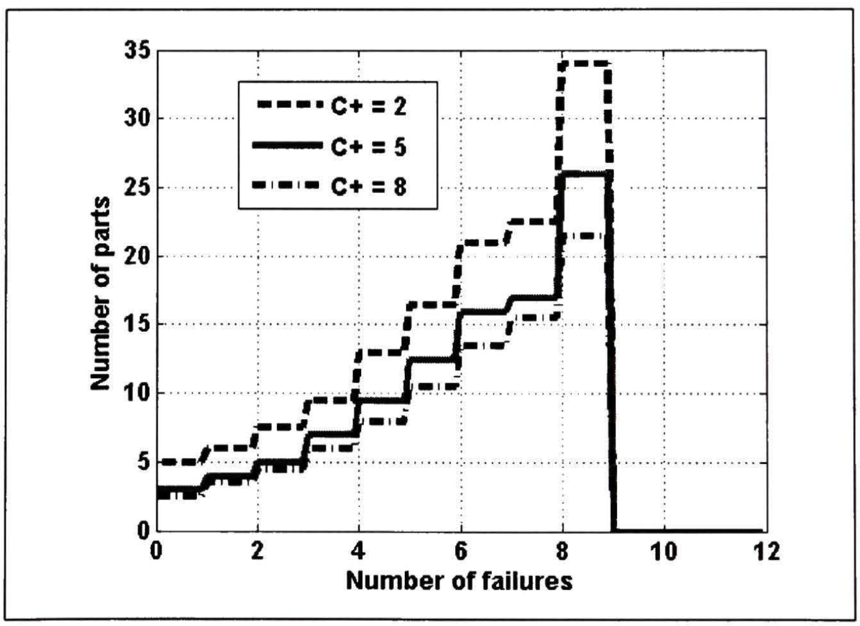


Figure 4.11 Sensitivity of maximum number of parts Zpn to hold in inventory to surplus cost.

In addition, figure 4.12 illustrates the fact that the oldest age at which parts do not need to be stocked after the n^{th} restart increases as inventory costs increases.

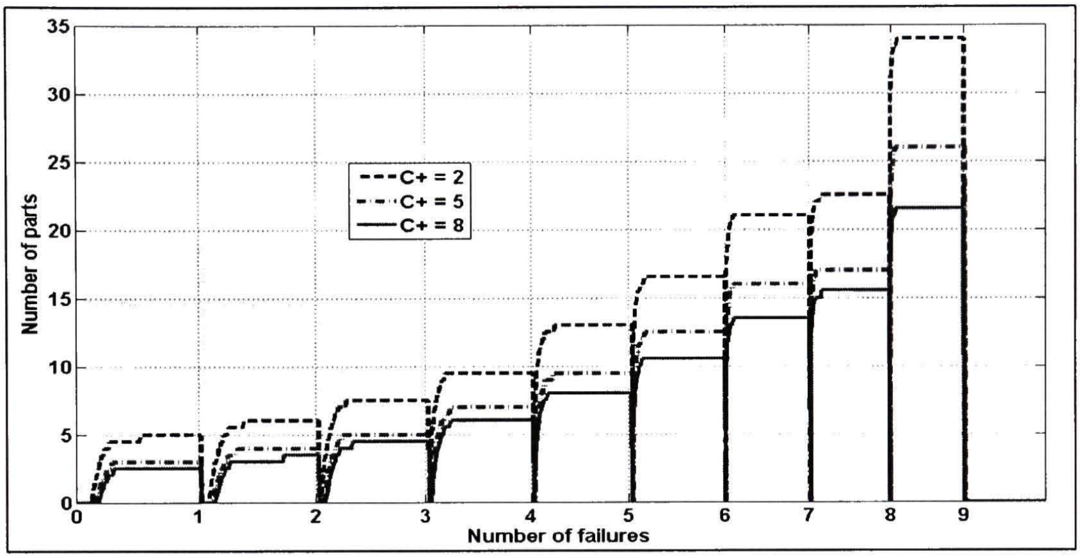


Figure 4.12 Sensitivity of maximum number of parts Zpn to hold in inventory to cost of surplus.

We see from figure 4.13 that breakdowns occur earlier when the repair intensity is higher, and later when it is lower.

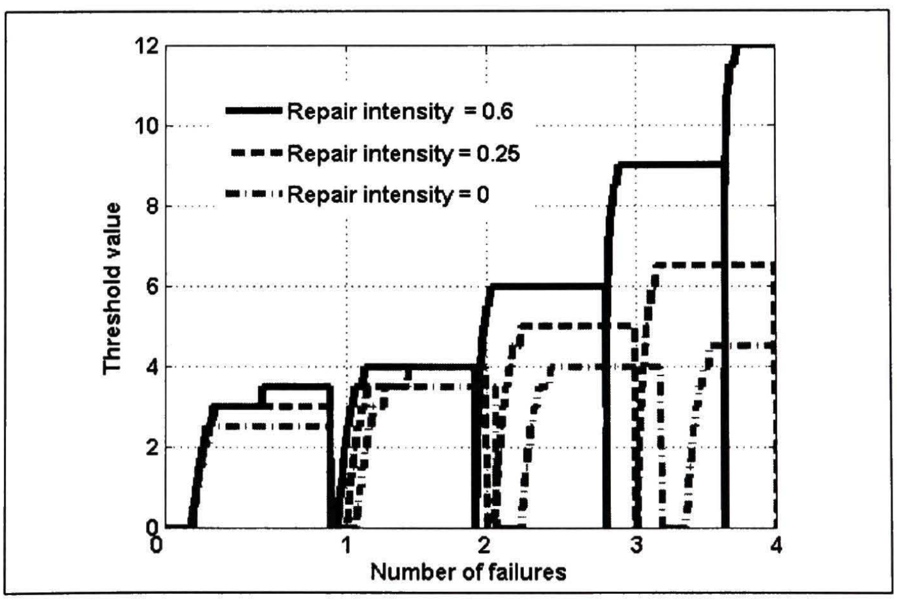


Figure 4.13 Sensitivity of maximum number of parts Zn to hold in inventory to repair intensity.

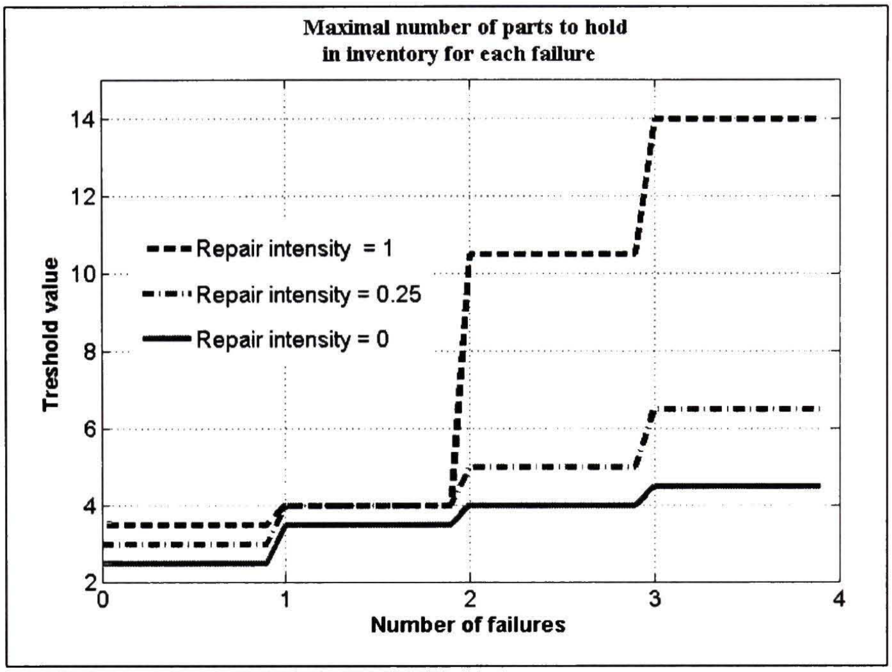


Figure 4.14 Sensitivity of maximum number of parts Zn to hold in inventory to repair intensity.

As indicated in figures 4.13 and 4.14, when the repair intensity increases, the number of parts to be held in inventory in order to hedge against breakdowns also increases. The case in figure 4.14, where the repair intensity is nil, corresponds to a perfect repair, and requires the lowest number of parts to be held in inventory. After a repair, the machine's age is almost the same as that of a new machine, but it takes more time to renew it from one failure to the next. When the repair intensity is 1, a minimal repair has being performed, and has no impact on the failure rate function. There is no improvement in the failure rate after a failure, and the number of parts to be held is at its highest value. A more realistic case is between minimal repair and complete repair, and has an intermediate level of number of parts to be held.

Traditional production control policies are based on a given threshold level for the number of parts to be held in order to hedge against breakdowns, as the system dynamic is considered to be the same after failures. In many situations, the dynamics of several variables change after breakdowns as a result of the machine degradation phenomenon. As shown in the above results, taking into account degradation with age after failure leads to a policy comprising several critical threshold values, which increase from one breakdown to another, and the general production policy for a given number of breakdowns is a policy characterized by four parameters: $Zp(n) = Zpn$; $A(n) = An$; $B(n) = Bn$; $S(n) = S_n$. These parameters are such that at the $(n+1)^{th}$ restart, the age of the machine is $A(n)$. While this machine age is less than $B(n)$, there is no need to stock parts; when it is situated between $B(n)$ and $S(n)$, the inventory should be brought up to $Zp(n)$, and this value maintained. After age $S(n)$ of the machine, it is no longer necessary to have parts in inventory.

4.12 Conclusions

In this paper, we have presented a hierarchical decision making approach in production and repair/replacement planning with imperfect repairs problems under uncertainties. We have shown that using reasonable assumptions, repair/replacement decisions can be based on the machine age and number of failures. Using such decisions, the operational level management can derive a production plan for the system. This work represents an important contribution to the literature on the production control of flexible manufacturing systems at several levels. Until now, production control researchers have always considered that from one breakdown to the next, either the machine does not age or it ages, but it is like a new machine after corrective or preventive maintenance. By letting the machine become new after maintenance or considering that the machine does not age, they obtained a number of parts to be held in inventory in order to hedge against breakdowns. This number of parts was always independent of the number of breakdowns occurring, and the machines never needed to be replaced. In reality however, production system machines must be replaced in the long run. Our work thus made it possible to take into account this reality while working on machines which after repair are not new, with repair times depending on the number of breakdowns. We have shown that the number of parts to hold in inventory must be adjusted to a higher level as the number of breakdowns increases or as the machine ages. We go from the traditional policy, involving only one high level threshold to a policy with several high threshold levels that depends on the number of breakdowns encountered. The various sensitivity analyses conducted prove the usefulness of the proposed approach.

Further extensions of the approach could be in addition to replacement, the use of preventive maintenance to increase the availability of the system. The demand rate facing the system in this work is assumed to be constant, but it could be a deterministic monotone increasing function of the time or be subject to random fluctuations. In the case where the demand rate is a deterministic monotone increasing function of the time, the production capacity of the system could be increased by purchasing a finite number of new machines over time while other are replaced.

Appendix 4.A

Resolution of equation (4.21)

To solve the system of equation (4.21) for each $\alpha \in \Omega$, let us consider

$$w(n,a)=\begin{cases} W_1(n,a,n+1) & \text{if at the } n^{th} \text{ failure a repair action is undertaken} \\ W_0(n,a,0) & \text{otherwise} \end{cases}$$

i) When $\alpha = 1$

- $Ind\{\alpha = 1\} = 1$
- $w(n,a) = 0$
- $G(\cdot) = h(x) = c^+x^+ + c^-x^-$ and
  $Q(a,n)V(\cdot,\varphi(a,\cdot,n),x,n')(\alpha)$
- $= Q(a,n)V(\cdot,\varphi(a,\cdot,n),x,n')(\alpha = 1)$
  $= q_{11}V(1,a,x,n) + q_{12}V(2,A_{n+1},x,n+1) + q_{13}V(3,0,x,0)$

Then, replacing in (4.21) gives

$$\rho V^h(1,a,x,n) = \underset{u\in\Gamma(1,S_n)}{Min} \left\{ h(x) + \frac{\partial}{\partial a}V(1,z,n) + \frac{\partial}{\partial x}V(1,z,n)(u(t)-d) \right.$$
$$\left. + q_{11}V^h(1,a,x,n) q_{12}V^h(2,A_{n+1},x,n+1) + q_{13}V^h(3,0,x,0) \right\}$$

$$= \underset{u\in\Gamma(1,S_n)}{Min} \left\{ h(x) + \frac{1}{h_a}\left[V^h(1,a+h_a,x,n) - V^h(1,a,x,n)\right] \right.$$
$$+ \frac{1}{h_x}\left[V^h(1,a,x+h_x,n) - V^h(1,a,x,n)\right]\left|u(t)-d\right| Ind\{(u(t)-d)\geq 0\}$$
$$- \frac{1}{h_x}\left[V^h(1,a,x,n) - V^h(1,a,x-h_x,n)\right]\left|u(t)-d\right| Ind\{(u(t)-d)< 0\}$$
$$\left. + q_{11}V^h(1,a,x,n) + q_{12}V^h(2,A_{n+1},x,n+1) + q_{13}V^h(3,0,x,0) \right\}$$

Let us now consider $V^h(1,a,x,n,u)=V^h(1,a,x,n)$ for a particular $u\in\Gamma(1,S_n)$

$$\left(\rho+\frac{1}{h_a}+\frac{1}{h_x}|u(t)-d|-q_{11}\right)V^h(1,a,x,n,u)$$
$$=h(x)+\frac{1}{h_a}V^h(1,a+h_a,x,n)+\frac{1}{h_x}V^h(1,a,x+h_x,n)|u(t)-d|\,Ind\{(u(t)-d)\geq 0\}$$
$$+\frac{1}{h_x}V^h(1,a,x-h_x,n)|u(t)-d|\,Ind\{(u(t)-d)<0\}+q_{12}V^h(2,A_{n+1},x,n+1)+q_{13}V^h(3,0,x,0)$$

That is

$$V^h(1,a,x,n,u)=\frac{1}{\left(\rho+\frac{1}{h_a}+\frac{1}{h_x}|u(t)-d|-q_{11}\right)}\left(\begin{array}{l} c^+x^++c^-x^-+\frac{1}{h_a}V^h(1,a+h_a,x,n)\\ +\frac{1}{h_x}|u(t)-d|\left(\begin{array}{l}V^h(1,a,x+h_x,n)\,Ind\{(u(t)-d)\geq 0\}\\ +V^h(1,a,x-h_x,n)\,Ind\{(u(t)-d)<0\}\end{array}\right)\\ +q_{12}V^h(2,A_{n+1},x,n+1)+q_{13}V^h(3,0,x,0)\end{array}\right)$$

ii) When $\alpha=2$

- $Ind\{\alpha=1\}=0$,
- $u(\cdot)=0$,
- $Ind\{(u(t)-d)\geq 0\}=Ind\{-d\geq 0\}=0$,
- $Ind\{(u(t)-d)<0\}=Ind\{-d<0\}=1$
- $w(n,a)=W_1(n,a,n+1)$,
- $G(\cdot)=c^+x^++c^-x^-+W_1(n,a,n+1)$ and
- $$\begin{aligned}Q(a,n)V(\cdot,\varphi(a,\cdot,n),x,n')(\alpha)&=Q(a,n)V(\cdot,\varphi(a,\cdot,n),x,n')(\alpha=2)\\&=q_{22}V(2,a,x,n)+q_{21}V(1,a,x,n)\end{aligned}$$

Then, replacing in (4.21) gives

$$\begin{aligned}\rho V^h(2,a,x,n)&=c^+x^++c^-x^-+W_1(n,a,n+1)+\frac{\partial}{\partial x}V(2,z,n)(u(t)-d)\\&\quad+q_{22}V^h(2,a,x,n)+q_{21}V^h(1,a,x,n)\\&=c^+x^++c^-x^-+W_1(n,a,n+1)-\frac{d}{h_x}\left[V^h(2,a,x,n)-V^h(2,a,x-h_x,n)\right]\\&\quad+q_{22}V^h(2,a,x,n)+q_{21}V^h(1,a,x,n)\end{aligned}$$

We then have

$$\left(\rho+\frac{d}{h_x}-q_{22}\right)V^h(2,a,x,n,u)$$
$$=c^+x^+ + c^-x^- + W_1(n,a,n+1) + \frac{d}{h_x}V^h(2,a,x-h_x,n)$$
$$+\, q_{22}V^h(2,a,x,n)+q_{21}V^h(1,a,x,n)$$

That is

$$V^h(2,a,x,n,u)=\frac{1}{\left(\rho+\frac{d}{h_x}-q_{22}\right)}\begin{pmatrix} c^+x^+ + c^-x^- + W_1(n,a,n+1) \\ +\, \frac{d}{h_x}V^h(2,a,x-h_x,n)+\, q_{21}V^h(1,a,x,n)\end{pmatrix}$$

iii) When $\alpha=3$

- $Ind\{\alpha=1\}=0$,
- $u(\cdot)=0$,
- $Ind\{(u(t)-d)\geq 0\}=Ind\{-d\geq 0\}=0$,
- $Ind\{(u(t)-d)<0\}=Ind\{-d<0\}=1$
- $w(n,a)=W_0(n,A_0,0)$,
- $G(\cdot)=c^+x^+ + c^-x^- + W_0(n,a,0)$ and
- $Q(a,n)V(\cdot,\varphi(a,\cdot,n),x,n')(\alpha)=Q(a,n)V(\cdot,\varphi(a,\cdot,n),x,n')(\alpha=3)$
  $=q_{33}V(3,a,x,n)+q_{31}V(1,a,x,n)$

Then, replacing in (4.21) gives

$$\rho V^h(3,a,x,n)=c^+x^+ + c^-x^- + W_0(n,a,0)+\frac{\partial}{\partial x}V(3,z,n)(u(t)-d)$$
$$+q_{33}V^h(3,a,x,n)+q_{31}V^h(1,a,x,n)$$
$$=c^+x^+ + c^-x^- + W_0(n,a,0)-\ \frac{d}{h_x}\left[V^h(3,a,x,n)-V^h(3,a,x-h_x,n)\right]$$
$$+\ q_{33}V^h(3,a,x,n)+q_{31}V^h(1,a,x,n)$$

Thus,

$$\left(\rho+\frac{d}{h_x}-q_{33}\right)V^h(3,a,x,n,u)$$
$$=c^+x^+ + c^-x^- + W_0(n,a,0)+\ \frac{d}{h_x}V^h(3,a,x-h_x,n)$$
$$+\ q_{33}V^h(3,a,x,n)+q_{31}V^h(1,a,x,n)$$

That is

$$V^{h}(3,a,x,n,u)=\frac{1}{\left(\rho+\dfrac{d}{h_x}-q_{33}\right)}\begin{pmatrix} c^{+}x^{+}+c^{-}x^{-}+W_{0}(n,a,0) \\ +\dfrac{d}{h_x}V^{h}(3,a,x-h_x,n)+q_{31}V^{h}(1,a,x,n) \end{pmatrix}$$

REFERENCES

Ansell J. I., Archibald T. W., and Thomas L.C. (2004). "*The elixir of life: using a maintenance, repair and replacement model based on virtual and operating age in the water industry*." IMA Journal of Management Mathematics, 15(2): pp. 151-60.

Boukas E.-K, and Haurie A. (1990). "*Manufacturing flow control and preventive maintenance: A stochastic control approach*." IEEE Transactions on Automatic Control, 35(9): pp. 1024-1031.

Boukas E. K. (1998). "*Hedging point policy improvement*." Journal of Optimization Theory and Applications, 97(1): pp. 47-70.

Coudert, T., Grabot B., and Archimede B. (2002). "*Production/maintenance cooperative scheduling using multi-agents and fuzzy logic*." International Journal of Production Research, 40(18): pp. 4611-4632.

Dempster M. A. H., Fisher M. L., Jansen L., Lageweg B. J., Lenstra J. K., Rinnooy Kan A. H. G. "*Analytical evaluation of hierarchical planning systems*." Operations Research, 29(4): pp. 707-716.

Gharbi A. and Kenne J.-P. (2005). "*Maintenance scheduling and production control of multiple-machine manufacturing systems*." Computers and Industrial Engineering, 48(4): pp. 693-707.

Gharbi A. and Kenne J. P. (2000). "*Production and preventive maintenance rates control for a manufacturing system: An experimental design approach*." International Journal of Production Economics, 65(3): pp. 275-287.

Kenne J.-P., Gharbi A. and Beit M. (2007). "*Age-dependent production planning and maintenance strategies in unreliable manufacturing systems with lost sale*." European Journal of Operational Research, 178(2): pp. 408-420.

Kenne J. P. and Boukas E. K. (2003). "*Hierarchical control of production and maintenance rates in manufacturing systems*." Journal of Quality in Maintenance Engineering, 9(1): pp. 66-82.

Kijima M. (1989). "*Some results for the quasi-stationary distribution in birth-death processes*." Research Reports on Information Sciences, Series B (Operations Research), 219: pp. 1-15.

Kimemia J. and Gershwin S. B. (1983). “*Algorithm for the computer control of a flexible manufacturing system.*” IIE Transactions (Institute of Industrial Engineers), 15(4): pp. 353-362.

Kushner H. and Dupuis P. G. (1992). “*Numerical methods for stochastic control problems in continuous time .*” Springer-Verlag. New York.

Love, C. E., Z. G. Zhang, M. A. Zitron and R. Guo (2000). “*Discrete semi-Markov decision model to determine the optimal repair/replacement policy under general repairs.*” European Journal of Operational Research, 125(2): pp. 398-409.

Love C. E., Zitron M.A. and Zhang Z.G. (1998). “*SMDP approach to optimal repair/replacement decisions for systems experiencing imperfect repairs.*” Journal of Quality in Maintenance Engineering, 4(2): pp. 131-149.

Makis V. and Jardine A. K. S. (1993). “*Note on optimal replacement policy under general repair.*” European Journal of Operational Research 69(1): pp. 75-82.

Mohafid A. et Castanier B. (2006). “*Optimisation de la maintenance imparfaite à efficacité aléatoire.*” 6e Conférence Francophone de MOdélisation et SIMulation - MOSIM’06 – Rabat- Maroc

Nakagawa T. and Kijima M. (1989). “*Replacement policies for a cumulative damage model with minimal repair at failure.*” IEEE Transactions on Reliability, 38(5): pp. 581-584.

Phelps R. I. (1983). “*Optimal policy for minimal repair.*” Journal of the Operational Research Society, 34(5): pp. 425-427.

Rishel R. (1975). “*Control of systems with jump markov disturbances.*” Jt Autom Control Conf, 15th, Proc, Jun 18-21 1974: pp. 246-249.

Sethi S. P. and Zhang Q. (1994). “*Hierarchical Decision Making in Stochastic Manufacturing* Systems.” Birkhâser.

Shin, I., Lim T. J. and Lie C. H. (1996). “*Estimating parameters of intensity function and maintenance effect for repairable unit.*” Reliability Engineering & System Safety, 54(1): pp. 1-10.

Sworder D. D. (1969). “*Feedback control of Class of linear Systems with Jump parameters.*” IEEE Transactions on Automatic Control, 14: pp. 8-14.

Vasant, P., Nagarajan R., and Yaacob S. (2004). “*Decision making in industrial production planning using fuzzy linear programming.*” IMA Journal Management Mathematics, 15(1): pp. 53-65.

Yong J. and Zhou X. Y. (1999). "*Stochastic Controls: Hamiltonian Systems and HJB Equations*." Applications of Mathematics (Stochastic Modelling and Applied Probability), Hardcover.

Zimmermann, H., (1990). "Fuzzy Set. *Theory and Its Applications*." second, revised edition. Kluwer Academic publishers. ISBN 0-7923-9075-X.

CHAPITRE 5

ARTICLE 4: PREVENTIVE MAINTENANCE AND REPLACEMENT POLICIES FOR DETERIORATING PRODUCTION SYSTEMS SUBJECT TO IMPERFECT REPAIRS

F. I. Dehayem Nodem [1], J. P. Kenne[1] and A. Gharbi[2]

[1]Mechanical Engineering Department, Production Technologies Integration Laboratory, University of Quebec,

[2]Automated Production Engineering Department, Production System Design and Control Laboratory, University of Quebec

École de technologie supérieure, 1100 Notre Dame Street West, Montreal, Quebec Canada H3C 1K3

This chapter is an article submitted to Reliability Engineering & System Safety on December 18, 2008. Ms. Ref. No.: RESS-D-08-00413.

Résumé

Cet article présente l'intégration des stratégies de maintenance préventive à la politique de réparation versus le remplacement d'un système manufacturier. Le système est sujet aux pannes à intensité croissante et à des temps de réparation après panne croissants. Afin de réduire les temps de réparation suite à une panne, il y a une incitation à effectuer la maintenance préventive sur la machine avant que la panne survienne. Le mode de la machine à tout moment peut être classifié comme étant en opération, en réparation, en remplacement ou en maintenance préventive. Les variables de décision du système sont l'âge auquel il faut cesser de réparer la machine et la remplacer à l'instant de survenue de la prochaine panne et le taux de maintenance préventive. Le problème de détermination des politiques de réparation versus le remplacement et de maintenance préventive est formulé comme un processus de décision semi-Markovien. Les méthodes numériques sont utilisées pour résoudre les conditions d'optimum et obtenir les politiques optimales qui minimisent le coût.

Le coût est constitué par le coût moyen de maintenance préventive, de réparation et de remplacement sur un horizon infini. Comme prévu, la décision de réparer ou de remplacer la machine suite à une panne de la machine est modifiée si une maintenance préventive a été effectuée avant la survenue de la panne. Un exemple numérique est donné et une analyse de sensibilité est réalisée pour illustrer l'approche proposée et à montrer l'impact de différents paramètres sur les politiques de commande ainsi obtenue.

Abstract

This paper presents the integration of preventive maintenance into the repair/replacement policy of a failure-prone manufacturing system. The system exhibits increasing failure intensity and increasing repair times. To reduce the subsequent repair time following a failure, there is an enticement to perform preventive maintenance on the machine before failure. The machine's mode at any time can be classified as either operating, in repair, in replacement or in preventive maintenance. The decision variables of the system are the repair/replacement switching age at the time of the machine's failure and the preventive maintenance rate. The problem of determining the repair/replacement and preventive maintenance policies is formulated as a semi-Markov decision process and numerical methods are given in order to compute optimal policies which minimize the average cost incurred by preventive maintenance, repair and replacement over an infinite planning horizon. As expected, the decisions to repair or to replace the machine upon a failure are modified by performing preventive maintenance. A numerical example is given and a sensitivity analysis is performed to illustrate the proposed approach and to show the impact of various parameters on the control policies thus obtained.

Keywords: Manufacturing systems; Numerical methods; Optimal control; damaging failures; replacement; repair; preventive maintenance.

5.1 Introduction

We consider a manufacturing system with one failure-prone machine. The machine deteriorates stochastically with age and the number of failures. For such a deteriorating system, it is quite reasonable to assume that the successive working times will become shorter and shorter and the repair times will become longer and longer (Leung, 2006; Jinyuan and Zehui, 2008) Due to the increasing repair times, the machine may finally be non-reparable after it experiences a certain number of failures. Therefore, a failure of the machine means that a decision must be made: to continue to repair at ever-increasing repair costs, or to replace the machine with a new one. However, there are alternatives that can counteract the deterioration of this type of system, and one of them is preventive maintenance.

Preventive maintenance is usually performed because it presents many advantages. Its effects include: increasing the availability of the system, decreasing the operating costs and improving the reliability of the system. Preventive maintenance could also serve to avoid failure in those cases where failure cannot be tolerated, and it can prevent unnecessary replacement.

There is extensive literature on systems that deteriorate due to the effects of ageing (Nakagawa and Kijima, 1989; Ansell *et al.*, 2004; Hariga *et al.*, 2006; Xiaojun *et al.*, 2007; Yu-Hung, 2008). This literature proposes maintenance models in order to counteract the ageing effect. Wang (2002) compared and classified maintenance models such that a decision maker can see which one is convenient to solve his or her specific problem. According to Wang (2002), maintenance can be divided into two major classes: corrective maintenance that occurs when the system fails and preventive maintenance that occurs when the system is operating. Replacement is qualified as maintenance that is perfect (Wang, 2002).

Many works deal with simultaneous repair, replacement and preventive maintenance of production systems. In some cases, what the authors call preventive maintenance appear to simply be the preventive replacement of the machine (Aven and Castro, 2008), since most of the time the machine is as good as new after preventive maintenance. In a manufacturing environment, some preventive interventions are performed on machines to improve their condition but that do not renew them. Various works considered this type of improvement by utilizing the concept of virtual age of the machine after intervention (Makis and Jardine, 1991; Love *et al.*, 1998; Love *et al.*, 2000; Doyen and Gaudoin, 2004; Hariga *et al.*, 2006; Kahle, 2007; Xiaojun *et al.*, 2007). The concept of virtual age of the machine, introduced by Kijima *et al.* (1988) and generalized by Kijima (1989), states that the repair (or preventive maintenance) serves to reset the age of the machine, which determines the revised failure intensity. The reset age is the virtual age of the machine. The reset age is the virtual age of the machine. Malik (1979) introduced the concept of an age reduction factor. By this concept, the machine's condition is improved through maintenance actions such as cleaning, lubrication and realignment. In Malik's model, each imperfect preventive maintenance changes the initial hazard rate value immediately following the preventive maintenance, but not all the way to zero (i.e., not new). Xiaojun *et al.* (2007) adopted a hybrid hazard rate recursion rule in order to combine the age reduction factor method with the hazard rate increase method. The aim of their combination method is to predict the evolution of a system's reliability in different maintenance cycles.

Malik (1979) proposed using expert judgment to estimate the improvement factor. According to Doyen and Gaudoin (2004), the age-reduction factors are estimated directly from the data for different types of repairs. They also provided an estimation of the reduction factor, obtained by simulation Doyen and Gaudoin (2004).

For major repairable systems, when the number of failures increases, the system failures become more severe (Lugtigheid *et al.*, 2008). The system may need to be de-installed and taken to the maintenance shop. Generally, it is disassembled, evaluated and repaired. The first two operations are required to determine which subsystems have failed and must be addressed. Lugtigheid *et al.* (2008) proposed replacing failed subsystems along with some non-failed parts, to take advantage of the fact that the system has already been uninstalled and taken out of operation. This sort of maintenance brings the state of the machine to somewhere between as good as new and the condition it was in immediately before the failure or preventive maintenance.

As we have mentioned above, and as many other works have concurred, the improvement is carried out in order to reduce the system failure intensity and/or the system age. The above-cited authors concluded that the time required for the repair(s) and preventive maintenance is negligible or not necessary to be taken into account, but this assumption is not realistic. The failures become more severe as their number increases; the machine must be uninstalled, taken to the maintenance shop, disassembled and evaluated all before being repaired. The repair times thus increase with the number of failures (Leung, 2006; Jinyuan and Zehui, 2008). The problem of integrating preventive maintenance into a repair/replacement policy has not yet been addressed. The proposed approach consists of integrating preventive maintenance with the repair/replacement policy of a deteriorating production machine.

The effect of preventive maintenance is to increase the system's availability by reducing the next mean repair time with a reduction factor that will be defined later in this paper. The proposed approach consists of developing a semi-Markov decision model in order to determine optimal preventive maintenance, repair and replacement policies for the system. Those policies should minimize the overall incurred costs over an infinite planning horizon. The preventive maintenance on the machine is performed after a random delay, as proposed by Boukas and Haurie (1990). The contribution of this paper is the integration of preventive maintenance to the repair/replacement determination model. We transpose the well-known age reduction and hazard rate increase factor methods to repair time reduction in order to improve the lifespan of the system. The contribution of this paper is the integration of preventive maintenance to the repair/replacement determination model. We transpose the well-known age reduction and hazard rate increase factor methods to repair time reduction in order to improve the lifetime of the system.

The paper is organized as follows. In Section 5.2, we present the problem statement. Optimality conditions are given in Section 5.3. Numerical methods are used in Section 5.4 to solve the optimality conditions obtained in Section 5.3. Numerical examples are presented in Section 5.5 and a sensitivity analysis is provided in section 5.6 to illustrate the usefulness of the proposed approach. Our conclusions are presented in Section 5.7.

5.2 Problem statement

The machine under consideration is subject to random failures, and may either be replaced, receive imperfect repair after failure or be sent to receive preventive maintenance while in working condition. Thus, it has four modes: up, in repair, in replacement and in preventive maintenance. These modes are denoted by $1, 2, 3$ and 4, respectively. The machine's mode changes from 'up' to 'preventive maintenance' with a transition rate of $q_{14}(.) = \omega(.)$. $q_{14}(.)$ is assumed to be a control variable. The machine's mode changes from 'preventive maintenance' to 'up' with a constant transition rate of q_{41}.

We consider that repair and replacement are two kinds of failure mode since they occur after a failure. The failure mode is denoted by F. Therefore, the machine's mode changes from 'up' to 'failure' at rate $q_{1F}(a(t))$, which is an increasing function of the machine's age $a(t)$. If the machine is to be repaired, then the transition rate from 'up' to 'repair' $q_{12}(a(t))$ will be equal to the failure rate $q_{1F}(a(t))$. Otherwise, the machine will be replaced. The corresponding transition rate from 'up' to 'replacement', $q_{13}(a(t))$, will be equal to the failure rate $q_{1F}(a(t))$. The machine's mode changes from 'repair' to 'up' with a transition rate of $q_{21}(n)$, which is a decreasing function of the number of failures n. The transition rate from 'replacement' to 'up' is described by q_{31}. The state-space diagram of this semi-Markov process is illustrated figure 5.1.

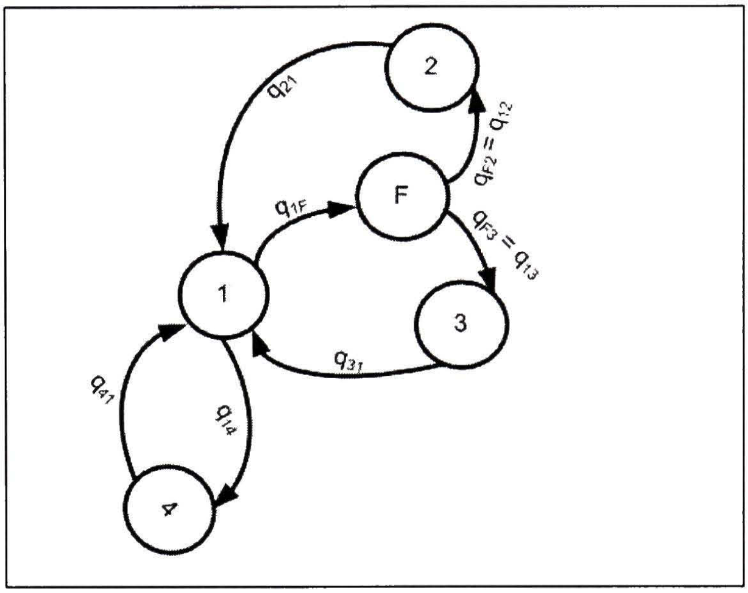


Figure 5.1 Machine state-space diagram.

It follows that

$$\begin{cases} q_{12}(a(t)) + q_{13}(a(t)) = q_{1F}(a(t)) \\ q_{12}(a(t)) = \left(1 - Ind\left\{q_{13}(a(t)) > 0\right\}\right) * q_{1F}(a(t)) \end{cases} \tag{5.1}$$

where the indicator function of any function $f(.)$ is defined by

$$Ind(f(.)) = \begin{cases} 1 & \text{if the } f(.) \text{ is realized} \\ 0 & \text{otherwise} \end{cases}$$

Equation (5.1) states that, when a failure occurs, the machine cannot be sent to repair and slated for replacement at the same time.

Let $T_{21}(n) = MTTR(n)$ be the mean repair time after the n^{th} failure and let us call $q^{m}{}_{21}(n)$ the repair rate and $MTTR^{m}(n)$ the mean repair time if preventive maintenance actions are triggered before the n^{th} failure. We assume that the machine has experienced its $(n-1)^{th}$ failure and has been repaired (Love *et al.*, 1998). As demonstrated in the literature, a monotone process such as an arithmetic-geometric approach is considered to be relevant, realistic and appropriate to the modelling of a deteriorating system maintenance problem, i.e., for modelling the survival time after the $(n-1)^{th}$ repair and the repair time after the n^{th} failure. For more details, we refer the reader to the work of Leung (2006), where some examples of arithmetic-geometric processes are provided.

5.2.1 Repair times reduction

We consider that the effect of preventive maintenance is to reduce the next repair time so that, if there is some preventive maintenance before the n^{th} failure, the mean time to repair at the n^{th} failure will be of the form $MTTR^m(n) = \phi(MTTR(n))$ with $0 < \phi(MTTR(n)) \le MTTR(n)$.

If the determination of the mean repair time depends on the complete history, we then lose the Markovian property of the system. Hence, $MTTR^m(n)$ depends only on the mean time to repair at the n^{th} failure.

From a practical point of view, when the machine is sent for preventive maintenance, non-failed but used subsystems are repaired. Hence, at the next failure, fewer subsystems are evaluated and repaired compared to the situation where no preventive maintenance had been performed before failure. Estimation of the repair time reduction factor can be based on preventive maintenance and the repair time's history data.

In addition, there are numerous studies of age reduction factor and hazard rate increase factor methods in the literature (Love *et al.*, 2000; Doyen and Gaudoin, 2004; Hariga *et al.*, 2006). Considering that the effect of those methods can result in reduced repair times, one could consider one of the following thwo reduction models Redf1 and Redf2:

Redf1 $MTTR^m(n) = MTTR(n) - \varepsilon_0$ where ε_0 is a given constant ;

Redf2 $MTTR^m(n) = \varepsilon_n MTTR(n)$ with $0 < \varepsilon_n \le 1$ after the n^{th} preventive maintenance

$MTTR^{m}(n) = \varepsilon_{g(n)} MTTR(n)$ with $\varepsilon_{g(n)} = \dfrac{g(n)}{g(n)+\eta}$ and $0 < \eta < 1$ after the n^{th} preventive maintenance. $g(.)$ is a given function (see (Hariga *et al.*, 2006) for more details).

Illustrative reduction factor example

Let us consider a manufacturing system in which the repair rate following an n^{th} failure is given by:

$q_{21}(n) = q_0 + q_1\left(1 - \left(\frac{n-1}{N}\right)^r\right)$ where q_1, q_0 and r are given constants. The values used in this example are presented in section 5.5. For a given manufacturing system, they can be determined from the repair time's history data.

The above-mentioned models Redf1 and Redf2 will have the effect of improving the availability of the equipment. For the purpose of illustration, we consider that repair times after the preventive maintenance is performed follow the reduction model Redf2. Let $\varepsilon_n = \varepsilon$ for all n.

The effect of Redf2 is to reduce the next repair time with a reduction factor ε such that, if preventive maintenance is performed before the n^{th} failure then, the mean repair time at the n^{th} failure will be $MTTR^{m}(n) = \varepsilon * MTTR(n)$, with $\varepsilon \in]0,1]$. Thus, there is a one-to-one correspondence between repair time without preventive maintenance (or no reduction) and the post-preventive maintenance repair time.

Figure 5.2 presents sample paths of the mean repair time for an increasing number of failures. The repair time is a function of the number of failures n. Each additional failure requires a longer repair time. Sample paths for different values of ε are shown in figure 5.2 presents sample paths of mean time to repair reduction for several failures number. The repair time of the machine is an increasing function of number of failures n. Thus, the consecutive repair times of the machine become longer and longer. Sample paths are in figure 5.2 for different values of ε.

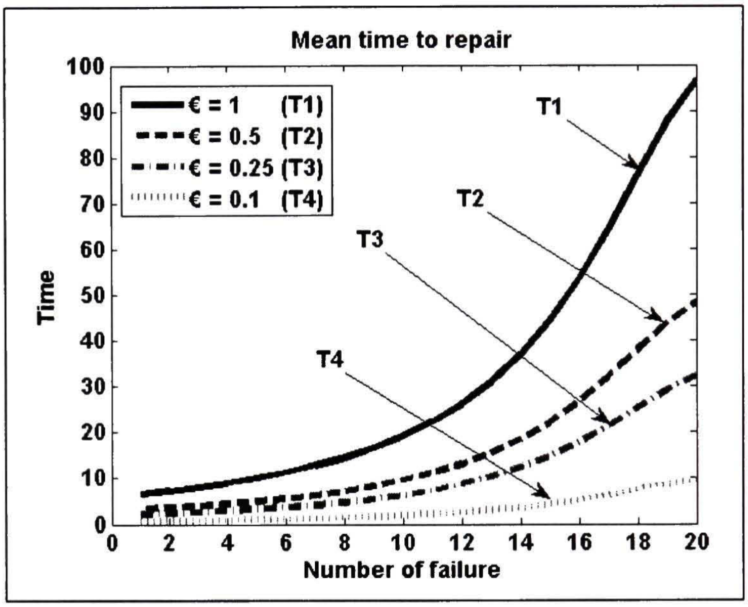


Figure 5.2 Mean time to repair after a preventive maintenance.

Figure 5.2 presents the path of the mean repair time for an increasing number of failures if preventive maintenance is performed before failures, for four ε values. Note that T_1 is also the post-preventive maintenance repairs time, with $\varepsilon = 1$. This corresponds to the case where there is preventive maintenance before failure, but the mean repair time at failure remains unchanged. This situation arises when preventive maintenance serves to reduce the probability of failures, with no effect on the severity of failures. T_2, T_3 and T_4 are the paths for $\varepsilon = 0.5$, $\varepsilon = 0.25$ and $\varepsilon = 0.1$ respectively, and show that the repair time is divided by 2, 4 and 10, respectively.

We can observe from figure 5.2 that the effect of the reduction factor on repair times is more noticeable when the machine experiences a greater number of failures. For the first few failures, curves T_2, T_3 and T_4 are almost the same. But for a large number of failures (18, for example), they are very distinctive. This can make a significant difference on the impact of performing preventive maintenance on a manufacturing system's availability and/or on overall costs.

Since we know that the other forms of reduction have the same effect of increasing equipment availability, in the remainder of this work, we will consider repair time reduction factor Redf2. We will also consider $\varepsilon_n = \varepsilon$ for all n, and call the corresponding model *arithmetic repair time reduction.*

5.2.2 Optimal control problem

The state of the system is described by the machine mode $\xi(t) \in \Omega = \{1,2,3,4\}$ with the transitions shown in figure 5.1, the number of failures $n(t)$ and the age of the machine $a(t)$.

The age of the machine $a(t)$ is an increasing function of chronological time and is described by the following differential equation:

$$\frac{da(t)}{dt} = \delta; \quad a(T) = 0 \tag{5.2}$$

where $0 < \delta \leq 1$ is a given constant and T the time of the last restart of the machine. If $\delta = 1$, then the age of the machine is the cumulative working time since the last restart of the machine.

Recall that the failure rate $q_{1F}(.)$ is an increasing function of a machine's age $a(t)$. It is reset to its initial value after a repair or preventive maintenance.

We assume that the following constraint holds for the preventive maintenance transition rate:

$$0 \le \omega(.) \le \omega_{\max} \tag{5.2}$$

where $\omega_{\max}$ is the maximum rate of preventive maintenance. The inverse of $\omega(.)$ represents the expected delay between the decision to perform preventive maintenance actions and the effective switch from operation mode to preventive maintenance mode.

We define $q_{12}^{s}(a) = q_{12}(a) \times Ind\{a \le s\}$ and $q_{13}^{s}(a) = q_{13}(a) \times Ind\{a > s\}$ if, after a failure occurs, the machine is replaced after age s. The parameter s is the repair/replacement switching threshold age. That is, on the n^{th} failure, if the age of the machine is above the threshold value s, the machine is replaced; if it is not, corrective maintenance is conducted.

Let $\{\xi(t):\ t \ge 0\}$ denote the stochastic process with value in Ω and $Q^{s}(\cdot) = \left(q_{\alpha\beta}^{s}(.)\right)$ denote a 4×4 matrix such that $q_{\alpha\beta}^{s}(a) \ge 0$ if $\alpha \ne \beta$, $q_{11}^{s}(.) = q_{11}(.)$; $q_{31}^{s}(.) = q_{31}(.)$; $q_{14}^{s}(.) = q_{14}(.)$; $q_{41}^{s}(.) = q_{41}(.)$; and $q_{\alpha\alpha}^{s}(.) = -\sum_{\alpha \ne \beta} q_{\alpha\beta}^{s}(.)$. We shall refer to

$$Q^{s}(\cdot) = \begin{bmatrix} q_{11}(.) & q_{12}^{s}(.) & q_{13}^{s}(.) & q_{14}(.) \\ q_{21}(.) & q_{22}(.) & 0 & 0 \\ q_{31}(.) & 0 & q_{33}(.) & 0 \\ q_{41}(.) & 0 & 0 & q_{44}(.) \end{bmatrix}$$ as the transition matrix of the semi-Markov chain

$\xi(\cdot)$

Let us define $G(\alpha, a, n)$ as the running cost of being in state α, at age a of a machine that has already had its n^{th} failure.

The expected discounted cost is given by:

$$J(\alpha,a,n,s,\omega)=E\left[\int_{0}^{\infty} e^{-\rho t}G(\cdot)dt\,/\;\xi(0)=\alpha,\ a(0)=a,\ n(0)=n\right] \tag{5.3}$$

where ρ is the discounted rate used to make the costs incurred at future dates less important than the cost incurred today.

Let $\Gamma=\{\pi=(s(.),\omega(.)):0\leq s(.)\leq M \text{ and } 0\leq \omega(.)\leq \omega_{\max}\}$.

Any plan $\pi=(s,\omega)\in\Gamma$ is called an admissible plan, and our problem is to minimize the expected discounted cost, given by equation (5.3), for any admissible plan.

The control variables are the threshold age s after which the machine should automatically be replaced at the next failure and the preventive maintenance rate ω.

Optimal policies are obtained by searching:

$$V(\alpha,a,n)=\min_{(s,\omega(\cdot))\in\Gamma} J(\alpha,a,n,\omega,s) \tag{5.4}$$

We define S_n as the optimal replacement policy before the n^{th} failure. That is, for each n, there exists an age S_n such that if $a(t)\geq S_n$, the optimal action is to replace the machine at the n^{th} failure.

Let N_m be the minimum number of failures that occur before a systematic replacement at the next failure, regardless the machine's age. Note that if $n=N_m$, then $S_n=0$; it is therefore sufficient to determine S_n instead of both variables.

Our objective is then to determine the age S_n before systematic replacement at the next failure and the preventive maintenance control $\omega(\cdot)$ that will minimize $J(\cdot)$.

As established in the literature (Makis and Jardine, 1993; Love *et al*., 2000), we can choose an upper bound on the age, M to be very large compared to S_1, beyond which the system is automatically replaced.

The value function $V(\alpha,a,n)$ satisfies specific properties called optimality conditions, which are presented in the next section.

5.3 Optimality conditions

In this section, we will show that under appropriate assumptions and lemma (see Rishel (1975)), the value function $V(\alpha,a,n)$ satisfies a set of coupled partial derivative equations derived from the application of the dynamic programming approach.

We make the following assumption on the repair, replacement and preventive maintenance instantaneous cost function $G(.)$.

Assumption (5.1)

The repair, replacement and preventive maintenance cost function $G(.)$ is a non negative convex function of a.

That is, for all $a_1,a_2 \in (0,\infty]$ and $t \in [0,1]$, $G(ta_1+(1-t)a_2,a,n) \le tG(a_1,a,n)+(1-t)G(a_2,a,n)$

Lemma (5.1)

Let f be a convex function. Then:

i) f is locally Lipschitz and therefore continuous, and

ii) f is differentiable

Proof:

For the proof, see (Sethi and Zhang, 1994) and Theorem 2.5.1 in (Clarke, 1983)

□

Given assumption (5.1) and Lemma (5.1), it follows that the value function $V(\alpha,a,n)$ is convex in a. Formally, $V(\alpha,a,n)$ satisfies the Hamilton-Jacobi-Bellman (HJB) equations given by eqn. (5.5).

$$\rho V(\alpha,a,n) = \underset{(s,\omega)\in\Gamma}{Min}\left\{ G(.) + \frac{\partial}{\partial a}V(\alpha,a,n)Ind\{\xi(t)=1\}\frac{da(t)}{dt} + \sum_{\beta\neq\alpha} q^s{}_{\alpha\beta}(a,n)\left[V(\beta,\varphi(a,\cdot,n),\psi(n)) - V(\alpha,a,n)\right] \right\} \quad (5.5)$$

where $\alpha \in \Omega$;

$$\varphi(a,\xi,n) = \begin{cases} 0 & \text{if } \xi(\tau^+)=1 \text{ and } \xi(\tau^-) = 2, 3 \text{ or } 4 \\ a(\tau^-) & \text{otherwise} \end{cases}$$ and

$$\psi(n) = \begin{cases} 0 & \text{if } \{\xi(\tau^+)=1 \text{ and } \xi(\tau^-)=3\} \\ n+1 & \text{if } \{\xi(\tau^+)=2,3 \text{ and } \xi(\tau^-)=1\} \\ & \text{or } \{\xi(\tau^+)=1 \text{ and } \xi(\tau^-)=2\} \\ n & \text{otherwise} \end{cases}$$

$\frac{\partial}{\partial a}V(\alpha,a,n)$ is the gradient of the value function $V(\alpha,a,n)$.

Theorem (5.1) (Uniqueness Theorem)

Assume $|G(\alpha,a,n)| \le C(1+|a|^k)$, and

$|G(\alpha,a_1,n) - G(\alpha,a_2,n)| \le C(1+|a_1|^k+|a_2|^k)|a_1-a_2|^k$ Then the HJB equation (5.5) has a unique viscosity solution.

Proof:

The proof is similar to the proof of theorem G.1 of (Sethi and Zhang, 1994) when replacing $V(a,.)$ by $V(.,a,.)$ and the function $\eta(a)$ by $\vartheta(a)=\exp(\frac{\rho}{\delta}(1+|a|^2)^{\frac{1}{2}})$

□

In the next section, we will construct a near optimal control policy for the problem through numerical methods based on Kushner's approach (Kushner and Dupuis, 1992).

5.4 Numerical approach

In this section, we solve the HJB equations (5.6) by approximating $V(\alpha,a,n)$ by approximating $V(\alpha,a,n)$ with a function $V^{h_a}(\alpha,a,n)$ and the first-order partial derivatives of the value function $\frac{\partial}{\partial a}V(\alpha,a,n)$ by: $\frac{\partial}{\partial a}V(\alpha,a,n)=\frac{1}{h_a}\left[V^{h_a}(\alpha,a+h_a,n)-V^{h_a}(\alpha,a,n)\right]$ where h_a is a discrete increment associated with state variable a.

Let define $Q_{h_a}^{\alpha}=\left|q_{\alpha\alpha}^{s}\right|$; $P_a(\alpha)=\begin{cases}\frac{\sigma}{h_a Q_{h_a}^{\alpha}} & \text{if } \alpha=1\\ 0 & \text{otherwise}\end{cases}$ and $p^{\beta}(\alpha)=\frac{q_{\alpha\beta}^{s}}{Q_{h_a}^{\alpha}}$

After a couple of manipulations, the HJB equations can be rewritten as follows:

$$V^{h_a}(\alpha,a,n)=\underset{(s,\omega)\in\Gamma}{Min}\left\{\frac{G(.)}{Q_{h_a}^{\alpha}(1+\frac{\rho}{Q_{h_a}^{\alpha}})}+\frac{1}{(1+\frac{\rho}{Q_{h_a}^{\alpha}})}(P_a(\alpha)V^{h_a}(\alpha,a+h_a,n)+\sum_{\beta\neq\alpha}p^{\beta}(\alpha)V^{h_a}(\alpha,a,n)\right\} \quad (5.6)$$

The next theorem shows that the value function $V^{h_a}(\alpha,a,n)$ is an approximation of $V(\alpha,a,n)$ for small size step h_a.

Theorem (5.2)

Let $V^{h_a}(\alpha,a,n)$ denote a solution to HJB equations (5.7). Assuming that there are constants Cg and Kg such that:

$0 \leq V^{h_a}(\alpha,a,n) \leq Cg(1+|a|^{Kg})$, then

$$\lim_{h_a \to 0} V^{h_a}(\alpha,a,n) = V(\alpha,a,n) \tag{5.7}$$

Proof:

This theorem can be proved similarly as in (Kenne and Boukas, 1998).

□

We use the policy improvement technique to obtain a solution of the approximating optimization problem. We refer the reader to (Kushner and Dupuis, 1992) for the algorithm of this technique. In the next section, we illustrate the approach developed in this paper, using a numerical example.

5.5 Numerical example

We consider the computational domain D defined by: $D = \{(a,n) : 0 \leq a \leq 100; 0 \leq n \leq 20\}$. The costs of preventive maintenance (C_m= 100) and replacement (C_{rem}= 20 000) are assumed to be constant. Costs are in dollars (US$).

Let C_{rep} be the cost of repair per unit of mean time. Thus the mean repair cost is $C_{rep} * T_{21}(n)$. Recall that $T_{21}(n)$ is the mean repair time before the n^{th} failure. Hence, the cost function G(.) is given by:

$$G(\alpha,a,n) = C_{rep} * T_{21}(n) * Ind\{\xi(t)=3\} + C_{rem} * Ind\{\xi(t)=2\} + C_m * Ind\{\xi(t)=4\}$$

Given the frequent utilization of the Weibull distribution in reliability engineering (Love *et al.*, 2000), we assume that the lifetime of a new machine follows a Weibull distribution. The parameter scale is $\lambda = 0.03$ and the shape parameter $\alpha = 2$. Table 5.1 summarizes other parameters used in this paper.

Table 5.1 Parameters of the numerical example №3

Parameter	N	q_0	q_1	q_{31}	r	δ
Value	20	0.01	0.14	10	2	1
Parameter	c_{rep}	h_a	ρ	q_{41}	ω_{max}	ε
Value	100	0.2	0.05	0.2	0.1	0.1

For $\delta = 1$, as in Table 5.1, the age of the machine is the chronological working time since the last restart of the machine. Recall that the lifetime distribution of a new machine follows a Weibull distribution with the density function $f(a) = \lambda^{\alpha} \alpha a^{\alpha-1} \exp(-(\lambda a)^{\alpha})$. Its cumulative distribution function is $F(a) = 1 - \exp(-(\lambda a)^{\alpha})$.

At any time, we consider that the system has experienced its $(n-1)^{th}$ failure, $n = 1,2,3,...$ and has been repaired. Since the repair restores the age of the machine to zero, the mean time to failure ($MTTF$) of the machine before the n^{th} failure is equal to the mean time to first failure and is given by $MTTF = (1/\lambda)\Gamma(1+\frac{1}{\alpha})$, where $\Gamma(.)$ is the gamma function.

Using the previous parameters, $MTTF = 29.5$ units of time. Thus, our selection of the upper bound for the control limits on age is $M = 100$. The probability that the machine will have its first failure before $M = 100$ is almost equal to 1, as illustrated by the graph of the distribution function in figure 5.3.

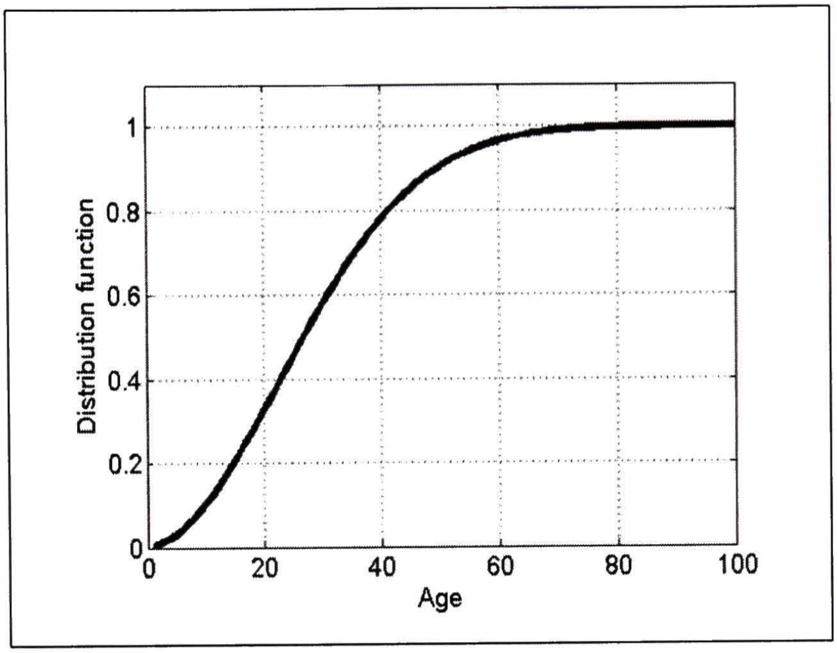


Figure 5.3 Distribution function of failures over time for a given failure number n.

This ensures that if the replacement threshold age at first failure of the machine is greater than $M = 100$, that machine will never be replaced after a first failure. In these circumstances, it is not necessary to ensure $M >> S_1$, as is the case in (Love *et al.* 2000), because M and S_1 appear to be infinity for the machine.

For $\varepsilon = 0.1$, there is a strong incentive to perform preventive maintenance because it will reduce the next repair time by a factor of 10. Moreover, if no preventive maintenance is carried out before a large number of failures, the mean time to repair for each failure will follow the path P1 of figure 5.4 below. When preventive maintenance is triggered once before each failure, then the path of the mean repair time follows P2.

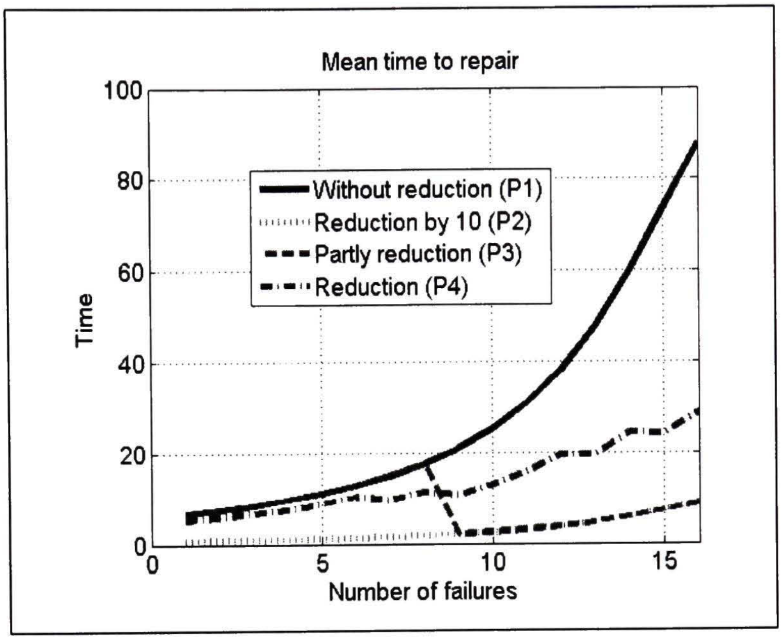


Figure 5.4 Mean time to repair and its possible reduction.

Note that in reality, preventive maintenance may not be performed before every failure. Thus, the real P3 path of the mean repair time could be composed of P1 and P2, as presented in figure 5.4. For relatively small reduction factors, preventive maintenance may be done more than once between failures. The corresponding reduced repair time follows P4.

The policy improvement technique is used to solve equation (5.7). The results obtained for the values given in Table 5.1 are presented in figures 5.5 to 5.7.

The repair/replacement policy S_n presented in figure 5.5 divides the plan $(a(.),n)$ into two zones: Zone 1 below S_n and Zone 2 above S_n. If a failure occurs when the system is located in Zone 1, the optimal action is to repair the machine. Otherwise, the optimal action is to replace the machine. Since the decisions to repair or to replace the machine are limited to instances of machine failure, figure 5.5 identifies the action to be undertaken if the location of the system at failure is known.

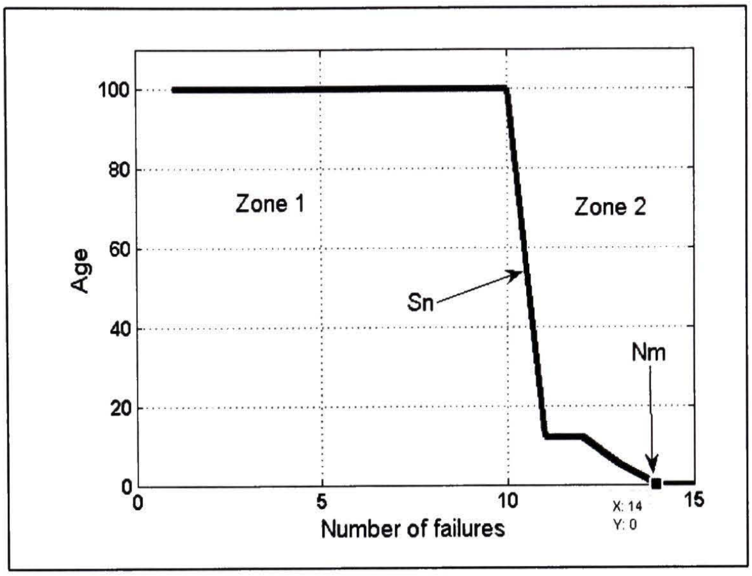


Figure 5.5 Repair/replacement policy.

The machine would have at least $N_m = 14$ failures before systematic replacement, regardless of its age, will occur at the next failure. Recall that the age $a = 100$ appears to be infinity for the machine. We can conclude from figure 5.5 that the machine will never be replaced before it has experienced at least 10 failures.

Let $R_n(a(t))$ denote a function with a value of 1 if a repair action is undertaken after the n^{th} failure occurs at age $a(t)$, and 0 if it does not. The above results enable us to illustrate the repair/replacement policy as: upon the n^{th} failure of the machine at age $a(t)$,

$$R_n(.) = \begin{cases} 1 & \text{if } a(.) \le S_n \\ 0 & \text{otherwise} \end{cases} \tag{5.8}$$

with S_n given in figure 5.3

Figure 5.6 below presents the preventive maintenance policy.

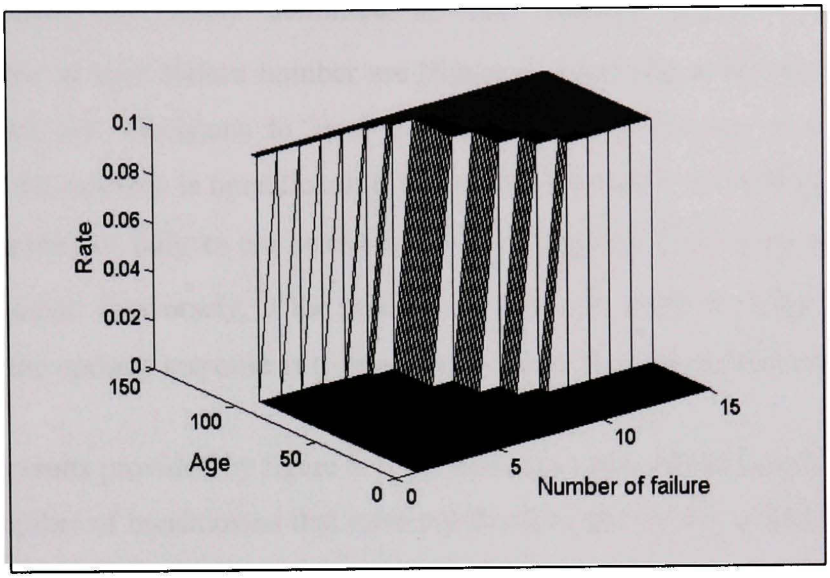


Figure 5.6 Preventive maintenance policy.

The preventive maintenance policy given by figure 5.6 will be represented in the remainder of the paper by its boundary, as shown in figure 5.7.

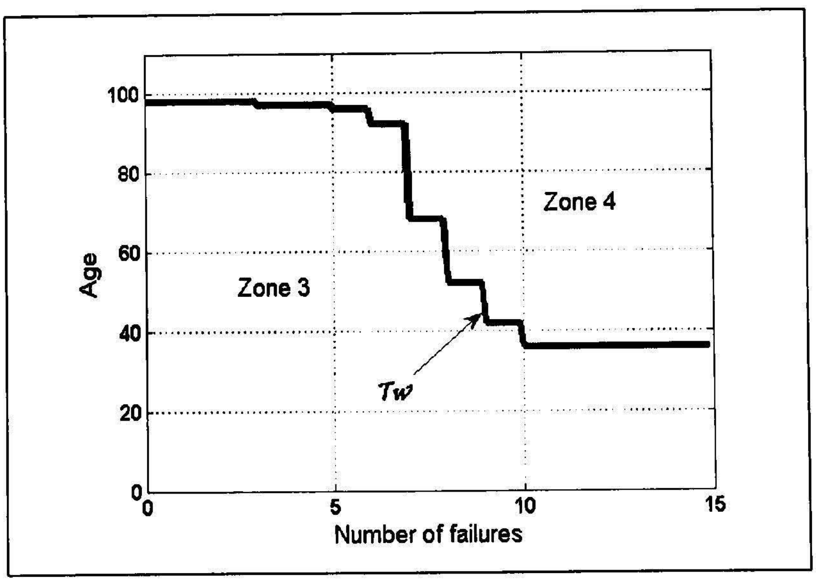


Figure 5.7 Boundary of the preventive maintenance policy.

Figure 5.7 identifies two zones delimited by the boundary $T_\omega(n)$. The preventive maintenance actions at each failure number are triggered according to the age limit policy described in figure 5.7. Decisions to send the machine to preventive maintenance are undertaken when the machine is operational. If the machine's state is located in Zone 3, that is, the age and number of failures are below the boundary $T_\omega(n)$, there is no need to call a maintenance specialist. Conversely, if the machine's state is in Zone 4, which is above the boundary $T_\omega(n)$, the optimal response is to trigger preventive maintenance actions.

According to the results provided by figure 5.7, the trend is to send old machines (determined by age and the number of breakdowns that have occurred) to preventive maintenance, which is the reality in manufacturing systems.

The preventive maintenance actions are triggered according to the age limit policy described in figure 5.7, which indicates that preventive maintenance should be performed at rate $\omega(.)$, with

$$\omega(.) = \begin{cases} 0 & \text{aif} \quad (.) \le T_\omega(.) \\ \omega_{\max} & \text{otherwise} \end{cases} \tag{5.9}$$

where $T_\omega(.)$ is the age limit for preventive maintenance before the next machine failure. For a given failure number n, $T_\omega(n)$ is provided by figure 5.7.

It can be observed that when the number of failures increases, the age limit $T_{\omega_n} = T_\omega(n)$ decreases, meaning that the machine is sent to preventive maintenance more often.

On the basis of the previous results, we can illustrate the repair/replacement and preventive maintenance policies by the vector $\left(S_n, N_m, T_{\omega_n}\right)$, where T_{ω_n} corresponds to the value of $T_\omega(.)$ for each n value in figure 5.7, and $N_m = \min\{\, n \ge 0 \text{ such that } S_n = 0\}$.

To see how these results are influenced by some of the parameters used in this paper, in the next section we present a sensitivity analysis performed according to the previous results.

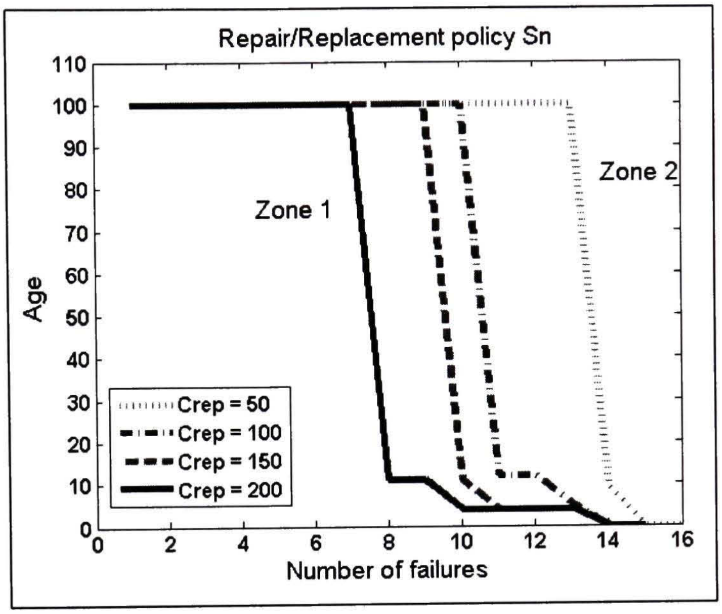
5.6 Result and sensitivity analysis

The previous section provided the repair/replacement and preventive maintenance policies (S_n, N_m, T_{ω_n}), which recommend replacing the machine at the $N_m{}^{th}$ failure or at failure after age S_n, whichever comes first, and not performing preventive maintenance before age T_{ω_n} is reached. The decisions to repair, to replace or to trigger preventive maintenance are based on the overall incurred cost. We analyze the sensitivity of those policies according to repair cost per unit of time, replacement cost and preventive maintenance cost in the first subsection. In the second subsection, we examine the sensitivity of the optimal policies to the reduction of the repair time when preventive maintenance is carried out.

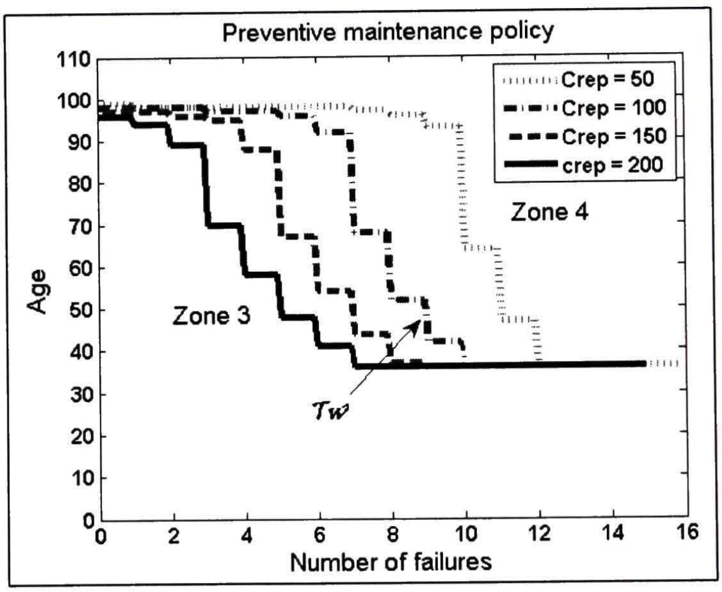
5.6.1 Sensitivity analysis on repair, replacement and preventive maintenance costs

In this subsection, we will perform sensitivity analysis on the repair cost per unit of time, the replacement cost and the cost of preventive maintenance.

When the repair cost per unit of time takes the four values 50, 100, 150 and 200, we obtain the results presented in figure 5.8.a and 5.8.b On the basis of figure 5.8.a we conclude that when repair costs increase, the machine is replaced earlier. According to figure 5.8.b, preventive maintenance actions are also triggered earlier.



a. S_n

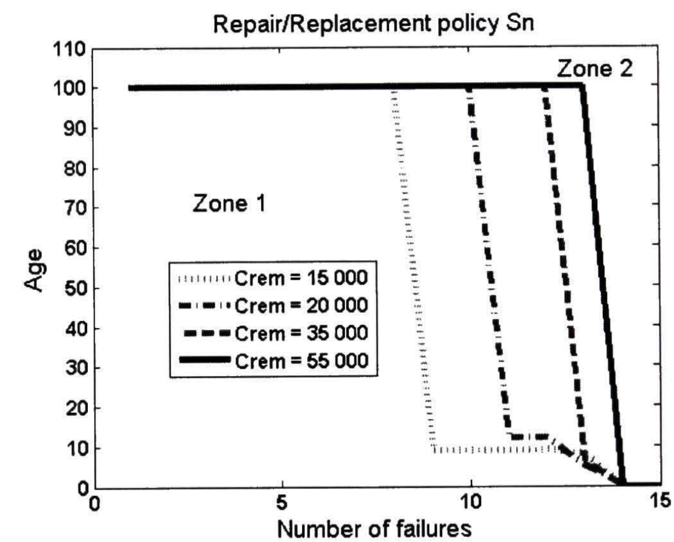


b. T_{ω_n}

Figure 5.8 Sensitivity of policies to the variation of repair cost per unit of time.

The trend is to reduce the repair time by recommending sending a machine to preventive maintenance earlier (Zones 2 and 4 increase).

As we can see from figure 5.9.a, Zone 1 increases as the machine replacement costs increase. That is, the higher the replacement cost, the higher the threshold age and the number of failures expected before systematic replacement



a. S_n

Figure 5.9 Sensitivity of policies on the variation of repair cost per unit of time.

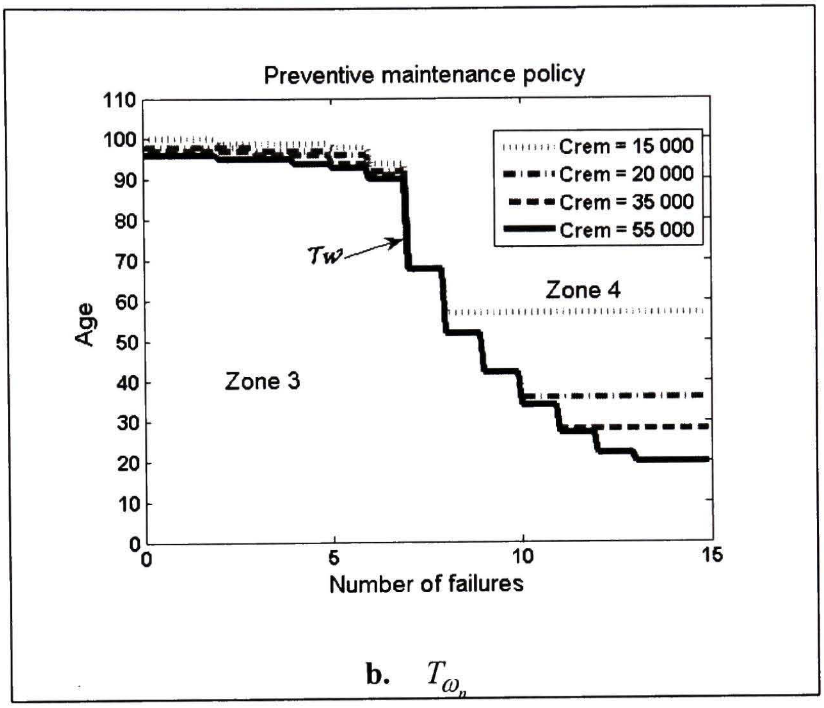
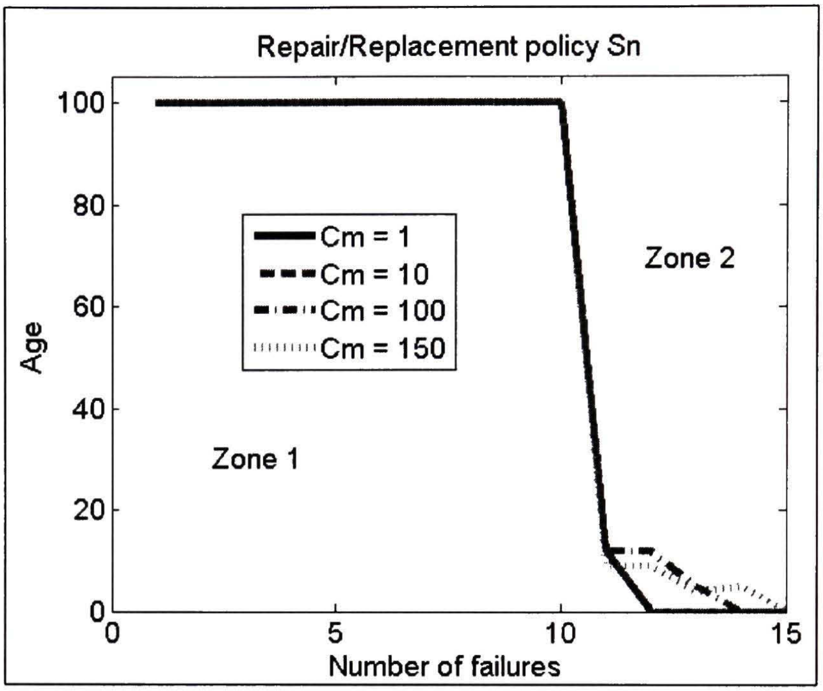


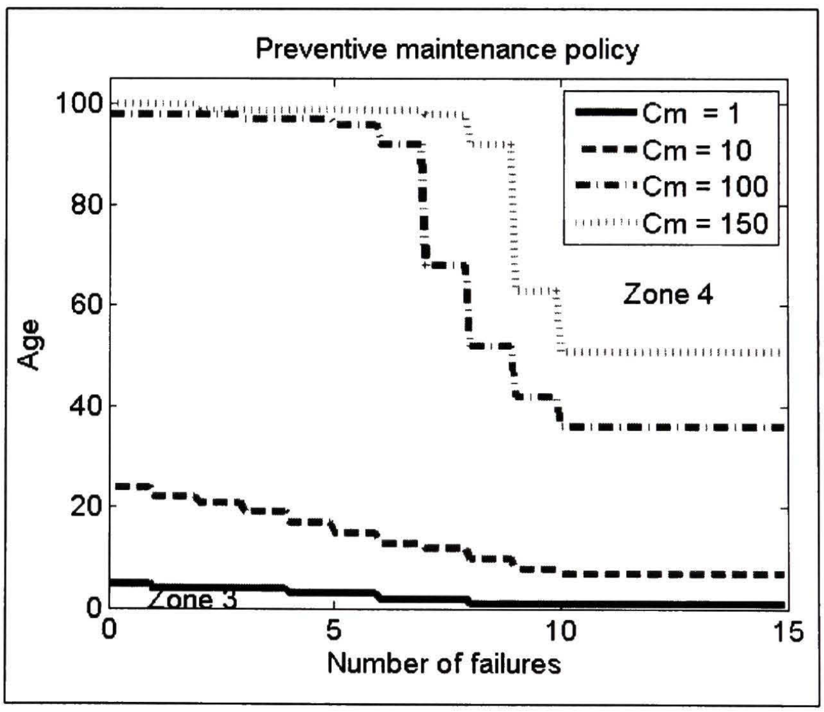
Figure 5.9 Sensitivity of policies on the variation of repair cost per unit of time (suite).

The corresponding preventive maintenance policies given by figure 5.9.b show that as replacement cost increases, preventive maintenance is triggered earlier. The size of Zone 4, where preventive maintenance is recommended, increases. However, it can be observed from both figures 5.9.a and figure 5.9.b that increasing the replacement cost over the range [15 000, 55 000] does not have a significant effect on the policies when the number of failures is less than 9, at least for the numerical examples considered in this study. Increasing replacement costs decrease Zones 2 and 3: replacement is postponed while preventive maintenance is triggered more often.

Since there is an incentive to perform preventive maintenance, we made a variation of preventive maintenance costs from low to moderate values as illustrated by the graphs in figures 5.10.a and 5.10.b.



a. S_n



b. T_{ω_n}

Figure 5.10 Variation of preventive maintenance cost per unit of time.

Decreasing preventive maintenance costs keeps Zone 1 virtually the same until the machine experiences its 10th failure. Conversely, it practically eliminates Zone 3. Figure 5.10.a shows that the repair and replacement policy is less sensitive to preventive maintenance cost variations. As indicated in figure 5.10.b decreasing preventive maintenance cost triggers more frequent preventive maintenance. These results are realistic because preventive maintenance costs are very small compared to replacement costs.

5.6.2 Sensitivity analysis on the variation of the reduction factor

In figure 5.11, the reduction factor (ε) takes four values: 0.1, 0.25, 0.5, 1. When $\varepsilon = 1$, the mean repair time remains unchanged after preventive maintenance. From figure 5.11, the machine experiences only 4 failures before systematic replacement at failure. In this case, $M >> S_1$. Recall that the mean time to failure is 29.5 units of time and $S_1 < 29.5$ units of time. Such a machine could be replaced after a first failure if its age is greater than 20 units of time.

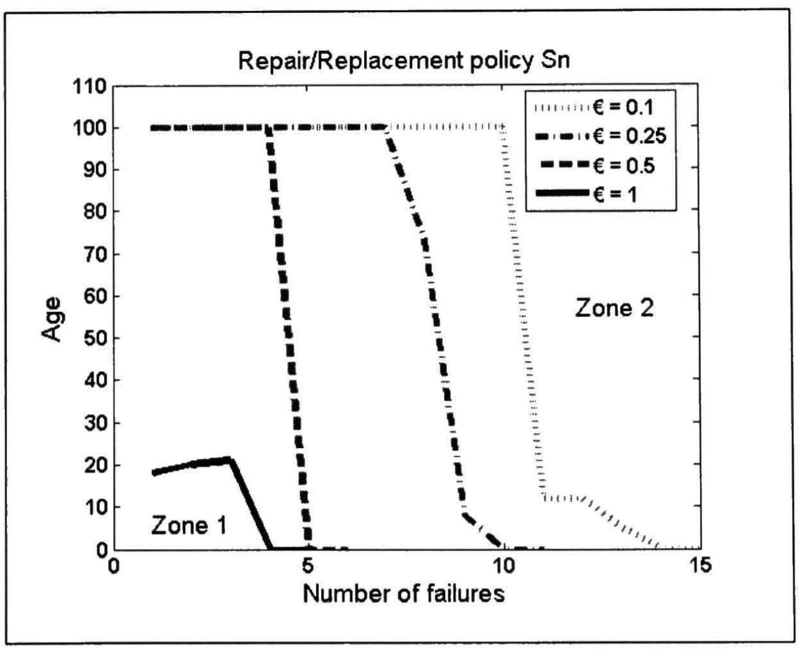
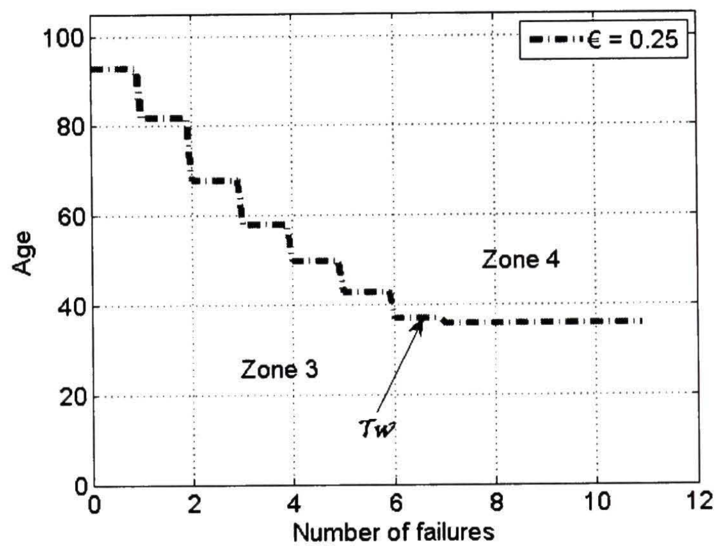


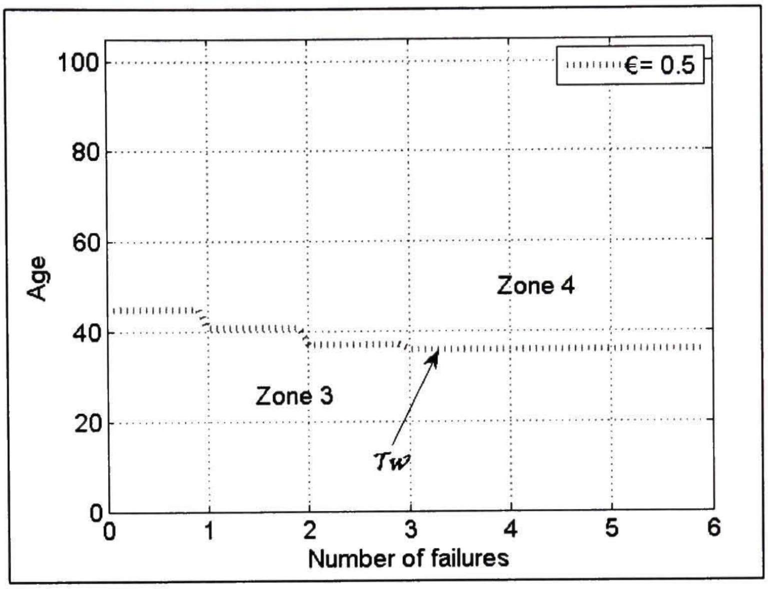
Figure 5.11 Repair/replacement policy for several values of the reduction factor.

When $\varepsilon = 0.1$, the mean repair time after a failure occurs is divided by 10 if preventive maintenance had been performed before failure. The machine experiences many more failures ($N_m = 14$) before systematic replacement after a failure. On the other hand, with such a system, the machine could never be replaced before it experiences 10 failures. The ε values between 0.1 and 1 confirm the trend that the smaller the ε, the more the machine experiences failures, and that performing preventive maintenance really increases the lifetime of the machine.

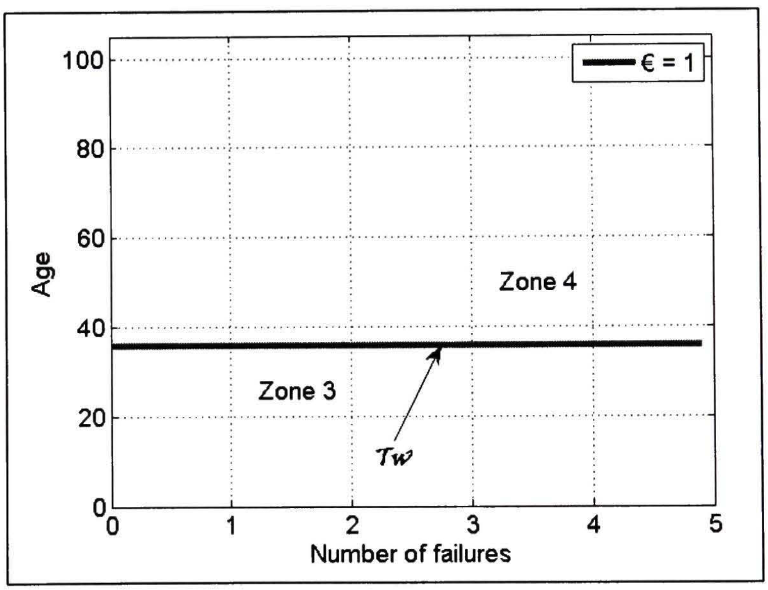


a. $\varepsilon = 0.25$

Figure 5.12 Preventive maintenance policy for several values of the reduction factor



b. $\varepsilon = 0.5$



c. $\varepsilon = 1$

Figure 5.12 Preventive maintenance policy for several values of the reduction factor (suite)

The preventive maintenance policies for $\varepsilon = 0.25$; 0.5; and $\varepsilon = 1$ are illustrated in figure 5.12.a-5.12.c. They show that although the machine is replaced early, preventive maintenance is somewhat recommended if the machine has reached a certain age.

The above sensitivity analysis confirms that the general repair/replacement and preventive maintenance policies (S_n, N_m, T_{ω_n}) are threshold-type, and they recommend replacing the machine at the $N_m{}^{th}$ failure or at failure after age S_n, whichever comes first, and to not perform preventive maintenance before age T_{ω_n}. If at the n^{th} failure, the age of the machine is greater than S_n or the failure number is above N_m, then undertake a repair.

The obtained results indicate that the optimal repair/replacement and preventive maintenance policy for the considered manufacturing system is characterized by the aforementioned three parameters (i.e. S_n, N_m, T_{ω_n}). Two of those parameters characterize the repair/replacement switching policy (i.e. $S_n; N_m$) and one parameter characterizes the preventive maintenance policy (i.e., T_{ω_n}). The overall control policy is given by equations (5.9-5.10) and figures 5.6-5.7, and is completely defined by the parameter values ($S_n; N_m$) for repair/replacement switching and T_{ω_n} for preventive maintenance policy.

Possible extensions of the results obtained here include production planning, corrective maintenance control and non-constant demand rate. A combination of control theory, simulation and experimental design could then be used to obtain a near-optimal control policy. The approach does not limit results to only those distributions used in the example presented in this work. Moreover, based on the parameterized control policy thus obtained, we could extend the proposed model to the case of manufacturing systems involving multiple products and multiple machines.

5.7 Conclusion

In this paper, we investigate the integration of preventive maintenance with the repair/replacement policy for a machine subject to failures and increasing repair times. Such a stochastic control problem is quite complex due to the machine's failure history. Modeling the system with a semi-Markov process allowed us to take into account the stochastic failure history, thereby adding a new dimension to the repair/replacement theory. Moreover, we introduce a reduction factor approach to decrease the repair times if preventive maintenance is performed before failures. We showed that implementing this type of preventive maintenance increases the lifetime of the machine. We illustrated the proposed approach using a numerical example and sensitivity analysis. The obtained results are particularly useful for industrial systems that experience losses due to increasing repair times.

REFERENCES

Ansell J. I., Archibald T. W., and Thomas L. C. (2004). "*The elixir of life: using a maintenance, repair and replacement model based on virtual and operating age in the water industry.*" IMA Journal of Management Mathematics, 15(2): pp. 151-60.

Aven T. and Castro I. T. (2008). "*A minimal repair replacement model with two types of failure and a safety constraint.*" European Journal of Operational Research, 188(2): pp. 506-515.

Boukas E.-K. and Haurie A. (1990). "*Manufacturing flow control and preventive maintenance: A stochastic control approach.*" IEEE Transactions on Automatic Control, 35(9): pp. 1024-1031.

Clarke F. (1983). "*Optimization and Non-Smooth Analysis.*" Wiley-Interscience: New York, NY.

Doyen L. and Gaudoin O. (2004). "*Classes of imperfect repair models based on reduction of failure intensity or virtual age.*" Reliability Engineering and System Safety, 84(1): pp. 45-56.

Hariga M. A., Azaiez M. N., and Daya M. B. (2006). "*A discounted integrated inspection-maintenance model for single deteriorating production facility.*" International Transactions in Operational Research, 13(4): pp. 353-64.

Jinyuan C. and Zehui L. (2008). "*An extended extreme shock maintenance model for a deteriorating system.*" Reliability Engineering and System Safety, 93(8): pp. 1123-1129.

Kahle W. (2007). "*Optimal maintenance policies in incomplete repair models.*" Reliability Engineering and System Safety, 92(5): pp. 563-565.

Kenne J.-P. and Boukas E. K. (1998). "*Optimal Control of the production and Corrective Maintenance of a Multiple-Machine Flowshop.*" Les Cahiers du Gerad, 1998.

Kijima M. (1989). "*Some results for repairable systems with general repair.*" Journal of Applied Probability, 26: pp. 89-102.

Kijima M., Morimura H., and Suzuki Y. (1988). "*Periodical replacement problem without assuming minimal repair.*" European Journal of Operational Research, 37(2): pp. 194-203.

Kushner H. and Dupuis P. G. (1992). "*Numerical methods for stochastic control problems in continuous time.*" in Springer-verlag. New York.

Leung K. N. F. (2006). "*A note on 'A bivariate optimal replacement policy for a repairable system.*" Engineering Optimization, 38(5): pp. 621-625.

Love C. E., Zhang Z. G., Zitron M. A., and Guo R.. (2000). "*Discrete semi-Markov decision model to determine the optimal repair/replacement policy under general repairs.*" European Journal of Operational Research, 125(2): pp. 398-409.

Love C. E., Zitron M. A., and Zhang Z. G. (1998). "*An SMDP approach to optimal repair/replacement decisions for systems experiencing imperfect repairs.*" Journal of Quality in Maintenance Engineering, 4(2): pp. 131-149.

Lugtigheid D., Banjevic D., and Jardine A. K. S. (2008). "*System repairs: When to perform and what to do?* " Reliability Engineering and System Safety, 93(4): pp. 604-615.

Makis V. and Jardine A. K. S. (1991). "*Optimal replacement of a system with imperfect repair.* Microelectronics and Reliability, 1991. 31(2-3): pp. 381-388.

Makis V. and Jardine A. K. S. (1993). "*A note on optimal replacement policy under general repair.*" European Journal of Operational Research, 69(1): pp. 75-82.

Malik M. A. K. (1979). "*Reliable preventive maintenance scheduling.*" AIIE Transactions, 11(3): p. 221-8.

Nakagawa T. and Kijima M. (1989). "*Replacement policies for a cumulative damage model with minimal repair at failure.*" IEEE Transactions on Reliability. 38(5): pp. 581-584.

Rishel R. (1975). "*Control of systems with jump markov disturbances.*" Jt Autom Control Conf, 15th, Proc, Jun 18-21, pp. 246-249.

Sethi S. P. and Zhang Q. (1994). "*Hierarchical Decision Making in Stochactic Manufacturing Systems.*" ed. W.U. Christpher I. Byrnes. Birkhâser.

Wang H. (2002). "*A survey of maintenance policies of deteriorating systems.*" European Journal of Operational Research, 139(3): pp. 469-489.

Xiaojun Z., Lifeng X., and Jay L. (2007). "*Reliability-centered predictive maintenance scheduling for a continuously monitored system subject to degradation.*" Reliability Engineering & System Safety, 92(4): pp. 530-534.

Yu-Hung C. (2008) "*Optimal age-replacement policy under an imperfect renewing free-replacement warranty.* " IEEE Transactions on Reliability, 57(1): pp. 125-133.

CHAPITRE 6

ARTCLE 5: SIMULTANEOUS CONTROL OF PRODUCTION, REPAIR/REPLACEMENT AND PREVENTIVE MAINTENANCE OF DETERIORATING MANUFACTURING SYSTEMS

F. I. Dehayem Nodem [1], J. P. Kenne[1] and A. Gharbi[2]

[1]Mechanical Engineering Department, Production Technologies Integration Laboratory, University of Quebec,

[2]Automated Production Engineering Department, Production System Design and Control Laboratory, University of Quebec

École de technologie supérieure, 1100 Notre Dame Street West, Montreal, Quebec Canada H3C 1K3

This chapter is an article submitted to Applied Stochastic Models in Business and Industry

Manuscript number: ASMB-09-68.

Résumé

Dans cet article, nous considérons le problème de détermination de la politique optimale de production, de réparation versus remplacement et d'entretien préventif d'un système de manufacturier en environnement incertain. La nature incertaine de l'environnement est due aux pannes et réparations aléatoires. Lorsqu'une panne survient, la machine est soit réparée ou remplacée. Le remplacement renouvelle la machine tandis que les réparations de machines sont imparfaites. L'imperfection d'une réparation se répercute sur le temps de réparations à la prochaine panne et augmente sa valeur. Le système est constitué d'une machine fabricant un seul type de produit. Lorsqu'elle est en opération, la machine peut être envoyée en maintenance préventive, dépendamment de son âge. L'âge de la machine reflète les activités effectives de production. Les variables de décision du système sont le taux de production des pièces, l'âge de commutation des réparations au remplacement de la machine si une panne survient, le nombre de pannes enregistrées par la machine avant remplacement systématique et le taux de maintenance préventive. L'objectif est de trouver les variables de décision qui préventive et de réparation/remplacement. Compte tenu de la prise en compte de l'historique des pannes, nous utilisons une formulation basée sur un processus de décision semi-markovienne et une approche basée sur l'utilisation de la solution viscosité pour obtenir les conditions d'optimalité. Un exemple numérique est fourni et par le biais d'analyses de sensibilité, nous montrons que les résultats obtenus sont utiles et la structure des politiques de commande obtenue est conservée.

Abstract

In this paper, we consider the problem of determining the optimal production, repair/replacement and preventive maintenance policies of a stochastic manufacturing system, with random breakdowns and repairs being at the root of the stochastic nature of the machine involved. When a failure occurs, the machine is either repaired or replaced; replacement renews the machine while repairs are imperfect. Consequently, the next repair times increase as the number of repairs increases. The machine produces one type of product, and during production, it can be sent for preventive maintenance, depending on its age, which for its part, reflects the effective production activities for which the machine can be used. The control variables of the system are the rate at which parts are produced, the repair/replacement switching age at failure, the allowable maximum number of failures before systematic replacement, and the preventive maintenance rate. Our objective is to choose admissible control variables that minimize the long-run average cost, including the surplus/backlog, repair/replacement and preventive maintenance costs. We use a formulation based on a semi-Markov decision process and viscosity solution approach to obtain the optimality conditions. We provide a numerical example, and through a couple of sensitivity analyses, show that the structure of the results obtained is maintained.

Keywords: Manufacturing systems, numerical methods, optimal control, damaging failures, production, replacement, repair, preventive maintenance.

6.1 Introduction

In manufacturing systems, machines are mostly repaired following a failure, and production must often be stopped in order to perform such repairs. Normally, the repair costs (in terms of time and financially) may be very high after the machine experiences a certain number of failures, in which case it becomes more reasonable to replace the machine with a new one. This often arises as successive operating times following repairs decrease more and more; similarly, it may also occur when consecutive repair times increase because repairs do not always return the machine to an 'as good as new' condition. Production activities are affected by such variations in operating times, and require appropriate planning. As is commonly the case, a preventive action or a replacement is adopted in order to increase the operating time of the machine and ensure continuous production, which is why in manufacturing environment, a machine's production activities, repairs, replacement and preventive maintenance are mutually interdependent. Indeed, many research projects have been performed to take this reality into account. However, until now, no work has been conducted which simultaneously provides the production planning, the repair/replacement and the preventive maintenance policies in place as repair times increase with the number of failures. Implementing such elements renders the planning task very challenging. For more details regarding what has been done to date, we refer the reader to the work of Gabriella *et al.* (2008). They present a general overview of models and discuss some sectors of interaction between maintenance and production. This work combines the production, repair/replacement and preventive maintenance control activities of a stochastic manufacturing system. The machine of the manufacturing system considered is subject to random breakdowns, and deteriorates with age and as the number of failures increases.

Many articles have been written on the interactions between production, corrective maintenance, preventive maintenance, replacement and machine deterioration. Chiu (2008) considered an optimization problem involving manufacturing systems with stochastic machine breakdowns and rework processes. In (Chiu, 2008), the machine deteriorates while producing, and produces defective items. Lai (2007) considered a periodical replacement model based on a cumulative repair-cost limit, whose concept uses information regarding all repair costs to decide whether the system is repaired or replaced. He classifies system failures under two categories: a minor failure assumed to be correctable by minimal repair, and a serious failure, where the system is completely damaged. Some authors consider technological changes (Nair and Hopp, 1992; Rajagopalan, 1998; Mercier, 2008), and study an optimal replacement policy for obsolete components with a general failure rate. Mercier (2008) tries to determine the optimal replacement strategy from among the purely preventive replacement strategy, the purely corrective strategy, and a mixture of both strategies.

The corrective maintenance normally performed in manufacturing is usually unplanned, and more often than not, brings production to a standstill (Gabriella *et al.*, 2008). During repair, production activities are interrupted, leading to a loss of production. To improve machine availability, many authors proposed the incorporation of corrective maintenance or its control when planning production (Kenne *et al.*, 2003; Jeong *et al.*, 2007). Corrective maintenance control can be incorporated using additional manpower or any tool that allows the maintenance team to reduce logistics and repair times (Viles *et al.*, 2007). Corrective maintenance results from reactive attitudes consisting of detecting and repairing breakdowns, and is therefore insufficient, as it could damage the financial health of a company (Viles *et al.*, 2007), which is why preventive maintenance, which could be carried out at different points in time, is particularly interesting.

Preventive maintenance is usually planned and aimed at increasing the reliability of a deteriorating system (Boukas and Haurie, 1990; Dellagi *et al.*, 2007) or decreasing the operating costs of repairable systems (Yuan, 2001). In their work, Boukas and Haurie (1990) include an age-dependent machine failure rate and allow the control to influence the jump rate of the machine from operation to preventive maintenance. Yao *et al.* (2005) study the joint preventive maintenance and production policies for an unreliable production-inventory system in which maintenance/repair times are non-negligible and random. In their work, they assume that failures are time-dependent, although a machine that deteriorates only when it is working might be more appropriate for production systems. Gharbi et al. (2007) assumed that failure frequencies can be reduced through preventive maintenance, and developed joint production and preventive maintenance policies depending on produced parts inventory levels.

Corrective and preventive maintenance times can lead to significant production losses if, for example, the production line is automated and the machines rigidly linked (Savsar, 2008). In many industries, including automobiles and semiconductors, buffer stocks are normally used to guarantee a continuous supply by the subsequent production machines during service interruptions due to repair or preventive maintenance (Chelbi and Ait-Kadi, 2004; El-Ferik, 2008). Many other authors have determined buffer stocks as a means of coping with machine failures and preventive maintenance times (Gharbi *et al.*, 2007; Chelbi and Ait-Kadi, 2004; Kenne and Nkeungoue, 2007). In their models, they assume that maintenance always restores the machine to a perfect condition or does not change the failure rate of the system, such that the machine performs similarly from one failure to another. However, this assumption is not realistic. A more realistic assumption is that the machine does not always return to a perfect condition following a corrective maintenance. Such a repair is called an imperfect repair (Liao, 2007).

The imperfect repair model has been well documented in the literature, as have been the corresponding repair and replacement policies for manufacturing systems (Kijima *et al.*, 1988; Love *et al.*, 2000; Pérès and Noyes, 2003). Unfortunately, most results obtained concern only cases where production and demand satisfaction are not taken into account. There are several reasons for this. One is that authors consider that repair, replacement and preventive maintenance take a small amount of time (Beichelt,1992; Makis and Jardine, 1993), and that they do not therefore significantly affect production activities. El-Ferik (2008) considered a joint determination of the economic lot size for an unreliable facility implementing an age-based maintenance policy with an increasing failure rate. In his model, the system is replaced after N production cycles and production rates are constant. Although the replacement time is average, repair times are presumed to be negligible. Pérès and Noyes (2003) classified values concerning 50 machines according to the increasing durations of repair times and specified informations about the nature of the failure (electric, mechanic, computer or other).We refer the reader to (Pérès and Noyes, 2003) for more details on the repair times for various classes and failure types. The other reason for not taking production and demand satisfaction into account is that such a control problem is very complex. Even if the machine is initially new or is new after each maintenance activity, it will clearly have different dynamics after each breakdown and repair. One way to deal with this complexity is to develop a hierarchical decision making model for the system. Some authors use a multi-agent and fuzzy logic-based method (Zimmermann, 1990; Vasant *et al.*, 2004), while others develop a semi-Markov decision process approach. As stated by Pérès and Torres (2002), under particular conditions involving exponential times, systems can be modeled as continuous-time Markov chains. However, that is not the case when the probability distributions of operating and repair times follow general distributions; a semi-Markov process can then be used for modeling the system. Sloan (2008) developed a semi-Markov decision process model of a single-stage production system with multiple products and multiple maintenance actions, which simultaneously determines maintenance and production schedules, while taking into account the fact that equipment conditions affect the yield of each product differently. As can be seen from previous works, available studies did not simultaneously take into account the effect of a preventive maintenance control, replacement, and the effect of increasing repair times with the number of failures.

Our aim in this work is to develop optimal strategies for manufacturing systems, which take into account two types of machine deteriorations. The first is the deterioration of a machine through age (i.e., failure distribution depends on the machine's age), while the second is the deterioration of a machine in accordance with the number of repairs (i.e., repair times increase with the number of failures). Thus, repair activities depend on the machine's repair history and Markovian decision processes are no longer appropriate for the model of the control problem. The proposed model will simultaneously provide the:

- Repair/replacement strategy based on the machine's age and number of failures above which the machine must be replaced if a failure occurs

- Preventive maintenance policy based on the machine's age at which preventive maintenance must be performed
- Production planning (rate) based on machine's age and the stock level of finished goods

The resulting simultaneous control approach consists in developing a Semi-Markov Decision Process (SMDP) in order to determine an optimal production, repair/replacement policy and preventive maintenance strategies for the system. Such policies are determined in order to minimize inventory, backlog, repair, replacement and preventive maintenance costs over an infinite planning horizon.

The paper is organized as follows: Section 6.2 presents the problem formulation, followed in Section 6.3 by optimality conditions. Numerical methods are used in Section 6.4 to solve the optimality conditions described by the Hamilton-Jacobi-Bellman (HJB) equations obtained in Section 6.3. Numerical examples are provided in Section 6.5. In Section 6.6, results and sensitivity analysis are presented to illustrate the usefulness of the proposed approach. We finally conclude in Section 6.7.

6.2 Problem formulation

We consider a failure-prone manufacturing machine that has experienced its n^{th} failure and has been repaired. A value for the number of failures $n = 0$ means that the machine is a new one and has experienced zero failure. The machine goes through a finite number of successive failures before replacement by a new one. Therefore, we can assume that there exist a maximum number of failures N that the machine can experience before replacement. Thus, $n = 0, 1, \ldots, N$.

The manufacturing system faces a constant demand rate d for its product over time. It produces parts at rate $u(t)$ such that the state equation of the stock level $x(t)$ is given by:

$$\frac{dx(t)}{dt} = u(t) - d, \quad x(0) = x_0 \tag{6.1}$$

Where x_0 is a given initial stock level.

We assume $0 \le u(t) \le U_m$ where U_m is the maximum production capacity of the machine.

As the machine produces part, its age increases. The cumulative age of the machine is described by the following differential equation:

$$\frac{da(t)}{dt} = f(u(t)), \quad a(0) = a, \quad a(T) = 0 \tag{6.2}$$

Where a is the given initial machine age and T the last restart time of the machine.

The machine modes can be classified as *operational*, denoted by 1, *under repair* denoted by 2, *under replacement* denote by 3 and *under preventive maintenance* denote by 4. The failure rate $q_{12}(a(t))$ and the replacement rate $q_{13}(a(t))$ of the machine are function of machine age $a(t)$. The machine is replaced at constant rate q_{31}.

Let $t_{21}(n)$ be the repair time after the n^{th} failure $n = 1,2,3...$ and $T_{21}(n)$ the expectation of $t_{21}(n)$. The sequence $\{t_{21}(n), n = 1,2,3,...\}$ forms a non decreasing arithmetic-geometric process with parameters $d_b \le 0$ and $0 < r_b \le 1$, and $E(t_{21}(n=1)) = T_{21}(n=1) \ge 0$. $T_{21}(n=1) = 0$ means that the repair time is negligible.

$$T_{21}(n) = \frac{T_{21}(n=1)}{(r_b)^{n-1}} - (n-1)d_b, \quad n = 1,2,3... \tag{6.3}$$

For more details on non decreasing repair times, see (Leung, 2006). The inverse of $T_{21}(n)$ is the repair rate $q_{21}(n)$.

We assume that the following constraints hold for preventive maintenance jump rate:

$$0 \le \omega(.) \le \omega_{\max} \tag{6.4}$$

where $\omega_{\max}$ is the maximum preventive maintenance rate.

The inverse of $\omega(.)$ represents the expected delay between the decision to trigger preventive maintenance actions and the effective switch from operational to preventive maintenance mode (Boukas and Haurie, 1990). The transition rate $\omega(.) = q_{14}$ from operational to preventive maintenance mode is assumed to be a control variable. The machine mode changes from preventive maintenance to operational mode at constant rate q_{41}. The preventive maintenance cost is fixed at C_m.

All other transition rates are equal to zero.

We assume that the machine will be able to meet the demand rate d over an infinite horizon and reach a steady state, at a given machine age a, if:

$$\overline{\alpha}(a).U_m > d \tag{6.5}$$

Where $\overline{\alpha}(.)$ is the availability of the machine given by: $\overline{\alpha}(.) = \dfrac{1}{1 + \dfrac{q_{21}}{q_{12}} + \dfrac{q_{31}}{q_{13}} + \dfrac{q_{41}}{q_{14}}}$

Thus, $\overline{\alpha}(.)$ depends on the decision variables as the transition rates are controlled through preventive maintenance. To increase the system availability, we consider that the transition rate $\omega(.) = q_{14}$ from operational to preventive maintenance mode is a control variable. By controlling $\omega(t)$, we act on the mean time before preventive maintenance. The system capacity is then described by a finite state semi-Markov chain that depends on the preventive maintenance policy.

Let us denote the mode of the machine by the variable $\xi(t)$ with value in $\Omega = \{1,2,3,4\}$ and the system state by the fourth variable $(\xi(t), a(t), x(t), n)$. We assume that the repair costs are functions of repair times. Each unit of repair time have a cost of C_r. Since repair times increase with the number of failures, this implies that repair costs depend on number of failures. Moreover, repair costs are bounded non-decreasing functions.

The machine failures are decision epochs for the two possible actions: repair or replace the machine. For consistency with Makis and Jardine (1993) and Love *et al.* (2000), we consider a constant replacement cost C_0.

Let $S_n(x) \geq 0$ be the age at which if a failure occurs, an action is chosen between repair and replacement. $S_n(.)$ is called the replacement switching age, and is assumed to be a control variable. For a given stock level, $S_n(.)$ has been shown in the literature to be of threshold type. Moreover, since each manufacturing system that deteriorates with age is to be replaced in the long term, we can naturally assume that there exists an upper bound on replacement age, M_{up}, which is very large as compared to $S_{n=1}(.)$, beyond which the machine is automatically replaced.

In manufacturing systems, machines are rarely replaced after the first breakdown. We therefore consider that the likelihood of a failure before the replacement age at first failure is almost equal to 1. Given that the machine will not reach the replacement age at first failure, it is unnecessary to determine the exact value of $S_{n=1}(.)$ if it exceeds a reasonable value of mean time to first failure. We will then choose a bound M sufficiently large as compared to the mean time to first failure instead of $S_{n=1}(.)$, thus ensuring that the probability of the machine to reach the bound before the failure occurs is effectively 0.

Let $a_n(t)$ be the age of the machine when the n^{th} failure occurs. For a given stock level and any state variable $(\xi(t), a(t), ., n)$, we can identify the action to be undertaken by the location of the system at failure. If at the n^{th} failure:

1. The number of failures $n < N$ and the age of the machine $a_n(t) < S_n(.)$, then the machine is to be repaired. That is $\xi(t) = 2$ and we will have the transition of the two variables from $(a_n(t), n)$ to $(a_{n+1}(t), n+1)$ at $(n+1)^{th}$ failure;
2. Otherwise, one of the following three conditions is respected:
   a. The number of failures $n \geq N$,
   b. The age of the machine $a_n(t) \geq S_n(.)$,
   c. The age of the machine $a_n(t) \geq M$,

   The machine is to be replaced. The machine is in mode 3, and the transition of the two variables $(a(t), n)$ will be from $(a_n(t), n)$ to $(a_1(t), 1)$ at the next failure, which is a first failure for the new machine.

The corrective maintenance of the machine is not perfect in the sense that the repair time increases with the number of failures. After corrective maintenance, the age of the machine is reset to its next lower level such that the machine has the same failure intensity as a new machine. But at the next failure, it will take much more time to repair the machine. Thus, repair activities depend on the machine's repair history. Since at each state $(\xi(t), a(t), ., n)$, a decision to either repair or replace must be taken, and the repair time increases with the number of failures, the resultant structure is a Semi-Markov Decision Process.

Let $\Gamma(.) = \{(u(t), S_n(x), \omega(t)) /\ 0 \leq u(.) \leq U_m;\ 0 \leq S_n(x) \leq M;\ \omega_{\min} \leq \omega \leq \omega_{\max};\ t \geq 0; x \in \mathbb{R}\}$

Given the initial stock level $x(0) = x$, the initial age $a(0) = a$ and the number of the failures already occurred, n, the objective of the study is to choose admissible controls $(u(.), s_n(.), \omega(.)) \in \Gamma(.)$ so as to minimize the total discounted cost given by:

$$J(\alpha,a,x,n,u(.),s_n(.),\omega(.)) = E\left[\begin{array}{l}\int_0^\infty e^{-\rho t}\left[h(x)+C_r(.)*Ind\{\xi(t)=2\}*T_{21}(n)+C_0*Ind\{\xi(t)=3\}+C_m*Ind\{\xi(t)=4\}\right]dt\,/ \\ \xi(0)=\alpha,\ a(0)=a,\ x(0)=x \text{ and the number failures already occured in the system is } n\end{array}\right] \tag{6.6}$$

where ρ is the discount rate used to get the costs incurred at future dates lower than those incurred today. $h(x)=c_s x^+ + c_p x^-$ is the cost of inventory/backlog. One of the goals of planning the production is to meet demand with minimal costs associated with inventory. It is better to have x be as close as possible to 0. The costs due to x are generally summarized in a convex function which has its minimum at $x=0$ and which grows as $|x| \to \infty$ (Gershwin, 1994). Thus, by definition, $h(x)=c_s x^+ + c_p x^-$ is a convex function. The constants c_s and c_p are used to penalize inventory and backlog, respectively. $x^+ = \max(0,\ x)$ and $x^- = \max(0, \text{-}x)$. The function $Ind\{(\cdot)\}$ equals 1 when the condition $(\cdot)$ is satisfied, and is 0 otherwise.

The value function of the optimal control problem is given by:

$$V(\alpha,a,x,n) = \min_{(u(.),s_n(.),\omega(.))\in\Gamma(\alpha,n)} J(\alpha,a,x,n,u(.),s_n(.),\omega(.)) \tag{6.7}$$

The value function $V(\alpha,a,x,n)$ satisfies specific properties called optimality conditions, which are presented in the next section.

6.3 Optimality conditions

In this section, we will show that under appropriate assumption and lemma, the value function $V(\alpha,a,x,n)$ satisfies a set of coupled partial derivative equations derived from the application of the dynamic programming approach.

We make the following assumption on the storage/shortage, repair/replacement and preventive maintenance instantaneous cost function:

$$G(\alpha,a,x,n)=G(.)=h(x)+C_r(.)*Ind\{\xi(t)=2\}*T_{21}(n)+C_0*Ind\{\xi(t)=3\}+C_m*Ind\{\xi(t)=4\}. \tag{6.8}$$

Assumption (6.1)

The cost function $h(x)$ is a non- negative convex function in x.

That is, for all $x_1,x_2\in\mathbb{R}$ and $\lambda_v\in[0,1]$, $G(.,\lambda_v x_1+(1-\lambda_v)x_2,.)\le\lambda_v G(.,x_1,.)+(1-\lambda_v)G(.,x_2,.)$. Moreover, $h(x)$ is strictly jointly convex in x if the inequality holding as an equality for some $\lambda_v\in[0,1]$ implies $x_1=x_2$.

Assumption (6.2)

Let C^{α} be the cost function defined by:

$$C^{\alpha}=C_r(.)Ind\{\alpha=2\}T_{21}(n)+C_0 Ind\{\alpha=3\}+C_m Ind\{\alpha=4\}$$

C^{α} is non-negative and twice differentiable in both intervals $[0,S_n]$ and $[S_n,\infty]$. Moreover, $C_r(.)Ind\{\xi(t)=2\}T_{21}(n)+C_0 Ind\{\xi(t)=3\}+C_m Ind\{\xi(t)=4\}$ is either strictly convex or linear in both interval $[0,S_n]$ and $[S_n,\infty]$.

Given assumption (6.1) and assumption (6.2), it follows that the instantaneous cost function $G(.)$ is locally Lipschitz, and therefore continuous in a and in x. Moreover, $G(.)$ is differentiable in a and in x.

Lemma (6.1)

i) For each (α,n), $V(\alpha,.,.,n)$ is a convex function on $\mathbb{R}^{+}\times\mathbb{R}$, and $V(\alpha,.,.,n)$ is strictly convex if $G(.)$ is so.

ii) $V(\alpha,a,x,n)$ is locally Lipschitz.

Proof:

To show i), it suffices to show that $J(\alpha,.,.,n,.,.,.)$ is jointly convex in (α,n) for each $(a,x)\in\mathbb{R}^{+}\times\mathbb{R}$.

For any initial values (a_1,x_1) and (a_2,x_2), for any admissible controls (u^1,S_n^1,ω^1) and (u^2,S_n^2,ω^2), let $(a^1(t),x^1(t))$ and $(a^2(t),x^2(t))$, $t\geq 0$ denotes the trajectories corresponding to $(a_1,x_1,u^1(.),S_n^1,\omega^1(.))$ and $(a_2,x_2,u^2(.),S_n^2,\omega^2(.))$. Then, for any $\lambda_v\in[0,1]$

$$
\begin{aligned}
&\lambda_v J(\alpha,a_1,x_1,n,u^1(.),S_n^1,\omega^1(.))+(1-\lambda_v)J(\alpha,a_2,x_2,n,u^2(.),S_n^2,\omega^2(.))\\
&=E\int_0^\infty e^{-\rho t}\left[\lambda_v G(\alpha,a^1(t),x^1(t),n,u^1(.),S_n^1,\omega^1(.))+(1-\lambda_v)G(\alpha,a^2(t),x^2(t),n,u^2(.),S_n^2,\omega^2(.))\right]dt\\
&\geq E\int_0^\infty e^{-\rho t}G(\alpha,a(t),x(t),n,u(.),S_n,\omega(.))dt
\end{aligned}
$$

where $u^1(.):=\lambda_v u^1(t)+(1-\lambda_v)u^2(t))$;

$S_n:=\lambda_v S_n^1+(1-\lambda_v)S_n^2$; $\omega^1(.):=\lambda_v\omega^1(t)+(1-\lambda_v)\omega^2(t))$

$a(t)$ and $x(t)$ denote the trajectories with initial values $\lambda_v a_1+(1-\lambda_v)a_2$, $\lambda_v x_1+(1-\lambda_v)x_2$ and control $(u(.),S_n,\omega(.))$. Thus,

$$\lambda_v J(\alpha,a_1,x_1,n,u^1(.),S_n^1,\omega^1(.))+(1-\lambda_v)J(\alpha,a_2,x_2,n,u^2(.),S_n^2,\omega^2(.))$$
$$\geq J(\alpha,\lambda_v a_1+(1-\lambda_v)a_2,\lambda_v x_1+(1-\lambda_v)x_2,n,\lambda_v u^1(.)+(1-\lambda_v)u^2(.),\lambda_v S_n^1+(1-\lambda_v)S_n^2,\lambda_v\omega^1(.)+(1-\lambda_v)\omega^2(.))$$

This means that $J(\alpha,.,.,n,.,.,.)$ is jointly convex. Therefore, $V(\alpha,a,x,n)$ is convex.

To show ii), let $(u(.),S_n,\omega(.))$ denote an admissible control, $z^1=(a^1(t),x^1(t))$ and $z^2=(a^2(t),x^2(t))$, the state trajectories under $(u(.),S_n,\omega(.))$ with initial values $z_1=(a_1,x_1)$ and $Z_2=(a_2,x_2)$, respectively. Then

$\left|z^1(t)-z^2(t)\right|\leq\left|z_1-z_2\right|$, $\left|z^1(t)\right|\leq C_g\left(1+\left|z_1\right|\right)$ and $\left|z^2(t)\right|\leq C_g\left(1+\left|z_2\right|\right)$ for some constant C_g.

In view of the local Lipschitz assumption, we can show that constants C_{g_1} and k_z exist independent of $u(.),S_n,\omega(.)$, z_1 and z_2 such that

$$\left|J(\alpha,z_1,n,u(.),S_n,\omega(.))-J(\alpha,z_2,n,u(.),S_n,\omega(.))\right|\leq C_{g_1}\left(1+|z_1|^{k_z}+|z_2|^{k_z}\right)|z_1-z_2|.$$

It follows that

$$\left|V(\alpha,z_1,n,u(.))-V(\alpha,z_2,n,u(.))\right|\leq\sup_{(u(.),S_n,\omega(.))\in\Gamma}\left|J(\alpha,z_1,n,u(.),S_n,\omega(.))-J(\alpha,z_2,n,u(.),S_n,\omega(.))\right|$$
$$\leq C_{g_1}\left(1+|z_1|^{k_z}+|z_2|^{k_z}\right)|z_1-z_2|$$

□

Given assumption (6.1), assumption (6.2) and Lemma (6.1), it follows that the value function $V(\alpha,a,x,n)$ is convex in (a,x).

The dynamic programming equations, also known as the Hamilton-Jacobi-Bellman (HJB) equations, associated with the optimal control problem are written formally as follows:

$$\rho V(\alpha,a,x,n)=\min_{(u(.),s_n(.),\alpha(.))\in\Gamma(\alpha,n)}\Big\{ G(.)+\frac{\partial}{\partial x}V(\alpha,a,x,n)\big(u(t)-d\big)+\frac{\partial}{\partial a}V(\alpha,a,x,n)Ind\{\xi(t)=1\}\frac{d\alpha(t)}{dt}$$
$$+\sum_{\beta\neq\alpha} q_{\alpha\beta}(a,n)\Big[V\big(\beta,0,x,\psi(n)\big)-V(\alpha,a,x,n)\Big]\Big\} \tag{6. 9}$$

where $\beta\in\Omega$;

$$\psi(n)=\begin{cases} 0 & \text{if } \left\{\xi\left(\tau^{+}\right)=1 \text{ and } \xi\left(\tau^{-}\right)=3\right\} \\ n+1 & \text{if } \left\{\xi\left(\tau^{+}\right)=2,3 \text{ and } \xi\left(\tau^{-}\right)=1\right\} \\ n & \text{otherwise} \end{cases}$$

$\frac{\partial}{\partial a}V(\alpha,a,x,n)$ and $\frac{\partial}{\partial x}V(\alpha,a,x,n)$ are partial derivatives of the value function $V(\alpha,a,x,n)$ with respect to a and to x in that order.

Lemma (6.2)

The value function $V(\alpha,a,x,n)$ defined in equation (7) is the unique viscosity solution to the HJB equations (9).

Proof:

The basic idea of proving Lemma (6.2) is similar to that of Lemma H.2. of (Sethi and Zhang, 1994) when replacing $V(x,\alpha)$ by $V(\alpha,z,n)$ and $\eta(x)$ by:

$$\vartheta(z)=\exp\left(\frac{\rho}{\sup_u |u-d|}\left(1+|z|^2\right)^{\frac{1}{2}}\right) \text{ with } z=(a,x).$$

□

In the following section, we implement a numerical approach and obtain an approximation of the control policy to this production, repair/replacement and preventive maintenance problem.

6.4 Numerical approach

In this section, we develop a numerical method to solve the HJB equations (6.9) by the adaptation of Kushner's technique (Kushner and Dupuis, 1992). Let h_x and h_a be the finite difference interval of variables x and a respectively. The main idea of the approach consists in approximating $V(\alpha,a,x,n)$ by a function $V^h(\alpha,a,x,n)$ and the first-order partial derivatives of the value function $\frac{\partial}{\partial x}V(\alpha,a,x,n)$ and $\frac{\partial}{\partial a}V(\alpha,a,x,n)$ using the following expressions:

$$\frac{\partial}{\partial x}V(\alpha,a,x,n)\times(u-d)=\begin{cases}\frac{1}{h_x}\left[V^h(\alpha,a,x+h_x,n)-V^h(\alpha,a,x,n)\right](u-d) & \text{if } (u-d)\geq 0\\ \frac{1}{h_x}\left[V^h(\alpha,a,x,n)-V^h(\alpha,a,x-h_x,n)\right](u-d) & \text{otherwise}\end{cases}$$

$$\frac{\partial}{\partial a}V(\alpha,a,x,n)\times f(u)=\frac{1}{h_a}\left[V^h(\alpha,a+h_a,x,n)-V^h(\alpha,a,x,n)\right]\times f(u)$$

where $h=(h_x,h_a)$

Let us define

$$Q_h^{\alpha}=\left|q_{\alpha\alpha}\right|+\frac{|u-d|}{h_x};\qquad P_x^{+}(\alpha)=\begin{cases}\frac{u-d}{h_x Q_h^{\alpha}} & \text{if } u-d>0\\ 0 & \text{otherwise}\end{cases};\qquad P_x^{-}(\alpha)=\begin{cases}\frac{d-u}{h_x Q_h^{\alpha}} & \text{if } u-d\leq 0\\ 0 & \text{otherwise}\end{cases};$$

$$P_a(\alpha)=\frac{f(u)}{h_a Q_h^{\alpha}} \text{ and } P^{\beta}(\alpha)=\frac{q_{\alpha\beta}}{Q_h^{\alpha}}$$

The following equations are the discrete dynamic programming equations obtained:

$$V^h(\alpha,a,x,n) = \underset{(u,\omega,s)\in\Gamma}{Min}\left\{\frac{G(.)}{Q_h^\alpha(1+\frac{\rho}{Q_h^\alpha})} + \frac{1}{(1+\frac{\rho}{Q_h^\alpha})}\left(\begin{array}{l} P_x^+(\alpha)V^h(\alpha,a,x+h_x,n)+P_x^-(\alpha)V^h(\alpha,a,x-h_x,n) \\ +P_a(\alpha)V^h(\alpha,a+h_a,x,n)+\sum\limits_{\beta\neq\alpha} P^\beta(\alpha)V^h(\alpha,a,x,n) \end{array}\right)\right\}$$

(6.10)

The system of equations (6.10) can be interpreted as the infinite horizon dynamic programming equation of a discrete-time, discrete-state decision process as in Boukas and Haurie (1990), Kenne *et al.* (2003), for production, repair/replacement and preventive maintenance planning problems. The discrete event dynamic programming obtained can be solved using either policy improvement or successive approximation methods.

The next theorem shows that the value function $V^h(\alpha,a,x,n)$ is an approximation of $V(\alpha,a,x,n)$ for small size h step.

Theorem 6.1

Let $V^h(\alpha,a,x,n)$ denote a solution to HJB equations (6.10). Assume that there are constants C_g and K_g such that:

$$0 \le V^h(\alpha,a,x,n) \le C_g(1+|x|^{K_g}+|a|^{K_g})$$

Then

$$\lim_{h\to 0} V^h(\alpha,a,x,n) = V(\alpha,a,x,n) \tag{6.11}$$

Proof:

The proof of this theorem can be obtained by extending the one presented in Yan and Zhang (1997).

□

In the next sections, we provide optimal policies for one numerical example.

6.5 Numerical example

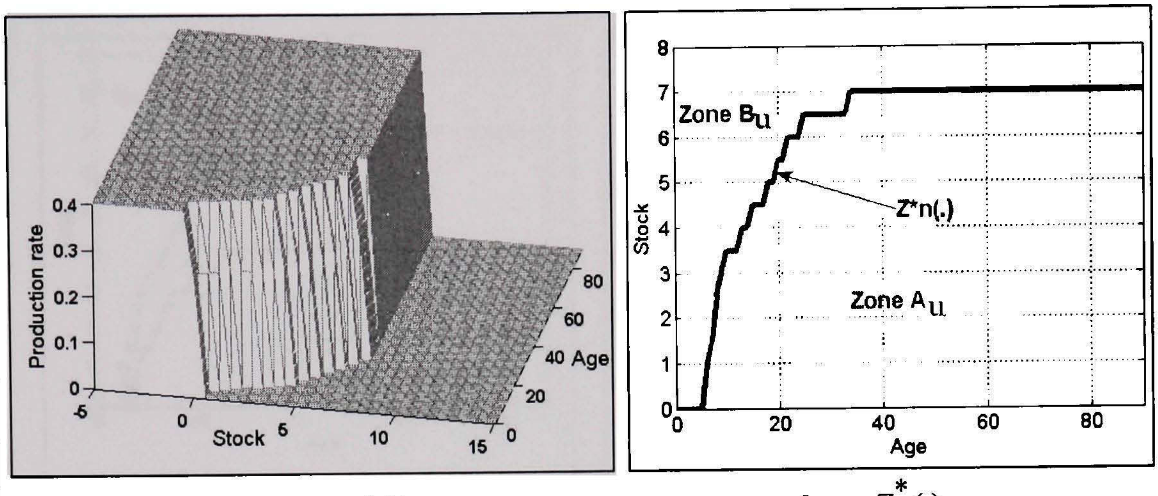
The computational domain D is defined by: $D=\{(a,n):0\le a\le 100;0\le n\le 20\}$. Assuming a Weibull distribution for the lifetime of a new machine, the scale parameter is $\lambda=0.05$ and the shape parameter is $\alpha=3$. Table 6.1 summarizes other parameters used in this paper.

Table 6.1 Parameters of the numerical example №4

Parameter	ρ	d_b	r_b	q_{31}	C_0	c_r	c_m	$T_{21}(1)$
Value	0.05	- 3	0.5	10	20 000	10	100	0.0001
Parameter	U_m	d	h_x	h_a	c^+	c^-	q_{41}	ω_{max}
Value	0.4	0.25	0.5	1	2	150	0.25	0.1

a) Production policy

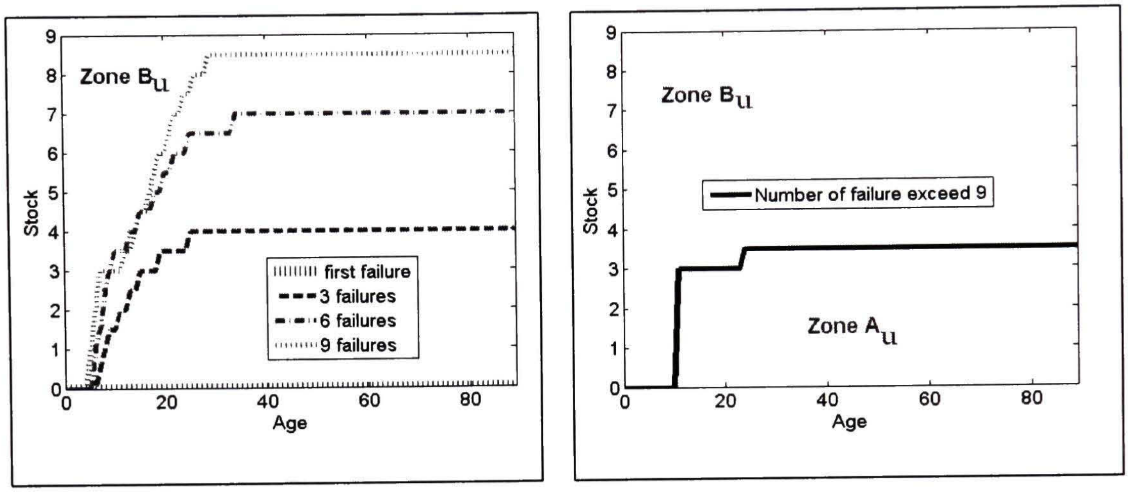
The optimal production policy $u^*(a,x,n)$, illustrated by the following figure 6.1.a indicates for a given number of failure n, for each stock level $x(t)$ and each age $a(t)$ of the machine, the rate at which parts are to be produced.



a $u*(.)$ b. $Z_n^*(.)$

Figure 6.1 Optimal production policy $u^*(.)$ and its boundary $Z_n^*(.)$.

To illustrate the optimal production policy given by figure 6.1.a, we use its boundary given by figure 6.1.b. The boundary of the optimal production policy is the optimal stock level $Z_n^*(.)$ such that, if the stock level in the system $x(.)$ is less than the optimal stock level, production should be at a maximum rate to reach the optimal stock level. Once the stock level in the system is equal to the optimal stock level, production should be at the demand rate. If the stock level in the system exceeds the optimal stock level, then we do not produce. The optimal production policy recommends production in zone A_u to reach the stock level $Z_n^*(.)$ and no production in zone B_u. The optimal stock level is given by the following Figures 6.2.a and 6.2.b for a couple of numbers of failures.



a. **Low failures number** b. **High failures number**

Figure 6.2 Optimal stock levels $Z_n^*(.)$.

As can be seen in figure 6.2.a, for this numerical example, when the machine has not yet had its first failure, while awaiting the first failure, the number of parts to hold in inventory is equal to zero. The corresponding curve is confused with the line $x = 0$ and zone A_u is inexistent. When the machine has experienced its n^{th} failure, the number of parts to hold in inventory increases and attains a maximal value at $\max(Z_n^*(.))$. This maximal number of parts to hold in inventory ($\max(Z_n^*(.))$) increases with the number of failures until a given maximal number of failures, as can be seen in figures 6.2.a and 6.2.b. The corresponding maximal number of failures in this example is 9, as shown in figure 6.2.b. Moreover, when the number of failures exceeds 9, zone A_u and zone B_u are constant.

After the machine has experienced its n^{th} failure, the optimal stock level trajectory highlights three parameters identified in the following figure 6.3.

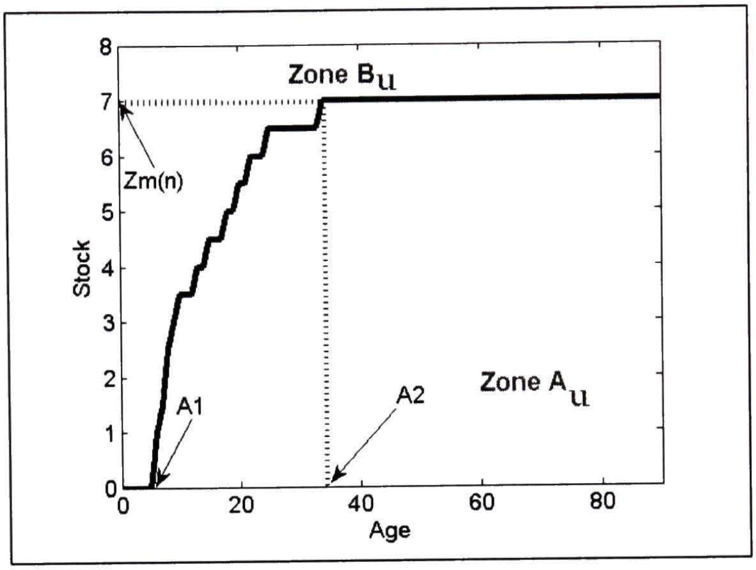


Figure 6.3 **Parameters of the production policy $Z_n^*(t)$.**

The production policy is characterized by three parameters $A_1(n)$, $A_2(n)$, $Z_m(n)$ such that, if the age of the machine $a(t)$ is less than $A_1(n)$, the number of parts to hold in inventory $Z_n^*(t)$ is zero. If the machine's age $a(t)$ is such that $A_1(n) \leq a(t) \leq A_2(n)$, then the number of parts to hold in inventory must be brought from zero to $Z_m(n)$. Once the machine's age reaches $A_2(n)$, the stock must be maintained at level $Z_m(n)$.

Figure 6.4 gives the production policy for several failure numbers. The number of parts to hold in inventory increases as $a(t)$ increases and as n increases.

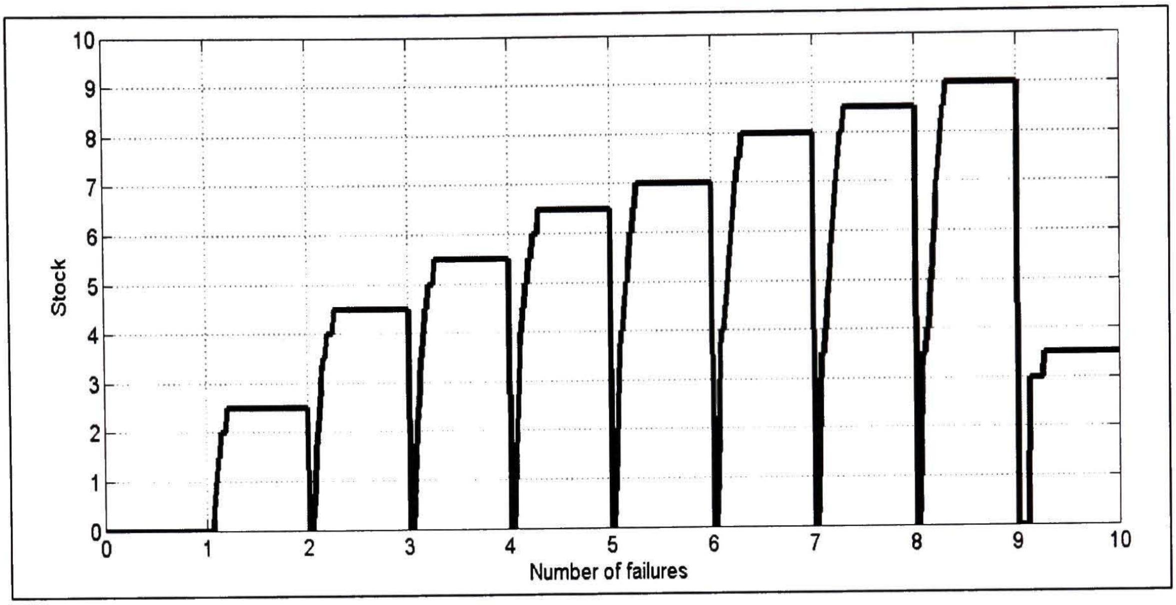


Figure 6.4 Number of parts to hold in inventory for each failure number.

The maximal number of parts to hold in inventory for several numbers of failures presented in figure 6.5 increases from one failure to the next. This is logical because if the machine is to be repaired, it will take a greater amount of time.

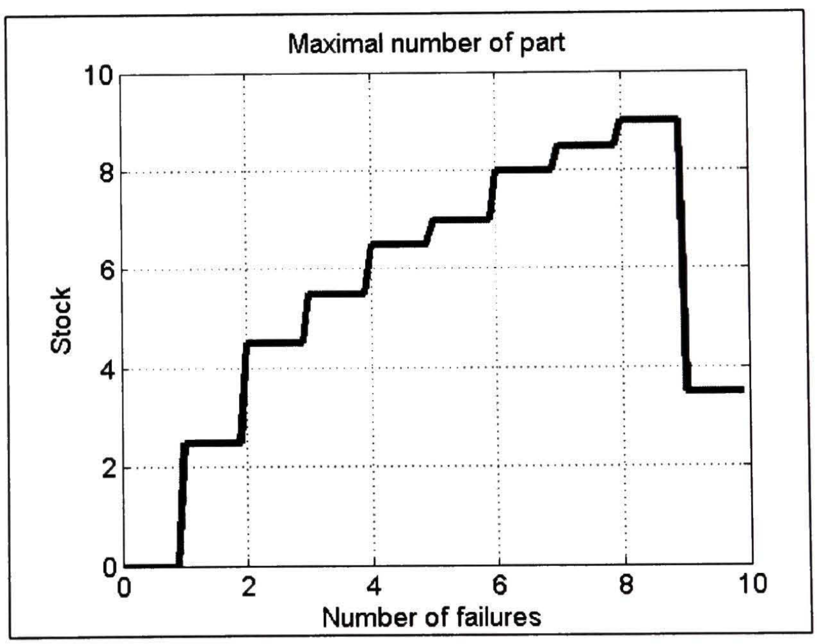


Figure 6.5 Maximal number of part $Z_m(.)$.

The optimal policy recommends holding a small number of parts in inventory after failure 9 because the machine is renewed when failure 10 occurs.

Before each failure n, the above production control policy (production rate) is summarized as follows:

$$u^*(\cdot) = \begin{cases} U_{\mathrm{m}} & \text{if } x(t) < z_n^*(.) \\ d & \text{if } x(t) = z_n^*(.) \\ 0 & \text{if } x(t) > z_n^*(.) \end{cases} \tag{6.12}$$

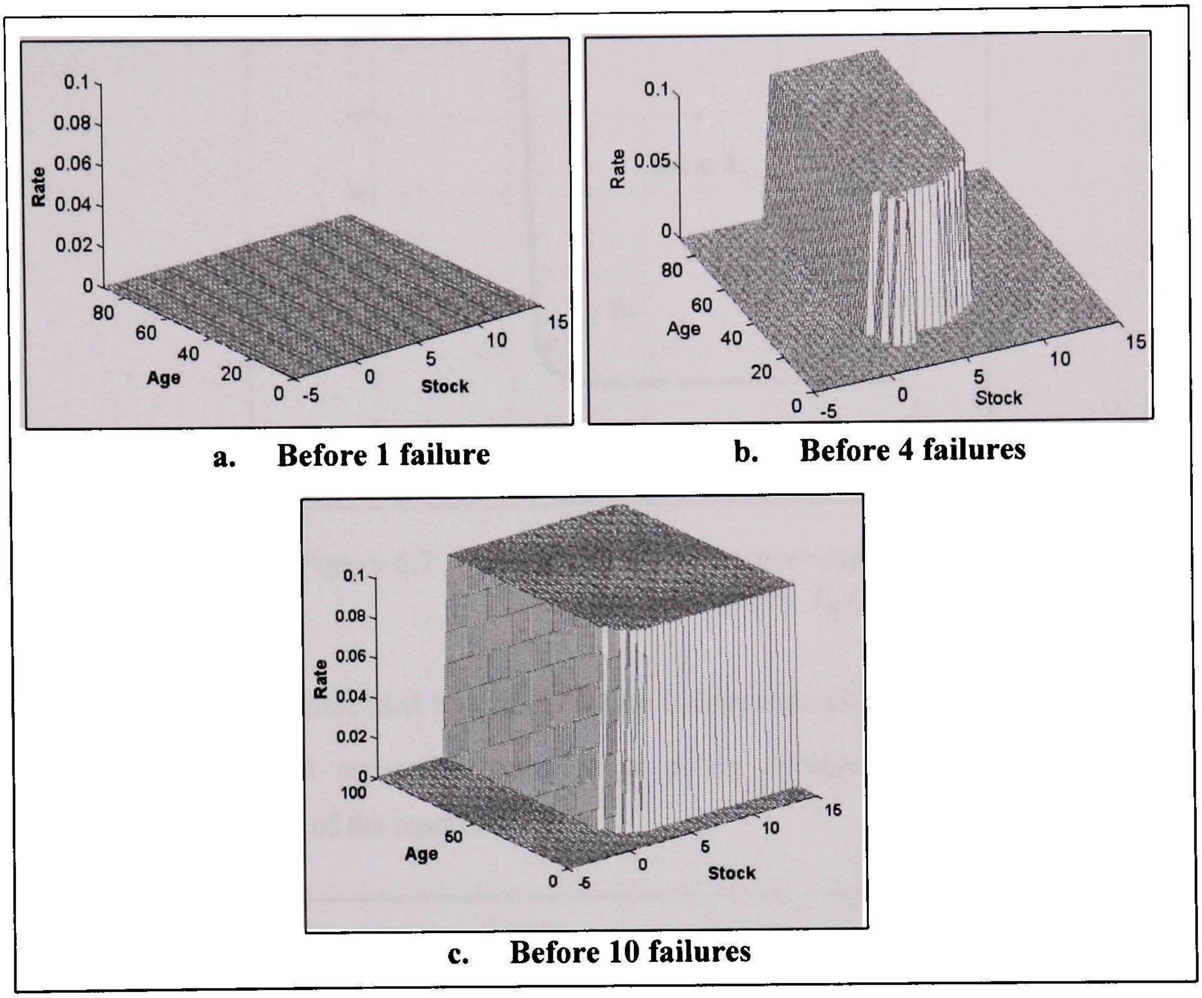
with

$$z_n^*(.) = \begin{cases} 0 & \text{if } a(t) \leq A_1(n) \\ \in \left]0, z_m(n)\right[& \text{if } A_1 < a(t) < A_2(n) \\ z_m(\mathrm{n}) & \text{otherwise} \end{cases} \tag{6.13}$$

It is important to note from equation (6.13) that when $a(t) > A_1(n)$, the stock level should be brought from 0 in order to reach $z_m(n)$ at $a(t) = A_2(n)$. The stock level is maintained at the value $Z_m(n)$ when $a(t) \geq A_1(n)$.

b) Preventive maintenance policy

The optimal preventive maintenance policy $\omega^*(a,x,.)$ for a given number of failures n is presented in the following figures 6.6.a, 6.6.b and 6.6.c. It shows that when the machine has not yet experienced its first failure (figure 6.6.a), preventive maintenance is not recommended. Conversely, when the number of failures already occurred increases (figure 6.6.b and figure 6.6.c), preventive maintenance is recommended. However, it depends on the age of the machine. We will analyze the evolution of the optimal preventive maintenance policy with age through the utilization of its boundary.



a. **Before 1 failure**

b. **Before 4 failures**

c. **Before 10 failures**

Figure 6.6 Preventive maintenance policy ω^*.

Throughout the rest of the paper, we will use the boundary of the optimal preventive maintenance present in figure 6.7 for a given number of failures n. The boundary $T_{\omega}(x,n)$ of the optimal preventive maintenance policy divided the area comprising the set of points (x,a) such that x is the stock and a the machine's age in two zones: Zone A_{ω} and Zone B_{ω}.

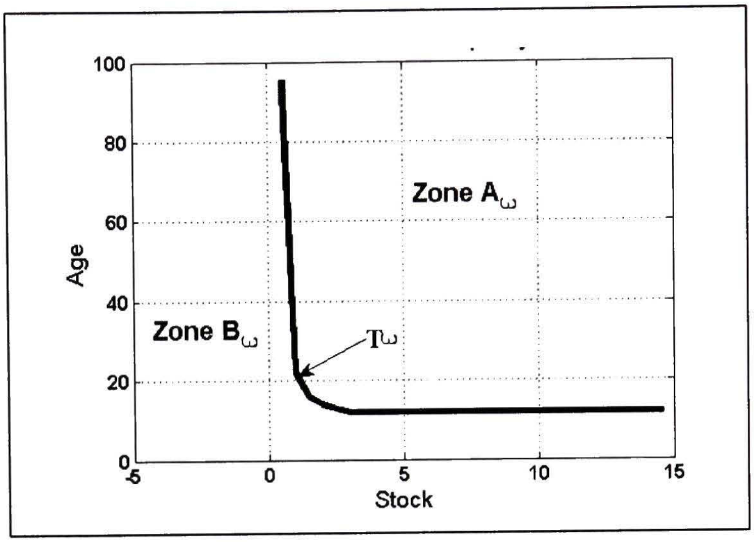


Figure 6.7 **Boundary of the preventive maintenance policy $T_\omega(.)$.**

In zone A_ω, it is recommended that the machine be preventively maintained. In zone B_ω conversely, the optimal preventive maintenance policy consists of not proceeding to a preventive maintenance of the machine.

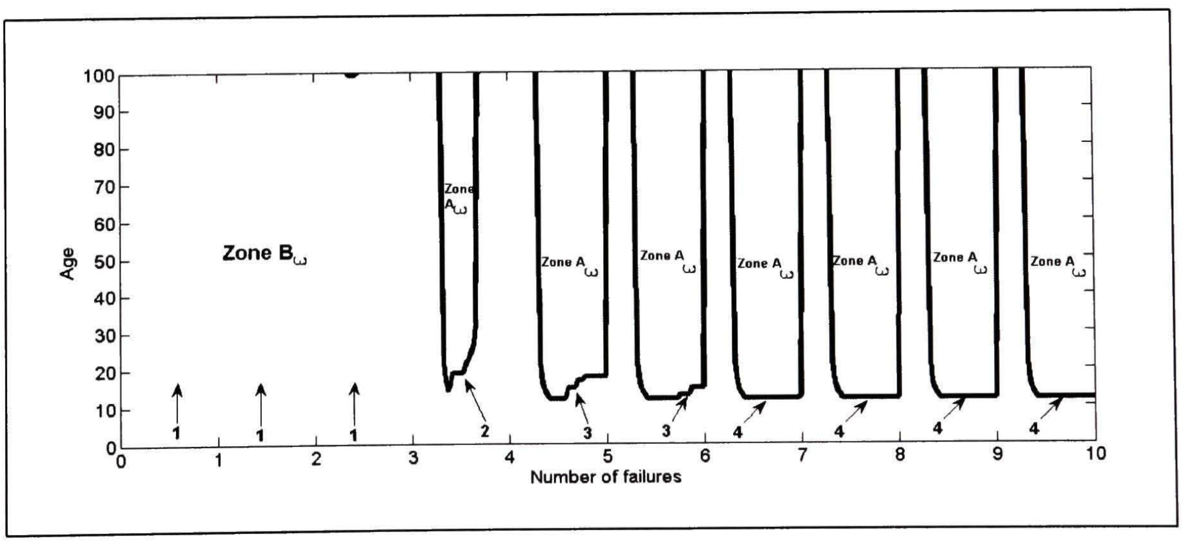


Figure 6.8 **Boundary of the preventive maintenance policy for several failures.**

As can be seen from figure 6.8 above, the preventive maintenance boundary appears and increases with the number of failures. In stage 1, indicates by arrows 1 in figure 6.8, the machine has experienced less than 3 failures. While waiting for the 3rd failure, zone A_{ω} is non-existent. The machine will not be sent out for preventive maintenance before the 3rd failures. Zone A_{ω} appears between failure 3 and failure 4, as indicated by arrow 2 in figure 6.8. It then increases, as indicated by arrow 3, and attains a constant value indicated by arrow 4, from failure 6 to failure 9. These results sound normal in that, normally in manufacturing systems, the preventive maintenance is more useful as the machine ages.

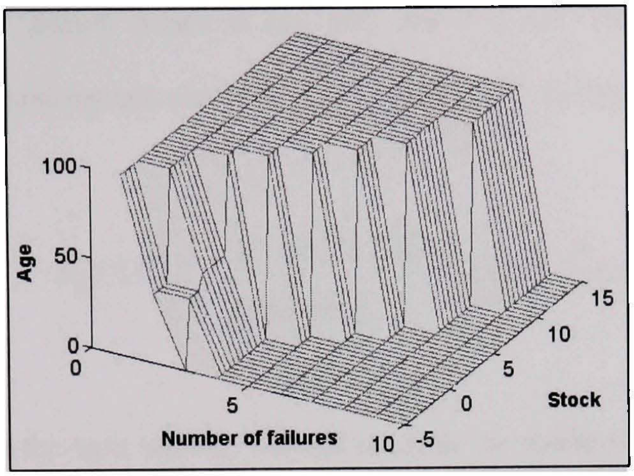
The preventive maintenance actions are triggered according to the age limit policy described in figure 6.6, 6.7 and 6.8. It states that preventive maintenance should be performed at rate $\omega^*(.)$, with

$$\omega^*(.) = \begin{cases} 0 & \text{if } a(t) <= T_{\omega}(.) \text{ that is, if the machine is in zone } B_{\omega} \\ \omega_{\max} & \textit{otherwise} \end{cases} \tag{6.14}$$

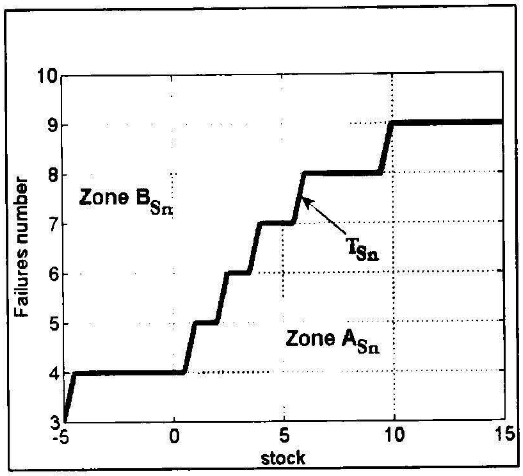
where $T_{\omega}(.)$ is the age limit for preventive maintenance before the n^{th} failure of the machine.

c) Repair/replacement policy $S_n^*(.)$

The action to be undertaken when a failure occurs can be identified by the location of the machine given by the following figures 6.9.a and 6.9.b.



a. $S_n^*(.)$



b. T_{Sn}

Figure 6.9 Repair /replacement policy and its Boundary.

From figure 6.9.a, the age $S_n^*(x)$, over which if a failure occurs, the machine should be replaced decreases with the number of failures and increases with the stock level in the system. The age $S_n^*(x)$ is equal to 0 when the machine has experiences 9 failures. It means that while waiting the 10th failure, the replacement age is 0, no matter what the stock level in the system is; that is, the machine will experience a maximum number of 9 failures before systematic replacement at next failure, no matter its age and the stock level in the system. We can then conclude that $N_m^* = 9$. Above the repair/replacement policy boundary given by figure 6.9.b, that is in zone B_{Sn}, the machine is systematically replaced if a failure occurs. Below the repair/replacement policy boundary, that is in zone A_{Sn} of figure 6.9.b, the machine is repaired if a failure occurs above replacement age $S_n^*(.)$.

Let $R_n(a(t), x(t))$ denote the function with value 1 if a repair action is undertaken after the n^{th} failure occurs at age $a(t)$ and 0 if not. The above results enable us to illustrate the repair/replacement policy as: upon the n^{th} failure of the machine at age $a(t)$,

$$R_n(.) = \begin{cases} 1 & \text{if } a(.) \le S_n^*(x) \\ 0 & \text{otherwise} \end{cases}, \text{ with } S_n^*(x) \text{ given in figure 5.21.a} \tag{6.15}$$

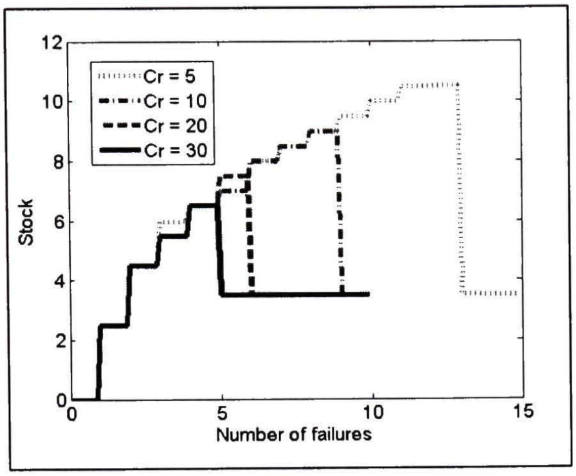
In the next section, we will confirm the structure of the obtained results through sensitivity analysis and illustrate the usefulness of the proposed approach.

6.6 Sensitivity and result analysis

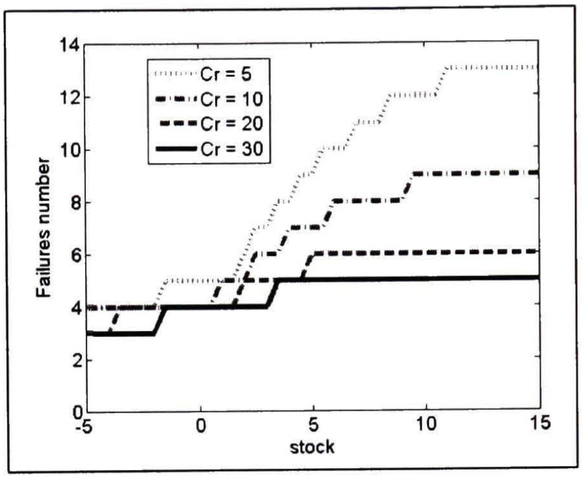
A sensitivity analysis is presented in this section to confirm the structure of the obtained results and illustrate the contribution of this paper.

6.6.1 Repair cost variation

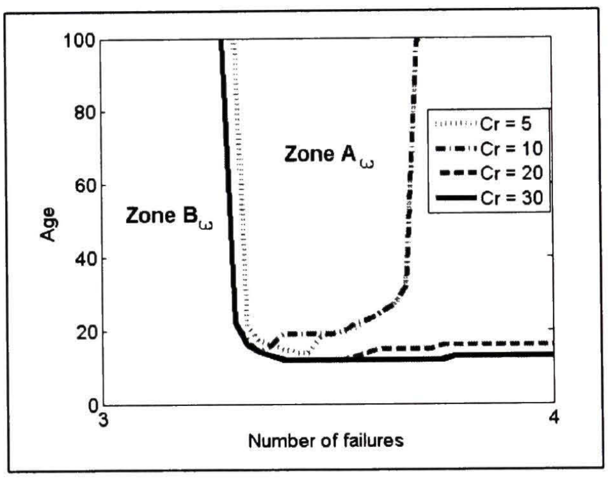
When the repair cost (Cr) changes and takes the values 5, 10, 20 and 30, we obtain the results in figure 6.10.a. Figure 6.10.a illustrates the maximal number of parts to hold in inventory before failure of the machine for four replacement cost values. The results show that variations in repair cost have no significant effect on the threshold value $Z_m(.)$ unless the machine is about to be replaced. This situation is explained if we refer to figure 6.10.b, which gives the repair/replacement policy. For a repair cost value of 30, the machine is systematically replaced after the 6th failure, as can be observed in figure 6.10.b.



a. $Z_m(n)$



b. **Boundary of $S_n(.)$**



c. $T_\omega(.)$ **for a given failure number** n

Figure 6.10 Sensitivity of policies to repair cost.

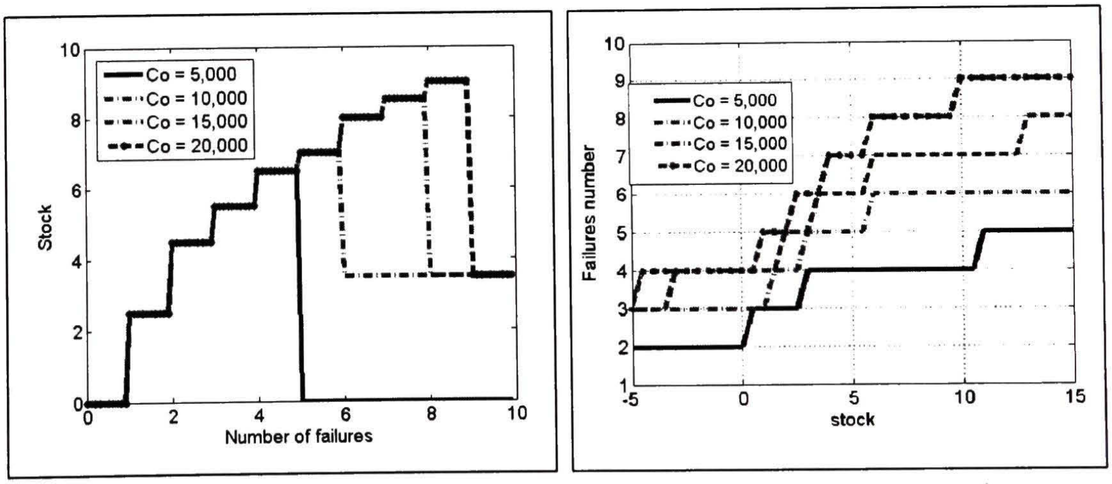
According to figure 6.10.b, when the repair cost varies and takes the values 5, 10, 20 and 30, the machine is systematically replaced after the 13th, 10th, 7th and 6th failure respectively. The trend is then to replace the machine early when repair costs increase. Thus, if the machine is to be replaced at the next failure, the number of parts to hold in inventory while waiting that failure is not high.

At the same time, preventive maintenance policy, as shown in figure 6.10.c is significantly affected. Area A_{ω} increases with the repair cost, that is, as repair costs increase, the preventive maintenance is recommended more often.

It is clear from the results above that repair costs significantly influence optimal policies overall. The machine is replaced earlier (it experiences a lower number of failures) when repair cost increases and the preventive maintenance area increases.

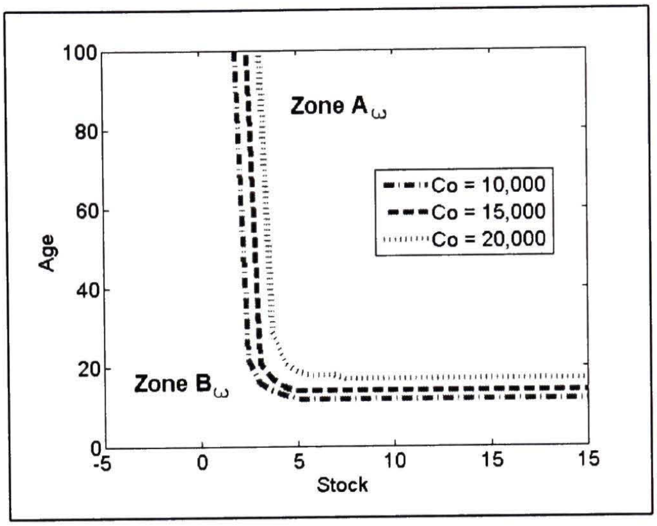
6.6.2 Replacement cost variation

For variations in replacement cost (Co), figure 6.11.a shows that it affects the optimal threshold stock level. Since the machine could not experience a certain number of failures as illustrated by figure 6.11.b, the threshold level is different from one replacement cost to the next. For a replacement cost value of 5,000, from the 5th failure, there is no need to stock parts, while for a replacement cost value of 20,000, products should be stocked up to 9 breakdowns.



a. $Z_m(n)$ b. **Boundary of** $S_n^*(.)$

Figure 6.11 Sensitivity of policies to replacement cost.



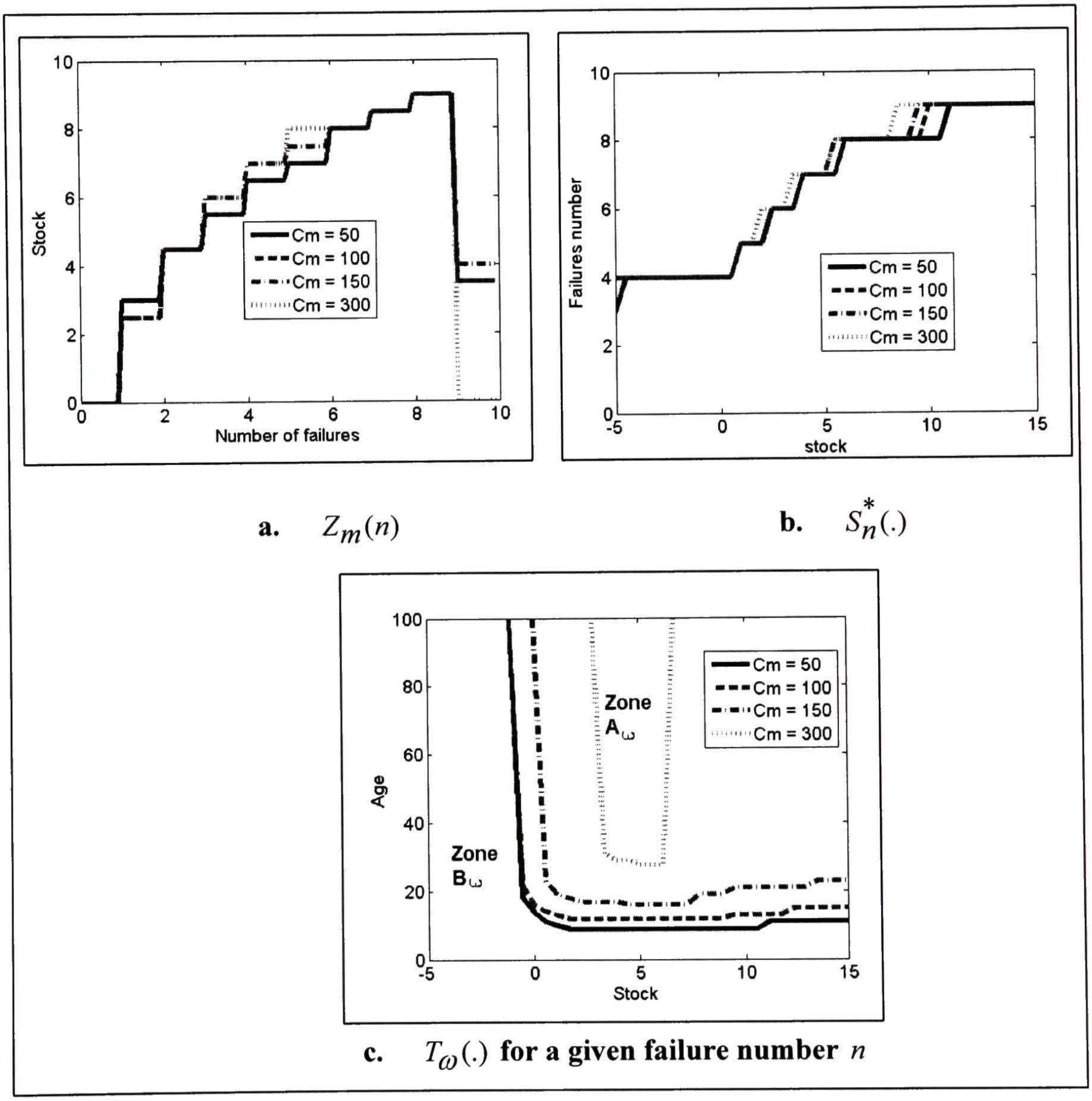
c. $T_\omega(.)$ **for a given failure number** n

Figure 6.11 Sensitivity of policies to replacement cost (Suite).

Further, when the replacement cost is 5,000, as can be seen from figure 6.11.b, the machine is systematically replaced at the 6th failure. If we take a look at figure 6.11.c, we notice that there are only three curves. The curve representing the value $C_0 = 5,000$ is absent. This is because no preventive maintenance is recommended when replacement cost is 5,000. That is logical in that, since the replacement is not expensive, it is better to replace the machine with a new one as compare to sending it to maintenance and expected another failure.

6.6.3 Preventive maintenance cost variation

Varying preventive maintenance costs (Cm) has a relatively small effect on the production and repair/replacement policies, presented in figure 6.12.a and 6.12.b. On the other hand, it significantly affects the preventive maintenance policy, as illustrated by figure 6.12.c., where we observe that for a high value of preventive maintenance cost ($C_m = 300$), the area where preventive maintenance of the machine is recommended is reduced. These conclusions on the influence of preventive maintenance cost on production and preventive maintenance policy are close to those obtained by (Kenne *et al.*, 2007). They observed, when formulating an analytical model for the joint determination of an optimal age-dependant buffer inventory and preventive maintenance policy in a production environment, that increasing preventive maintenance costs reduces preventive maintenance frequency and have no significant effect on the staircase trend of the production policy.



a. $Z_m(n)$ b. $S_n^*(.)$

c. $T_\omega(.)$ **for a given failure number** n

Figure 6.12 Sensitivity of policies to preventive maintenance cost.

The results obtained indicate that the optimal production, repair/replacement and preventive maintenance policy for the manufacturing system considered is characterized by a special structure of the control policy. Such a policy is defined by the above-mentioned three parameters for production (i.e. $Z_m(n)$; $A_1(n)$; $A_2(n)$), two parameters for the repair/replacement switching policy (i.e. $S_n(x)$; N_m) and one parameter for preventive maintenance policy (i.e. $T_\omega(x,n)$). The overall control policy given by equations (6.12), (6.14) and (6.15) is completely defined by values of parameters ($Z_m(n)$; $A_1(n)$; $A_2(n)$), ($S_n(x)$; N_m) and $T_\omega(x,n)$ for production, repair/replacement switching and preventive maintenance policy.

For a given stock level, the structure of the optimal repair/replacement policy is similar to that obtained by Makis and Jardine (1993) and Love *et al.* (2000). It states that if at the n^{th} failure, the age of the machine is above a certain critical value, replace the machine at the next failure. Otherwise, an imperfect repair should be undertaken. Note that the notion of imperfect repair in this work differs from the notion of imperfect repair used by those authors. However, we can easily include the type of imperfect repair considered in their work by not letting the repair and preventive maintenance activities reset the age of the machine at level 0. Thus, the repair/replacement policy present in this work is more realistic and appropriate for manufacturing systems because it takes into account the stock level in the system, the rate at which to produce parts and preventive maintenance possibility.

Through the observations from the sensitivity analysis, it clearly appears that the proposed approach, based on a simultaneous control of production, repair/replacement and preventive maintenance strategies, provides more realistic results in the context of stochastic manufacturing systems.

6.7 Conclusions

In this paper, a production, repair/replacement and preventive maintenance planning problem in manufacturing systems has been studied. The objective of the study was to determine how to produce while the machine is in operation, when to replace versus repair the machine if a failure occurs and when to perform a preventive maintenance, if any, in order to reduce the overall incurred cost. Our formulation included mode- and control-dependent jump rates, and allows us to represent the increasing probability of failures due to machine aging and increasing repair time resulting of imperfect repair activities. We used reset functions that permit repair and preventive maintenance activities to restore the machine's age to level 0, and replacement, which provides a new machine. Thus, we extended the repair and replacement switching age control model to the case where production and preventive maintenance rate are controlled. Because machine repair activities depend on the repair history, we have proposed the utilization of a semi-Markov decision process to determine the optimal policies to be used. We provided a numerical example, and through a couple of sensitivity analyses, show that the structure of results obtained is maintained.

REFERENCES

Beichelt F. (1992). "*A general maintenance model and its application to repair limit replacement policies.*" Microelectronics and Reliability, 32(8): pp. 1185-1196.

Boukas E.-K. and Haurie A. (1990). "*Manufacturing flow control and preventive maintenance: A stochastic control approach.*" IEEE Transactions on Automatic Control, 35(9): pp. 1024-1031.

Chelbi A. and Ait-Kadi D. (2004). "*Analysis of a production/inventory system with randomly failing production unit submitted to regular preventive maintenance.*" European Journal of Operational Research, 156(3): pp. 712-18.

Chiu S. W. (2008). "*An optimization problem of manufacturing systems with stochastic machine breakdown and rework process.*" Applied Stochastic Models in Business and Industry, 24: pp. 203-219.

Dellagi S., Rezg N. and Xie X. (2007). "*Preventive maintenance of manufacturing systems under environmental constraints.*" International Journal of Production Research, 45(5): pp. 1233-1254.

El-Ferik S., 2008. "*Economic production lot-sizing for an unreliable machine under imperfect age-based maintenance policy.*" European Journal of Operational Research, 186(1): pp. 150-163.

Gabriella B., Rommert D. and Robin P. N. (2008). "*Maintenance and Production: A Review of Planning Models.*" Complex System Maintenance Handbook. S. London.

Gershwin, S. B. (1994). "*Manufacturing systems engineering.*" Englewood Cliffs, NJ: prentice-Hall.

Gharbi A., Kenne J. P. and Beit M. (2007). "*Optimal safety stocks and preventive maintenance periods in unreliable manufacturing systems.*" International Journal of Production Economics, 107(2): pp. 422-434.

Jeong I. J., Leon V. J. and Villalobos J. R. (2007). "*Integrated decision-support system for diagnosis, maintenance planning, and scheduling of manufacturing systems.*" International Journal of Production Research, 45(2): pp. 267-285.

Kenne J. P. and Nkeungoue L. J. (2008). "*Simultaneous control of production, preventive and corrective maintenance rates of a failure-prone manufacturing system.*" Applied Numerical Mathematics, 58(2): pp. 180-194.

Kenne J. P., Boukas E. K. and Gharbi A. (2003). "*Control of production and corrective maintenance rates in a multiple-machine, multiple-product manufacturing system.*" Mathematical and Computer Modelling, 38(3-4): pp. 351-65.

Kijima M., H. Morimura and Suzuki Y. (1988). "*Periodical replacement problem without assuming minimal repair.*" European Journal of Operational Research, 37(2): pp. 194-203.

Kushner H. and Dupuis P. G. (1992). "*Numerical methods for stochastic control problems in continuous time.*" Springer-verlag. New York.

Lai M-T. (2007). "*A periodical replacement model based on cumulative repair-cost limit.*" Applied Stochastic Models in Business and Industry, 23: pp. 455-464.

Liao G.-L. (2007). "*Optimal production correction and maintenance policy for imperfect process.*" European Journal of Operational Research, 182(3): pp. 1140-1149.

Leung K. N. F. (2006). "*A note on 'A bivariate optimal replacement policy for a repairable system.*" Engineering Optimization, 38(5): pp. 621-625.

Love C. E., Zhang Z. G., Zitron M. A. and Guo R. (2000). "*Discrete semi-Markov decision model to determine the optimal repair/replacement policy under general repairs.*" European Journal of Operational Research, 125(2): pp. 398-409.

Makis V. and Jardine A. K. S. (1993). "*A note on optimal replacement policy under general repair.*" European Journal of Operational Research, 69(1): pp. 75-82.

Mercier S., (2008). "*Optimal replacement policy for obsolete components with general failure rates*". Applied Stochastic Models in Business and Industry, 24: pp. 221-235.

Nair S.K. and Hopp W.J., (1992). "*A model for equipment replacement due to technological obsolescence.*" European Journal of Operational Research, 63(2): pp. 207-221.

Pérès F. and Noyes D., (2003). "*Evaluation of a maintenance strategy by the analysis of the rate of repair*". Quality and Reliability Engineering International, 19: pp. 129-148.

Pérès-Ocon R. and Torres-Castro I., (2002). "*A reliability semi-Markov model involving geometric processes*". Applied Stochastic Models in Business and Industry, 18: pp. 157-170.

Rajagopalan, S., M. R. Singh and T. E. Morton (1998). "*Capacity expansion and replacement in growing markets with uncertain technological breakthroughs.*" Management Science, 44(1): pp. 12-30

Savsar M. (2008). "*Modelling of multi-stage production lines with maintenance operations.*" International Journal of Computer Integrated Manufacturing, 21(4): pp. 396-406.

Sethi S. P. and Zhang Q. (1994). "*Hierarchical Decision Making in Stochastic Manufacturing.*" Systems, Birkhâser.

Sloan T., (2008). "*Simultaneous determination of production and maintenance schedules using in-line equipment condition and yield information.*" Naval research Logistics, 55(2): pp. 116-119.

Vasant, P., Nagarajan R., and Yaacob S. (2004). "*Decision making in industrial production planning using fuzzy linear programming.*" IMA Journal Management Mathematics 15(1): pp. 53-65.

Viles E., Puente D., Alvarez M. J. and Alonso F. (2007). "*Improving the corrective maintenance of an electronic system for trains.*" Journal of Quality in Maintenance Engineering, 13(1): pp. 75-87.

Yan H. and Zhang Q. (1997). "*Numerical method in optimal production and setup scheduling to stochastic manufacturing systems.*" IEEE Transactions on Automatic Control, 42: pp. 1452-1455.

Yao X., Xie X., Fu M. C. Marcus S. (2005). "*Optimal Joint Preventive Maintenance and Production Policies.*" Naval Research Logistics, 52(7): pp. 668-681.

Yuan L. Z., Richard C. M. Yam and Zuo M. J. (2001). "*Optimal replacement policy for a deteriorating production system with preventive maintenance.*" International Journal of Systems Science, 32(10): pp. 1193-1198.

Zimmermann, H., (1990), Fuzzy Set. "*Theory and Its Applications.*" Second, revised edition. Kluwer Academic publishers. ISBN 0-7923-9075-X, 1990.

CONCLUSION GÉNÉRALE

Cette thèse propose une nouvelle approche de formulation et de résolution du problème de commande optimale stochastique des systèmes manufacturiers dynamiques. L'approche proposée tient compte du vécu de la machine en intégrant dans le processus d'optimisation la notion d'usure de la machine. Cette approche a permis de trouver la stratégie optimale de gestion de capacité, de gestion de production et de contrôle de maintenance des machines en quatre parties.

Dans la première partie, l'introduction des stratégies de maintenance corrective afin d'améliorer la disponibilité des machines est combinée à la gestion de capacité et de production du système. Le système est modélisé par une chaîne de Markov non homogène. Le problème d'optimisation est résolu à l'aide des méthodes numériques et des exemples sont fournis. Les diverses analyses de sensibilité effectuées confirment la structure des politiques obtenues.

Dans la seconde partie, nous introduisons la dégradation progressive de la machine à mesure qu'elle fabrique des pièces et à mesure qu'elle subit des réparations. Les temps de réparations deviennent de plus en plus longs quand le nombre de pannes augmente. Compte tenu de la prise en compte de l'historique des réparations, le système est modélisé par un processus semi-Markovien et le problème d'optimisation résolu à l'aide des méthodes numériques. Les résultats obtenus montrent que les politiques doivent être ajustées pour tenir compte de la dégradation par rapport aux approches classiques de la littérature. D'autre part, dans les approches classiques de la littérature sur les politiques de réparation versus le remplacement, le seuil critique de remplacement de la machine dépend uniquement du nombre de pannes que le système a subit jusqu'à date. Nous avons montré que ces résultats ne sont valables que si le niveau de stock dans le système est fixe ou que les réparations sont instantanées. En contexte de gestion de production dans lequel le stock fluctue, le seuil critique de remplacement de la machine dépend également du nombre de pièces en stock.

Dans la troisième partie du travail, nous avons maintenu les dégradations affectant la machine à la phase deux et avons introduit une troisième dimension de la dégradation. L'imperfection des réparations se traduit souvent par l'âge cumulé que prend la machine après chaque phase d'opération suivi d'une intervention, appelée âge virtuel de la machine. En introduisant la notion d'âge virtuel de la machine dans la troisième partie du travail, le système a été modélisé une fois de plus par un processus semi-Markovien et le problème résolu par une approche hiérarchique de prise de décisions et des méthodes numériques. Un exemple numérique a permis d'illustrer l'approche proposée. Nous avons montré que si après réparation la machine n'a pas un âge nul, les seuils critiques doivent être ajustés pour en tenir compte. Diverses analyses de sensibilité ont été effectuées et ont confirmé la structure des politiques de commande optimale obtenue.

Nous ne pouvions conclure nos travaux sans explorer l'impact de l'introduction des stratégies de maintenance préventive sur les politiques obtenues aux parties deux et trois. Compte tenu du fait que les temps de réparation s'allongent au fil des réparations, nous avons dans la quatrième partie de cette thèse introduit les stratégies de maintenance préventive pour contrer ce phénomène. Nous avons appliqué la notion d'âge et de réduction de l'intensité de défaillance de la machine à la réduction des temps de réparation de la machine en utilisant les stratégies de maintenance préventive de la machine. Nous avons montré que l'introduction des stratégies de maintenance préventive pour réduire les temps de réparation et améliorer la disponibilité du système permet d'améliorer la durée de vie de la machine.

TRAVAUX FUTURS

Nos travaux futurs viseront dans un premier temps à compléter le tableau 1.1 car il comporte encore des cases vides. Chacune de ces cases correspond à une possibilité de développement futur de cette thèse.

D'autres prochains développements du travail effectué dans cette thèse concernent:

- La planification de production, de réparation et du remplacement d'un système manufacturier à deux états opérationnels: parfait et dégradé (dégradation par diminution du taux de production). Nous pourrons ainsi introduire les stratégies de contrôle de la qualité du produit fabriqué;

- La planification de production, de réparation et de maintenance préventive et expansion de capacité en considérant l'âge de la machine, sans remise de l'âge à zéro après intervention sur la machine : achat d'une deuxième machine après un 1er niveau de dégradation de la première, suppression de la première machine à un 2e niveau de vieillissement;

- La planification de production, de maintenance corrective, de maintenance préventive et expansion de capacité en considérant l'âge de la machine:
  - Dégradation affectant l'intensité de défaillance (diminution du temps moyen de production),
  - Dégradation par diminution du taux de production,
  - Dégradation par diminution du temps moyen et du taux de production,
  - Amélioration de la disponibilité des équipements par le contrôle de la maintenance corrective,
  - Considération de la nature de la demande: constante, monotone strictement croissante, aléatoire,
  - Considération de la réduction de capacité,
  - Considération de la forme d'expansion de capacité: Acquisition d'équipement, sous-traitance, location;

- La prise en compte des incertitudes affectant la demande;

- Utiliser la structure des lois de commande obtenue dans cette thèse pour étendre les problèmes résolus à des cas de systèmes manufacturiers plus larges, impliquant plusieurs machines et plusieurs produits. Une approche basée sur les plans d'expérience, modèles de simulation et surfaces de réponse pourra être utilisée;

- La simulation sur des cas réels de systèmes manufacturiers.

Lorsque les machines des systèmes manufacturiers ont une durée de vie importante (15 ans par exemple), il est difficilement concevable de remplacer cette machine par une nouvelle qui aurait les mêmes caractéristiques sans avoir à son actif une performance supplémentaire. Nous envisageons donc pour nos prochains travaux des remplacements de machines désuètes par des machines neuves ou des machines usagées mais plus performantes en termes de capacité.

BIBLIOGRAPHIE

Akella, R., Kumar P. R. (1986). "*Optimal control of production rate in a failure prone manufacturing system.*" IEEE Transactions on Automatic control, 31(2): pp. 116-126.

Anne-Sylvie, C., I.-R. Floru, C. Azzaro-Pantel, L. Pibouleau and S. Domenech (2003). "*Optimization of preventive maintenance strategies in a multipurpose batch plant: Application to semiconductor manufacturing.*" Computers and Chemical Engineering, 27(4): pp. 449-467.

Ansell, J. I., T. W. Archibald and L. C. Thomas (2004). "*The elixir of life: using a maintenance, repair and replacement model based on virtual and operating age in the water industry.*" IMA Journal of Management Mathematics, 15(2): pp. 151-60.

Aven, T. and I. T. Castro (2008). "*A minimal repair replacement model with two types of failure and a safety constraint.*" European Journal of Operational Research, 188(2): pp. 506-515.

BAILLET, P. (1994). "*Contribution à l'amélioration de la réactivité dans les systèmes de production notamment pour la mise en oeuvre des concepts de décentralisation des fonctions de décision.*" Marseille, Université d'Aix-Marseille III.

Balduzzi, F. (2001). "*Hybrid system model of production work-cells and its optimal control subject to logical constraints.*" 8th International Conference on Emerging Technologies and Factory Automation (ETFA 2001), Oct 15-18 2001, Antibes-Juan les Pins, Institute of Electrical and Electronics Engineers Inc.

Basnet, C. and J. H. Mize (1995). "*Rule based, object-oriented framework for operating flexible manufacturing systems.*" International Journal of Production Research, 33(5): pp. 1417-1431.

Beichelt, F. (1992). "*A general maintenance model and its application to repair limit replacement policies.*" Microelectronics and Reliability, 32(8): pp. 1185-1196.

Bhattacharya, A. K. and J. L. Coleman (1994). "*Linking manufacturing strategy to the design of a customized hybrid production control system.*" Computer Integrated Manufacturing Systems, 7(2): pp. 134-141.

Bironneau, L. (2000). "*Le choix des méthodes et outils de pilotage de la production en milieu industriel.*" France, Centre de Recherche Rennais en Economie et Gestion, Institut de Gestion de Rennes, Université de Rennes 1.

Bitran G. R., Haas E. A. and Matsuo H. (1986). "*Production planning of style goods with high set-up costs and forecast revisions.*" Operations Research, 34(2): pp. 226-236

Boukas, E.-K. and A. Haurie (1990). "*Manufacturing flow control and preventive maintenance: A stochastic control approach.*" IEEE Transactions on Automatic Control, 35(9): pp. 1024-1031.

Boukas, E. K. (1997). "*Production and corrective maintenance control for failure prone production systems.*" San Diego, CA, USA, IEEE.

Cakanyildirim M., Roundy R. and Wood S.C. (1999). "*Machine purchasing strategies under demand-and technology-driven uncertainties.*" Technical paper no. 1250, SORIE, Cornell University, Ithaca, NY.

Cassady, C. R., I. M. Iyoob, K. Schneider and E. A. Pohl (2005). "*A generic model of equipment availability under imperfect maintenance.*" IEEE Transactions on Reliability, 54(4): pp. 564-571.

Cassandras, C. G., Q. Liu, K. Gokbayrak and D. L. Pepyne (1999). "*Optimal control of a two-stage hybrid manufacturing system model.*" The 38th IEEE Conference on Decision and Control (CDC), Dec 7-Dec 10 1999, Phoenix, AZ, USA, IEEE, Piscataway, NJ, USA.

Chelbi, A. and D. Ait-Kadi (2004). "*Analysis of a production/inventory system with randomly failing production unit submitted to regular preventive maintenance.*" European Journal of Operational Research, 156(3): pp. 712-718.

Chiodini, V. (1986). " *Knowledge based system for dynamic manufacturing replanning.*" Real-Time Optimization in Automated Manufacturing Facilities, Proceedings of a Symposium., Gaitherburg, MD, USA, NBS, Gaithersburg, MD, USA.

Clarke, F., Ed. (1983). "*Optimization and Non-Smooth Analysis.*" New York, NY., Wiley-Interscience.

Dellagi S., Rezg N. and Xie X. (2007). "*Preventive maintenance of manufacturing systems under environmental constraints.*" International Journal of Production Research, 45(5): pp. 1233-1254.

Dempster M. A. H., Fisher M.L., Jansen L., Lageweg B.J., Lenstra J.K. and Rinnooy Kan A.H.G. (1981). "*Analytical evaluation of hierarchical planning systems.*" Operations Research, 29(4): pp. 707-716

Doyen, L. and O. Gaudoin (2004). "*Classes of imperfect repair models based on reduction of failure intensity or virtual age.*" Reliability Engineering and System Safety, 84(1): pp. 45-56.

Draghici G. (1997). "*Production compétitive basée sur l'ingénierie intégrée.*" Les Cahiers des Enseignements Francophones en Roumanie, Service Culturel de l'Ambassade de France en Roumanie, Bucarest.

Elhafsi, M. and S. X. Bai (1996). "*Optimal production control of a dynamic two-product manufacturing system with setup costs and setup times.*" Journal of Global Optimization, 9(2): pp. 183-216.

Gabriella, B., D. Rommert and P. N. Robin (2008). "*Maintenance and Production: A Review of Planning Models.*" Complex System Maintenance Handbook. S. London.

George, D., B. Nicolae and F. Ioana (1998). "*La modélisation et la simulation en vue de la conduite des systèmes de production.*" Universitatea Politehnica din Timisoara.

Gershwin S. B. (1989). "*Hierarchical flow control: a framework for scheduling and planning discrete events in manufacturing systems.*" Proceedings of the IEEE, 77(1): pp. 195-209

Gershwin, S. B. (1994). "*Manufacturing systems engineering.*" Englewood Cliffs, NJ: prentice-Hall.

Gharbi, A. and J. P. Kenne (2003). "*Optimal production control problem in stochastic multiple-product multiple-machine manufacturing systems.*" IIE Transactions (Institute of Industrial Engineers), 35(10): pp. 941-952.

Gharbi, A. and J. P. Kenne (2005). "*Maintenance scheduling and production control of multiple-machine manufacturing systems.*" Computers & Industrial Engineering, 48(4): 693-707.

Gharbi, A., J. P. Kenne and M. Beit (2007). "*Optimal safety stocks and preventive maintenance periods in unreliable manufacturing systems.*" International Journal of Production Economics, 107(2): pp. 422-434.

Hariga, M. A., M. N. Azaiez and M. B. Daya (2006). "*A discounted integrated inspection-maintenance model for single deteriorating production facility.*" International Transactions in Operational Research, 13(4): pp. 353-364.

Haurie, A., P. L'Ecuyer and C. van Delft (1990). "*Optimization by simulation in manufacturing flow control.*" Proceedings of Rensselaer's Second International Conference on Computer Integrated Manufacturing, 21-23 May 1990, Troy, NY, USA, IEEE Comput. Soc. Press.

Jeong, I. J., V. J. Leon and J. R. Villalobos (2007). "*Integrated decision-support system for diagnosis, maintenance planning, and scheduling of manufacturing systems.*" International Journal of Production Research, 45(2): pp. 267-285.

Jinyuan, C. and L. Zehui (2008). "*An extended extreme shock maintenance model for a deteriorating system.*" Reliability Engineering and System Safety, 93(8): pp. 1123-1129.

Jones, A. and A. Saleh (1990). "*A multi-level/multi-layerarchitecture for intelligent shopfloor control.*" Inter-national Journal of Computer Integrated Manufacturing, 3: pp. 60-70.

Kahle, W. (2007). "*Optimal maintenance policies in incomplete repair models.*" Reliability Engineering and System Safety, 92(5): pp. 563-565.

Kenne, A. and A. Gharbi (2004). "*A simulation optimization based control policy for failure prone one-machine, two-product manufacturing systems.*" Computers & Industrial Engineering, 46(2): pp. 285-292.

Kenne, J.-P. and E. K. Boukas (1998). "O*ptimal Control of the production and Corrective Maintenance of a Multiple-Machine Flowshop.*" Les Cahiers du Gerad.

Kenne, J. P. and E. K. Boukas (2003). "*Hierarchical control of production and maintenance rates in manufacturing systems.*" Journal of Quality in Maintenance Engineering, 9(1): pp. 66-82.

Kenne, J. P., E. K. Boukas and A. Gharbi (2003). "*Control of production and corrective maintenance rates in a multiple-machine, multiple-product manufacturing system.*" Mathematical and Computer Modelling, 38(3-4): pp. 351-365.

Kenne, J. P. and A. Gharbi (1999). "*Experimental design in production and maintenance control problem of a single machine, single product manufacturing system.*" International Journal of Production Research, 37(3): pp. 621-637.

Kenne, J. P. and A. Gharbi (2000). "*Production planning problem in manufacturing systems with general failure and repair time distributions.*" Production Planning and Control, 11(6): pp. 581-588.

Kenne, J. P. and A. Gharbi (2001). "*A simulation optimization approach in production planning of failure prone manufacturing systems.*" Journal of Intelligent Manufacturing, 12(5-6): pp. 421-431.

Kenne, J. P. and A. Gharbi (2004). "*Stochastic optimal production control problem with corrective maintenance.*" Computers & Industrial Engineering, 46(4): pp. 865-875.

Kenne, J. P., A. Gharbi and M. Beit (2007). "*Age-dependent production planning and maintenance strategies in unreliable manufacturing systems with lost sale.*" European Journal of Operational Research, 178(2): pp. 408-420.

Kenne, J. P. and L. J. Nkeungoue (2008). "*Simultaneous control of production, preventive and corrective maintenance rates of a failure-prone manufacturing system.*" Applied Numerical Mathematics, 58(2): pp. 180-194.

Kijima, M. (1989). "*Some results for repairable systems with general repair.*" Journal of Applied Probability, 26: pp. 89-102.

Kijima, M. (1989). "*Some results for the quasi-stationary distribution in birth-death processes.*" Research Reports on Information Sciences, Series B (Operations Research), 219: pp. 1-15.

Kijima, M., H. Morimura and Y. Suzuki (1988). "*Periodical replacement problem without assuming minimal repair.*" European Journal of Operational Research, 37(2): pp. 194-203.

Kijima, M. and T. Nakagawa (1992). "*Replacement policies of a shock model with imperfect preventive maintenance.*" European Journal of Operational Research, 57(1): pp. 100-110.

Kijima, M. and Y. Suzuki (1986). "*Periodical replacement problem without assuming the minimal repair.*" Research Reports on Information Sciences, Series B (Operations Research), 186: pp. 1-24.

Kimemia, J. and S. B. Gershwin (1983). "*Algorithm for the computer control of a flexible manufacturing system.*" IIE Transactions, 15(4): pp. 353-362.

Kubota F. Sato S. and Nakano M. (1999). "*Enterprise modeling and simulation platform integrating manufacturing system design and supply chain.*" Paper presented at the 1999 IEEE International Conference on Systems, Man, and Cybernetics 'Human Communication and Cybernetics', Oct 12-Oct 15 1999, Tokyo, Jpn

Kushner, H. and P. G. Dupuis (1992). "*Numerical methods for stochastic control problems in continuous time.*" Springer-verlag. New York.

Lazarescu, I., G. Draghici, A. E. Moudni and N. Zerhouni (1998). "*Modelisation et simulation de systemes de production hybrides a l'aide de reseaux de petri.*" Comunicarile celei de-a VIII-a Conferinte Internationale de Inginerie Manageriala si Tehnologica, Timisoara.

Leung, K. N. F. (2006). "*A note on 'A bivariate optimal replacement policy for a repairable system.*" Engineering Optimization, 38(5): pp. 621-625.

Liao, G.-L. (2007). "*Optimal production correction and maintenance policy for imperfect process.*" European Journal of Operational Research, 182(3): pp. 1140-1149.

Liberatore, G., S. Nicosia and P. Valigi (1995). "*Dynamic allocation of kanbans in a manufacturing system using perturbation analysis.*" Proceedings 1995 INRIA/IEEE Symposium on Emerging Technologies and Factory Automation. ETFA'95, 10-13 Oct. 1995, Paris, France, IEEE Comput. Soc. Press.

Liberopoulos, G. and M. Caramanis (1994). "*Production control of manufacturing systems with production rate-dependent failure rates.*" Automatic Control, IEEE Transactions, 39(4): pp. 889-895.

Liu, J., P. Li and C. Song (2004). "*Production and corrective maintenance control problem for failure prone systems with demand uncertainty.*" Hangzhou, China, Institute of Electrical and Electronics Engineers Inc., Piscataway, United States.

Love, C. E., Z. G. Zhang, M. A. Zitron and R. Guo (2000). "*Discrete semi-Markov decision model to determine the optimal repair/replacement policy under general repairs.*" European Journal of Operational Research, 125(2): pp. 398-409.

Love C. E., Zitron M. A. and Zhang Z. G. (1998). "*An SMDP approach to optimal repair/replacement decisions for systems experiencing imperfect repairs.*" Journal of Quality in Maintenance Engineering, 4(2): pp. 131-149.

Lugtigheid, D., D. Banjevic and A. K. S. Jardine (2008). "*System repairs: When to perform and what to do*? " Reliability Engineering and System Safety, 93(4): pp. 604-615.

Makis, V. and A. K. S. Jardine (1991). "*Optimal replacement of a system with imperfect repair.*" Microelectronics and Reliability, 31(2-3): pp. 381-388.

Makis, V. and A. K. S. Jardine (1993). "*A note on optimal replacement policy under general repair.*" European Journal of Operational Research, 69(1): pp. 75-82.

Malik, M. A. K. (1979). "*Reliable preventive maintenance scheduling.*" AIIE Transactions, 11(3): pp. 221-228.

Manz, E. M., J. Haddock and J. Mittenthal (1989). "*Optimization of an automated manufacturing system simulation model using simulated annealing.*" Winter Simulation Conference Proceedings (Cat. No.89CH2778-9), 4-6 Dec. 1989, Washington, DC, USA, SCS.

Megartsi, R. (1996). "*Analyse des systèmes de production.*" DEA de Productique et d'Informatique. < http://riad.megartsi.free.fr/doc/dea.pdf >. Consulté le 5 avril 2004.

Nakagawa, T. and M. Kijima (1989). "*Replacement policies for a cumulative damage model with minimal repair at failure.*" IEEE Transactions on Reliability, 38(5): pp. 581-584.

Nakagawa, T. and M. Kowada (1983). "*Analysis of a system with minimal repair and its application to replacement policy.*" European Journal of Operational Research, 12(2): pp. 176-182.

Ohnishi, M., T. Morioka and T. Ibaraki (1994). "*Optimal minimal-repair and replacement problem of discrete-time Markovian deterioration system under incomplete state information.*" Computers & Industrial Engineering, 27(1-4): pp. 409-412.

Older G.J. and Suri R. (1980). "*Time optimal of part-routing in a manufacturing system with failure prone machines.*" In Proc. 19th IEEE Conf. Decis. Contr., Alburquerque, NM, pp. 722–727.

Phelps, R. I. (1981)."*Replacement policies under minimal repair.*" Journal of the Operational Research Society, 32(7): pp. 549-554.

Rajagopalan, S., M. R. Singh and T. E. Morton (1998). "*Capacity expansion and replacement in growing markets with uncertain technological breakthroughs.*" Management Science, 44(1): pp. 12-30.

Rishel, R. (1975). "*Control of systems with jump markov disturbances.*" Jt Autom Control Conf, 15th, Proc, Jun 18-21: pp. 246-249.

Sader, B. H. and C. D. Sorensen (2003). "*Deterministic and stochastic dynamic modeling of continuous manufacturing systems using analogies to electrical systems.*" Proceedings of the 2003 Simulation Conference: Driving Innovation, Dec 7-10 2003, New Orleans, LA, United States, Institute of Electrical and Electronics Engineers Inc.

Savkin A. V. (2003). "*Optimal Distributed Real-Time Scheduling of Flexible Manufacturing Networks Modeled as Hybrid Dynamical Systems.*" 42nd IEEE Conference on Decision and Control, Dec 9-12 2003, Maui, HI, United States, Institute of Electrical and Electronics Engineers Inc.

Savsar M. (2008). "*Modelling of multi-stage production lines with maintenance operations.*" International Journal of Computer Integrated Manufacturing, 21(4): pp. 396-406.

Schneeweiss C. and Schröder H. (1992). "*Planning and scheduling the repair shops of the Deutsche Lufthansa AG: A hierarchical approach.*" Production and Operations Management, 1: pp. 22-33.

Sethi S. P., Taksar M. I. and Zhang Q. (1992). "*Capacity and production decisions in stochastic manufacturing systems: an asymptotic optimal hierarchical approach.*" Production and Operations Management, 1(4): pp. 367-392

Sethi S. P., Taksar M. and Zhang Q. (1995). "*Hierarchical capacity expansion and production planning decisions in stochastic manufacturing systems.*" Journal of Operations Management, 12(3-4): pp. 331-352.

Sethi S. P. and Zhang Q. (1994). "*Hierarchical Decision Making in Stochactic Manufacturing Systems.*" Birkhâser.

Shirmohammadi A. H., Zhang Z. G. and Love E. (2007). "*A computational model for determining the optimal preventive maintenance policy with random breakdowns and imperfect repairs.*" IEEE Transactions on Reliability, 56(2): pp. 332-339.

Stecke, K. E. and J. J. Solberg (1981). "*Loading and control policies for a flexible manufacturing system.*" International Journal of Production Research, 19(5): pp. 481-490.

Thesen, A. (1999). "*Some simple, but efficient, push and pull heuristics for production sequencing for certain flexible manufacturing systems.*" International Journal of Production Research, 37(7): pp. 1525-1539.

Valdez-Flores C. and Feldman R. (1989). "*A survey of preventive maintenance models for stochastically deteriorating single-unit systems.*" Naval Research Logistics Quarterly, 36: pp. 419–446.

Viles, E., D. Puente, M. J. Alvarez and F. Alonso (2007). "*Improving the corrective maintenance of an electronic system for trains.*" Journal of Quality in Maintenance Engineering, 13(1): pp. 75-87.

Wang, H. (2002). "*A survey of maintenance policies of deteriorating systems.*" European Journal of Operational Research, 139(3): pp. 469-489.

Williams, D. and A. Wirth (1996). "*New heuristic for a single machine scheduling problem with set-up times.*" Journal of the Operational Research Society, 47(1): pp. 175-180.

Xiaojun, Z., X. Lifeng and L. Jay (2007). "*Reliability-centered predictive maintenance scheduling for a continuously monitored system subject to degradation.*" Reliability Engineering & System Safety, 92(4): pp. 530-534.

Yan H. & Zhang Q. (1995). "*A numerical method in optimal production and setup scheduling of stochastic manufacturing systems: summary of results.*" Paper presented at the Proceedings of 1995 34th IEEE Conference on Decision and Control, 13-15 Dec. New Orleans, LA, USA

Yan H. and Zhang Q. (1997). "*Numerical method in optimal production and setup scheduling to stochastic manufacturing systems.*" IEEE Transactions on Automatic Control, 42: 1452-1455.

Yu-Hung, C. (2008). "*Optimal age-replacement policy under an imperfect renewing free-replacement warranty.*" IEEE Transactions on Reliability, 57(1): pp. 125-133.

Yuan Lin, Z. (1994). "*A bivariate optimal replacement policy for a repairable system.*" Journal of Applied Probability, 31(4): pp. 1123-1127.

Yuan, L. Z., Richard C. M. Yam and Zuo M. J. (2001). "*Optimal replacement policy for a deteriorating production system with preventive maintenace.*" International Journal of Systems Science, 32(10): pp. 1193-1198.

Yuan, L. Z., Richard C. M. Yam and Zuo M. J. (2007). "*A bivariate optimal replacement policy for a multistate repairable system.*" Reliability Engineering and System Safety, 92(4): pp. 535-542

Zhang F., Roundy R., Cakanyildirim M. and Huh W. T. (2004). "*Optimal capacity expansion for multi-product, multi-machine manufacturing systems with stochastic demand.*" IIE Transactions (Institute of Industrial Engineers), 36(1): pp. 23-36.