Resilient Peer-to-Peer Energy Trading: Addressing Price Uncertainty and Communication Delays in Decentralized Markets

by

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Négociation Résiliente d'Énergie Pair-à-Pair : Gérer l'Incertitude des Prix et les Retards de Communication sur les Marchés Décentralisés

Mostafa YAGHOUBI

RÉSUMÉ

Cet article présente un cadre robuste et entièrement décentralisé pour la négociation d'énergie pair-à-pair (P2P), conçu pour les réseaux électriques intelligents, afin de relever des défis critiques tels que l'incertitude des prix sur le marché de gros et les retards dans les réseaux de communication. Ce cadre permet aux prosommateurs, équipés de systèmes de stockage d'énergie et participant à des programmes de réponse à la demande (DR), de réaliser des transactions énergétiques dynamiques avec des acteurs locaux et des fournisseurs d'énergie. Grâce à une approche d'optimisation robuste, le modèle atténue efficacement les risques liés à la volatilité des prix de détail, garantissant une prise de décision fiable dans des conditions de marché incertaines. Les retards de communication sont explicitement pris en compte pour évaluer leur impact sur la convergence du marché, l'efficacité des décisions et la stabilité opérationnelle. En utilisant l'algorithme Méthode Rapide de Direction Alternée des Multiplicateurs, le cadre proposé permet une compensation décentralisée du marché sans nécessiter de nœud de supervision central, préservant ainsi la confidentialité et assurant une évolutivité. Des mécanismes d'interaction flexibles entre acheteurs, vendeurs et fournisseurs améliorent le bien-être social grâce à une participation optimisée aux marchés locaux et de détail. Les résultats de simulation valident l'efficacité du cadre dans divers scénarios, incluant des conditions de tarification déterministes et stochastiques, avec et sans retards de communication. Les résultats démontrent la robustesse du modèle proposé pour maximiser le bien-être social, réduire la dépendance au réseau et relever les défis opérationnels réels des marchés énergétiques décentralisés, contribuant ainsi au développement de systèmes de négociation d'énergie P2P évolutifs et durables.

Mots-clés: marché pair-à-pair, délais de communication, optimisation décentralisée robuste, méthode de multiplicateurs à direction alternée rapide, prosommateur, incertitude des prix

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ABSTRACT

This paper presents a robust, fully decentralized peer-to-peer (P2P) energy trading framework tailored for smart grids, that address critical challenges such as wholesale market price uncertainty and communication network delays. The framework enables prosumers, equipped with energy storage systems and participating in demand response (DR) programs, to engage in dynamic energy transactions with local participants and retailers. By employing a robust optimization approach, the model effectively mitigates risks associated with retail price volatility, ensuring reliable decision-making under uncertain market conditions. Communication delays are explicitly incorporated to assess their impact on market convergence, decision efficiency, and operational stability. By using the Fast-Alternating Direction Method of Multipliers (FADMM) algorithm, the proposed framework achieves decentralized market clearing without the need for a central supervisory node, thereby preserving privacy and ensuring scalability. Flexible interaction mechanisms between buyers, sellers, and retailers enhance social welfare through optimized local and retail market participation. Simulation results validate the framework's efficacy under diverse scenarios, including deterministic and stochastic pricing conditions, with and without communication delays. The findings demonstrate the robustness of the proposed model in maximizing social welfare, reducing grid dependency, and addressing real-world operational challenges in decentralized energy markets, contributing to the advancement of scalable and sustainable P2P energy trading systems.

Keywords: Peer-to-Peer Market, Communication Delays, Robust Decentralized Optimization, Fast Alternating Direction Method of Multipliers (FADMM), Prosumer, Price Uncertainty

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LIST OF ABBREVIATIONS

P2P	Peer-to-Peer
DR	Demand Response
FADMM	Fast Alternating Direction Method of Multipliers
DER	Distributed Energy Resources
RES	Renewable Energy Sources
ESS	Energy Storage System
DA	Day-Ahead Market
KKT	Karush-Kuhn-Tucker
PR	Price Resilience
LP	Linear Programming
SOC	State of Charge
DRP	Demand Response Program
DAPP	Distributed Adaptive Primal Problem
MISO	Mixed-Integer Second-Order Programming
FOP	Feasible Operating Point
NOM	Network Optimization Model

LIST OF SYMBOLS AND UNITS OF MEASUREMENTS

λ_{ij}	Price bid from seller i to buyer j.
λ_{ir}	Price bid from seller i to retailer r.
λ_{rj}	Price bid from retailer r to buyer j.
λ_{DA}	Wholesale market price.
x_{ij}	Energy traded from seller i to buyer j.
x_{ir}	Energy traded from seller i to retailer r.
y_{jr}	Energy traded from buyer j to retailer r.
z_r	Energy traded by retailer r with the wholesale market.
y_{jt}	Energy demand of buyer j at time t.
x_{it}	Energy production of seller i at time t.
z_{rt}	Energy generation or trading decision of retailer r at time t.
Γ_λ	Uncertainty budget for wholesale price variations.
η_{ch}	Charging efficiency of storage systems.
η_{dch}	Discharging efficiency of storage systems.
E_{jt}	Stored energy level of buyer j at time t.
$E_{\min}, E_{\max}$	Minimum and maximum energy storage limits.
b_{ch}, b_{dch}	Binary variables representing charging or discharging mode.
$U(y_{jt})$	Utility function of buyer j for consuming energy.
$C(x_{it})$	Cost function of seller i for producing energy.

$C(z_{rt})$	Cost function of retailer r for generating or trading energy.
k_{ij}	Communication delay between seller i and buyer j.
k_{ir}	Communication delay between seller i and retailer r.
k_{rj}	Communication delay between retailer r and buyer j.
$G(t)$	Communication network graph at time t.
ρ	Penalty parameter in the FADMM algorithm.

INTRODUCTION

Over the past decade, the integration of distributed energy resources (DERs) has revolutionized the landscape of energy generation, distribution, and consumption. This transition has empowered consumers to become active prosumers—entities that can both produce and consume energy—enabling more localized and decentralized energy systems. Despite its benefits, this paradigm shift has introduced challenges such as grid instability, coordination complexities, and efficient energy utilization among prosumers (O. Jogunola et al., 2017; Y. Wang et al., 2019). To address these challenges, decentralized peer-to-peer (P2P) energy trading frameworks have gained prominence as viable solutions. These frameworks allow prosumers to trade surplus energy directly, avoiding reliance on centralized intermediaries. This decentralization enhances system flexibility and resilience by enabling localized energy optimization and reducing dependency on grid infrastructure. Furthermore, the integration of demand response (DR) programs and energy storage systems within prosumer models facilitates strategic energy management, enabling actions like charging during low-price periods and discharging during peak-demand intervals (W. Tushar et al., 2020; D. Gregoratti and J. Matamoros, 2015). A critical factor in the successful implementation of P2P energy trading systems is the role of communication infrastructure. Effective communication enables seamless transactions between prosumers, but it is often hindered by network latency, jitter, and other environmental disruptions. These challenges are particularly significant in distributed consensus algorithms, where communication delays can disrupt convergence and lead to inefficiencies in market operations (Z. Zhang and M.-Y. Chow, 2011; O. Jogunola et al., 2021).

Another major challenge in decentralized energy markets is retail price uncertainty, driven by volatile wholesale market dynamics. Retail price fluctuations directly impact prosumer decision-making, creating instability in energy markets. Robust optimization techniques have been introduced to mitigate these risks, allowing prosumers to navigate market uncertainties while ensuring economic efficiency (M. Mehdinejad et al., 2020; H. Zhang et al., 2016).

Advancements in distributed optimization algorithms have further facilitated the development of robust decentralized energy markets. Among these, the Fast-Alternating Direction Method of Multipliers (FADMM) has emerged as an effective tool for decentralized market clearing. Unlike centralized methods that require aggregating participant data and compromising privacy, FADMM preserves scalability and privacy by eliminating the need for a supervisory node. These characteristics make FADMM an ideal approach for decentralized energy systems (S. Boyd et al., 2011; Z. Zeyu and H. Wang, 2023).

In this context, this paper proposes a robust and fully decentralized P2P energy trading framework tailored for smart grids. The framework incorporates robust optimization to address price uncertainty and models communication delays to evaluate their impact on market performance. By leveraging FADMM for market clearing, the proposed framework ensures robustness, privacy, and resilience, aiming to enhance social welfare and promote the adoption of decentralized energy trading platforms.

CHAPTER 1

RELATED WORKS

The emergence of peer-to-peer (P2P) energy trading systems, propelled by advancements in distributed energy resources (DERs), has transformed traditional energy markets. These systems aim to address challenges such as storage optimization, demand response (DR) program participation, communication constraints, and uncertainties posed by upstream market prices. While P2P frameworks empower prosumers to engage in direct energy transactions, several complexities persist, particularly in designing fully decentralized systems that maintain privacy, scalability, and robustness.

Numerous studies have explored decentralized market models to overcome the limitations of centralized frameworks. For instance, a fully decentralized P2P market using a primal-dual gradient approach was proposed, enabling participants to negotiate bilaterally over prices and energy volumes (M. Khorasany et al., 2020). However, this approach is limited to convex models and lacks comprehensive robustness against uncertainties. Similarly, a consensus-based approach was introduced to address product differentiation in decentralized P2P markets (E. Sorin et al., 2018), while another study leveraged blockchain technology to enhance privacy and transaction security (A. Esmat et al., 2021). Advanced frameworks incorporating storage systems and DR programs have demonstrated potential in optimizing prosumer energy use. A hierarchical and decentralized P2P trading system utilized home batteries and shiftable appliances to facilitate transactions (M. Elkazaz et al., 2021). In parallel, a blockchain-based virtual power plant was employed to support secure and decentralized energy trading among prosumers (T. Cioara et al., 2021).

Uncertainty management has gained increasing attention in P2P energy markets. A study on load uncertainty integrated carbon emissions and transaction costs, presenting a robust model for grid-connected prosumers (H. T. Doan et al., 2022). Another investigation addressed demand and supply uncertainties by developing efficient trading mechanisms, employing novel allocation methods to mitigate the impacts of unreliable prosumers on energy markets (A. Anastopoulou et al., 2021).

Despite these advancements, the integration of communication delays and upstream market price uncertainties remains an open challenge. Recent efforts have sought to address specific aspects of these issues. For example, a dynamic pricing model for electric vehicle (EV) charging stations was introduced, enhancing both local market participation and profit maximization (S. Aznavi et al., 2020). Similarly, a cost-sharing mechanism for energy communities under uncertain pricing scenarios highlighted the importance of robust optimization techniques (M. Grzanić et al., 2021). Another study proposed a framework for distributed energy trading, incorporating communication delay models and emphasizing the trade-offs between local and global trading under constrained communication scenarios (K. Anoh and B. Adebisi, 2017).

Communication delays significantly impact decentralized energy systems. A study analyzed the convergence of asynchronous P2P markets under communication delays, demonstrating the scalability and robustness of an ADMM-based algorithm (A. Dong et al., 2022). The study proposed Gaussian and advanced delay models, showing that asynchronous methods improve efficiency and reduce convergence time in markets with variable communication times. Additionally, a Distributed Adaptive Primal (DAP) algorithm was introduced to address communication reliability in P2P trading over unreliable communication links. This algorithm reduces delays by 20% compared to conventional methods, effectively optimizing performance in congested network topologies (O. Jogunola et al., 2018). Furthermore, a hybrid wireless sensor network (WSN)-based framework enhanced communication reliability for P2P trading (K. Anoh et al., 2018), addressing latency and bandwidth challenges in real-time decentralized systems.

Existing studies often fail to simultaneously address communication delays and price uncertainties within a unified framework. While one study focused on optimal routing to mitigate network congestion (O. Jogunola et al., 2020), it did not account for market price volatility. Another study developed a robust decentralized P2P market-clearing algorithm but did not fully integrate communication delay modeling (M. Mehdinejad et al., 2022). This study builds upon these findings by presenting a novel framework for decentralized P2P energy trading. The framework integrates robust optimization techniques to manage wholesale market price uncertainties and incorporates delay-tolerant market-clearing mechanisms that address communication constraints. By simultaneously tackling price volatility and communication delays, the proposed model enhances the robustness, reliability, and efficiency of decentralized energy markets, effectively bridging critical gaps identified in the literature.

1.1 Novelties and Contributions

This paper contributes to the existing body of research by proposing an innovative framework that addresses the dual challenges of communication constraints and price uncertainties in decentralized P2P energy markets. The key novelties and contributions are summarized as follows:

1. **Fully Decentralized Market Clearing**: A novel P2P energy market model is developed that eliminates dependence on centralized nodes, enabling autonomous prosumer participation while incorporating delay-tolerant mechanisms for robust and scalable operations.
2. **Robust Optimization for Price Volatility**: The model introduces robust optimization techniques to address wholesale market price uncertainties, enhancing the reliability of decision-making processes for prosumers.
3. **Delay-Tolerant Algorithms**: A decentralized Fast Alternating Direction Method of Multipliers (FADMM) is employed to ensure efficient market clearing under communication constraints, improving the system's resilience to delays.
4. **Unified Framework**: The proposed framework uniquely integrates communication network constraints and market price uncertainties, achieving robustness, reliability, and operational efficiency in dynamic energy markets.

1.2 Structure of the Paper

The remainder of this paper is structured as follows:

Chapter 2: Introduces the conceptual framework and mathematical formulation of the proposed P2P energy trading market, detailing its decentralized architecture and operational mechanisms.

Chapter 3: This section presents numerical studies and simulation results, including case studies analyzing the proposed framework under scenarios with and without communication delays and price uncertainties.

Chapter 4: Presents the conclusions of this paper and future works.

CHAPTER 2

SYSTEM MODEL AND METHODOLOGY (P2P ENERGY TRADING PLATFORM)

This chapter introduces the conceptual framework and mathematical formulation of the proposed peer-to-peer (P2P) energy trading market. It provides a detailed exploration of the decentralized architecture and operational mechanisms that underpin the platform, emphasizing its role in enabling efficient and equitable energy transactions. By addressing key challenges such as price uncertainty and communication delays, this model aims to foster a robust, scalable, and transparent energy trading system within smart grids. The methodology outlined in this chapter lays the groundwork for implementing a practical and innovative approach to decentralized energy management.

2.1 Peer-to-Peer Energy Trading Platform

2.1.1 Market Overview and Problem Definition

In the proposed market platform, it is assumed that prosumers, as local market participants, are categorized into two groups: producers and consumers. Their roles remain fixed throughout the scheduling period. Specifically, local prosumers who sell their surplus energy act as sellers throughout the scheduling period, while those who purchase energy to compensate for their shortage act as buyers. In other words, local market participants in the proposed market are defined based on their net production or demand, which is determined by the aggregate production and consumption of all their assets. Retailers operating within the prosumer neighborhood simultaneously assume the roles of both buyers and sellers. In addition to engaging with local participants, retailers are also capable of energy exchanges with the network under the wholesale market structure. The market participants in the proposed market have flexible production and consumption capabilities, and the market is settled through peer-to-peer (P2P) interactions between all participants. The structure of the proposed market corresponds to a forward market, focusing on market settlement for a 24-hour scheduling period.

As illustrated in Figure 2.1, buyers, upon receiving price bids from local sellers (λ_{ij}) and active retailers within their geographical area (λ_{rj}), determine both their overall demand (y_j) and the specific quantities they wish to purchase from each seller (y_{ji}) and retailer (y_{jr}), aligned with their respective objectives. Meanwhile, local sellers, after sending their price bids to local buyers (λ_{ij}) and retailers operating in their geographical area (λ_{ir}), determine their total production (x_i) and the amount to sell to each individual local buyer (x_{ij}) and retailer (x_{ir}) based on the buyers' announced demands. Unlike the local buyers and sellers, retailers play a dual role: they offer prices to buyers (λ_{rj}), receive price bids from local sellers (λ_{ir}), and account for the wholesale market price (λ^{DA}). According to their strategic goals objectives, retailers determine their total self-generation level (z_r), the amount they will sell to individual buyers (z_{rj}), the quantity to procure from sellers (z_{ri}), and the volume of energy to trade (either buying or selling)) with the wholesale market ($z_{r,g}$).

Taking these interactions into account, the proposed model offers a fully decentralized bilateral energy market to enable peer-to-peer (P2P) trading among local players, including producers and retailers. Each market player engages in energy transactions with local sellers and retailers transparently, while maintaining data privacy. Moreover, a decentralized algorithm has been developed to clear the market for all players.

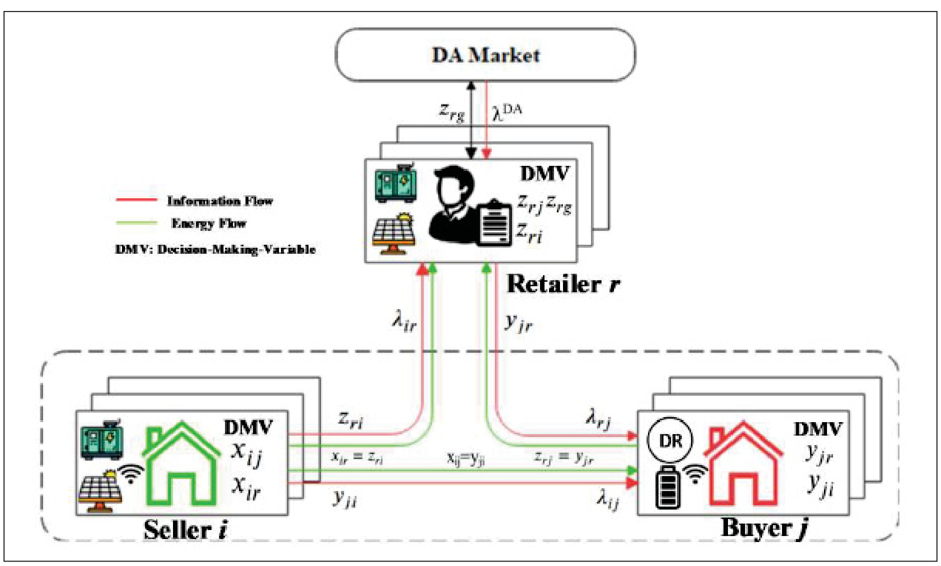

Figure 2.1 P2P Trading Platform

2.2 Mathematical Formulation of the Proposed Market

A market with $\mathcal{N}$ participants is considered, consisting of $\mathcal{N}_S$=$\{1,...,N_S\}$ local sellers, $\mathcal{N}_S$=$\{1,...,N_S\}$ local buyers, and $\mathcal{N}_S$=$\{1,...,N_S\}$ retailers operating in the target area, where $\mathcal{N}_B \cap \mathcal{N}_S \cap \mathcal{N}_R = \emptyset$. It is assumed that the roles of both local buyers and sellers remain constant throughout the scheduling period.

2.2.1 Buyers and Sellers Welfare with Retailer Revenue Consideration

In the proposed peer-to-peer (P2P) market model, consumers (buyers) can engage in simultaneous negotiations with both producers (sellers) and retailers over multiple trading periods. This dynamic allows consumers to purchase power from different retailers and local producers at varying marginal prices for each trade during each time period. Consequently, each consumer has a distinct evaluation and welfare for engaging in energy transactions across multiple time frames.

The welfare maximization of consumers can be expressed as:

$$\max_{y_{jt}} \quad \sum_{t=1}^{T} U(y_{jt}) - \sum_{t=1}^{T} \sum_{r \in N_R} \lambda_{rjt} y_{jrt} - \sum_{t=1}^{T} \sum_{i \in N_S} \lambda_{ijt} y_{jit} \tag{2.1}$$

$$U(y_{jt}) = \begin{cases} \omega_j y_{jt} - \delta_j y_{jt}^2 & y_{jt} < \frac{\omega_j}{2\delta_j} \\ \frac{\omega_j^2}{2\delta_j} & y_{jt} \geq \frac{\omega_j}{2\delta_j} \end{cases} \tag{2.2}$$

$$y_{jt} = \sum_{i=1}^{N_S} y_{jit} + y_{jt}^{\text{dch}} - y_{jt}^{\text{ch}} - y_{jt}^{\text{DR}} \tag{2.3}$$

$$\sum_{i=1}^{N_S} y_{jit} + y_{jt}^{\text{dch}} = y_{jt}^{\text{ch}} + D_{jt} + y_{jt}^{\text{DR}} \tag{2.4}$$

$$-DR^{\max} D_{jt} \leq y_{jt}^{\text{DR}} \leq DR^{\max} D_{jt} \tag{2.5}$$

$$\sum_{t=1}^{T} y_{jt}^{\text{DR}} = 0 \tag{2.6}$$

$$E_{jt} = E_{jt-1} + \eta^{\text{ch}} y_{jt}^{\text{ch}} - \frac{y_{jt}^{\text{dch}}}{\eta^{\text{dch}}} \tag{2.7}$$

$$0 \leq y_{jt}^{\text{ch}} \leq b_{jt}^{\text{ch}} y_j^{(\text{ch,max})} \tag{2.8}$$

$$0 \leq y_{jt}^{\text{dch}} \leq b_{jt}^{\text{dch}} y_j^{(\text{dch,max})} \tag{2.9}$$

$$E_j^{min} \leq E_{jt} \leq E_j^{max} \tag{2.10}$$

$$b_{jt}^{\text{ch}} + b_{jt}^{\text{dch}} \leq 1 \tag{2.11}$$

$$y_{jit}, y_{jt}^{\text{ch}}, b_{jt}^{\text{dch}}, E_{jt} \geq 0$$

$$b_{jt}^{\text{ch}}, b_{jt}^{\text{dch}} \in \{0, 1\}$$

$$y_{jt}^{\text{DR}} \text{ is free}$$

The welfare WB_j in (2.1), represents the net welfare of consumer j over the entire time horizon T. Net welfare is defined as the difference between the utility derived from energy consumption and the costs paid for energy transactions with both retailers and producers.

In (2.1), $U\left(y_{jt}\right)$ is the utility of consumer j from energy consumption at time t, while the second and third terms represent the total costs incurred by consumer j when trading energy with retailer r and producer i over multiple time periods. The welfare function captures the temporal nature of energy transactions, with marginal prices λ_{rjt} and λ_{ijt} varying across periods and parties. Here, ω_j and δ_j are positive parameters that determine how consumer j responds to price changes and represent private information. The utility function for consumer j, $U\left(y_{jt}\right)$ in (2.2), can be modeled as a piecewise quadratic function that represents satisfaction from consuming energy, given their ability to adjust demand based on price fluctuations (X. Song and J. Qu, 2014). The function follows these key properties:

1. $U_j\,(0) = 0$: No energy consumption results in zero utility, meaning that without energy use, the consumer derives no benefit.
2. $U_j^{'}(y_j) \geq 0$: The utility function is non-decreasing, meaning that consumers will continue to consume energy as long as it brings them benefit.

3. $U_j''(y_j) \leq 0$: The utility function is concave, indicating that as consumption increases, the satisfaction derived from additional energy begins to decrease, leading to a saturation point.

In (2.3), the total energy demanded by consumer j at time t, y_{jt} is determined by the sum of energy purchased from producers and retailers. This equation describes the balance of energy between producers, retailers, and consumer demand. Additionally, the energy consumed must respect the (2.4) bounds. The constraints related to demand response (DR) participation are outlined in equations (2.5)-(2.6). Equation (2.5) places a limit on the level of participation in the DR program, while equation (2.6) ensures that no load shedding occurs during the scheduling period.

The equality and inequality constraints concerning the electrical storage of buyer j are detailed in equations (2.7)-(2.11). Specifically, equation (2.7) defines the hourly energy level of storage, while the constraints governing the charging and discharging of electrical storage are described in equations (2.8) and (2.9). Finally, equation (2.10) sets limits on the minimum and maximum energy levels that can be stored. At each hour, the storage system can function in either charging mode (b_{jt}^{ch}= 1) or discharging mode (b_{jt}^{dch}= 1), as described in equation (2.11).
Complementing the buyer's role, sellers actively contribute to the P2P trading ecosystem by engaging directly with buyers. In this fully decentralized P2P framework, local sellers negotiate transaction terms, including energy volumes and prices that reflect varying marginal costs, enabling flexibility and adaptability. The welfare of the seller i, WS_i over a period of T is represented by equation (2.12).

$$\max_{x} \quad WS_i = \sum_{t=1}^{T} \sum_{j \in N_B} \lambda_{ijt} x_{ijt} + \sum_{t=1}^{T} \sum_{r \in N_R} \lambda_{irt} x_{irt} - \sum_{t=1}^{T} C(x_{it}) \tag{2.12}$$

$$C(x_{it}) = \alpha_i x_{it}^2 + \beta_i x_{it} + \gamma_i \tag{2.13}$$

$$x_{it} = \sum_{j} x_{ijt} + \sum_{r} x_{irt} \tag{2.14}$$

$$x_{it}^{\min} \leq x_{it} \leq x_{it}^{\max} \tag{2.15}$$

In (2.12), the first two terms represent the revenue generated from selling energy to consumers j and retailers r, while the third term accounts for the production cost of the energy sold. The revenue is based on the marginal prices λ_{ijt} and λ_{irt} for transactions with consumers and retailers, respectively. The cost function $C\left(x_{it}\right)$, shown in equation (2.13), describes the cost of producing x_{it} units of energy by the seller's local generator (J. J. Grainger, W. D. Stevenson et al., 2003). It is modeled as a convex quadratic function, where the parameters α_i , β_i , and γ_i are pre-determined constants that represent the specific cost structure of seller i. (2.14) ensures that the total energy produced by the seller i is distributed among the buyers and retailers. Finally, (2.15) sets the upper and lower bounds for the seller's energy production, ensuring that it stays within the minimum and maximum production limits. Building on the interactions of sellers and buyers, retailers r serve a pivotal role in the proposed P2P energy market, engaging in transactions with both local producers and consumers while simultaneously managing energy trades with the wholesale market. The revenue of retailer r, given in (2.16), is derived from multiple sources.

$$\max_{z} \quad \text{Revenue}_r = \sum_{t=1}^{T} \lambda_t^{\text{DA}} z_{rt,g} + \sum_{t=1}^{T} \sum_{j} \lambda_{rjt} z_{rjt} - \sum_{t=1}^{T} \sum_{i} \lambda_{irt} z_{rit} - \sum_{t=1}^{T} C(z_{rt}) \tag{2.16}$$

s.t.

$$z_{rt} + \sum_{i} z_{rit} = z_{rt,\,g} + \sum_{j} z_{rjt} \tag{2.17}$$

$$C\left(z_{rt}\right) = \alpha_r z_{rt}^2 + \beta_r z_{rt} + \gamma_r \tag{2.18}$$

$$z_{rt}^{min} \leq z_{rt} \leq z_{rt}^{max} \tag{2.19}$$

$$-z_{rt}^{max} \leq z_{rt,\,g} \leq z_{rt}^{max} \tag{2.20}$$

$$x_{ijt} = y_{jit} \qquad : \lambda_{ijt} \tag{2.21}$$

$$x_{irt} = z_{rit} \qquad : \lambda_{irt} \tag{2.22}$$

$$z_{rjt} = y_{jrt} \qquad : \lambda_{rjt} \tag{2.23}$$

The first term in (2.16) accounts for the revenue (or cost) from energy transactions with the wholesale market, where $z_{rt,g}$ is the traded energy and λ_t^{DA} represents the corresponding price. If $z_{rt,g}$ is positive, it represents revenue; if negative, it is regarded as a cost. The second term in (2.16) describes the revenue earned from selling energy to local consumers j, while the third term indicates the cost of purchasing energy from local producers i. The last term in (2.16) accounts for the retailer's energy production cost. (2.17) ensures that the total energy traded by the retailer with the grid, local producers, and consumers remains balanced. The sum of the energy generated by the retailer and the energy purchased from producers must equal the total energy sold to both consumers and the wholesale market. This balance ensures that the retailer does not over-trade or under-trade energy in any given period. If $z_{rt}> 0$, it indicates that the retailer has used its own generator to supply energy in the market. The cost of operating this generator is given by the quadratic cost function $C\left(z_{rt}\right)$.

The cost function $C\left(z_{rt}\right)$, defined in (2.18) is a convex and quadratic function that describes the retailer's cost for generating energy. The coefficients α_r , β_r , and γ_r are specific to each retailer and reflect their production capabilities and associated risks. This function mirrors the cost functions used by local producers, ensuring that energy production costs increase with the amount of energy generated. The constraint in (2.19) bounds the energy produced by the retailer within a minimum and maximum range, while (2.20) defines the bounds for energy traded with the wholesale market. Specifically, the term $z_{rt,\ g}$ must lie within the range $z_{rt}^{min}{\le}z_{rt}{\le}z_{rt}^{max}$, meaning that the retailer cannot exceed a certain threshold of energy trade, whether buying or selling, with the wholesale market.

Equations (2.21), (2.22), and (2.23) impose the supply and demand constraints on the energy exchanged between buyers, sellers, and retailers in the P2P trading system. These ensure that energy flows align with the agreed prices and quantities during the trades. For example, (2.21) ensures that the energy sold by producer i to buyer j matches the amount that buyer j purchases. Similarly, (2.22) governs the energy traded between producers and retailers, and (2.23) applies to transactions between retailers and consumers. In these equations, λ_{ijt}, λ_{irt}, λ_{rjt} describes the marginal cost (agreed price) for each transaction between the players.

2.2.2 Mathematical Modeling of the Optimization Problem

The objective of the proposed problem is to maximize the overall social welfare of the local participants (prosumers) in the energy trading market. The optimization problem is formulated to simultaneously maximize the welfare of all local consumers and the revenue of retailers. The combined objective function is expressed as:

$$\max_{x,y,z} \quad \sum_{t=1}^{T} \left(\sum_{j=1}^{N_B} U(y_{jt}) - \sum_{i=1}^{N_S} C(x_{it}) + \sum_{r=1}^{N_R} \left(\lambda_t^{\mathrm{DA}} z_{rt,g} - C(z_{rt}) \right) \right) \tag{2.24}$$

s.t.

$$\textit{Equations } (2.2) \textit{ to } (2.11),\ (2.13) \textit{ to } (2.15),\ (2.17) \textit{ to } (2.20),\ (2.21) \textit{ to } (2.23)$$

In this paper, the objective function aims to maximize the overall social welfare by optimizing the amount of energy trading between buyers, sellers, and retailers. This is achieved by balancing consumer welfare and retailer revenue while considering the associated costs and constraints. The optimization problem consolidates the objective functions of both sellers and buyers to ensure maximum benefit for all participants in the local energy market.

2.2.3 Design of a Decentralized Market-Clearing Algorithm (FADMM)

The optimization problem in equation (2.24) can be solved using a centralized approach; however, implementing such an approach requires a centralized controller with full knowledge of all market participants' characteristics, which compromises their privacy. The centralized controller would need access to all participants' data to solve the problem. To mitigate this, a decentralized optimization algorithm is proposed in this paper, where all data and computations related to each participant (agent) are handled locally by the participants themselves. The optimization problem is divided into multiple sub-problems using the dual decomposition principle (M. Zhu and S. Martínez, 2011), and these sub-problems are then solved in a distributed manner. In this method, each agent, with limited information about other agents, works to optimize their own sub-problem as part of the larger problem.

The Lagrangian of the main problem (2.24) is formulated by relaxing the global (coupled) and local constraints, with Lagrange multipliers applied as follows:

$$\begin{aligned} L = \sum_{t=1}^{T}\Bigg(\sum_{j=1}^{N_B} U(y_{jt}) + \sum_{r=1}^{N_R} \lambda^{\mathrm{DA}} z_{rt,g} - \sum_{i=1}^{N_S} C_i(x_{it}) - \sum_{r=1}^{N_R} C_r(z_{rt})\Bigg) + \sum_{t=1}^{T}\Bigg(\sum_{i=1}^{N_B}\sum_{j=1}^{N_S} \lambda_{ijt}(x_{ijt} - y_{jit}) \\ + \sum_{i=1}^{N_S}\sum_{r=1}^{N_R} \lambda_{irt}(x_{irt} - z_{rit}) + \sum_{r=1}^{N_R}\sum_{j=1}^{N_B} \lambda_{rjt}(z_{rjt} - y_{jrt}) - \sum \mu_g - \sum_{i=1}^{N_B}\sum_{j=1}^{N_S} \rho\|x_{ijt} - y_{jit}\|_2^2 \\ - \sum_{i=1}^{N_S}\sum_{r=1}^{N_R} \rho\|x_{irt} - z_{rit}\|_2^2 - \sum_{r=1}^{N_R}\sum_{j=1}^{N_B} \rho\|z_{rjt} - y_{jrt}\|_2^2\Bigg) \end{aligned} \tag{2.25}$$

The Lagrangian supremum over the variables of the main problem yields the following dual function:

$$\begin{aligned} \Upsilon = \sup_{x,y,z}\Bigg\{L = \sum_{i\in N_S,\ t\in T}\Bigg(\arg \max_{x_{it}^{\min}\le x_{it}\le x_{it}^{\max}} S_i\Bigg) + \sum_{j\in N_B,\ t\in T}\Bigg(\arg \max_{y_{jt}^{\min}\le y_{jt}\le y_{jt}^{\max}} B_j\Bigg) \\ + \sum_{r\in N_R,\ t\in T}\Bigg(\arg \max_{z_{rt}^{\min}\le z_{rt}\le z_{rt}^{\max}} R_r\Bigg)\Bigg\} \end{aligned} \tag{2.26}$$

In (2.26), $\mathfrak{s}_i$ represents a sub-problem of the main problem ((2.24)), which is locally maximized by seller i in the context of energy trading with buyers and active retailers. Similarly, $\mathfrak{B}_j$ is a sub-problem that is locally maximized by buyer j for energy trading with local sellers and active retailers at buyer j location. Likewise, $\mathcal{R}_r$ is a sub-problem that is locally maximized by retailer r for energy trading with the upstream grid, as well as with local buyers and sellers. The sub-problems $\mathfrak{s}_i$, $\mathfrak{B}_j$, and $\mathcal{R}_r$ are defined as follows.

$$S_i \triangleq \sum_{t=1}^{T}\left(\sum_{j\in N_B}\lambda_{ijt}x_{ijt}+\sum_{r\in N_R}\lambda_{irt}x_{irt}-C(x_{it})-\sum\mu_{it}g_{it}-\sum_{j=1}^{N_B}\rho\|x_{ijt}-y_{jit}\|_2^2-\sum_{r\in N_R}\rho\|x_{irt}-z_{rit}\|_2^2\right) \tag{2.27}$$

$$B_j \triangleq \sum_{t=1}^{T}\left(U_j(y_{jt})-\sum_{r\in N_R}\lambda_{rjt}y_{jrt}-\sum_{i\in N_S}\lambda_{ijt}y_{jit}-\sum\mu_{jt}g_{jt}-\sum_{i=1}^{N_S}\rho\|x_{ijt}-y_{jit}\|_2^2-\sum_{r\in N_R}\rho\|z_{rjt}-y_{jrt}\|_2^2\right) \tag{2.28}$$

$$R_r \triangleq \sum_{t=1}^{T}\left(\lambda^{\mathrm{DA}}z_{rt,g}+\sum_{j}\lambda_{rjt}z_{rjt}-\sum_{i}\lambda_{irt}z_{rit}-C(z_{rt})-\sum\mu_{rt}g_{rt}-\sum_{i=1}^{N_S}\rho\|x_{irt}-z_{rit}\|_2^2-\sum_{j=1}^{N_B}\rho\|z_{rjt}-y_{jrt}\|_2^2\right) \tag{2.29}$$

Equations (2.30)-(2.32) define the optimization problems for the local buyer, local seller, and retailer, respectively, as part of the decentralized market design. Each of these problems can be solved by maximizing the objectives formulated in Equations (2.27) to (2.29), which represent welfare and revenue functions for the respective participants. While methods like the interior-point algorithm (M. Khorasany et al., 2011) can be employed to achieve optimal solutions for each agent, solving these problems independently for every participant can be computationally expensive and may pose challenges in scaling efficiently to real-time market conditions.

$$\max_{y_j} \quad B_j \tag{2.30}$$

s.t.

$$\mathit{Equation}\ (2.2)-(2.11)$$

$$\max_{\mathrm{x_i}} \quad \mathfrak{s}_\mathrm{i} \tag{2.31}$$

s.t.

$$\textit{Equation}\ (2.13) - (2.15)$$

$$\max_{\mathrm{z_r}} \quad \mathcal{R}_{\mathrm{r}} \tag{2.32}$$

s.t.

$$\textit{Equation}\ (2.17) - (2.20)$$

Equations (2.33)-(2.35) describe the iterative update of the dual variables λ_{ijt}, λ_{irt}, λ_{rjt} in the optimization process using the standard ADMM. The updates are performed to minimize the dual residuals and ensure convergence of the decentralized energy market-clearing algorithm. The penalty parameter ρ controls the step size in these updates, ensuring a balance between convergence speed and stability. Equations (2.36)-(2.38) impose convergence criteria for the ADMM iterations. The stopping condition is defined by the error tolerance (ε), which determines whether the difference between successive iterations falls within an acceptable range. Meeting these criteria ensures that the optimization process converges to a feasible and optimal solution for all players in the market.

$$\lambda_{ijt}^{k} = \lambda_{ijt}^{k-1} - \rho(x_{ijt}^{k} - y_{jit}^{k}) \tag{2.33}$$

$$\lambda_{irt}^{k} = \lambda_{irt}^{k-1} - \rho(x_{irt}^{k} - z_{rit}^{k}) \tag{2.34}$$

$$\lambda_{rjt}^{k} = \lambda_{rjt}^{k-1} - \rho(z_{rjt}^{k} - y_{jrt}^{k}) \tag{2.35}$$

$$|\lambda_{ijt}^{k} - \lambda_{ijt}^{k-1}| \leq \epsilon \tag{2.36}$$

$$|\lambda_{irt}^{k} - \lambda_{irt}^{k-1}| \leq \epsilon \tag{2.37}$$

$$|\lambda_{rjt}^{k} - \lambda_{rjt}^{k-1}| \leq \epsilon \tag{2.38}$$

To accelerate the convergence of the standard ADMM, instead of using a constant step size ρ for updating the dual variable, α^k is used. This coefficient is obtained through the following relations (M. H. Ullah and J.-D. Park, 2021).

$$\mu^k = \frac{1 + \sqrt{1 + 4(\mu^{k-1})^2}}{2} \tag{2.39}$$

$$\alpha^k = \frac{\mu^{k-1} - 1}{\mu^k} \tag{2.40}$$

Therefore, (2.33) to (2.35) are modified as follows:

$$\lambda_{ijt}^{k} = \lambda_{ijt}^{k-1} - \alpha^k (x_{ijt}^{k} - y_{jit}^{k}) \tag{2.41}$$

$$\lambda_{irt}^{k} = \lambda_{irt}^{k-1} - \alpha^k (x_{irt}^{k} - z_{rit}^{k}) \tag{2.42}$$

$$\lambda_{rjt}^{k} = \lambda_{rjt}^{k-1} - \alpha^k (z_{rjt}^{k} - y_{jrt}^{k}) \tag{2.43}$$

2.2.4 Uncertainty Modeling of Energy Price (Pool Market)

In energy markets, uncertainty in price predictions poses a significant challenge, especially for buyers and retailers who depend on fluctuating wholesale market prices. Robust optimization is a suitable method for modeling this uncertainty when only limited data on price variations is available. This approach uses an uncertainty set, denoted as U, to define the potential range of price deviations. For local buyers, whose purchase prices from the retail market are tied to wholesale prices, incorporating such an uncertainty set ensures a more reliable model. This paper utilizes a polyhedral uncertainty set U, which is a structured representation of price variability as described in detail in reference (P. Xiong et al., 2017). The uncertainty set U for modeling wholesale market price variations is defined as follows:

$$U_\lambda = \left\{ \lambda_t^{DA} \in \mathbb{R}^+ : \Gamma_\lambda \leq \frac{\sum \lambda_t^{DA}}{\sum \overline{\lambda}_t^{DA}} \leq \overline{\Gamma}_\lambda \, \forall t, \lambda_t^{DA} \in \left[\lambda_t^{DA,\min}, \lambda_t^{DA,\max} \right] \right\} \tag{2.44}$$

The wholesale market price range is defined by $\lambda_t^{DA,max}$ as the upper limit and $\lambda_t^{DA,min}$ as the lower limit. These bounds are applied using uncertainty budgets (Γ) on the wholesale market price, with $\overline{\Gamma}_\lambda$ and $\underline{\Gamma}_\lambda$ controlling the level of conservatism for U_λ. The uncertain parameter ($\widetilde{\lambda}_t^{DA}$) appears in the retailer's objective function, which can be reformulated as a constraint. This formulation can be expressed as follows:

$$\max_z \text{ Revenue}_r \tag{2.45}$$

s.t.

$$\text{Revenue}_r + \sum_{t=1}^{T} \tilde{\lambda}_t^{\text{DA}} \leq \sum_{t=1}^{T} \sum_{j} \lambda_{rjt} z_{rjt} \quad - \sum_{t=1}^{T} \sum_{i} \lambda_{irt} z_{rit} - \sum_{t=1}^{T} C(z_{rt}) \tag{2.46}$$

Equation (2.17) – (2.20)

By applying this modification, the retailer's model r becomes a robust, hard worse case model, commonly known as the max-min method. The uncertainty set U_λ for the parameter $\widetilde{\lambda}_t^{DA}$ can be expressed as follows:

$$\tilde{\lambda}_t^{\text{DA}} = \bar{\lambda}_t^{\text{DA}} + \xi_{jt} \hat{\lambda}_t^{\text{DA}}, \quad \xi_{rt} \in [-1, 1] \tag{2.47}$$

In this expression, $\overline{\lambda}_t^{DA}$ represents the nominal value, $\widehat{\lambda}_t^{DA}$ denotes the constant fluctuation, and ξ_{rt} is the stochastic variable associated with the uncertain wholesale market price, $\widetilde{\lambda}_t^{DA}$.

Therefore, (2.46) is reformulated as following expression:

$$\text{Revenue}_r + \sum_{t=1}^{T} \bar{\lambda}_t^{\text{DA}} z_{rt,g} + \sum_{t=1}^{T} \xi_{rt} \hat{\lambda}_t^{\text{DA}} z_{rt,g} \leq \sum_{t=1}^{T} \sum_{j} \lambda_{rjt} z_{rjt} - \sum_{t=1}^{T} \sum_{i} \lambda_{irt} z_{rit} - \sum_{t=1}^{T} C(z_{rt}) \tag{2.48}$$

Adopting the hard worst case, we derive the following:

$$\text{Revenue}_r + \sum_{t=1}^{T} \bar{\lambda}_t^{\text{DA}} z_{rt,g} + \max_{\xi_{rt}} \left(\sum_{t=1}^{T} \xi_{rt} \hat{\lambda}_t^{\text{DA}} z_{rt,g} \right) \leq \sum_{t=1}^{T} \sum_{j} \lambda_{rjt} z_{rjt} - \sum_{t=1}^{T} \sum_{i} \lambda_{irt} z_{rit} - \sum_{t=1}^{T} C(z_{rt}) \tag{2.49}$$

A deviation at the upper bounds of the range represents the occurrence of the worst-case uncertainty. In this expression, obtaining a robust counterpart $\max_{\xi_{rt}} \sum_{t=1}^{T} \xi_{rt} \widehat{\lambda}_t^{DA} z_{rt,\, g}$ within the uncertainty set U_λ is essential. This expression can be equivalently reformulated into the following problem (Z. K. Li et al., 2011):

$$\max_{\xi_{rt}} \quad \sum_{t=1}^{T} \xi_{rt} \hat{\lambda}_t^{\text{DA}} z_{rt,g} \tag{2.50}$$

$$\begin{aligned} \text{s.t.} \quad & \xi_{rt} \leq 1 \quad \forall t \quad : \varpi_{rt} \\ & \sum_{t=1}^{T} \xi_{rt} \leq \Gamma_r \quad : \beta_r \\ & \xi_{rt} \geq 0 \quad \forall t \end{aligned}$$

Here, ϖ_{rt} and β_r are the dual coefficients of first and second constraints, respectively. The KKT (Karush-Kuhn-Tucker) conditions are applied to derive the robust counterpart. As a result, the dual of the problem defined by (2.50) can be obtained as follows:

$$\min \quad \beta_r \Gamma_r + \sum_{t=1}^{T} \varpi_{rt} \tag{2.51}$$

$$\text{s.t.} \quad \varpi_{rt} + \beta_r \geq \hat{\lambda}_t^{\text{DA}} z_{rt,g} \quad \forall t$$

$$\varpi_{rt} \geq 0 \quad \forall t$$

Given that the optimal solution of the primary problem (2.50) is equivalent to that of its dual (2.51), the robust counterpart of relation (2.49) can thus be derived as follows:

$$\text{Revenue}_r + \sum_{t=1}^{T} \bar{\lambda}_t^{\text{DA}} z_{rt,g} + \beta_r \Gamma_r + \sum_{t=1}^{T} \varpi_{rt} \leq \sum_{t=1}^{T} \sum_{j} \lambda_{rjt} z_{rjt} - \sum_{t=1}^{T} \sum_{i} \lambda_{irt} z_{rit} - \sum_{t=1}^{T} C(z_{rt}) \tag{2.52}$$

$$\text{s.t.} \quad \varpi_{rt} + \beta_r \geq \hat{\lambda}_t^{\text{DA}} z_{rt,g} \quad \forall t$$

$$\varpi_{rt} \geq 0 \quad \forall t$$

Therefore, the retailer's problem, considering the uncertainty of the upstream (wholesale) market price, can be obtained as follows:

$$\text{Revenue}_r + \sum_{t=1}^{T} \bar{\lambda}_t^{\text{DA}} z_{rt,g} + \beta_r \Gamma_r + \sum_{t=1}^{T} \varpi_{rt} \leq \sum_{t=1}^{T} \sum_{j} \lambda_{rjt} z_{rjt} - \sum_{t=1}^{T} \sum_{i} \lambda_{irt} z_{rit} - \sum_{t=1}^{T} C(z_{rt}) \tag{2.53}$$

$$\text{s.t.} \quad \varpi_{rt} + \beta_r \geq \hat{\lambda}_t^{\text{DA}} z_{rt,g} \quad \forall t$$

$$\varpi_{rt} \geq 0 \quad \forall t$$

$$\text{Constraints} \quad (2.17) \;\; to \;\; (2.20)$$

2.2.5 Decentralized energy market clearing with FADMM under wholesale market uncertainty

The decentralized energy market clearing algorithm using FADMM is designed to address privacy concerns by eliminating the need for a central supervisory node, which can pose a risk to the confidentiality of participants' information. The FADMM approach operates independently of the players' models, ensuring that buyers/sellers' data remains private and protected. The only modification required is in the retailer's model, which is adjusted as follows:

$$
\begin{aligned}
\max_{z_r} \quad & \text{Revenue}_r && (2.54) \\
\text{s.t.} \quad & \text{Revenue}_r + \sum_{t=1}^{T} \bar{\lambda}_t^{\text{DA}} z_{rt,g} + \beta_r \Gamma_r + \sum_{t=1}^{T} \varpi_{rt} \leq \sum_{t=1}^{T} \sum_{j} \lambda_{rjt} z_{rjt} - \sum_{t=1}^{T} \sum_{i} \lambda_{irt} z_{rit} - \sum_{t=1}^{T} C(z_{rt}) \\
& - \sum_{t=1}^{T} \sum_{i=1}^{N_S} \rho \| x_{irt} - z_{rit} \|_2^2 - \sum_{t=1}^{T} \sum_{j=1}^{N_B} \rho \| z_{rjt} - y_{jrt} \|_2^2 \\
& \varpi_{rt} + \beta_r \geq \hat{\lambda}_t^{\text{DA}} z_{rt,g} \quad \forall t \\
& \varpi_{rt} \geq 0 \quad \forall t \\
& \text{Constraints} \quad (2.10) \ \ to \ \ (2.13)
\end{aligned}
$$

2.3 Incorporating communication delay in P2P energy trading platform

In real-world peer-to-peer (P2P) energy trading systems, communication delays between prosumers and retailers can significantly affect market efficiency and optimization processes. This section introduces the modeling of signal delay into the existing P2P energy trading platform, aiming to provide a more accurate representation of communication constraints and their impact on market dynamics. This modeling helps to better understand the effects of these delays on optimal market performance and provides solutions to mitigate these issues.

2.3.1 Modeling communication delay

In the proposed platform for modeling energy exchange between prosumers and retailers, price bids, supply, and demand information are transmitted over a communication network. To account for delays in this network, we introduce time delays k_{ij}, k_{ir}, and k_{rj} to represent the transmission delays for different exchanges. Specifically, k_{ij} models the delay between local sellers and buyers, k_{ir} represents the delay between a local seller and an active retailer in the same geographical area, and k_{rj} captures the delay between a local buyer and the retailer.
The network is modeled as a time-varying graph G(t) = (V, E(t)), where V represents the set of prosumers and retailers, and E(t) denotes the set of active communication links at time t. Each communication link has an associated delay: k_{ij} for links between local buyers and sellers, k_{ir} for links between a local seller and a retailer, and k_{rj} for links between a retailer and a local buyer. This implies, for instance, that bids (including price, supply, and demand) sent from node i at time t reach node j at time t+k_{ij}. In general, communication delays are defined as follows:

$$G(t) = (V, E(t)) \tag{2.55}$$

$$\forall (i,j), (i,r), (r,j) \in E(t), \quad k_{ij}, k_{ir}, k_{rj} \geq 0$$

2.3.2 Impact of communication delay on price updates

In a no-delay model, the local buyer j receives a price bid λ^k_{ijt} from the local seller i and a price bid λ^k_{rjt} from the retailer r at time t (equivalent to iteration k). The buyer uses these prices to determine their energy demand. Similarly, the local retailer receives a price bid λ^k_{irt} from the local seller i and a purchase bid y^k_{jrt} from the local buyer j to guide its buying and selling strategies. The local seller i also decides on maximizing its objective function based on the demand bids z^k_{rit} from the retailer r and y^k_{jrt} from the local buyer j. However, introducing communication delays renders these prices and demand bids outdated by the time they are received. To accommodate this delay, the price update mechanism is modified to reflect delayed information. To simplify and better analyze the impact of delays on energy exchanges and price discovery, we model the delay on the supplier's price bids in the centralized calculations. Since in this market the bilateral price updates fall under the role of the selling agent, the pricing signals in the FADMM algorithm are modeled as follows:

$$\lambda_{ijt}^{k} = \lambda_{ijt}^{k-1} - \rho \left(x_{ijt}^{k} - y_{jit}^{k-k_{ij}} \right) \tag{2.56}$$

$$\lambda_{irt}^{k} = \lambda_{irt}^{k-1} - \rho \left(x_{irt}^{k} - z_{rit}^{k-k_{ir}} \right) \tag{2.57}$$

$$\lambda_{rjt}^{k} = \lambda_{rjt}^{k-1} - \rho \left(z_{rjt}^{k} - y_{jrt}^{k-k_{rj}} \right) \tag{2.58}$$

For instance, Equation (2.56) models the delay in the price signal received by buyer j from seller i with a delay factor k_{ij}. Incorporating this delay into the Lagrangian update process ensures that the decentralized algorithm accounts for the fact that energy production and consumption decisions are based on delayed information. This delay can impact the algorithm's convergence rate and the accuracy of the market-clearing process. Three critical factors were considered when evaluating the impact of communication delays in the proposed decentralized P2P energy trading framework:

1. **Convergence Time:** The simulation results demonstrated that communication delays significantly influence the convergence rate of the decentralized Fast Alternating Direction Method of Multipliers (FADMM) algorithm. Delayed signals cause decisions to be made based on outdated information, resulting in slower convergence toward equilibrium. Additionally, the presence of delays led to convergence within a broader neighborhood around the optimal solution, prolonging the overall time required for the market to stabilize.
2. **Market Efficiency:** Communication delays affect the accuracy of energy allocation and pricing decisions, leading to temporary mismatches between supply and demand. The simulations indicated that delays introduce inefficiencies in energy transactions, as buyers and sellers cannot react immediately to updated market conditions. Consequently, suboptimal energy trading decisions were observed during periods of high delays, highlighting the sensitivity of market efficiency to communication performance.

3. **System Robustness:** The robustness of the decentralized market-clearing process was assessed by simulating different levels of communication delay. The results revealed that, although market performance deteriorates with increased delays, the proposed decentralized framework maintains operational stability under moderate delay conditions. These findings provide valuable insights into the scalability of the system and highlight the potential need for adaptive mechanisms to mitigate the adverse effects of severe communication delays in future implementations.

CHAPTER 3

SIMULATION RESULTS

This section discusses the simulation outcomes to demonstrate the functionality of the proposed decentralized market model and assesses the effectiveness of the solution approach. The numerical simulations were performed using the General Algebraic Modeling System (GAMS) on a PC with an Intel(R) Core (TM) i3-2330M processor running at 2.20 GHz and 6 GB of RAM. To evaluate the performance of the proposed decentralized method, results from a centralized approach were used as a benchmark for comparison.

3.1 Test Platform

The numerical simulations were conducted on a small residential distribution network consisting of six households acting as buyers, four households with small-scale generation as sellers, and three retailers. The relevant parameters for the sellers, buyers, and retailers are presented in Tables 3.1, 3.2, and 3.3, with local buyer and seller data extracted from (M. Seyfi et al., 2021). The cost function coefficients and production capacity limits for each retailer were randomized within the ranges: α_r= $[0.09, 0.1]$, β_r= $[3, 10]$, z_{rt}^{max}= $[3, 10]$, z_{rt}^{min}= 0 as detailed in Table 3.4. The FADMM algorithm was applied with a step size ρ=0.05 and a stopping criterion of 0.001. Two case studies with different operational scenarios are analyzed in this paper.

Table 3.1 Input parameters of the local sellers

Producer	α_i(¢/kWh^2)	β_i(¢/kWh)
1	0.092	4.899
2	0.098	4.456
3	0.096	5.274
4	0.093	8.566

Table 3.2 Input parameters of the local buyers

Consumer	α_i(¢/kWh^2)	β_i(¢/kWh)
1	0.091	10.653
2	0.095	13.199
3	0.100	10.798
4	0.096	11.250
5	0.100	13.345
6	0.098	12.177

Table 3.3 Input parameters of the local retailers

Retailer	α_r(¢/kWh^2)	β_r(¢/kWh)
1	0.094	4.051
2	0.094	7.124
3	0.091	8.816

In this paper, two case studies are conducted in this paper to evaluate the performance of the proposed P2P energy trading framework under varying conditions:

1. **Case Study 1:** Model Without Considering Communication Delay. This case study evaluates decentralized market performance under two pricing scenarios. **Scenario 1**: P2P energy trading among prosumers with deterministic pricing, highlighting buyer strategies in stable market conditions. **Scenario 2**: P2P energy trading with stochastic pricing, demonstrating the model's robustness and adaptability to price fluctuations. For comparison, key performance metrics of the centralized approach are summarized in Table 3.5.
2. **Case Study 2:** Model with Communication Delay. This case study assesses the decentralized model's performance under both deterministic and stochastic pricing conditions, focusing on the impact of communication delays on market convergence, efficiency, and robustness.

3.2 Case Study 1

In this case study, we evaluate P2P energy transactions among prosumers under decentralized operation without considering communication delays in the optimization models. The performance of the decentralized approach is analyzed under two pricing scenarios: deterministic (without price uncertainty) and stochastic (with price uncertainty). Results for the centralized approach are presented in Table 3.4 for comparative analysis. For convergence analysis, Figure 3.1 visualizes the convergence process of the decentralized approach for energy transactions at 15:00. The iterative adjustments by buyers and sellers demonstrate successful convergence, highlighting the robustness and operational feasibility of the decentralized market model. This adaptive behavior underscores the competitive dynamics and strategic flexibility inherent in the proposed framework.

As described in the proposed P2P platform, buyers are equipped with energy storage systems (batteries) to enhance their economic benefits. They strategically discharge stored energy to meet part of their load during periods of high retail market prices and recharge their batteries when prices are lower. For illustration of decentralized approach under deterministic pricing (scenario 1), Figure 3.2 shows the power balance of buyers, highlighting their strategic use of energy storage systems. It is evident that buyers charge their batteries during low-price periods (e.g., hours 1–5 and 10–15) and discharge them during high-price periods (e.g., hours 6–9 and 17–21) to minimize costs. These behaviors validate the economic efficiency of the decentralized model under predictable market conditions. When comparing this to the power balance in Figure 3.3 depicting decentralized approach under stochastic pricing (scenario 2), a notable shift can be observed in the buyers' strategy, showcasing increased caution in energy management. Buyers maintain their charging strategy during low-price periods but show more conservative behavior during periods of price uncertainty by using stored energy more efficiently and relying more on the demand response program. This behavior helps mitigate the effects of fluctuating prices and supports their welfare by reducing reliance on higher-priced market transactions.

These observations underline the flexibility and strategic behavior of buyers in response to deterministic and uncertain price scenarios, validating the effectiveness of the proposed decentralized market framework in adapting to various market conditions while ensuring energy balance and enhancing buyer welfare.

Table 3.4 presents a comprehensive comparison of the performance between deterministic and robust optimization methods for both centralized and decentralized approaches. The total social welfare under deterministic conditions in the decentralized approach is 2856.6461, which is slightly higher than in the robust condition, where it measures 2847.0226. This indicates a decrease in total social welfare of approximately 0.34% when transitioning from deterministic to robust optimization.

This reduction in welfare highlights the impact of incorporating price uncertainty, which typically leads to more conservative behavior among buyers to safeguard against price fluctuations. The results for the robust decentralized approach show that while local market transactions (73.1999) remain comparable to those in deterministic settings (73.2000), there is an increase in negative pool market trade (-41.6904 compared to -39.4203), implying reduced reliance on external market purchases under uncertainty. The individual metrics, such as XirTo , XijTo, and YirTo reflect adaptive strategies among participants. Notably, XijTo in the decentralized robust approach (51.9755) is higher than in the deterministic centralized approach (37.1362), suggesting a shift towards increased local transactions despite the cautious adjustments due to uncertainty. Overall, while the robust approach ensures greater resilience and adaptability to market price variations, it comes at the expense of slightly reduced total social welfare. This outcome aligns with expectations from robust optimization practices where ensuring protection against uncertain conditions often leads to marginally lower economic outcomes but provide enhanced security and predictability for market participants.

Table 3.4 Test comparison results for deterministic and robust method

Uncertainty Method	Deterministic		Robust	
Approach	**Central**	**Decentralized**	**Central**	**Decentralized**
Local Market Trade	72.9714	73.2000	72.9705	73.1999
Pool Market Trade	-38.8836	-39.4203	-41.1522	-41.6904
XirTo	35.8352	21.1698	34.0201	21.2244
XijTo	37.1362	52.0301	38.9504	51.9755
YjrTo	289.4020	274.5080	287.4261	274.4010
Total Social Welfare	2855.3190	2856.6461	2845.6954	2847.0226

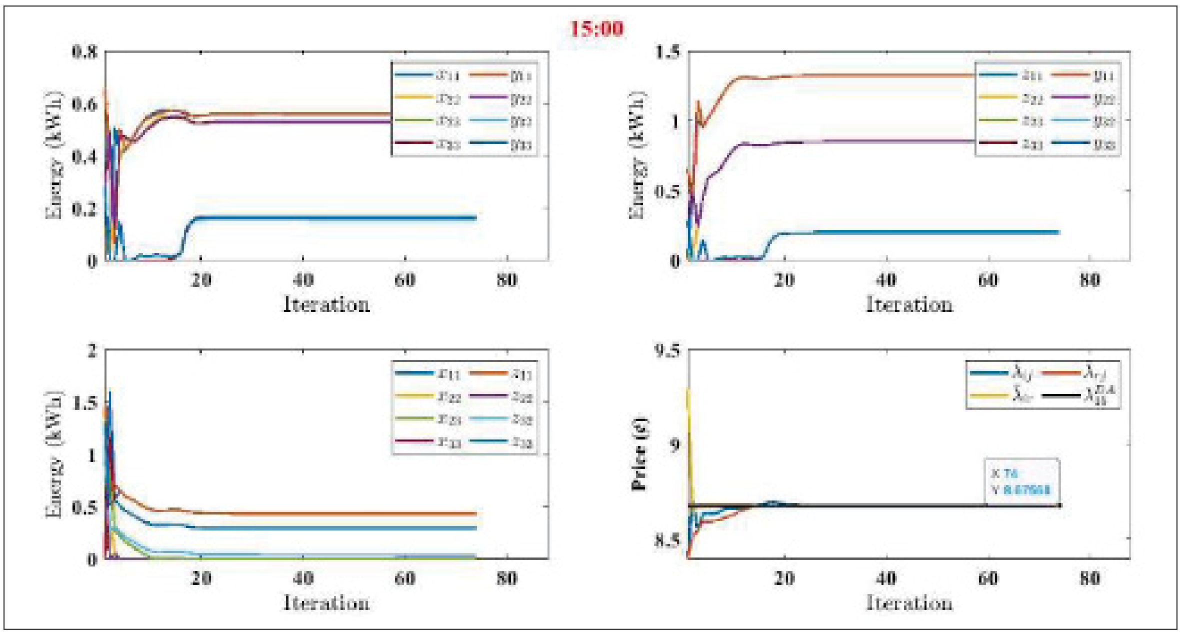

Figure 3.1 Convergence of the proposed method for P2P energy transactions among prosumers at 15:00 in Case Study 1

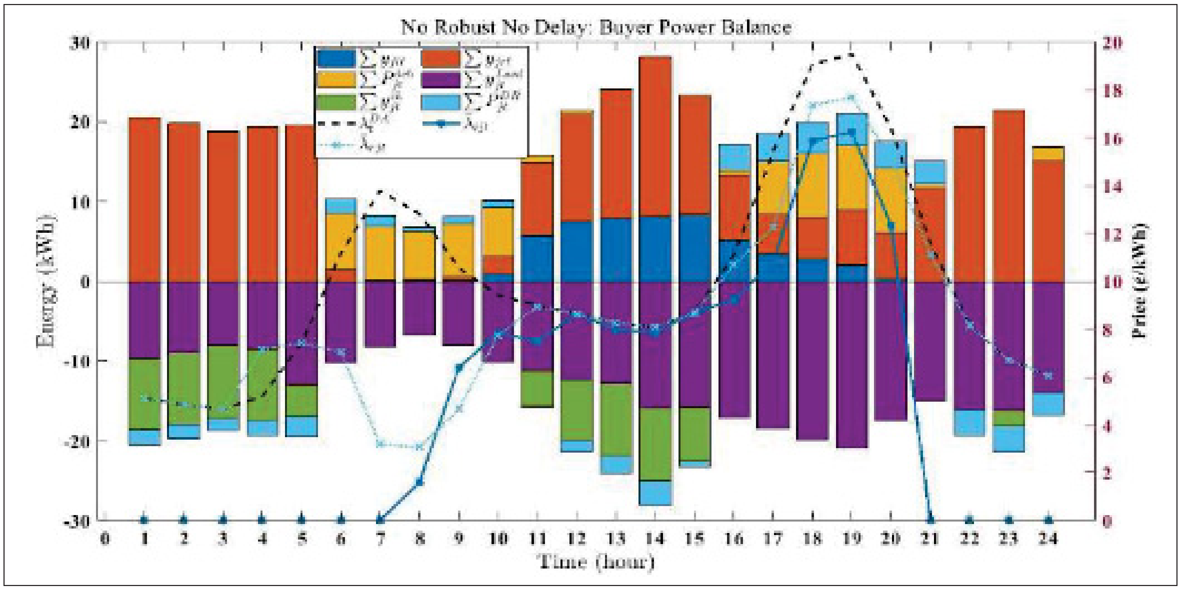

Figure 3.2 Power balance of local buyers, retail market nominal price, and average local transaction price (No robust No delay)

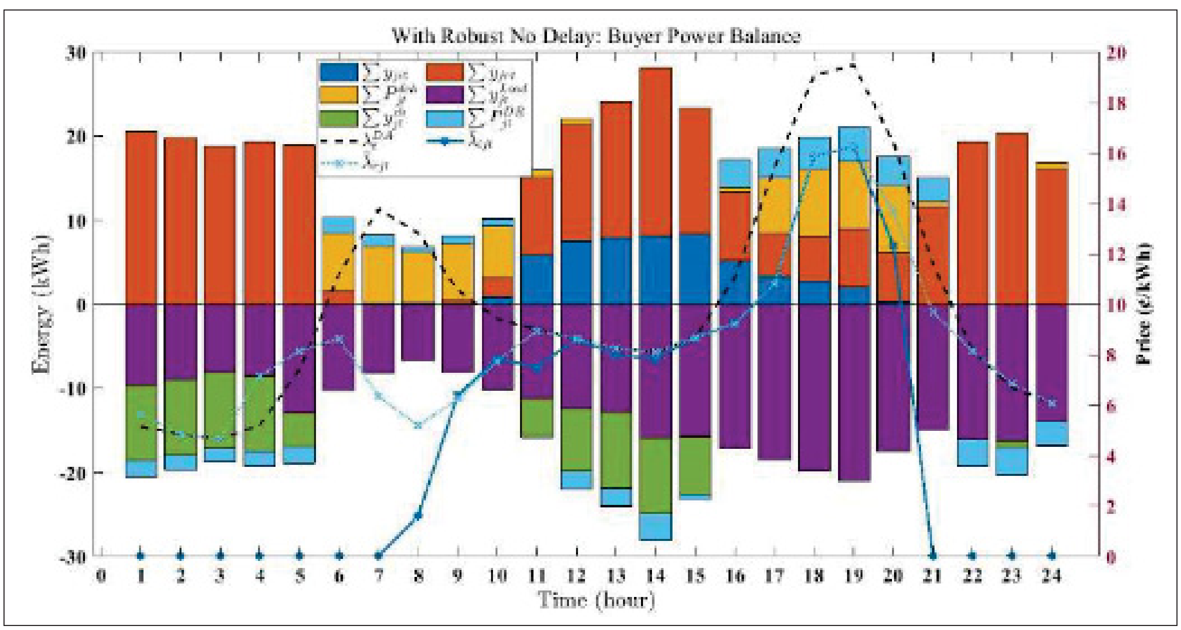

Figure 3.3 Power balance of local buyers, retail market nominal price, and average local transaction price (With robust No delay)

It is important to address the observation that the decentralized approach achieves slightly higher social welfare (2856.6461) compared to the centralized approach (2855.3190), which may appear counterintuitive. This difference arises due to the stopping conditions and the iterative nature of the Fast Alternating Direction Method of Multipliers (FADMM) algorithm used in the decentralized optimization process. Specifically, with a convergence tolerance (ε) of 0.01, the decentralized method yields a social welfare of 2856.6461, which decreases to 2855.3190 when ε is tightened to 0.001 or 0.0001, closely aligning with the centralized result. This progression indicates that the initial higher value at a looser tolerance reflects a relaxation from the upper bound during the early convergence stages, with the decentralized solution approaching the global optimum as iterations increase —- guaranteed by the problem's convexity. To further clarify, Figure 3.4 shows the energy (kW) over time for different stopping conditions (ε = 0.01, 0.001, 0.0001) alongside the centralized result, demonstrating that the decentralized approach converges toward the centralized value with stricter tolerances, highlighting that the slight initial advantage is a practical artifact of the stopping criterion rather than a fundamental superiority.

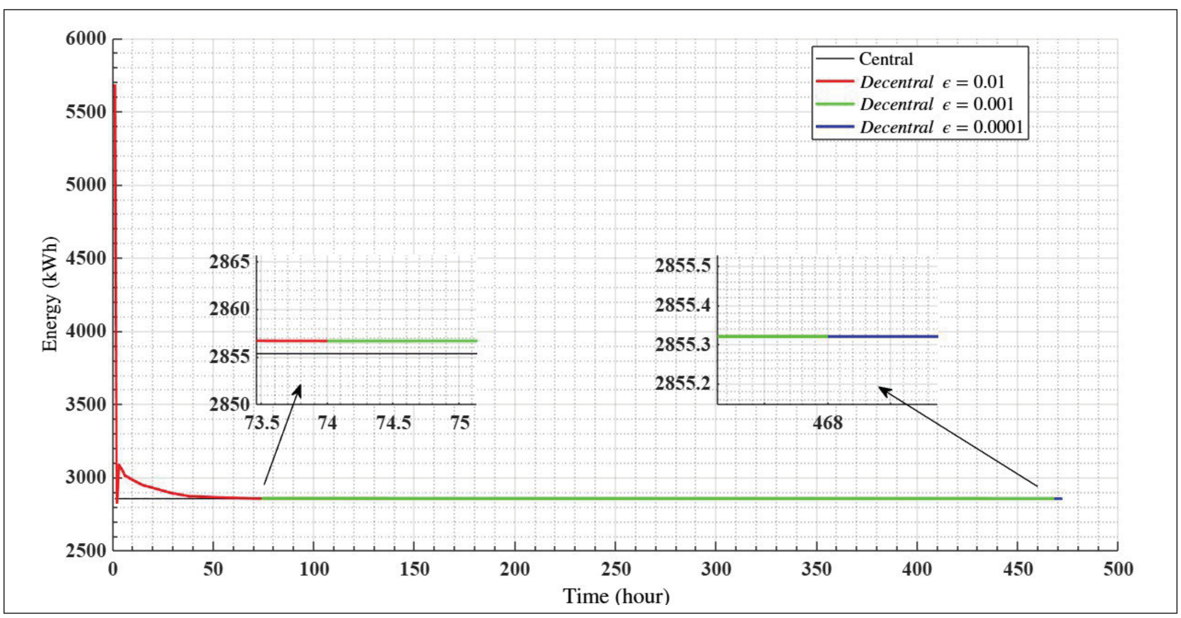

Figure 3.4 Energy (kW) Profiles for Decentralized vs. Centralized Approaches with Varying Tolerances (ε = 0.01, 0.001, 0.0001) at 15:00 in Case Study 1

3.3 Case Study 2

In this case study, P2P energy transactions among prosumers are analyzed under a decentralized framework while incorporating communication signal delays into the optimization models. The analysis is conducted under two pricing scenarios: deterministic (without price uncertainty) and stochastic (with price uncertainty). Comparative performance results for the decentralized approach are presented in Table 3.5. The convergence behavior of the decentralized approach at 15:00 is illustrated in Figures 3.5 and 3.6, representing deterministic and stochastic conditions, respectively. Under deterministic pricing (Figure 3.5), the method demonstrates efficient convergence, with prosumers iteratively adjusting their transactions to reach an optimal state. When incorporating price uncertainty (Figure 3.6), the model shows slightly slower convergence due to the added complexity of managing uncertainty. Nonetheless, the proposed framework successfully adapts to both scenarios, validating its operational feasibility under delayed communication signals.

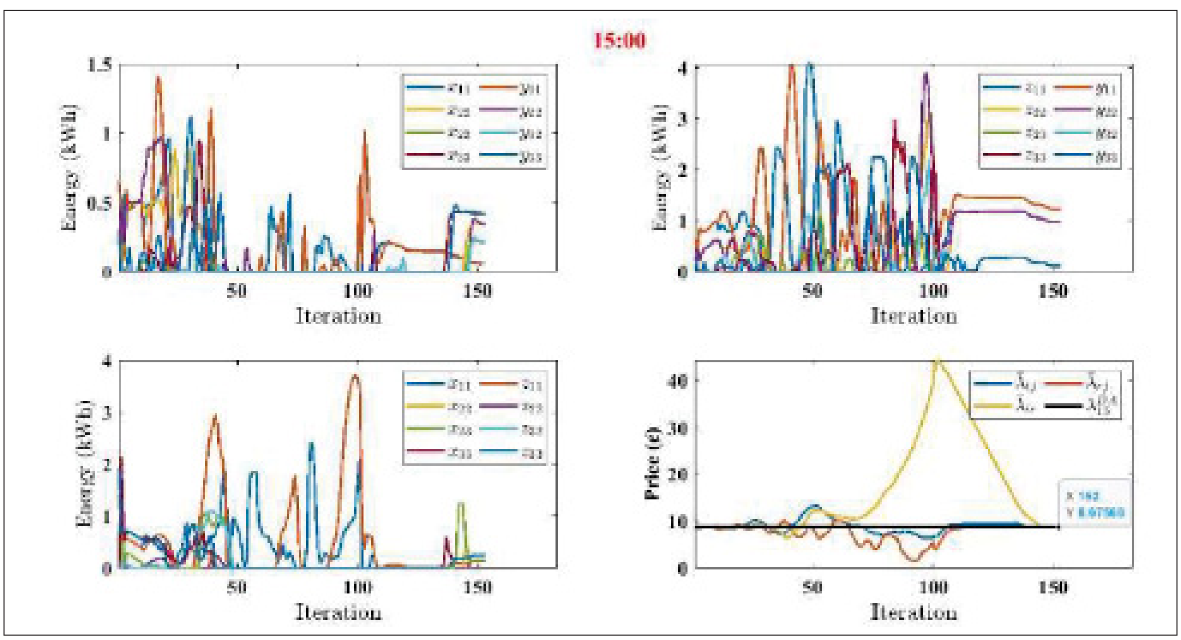

Figure 3.5 Convergence of the proposed method for P2P energy transactions among prosumers at 15:00 (No robust) in Case Study 2

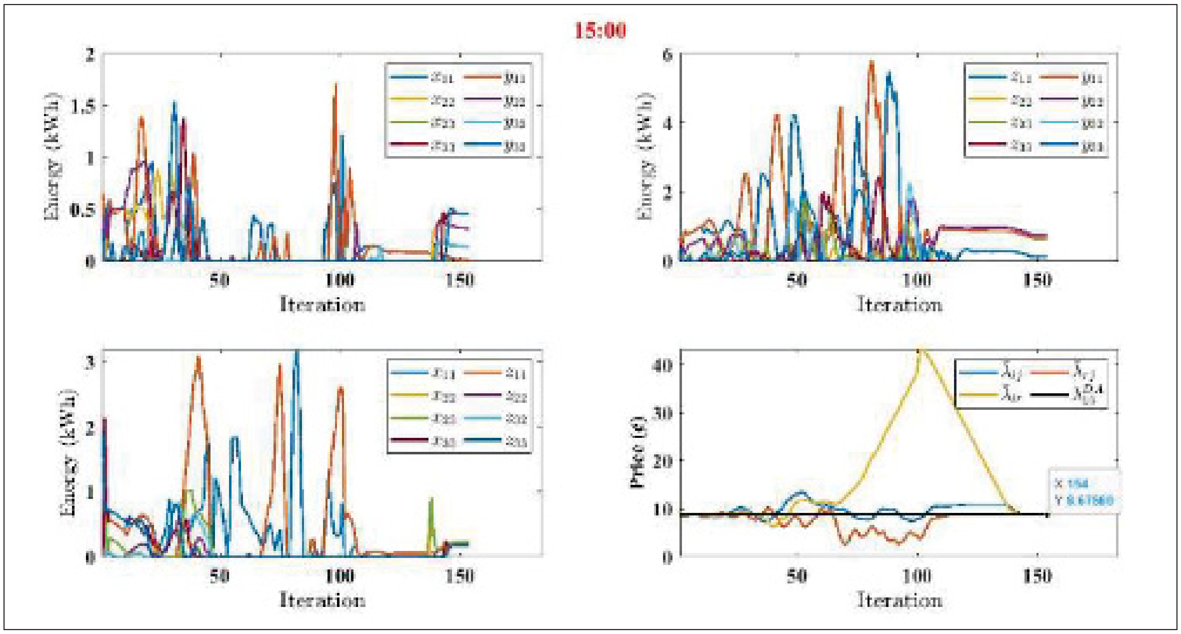

Figure 3.6   Convergence of the proposed method for P2P energy transactions among prosumers at 15:00 (with robust) in Case Study 2

Figures 3.7 and 3.8 provide insights into the power balance of buyers under deterministic and stochastic pricing conditions with communication delays. It is obvious in Figure 3.6, buyers strategically use their energy storage systems to charge during low-price periods and discharge during high-price intervals, similar to Case Study 1. However, communication delays slightly influence the timing and magnitude of transactions, introducing minor inefficiencies in energy utilization. Also, it is evident in Figure 3.7 under uncertain pricing conditions, buyers adopt a more cautious approach by increasing their reliance on stored energy and demand response programs. This strategy mitigates the risks associated with price fluctuations and helps maintain economic efficiency despite the challenges posed by delayed communication.

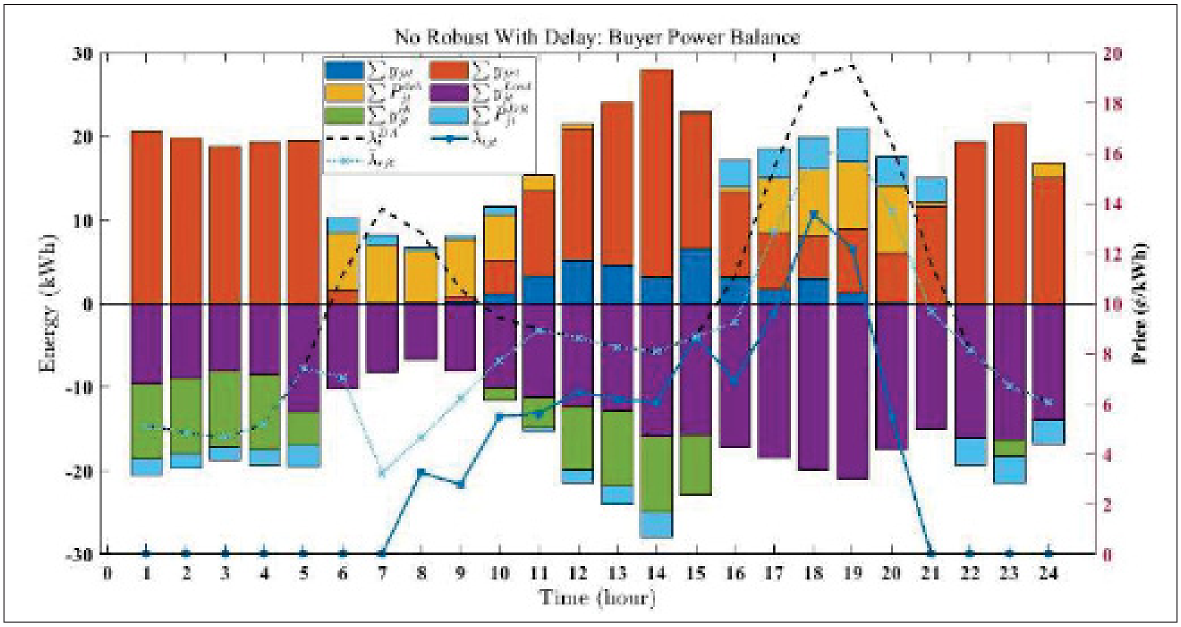

Figure 3.7 Power balance of local buyers, retail market nominal price, and average local transaction price (No robust with delay) in Case Study 2

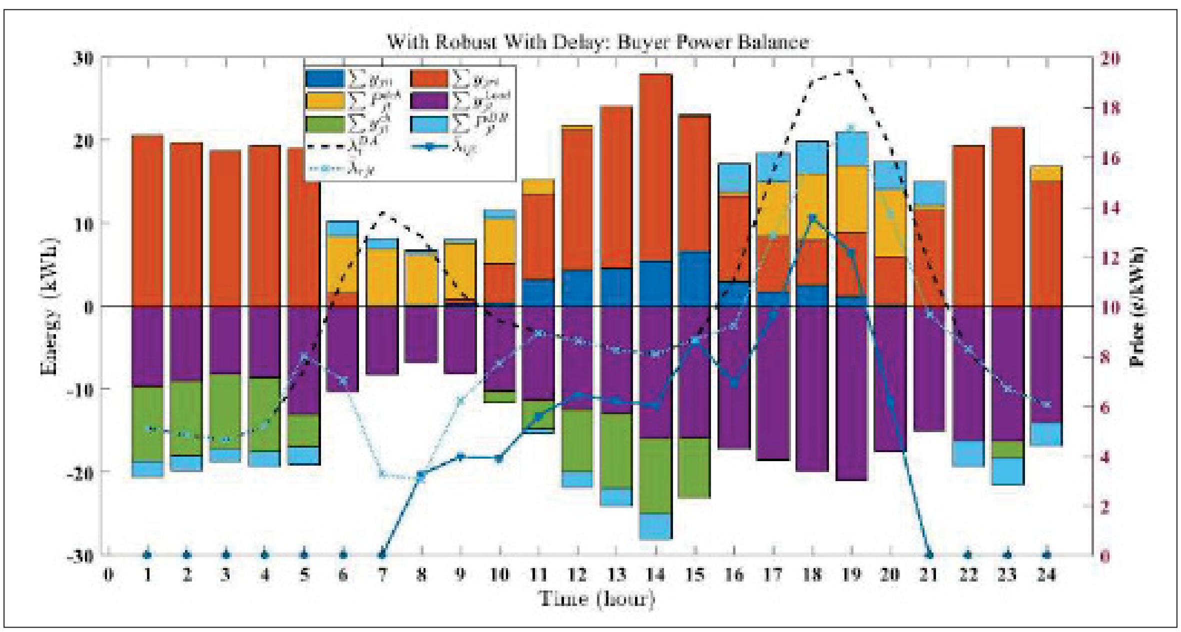

Figure 3.8 Power balance of local buyers, retail market nominal price, and average local transaction price (With robust with delay) in Case Study 2

Table 3.5 Test comparison results for deterministic and robust method (decentralized approach)

Uncertainty Method	**Deterministic**	**Robust**
Local Market Trade	82.0694	81.1236
Pool Market Trade	-104.5305	-108.1978
XirTo	48.9960	46.2590
XijTo	33.0734	34.8647
YjrTo	293.6205	291.5679
Total Social Welfare	5609.8428	5562.3018

Table 3.5 provides a detailed comparison of key performance metrics under deterministic and robust optimization methods in the decentralized approach. The deterministic scenario achieves higher total social welfare (5609.8428) compared to the robust scenario (5562.3018). This reduction reflects the conservative strategies employed to safeguard against uncertainty, a trade-off inherent in robust optimization. For comparison of local market trade (XijTo), robust optimization shows a marginal increase in local market trade (34.8647) compared to deterministic conditions (33.0734), indicating a shift towards local transactions to reduce reliance on external markets. For comparison of pool market trade (YjrTo), there is a notable increase in negative pool market trade under robust conditions (-108.1978) compared to deterministic conditions (-104.5305), suggesting reduced dependency on external market purchases. Case Study 2 highlights the flexibility and robustness of the proposed decentralized framework under communication delays. The results demonstrate that while delays introduce minor inefficiencies, the model remains effective in adapting to varying pricing conditions. The incorporation of robust optimization enhances the system's resilience to market uncertainties, albeit at the cost of reduced total social welfare. These findings underscore the proposed framework's applicability in real-world P2P energy trading scenarios characterized by communication constraints and dynamic market conditions.

Consequently, following the analysis presented in Case Study 2, Figures 3.9 and 3.10 provide further insights into how energy storage systems respond to different combinations of price uncertainty and communication delays, highlighting prosumers' strategic adjustments in energy management. Obviously, Figure 3.8 illustrates the mean State of Charge (SOC) of energy storage systems under four scenarios. The SOC patterns indicate the strategic energy management adopted by prosumers. In scenarios without robustness, the SOC follows a predictable pattern, where storage systems are charged during low-price periods (e.g., hours 1–5 and 10–15) and discharged during high-price periods (e.g., hours 6–9 and 17–21). Incorporating robustness into the model results in a more conservative charging and discharging behavior, with reduced peaks in SOC. The presence of communication delays further emphasizes cautious energy usage, where prosumers prioritize minimizing their reliance on uncertain market conditions and adapting to slower information exchanges. Figure 3.9 depicts the mean changes in SOC (ΔSOC), highlighting the charging and discharging dynamics across the same four scenarios. In the "No Robust No Delay" scenario, the ΔSOC exhibits pronounced charging during low-price periods and discharging during high-price intervals. With robustness, the magnitude of ΔSOC is reduced, reflecting more balanced and cautious energy transactions. The introduction of communication delays impacts the timing and magnitude of ΔSOC, particularly in the "with Robust with Delay" scenario, where the changes are smoother and more distributed across time. These observations underscore the interplay between price uncertainty, communication delays, and prosumer strategies in optimizing their energy storage operations.

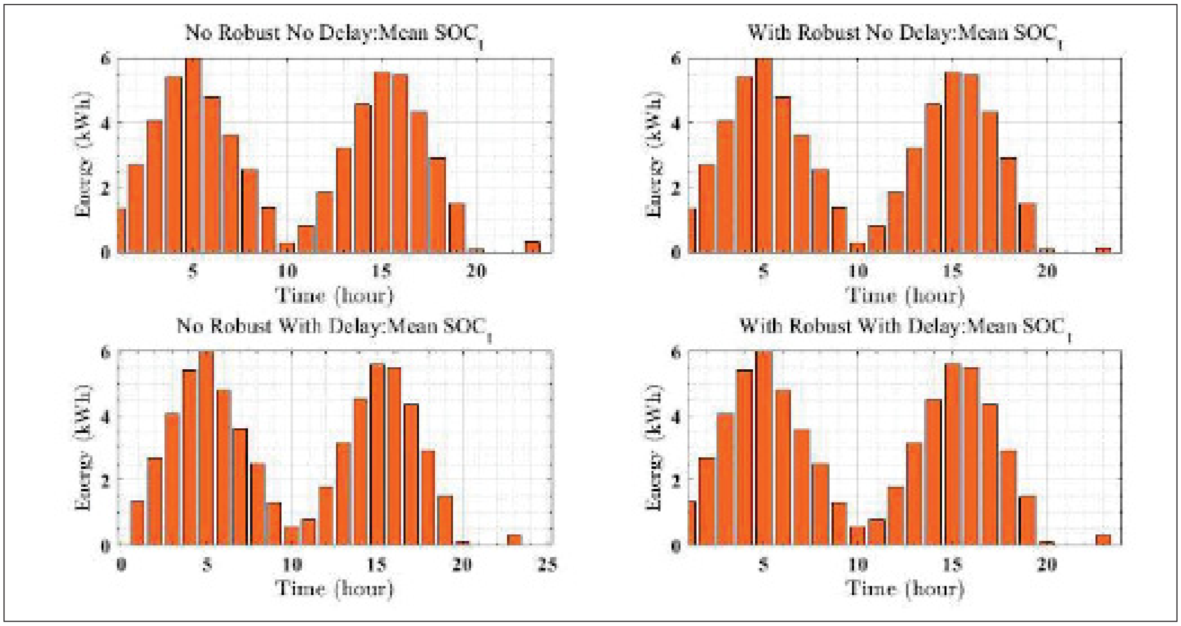

Figure 3.9   Mean State of Charge (SOC) Profiles Under Various Scenarios.

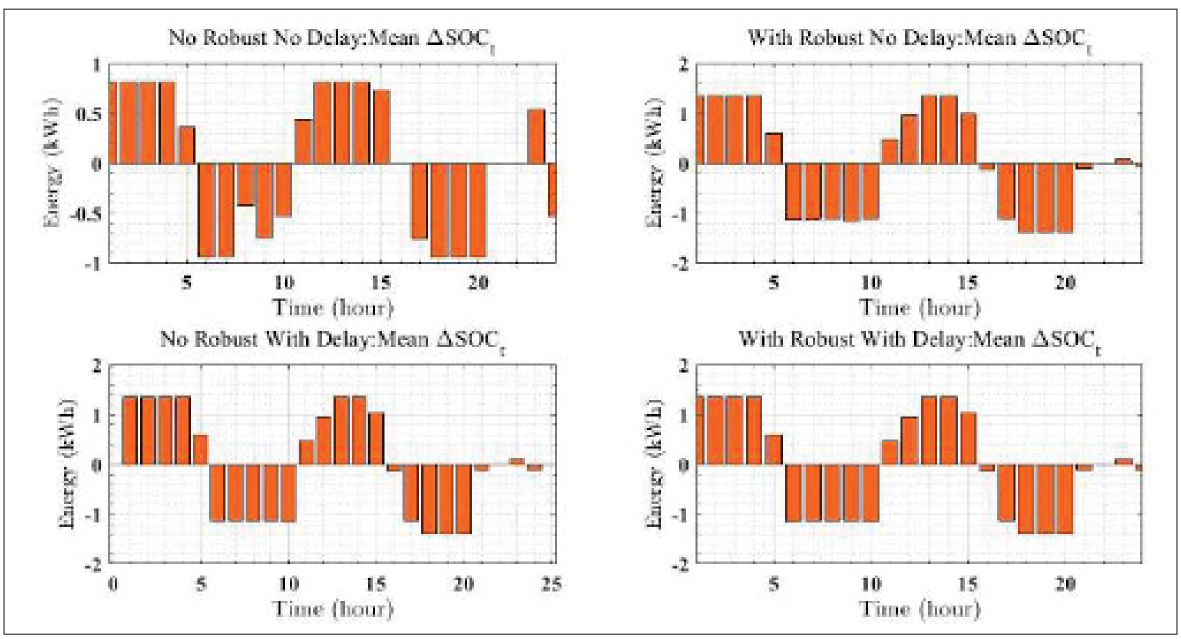

Figure 3.10   Mean Changes in State of Charge (ΔSOC) Under Various Scenarios.

CONCLUSION AND FUTURE WORKS

4.1 Conclusion

In this paper, we proposed a robust, fully decentralized peer-to-peer (P2P) energy trading framework tailored to address critical challenges in smart grids, such as wholesale market price uncertainty and communication delays in decentralized market operations. The framework enables prosumers—, equipped with energy storage systems and participating in demand response (DR) programs,—to dynamically engage in energy trading with local participants and retailers. By employing robust optimization techniques, the model effectively mitigates risks associated with retail price volatility, ensuring reliable decision-making under uncertain market conditions. Communication delays are explicitly modeled and analyzed, demonstrating their impact on market convergence, transaction efficiency, and operational stability. By using the Fast-Alternating Direction Method of Multipliers (FADMM) algorithm, the proposed approach achieves decentralized market clearing without requiring a central supervisory node, preserving privacy while maintaining scalability. Simulation results validated the framework's effectiveness across various scenarios, including deterministic and stochastic pricing conditions, both with and without communication delays. Under deterministic pricing, buyers leveraged energy storage to reduce grid dependency by charging during low-price periods and discharging during high-price intervals, enhancing overall welfare. The introduction of price uncertainty prompted more cautious energy management strategies, resulting in a slight reduction in social welfare by 0.34% compared to the deterministic approach. Furthermore, incorporating communication delays demonstrates that the decentralized market-clearing process maintains convergence despite delayed information exchanges, leading to smoother and more distributed energy transactions. Comparative analyses between the deterministic and robust models, with and without delays, underscores the framework's capability to maximize social welfare while ensuring system stability, adaptability, and scalability. These findings validate the operational feasibility and practicality of the proposed decentralized approach in dynamic, real-world energy trading environments, offering a scalable and sustainable solution for future smart grid ecosystems.

4.2 Future Works

Based on this study, the authors suggest several avenues for future research to further enhance the proposed framework:

- Future studies could explore advanced network communication structures, such as adaptive or hybrid networks, where prosumers dynamically adjust their communication paths based on network conditions to further enhance market scalability and robustness.
- Extending the current model to incorporate real-time pricing and demand-response signals could improve the adaptability of prosumers in highly volatile energy markets, ensuring faster convergence and improved market efficiency.
- Investigating the integration of P2P energy trading with multi-energy systems, including heating, cooling, and gas networks, would broaden the applicability of the proposed framework to diverse energy infrastructures.
- While this study focused on a small-scale P2P market, future work could examine the performance of the framework in large-scale networks with thousands of prosumers, addressing computational and communication challenges.

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